Contents

ISBN 978-1-84890-253-4

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9 781848 902534

IFCoLog

9 781848 902534

7.44 x 9.69
246 mm x 189 mm

Volume 4 ● Issue 8 ● September 2017

The IfCoLog Journal of Logics and their Applications

1.673
42.49mm

ISSN PRINT 2055-3706
ISSN ONLINE 2055-3714

The IfCoLog Journal of Logics and their Applications

Volume 4 ● Issue 8 ● September 2017

Special Issue Formal Argumentation

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7.44 x 9.69
246 mm x 189 mm

Content Type: Black & White
Paper Type: White
Page Count: 844
File Type: InDesign
Request ID: CSS2236031

Perfect Bound Cover Template

Lightning Source

Document Size: 19" x 12"
305 x 483mm

IFCoLog Journal of Logics and their Applications

Volume 4, Number 8

September 2017

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ISBN 978-1-84890-253-4
ISSN (E) 2055-3714
ISSN (P) 2055-3706

College Publications
Scientific Director: Dov Gabbay
Managing Director: Jane Spurr

http://www.collegepublications.co.uk

Printed by Lightning Source, Milton Keynes, UK

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Contents

ARTICLES

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This special issue contains the journal version of thirteen contributions to the first volume of the Handbook of Formal Argumentation (HOFA), which will appear at College Publications. The HOFA initiative aims at producing a series of volumes providing a comprehensive coverage of both the state of the art and future research perspectives in the lively interdisciplinary field of formal argumentation. It is meant to be an open community effort and a service to current and future students and researchers interested in this field.

Some authors changed the title in the journal version compared to their article in the handbook when they felt this appropriate. Other papers kept the same title. We invite the readers to buy the forthcoming HOFA, which consists of 19 chapters, for a full view. Please visit the website for more information and feel free to send us comments, suggestions and proposals. http://formalargumentation.org/

The articles in this special issue and the handbook series reflect the development of formal argumentation theory in the last decades, with a special emphasis on the role played by the theory of abstract argumentation introduced by Dung in 1995. The graph based framework and language introduced by Dung constitutes a turning point for the modern stage of formal argumentation theory.

It gave rise to many further developments both in theory and in application and should be a focal point of reference for any study of argumentation, even if (especially if) it is critical about it. The contributions in this special issue highlight the main innovations of this new stage of formal argumentation theory. Dung's graph based theory is integrated within structured approaches using abstract rules and assumptions, and extensions of the graph based representation have been studied as abstract dialectical frameworks. Argumentation as inference developed by Dung has been complemented by argumentation as dialogue, based on argumentation semantics as formal discussion, and argumentation schemes. In addition, computational problems have been studied, including their complexity, and implementations have been built. Formal analysis is based on a principle based approach to formal argumentation, including the use of rationality postulates to evaluate argumentation semantics. The relations between formal argumentation and other areas of formal reasoning, in particular logic, has been studied.

The articles in this special issue give a survey of the area and may also contain a more personal view. For the survey part, at least the work reported in the COMMA conference series is discussed. Instead of just a historical overview, which is restricted to the first two articles, the authors also address new developments, open topics and emerging areas. We appeal to all disciplines, including logic, computer science, law, philosophy, and linguistics. Maybe the most pressing question is how this theory of formal argumentation, developed from the area of non-monotonic logic and artificial intelligence, can be used as the foundations for informal argumentation in areas such as linguistics and law. Future volumes of the handbook series will consider extensions of Dung's theory, including numerical ones, dynamics and update, dialogue, and applications, for example in artificial intelligence, computer science, linguistics or legal reasoning.

Received 26 September 2017

Argumentation Theory in Formal and Computational Perspective

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Abstract

Argumentation has been studied since Antiquity. Modern argumentation theory took inspiration from these classical roots, with Toulmin's 'The Uses of Argument' (1958) and Perelman and Olbrechts-Tyteca's 'The New Rhetoric' (1969) as representants of a neo-classical development. In the 1970s, a significant rise of the study of argumentation started, often in opposition to the logical formalisms of those days that lacked the tools to be of much relevance for the study of argumentation as it appears in the wild. In this period, argumentation theory, rhetoric, dialectics, informal logic, and critical thinking became the subject of productive academic study. Since the 1990s, innovations in artificial intelligence supported a formal and computational turn in argumentation theory, with ever stronger interaction with non-formal and non-computational scholars. The present article sketches argumentation and argumentation theory as it goes back to classical times, following the developments before and during the currently ongoing formal and computational turn.

1 Introduction

Argumentation has been studied since Antiquity. Several 20$^{\text{th}}$ century developments in the study of argumentation (in particular since the 1950s) were initiated by concerns that the formal methods of the time, especially classical formal logic, were not fully adequate for the study of argumentation. In recent years, such concerns have been addressed, and partially answered, using innovations in formal and computational methods, in particular in computer science and in artificial intelligence.

We can speak of a formal and computational turn in the study of argumentation. This article sketches argumentation and argumentation theory as it goes back to classical times, following the developments before and during the currently ongoing formal and computational turn. While doing so, we explain what the study of argumentation, generally known as *argumentation theory*, involves. Our exposé is based on the *Handbook of Argumentation Theory* that we recently co-authored with Bart Garssen, Erik C.W. Krabbe, A. Francisca Snoeck Henkemans and Jean Wagemans [Amgoud *et al.*, 2008, in particular Chapters 1 and 11].[1]

In Section 2, 'Argumentation and argumentation theory before the formal and computational turn', we define argumentation in the way this concept has been used in argumentation theory before the formal and computational turn; starting from this definition we explain what argumentation theory is about and describe its main aims. We introduce crucial concepts that play a major role in argumentation theory, and give an overview of prominent theoretical approaches. In Section 3, 'Formal and computational argumentation theory: precursors and first steps', we start the discussion of formal and computational approaches to argumentation by addressing precursors and first steps made, in particular in non-monotonic logic and defeasible reasoning. Section 4, 'Argumentation and the structure of arguments in formal and computational perspective', is about the formalization of argument attack, the structure of arguments, argument schemes and dialogue. In Section 5, 'Specific kinds of argumentation in formal and computational perspective', we discuss argumentation with rules, cases, values and evidence. We conclude the article by looking back at the formal and computational turn in argumentation theory using the crucial concepts of argumentation theory before that turn, and by an outlook into the future of argumentation theory.[2]

[1]Relevant journals include: Artificial Intelligence, Artificial Intelligence and Law, Autonomous Agents and Multi-Agent Systems, Computational Intelligence, International Journal of Cooperative Information Systems, International Journal of Human-Computer Studies, Journal of Logic and Computation, and The Knowledge Engineering Review. Contributions have also been made to journals that deal primarily with argumentation, such as Argumentation and Informal Logic. A journal devoted explicitly to the interdisciplinary area of AI is Argument and Computation. The biennial conference series COMMA is devoted to the study of computational models of argument. The first was held in Liverpool in 2006, followed by conferences in Toulouse (2008), Desenzano del Garda (2010), Vienna (2012), Pitlochry (2014), and Potsdam (2016). See `http://www.comma-conf.org/`. ArgMAS (Argumentation in Multi-Agent Systems) and CMNA (Computational Models of Natural Argument) are related workshops.

[2]The article has been written as a chapter for the *Handbook of Formal Argumentation* (Volume 1: Foundations) (`http://formalargumentation.org/`).

2 Argumentation and argumentation theory before the formal and computational turn

Argumentation, a phenomenon we are all familiar with, arises in response to, or in anticipation of, a real or imagined difference of opinion. It comes into play in cases when people start defending a view they assume not to be shared by others. Not only the need for argumentation, but also the requirements argumentation has to fulfil and the structure of argumentation are connected with a context in which doubt, potential opposition, and perhaps also objections and counterclaims arise.

A definition of argumentation suitable to be used in argumentation theory should connect with commonly recognized characteristics of argumentation. It is important to realize however that there are striking differences between the meaning of the pivotal word 'argumentation' in English usage and the meaning of its lexical counterparts in other languages.[3] A first relevant difference is that the meaning of argumentation in the latter naturally includes both argumentation as a process and argumentation as a product. Second, unlike the English word 'argumentation', its non-English counterparts pertain exclusively to a constructive effort to convince the addressee of the acceptability of one's standpoint, so that argumentation is immediately associated with reasonableness.[4] Third, in the non-English counterparts 'argumentation' is taken to refer only to the constellation of propositions put forward in defence of a standpoint without including the standpoint,[5] so that standpoint and argumentation are viewed as separate entities, which facilitates the study of their relationship [van Eemeren & Grootendorst, 1984, p. 18]. Note that—as we will see below—since the formal and computational turn discussed below, attention for argumentation that goes against a standpoint has increased.

Next to the meaning of the non-English counterparts, which captures some vital characteristics, there are also some general characteristics of argumentation that are independent of any specific language that are taken into account in defining the term argumentation in argumentation theory. To begin with, argumentation is a *communicative act complex*,[6] whose structural design reflects the functional intent of the communicative moves that are made. Next, argumentation is an *interactional act*

[3]For instance, in French 'argumentation,' in German 'Argumentation,' in Italian 'argomentazione,' in Portuguese 'argumentação,' in Spanish 'argumentación,' in Dutch 'argumentatie,' and in Swedish 'argumentation.'

[4]This does not mean, of course, that in practice argumentation cannot be abused, so that there is no matter of acting reasonably.

[5]According to Tindale [1999, p. 45], it is 'the European fashion' to refer to the premises of an argument as the argumentation and to the conclusion by using another term, such as standpoint.

[6]Because argumentation can also be non-verbal, for instance, visual, it is defined here—more generally—as a 'communicative' rather than a 'verbal' ('linguistic') act complex.

complex directed at eliciting a response that indicates acceptance of the standpoint that is defended, so that it is always part of an explicit or implicit dialogue with the addressee. Further, as a rational activity of reason, argumentation involves putting forward a constellation of propositions the arguer can be held accountable for, so that it is not just an expressive but creates commitments. Finally, in making an appeal to common critical standards of reasonableness in trying to convince the addressee, the arguer approaches the addressee as a rational judge who judges reasonably.[7]

Based on these starting points, defining argumentation starts from ordinary usage and is next made more precise and explicit in order to adequately serve its purpose in argumentation theory:

Argumentation is a communicative and interactional act complex aimed at resolving a difference of opinion with the addressee by putting forward a constellation of propositions *the arguer can be held accountable for* to make the standpoint at issue acceptable to *a rational judge who judges reasonably*.[8]

Argumentation theory is the umbrella term used to denote the study of argumentation in all its manifestations and varieties, irrespective of the intellectual backgrounds, primary research interests and angles of approach of the theorists. Other general labels, such as informal logic and rhetoric, refer to specific theoretical perspectives on the study of argumentation (and usually also include other research interests than argumentation).

Because the standpoints at issue in a difference of opinion and the argumentation advanced to support them can pertain to all walks of life and all kinds of subjects, the scope of argumentation theory is very broad. It ranges from argumentative discourse in the public and the professional sphere to argumentative discourse in the personal or private sphere. The types of standpoints supported by argumentation may vary from descriptive standpoints to evaluative and prescriptive standpoints. It is in particular worth noting that argumentation is certainly not used only for truth-finding and truth-preservation.[9]

Scholars are often drawn to studying argumentation by their practical interest in improving the quality of argumentative discourse where this is called for. In order

[7]Although the terms rational and reasonable are often used interchangeably, we think that it is useful to make a distinction between acting 'rationally' in the sense of using one's faculty of reason and acting 'reasonably' in the sense of utilizing one's faculty of reason in an appropriate way.

[8]The term argumentation refers to the whole constellation of propositions put forward in defence of the standpoint. Because each of the propositions constituting the constellation has its own share in providing grounds for accepting the standpoint at issue, in principle, these propositions by themselves also have an argumentative function. This is expressed terminologically by calling them the reasons that make up the argumentation as a whole.

[9]Generally, in discussing a claim to acceptance, argumentation has in fact no major role to play when a decisive solution can readily be offered by other means.

to be able to realize this ambition, they have to combine an empirical orientation towards how argumentative discourse is conducted with a critical orientation towards how it should be conducted. To give substance to this challenging combination, they need to carry out a comprehensive research programme that ensures that argumentative discourse will not only be examined descriptively as a specimen of verbal communication and interaction ('pragmatics') but also be measured against normative standards of reasonableness ('normative pragmatics') [van Eemeren, 1990].

In order to combine critical and empirical insights systematically, in argumentation theory argumentation scholars make it their business to bridge the gap between the normative dimension and the descriptive dimension of argumentative discourse. The complex problems that are at stake are to be solved with the help of a research programme with five interrelated components [van Eemeren & Grootendorst, 2004, pp. 9–41].[10] On the one hand, the programme has a *philosophical* component, in which a philosophy of reasonableness is developed, and a *theoretical* component, in which, starting from this philosophy, a model for argumentative discourse is designed. On the other hand, the programme has an *empirical* component, in which argumentative reality as it manifests itself in communicative and interactional exchanges is investigated. Next, in the pivotal *analytical* component of the research programme, the normative and the descriptive dimensions are systematically linked together by a theoretically motivated and empirically justified reconstruction of argumentative discourse. Finally, in the *practical* component the problems that occur in the various kinds of argumentative practices are identified, and methods are developed to tackle these problems.

In developing a philosophy of reasonableness argumentation theorists reflect in the philosophical component upon the rationale for the view of reasonableness that is to underlie their theoretical approach. Depending on the conception of reasonableness they favour, in the theoretical component standards for the validity, soundness or appropriateness of argumentation are adopted and theoretical models are developed based on these conceptions. Because the model of argumentation is in this case a normative instrument for assessing the quality of argumentation put forward in argumentative reality, the model constitutes a point of orientation for the empirical research that is to be carried out in argumentation theory but does not constitute a test of the model. The model indicates which factors and processes are worth investigating and to what extent the norms prevailing in argumentative reality agree with the theoretical standards, but deviations are not necessarily an indication of any wrongness in the model.[11]

[10] The five components of a fully-fledged research programme in argumentation theory were introduced in van Eemeren [1987].

[11] Only in case of a purely descriptive theory the empirical research could be aimed at testing

Analytical research in argumentation theory is aimed at the reconstruction of argumentative discourse as it occurs in argumentative reality from the perspective of the model of argumentation that is chosen as the theoretical starting point. Whichever theoretical background they may have, argumentation theorists engaging in analytical research need to develop appropriate tools and methods for reconstructing argumentative discourse. Practical research in argumentation theory, finally, is aimed at analyzing the (spoken and written) argumentative practices that can be distinguished in the various communicative domains from the perspective of argumentation theory and developing instruments for intervention in argumentative discourse where this is due. The instruments for enhancing the quality of argumentative practices may consist of designs for the formats of communicative activity types or of methods for improving arguers' skills in analysing, evaluating and producing argumentative discourse.

In the end, the general objective of argumentation theory is a practical one: to provide adequate instruments for analysing, evaluating and producing argumentative discourse. Ultimately the *raison d'être* of the other components of the research programme carried out in argumentation theory is that they enable the systematic development of such instruments. When taken together, philosophical and theoretical insights into argumentative discourse, analytically connected with empirical insights, are to lead to methodical applications of argumentation theory to the various kinds of argumentative practices.

In pursuing their objective of improving the analysis, evaluation and production of argumentative discourse, argumentation theorists take account of the point of departure of argumentation, consisting of the explicit and implicit material and procedural premises that serve as the starting point, and the layout of the argumentation displayed in the constellation of propositions explicitly or implicitly advanced in support of the standpoints at issue. Both the point of departure and the layout of argumentation are to be judged by appropriate standards of evaluation that are in agreement with all requirements a rational judge who judges reasonably should comply with. This means that the descriptive and normative aims of argumentation theory as a discipline can be specified as follows:[12]

1. Giving a descriptive account of the components of argumentative discourse that constitute together the point of departure of argumentation;

the model, but so far no fully-fledged argumentation theory without a critical dimension has been developed.

[12]The descriptive aims of argumentation theory are often associated with the 'emic' study of what is involved in justifying claims and what are good reasons for accepting a claim viewed from the 'internal' perspective of the arguers while the normative aims are associated with the 'etic' study of these matters viewed from the 'external' perspective of a critical theorist.

2. Giving a normative account of the standards for evaluating the point of departure of argumentation that are appropriate to a rational judge who judges reasonably;

3. Giving a descriptive account of the components of argumentative discourse that constitute together the layout of argumentation;

4. Giving a normative account of the standards for evaluating argumentation as it is laid out in argumentative discourse that are appropriate to a rational judge who judges reasonably.

2.1 Crucial concepts

Certain theoretical concepts are indispensable in developing instruments for methodically improving the quality of the analysis, evaluation and production of argumentative discourse. Among them are the notions of 'standpoint,' 'unexpressed premise,' 'argument scheme,' 'argumentation structure,' and 'fallacy.' All of them are immediately connected with central problem areas in argumentation theory.

Standpoints We use the term standpoint (or point of view) to refer to what is at issue in argumentative discourse in the sense of what is being argued about.[13] In advancing a standpoint the speaker or writer assumes a positive or negative position regarding a proposition. Because advancing a standpoint implies undertaking a positive or negative commitment, in view of the aim of resolving a difference of opinion, whoever advances a standpoint is obliged to defend their standpoint if challenged to do so by the listener or reader. The standpoints at issue in a difference of opinion can be descriptive, evaluative or prescriptive, but in all cases they can be reconstructed as a claim to acceptability (in case of a positive standpoint) or unacceptability (in case of a negative standpoint) regarding the proposition the standpoint pertains to.[14]

Unexpressed premises Unexpressed premises are often pivotal missing links in transferring acceptance from the premises that are explicitly put forward in the argu-

[13]The terms claim, conclusion, thesis and debate proposition are used to refer from different theoretical angles to virtually the same concept as the term standpoint. Terms such as belief, opinion and attitude usually refer to related concepts that are in relevant ways different from a standpoint.

[14]For an overview of the various approaches to standpoints see Houtlosser [2001].

mentation to the standpoint that is defended.[15] Such partly implicit argumentation, which is quite usual in ordinary argumentative discourse, is called enthymematic. The identification of elements left implicit in enthymematic argumentation is in practice usually unproblematic, but in some cases it can be a problem. According to most argumentation theorists, then carrying out a logical analysis does not suffice. Starting from a logical analysis, a pragmatic analysis needs to be carried out in which the analyst tries to identify the unexpressed premise by determining on the basis of the available contextual and background information to which implicit proposition the arguer can be held committed to.[16]

Argument schemes An argument(ation) scheme is an abstract characterization of the way in which in a particular type of argumentation a reason used in support of a standpoint is related to that standpoint in order to bring about a transfer of acceptance from that reason to the standpoint. Depending on the kind of relationship established in the argument scheme, specific kinds of evaluative questions—usually referred to as critical questions—are to be answered in evaluating the argumentation. These critical questions capture the specific pragmatic rationale for bringing about the transition of acceptance.[17]

Argumentation structures The argumentation structure of a piece of argumentative discourse characterizes the 'external' organization of the argumentation that is advanced: how do the reasons put forward in a particular argumentation hang together and in what way exactly do they relate to the standpoint at issue? In argumentation theory, various ways of combining reasons have been distinguished that characterize the different kinds of argumentation structures that can be instrumental in defending a standpoint.[18]

[15]Depending on the theoretical background of the theorists, other terms are used to refer to an unexpressed premise: implicit, suppressed, tacit, and missing premise, reason or argument, but also warrant, implicature, supposition, and even assumption, inference and implication.

[16]For an approach in which a logical analysis is used as a heuristic tool in carrying out a pragmatic analysis see van Eemeren and Grootendorst [1992, pp. 64–67]; [2004, pp. 117–118]. For the various kinds of resources that can be used in accounting for the reconstruction see van Eemeren [2010, pp. 16–29]

[17]For an overview of the study of argument schemes, see Garssen [2001]; for attempts at formalization and the computational implications, see Walton, Reed and Macagno [2008, Ch. 11 and 12]. A recent development is the study of what have been called prototypical argumentative patterns. These consist of constellations of argumentative moves in which a particular argument scheme or combination of argument schemes is used [van Eemeren, 2017].

[18]Different terminological conventions have been developed for naming the combinations of reasons and the divisions of the various types of structures are not always exactly the same. For an overview of the study of argumentation structures see Snoeck Henkemans [2001].

Fallacies The difference of opinion at issue in argumentative discourse will not be resolved satisfactorily if contaminators of the argumentative exchange enter the discourse that are not detected. Such contaminators, which may be so treacherous that they go unobserved in the argumentative exchange, are known as fallacies. Virtually every normative theory of argumentation includes a treatment of the fallacies. The degree to which a theory of argumentation makes it possible to give an adequate treatment of the fallacies can even be considered as a litmus test of the quality of the theory.[19]

2.2 Prominent theoretical approaches

Ancient dialectic and rhetoric—in combination with syllogistic logicare the forbears of modern argumentation theory.[20] The Aristotelian concept of dialectic is best understood as the art of inquiry through critical dialogue. In a dialogue that is dialectical in the Aristotelian sense the adequacy of any particular claim is supposed to be cooperatively assessed by eliciting premises that might serve as commonly accepted starting points, then drawing out implications from those starting points and determining their compatibility with the claim in question. Where contradictions emerge, revised claims might be put forward to avoid such problems. This method of regimented opposition amounts to a pragmatic application of logic, a collaborative method of putting logic into use so as to move from conjecture and opinion to more secure belief.

Aristotle's rhetoric deals with the principles of effective persuasion leading to assent or consensus. It bears little resemblance to modern-day persuasion theories heavily oriented to the analysis of attitude formation and attitude change but largely indifferent to the problem of the invention of persuasive messages [Eagly & Chaiken, 1993; O'Keefe, 2002]. In Aristotle's rhetoric, the emphasis is on the production of effective argumentation for an audience when the subject matter does not lend itself to a logical demonstration of certainty. When it comes to logical demonstration, the syllogism is the most prominent form; the enthymeme, thought of as an incomplete syllogism whose premises are acceptable to the audience, is its rhetorical counterpart. As yet, there is no unitary theory of argumentation available that encompasses the dialectical and rhetorical dimensions of argumentation and is universally accepted. The current state of the art in the argumentation theory (as it developed before the

[19]For a more detailed overview of the study of fallacies see van Eemeren [2001].

[20]Although ancient dialectic and rhetoric are often discussed as if both of them were unified wholes, contributions to their development have been made by various scholars and their views were by no means always in harmony. In order to be accurate, we must therefore always indicate precisely to whose views exactly we are referring.

recent formal and computational turn) is characterized by the co-existence of a variety of theoretical perspectives and approaches, which differ considerably from each other in conceptualization, scope and theoretical refinement. Every fully-fledged theoretical approach to argumentation represents in fact a particular specification of what it means for a rational judge to judge reasonably and provides a definition of (crucial aspects of) the type of validity favoured by the theorist.

Some argumentation theorists, especially those having a background in linguistics, discourse analysis or rhetoric, have a goal that is primarily (and sometimes even exclusively) descriptive. They are interested in finding out how in argumentative discourse speakers and writers try to convince or persuade others. Other argumentation theorists, often inspired by logic, philosophy or insights from law, study argumentation primarily for normative purposes. They are interested in developing validity or soundness criteria that argumentation must satisfy in order to qualify as rational or reasonable. Currently, however, most argumentation theorists seem to recognize that argumentation research has a descriptive as well as a normative dimension and that in argumentation theory both dimensions must be combined.[21]

Most modern approaches to argumentation are strongly affected by the perspectives on argumentation developed in Antiquity. Both the dialectical perspective (which nowadays usually incorporates the logical dimension) and the rhetorical perspective are represented prominently. Approaches to argumentation that are dialectically oriented tend to focus primarily on the quality of argumentation in defending standpoints in regulated critical dialogues. They put an emphasis on guarding the reasonableness of argumentation by means of regimentation. It is noteworthy that in the rhetorically oriented approaches to argumentation putting an emphasis on factors influencing the effectiveness of argumentation, effectiveness is usually viewed as a 'right to acceptance' that speakers or writers are, as it were, entitled to on the basis of the qualities of their argumentation rather than in terms of actual persuasive effects.[22]

[21]The infrastructure of the field of argumentation theory in terms of academic associations, journals and book series reflects to some extent the existing division in theoretical perspectives. The American Forensic Association (AFA), associated with the National Communication Association, and its journal *Argumentation & Advocacy* concentrate on argumentation, communication and debate. The Ontario Society for the Study of Argumentation (OSSA), the Association of Informal Logic and Critical Thinking (AILACT) and the electronic journal *Informal Logic* focus on informal logic. The International Society for the Study of Argumentation (ISSA), the journals *Argumentation* and *Journal of Argumentation in Context*, and the accompanying book series Argumentation Library and Argumentation in Context aim to cover the whole spectrum of argumentation theory. Other international journals relevant to argumentation theory are *Philosophy and Rhetoric*, *Logique et Analyse*, *Controversia*, *Pragmatics and Cognition*, *Argument and Computation*, and *Cogency*.

[22]Research aimed at examining the actual effectiveness of argumentation is usually called persuasion research. In practice, it generally amounts to quantitative empirical testing of the ways in

In modern argumentation theory a remarkable revival has taken place of both dialectic and rhetoric. Unlike in Aristotle's approach, however, there is a wide conceptual gap between the two perspectives on argumentation, going together with a communicative gap between their protagonists. In recent times, some argumentation scholars have come to the conclusion that the dialectical and rhetorical views on argumentation are not per se incompatible. It has even been argued that re-establishing the link between dialectic and rhetoric will enrich the analysis and evaluation of argumentative discourse [van Eemeren, 2010, especially Ch. 3].

In giving a brief overview of the current theoretical approaches, we first turn to two 'neo-classical' proposals developed in the 1950s: the Toulmin model and the 'new rhetoric'. In dealing with argumentation both aim to counterbalance the formal approach that modern logic provides for dealing with analytic reasoning.

In *The uses of argument*, first published in 1958, Toulmin [2003] reacted against the then dominant logical view that argumentation is just another specimen of the reasoning that the formal approach is qualified to deal with. As an alternative, he presented a model of the 'procedural form' of argumentation aimed at capturing the functional steps that can be distinguished in the defence of a standpoint by means of argumentation. The procedural form of argumentation is, according to Toulmin, 'field-independent', meaning that the steps that are taken are always the same, irrespective of the subject that is being discussed.[23]

In judging the validity of argumentation, Toulmin gives the term validity a different meaning than it has in formal logic. The validity of argumentation is in his view primarily determined by the degree to which the (usually implicit) warrant that connects the data advanced in the argumentation with the claim at issue is acceptable—or, if challenged, can be made acceptable by a backing. What kind of backing may be required in a particular case depends on the field to which the standpoint at issue belongs. This means that the criteria used in evaluating the validity of argumentation are in Toulmin's view 'field-dependent'. Thus, Toulmin puts the validity criteria for argumentation in an empirical and historical context.

In their monograph *The new rhetoric*, also first published in 1958, Perelman and Olbrechts-Tyteca [1969] regard argumentation—in line with classical rhetoric—as sound if it adduces or reinforces assent among the audience to the standpoint at issue. The audience addressed may be a 'particular' audience consisting of a specific person or group of people, but it can also be the 'universal' audience—the (real or imagined) audience that, in the arguer's view, embodies reasonableness.

which argumentation and other means of persuasion lead to changes of attitude in the recipients [O'Keefe, 2002].

[23]It is noteworthy that Toulmin's model of the argumentative procedure is in fact conceptually equivalent to the extended syllogism known in Roman-Hellenistic rhetoric as epicheirema.

Besides an overview of the elements of agreement that can in argumentation serve as points of departure (facts, truths, presumptions, values, value hierarchies and topoi[24]), Perelman and Olbrechts-Tyteca provide an overview of the argument schemes that in the layout of argumentation can be used to convince or persuade an audience. The argument schemes they distinguish remain for the most part close to the classical topical tradition. Apart from argumentative techniques of 'association', in which these argument schemes are employed, Perelman and Olbrechts-Tyteca also distinguish an argumentative technique of 'dissociation.' Dissociation divides an existing conceptual unity into two separate conceptual unities.

In spite of obvious differences between Toulmin's approach to argumentation and that of Perelman and Olbrechts-Tyteca, there are also some striking commonalities. Starting from an interest in the justification of views by means of argumentative discourse, both emphasize that values play a part in argumentation, both reject formal logic as a theoretical tool, and both turn for an alternative model to juridical procedures. A theoretical connection between the Toulmin model and the new rhetoric could be made by viewing the various points of departure distinguished in the new rhetoric as representing different types of data in the Toulmin model and its argument schemes as different types of warrants or backings.

Of the approaches to argumentation that have been developed more recently, formal dialectic, coined and instigated by Hamblin [1970], remains closest to formal logic, albeit logic in a dialectical garb. The scholars responsible for the revival of dialectic in the second part of the twentieth century treat argumentation as part of a formal discussion procedure for resolving a difference of opinion by testing the tenability of the 'thesis' at issue against challenges. Apart from the ideas about formal dialectic articulated by Hamblin, in designing such a procedure they make use of the 'dialogue logic' of the Erlangen School [Lorenzen & Lorenz, 1978], but also from insights advanced by Crawshay-Williams [1957]; Næss [1966]. The most complete proposal was presented by Barth and Krabbe [1982] in *From axiom to dialogue*. Their formal dialectic describes systems for determining by means of a regimented dialogue game between the proponent and the opponent of the thesis whether the proponent's thesis can be maintained given the premises allowed as 'concessions' by the opponent.

Building on the proposals for a dialogue logic made by the Erlangen School, Barth and Krabbe's formal dialectic offers a translation of formal logical systems into formal rules of dialogue. In *Commitment in dialogue*, Walton and Krabbe [1995] integrate the proposals of the Erlangen School with the more permissive kind of dialogues promoted in Hamblin's [1970] dialectical systems. After having provided a

[24]Perelman and Olbrechts-Tyteca use the Latin equivalent loci.

classification of the main types of dialogue, they discuss the conditions under which in argumentation commitments should be maintained or may be retracted without violating any of the rules of the type of dialogue concerned.

Related approaches can be found in some of the proposals made by formal and informal logicians. Out of dissatisfaction with the treatment of argumentation in logical textbooks, and inspired by the Toulmin model (and to a much lesser extent the new rhetoric), a group of Canadian and American philosophers have propagated since the 1970s an approach known as informal logic. The label informal logic refers in fact to a collection of logic-oriented normative approaches to the study of reasoning in ordinary language which remain closer to the practice of argumentation than is usually the case in formal logic. Informal logicians aim in the first place at developing adequate norms for interpreting, assessing and construing argumentation.

Since 1978, the journal *Informal Logic*,[25] started and edited by Blair and Johnson (later joined by others), has been the speaking voice of informal logic and the connected educational reform movement dedicated to 'critical thinking'. In their textbook *Logical self-defense*, Johnson and Blair [2006] have indicated what they have in mind when they speak of an informal logical alternative to formal logic. They explain that the premises of an argument have to meet the criteria of 'acceptability', 'relevance' and 'sufficiency'. Other informal logicians have adopted these three criteria, albeit sometimes under slightly different names (e.g., [Govier, 1987]).

Freeman [2005] provides, from an epistemological perspective on informal logic, a comprehensive theory of premise acceptability. Generally, however, informal logicians remain in the first place interested in the premise-conclusion relations in arguments (e.g., [Walton, 1989]). Most of them maintain that argumentation should be valid in some logical sense, but generally they do not stick to the formal criterion of deductive validity. Woods and Walton [1989] claim that each fallacy requires its own theoretical treatment, which leads them to applying a variety of logical systems in their theoretical treatment of the fallacies. Johnson [2000] also takes a predominantly logical approach, but he complements this approach with a 'dialectical tier', where the arguer discharges his or her dialectical obligations, for instance, by anticipating objections, and dealing with alternative positions. In Finocchiaro's contributions to informal logic, too, the logical and the dialectical approach are combined, albeit that the emphasis is more strongly on the dialectical dimension, and historical and empirical dimensions are added (e.g., [Finocchiaro, 2005]). The rhetorical perspective has received less attention from informal logicians. A notable exception is Christopher Tindale [1999; 2004].

In modern times, the study of rhetoric has fared considerably better in the United

[25] At first named Informal Logic Newsletter.

States than in Europe. Not only has classical rhetoric from the nineteenth century onwards been represented in the academic curriculum, but also has the development of modern rhetorical approaches been more prolific. In the last decades of the twentieth century, the image that rhetoric had acquired of being irrational and even anti-rational has been revised. Paying tribute to Perelman and Olbrechts-Tyteca's new rhetoric, in various countries various scholars have argued for a rehabilitation of the rhetorical approach. In spite of the unlimited extension in the United States in the 1960s of the scope of Big Rhetoric 'to the point that everything, or virtually everything, can be described as 'rhetorical' [Swearingen & Schiappa, 2009, p. 2], Wenzel [1987] emphasized the rational qualities of rhetoric. In Europe, Reboul [1990] and Kopperschmidt [1989a] argued at about the same time for giving rhetoric its rightful position in the study of argumentation beside dialectic.

Although all of them may be described as rhetoricians in the broad sense, the American scholars from the field of (speech) communication currently engaged in argumentation theory do not share a clearly articulated joint perspective. Their most obvious common feature is a concern with the connection between claims and the people engaged in some kind of argumentative practice. The American debate tradition in particular has had an enormous influence on American argumentation studies. More or less outside the immediate debate tradition, Zarefsky [2006; 2009], Leff [2003] and Schiappa [2002] have contributed profound historical rhetorical analyses. Fahnestock [1999; 2009] dealt theoretically with rhetorical figures and stylistics.

Concentrating on the public features of communicative acts, Jackson and Jacobs [1982] initiated a research programme to study argumentation in informal conversations. Their joint research aims at understanding the reasoning processes by which individuals make inferences and resolve disputes in ordinary conversation. A related empirical angle in American argumentation research is the study of argument in natural settings, such as school board meetings, counseling sessions and public relations campaigns, to produce 'grounded theory'—a theory of the specific case.

