Structural Modal Identification Toolsuite (SMIT)

User's Guide

Release 1.0

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Structural Modal Identification Toolsuite

User's Guide

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Introduction

Over the past decade, advances in micro-electro-mechanical systems (MEMS) technology have paved the way for the development of multifunctional sensor nodes that make up a wireless sensor network (WSN) [1-3]. These nodes are less expensive to install and maintain than wired sensor networks, and can also collect data at a higher spatial and temporal resolution [4, 5].

A significant application in the civil engineering industry is the use of WSNs for structural health monitoring (SHM). Researchers can use data collected by strategically-placed sensors to identify a structural modal parameters, i.e. natural frequencies, damping ratios, and mode shapes [6-8]. Structural damage affects structural properties; therefore, these parameters may be traced over time to monitor the health of the system [9]. The center for Advanced Technology for Large Structural Systems (ATLSS) at Lehigh University housed an experiment to evaluate the performance of a WSN by comparing (1) the quality of influence coefficients and (2) the rate of convergence of estimated parameters of a scaled specimen of a steel beam-column connection that was subjected to harmonic excitations. The performance of the WSN was deemed acceptable and fairly capable of detecting structural damage. The WSN was tested against a wired network and detected damage just as well as the wired sensor network [10].

System identification algorithms make it possible to estimate modal parameters based on a vibrational response to a disturbance. SMIT collects the following information in order to construct a stabilization diagram and mode shape plots where the user can graphically identify true structural modes:

- Multi Input and Multi Output (MIMO) data files (output-only, in some cases)
- Geometric information (simple shear, simple bridge, etc.)
- System identification method
- Sampling frequency
- Filtering method
- Tolerance of modal parameters

There are many system identification methods, but it is not efficient to implement these techniques by hand. Although several system identification software programs have been developed, there is currently no program that allows users to customize required information while also being able to estimate the modal parameters of structural systems. SMIT meets the need for a comprehensive computer program that simplifies the system identification process and allows for comparison of methodology.

MATLAB was the program chosen for the development of SMIT because it meets the following criteria: (1) ability to process eigenvalue/eigenvector and singular value decomposition (SVD) functions, (2) ability to formulate large matrices and create variables without pre-allocation of memory, (3) possession of build-in functions for creating graphical results, and (4) compatibility with all major operating systems, including Microsoft Windows, Apple Mac OS, and other UNIX systems [11].

Getting Started

Product Overview

Why use Structural Modal Identification Toolsuite (SMIT)?

Use SMIT if you want to...

- Plot a stabilization diagram
- Inspect mode shapes
- Estimate natural frequency and damping ratio
- Display Power Spectral Density (PSD) of output response

SMIT allows users to...

- Choose between Input/Output and Output-Only systems
- Customize structural geometries
- Compare results from several identification methods
- Apply various filtering methods (or no filtering at all)
- Designate the model order specification of Hankel matrix
- Limit bounds for frequency, damping ratio, and MAC
- Select which mode shapes they wish to view

SMIT is different from other system identification programs because it...

- Accommodates both Input/Output and Output-Only systems
- Comes with a user-friendly interface
- Requires less computer memory and time to run than similar programs

About System Identification

What is system identification?

System identification is the process of constructing mathematical models of dynamic systems based on analysis of the input and output signals.

What process does SMIT use to perform system identification?

SMIT allows users to choose a method among ten system identification algorithms (five for Input/Output and other five for Output-Only). The theoretical background of the investigated system identification methods is based on the state space model, which expresses the dynamic equations of motion in the first order differential equations as,

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i \\ y_i &= Cx_i + Du_i \end{aligned} \tag{1}$$

In Eq. (1), x_i is the state vector at i^{th} time step, u_i is the input vector at r locations, and y_i is the output vector at n locations on the structure (nodes). See appendix 1 for more information about the state space model. The coefficients [A, B, C, D] are called the discretized state, input, output, and feed-through matrices, respectively.

The purpose of each system identification method is to estimate the modal parameters equivalent to the above coefficient matrices using sensor data. The solution of Eq. (1) is given in appendix 2 which shows the relationship between the output response and coefficient matrices.

How do SMIT's ten different system identification methods compare to one another?

The current version of SMIT includes five methods for analyzing input/output systems and five methods for analyzing output-only systems as summarized in Table 1

Table 1: SMIT's system identification methods by data type applicability. Input/Output systems are denoted "I/O", and Output-Only systems are denoted "OO".

ERA		ARMAX		SRIM		N4SID	
I/O	OO	I/O	OO	I/O	OO	I/O	OO
ERA (impulse response only)	ERA-OKID-OO	ARX	AR	SRIM	X	N4SID -IO	N4SID -OO
	ERA-NExT						
ERA-OKID-IO	ERA-NExT-AVG						

The descriptions of methods retrieved from [12] in each category of algorithms (I/O and OO) are presented in a separate section in this manual.

ERA / ERA-NExT

The original ERA method [13] uses impulse response (i.e. Markov parameter) in order to exclude the input and feedthrough matrices in Eq. (1). The Natural Excitation Technique [14] simulates impulse response of a system due to a white noise input load, by placing auto- and cross-correlation functions in terms of time lag. The Hankel matrix, composed of Markov parameters corresponding to the sequential time lag, estimates the transition state and output matrices using Singular Value Decomposition (SVD) applied to the Hankel matrix.

ERA-NExT-AVG

A modified NExT algorithm, known as ERA-NExT-AVG, proposed in [15], uses a coded average of row vectors in each Markov parameter for original NExT. The coding scheme is applied to avoid eliminating the real anti-symmetric modes by adding averaging responses from every sensing location. The main focus of this method is to improve the efficiency of the ERA-NExT method. The computational cost is reduced tremendously by taking Markov parameters in a vector form instead of a square matrix with a comparable level of accuracy.

ERA-OKID (ERA-OKID-IO / ERA-OKID-OO)

The observer/Kalman filter is used to extend the applicability of ERA method to the identification of systems with unknown initial conditions and input terms [16, 17]. The observer gain inserted into Eq. (1) derives the transformed state space model, which possibly eliminates the input and feedthrough matrices for large model orders. The system Markov parameters are then obtained by taking the convolution sum of the previous output data and observer Markov parameters.

ARMAX (ARX / AR)

The Auto-Regressive model with eXogenous terms (ARX) expresses the current step of the output in terms of the current input, and the previous input and output [5, 18]. Assuming that the input terms are uniformly distributed in the frequency domain, the current output of a system is determined by the convolution sum between the coefficient matrix and the previous output. For the canonically arrayed input/output, the coefficient matrix (i.e. the companion matrix) can be obtained, which plays the role of the transition state matrix.

SRIM

The System Realization using Information Matrix (SRIM), which is defined by using correlation functions of the input and output data, is another modification of the ERA method [19]. The stationary state space model equation shares equivalent modal information with the modified model by taking correlation functions. It is noted that the information matrix contains all structural parameters since the correlation functions are known to be positive semi-definite or semi-definite, according to the rank of the original data.

N4SID (N4SID-IO / N4SID-OO)

Numerical algorithms for Subspace State Space System Identification (N4SID) use projections of the future output onto the previous input/output data [20]. Theoretically, the optimal prediction of future output is acquired when the error between projection and measured future output is minimized and equivalent to the combination of state space models. Two successive state sequences, obtained by the predictions of input and output data, are used to derive the state space model.

