

# AF Cooperative CDMA Outage Probability Analysis in Nakagami- $m$ Fading Channels

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**Abstract**—The performance of amplify-and-forward (AF) cooperative code-division multiple-access (CDMA) systems is analyzed over independent nonidentical (i.n.i.) Nakagami- $m$  fading channels. In the underlying AF relaying scheme, each relay applies interference suppression to the received signal to mitigate the effect of multiple-access interference (MAI) where a soft estimate is obtained before signal amplification takes place. The outage probability of the system is analyzed using the cumulative distribution function (cdf) of the total signal-to-noise ratio (SNR) at the base station. In that, we derive a simplified yet tight lower bound for the AF relaying system. We also define an approximation for the probability density function (pdf) of the total SNR, which enables us to derive an asymptotic outage probability of the system. The derived asymptotic outage probability is used to evaluate the achievable diversity order for various system parameters. Simulations are presented to verify the accuracy of our analytical results.

**Index Terms**—Code-division multiple-access (CDMA), cooperative communications, Nakagami fading, outage probability.

## I. INTRODUCTION

**I**N WIRELESS networks, cooperative communications offer the ability to implement spatial diversity, which improves the performance over fading channels without the need for multiple antennas at the transmitter and/or receiver sides. It has been shown that user cooperation diversity with multirelays, a new form of space diversity, has the potential to enhance the channel capacity and the reliability of wireless communication systems [1], [2]. Within the research on user cooperation diversity, amplify-and-forward (AF) is a simple cooperative scheme in which the relay (i.e., cooperative user) transmits an amplified version of the received partner's signal to the base station [3].

Recently, multiuser cooperative systems, such as the multicarrier code-division multiple access (MC-CDMA) and the optical CDMA, have been implemented based on the direct-sequence CDMA (DS-CDMA) approach. Cooperative DS-CDMA, as one common network of user cooperation diversity, has been investigated using nonorthogonal spreading codes for synchronous networks over Rayleigh fading channels in [4].

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In [4], the authors employed a modified minimum-mean-square-error detector as a multiuser detector (MUD) for DS-CDMA cooperative networks to mitigate the effect of multiple-access interference (MAI). The performance of multiuser AF relay networks has also been studied in [5] where Liu *et al.* derived an approximation for the outage probability over Rayleigh fading channels. The joint iterative power allocation and the interference cancellation for DS-CDMA systems using AF relaying were analyzed in [6]. In their work, the authors determined the required power level for a multirelay system. In [7], performance analysis of incremental-relaying cooperative diversity networks over Rayleigh fading channels is presented where a complete analytical method is introduced to obtain closed-form expressions for the error rate, outage probability, and the average achievable rate using decode-and-forward (DF) and AF over independent nonidentical (i.n.i.) Rayleigh fading channels.

Given field measurements, it is known that the Rayleigh fading model cannot represent the statistical characteristics of many wireless environments. In that context, the Nakagami fading model is well-known as a generalized distribution, where many fading environments can be modeled [8]. In [9], a performance analysis of time-division multiple-access relay protocols over independent and identically distributed (i.i.d.) Nakagami- $m$  fading channels is presented. Atapattu *et al.* [9] considered an Alamouti-coded system with two-stage protocols and fixed-gain AF. On the other hand, Fang *et al.* [10] evaluated the bit error rate (BER) of a cooperative downlink transmission scheme for DS-CDMA systems over Nakagami- $m$  fading channels to achieve cooperative diversity. In their work, they also proposed transmitter zero forcing at the base station for suppressing the downlink multiuser interference. In [11], Suraweera *et al.* analyzed the outage probability of cooperative relay networks over i.i.d. Nakagami- $m$  fading channels. Along the same lines, Yang *et al.* [12] studied the performance of downlink multiuser relay networks using single-relay AF where the outage probability was evaluated at a high SNR. In a single-user system, performance analysis for opportunistic DF with a selection combining receiver at the destination has been evaluated in terms of the outage probability over nonidentical Nakagami- $m$  fading channels in [13]. In [14], the performance of cooperative CDMA systems using adaptive DF relaying over Nakagami- $m$  fading channels was investigated. In [14], a closed-form expression for the outage probability using the moment-generating function (MGF) for the total SNR at the base station was obtained.

In the literature, there have been many works on the analysis of DF and AF relaying, particularly in the case of Rayleigh

fading. Here, different from previous works, we derive a new lower bound and asymptotic expression for the outage probability of cooperative diversity in DS-CDMA using AF relaying over Nakagami- $m$  fading with maximal-ratio-combining receiver at the base station. In that, an approximate probability density function (pdf) for the total SNR at the base station is determined at high SNR. This approximation is then used to evaluate the asymptotic outage probability of the system. We consider asynchronous transmission where we study the effect of MAI and intersymbol interference at both the relay and the base station for AF cooperative CDMA systems. To overcome the effect of MAI, a MUD at both the relay and the base station is employed to convert the inter-user channels into parallel interference-free fading channels. Here, different from previous works, our AF scheme employs MAI cancellation to obtain a soft estimate of the transmitted data for the source.

The rest of this paper is organized as follows. In Section II, we introduce the system model. The outage probability for AF relaying is analyzed in Section III. Section IV provides some simulation and numerical results. Finally, conclusions are drawn in Section V.

## II. SYSTEM MODEL

An uplink  $K$ -user asynchronous nonorthogonal DS-CDMA system over i.n.i. Nakagami- $m$  fading channel is considered. A half-duplex system is assumed, and each user is equipped with a single antenna. A single-relay cooperation system is studied where the group of users  $\{1, 2, \dots, K\}$  is arranged into groups, each of two cooperating partners. The cooperation takes place in two time slots. In the first time slot, each user  $s$  broadcasts its signal to its partner  $r$  and to station  $b$ . In the second time slot, the partner amplifies the received signal and retransmits a soft estimate of the output of the MUD in an attempt to cancel the effect of MAI.

