Differential Unitary Space-Time Signal Design Using Matrix Rotation Structure

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***Abstract*—We consider the design of matrix rotation based space-time signals based on the design criterion of minimizing the union bound on block error probability. We further propose to design the signal parameters via noninteger searching to get better signals. Superior performance of our improved design over the previous design are demonstrated through numerical calculations and performance simulations. With our proposed design for two transmit antennas and one or two receive antennas, we achieve the coding gain of about 1 dB over that of the previous design.**

***Index Terms*—Differential unitary space-time modulation, multiple-antenna communication systems, union bound.**

I. Introduction

WITHOUT the knowledge of channel state information at either the transmitter or the receiver, differential unitary space-time (DUST) modulation scheme [4], [5] was proposed for a multiple input multiple output (MIMO) system under slow Rayleigh flat-fading channels. The proposed design of unitary space-time signals is based on minimizing pairwise block error probability (PBEP). In particular, at asymptotically high signal-to-noise ratio (SNR), the PBEP performance of a good DUST constellation is determined by the so-called diversity product [5].

Based on the design criterion of maximizing the diversity product, a large number of DUST codes have been proposed, for example, diagonal codes or cyclic group codes [4] and [5], generalized quaternion codes or dicyclic group codes [4], fixed-point-free group codes [7] using representation theory, and a nongroup signal constellation called parametric codes [8]. The parametric codes was specifically designed for two transmit antenna communication systems. Recently, the matrix rotation based (MRB) space-time signals [12], with a similar concept as [8], have been proposed for a MIMO system with an even number of transmit antennas.

It was argued recently in [13] that the main target of the performance evaluation is block error probability (BEP), not the PBEP. The codes optimized over the worst-case PBEP do not guarantee the optimum performance in terms of the BEP. Thus, in [13], a code design criterion of minimizing the union bound on BEP was proposed, and some new cyclic codes were obtained.

In this letter, we design the MRB space-time signals by using the design criterion of minimizing the union bound on BEP. Moreover, we propose to search noninteger parameters for the MRB signal scheme. The search method and computational complexity reduction are also presented. The merit of the proposed design is demonstrated by numerical calculations and performance simulations.

II. Channel Model and DUST Scheme

We consider a MIMO system with M_T transmit antennas and M_R receive antennas. The channel coefficients are assumed unknown to neither the transmitter nor the receiver. For T consecutive time intervals, the received signals are formulated in matrix form as

$$\mathbf{Y}_{\boldsymbol{\tau}} = \sqrt{\rho}\mathbf{S}_{\boldsymbol{\tau}}\mathbf{H}_{\boldsymbol{\tau}} + \mathbf{W}_{\boldsymbol{\tau}}, \quad \tau = 0, 1, \ldots, \tag{1}$$

where τ is the time index of block transmissions, $\mathbf{Y}_{\boldsymbol{\tau}}$ is the $T \times M_R$ received signal matrix, $\mathbf{S}_{\boldsymbol{\tau}}$ is the $T \times M_T$ transmitted signal matrix, and the $M_T \times M_R$ fading-coefficient matrix $\mathbf{H}_{\boldsymbol{\tau}}$ and the $T \times M_R$ additive noise matrix $\mathbf{W}_{\boldsymbol{\tau}}$ have complex Gaussian elements with $\mathcal{CN}(0,1)$ distributed. The transmitted signal is normalized to have unit energy during one transmission period to ensure that ρ is the averaged SNR per receiver, i.e., $\mathrm{E}[\sum_{i=1}^{M_T} |s_t^i|^2] = 1$, where E represents expectation operator.

In the following, we will assume square-size transmitted signal matrices, i.e., $T = M_T$. The transmission process follows the fundamental differential transmitter equation [5]

$$\mathbf{S}_{\tau} = \begin{cases} \boldsymbol{\Phi}_{z_\tau}\mathbf{S}_{\tau-1}, & \tau = 1, 2, \ldots \\ \mathbf{I}_{M_T \times M_T}, & \tau = 0 \end{cases} \tag{2}$$

where $\mathbf{I}_{M_T \times M_T}$ is an $M_T \times M_T$ identity matrix. $z_\tau \in \{0, 1, \ldots, L-1\}$ denotes an integer index of a distinct unitary matrix signal $\boldsymbol{\Phi}_{z_\tau}$ drawn from a signal constellation $\mathcal{V}$ of size $L = 2^{RM_T}$ with R represents the information rate in bits per second per hertz (b/s/Hz).

