Status and possible improvements of electroweak effective couplings for future precision experiments.

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Abstract

I will report on my new package "alphaQED", which besides the effective fine structure constant $\alpha_{\rm em}$ also allows for a fairly precise calculation of the $SU(2)_L$ gauge coupling α_2. I will briefly review the role, future requirements and possibilities.

Outline of Talk:

- Motivation
- Status: recent package `alphaQED`
- A new evaluation of the $SU(2)_L$ running coupling α_2
- Prospects for future improvements

Motivation

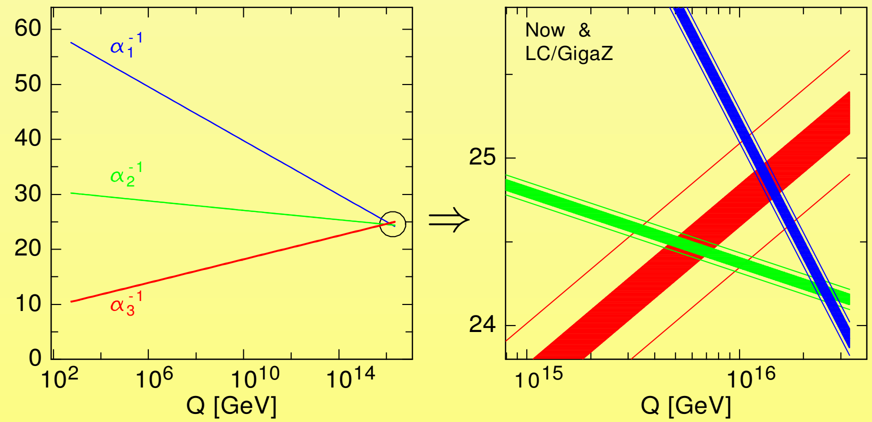
Precise SM predictions require to determine the $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$ SM gauge couplings $\alpha_{\rm em}$, α_2 and $\alpha_s \equiv \alpha_3$ (QCD) as accurately as possible

**** a theory can not be better than its input parameters ****

$\Rightarrow$ precision limitations due to non-perturbative hadronic contributions $\Leftarrow$

❖beyond SM physics gauge coupling unification?

from Zerwas



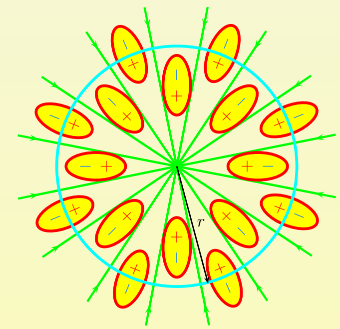
$\alpha_s = 0.1183 \pm 0.0027$ vs ± 0.0009

1. Introduction

Non-perturbative hadronic effects in electroweak precision observables, main effect via

effective fine-structure "constant" $\alpha(E)$

(charge screening by vacuum polarization)



Of particular interest:

$$\alpha(M_Z) \text{ and } a_\mu \equiv (g-2)_\mu/2 \Leftrightarrow \alpha(m_\mu)$$

- ❖ electroweak effects (leptons etc.) calculable in perturbation theory
- ❖ strong interaction effects (hadrons/quarks etc.) perturbation theory fails

$\Longrightarrow$ **Dispersion integrals over e^+e^-–data**

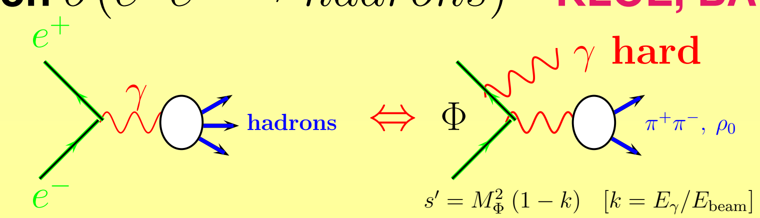
encoded in $$R_\gamma(s) \equiv \frac{\sigma(e^+e^-\to\gamma^*\to\text{hadrons})}{\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)}$$

Errors of data $\Longrightarrow$ theoretical uncertainties !!!

The art of getting precise results from non-precision measurements !

New challenge for precision experiments on $\sigma(e^+e^- \to hadrons)$ **KLOE, BABAR, ...**

σ_{hadronic} via radiative return:



Energy scan Photon tagging

2. $\alpha(M_Z)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

$\alpha\ ,\ G_\mu, M_Z$ **most precise input parameters**

50% non-perturbative relationship

$\Updownarrow$

precision predictions $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$\begin{array}{lcccl}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 & \times & 10^{-4} \quad (\mathrm{present: lost}\ 10^5\ \mathrm{in\ precision!}) \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 5.3 & \times & 10^{-5} \quad (\mathtt{ILC\ requirement})
\end{array}$$

LEP/SLD: $\sin^2\Theta_{\mathrm{eff}} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm \boxed{0.00017}$

$$\delta\Delta\alpha(M_Z) = 0.00036 \quad \Rightarrow \quad \delta\sin^2\Theta_{\mathrm{eff}} = \boxed{0.00013}$$

affects Higgs mass bounds, precision tests and new physics searches!!!

For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ Lattice-QCD

Indirect

Higgs boson mass "measurement"

$m_H = 87^{+35}_{-26}$ **GeV**

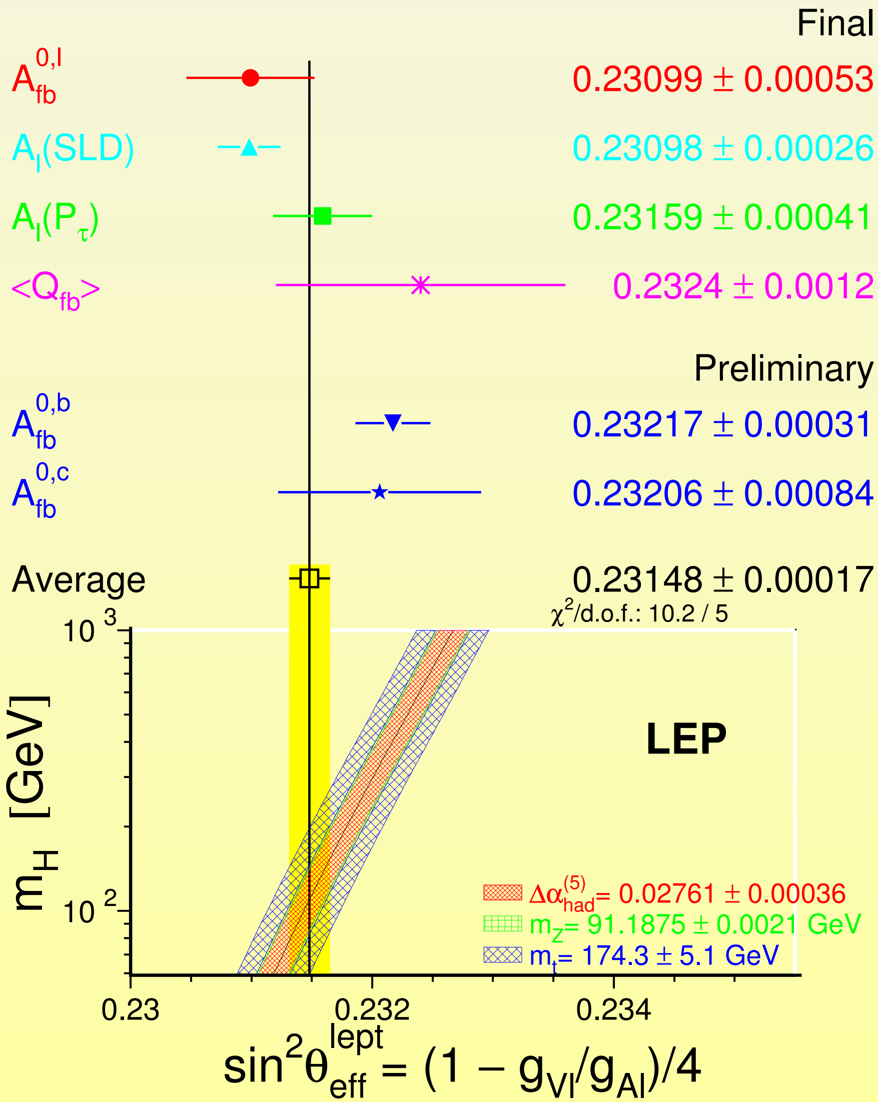
CDF/D0 exclude 160-170 GeV 95% C.L.

Direct lower bound:

$m_H > 114$ **GeV at 95% CL**

Indirect upper bound:

$m_H < 186$ **GeV at 95% CL**



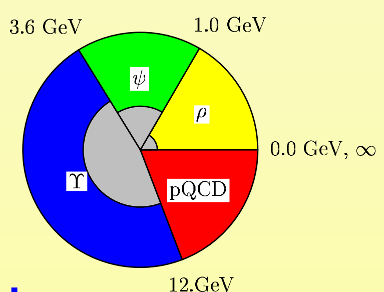
What is the point once m_H has been measured by the LHC? $\Rightarrow \sin^2\Theta_{\rm eff}$ turns into an excellent monitor for new physics!