The independent fading channel coefficients between partners and the base station are defined as follows: source-relay  $h_{sr}$ , source-base station  $h_{sb}$ , and relay-base station  $h_{rb}$ . All are modeled as i.n.i. Nakagami- $m$  random variables (RVs). The instantaneous SNRs of different links are denoted by  $\gamma_{sr} = |h_{sr}|^2(E_s/N_o)$ ,  $\gamma_{sb} = |h_{sb}|^2(E_s/N_o)$ , and  $\gamma_{rb} = |h_{rb}|^2(E_s/N_o)$ , where  $E_s$  is the transmitted signal energy,  $N_o$  is the noise spectral energy, and  $|h_{sr}|^2$ ,  $|h_{sb}|^2$ , and  $|h_{rb}|^2$  are gamma distributed RVs with a pdf given by

$$p_{\gamma_{ij}}(x) = \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})} x^{m_{ij}-1} \exp(-xB_{ij}) \quad (1)$$

where  $m_{ij} > 0.5$  is the fading parameter,  $\Gamma(\cdot)$  is the gamma function [15, eq. (8.310, 1)],  $B_{ij} = m_{ij}/\bar{\gamma}_{ij}$  with average SNR, and  $\bar{\gamma}_{ij} = \mathbb{E}\langle |h_{ij}|^2 \rangle E_s/N_o$  and  $\mathbb{E}\langle \cdot \rangle$  denote expectation. The cdf of  $\gamma_{ij}$  is then given by

$$F_{\gamma_{ij}}(x) = \frac{\gamma(m_{ij}, xB_{ij})}{\Gamma(m_{ij})} \quad (2)$$

where  $\gamma(\cdot, \cdot)$  is the lower incomplete gamma function [15, eq. (8.350, 1)].

## III. OUTAGE PROBABILITY ANALYSIS

A repetition-based cooperative diversity scenario is applied where the relay amplifies the received signal and transmits a soft estimate of the desired user signal to the base station [3]. The allowable degrees of freedom of the cooperative system, i.e.,  $K/2N$ , depend on the total number of users  $K$ , length of the available spreading code  $N$ , and the factor 1/2, which refers to the bandwidth expansion required for relaying due to the half-duplex constraint [16]. In the following, the performance of the cooperative diversity protocol under diversity combining is investigated. Let us first consider the direct link between the source  $s$  and the base station  $b$ , where the mutual information is defined by [16]

$$I_{sb} = \frac{K}{2N} \log_2 \left( 1 + \frac{2N\gamma_{sb}}{K^2 [\mathbf{R}_b^{-1}]_{s,s}} \right), \quad s \in \{1, \dots, K\} \quad (3)$$

where  $\mathbf{R}_b$  is the  $(Kf \times Kf)$  cross-correlation matrix at the base station receiver, which is defined as [17]

$$\mathbf{R}_b = \begin{bmatrix} \rho_{1,1} & \cdots & \rho_{1,2} & \cdots & \rho_{1,K} \\ \rho_{2,1} & \cdots & \rho_{2,2} & \cdots & \rho_{2,K} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_{K,1} & \cdots & \rho_{K,2} & \cdots & \rho_{K,K} \end{bmatrix} \quad (4)$$

with  $\rho_{ij}$  being the cross correlation between any two spreading codes  $C_i$  and  $C_j$ .  $[\mathbf{R}_b^{-1}]_{ij}$  represents the  $i$ th row and the  $j$ th column elements of the inverse of the cross-correlation matrix  $\mathbf{R}_b^{-1}$ , and  $f$  is the frame length. In (3),  $2N/K^2$  is the normalized discrete-power constraint [16]. Given that outage will occur when the spectral efficiency  $\mathfrak{R}$  exceeds the mutual information  $I_{sb}$ , one can define the outage probability of the noncooperative system as

$$P_{\text{out}} = P_r[I_{sb} < \mathfrak{R}]. \quad (5)$$

From (3) and (5), we have

$$\begin{aligned} P_{\text{out}} &= P_r \left[ \frac{K}{2N} \log_2 \left( 1 + \frac{2N\gamma_{sb}}{K^2 [\mathbf{R}_b^{-1}]_{s,s}} \right) < \mathfrak{R} \right] \\ &= P_r[\gamma_{sb} < \gamma_{\text{th-sb}}] \end{aligned} \quad (6)$$

where  $\gamma_{\text{th-sb}} = (2^{2N\mathfrak{R}/K} - 1)/(2N/K^2[\mathbf{R}_b^{-1}]_{s,s})$ . From (6), one can notice that  $P[\gamma_{sb} < \gamma_{\text{th-sb}}]$  is the cdf of  $\gamma_{sb}$ , which is given by (2) as

$$P_{\text{noncoop}} = P_r[\gamma_{sb} < \gamma_{\text{th-sb}}] = \frac{\gamma(m_{sb}, B_{sb}\gamma_{\text{th-sb}})}{\Gamma(m_{sb})}. \quad (7)$$

### A. Outage Probability Lower Bound

In AF, the relay (i.e., cooperative user) transmits an amplified version of the received partner's signal after linear filtering using a decorrelator detector (DD) as a MUD. Here, the average mutual information on AF relaying in DS-CDMA using the single-relay cooperative system is derived.

