Approximating Radio Maps

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Abstract

Given a terrain T and an antenna A located on it, we would like to approximate the *radio map* of A over T, namely, to associate the signal strength of each point $p \in T$ as received from A. Several facility location algorithms, which involve locating large scale wireless networks ($WiMAX$), use approximated *radio maps* algorithms. In fact, computing *radio maps* is often the runtime bottle-neck of such facility location algorithms. This article suggests a new radar-like algorithm (RLA) for approximating *radio maps*. We also report on experiments performed to compare between the suggested new algorithm, and other well-known methods. The main conclusion is that the new algorithm (RLA) is significantly better than the others, i.e. its running time is 3-5 times faster for the same approximation accuracy.

1 Introduction

The rapid development in wireless communication technology has dramatically reduced costs and increased the number of users. This development has increased the demand for wireless communication while supporting better throughput and quality of service. To withstand the increasing demand, more antennas should be installed on urban and rural regions. However, the cost of installing antennas accounts for a significant fraction of the wireless communication hardware system. For that reason, communication companies are interested in installing a minimal number of antennas that provide the required throughput and quality of service. Solving such optimization problems, requires efficient methods to approximate *radio maps*. In addition, recently new wireless broadband technologies (namely $WiMAX$ or 802.16) can support 50-kilometer connections and more. Meaning, single antenna can cover large area of sparse population. Hence accurate *radio maps* approximations are required for optimizing such antenna location.

The problem of approximating *radio map* of an antenna over a terrain was studied extensively [4, 9]. This problem is a generalization of the well known problem of computing the visible regions from a point p over a terrain [1, 6]. A point p is either visible or not from a viewpoint, while the signal strength at a point p (sent from an antenna A) is a real value that is base on the visibility, distance, and terrain between p and A. Computing *radio maps* is often the runtime bottle-neck of wireless facility location algorithms as a result of its high computational complexity. For that reason, it is desirable to develop fast approximation algorithms.

In this paper we present a new approach to approximate *radio maps* which is based on a radar-like radial sweep-line algorithm centered at the antenna position. Our experimental results show that the new radar-like approach is significantly faster than existing methods.

In the rest of this paper, we first overview the basis of RF (diffraction) models for predicting a signal strength at a point followed by several sampling algorithms for predicting the signal strength at every point in a region i.e. approximating *radio map*. Then we present the pipeline algorithm to compute a set of samples along a ray followed by the new RLA for approximating the *radio map*. Next we present a large-scale experiment comparing the new radar-like method with previous *radio map* approximations. Finally we draw some conclusions and directions for future work.

2 Background and Related works

In this section, we briefly overview RF (diffraction) models and the framework for predicting the signal strength for every point in a region.

2.1 Computing the signal strength at a point

Radio map approximation algorithms are based on models that predict the signal strength at a query point on a terrain, i.e. given a terrain T, an antenna position A and a receiver at R (on T), predict the signal strength coming from A at R. RF propagation models[1] for large scale rural areas usually use the following frame work to predict the signal strength at R: (i) Compute the *cross-section* between A and R by projecting the terrain points between A and R on a 2D plane where X-value is the distance from A and Y-value is the height. (ii) Simplify the *cross-section* to a small (usually constant) set of points. This set often includes the dominant points of the *cross-section* convex hull. (iii) Use the simplified *cross-section* to approximate the signal strength at R.

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[1] This paper only addresses RF models based on *cross-section* for large scale rural regions and not urban RF models.

In this paper we assume the use of Single knife edge (Bullington), Multiple knife edge (Epstein and Peterson, Deygout) propagation models. For detailed description and formulation of these models we refer the interested readers to [4].

2.2 Computing a radio map using samples

Several approaches have used various sampling techniques to approximate *radio map* over the surface of a given terrain. These samples, which we denote as the *sample set* (SP), is selected randomly, grid-based points, or the points survive a terrain simplification algorithm [2, 7] (these points, usually fairly represent the original terrain). Acceptable sample sets SP usually consists of dominant points (points remain after simplification) as well as some additional grid points to ensure flat regions are not under sampled. This is done because the wave propagation is not linear and its behavior may be difficult to predict.