A Toulminian concept that has strongly influenced American argumentation scholarship is the notion of 'field'. Toulmin [1972] describes fields as 'rational enterprises', which he equates with intellectual disciplines, and explores how the nature of reasoning differs from field to field. This treatment led to vigorous discussion about what defines a 'field of argument': subject matter, general perspective, world-view, or the arguer's purpose—to mention just a few of the possibilities. The concept of fields of argument encouraged recognition that the soundness of arguments is not something universal and necessary, but context-specific and contingent. Instead of the term fields, Goodnight prefers the term spheres, referring to 'the grounds upon which arguments are built and the authorities to which arguers appeal' [1982, p. 216]. He uses 'argument' to mean interaction based on dissensus, so that the grounds of arguments lie in doubts and uncertainties. In a similar vein as Habermas [1984], Goodnight [2012] distinguishes between three spheres of argument: the 'personal' (or 'private') sphere, the 'public' sphere, and the 'technical' sphere.

Meanwhile, starting in the 1970s, in Europe a descriptive approach has developed in which argumentation is viewed as a linguistic phenomenon that not only manifests itself in language use, but is also inherent in most language use. In a number of publications (almost exclusively in French), the protagonists of this approach, Ducrot and Anscombre, have presented a linguistic analysis to show that almost all verbal utterances lead the listener or reader—often implicitly—to certain conclusions, so that their meaning is crucially argumentative. In *L'argumentation dans la langue* [Anscombre & Ducrot, 1983] they refer to the theoretical position they adopt as radical argumentativism. Their approach is characterized by a strong interest in words that can serve as argumentative 'operators' or 'connectors', giving linguistic utterances a specific argumentative force and argumentative direction (e.g., 'only', 'no less than', 'but', 'even', 'still', 'because', 'so'). Anscombre[1994] observes that the argumentative principles that are at issue here are on a par with the topoi from classical rhetoric.

It has become a tradition among a substantial group of European researchers, primarily based in the French-speaking world, to approach argumentation from a descriptive linguistic angle. Some of them continue the approach started by Ducrot and Anscombre. Others, such as Plantin [1996] and Doury [1997], build on this approach but are also—and often more strongly—influenced by conversation analysis and discourse analysis. Other researchers, based in Switzerland, who favour a linguistic approach, but allow also for normativity, are Rigotti [2009], Rocci [2009], and Greco Morasso [2011]. They combine their linguistic approach with insights from other approaches, such as pragma-dialectics.

The pragma-dialectical theory of argumentation developed in Amsterdam combines a dialectical and a rhetorical perspective on argumentation and is both normative and descriptive. As van Eemeren and Grootendorst [1984] explain, pragma-dialecticians view argumentation as part of a discourse aimed at resolving a difference of opinion on the merits by methodically testing the acceptability of the standpoints at issue. The dialectical dimension of the approach is inspired by normative insights from critical rationalism and formal dialectics, the pragmatic dimension by descriptive insights from speech act theory, Gricean pragmatics and discourse analysis.

The various stages argumentative discourse must pass through to resolve a difference of opinion on the merits by a critical exchange of speech acts are in the pragma-dialectical theory laid down in an ideal model of a critical discussion [van Eemeren & Grootendorst, 2004)] Viewed analytically, there should be a 'confrontation stage', in which the difference of opinion comes about, an 'opening stage', in which the point of departure of the discussion is determined, an 'argumentation stage', in which the standpoints at issue are defended against criticism, and a 'concluding stage', in which it is determined what the result of the discussion is. The model of a critical discussion defines the nature and the distribution of the speech acts that have a constructive role in the various stages of the resolution process. In addition, the standards of reasonableness authorizing the performance of particular speech acts in the various stages of a critical discussion are laid down in a set of dialectical rules for critical discussion. Any violation of any of the rules amounts to making an argumentative move that is an impediment to the resolution of a difference of opinion on the merits and is therefore fallacious [van Eemeren & Grootendorst, 1992].[26]

Because argumentative discourse generally diverges for various reasons from the ideal of a critical discussion, in the analysis of the discourse a reconstruction is required to achieve an analytic overview of all those, and only those, speech acts that play a potential part in resolving a difference of opinion on the merits. Van Eemeren, Grootendorst, Jackson and Jacobs [1993] emphasize that the reconstruction should be guided by the theoretical model of a critical discussion and faithful to the commitments that may be ascribed to the arguers on the basis of their contributions to the discourse. Because the reconstruction of argumentative discourse as well as its evaluation can be made more pertinent, more precise, and also better accounted for if, next to the maintenance of dialectical reasonableness, the simultaneous pursuit of rhetorical effectiveness is taken into account, van Eemeren and Houtlosser [2002] developed the notion of strategic manoeuvring. This notion makes it possible to integrate relevant rhetorical insights systematically in the pragma-dialectical analysis and evaluation [van Eemeren, 2010].

3 Formal and computational argumentation theory: precursors and first steps

Today much research addresses argumentation using formal and computational methods. Precursors can be found in the fields of non-monotonic logic and logic programming, and first steps were made by philosophers addressing defeasible reasoning.

[26]The extent to which the rules for critical discussion are capable of dealing with the defective argumentative moves traditionally designated as fallacies is viewed as a test of their 'problem-solving validity'. For experimental empirical research of the 'intersubjective acceptability' of the rules for critical discussion that lends them 'conventional validity' see van Eemeren, Garssen and Meuffels [2009].

3.1 Non-monotonic logic

A relevant field predating the formal and computational study of argumentation is non-monotonic logic [Antonelli, 2010]. A logic is non-monotonic when a conclusion that, according to the logic, follows from certain premises need not always follow when premises are added. In contrast, classical logic is monotonic. For instance, in a standard classical analysis, from premises 'Edith goes to Vienna or Rome' and 'Edith does not go to Rome', it follows that 'Edith goes to Vienna', irrespective of possible additional premises. The standard example of non-monotonicity used in the literature of the 1980s concerns the flying of birds. Typically, birds fly, so if you hear about a bird, you will conclude that it can fly. However, when you next learn that the bird is a penguin, you retract your conclusion. In a non-monotonic logic, a balance can be sought between the advantage of drawing a tentative conclusion, which is usually correct, and the risk of having to withdraw the conclusion in light of new information.

A prominent proposal in non-monotonic logic is Raymond Reiter's [1980] logic for default reasoning, using default rules. Reiter's first example of a default rule expresses that birds typically fly:

$$\mathrm{BIRD}(x) : \mathrm{M}\ \mathrm{FLY}(x)\ /\ \mathrm{FLY}(x)$$

The default rule expresses that, if x is a bird, and it is consistent to assume that x can fly, then by default one can conclude that x can fly. Other influential logical systems for non-monotonic reasoning include circumscription, auto-epistemic logic, and non-monotonic inheritance; each of them discussed in the representative overview of the study of non-monotonic logic at its heyday by Gabbay, Hogger and Robinson [1994].

3.2 Logic programming

A development related to non-monotonic logic is logic programming. The general idea underlying logic programming is that a computer can be programmed using logical techniques. In this view, computer programs are not only considered procedurally as recipes for how to achieve the program's aims, but also declaratively, in the sense that the program can be read like a text, for instance, as the rule-like knowledge needed to answer a question. In the logic programming language Prolog (the result of a collaboration between Colmerauer and Kowalski; see [Kowalski, 2011]), these are examples of facts and a rule [Bratko, 2001]:

parent(pam, bob)
female(pam)
mother(X, Y) :- parent(X, Y), female(X)

This small logic program represents the facts that Pam is Bob's parent, and that Pam is female, and the rule that someone's mother is a female parent. Given this Prolog program, a computer can as expected derive that Pam is Bob's mother. In the interpretation of logic programs, the closed world assumption plays a key role: a logic program is assumed to describe all facts and rules about the world. For instance, in the program above it is assumed that all parent relations are given, so 'parent(tom, bob)' cannot be derived. By what is called negation as failure, it will be considered false that Tom is Bob's parent. If we add 'parent(tom, bob)' it becomes derivable that Tom is Bob's parent, showing the connection between logic programming's negation as failure and non-monotonic logic.

3.3 Themes and impact of non-monotonic logics

The study of non-monotonic logics gave hope that logical tools would become more relevant for the study of natural reasoning. To some extent this hope has been fulfilled, since certain themes that before were at the boundaries of logic, were now placed in the centre of attention. Examples of such themes are defeasible inference, consistency preservation, and uncertainty. In the handbook edited by Gabbay, Hogger and Robinson [1994], Donald Nute discusses defeasible inference that can be blocked or defeated in some way [Nute, 1994, p. 354]. Interestingly, Donald Nute speaks of the presentation of sets of beliefs as reasons for holding other beliefs as advancing arguments. David Makinson [1994, p. 51] describes consistency preservation as the property that the conclusions drawn on the basis of certain premises can only be inconsistent in case the premises are inconsistent. Henry Kyburg [1994, p. 400] distinguishes three kinds of inference involving uncertainty: classical, deductive, valid inference about uncertainty; an 'inductive' kind where a conclusion can be false even when the premises are true (hence distinct from the idea of induction as going from the specific to the general, and closer to what today is often called 'defeasible'); and a kind of inference with uncertainty that gives probabilities of particular statements.

The study of non-monotonic logic has been very successful as a research enterprise, and coincided with innovations in computer programming in the form of logic-based languages such as Prolog, and to commercial applications: today's knowledge-based expert systems—in wide-spread use—often include some elementary form of non-monotonic reasoning.

At the same time, non-monotonic logic did not fulfil all expectations of the artificial intelligence community in which it was initiated. Matthew Ginsberg [1994], for instance, notes—somewhat disappointedly—that the field put itself "in a position where it is almost impossible for our work to be validated by anyone other than a member of our small subcommunity of Artificial Intelligence as a whole" [1994, p. 28–29]. His diagnosis of this issue is that attention shifted from the key objective of building an intelligent artefact to the study of simple examples and mathematics. This leads him to plead for a more experimental, scientific attitude as opposed to a theoretical, mathematical focus.

3.4 Defeasible reasoning

In 1987, the publication of John Pollock's paper 'Defeasible reasoning' in *Cognitive Science* marked a turning point. The paper emphasized that the philosophical notion of 'defeasible reasoning' coincides with what in AI is called 'non-monotonic reasoning.' As philosophical heritage for the study of defeasible reasoning, Pollock [1987] refers to works by Roderick Chisholm (going back to 1957) and himself (earliest reference in 1967). Ronald Loui [1995] places the origins of the notion of 'defeasibility' a decade earlier, namely in 1948 when the legal positivist H. L. A. Hart presented the paper 'The ascription of responsibility and rights' at the Aristotelian Society [Hart, 1951]. Although Toulmin [1958/2003] rarely uses the term defeasible in *The uses of argument*, he is obviously an early adopter of the idea of defeasible reasoning, but he is not mentioned by Pollock [1987]. Like Pollock, he mentions Hart, but also another philosopher, David Ross, who applied the idea to ethics, recognizing that moral rules may hold prima facie, but can have exceptions.

In Pollock's approach [1987], 'reasoning' is conceived as a process that proceeds in terms of reasons. Pollock's reasons correspond to the constellations of premises and a conclusion which argumentation theorists and logicians call (elementary) arguments. Pollock distinguishes two kinds of reasons:

1. A reason is *non-defeasible* when it logically implies its conclusion;

2. A reason P for Q is *prima facie* when there is a circumstance R such that $P \wedge R$ [where '$\wedge$' denotes logical conjunction] is not a reason for the reasoner to believe Q. R is then a *defeater* of P as a reason for Q.

Note how closely related the idea of a prima facie reason is to non-monotonic inference: Q can be concluded from P, but not when there is additional information R.

Pollock's standard example is about an object that looks red. 'X looks red to John' is a reason for John to believe that X is red, but there can be defeating circumstances, for instance, when there is a red light illuminating the object. See Figure 1.

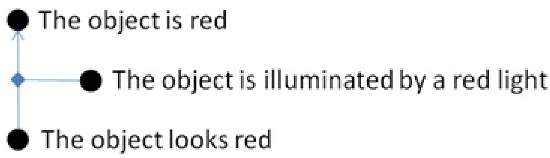

Figure 1: Pollock's red light example

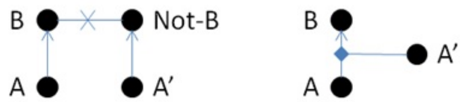

Figure 2: A rebutting defeater and an undercutting defeater

Pollock argues for the existence of two kinds of defeaters: 'rebutting' and 'undercutting defeaters.' A defeater is rebutting when it is a reason for the opposite conclusion (Figure 2, left). Undercutting defeaters attack the connection between the reason and the conclusion, and not the conclusion itself (Figure 2, right). The example about looking red concerns an undercutting defeater since when there is a red light it is not attacked that the object is red, but merely that the object's looking red is a reason for its being red.

A key element in Pollock's work on defeasible reasoning is the development of a theory of warrant. Pollock uses the term warrant as follows: a proposition is warranted in an epistemic situation if and only if an ideal reasoner starting in that situation would be justified in believing the proposition. Here justification is based on the existence of an undefeated argument with the proposition as conclusion. Pollock has developed his theory of warrant in a series of publications which formed the basis of his 1995 book *Cognitive Carpentry*. As a background for his approach to the structure of defeasible reasoning, Pollock provides a list of important classes of specific reasons: reasons based on logical deduction, perception, memory, statistics, or induction. Pollock's theory is embedded in what he called the OSCAR project [Pollock, 1995]. This project aims at the implementation of a rational agent. In the project Pollock addresses both theoretical (epistemic) and practical reasoning.[27]

In a theory of defeasible reasoning based on arguments that can defeat each other, such as Pollock's, the question needs to be considered which arguments can defeat which other arguments. Different forms of argument defeat can be distinguished:

[27]See Hitchcock [2001; 2002] for a survey and a discussion of the OSCAR project for those interested in argumentation. Hitchcock also gives further information about Pollock's work on practical reasoning, i.e., reasoning concerning what to do.

1. An argument can be *undermined*. In this form of defeat, the premises or assumptions of an argument are attacked.[28] Cf. the denial of the premises of an argument.

2. An argument can be *undercut*. In this form of defeat, the connection between a (set of) reason(s) and a conclusion in an argument is attacked. Cf. Pollock's undercutting defeaters.

3. An argument can be *rebutted*. In this form of defeat, an argument is attacked by giving an argument for an opposite conclusion. Cf. Pollock's rebutting defeaters.

4. An argument can be defeated by *sequential weakening*. Then each step in an argument is correct, but the argument breaks down when the steps are chained. An example is an argument based on the sorites paradox [Verheij 1996a, p. 122f.]:

    This body of grains of sand is a heap.
    So, this body of grains of sand minus 1 grain is a heap.
    So, this body of grains of sand minus 2 grains is a heap.
    ...
    So, this body of grains of sand minus n grains is a heap.

5. An argument can be defeated by parallel strengthening. This kind of defeat is associated with what has been called the 'accrual of reasons.' When reasons can accrue, it is possible that different reasons for a conclusion are together stronger than each reason separately. For instance, having robbed someone and having injured someone can be separate reasons for convicting someone. But when the suspect is a minor first offender, these reasons may each by itself be rebutted. On the other hand when a suspect has both robbed someone and also injured that person, the reasons may accrue and outweigh the fact that the suspect is a minor first offender. The argument for not punishing the suspect based on the reason that he is a minor first offender is defeated by the 'parallel strengthening' of the two arguments for punishing him.

[28]This form of defeat is the basis of Bondarenko *et al.* [1997]. We shall here not elaborate on the distinction between premises and assumptions. One way of thinking about assumptions is to see them as defeasible premises.

Building on experiences in the ASPIC project,[29] the recent state-of-the-art ASPIC+ system for the formal modelling of defeasible argumentation [Prakken, 2010][30] uses the first three kinds of defeat. The final two kinds of defeat are distinguished by Verheij [1996a, p. 122f.]. Pollock considered the accrual of reasons to be a natural idea, but argued against it [1995, p. 101f.]. More recent discussions of the accrual of reasons are to be found in Prakken [2005]; Gómez Lucero *et al.* [2009; 2013], and D'Avila Garcez *et al.* [2009, p. 155f.].

4 Argumentation and the structure of arguments in formal and computational perspective

4.1 Abstract argumentation

Phan Minh Dung's 1995 paper 'On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games' in the journal *Artificial Intelligence* [Dung, 1995] reformed the formal study of non-monotonic logic and defeasible reasoning. By his focus on argument attack as an abstract formal relation, Dung gave the field of study a mathematical basis that inspired many new insights. Dung's approach and the work inspired by it are generally referred to as abstract argumentation.

Dung's paper is strongly mathematically oriented, and has led to intricate formal studies. However, the mathematical tools used by Dung are elementary, hence various concepts studied by Dung can be explained without going into much formal detail.

The central innovation of Dung's 1995 paper is that he started the formal study of the attack relation between arguments, thereby separating the properties depending exclusively on argument attack from any concerns related to the structure of the arguments. Mathematically speaking, the argument attack relation is a directed graph, the nodes of which are the arguments, whereas the edges represent that one argument attacks another. Such a directed graph is called an argumentation framework. Figure 3 shows an example of an argumentation framework, with the dots representing arguments, and the arrows (ending in a cross to emphasize the

[29] The ASPIC project (full name: Argumentation Service Platform with Integrated Components) was supported by the EU 6th Framework Programme and ran from January 2004 to September 2007. In the project, academic and industry partners cooperated in developing argumentation-based software systems.

[30] Prakken [2010] speaks of ways of attack, where argument defeat is the result of argument attack.

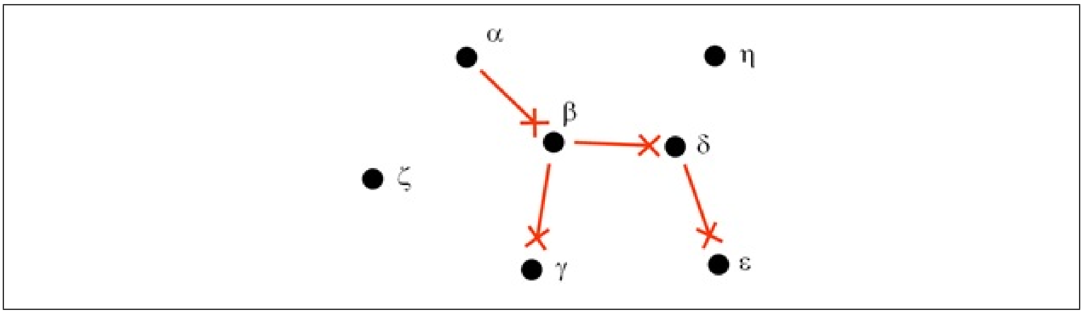

Figure 3: An argumentation framework representing attack between arguments

attacking nature of the connection[31]) representing argument attack.

In Figure 3, the argument α attacks the argument β, which in turn attacks both γ and δ, etc.

Dung's paper consists of two parts, corresponding to two steps in what he refers to as an 'analysis of the nature of human argumentation in its full generality' [Dung, 1995, p. 324]. In the first step, Dung develops the theory of argument attack and how argument attack determines argument acceptability. In the second part, he evaluates his theory by two applications, one consisting of a study of the logical structure of human economic and social problems, the other comprising a reconstruction of a number of approaches to non-monotonic reasoning, among them Reiter's and Pollock's. Notwithstanding the relevance of the second part of the paper, the paper's influence is largely based on the first part about argument attack and acceptability.

In Dung's approach, the notion of an 'admissible set of arguments' is central. A set of arguments is admissible if two conditions obtain:

1. The set of arguments is conflict-free, i.e., does not contain an argument that attacks another argument in the set (nor self-attacking arguments).

2. Each argument in the set is acceptable with respect to the set, i.e., when an argument in the set is attacked by an argument (which by (1) cannot be in the set itself), the set contains an argument that attacks the attacker.

In other words, a set of arguments is admissible if it contains no conflicts and if the set also can defend itself against all attacks. An example of an admissible set of arguments for the framework in Figure 3 is $\{\alpha, \gamma\}$. Since α and γ do not attack one another the set is conflict-free. The argument α is acceptable with respect to the set since it is not attacked, so that it needs no defence. The argument γ is

[31]This is especially helpful when also supporting connections are considered; see Section 4.2.

also acceptable with respect to $\{\alpha, \gamma\}$: the argument γ needs a defence against the attack by β, which defence is provided by the argument α, α being in the set. The set $\{\alpha, \beta\}$ is not admissible since it is not conflict-free. The set $\{\gamma\}$ is not admissible since it does not contain a defence against the argument β, which attacks argument γ.

Admissible sets of arguments can be used to define argumentation notions of what counts as a proof or a refutation.[32] An argument is '(admissibly) provable' when there is an admissible set of arguments that contains the argument. A minimal such set can be regarded as a kind of 'proof' of the argument, in the sense that the arguments in such a set are just enough to successfully defend the argument against counterarguments. An argument is '(admissibly) refutable' when there is an admissible set of arguments that contains an argument that attacks the former argument. A minimal such set can be regarded as a kind of 'refutation' of the attacked argument.

Dung speaks of the basic principle of argument acceptability using an informal slogan: the one who has the last word laughs best. The argumentative meaning of this slogan can be explained as follows. When someone makes a claim, and that is the end of the discussion, the claim stands. But when there is an opponent raising a counterargument attacking the claim, the claim is no longer accepted—unless the proponent of the claim provides a counterattack in the form of an argument attacking the counterargument raised by the opponent. Whoever has raised the last argument in a sequence of arguments, counterarguments, counter-counterarguments, etc., is the one who has won the argumentative discussion.

Formally, Dung's argumentation principle 'the one who has the last word laughs best' can be illustrated using the notion of an 'admissible set of arguments'. In Figure 3, a proponent of the argument γ has the last word and laughs best, since the only counterargument β is attacked by the counter-counterargument α. Formally, this is captured by the admissibility of the set $\{\alpha, \gamma\}$.

Although the principle of argument acceptability and the concept of an admissible set of arguments seem straightforward enough, it turns out that intricate formal puzzles loom. This has to do with two important formal facts:

1. It can happen that an argument is both admissibly provable and refutable.

2. It can happen that an argument is neither admissibly provable nor refutable.

The two argumentation frameworks shown in Figure 4 provide examples of these two facts. In the cycle of attacks on the left, consisting of two arguments α and β,

[32] In the following, we make use of terminology proposed by Verheij [2007].

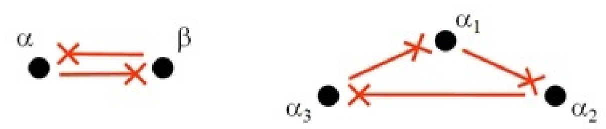

Figure 4: Arguments attacking each other in cycles

each of the arguments is both admissibly provable and admissibly refutable. This is a consequence of the fact that the two sets $\{\alpha\}$ and $\{\beta\}$ are each admissible. For instance, $\{\alpha\}$ is admissible since it is conflict-free and can defend itself against attacks: the argument α itself defends against its attacker α. By the admissibility of the set $\{\alpha\}$, the argument α is admissibly probable, and the argument β admissibly refutable.

The cycle of attacks on the right containing three arguments α_1, α_2 and α_3 is an example of the second fact above, the fact that it can happen that an argument is neither admissibly provable nor refutable. This follows from the fact that there is no admissible set that contains (at least) one of the arguments α_1, α_2 or α_3. Suppose that the argument α_3 is in an admissible set. Then the set should defend α_3 against the argument α_2, which attacks α_3. This means that α_1 should also be in the set, since it is the only argument that can defend α_3 against α_2. But this is not possible, because then α_1 and α_3 are both in the set, introducing a conflict in the set. As a result, there is only one admissible set: the empty set, which contains no arguments at all. We conclude that no argument is admissibly provable or admissibly refutable.

A related formal issue is that when two sets of arguments are admissible, it need not be the case that their union is admissible. The framework on the left in Figure 4 is an example. As we saw, the two sets $\{\alpha\}$ and $\{\beta\}$ are both admissible, but their union $\{\alpha, \beta\}$is not, since it contains a conflict. This has led Dung to propose the notion of a preferred extension of an argumentation framework, which is an admissible set that is as large as possible, in the sense that adding elements to the set makes it not admissible. The framework in Figure 3 has one preferred extension: the set $\{\alpha, \gamma, \delta, \zeta, \eta\}$. The framework in Figure 4 on the left has two preferred extensions $\{\alpha\}$ and $\{\beta\}$, the one on the right has one: the empty set.

Some preferred extensions have a special property, namely that each argument that is not in the set is attacked by an argument in the set. Such an extension is called a stable extension. Stable extensions are formally defined as conflict-free sets that attack each argument not in the set. It follows from this definition that a stable extension is also a preferred extension.

The preferred extension $\{\alpha, \gamma, \delta, \zeta, \eta\}$ of the framework in Figure 3, for instance, is stable, since the arguments β and ϵ, which are the only ones that are not in the set, are attacked by arguments in the set, α and δ, respectively. The preferred extensions $\{\alpha\}$ and $\{\beta\}$ of Figure 4 (left) are also stable. The preferred extension of Figure 4 (right), the empty set, is not stable, since none of the arguments α_1, α_2 and α_3 is attacked by an argument in the set. This example shows that there exist preferred extensions that are not stable. It also shows that there are argumentation frameworks that do not have a stable extension. In contrast, every argumentation framework has at least one preferred extension (which can be the empty set).

The concepts of preferred and stable extension of an argumentation framework can be regarded as different ways to interpret a framework, and therefore they are often referred to as 'preferred semantics' and 'stable semantics.' Dung [1995] proposed two other kinds of semantics: 'grounded semantics' and 'complete semantics,' and following his paper several additional kinds of semantics have been proposed (see Baroni *et al.* [2011], for an overview). By the abstract nature of argumentation frameworks, formal questions about the computational complexity of related algorithms and formal connections with other theoretical paradigms came within reach (see, e.g., [Dunne & Bench-Capon, 2003; Dunne, 2007; Egly *et al.*, 2010]).

Dung's original definitions are in terms of mathematical sets. An alternative way of studying argument attack is in terms of labelling. Arguments are marked with a label, such as 'Justified' or 'Defeated' (or IN/OUT, +/-, 1/0, 'Warranted'/'Unwarranted,' etc.), and the properties of different kinds of labelling are studied in the field. For instance, the notion of a stable extension corresponds to the following notion in terms of labelling:

> A *stable labelling* is a function that assigns one label 'Justified' or 'Defeated' to each argument in the argumentation framework such that the following property holds: an argument α is labelled 'Defeated' if and only if there is an argument β that attacks α and that is labelled 'Justified.'

A stable extension gives rise to a stable labelling by labelling all arguments in the extension 'Justified' and all other arguments 'Defeated.' A stable labelling gives rise to a stable extension by considering the set of arguments labelled 'Justified.'

The idea of labelling arguments can be thought of in analogy with the truth functions of propositional logic, where propositions are labelled with truth-values 'true' and 'false' (or 1/0, t/f, etc.). In the formal study of argumentation, labelling techniques predate Dung's abstract argumentation [1995]. Pollock [1994] uses labelling techniques in order to develop a new version of a criterion that determines warrant.

Verheij [1996b] applied the labelling approach to Dung's abstract argumentation frameworks. He uses argument labelling also as a technique to formally model which arguments are taken into account: in an interpretation of an abstract argumentation framework, the arguments that are assigned a label can be regarded as the ones taken into account, whereas the unlabelled arguments are not considered. Using this idea, Verheij defines two new kinds of semantics: the 'stage semantics' and the 'semi-stable semantics.'[33] Other authors using a labelling approach are Jakobovits and Vermeir [1999] and Caminada [2006]. The latter author translated each of Dung's extension types into a mode of labelling.

As an illustration of the labelling approach, we give a labelling treatment of the grounded extension of an argumentation framework as defined by Dung.[34] Consider the following procedure in which gradually labels are assigned to the arguments of an argumentation framework:

1. Apply the following to each unlabelled argument α in the framework: if the argument α is only attacked by arguments that have been labelled 'Defeated' (or perhaps is not attacked at all), label the argument α as 'Justified.'

2. Apply the following to each unlabelled argument α in the framework: if the argument α is attacked by an argument that has been labelled 'Justified,' label the argument α as 'Defeated.'

3. If step 1 and/or step 2 have led to new labelling, go back to step 1; otherwise stop.

When this procedure is completed (which always happens after a finite number of steps when the argumentation framework is finite), the arguments labelled 'Justified' constitute the grounded extension of the argumentation framework. Consider, for instance, the framework of Figure 3. In the first step, the arguments α, ζ and η are labelled 'Justified.' The condition that all arguments attacking them have been 'Defeated' is vacuously fulfilled, since there are no arguments attacking them. In the second step the argument β is labelled 'Defeated', since α has been labelled 'Justified.' Then a second pass of step 1 occurs and the arguments γ and δ are labelled 'Justified,' since their only attacker β has been labelled 'Defeated.' Finally, the argument ϵ is labelled 'Defeated,' since δ has been labelled 'Justified.' The arguments α, γ, δ, ζ and η (i.e., those labelled 'Justified') together form the grounded

[33] In establishing the concept Verheij [1996b] used the term admissible stage extensions. The now standard term semi-stable extension was proposed by Caminada [2006].

[34] Dung's own definition of grounded extension, which does not use labelling, is not discussed here.

extension of the framework. Every argumentation framework has a unique grounded extension. In the framework of Figure 3, the grounded extension coincides with the unique preferred extension that is also the unique stable extension. The framework in Figure 4 (left) shows that the grounded extension is not always a stable or preferred extension. Its grounded extension is here the empty set, but its two preferred and stable extensions are not empty.

4.2 Arguments with structure

Abstract argumentation, discussed in the previous subsection, focuses on the attack relation between arguments, abstracting from the structure of arguments. We now discuss various themes related to the structure of arguments for and against conclusions, and how it has been studied: arguments and specificity, the comparison of conclusive force, arguments with prima facie assumptions, arguments and classical logic, and the combination of support and attack.

Argument specificity An early theme in the formal study of argumentation was that of 'argument specificity' in relation to the resolution of a conflict between arguments. The key idea connecting arguments and specificity is that when two arguments are conflicting, with one of them being based on more specific information, the more specific argument wins the conflict, and defeats the more general argument.

Guillermo Simari and Ronald Loui [1992] have provided a mathematical formalization of this connection between arguments and specificity, taking inspiration from Poole's [1985] work in non-monotonic logic, and connecting to Pollock's work on argumentative warrant. In their proposal, an argument is a pair (T, h), with T being a set of defeasible rules that are applied to arrive at the argument's conclusion h given the argument's premises (formalized in the background knowledge). Arguments are assumed to be consistent, in the sense that no contradiction can be derived (not even defeasibly). Also arguments are assumed to be minimal, in the sense that all rules are needed to arrive at the conclusion. Formally, for an argument (T, h), it holds that when T' is the result of omitting one or more rules in T, the pair (T', h) is not an argument. Two arguments (T, h) and (T', h') disagree when h and h' are logically incompatible, given the background knowledge. An argument (T, h) counter-argues an argument (T', h') if (T, h) disagrees with an argument (T'', h'') that is a sub-argument of (T', h'), i.e., T'' is a subset of T'. An argument (T, h) defeats an argument (T', h') when (T, h) disagrees with a sub-argument of (T', h') that is strictly less specific. Simari and Loui's approach has been developed further—with applications in artificial intelligence, multi-agent systems, and logic by the Bahia Blanca group, led by Simari (e.g., [García & Simari, 2004; Chesñevar *et al.*, 2004; Falappa *et al.*, 2002]). García and Simari [2004] show the close connection between argumentation and logic programming that was also an inspiration for Dung [1995].

Conclusive force A second theme connected to arguments and their structure is conclusive force. Arguments that have more conclusive force will survive a conflict more easily than arguments with less conclusive force. One idea that connects conclusive force with argument defeat is the weakest link principle, which Pollock characterizes as follows:

The degree of support of the conclusion of a deductive argument is the minimum of the degrees of support of its premises [1995, p. 99].

Pollock presents the weakest link principle as an alternative to a Bayesian approach, which he rejects. Gerard Vreeswijk [1997] has proposed an abstract model of argumentation with defeasible arguments that focuses on the comparison of the conclusive force of arguments. In his model, conclusive force is not modelled directly but as an abstract comparison relation that expresses which arguments have more conclusive force than which other arguments. Vreeswijk defines an abstract argumentation system as a triple $(L, R, \leq)$, where L is a set of sentences expressing the claims made in an argument, R is a set of defeasible rules allowing the construction of arguments, and $\leq$ represents the conclusive force relation between arguments. The rules come in two flavours: strict and defeasible. Arguments are constructed by chaining rules. A set of arguments Σ is a defeater of an argument α if Σ and α are incompatible (i.e., imply an inconsistency), and α is not an underminer of Σ. An argument α is an underminer of a set of arguments Σ if Σ contains an argument β that has strictly lower conclusive force than α. Whereas Dung's [1995] system is abstract by its focus on argument attack, Vreeswijk's proposal is abstract in particular also because the conclusive force relation is left unspecified. Vreeswijk gives the following examples of conclusive force relations:

1. *Basic order.* In this order, a strict argument has more conclusive force than a defeasible argument. In a strict argument, no defeasible rule is used.

2. *Number of defeasible steps.* An argument has more conclusive force than another argument if it uses less defeasible steps. Vreeswijk remarks that this is not a very natural criterion, but it can be used to give formal examples and counterexamples.

3. *Weakest link.* Here the conclusive force relation on arguments is derived from an ordering relation on the rules. An argument has more conclusive force than another if its weakest link is stronger than the weakest link of the other.

4. *Preferring the most specific argument.* Of two defeasible arguments, one has more conclusive force than the other if the first has the premises of the second among its conclusions.

Prima facie assumptions A third theme related to arguments and their structure is arguments with prima facie assumptions. In particular, the defeat of arguments can be the result of prima facie assumptions that are successfully attacked. In their abstract, argumentation-theoretic approach to default reasoning, Bondarenko, Dung, Kowalski, and Toni [1997] use such an approach. Using a given deductive system (L, R) that consists of a language L and a set of rules R, so-called 'deductions' are built by the application of rules. Given a deductive system (L, R), an assumption-based framework is then a triple $(T, Ab, \text{Contrary})$, where T is a set of sentences expressing the current beliefs, Ab expresses assumptions that can be used to extend T, and Contrary is a mapping from the language to itself that expresses which sentences are contraries of which other sentences. Bondarenko and colleagues define a number of semantics (similar to Dung's 1995 in the context of abstract argumentation). For instance, a stable extension is a set of assumptions Δ such that the following properties hold:

1. Δ is closed, meaning that Δ contains all assumptions that are logical consequences of the beliefs in T and Δ itself.

2. Δ does not attack itself, meaning that there is no deduction from the beliefs in T and Δ with a contrary of an element of Δ as conclusion.

3. Δ attacks each assumption not in Δ, meaning that, for every assumption outside Δ, there is a deduction from T and Δ with a contrary of that assumption as conclusion.