We combine two consecutive received signal matrices using (1) and (2) and assume that the channel coefficients are almost constant over two consecutive blocks, i.e., $\mathbf{H}_\tau \approx \mathbf{H}_{\tau-1}$. We obtain the fundamental differential receiver equation

$$\mathbf{Y}_{\boldsymbol{\tau}} = \boldsymbol{\Phi}_{z_\tau}\mathbf{Y}_{\tau-1} + \sqrt{2}\mathbf{W}'_{\boldsymbol{\tau}}, \tag{3}$$

where $\mathbf{W}'_\tau = (1/\sqrt{2})(\mathbf{W}_\tau - \boldsymbol{\Phi}_{z_\tau}\mathbf{W}_{\tau-1})$ is an $M_T \times M_R$ additive independent noise matrix with $\mathcal{CN}(0,1)$ distributed elements. The differential decoder performs maximum likelihood decoding and the decision rule can be expressed as [5]

$$\hat{z}_\tau^{ML} = \arg\min_{l \in \mathbb{Z}_l} \|\mathbf{Y}_\tau - \boldsymbol{\Phi}_l\mathbf{Y}_{\tau-1}\|_F \tag{4}$$

where $\mathbb{Z}_l = \{0, 1, \ldots, L-1\}$ and $\|\cdot\|_F$ is the Frobenius norm.

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It has been shown in [13] that the exact expression of the PBEP of mistaking $\mathbf{\Phi}_l$ for $\mathbf{\Phi}_{l'}$ is

$$P(\mathbf{\Phi}_l \to \mathbf{\Phi}_{l'}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{m=1}^{\Delta_H} \left(1 + \frac{\gamma\lambda_m}{4\sin^2\theta}\right)^{-M_R} d\theta \qquad (5)$$

where $\gamma = (\rho^2/(1+2\rho))$, λ_m is the mth eigenvalue of the matrix $\mathbf{C}_S = (\mathbf{\Phi}_l - \mathbf{\Phi}_{l'})(\mathbf{\Phi}_l - \mathbf{\Phi}_{l'})^\dagger$ and Δ_H is the rank of $\mathbf{C}_S$.

III. Design Method

A. The MRB Space-Time Signal Scheme

For asymptotically high SNR, the Chernoff bound of (5) depends on the product of nonzero eigenvalues of $\mathbf{C}_s$. This leads to a design criterion that aims to maximize the following diversity product [5]:

$$\zeta = \frac{1}{2}\min_{l \neq l' \in \mathbb{Z}_l} |\det(\mathbf{\Phi}_l - \mathbf{\Phi}_{l'})|^{1/M_T}. \qquad (6)$$

Many DUST signal constellations, such as [4]–[8], [11], and [12], were designed based on the performance measure in (6). Recently, in [12], the MRB space-time signal scheme was introduced particularly for communication systems with even number of transmit antennas. Assume a system with M_T transmit antennas and a unitary signal constellation of size L, a set of MRB space-time signals is defined as [12]

$$\mathcal{V} = \{\mathbf{\Phi}_l(\mathbf{K}) : l = 0, 1, \ldots, L-1\}, \qquad (7)$$

where each $\mathbf{\Phi}_l$ is a unitary matrix depending on the parameters $\mathbf{K} = \{k_{11}, \ldots, k_{1M_T}; k_2\}$ whose elements are integer numbers from $\mathbb{Z}_l$. Specifically, denote $\mathbf{j} = \sqrt{-1}$ and $\theta_L = 2\pi/L$, then for any $l = 0, 1, \ldots, L-1$, $\mathbf{\Phi}_l$ is given by

$$\mathbf{\Phi}_l(\mathbf{K}) = \mathbf{\Lambda}^l \cdot [\mathbf{I}_N \otimes \mathbf{\Psi}(k_2\theta_L)]^l \qquad (8)$$

where

$$\mathbf{\Lambda} = \mathrm{diag}\left(e^{\mathbf{j}\theta_L k_{11}}, \ldots, e^{\mathbf{j}\theta_L k_{1M_T}}\right)$$
$$\mathbf{\Psi}(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

where $\mathbf{I}_N$ is the identity matrix of size $N \times N$ with $N = M_T/2$ and $\otimes$ represents the tensor product.