Upon the selection of the sample set the sampling based approaches compute the signal strength using RF propagation model (see [4]) at each sample point and the signal strength at any other point (on the terrain surface) is computed by extrapolating adjacent samples. A common method to approximate the signal strength over a region (represented by SP) is to triangulate the points in SP using their XY-coordinates. Given a query point q, find the triangle t on which q resides. The signal strength of q is given by the intersection value (Z – signal) of q and the 3D plane represented by t.

3 Our Approach

Motivated by radar-like visibility algorithms [1], we have developed a novel approach to approximate *radio maps* over a terrain. Our approach simulates a radar scanning scheme using an efficient pipeline technique to compute sample-points and signal strength along a radar ray that represents a *cross-section* of the terrain. We will first overview our efficient pipeline technique.

3.1 Pipeline signal computation

Computing the *cross-section* between the receiver and the transmitter and its convex hull is an essential stage of typical RF model prediction algorithms. The time complexity of these operations is not small. Furthermore, these operations are performed many times over the execution of the approximation algorithm. For that reason, these operations determine the complexity of any RF model implementation. Therefore, reducing the complexity of these two operations could directly accelerate computing the signal strength over the given terrain. This discussion has motivated the development of the pipeline technique.

The main idea is based on computing several sample points along a single *cross-section*. For a given transmitter A and receiver B, instead of just computing the signal strength at B (coming from A), with just a minor overhead, the signal strength at other points along the *cross-section* AB could be computed. Our pipeline method is based on efficient 2D-terrain simplification heuristics that we will described next.

Several approaches to measure the distance between two x-monotone polygonal chains $S_1 = [u_1, \ldots, u_n]$ and $S_2 = [v_1, \ldots, v_m]$ have been developed. We have chosen to use the following distance metrics:

- **Maximal Vertical Distance** $= max(d_1, d_2)$ where d_1=max($\{\text{dist}(u_i, S_2) : i \in \{1, \ldots, n\}\}$ and d_2=max($\{\text{dist}(v_i, S_1) : i \in \{1, \ldots, m\}\}$
- **Average Distance** $= \frac{1}{|S|} \cdot (A(S_1 \cup S_2) - A(S_1 \cap S_2))$ where $A(S_1 \cup S_2)$ is the area below one of the chains, $A(S_1 \cap S_2)$ is the area below both the chains, and $|S|$ is the average length of the chains.
- **Root Mean Square** resembles *average distance*, with more sensitivity to large vertical distances [3].

Several versions of finding the smallest $\epsilon - bound$ approximation of a 2D-terrain were proved to be NP-hard [5], other versions of this problem have polynomial optimal algorithms [5] but their runtime is impractical for actual applications. Therefore, many heuristics were suggested, which may not be optimal. However, these heuristics are often very efficient and provide acceptable approximations.

We have adopted a sample-budget based simplification algorithm to select sample points from a give *cross-section*. One part of the sample points is selected uniformly over the entire *cross-section* and the rest of the sample points are selected using the following subdivision algorithm. The subdivision algorithm starts with the two extreme points of the *cross-section* and adds sample points in an iterative manner. At each iteration it adds the point which has the maximal vertical distance from the currently selected sample. The algorithm stops when it reaches the predefined budget. Note that the simplified chain contains the convex-hull of the original chain.

All error metrics mentioned earlier, could be used to define the error distance between a *cross-section* and its simplification. however, it seems that the signal strength is highly correlated with the visibility (or angle of blocking), and trying to minimize the maximal vertical error leads to the best signal strength approximation.

3.2 The Radar-Like Algorithm

In this section we present the radar-like generic algorithm and a measure of resemblance required to transform the generic algorithm into a RLA.

Let *signal-section*(T, p, θ) be the signal strength over some sampling set of points along the ray emanating from p and forming an angle θ with the positive x-axis. Namely, *signal-section*(T, p, θ) computes a simplified *signal-section* , represented by the projections of the signal portions of a sampling set SP over a *cross-section* of T in the direction specified by this ray.