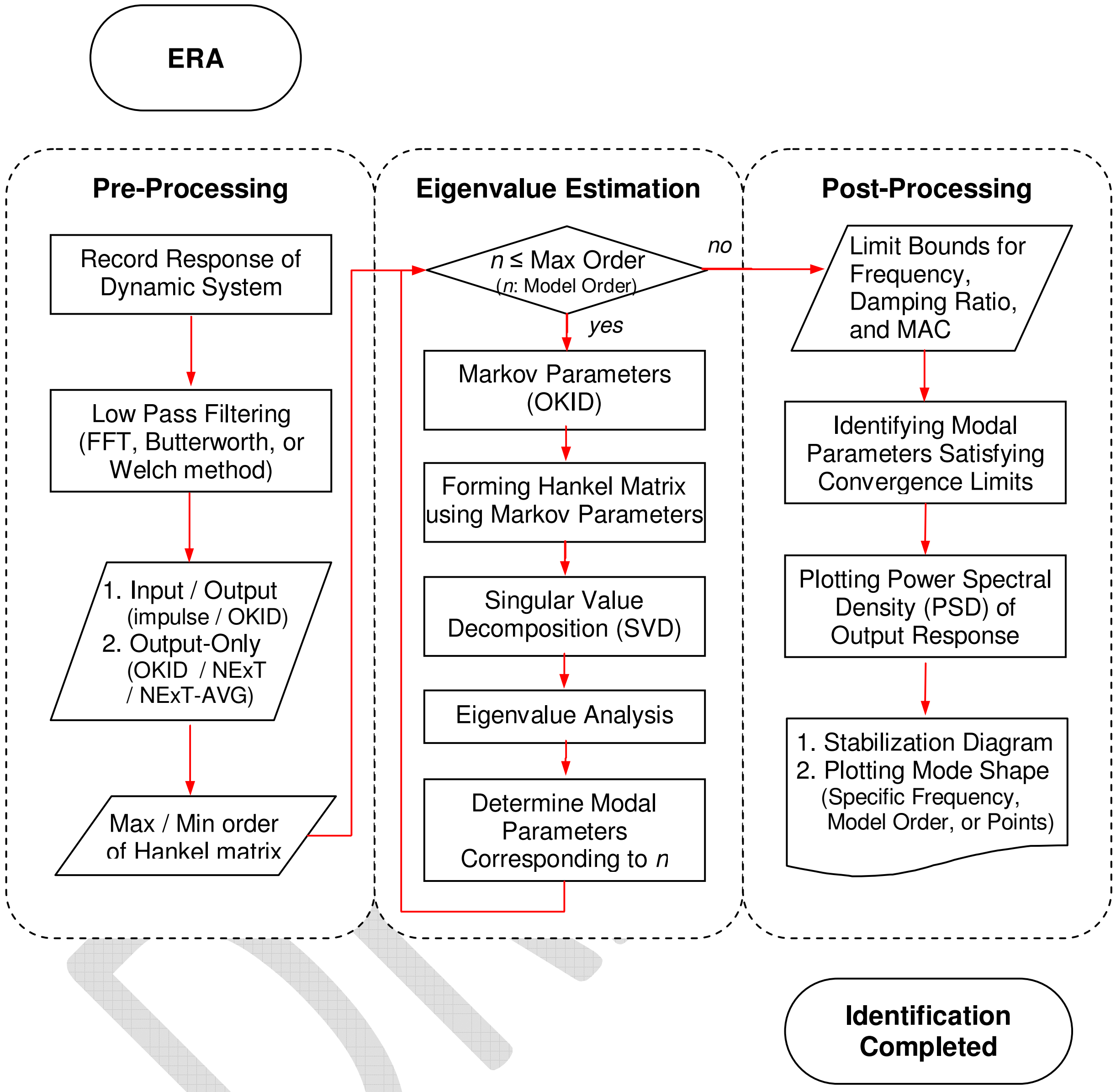


Figure 1: Flow chart illustrating the computational process behind the ERA method

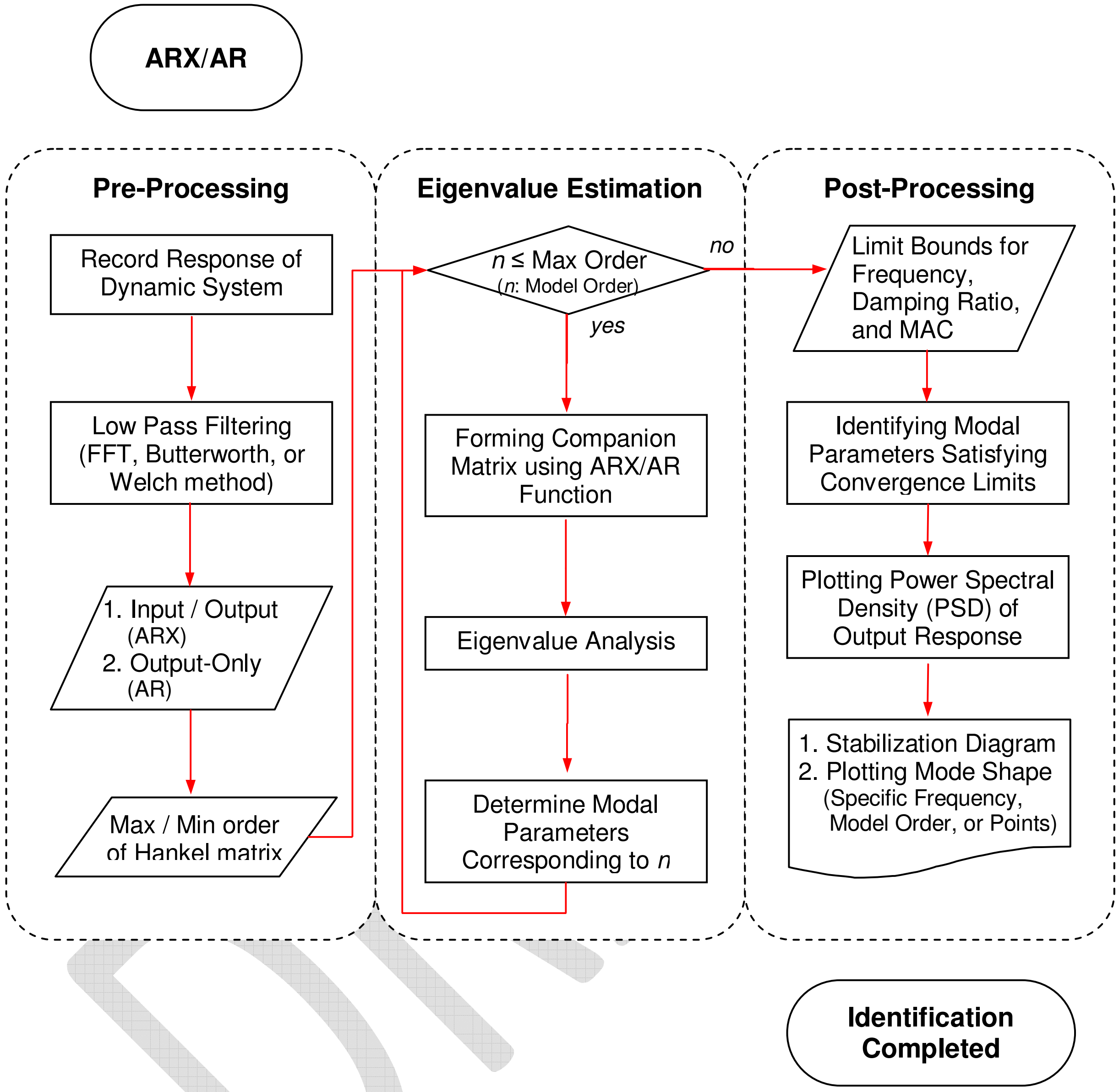


Figure 2: Flow chart illustrating the computational process behind the ARX/AR method

