# Wireless Network Utility Maximization

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**ABSTRACT**

*We describe Wireless Network Utility Maximization, WNUM, and compare its performance to traditional NUM along the dimensions of rate, delay and reliability under flat fading. Both coded and uncoded links are considered as are networks with interfering links. In each case, WNUM is shown to offer superior performance in simulations operating under Rayleigh fading due to its ability to adapt to changing channel conditions. A general method for finding adaptive optimal policies is presented that is sample-based and that makes no assumptions about the distribution of channel states.*

## INTRODUCTION

The network centric battlefield interconnects many different tactical elements with different data rate requirements, priorities, bandwidth allocations, and radio capabilities. Managing and controlling mobile ad-hoc networks (MANET's) with such diverse elements presents many new technical challenges, resulting in much research on optimizing the performance of these networks. In particular, much recent research has investigated the application of Network Utility Maximization (NUM) [1, 2] to optimize MANETs. While NUM has been shown to be a powerful optimization tool in complex wired networks, recent results have shown NUM performance to be disappointing when applied to MANETs.

We conjecture that this disappointing performance results from several fundamental limitations of the existing NUM framework in the dynamic lossy environments inherent to MANETs. In particular, NUM does not capture the effects of lossy randomly-varying RF channels nor does it include explicit reliability mechanisms to address this fundamental wireless issue. In addition, NUM does not readily capture (physical) time dynamics such as changes in the RF environment or traffic on the network. This is particularly acute in tactical environments where interference and mobility are important or where short-lived time critical traffic shares the network with long lived elastic flows. To address these limitations, we extend NUM to dynamic wireless environments by explicitly incorporating assumptions about the physical channel such as time varying fading, noise, node mobility, and link reliability and traffic characterization. We term this framework Wireless NUM (WNUM.)

In this paper we specialize Wireless NUM to networks with random channel variations and network time dynamics. WNUM yields adaptive policies that optimally manage the network. Our results show that WNUM offers significant performance improvements for MANETs over that obtained with traditional NUM techniques. We characterize performance by comparing the inherent trade-offs between throughput, delay and reliability in wireless networks using NUM and WNUM. WNUM policies are sample-based and make no parametric assumptions about the distribution of channel states. We describe a method to find optimal policies and extend our work described in [3, 4] to networks of interfering links using convolutional codes.

## SYSTEM MODEL

There are $M$ logical source/destination pairs and $L$ links in the network. Each source and destination pair is associated with an upper layer protocol stack. The flow of information over the network from a logical source to a logical destination, possibly over multiple links, is termed an information flow. Flows from different sources $m$ may traverse the same link $l$. The routing of information flows over links is described by the routing matrix $A$, where $A_{lm} = 1$ if information on flow $m$ traverses link $l$ and is otherwise zero.

A single link is modeled in Figure 1. Packets are injected into the link buffer by the upper layer protocol stack at information rate $r_m$ and are removed and transmitted by the wireless link at rate $(1-BER)\theta_l R_l$, where $R_l$ is the link rate, $0 < \theta_l \leq 1$ is the code rate, BER is the bit error rate. The code rate is the ratio of information bits sent over the link

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to the total number of bits (coded bits) sent over the link. Because $(1-BER) \approx 1$ we suppress this term.

The channel is modeled by a channel state (gain) matrix $G \in \mathbf{R}^{L\times L}$, where $G_{ij}$ is the power gain from the transmitter on link $j$ to the receiver on link $i$. The vector of transmitter powers is given by $S \in \mathbf{R}^{L}$. Each transmitter has an average power budget $\bar{S}$. For concreteness the link rate function is assumed to be of the form [5]

$$R_l(S,G) = \log\left(1 + \frac{\phi K G_{ii} S_l}{\sum_{j\neq l} G_{lj} S_j + N}\right) \quad l = 1,\ldots,L \qquad (1)$$

where $K = -1.5/\log(5BER)$ scales the received power to meet an instantaneous $BER$ ceiling, $\phi$ is the coding gain associated with a choice of convolutional code and $N$ is receiver noise.

The distribution of $G \sim p(G)$ is stationary and ergodic and is unknown to the network. We assume the channel state is estimated without error and is known at the set of transmitters. Because the channel is randomly varying, the link rates can also vary, resulting in congestion and queuing delay at the link buffers.