B. Improved MRB Space-Time Signal Design

It has been argued in [13] that constellation design that achieve maximum diversity product ζ may not be appropriate, especially at a medium range of SNRs. Hence, it was suggested in [13] that DUST codes should be designed based on the design criterion of minimizing the union bound on BEP. Specifically, assuming that all the L space-time signals, $\mathbf{\Phi}_l$, are equally likely transmitted, the performance measure of BEP is approximated by the union bound

$$\mathrm{BEP} \leq \frac{2}{L}\sum_{l=0}^{L-2}\sum_{l'=l+1}^{L-1} P(\mathbf{\Phi}_l \to \mathbf{\Phi}_{l'}) \triangleq \mathrm{PUB} \qquad (9)$$

where $P(\mathbf{\Phi}_l \to \mathbf{\Phi}_{l'})$ is specified in (5).

We now consider to design the MRB space-time signals using the design criterion of minimizing the union bound in (9). For demonstration purposes, we focus on a constellation design method for two transmit antennas. Similar procedures can be applied to constellation design for higher number of transmit antennas. In the case of $M_T = 2$, the code structure of the MRB signal scheme is

$$\mathbf{\Phi}_l = \begin{bmatrix} e^{\mathbf{j}\theta_L l\tilde{k}_{11}} & 0 \\ 0 & e^{\mathbf{j}\theta_L l\tilde{k}_{12}} \end{bmatrix} \cdot \mathbf{\Psi}(l\tilde{k}_2\theta_L) \qquad (10)$$

where $l = 0, 1, \ldots, L-1$. The MRB space-time signals in (10) are determined by three parameters $\tilde{\mathbf{K}} = \{\tilde{k}_{11}, \tilde{k}_{12}; \tilde{k}_2\}$. Our design goal is to find a set of parameters $\tilde{\mathbf{K}}$ that minimize the PUB in (9).

Moreover, in our design, we relax the set of parameter $\tilde{\mathbf{K}}$ in (10) to be noninteger numbers, i.e., $\tilde{\mathbf{K}} = \{\tilde{k}_{11}, \tilde{k}_{12}; \tilde{k}_2 \,|\, 0 < \tilde{k}_{11}, \tilde{k}_{12}, \tilde{k}_2 < L\}$. With such extension, we increase the set of search parameters which allow us to have more chance to obtain better signals. Note that all of the previous designs in [4]–[8], and [11]–[13], the set of signal parameters is confined to the set integer numbers. Actually, such a requirement is not necessary in the DUST modulation scheme.

C. Search Method

For any number of constellation size $L \geq 2$ and given values of M_T, M_R, and SNR (ρ) of interest, we perform an exhaustive computer search for the best set of noninteger parameters $\tilde{\mathbf{K}}$ that minimize the PUB. We target constellation performances in the range of 10^{-4} to 10^{-7} which correspond to operating SNRs between 20 and 30 dB, depending on L, M_T and M_R. With the symmetrical property of the full-rotation matrix $\Psi(\theta)$, we found that the summation of the best parameters $\tilde{k}_{11}$ and $\tilde{k}_{12}$ are approximately L in many cases of computer searching. Simplifying the search algorithm below is sufficient to find the best set of parameters $\tilde{\mathbf{K}}$, as follows:

$$0 < \tilde{k}_{11} \leq L/2, \quad (L/2 + \tilde{k}_{11}) \leq \tilde{k}_{12} < L, \quad 0 < \tilde{k}_2 \leq L/2.$$

For a signal constellation of small size, i.e., $L = 4$, we use step size 0.001 for each of the parameters in $\tilde{\mathbf{K}}$. For other constellation sizes, due to the complexity of the search space, we limit our search to a searching step of 0.1.