The received signal during an  $i$ th data symbol duration at the base station in the first transmission phase is given by [17]

$$r_{b1}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K x_s(i) C_s(t - \tau_s - iT_b) h_{sb}(i) + n_{b1}(t) \quad (8)$$

$iT_b \leq t < (i+1)T_b$

where  $x_s(i) \in \{1, -1\}$  is the  $i$ th data symbol of user  $s$ ,  $C_s(t)$  is the spreading code of user  $s$  with spreading gain  $N = T_b/T_c$ ,  $T_b$  is the bit period,  $T_c$  is the chip period, and  $\tau_s$  is the random transmit delay of the  $s$ th user, which is assumed to be uniformly distributed along the symbol period.  $n_{b1}(t)$  is the additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_n^2 = N_o$ . The received signal at relay  $r$  can be written as

$$r_r(t) = \sum_{i=0}^{f-1} \sum_{s=1, s \neq r}^K x_s(i) C_s(t - \tau_s - iT_b) h_{sr}(i) + n_r(t) \quad (9)$$

where  $n_r(t)$  is an AWGN with zero mean and variance  $\sigma_n^2 = N_o$ . In the second phase, each cooperating user transmits the amplified version of the received signal to the base station, which is expressed as

$$r_{b2}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^K r_r(t - D_s - \tau_r) \cdot h_{rb}(i) \cdot \beta + n_{b2}(t) \quad (10)$$

where  $r$  is the relay cooperating with user  $s$ ,  $n_{b2}(t)$  is the AWGN with zero mean and variance  $\sigma_n^2 = N_o$ , and  $D_s$  is the transmission delay during the second transmission period.  $\beta$  is the amplification factor for the AF scheme at the relay node, which is expressed as [3]

$$\beta = \sqrt{\frac{E_s}{E_s |h_{sr}|^2 + N_o}} \quad (11)$$

where  $E_s$  is the transmitted signal energy.

The output of the bank of matched filters at the base-station receiver and the relay, and by dropping the time index from modules (8)–(10), can be defined in vector form for the first phase as

$$\mathbf{y}_{b1} = \mathbf{R}_b \mathbf{H}_{s,b} \mathbf{x} + \mathbf{n}_{b1} \quad (12)$$

where  $\mathbf{x}$  is the  $(Kf \times 1)$  user data vector of the  $K$  total users defined as  $\mathbf{x} = [x_1(1), \dots, x_1(f), \dots, x_K(1), \dots, x_K(f)]^T$ , and  $\mathbf{H}_{s,b}$  is the  $(Kf \times Kf)$  channel coefficient matrix between the source and the base station defined as

$$\mathbf{H}_{s,b} = \begin{bmatrix} h_{1,b}(1) & 0 & \dots & \dots & 0 \\ \vdots & \dots & \dots & \dots & \vdots \\ 0 & \dots & h_{1,b}(f) & \dots & 0 \\ \vdots & \dots & \dots & \dots & \vdots \\ 0 & \dots & \dots & \dots & h_{K,b}(f) \end{bmatrix}. \quad (13)$$

Under the assumption of frequency-flat fading, we use a block-fading channel model, assuming that different channels experience roughly the same fading conditions (or equivalently the same channel coefficients) over the whole duration of each

frame  $f$ .  $\mathbf{n}_{b1}$  is the  $(Kf \times 1)$  AWGN vector with i.i.d. elements each of zero mean and variance  $\sigma_n^2 = N_o$ . Similarly, the output of the bank of matched filters at the relay can be defined as

$$\mathbf{y}_r = \mathbf{R}_r \mathbf{H}_{s,r} \mathbf{x} + \mathbf{n}_r, \quad r \in \{1, \dots, K\} \quad (14)$$

where  $\mathbf{R}_r$  is the  $(Kf \times Kf)$  relay cross-correlation matrix;  $\mathbf{H}_{s,r}$  is the  $(Kf \times Kf)$  channel coefficient matrix between the sources and the relay, with  $\rho_{K,K} = 0$  and  $h_{K,K} = 0 \forall s = r$  in  $\mathbf{R}_r$  and  $\mathbf{H}_{s,r}$ , respectively, i.e.,

$$\mathbf{R}_r = \begin{bmatrix} \rho_{1,1} & \dots & \rho_{1,2} & \dots & \rho_{1,K} \\ \rho_{2,1} & \dots & \rho_{2,2} & \dots & \rho_{2,K} \\ \vdots & \dots & \dots & \dots & \vdots \\ \rho_{K,1} & \dots & \rho_{K,2} & \dots & \rho_{K,K} \end{bmatrix} \quad (15)$$

$$\mathbf{H}_{s,r} = \begin{bmatrix} h_{1,r} & 0 & \dots & \dots & 0 \\ 0 & \dots & h_{2,r} & \dots & 0 \\ \vdots & \dots & \dots & \dots & \vdots \\ 0 & \dots & \dots & \dots & h_{K,r} \end{bmatrix} \quad (16)$$

and  $\mathbf{n}_r$  is the AWGN vector with zero mean and variance  $\sigma_n^2 = N_o$ . The output of the bank of matched filters at the base station in the second transmission phase is given by

$$\mathbf{y}_{b2} = \mathbf{R}_b \mathbf{H}_{r,b} (\mathbf{y}_r) \beta + \mathbf{n}_{b2} \quad (17)$$

with  $\mathbf{H}_{r,b}$  being the channel coefficient matrix between the relays and the base station, and  $\mathbf{n}_{b2}$  being the AWGN vector with zero mean and variance  $\sigma_n^2 = N_o$ . Employing a DD at the base station, we have

$$\mathbf{z}_{b1} = (\mathbf{R}_b^{-1}) \mathbf{y}_{b1} = \mathbf{H}_{s,b} \mathbf{x} + \mathbf{R}_b^{-1} \mathbf{n}_{b1}. \quad (18)$$

In addition, considering the MUD at the relay side (i.e., no decoding is done), the soft data estimate at the decorrelator output is given by

$$\mathbf{z}_r = (\mathbf{R}_r^{-1}) \mathbf{y}_r = \mathbf{H}_{s,r} \mathbf{x} + \mathbf{R}_r^{-1} \mathbf{n}_r. \quad (19)$$