The system can adapt to changing channel conditions by estimating $G$ and adapting parameters such as transmit power $S = S(G)$, transmitter link rate $R = R(S(G),G)$ and the upper layer information rate $r = r(G)$.

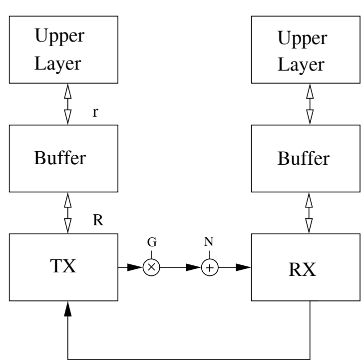


Figure 1: Single Link Model

The performance of upper layer protocols are modeled as utility functions. Utility functions are strictly concave increasing functions of the information rate. We consider the following parameterized family of utility functions [6, 7] often used in the literature:

$$U(r) = \begin{cases} \frac{r^{1-\alpha}}{1-\alpha} & \alpha > 0 \quad \alpha \neq 1 \\ \ln(r) & \alpha = 1. \end{cases} \qquad (2)$$

## NUM PERFORMANCE

In this section we briefly describe NUM. The canonical NUM problem is to find the optimal source rate $r$ that maximizes overall network utility of a network of links. Each link is assumed to have an associated buffer. Formally the NUM problem can be expressed as

$$\begin{array}{ll} \underset{r\geq 0}{\text{maximize}} & \sum_m U_m(r_m) \\ \text{subject to} & Ar \leq \bar{R} \end{array} \qquad (3)$$

where $A$ describes the fixed typology of the network. The operation of the network is described as an optimization algorithm seeking to solve this problem.

The links are assumed to have fixed, error free link rates, $\bar{R}$. This ignores the effects of the randomly time varying wireless channel. In a fading context, this implies that the fading can be inverted via power control or that the network will experience outages when channel conditions fall below those necessary to support this fixed transmission rate. Such outages reduce throughput and increase average delay.

Figure 2 shows the effect of outages on NUM network throughput as a function of average SNR and under Rayleigh fading. At 10% outage probability and an average SNR=20 dB, the effective throughput is only 30% of what is anticipated at this SNR. At moderate to low SNR's the effective throughput is further reduced. This reduction also increases the average backlog at link buffers.

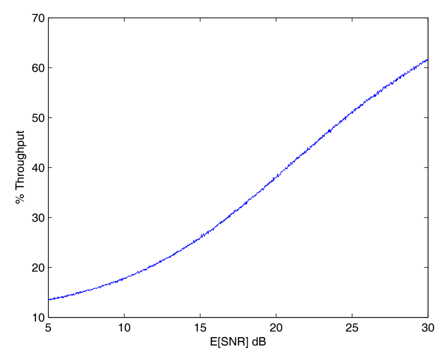


Figure 2: NUM Effective Rate vs. Ave SNR

## WNUM

In this section we describe WNUM and the method for finding optimal control policies when the channel is randomly time varying and where the distribution of channel states is unknown. WNUM extends NUM (3) by formally introducing random channel (or other network component) variations and modifying the performance metrics and constraints to be averages. The idea is to find adaptive rate and power policies that maximize the average utility of the network, under constraints on information rates, link rates and average power transmitted and to explore the delay, code rate and throughput trade offs of these optimal policies. By policies we mean rate and power functions that optimally adapt to changes in the channel state, and we write $S(G_t)$, $r(G_t)$, $R(S(G_t),G_t)$ for the respective transmitter power, source rate and link rate policies.

Conceptually the problem can be stated as

$$\begin{array}{ll} \underset{r(G_t),S(G_t)}{\text{maximize}} & \lim \frac{1}{T}\int_T \sum_l U_l(r_l(G_t))dt \\ \text{subject to} & \\ & \lim \frac{1}{T}\int_T Ar(G_t)dt \leq \underline{\lim}\, \frac{1}{T}\int_T R(S(G_t),G_t)dt \\ & \lim \frac{1}{T}\int_T S_l(G_t)dt = \bar{S_l} \end{array} \tag{4}$$

where $A$ is an incidence matrix routing information flows $r$ across links. The objective function is the time average of the instantaneous utility of the network. The utility is assumed to be a function of the information rate $r(G_t)$, the rate at which the upper layers of the protocol stack inject packets or bits into the network. The first constraint is a buffer constraint, which requires that the average arrival rate to the buffer $r(G_t)$ must be less than the departure rate from that buffer $\phi R(S(G_t),G_t)$. The second constraint requires that averaged transmitter power $S(G_t)$ cannot exceed a maximum.