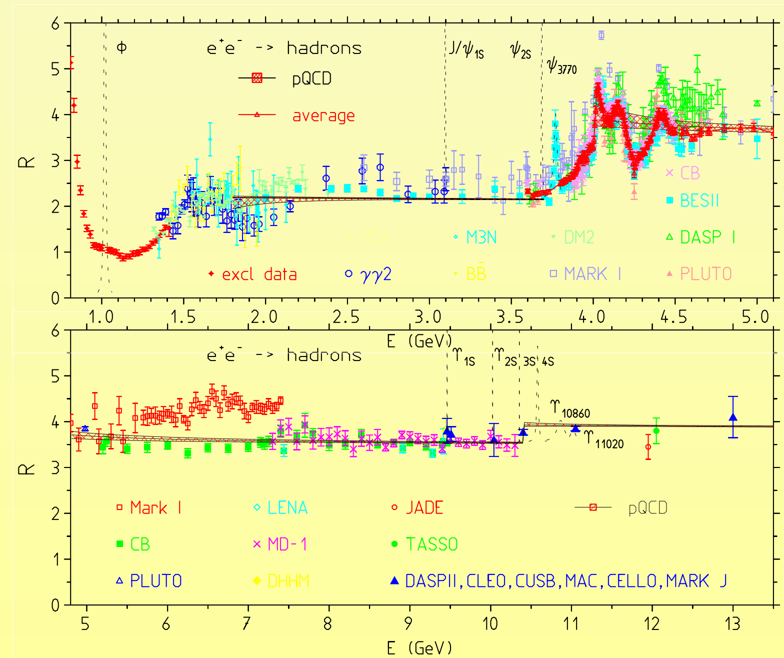
3. Evaluation of $\alpha(M_Z)$

Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s)$ can be evaluated in terms of $\sigma(e^+e^- \to \text{hadrons})$ data via dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\;\mathrm{P}\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

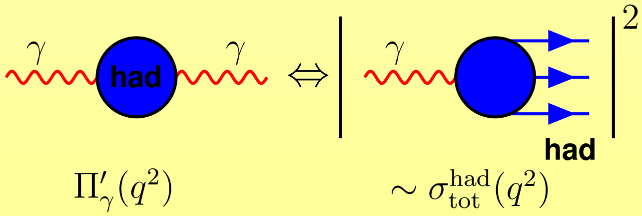
Compilation: FJ 08
Theory = pQCD: Gorishny et al. 91, Chetyrkin et al. 97





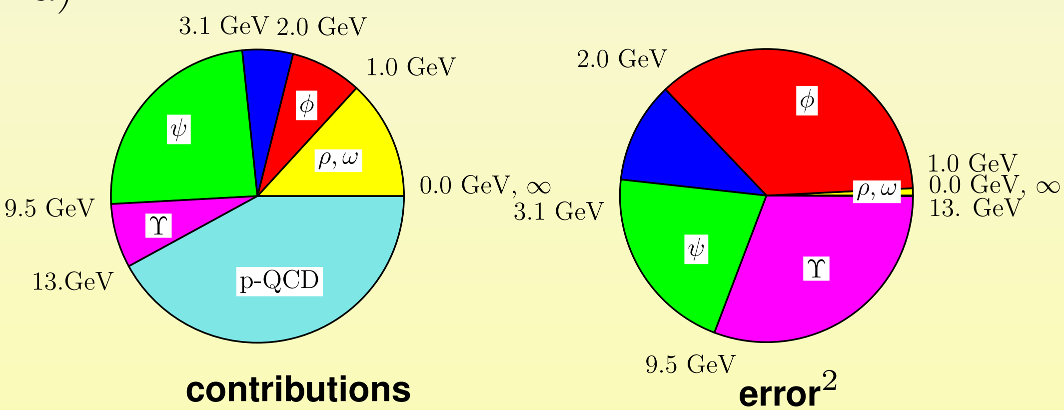
where

$$R_\gamma(s) \equiv \frac{\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$$



present distribution of contributions and errors

a)



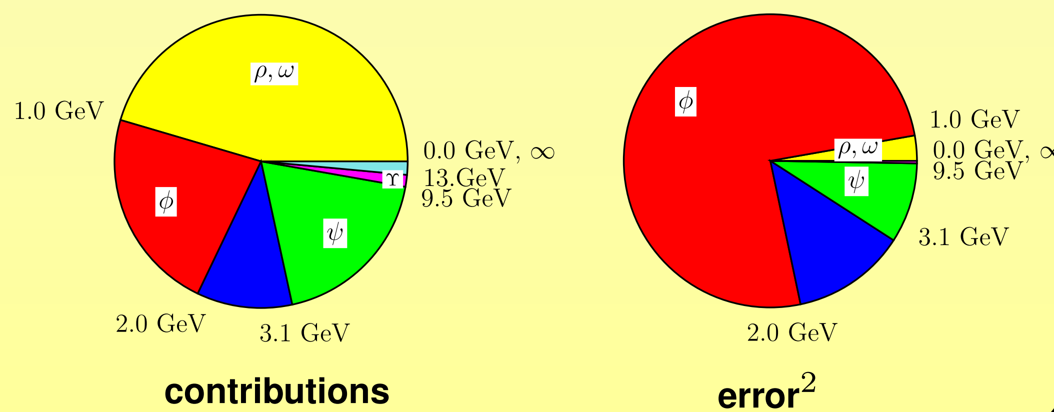
direct integration of data

$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$$

b)

"Adler function controlled"

$\Delta\alpha^{(5)}_{\rm had}(-M_0^2)^{\rm data}$ **($M_0 = 2.5$ GeV)**



The coupling α_2, M_W and $\sin^2\Theta_f$

How to measure α_2:

❖charged current channel M_W ($g \equiv g_2$):

$$M_W^2 = \frac{g^2 v^2}{4} = \frac{\pi\,\alpha_2}{\sqrt{2}\,G_\mu}$$

❖neutral current channel $\sin^2\Theta_f$

In fact here running $\sin^2\Theta_f(E)$: LEP scale $\Longleftrightarrow$ low energy $\nu_e e$ scattering

$$\sin^2\Theta_e = \left\{\frac{1-\Delta\alpha_2}{1-\Delta\alpha} + \Delta_{\nu_\mu e,\mathrm{vertex+box}} + \Delta\kappa_{e,\mathrm{vertex}}\right\}\sin^2\Theta_{\nu_\mu e}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel.

Above result allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self energies, as

$$
\begin{aligned}
\Pi^{\gamma\gamma} &= e^2\,\hat{\Pi}^{\gamma\gamma}\;;\quad \Pi^{Z\gamma} = \frac{eg}{c_\Theta}\,\hat{\Pi}_V^{3\gamma} - \frac{e^2\,s_\Theta}{c_\Theta}\,\hat{\Pi}_V^{\gamma\gamma}\;;\\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2}\,\hat{\Pi}_{V-A}^{33} - 2\,\frac{e^2}{c_\Theta^2}\,\hat{\Pi}_V^{3\gamma} + \frac{e^2\,s_\Theta^2}{c_\Theta^2}\,\hat{\Pi}_V^{\gamma\gamma}\\
\Pi^{WW} &= g^2\,\hat{\Pi}_{V-A}^{+-}
\end{aligned}
$$

with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\hat{\pi}(s)$.

Leading hadronic contributions:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -e^2\,\left[\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)\right]$$

$$\Delta\alpha^{(5)}_{2\,\rm had}(s) = -\frac{e^2}{s^2_\Theta}\left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right]$$

which exhibit the leading hadronic non-perturbative parts, i.e. the ones involving the photon field via mixing.

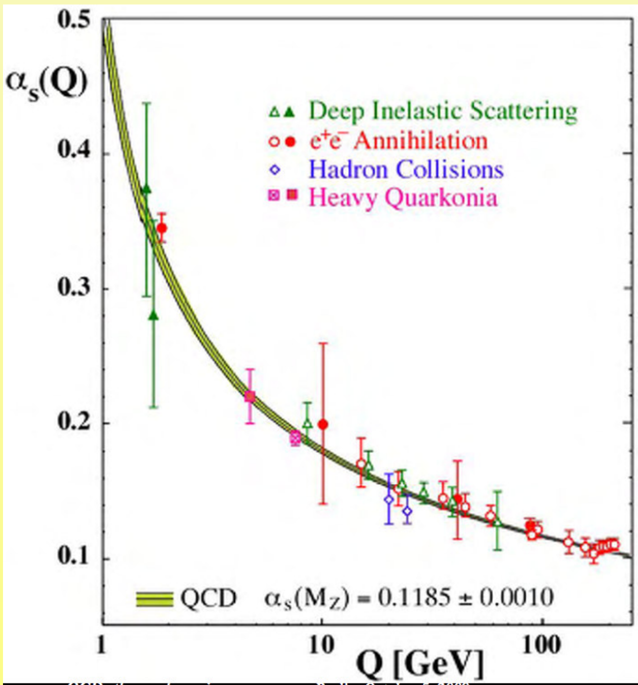
Note: gauge boson SE potentially very sensitive to New Physics (oblique corrections)

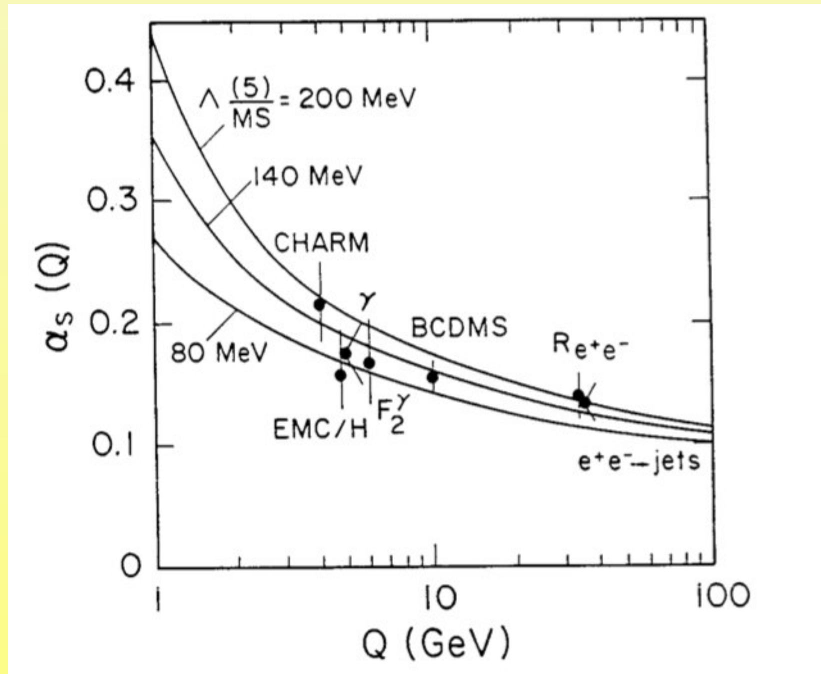
➠new physics may be obscured by non-perturbative hadronic effects; need to fix this!

Remark on the QCD coupling α_s

❖Asymptotic freedom: weak coupling at high energies

❖always needed for evaluating perturbative windows/tails !!!