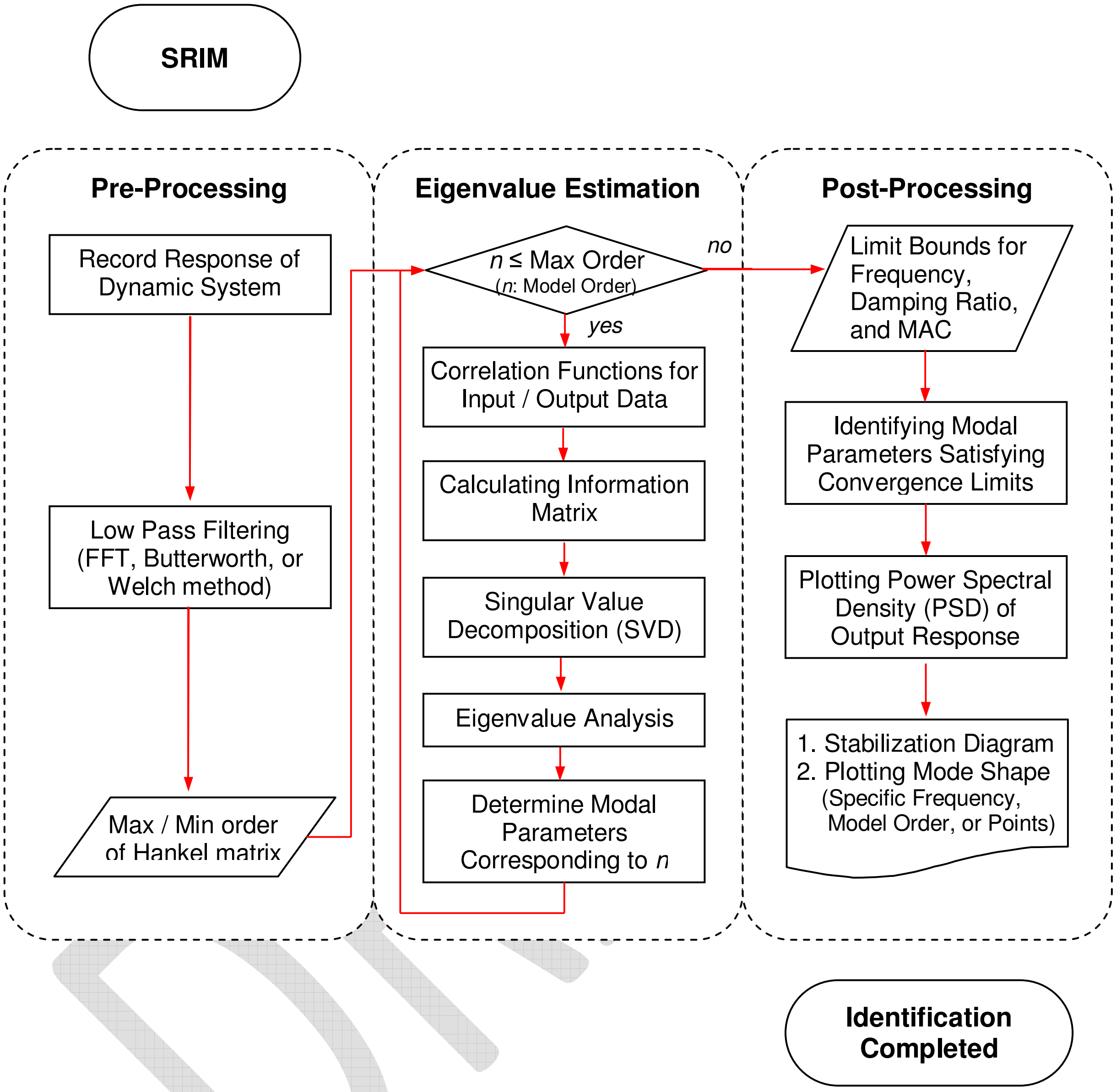


Figure 3: Flow chart illustrating the computational process behind the SRIM method

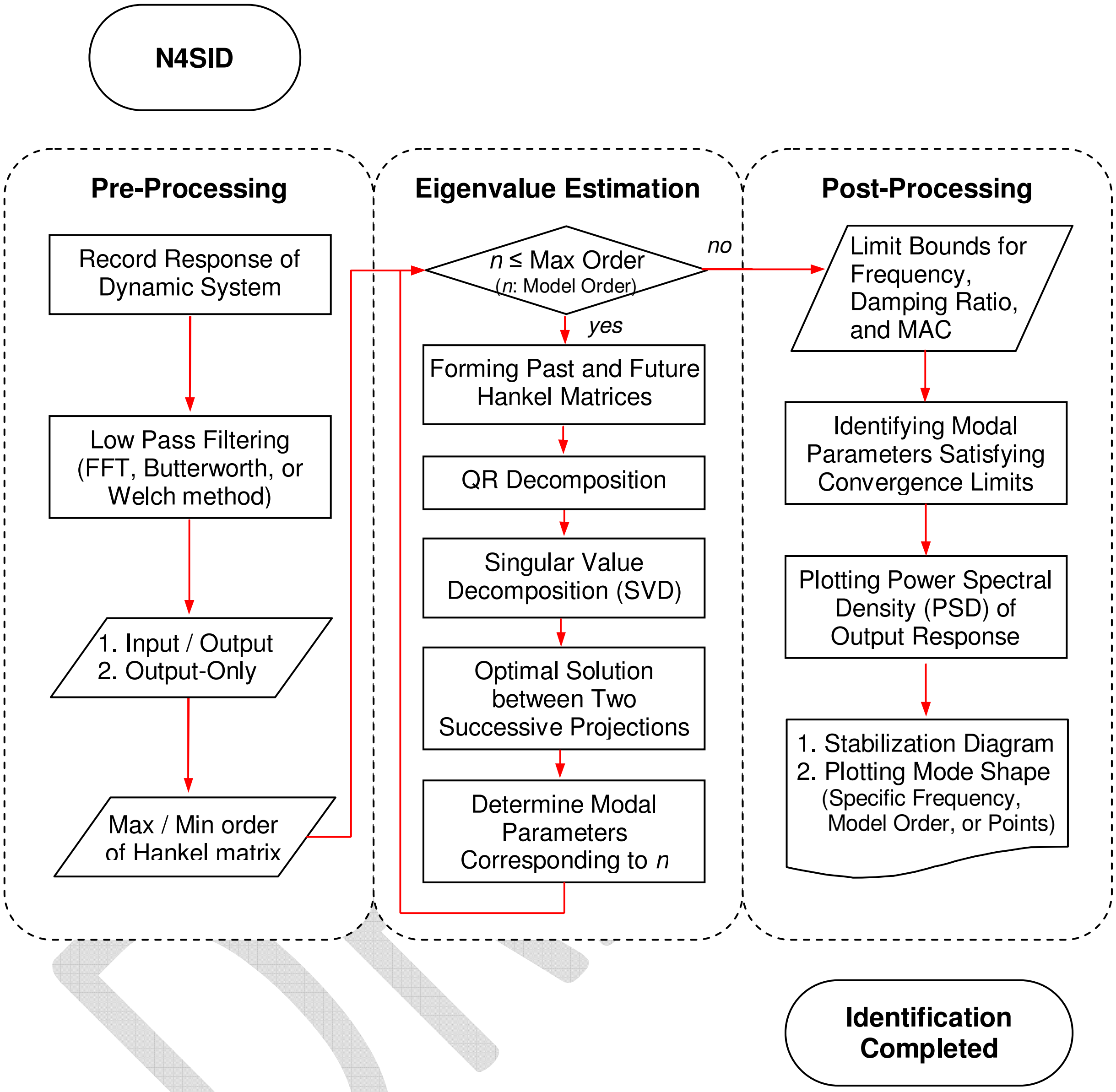
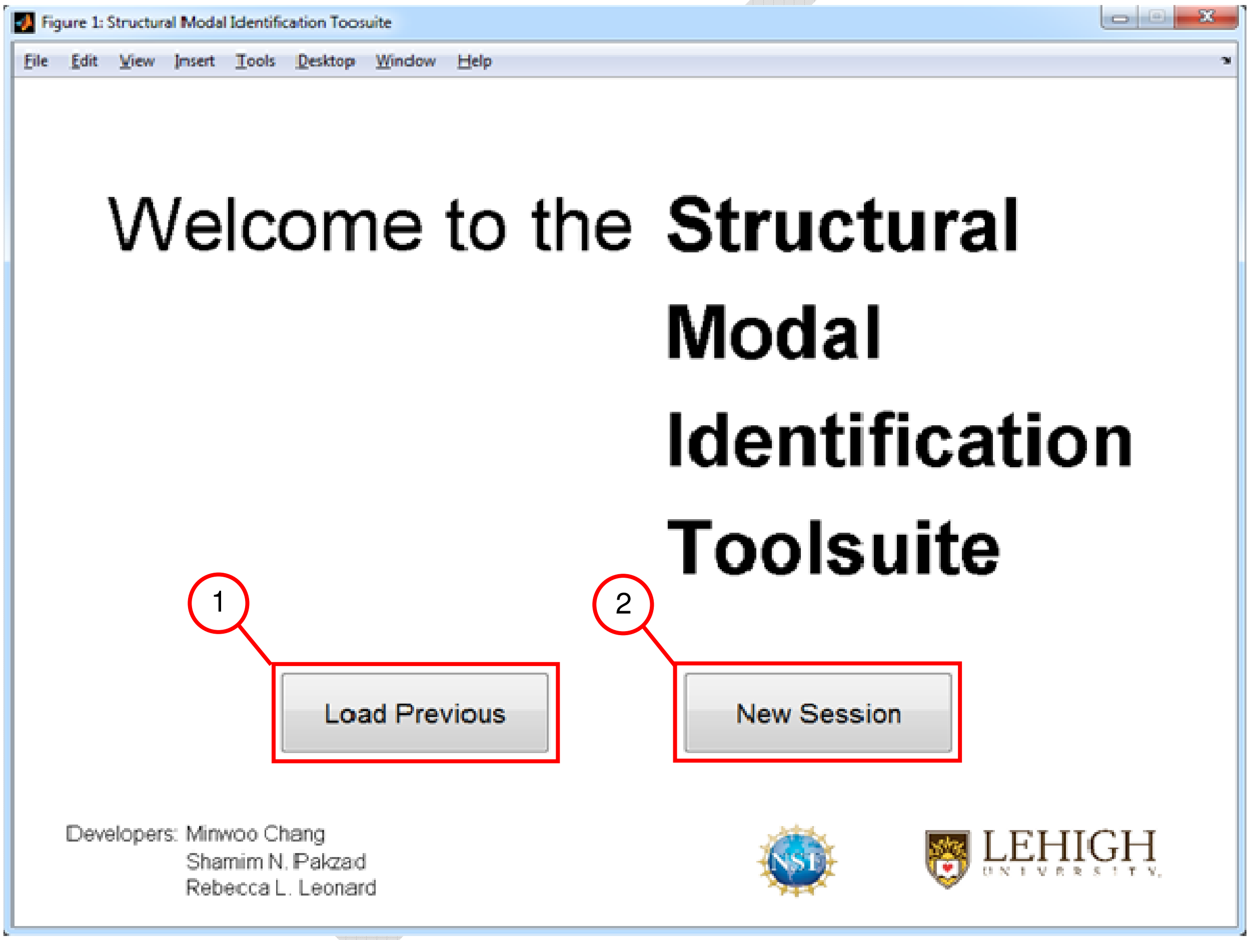