Under conditions of stationarity and ergodicity we can reexpress (4) as the following formal WNUM problem:

$$\begin{array}{lll} \underset{r(G),S(G)\geq 0}{\text{maximize}} & \mathbf{E}[\sum_l U_l(r_l(G))] & \\ \text{subject to} & & \\ & \mathbf{E}[S_l(G)] = \bar{S_l} & l=1,\ldots,L \\ & \mathbf{E}[Ar] \leq \mathbf{E}[R(S(G),G)] & \end{array} \tag{5}$$

The optimization is over the policies $r(G)$ and $S(G)$ and indirectly the link rate $R(S(G),G)$.

## Method of Solution

Full Recourse Optimization with Expected Constraints (FROEC) is used to solve (5). FROEC is an on line discrete time approach to optimization. It takes as its input the sequence of channel states seen by the network and produces as its output estimates of the optimal Lagrange multipliers and optimal policy values. The time index is $k$, and we indicate the estimates of optimal Lagrange multiplier $\lambda^*$ by $\lambda^k$. Policy values are denoted by $r^k = r(G^k,\lambda^k)$, $S^k = S(G^k,\lambda^k)$, and $R^k = R((G^k,\lambda^k),G^k)$, e.g. $S^k$ is the value of the power policy at channel state $G^k$ and $\lambda^k$. FROEC does not assume knowledge of $p(G)$ and under suitable conditions adjusts to changes in the channels empirical distribution.

FROEC solves the dual problem to (5). The dual function is first defined as

$$g(\lambda) = \underset{r(G)\geq 0,S(G)\geq 0}{\text{argmax}} L(r(G),S(G),\lambda) \tag{6}$$

where

$$\begin{aligned} L(r(G),S(G),\lambda) \;=\; & \mathbf{E}[U(r(G)) \\ & -\lambda_q(r(G) - \underline{R}(S(G),G)) \\ & -\lambda_s(S(G) - \bar{S}))] \end{aligned} \tag{7}$$

and $\lambda = [\lambda_q^T, \lambda_s^T]^T$ is the vector of Lagrange multipliers.

The dual problem is

$$\underset{\lambda\geq 0}{\text{minimize}} \quad g(\lambda). \tag{8}$$

The FROEC approach samples the channel and generates a sequence of stochastic subgradients to $g(\lambda)$. These in turn are used to optimize (8). The FROEC algorithm has three steps. In the first step, the channel is estimated at time $k$ and policy values calculated:

$$[r^k, S^k] = \underset{r\geq 0,S\geq 0}{\text{argmax}} \left[U(r) - \lambda_q^k(r - R(S,G^k)) - \lambda_s^k(S-\bar{S})\right]. \tag{9}$$

The second step calculates stochastic subgradients:

$$\delta g = -\left[\begin{array}{c} (r^k - \theta R^k) \\ (S^k - \bar{S}) \end{array}\right] \tag{10}$$

which is a vector composed of the "slack" in the constraints evaluated at the current policy estimates. In the third step, the $\lambda^k$ are updated using the subgradient recursion

$$\lambda^{k+1} = \left[\lambda^k + \Delta_k \delta g\right]^+ \tag{11}$$

where $[]^+$ is the positivity operator and the step size $\Delta_k$ is a sequence of positive constants.

The top portion of equation (11) can be rewritten as

$$\begin{aligned} \lambda_q^{k+1} &= [\lambda_q^k + \Delta_k\left(r(G^k,\lambda^k) - \theta R(S(G^k,\lambda^k),G^k)\right)]^+ \\ &= [\lambda_q^0 + \textstyle\sum_{l=1}^k T^l\left(r(G^l,\lambda^l) - \theta R(S(G^l,\lambda^l),G^l)\right)]^+ \\ &= [\lambda_q^0 + \textstyle\sum_{l=1}^k \left(A^l - D^l\right)]^+ \end{aligned} \tag{12}$$

where we interpret $A^l$ as the packet workload injected into the buffer and $D^l$ as the packet workload (information) transmitted by the link, $T^l$ as the time duration of the $l^{\text{th}}$ time period and $\lambda_q^0$ as the initial value of the the estimated Lagrange multiplier. When $T^l = T$ and the step size is fixed $\Delta_k = \Delta$, then $\lambda_q^k$ can be interpreted as proportional to the number of packets queued at the buffer. The initial buffer workload is $\lambda_q^0$. A packet arriving at the buffer will be delayed by the packet workload in front of it.