For large-signal constellation sizes, we further reduce computational complexity by applying the inequality [9]

$$\prod_{i=1}^{\Delta_H}(1 + x_i) \geq (1 + x_{gm})^{\Delta_H} \qquad (11)$$

for $x_i > 0$ and $x_{gm} = (\prod_{i=1}^{\Delta_H} x_i)^{1/\Delta_H}$ into (5) to get the approximated PBEP in closed form as [13]

$$P(\mathbf{\Phi}_l \to \mathbf{\Phi}_{l'}) \lesssim \frac{1}{2}\{1 - \Gamma(k, \alpha_1)\} \qquad (12)$$

where the expression of $\Gamma(k, \alpha_1)$ is

$$\Gamma(k, \alpha_1) = \alpha_1 \sum_{k=0}^{M_T M_R - 1} \binom{2k}{k}\left(\frac{1 - \alpha_1^2}{4}\right)^k \qquad (13)$$

TABLE I
COMPARISON OF CONSTELLATION PARAMETERS AND UNION BOUNDS FOR THE MRB SPACE-TIME SIGNAL DESIGN WITH $M_T = 2$ TRANSMIT ANTENNAS

$[L, M_R]$	New Parameters $\tilde{\mathbf{K}}$	New Parameters PUB	Original Parameters $\mathbf{K}$	Original Parameters PUB
[4, 1]	[0.389, 3.611, 1.338]	$5.1786e^{-4}$	[1, 1, 0]	$6.4076e^{-4}$
[4, 2]	[1.616, 2.384, 0.692]	$3.5227e^{-7}$	[1, 1, 0]	$6.4036e^{-7}$
[8, 1] †	[2, 6, 5]	$1.8647e^{-5}$	[3, 5, 2]	$1.8647e^{-5}$
[8, 2]	[2, 6, 5]	$1.8795e^{-6}$	[3, 5, 2]	$1.8795e^{-6}$
[16, 1] †	[1.5, 14.5, 7.5]	$6.9510e^{-5}$	[3, 9, 4]	$7.7840e^{-5}$
[16, 2]	[2, 10, 9]	$1.3671e^{-5}$	[3, 9, 4]	$1.3671e^{-5}$
[32, 1] †	[4, 28, 15]	$2.3474e^{-4}$	[3, 5, 8]	$2.6699e^{-4}$
[32, 2]	[9.1, 30.7, 8.2]	$1.1025e^{-4}$	[3, 5, 8]	$1.2863e^{-4}$
[64, 1] †	[2.5, 51.5, 8.5]	$7.9933e^{-4}$	[3, 21, 2]	$1.1197e^{-3}$
[64, 2]	[26.5, 59.5, 0.5]	$5.4148e^{-4}$	[3, 21, 2]	$1.2310e^{-3}$

Note: † indicates an operating SNR at 30 dB and 20 dB otherwise.

with $\lambda_{gm} = (\Delta_P)^{1/M_T}, \Delta_P = \prod_{m=1}^{M_T} \lambda_m$, and $\alpha_1 = \sqrt{((\gamma\lambda_{gm})/(4+\gamma\lambda_{gm}))}$. It was shown in [13] that the approximated PBEP in (13) is asymptotically tight to its exact value at high SNR. Explicitly, for $M_T = 2$ and $M_R = 1$, we have

$$\text{PUB} \approx \frac{2}{L}\sum_{l=0}^{L-2}\sum_{l'=l+1}^{L-1}\left(0.5 - 0.75\alpha_1 + 0.25\alpha_1^3\right). \quad (14)$$

Similarly, in the case of $M_T = 2$ and $M_R = 2$

$$\text{PUB} \approx \frac{2}{L}\sum_{l=0}^{L-2}\sum_{l'=l+1}^{L-1}\boldsymbol{\Upsilon}(\alpha_1) \quad (15)$$

where $\boldsymbol{\Upsilon}(\alpha_1) = 0.5 - 1.094\alpha_1 + 1.094\alpha_1^3 - 0.656\alpha_1^5 + 0.156\alpha_1^7$. Instead of performing numerical integrations, the approximated PUB in (14) and (15) require only algebraic computations, which reduce execution times considerably.

