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Tightening piecewise McCormick relaxations for bilinear problems

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ABSTRACT

We address nonconvex bilinear problems where the main objective is the computation of a tight lower bound for the objective function to be minimized. This can be obtained through a mixed-integer linear programming formulation relying on the concept of piecewise McCormick relaxation. It works by dividing the domain of one of the variables in each bilinear term into a given number of partitions, while considering global bounds for the other. We now propose using partition-dependent bounds for the latter so as to further improve the quality of the relaxation. While it involves solving hundreds or even thousands of linear bound contracting problems in a pre-processing step, the benefit from having a tighter formulation more than compensates the additional computational time. Results for a set of water network design problems show that the new algorithm can lead to orders of magnitude reduction in the optimality gap compared to commercial solvers.

1. Introduction

The simplest and perhaps most common type of constraint in Chemical Engineering is the blending equation, in which the property of a product resulting from a mix of materials is estimated as a weighted sum by flowrates of the properties of the components. The products of flowrates by properties form bilinear terms that are nonconvex and therefore give rise to multiple local solutions. Blending constraints arise in crude oil operations in refineries (Lee et al., 1996; Jia et al., 2003; Yadav and Shaik, 2012), in the blending of different distilled fractions such as gasoline and diesel (Moro et al., 1998; Jia and Ierapetritou, 2003; Kolodziej et al., 2013b), in the design of distributed wastewater treatment systems (Galan and Grossmann, 1998; Meyer and Floudas, 2006; Teles et al., 2012), integrated water (Karuppiah and Grossmann, 2006; Faria and Bagajewicz, 2012; Rubio-Castro et al., 2013) and mass and property integration networks (Nápoles-Rivera et al., 2010). Bilinear terms also arise in the trim loss problem in paper plants (Harjunkoski et al., 1998; Zorn and Sahinidis, 2013) and in the operation of hydro energy systems (Catalão et al., 2011; Castro and Grossmann, 2014).

In order to find rigorous global optimal solutions to bilinear problems, which can be of the nonlinear (NLP) or mixed-integer nonlinear (MINLP) type, alternative algorithms can be used that have in common the generation of linear (LP) or mixed-integer linear (MILP) relaxations of the original problem. Assuming that we are dealing with a minimization problem, a relaxation provides a lower bound, any feasible solution provides an upper bound and global convergence is achieved when bounds lie within a specified tolerance. It is thus critical to derive tight relaxations to improve the lower bound and also, indirectly, to provide initialization points leading to better solutions.

In the standard McCormick (1976) relaxation, each bilinear term is replaced by a new variable and four sets of linear inequality constraints are added to the formulation. In these, the new variable is related to the two variables forming the bilinear term and their lower and upper bounds. As the tighter the lower and upper bounds, the higher the quality of the relaxation, most global optimization solvers do variable bounding to reduce the search space of the relaxed problem. For instance, the new solver GloMIQO (Misener and Floudas, 2013b) uses different techniques such as interval arithmetic, reduced cost and optimality-based bound contraction. In the latter, which is the subject of this paper, a sequence of minimization and maximization problems are solved for each nonlinearly participating variable to find the tightest possible bounds. Faria and Bagajewicz (2011) proposed a global optimization algorithm based on a more thorough optimality-based bound contraction procedure for one of the variables of each bilinear term. It consists of successive contracting steps that first involve solving a relaxation problem to compute reference values for the bilinear participating variables and then on solving multiple auxiliary linear

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