Similarly the lower portion of (11) can be rewritten as

$$\begin{aligned} \lambda_s^{k+1} &= \left[\lambda_s^k + \Delta_k(S(G_k) - \bar{S})\right]^+ \\ &= \left[\lambda_s^0 + \textstyle\sum_{l=1}^k (E(G^l) - \bar{E})\right]^+ \end{aligned} \tag{13}$$

where $E(G^k)$ is the energy used by the transmitter at time $k$ when the channel is in state $G^k$, and $\bar{E}$ is the average energy spent by the transmitter per transmission. When $\Delta_k = \Delta$, the Lagrange multiplier $\lambda_s^k$ is proportional to the total deviation from $\bar{E}$ spent by the transmitter up to time $k$. If on average the transmitter has exceeded its energy budget $\lambda_s^k$ will be large and conversely.

The ratio $\frac{\lambda_q^k}{\lambda_s^k}$ measures the relative cost of queue backlog to energy spent at time $k$. We term this ratio energy normalized backlog and it is the estimated energy cost per packet to transmit data at time $k+1$.

## Convergence

The convergence properties of (11) depends on the sequence $\{\Delta_k\}$. When $\Delta_k = \Delta$, the estimated Lagrange multiplier probabilistically converges to a region centered around the optimal value [8]. If we define $e(k) = ||\lambda^k - \lambda^*||$ then

$$P[e(k) \geq \varepsilon | \lambda^0] \leq A_1(\Delta) + A_2(\lambda^0) \exp(-h(\Delta)k) \qquad (14)$$

where $\lambda^0$ is the initial guess for $\lambda$, and $A_1 \to 0$, $h(\Delta) \to 0$, as $\Delta \downarrow 0$. In steady state, the fixed step size approach only approximately meets the constraints, but the approximation can be made very tight for small enough $\Delta$.

With a fixed $\Delta$ the queue length converges to a region centered on the average queue length $\lambda_q^*$ associated with the optimal policy. As channel samples vary and the system responds, the queue lengths will randomly drift within this region. Similarly, $\lambda_s^k$ will converge to a region centered about the optimal value of the Lagrange multiplier $\lambda_s^*$, and also will drift within this region as channel conditions vary.

## COMPARING WNUM TO NUM

In this section we compare WNUM and NUM performance for single links. We compare performance for a range of instantaneous $BER$ requirements when a convolutional code is not used. In the next section we compare WNUM and NUM when codes of varying rate are used. The focus is on comparing the average rate, delay, and reliability trade offs. In all cases WNUM yields substantially improved performance, since it adapts to changes in the wireless environment.

Figures 3-5 show the rate and delay performance of NUM under different BER targets and compares them to the performance of WNUM.

The optimal WNUM policies for single links can be found analytically using FROEC from (9). The optimal adaptive policies are the following:

$$S(G^k, \lambda^k) = \begin{cases} \left(\frac{\lambda_q^k}{\lambda_s^k} - \frac{N}{G^k}\right) & \frac{\lambda_s^k}{\lambda_q^k} < \frac{G^k}{N} \quad \lambda_q, \lambda_s^k > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (15)$$

