A FUZZY APPROACH FOR MEDICAL EQUIPMENT REPLACEMENT PLANNING

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KEYWORDS
Maintenance, medical equipment, replacement analysis; patient satisfaction

ABSTRACT

Management of hospital service is continuously faced with the reduction of costs by introducing new operational criteria in allocating both technical and human resources.
Managing thousands of medical devices is time-consuming and costly. An effective maintenance management can result in a significant reduction of operational costs. However, medical devices are often not properly managed and this leads to their rapid obsolescence. An accurate process of equipment state appraisal is required. On one hand, in case of a increasing failure rate of medical devices an increase of both break-down maintenance costs and risk of unfavourable events are expected. On the other, if devices are replaced too early, an increase of purchase costs will occur. A trade off between cost policies and a high quality level of service is a common management problem.
In this paper, a fuzzy inference model is proposed to identify the equipment to replace in order to achieve the goals of reducing expenditure in a hospital structure and to increase patient and medical staff satisfaction. The model considers both linguistic and quantitative parameters, estimated in an objective way, in order to include many of the factors that actually influence replacement decisions with an approach that is unique yet simple to use in the healthcare context.
The model is applied to a full case study. The results obtained from a test conducted on 100 medical devices encourage the use of the model for decisions regarding replacement planning.

INTRODUCTION

The constant development of medical science and the related diagnostic, therapeutic and rehabilitative methods requires many financial resources for the procurement and maintenance of biomedical technologies.
The hospital structure becomes a place where a lot of advanced technology is available for specialised personnel in order to assure a high quality level of healthcare service performance.
The high concentration of such technology involves a great amount of human and technical resources that requires concerted management effort in planning and control of their use to avoid an uncontrollable increase in costs.
In this context the maintenance management of medical equipment play a key role, since the lives of patients often depend on its functioning correctly, and in other cases on the services provided by the health system.
Consequently, their management allows for a reduction in efficiency, such as unpredictable downtime or the increase of technical repairs, guaranteeing the increase in the productivity of clinical services an in customer and medical staff satisfaction.
However, in many cases, biomedical equipment is not properly managed, maintenance is only carried out in emergency situations leading to their rapid obsolescence.
The sector of biomedical technologies is subdivided in to the following areas:
- life support devices that include intensive care equipment characterised by the capability to damage a patient seriously in case of malfunction;
- therapeutic or rehabilitative devices that include surgical therapy, operations with a low level of invasiveness, non invasive therapy, artificial organs and protheses rehabilitation and support;
- diagnostic devices for functional evaluation, bio-images and clinical diagnosis;
- other devices including laboratory analysis equipment and other support devices, such as centrifuge, laboratory instruments, computers and related tools.

As can be seen from the proposed classification, there are thousands of medical devices in a hospital structure; the application fields are widespread and their management consumes a large quantity of time and money to avoid errors or omissions which could be fatal for the health of the patient.
From a management point of view, one problem which creates notable criticism is the procurement and the disposal of equipment, which requires effort in defining the procedures for removing devices and for adequate replacement plans.

Currently, procurement and technology updating planning is an activity carried out in vastly different ways depending on the system. The methodology for replacing devices depends on the information that the hospital structure has.
A lack of information on the real condition of equipment leads to choices based on the experiences of their operators or on the budget contingencies. This means that in some cases the renewal of devices is often premature, unnecessary or even unsuitable or too late due to a lack of appropriate planning.
On the other hand, when information is available from an adequate monitoring of devices, the tendency is to concentrate only on equipment which is broken without carrying out a comprehensive evaluation of the state of devices.
It could be said that the situation is still that of ten years ago, as described in (Fennigkoh, 1992): "even though the concept of replacement planning is clearly established, it is not yet been applied through out the health system. In particular replacement planning of biomedical devices has so far received only minimum attention. Very few hospitals have the formal mechanism to define a device replacement program. A lack of usable models leads to procurement which is premature, inappropriate or simply unnecessary".
From this, the necessity to provide health systems with formal replacement planning models can be seen. An analysis of replacement is one of the principal problems facing the economic engineering and many mathematical models have been proposed in literature to support decisions about the best time to substitute a functioning device, starting with the dynamic programming model proposed by (Belmann 1955).
The problem is generally faced an economic point of view, through the introduction of cost objective functions, the variables of which are the cost expression of all the events which could influence the life cycle cost of a device.