Verheij [2003a] has also developed an assumption-based model of defeasible argumentation. In contrast with Bondarenko *et al.* [1997], in Verheij's system, the rules from which arguments are constructed are part of the prima facie assumptions. Technically, the rules have become conditionals of the underlying language. As a result, it can be the issue of an argument whether some proposition supports another proposition. In this way, Pollock's undercutting defeaters can be modelled as an attack on a conditional. Pollock's example of an object that looks red (Section 3.4) is formalized using two conditional sentences:

$$\texttt{looks\_red} \leadsto \texttt{is\_red}$$
$$\texttt{red\_light} \leadsto \times(\texttt{looks\_red} \leadsto \texttt{is\_red})$$

The first expresses the conditional prima facie assumption that if something looks red, it is red. The second expresses an attack on this prima facie assumption: when there is a red light illuminating the object, it no longer holds that if the object looks red, it is red. The sentences illustrate the two connectives of the language: one to express the conditional ($\leadsto$), the other to express what is called dialectical negation ($\times$). The two conditional sentences correspond exactly to two graphical elements in Figure 1: the first to the arrow connecting the reason and the conclusion, the second, nested, conditional to the arrow (ending in a diamond) that expresses the attack on the first conditional. This isomorphism between formal structures of the language and graphical elements has been used for the diagrams supported by the argumentation software ArguMed [Verheij, 2005b; see Section 4.5]).

The use of assumptions raises the question how they are related to an argument's ordinary premises. Assumptions can be thought of as the defeasible premises of an argument, and as such they are akin to defeasible rules[35] with an empty antecedent. The Carneades framework [Gordon *et al.*, 2007] distinguishes three kinds of argument premises: ordinary premises, presumptions (much like the prima facie assumptions discussed here) and exceptions (which are like the contraries of assumptions).

Arguments and classical logic A fourth theme connected to arguments and their structure is how they are related to classical logic. In particular, the relation between classical logic and defeasible argumentation remains a puzzle. Above we already saw different attempts at combining elements of classical logic and defeasible argumentation. In Pollock's system, classical logic is one source of reasons. Often conditional sentences ('rules') are used to construct arguments by chaining them (e.g., [Vreeswijk, 1997]). Chaining rule applications is closely related to the inference rule modus ponens of classical logic. Verheij's [2003a] system gives conditionals which validate modus ponens a central place. Bondarenko *et al.* [1997] allow generalized rules of inference by their use of a contingent deductive system as starting point.

Besnard and Hunter [2008] have proposed to formalize arguments in classical logic entirely. For them, an argument is a pair (Φ, α), such that Φ is a set of sentences and α is a sentence, and such that Φ is logically consistent, Φ logically

[35] Some would object to the use of the term rules here. Rules are here thought of in analogy with the inference rules of classical logic. An issue is then that, as such, they are not expressed in the logical object language, but in a meta-language. In the context of defeasible reasoning and argumentation (and also in non-monotonic logic), this distinction becomes less clear. Often there is one logical language to express ordinary sentences, a second formal language (with less structure and/or less semantics, and therefore not usually referred to as 'logical') used to express the rules, and the actual meta-language that is used to define the formal system.

entails α (in the classical sense), and Φ is a minimal such set. (Note the analogy with the proposal by Simari and Loui [1992], discussed earlier.) Φ is the support of the argument, and α the claim. They define defeaters as arguments that refute the support of another argument. More formally, a defeater for an argument (Φ, α) is an argument (Ψ, β), such that β logically entails the negation of the conjunction of some of the elements of Φ. An undercut for an argument (Φ, α) is an argument (Ψ, β) where β is equal to (and not just entails) the negation of the conjunction of some of the elements of Φ. A rebuttal for an argument (Φ, α) is an argument (Ψ, β) such that $\beta \leftrightarrow \neg\alpha$ is a tautology. Besnard and Hunter give the following example [2008, p. 46]:

p	Simon Jones is a Member of Parliament.
p $\rightarrow$ ¬q	If Simon Jones is a Member of Parliament, then we need not keep quiet about details of his private life.
r	Simon Jones just resigned from the House of Commons.
r $\rightarrow$ ¬p	If Simon Jones just resigned from the House of Commons, then he is not a Member of Parliament.
¬p $\rightarrow$ q	If Simon Jones is not a Member of Parliament, then we need to keep quiet about details of his private life.

Then ({p, p $\rightarrow$ ¬q}, ¬q) is an argument with the argument (r, r $\rightarrow$ ¬p, ¬p) as an undercut and the argument (r, r $\rightarrow$ ¬p, ¬p $\rightarrow$ q, q) as a rebuttal.

Besnard and Hunter focus on structural properties of arguments, in part because of the diversity of proposals for semantics (see Section 4.1). For instance, when they discuss these systems, they note that the semantic conceptualization of such systems is not as clear as the semantics of classical logic, which is the basis of their framework [p. 221, also p. 226]. At the same time, they note that knowledge representation can be simpler in systems based on defeasible logic (see below) or inference rules.

Combining support and attack A fifth and final theme discussed here in connection with arguments and their structure is how support and attack are combined. In several proposals, support and attack are combined in separated steps. In the first step, argumentative support is established by constructing arguments for conclusions from a given set of possible reasons or rules (of inference). The second step determines argumentative attack. Attack is, for instance, based on defeaters or on

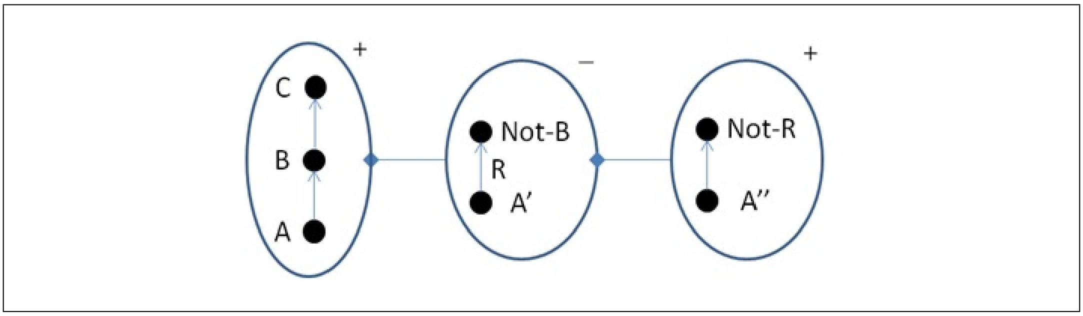

Figure 5: Supporting arguments that attack each other

the structure of the supporting arguments in combination with a preference relation on arguments. In the third and final step, it is determined which arguments are warranted or undefeated. We already saw that several criteria have been proposed (e.g., Pollock's gradual development of criteria for argumentative warrant, and Dung's abstract argumentation semantics).

An example of this modelling style is depicted in Figure 5. Three supporting arguments are shown. The first on the left shows that A supports B, which in turn supports C. In the middle of the figure, this argument is attacked by a second argument, which reasons from A' for Not-B (hence against B). This argument is in turn attacked by a third argument, which reasons from A'' against the support relation R between A' and Not-B. Using the terminology of Section 3.4, the first subargument of the first argument is rebutted by the second, which is undercut by the third. The arguments are marked with a + sign when they are warranted, and a − sign when they are defeated (which can be thought of as a variant of the labelling approaches of Section 4.1). The argument on the right is warranted, since it is not attacked. As a result, the middle argument is defeated, since it is attacked by a warranted argument. The left argument is then also warranted, since its only attacker is defeated. (See the procedure for computing the grounded extension of an argumentation framework discussed in Section 4.1.)

In this approach, the relation with Dung's abstract argumentation is that we can abstract from the structure of the supporting arguments resulting in an abstract argumentation framework. For the three arguments in Figure 5, we obtain the abstract framework shown in Figure 6. In this example, the argumentation semantics is unproblematic at the abstract argument attack level since the grounded extension coincides with the unique preferred extension that is also stable. Special care is needed to handle parts of arguments. For instance, the middle argument has the premise A', which is not attacked, and should therefore remain undefeated.

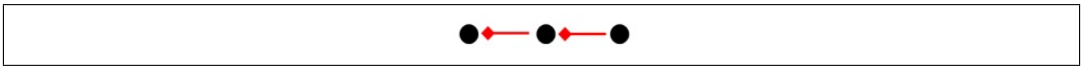

Figure 6: The abstract argumentation framework associated with the example of Figure 5

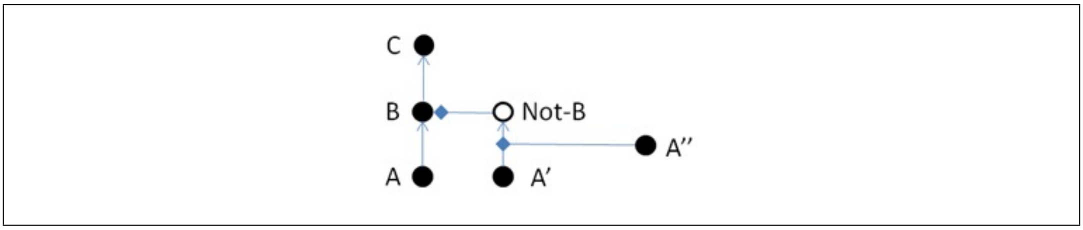

Figure 7: Arguments supporting and attacking conclusions

This type of combining support and attack is used in the ASPIC+ model [Prakken, 2010]. A second approach does not separate support and attack when combining them. Arguments are constructed from reasons for and against conclusions, which in turn determine whether a conclusion follows or not. Figure 7 models the same argumentative information as Figure 5, but now using this second approach.

Here the reason A'' undercuts the argument from A' to Not-B, so Not-B is not supported (indicated by the open circle). As a result, Not-B does not actually attack B, which is therefore justified by A and in turn justifies C.

In this approach, for instance, conditional sentences are used to express which reasons support or attack which conclusions. An example is Nute's defeasible logic [Nute, 1994; Antoniou *et al.*, 2001], which uses conditional sentences for the representation of strict rules and defeasible rules, and for defeater rules, which can block an inference based on a defeasible rule. Algorithms for defeasible logic have been designed with good computational properties. Another example of the approach is Verheij's DefLog [2003a], in which a conditional for the representation of support is combined with a negation operator for the representation of attack. A related proposal extending Dung's abstract argumentation frameworks by expressing both support and attack is bipolar argumentation [Cayrol & Lagasquie-Schiex, 2005; Amgoud *et al.*, 2008]. For DefLog and bipolar argumentation, generalisations of Dung's stable and preferred semantics are presented. DefLog has been used to formalize Toulmin's argument model [Verheij, 2005a].

A special case of the combination of support and attack occurs when the support and attack relations can themselves be supported or attacked. Indeed it can be at issue whether a reason supports or attacks a conclusion. The four ways of arguing

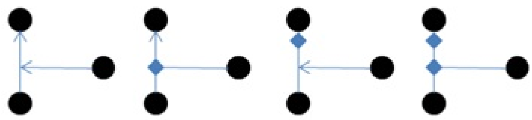

Figure 8: The four ways of arguing about support and attack

about support and attack are illustrated in Figure 8, from left to right: support of a support relation, attack of a support relation, support of an attack relation, and attack of an attack relation, respectively.

For instance, Pollock's undercutting defeaters can be thought of as attacks of a support relation (second from the left in Figure 8). In Verheij's DefLog [2003a; 2005b], the four ways are expressed using nested conditional sentences, in a way that extends the expressiveness of Dung's frameworks. Modgil [2009] has studied attacks of attacks (rightmost in 11) in a system that also extends Dung's expressiveness.

4.3 Formalizing argument schemes

Argumentation formalisms can only come to life when arguments are built from meaningful reasons. We already mentioned that Pollock made explicit which kinds of reasons he considered: deductive reasons, perception, memory, statistical syllogism, and induction.

An approach to the specification of meaningful kinds of reasons to construct arguments from is that of argument schemes, as they have been studied in argumentation theory. Argument schemes were already distinguished by Perelman and Olbrechts-Tyteca [1969].[36] In today's artificial intelligence research on argumentation, Douglas Walton's approach to argumentation schemes (his terminology) has been widely adopted (e.g., [Walton *et al.*, 2008].

Argument schemes can be thought of as analogues of the rules of inference of classical logic. An example of a rule of inference is, for instance, the following version of modus ponens:

> P
> If P, then Q
> Therefore: Q

[36]Although the term schème argumentative [argumentative scheme] was already used by Perelman and Olbrechts-Tyteca, according to Garssen [2001], van Eemeren *et al.* [1978; 1984] used the notion of argument(ation) scheme for the first time in its present sense. See also [van Eemeren and Grootendorst, 1992; Kienpointner, 1992; Walton, 1996; Walton *et al.*, 2008].

Whereas logical rules of inference, such as modus ponens, are abstract, strict, and (usually) considered to have universal validity, argumentation schemes are concrete, defeasible, and context-dependent. An example is the following scheme for witness testimony:

Witness A has testified that P.
Therefore: P

The use of this scheme is defeasible, as can be made explicit by asking critical questions, for instance:

Wasn't A mistaken?
Wasn't A lying?

A key reason why argument schemes have been taken up in artificial intelligence is that the critical questions associated with them correspond to defeating circumstances. For instance, the question whether A was mistaken gives rise to the defeater 'A was mistaken'.

Bex, Prakken, Reed and Walton [2003] applied the concept of 'argumentation schemes' to the formalization of legal reasoning from evidence. An example of a scheme in that paper (taken from [Walton, 1996]) is the following.

Argument from expert opinion
Source E is an expert in domain D.
E asserts that proposition A is known to be true (false).
A is within D.
Therefore, A may plausibly be taken to be true (false).

This scheme has the following critical questions:

1. Expertise question: How credible is E as an expert source?
2. Field question: Is E an expert in D?
3. Opinion question: What did E assert that implies A?
4. Trustworthiness question: Is E personally reliable as a source?
5. Consistency question: Is A consistent with what other experts assert?
6. Backup evidence question: Is E's assertion based on evidence?

The authors elaborate on how these and other argumentation schemes related to evidential reasoning can be formalized.

From the perspective of artificial intelligence, the work on argumentation schemes of Walton and his colleagues can be regarded as contributions to the theory of knowledge representation. Gradually, a collection of argumentation schemes is being developed. When appropriate, a scheme is added, and existing schemes are adapted, for instance, by refining the scheme's premises or critical questions. This knowledge representation point of view is developed by Verheij [2003b], who like Bex *et al.* [2003] formalizes argumentation schemes as defeasible rules of inference. He notes that in Walton's work argumentation schemes sometimes take the form of small derivations, or sequences of argumentation schemes; or even of a small prototypical dialogue. To streamline the work on knowledge representation, Verheij proposes to treat argumentation schemes as consisting of four elements: Conclusion, Premises, Conditions of use, and Exceptions. The Exceptions correspond to answers to the critical questions of an argumentation scheme. By this representation format, it is also possible to consider different roles of critical questions: critical questions concerning a conclusion, a premise, a condition of use, or an exception.

Reed and Rowe [2004) have incorporated argumentation schemes in their Araucaria tool for the analysis of argumentative texts. Rahwan *et al.* [2007] have proposed formats for the integration of argumentation schemes in what is called the Semantic Web. The vision underlying the Semantic Web is that, when information on the Internet is properly tagged, it becomes possible to add meaning to such information that can be handled by a machine. For instance, when the Conclusion, Premises, Conditions of use, and Exceptions of an argumentation scheme are marked as such, software can be built that can handle these different elements of a scheme appropriately. Gordon, Prakken and Walton [2007] have integrated argumentation schemes in their Carneades model.

A fundamental issue concerning argumentation schemes is how to evaluate a scheme or set of schemes. When is a scheme good, under which circumstances? When is an adaptation appropriate? This issue is, for instance, discussed in Reed and Tindale [2010].

4.4 Formalizing argumentation dialogues

One reason why Toulmin's [2003] *The uses of argument* remains a thought-provoking study is his starting point that argument should be considered in its natural, critical, and procedural context. This starting point led him to propose that logic, in the sense of the theory of good argument, should be treated as 'generalized jurisprudence,' where a critical and procedural perspective on good argument is the norm. The critical and procedural sides of arguments come together in the study of argumentation dialogues.

The following is a fragment, taken from McBurney and Parsons [2002a], of an argumentation dialogue concerning the sale of a used car between a buyer (B) and seller (S), illustrating the study of argumentative dialogue in a computational setting:

> S: BEGIN(PERSUASION(Make);
> PERSUASION(Condition_of_Engine);
> PERSUASION(Number_of_Owners))
> S requests a sequence of three Persuasion dialogues over the purchase criteria Make, Condition of the Engine, and Number of Owners.
> B:
> AGREE(PERSUASION(Make);PERSUASION(Condition_of_Engine);
> PERSUASION(Number_of_Owners)) PERSUASION Dialogue 1 in the sequence of three opens.
> S: Argues that 'Make' is the most important purchase criterion, within any budget, because a typical car of one Make may remain in better condition than a typical car of another Make, even though older.
> B: Accepts this argument.
> PERSUASION Dialogue 1 closes upon acceptance of the proposition by B. PERSUASION Dialogue 2 opens.
> S: Argues that that 'Condition_of_Engine' is the next most important purchase criterion.
> B: Does not accept this. Argues that he cannot tell the engine condition of any car without pulling it apart. Only S, as the Seller, is able to tell this. Hence, B must use 'Mileage' as a surrogate for 'Condition_of_Engine.'
> PERSUASION Dialogue 2 closes with neither side changing its views: B does not accept 'Condition_of_Engine' as the second criterion, and S does not accept 'Mileage' as the second criterion. PERSUASION Dialogue 3 opens.

The fragment shows how dialogues about certain topics are opened and closed in relation to the arguments provided.

The formal and computational study of argumentation dialogues has primarily been performed in the fields of AI and law and of multi-agent systems, as addressed below.

In the field of AI and law, argumentation dialogues have been studied extensively (see [Bench-Capon *et al.*, 2004; 2009]). Ashley's [1990] HYPO, to be discussed more extensively in Section 5.2, takes a 3–ply dialogue model as starting point, in which a proponent makes a claim, which can be attacked by an opponent, and then defended by the proponent. An early AI and law conception of argumentation dialogue is

Thomas Gordon's [1993; 1995] *Pleadings game.* Gordon formalizes the pleading in a US-style civil law process, which is aimed at determining the legal and factual issues of a case. In the Pleadings Game, a proponent and opponent (in this setting referred to as 'plaintiff' and 'defendant') can concede, deny and defend claims, and also declare defeasible rules. Players can discuss the validity of a defeasible rule. Players are committed to the consequences of their claims, as prescribed by a non-monotonic logic underlying the Pleadings Game.

Other dialogue models of argumentation in AI and law have been proposed by Prakken and Sartor [1996; 1998], Hage *et al.* [1993], and Lodder [1999]. In Prakken and Sartor's approach [1996; 1998], dialogue models are presented as a kind of proof theory for their argumentation model. Prakken and Sartor interpret a proof as a dialogue between a proponent and opponent. An argument is justified when there is a winning strategy for the proponent of the argument. Hage *et al.* [1993] and Lodder [1999] propose a model of argumentation dialogues with the purpose of establishing the law in a concrete case. They are inspired by the idea of law as a pure procedure (though not endorsing it): when the law is purely procedural, there is no criterion for a good outcome of a legal procedure other than the procedure itself.

Some models emphasize that the rules of argumentative dialogue can themselves be the subject of debate. An actual example is a parliamentary discussion about the way in which legislation is to be discussed. In philosophy, Suber has taken the idea of self-amending games to its extreme by proposing the game of Nomic, in which the players can gradually change the rules.[37] Proposals to formalize such meta-argumentation include Vreeswijk [2000] and Brewka [2001], who have proposed formal models of argumentative dialogues allowing self-amendments.[38]

In an attempt to clarify how logic, defeasibility, dialogue and procedure are related, Henry Prakken [1997, p. 270f.] proposed to distinguish four layers of argumentation models. The first is the logical layer, which determines contradiction and support. The second layer is dialectical, which defines what counts as attack, counterargument, and also when an argument is defeated. The third layer is procedural and contains the rules constraining a dialogue, for instance, which moves parties can make, when parties can make a move, and when the dialogue is finished. The fourth and final layer is strategic. At this layer, one finds the strategies and heuristics used by a good, effective arguer.

Jaap Hage [2000] addresses the question of why dialogue models of argumentation became popular in the field of AI and law. He gives two reasons. The first is that legal reasoning is defeasible, and dialogue models are a good tool to study

[37] `http://en.wikipedia.org/wiki/Nomic`. See also Hofstadter [1996, chapter 4].

[38] See also the study of Nomic by Vreeswijk [1995a].

defeasibility. The second reason is that dialogue models are useful when investigating the process of establishing the law in a concrete case. Hage recalls the legal theoretic discussion about the law as an open system, in the sense that there can be disagreement about the starting points of legal arguments. As a result, the outcome of a legal procedure is indeterminate. A better understanding of this predicament can be achieved by considering the legal procedure as an argumentative dialogue.

Hage [2000] then discusses three functions of dialogue models of argumentation in AI and law. The first function is to define argument justification, in analogy with dialogical definitions of logical validity as can be found in the work by Lorenzen and Lorenz [1978]. In this connection, Hage refers to Barth and Krabbe's notion of the 'dialectical garb' of a logic as opposed to an axiomatic, inferential or model-theoretic garb [Barth & Krabbe, 1982, pp. 7–8]. Hage generalizes the idea of dialectical garb to what he refers to as battle of argument models of defeasible reasoning in which arguments attack each other, such as Loui's [1987], Pollock's [1987; 1994], Vreeswijk's [1993], Dung's [1995], and Prakken and Sartor's [1996]. Battle of argument models can or cannot be presented in a dialectical garb. In their dialectical garb, such models define the justification of an argument in terms of the existence of a winning strategy in an argumentative dialogue game.

The second function of dialogue models of argumentation that is distinguished by Hage is to establish shared premises. Proponent and opponent enter into a dialogue that leads to a shared set of premises. The conclusions that follow from these shared premises can be regarded as justified. In this category, Hage discusses Gordon's Pleadings Game, which we discussed above. Hage makes connections to legal theory, in particular to Alexy's [1978] procedural approach to legal justification, and the philosophy of truth and justification, in particular Habermas's [1973] consensus theory of truth, and Schwemmer's approach to justification, in which the basis of justification is only assumed as long as it is not actually questioned [Schwemmer & Lorenzen, 1973].

As a third and final function of dialogue models of argumentation in AI and law, Hage discusses the procedural establishment of law in a concrete case. In this connection, he discusses mediating systems, which are systems that support dialogues, instead of evaluating them. He uses Zeno [Gordon & Karacapilidis, 1997], Room 5 [Loui *et al.*, 1997] (see also Section 4.5) and DiaLaw [Lodder, 1999] as examples. Hage argues that regarding the law as purely procedural is somewhat counterintuitive, since there exist cases in which there is a clear answer, which can be known even without actually going through the whole procedure. Hage speaks therefore of the law as an imperfect procedure, in which the correctness of the outcome is not guaranteed.

Outside the field of AI and law, one further function of dialogue models of argumentation has been emphasized, namely that a dialogue perspective on argumentation can have computational advantages. For instance, argumentative dialogue can be used to optimize search, for instance, by cutting off dead ends or focusing on the most relevant issues. Vreeswijk [1995b] takes this assumption as the starting point of a paper:

> If dialectical concepts like argument, debate, and resolution of dispute are seemingly so important in practical reasoning, there must be some reason as to why these techniques survived as rulers of commonsense argument. Perhaps the reason is that they are just most suited for the job [Vreeswijk, 1995b, p. 307].

Vreeswijk takes inspiration from a paper by Loui [1998], which circulated in an earlier version since 1992. Loui emphasises the relevance of protocol, the assignment of burdens to parties, termination conditions, and strategy. A key idea is that argumentation dialogues are well-suited for reasoning in a setting of bounded resources (see also [Loui & Norman, 1995]).

Inspired by the computational perspective on argumentation, approaches to argumentative dialogue have been taken up in the field of multi-agent systems.[39] The focus in that field is on the interaction between autonomous software agents that pursue their own goals or goals shared with other agents. Since the actions of one agent can affect those of another, beyond control of an individual agent or the system as a whole, the kinds of problems when designing multi-agent software systems are of a different nature than those in the design of software where control can be assumed to be centralized. Computational models of argumentation have inspired the development of interaction protocols for the resolution of conflicts among agents and for belief formation. The typology of argumentative dialogue that has been proposed by Douglas Walton and Erik Krabbe [1995] has been especially influential.[40] In this typology, seven dialogue types are distinguished:

1. Persuasion, aimed at resolving or clarifying an issue;
2. Inquiry, aimed at proving (or disproving) a hypothesis;
3. Discovery, aimed at choosing the best hypothesis for testing;
4. Negotiation, aimed at a reasonable settlement all parties can live with;

[39] For an overview of the field of multi-agent systems see the textbook by Wooldridge [2009], which contains a chapter entitled 'Arguing.'

[40] The 2000 Symposium on Argument and Computation at Bonskeid House Perthshire, Scotland, organized by Reed and Norman, has been a causal factor. See Reed and Norman [2004].

5. Information-seeking, aimed at the exchange of information;
6. Deliberation, aimed at deciding the best available course of action;
7. Eristic, aimed at revealing a deeper basis of conflict.

In particular, the persuasion dialogue, starting with a conflict of opinion and aimed at resolving the issue by persuading a participant, has been extensively studied. An early persuasion system—focusing on persuasion in a negotition setting—is Sycara's Persuader system [1989]. Persuader, developed in the field of what was then called Distributed AI, uses the domain of labour negotiation as an illustration. An agent forms a model of another agent's beliefs and goals, and determines its actions in such a way that it influences the other agent. For instance, agents can choose a so-called 'threatening argument,' i.e., an argument that is aimed at persuading another agent to give up a goal. Here it is notable that in Walton and Krabbe's typology negotiation is a dialogue type different from persuasion.

Prakken [2006; 2009] gives an overview and analysis of dialogue models of persuasion. In a dialogue system, dialogues have a goal and participants. It is specified which kinds of moves participants can make, for instance, making claims or conceding. Participants can have specific roles, for instance, Proponent or Opponent. The actual flow of a dialogue is constrained by a protocol, consisting of rules for turn-taking and termination. Effect rules determine how the commitments of participants change after each dialogue move. Outcome rules define the outcome of the dialogue, by determining, for instance, in persuasion dialogues who wins the dialogue. These elements are common to all dialogue types. By specifying or constraining the elements, one generates a system of persuasion dialogue. In particular, the dialogue goal of persuasion dialogue consists of a set of propositions that are at issue and need to be resolved. Prakken formalizes these elements and then uses his analytic model to discuss several extant persuasion systems, among them Mackenzie's [1979] proposals, and Walton and Krabbe's [1995] model of what they call Permissive persuasion dialogue.

Sycara's Persuader system [1989] is a persuasion system applied to labour negotiation. Parsons, Sierra and Jennings [1998] also speak of negotiation as involving persuasion. Their model uses the Belief-Desire-Intention model of agents [Rao & Georgeff, 1995] and specifies logically how the beliefs, desires and intentions of the agents influence the process of negotiation.[41] Dignum, Dunin-Kęplicz and Verbrugge [2001] have studied the role of argumentative dialogue for the forming of coalitions

[41] A systematic overview of argumentation dialogue models of negotiation has been provided by Rahwan *et al.* [2003].

of agents that create collective intentions. Argumentation about what to do rather than about what is the case has been studied in a dialogue setting by Atkinson and colleagues [Atkinson *et al.*, 2005; 2006; Atkinson & Bench-Capon, 2007]. In this connection, it is noteworthy that Pollock's OSCAR model [1995] is an attempt to combine theoretical reasoning—about what to believe—with practical reasoning—about what to do—, though in a single agent, non-dialogical setting. Amgoud [2009] discusses the application of dialogical argumentation to decision making (see also [Girle *et al.*, 2004]). Deliberation has been studied by McBurney *et al.* [2007].

Several attempts have been made to systematize the extensive work on argumentation dialogue. Bench-Capon *et al.* [2000], for instance, propose a formal method for modelling argumentation dialogue. Prakken [2005b] provides a formal framework that can be used to study argumentation dialogue models with different choices of underlying argument model and reply structures. McBurney and Parsons [2002a; 2002b; 2009] have developed an abstract theory of argumentative dialogue in which syntactic, semantic, and pragmatic elements are considered.

4.5 Argumentation support software

When studying argumentation from an artificial intelligence perspective, it can be investigated how software tools can perform or support argumentative tasks. Some researchers in the field of argumentation in AI have openly addressed themselves to building an artificial arguer. The most prominent among them is John Pollock (see also Section 3.4), who titled one of his books about his OSCAR project ambitiously *How to build a person* [Pollock, 1989].[42] Most researchers however have not aimed at realizing the grand task of addressing the so-called 'strong AI' problem of building an intelligent artefact that can perform any intellectual task a human being can. Instead of building software mimicking human argumentative behaviour, the more modest aim of supporting humans performing argumentative tasks was chosen. A great deal of research has been aimed at the construction of argumentation support software. Here we discuss three recurring themes: argument diagramming in software, the integration of rules and argument schemes, and argument evaluation.[43]

Argument diagramming in software The first theme discussed is argument diagramming in software. In the literature on argumentation support software, much attention has been paid to argument diagramming. Different kinds of argument diagramming styles have been proposed, many inspired by non-computational research

[42]The book's subtitle adds modestly: A Prolegomenon.

[43]The reviews by Kirschner *et al.* [2003], Verheij [2005b], and Scheuer *et al.* [2010] provide further detail about argumentation support software.

on argument diagrams. We shall discuss three styles: boxes and arrows, boxes and lines, and nested boxes.

The first style of argument diagramming uses boxes and arrows. Argumentative statements are enclosed in boxes, and their relations indicated by arrows. A common use of arrows is to indicate the support relation between a reason and a conclusion. An example of a software tool that uses boxes and arrows diagrams is the Araucaria tool by Chris Reed and Glenn Rowe [2004] (Figure 9[44]). The Araucaria tool has been designed for the analysis of written arguments. Vertical arrows indicate reasons and their conclusions, and horizontal bi-directional arrows indicate conflicts between statements. The Araucaria software was one step in the development by the Dundee Argumentation Research Group, led by Reed, of open source argumentation software. For this purpose, a representation format, called the Argument mark-up language (AML), has been developed that allows for the exchange of arguments and their analyses using contemporary Internet technology. The format also allows for the exchange of sets of argument schemes (see Section 4.3) that can be used for argument analysis. Connected developments concerning machine-readable argument representation formats are the Argument interchange format [Chesñevar *et al.*, 2006] and ArgDF, a proposal for a language allowing for a World wide argument web [Rahwan *et al.*, 2007]. One aim of the latter work is to develop classification systems for arguments, using ontology development techniques in Artificial Intelligence. In AI, an 'ontology' is a systematic conceptualization of a domain, often taking the form of a hierarchical system of concepts and their relations.

Another example of a system using boxes and arrows is the Hermes system [Karacapilidis & Papadias, 2001], an extension of the Zeno system [Gordon & Karacapilidis, 1997]. Both Hermes and Zeno have been inspired by the IBIS approach. In IBIS, an abbreviation of Issue-Based Information Systems [Kunz & Rittel, 1970], problems are analysed in terms of issues, questions of fact, positions, and arguments. The focus is on what Rittel and Webber [1973] call wicked problems: problems with no definitive formulation, and no definitive solutions. Hence a goal of IBIS and systems such as Hermes and Zeno is to support the identification, structuring and settling of issues.

The second style of argument diagramming uses boxes and lines. In a boxes and lines style of argument diagramming, argumentative statements are depicted in boxes and their relations are indicated by (undirected) lines between them. This diagramming style abstracts from the directionality between statements, for instance, from a reason to a conclusion, or from a cause to an event. An example of a tool using the boxes and lines style is the Belvedere system [Suthers *et al.*, 1995; Suthers,

[44]Source: `http://staff.computing.dundee.ac.uk/creed/araucaria/`.

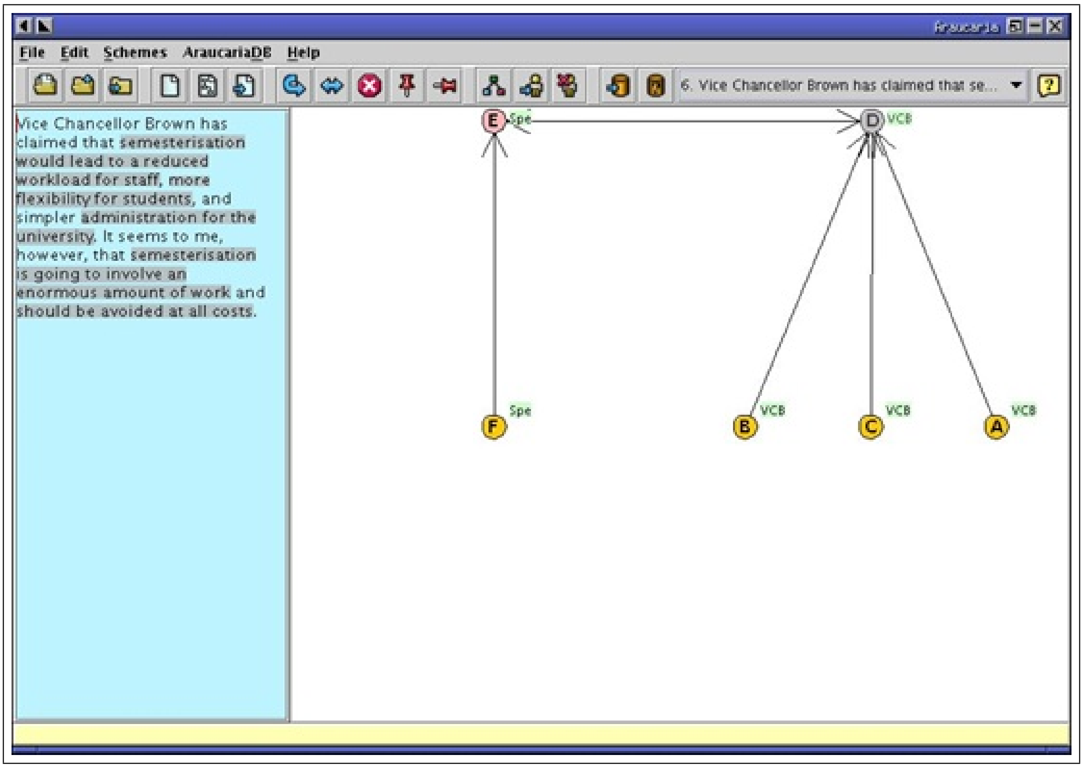

Figure 9: Boxes and arrows diagramming: The Araucaria system

1999]. A goal of the system was to stimulate the critical discussion of science and public policy issues by middle school and high-school students, taking the cognitive limitations of the intended users into account. Such limitations include difficulty in focusing attention, lack of domain knowledge, and lack of motivation. In early versions, the diagrams were richly structured: there were links for support, explanation, causation, conjunction, conflict, justification, and undercutting. Link types could be distinguished graphically and by label. To prevent unproductive discussions about which structure to use, the graphical representation was significantly simplified in later versions [Suthers, 1999]. Two types of statements were distinguished: data and hypotheses; and two link types: expressing a consistency and an inconsistency relation between statements. Figure 10[45] shows an example of a Belvedere screen using an even further simplified format with one statement type and one link type.