Table I shows our parameter search results for signal constellation size $L = 4, 8, 16, 32$, and 64. For consistent of the predetermined operating SNRs, we chose to design the MRB signals at either 20 dB or 30 dB as indicated. To illustrate the coding advantages of our design, we list PUBs of the obtained codes in [12] that optimized the diversity product compare to PUBs from our design. We observe that for $L = 4, 32$, and 64, the union bounds of the new designs are less than that of the original designs. Depending on a predetermined operating SNR, the constellation parameters can be different for system with one or two receive antennas. In the case of $L = 8$ and 16, although the obtained parameters are different from those in [12], the union bounds of them are almost the same.

Note that if we constrain the searching over integer parameters using the PUB in (9) or the approximated PUB in (14) or (15), we obtained the same PUBs as those with original parameters for almost every constellation sizes. Except for constellation size $L = 64$ with $M_R = 1$, the resulting PUB is $8.6426e^{-4}$, which is better than that of original design, but it is higher than the one from noninteger searching.

IV. SIMULATION RESULTS

We simulated the DUST modulation schemes for two transmit and one or two receive antennas. The channel fading coefficients are assumed to be independent between antennas, but time correlated according to Jakes' model [15], in which the Doppler frequency is $f_D = 75$ Hz and normalized fading parameter is $f_D T_s = 0.0025$, where T_s is the sampling period.

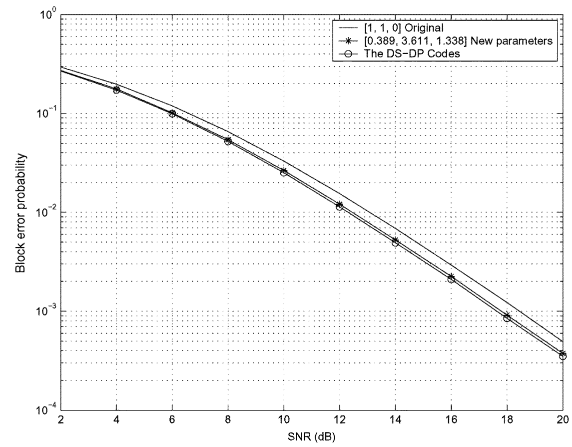

Fig. 1. Performance for $L = 4, M_T = 2$, and $M_R = 1$.

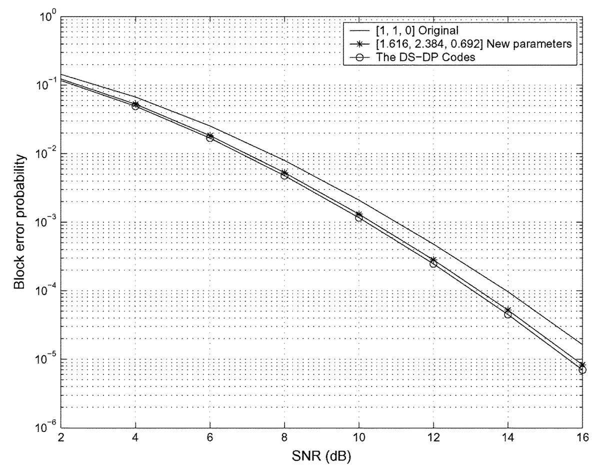

Fig. 2. Performance for $L = 4, M_T = 2$, and $M_R = 2$.