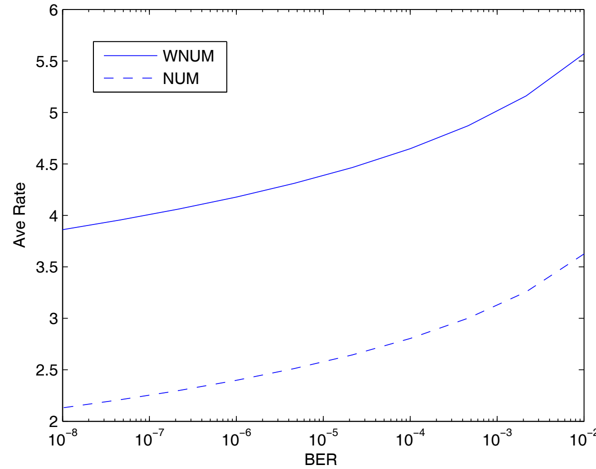


Figure 3: Rate vs. BER Threshold

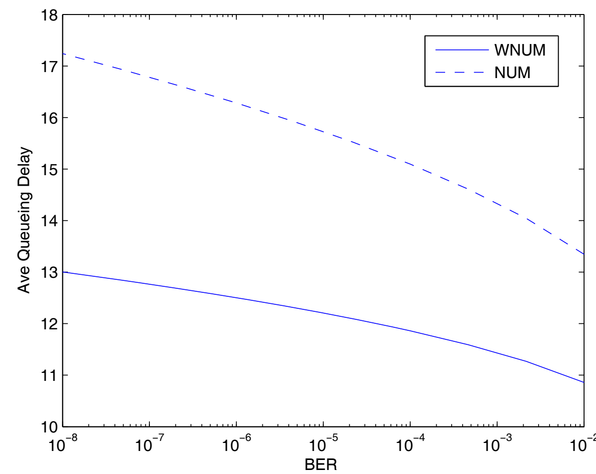


Figure 4: Delay vs. BER Threshold

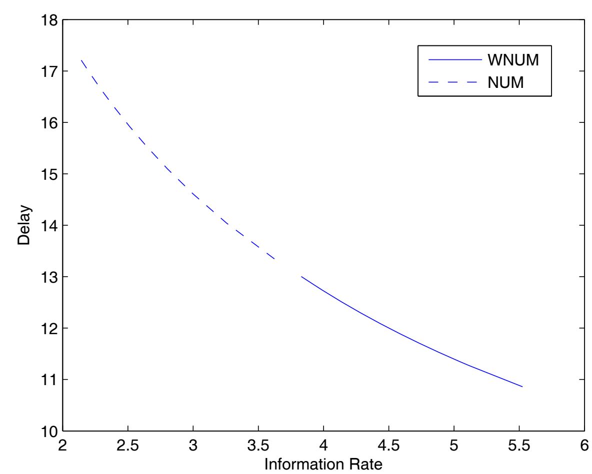


Figure 5: Delay vs. Information Rate

$$R(G^k,\lambda^k) = \begin{cases} \log\left(1+K\frac{\lambda_q^k}{\lambda_s^k}\frac{G^k}{N}\right) & \frac{\lambda_s^k}{\lambda_q^k} < \frac{KG^k}{N} \quad \lambda_s^k > 0 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

$$r(G^k,\lambda_q^k) = r(\lambda_q^k) = [\dot{U}]^{-1}(\lambda_q^k). \tag{17}$$

The optimal power and link rate policies are functions of the energy normalized backlog, balancing the energy cost per packet to send the next packet with the noise normalized channel gain. The transmitter stops transmitting if noise normalized channel conditions are poor enough. This threshold, however, changes over time and is determined by the the current value of the energy normalized backlog. Together these policies control the rate at which packets are removed from the queue. The upper layer protocols inject packets at an instantaneous rate determined by the backlog of the queue. This rate is independent of the current channel state. Since the arrival and service rates of the links' queue are randomly time varying through the $G^k$, link congestion will occur.

Average rate and delay are functions of the distribution of the channel states and cannot be readily expressed in closed form, so numerical techniques are used. Figures 3-5 show the performance of WNUM and NUM under Rayleigh fading with an average SNR=20 dB.

WNUM rate performance is twice NUM rate performance at $BER = 10^{-6}$. Figures 3 and 4 show the average link rate and delay for a range of instantaneous BER (reliability) requirements. As intuition would suggest, the average link rate increases for both WNUM and NUM with decreasing reliability, since $K$ is an increasing function of the BER target. WNUM delay performance is approximately 30% better than NUM delay performance. The average queuing delay decreases for both WNUM and NUM with decreasing reliability since the (net of errors) link transmission rate increases.

As shown in Figure 5, WNUM's rate and delay trade off curve is superior to NUM's. Not surprisingly both curves are convex [] in the information rate, since packet arrivals to the queue buffer and their departures are functions of the random channel gain $G$.

As shown in Figure 6 the average power cost, $\lambda_s^k$, declines with less stringent reliability requirements. The reason is intuitive; a less stringent BER requirement is equivalent to a greater power budget, $\bar{S}$, which in turn decreases the power cost. Figure 7 displays the energy normalized backlog, ENB. The ENB remains essentially constant(+/- 2%) over the BER range considered. There are no NUM equivalents and so are not compared.