Figure 4: Flow chart illustrating the computational process behind the N4SID method

Using SMIT

Welcome

Begin by typing ‘SMIT’ into the MATLAB command window and pressing ENTER.



Welcome to the Structural Modal Identification Toolsuite

Developers: Minwoo Chang
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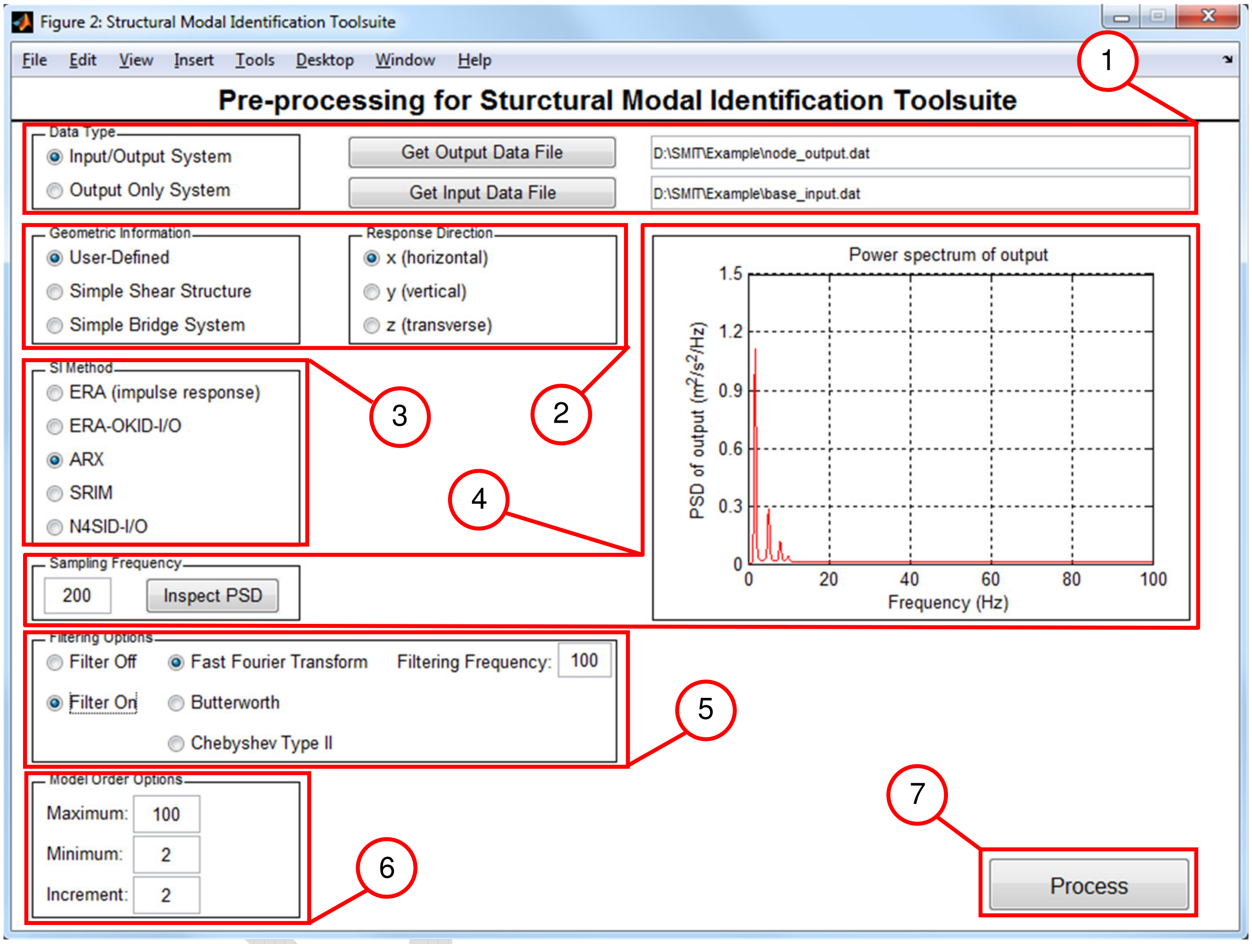
Figure 5: SMIT welcome screen that is displayed when “SMIT” is typed into the MATLAB command window

1. The **Load Previous** pushbutton will open a browser window where the user can open a previous system identification result (.mat) file. You will then continue to the **Post-Processing** window.

2. The **New Session** pushbutton will open the **Pre-Processing** window and begin a new system identification session.

Pre-Processing



● Input/Output System

D:\SMIT\Example\node_output.dat

D:\SMIT\Example\base_input.dat

● User-Defined

● x (horizontal)

● ARX

Sampling Frequency: 200

● Filter On

● Fast Fourier Transform

Filtering Frequency: 100

Maximum: 100

Minimum: 2

Increment: 2

Figure 6: Pre-processing window that is displayed when “New Session” is selected from the welcome screen shown in Figure 5.