In the cooperative phase, the decorrelator output at the base station is given by

$$\mathbf{z}_{b2} = (\mathbf{R}_b^{-1}) \mathbf{y}_{b2} = \mathbf{H}_{r,b} \mathbf{H}_{s,r} \beta \mathbf{x} + \mathbf{H}_{r,b} \beta \mathbf{R}_r^{-1} \mathbf{n}_r + \mathbf{R}_b^{-1} \mathbf{n}_{b2}. \quad (20)$$

Considering the  $i$ th data bit of the  $s$ th user,  $s \in \{1, \dots, K\}$ , the decorrelator output at the base station for the first and second transmission phases of (18) and (20) can be rewritten, respectively, as

$$[z_s(i)]_{b1} = h_{s,b} x_s(i) + N_{b1} \quad (21)$$

$$[z_s(i)]_{b2} = h_{r,b} h_{s,r} \beta x_s(i) + h_{r,b} \beta N_r + N_{b2} \quad (22)$$

where  $[z_s(i)]_{b1}$  and  $[z_s(i)]_{b2}$  are the decision statistics corresponding to the  $i$ th bit for user  $s$ , and  $N_{b1} = [\mathbf{R}_b^{-1}]_{s,s} \mathbf{n}_{b1}$ ,  $N_r = [\mathbf{R}_r^{-1}]_{r,r} \mathbf{n}_r$ , and  $N_{b2} = [\mathbf{R}_b^{-1}]_{s,s} \mathbf{n}_{b2}$  are the  $s$ th element in the corresponding vector.

Now, the average mutual information for AF CDMA transmission can be derived using a vector representation of (21) and (22) as

$$\underbrace{\begin{bmatrix} [z_s(i)]_{b1} \\ [z_s(i)]_{b2} \end{bmatrix}}_{\mathbf{z}} = \underbrace{\begin{bmatrix} h_{sb} \\ h_{rb}\beta h_{sr} \end{bmatrix}}_{\mathbf{h}} x_s(i) + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ h_{rb}\beta & 0 & 1 \end{bmatrix}}_{\mathbf{G}} \cdot \underbrace{\begin{bmatrix} N_r \\ N_{b1} \\ N_{b2} \end{bmatrix}}_{\mathbf{N}}. \quad (23)$$

Since the channel is assumed to be memoryless, the average mutual information  $I_{AF}$  can be defined as [3]

$$I_{AF} \leq I(x, z) \leq \log_2 \det \left( I + (E_s h h^H) (G \mathbb{E}\langle N N^H \rangle G^H)^{-1} \right) \quad (24)$$

where the covariance of the noise  $\mathbb{E}\langle N N^H \rangle = \text{diag}(N_d [\mathbf{R}_r^{-1}]_{r,r}, N_o [\mathbf{R}_b^{-1}]_{s,s}, N_o [\mathbf{R}_r^{-1}]_{r,r})$ . Then

$$\begin{aligned} h h^H &= \begin{bmatrix} |h_{sb}|^2 & h_{sb} h_{rb}^H \beta h_{sr}^H \\ h_{sb}^H h_{rb} \beta h_{sr} & |h_{rb}|^2 \beta^2 |h_{sr}|^2 \end{bmatrix} \\ (G \mathbb{E}\langle N N^H \rangle G^H)^{-1} &= \begin{bmatrix} \frac{1}{N_o [\mathbf{R}_b^{-1}]_{s,s}} & 0 \\ 0 & \frac{1}{|h_{rb}|^2 \beta^2 N_o [\mathbf{R}_r^{-1}]_{r,r} + N_o [\mathbf{R}_r^{-1}]_{r,r}} \end{bmatrix} \end{aligned}$$

and det of (24) is calculated as

$$\begin{aligned} \det \left( I + (E_s h h^H) (G \mathbb{E}\langle N N^H \rangle G^H)^{-1} \right) &= 1 \\ &+ \frac{E_s |h_{sb}|^2}{N_o [\mathbf{R}_b^{-1}]_{s,s}} \frac{E_s |h_{rb}|^2 \beta^2 |h_{sr}|^2}{|h_{rb}|^2 \beta^2 N_o [\mathbf{R}_r^{-1}]_{r,r} + N_o [\mathbf{R}_r^{-1}]_{r,r}}. \quad (25) \end{aligned}$$

Substituting (11) into (25), and after many algebraic manipulations, we have

$$\begin{aligned} \det \left( I + (E_s h h^H) (G \mathbb{E}\langle N N^H \rangle G^H)^{-1} \right) &= 1 + \frac{E_s}{N_o} \frac{|h_{sb}|^2}{[\mathbf{R}_b^{-1}]_{s,s}} + \frac{\frac{E_s}{N_o} \frac{|h_{rb}|^2}{[\mathbf{R}_r^{-1}]_{r,r}} \cdot \frac{E_s}{N_o} \frac{|h_{sr}|^2}{[\mathbf{R}_r^{-1}]_{r,r}}}{\frac{E_s}{N_o} \frac{|h_{rb}|^2}{[\mathbf{R}_r^{-1}]_{r,r}} + \frac{E_s}{N_o} \frac{|h_{sr}|^2}{[\mathbf{R}_r^{-1}]_{r,r}} + 1}. \quad (26) \end{aligned}$$