The third style of argument diagramming uses nested boxes. In this style, too, the argumentative statements are enclosed in boxes, but their relationships are indi-

[45]Source: `http://belvedere.sourceforge.net/`.

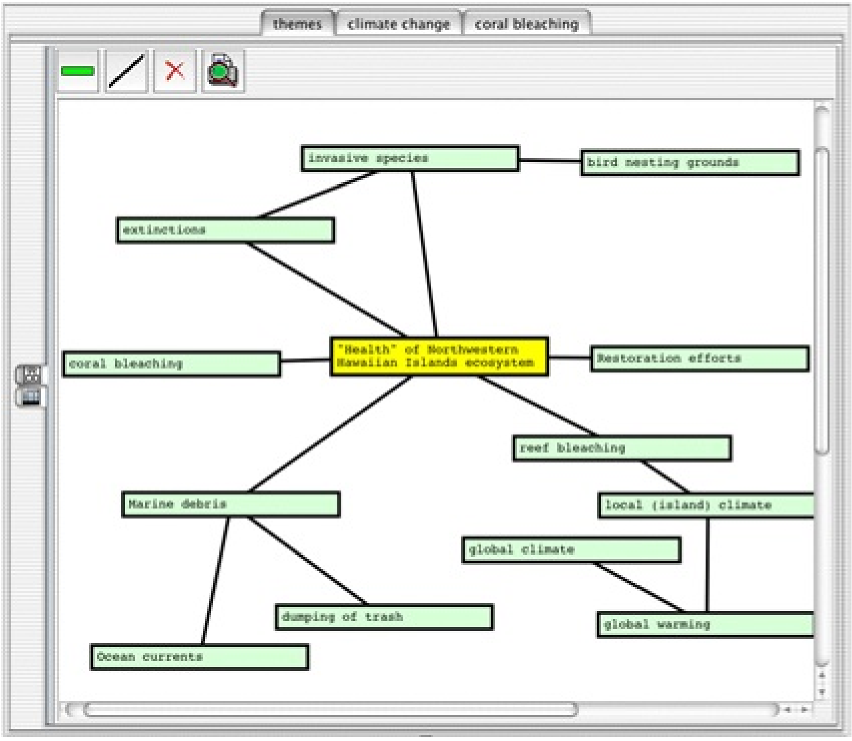

Figure 10: Boxes and lines diagramming: the Belvedere 4.1 system

cated by the use of nesting. An example of the use of nested boxes is the Room 5 tool designed by Loui, Norman and a group of students [Loui *et al.*, 1997]. The Room 5 system aimed at the collaborative public discussion of pending Supreme Court cases. It was web-based, which is noteworthy as the proposal predates Google and Wikipedia. In its argument-diagramming format, a box inside a box expresses support, and a box next to a box indicates attack. In the argument depicted in the Room 5 screen shown in Figure 11[46], for instance, the punishability of John is supported by the reason that he has stolen a CD, and attacked by the reason that he is a minor first offender.

The integration of rules and argument schemes A second theme concerning the design of argumentation support software is the integration of rules and argument schemes. The integration of rules and argument schemes in argument diagramming software has been addressed in different ways: by the use of schematic

[46]Screenshot of Room 5, as shown in Verheij [2005b]. See also Bench-Capon *et al.* [2012].

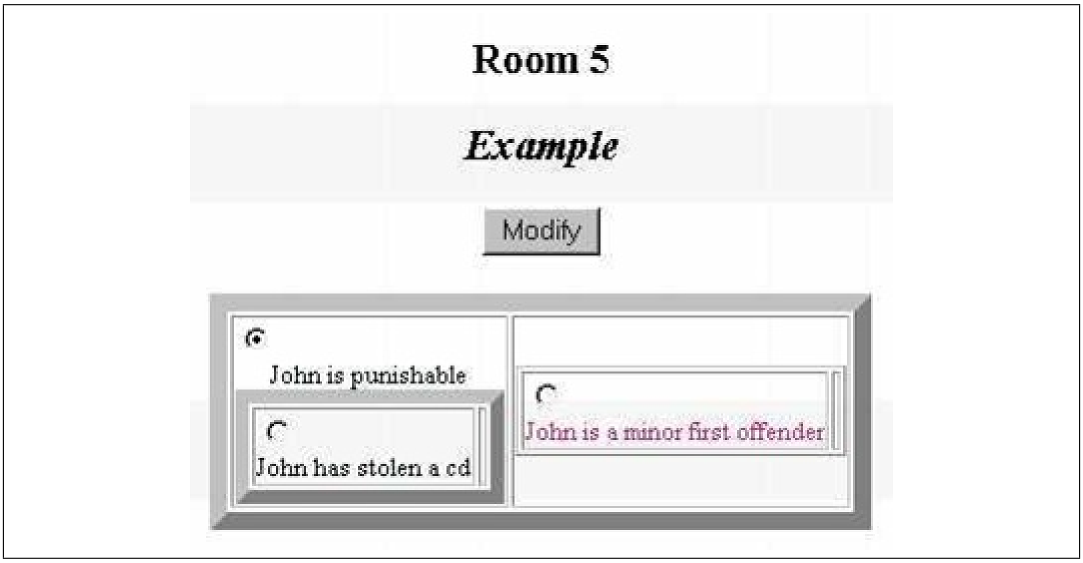

Figure 11: Nested boxes diagramming: the Room 5 system

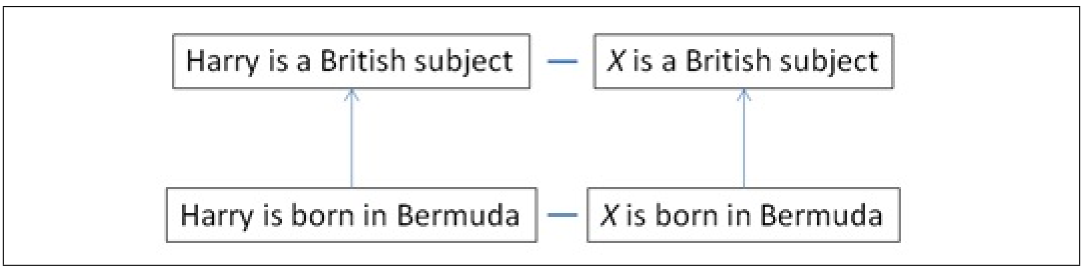

Figure 12: An elementary argument step as an instance of a schematic argument

arguments, conditional sentences, nested arrows and rule nodes. Consider, for instance, the elementary argument that Harry is a British subject because he is born in Bermuda (borrowed from Toulmin), and its underlying rule (or 'warrant' in Toulmin's terminology) that people born in Bermuda are British subjects.

A first approach is to consider such an argument as an instance of a scheme that abstracts from the person Harry in the argument. In Figure 12, an associated schematic argument is shown to the right of the argument about Harry. In the schematic argument, X appears as a variable that serves as the placeholder of someone's name. In software, the schematic argument is normally not shown graphically.

A second approach uses conditional sentences. The conditional sentence that expresses the connection between reason and conclusion is made explicit as an auxiliary

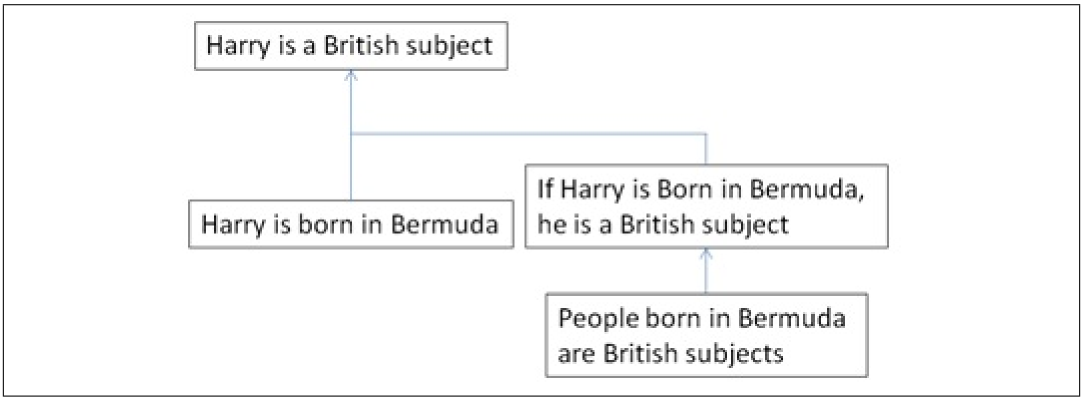

Figure 13: Using a conditional sentence

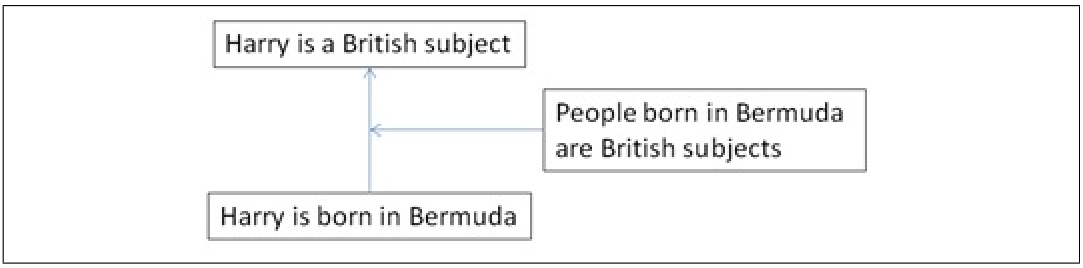

Figure 14: Nested arrows

premise. This conditional sentence can then be supported by further arguments, such as a warrant (as in Figure 13) or a backing. This approach is, for instance, proposed in the user-friendly Rationale[47] tool developed by van Gelder and his collaborators [van Gelder, 2007].

A third approach uses nested arrows. The arrows are treated as graphical expressions of the connection between the reason and conclusion, and can hence be argued about. In Figure 14, for instance, the warrant has been supplied as support for the connection between reason and conclusion. This approach has a straightforward generalisation when support and attack are combined (Section 4.2). The ArguMed tool developed by Verheij [2005b] uses this approach.

A variation of the nested arrows approach uses rule nodes (Figure 15), instead of nested arrows. The AVERs tool [van den Braak *et al.*, 2007] uses this approach.

[47] `http://rationale.austhink.com/`.

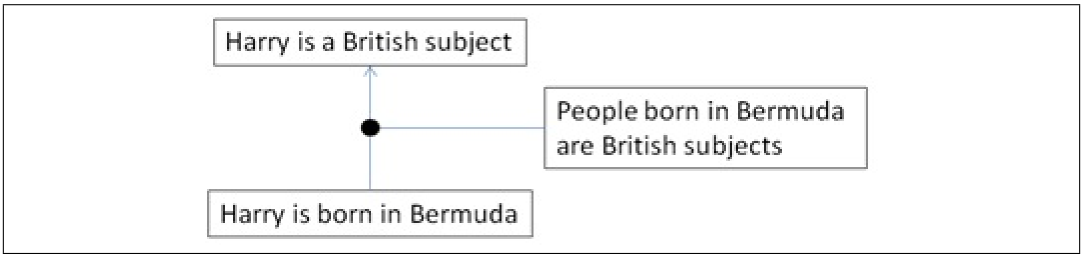

Figure 15: Rule nodes

Argument evaluation The third and final theme that we discuss in connection with the design of argumentation support software is argument evaluation. In argumentation software, different strategies for argument evaluation have been implemented. Some tools choose to leave argument evaluation as a task for the user of the system. For instance, in the Rationale system [van Gelder, 2007] a user can indicate which claims follow or do not follow given the reasons in the diagram. Specific graphical elements are used to show the user's evaluative actions.

In several other systems, some form of automatic evaluation has been implemented. Automatic evaluation algorithms can be logical, or numeric.

Logical evaluation algorithms in argumentation support tools have been grounded in versions of argumentation semantics (see Section 4.1). For instance, ArguMed [Verheij, 2005b] computes a version of stable semantics. Consider, for instance, Pollock's example of an undercutting defeater about red lights (see Section 3.4). ArguMed's evaluation algorithm behaves as expected: when the reason that the object looks red is assumed, the conclusion that the object is red will be justified, but that will no longer be the case when the defeater is added that the object is illuminated by a red light. A typical property of logical evaluation algorithms is reinstatement: when a defeating attacker of an initial argument is successfully attacked, the initial argument will no longer count as defeated and therefore be reinstated.

Numeric evaluation algorithms have been based on the numeric weights of the reasons supporting and attacking conclusions. A weight-based numeric evaluation algorithm has, for instance, been implemented in the Hermes system [Karacapilidis & Papadias, 2001]. In Hermes, positions can be assigned a numeric score by adding the weights of active pro-positions and subtracting the weights of active con-positions. A proof standard can be used to determine an activation label of a position. In the proof standard called Preponderance of evidence, for instance, a position is active when the active pro-positions outweigh the active con-positions.

A numeric evaluation algorithm of a different kind has been implemented in the so-called 'Convince me' system [Schank, 1995]. It uses ECHO, which is a connectionist version of Thagard's [1992] theory of explanatory coherence. In Convince me, statements are assigned numerical values by a step-wise constraint satisfaction algorithm. In the algorithm, incremental changes of the default weights of a statement are made by considering the excitatory and inhibitory links connected to a statement. When changes become too small to be taken into account (or computation is taking too long), the algorithm stops.

5 Specific kinds of argumentation in formal and computational perspective

In this section, we discuss specific kinds of argumentation using rules, cases, values and evidence. We end the section with applications and case studies.

5.1 Reasoning with rules

We already saw examples showing the close connections between argumentation research in artificial intelligence and legal applications. Since argumentation is an everyday task of professional lawyers this is not unexpected. An institutional reason however is that there exists an interdisciplinary research field, called artificial intelligence and law,[48] in which because of the nature of law the topic of argumentation has been given a great deal of attention. Early work in that field (e.g., [McCarty, 1977; Gardner, 1987]) already showed the intricacies and special characteristics of legal argumentation. Thorne McCarty [1977] attempted to formalize the detailed reasoning underlying a US Supreme Court case. Anne Gardner [1987] proposed a system aimed at what she called issue spotting. In a legal case, there is an issue when no rule applies or when conflicting rules apply and the conflict cannot be resolved. In this section, we pay special attention to the work inspired by developments in non-monotonic logic that has been carried out, mostly in the mid-1990s, regarding reasoning with (legal) rules.

Henry Prakken's [1997] book Logical tools for modelling legal argument provides an extensive and careful treatment of the contributions of techniques from non-monotonic logic to the formal modelling of legal reasoning.[49] The formal tools presented by Prakken have gradually evolved into the ASPIC+ model already mentioned [Prakken, 2010]. Parts of the material were developed in close collaboration

[48]The primary journal of the field of AI & law is *Artifical Intelligence and Law*, with the biennial ICAIL and annual JURIX as the main conferences.

[49]The book is based on Prakken's [1993] doctoral dissertation.

with Sartor (e.g., [Prakken & Sartor, 1996; 1998]; see also the excellent resource [Sartor, 2005]).

The following example shows how Prakken models a case in contract law [1997, p. 171]. The example concerns the defeasible rule that contracts only bind the contracting parties (d_1), and a defeasible, possibly contravening, rule specifically for contracts that concern the lease of a house, saying that such contracts also bind future owners of the house (d_2). Another exception is added by a defeasible rule saying that, even in the case of a house lease, when a tenant agrees to make such a stipulation only the contracting parties are bound (d_3). The factual statements f_1 and f_2 say respectively (1) that a house lease is a special kind of contract and (2) that binding only the contracting parties and binding also future owners of a house do not go together.

d_1 : x is a contract $\Rightarrow$ x only binds its parties
d_2 : x is a lease of house $y \Rightarrow x$ binds all owners of y
d_3 : x is a lease of house $y\wedge$ tenant has agreed in x that x only binds its parties $\Rightarrow$ x only binds its parties
f_1 : $\forall x \forall y$ (x is a lease of a house $y \rightarrow x$ is a contract)[50]
f_2 : $\forall x \forall y \neg$ (x only binds its parties $\wedge x$ binds all owners of y)

When there is a contract about the lease of a house, there is an apparent conflict, since both d_1 and d_2 seem to apply. In the system, the application of d_2 blocks the application of d_2, using a mechanism of specificity defeat (see Section 4.2). In a case where also the condition of d_3 is fulfilled, namely when the tenant has agreed that the lease contract only binds the contracting parties, the application of rule d_3 blocks the application of rule d_2, which in that case does no longer block the application of d_1.

Prakken uses elements from classical logic (for instance, classical connectives and quantifiers) and non-monotonic logic (defeasible rules and their names), and shows how they can be used to model rules with exceptions, as they occur prominently in the law. He treats, for instance, the handling of explicit exceptions, preferring the most specific argument, reasoning with inconsistent information, and reasoning about priority relations.

In the same period, Hage developed Reason-based logic ([Hage, 1997]; see also [Hage, 2005)].[51] Hage presents Reason-based logic as an extension of first-order predicate logic in which reasons play a central role. Reasons are the result of the

[50]'$\forall x$...' stands for 'for every entity x it holds that ...'. Similarly, for '$\forall y$...'

[51]Reason-based logic exists in a series of versions, some introduced in collaboration with Verheij (e.g., [Verheij, 1996a]).

application of rules.[52] Treating them as individuals allows the expression of properties of rules. Whether a rule applies depends on the rule's conditions being satisfied, but also on possible other reasons for or against applying the rule. Consider, for instance, the rule that thieves are punishable:

punishable: thief(x) $\Rightarrow$ punishable(x)

Here 'punishable' before the colon is the rule's name. When John is a thief (expressed as thief(john)), the rule's applicability can follow:

Applicable(thief(john) $\Rightarrow$ punishable(john))

This gives a reason that the rule ought to be applied. If there are no reasons against the rule's application, this leads to the obligation to apply the rule. From this it will follow that John is punishable.

A characteristic aspect of Reason-based logic is that it models the weighing of reasons. In this system, there is no numerical mechanism for weighing; rather it can be explicitly represented that certain reasons for a conclusion outweigh the reasons against the conclusion. When there is no weighing information the conflict remains unresolved and no conclusion follows.

Like Prakken, Hage uses elements from classical logic and non-monotonic logic. In his theory, because of the emphasis on philosophical and legal considerations, the flavour of Reason-based logic is less that of pure logic, but comes closer to representing the ways of reasoning in the domain of law. Where Prakken's book remains closer to the field of AI, Hage's book reads more like a theoretical essay in philosophy or law.

Reason-based logic has been applied, for instance, to a well-known distinction made by the legal theorist Dworkin [1978]: whereas legal rules seem to lead directly to their conclusion when they are applied, legal principles are not as direct, and merely give rise to a reason for their conclusion. Only a subsequent weighing of possibly competing reasons leads to a conclusion. Different models of the distinction between rules and principles in Reason-based logic have been proposed. Hage [1997] follows Dworkin and makes a strict formal distinction, whereas Verheij *et al.* [1998] show how the distinction can be softened by presenting a model in which rules and principles are the extremes of a spectrum.

Loui and Norman [1995] have argued that there is a calculus associated with what they call the compression of rationales, i.e., the combination and adaptation

[52] We shall simplify Hage's formalism a bit by omitting the explicit distinction between rules and principles.

of the rules underlying arguments which are akin to Toulmin's warrants. They give the following example of a compression of rules (rationales). When there is a rule 'vehicles used for private transportation are not allowed in the park' and also a rule 'vehicles are normally for private transportation,' then a two-step argument based on these two rules can be shortened when the so-called compression rationale 'no vehicles in the park,' based on these two rules, is used.

5.2 Case-based reasoning

Reasoning with rules (Section 5.1) is often contrasted with case-based reasoning. Whereas the former is about following rules that describe existing conditional patterns, the latter is about finding relevantly similar examples that, by analogy, can suggest possible conclusions in new situations. In the domain of law, rule-based reasoning is associated with the application of legal statutes, and case-based reasoning with the following of precedents. The contrast can be appreciated by looking at the following two examples.

> *Art. 300 of the Dutch Criminal Code*
> 1. Inflicting bodily harm is punishable with up to two years of imprisonment or a fine of the fourth category.
> 2. When the fact causes grievous bodily harm, the accused is punished with up to four years of imprisonment or a fine of the fourth category.
> 3. [...]

> *Dutch Supreme Court July 9, 2002, NJ 2002, 499*
> Theft requires the taking away of a good. Can one steal an already stolen car? The Supreme Court's answer is: yes.

The first example is an excerpt from a statutory article expressing a material rule of Dutch criminal law, stating the kinds of punishment associated with inflicting bodily harm. The levels of punishment depend on specific conditions, with more severe bodily harm being punishable with longer imprisonment. The second example is a (very) brief summary of a Supreme Court decision. In this case, an already stolen car was stolen from the thief. One of the statutory requirements of the crime theft is that a good is taken away, and here the car was already taken away from the original owner of the car. The new legal question was addressed whether stealing from the original thief can count as theft from the car's owner. In other words, can an already stolen car still be taken away from the original owner? Here the Supreme Court decided that stealing a stolen car can count as theft since the original ownership is the deciding criterion; it does not matter whether a good is actually in the control of the owner at the time of theft. When used as a precedent, this Supreme Court decision has the effect that similar cases are decided alike.

In case-based reasoning, the stare decisis doctrine is leading: when deciding a new case one should not depart from an earlier, relevantly similar decision, but decide analogously. In the field of AI and law, Kevin Ashley's HYPO system [1990] counts as a milestone in the study of case-based reasoning.[53] In HYPO, cases are treated as sets of factors, where factors are generalised facts pleading for or against a case. Consider the following example about an employee who has been dismissed by his employer, and aims to void (i.e., cancel) the dismissal.[54]

Issue:
  Can a dismissal be voided?

Precedent case:
+ The employee's behaviour was always good.
- There was a serious act of violence.
Outcome:
+ (voided)

Current case:
+ The employee's behaviour was always good.
- There was a serious act of violence.
+ The working atmosphere was not affected.
Outcome:
?

There is a precedent case with one factor pleading for voidance (the good behaviour), and one pleading against voidance (the violence). In this precedent case, it was decided that voidance was in place. In the current case, the same factors apply, but there is also one additional factor pleading for voidance, namely that the working atmosphere was not affected. One could say that the decision taken in the precedent case is even more strongly supported in the current case. As a result, in HYPO and similar systems the suggested conclusion is that also in the current case voidance of the dismissal would be called for.

[53]See also Rissland and Ashley [1987], Ashley [1989], and Rissland and Ashley [2002].
[54]The example is inspired by the case material used by Roth [2003].

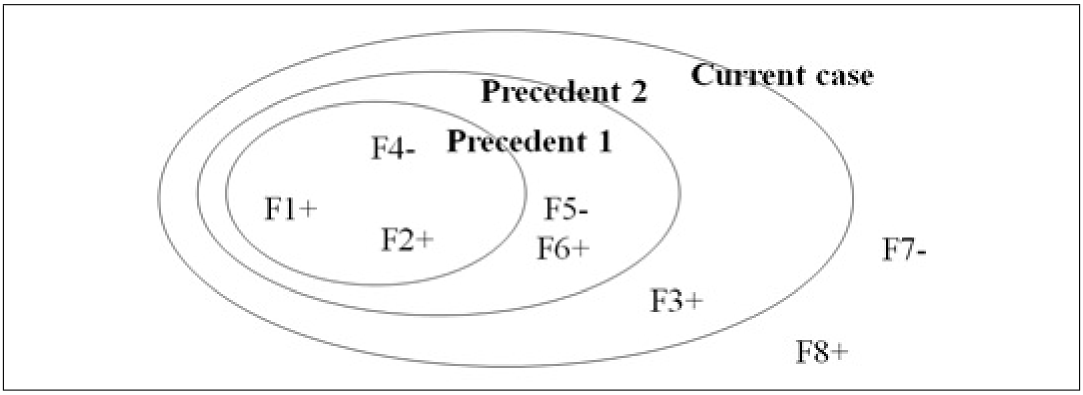

Figure 16: Factors in two precedent cases and the current case

The example in Figure 16 shows that factors can be handled formally without knowing what they are about. There is a first precedent with pro-factors F1 and F2 and a con-factor F4. The second precedent has as additional factors a con-factor F5 and a pro-factor F6. The current case has all these factors and one more pro-factor F3. The domain also contains con-factor F7 and pro-factor F8 which do not apply to these cases.

Assume now that the first precedent was decided negatively, and the second positively. The second precedent is more on point, in the sense that it shares more factors with the current case than the first precedent. Since the current case even has an additional pro-factor, it is suggested that the current case should be decided positively, in analogy with precedent 2. Precedents do not always determine the outcome of the current case. For instance, if the second precedent had been decided negatively, there would be no suggested outcome for the current case, since pro-factor F3 may be or may not be strong enough to turn the case.

Another formal example is shown in Figure 17. When both precedents have been decided positively, the suggested outcome for the current case is also positive. Precedent 1 can be followed because its support for a positive decision is weaker than that of the current case: the precedent has an additional con-factor, and the current case an additional pro-factor. Precedent 2 cannot be followed since F8 may be or may not be a stronger pro-factor than F3.

HYPO's aim is to form arguments about the current case, without determining a decision. This is made explicit in its model of 3–ply arguments. In HYPO's 3–ply model, the first argument move ('ply'), by the Proponent, is the citing of a precedent case in analogy with the current case. The analogy is based on the shared factors. The second argument move, by the Opponent, responds to the analogy, for instance,

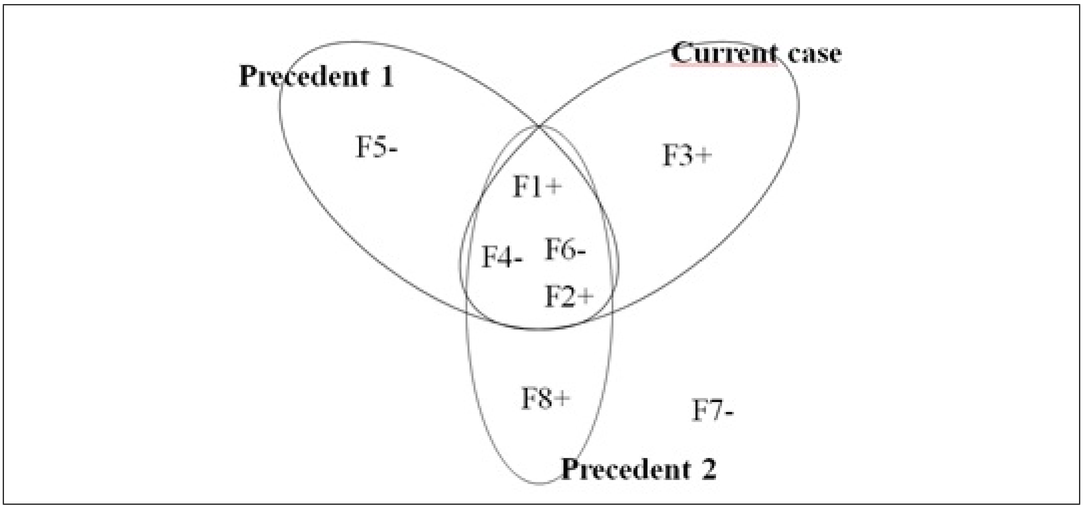

Figure 17: A different constellation of precedents

by distinguishing between the cited precedent case and the current case, pointing out differences in relevant factors, or by citing counterexamples. The third argument move, again by the Proponent, responds to the counterexamples, for instance, by making further distinctions.

HYPO's factors not only have a side (pro or con) associated with them, but can also come with a dimension pertaining in some way to the strength of the factor. This allows the citation of cases that share a certain factor, but have this factor with a different strength. For instance, by the use of dimensions, the good behaviour of the employee (of the first informal example) can come in gradations, say from good, via very good to excellent.

Vincent Aleven extended the HYPO model by the use of a factor hierarchy that allowed modelling of factors with hierarchical dependencies [Aleven, 1997; Aleven & Ashley, 1997a; 1997b]. For instance, the factor that one has a family to maintain is a special case of the factor that one has a substantial interest in keeping one's job. Inspired by Verheij's DefLog model [2003a], which allowed for reasoning about support and attack (Section 4.2), Roth [2003] developed case-based reasoning based on what he referred to as an entangled factor hierarchy, in order to expand the possible argumentative moves (Figure 18). For instance, the relevance of the factor that one has a family to maintain is strengthened by one's having children that go to university and weakened by one's having a wife with a good income. A factor hierarchy allows new kinds of argument moves by making it possible to downplay or emphasize a distinction. For instance, the factor of having a family to maintain

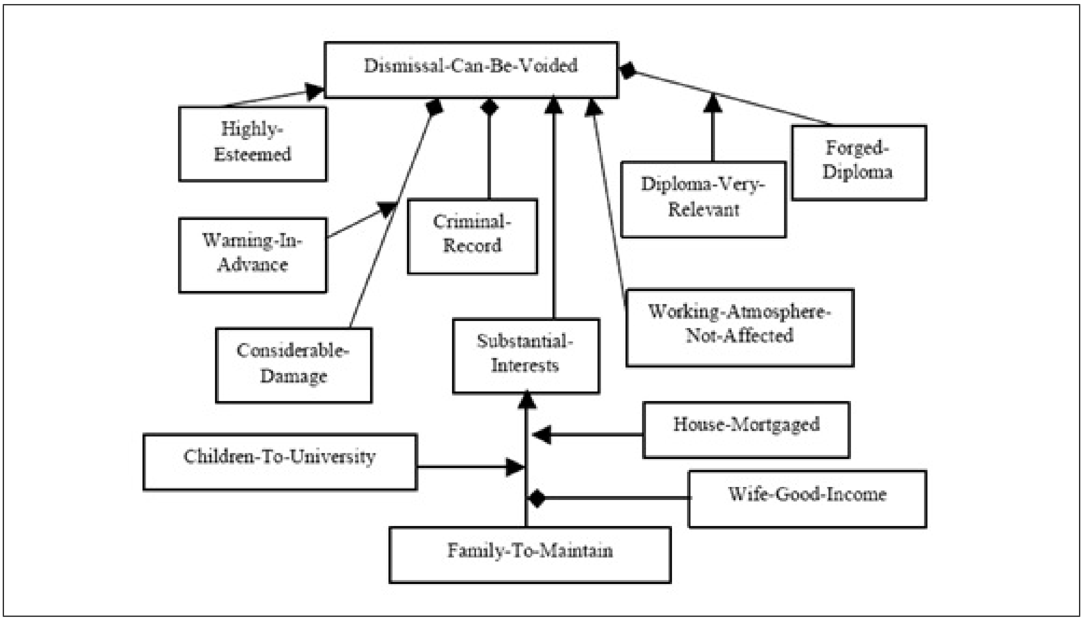

Figure 18: An entangled factor hierarchy [Roth, 2003]

can be downplayed by pointing out that one has a partner with a good income, or emphasized by mentioning that one has children going to university.

Proposals have been made to combine case-based and rule-based reasoning. For instance, Branting's GREBE model [1991; 2000] aims to generate explanations of decisions in terms of rules and cases. Both rules and cases can serve as warrants for a decision. Branting extends Toulmin's approach to warrants by using a so-called warrant reduction graph, in which warrants can be special cases of other warrants. Prakken and Sartor [1998] have applied their model of rule-based reasoning ([Prakken & Sartor, 1996]; see also Section 5.1) to the setting of case-based reasoning. Analogizing and distinguishing are connected to the deletion and addition of rule conditions that describe past decisions.

5.3 Values and audiences

Trevor Bench-Capon [2003] has developed a model of the values underlying arguments.[55] In this endeavour he refers to Perelman and Olbrechts-Tyteca's new rhetoric:

[55]In AI and law, the importance of the modelling of the values and goals underlying legal decisions was already acknowledged by Berman and Hafner [1993].

> If men oppose each other concerning a decision to be taken, it is not because they commit some error of logic or calculation. They discuss apropos the applicable rule, the ends to be considered, the meaning to be given to values, the interpretation and characterisation of facts [Perelman & Olbrechts-Tyteca, 1969, p. 150].

Because of the character of real-life argumentation, it is not to be expected that cases will be conclusively decided. Bench-Capon therefore aims to extend formal argumentation models by the inclusion of the values of the audiences addressed. This allows him to model the persuasion of an audience by means of argument.

Bench-Capon [2003] uses Dung's [1995] abstract argumentation frameworks as a starting point. He defines a value-based argumentation framework as a framework in which each argument has an associated (abstract) value. The idea is that values associated with an argument are promoted by accepting the argument. For instance, in a parliamentary debate about a tax raise it can be argued that accepting the raise will promote the value of social equality, while the value of enterprise is demoted. In an audience-specific argumentation framework, the preference ordering of the values can depend on an audience. For instance, the Labour Party may prefer the value of social equality, and the Conservative Party that of enterprise.

Bench-Capon continues to model defeat for an audience: an argument A defeats an argument B for audience *a* if A attacks B and the value associated with B is not preferred to the value associated with A for audience *a*. In his model, an attack succeeds, for instance, when the arguments promote the same value, or when there is no preference between the values. Dung's notions of argument acceptability, admissibility and preferred extension are then redefined relative to audience attack.

Bench-Capon uses a value-based argumentation framework with two values 'red' and 'blue' as an example (Figure 19). The underlying abstract argumentation framework is the same as that in Figure 6. In its unique preferred extension (which is also grounded and stable), A and C are accepted and B is rejected. For an audience preferring 'red,' defeat for the audience coincides with the underlying attack relation. In the preferred extension for an audience preferring 'red,' therefore, A and C are accepted and B is rejected. However, for an audience preferring 'blue,' A does not defeat B. But for such an audience B still defeats C. For a 'blue'-preferring audience, A and B are accepted and C is not.