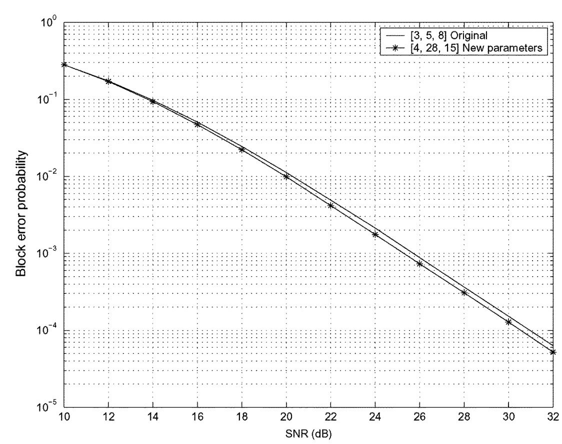

Fig. 3. Performance for $L = 32, M_T = 2$, and $M_R = 1$.

Figs. 1 and 2 show performances of the MRB signals with constellation size $L = 4$, i.e., $R = 1$ b/s/Hz. We observe that our new codes achieve coding advantages of about 0.75–1 dB over the codes designed in [12] at the BEP range 10^{-3}–10^{-4}. Moreover, we compare our signal performances to those of a code with optimum diversity sum and product in [8]; the so-called DS-DP codes. Simulation results show that the performances of

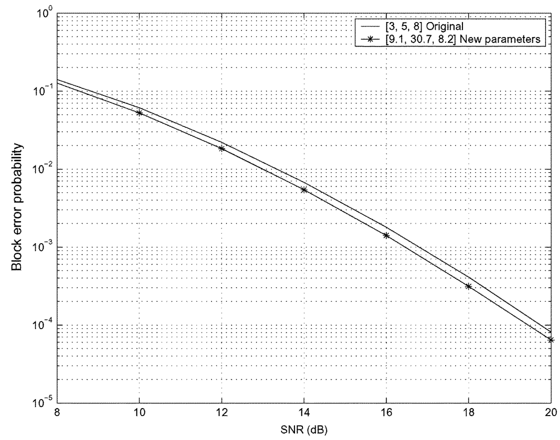

Fig. 4. Performance for $L = 32, M_T = 2$, and $M_R = 2$.

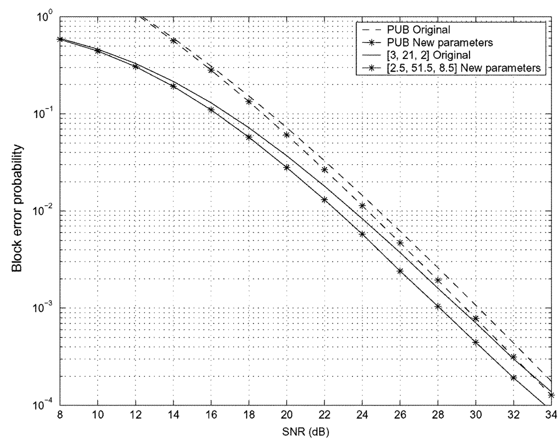

Fig. 5. Performance for $L = 64, M_T = 2$, and $M_R = 1$.

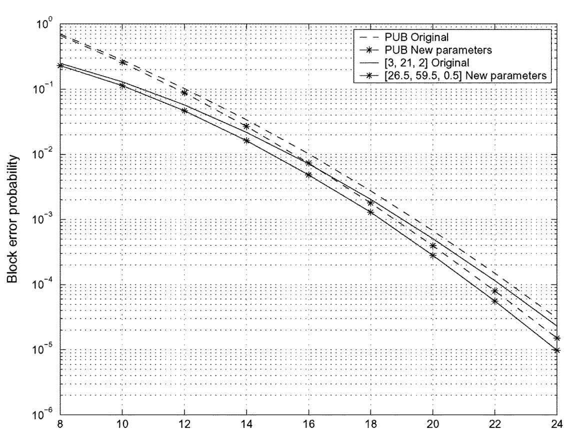

Fig. 6. Performance for $L = 64, M_T = 2$, and $M_R = 2$.

the MRB codes with new parameters are close to the DS-DP code performances. Although the DS-DP code provide slightly better performance, it is a hand-crafted signal constellation derived from Sphere Packings.

The BEP performances of constellation size $L = 32$ ($R = 2.5$ b/s/Hz) are illustrated in Figs. 3 and 4 for $M_R = 1$ and 2, respectively. In comparison with the original parameters, our new parameters yields better performance in both cases. This confirms the merit of the design criterion in (9).