The choice of independent variables of the model has been a widespread subject of study and research. Above all, it is possible to distinguish between deterministic and stochastic models. In the first case, events are considered deterministic and all the possible uncertainties linked to the exact value of independent variables are then taken into consideration through sensitivity analysis. This approach, despite showing a degree of computational complexity, does not allow for an effective identification of the mathematical relation which links the variables of the problem to the objective function.
In the case of the stochastic model, the variables of the problem are distributed in line with a probability density function and the optimal time to remove the device is provided by a probabilistic point of view. An alternative to the stochastic model can be found in fuzzy models (Chang, 2005).
Furthermore, the models can be distinguished by considering different variables which influence replacement decisions.
Classic models only consider equipment deterioration and estimate the total life cycle costs of each component.
Although the calculation theories for the related costs are reasonable, the actual money spent is only one of the factors that should be taken into account when choosing replacement strategies.
However, these approaches could fail in many situations and are not used in practice. The findings of the survey, proposed by Christer and Walker, 1987, to evaluate the performance of the medium and big firms in replacement decisions, show that the classic approach based on the cost model is only appropriate for trivial devices. In the case of critical and more expensive devices, non-economical factors influencing the decisions are considered by decision makers, such as market expansion and contraction and the workers' attitude to change.
Meyer, 1993, highlights the importance of taking into account not only technical parameters related to each device but also their compliance with business strategies and technological changes. Indeed, devices characterised by a high technological obsolescence rate are subject to more rapid replacement.
Chang, 2005, introduces two different types of parameters: intrinsic parameters based on technical obsolescence of each device and extrinsic parameters related to the market and cost obsolescence; they are measured, respectively, by the loss of revenues and the increase of operational costs to use the existing device instead of a newer one.
In order to overcome the limits of the classic life cycle cost models, some authors propose additional economical parameters. For example, in Christer and Scarf, 1994, penalty costs are introduced to include the monetary value of the opportunity to replace a device with a newer one; moreover, the effects of the usage on equipment life are included in the cost model. In Hritonenko and Yatsenko, 2007, an analysis of the effects of technological obsolescence on costs is provided.
Nevertheless, these attempts fail in contexts when non-economical factors prevail over economical factors such as in the case of medical devices where factors related to patient and medical staff satisfaction and technological obsolescence are prominent and cost models could fail in estimating the ideal age to replace equipment.
This problem could be handled using multicriteria decision models able to consider both quantitative and linguistic variables using a unique approach (Schnadt-Kirschner R. 1995; Bevilacqua and Braglia, 2000; Wang et al., 2007). Nevertheless, these models are characterised by a time consuming process of appraisal based on more subjective expert judgements.
In this paper, a fuzzy inference model is proposed to identify the equipment to replace in order to achieve the goals of reducing expenditure in a hospital structure and to increase patient and medical staff satisfaction. The model considers both linguistic and quantitative parameters, estimated in an objective way, in order to include many of the factors that actually influence replacement decisions with an approach that is unique yet simple to use in the healthcare context.
In section 2, the model is described from a theoretical point of view. In the third section, results from the application of the model on 100 devices belonging to the hospital "Casa Sollievo della Sofferenza - S. Giovanni Rotondo" are shown. Finally, there are the summary and the conclusions.

THE PROPOSED MODEL

The theoretical model

The equipment procurement and replacement policy plays a strategic role in hospital management since it significantly influences the competitiveness of healthcare firms. The choice of each “patient -customer” is strongly influenced by the technological level of the hospital. This implies the presence of an accurate process of appraisal in replacement decisions.

In our model the replacement analysis is based both on technical deterioration, patient satisfaction and the medical staff satisfaction criteria. The model structure (see figure 1) includes seven different input parameters and one output.