Bench-Capon illustrates value-based argumentation by considering the case of a diabetic who almost collapses into a coma by lack of insulin, and therefore takes another diabetic's insulin after entering her house. He analyses the case by discussing the roles of the value of property right infringement as opposed to that of saving one's life.

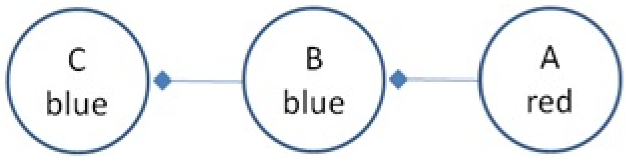

Figure 19: A value-based argumentation framework with two values (adapted from [Bench-Capon, 2003])

Bench-Capon and Sartor [2003] have used the value-based perspective in a treatment of legal reasoning that combines rule-based and case-based reasoning (see Sections 5.1 and 5.2). Legal reasoning takes the form of constructing and using a theory that explains a decision in terms of the values promoted and demoted by the decision. Precedent decisions have the role of revealing preferences holding between factors. This is similar to the role of precedents in HYPO that reveal how the factors in a precedent case are weighed. In Bench-Capon and Sartor's approach, the factor preferences in turn reveal preferences between values. The resulting preferences can then be used to decide new cases.

5.4 Burden of proof, evidence, and argument strength

Some arguments are more successful than others. An argument can meet or not meet the burden of proof fitting the circumstances of the debate. An argument can be founded on better evidence than another. An argument can also be stronger than another. In this section, we address the topics of burden of proof, evidence and argument strength.

Burden of proof and evidence The topic of burden of proof is strongly connected to the dialogical setting of argumentation. A burden of proof is assigned to a party in an argumentative dialogue when the quality of the arguments produced in the dialogue depends in part on whether the arguments produced by that party during the dialogue meet certain constraints. Such constraints can be procedural, for instance, requiring that a counterargument is met by a counterattack, or material, for instance, requiring that an argument is sufficiently strong in the light of the other arguments. Constraints of the latter, material, non-procedural type are also referred to as proof standards.

The topic of burden of proof is especially relevant in the law, as argumentation in court is often constrained by burden of proof constraints. As a result, in legal theory the topic has been studied extensively. The topic has also been addressed in

AI approaches to argumentation, in particular by researchers connected to the field of AI and law (see also Section 4.4). In the Carneades argumentation model [Gordon *et al.*, 2007], for instance, statements are categorized using three proof standards:

> *SE (Scintilla of Evidence).* A statement meets this standard if and only if it is supported by at least one defensible pro argument.
> *BA (Best Argument).* A statement meets this standard if and only if it is supported by some defensible pro argument with priority over all defensible con arguments.
> *DV (Dialectical Validity).* A statement meets this standard if and only if it is supported by at least one defensible pro argument and none of its con arguments are defensible.

A theme related to proof standards is argument accrual. What happens when there are several arguments for a conclusion? See Section 4.2, where research addressing the relation between argument defeat and accrual is discussed.

AI models of argumentation have been helpful in clarifying distinctions made in legal theory. Prakken and Sartor in particular have in a series of articles [Prakken & Sartor, 2007; 2009] contributed to the explication of different forms of burden of proof. They distinguish a burden of persuasion, a burden of production, and a tactical burden. A burden of persuasion requires that a party proves a statement to a specified degree (the standard of proof) or runs the risk of losing on the issue at the end of the debate. A burden of production has been assigned to a party when the party is required by law to provide evidence for a certain claim. Burdens of persuasion and burdens of production are assigned by the applicable law. The tactical burden of proof depends on a party's own assessment of whether sufficient grounds have been adduced about a claim made by the party. Prakken and Sartor connect these different notions to a formal dialogue model of argumentation.

Probability and other quantitative approaches to argument strength Argument strength can be considered by using quantitative approaches. For instance, a conditional probability $p(H|E)$, expressing the probability of a hypothesis H given the evidence E, can be interpreted as a measure of the strength of the argument for the hypothesis based on the evidence. The idea is that higher values of $p(H|E)$ make H more strongly supported when given E. This interpretation of argument strength is associated with what is called Bayesian epistemology [Talbott, 2011]. Bayesian epistemology provides in the following way an interpretation of the relevance of additional evidence, say E': additional evidence E' strengthens the argument E for H when $p(H|E \wedge E') > p(H|E)$. In this interpretation, Bayes' theorem:

$$p(H|E) = p(E|H) \times p(H)/p(E)$$

connects the strength of the argument from E to H and that of the argument from H to E, thereby reversing the direction of the arrow. This relation is helpful, when the values of $p(E|H), p(H)$ and $p(E)$ are available, or when they are more easily established than $p(H|E)$ itself. Bayesian epistemology also provides a perspective on the comparison of hypotheses given additional evidence. When there are two hypotheses H and H', the odds form of Bayes' theorem can be used to update the odds of the hypotheses in light of new evidence E. The following relation shows how the prior odds $p(H)/p(H')$ is connected to the posterior odds $p(H|E)/p(H'|E)$:

$$p(H|E)/p(H'|E) = (p(H)/p(H')) \times (p(E|H)/p(E|H'))$$

This formal relation is helpful when the prior odds $p(H)/p(H')$, and the values of $p(E|H)$ and $p(E|H')$ are available.

Pollock has argued against a probabilistic account of argument strength (e.g., [Pollock, 1995; 2006; 2010]), referring to this position as 'generic Bayesianism' or 'probabilism.' Pollock argues that in a probabilistic account we would be justified in believing a mathematical theorem even before it is proven. This is especially absurd in cases such as Fermat's last theorem, which remained a conjecture for centuries before Wiles finally could complete a proof in the 1990s. Fitelson [2010] defends a probabilistic account against this and other criticisms advanced by Pollock.

Zukerman, McConachy and Korb [1998] have discussed the possibility of generating arguments from Bayesian networks, which are a widely studied tool for the representation of probabilistic information. Riveret *et al.* [2007] consider success in argument games in connection with probability. Dung and Thang [2010] have presented an approach to probabilistic argumentation in the setting of dispute resolution. Verheij [2012; 2017] has proposed a formal theory of defeasible argumentation in which logical and probabilistic properties are connected. Hunter [2013] discusses a model of deductive argumentation with uncertain premises. Verheij *et al.* [2016] discuss connections between arguments, scenarios and probabilities as normative tools in forensic reasoning with evidence.

Evidence and inference to the best explanation When an argument is aimed at establishing the truth, empirical evidence can be used to support alleged facts. For instance, a witness's testimony can provide evidence for the claim that the suspect was at the scene of a crime, a clinical test can provide evidence against a medical diagnosis, and the outcome of a laboratory experiment can be evidence confirming (or falsifying) a psychological phenomenon. The conclusions based on the available evidence can be regarded as hypothetical explanations for the occurrence of the evidence. As a result, reasoning on the basis of evidence is a specimen of what Peirce referred to as abductive reasoning, or inference to the best explanation: reasoning that goes from data describing something to a hypothesis that best explains or accounts for the data [Josephson & Josephson, 1996, p. 5]. Josephson and Josephson conceive of inference to the best explanation as a kind of argument scheme (see Section 4.3):

> D is a collection of data (facts, observations, givens).
> H explains D (would, if true, explain D).
> No other hypothesis can explain D as well as H does.
> Therefore, H is probably true.
> [Josephson & Josephson, 1996, p. 5]

The explanatory connection between D and H is often regarded as going against the causal direction. For instance, a causal, expectation-evoking rule 'If there is a fire, then there is smoke' can be used to infer, or argue for, the effect 'there is smoke' after observing the cause 'there is fire.' The causal rule has an evidential, explanation-evoking counterpart, 'If there is smoke, then there is a fire,' that can be used to infer (argue for) the explanation 'there is a fire' after observing 'there is smoke.' Arguments based on causal or evidential rules are typically defeasible: not all fires generate smoke, and not all smoke stems from a fire.

In artificial intelligence, the distinction between causal and evidential rules has been emphasized by Pearl [1988, p. 499f.]. He argues that special care is needed when mixing causal and evidential reasoning. To make his point, Pearl uses the following examples:

> Bill showed slight difficulties standing up, so I believed he was injured.
> Harry seemed injured, so I believed he would be unable to stand up.

The former uses the evidential pathway from the observation of Bill's difficulties in standing up to the explanation that he is injured, and the latter the reverse causal pathway from the observation of Harry's injuries to the effect that he is unable to stand up. The question is then addressed whether it is likely that Bill or Harry are likely to be drunk, drunkenness being a second cause for difficulties in standing up, independent from injury. Both Bill's and Harry's intoxicated state could be argued for using the evidential rule 'If someone has difficulties standing up, then he may be drunk.' However, for Bill the conclusion that he may be drunk seems more likely than for Harry, since for Bill both explanations for his difficulties in standing up, namely injury or being drunk, seem to be reasonable, whereas for Harry drunkenness is a less likely hypothesis now that an injury has been observed. The distinction between causal and evidential rules has played a central role in Pearl's thinking about causality [Pearl, 2000/2009], which relates to the probabilistic modelling tool of Bayesian Networks (see [Jensen & Nielsen, 2007; Kjaerulff & Madsen, 2008]). Bayesian Networks have been connected to the modelling of argumentation with legal evidence by Hepler *et al.* [2007] and by Fenton *et al.* [2012] (see also [Taroni *et al.*, 2006]). Vlek *et al.* [2014; 2016] discuss the design and understanding of Bayesian Networks for evidential reasoning using scenarios. Timmer *et al.* [2017] discuss an algorithm to extract argumentative information from a Bayesian Network modeling hypotheses and evidence. Verheij [2017] investigates connections between arguments, scenarios and probabilities in one formal model.

The distinction between causal and evidential rules has also been used in the formalized hybrid argumentative-narrative model of reasoning with evidence developed by Bex and his colleagues [Bex *et al.*, 2010; Bex, 2011]. In this model, the elements of a scenario, or narrative, describing how a crime may have been committed, can be supported by arguments grounded in the available evidence. Causal connections between the elements of a scenario contribute to its coherence. It is possible that more than one scenario is available, each scenario with different evidential support and a different kind of coherence. Bex and Verheij [2012] have developed the argumentative-narrative model in terms of argument schemes and their associated critical questions (see Section 4.3).

5.5 Applications and case studies

A first reason for the popularity of argumentation research in the field of artificial intelligence is that it has led to theoretical advances. A second reason is that the theoretical advances have been corroborated by a variety of interesting applications and case studies, including advances in natural language processing. We give some examples.

Fox and Das [2000] provided a book-length study of AI technology in medical diagnosis and decision making, with much emphasis on the argumentative aspects (see also Fox and Modgil, 2006, where argumentation-based decision making is used to extend the Toulmin model). Aleven and Ashley [1997a; 1997b] developed a case-based argumentation tool that was empirically tested for its effects on learning. Buckingham Shum and Hammond [1994] approached the design of artefacts such as software as an argumentation problem. Grasso *et al.* [2000] worked on argumentative conflict resolution in the context of health promotion. Teufel [1999] has worked on the problem of automatically estimating a sentence's role in argumentation, using a model of seven text categories called argumentative zones. Mochales Palau and

Moens [2009] developed software for the mining of argumentative elements in legal texts. Hunter and Williams [2010] investigated the aggregation of evidence in a healthcare setting. Grasso [2002] and Crosswhite *et al.* [2004] have worked on the computational modelling of rhetorical aspects of argument. Reed and Grasso [2007] have collected argumentation-oriented research using natural language techniques. They discuss, for instance, the generation of argumentative texts as studied by Elhadad [1995], Reed [1999], Zukerman *et al.* [1998], and Green [2007].

Rahwan and McBurney [2007] edited a special issue on argumentation technology of the journal *IEEE Intelligent Systems.* Application areas addressed in the issue are medical decision-making, emotional strategies to persuade people to follow a healthy diet, ontology engineering, discussion mediation, and web services. In the 2012 edition of the COMMA conference proceedings series on the computational modelling of argument, a separate section was devoted to innovative applications. The topics included: automatic mining of arguments in opinions, a learning environment for scientific argumentation, semi-automatic analysis of online product reviews, argumentation with preferences in the setting of eco-efficient biodegradable packaging, hypothesis generation from cancer databases, sense making in policy deliberation, music recommendation, and argumentation about firewall policy. For applications focusing on argumentation support and facilitation, the reader is referred to Section 4.5.

In the domain of AI and law theories and systems were developed and tested by the use of case studies. For instance, McCarty [1977; 1995] analysed a seminal case in US tax law (Eisner v. Macomber, 252 U.S. 189 [1920]). In that case, the US Supreme Court decided that a federal rule of tax law was invalid. McCarty's aims were set high, namely to build a software implementation that could handle a number of elusive, argumentative aspects of legal reasoning, illustrated in the majority opinion and dissenting opinions concerning the issues in this case. Quoting McCarty [1995]:

1. Legal concepts cannot be adequately represented by definitions that state necessary and sufficient conditions. Instead, legal concepts are incurably 'open-textured'.

2. Legal rules are not static, but dynamic. As they are applied to new situations, they are constantly modified to 'fit' the new 'facts'. Thus the important process in legal reasoning is not theory application, but theory construction.

3. In this process of theory construction, there is no single 'right answer'. However, there are plausible arguments, of varying degrees of persuasiveness, for each alternative version of the rule in each new factual situation.

Berman and Hafner [1993] studied the 1805 Pierson v. Post case concerning the ownership of a dead fox chased by Post, but killed and taken by Pierson. They emphasize the teleological aspects of legal argumentation, in which the goals of legal rules and decisions are taken into account. Bex [2011] used the Anjum case, a Dutch high media profile murder case, to test his proposal for a hybrid argumentative-narrative model of reasoning with evidence. Atkinson [2012] edited an issue of the journal *Artificial Intelligence and Law* on the modelling of a 2002 case about the ownership of a baseball, representing possibly value in the order of a million dollars, being the one that Barry Bonds hit when he broke the record of home-runs in one season (Popov v. Hayashi).

6 Conclusion

In the previous sections, we have introduced argumentation and argumentation theory as a field of study that goes back to classical times, passing through a neo-classical and anti-formal period in the second half of the 20^{nd} century, and since the final decade of the 2^{nd} millenium going through a formal and computational turn.

In Section 2, we discussed crucial concepts that have been indispensable in the study of argumentation before the recent formal and computational turn: standpoints, unexpressed premises, argument schemes, argumentation structures, and fallacies. All of these also played—and still play—a significant role in current formal and computational approaches to argumentation.

Standpoints occur in formal and computational work as the conclusions of arguments—possibly intermediate—and as the commitments of the players in a computational dialogue game. Recently we see a move towards standpoints with a complex structure, in work that allows a complex hypothesis (such as a plan or a scenario) as the conclusion of an argument.

Unexpressed premises have been studied in the context of manually analyzing argumentative texts in software tools. In today's research on argument mining, attempts are made to automatically understand argumentative texts, and we see that the ubiquity of unexpressed elements in argumentative discourse provides a significant hurdle.

Argument schemes have been the source of much interaction between the non-formal and formal/computational research communities. This is not a coincidence as argument schemes can be regarded as being intermediate between non-formal and the formal: argument schemes are formal in the sense that they have a well-organized structure, including elements such as premises, conclusions and critical questions; and argument schemes are non-formal in the sense that they handle just about every

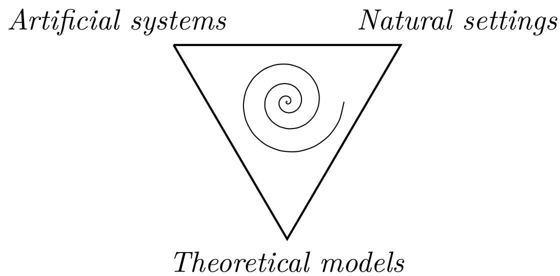

Figure 20: Perspectives on argumentation

area of human reasoning, whether legal, medical, or common-sense. Because of their intermediate position, argument schemes have been referred to as semi-formal.

Argumentation structures have been extensively studied both in non-formal and in formal research into argumentation theory. Today's argumentation logics and argumentation diagramming tools provide carefully designed structuring tools that fit the non-formal theory well, and that have been applied to argument analysis and design. In the study of argumentation structures, we see perhaps most convincingly that the anti-logical period in argumentation theory of the second half of the 20$^{\text{nd}}$ century is now superseded by a fruitful interaction between formal and non-formal methods.

Fallacies have received mostly indirect attention in the formal and computational study of argumentation, in particular because the mirror image of fallacies—correct argumentation—is and always has been in the center of formal attention. Much progress has been made in the characterization of typically argumentative versions of validity, initially distancing from classical formal theories, and nowadays gradually returning to an integration with classical logic and standard probability theory, this time while engaging with the needs of actual human argumentation as uncovered in argumentation theory.

We hope that it has become clear that there are a great many issues that can be fruitfully researched if argumentation and artificial intelligence scholars cooperate (cf. the research programme initiated by Reed & Norman [2004]). The distinction between non-formal and formal argumentation theory becomes ever more blurred, and argumentation theory is ever further turning into an interdisciplinary enterprise, integrating insights from different perspectives (see Figure 20).

In the *theoretical models* perspective, the focus is on theoretical (possibly non-formal) and formal models of argumentation, for instance, extending the long tradition of philosophical and formal logic. In the *artificial systems* perspective, the aim is to build computer programmes that model or support argumentative tasks, for instance, in online dialogue games or in knowledge-based systems (computer programmes that reproduce the reasoning of an expert, for instance, in the law or in medicine). The *natural settings* perspective helps to ground research by concentrating on argumentation in its natural form, for instance, in the human mind or in an actual debate. We are curious where the continuing synergy between these perspectives will bring our understanding of argumentation, this utterly human characteristic of civilized coexistence.

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Received 14 July 2017

Historical Overview of Formal Argumentation

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Abstract

This article gives an overview of the history of formal argumentation in terms of a distinction between argumentation-based *inference* and argumentation-based *dialogue*. Systems for argumentation-based *inference* are about which conclusions can be drawn from a given body of possibly incomplete, inconsistent of uncertain information. They ultimately define a nonmonotonic notion of logical consequence, in terms of the intermediate notions of argument construction, argument attack and argument evaluation, where arguments are seen as constellations of premises, conclusions and inferences. Systems for argumentation-based *dialogue* model argumentation as a kind of verbal interaction aimed at resolving conflicts of opinion. They define argumentation protocols, that is, the rules of the argumentation game, and address matters of strategy, that is, how to play the game well. For both aspects of argumentation the main formal and computational models are reviewed and their main historical influences are sketched. Then some main applications areas are briefly discussed.

1 Introduction

This article gives an overview of the history of formal argumentation. There are two ways to write such an overview. One is to describe all significant research that has been done, while another is to give insight into the historical developments underlying the current state of the art. In this article I will do the latter. This will inevitably lead to a stronger focus on the early developments and a less detailed description of later research.

The historical overview is given in terms of a distinction between argumentation-based *inference* and argumentation-based *dialogue*. Systems for argumentation-based *inference* are about which conclusions can be drawn from a given body of possibly incomplete, inconsistent of uncertain information. They ultimately define a nonmonotonic notion of logical consequence, in terms of the intermediate notions of argument construction, argument attack and argument evaluation, where arguments are seen as constellations of premises, conclusions and inferences. Systems for argumentation-based *dialogue* model argumentation as a kind of verbal interaction aimed at resolving conflicts of opinion. They define argumentation protocols (the rules of the argumentation game) and address matters of strategy (how to play the game well). While accounts of argumentation as inference assume a single static and global body of information from which the arguments and attacks are constructed, in studies of argumentation as dialogue this information is dynamic (it can change during a dialogue) and distributed over the dialogue's participants. Models of argumentation as inference can be embedded in models of argumentation as dialogue in two complementary ways: at each stage of a dialogue they can be 'globally' applied to the 'current' body of information; and within each dialogue participant they can be 'locally' applied as the participant's internal reasoning model.

Like all informal distinctions, the distinction between argumentation as inference and argumentation as dialogue breaks down at some point, and therefore I will also discuss work that cannot easily be classified as belonging to either inference or dialogue, especially work on argumentation dynamics that abstracts from agent-related and dialogical aspects. Another way in which a strict distinction between inference and dialogue causes problems for a historical overview is that some historical influences cannot clearly be described as influencing just models of inference or just models of dialogue. Some work has instead more generally promoted the idea of dialectics as constructing, criticising and comparing arguments, whether in an inferential or in a dialogical setting. One such historical influence was the development of dialogue logic [Lorenzen and Lorenz, 1978], which gives a game-theoretic formulation of the semantics of logical constants in terms of a dispute between a proponent and an opponent of a claim, plus a game-theoretic notion of logical consequence as the existence of a winning strategy for the proponent. This predates modern argument games for argumentation-based inference and also influenced the development of formal dialogue systems for argumentation. Having said so, in dialogue logic these ideas were only used to reformulate existing monotonic notions of logical consequence, so dialogue logic cannot be said to model genuine argumentation.

Another historical influence that is not confined to either inference or dialogue is early AI & Law work on the computational modelling of legal argument. Among the earliest work in AI and law on legal argument was the TAXMAN II project of [McCarty, 1977; McCarty, 1995]). According to McCarty [1995], p. 285 "The task for a lawyer or a judge in a "hard case" is to construct a theory of the disputed rules that produces the desired legal result, and then to persuade the relevant audience that this theory is preferable to any theories offered by an opponent". Other influential early systems were the HYPO system [Rissland and Ashley, 1987; Ashley, 1990] and its successor the CATO system [Aleven and Ashley, 1991; Aleven, 2003]. These systems were meant to model how lawyers in common-law jurisdictions make use of past decisions when arguing a case. They did not compute an 'outcome' or 'winner' of a dispute; instead they were meant to generate debates as they could take place between 'good' common-law lawyers. Several researchers who later contributed to the general formal study of argumentation originate from AI & Law, such as Trevor Bench-Capon, Tom Gordon, Giovanni Sartor, Bart Verheij and myself.

The remainder of this article is divided into two main sections on, respectively, argumentation-based inference (Section 2) and dialogue (Section 3). Then some main applications areas are briefly discussed in Section 4 and some concluding remarks are made in Section 5.

2 Formal and computational models of argumentation-based inference

Nowadays, many systematic introductions to argumentation start with Dung's [1995] theory of abstract argumentation frameworks, which takes the notions of argument and attack as primitive, i.e., nothing is assumed about about the structure of arguments or the nature of attack. Yet there had been quite some formal work on argumentation-based inference before Dung's landmark 1995 paper, and all this early work specified the structure of arguments and the nature of attack. The seminal paper in this respect was [Pollock, 1987]. Many ideas developed in this early body of work are still important today. The focus in this early work on structured argumentation agrees with the usual approaches in informal argumentation, which do not have arguments as the primitive notion but concepts like claims, reasons and grounds. For example, Walton [2006a], p. 285 defines the term 'argument' as "the giving of reasons to support or criticize a claim that is questionable, or open to doubt".

In this section first the three main historical sources of influence are sketched, namely, philosophy, nonmonotonic logic & logic programming, and informal logic & argumentation theory. Then the two seminal bodies of work are discussed in more more detail, John Pollock's argumentation-based system for defeasible reasoning and Phan Minh Dung's theory of abstract argumentation frameworks. Their works have inspired much research on, respectively, structured and abstract approaches to argumentation-based inference, which will subsequently be discussed.

2.1 Main historical influences

The formal and computational study of argumentation-based inference is generally regarded as a subfield of AI, originating from the study of nonmonotonic logic. However, there are two main other historical influences.

2.1.1 Philosophy

Arguably, the first mature formal system for argumentation-based inference was proposed by Pollock [1987][1]. John Pollock (1940-2009) was an influential American philosopher who made important contributions to various fields, including epistemology and cognitive science. In the last 25 years of his life he also contributed to artificial intelligence, starting with his classic 1987 paper on defeasible reasoning. Many important topics in the formal study of argumentation-based inference were first studied by Pollock, or first studied in detail, such as argument structure, the nature of defeasible reasons, the interplay between deductive and defeasible reasons, rebutting versus undercutting defeat, argument strength, argument labellings, self-defeat, and resource-bounded argumentation.

Pollock's work on formal argumentation was heavily influenced by the idea of defeasible reasons as developed in moral philosophy by Ross [1930] in his notion of *prima facie* moral rules, in epistemology by Chisholm [1957], Rescher [1977] and Pollock himself [1970, 1974], and as applied to practical reasoning by Raz [1975]. The term 'defeasibility' originates from legal philosophy, in particular from Hart [1949] (see the historical discussion in Loui [1995]). Hart observed that legal concepts are defeasible in that the conditions for when a fact situation classifies as an instance of a legal concept (such as 'contract'), are only ordinarily, or presumptively, sufficient. If a party in a law suit succeeds in proving these conditions, this does not have the effect that the case is settled; instead, legal procedure is such that the burden of proof shifts to the opponent, whose turn it then is to prove exceptional facts which, despite the facts proven by the proponent, nevertheless prevent the claim from being granted. For instance, insanity of one of the contracting parties is an exception to the legal rule that an offer and an acceptance constitute a binding contract. The notion of burden of proof was also studied by [Rescher, 1977], in the context of epistemology. Among other things, Rescher claimed that a dialectical model of scientific reasoning can explain the rational force of inductive arguments: they must be accepted if they cannot be successfully challenged in a properly conducted scientific dispute.

Pollock's work on formal argumentation originated as an attempt to make formal

[1]Several paragraphs in this subsection are, some with minor modifications, taken from Prakken and Horty [2012].

sense of the intuitive notion of defeasible reasoning that seemed to be at work in these papers and books. In fact, the task had been attempted before. There is an early paper by Chisholm [1974], a heroic effort whose failure is no surprise given the limited tools available at the time. Still, in spite of the blossoming of philosophical logic in the 1960's and 1970's, the logical study of defeasible reasoning had received almost no attention at all. It is fair to say that Pollock, working in isolation, was the first philosopher working in the field of philosophy, as opposed to computer science, to outline an adequate framework for defeasible reasoning.

2.1.2 Nonmonotonic logic and logic programming

The first AI systems for argumentation-based inference were not influenced by the above-discussed philosophical developments. Instead, they were presented as new ways to do nonmonotonic logic. Nonmonotonic logic had become fashionable around 1980 and a variety of approaches was being pursued. By the late 1980's, the field of nonmonotonic logic had been recognized as an important subfield of artificial intelligence. The field was motivated by the fact that commonsense reasoning often involves incomplete or inconsistent information, in which cases logical deduction is not a useful reasoning model. If information is incomplete, then nothing useful can be deductively derived, while if it is inconsistent, then anything is deductively implied. Nonmonotonic logics allow 'jumping to conclusions' in the absence of information to the contrary. The canonical example is 'birds typically fly, Tweety is bird, therefore (presumably) Tweety can fly'. This inference holds as long as no information is available that Tweety is not a typical bird with respect to flying, such as a penguin. Nonmonotonic logic can also model the derivation of useful conclusions from inconsistent information, namely, by focusing on consistent subsets of the inconsistent information. Several years after the first nonmonotonic logics were proposed in the now famous special issue on nonmonotonic logic of the *Artificial Intelligence* journal [Bobrow, 1980], the idea arose in this field that nonmonotonic inference can be modelled as the competition between arguments.

The earliest nonmonotonic reasoning systems with an argumentation flavour include the work of Touretzky [1984; 1986] on inheritance systems, later developed along with several collaborators [Horty *et al.*, 1990]. Inheritance systems model reasoning about how objects inherit properties from the classes to which they belong. They are nonmonotonic since the inheritance of properties of classes by subclasses can be blocked by exceptions. For example, penguins do not inherit from birds the property of being able to fly. Although the work on inheritance systems did not use argumentation terms, such systems still have all the characteristics of argumentation systems. To start with, inheritance paths effectively are arguments. For example, the conclusion that Tweety the penguin can fly can be drawn via the path 'Penguins are birds and birds can fly' while the conclusion that Tweety the Penguin cannot fly can be drawn via the inheritance path 'Penguins cannot fly'. Inheritance systems also have various notions of conflict between inheritance plus definitions of whether a path is 'permitted' given its conflict relations with other paths. While the technical solutions devised in this work are now somewhat outdated, the work on inheritance paths has clearly influenced the development of the first AI argumentation systems. Among other things, the publications in inheritance are great sources of relevant examples.

An influential figure in the early days was Ron Loui. His [1987] paper was, although technically still preliminary, influential in promoting the idea of formulating nonmonotonic logic as argumentation. With Guillermo Simari he developed a a technically mature version of his ideas [Simari and Loui, 1992]. Several other of his papers more generally promoted the idea of computational dialectics and were thus also relevant for dialogue models of argumentation. The fullest exposé of these ideas is [Loui, 1998], which circulated among researchers for several years until it was finally published in 1998.

Other relevant early work was the work of Nute [1988], later developed into so-called Defeasible Logic [Nute, 1994]. This approach is in spirit very close to argumentation but while in argumentation approaches conflict and defeat happen between arguments, in Defeasible Logic they happen between rules. For this reason the work on Defeasible Logic has diverged somewhat from the field of computational argument, although some work on the former has studied the formal relation with argumentation approaches. In particular, [Governatori *et al.*, 2004] studied to which extent defeasible logics can be reformulated in terms of Dung's theory of abstract argumentation frameworks.

Finally, the field of logic programming was influential since the idea arose to give semantics to negation as failure in argumentation-theoretic terms. If *not* P is assumed to hold because of the failure to derive P, then a derivation of P can be regarded as an attack on any derivation using *not* P. In other words, a logic-programming derivation can be regarded as a competition between arguments and counterarguments. Work on this idea of e.g. Geffner [1991] and Kakas *et al.* [1992] was a main source of inspiration of Dung's landmark [1995] paper on abstract argumentation frameworks.

2.1.3 Informal logic and informal argumentation theory

One would expect that the fields of informal logic and argumentation theory (which are often regarded as a single field) were also important historical influences on argumentation-based models of inference. However, in fact their influence has been relatively modest. In particular, the work of Toulmin [1958] and the resulting work on argumentation schemes was until around 2000 hardly linked to computational argument. An important event here was the 2000 Bonskeid Symposium on Argument and Computation in the Scottish mountains, organised by Tim Norman and Chris Reed, at which researchers from various formal and informal fields met in an informal setting. Various interdisciplinary collaborations resulted from this event, partly reported in [Reed and Norman, 2003].

Yet these fields originated from similar concerns about deductive logic as those that gave rise to the field of nonmonotonic logic in AI, namely, the inadequacy of deductive logic as a model of 'ordinary' reasoning. Stephen Toulmin, whose 1958 book *The Uses of Argument* is generally regarded as the origin of informal logic and argumentation theory, criticised the logicians of his days for neglecting many features of ordinary reasoning. In his well-known pictorial scheme for arguments (see Figure 1) he left room for "rebuttals" of an argument on the basis of exceptions to the "warrant" connecting the arguments "data" to its "claim". The idea of rebuttals is clearly related to Hart's [1949] ideas on exceptional circumstances that can defeat the application of a legal concept.

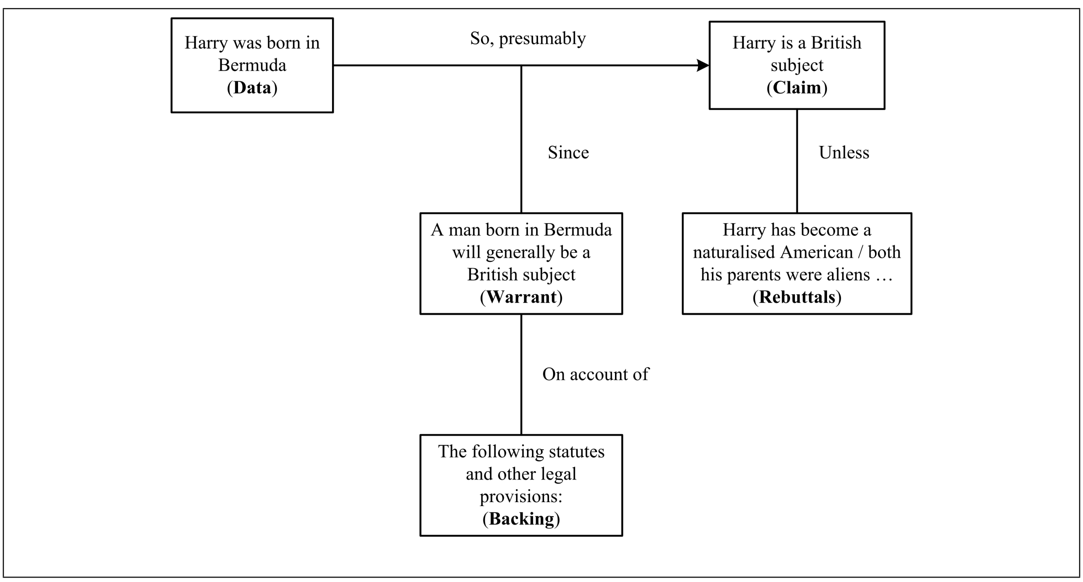

Figure 1: Toulmin argument scheme and an instance

Toulmin's notion of a warrant was in informal logic and argumentation theory generalised into rich classifications of argument schemes for presumptive forms of reasoning, while his notion of a rebuttal was generalised into lists of critical questions attached to argument schemes [Walton, 1996]. The idea of argumentation schemes with critical questions has since the above-mentioned Bonskeid 2000 event often been used in formal and computational models of argumentation-based inference and dialogue.

Toulmin also argued that outside mathematics the validity of an argument does not depend on its syntactic form but on whether it can be defended in a properly conducted dispute, and that the task of logicians is to study the criteria for properly conducted disputes. This became an important and very influential idea, as further discussed below in Section 3 on argumentation-based dialogue. However, it also had an unfortunate effect. For decades, informal logic and argumentation theory rejected any use of formal methods in the study of ordinary reasoning, based on a mistaken equation of formal methods with deductive logic. As we now know after more than 35 years of research on nonmonotonic logic, belief revision and computational argument, many features of non-mathematical reasoning that Toulmin and his successors analysed can be formalised. For example, the AI work on argumentation schemes since 2000 has shown that reasoning with such schemes can to a large extent be formalised in modern argumentation logics.