Status of α_s [Bethke 2009](left) compared with 1989 pre LEP status (right) $\alpha_s^{(5)}(M_Z) = 0.11 \pm 0.01$ (corresponding to $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 140 \pm 60$ MeV).[Altarelli 89].

❖perturbative QCD (pQCD) applies for $M_\tau \leq \sqrt{s}$

Fortunately in good shape thanks to LEP, HERA, Tevatron etc as well as heroic efforts by theorists

❖running α_s in $\overline{\mathrm{MS}}$ to 4 loops exact with matching, perturbative $R(s)$

e.g. `RHAD` package for pQCD Harlander, Steinhauser 2002/2009

My alphaQED and alpha2SM packages

Download link: ⁎⋙ [alphaQED.tar.gz]

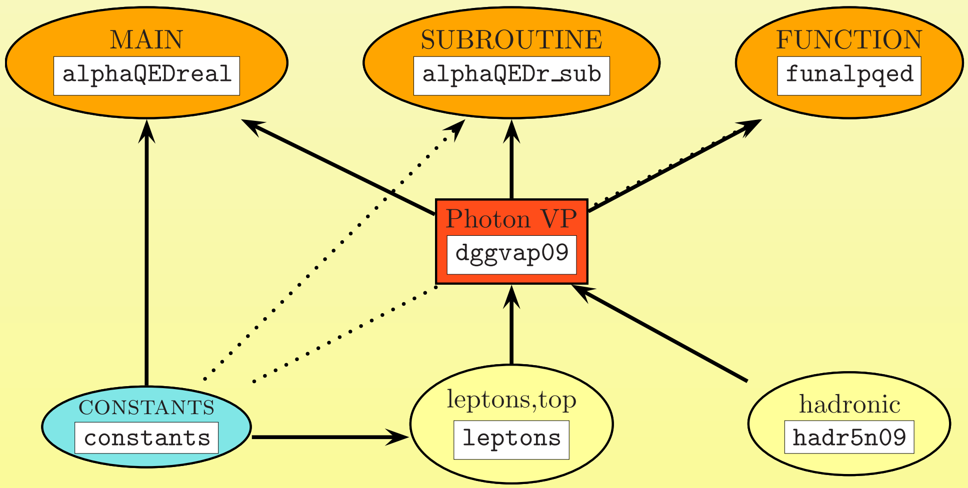
The package for calculating the effective electromagnetic fine structure constant is available in two versions:

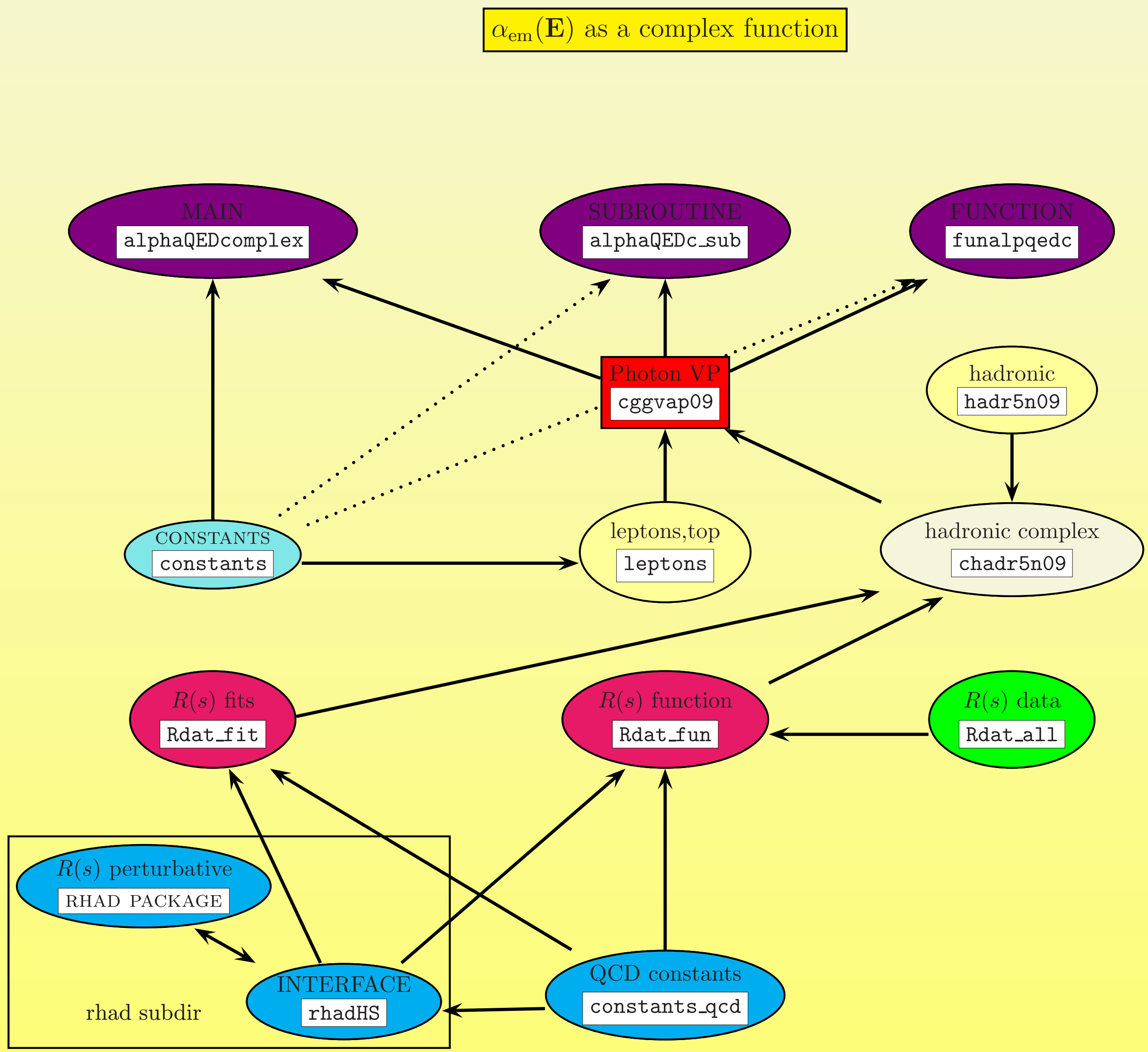
➠alphaQEDreal [FUNCTION funalpqed] providing the real part of the subtracted photon vacuum polarization including hadronic, leptonic and top quark contributions as well as the weak part (relevant at ILC energies)

➠alphaQEDcomplex [FUNCTION funalpqedc] provides in addition the corresponding imaginary parts.

➠corresponding options for $SU(2)_L$ coupling $\alpha_2 = g^2/4\pi$ alpha2SMreal and alpha2SMcomplex

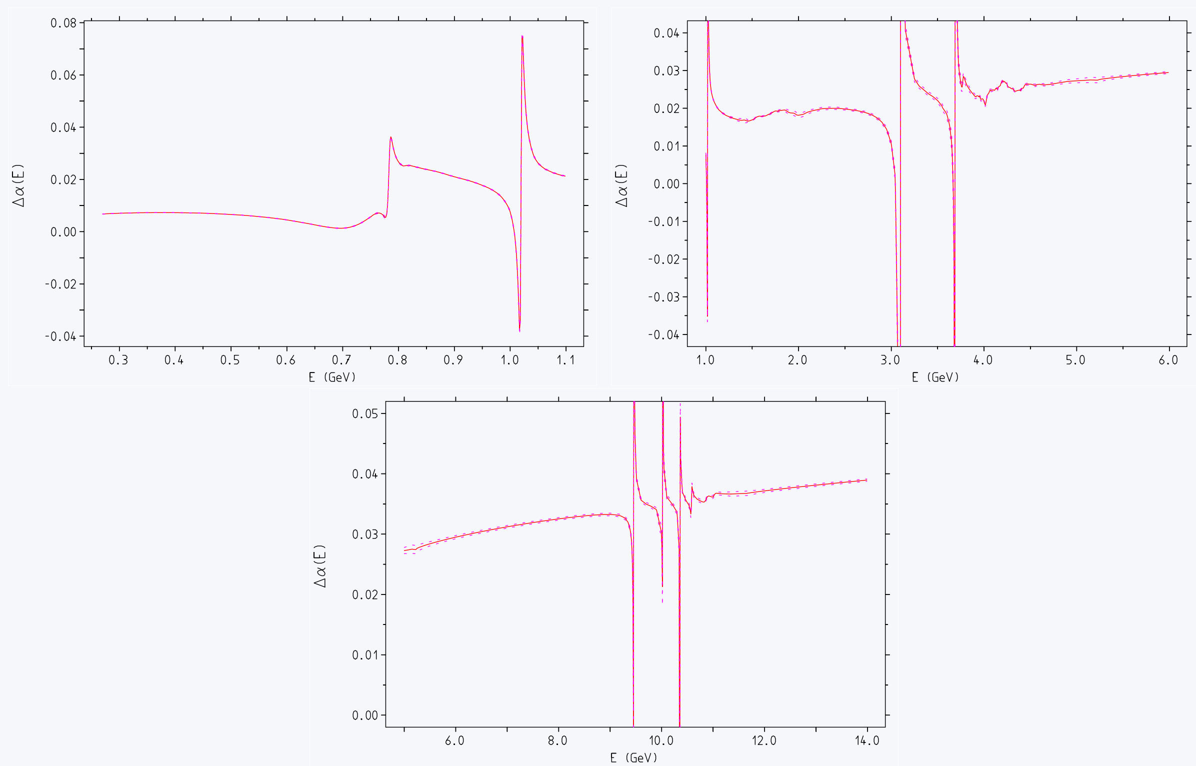
$\alpha_{\mathrm{em}}(\mathbf{E})$ as a real function