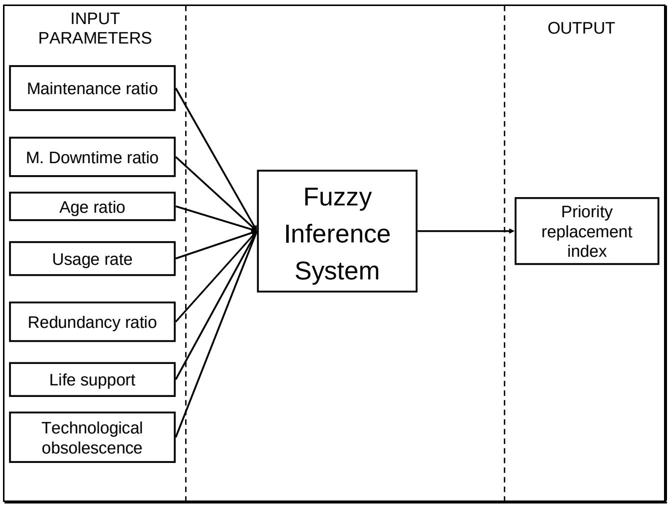

Figure 1: The model structure

The technical deterioration of each device is influenced by the paramters related to the characteristics of the equipment itself and to the aging processes: mean downtime, maintenance costs and age of each device.

Inputs of our model related to the abovementioned parameters are calculated by considering functioning devices as a benchmark.

The mean downtime of the *kth* device is compared with the average downtime (DT_k) of the other similar **n** devices (DT_i) belonging to the hospital structure.

The formula used for the input parameter **mean downtime ratio** (MDT) is:

$$MDT_k = \frac{DT_k}{\frac{\sum_{i=1}^{n} DT_i}{n}} \qquad (1)$$

The data necessary to evaluate the mean downtime are difficult to obtain, since many hospitals do not apply formal procedures to monitor and record them. The presence of a computer maintenance information system facilitates the evaluation of the mean downtime even if erroneous data records frequently reduce the effectiveness of each analysis.

The maintenance ratio (**MR**) is calculated by considering the total amount of maintenance costs of each device spent in the last three years divided by its purchase cost.

In the maintenance expenditure the repair costs and the preventive maintenance costs should be included. The spare parts costs are included in the maintenance expenditure only if they are related to a failure and do not depend on daily use.

Alternatively, if the contract includes guarantees, the extra fee paid to the buyer should be considered in the maintenance expenditure. This hypothesis is in compliance with the behaviour of hospitals that have “full risk contracts” for some equipment and therefore the incentive to replace devices too early.

The device **age ratio** (**AR**) is calculated by comparing the current age of each device with its expected life calculated as a mean of the life of other similar devices disposed by the hospital structure. If these data are not available benchmark values could be founded in literature. In our case study (see section 3) we referred to Lamberti and Rainer, 1998 which consider 4, 6 and 12 years of expected life, respectively for laboratory, electrophysiology and radiology devices.

Moreover, a measure of the usage of each medical device is significant in the aging process since medical technologies have many different applications and are used with a different intensity depending on the patients’ needs: some devices, such as operating theatre devices are frequently used, other, such as defibrillators, are used only in specific situations.

These data, even if they could significantly influence equipment deterioration, are hard to record, as they are a result of a lack of effort of operators in checking and registering these data; moreover an automatic system able to capture the actual usage of each device is not simple to set up.

In order to simplify the application of our model in a real context, we have introduced a score system for the evaluation of the usage, as reported in table 1.

Table 1: The scoring system for the **Usage rate (UR)**

Usage frequency rate	Score
More than 6 hours per day	3
[0:6] hours per day	2
not daily use	1

The abovementioned parameters are significant for both medical staff and patient satisfaction since a lack of availability or an increase of failure rate compromises the efficiency and effectiveness of each medical performance.

However, the influence of the equipment’s proper operation on patient safety should be taken in consideration.

A failure of medical equipment is frequently associated with consequences on patient care, requiring perhaps different treatment, or greater medical resources, or worse. Patient care consequences need to be considered and allowed for in some way.

In our model we have classified each device in 4 categories and we have assigned the scoring system reported in table 2.

Table 2: The scoring system for the **Life support index (LS)**

devices	Score
life support	4
therapeutic	3
diagnostic	2
other	1

The magnitude of the consequence of a failure of a device depends on its redundancy level. In every hospital, the medical staff requires a determined level of equipment redundancy in order to assure a specific level of safety for the patients and a fixed level of quality service according to patient demand and needs.