## WNUM CODED LINKS

In this section we investigate the performance of WNUM wireless networks using convolutional codes with a range of

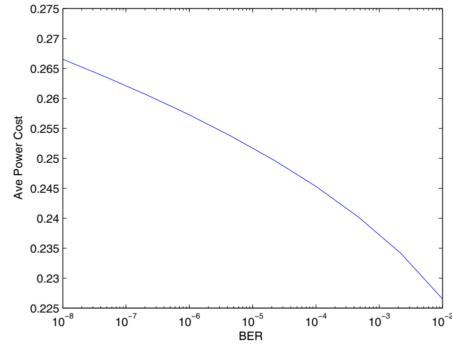


Figure 6: Power Cost vs. BER Threshold

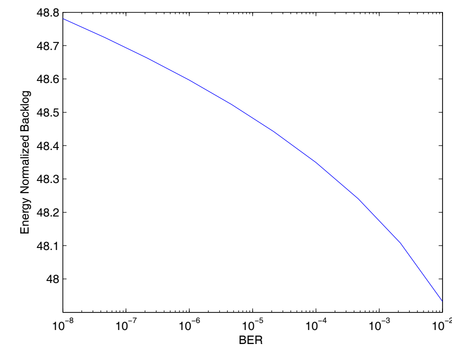


Figure 7: Energy Normalized Backlog vs. BER Threshold

different coding gains $\phi$ and coding rates $\theta$. At a given BER threshold, WNUM uses any excess available power to increase the link transmission rate, thus an increase in coding gain can translate into an increase in link rate.

### Single Link Case

The optimal WNUM policies can be found analytically using FROEC and are similar to the uncoded case:

$$S(G^k,\lambda^k) = \begin{cases} \left(\frac{\theta\lambda_q^k}{\lambda_s^k} - \frac{N}{\phi KG^k}\right) & \frac{\lambda_s^k}{\theta\lambda_q^k} < \frac{\phi KG^k}{N} \quad \lambda_q,\lambda_s^k > 0 \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

$$R(G^k,\lambda^k) = \begin{cases} \log\left(1+K\frac{\theta\lambda_q^k}{\lambda_s^k}\phi\frac{G^k}{N}\right) & \frac{\lambda_s^k}{\theta\lambda_q^k}N < \frac{\phi KG^k}{N} \quad \lambda_s^k > 0 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

$$r(G^k,\lambda_q^k) = r(\lambda_q^k) = [\dot{U}]^{-1}(\lambda_q^k). \tag{20}$$

At a given BER target, equation (19) increases the link rate with increasing coding gain or coding rate. The transmission cut-off threshold is shifted by the product of the coding gain and coding rate.

Figures 8-12 display the rate, delay, reliability trade offs for a coded link using the optimal policies. In all cases

WNUM performance significantly exceeds NUM performance. As shown in Figure 8, average link rates are essentially constant with increasing coding rate, but information rates increase, since the coding rate scales the link rate. The link rates are almost constant since $\phi$ does not vary greatly in this example. The average queuing delay shown in Figure 9 also decreases with increasing code rate as in the uncoded link case. The power cost, Figure 11, increases with increasing code rate. This is an indirect result of the code rate scaling the link rate function. The ENB grows with the coding rate, reflecting the queuing delay curve. As in the uncoded case, ENB is relatively constant for each BER target.