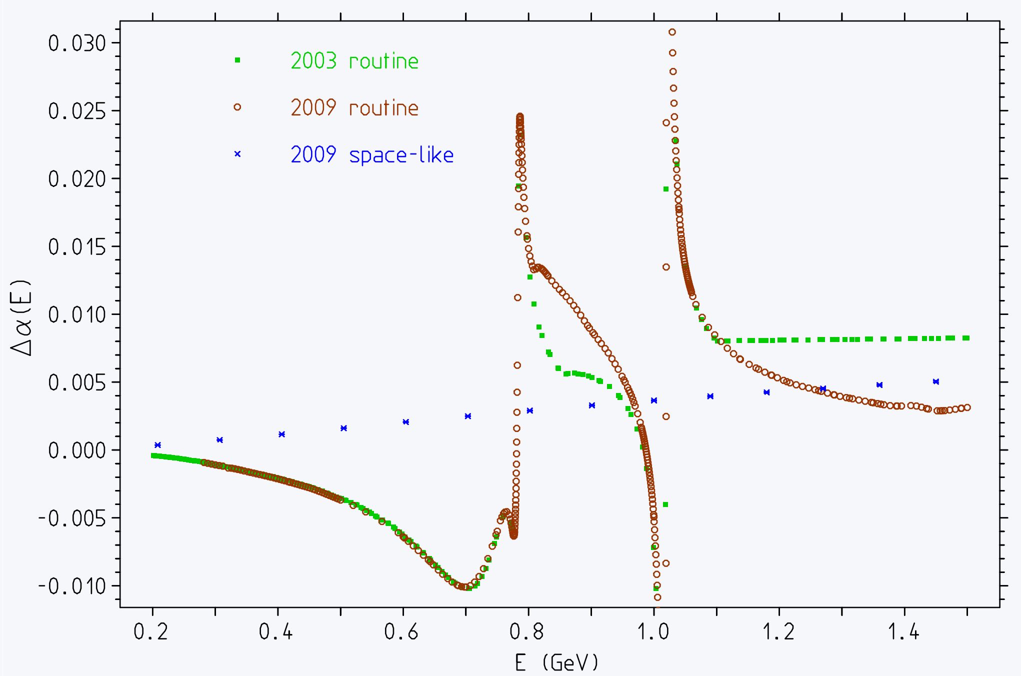



$\alpha_{\rm em}(\mathbf{E})$ as a complex function
MAIN
alphaQEDcomplex
SUBROUTINE
alphaQEDc_sub
FUNCTION
funalpqedc
Photon VP
cggvap09
hadronic
hadr5n09
CONSTANTS
constants
leptons,top
leptons
hadronic complex
chadr5n09
$R(s)$ fits
Rdat_fit
$R(s)$ function
Rdat_fun
$R(s)$ data
Rdat_all
$R(s)$ perturbative
RHAD PACKAGE
INTERFACE
rhadHS
QCD constants
constants_qcd
rhad subdir

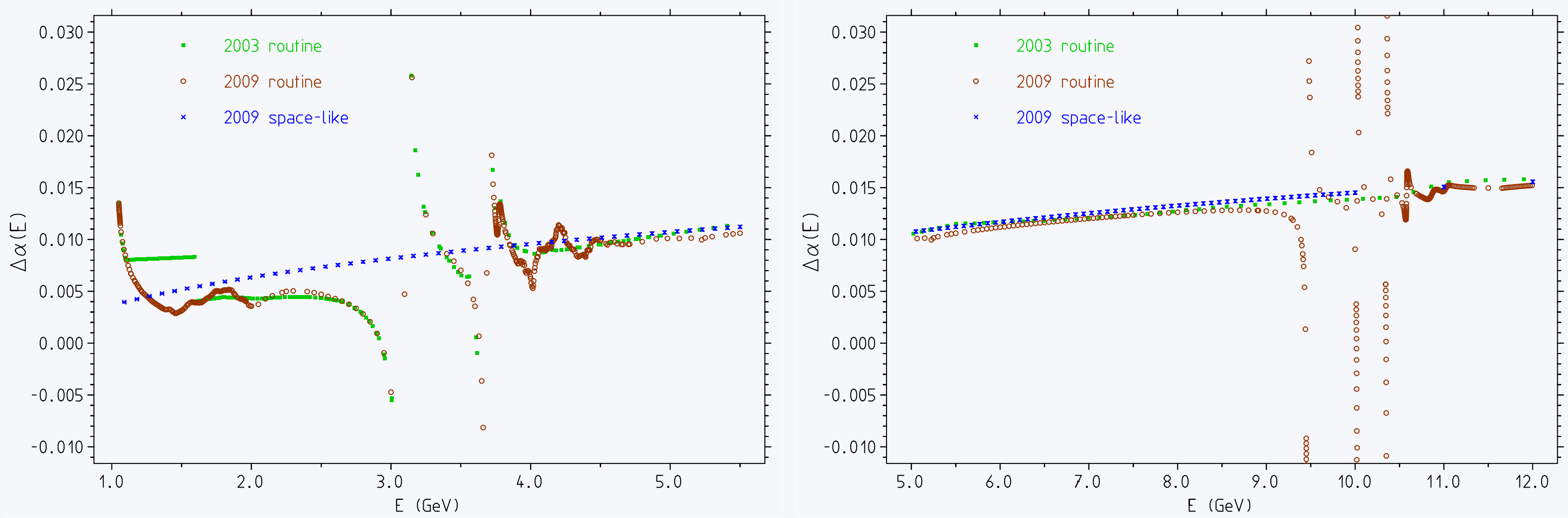
This is much more complex as it requires $R(s)$ in addition to $\alpha(s)$ (usually taken to be real). This requires first to install the `rhad` package written by Harlander and Steinhauser (FORTRAN package version rhad-1.01 (March 2009 issue))

Sample Plots:

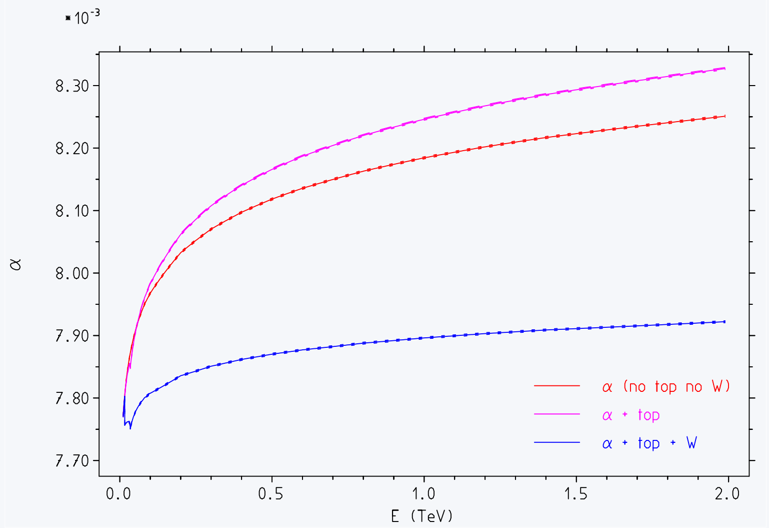




0.030
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
Δα(E)
2003 routine
2009 routine
2009 space-like
0.2
0.4
0.6
0.8
1.0
1.2
1.4
E (GeV)

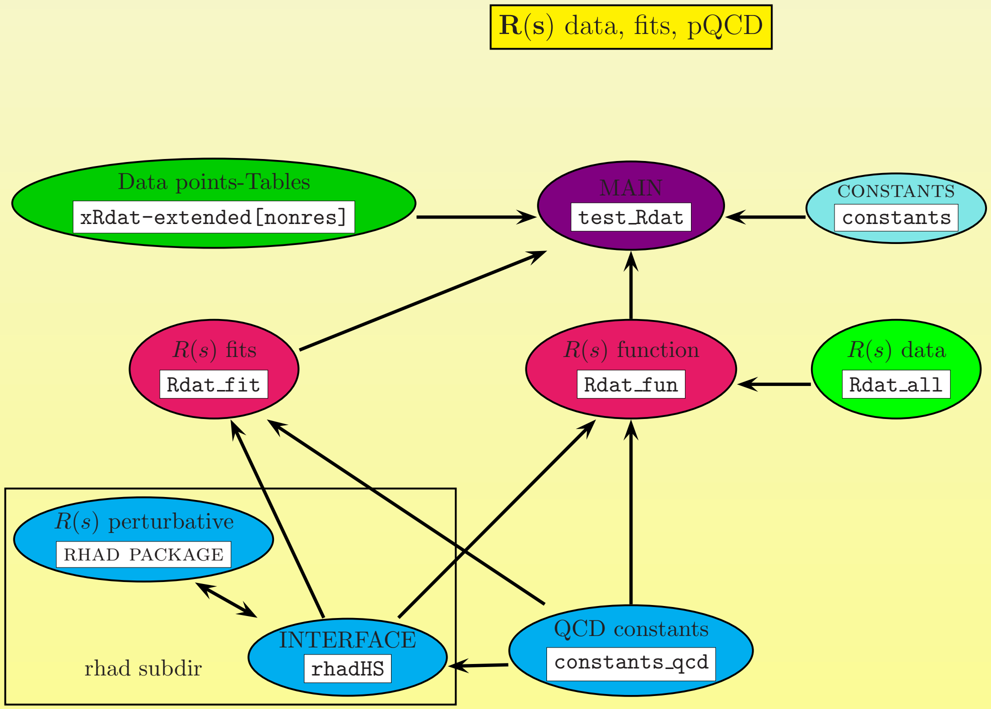


Note that the smooth space-like effective charge agrees rather well with the non-resonant “background” above the Φ (kind of duality)