For $L = 64$ or $R = 3$ b/s/Hz with $M_T = 2, M_R = 1$, and $M_R = 2$, the constellation performances are shown in Figs. 5 and 6, respectively. We observe coding gains of 1 dB at a BEP of 10^{-3} and 10^{-4} over the previous design in [12] for the single receive antenna system and the system with two receive antennas, respectively. Also in the figures, we show the tightness of the PUB curve, which is numerically evaluated from (9), to the true constellation performance. As expected, the PUB curve converges to the true performance at high SNR in both figures.

V. Conclusion

We improved the MRB signal design for the DUST modulation system by using the design criterion of minimizing union bound on block error probability. Furthermore, we relaxed the parameter search from integers to nonintegers to get better codes. By taking advantage of symmetric property of the full-rotation matrix, we remarkably reduced search space for the best signal constellation. The approximated union bound was applied to further reduce computation time. Simulation results showed the performance improvement of the obtained signals, for example, about 0.75 dB for constellation size $L = 4$ and about 1 dB for $L = 64$, which support our numerical calculations.

References

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 3, pp. 744–765, Mar. 1998.

[2] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communications in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 46, no. 3, pp. 543–564, Mar. 2000.

[3] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity," *IEEE J. Select. Areas Commun.*, vol. 18, no. 7, pp. 1169–1174, Jul. 2000.

[4] B. L. Hughes, "Differential space-time modulation," *IEEE Trans. Inform. Theory*, vol. 46, no. 11, pp. 2567–2578, Nov. 2000.

[5] B. M. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *IEEE Trans. Commun.*, vol. 48, no. 12, pp. 2041–2052, Dec. 2000.

[6] B. L. Hughes, "Optimal space-time constellation from groups," *IEEE Trans. Inform. Theory*, vol. 49, no. 2, pp. 401–410, Feb. 2003.

[7] A. Shokrollahi, B. Hassibi, B. M. Hochwald, and W. Sweldens, "Representation theory for high-rate multiple-antenna code design," *IEEE Trans. Inform. Theory*, vol. 47, no. 9, pp. 2335–2367, Sep. 2001.

[8] X.-B. Liang and X.-G. Xia, "Unitary signal constellations for differential space-Time modulation with two transmit antennas: Parametric codes, optimal designs, and bounds," *IEEE Trans. Inform. Theory*, vol. 48, no. 8, pp. 2291–2322, Aug. 2002.

[9] M. K. Byun and B. G. Lee, "New bounds of pairwise error probability for space-time codes in Rayleigh fading channels," in *Proc. IEEE WCNC*, Mar. 2002, pp. 89–93.

[10] H. F. Lu, Y. Wang, P. V. Kumar, and K. M. Chugg, "On the performance of space-time codes," in *Proc. IEEE Inform. Theory Workshop*, Oct. 2002, pp. 49–52.

[11] J. Wang, M. P. Fitz, and K. Yao, "Differential unitary space-time modulation for a large number of receive antennas," in *Proc. Signals, Systems and Computers Conf.*, vol. 1, Nov. 2002, pp. 565–569.

[12] C. Shan, A. Nallanathan, and P. Y. Kam, "Signal constellation for differential unitary space-time modulation with multiple transmit antennas," in *Proc. 57th IEEE Semiannual Vehicular Technology Conf.*, vol. 1, Apr. 2003, pp. 713–716.

[13] J. Wang, M. K. Simon, and K. Yao, "On the optimum design of differential unitary space-time modulation," in *Proc. Global Telecommun. Conf*, vol. 4, Dec. 2003, pp. 1968–1972.

[14] T. Himsoon, W. Su, and K. J. R. Liu, "Matrix rotation based signal design for differential unitary space-time modulation," in *Proc. IEEE WCNC*, vol. 3, Atlanta, GA, Mar. 2004, pp. 1446–1450.

[15] W. C. Jakes, *Microwave Mobile Communications*. Piscataway, NJ: IEEE Press, 1993.