It is shown that (26) is increasing in  $\beta$ ; therefore, the amplification factor for the AF in (11) should be active [3]. After substitution and algebraic manipulations, using  $\gamma_{ij} = |h_{ij}|^2 (E_s/N_o)$  and the normalized discrete-power constraint as in [16],  $I_{AF}$  can be finally written as

$$\begin{aligned} I_{AF} &= \frac{K}{2N} \log_2 \\ &\times \left( 1 + \frac{2N\gamma_{sb}}{K^2 [\mathbf{R}_b^{-1}]_{s,s}} + \frac{\frac{2N\gamma_{sr}}{K^2 [\mathbf{R}_r^{-1}]_{r,r}} \cdot \frac{2N\gamma_{rb}}{K^2 [\mathbf{R}_r^{-1}]_{r,r}}}{\frac{2N\gamma_{sr}}{K^2 [\mathbf{R}_r^{-1}]_{r,r}} + \frac{2N\gamma_{rb}}{K^2 [\mathbf{R}_r^{-1}]_{r,r}} + 1} \right). \quad (27) \end{aligned}$$

For simplicity, we define  $X_{sb} = (2N\gamma_{sb})/(K^2 [\mathbf{R}_b^{-1}]_{s,s})$ ,  $X_{sr} = (2N\gamma_{sr})/(K^2 [\mathbf{R}_r^{-1}]_{r,r})$ , and  $X_{rb} = (2N\gamma_{rb})/(K^2 [\mathbf{R}_r^{-1}]_{r,r})$ , where the total SNR at the base station is

$$X_{\text{total}} = X_{sb} + \frac{X_{sr} X_{rb}}{X_{sr} + X_{rb} + 1}. \quad (28)$$

Using the lower bound defined in [18], we have

$$X_{\text{total}} \leq X_{\text{up}} = X_{sb} + \min(X_{sr}, X_{rb}). \quad (29)$$

Note that the SNR value  $X_{\text{up}}$  in (29) is analytically more tractable than the exact value in (28). This will simplify the derivation of the outage probability, and as shown in [18], it represents an accurate measure in medium and high SNRs.

Given the cdf of  $X_{rb}$  defined in (2), the cdf of  $\min(X_{sr}, X_{rb})$  can be defined as

$$\begin{aligned} F_{\min(X_{sr}, X_{rb})}(\gamma_{\text{th}}) &= 1 - P[X_{sr} > \gamma_{\text{th}} \text{ and } X_{rb} > \gamma_{\text{th}}] \\ &= 1 - \frac{\Gamma(m_{sr}, B_{sr} \gamma_{\text{th}})}{\Gamma(m_{sr})} \frac{\Gamma(m_{rb}, B_{rb} \gamma_{\text{th}})}{\Gamma(m_{rb})}. \quad (30) \end{aligned}$$

After some manipulation, the cdf of the sum of gamma RVs is derived, as shown in the Appendix, and expressed as

$$\begin{aligned} F_{X_{\text{total}}}(y) &= \frac{\gamma(m_{sb}, \gamma B_{sb})}{\Gamma(m_{sb})} - \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} \exp(-y(B_{sr} + B_{rb})) \\ &\times \int_0^y \left( x^{(m_{sb}-1)} \exp(-x(B_{sb} - B_{sr} - B_{rb})) \sum_{n=0}^{m_{sr}-1} \right. \\ &\times \frac{(B_{sr})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} x^k \\ &\times \left. \sum_{i=0}^{m_{rb}-1} \frac{(B_{rb})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f y^{i-f} x^f \right) dx. \quad (31) \end{aligned}$$

Finally, the  $P_{\text{out}}$  for the AF scheme can be determined numerically by solving the integral in (31) using mathematical software such as Maple and Mathematica. In the case of Nakagami- $m$  fading channels, the closed-form expression for the pdf of the total SNR is intractable to derive. In what follows, we provide a closed-form expression for the asymptotic outage probability.

### B. Approximate Distributions of the Total SNR

The asymptotic outage probability is traditionally obtained using the pdf of the indirect link between  $s \rightarrow r$  and  $r \rightarrow b$ . Due to the intractability of the pdf of  $X_{\text{total}}$  given in (28), it becomes difficult to obtain a closed-form expression for the outage probability in the AF case. Instead, we employ an approximation of this distribution at high SNR. This approximation, as will be shown, will enable us to derive a closed-form expression for the outage probability of the underlying system.

At high SNR, (1) can be approximated as

$$p_{\gamma_{ij}}(\gamma) \approx \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})} \gamma^{m_{ij}-1} + H.O. \quad (32)$$

where  $H.O.$  stands for high-order terms. Applying Laplace transform to (32), and with the help of [19], the MGF is given by

$$M_{\gamma_{ij}}(s) = \frac{B_{ij}^{m_{ij}}}{s^{m_{ij}}}. \quad (33)$$



Using the method in [18], the pdf of the approximate distribution for link  $s \rightarrow r \rightarrow b$  can be written as

$$f_{\gamma_{s,r,b}}(\gamma) \approx f_{\gamma_{s,r}}(\gamma) + f_{\gamma_{r,b}}(\gamma) \quad (34)$$

where  $f_{\gamma_{s,r}}(\gamma)$  and  $f_{\gamma_{r,b}}(\gamma)$  can be approximated at high SNR as in (32), i.e.,

$$f_{\gamma_{s,r}}(\gamma) \approx \frac{B_{sr}^{m_{sr}}}{\Gamma(m_{sr})} \gamma^{m_{sr}-1} + H.O. \quad (35)$$

$$f_{\gamma_{r,b}}(\gamma) \approx \frac{B_{rb}^{m_{rb}}}{\Gamma(m_{rb})} \gamma^{m_{rb}-1} + H.O. \quad (36)$$