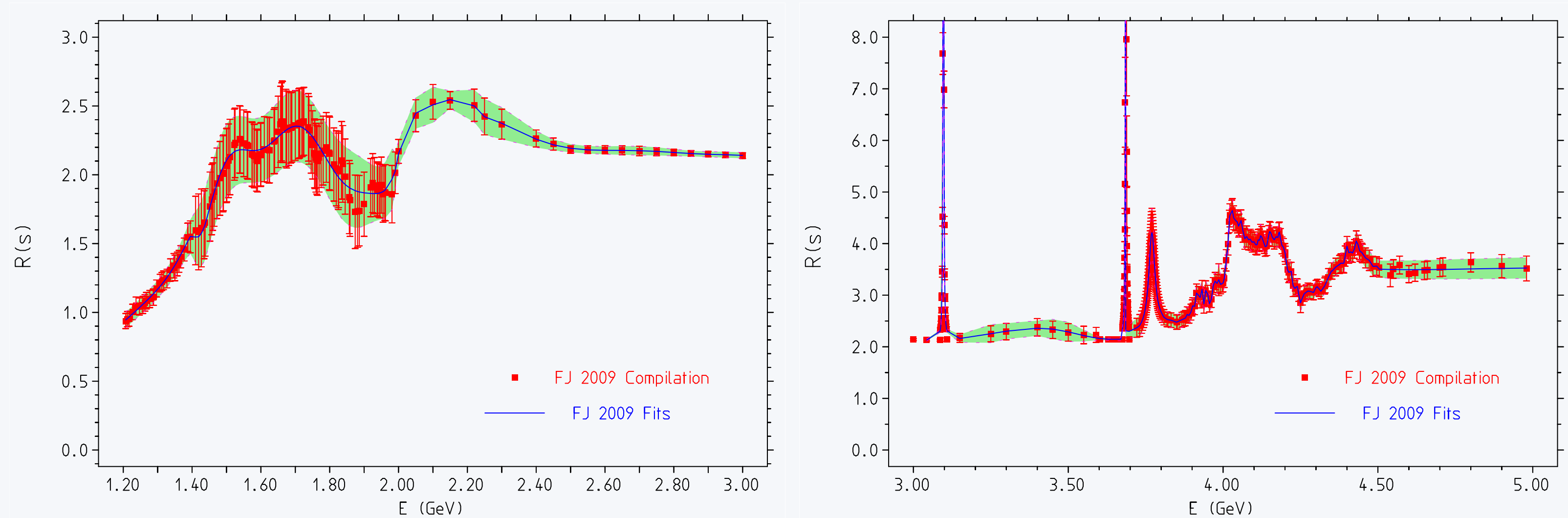


The top and the W-boson 1-loop contributions at high energies.

Sample program `test_Rdat.f` for extracting $R(s)$ data, fits and pQCD calculation.



Sample results:



$R(s)$ $e^+e^- \to$ hadrons data vs. Chebyshev polynomial fits
[no fit for $\psi_3 \dots \psi_6$ region yet]

Complex vs. real α VP correction

- Usually adopted VP subtraction corrections: $\alpha(s) \to \alpha$

  $R(s)$ corrected by $(\alpha/\alpha(s))^2 = |1 - \mathrm{Re}\,\Pi'(s)|^2$ ($\Pi'(0)$ subtracted)

- more precisely, should subtract $|1 - \Pi'(s)|^2 = \alpha/|\alpha_c(s)|)^2$
  where $\alpha_c(s)$ complex version of running α

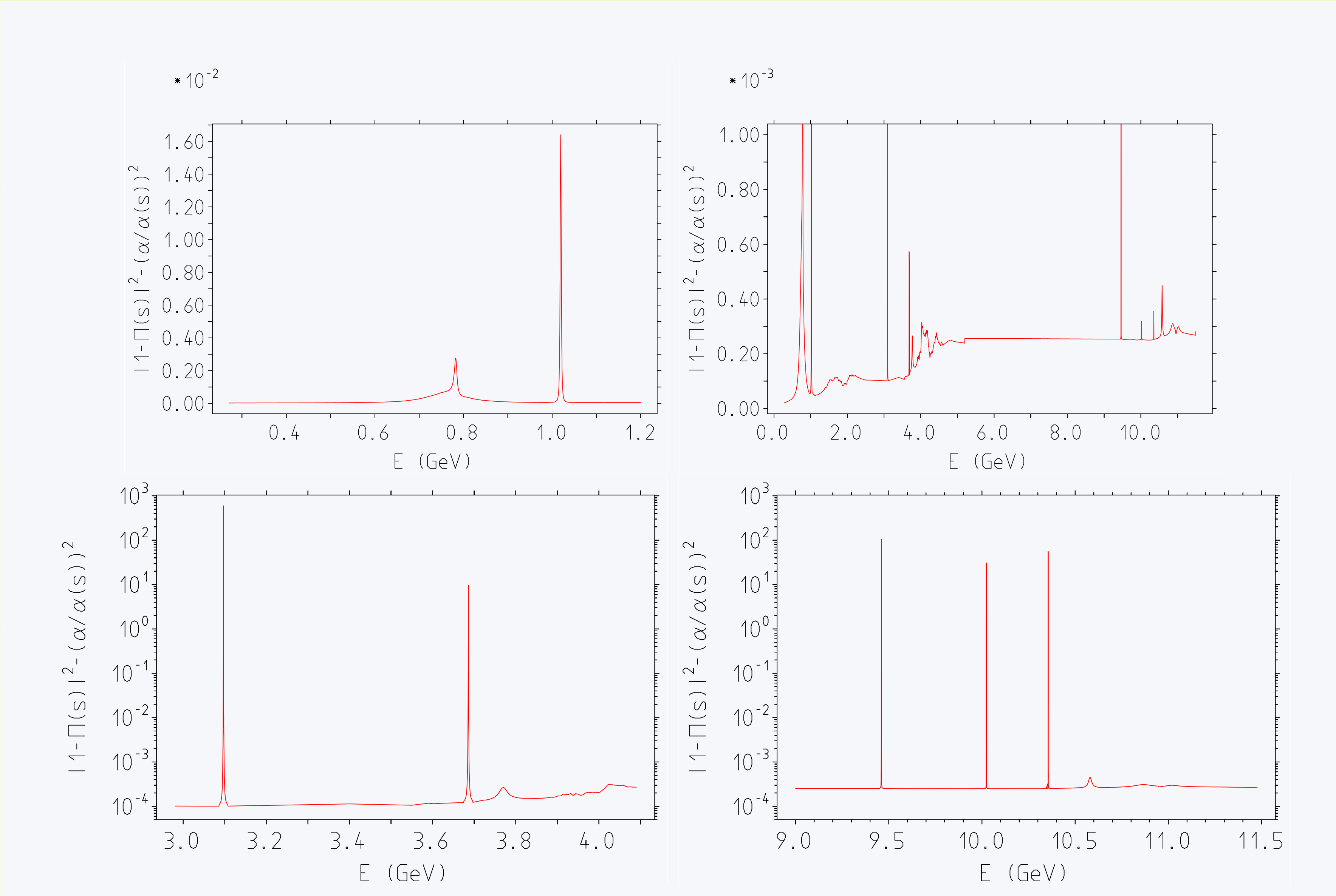
- complex version what the Novosibirsk CMD-2 Collaboration has been using in more recent analyzes [code available from Fedor Ignatov ✲⋙]

- Typically, corrections

$$1 - |1 - \Pi'(s)|^2/(\alpha/\alpha(s))^2$$

❒ non-resonance regions corrections $\lesssim$ 0.1 %

❒ at resonances where corrections $\sim 1/\Gamma_R$



* 10^-2
|1-Π(s)|^2-(α/α(s))^2
E (GeV)
* 10^-3
|1-Π(s)|^2-(α/α(s))^2
E (GeV)
|1-Π(s)|^2-(α/α(s))^2
E (GeV)
|1-Π(s)|^2-(α/α(s))^2
E (GeV)

Note: imaginary parts from narrow resonances, $\mathrm{Im}\,\Pi'(s)) = \frac{\alpha}{3} R(s) = \frac{3}{\alpha} \frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

$$|1 - \Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\mathrm{Im}\,\Pi'(s))^2$$

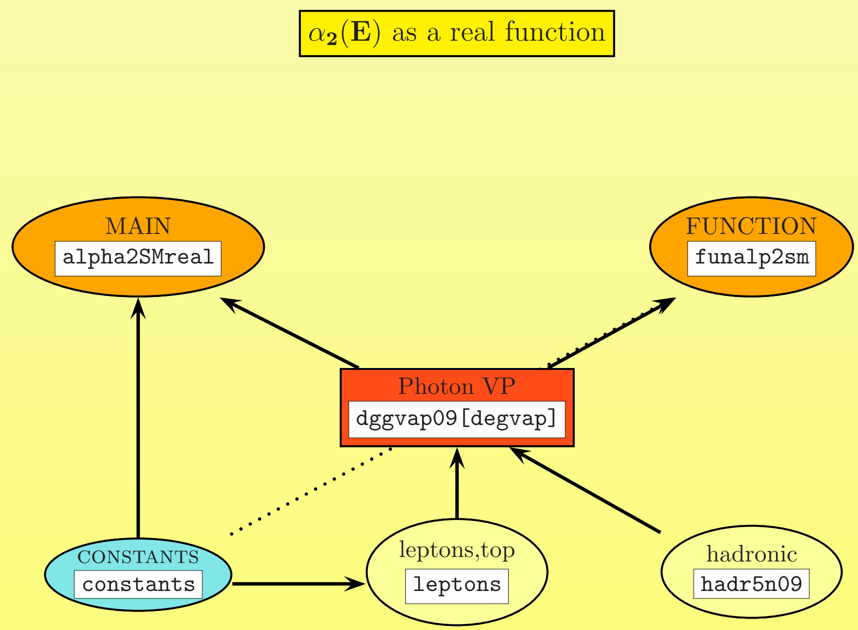
at $\sqrt{s} = M_R$ is given by

1.23×10^{-3} [ρ], 2.76×10^{-3} [ω], 1.56×10^{-2} [ϕ], 594.81 [J/ψ], 9.58 [ψ_2], 2.66×10^{-4} [ψ_3], 104.26 [Υ_1], 30.51[Υ_2], 55.58 [Υ_3]