If the actual number of devices is less than the number fixed by the medical staff, a critical situation occurs and the correct functioning of each device is important for patient satisfaction and safety. The input parameter **redundancy ratio** (**RR**) for th kth device is defined as:

$$RR_k = \frac{(\text{actual redundancy level})_k}{(\text{the requested redundancy level})_k} \qquad (2)$$

The last input parameter is the technological obsolescence (**TO**). This parameter plays a key role both for the patient and the medical staff satisfaction: the medical staff members using high technological devices improve the efficiency and the effectiveness of their performance with a considerable advantage for the patient satisfaction.

In our model, the score assigned to each device is 1 if the medical staff indicates the presence of a newer and a more technologically advanced device able to create advantages in the efficiency and in the effectiveness of the medical performances; 0 otherwise.

The output of the model is the priority replacement index (**PRI**) that provides a quantification of the urgency in the equipment replacement.

Each device has a score from 1 to 30. If the device has a score between 0 and 10 the priority of replacement is low. If the score is between 10 and 20 then the priority replacement index is medium and the device becomes "to be monitored". Finally, if the device has a value between 20 and 30, the model suggests its immediate replacement.

The fuzzy model

The selected modeling approaches followed in order to develop an expert system able to predict the PRI index were three: we firstly used a Fuzzy Inference System (FIS) based on a pre-established set of rules, then we trained an artificial neural network on the same seven variables in order to compare their performances and we set up a neuro-fuzzy system to find the most relevant features.

Fuzzy models, as well as other computational intelligence based algorithms (Pinto et al, 2005), have been successfully employed in fault detection expert systems in the industrial environment.(Lo et al, 2007) (Ichtev et al, 2001). The advantage provided by Fuzzy theory in this field is pretty important: it is sometimes argued, in fact, that domain knowledge owned by experts is a valuable information often neglected in this kind of engineering analyses. This is due to many factors, first of all the difficulty in translating domain specific knowledge in machine interpretable rules. Fuzzy theory, developed by Zadeh in 1965 (Zadeh, 1965) makes this process trivial. Using concept fuzzification through membership functions and fuzzy rules/operators it is possible to translate in a machine interpretable language rules like "IF it's cold THEN don't go out", whose conditions' truth couldn't otherwise be interpreted by computers. For our experiment we used a Mamdami Fuzzy model characterized by seven input variables and only one output being the coefficient that suggests if a certain machine should be changed. The model with membership functions pertaining the output variable is presented in Figure 2.

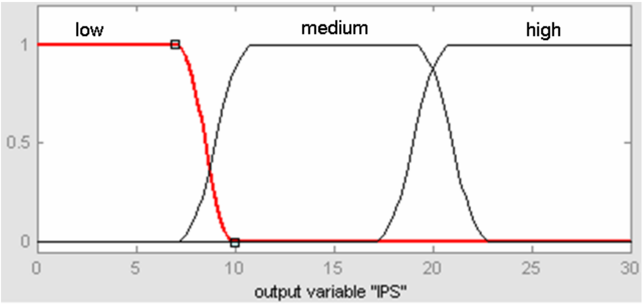

Figure 2. Fuzzy model developed. Membership functions shown are related to the output variable

After the Fuzzy Inference System has been completed the neural network model has been designed. Artificial neural networks had a great success in the scientific community due to their flexibility and their potentialities in predicting highly complex phenomena (Chattopadhyay et al, 2006). A preliminary preprocessing stage has been carried out in order to normalize the data in the [0-1] range in order to optimize the matching between the data domains and transfer functions ones'. A seven input-one output architecture with logistic transfer functions has been designed (Fig. 3).

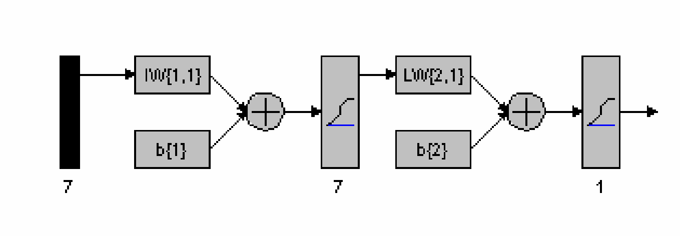

Figure 3 Neural network architecture

The neural network (ANN) has been trained using 60% of the whole available dataset using the conjugate gradient backpropagation algorithm with Powell-Beale restarts (Powell, 1977). The remaining 40% has been used as follows: 20% as testing (used for testing generalization capabilities of the network on-line, in order to stop training when overfitting phenomena start to occur) and 20% as validation (in order to obtain an estimate on the predictive capabilities of the ANN on unseen data). Stopping criterion was set to SSE equal to 1e-4 or number of epochs smaller than 5000. Learning curves are presented in Figure 4