Examining (34) when  $E_s/N_o \rightarrow \infty$ , a simplified expression for the  $f_{\gamma_{s,r,b}}(\gamma)$  is given by

$$f_{\gamma_{s,r,b}}(\gamma) \approx \Psi \gamma^{m_r-1} + H.O. \quad (37)$$

where  $m_r = \min(m_{sr}, m_{rb})$ , and

$$\Psi = \begin{cases} \frac{B_{sr}^{m_{sr}}}{\Gamma(m_{sr})}, & m_{sr} < m_{rb} \\ \frac{B_{rb}^{m_{rb}}}{\Gamma(m_{rb})}, & m_{sr} > m_{rb} \\ \frac{1}{\Gamma(m_r)} (B_{sr}^{m_r} + B_{rb}^{m_r}), & m_{sr} = m_{rb} = m_r. \end{cases} \quad (38)$$

Hence, the pdf of the SNR is given by

$$f_{\gamma_{AF}}(\gamma) \approx f_{\gamma_{s,b}}(\gamma) + f_{\gamma_{s,r,b}}(\gamma) \quad (39)$$

and the corresponding MGF is given by

$$M_{AF}(s) = M_{sb}(s) \cdot M_{s,r,b}(s). \quad (40)$$

Using (33), we have

$$M_{AF}(s) = B_{sb}^{m_{sb}} \frac{\Psi \Gamma(m_r)}{s^{m_{sb}+m_r}}. \quad (41)$$

Finally,  $P_{out}$  is defined as  $\mathcal{L}^{-1}\{(M_{AF}(s))/s; t\}|_{t=\gamma_{th}}$ , i.e.,

$$P_{out}(s) = B_{sb}^{m_{sb}} \Psi \Gamma(m_r) \frac{\gamma_{th}^{(m_{sb}+m_r)}}{\Gamma(m_{sb}+m_r+1)}. \quad (42)$$

From (42), one can see that the achieved diversity order is  $(m_{sb} + m_r)$  with  $m_r = \min(m_{sr}, m_{rb})$  defined before, whereas the achieved diversity order for the DF case presented in [14] is given by  $(m_{sb} + m_{rb})$ .

#### IV. NUMERICAL RESULTS AND DISCUSSIONS

In what follows, we present our analytical and simulation results for the outage probability. In all results, we set the spreading gain to  $N = 31$  for a system with number of users  $K = 16$ . Without loss of generality, we assume that the spectral efficiency  $\mathfrak{R} = 1$  bit/sec/Hz. A DD is used to mitigate the effect of MAI.

Fig. 1 shows  $P_{out}$  for both noncooperative and cooperative AF relaying over i.i.d. Nakagami- $m$  fading channel. It is clear that the outage probability derived is tight at medium and high SNRs. As shown in Fig. 1, the AF cooperative CDMA system offers a considerable outage probability gain compared

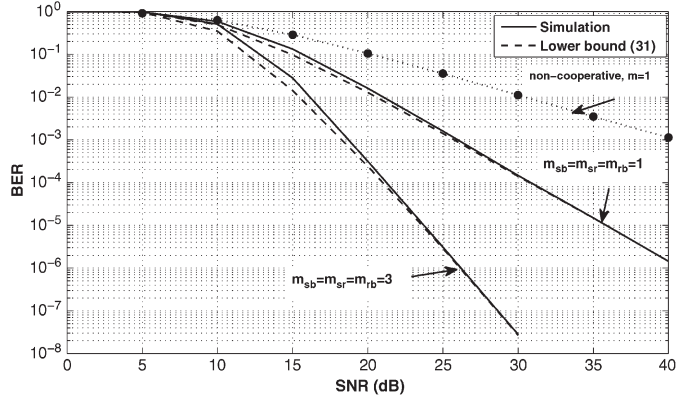


Fig. 1. Outage probability for cooperative AF DS-CDMA over the i.i.d. Nakagami- $m$  fading channel.

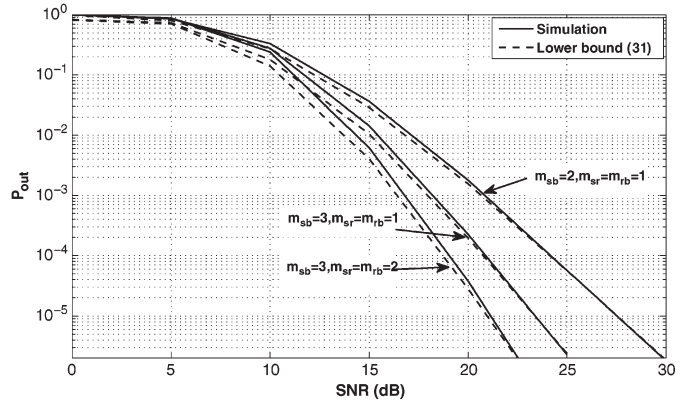


Fig. 2. Outage probability for cooperative AF DS-CDMA with different fading figures  $m$  and considering i.n.i. Nakagami fading channels.

with the noncooperative system. In Fig. 2, we present the outage probability of the AF scheme over the i.n.i. Nakagami- $m$  fading channel. As noted, when  $m_{ij}$  increases, the system performance is improved. In Figs. 1 and 2, one can see the improved accuracy of the derived lower bound given by (28) for the outage probability as the SNR increases.

In Fig. 3, the asymptotic outage probability given by (42) for the AF scheme is investigated. Fig. 3 confirms the accuracy of the approximation at medium and high SNRs, where the system performance is improved when the fading parameter  $m_{ij}$  becomes large. Fig. 3 also demonstrates the variation in the diversity order obtained from (42).