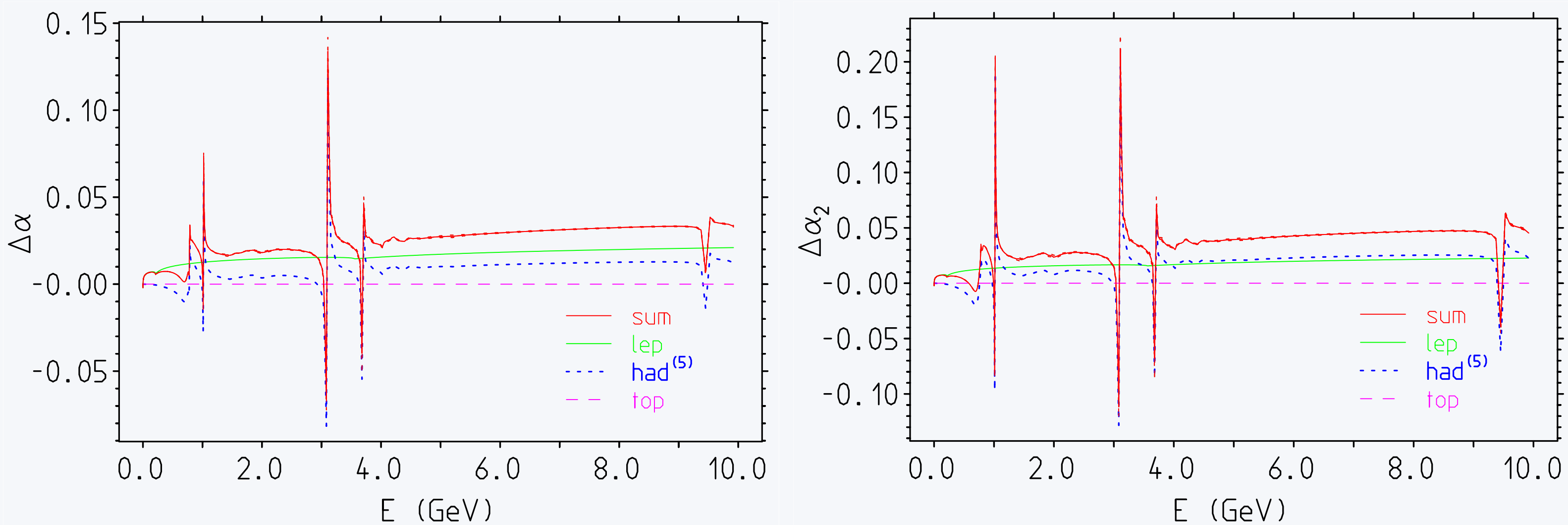
Standard Model $SU(2)_L$ coupling $\alpha_2 = g^2/4\pi$

Non-perturbative hadronic (data-driven) provided by

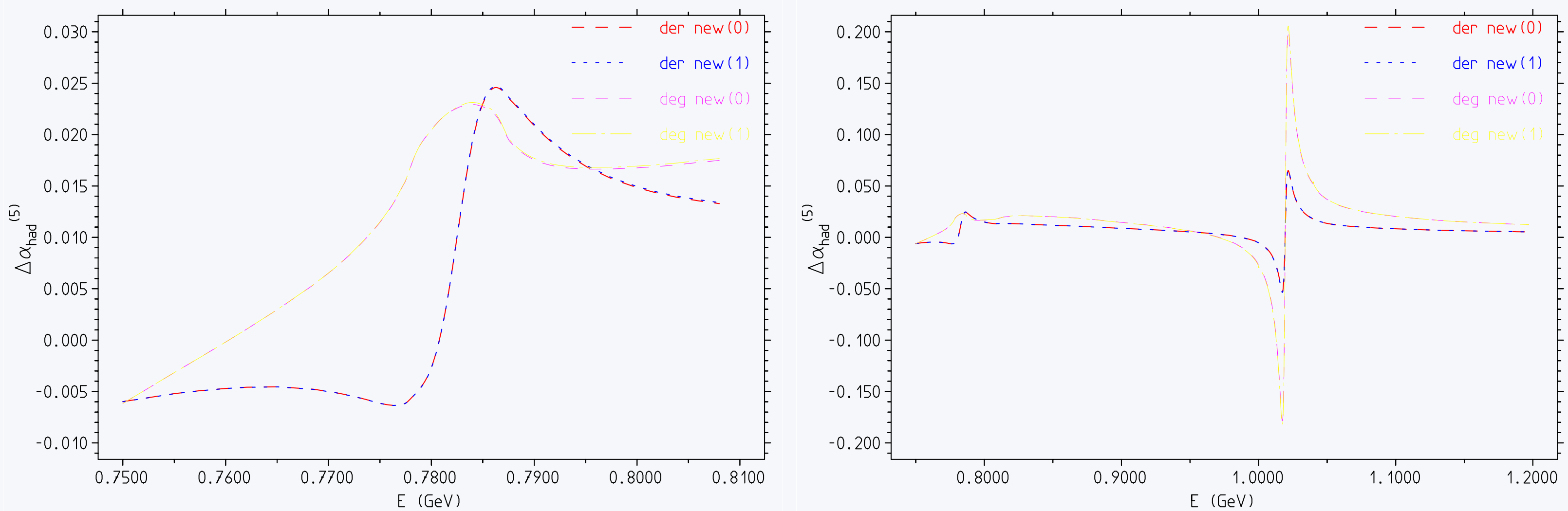
```
call hadr5n09(e,st2,der,errder,deg,errdeg)
```



Sample results:



Comparison of $\Delta\alpha$ and $\Delta\alpha_2$ in the time-like region.



Comparison of $\Delta\alpha$ and $\Delta\alpha_2$ in the ρ and ϕ region. It is the ω contribution, missing in $\Delta\alpha_2$, which produces the characteristic "bump" in $\Delta\alpha$ ($\rho - \omega$ mixing) near the ρ.

Remark: the hadronic shift $\Delta\alpha^{(5)}_{2\,\mathrm{had}}$ is calculated based on e^+e^--annihilation data using the flavor separation procedure proposed and investigated in *Hadronic Contributions to Electroweak Parameter Shifts* Z. Phys. C **32** (1986) 195.

Perspectives to reduce uncertainties in estimates of effective fine structure constant

- experiment side: new more precise measurements of $R(s)$
  see Graziano Venanzoni's talk (next talk)

- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\alpha(M_Z^2) = \alpha(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right] + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]$$
$$= \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

where the space-like $-s_0$ is chosen such that pQCD is well under controlled for

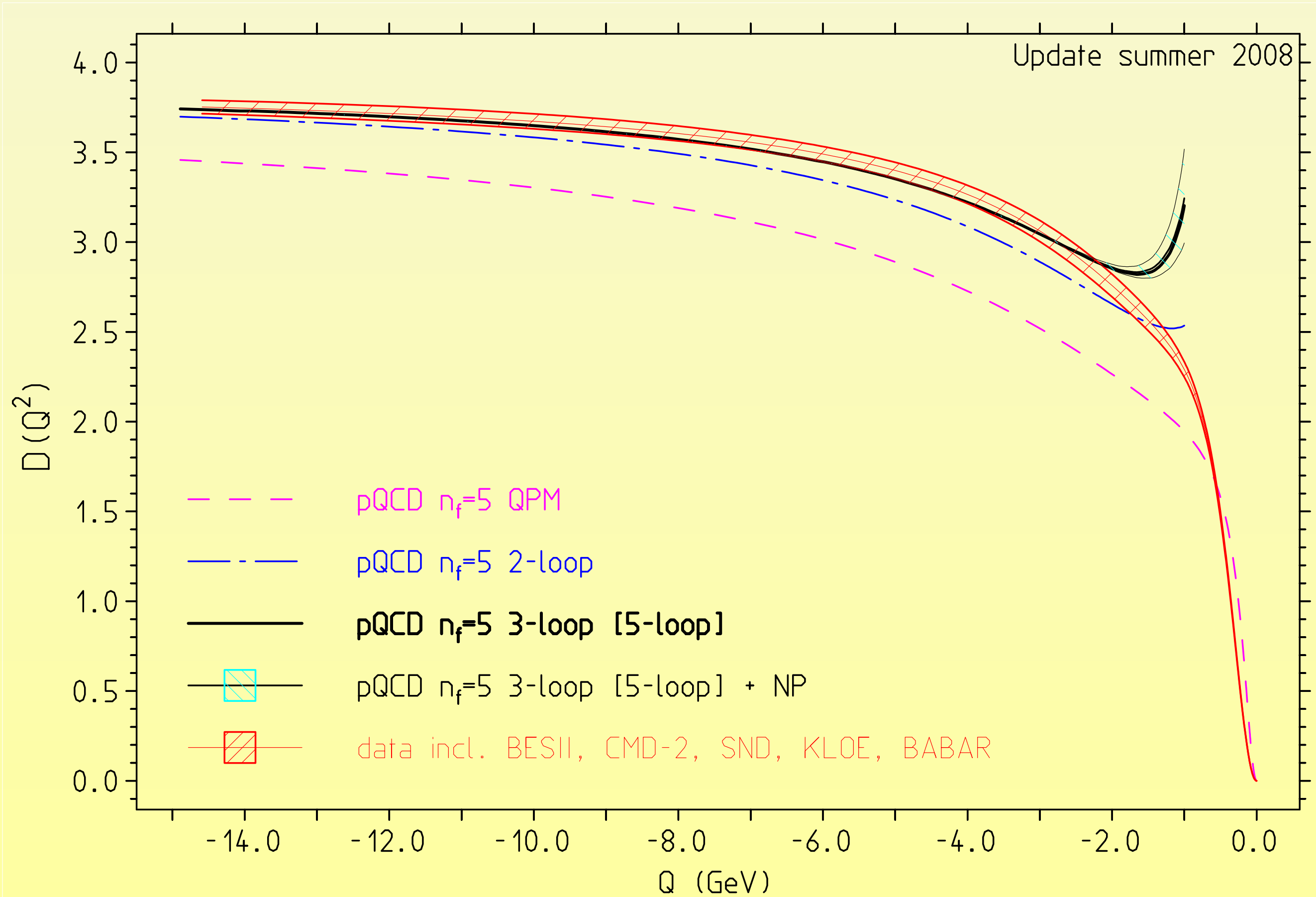
$-s < -s_0$. The monitor to control the applicability of pQCD is the Adler function

$$D(Q^2 = -s) = -(12\pi^2)\, s\, \frac{\mathrm{d}\Pi'_\gamma(s)}{\mathrm{d}s} = Q^2 \int\limits_{4m_\pi^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}$$

which on the one hand for, Q^2 not too small, can be calculated in pQCD on the other hand it can be evaluated non-perturbatively in the standard manner using data at lower energies plus pQCD for perturbative regions and the perturbative tail.

"Experimental" Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F.J., Kataev, Veretin 98, FJ 08 update) theory based on results by Chetyrkin, Kühn et al.