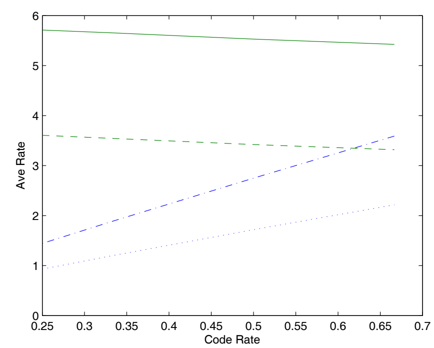


Figure 8: Rates vs. Code Rate

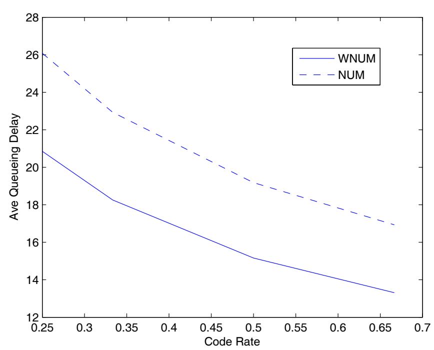


Figure 9: Delay vs. Code Rate

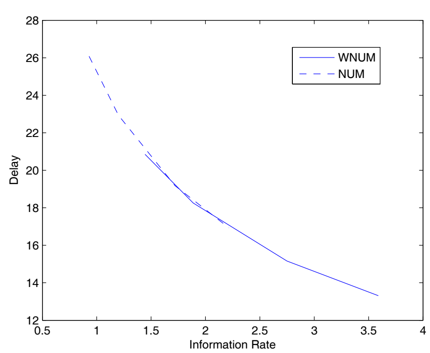


Figure 10: Delay vs. Information Rate

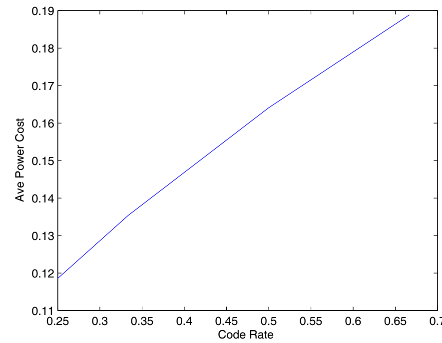


Figure 11: Power Cost vs. Code Rate

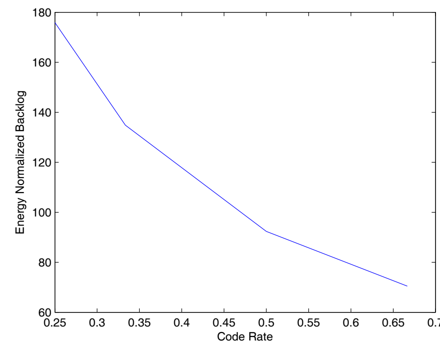


Figure 12: Energy Normalized Backlog vs. Information Rate

## Multiple Interfering Links

In this section we investigate the rate, delay and reliability trade offs for networks of interfering links. The relative performance between WNUM and NUM is similar to the single link case and for brevity is not discussed here. For simplicity we assume the number of links is equal to the number of sources, $L = M$, and that each flow traverses exactly one link. The multi-hop case is solved in a similar fashion. Equation (5) is not a convex function for $L \geq 2$ and globally optimal policies may not exist. It can be made convex by assuming $SIR_l >> 1$ and transforming the variables $S_l = \exp(x_l)$, $G_{ij} = \exp(g_{ij})$, $N = \exp(n)$, $\theta = \exp(\gamma)$ where $x_l$ $g_{ij}$ and $n$ are proportional to transmitter power, channel gain, and noise in dB. The link rate model can now be expressed as

$$R_i(G, S(G)) = -\ln(\mathrm{e}^{(-x_i - g_{ii} - \gamma)}(\textstyle\sum_{j \neq i} \mathrm{e}^{(x_j + g_{ij})} + \mathrm{e}^{n})). \tag{21}$$

The set of possible (positive) link rates is bounded and convex. As the transmitter power levels in the system grow, the vector of link rates asymptotically approaches the rate surface. In this situation, due to inter-link interference the coding gain may not result in a significant improvement in link performance at a fixed BER target.

Analytical expressions for the optimal policies are difficult to find but can be expressed algorithmically. In this setting it is useful to describe the behavior of these policies. Optimal power policies can be thought of as allocating power across channel states such that the average performance of the network is maximized. For each state, the rate policy maximizes the link rate performance of the interfering network, arriving at an operating point inside the rate region. For large SINR the operating point will lie near the rate surface. Changes in convolutional codes will shift this operating point. As a consequence of the mapping from power to rate, changes in coding gain may not significantly effect the the operating point of the network.

Figures 14-17 depict an example network's performance as a function of different convolutional codes. Each link operates using identical codes in this example. We consider the case of $L = M = 6$ links and sources and $\overline{S_l} = 1$. The channel state matrix is drawn iid Rayleigh, with diagonal elements scaled to yield an average SINR of 20 dB over the set of links. Figure 13 shows the link rates (upper curves) and information throughput (lower curves) for each of the six links for a variety of coding rates. The link rates remain relatively constant indicating the that the the network may be operating operating near its rate surface. The information rates increase with the coding rate since they are scaled versions of the link rates $\theta R$.