1. The **Data Type** panel is where you will designate whether you would like to process input and output data files or an output data file only. Changes in the radio button selection will cause the choices in the **SI Method** panel (3) to reflect the applicable system identification method choices.

2. Within the **Geometric Information** panel, select the structural geometry that best reflects your system. The user-defined geometry information is stored in three editable* text files: [connectivity.txt, node_info.txt, and sensor_location.txt]. The **Response Direction** panel is only visible when the User-Defined radio button is selected. The response directions for simple shear structures and simple bridge systems are already set.

* See **Defining Your Own Geometries** on page 14 for information on how to manipulate these files.

3. The **SI Method** panel is where you will select which system identification you would like to use to construct a mathematical models of your dynamic system. Please refer to the **About System Identification** section in the **Getting Started** chapter if you need help choosing a system identification method.

4. In the **Sampling Frequency** panel, input the sampling frequency at which your sensors collected data. You may also **Inspect PSD** (the power spectral density) of the input response by activating the pushbutton within this panel. For more information on reading a PSD graph, please refer to the chapter on **Interpreting Results**.

5. SMIT's **Filtering Options** panel allows you to choose whether or not you would like to filter your data and which method you would like to use to do so. The Filter On radio button must be selected in order for the second and third columns to be visible. Also, alternating between filtering methods in the second column will cause objects in the third column to appear and disappear, depending on what information is required for that filtering method.

Filtering Options

Fast Fourier Transform: You can control the maximum frequency content (i.e. cutoff frequency), which is smaller than the nyquist frequency of the inserted output signal. All other contribution from outside of the cutoff frequency is set to be zero.

Butterworth: You must specify the cutoff frequency and filtering order. This filter is designed to have a frequency response as flat as possible. SMIT uses the built-in function ‘butter.m’ in the signal processing toolbox library in Matlab.

Chebyshev Type II: You must specify the cutoff frequency, filtering order, and amplitude loss of a frequency response. The Chebyshev Type II filter is designed to have a steeper roll-off and stopband ripple than the Butterworth filter. SMIT uses the built-in function ‘cheby2.m’ in the signal processing toolbox library in Matlab.

* *For information about input limitations regarding these filtering methods, please see the chapter on* ***Troubleshooting****.*

6. The **Model Order Options** panel is where you will define the *y*-axis of the stabilization diagram that is available in the **Post-Processing** window. Keep in mind that increasing the number of model order values to be processed will increase the time required to complete the computation. Additionally, higher model orders take longer to process than do lower model orders.

7. The **Process** pushbutton will begin the system identification process. A **Progress Bar** will appear to let you know what model order is currently being processed as well as the percentage of the task that has been completed.

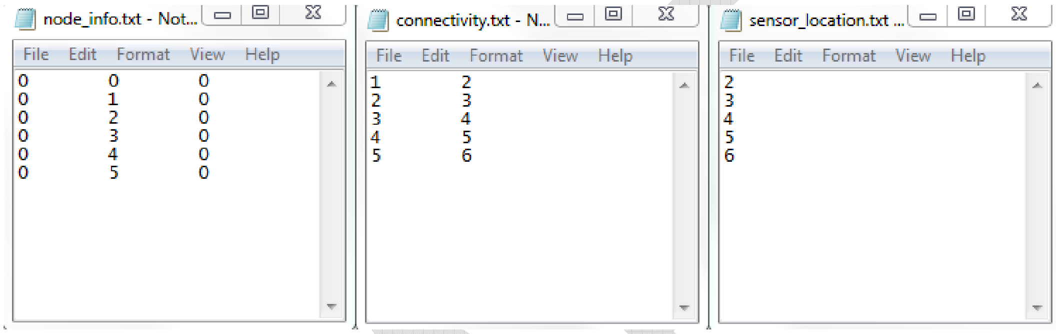
Pre-Processing Notes

Defining Your Own Geometries

- Store the geometric information based on the 3D coordinate system in node_info.txt. The order of a row indicates the node number, and each number in a row corresponds to the coordinates in the *x*, *y*, and *z* axes, respectively.
- Create the members of the structure by connecting two nodes in connectivity.txt.

- Match the sensor location and output data in sensor_location.txt. Choose the nodes which order follows the column of output data. The direction of output data should be defined among global *X* (horizontal), *Y* (vertical), and *Z* (transverse).

For instance, the following figure is the geometric information used in this example which is composed of six nodes and five members and measures vibration response from five locations.



```
0	0	0
0	1	0
0	2	0
0	3	0
0	4	0
0	5	0
```

```
1	2
2	3
3	4
4	5
5	6
```

```
2
3
4
5
6
```

Figure 7: Geometric information of 5-DOF shear model (node_info.txt, connectivity.txt, and sensor_location.txt)

The following figure is displayed on your screen when you start eigenvalue estimation procedure by clicking 'Process' button in the pre-processing window.

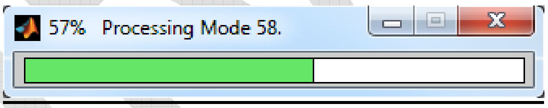
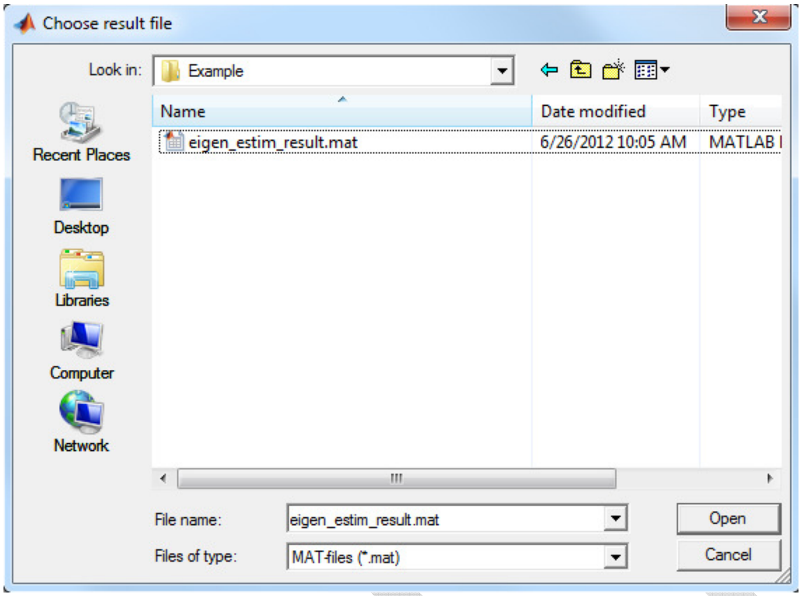


Figure 8: Progress bar captured during the middle of eigenvalue estimation

Once the system identification process is complete, SMIT automatically generates the 'eigen_estim_result.mat' which is a data storage used for the **Post-Processing** procedure. This file can be renamed and relocated depending on the user's choice and be able to load this result at a later time by selecting the **Load Previous** pushbutton in the **Welcome** window.