To examine the effect of MAI on the AF scheme, we consider the three following scenarios. In the first system scenario, namely multiuser detection at the base station (MUD-BS), the MUD is only applied at the base station. In the second scenario, i.e., *MUD-R-BS*, the MUD is applied at both the relay and the base station. In the third scenario, conventional detection, referred to as *conventional R-BS*, only matched filter detection is employed at the relay and the base station. Fig. 4 depicts the average BER performance of cooperative CDMA over Nakagami- $m$  fading channels for the AF scheme when considering the three aforementioned systems. As noted from these results, the optimum performance will be achieved when employing interference cancellation at both the relay and the

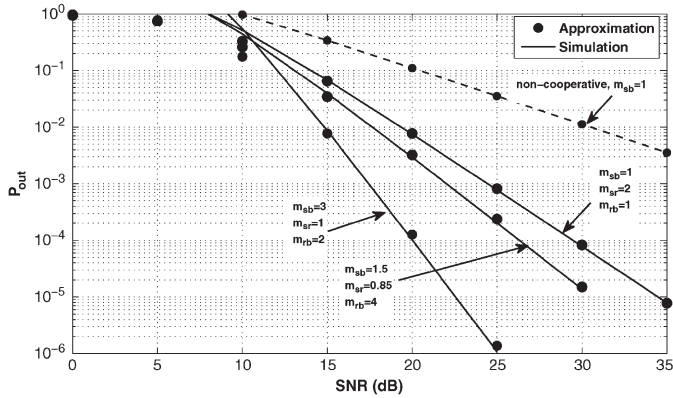


Fig. 3. Asymptotic outage probability for cooperative AF DS-CDMA with different fading figures  $m$  and considering i.n.i Nakagami fading channels.

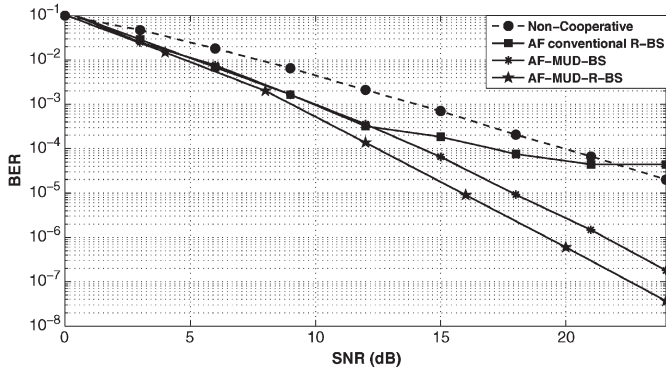


Fig. 4. BER comparison between cooperative AF and DF DS-CDMA with fading figures  $m_{ij} = 1.5$ .

base station, whereas the case of MUD-BS only suffers from SNR loss due to the amplification of unwanted interfering signals. In both systems, full diversity is achieved for the AF scheme. On the other hand, the BER of the conventional matched filter (conventional R-BS) exhibits an error floor at a certain SNR level due to the aggregate effect of multiuser interference at the relay and the base station. These results demonstrate that the mitigation of multiuser interference is essential to reach the diversity advantage of the cooperative system.

V. CONCLUSION

The performance of AF cooperative diversity in a DS-CDMA system under the diversity combining of the relayed information at the base station over Nakagami- $m$  fading channels is investigated. The cooperative system studied employs multiuser detection to mitigate the effect of multiuser interference at both the base station and the relay sides. A lower bound on the outage probability of the AF cooperative system is derived. In addition, an asymptotic outage probability expression is derived where we evaluated the overall system diversity under different system settings. Moreover, the impact of MAI on AF is studied, where the AF relaying is shown to achieve a satisfactory diversity gain without multiuser detection at the relay terminal.

APPENDIX  
PROOF OF EQUATIONS (31)

In this Appendix, we derive  $F_{X_{total}}(y)$  of (31). We have

$$I_{AF} = \frac{K}{2N} \log \left( 1 + X_{sb} + \frac{X_{sr}X_{rb}}{X_{sr} + X_{rb} + 1} \right). \quad (43)$$

We can rewrite (43) as

$$I_{AF} = b \log (1 + (X_{sb} + X_{min})) \quad (44)$$

where  $b = K/2N$  and  $X_{min} = \min(X_{sr}, X_{rb})$ . The outage probability then is written as

$$P_{out} = P_r [I_{AF} < \mathfrak{R}] = P_r [b \log (1 + (X_{sb} + X_{min})) < \mathfrak{R}] \quad (45)$$

which yields

$$P_{out} = P_r [X_{sb} + X_{min} < y] \quad (46)$$

where  $y = 2^{\mathfrak{R}/b} - 1$ . Having  $X_{sb} \sim \mathcal{G}(m_{sb}, \gamma_{sb}^-)$  and  $X_{min} \sim \mathcal{G}(m_{min}, \gamma_{min}^-)$  as two gamma RVs, we derived the cdf of the sum of two gamma RVs to get the outage probability as

$$\begin{aligned} P_{out} &= F_{X_{total}}(y) = P_r [X_{sb} + X_{min} < y] \\ &= \int_0^y P_r [X_{sb} + X_{min} < y | X_{sb} = x] \cdot f_{X_{sb}}(x) dx \\ &= \int_0^y P_r [X_{min} < y - x | X_{sb} = x] \cdot f_{X_{sb}}(x) dx \\ &= \int_0^y F_{X_{rb}}(y - x) f_{X_{sb}}(x) dx. \end{aligned} \quad (47)$$

Then, we can get

$$F_{X_{total}}(y) = \int_0^y F_{X_{min}}(y - x) f_{X_{sb}}(x) dx, \quad \text{for } y \geq 0. \quad (48)$$

As in (1), the pdf of  $X_{sb}$  is

$$f_{X_{sb}}(x) = \frac{B_{sb}^{m_{sb}}}{\Gamma(m_{sb})} x^{m_{sb}-1} \exp(-x B_{sb}) \quad (49)$$

and as in (2), the cdf of  $X_{min}$  is

$$\begin{aligned} F_{X_{min}}(y) &= 1 - P[X_{sr} > y \text{ and } X_{rb} > y] \\ &= 1 - \frac{\Gamma(m_{sr}, B_{sr}y)}{\Gamma(m_{sr})} \cdot \frac{\Gamma(m_{rb}, B_{rb}y)}{\Gamma(m_{rb})}. \end{aligned} \quad (50)$$

$$\begin{aligned} F_{X_{min}}(y - x) &= 1 - \frac{\Gamma(m_{sr}, B_{sr}(y - x))}{\Gamma(m_{sr})} \\ &\quad \cdot \frac{\Gamma(m_{rb}, B_{rb}(y - x))}{\Gamma(m_{rb})}. \end{aligned} \quad (51)$$