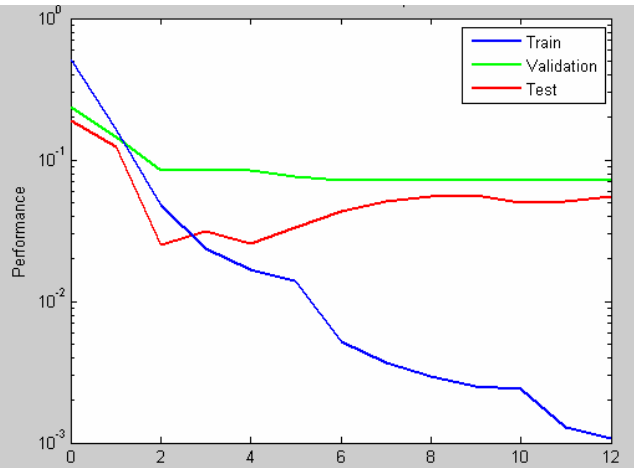

Figure 4. Neural network training, testing and validation curves

After these two approaches have been completed a hybrid neuro-fuzzy strategy has been faced using the ANFIS interface provided by MATLAB (v. 7.4.0). ANFIS is a adaptive neuro-fuzzy inference system that, using a given input/output data set, constructs a fuzzy inference system (FIS) whose membership function parameters are tuned (adjusted) using either a backpropagation algorithm alone or in combination with a least squares type of method.
This adjustment allows fuzzy systems to learn from the data they are modeling. The neuro-adaptive learning method works similarly to that of neural networks. Then, the Fuzzy Logic component computes the membership function parameters that best allow the associated fuzzy inference system to track the given input/output data.

THE CASE STUDY

The Italian context

In Italy, the need to reduce health expenditure has made it necessary to define new management criteria based on a more efficient and rational use of health resources. This need is particularly acute in the sector of biomedical technologies, which, together with other factors, is chiefly responsible for the continued and limitless increase in health expenditure.
Obsolescence often results in the need to check non-rational costs and inefficient healthcare asset management.
In particular, one of the major problems affecting the national health system is the obsolescence of equipment in Italian hospitals and clinics. This problem is already well known and re-emerged following the "Report on the age of electro-medical equipment installed in healthcare structures" carried out by ANIE in 2005.
The study was carried out through a census of the diagnostic equipment, installed by companies which are part of ANIE
and were still in use in a total of 872 public hospital structures.
The equipment typologies analysed are radiology diagnostic systems, such as x-ray machines, remote controlled systems, mammography devices, TAC, ultrasound machines, angiography machines.
The survey underlined a progressive obsolescence of the devices installed. Almost all types of equipment showed an average age which was notably higher than their expected lifespan.
An example is that of equipment for radiological diagnosis which in 69% of cases exceeded 10 years, followed by remote controlled systems (58%), angiography devices (48%), mammography machines (42%). Moreover the geographic data shows that in the south of Italy the obsolescence is greater than in the centre and in the north.
The research also confirmed the seriousness of the phenomenon of aging diagnostic equipment in national healthcare structures.
Of particular concern were the data about equipment used for more than 10 years but still in use in public hospitals (this phenomenon is particularly serious for ultrasound machines, TAC and radiological devices).
In general the Italian statistics show a considerable aging of technological devices in almost all equipment.
Retaining obsolete devices can have extremely negative consequences, among which, the increasing deterioration of the device (with resulting maintenance costs and risk of accidents due to devices that are out of order), an increase in operational costs and finally performance which is not in line with current technological standards.
Unfortunately, accidents in hospitals involving patients which result from malfunctioning devices are all too commonplace and it is essential to provide health structures with suitable management tools, such as that proposed in this paper.