Choose result file

Look in: Example

Name	Date modified	Type
eigen_estim_result.mat	6/26/2012 10:05 AM	MATLAB I

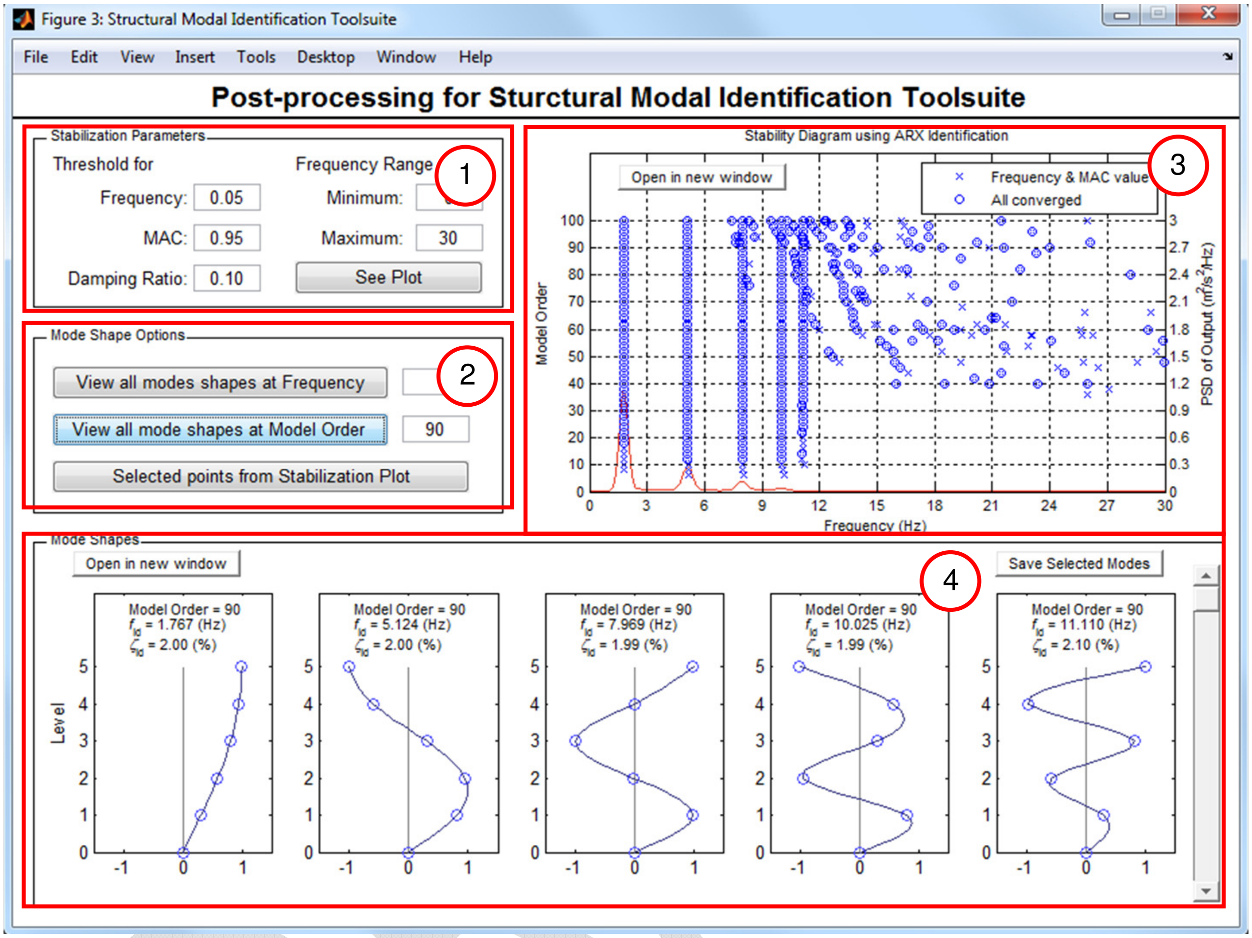
File name: eigen_estim_result.mat

Files of type: MAT-files (*.mat)

Figure 9: Browser window when you select Load Previous button in Welcome window

.

Post-Processing



Threshold for

Frequency: 0.05

MAC: 0.95

Damping Ratio: 0.10

Frequency Range

Minimum:

Maximum: 30

View all mode shapes at Model Order: 90

Figure 10: Post-processing window that displays either (1) after you load previous system identification results, or (2) after the system identification process is completed during a new session.

1. This panel houses all **Stabilization Parameters**. Press the **See Plot** pushbutton to display the **Stabilization Diagram** (3).

Stabilization Parameters

Set the convergence thresholds for

- Frequency
- Damping ratio
- Modal Assurance Criteria (MAC) value

The identified modal parameters for a n^{th} model order are compared to the ones for $(n-1)^{th}$ model order. If the difference is less than what you assigned on the post-processing figure, the corresponding modal parameter is regarded as identified.

Indicate the desired
· Minimum frequency
· Maximum frequency
You can control the range of frequency in order to look at the specific region of the stabilization diagram. The modal parameters inside of the region what you specified are identified only.

2. In the **Mode Shape Options** panel, you can select which mode shapes you would like to view. These points will be ordered pairs, (Frequency (Hz), Model Order). You have the option of seeing all mode shapes at a particular frequency or model order, or even selecting specific points from the **Stabilization Diagram** (3).

   To **view all mode shapes at a particular frequency**, enter a value (Hz) into the edit text box to the right of the **View all mode shapes at Frequency** pushbutton. You can either guess a frequency value or click any point on the stabilization diagram to view the frequency (*x*) and model order (*y*) values at that point. Once you have entered a frequency, click the pushbutton to display the mode shapes in the bottom half of the screen. Follow this same process to **view all mode shapes at a particular model order**

   In order to **select points from the stabilization diagram**:
   1) Click the Select points from Stabilization Plot pushbutton.
   2) Select as many points as you wish from the stabilization diagram to the right.
   3) Right click, and select ‘Export Cursor Data to Workspace’.
   4) Enter the variable name (use default) and press OK.
   5) Click the pushbutton ‘Select points from Stabilization plot’.

The mode shapes should then appear in the **Mode Shapes** panel, below.

3. The **Stabilization Diagram** allows you to graphically distinguish true modes from noise. True modes are likely located at frequencies that display a relatively solid vertical line. If you would like to open the stabilization diagram in a new, larger window, simply click the ‘Open in new window’ pushbutton, located in the upper-left corner of the plot. For more information on reading a stabilization diagram, see the chapter on **Interpreting Results**.

4. The **Mode Shapes** panel is where you will view your selected mode shapes. The scroll bar on the right-hand side of the panel allows you to easily browse through all mode shape plots. If you would like to open the mode shape plots in a new, larger window, simply click the ‘Open in new window’ pushbutton, located in the upper-left corner of the plot. For more information on reading mode shape plots, see the chapter on **Interpreting Results**.

Interpreting Results

Power Spectral Density (PSD) Diagram

You can view a PSD diagram in SMIT's pre-processing and post-processing windows. In the pre-processing window, the PSD diagram will appear by itself when you click the 'Inspect PSD' pushbutton. In the post-processing window, the PSD diagram will appear on the same plot with the stabilization diagram.

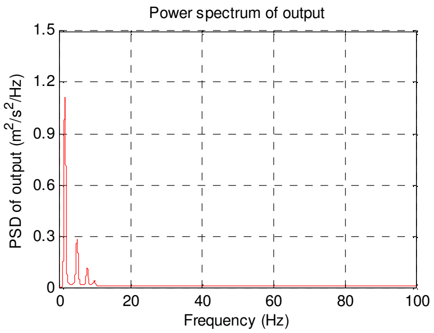


Figure 11: An example power spectral density (PSD) diagram as it would appear in the SMIT pre-processing window.

The PSD diagram shows how the power of your signal (in terms of $m^2/s^2/Hz$) is distributed with frequency. The Welch's method is used to quantify the PSD of the signal in SMIT. The range of frequency is automatically determined as a nyquist frequency of sampling frequency. During the pre-processing procedure, you can specify the cutoff frequency, which may cover major modal contribution to the response of a system, based on the peaks of the PSD diagram. The PSD diagram shown in the post-processing procedure allows users to confirm the structural modes depending on the identified data overlapped in the stabilization diagram. The PSD diagram on the post-processing procedure is the consequence of the filtering if it is turned on in the pre-processing procedure.

Stabilization Diagram

You can view a stabilization diagram in the SMIT post-processing window. As stated earlier, the stabilization diagram will allow you to graphically distinguish true modes from noise. True modes are likely located at frequencies that display a relatively solid vertical line.