From [15, eq. (8.352, 6)], we have

$$\Gamma(n, x) = (n-1)! \exp(-x) \sum_{m=0}^{n-1} \frac{x^m}{m!}. \quad (52)$$

Then, using (52), we can rewrite (51) as

$$\begin{aligned} F_{X_{\min}}(y-x) &= 1 - \frac{1}{\Gamma(m_{\text{sr}})} \cdot (m_{\text{sr}}-1)! \cdot \frac{1}{\Gamma(m_{\text{rb}})} \\ &\times (m_{\text{rb}}-1)! \exp(-(y-x)B_{\text{sr}}) \exp(-(y-x)B_{\text{rb}}) \\ &\times \sum_{n=0}^{m_{\text{sr}}-1} \frac{(B_{\text{sr}}(y-x))^n}{n!} \sum_{i=0}^{m_{\text{rb}}-1} \frac{(B_{\text{rb}}(y-x))^i}{i!}. \end{aligned} \quad (53)$$

From [15, eq. (8.339, 1)], we have  $\Gamma(m_{\text{sr}}) = (m_{\text{sr}}-1)!$  and  $\Gamma(m_{\text{rb}}) = (m_{\text{rb}}-1)!$ . Therefore, we can rewrite (53) as

$$\begin{aligned} F_{X_{\min}}(y-x) &= 1 - \exp(-y(B_{\text{sr}} + B_{\text{rb}})) \\ &\times \exp(x(B_{\text{sr}} + B_{\text{rb}})) \sum_{n=0}^{m_{\text{sr}}-1} \frac{(B_{\text{sr}})^n}{n!} (y-x)^n \\ &\times \sum_{i=0}^{m_{\text{rb}}-1} \frac{(B_{\text{rb}})^i}{i!} (y-x)^i. \end{aligned} \quad (54)$$

Using the binomial expansion as in [15, eq. (1.111)], where  $(a+x)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$ , (54) can be written as

$$\begin{aligned} F_{X_{\min}}(y-x) &= 1 - \exp(-y(B_{\text{sr}} + B_{\text{rb}})) \\ &\times \exp(x(B_{\text{sr}} + B_{\text{rb}})) \sum_{n=0}^{m_{\text{sr}}-1} \frac{(B_{\text{sr}})^n}{n!} \sum_{k=0}^n \binom{n}{k} \\ &\times (-1)^k x^k y^{n-k} \sum_{i=0}^{m_{\text{rb}}-1} \frac{(B_{\text{rb}})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f x^f y^{i-f}. \end{aligned} \quad (55)$$

By substituting (55) into (48) and using (49), we get

$$\begin{aligned} F_{X_{\text{total}}}(y) &= \int_0^y \left( 1 - \exp(-y(B_{\text{sr}} + B_{\text{rb}})) \right. \\ &\times \exp(x(B_{\text{sr}} + B_{\text{rb}})) \sum_{n=0}^{m_{\text{sr}}-1} \frac{(B_{\text{sr}})^n}{n!} \\ &\times \sum_{k=0}^n \binom{n}{k} (-1)^k x^k y^{n-k} \sum_{i=0}^{m_{\text{rb}}-1} \frac{(B_{\text{rb}})^i}{i!} \\ &\times \left. \sum_{f=0}^i \binom{i}{f} (-1)^f x^f y^{i-f} \right) f_{X_{\text{sb}}}(x) dx \end{aligned}$$

$$\begin{aligned} &= \int_0^y f_{X_{\text{sb}}}(x) dx - \int_0^y f_{X_{\text{sb}}}(x) \exp(-y(B_{\text{sr}} + B_{\text{rb}})) \\ &\times \exp(x(B_{\text{sr}} + B_{\text{rb}})) \sum_{n=0}^{m_{\text{sr}}-1} \frac{(B_{\text{sr}})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k \\ &\times x^k y^{n-k} \sum_{i=0}^{m_{\text{rb}}-1} \frac{(B_{\text{rb}})^i}{i!} \sum_{f=0}^i \binom{i}{f} (-1)^f x^f y^{i-f} dx. \end{aligned} \quad (56)$$

Finally, the total cdf can be written after some manipulation as

$$\begin{aligned} F_{X_{\text{total}}}(y) &= \frac{\gamma(m_{\text{sb}}, \gamma B_{\text{sb}})}{\Gamma(m_{\text{sb}})} - \frac{B_{\text{sb}}^{m_{\text{sb}}}}{\Gamma(m_{\text{sb}})} \\ &\times \exp(-y(B_{\text{sr}} + B_{\text{rb}})) \int_0^y \\ &\times \left( x^{(m_{\text{sb}}-1)} \exp(-x(B_{\text{sb}} - B_{\text{sr}} - B_{\text{rb}})) \right. \\ &\times \sum_{n=0}^{m_{\text{sr}}-1} \frac{(B_{\text{sr}})^n}{n!} \sum_{k=0}^n \binom{n}{k} (-1)^k y^{n-k} x^k \\ &\times \left. \sum_{i=0}^{m_{\text{rb}}-1} \frac{(B_{\text{rb}})^i}{i!} \times \sum_{f=0}^i \binom{i}{f} (-1)^f y^{i-f} x^f \right) dx. \end{aligned} \quad (57)$$

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