The generic algorithm first sweeps the terrain T clockwise with a fix rotation angle δ_θ. For each rotation angle (θ) it computes and stores the *signal-section*(T, p, θ). Then, it iteratively finds the two most distant consecutive *signal-sections* and refine them using an intermediate *signal-section*. The pseudo-code of the generic algorithm is presented in the frame below.

```
Given a triangulation T representing a terrain
and an antenna location p (on or above T):

θ ← 0
δ_θ ← some constant small angle
S_1 ← signal-section(T, p, θ)
PQ ← new priority queue
while (θ < 360)
   S_2 ← signal-section(T, p, θ + δ_θ)
   Pz ← pizza-slice(S_1, S_2)
   add Pz to PQ
   S_1 ← S_2
   θ ← θ + δ_θ
i ← 0
while (i ≤ BUDGET)
   Pz ← the most distant pizza-slice in PQ
   compute a middle-angle signal-section (cutting Pz)
   divide Pz into two pizza-slices: Pz_1, Pz_2
   remove Pz and add Pz_1 and Pz_2 from/to PQ
   i = i + 1
```

In the algorithm above **pizza slice** refers to two consecutive *signal-sections* (see Figure 1), the distance associated with a *pizza slice* is the *distance* between the two *signal-sections* composing it. The *distance* between the two consecutive *signal-sections* S_1 and S_2 is computed by one of the distance metrics suggested in section 3.1. This *distance* value is then multiplied by δ, the angle difference between S_1 and S_2. The role of δ here is to correlate between the area and the distance of a *pizza-slice*.

We define **Fixed radar** as an RLA of zero budget, **Adaptive radar** as an RLA of a significant size budget, and **Advanced radar** as a fine tuned RLA. Fine-tuned RLA means that the $BUDGET$, δ_θ, and parameter which influence the *signal-section* are adjusted to generate better approximation.

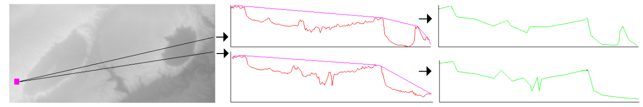

Figure 1: Two consecutive *cross-sections* (*Pizza-slice*), are simplified using the max-vertical metric, that reduces the number of vertices in the 2D-terrain from approximated 300 vertices to 20.

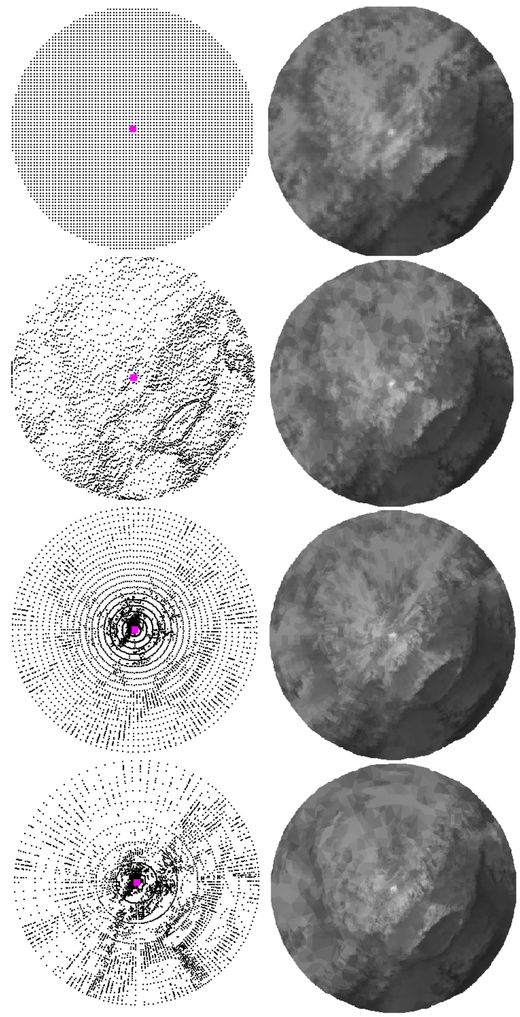

Figure 2: *Radio-maps* as computed by the various methods: 5000 samples each (high to low): grid, terrain simplification, fixed radar and sensitive radar. The corresponding extrapolation is given to the right of each sample-set. The antenna is in the center and the brightness represents the strength of the signal. Observe that sensitive radar (lowest) has the best edge detection.