⇒ **pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.5\,\text{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2\,\frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.5\,\text{GeV})^2$: **(FJ 98/10)**

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.007337 \pm 0.000090 \\
\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027460 \pm 0.000134 \\
\Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027498 \pm 0.000135
\end{aligned}$$

- **shift $+0.000008$ from the 5-loop contribution**
- **error ± 0.000103 added in quadrature form perturbative part**

QCD parameters: $\alpha_s(M_Z) = 0.1189(20)$,

$m_c(m_c) = 1.286(13)\ [M_c = 1.666(17)]$ **GeV**,

$m_b(m_c) = 4.164(25)\ [M_b = 4.800(29)]$ **GeV**

based on a complete 3–loop massive QCD analysis (Kühn et al 2007)

Present situation: (after KLOE & BaBar)

$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.027510 \pm 0.000218$$

$$0.027498 \pm 0.000135 \quad \text{Adler}$$

$$\alpha^{-1}(M_Z^2) = 128.961 \pm 0.030$$

$$128.962 \pm 0.018 \quad \text{Adler}$$

The virtues of Adler function approach are obvious:

- no problems with physical threshold and resonances
- pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).
- no manipulation of data, no assumptions about global or local duality.
- non–perturbative “remainder” $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!

Future: ILC requirement: improve by factor 10 in accuracy

- direct integration of data: 58% from data 42% p-QCD

$$\Delta\alpha^{(5)\,{\rm data}}_{\rm had} \times 10^4 = 162.72 \pm 4.13 \quad (2.5\%)$$

  1% overall accuracy ± 1.63

1% accuracy for each region (divided up as in table)
added in quadrature: ± 0.85
Data: [4.13] vs. [0.85] $\Rightarrow$ improvement factor 4.8

$$\Delta\alpha^{(5)\,\mathrm{pQCD}}_{\mathrm{had}} \times 10^4 = 115.57 \pm 0.12 \text{ (0.1\%)}$$

Theory: no improvement needed !

- integration via Adler function: 26% from data 74% p-QCD

$$\Delta\alpha^{(5)\,\mathrm{data}}_{\mathrm{had}} \times 10^4 = 073.61 \pm 1.68 \text{ (2.3\%)}$$

1% overall accuracy ± 0.74
1% accuracy for each region (divided up as in table)
added in quadrature: ± 0.41
Data: [2.25] vs. [0.46] $\Rightarrow$ improvement factor 4.9 (Adler vs Adler)
[4.13] vs. [0.46] $\Rightarrow$ improvement factor 9.0 (Standard vs Adler)

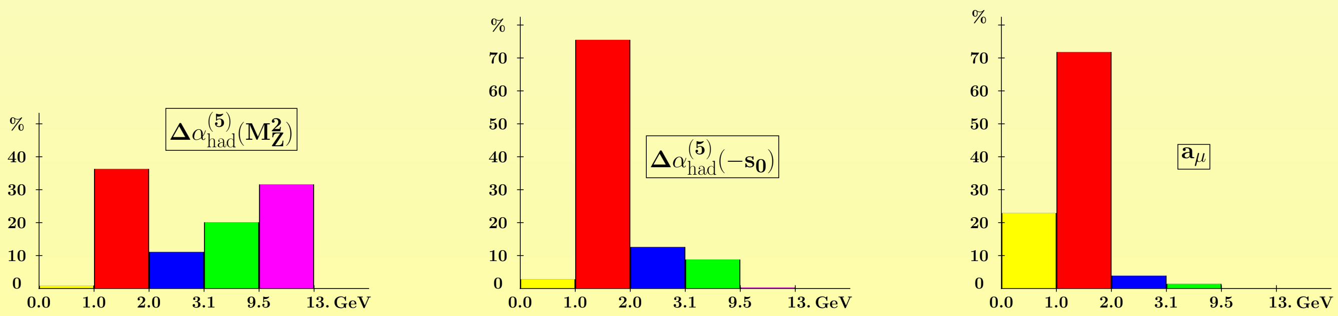
$$\Delta\alpha^{(5)\,\mathrm{pQCD}}_{\mathrm{had}} \times 10^4 = 204.68 \pm 1.49 \text{ (0.7\%)}$$

Theory: (QCD parameters) has to improve by factor 10 ! $\longrightarrow \pm 0.20$

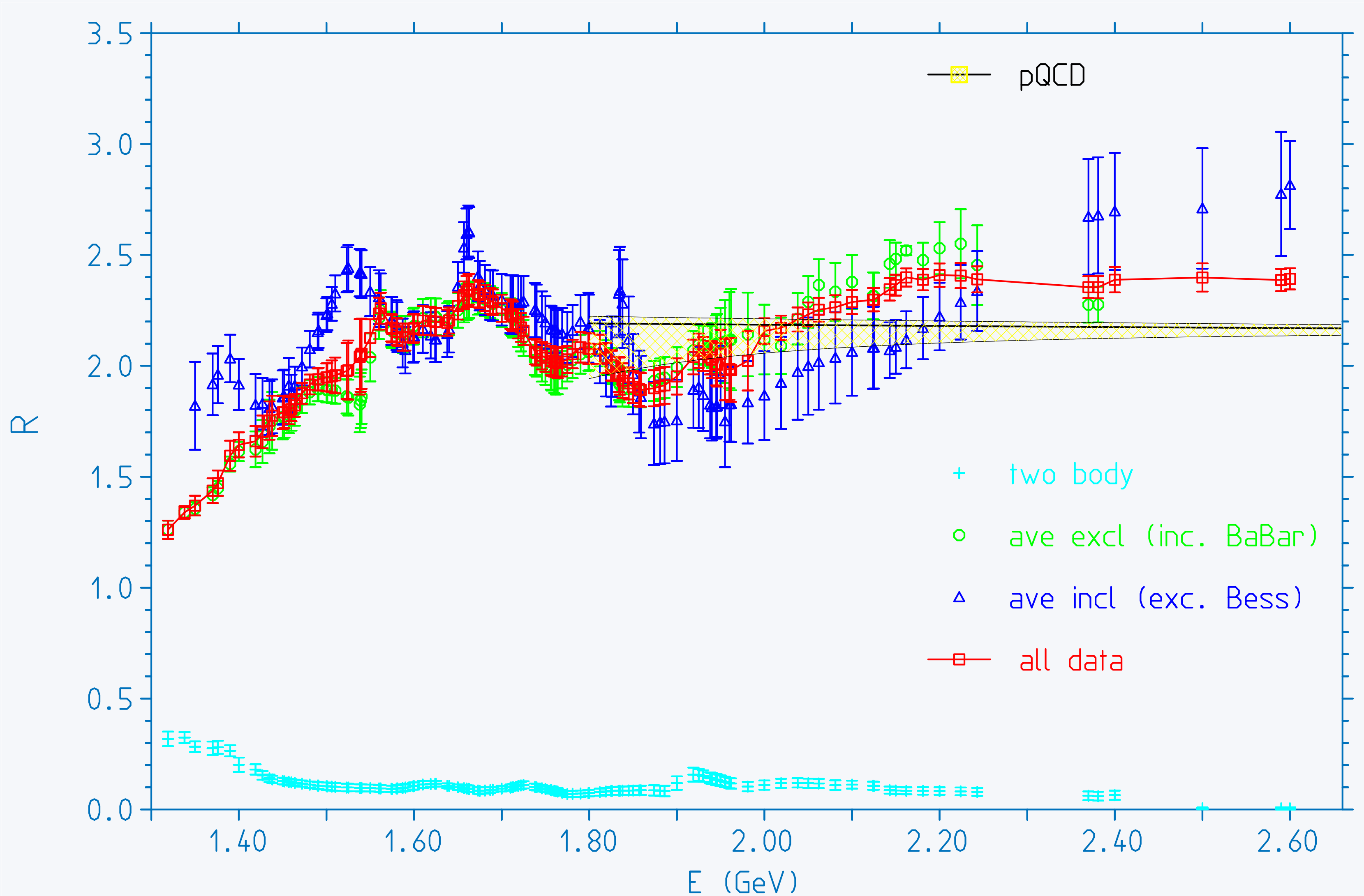
Requirement may be realistic:

- **pin down experimental errors to 1% level in all non-perturbative regions up to 10 GeV**
- **switch to Adler function method**
- **improve on QCD parameters, mainly on m_c and m_b**

DAFNE-2 in conjunction with Adler function approach



Unique chance for DAFNE-2 to improve precision of $\alpha_{\rm eff}(E)$ substantially! In conjunction with improvement of QCD parameters (lattice QCD!). Mandatory for ILC project, but in many other places e.g. $g-2$ of the muon.



pQCD
two body
ave excl (inc. BaBar)
ave incl (exc. Bess)
all data
R
E (GeV)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1.40
1.60
1.80
2.00
2.20
2.40
2.60

DAFNE-2 challenge

- rich and challenging physics program ahead
- key issues: $R(s)$ measurements **inclusive vs exclusive**
- **two photon physics**: urgently need more data to constrain $\pi^0 \to \gamma\gamma$ form-factor (present status BaBar vs CLOE/CELLO confusing)
- urgently need to investigate hadronic **Final State Radiation** mechanism
- one big advantage: $e^+e^- \to$ hadrons clean environment clear answers
- big chance to contribute substantially to precision physics: improving **$\alpha_{\rm eff}$!** If not a factor 5 **a factor 2 is big progress** already now! Example **Higgs bound from $\sin^2\theta_{\rm eff}$** already now a big step forward! Allows to get more out from LEP data a posteriori!

Take the Chance, Do it!

ILC community should actively support these activities
as integral part of LC precision physics!!!
Be engaged!

Remember: tremendous progress since middle of 90's

- Novosibirsk VEPP-2M: MD-1, CMD2, SND
- Beijing BEPC: BES II
- Cornell CESR: CLEO
- Frascati DAFNE: KLOE
- Stanford SLAC PEP-II: BaBar

Many analyzes exploiting these results: Davier et al, Teubner et al., Burkhardt, Pietrzyk, Yndurain et al....

Indispensable for Muon g-2, indirect LEP Higgs mass constraint etc. and future precision test at ILC and new physics signals in precision observables.