2.2 Seminal work

I now discuss the two seminal contributions in the field, the ones of Pollock [1987] and Dung [1995]. These two papers successively introduced the two key ideas of the formal study of argumentation-based inference. Pollock introduced the notion of a defeasible reason, while Dung showed that argument evaluation can be formalised by assuming just two primitive notions of argument and attack. Neither of these ideas on their own define the field; it is their combination that makes the argumentation way of doing nonmonotonic logic so powerful.

2.2.1 Pollock's work

As said above, arguably, the first mature formal system for argumentation-based inference was proposed by Pollock [1987][2]. In fact, this work became close to being one of the first nonmonotonic logics at all. Concerning his 1987 paper, Pollock later wrote that he first developed the idea in 1979, but that he did not initially publish it because, as he says, "being ignorant of AI, I did not think anyone would be interested." [Pollock, 2007b, p. 469]. If Pollock had published this idea when

[2]Several parts of this subsection are reused or adapted from Prakken and Horty [2012].

it first occurred to him, the result would have been not only the first argument-based theory of defeasible reasoning, but one of the first systems of any kind for nonmonotonic reasoning.

I now discuss Pollock's system in some more detail, to illustrate that it introduced several fundamental ideas into our field. As usual in logic, arguments in Pollock's approach are inference graphs, in which a final conclusion is inferred from the premises via intermediate conclusions. Noe that when an argument uses no premise more than once, the graph is a tree. What is unusual is Pollock's ideas on how conclusions can be supported by premises. The 'classic' logicians' view attacked by Toulmin [1958] had been that all arguments should be deductively valid, that is, the truth of their premises should guarantee the truth of their conclusion, and that the only source of fallibility of good arguments is their premises. Influenced by Toulmin, the fields of informal logic and argumentation theory had already questioned this view and argued that arguments that fail to meet this standard of inferential perfection can still be good, as long as they withstand critical scrutiny. Pollock [1987] gave us the tools to formalise this new account, with his notion of a defeasible reason.

In Pollock's approach, the inference rules (in his terminology "reasons") used to construct arguments come in two kinds: *deductive* and *defeasible* reasons (in his early work called "conclusive' and "prima facie" reasons). An argument can be defeated on its applications of defeasible reasons, which can happen in two ways. *Rebutting* defeaters attack the conclusion of a defeasible inference by supporting a conflicting conclusion. For example, 'Tweety can fly since it is a bird and birds typically fly' can be attacked by 'Tweety cannot fly since Tweety is a penguin and penguins cannot fly'. *Undercutting* defeaters instead attack the defeasible inference itself, without supporting a conflicting conclusion. For example: if the object looks red, this is a reason for concluding, defeasibly, that the object is red; but the presence of red illumination interrupts the reason relation without suggesting any conflicting conclusion. Pollock formalized several defeasible reasons that he found important in human cognition, such as reasons for perception, memory, induction, the statistical syllogism and temporal persistence, as well as undercutting defeaters for these reasons.

Pollock's notion of a defeasible reason is clearly related to argumentation theory's notion of an argumentation scheme: such schemes are defeasible reasons while many of their critical questions can be regarded as pointers to undercutting defeaters and other questions as pointers to rebutting defeaters or premise attacks.

Consider by way of example of Pollock's notions of reason, argument and conflict the following version of the Tweety example. Figure 2 contains two rebutting arguments for the conclusions that Tweety flies, respectively, does not fly, and an undercutting argument defeating the argument that Tweety flies. In this figure, de-

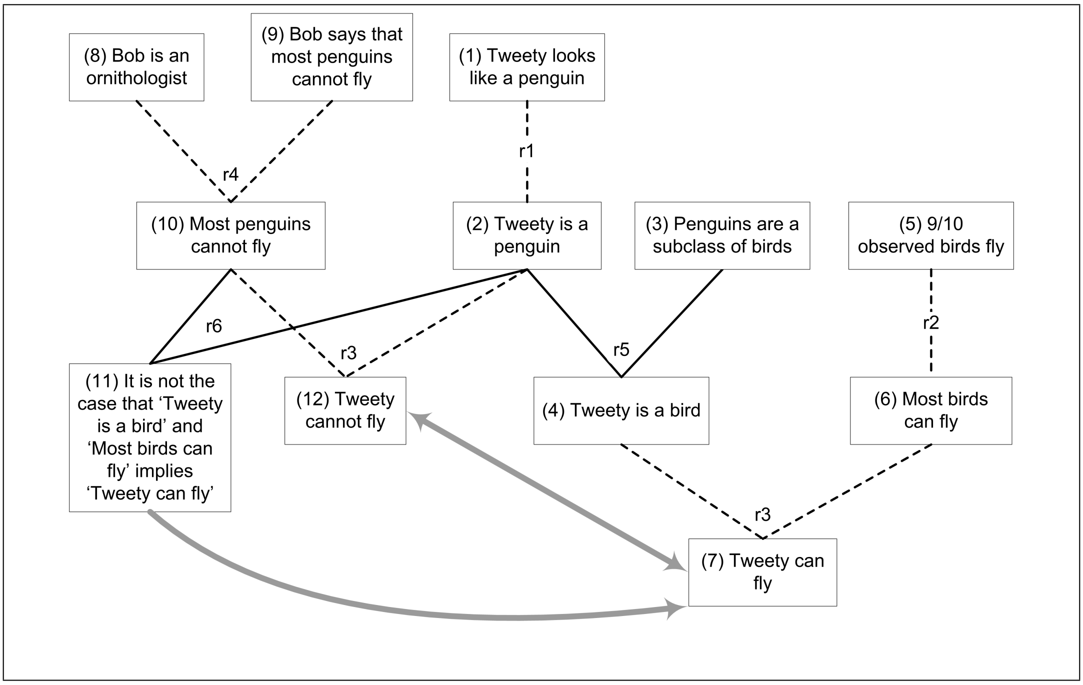

Figure 2: An example

ductive, respectively defeasible inferences are visualized with, respectively, solid and dotted lines without arrow heads, while defeat relations are displayed with arrows. The figure assumes four defeasible inference rules, informally paraphrased as follows:

- r_1: That an object looks like having property P is a defeasible reason for believing that the object has property P
- r_2: That n/m observed P's are Q's (where $n/m > 0,5$) is a defeasible reason for believing that most P's are Q's
- r_3: That most P's are Q's and x is a P is a defeasible reason for believing that x is a Q
- r_4: That an ornithologist says φ about birds is a defeasible reason for believing φ

Rule r_1 expresses that perceptions yield a defeasible reason for believing that what is perceived to be the case is indeed the case, rule r_2 captures enumerative induction, while r_3 expresses the statistical syllogism. Rule r_4 can be seen as a special case of the argumentation scheme from expert testimony; cf. [Walton, 1996].

Moreover, the figure assumes an obvious strict inference rule plus an undercutting defeater for r_3:

r_5: That P's are a subclass of Q's and a is a P is a deductive reason for believing that a is a Q

r_6: That x is an R, most R's are not Q's and R's are a subclass of P's is a deductive reason for believing $\neg r_3$

Rule r_6 is a special case of Pollock's "subproperty defeater" of the statistical syllogism, which says that conflicting statistical information about a subclass undercuts the statistical syllogism for the superclass.

Defeasible reasons should not be confused with nonmonotonic consequence notions. It is possible to design argumentation logics with nonmonotonic consequence notions in which nevertheless all arguments have to be deductively valid. For example, in classical argumentation arguments are classical implication relations from consistent subsets of a possibly inconsistent body of information and the only source of fallibility of arguments is their premises. Recent portrayals of Pollock's approach as 'deductive' [Hunter and Woltran, 2013] do no justice to his approach, given that Pollock strongly emphasised that "It is logically impossible to reason successfully about the world around us using only deductive reasoning. All interesting reasoning outside mathematics involves defeasible steps." [Pollock, 1995, p.41]. Pollock thus clearly rejected the conventional view that all arguments have to be deductively valid.

Defeasible reasons should also not be confused with deductive inference rules with assumption-type premises. Thinking otherwise would have the odd consequence that even the classically valid rules of inference become defeasible when applied to assumptions.

Once arguments can employ defeasible reasons, the support relation between their premises and conclusion can have varying strength. Pollock's 1987 system did not yet include a notion of strength but Pollock later took the notion of strength of arguments very seriously. Since his systems were meant for epistemic reasoning, he always formulated strength of reasons in terms of numerical degrees of belief. In his 1994 system, rebutting and undercutting arguments only succeed in defeating their target if the degree of belief of their conclusions is not lower than that of the attacked argument.

Finally, Pollock was well aware that just defining notions of argument and defeat are not enough and he spent much effort in designing well-behaved notions of argument acceptability. His two earliest definitions predate much current work on argumentation-based semantics. His 1987 proposal was by Dung [1995] proven to be an instance of Dung's grounded semantics, while his 1994 labelling definition predates the currently popular labeling approach to abstract argumentation and was by Jakobovits [2000] proven to be an instance of Dung's preferred semantics.

2.2.2 Dung's abstract argumentation frameworks

Dung's landmark 1995 paper is the origin of the second main idea of our field, namely, that argument evaluation can be formalised by assuming just two primitive notions of argument and attack. With just these two notions, Dung was able to develop an extremely rich and elegant abstract theory of argument evaluation. As apparent from this historic overview, Dung was not the first to study argument evaluation nor the first to provide well-behaved definitions. His great contribution was twofold: he showed that particular definitions of argument evaluation conformed to simple abstract patterns, and he showed that the same patterns are also implicit in other nonmonotonic logics, in logic programming and even in cooperative game theory. Exaggerating a little, one could say that while Pollock arguably was the father of argumentation in AI, Dung was the midwife, who smoothened its delivery into mainstream AI. His 1995 AI Journal paper was not the first work on argumentation-based inference, but its influence has been enormous, now being the de facto standard in the field. It is fair to say that Dung [1995] has made argumentation respectable in mainstream AI.

Nevertheless, the historic roots of Dung's 1995 paper should not be forgotten. As mentioned in the introduction to Section 2, all early work on argumentation-based inference specified the structure of arguments and the nature of attack (often called 'defeat'). Even Dung in his landmark 1995 paper stood in this tradition. Dung did two things: he developed the new idea of abstract argumentation frameworks, and he used this idea to reconstruct and compare a number of then mainstream nonmonotonic logics and logic-programming formalisms, namely, default logic [Reiter, 1980], Pollock's [1987] argumentation system and several logic-programming semantics. However, these days the second part of his paper, and also the third part on relations with cooperative game theory, is largely forgotten and his paper is almost exclusively cited for its general theory of abstract argumentation frameworks.

A historic overview of work on argumentation-based inference would not be complete without listing Dung's simple and elegant basic notions. An *abstract argumentation framework* (AF) is a pair $\langle AR, attacks \rangle$, where AR is a set arguments and $attacks \subseteq AR \times AR$ is a binary relation. The theory of AFs then addresses how sets of arguments (called *extensions*) can be identified which are internally coherent and defend themselves against attack. A key notion here is that of an argument being *acceptable with respect to* a set of arguments: $A \in AR$ is acceptable with respect to $S \subseteq AR$ if for all $A \in S$: if $B \in AR$ attacks A, then some $C \in S$ attacks B (nowadays it is more usual to say that $A \in AR$ is defended by $S \subseteq AR$). Then relative to a given AF various types of extensions can be defined as follows (here E is *conflict-free* if no argument in E attacks an argument in E):

- E is *admissible* if E is conflict-free and each argument in E is acceptable with respect to E;
- E is a *complete extension* if E is admissible and each argument that is acceptable with respect to E belongs to E;
- E is a *preferred extension* if E is a maximal (with respect to set inclusion) admissible set;
- E is a *stable extension* if E is conflict-free and attacks all arguments outside it;
- E is a *grounded extension* if E is the least fixpoint of operator F, where $F(S)$ returns all arguments acceptable to S.

Dung showed that the grounded extension is always unique but that there can be multiple extensions of the other types. Dung also showed that every stable extension is preferred but not vice versa, that the grounded extension is contained in every other extension, and that all extensions of any type are complete.

To illustrate how abstract argumentation frameworks can be instantiated, consider again Figure 2. There are three arguments. In fact, there are more arguments, since each of the three arguments we consider has several subarguments. However, none of these is attacked, so they can be ignored for simplicity. The two rebutting arguments for the conclusions that Tweety can fly, respectively, cannot fly attack each other, while the undercutting argument attacks the argument that Tweety flies. The resulting argumentation framework is shown in Figure 3. In this case the four semantics coincide: the set with the undercutting argument and the argument that Tweety cannot fly is the grounded extension, while it is also the unique complete, stable and preferred extension (the grey colourings indicate extension membership). To see why it is preferred, observe that the undercutting argument defends the argument that Tweety cannot fly against its rebutting attacker that Tweety can fly.

To illustrate that argumentation frameworks can have multiple extensions, consider the simpler example in Figure 4 where the undercutting argument has been deleted from the AF of Figure 3. In grounded semantics the extension is empty

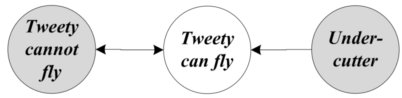

Figure 3: An abstract argumentation framework

(case a) but in preferred and stable semantics there are two extensions, depending on whether the argument that Tweety can (case b) or cannot fly (case c) is accepted. Finally, all three extensions are complete.

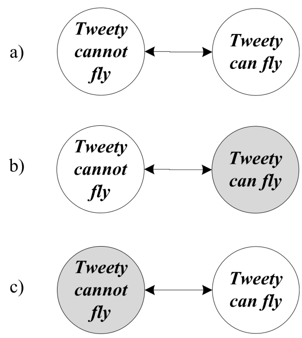

Figure 4: A simpler abstract argumentation framework and three extensions

These examples point at a minor source of terminological confusion, since they use Dung's term 'attack' while Pollock always used 'defeat'. When Dung's 1995 paper appeared, 'defeat' was the standard term, not just in Pollock's work but essentially in all early work on argumentation-based inference. Current work on the $ASPIC^+$ framework [Prakken, 2010; Modgil and Prakken, 2013; Modgil and Prakken, 2014] also uses 'defeat' and reserves the term 'attack' for more basic, purely syntactical forms of conflicts between arguments. Defeat is then successful attack according to some notion of argument strength or preference, an idea present in much early work on argumentation-based inference, although usually not employing the term 'attack'. Thus it is not $ASPIC^+$'s attack relation but its defeat relation which instantiates Dung's notion of attack.

2.3 Other early work

Initial ideas In the same year in which Pollock published his seminal paper, Loui [1987] appeared as arguably the first AI paper that explicitly proposed to design nonmonotonic logics in the argumentation way. In 1992, Simari and Loui fully formalized Loui's [1987] initial ideas, which work in turn led to the development of Defeasible Logic Programming [Garcia *et al.*, 1998; Garcia and Simari, 2004]. One year later, Konolige [1988] proposed an argumentation approach as a solution to the famous Yale Shooting problem in logic-based specifications of dynamic systems [Hanks and McDermott, 1986]. Although his formalism was still rather rudimentary, Konolige's discussion anticipates many issues and distinctions of later work, so that his paper can be regarded as one of the forerunners of the study of argumentation-based inference.

Argumentation as a proof theory for preferential entailment Around 1990, some papers proposed argumentation as a proof theory for model-theoretic notions of nonmonotonic consequence (preferential entailment). Baker and Ginsberg [1989] did this for a minimal-model semantics of prioritised circumscription, while Geffner [1992] and Geffner and Pearl [1992] did the same for their 'conditional entailment' semantics for default reasoning. The basic idea is that (1) given a propositional or first-order theory, an argument is a set or conjunction of assumptions consistent with the theory and that combined with the theory yields conclusions; and (2) arguments can be attacked by arguments for the negation of the attacked argument or one of its assumptions. This idea later became the basis for assumption-based argumentation [Bondarenko *et al.*, 1997], to be discussed in Section 2.4. Although the idea to found argumentation-based inference on preferential entailment is very interesting, it has since then not been further pursued.

Abstract argumentation systems Lin and Shoham [1989] were the first to propose the idea of abstraction in structured argumentation. They developed the notion of abstract argumentation structures with strict and defeasible rules and they showed how a number of existing nonmonotonic logics could be reconstructed as such structures. Gerard Vreeswijk further developed these ideas into his abstract argumentation systems [Vreeswijk, 1991; Vreeswijk, 1993b; Vreeswijk, 1997]. Since several of Vreeswijk's ideas are included in today's $ASPIC^+$ framework, it is worthwhile summarising some of his definitions. Like Lin & Shoham, Vreeswijk defined arguments in terms of an unspecified logical language $\mathcal{L}$, only assumed to contain the symbol $\perp$, denoting 'falsum' or 'contradiction,' and two unspecified sets of strict ($\rightarrow$) and defeasible ($\Rightarrow$) inference rules defined over $\mathcal{L}$. In addition, he defined the main elements that are missing in Lin & Shoham's system, namely, notions of conflict and defeat between arguments. Vreeswijk defined arguments as follows:

Definition 2.1. *An* argument σ *is:*

1. φ *if* $\varphi \in \mathcal{L}$*; in that case:* $\texttt{Prem}(\sigma) = \{\varphi\}$, $\texttt{Conc}(\sigma) = \varphi$, $\texttt{Sent}(\sigma) = \{\varphi\}$*;*

2. $\sigma_1, \dots \sigma_n \rightarrow \varphi$ *where* $\sigma_1, \dots, \sigma_n$ *is a finite, possibly empty sequence of arguments such that* $\texttt{Conc}(\sigma_1) = \varphi_1, \dots, \texttt{Conc}(\sigma_n) = \varphi_n$ *for some strict rule* $\varphi_1, \dots, \varphi_n \rightarrow \varphi$*, and* $\varphi \notin \texttt{Sent}(\sigma_1) \cup \dots \cup \texttt{Sent}(\sigma_n)$*; in this case:* $\texttt{Prem}(\sigma) = \texttt{Prem}(\sigma_1) \cup \dots \cup \texttt{Prem}(\sigma_n)$, $\texttt{Conc}(\sigma) = \psi$, $\texttt{Sent}(\sigma) = \texttt{Sent}(\sigma_1) \cup \dots \cup \texttt{Sent}(\sigma_n) \cup \{\varphi\}$*;*

3. $\sigma_1, \dots \sigma_n \Rightarrow \varphi$ *where* $\sigma_1, \dots, \sigma_n$ *is a finite, possibly empty sequence of arguments such that* $\texttt{Conc}(\sigma_1) = \varphi_1, \dots, \texttt{Conc}(\sigma_n) = \varphi_n$ *for some defeasible rule* $\varphi_1, \dots, \varphi_n \Rightarrow \varphi$*, and* $\varphi \notin \texttt{Sent}(\sigma_1) \cup \dots \cup \texttt{Sent}(\sigma_n)$*; with the further attributes defined as in (2).*

Note that this definition, unlike most other definitions of arguments in the formal literature, excludes circular arguments.

Vreeswijk's notion of conflicts between arguments is unusual in that a counterargument is a *set* of arguments: a set Σ of arguments is *incompatible* with an argument τ iff the conclusions of $\Sigma \cup \{\tau\}$ give rise to a strict argument for $\bot$. While unusual, there is nothing obviously wrong with this kind of definition. The reason why currently conflict is usually defined as a relation between individual arguments is probably that such definitions better fit with Dung's theory of abstract argumentation frameworks. Vreeswijk's approach might fit better with generalisations of Dung's theory that allow attacks from sets of arguments to arguments [Bochman, 2003; Nielsen and Parsons, 2007b]. Recently, Baroni *et al.* [2015] have combined the *ASPIC*$^+$ framework with a Vreeswijk-style definition of conflict.

Conflicts can in Vreeswijk's approach be resolved with any reflexive and transitive ordering on arguments that the user likes to adopt. A set of arguments Σ is *undermined* by an argument τ if $\sigma < \tau$ for some $\sigma \in \Sigma$. Then a set of arguments Σ is a *defeater* of σ if Σ is incompatible with σ and not undermined by it.

Finally, Vreeswijk defined argument acceptability ("warrant") with a definition that is close but not equivalent to Dung's [1995] stable semantics. In light of the modern theory of abstract argumentation frameworks, Vreeswijk's definition of warrant is, unlike the rest of his approach, somewhat premature. This is understandable, since Vreeswijk developed his approach before 1995.

Logic-programming approaches The work on argumentation semantics for logic-programming's negation as failure did not only inspire Dung to develop his theory of abstract argumentation frameworks but also gave rise to logic-programming systems for argumentation with explicit negation. Two early papers here were Dung [1993] and Dimopoulos and Kakas [1995]. The first of these papers was in turn a source of inspiration for Prakken and Sartor's [Prakken and Sartor, 1997] argument-based logic programming system with defeasible priorities. Theirs was arguably the first system that was explicitly designed as an instance of Dung's [1995] approach. Strictly speaking, it was technically based not on Dung [1995] but on Dung [1993], but a reformulation in terms of abstract argumentation is trivial. Like all other work reviewed so far, it distinguished between strict and defeasible inference rules. Unlike Dimopoulos and Kakas [1995] but like Dung [1995], its language had both explicit negation and negation as failure, with corresponding "rebutting" attacks on defeasibly derived conclusions and "undercutting" attacks on negation-as-failure premises. One innovative feature was that it allowed argumentation about preferences inside the argumentation system, while another innovative feature was that the system had the first published argument game meant as a proof theory for the semantics of abstract argumentation frameworks (for more on argument games see Section 2.5.2 below).

Defeasible vs. plausible reasoning As apparent from the overview so far, until 1993 almost all accounts of argumentation-based inference made a distinction between deductive (or 'strict') and defeasible inference rules, introduced in philosophy by Pollock [1970; 1974] and in AI by Pollock [1987] and Touretzky [1984]. This approach is still being pursued today, notably in Defeasible Logic Programming, Defeasible Logic and the $ASPIC^+$ framework. In this approach a special definition of arguments is needed that regulates the interplay between strict and defeasible reasons (such as the above one of Vreeswijk [1993b; 1997]), since with two kinds of inference rules one cannot rely on a single given logical consequence notion to specify how conclusions are supported by premises. Around 1993 an alternative approach to structured argumentation emerged, according to which arguments are constructed in a single given deductive logic, obviating the need of a separate definition of an argument beyond being a premises-conclusion pair. In understanding and relating the two approaches, the philosophical distinction between *plausible* and *defeasible* reasoning is relevant; cf. Rescher [1976; 1977] and Vreeswijk [1993b], Ch. 8. Following Rescher, Vreeswijk described plausible reasoning as sound (i.e., deductive) reasoning on an uncertain basis and defeasible reasoning as unsound (but still rational) reasoning on a solid basis. In other words, argumentation models of plausible reasoning locate all fallibility of an argument in its premises, while argumentation models of defeasible reasoning locate all fallibility in its defeasible inferences. Thus plausible-reasoning approaches effectively view argumentation as a kind of inconsistency handling, since in these approaches conflicts between arguments can only arise if the knowledge base is inconsistent. By contrast, in defeasible-reasoning approaches conflicts can arise from consistent knowledge bases, since in those approaches it is the application of defeasible rules that makes an argument fallible.

Two groups in particular initiated the plausible-reasoning approach to argumentation, respectively at Queen Mary's University in London and at INRIA in Toulouse. Elvang-Göransson *et al.* [1993] conceived of arguments as premise-conclusion pairs (δ, p) where δ is a subset of a possibly inconsistent database Δ and there exists a natural-deduction proof of p from δ. Arguments can be attacked in two ways: an argument (δ', q) *rebuts* (δ, p) if q is logically equivalent to $\neg p$ and it *undercuts* it if q is logically equivalent to $\neg r$ for some $r \in \delta$. Note that Elvang-Göransson *et al.* thus introduced a terminological confusion into the literature that exists until today. While they fully adopted Pollock's [1974; 1987] terminology, they only partly adopted its meaning, since Pollock used the term 'undercutter' not for premise attack but for attack on the application of a defeasible inference rule. Today, Pollock's meaning of the term 'undercutter' is adopted in the $ASPIC^+$ framework and Dung's recent work on structured argumentation frameworks, while Elvang-Göransson *et al.*'s meaning is fashionable in work on classical and Tarskian argumentation.

Elvang-Göransson *et al.* classified arguments into five classes of increasing degrees of acceptability: arguments, consistent arguments (i.e., arguments with consistent premises), non-rebutted consistent arguments, non-rebutted and non-undercut consistent arguments, and "tautological" arguments (i.e., arguments with an empty set of premises). In light of modern work this definition of argument acceptability seems somewhat ad-hoc. Among other things, it does not model the notions of defense and admissibility that are so beautifully modelled by Dung [1995]. The ideas of Elvang-Göransson *et al.* were further developed by Krause *et al.* [1995], replacing classical logic by intuitionistic logic as the underlying logic and adding notions of argument structure and argument strength.

Around the same time as Elvang-Göransson *et al.*, Benferhat *et al.* [1993] proposed a similar system, containing what now is the standard definition of an argument in this approach, adding to Elvang-Göransson *et al.*'s definition the requirements that the set of premises is consistent and subset-minimal:

Definition 2.2. *Given a database Σ, a set $\Sigma_i \subseteq \Sigma$ is an argument for a formula φ iff:*

1. $\Sigma_i \nvdash \bot$*; and*
2. $\Sigma_i \vdash \varphi$*; and*
3. *for all* $\psi \in \Sigma_i$*:* $\Sigma_i \setminus \{\psi\} \nvdash \varphi$

Here, $\vdash$ denotes classical propositional consequence. Benferhat *et al.* did not define explicit notions of attack. Instead they defined φ to be an *argumentative consequence* of Σ if given Σ there exists an argument for φ but not for $\neg\varphi$. They also studied alternative consequence notions and their relations, and refined their system with a preference relation on the database. Their approach was related to abstract argumentation by Cayrol [1995], who among other things proved that with Elvang-Göransson *et al.*'s undercutting relation as the attack relation, the stable extensions given a database are in a one-to-one correspondence with the database's maximal consistent subsets. This result was later generalised by Amgoud and Besnard [2013] for any abstract Tarskian logic and by Modgil and Prakken [2013] in the context of the $ASPIC^+$ framework.

The ideas of Elvang-Göransson *et al.* and Benferhat *et al.* were picked up by e.g. Amgoud and Cayrol [1998] and Besnard and Hunter [2001] and evolved into classical, or classical-logic argumentation e.g. [Besnard and Hunter, 2008; Gorogiannis and Hunter, 2011] and its generalisations to deductive [Besnard and Hunter, 2014] and abstract Tarskian argumentation [Amgoud and Besnard, 2013], to be further discussed below.

2.4 Structured argumentation: developments until now

While until 1995 work on structured argumentation had specific and sometimes ad-hoc definitions of argument evaluation, since 1995 most work on structured argumentation adopts Dung's approach or at least explicates the relation with it. Work that adopts Dung's approach does so by giving definitions of the structure of arguments and the nature of attack. Thus abstract argumentation frameworks are generated, so that arguments can be evaluated according to one of the abstract argumentation semantics and their acceptability status can be used to define nonmonotonic consequence notions for their statements. However, there is also work that deviates from Dung's approach. In this section I will give an overview of these research strands.

2.4.1 Argumentation models of plausible reasoning

Current argumentation models of plausible reasoning are essentially of two kinds.

Assumption-based argumentation Around the same time as argumentation was proposed as a way of inconsistency handling in classical logic, assumption-based argumentation (ABA) emerged from attempts to give an argumentation-theoretic semantics to logic-programming's negation as failure [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997]. Like the classical-logic approaches, ABA also assumes a unique 'base logic', which in ABA is called a "deductive system", consisting of set of inference rules defined over some logical language. Given a set of so-called 'assumptions' formulated in the logical language, arguments are then deductions of claims using rules and supported by sets of assumptions. Contrary to in classical and abstract argumentation, the premises of ABA arguments, i.e., its assumptions, do not have to be consistent. ABA leaves both the logical language and set of inference rules unspecified in general, so it is like Vreeswijk's [1993b; 1997] approach and the later $ASPIC^+$ framework, an abstract framework for structured argumentation. However, unlike these approaches, ABA only allows attacks on an argument's assumptions, so that ABA's rules are effectively equivalent to Vreeswijk's and $ASPIC^+$'s strict inference rules (as formally confirmed in [Prakken, 2010]).

In order to express conflicts between arguments, ABA makes like Vreeswijk a minimum assumption on the logical language, which in ABA is that each assumption in the logical language has a *contrary.* That b is a contrary of a, written as $b = \overline{a}$, informally means that b contradicts a. An argument using an assumption a is then attacked by any argument for conclusion $\overline{a}$. Contrary relations do not have to be symmetric. This feature allows an argumentation-theoretic semantics for negation as failure (*not*) by for every formula *not* p letting $p = \overline{not\ p}$ but not vice versa. However, ABA's application is not limited to logic programming; in the landmark ABA paper [Bondarenko *et al.*, 1997], it is instantiated with various nonmonotonic logics, including default logic, circumscription and Poole's [1989] Theorist system.

Although ABA and Dung's approach clearly have commonalities, ABA as originally formulated by Bondarenko *et al.* [1997] does not generate abstract argumentation frameworks. Instead, its extensions are (in some sense maximal) sets of assumptions, induced by transforming attack relations between arguments to attack relations between sets of assumptions. Only ten years later was ABA given an explicit Dungean formulation by Dung *et al.* [2007]. Currently, there is some controversy about whether the correspondence holds for all current abstract argumentation semantics or not; cf. Gabbay [2015] and Caminada [2015].

ABA was originally used theoretically as a framework for nonmonotonic logic. Over the years, the focus has shifted somewhat to developing algorithms and implementations and to applying these to a wide range of reasoning and decision problems.

An interesting variant of assumption-based argumentation is Verheij's [2003] DefLog system. Verheij assumes a logical language with just two connectives, a unary connective $\times$ which informally stands for 'it is defeated that' and a binary connective $\rightsquigarrow$ for expressing defeasible conditionals. Verheij then assumes a single inference scheme for this language, namely, modus ponens for $\rightsquigarrow$. A set of sentences

T is said to *support* a sentence φ if φ is in T or follows from T by repeated application of $\leadsto$-modus ponens. Moreover, T is said to *attack* φ if T supports $\times\varphi$. Verheij then considers partitions (J, D) of sets of sentences Δ such that J (the "justified" sentences) is conflict-free and attacks every sentence in D (the "defeated" sentences). As observed by Verheij, DefLog can be encoded as an ABA instance with stable semantics by setting ABA's assumptions to Δ, defining the ABA ABF contrary mapping as $\times\varphi = \overline{\varphi}$ for any φ and letting ABA's set of rules be generated by the modus scheme for $\leadsto$.

Classical, deductive and Tarskian argumentation The initial work of Elvang-Göransson *et al.* [1993] and Benferhat *et al.* [1993] led to a family of approaches usually called 'classical' or 'deductive' argumentation [Amgoud and Cayrol, 2002; Besnard and Hunter, 2001; Kaci *et al.*, 2007; Besnard and Hunter, 2008; Amgoud and Vesic, 2010; Kaci, 2010]. The first name refers to instances with as base logic classical propositional or first-order logic, while the term 'deductive argumentation' is used for approaches that abstract from particular base logics, as long as they are "deductive". Often the term 'deductive' is here used in an informal sense. For example, Besnard and Hunter [2014] describe a deductive inference as an inference that is "infallible in the sense that it does not introduce uncertainty". This agrees with Pollock's notion of a deductive reason. Recently Amgoud and Besnard [2010; 2013] gave a precise interpretation by assuming that the base logic satisfies the properties of a so-called Tarskian abstract logic.

In all these approaches arguments are, as in Benferhat *et al.* [1993] for the special case of classical propositional logic, premises-conclusion pairs such that the premises are, according to the base logic, consistent and subset-minimal sets logically implying their conclusion. Unlike in many other approaches, these approaches do not commit to specific definitions of argument attack but explore the consequences of various definitions, all exhibiting some form of premise- and/or conclusion attack. Given that these approaches locate all fallibility of arguments in their premises, one might expect that definitions that only allow premise attack are the best-behaved. This was formally confirmed by Gorogiannis and Hunter [2011] and Amgoud and Besnard [2013] who, for respectively classical and Tarskian argumentation, showed that when abstract argumentation frameworks are generated, only particular forms of premise attack fully guarantee the consistency of the conclusion sets of extensions of abstract argumentation frameworks.

Until these investigations, research in this strand was not much concerned with argument evaluation. Instead, other properties were studied, such as relations between kinds of attack, and the formalisms were used as a tool for investigating dialogue-related questions, such as enthymemes [Black and Hunter, 2012] and persuasive force of arguments [Hunter, 2004]. See for further details e.g. Besnard and Hunter [2008; 2014].

2.4.2 Argumentation models of defeasible reasoning

Defeasible Logic Programming Defeasible Logic Programming, or DeLP [Garcia *et al.*, 1998; Garcia and Simari, 2004] is a further development of Simari and Loui's [1992] argumentation system with strict and defeasible rules. While Simari and Loui only allowed specificity as a source of preferences, DeLP allows any preference ordering. DeLP's logic-programming rules can contain both explicit negation and negation as failure. It is noteworthy that while the consequence notion of Simari and Loui's system is equivalent to Dung's [1995] grounded semantics, DeLP as described by Garcia *et al.* [1998] and Garcia and Simari [2004] does not conform to any of Dung's semantics. Instead, it is based on the notion of a dialectical tree, which essentially captures all ways in which a proponent and an opponent of a claim can have a debate about the claim by defeating each other's arguments. This notion is very similar to the notion of an argument game as a proof theory for the semantics of abstract argumentation frameworks (see further Section 2.5.2). However, while the constraints on argument games are based on the semantics for abstract argumentation frameworks, DeLP's constraints on dialectical trees are based on intuitions concerning concrete examples.

A unifying approach: the *ASPIC*$^+$ framework The *ASPIC*$^+$ framework [Prakken, 2010; Modgil and Prakken, 2013; Modgil and Prakken, 2014] unifies plausible and defeasible reasoning. Its main sources of inspiration are the systems of Pollock [1987; 1994; 1995] and Vreeswijk [1993b; 1997], which model defeasible reasoning. However, *ASPIC*$^+$ adds to these systems the possibility to attack an argument's premises, which makes it also suitable for modelling plausible reasoning. Apart from this, *ASPIC*$^+$ adopts Pollock's distinction between deductive (strict) and defeasible inference rules, Vreeswijk's definition of an argument and Pollock's notions of rebutting and undercutting attack, with the exception that in *ASPIC*$^+$, unlike in Pollock's systems, undercutting attack succeeds as defeat irrespective of preferences. Also, like Vreeswijk, *ASPIC*$^+$ abstracts from particular logical languages, sets of inference rules and argument orderings. Unlike Vreeswijk's particular method of argument evaluation, *ASPIC*$^+$ generates abstract argumentation frameworks, so that any semantics for such frameworks can be used to evaluate arguments.