The case study

The proposed model was applied to the devices of the hospital "Casa Sollievo della Sofferenza" of S. Giovanni Rotondo (FG), founded by S.Pio from Pietrelcina in the 1956.
The hospital has the status of scientific institute for hospitalisation an treatment of national importance in the sector of genetic diseases; as a result of this recognition, this insitute continues to operate in the sector of scientific research.
Therefore, *"Casa Sollievo della Sofferenza"* has extremely complex technological equipment both in qualitative terms (large devices for bio-images, innovative tools both for research and routine clinical analysis) as well as quantitatively, with more than 6000 devices.
This equipment is characterised by both a high level of economic renewal and a high level of integration with the technical resources of the hospital, such as IT systems.
The management of these devices is the responsibility of the clinic engineering department within the hospital.

The task was to guarantee the safe, suitable and cost efficient use of technological equipment. This department was essential in acquiring the data necessary to apply the model analysed here.

The hospital database contains data concerning the downtime of each device, the maintenance costs, the age of equipment and it is continuously released by the clinical engineering department according to the information obtained by the other medical departments.

Each process of appraisal is coordinated by this department and involves either the medical staff members that provide their recommendations.

The model was applied to 100 devices belonging to the three groups of operating theatre. The sample selected is representative of the heterogeneity of the devices placed in the hospital structure (fig.5).

The data collected for each device, according to the information needs of the proposed model, was stored in the hospital management information system and/or was provided by the Clinical engineering Department.

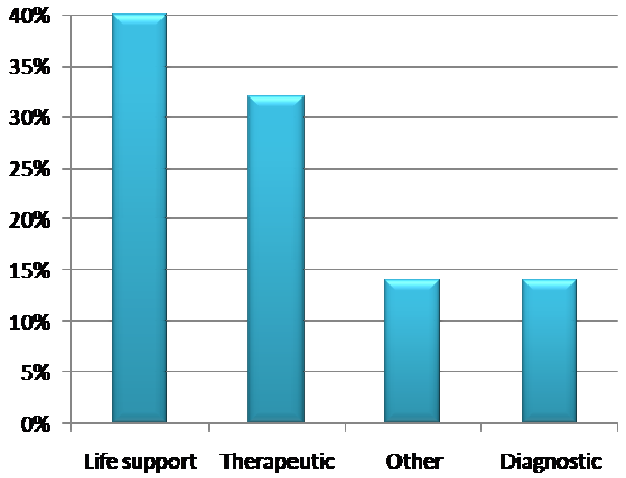

Figure 5: A classification of the sample devices

RESULTS AND DISCUSSIONS

The application of the proposed model to the abovementioned devices, provides the results shown in figure 6.

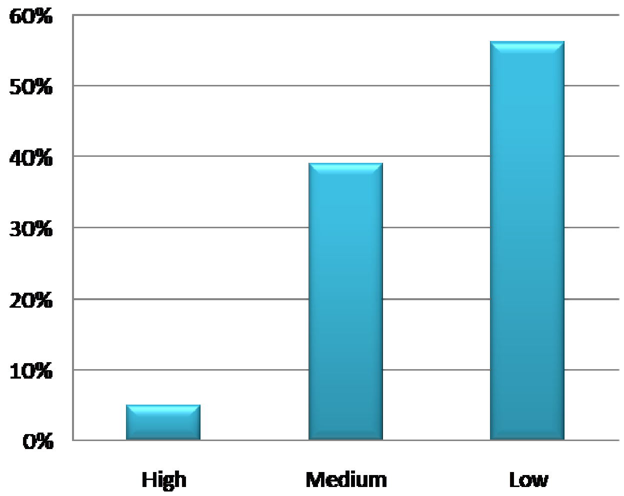

Figure 6. Output results (priority replacement index) provided by the model

These data indicate that only few devices are critical (5% of the total number of devices) and have to be replaced. Other devices have are equally distributed between a medium level RPI (39%) and a low level of RPI (56%)

In particular we carried out two different analysis using a FIS and a Supervised Neural Network in order to assess the accuracy of FIS model. Results returned by Fuzzy Inference System and Artificial Neural Network resulted to be highly overlapping with a K-Statistic equal to 0.87. This is a quite interesting aspect since these are two different learning schemes that, however, predict quite similar outcomes for the same machines. Being the supervised method the closest to the real process under investigation, here we present regression analyses carried out on target/predictions of the ANN on training, testing and validation sets (Figure 7).