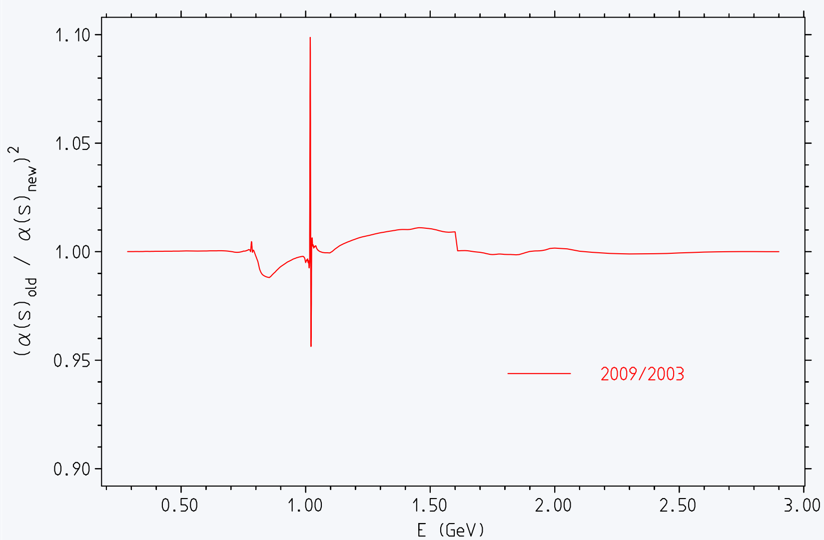
The story must go on!

⋆ Backup slides

How much pQCD?

Method	range [GeV]	pQCD		
Standard approach:	5.2 - 9.5	33.50(0.02)		
My choice	13.0 - ∞	115.69(0.04)	$\rightarrow$	149.19 (0.06)
Standard approach:	2.0 - 9.5	72.09(0.07)		
Davier et al.	11.5 - ∞	123.24(0.05)	$\rightarrow$	195.33 (0.12)
Adler function controlled:	5.2 - 9.5	3.92(0.00)		
	13.0 - ∞	1.09(0.00)		
	$-\infty - -2.5$	201.23(1.03)		
	$-M_Z \rightarrow M_Z$	0.38(0.00)	$\rightarrow$	206.62 (1.03)

Correction factor $(\alpha_{\rm old}(s)\,/\alpha_{\rm new}(s))^2$ applying to $R(s)$ [new routine vs. old one]. The sharp peak is due to the updated ϕ mass and width.



November 2010 Update

Most recent update: incl. KLOE I, BaBar and KLOE II

a_μ:

Energy range	$a_\mu^{\rm had}[\%](\rm error) \times 10^{10}$	rel. err.	abs. err.
ρ, ω $(E < 2M_K)$	540.49 [78.0](2.26)	0.4 %	22.9 %
$2M_K < E < 2$ GeV	102.19 [14.7](4.00)	3.9 %	71.8 %
2 GeV $< E < M_{J/\psi}$	21.63 [3.1](0.93)	4.3 %	3.9 %
$M_{J/\psi} < E < M_\Upsilon$	26.12 [3.8](0.57)	2.2 %	1.4 %
$M_\Upsilon < E < E_{\rm cut}$	1.38 [0.2](0.07)	5.1 %	0.0 %
$E_{\rm cut} < E$ pQCD	1.53 [0.2](0.00)	0.0 %	0.0 %
$E < E_{\rm cut}$ data	691.81 [99.8](4.72)	0.7 %	100.0 %
total	693.34 [100.0](4.72)	0.7 %	100.0 %

final state	range (GeV)	res (stat) (syst) [tot]	rel	abs
ρ	(0.28, 0.99)	503.49 (0.67) (1.86)[1.98]	0.4%	17.4%
ω	(0.42, 0.81)	37.00 (0.44) (1.00)[1.09]	3.0%	5.3%
ϕ	(1.00, 1.04)	35.20 (0.49) (0.81)[0.95]	2.7%	4.0%
J/ψ		8.51 (0.40) (0.38)[0.55]	6.5%	1.4%
Υ		0.10 (0.00) (0.01)[0.01]	6.7%	0.0%
had	(0.99, 2.00)	66.99 (0.22) (3.91)[3.92]	5.8%	68.1%
had	(2.00, 3.10)	21.63 (0.12) (0.92)[0.93]	4.3%	3.8%
had	(3.10, 3.60)	3.77 (0.03) (0.10)[0.10]	2.8%	0.0%
had	(3.60, 9.46)	13.83 (0.04) (0.05)[0.06]	0.4%	0.0%
had	(9.46,13.00)	1.28 (0.01) (0.07)[0.07]	5.4%	0.0%
pQCD	$(13.0,\infty)$	1.53 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	691.81 (1.06) (4.63)[4.75]	0.7%	0.0%
total		693.34 (1.06) (4.63)[4.75]	0.7%	100.0%

Table 1: Results for $a_{\mu\,\mathrm{had}} \times 10^{10}$.

$\Delta\alpha^{(5)}_{\rm had}(-2.5\ \mathrm{GeV})$

Contributions and uncertainties In red the results relevant for VEPP-2000/DAFNE.

Energy range	$\Delta\alpha^{(5)}_{\rm had}[\%](\mathrm{error}) \times 10^4$	rel. err.	abs. err.
$\rho, \omega\ (E < 2M_K)$	33.35 [45.5](0.15)	0.5 %	2.8 %
$2M_K < E < 2$ GeV	16.46 [22.4](0.79)	4.8 %	75.5 %
2 GeV $< E < M_{J/\psi}$	7.72 [10.5](0.32)	4.2 %	12.6 %
$M_{J/\psi} < E < M_\Upsilon$	13.81 [18.8](0.27)	1.9 %	8.8 %
$M_\Upsilon < E < E_{\rm cut}$	0.95 [1.3](0.05)	5.1 %	0.3 %
$E_{\rm cut} < E$ pQCD	1.09 [1.5](0.00)	0.0 %	0.0 %
$E < E_{\rm cut}$ data	72.28 [98.5](0.90)	1.3 %	100.0 %
total	73.37 [100.0](0.90)	1.2 %	100.0 %

final state	range (GeV)	res (stat) (syst) [tot]	rel	abs
ρ	(0.28, 0.99)	30.69 (0.04) (0.12)[0.13]	0.4%	2.0%
ω	(0.42, 0.81)	2.66 (0.03) (0.07)[0.08]	3.0%	0.7%
ϕ	(1.00, 1.04)	4.00 (0.06) (0.09)[0.11]	2.7%	1.4%
J/ψ		3.95 (0.19) (0.18)[0.26]	6.6%	8.2%
Υ		0.07 (0.00) (0.00)[0.00]	6.7%	0.0%
had	(0.99, 2.00)	12.45 (0.05) (0.78)[0.78]	6.3%	74.4%
had	(2.00, 3.10)	7.72 (0.04) (0.32)[0.32]	4.2%	12.4%
had	(3.10, 3.60)	1.77 (0.01) (0.05)[0.05]	2.8%	0.3%
had	(3.60, 9.46)	8.09 (0.02) (0.03)[0.04]	0.5%	0.2%
had	(9.46,13.00)	0.88 (0.01) (0.05)[0.05]	5.5%	0.3%
pQCD	(13.0,∞)	1.09 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	72.28 (0.21) (0.88)[0.91]	1.3%	0.0%
total		73.37 (0.21) (0.88)[0.91]	1.2%	100.0%

Table 2: Results for $\Delta\alpha^{(5)}_{\rm had}(-2.5\ \mathrm{GeV}) \times 10^4$.

$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z)$

Energy range	$\Delta\alpha^{(5)}_{\mathrm{had}}[\%](\mathrm{error}) \times 10^4$	rel. err.	abs. err.
$\rho, \omega\ (E < 2M_K)$	36.29 [13.2](0.17)	0.5 %	0.8 %
$2M_K < E < 2$ GeV	21.73 [7.9](1.11)	5.1 %	36.3 %
2 GeV $< E < M_{J/\psi}$	15.33 [5.6](0.62)	4.0 %	11.1 %
$M_{J/\psi} < E < M_{\Upsilon}$	66.56 [24.2](0.83)	1.2 %	20.1 %
$M_{\Upsilon} < E < E_{\mathrm{cut}}$	19.49 [7.1](1.04)	5.3 %	31.6 %
$E_{\mathrm{cut}} < E$ pQCD	115.69 [42.1](0.04)	0.0 %	0.0 %
$E < E_{\mathrm{cut}}$ data	159.41 [57.9](1.85)	1.2 %	100.0 %
total	275.10 [100.0](1.85)	0.7 %	100.0 %

final state	range (GeV)	res (stat) (syst) [tot]	rel	abs
ρ	(0.28, 0.99)	33.37 (0.05) (0.13)[0.14]	0.4%	0.6%
ω	(0.42, 0.81)	2.92 (0.04) (0.08)[0.09]	3.0%	0.2%
ϕ	(1.00, 1.04)	4.67 (0.07) (0.11)[0.13]	2.7%	0.5%
J/ψ		11.14 (0.53) (0.58)[0.79]	7.1%	18.1%
Υ		1.18 (0.05) (0.06)[0.08]	6.9%	0.2%
had	(0.99, 2.00)	17.06 (0.07) (1.11)[1.11]	6.5%	36.1%
had	(2.00, 3.10)	15.33 (0.08) (0.61)[0.62]	4.0%	11.1%
had	(3.10, 3.60)	4.93 (0.03) (0.13)[0.14]	2.8%	0.5%
had	(3.60, 9.46)	50.49 (0.11) (0.20)[0.22]	0.4%	1.4%
had	(9.46,13.00)	18.32 (0.24) (1.01)[1.04]	5.7%	31.2%
pQCD	(13.0,∞)	115.69 (0.00) (0.04)[0.04]	0.0%	0.0%
data	(0.28,13.00)	159.41 (0.61) (1.75)[1.85]	1.2%	0.0%
total		275.10 (0.61) (1.75)[1.85]	0.7%	100.0%

Table 3: Results for $\Delta\alpha^{(5)}_{\rm had}(M_Z) \times 10^4$.