A preliminary version of *ASPIC*$^+$ was developed during the EC-sponsored ASPIC project, which ran from 2004 to 2007. This version was used by Caminada and Amgoud [2007] as a vehicle for proposing the idea of rationality postulates for structured argumentation. The first publication focusing on $ASPIC^+$ as a framework for structured argumentation was Prakken [2010]. Modgil and Prakken [2013] proposed some small modifications and variations and proved further results on the framework and its relation with other work. Recently, several other variations of the $ASPIC^+$ framework have been studied.

Its abstract nature makes that $ASPIC^+$ can be instantiated in many different ways and captures a number of other approaches as special cases. For example, Prakken [2010] proves that Dung *et al.*'s [2007] version of assumption-based argumentation can be reconstructed as a special case of $ASPIC^+$ with only strict inference rules, no unattackable premises and no preferences. And Modgil and Prakken [2013] reconstruct two forms of classical argumentation as studied by Gorogiannis and Hunter [2011] as the special case with only strict rules, being all valid classical inferences from finite sets, no unattackable premises, no preferences and the constraint that an argument's premises are classically consistent and subset-minimal. They then generalise this reconstruction with a preference relation on the knowledge base and prove that the resulting stable extensions are in a one-to-one correspondence with Brewka's [1989] preferred subtheories. Thus they also extend Cayrol's [1995] similar result without preferences for maximal consistent subsets.

Not only $ASPIC^+$ but also assumption-based argumentation is an abstract model of structured argumentation. Compared to ABA, $ASPIC^+$ is more complex, with its two kinds of inference rules, its three kinds of attack and its explicit preferences to distinguish between attack and defeat. As stated by Toni [2014], the philosophy behind ABA is instead to translate preferences and defeasible rules into ABA rules plus ABA assumptions, so that rebutting and undercutting attack and the application of preferences all reduce to premise attack. This approach has its merits but it is an open question whether $ASPIC^+$ can in its full generality be translated into ABA. Currently there are only partial answers to this question. Dung and Thang [2014] prove for the case without preferences that defeasible $ASPIC^+$ rules can be translated to ABA rules with assumption premises. Moreover, in an early paper, Kowalski and Toni [1996] give a partial method for encoding rule preferences with explicit assumption premises. However, it remains to be seen whether this can be done for any argument ordering. Moreover, $ASPIC^+$ representations of examples are often arguably closer to natural-language than ABA presentations, in which every conflict has to be translated to premise attack and every preference statement to explicit exceptions. If the aim is to formalise modes of reasoning in a way that corresponds with human modes of reasoning and debate, then there is some merit in having a theory with explicit notions of rebutting and undercutting attack and preference application.

2.4.3 The study of rationality postulates

An important recent development is the introduction by Caminada and Amgoud [2005; 2007] of the idea of rationality postulates for structured argumentation. According to Caminada and Amgoud, all systems of structured argumentation that have notions of negation, strict rules and subarguments should satisfy the following properties:

> **Sub-argument Closure**: For any argument A in E, all sub-arguments of A are in E.
>
> **Closure under Strict Rules**: If E contains arguments with conclusions $\alpha_1, \ldots . \alpha_n$, then any arguments obtained by applying only strict inference rules to these conclusions, are in E.
>
> **Direct Consistency**: The set of conclusions of all arguments in E are directly consistent, i.e., it contains no pair of formulas φ and $\neg\varphi$.
>
> **Indirect Consistency**: The set of conclusions of all arguments in E are indirectly consistent, i.e., its closure under strict rules is directly consistent.

ASPIC$^+$ unconditionally satisfies closure under subarguments. Whether *ASPIC*$^+$ satisfies closure under strict rules and the consistency postulates depends on whether the non-attackable premises are consistent, on structural properties of the strict rules and on properties of the argument ordering [Caminada and Amgoud, 2007; Prakken, 2010; Modgil and Prakken, 2013]. These results on *ASPIC*$^+$ directly generalise to systems that can be reconstructed within *ASPIC*$^+$, such as assumption-based argumentation and several forms of classical and deductive argumentation with preferences. Recently, Dung and Thang [2014] identified alternative and partly weaker sufficient conditions for satisfying strict closure and consistency.

Three further rationality postulates were proposed by Caminada *et al.* [2012] and are about the extent to which contradictions can trivialise the set of conclusions. These postulates have been further studied by Wu and Podlaszewski [2015].

Although Caminada and Amgoud defined their postulates for rule-based systems, they can be straightforwardly adapted to systems that define argument structure in terms of consequence notions instead of inference rules, such as classical and deductive argumentation. In particular the consistency postulates have been studied for these approaches [Gorogiannis and Hunter, 2011; Amgoud and Besnard, 2013]. One insight here (of which the core is already in Caminada and Amgoud [2007]) is that satisfaction of the consistency postulates partly depends on the definitions of attack and defeat. Building on this idea, Dung [2014; 2016] proposes several desirable properties for defeat relations (which in line with his 1995 paper he calls 'attack' relations) and studies their effect on satisfaction of the consistency postulates.

Finally, the recent research on rationality postulates is reminiscent of work in other areas of nonmonotonic logic on general properties of nonmonotonic consequence notions [Gabbay, 1985; Kraus *et al.*, 1990; Makinson, 1994]. One much discussed property in that body of work is *cautious monotony.* Informally, this property is that if φ and ψ are implied by a knowledge base and φ is added to the knowledge base, then ψ is still implied by the new knowledge base. Recently, Dung [2014; 2016] has argued that this property should hold for credulous argumentation-based inference, i.e., for membership of at least one extension. By contrast, Prakken and Vreeswijk [2002], Section 4.4 argue that satisfaction of this property is not desirable in general, since strengthening a nonmonotonic conclusion to an indisputable fact can give arguments using the fact the power to defeat other arguments that they did not have before; and this may well result in the loss of the extension from which the conclusion was promoted to an indisputable fact.

2.4.4 Preferences and argument strength

An important element in many argumentation systems is the use of some notion of preference or strength to resolve conflicts between arguments. In Dungean terms, this boils down to defining his attack relation in terms of a more basic, non-evaluative notion of conflict between arguments and some binary preference relation on arguments. As noted above, most work before Dung [1995] used the term 'defeat' instead of 'attack' while much work after 1995 explicitly renamed Dung's attack relation to 'defeat' in order be able to call the more basic, non-evaluative notion of conflict 'attack'. This is what I will also do in this section. The use of preferences then amounts to checking which attacks succeed as defeats.

Arguably the first systems embodying some form of argument preference were the inheritance systems of Touretzky [1984] and Horty *et al.* [1990], which used syntactic specificity checks on inheritance paths to let inheritance paths from more specific classes defeat conflicting inheritance paths form more general classes. Loui [1987] and Simari and Loui [1992] also used specificity for conflict resolution.

Although Pollock's earliest system, from 1987, did not yet include a notion of strength, Pollock later took the notion of strength of arguments very seriously. Since his systems were meant for epistemic reasoning, he always formulated strength in terms of numerical degrees of belief. His approach here was non-standard. Against Bayesian approaches, he argued that degrees of belief and justification do not conform to the laws of probability theory. In his [1994, 1995], Pollock used a weakest-link approach to compute the strength of arguments: given numerical strengths of reasons (where deductive reasons have infinite strength), the strength of an argument's conclusion is the minimum of the strengths of the reason with which the conclusion is derived and the strengths of the intermediate conclusions to which this reason is applied. While thus arguments can have various strengths, defeat is still an all-or-nothing matter in that defeaters that are weaker than their target cannot affect the status of their target at all. This allows a reconstruction of Pollock's [1994, 1995] approach in terms of Dung's theory of abstract argumentation frameworks. Later, in his [2002, 2007a, 2010] Pollock explored the idea that weaker defeaters can still weaken the justification status of their stronger targets. To formalize this, he now made the justification status of statements a matter of numerical degree, being a function of the strengths of both supporting and defeating arguments. Thus in his latest work he deviated from a Dungean approach.

Similar to Pollock's [1994; 1995] way to use degrees of belief is Chesñevar *et al.*'s [2004] use of possibilistic logic in the context of Defeasible Logic Programming. In this paper, possibilistic strengths are added to rules, which are propagated through arguments according to possibilistic logic. Then the propagated strengths are used to resolve attacks into defeats.

Other early work resolved attacks with qualitative preference relations on premises or inference rules. One of the first argumentation models of defeasible reasoning with rule preferences from arbitrary sources was Prakken [1993], developed into Prakken and Sartor [1997]. One of the first argumentation models of plausible reasoning with prioritized knowledge bases was Benferhat *et al.* [1993]. Amgoud and Cayrol [1998; 2002] combined Benferhat *et al.*'s idea of prioritised knowledge bases and Cayrol's [1995] Dungean modelling of classical argumentation with Prakken and Sartor's way to distinguish between attack and defeat in Dung's grounded semantics and their argument game for it. Later papers included preferences in classical argumentation in other ways; e.g. Amgoud and Vesic [2010] and Kaci [2010].

Vreeswijk [1993a; 1997] was the first to include a binary argument ordering as primitive in his approach. The $ASPIC^+$ framework adopts this idea and several papers on $ASPIC^+$ study instantiations with qualitative preference relations on defeasible rules and attackable premises, building on the work of Benferhat *et al.* [1993], Prakken and Sartor [1997] and their successors. Recently, Dung [2014; 2016] has also contributed to this study.

Since there is not a unique kind of content of arguments, there is also not a unique kind of argument preference. In epistemic reasoning, argument preferences are often based on probabilistic considerations, degrees of belief, or on credibility estimates of information sources. In argumentation as decision making they have been based on preferences for decision outcomes. In normative (legal or moral reasoning) they have been derived from hierarchical relations between elements of normative systems.

In addition, some have modelled argumentation *about* preference relations within argumentation logics. One of the first proposals of this kind was made by Prakken and Sartor [1997]. Modgil [2009] extended abstract argumentation frameworks with the possibility to attack attacks. Modgil then, among other things, showed that Prakken and Sartor's proposal can be reconstructed as an instance of his 'extended argumentation frameworks'.

One question here is whether preference relations logically behave the same regardless of their source. Dung [2016] seems to answer this question affirmatively, while Modgil and Prakken [2014] suggest that the right way to use preferences may depend on the kind of content of arguments, for example, on whether the reasoning is epistemic, normative or about decision making.

2.5 Abstract argumentation: developments into now

In the first years after publication of Dung's landmark paper it gave rise to two kinds of follow-up work. Some continued to use AFs as Dung did in his paper, namely, to reconstruct and compare existing systems for structured argumentation as instances of AFs. In line with this was work on developing new systems for structured argumentation as instances of AFs. Others further developed the theory of abstract argumentation frameworks in the form of proof of properties (such as complexity results), reformulations (e.g. in terms of labellings), argument games as a proof theory, and algorithms. Somewhat later a third kind of follow-up work emerged, namely, extending AFs with new elements without specifying the structure of arguments. I now briefly review these three bodies of work.

2.5.1 Instantiating abstract argumentation frameworks

Some continued Dung's work on reconstructing and comparing existing systems for structured argumentation as instances of AFs. For example, Jakobovits [2000] and Jakobovits and Vermeir [1999b] showed that Pollock's [1994; 1995] system for defeasible reasoning has preferred semantics and Cayrol [1995] related various forms of classical argumentation to Dung's stable semantics and (with Amgoud in [Amgoud and Cayrol, 2002]) to Dung's grounded semantics for AFs. More recent work in this vein is Gorogiannis and Hunter [2011] and Amgoud and Besnard [2013].

Others developed new systems for structured argumentation as an instantiation of abstract argumentation frameworks. As described above, possibly the first system developed in this way was Prakken and Sartor's [1997] system for argumentation-based logic programming. More recently, the $ASPIC^+$ framework was designed in this way.

2.5.2 Developing the theory of abstract argumentation frameworks

Labellings A few years after Dung introduced his extension-based approach to abstract argumentation, an alternative labelling-based approach became popular, based on the following definition:

> A *labelling* of an $AF = \langle AR, attacks \rangle$ assigns to zero or more members of AR either the status *in* or *out* (but not both) such that:
>
> 1. an argument is *in* iff all arguments attacking it are *out*.
> 2. an argument is *out* iff it is attacked by an argument that is *in*.
>
> Let $In = \{A \in AR \mid A \text{ is } in\}$ and $Out = \{A \in AR \mid A \text{ is } out\}$ and $Undecided = AR \setminus (In \cup Out)$. Then
>
> 1. A labelling is *stable* if $Undecided = \emptyset$.
> 2. A labelling is *preferred* if *Undecided* is minimal (wrt set inclusion)
> 3. A labelling is *grounded* if *Undecided* is maximal (wrt set inclusion)
> 4. Any labelling is *complete*.

These notions coincide with Dung's extension-based definitions as follows. Let $S \in$ {stable, preferred, grounded, complete}. Then (In, Out) is an S-labelling iff In is an S-extension.

To illustrate the labelling definition, in Figure 3 the grey-white colourings correspond to the *in*-*out* labels in the unique stable/preferred/grounded/complete labelling. In Figure 4(b,c) the grey-white colourings correspond to the *in*-*out* labels of the two stable-and-preferred labellings but in Figure 4(b,c) both arguments are undecided.

Actually, Pollock was a source of inspiration here too, since he used a labelling definition in his [1994; 1995] system. Pollock was possibly in turn inspired by Doyle's [1979] justification-based truth maintenance systems. Pollock's 1994 system was, as just noted, by Jakobovits [2000] proved to be an instance of Dung's preferred semantics. Jakobovits' PhD thesis contains an in-depth investigation of the labelling approach, summarised by Jakobovits and Vermeir [1999b]. Other early work on labellings was done by Verheij [1996] and the labelling approach was finally popularised by Caminada [2006].

Argument games Both the extension- and the labelling-based approach can be regarded as a semantics of argumentation-based inference in that the main focus is on characterising properties of *sets* of arguments, without specifying procedures for determining whether a given argument is a member of the set. The proof theory of argumentation-based inference amounts to specifying such procedures. An elegant form of such a proof theory is that of an *argument game* between a proponent and an opponent of an argument. The precise rules of the game depend on the semantics the game is meant to capture. The rules should be chosen such that the existence of a winning strategy (in the usual game-theoretic sense) for the proponent of an argument corresponds to the investigated semantic status of the argument, for example, 'being in the grounded' or 'being in at least one (or in all) preferred extensions'.

To give an idea, the following game is sound and complete for grounded semantics in that the proponent of argument A has a winning strategy just in case A is in the grounded extension. The proponent starts a game with an argument and then the players take turns, trying to defeat the previous move of the other player. In doing so, the proponent must strictly defeat the opponent's arguments while he is not allowed to repeat his own arguments. A game is terminated if it cannot be extended with further moves. The player who moves last in a terminated game wins the game. Thus the proponent has a winning strategy if he has a way to make the opponent run out of moves (from the implicitly assumed AF) whatever choice the opponent makes.

The idea of argument games had been around since the beginning of the formal study of argumentation (see e.g. Vreeswijk [1993a]) but they were not formally linked to argumentation-based semantics until the mid 1990s. Dung [1995] refers to a technical report [Dung, 1992] that was never formally published and in which he proposed argument games for two logic-programming semantics. Prakken and Sartor [1997] proposed an argument game for their logic-programming instantiation of Dung's grounded semantics. Arguably the first publication on argument games for abstract argumentation semantics was Prakken [1999], who proposed the above game for grounded semantics as an abstraction of the game of Prakken and Sartor. Vreeswijk and Prakken [2000] proposed argument games for preferred semantics, which were further developed and studied by Dunne and Bench-Capon [2003].

New semantics and general study of semantics While Dung [1995] originally proposed four semantics for abstract argumentation frameworks, in later years several alternative semantics were proposed; cf. Baroni *et al.* [2011a]. A related development is the study of general characterisations of types of semantics and their properties and relations, initiated by Baroni and Giacomin [2007] and further pursued by e.g. Dvorak and Woltran [2011] and Baroni *et al.* [2014]. Baroni and Giacomin [2007] also had a normative aim, namely, to propose a set of principles for the evaluation of semantics for abstract argumentation frameworks. Thus their work can be seen as an abstract counterpart of Caminada and Amgoud's [2007] introduction of rationality postulates for structured argumentation formalisms (see Section 2.4.3 above).

Complexity results and algorithms The graph-based format of abstract argumentation frameworks naturally lends itself to studies of computational complexity. A leading figure here has been Paul Dunne [Dunne and Bench-Capon, 2002; Dunne and Bench-Capon, 2003; Dunne, 2007].

Algorithms for proof theories for abstract argumentation frameworks were proposed by e.g. [Cayrol *et al.*, 2003; Vreeswijk, 2006; Verheij, 2007]. Early work on algorithms for enumerating extensions or labellings is reviewed by [Modgil and Caminada, 2009]. An interesting strategy for developing algorithms is encoding argumentation frameworks in some other formalism and to utilise algorithms for the other formalism. For example, [Besnard and Doutre, 2004] encoded abstract argumentation frameworks in propositional logic in order to apply model-checking and SAT solver techniques. They also proposed an equation checking approach, which was later further developed by [Gabbay, 2011]. Some other examples of this approach are Grossi's [2010] encoding of abstract argumentation frameworks in modal logic and Egly *et al.*'s [2010] encoding in answer set programming.

2.5.3 Adding new elements to abstract argumentation frameworks

A third research strand in the abstract approach to argumentation is to extend AFs with new elements without specifying the structure of arguments. In this subsection I briefly discuss various ways in which this has been done.

Adding preferences or values [Amgoud and Cayrol, 1998] added to abstract argumentation frameworks a a preference relation on AR, resulting in *preference-based argumentation frameworks* ($PAFs$), which are a triple $\langle AR, attacks, \preceq \rangle$. An argument A then *defeats* an argument B if A attacks B and $A \not\prec B$. Thus each PAF generates an AF of the form $\langle AR, defeats \rangle$, to which Dung's theory of abstract argumentation frameworks can be applied.

[Bench-Capon, 2003] proposed a variant of idea called *value-based argumentation frameworks* ($VAFs$), in which each argument is said to promote some value. The notion of value should be taken here not in a numerical sense but in the sense of, for example, legal, moral or societal values, such as welfare, equality, fairness, certainty of the law, freedom of speech, privacy, and so on. Attacks are in $VAFs$ resolved in terms of one or more orderings on the values. These value orderings are assumed to be provided by an audience evaluating the arguments.

Adding abstract support relations There have been several recent proposals to extend Dung's [1995] well-known abstract argumentation frameworks (AFs) with abstract support relations, such as Cayrol and Lagasquire-Schiex's [2005b; 2009; 2013] Bipolar Argumentation Frameworks (BAFs), the work of [Martinez *et al.*, 2006] and Oren and Norman's [2008] Evidential Argumentation Systems (EASs). Various semantics for such frameworks have been defined, claimed to capture different notions of support. For example, [Martinez *et al.*, 2006] want to abstract from subargument relations in systems for structured argumentation. [Boella *et al.*, 2010a] study semantics of what they call "deductive" support, which satisfies the constraint that if A is acceptable and A is a deductive support of B, then B is acceptable. [Nouioua and Risch, 2011] consider "necessary support", which satisfies the constraint that if B is acceptable and A is a necessary support of B, then A is acceptable.

Other additions Both [Bochman, 2003] and [Nielsen and Parsons, 2007b] generalised Dung's attack relation to a relation from *sets* of arguments to arguments. As noted above, [Modgil, 2009] extended abstract argumentation frameworks with attacks on attacks, as an abstraction of earlier proposals to model reasoning about priorities in nonmonotonic logics. [Coste-Marquis *et al.*, 2006] added constraints to argumentation frameworks in the form of propositional encodings of properties of extensions. Finally, [Dunne *et al.*, 2011] added weights to attacks, the idea being that attacks that are of insufficient weight (modelled by a "weight budget") can be ignored.

A word of caution Although it is tempting to extend abstract argumentation frameworks with additional elements, a word of caution is in order. One should resist the temptation to think that for any given argumentation phenomenon the most principled analysis is at the level of abstract argumentation frameworks. In fact, it often is the other way around, since at the abstract level crucial notions like claims, reasons and grounds are abstracted away.

An example where this leads to problems is the way preferences are used in $PAFs$ and $VAFs$ to resolve attacks. As shown in work on structured argumentation with preferences (e.g. Pollock's or Vreeswijk's system, $ASPIC^+$ or DeLP), the structure of arguments is crucial in determining how preferences must be applied to attacks. Consider the following semi-formal example adapted from [Prakken, 2012; Modgil and Prakken, 2013], which can easily be formalised in any of the above-mentioned systems for structured argumentation.

$A = p$
$B_1 = \neg p$
$B_2 = \neg p$, therefore, *presumably*, q

Here p and $\neg p$ are default assumptions. Note that B_1 is a subargument of B_2, so B_2 includes B_1 as part of itself. The arguments with their internal structure and their direct attack relations are displayed in Figure 5. In any of the above systems for

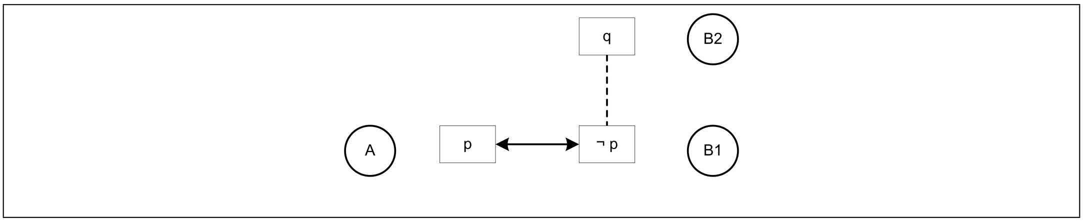

Figure 5: Argument structure and direct attack

structured argumentation we then have that A and B_1 directly attack each other while, moreover, A indirectly attacks B_2, since it directly attacks B_2's subargument B_1. So we have the abstract argumentation framework displayed in Figure 6(a).

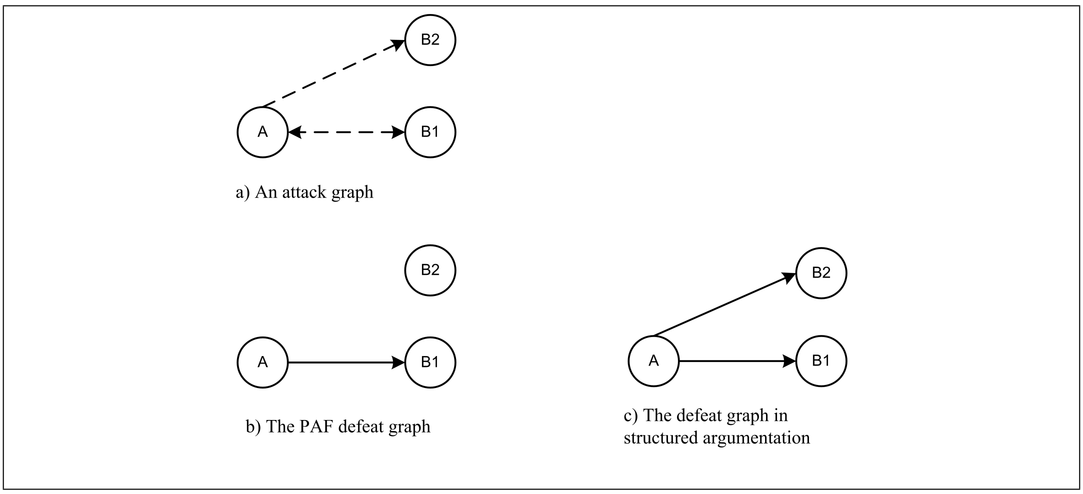

Figure 6: The abstract attack and defeat graphs

Assume next that A is preferred over B_1 and B_2 is preferred over A. Such an ordering could, for instance, be the result of comparing arguments according to their last fallible elements. A *PAF* modelling then generates the following single defeat relation: A defeats B_1; see Figure 6(b). Then we have a single extension (in whatever semantics), namely, $\{A, B_2\}$. So not only A but also B_2 is justified.

However, this violates Caminada and Amgoud's [2007] rationality postulate of subargument closure of extensions, since B_2 is in the extension while its subargument B_1 is not. ([Prakken, 2012] also discusses examples in which the postulate of indirect consistency is violated.) The cause of the problem is that the *PAF* modelling of this example cannot recognise that the reason why A attacks B_2 is that A directly attacks B_1, which is a subargument of B_2. So the *PAF* modelling fails to capture that in order to check whether A's attack on B_2 succeeds, we should compare A not with B_2 but with B_1. Now since $B_2 \prec A$ we also have that A defeats B_2; see Figure 6(c). So the single extension (in whatever semantics) is $\{A\}$, so closure under subarguments is respected.

This shows that *PAFs* (and also *VAFs*) only behave correctly under the assumption that all attacks are direct. We can conclude that for a principled analysis of the use of preferences to resolve attacks, the structure of arguments must be made explicit.

More generally, this analysis shows that in proposing an abstract model of argumentation, it is important to be aware what is abstracted from. Yet in the study of abstract support relations there is, unlike with Dung's original abstract frameworks, hardly any formal study of the relation between the abstract and the structured level. In consequence, it remains unclear what exactly is being modelled. One of the few studies in this vein is my [Prakken, 2014], in which I studied to what extent bipolar and evidential abstract frameworks can be interpreted as abstracting from the inferential relations in structured argumentation, as captured in $ASPIC^+$ by its subargument relations. I obtained mixed results. A form of BAFs that by [Boella *et al.*, 2010a] was claimed to be suitable for "deductive support" turned out to have no relation with classical-logic approaches to structured argumentation but Oren and Norman's [2008] evidential frameworks turned (for preferred semantics) out to be a suitable abstraction of $ASPIC^+$'s subargument relation. The same holds (for all four of Dung's [1995] semantics) for Dung and Thang's [Dung and Thang, 2014] proposal. They add a binary support relation to abstract argumentation frameworks with the sole additional constraint that if B supports C and A attacks B then A also attacks C, and they then evaluate arguments as in [Dung, 1995] by only taking the thus constrained attack relation into account. The resulting system conforms to Nouioua and Risch's [2011] notion of "necessary support". Apart from these results it is still an open question what abstract models of argumentation with support relations abstract from.

These discussions lead me to propose a (to some readers possibly controversial) methodological guideline that every new proposal for extending abstract argumentation frameworks should in the same paper be accompanied by at least one non-trivial instantiation in order to demonstrate the significance of the new extension. Work that respects this guideline, respects the historic origins of the abstract study of argumentation, since the prime example of how this guideline can be applied is [Dung, 1995], who instantiated his frameworks with four nonmonotonic logics. A more recent example is [Modgil, 2009], who showed that his 'extended argumentation frameworks', which extend abstract argumentation frameworks with attacks on attacks, can be instantiated with Prakken and Sortor's [1997] modelling of reasoning about preferences.

2.6 Further developments

I now briefly sketch some important further developments in the formal study of argumentation as inference.

2.6.1 More recent graph-based approaches

Since 2007 several graph-based approaches have been proposed, in which not arguments and their relations but statements and their relations are the main focus of attention. This idea also goes back to the work of Pollock, since the system of [Pollock, 1994] is strictly speaking not formalised in terms of arguments but in terms of so-called 'inference graphs', in which nodes are connected either by inference links (applications of inference rules) or by defeat links. The nodes are 'lines of argument', which are propositions plus an encoding of the argument lines from which they are derived. Nodes are evaluated in terms of the recursive structure of the graph. As noted above, [Jakobovits and Vermeir, 1999b] proved that Pollock's system can be given an equivalent formulation as an instance of Dung's abstract argumentation frameworks with preferred semantics.

[Gordon *et al.*, 2007] proposed the Carneades framework 'of argument and burden of proof'. Carneades' main structure is that of an argument graph, which, despite its name, is similar to Pollock's inference graphs. Statement nodes are linked to each other via argument nodes, which record the inferences from one or more nodes to another. This notion of an argument does not have the usual recursive structure in systems for structured argumentation but instead stands for a single inference step. Unlike Pollock, Carneades does not express conflicts as a special type of link between statement nodes. Instead, inferences (i.e., arguments) can be either pro or con a statement. The evaluation of statements in an argument graph is, as with Pollock's inference graphs, defined in terms of the recursive structure of the graph. Statements are acceptable if they satisfy their 'proof standard'. The general framework abstracts from their nature but [Gordon *et al.*, 2007] give several examples of proof standards.

Inspired by Carneades, [Brewka and Woltran, 2010] proposed their *Abstract Dialectical Frameworks*, which are directed graphs in which nodes are arguments, statements or positions which can be accepted or not and the links represent dependencies between arguments. The dialectical status (accepted or rejected) of a node depends on the status of its parents as specified in an acceptance condition for the node. [Brewka and Woltran, 2010] present ADFs as generalisations of abstract argumentation frameworks. In a purely technical sense they are, but so are assumption-based argumentation, Deflog and $ASPIC^+$, which can all represent AFs as a special case. For example, in assumption-based argumentation arguments from the AF can be made assumptions and an assumption can be said to be a contrary of another assumption if it attacks it in the AF. So far, applications of ADFs have instead interpreted the nodes as statements, e.g. [Strass, 2013], thus making ADFs more similar to Pollock's inference graphs. Future research should shed more light on the potential of ADFs as generalisations of abstract argumentation frameworks in a conceptual sense also.

2.6.2 Decision making as argumentation

While most early work on argumentation-based inference was on epistemic reasoning (what is the case?), in recent years there has been much attention for practical reasoning (what should we do?). Among the first papers on this topic was [Fox and Parsons, 1997], motivated by medical decision making.

One strand of work was initiated by Grasso *et al.*'s [2001] design for a nutrition advice system and Bench-Capon's [2003] formal work on value-based argumentation frameworks. Both works were influenced by Perelman and Olbrechts-Tyteca's [1969] idea (further discussed in Section 3.1) that whether an argument in ordinary discourse is good does not depend on its logical form but on whether it is capable of persuading the addressed audience, which in turn depends on the extent to which it takes the audience's "values" into account. The work on value-based argumentation frameworks was further developed by e.g. [Atkinson, 2005; Atkinson and Bench-Capon, 2007], which instantiated value-based argumentation frameworks with an argumentation scheme approach inspired by Walton's [Walton, 2006b] schemes for practical reasoning. This work has among other things been applied to legal interpretation [Atkinson *et al.*, 2005a], seen as a decision problem in which the various interpretation options promote or demote various legal or societal values.

Another strand of work is the work of Amgoud and others, e.g. [Amgoud *et al.*, 2005; Amgoud, 2009; Amgoud and Prade, 2009], which combines argumentation models of the inferential aspects of argumentation with models of qualitative decision theory for the choice aspects of decision making.

To compare and contrast the various bodies of work, note that decision making has various aspects: identifying possible decision options in the form of possible actions, identifying the decision criteria (preferences, desires, goals, values), determining the consequences of actions with respect to these criteria and choosing between the decision options. There is consensus that all these aspects up to the choice problem can be modelled as argumentation as inference, but there is no consensus whether they can be modelled as instantiating the above-discussed general models of structured argumentation or whether special argumentation formats should be developed. Examples of the former approach are [Kakas and Moraitis, 2003], who model decision-making arguments in Dimopoulos and Kakas's [1995] logic-programming system for argumentation, [van der Weide *et al.*, 2011; van der Weide, 2011], who model practical-reasoning arguments in a combination of $ASPIC^+$ with Wooldridge *et al.*'s [2006] formal model of meta-argumentation, and [Fan and Toni, 2013], who model decision making in assumption-based argumentation. Examples of the latter approach are [Amgoud, 2009], who proposes specific formats for decision-making arguments, and [Atkinson, 2005; Atkinson and Bench-Capon, 2007], who define special argument-schemes for practical-reasoning.

Another issue is whether argumentation-based decision-making can be fully modelled as argumentation-based inference. [Amgoud, 2009, p. 318] claims that decision making goes beyond inference when a choice has to be made between the decision options; all argumentation can do according to her is generating the decision options that have justified epistemic subarguments or assumptions. Accordingly, [Amgoud and Prade, 2009] model the choice between epistemically justified decision options outside their argumentation model in terms of models of qualitative decision theory. By contrast, the above-mentioned work in $ASPIC^+$ and assumption-based argumentation tries to model choice through the general conflict-resolution mechanisms of argumentation-based inference, such as $ASPIC^+$'s argument ordering.

2.6.3 Argumentation combined with probability theory

A recent trend is the combination of argumentation-based inference with probability theory. This is not surprising, since argumentation has from the early days on been proposed as a model for reasoning under uncertainty. Yet systematic studies of the combination of argumentation with probability were sparse until recently.

Argumentation has been combined with probability theory for three different kinds of purposes. First, there has been some work in which probabilistic models are the object of argumentative discourse, such as [Nielsen and Parsons, 2007a], who model how Bayesian networks can be jointly constructed in an argumentation process. In all other work the uncertainty does not concern the probabilistic but the argumentation model. Two approaches can be distinguished, depending on whether the uncertainty is *in* or *about* the arguments. When the uncertainty is in the arguments, probabilities are *intrinsic* to an argument in that they are used for capturing the strength of an argument given uncertainty concerning the truth of its premises or the reliability of its inferences. An example is default reasoning with probabilistic generalisations, as in *The large majority of Belgian people speak French, Mathieu is Belgian, therefore (presumably) Mathieu speaks French.* Clearly, if all premises of an argument are certain and it only makes deductive inferences, the argument should be given maximum probabilistic strength. [Hunter, 2013] calls this use of probability the *epistemic* approach.

When the uncertainty is about the arguments, probabilities are *extrinsic* to an argument in that they are used for expressing uncertainty about whether arguments are accepted as existing by some arguing agent. [Hunter, 2014] gives the example of a dialogue participant who utters an enthymeme and where the listener can imagine two reasonable premises that the speaker had in mind: the listener can then assign probabilities to these options, which translate into probabilities on which argument the speaker meant to construct. This uncertainty has nothing to do with the intrinsic strengths of the two candidate completed arguments: one might be stronger than the other while yet the other is more likely the argument that the speaker had in mind. [Hunter, 2013] calls this use of probability the *constellations* approach. Note that in this approach even deductive arguments from certain premises can have less than maximal strength.