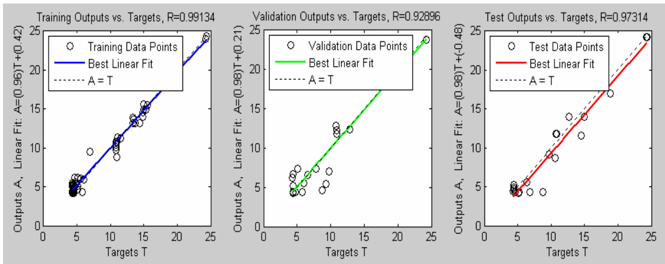

Figure 7. Regression analysis carried out on target/predictions on training, testing and validation. On the top of the figures the agreement between target and predictions.

As it can be seen by the levels of accuracies reached in training and validation sets, both learning and generalization process result to be optimized. After having verified the accuracy levels reached by both the systems we carried out a sensitivity analysis in order to find most predictive values. The results obtained by this analysis put in evidence that higher perturbation in “usage rate” values led to higher instability in output predictions. We tried to perform the same analysis using a the hybrid neuro-fuzzy system presented in the “Algorithms” section. We carried out this analysis using the exhaustive search approach due to the small number of variable. Results of this analysis are reported in Fig. 8.

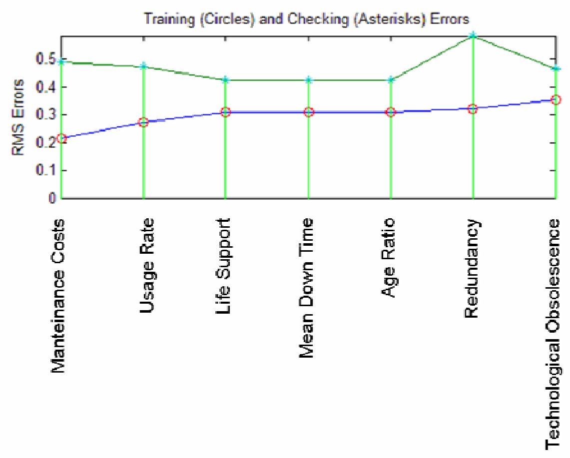

Figure 8. Exhaustive search through the variable space. An ANFIS model has been trained for 1 epoch using each variable and errors on training and validation set have been associated to each input

Conclusions could be drawn from the analysis of the performance reached by the hybrid neuro-fuzzy system developed: both "maintenance ratio" and "usage rate" variables show superior predictive power when compared to other variables. This enforces the results of the sensitivity analysis previously carried out and pushes the interest for further research in this field.

CONCLUSIONS

Replacement analysis is one of the key issues in hospital management. Managing thousands of medical devices is time and cost consuming and in some cases this may result in errors which lead to accidents with potentially fatal consequence for patients. For example, in the context of Italy, patient deaths caused by equipment inefficiency are still commonly reported.

A formal process for appraising medical equipment replacement is needed in order to avoid any risk for patient safety. Replacement analysis based on cost models could fail in the healthcare context since non-economical factors, such as technological obsolescence, medical staff satisfaction and patient safety, are all vitally important in the replacement decision.

Furthermore, scoring models based only on expert judgments could be inappropriate since they are subject to human error and in some situations require a detailed and complex process of appraisal.

In this paper, a ranking procedure of medical device replacement based on Fuzzy Inference Systems (FIS) is proposed in order to include both quantitative and qualitative parameters influencing replacement decisions in a unique and simple process. The model is designed in order to be applied even in hospitals where there is a lack of available data. Moreover, the capability of the model could increase significantly if it is integrated with a computer management system.

To assess the accuracy of the proposed FIS we tried to validate it by using a supervised neural network and to estimate the parameters relevance we implemented a Neuro Fuzzy (ANFIS) approach.

The model, tested on a full case study, proves its effectiveness in the identification of the more critical devices that should be replaced. Obtained results have shown that previously neglected variable such as usage rate and patient safety are relevant for replacement decisions. Future research will focus on a more extensive campaign of experiments in order to improve the fuzzy model performance.

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