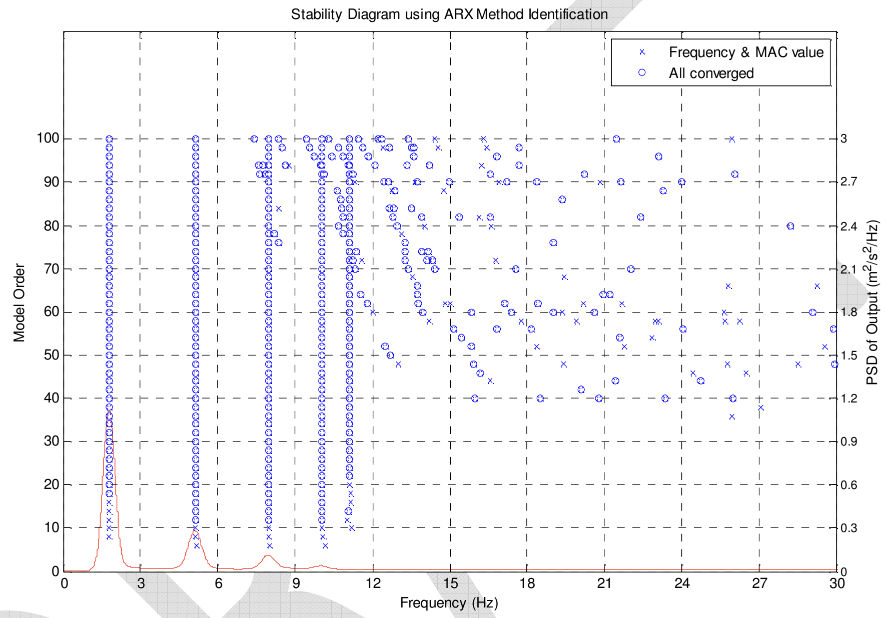


Figure 12: An example stabilization diagram, coupled with a PSD diagram, as it would appear in the SMIT post-processing window.

This stabilization diagram conveys that there are five true structural modes, occurring at the following approximate frequencies: 1.8, 5.1, 7.9, 10.0, and 11.1 Hz. The following figures are variations of the stabilization diagram shown in Figure 12, obtained by manipulating parameters in the post-processing window.

The alterations shown in Figures 13-17 are all ways of eliminating unwanted points from your stabilization diagram. To expand the number of points that will appear on your plot, simply do the opposite of what was done above, i.e. increase maximum frequency, increase convergence frequency, decrease MAC, and increase damping ratio.

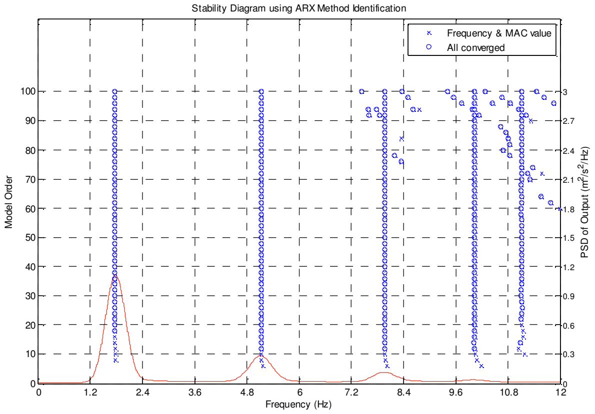


Figure 13: Maximum frequency reduced from 30 Hz to 12 Hz

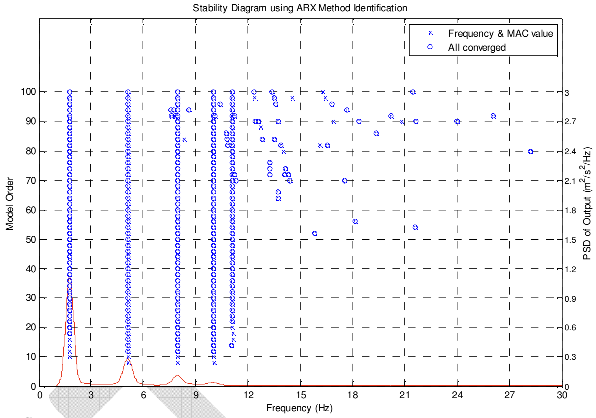


Figure 14: Convergence frequency reduced from 0.05 Hz to 0.01 Hz

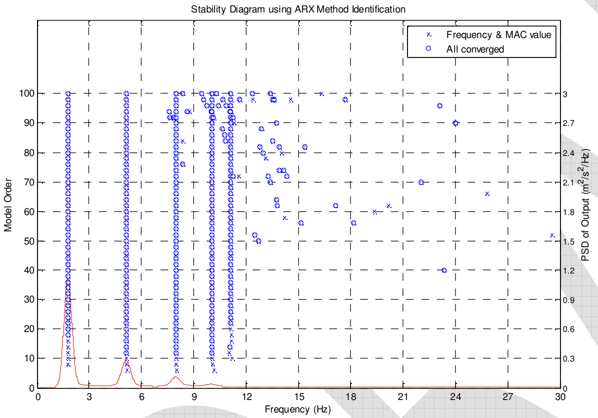


Figure 15: MAC value increased from 0.95 to 0.99.

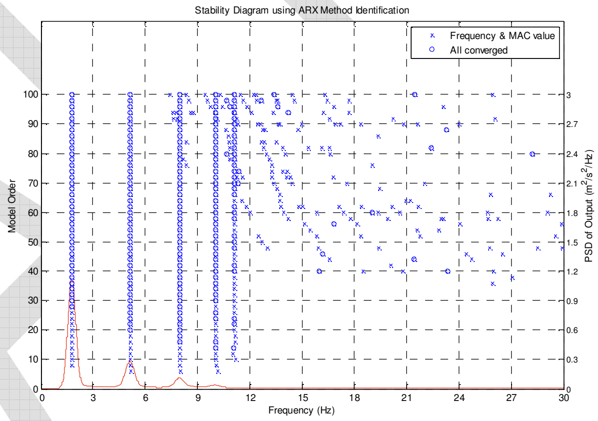


Figure 16: Convergence damping ratio decreased from 0.1 to 0.01.

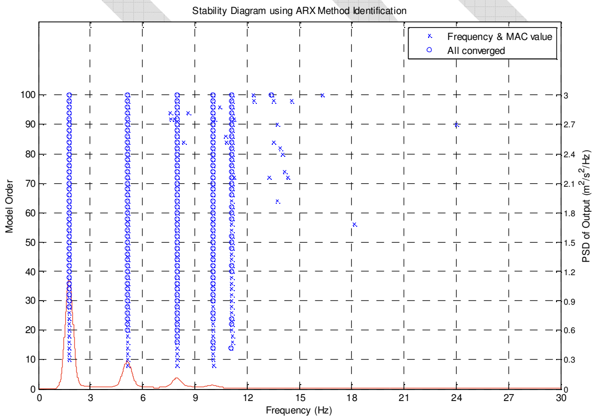


Figure 17: Stabilization diagram with all methods described in Figures 14-16

Mode Shape Plot

Mode shape plots that you select will appear in the Mode Shapes panel in the bottom half of the post-processing window. Each mode shape plot displays the model order and frequency values (essentially, the *x* and *y* coordinates on the stabilization diagram) and the estimated damping ratio.