Figure 14 shows the average delay vs. code rate for each of the six flows. As in the single link case the average queuing delay of packets decreases with increasing code rate. As the rate increases so does the information transfer rate of the link, reducing average delay. Figure 15 shows the trade off between average information rate and average queuing delay for each of the six links. Because the links interfere with each other, the transmission rates of the links are entangled, thereby entangling the queuing delays of the links. As expected for queues of this type with random service rates, the trade off is convex. The individual link curves overlap due the symmetry of this example.

Figure 16 and 17 display the average power cost and ENB for the links. As in the single link case, the average power cost increases with the coding rate and is an indirect result of the code rate scaling the link rate function. The ENB decreases with increasing code rate, since more packets can be sent for a given energy cost.

## CONCLUSIONS

We describe WNUM and compare its performance to NUM along the dimensions of rate, delay and reliability under flat fading. At typical SINR's simulations indicate that rate performance is improved by greater than 50% and average de-

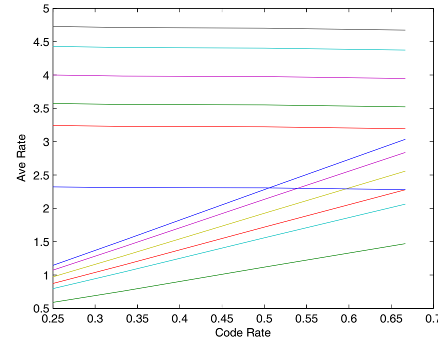


Figure 13: Rates vs. Code Rate

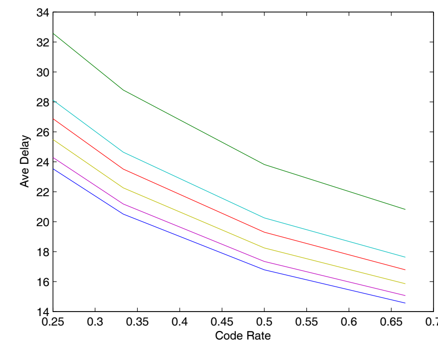


Figure 14: Delay vs. Code Rate

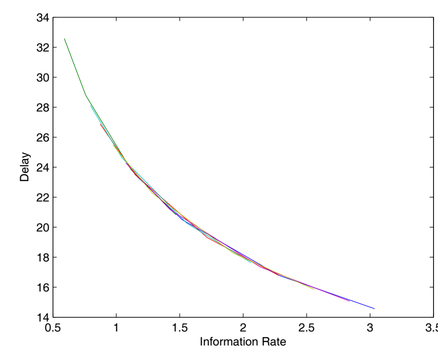


Figure 15: Delay vs. Information Rate

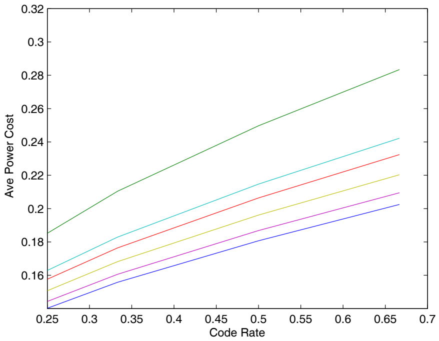


Figure 16: Power Cost vs. Code Rate

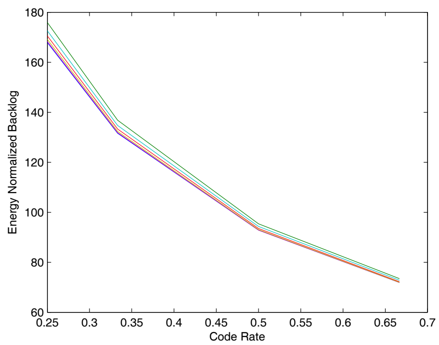


Figure 17: Energy Normalized Backlog vs. Code Rate

lay by 30%. We consider simple uncoded links and networks using convolutional codes. WNUM uses optimal policies to adapt to changing channel conditions by adjusting network resources. We present the optimal adaptive control policies for WNUM in the single link case and describe a FROEC based algorithm for the multiple interfering link case. These policies are sample-based and make no assumptions about the distribution of channel states. NUM does not model the physical layer and consequently is unable to exploit good channel conditions or respond to poor channel conditions, resulting in relatively inferior performance.

Future research work includes extending this formulation to broader types of reliability mechanisms and extending the formulation to traffic with QoS requirements

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