4 Experimental Results

We have implemented our algorithm using Java over Windows XP. Then, have performed several tests on various datasets using our unoptimized implementation and have received encouraging results. In addition, we have tested the performance and results of our implementation against different *radio map* approximation algorithms. We have compared basic sampling algorithms that include grid, random, and simplification based sampling with the three version of the radar-like algorithms– fixed, sensitive, and advanced. All *RF* propagation models were tested and calibrated according to Hexagon NIR application [8].

The error of the *radio map* $RM(T, A)$ is approximated using a random sample set S that satisfies :
$RM(T, A, S) = \frac{1}{|S|} \cdot \sum_{s_i \in S} |sig(T, A, s_i) - RM(T, A, s_i)|$
where $sig(T, A, s)$ is the exact signal at s, and $RM(T, A, s)$ is the approximated signal at s (using the *radio map* $RM(T, A)$) and $1000 \leq |S| \leq 5000$.

4.1 Experiment details

The experiment consisted of 17 different high resolution elevation maps representing various terrain types (i.e. flat, hills, mountains, lakes, dunes etc'). Each map represents rectangular area of 100×100 km^2, and includes 10^6 vertices. For each terrain 50 random antenna locations were chosen. For each viewpoint p, the six approximation algorithms were applied 15 times using a combination of the heights 10, 20, and 50 meters above the surface of T and radius of 5, 10, 15, 20, and 30 km. For each approximated *radio map* the associated error was computed according to the above-mentioned error measures. In addition, we have performed the tests for various sample sizes: 1000, 2500, 5000, 10000, 20000. These experiments were conducted using a PC machine with AMD Sempron 1.6GHz CPU and 512MB memory running Windows XP.

4.2 Results summary

Part of our experimental results are reported in this section. Table 1 shows the average runtime, in milliseconds, for each of the six methods as a function of the sampling size (*ps*). Table 2 presents the average error ratio for a given sample size (normalized to the Random method results).

Ave Time	ps=1000	ps=2500	ps=5000	ps=10000	ps=20000
Random	58.1	144.5	287.3	569.8	1127.4
Grid	56.4	137.1	275.9	547.7	1087.3
TS	58.2	144.0	287.8	570.2	1130.5
F-Radar	19.2	46.6	87.1	184.8	392.1
S-Radar	20.6	47.1	89.8	191.2	406.6
A-Radar	20.8	47.3	90.2	191.9	408.2

Table 1: Average runtime (milliseconds) for constructing a *radio map* of radius 10 km.

Ave Error	ps=1000	ps=2500	ps=5000	ps=10000	ps=20000
Random	1.00	1.00	1.00	1.00	1.00
Grid	0.98	0.97	0.96	0.96	0.97
TS	0.92	0.91	0.9	0.89	0.9
F-Radar	1.01	1.00	1.01	1.02	1.01
S-Radar	0.95	0.93	0.91	0.91	0.93
A-Radar	0.91	0.89	0.88	0.88	0.90

Table 2: Average error size (normalized to the random method) of each *radio map* method (of radius 10 km).

5 Conclusions and future work

We have presented a new approach to approximate *radio maps* that is based on a new *RLA*. In addition, we have shown that our new approach provides better results, in terms of accuracy and efficiency, than the basic sampling methods (grid, random, and terrain simplification). For any sample size and range, computing *radio maps* using our *RLA* is 3 to 5 times faster than other methods, and yet *radio maps* computed by *RLA* were (on average) more accurate.

Further research may include additional heuristics and fine-tuning of the ones suggested here. In particular the *RLA* property of having radial order over the *pizza-slices* can be used to compute an alternative extrapolation method (avoid computing a triangulation).

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