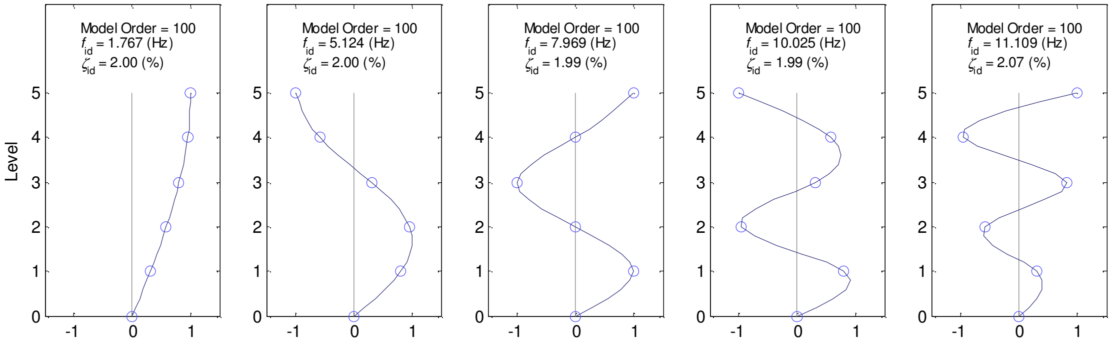


Figure 18: Identified mode shapes for a 5-DOF structure, corresponding to the model order 50

Figure 18 is an example of what you can expect when viewing all mode shapes at a particular model order. In this example, there are 5 true modes. Note that the number of times the plot crosses the vertical axis is equal to the mode number. Mode shape plots are a way of examining points of resonance on a structure.

Troubleshooting

Pre-Processing

Table 2: Empty form error messages that may appear when *Process* is selected in the pre-processing window.

Error Message	Solution
Please select an output file.	SMIT will not proceed until all necessary forms have been filled. If you receive any of these error messages, resolve the problem by completing the indicated form before selecting *Process* again.
Please select an input file.	
Please enter a sampling frequency.	
Please enter a filtering frequency.	
Please enter a filtering order.	
Please enter an amplitude loss.	
Please enter a maximum model order number.	
Please enter a minimum model order number.	
Please enter a model order increment value.	

Table 3: Invalid value error messages that may appear when *Process* is selected in the pre-processing window.

Error Message	Solution
Please enter a valid filtering frequency value.	The filtering frequency value must be a positive integer and cannot exceed half of the value of the sampling frequency
Please enter a valid filtering order value.	The filtering order value must be a positive integer.
Please enter a valid amplitude loss value.	The amplitude loss value must be a positive integer.
Please enter a valid maximum model order value.	The maximum model order value must be a positive integer.
Please enter a valid minimum model order value.	The minimum model order value must be a positive integer and cannot exceed the maximum model order value.
Please enter a valid model order increment value.	The model order increment value must be a positive integer and evenly divide into the difference between maximum and minimum model order values.

Post-Processing

Table 4: Empty form error messages that may appear in the post-processing window.

Error Message	Solution
Please enter a convergence frequency.	SMIT will not proceed until all necessary forms have been filled. If you receive any of these error messages, resolve the problem by completing the indicated form before selecting *See Plot* again.
Please enter a MAC value.	
Please enter a damping ratio.	
Please enter a minimum frequency.	
Please enter a maximum frequency.	
Please enter a frequency.	Enter a frequency and select *See all mode shapes at Frequency* again.
Please enter a model order.	Enter a model order and select *See all mode shapes at Model Order* again.

Table 5: Invalid value error messages that may appear in the post-processing window.

Error Message	Solution
Please enter a valid convergence frequency.	The convergence frequency value must be between 0 and 1.
Please enter a valid MAC value.	The MAC value must be between 0 and 1.
Please enter a valid damping ratio.	The damping ratio value must be between 0 and 1.
Please enter a valid minimum frequency.	The minimum frequency value must be a positive integer and cannot exceed the maximum frequency value.
Please enter a valid maximum frequency.	The maximum frequency value must be a positive integer and cannot exceed the filtering frequency value entered in the pre-processing window.
Please enter a valid frequency.	The frequency you choose must obey the following inequality: Min. Freq. ≤ Frequency ≤ Max. Freq. Where Min. Freq. and Max. Freq. are the minimum and maximum frequency values that you entered in the Stabilization Parameters panel, respectively, and Frequency is the frequency value at which you want to view all mode shape plots.
Please enter a valid model order.	The model order you choose must obey the following inequality: Min. MO ≤ Model Order ≤ Max. MO Where Min. MO and Max. MO are the minimum and maximum model order values that you entered in the Model Order Options panel in the pre-processing window, respectively, and Model Order is the model order at which you want to view all mode shape plots.
Please follow proper selection procedure.	This error message appears if you do not follow the proper procedure when selecting points from the stabilization diagram. Please see page 19 to review these instructions.

Appendix

Appendix 1

For N degree of freedom linear system, the equation of motion can be expressed as,

$$M\ddot{z}(t) + C\dot{z}(t) + Kz(t) = B_1 u(t) \tag{A.1}$$

where, M , C, and $K \in \Re^{N\times N}$ are the mass, damping, and stiffness matrix, respectably; N is the number of DOF, $B_1 \in \Re^{N\times r}$ is the excitation influence matrix corresponding to the time series input vector $u(t) \in \Re^{r}$; r is the number of input nodes; $z(t)$ is the displacement response from N locations. Introducing state vector $x(t) = [z(t)^T \quad \dot{z}(t)^T]^T$, Eq. (A.1) can be converted into the state-space form as,

$$\dot{x}(t) = A_c x(t) + B_c u(t) \tag{A.2}$$

In Eq. (A.2), $A_c = \begin{bmatrix} O & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ and $B_c = \begin{pmatrix} O \\ M^{-1}B_1 \end{pmatrix}$; the subscript $(\cdot)_c$ denotes that the given system is under the continuous time domain.

Considering that the limited number of measurement is available and the state vector is difficult to observe directly, the n dimensional output vector $y(t)$ for observer system is introduced as:

$$y(t) = C_d z(t) + C_v \dot{z}(t) + C_a \ddot{z}(t) \tag{A.3}$$

In Eq. (A.3), C_d , C_v , and $C_a \in \Re^{n\times N}$ are the output coefficient matrices for the displacement, velocity, and acceleration of the system, respectively; n is the number of output. State vector $x(t)$ can be converted into the output vector $y(t)$ as follows continuous state-space equation.

$$y(t) = C_c x(t) + D_c u(t) \tag{A.4}$$

In Eq. (A.4), $C_c = [C_d - C_a M^{-1}K \quad C_v - C_a M^{-1}C]$ and $D_c = C_a M^{-1} B_2$.

The Eq. (A.2) and (A.4) are discretized into the following equations as

$$x_{i+1} = Ax_i + Bu_i \tag{A.5a}$$

$$y_i = Cx_i + Du_i \tag{A.5b}$$

where, the subscript denotes time steps ($t_i = i \cdot \Delta t$) and Δt is the sampling period; the coefficient matrices are defined as:

$$A = e^{A_c \Delta t} \tag{A.6a}$$

$$B = (A - I) \cdot A_c^{-1} \cdot B_c \tag{A.6b}$$

$$C = C_c \tag{A.6c}$$

$$D = D_c \tag{A.6d}$$

Appendix 2

Applying Laplace transform of the Eq. (A.2) yields to

$$sX(s) - X(0) = A_c X(s) + B_c U(s) \tag{A.7}$$

If $X(s)$ terms are collected to the left side and the other terms to the right side, then

$$(sI - A_c) X(s) = X(0) + B_c U(s) \tag{A.8}$$

In Eq. (A.6), I is an identity matrix corresponding to the size of A_c.

The solution for $X(s)$ can be obtained by multiplying inverse matrix of $(sI - A_c)$ both sides of the equation which yields to

$$X(s) = (sI - A_c)^{-1} X(0) + (sI - A_c)^{-1} B_c U(s) \tag{A.9}$$

The general solution can be obtained by taking the inverse Laplace transform and initializing initial time step to t_0 as:

$$x_t = e^{A_c(t - t_0)} x_{t_0} + \int_{t_{-}}^{t} e^{A_c(\tau - t)} B_c u_\tau d\tau \text{, } (t \geq t_0) \tag{A.10}$$

The discretized for of Eq. (10) is given below:

$$x_i = A^i x_0 + \sum_{\tau=0}^{i} A^\tau B u_{i-\tau-1} \tag{A.11}$$

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