The intrinsic, or epistemic approach can be applied in two ways: by simply computing probability values of conclusions or by using such probabilities to resolve attacks into defeats. Computing probability values of conclusions is done in early work by [Haenni *et al.*, 2000]. Their argumentation model is a rather specific one for diagnosis and has no clear relations with more general structured and abstract models of argumentation. More recent work in this vein is [Dung and Thang, 2010], who within assumption-based argumentation allow rules to be labelled with probabilities. As noted above in Section 2.4.4 [Pollock, 2002; Pollock, 2007a; Pollock, 2010] (using a non-standard account of probability) made the justification status of statements a matter of numerical degree, being a function of the strengths of both supporting and defeating arguments.

Other examples of the intrinsic/epistemic approach are methods for extracting arguments from (qualitative or quantitative) Bayesian networks. Older work in this vein is [Parsons, 1998a; Parsons, 1998b], using a logic similar to the one of [Krause *et al.*, 1995] and [Williams and Williamson, 2006], using the logic of [Prakken and Sartor, 1997]. In this work no probability values of conclusions are computed:

Parsons just generates arguments while Williams & Williamson generate abstract argumentation frameworks using rebutting attack without preferences. Recent work is of [Timmer *et al.*, 2017], who generates arguments in $ASPIC^+$ and resolves their rebutting attacks with probabilistic strengths of arguments. The latter is also done in two pieces of work using alternatives for standard probability theory, viz. Pollock's [1994; 1995] use of degrees of belief and Chesñevar *et al.*'s [2004] use of possibilistic logic (both discussed above in Section 2.4.4).

The extrinsic/constellations approach has been largely applied to abstract argumentation frameworks, as in [Li *et al.*, 2012] and [Hunter, 2014] (but see the early work of [Riveret *et al.*, 2007; Riveret *et al.*, 2008] using the logic of [Prakken and Sartor, 1997]). For a recent overview see [Hunter and Thimm, 2016]. In this approach, probabilities can unlike in the intrinsic approach, also be attached to, for instance, legal rules or moral value judgements. Another difference is that the extrinsic use of probability is defined on top of a separate model of argumentation existing independently of the probabilistic model, while in the second use probability is part of the argumentation model itself.

Assigning probabilities to arguments in the abstract is problematic, since in probability theory probabilities are assigned to the truth of statements or to outcomes of events, and an argument is neither a statement nor an event. What is required here is a precise specification of what the probability of an argument means. If it corresponds to the degree of justification of the argument's justification, then this should arguably be specified at the level of structured argumentation. For a preliminary attempt to do so in the context of classical-logic argumentation see [Hunter, 2013]. If the probability of an argument corresponds to the probability of a statement about the argument, then the nature of that statement should be made clear. More generally, here too the need arises to be explicit about what is abstracted from, in this case in abstract models of probabilistic argumentation.

2.6.4 Argumentation dynamics

A development that is in the border area of inference and dialogue is the logical study of the dynamics of argumentation, insofar as it abstracts from agent aspects and the dialogical setting. For example, [Coste-Marquis *et al.*, 2007] study the merging of abstract argumentation frameworks, with attention for the resolution of conflicts between the merged frameworks. Also, much work has recently been done on the nature and effects of change operations on a given argumentation state [Modgil, 2006; Rotstein *et al.*, 2008; Baumann and Brewka, 2010; Baroni *et al.*, 2011b]. Among other things, enforcing and preservation properties are studied. Enforcement concerns the extent to which desirable outcomes can or will be obtained by changing an argumentation state, while preservation is about the extent to which the current status of arguments is preserved under change. Quite recently, the revision of argumentation frameworks has been studied analogously to revising belief sets or bases in belief revision, i.e. as incorporating new or deleting old elements while keeping the changes minimal [Coste-Marquis *et al.*, 2014].

Almost all current work on argumentation dynamics concerns abstract argumentation frameworks. In particular the following operations have been studied: addition or deletion of (sets of) arguments (e.g. [Baumann and Brewka, 2010; Cayrol *et al.*, 2010; Baumann, 2012b; Baumann, 2012a]) and addition or deletion of (sets of) attack relations (e.g. [Modgil, 2006; Boella *et al.*, 2010b; Baroni *et al.*, 2011b; Bisquert *et al.*, 2013]). Deleting attacks can be seen as an abstraction from the use of preferences to resolve attacks into defeats.

This current abstract work on argumentation dynamics abstracts from the structure of arguments and the nature of their conflicts, which is a significant limitation. See e.g. [Modgil and Prakken, 2012], who for this reason propose a model of preference dynamics in $ASPIC^+$. For example, abstract models of argumentation dynamics do not recognise that some arguments are not attackable (such as deductive arguments with certain premises) or that some attacks cannot be deleted (for example between arguments that were determined to be equally strong), or that the deletion of one argument implies the deletion of other arguments (when the deleted argument is a subargument of another, as in Figure 6 above), or that the deletion or addition of one attack implies the deletion or addition of other attacks (for example, attacking an argument implies that all arguments of which the attacked argument is a subargument are also attacked; in Figure 6 above attacking B_1 implies attacking B_2). All this means that formal results about the abstract model may only be relevant for specific cases and may fail to cover many realistic situations in argumentation. To give a very simple example, in models that allow the addition of arguments and attacks, any non-selfattacking argument can be made a member of every extension by simply adding non-attacked attackers of all its attackers. However, this result at the abstract level does not carry over to instantiations in which not all arguments are attackable. Here, too, the importance shows of being aware what the model abstracts from.

2.6.5 Other work

I end this section on argumentation-based inference with a very brief review of some other relevant work (without any hope of being complete).

[Wooldridge *et al.*, 2006] proposed a formalism for meta-argumentation, supporting a hierarchical formalisation of logic-based arguments. At each level of the hierarchy, arguments, statements and positions can refer to arguments, statements and positions at lower levels. This is achieved by using a hierarchical first-order meta-logic, a type of first-order logic in which individual terms in the logic can refer to terms in another language. One application of this formalism is van der Weide's [2011] model of reasoning about preferences in argumentation about decision making.

Finally, a recent trend is to develop gradual notions of argument acceptability in terms of structural properties of abstract argumentation frameworks [Cayrol and Lagasquie-Schiex, 2005a; Grossi and Modgil, 2015].

3 Formal and computational models of argumentation-based dialogue

So far we have discussed argumentation as a form of (nonmonotonic) inference. However, argumentation can also be seen as a form of dialogue, in which two or more agents aim to resolve a conflict of opinion by verbal means. When argumentation is viewed as a kind of dialogue between 'real' agents (whether human or artificial), new issues arise, namely, the *distributed* nature of information (over the agents), the *dynamic* nature of information, since agents do not reveal everything they believe from the start and since they can learn from each other, and *strategic* issues, since agents will have their internal preferences, desires and goals. At first sight, it might be thought that the argument games for argumentation-based semantics discussed above in Section 2.5.2 are dialogical models of argumentation. However, this is not the case, since they are not meant for discussions between real agents but as a proof theory. There is no dynamics, no distributed information and the notions of a proponent and an opponent are just proof-theoretic metaphors, not real agents with preferences, desires and goals.

Research on argumentation-based dialogue divides into research on communication languages and protocols (their formal definition and study of their properties) and research on agent behaviour in argumentation dialogues (strategies, tactics, heuristics). Some work studies the combination of a protocol and agent behaviour within that protocol. The main idea of work on argumentation protocols is that such protocols should promote fair and effective resolution of conflicts of opinion. In work on argumentative agent design the agents are assumed to adhere to this purpose of the dialogue but within the rules of the protocol they can pursue their own interests and objectives.

Research on argumentation-based dialogue is often done against the background of Walton's [1984] classification of dialogues into six types according to their goal (see also e.g. [Walton and Krabbe, 1995]). *Persuasion* aims to resolve a difference of opinion, *negotiation* tries to resolve a conflict of interest by reaching a deal, *information seeking* aims at transferring information, *deliberation* wants to reach a decision on a course of action, *inquiry* is aimed at "growth of knowledge and agreement" and *quarrel* is the verbal substitute of a fight. This classification is not meant to be exhaustive and leaves room for dialogues of mixed type. Persuasion can, given its purpose, be seen as 'pure' argumentation and is often embedded in other dialogue types in that dialogues of other types may shift to persuasion if a conflict of opinion arises. For example, in information-seeking a conflict of opinion could arise on the credibility of a source of information, in deliberation the participants may disagree about likely effects of plans or actions and in negotiation they may disagree about the reasons why a proposal is in one's interest; also, in all three cases the participants may disagree about relevant factual matters.

The formal study of argumentation-based dialogue is less substantial and less advanced than the formal study of argumentation-based inference. Unlike with inference, it largely consists of a variety of different approaches and individual systems, with few unifying accounts or general frameworks. For these reasons this section is shorter than Section 2 on argumentation-based inference.

3.1 Main historical influences

As noted in Section 2.1.3, [Toulmin, 1958] claimed that outside mathematics the validity of an argument does not depend on its syntactic form but on whether it can be defended in a properly conducted dispute. It might be argued that Toulmin thus anticipated argumentation-based inference, especially in argument-game form. However, more importantly, he thus planted the seed of an idea that later became prominent in informal logic and argumentation theory, namely, that arguments can only be evaluated in the context of a dialogue. Toulmin's call to logicians of his days to study the criteria for properly conducted disputes can be regarded as a call to study dialogue protocols for argumentation.

The Belgian philosopher Chaïm Perelman also emphasised the dialogical nature of argument evaluation. However, he did not address protocol but strategy, in arguing that arguments in ordinary discourse should not be evaluated in terms of their syntactic form but on their rhetorical potential to persuade an audience [Perelman and Olbrechts-Tyteca, 1969]. In particular, an argument is more persuasive the more it takes the audience's "values" into account. For example, an argument that governments should not tap internet communications of their citizens since this infringes on their privacy is not very persuasive to an audience that values security over privacy. While initially Perelman's work was only influential in informal logic and argumentation theory, it was around 2000 taken up by AI researchers in both inferential and dialogical models of argumentation about action selection, starting with Grasso *et al.*'s [2001] design of a nutrition advice system and Bench-Capon's [2003] formal work on value-based argumentation frameworks (the latter was discussed above in Section 2.6.2). Other work more generally aimed at characterising the persuasive force of arguments in terms of the similarity of an argument with the beliefs of a typical audience [Hunter, 2004].

While argumentation logics define notions of consequence from a given body of information, dialogue systems for argumentation regulate disputes between real agents. Systems for persuasion dialogues were already studied in medieval times [Angelelli, 1970]. The modern study of formal dialogue systems for persuasion probably started with two publications by Charles Hamblin [1970, 1971], who coined the term 'formal dialectic', and was also inspired by speech act theory in philosophy [Searle, 1969] and dialogue logic [Lorenzen and Lorenz, 1978]. It should be noted that formal systems for persuasion dialogue differ from dialogue logics in one crucial respect. Dialogue logic aims to define the semantics of logical operators in terms of rules of attack and defence. Accordingly the purpose of a dialogue is to determine whether a proposition is implied by a given set of propositions and the roles of proponent and opponent are just logical metaphors, just as in the logical argument games discussed above in Section 2.5.2. By contrast, the purpose of a persuasion dialogue is to resolve a conflict of opinion between real agents, who can ask for and provide substantive reasons for their claims.

Initially, formal systems for argumentation as dialogue were studied only within philosophical logic and argumentation theory; see, for example, [Mackenzie, 1979; Mackenzie, 1990; Woods and Walton, 1978; Walton and Krabbe, 1995]. From the early nineties the study of argumentation dialogues was taken up in several fields of computer science. In Artificial Intelligence logical models of commonsense reasoning have been extended with formal models of persuasion dialogue as a way to deal with resource-bounded reasoning [Loui, 1998; Brewka, 2001]. Persuasion dialogues have also been used in the design of intelligent tutoring systems [Moore, 1993; Yuan, 2004] and were proposed as an element of computer-supported collaborative argumentation [Maudet and Moore, 1999]. In AI & law formal dialogue systems for persuasion were developed as a model of procedural justice in the sense of e.g. [Alexy, 1978]. See, for example, [Gordon, 1994; Hage *et al.*, 1993; Bench-Capon, 1998; Bench-Capon *et al.*, 2000; Lodder, 1999; Prakken, 2001a; Prakken, 2008]. Finally, in the field of multi-agent systems dialogue systems have been incorporated into models of rational agent interaction based on the observation that many kinds of agent interaction (such as negotiation and group decision making) involve argumentation. Accordingly, interaction protocols for various dialogue types involving argumentation have been designed [Parsons and Jennings, 1996; Kraus *et al.*, 1998; Parsons *et al.*, 1998; Amgoud *et al.*, 2000a; McBurney and Parsons, 2002; Parsons *et al.*, 2002; Parsons *et al.*, 2003].

Most dialogue systems for argumentation are formulated in an informal mathematical metalanguage, but some have studied the full formalisation of protocols in logical action languages, such as [Brewka, 2001] in the situation calculus, [Bodenstaff *et al.*, 2006] in the event calculus and [Artikis *et al.*, 2007] in C^{++}.

3.2 General remarks on dialogue systems for argumentation

Persuasion is usually modelled as a two-party dialogue between a proponent and an opponent of an initial claim. Essentially, dialogue systems define a *communication language* (the well-formed utterances) and a *protocol* (when a well-formed utterance may be made and when the dialogue terminates). The communication language consists of a set of locutions applied to statements or arguments expressed in a logical language according to some adopted monotonic or nonmonotonic logic. If this logic is nonmonotonic, it can but need not be an argumentation logic.

Dialogue systems define the principles of coherent dialogue. [Carlson, 1983] defined coherence in terms of the purpose of a dialogue. According to him, whereas logic defines the conditions under which a proposition is true, dialogue systems define the conditions under which an utterance is appropriate, and this is the case if the utterance furthers the purpose of the dialogue in which it is made. Thus according to Carlson the principles governing the meaning and use of utterances should not be defined at the level of individual speech acts but at the level of the dialogue in which the utterance is made. This justifies why most work on argumentation dialogues, like Carlson, takes a game-theoretic approach to dialogues, where speech acts are viewed as moves in a game and rules for their appropriateness are formulated as rules of the game. [Loui, 1998] distinguished between *effectiveness* and *fairness* of dialogue systems. Effectiveness means that the protocol furthers the purpose of the dialogue (in the case of persuasion that the conflict of opinion is resolved). Some aspects of effectiveness are efficiency (how long are dialogues and is there a guarantee of termination?) and relevance (is every move relevant to the dialogue topic?). Fairness means that the participants have a fair opportunity to argue their case. Some aspects of fairness are that the participants always have the opportunity to move relevant moves and that the outcome of a dialogue agrees with the parties' commitments.

Communication language Here are some common speech acts that can be found in the literature on persuasion dialogues, with their informal meaning and the various terms with which they have been denoted in the literature.

- *claim* φ (assert, statement, ...). The speaker asserts that φ is the case.
- *why* φ (challenge, deny, question, ...) The speaker challenges that φ is the case and asks for reasons why it would be the case.
- *concede* φ (accept, admit, ...). The speaker admits that φ is the case.
- *retract* φ (withdraw, no commitment, ..) The speaker declares that he is not committed (any more) to φ. Retractions are 'really' retractions if the speaker is committed to the retracted proposition, otherwise it is a mere declaration of non-commitment (e.g. in reply to a question).
- φ *since* S (argue, argument, ...) The speaker provides reasons why φ is the case. Some protocols do not have this move but require instead that reasons be provided by a *claim* φ or *claim* S move in reply to a *why* ψ move (where S is a set of propositions). Also, in some systems the reasons provided for φ can have structure, for example, of a proof three or a deduction.
- *question* φ (...) The speaker asks another participant's opinion on whether φ is the case.

Structural degrees of freedom Dialogue systems can vary in their structural properties in several ways [Loui, 1998]: whether players can reply just once to the other player's moves or may try alternative replies (*unique- vs. multi-reply protocols*); whether players can make just one or may make several moves before the turn shifts (*unique- vs. multi-move protocols*); and whether the turn shifts as soon as the player-to-move has made himself the winning side or may shift later (*immediate- vs. non-immediate-reply protocols*). According to [Loui, 1998], the desired degree of structural 'strictness' of a dialogue system depends on the context of a dialogue. In contexts with little time and resources a unique-move, unique- and immediate reply protocol may be best, to force the participants not to waste resources, while in other contexts with more time and resources it is better to allow the participants more freedom to explore alternatives and return to earlier choices.

Commitments An important notion in systems for argumentation dialogue is that of propositional *commitments* [Walton and Krabbe, 1995]. Commitments are an agent's publicly declared points of view about a proposition, which may or may not agree or coincide with the agent's internal beliefs. An example of where they often do not agree is criminal trial, where the accused may very well publicly defend his innocence while he knows he is guilty. Commitments are typically incurred by stating claims or arguments, while they are typically lost by retracting a claim or argument. Commitments can serve several purposes in dialogue systems. One role is in enforcing a participant's dialogical consistency, for instance, by requiring him to keep his commitments consistent at all times or to make them consistent upon demand, or to defend one's commitments when challenged or else give them up. Another role of commitments is to determine termination and outcome of a dialogue. For example, persuasion dialogues can be defined to terminate if the opponent is committed to the proponent's main claim or the proponent is not committed any more to the main claim.

3.3 Some work on systems for persuasion dialogue

Since persuasion is 'pure' argumentation, I now review some historically important work on systems for persuasion dialogue in more detail. Then I will more briefly review work that embeds argumentation in systems for other kinds of dialogues.

3.3.1 Mackenzie [1979]

Mackenzie's [1979] system has been historically influential especially for its set of locutions. His system has the *claim*, *why*, *concede* and *retract* locutions. The logical language is that of propositional logic but the logic is not full PL but instead a restricted notion of "immediate consequence", to capture resource-bounded reasoning (e.g. p, $p \rightarrow q$ and $q \rightarrow r$ immediately imply q but not r). Arguments are moved implicitly, by replying to a *why* move with a *claim*. An argument may be incomplete but its mover becomes committed to the material implication *premises* $\rightarrow$ *conclusion*. In addition, Mackenzie has a *question* speech act, which asks the hearer to declare a standpoint with respect to a proposition, and a *resolve* speech act for demanding resolution of conflicts in or logical implication by commitments. Mackenzie does not define outcomes or termination of dialogues. This makes his system underspecified as to the dialogue purpose, so that it can be extended to various types of dialogues. The protocol is unique-move and unique-reply but it nevertheless hardly enforces coherence of dialogues. Only the moves required after *why* and *question* and the use of the *resolve* move are constrained; the participants may freely exchange unrelated claims, and may freely challenge, retract or question. For instance, the following dialogue is legal:

$$P\text{: } \textit{claim } p,\ O\text{: } \textit{claim } q,\ P\text{: } \textit{question } r,\ O\text{: } \textit{claim } \neg r,\ P\text{: } \textit{retract } s.$$

3.3.2 Walton & Krabbe [1995]

[Walton and Krabbe, 1995] developed the ideas of [Mackenzie, 1979] and also [Woods and Walton, 1978] into a full system for persuasion dialogues. To Mackenzie's locutions they added an explicit *since* locution for moving arguments. In their system, the only way to attack an argument is by challenging its premises, so the underlying logic is monotonic. The dialogues allowed by [Walton and Krabbe, 1995] are much more focused than Mackenzie's, since moves in a new turn must reply to a move in the previous turn of the other player. So, for instance, in the just-given example dialogue in Mackenzie's system, O's *claim* q move is not allowed and O must instead either concede or challenge p. This constraint also makes backtracking and postponement of replies impossible. Apart from this, the protocol allows that more than one move is made in one turn and alternative arguments for the same challenged proposition are moved. However, each move from the last turn must be replied-to (though other moves may be made as well),

Commitments are used by the protocol to enforce a participant's dialogical coherence. For example, if a participant's commitments logically imply an assertion of the other participant but do not contain that assertion, then the initial participant must either concede the assertion or retract one of the implying commitments.

The following example illustrates how the system deals with implicit premises:

> P_1: *claim* this car is safe O_1: *why* is this car safe?; P_2: this car is safe *since* it has an airbag; P_2: safe *since* airbag.

Now the opponent must either challenge or concede both the explicit premise that the car has an airbag and the implicit premise 'if the car has an airbag, then it is safe'.

3.3.3 Gordon's Pleadings Game

Gordon's work on the *Pleadings Game* [Gordon, 1995] is seminal AI & Law work on the modelling of legal procedures as dialogue games. The game was intended as a normative model of civil pleading in Anglo-American legal systems, where the participants aim to identify the issues to be decided in court. The underlying logic is a nonmonotonic one, viz. Geffner and Pearl's [1992] conditional entailment, which as discussed above in Section 2.3 has a model-theoretic semantics and an argument-based proof theory. The game contains speech acts for conceding and challenging a claim, for stating and conceding arguments, and for challenging challenges of a claim. The latter has the effect of leaving the claim for trial. The Pleadings Game can be argued to have an implicit distinction between attacking and surrendering replies (as later made explicit in [Prakken, 2005]) in its distinction between three kinds of moves that have been made during a dialogue: the *open moves*, which have not yet been replied to, the *conceded moves*, which are the arguments and claims that have been conceded, and the *denied moves*, which are the claims and challenges that have been challenged and the arguments that have been attacked with counterarguments. The protocol is multi-move but unique-reply. At each turn a player must respond in some allowed way to every open move of the other player that is still 'relevant' (in a sense similar but not identical to that of [Prakken, 2005]), and may reply to any other open move. If no allowed move can be made, the turn shifts to the other player, except when this situation occurs at the beginning of a turn, in which case the game terminates. Move legality is further defined by specific rules for the various speech acts, which are mostly standard.

The result of a terminated game is twofold: a list of issues identified during the game (i.e., the claims on which the players disagree), and a winner, if there is one. Winning is defined relative to the background theory constructed during a game. If issues remain, there is no winner and the case must be decided by the court. If no issues remain, then the plaintiff wins iff his main claim is defeasibly implied by the final background theory, while the defendant wins otherwise.

3.3.4 Deriving locutions from argument schemes

The Toulmin Diagram Game (TDG) of [Bench-Capon, 1998; Bench-Capon *et al.*, 2000] was intended to produce more natural dialogues than the "stilted" ones produced by systems such as those reviewed thus far. To this end, its speech acts are based on an adapted version of Toulmin's [1958] well-known argument scheme. In this scheme a *claim* is supported by *data*, which support is *warranted* by an inference license, which possibly has *presuppositions*, and which is *backed* by grounds for its acceptance; finally, a claim can be attacked with a *rebuttal*, which itself is a claim and thus the starting point of a counterargument. Arguments can be chained by regarding data also as claims, for which further data can be provided.

The locutions of TDG's communication language correspond to the elements of this scheme, as shown in Table 1. For ease of comparison, this table has an explicit reply structure as in [Prakken, 2005], to be discussed below, although the original TDG system leaves this structure implicit in its protocol.

The idea to generate natural dialogues by defining the communication language in terms of some argumentation scheme was later applied to practical reasoning by [Atkinson *et al.*, 2005b; Atkinson *et al.*, 2006], who embedded Atkinson's [2005] argumentation scheme for practical reasoning in a dialogue system for persuasion over action.

Locutions	Attacks	Surrenders
claim φ	*why* φ	*concede* φ
why φ	*supply* $data_\varphi$ ψ	*retract* φ
concede φ		
supply $data_\psi$ φ	so_ψ φ *why* φ	*concede* φ
so_ψ φ	*supply warrant* $\psi \Rightarrow \varphi$	
supply warrant w	*presupposing* w *on account of* w	*OK* w
presupposing w	*supply* $presupposition_w$ φ	*retract* w
on account of w	*supply* $backing_w$ b	*retract* w
supply $backing_w$ b		

Table 1: Attackers and surrenders in TDG

3.4 Later formal work

All systems reviewed so far are either philosophically motivated or geared towards application domains, and none of them were formally investigated on their properties. This changed in later AI work on dialogue systems for argumentation, some of which I will now discuss.

3.4.1 Parsons, Wooldridge & Amgoud [2003]

[Parsons *et al.*, 2002; Parsons *et al.*, 2003] were among the first to undertake a systematic formal study of argumentation as dialogue. They proposed dialogue systems for various types of dialogues involving argumentation and formally investigated them on various kinds of properties. In all of them the underlying logic is nonmonotonic, namely, Amgoud and Cayrol's [2002] system for classical-logic argumentation with grounded semantics. In this section I discuss their system for persuasion dialogues. Its communication language consists of claims, challenges, concessions and questions. Arguments are moved implicitly as *claim* replies to *why* moves (where sets of propositions may be claimed). The protocol has a rigid, unique-move and unique-reply nature, except that each premise of an argument may be responded to in turn. Unlike the above work, [Parsons *et al.*, 2003] make several assumptions on agent behaviour. Participants have their own, possibly inconsistent belief base and they are assumed to adopt an assertion and acceptance attitude, which they must respect throughout the dialogue. Moreover, claims moved in support of other claims must be from the participant's internal belief base.

[Parsons *et al.*, 2003] distinguish the following assertion attitudes: a *confident* agent can assert any proposition for which he can construct an argument, a *careful* agent can do so only if he can construct such an argument and cannot construct a stronger counterargument and a *thoughtful* agent can do so only if he can construct an acceptable argument for the proposition (according to grounded semantics). The corresponding acceptance attitudes also exist: a *credulous* agent concedes a proposition if he can construct an argument for it, a *cautious* agent does so only if in addition he cannot construct a stronger counterargument and a *skeptical* agent does so only if he can construct an acceptable argument for the proposition. In verifying these attitudes, each player must reason with its own beliefs and the commitments of the other side.

Consider the following example, where the proponent P believes p and $p \rightarrow q$, the opponent believes r and $r \rightarrow \neg q$ and all formulas are of equal preference. If P starts with *claim* q, then O must, depending on its dialogical attitudes, concede q if possible, otherwise claim $\neg q$ if possible, otherwise challenge q. If O is credulous or cautious, then perhaps surprisingly she must concede, since she has to reason with P's commitment p so she can construct a trivial argument for q, namely, $\{q\} \vdash q$. In both cases the dialogue terminates with agreement. By contrast, if O is skeptical, she has to challenge q. Then P has to move *claim* $\{p, p \rightarrow q\}$. Then O, being skeptical, must challenge both p and $p \rightarrow q$. The proponent then has to reply with *claim* $\{p\}$ and *claim* $\{p \rightarrow q\}$, after which the dialogue terminates without agreement, because the players are not allowed to repeat their moves, while O's acceptance attitude tells her to repeat her last two challenges.

[Parsons *et al.*, 2003] investigate various properties of the protocols and their outcomes. Some results are on whether termination of dialogues is guaranteed. Other results are on the computational complexity of the various aspects. Yet other results concern possible agent behaviours. For example, they studied the extent to which one agent can mislead the other agent by making her concede a proposition he himself does not believe. They thus were among the first to address issues of trust in argumentation dialogue.

A very interesting aspect of this work is the definition of the various dialogical attitudes. However, these notions are perhaps better seen as aspects of strategy than of protocol, since if they are referred to by the protocol, an outside observer cannot verify protocol compliance, which is often regarded as a drawback of communication protocols.

Locutions	Attacks	Surrenders
claim φ	*why* φ	*concede* φ
φ *since* S	*why* $\psi (\psi \in S)$ φ' *since* S' (φ' *since* S' defeats φ *since* S)	*concede* ψ $(\psi \in S)$ *concede* φ
why φ	φ *since* S	*retract* φ
concede φ		
retract φ		

Table 2: An example communication language in Prakken's framework

3.4.2 Prakken [2005]

In [Prakken, 2005] a framework for specifying two-party persuasion dialogues is presented, which is then instantiated with some example protocols. The aim of this work was to allow a more general study of properties of dialogue systems of argumentation than the work reviewed so far. To this end, the framework largely abstracts from the logical language, the logic and the communication language, except that the communication language has to have an explicit reply structure and that underlying logic is assumed to be a system that is much like a preliminary version of $ASPIC^+$. Moreover, different protocols were defined, all extending a partial core protocol.

A main motivation of the framework was to ensure focus of dialogues while yet allowing for freedom to move alternative replies and to postpone replies. This was achieved with two main features of the framework. Firstly, an explicit reply structure on the communication language is assumed (implicit in several other systems), where each move either *attacks* or *surrenders to* its target. An example $\mathcal{L}_c$ of this format is displayed in Table 2. Secondly, winning is defined for each dialogue, whether terminated or not, and it is defined in terms of a notion of *dialogical status* of moves. The *dialogical status* of a move is recursively defined as follows, exploiting the tree structure of dialogues. A move is *in* if it is surrendered or else if all its attacking replies are *out*. This implies that a move without replies is *in*. And a move is *out* if it has a reply that is *in*. Actually, this has to be refined to allow that some premises of an argument are conceded while others are challenged; see [Prakken, 2005] for the details. Then a dialogue is (currently) won by the proponent if its initial move is *in* while it is (currently) won by the opponent otherwise.

Together, these two features of the framework allow for a notion of relevance that ensures focus while yet leaving the desired degree of freedom (generalised from [Prakken, 2001b]): a move is *relevant* just in case making its target *out* would make the speaker the current winner. Termination is defined as the situation that a player is to move but has no legal moves. The players can also agree to terminate a dialogue.

Consider by way of example the following dialogue in a protocol that allows replies to all moves of the other player but only if the move is relevant.

P_1:	*claim p*	
O_1:	*why p*	(replying to P_1)
P_2:	*p since q*	(replying to O_1)
O_2:	*why q*	(replying to P_2)
P_3:	*p since r*	(replying to O_1)

At this point a reply to P_2 is irrelevant, since P_2 is *out*, so replying to it cannot change the status of P_1. Note that the dialogue can only terminate after either P has replied to O_1 with *retract p* or O has replied to P_1 with *concede p*. In all other cases, legal moves can always be made.

3.4.3 Argument games as dialogue systems

Argument games for abstract argumentation semantics were above in Section 2.5.2 discussed as a proof theory for abstract argumentation semantics. However, they have also been studied as genuine dialogue games for disagreeing agents, by dropping the assumption that all arguments are taken from a fixed and globally known argumentation framework [Loui, 1998; Jakobovits and Vermeir, 1999a; Jakobovits, 2000; Prakken, 2001b]. If this assumption is dropped, the properties of the game can change. A positive change is proven by Jakobovits, viz. that certain dynamic argument-game protocols prevent the construction of AFs containing odd loops (it is well known that such theories may have no extensions). A negative result is proven by [Prakken, 2001b], viz. that the dynamified game for grounded semantics loses soundness with respect to the joint framework constructed during a dialogue. However, if the game is changed by allowing any relevant reply (in the sense of [Prakken, 2005]) to any earlier move of the other side, then soundness is restored.

While the study of argument games as dialogue systems is theoretically very interesting, their very simple logic and communication language make that they cannot be a realistic model of persuasion dialogue.

3.5 Persuasion embedded in other types of dialogues

I now briefly review work that embeds argumentation in a dialogue system for other types of dialogues.

3.5.1 Negotiation

Much work on embedding argumentation into negotiation protocols is motivated by the claim that argumentation can be beneficial to negotiation. From the point of view of the negotiating agents, adding reasons for a proposal could increase the chance of acceptance. This was the idea of Sycara's [1985; 1990] early work on modelling threats and reward in labour negotiation. For example 'if you do not accept our offer, we will go on strike' (a threat) or 'if you accept that you have to work during the weekends, you will receive an increase in salary' (a reward). This idea was generalised by [Parsons *et al.*, 1998] and [Kraus *et al.*, 1998] for BDI-style agents, that is, agents that form their intentions to act according to their beliefs and, possibly prioritised, desires [Rao and Georgeff, 1991]. The general idea is that the other agent should be made to change its beliefs or preferences in such a way that it will form the intention to accept or make an offer that the initial agent wants.

From the perspective of protocol design the idea is that if negotiating agents exchange and discuss reasons for their proposals and rejections, the negotiation process may become more efficient and the negotiation outcome may be of higher quality. If an agent explains why he rejects a proposal, the other agent knows which of her future proposals will certainly be rejected so she will not waste effort at such proposals. Thus efficiency is promoted. In such exchanges, reasons are not only exchanged, they can also become the subject of debate. Suppose a car seller offers a Peugeot to the customer but the customer rejects the offer on the grounds that French cars are not safe enough. The car seller might then try to persuade the customer that he is mistaken about the safety of French cars. If she succeeds in persuading the customer that he was wrong, she can still offer her Peugeot. Thus the quality of the negotiation is promoted, since the buyer has revised his preferences to bring them in agreement with reality. This example illustrates that a negotiation dialogue (where the aim is to reach a deal) sometimes contains an embedded persuasion dialogue (where the aim is to resolve a conflict of opinion).

Since all this is about giving reasons for or against acting in a certain way, the kind of argumentation that is involved is, inferentially speaking, argumentation about decision options (see Section 2.6.2 above), although it can, as the car sales example shows, also shift to epistemic argumentation about the underlying facts. The early work of [Sycara, 1985; Sycara, 1990] and [Kraus *et al.*, 1998] applied informal rhetorical models of argumentation. Later work incorporated formal inferential models of argumentation in negotiation protocols. For example, [Parsons *et al.*, 1998] embed Krause *et al.*'s [1995] logic of argumentation, [Amgoud *et al.*, 2000b] embed Amgoud and Cayrol's [1998] models of classical argumentation with preferences, [Amgoud and Prade, 2004] incorporate the model summarised by [Amgoud, 2009], and [van Veenen and Prakken, 2006] combine Wooldridge and Parson's [2000] negotiation protocol with one of Prakken's [2005] persuasion protocols, thus also embedding its preliminary version of $ASPIC^+$.

3.5.2 Deliberation

The purpose of deliberation is to agree on a course of action. It differs from persuasion over action, as modelled in e.g. [Atkinson *et al.*, 2005b; Atkinson *et al.*, 2006] in that at the start of a deliberation dialogue there typically just is a problem and no proposed solutions yet. It differs from negotiation in that deliberating agents are assumed not to be self-interested but collaborative, sharing the goals of the group or community they are part of. The group may be small, such as a few people choosing a restaurant for dinner, it may be big, such as in parliamentary debate, and it may be huge, such as in public debate about political or societal issues. Clearly, different settings require different kinds of protocols.

Embedding argumentation in deliberation has much the same benefits as embedding it in negotiation: for the agents it may increase the chance of acceptance of their proposals, and for the dialogue it may increase the quality of the outcome. Research on deliberation with argumentation started later than research on argumentation-based negotiation and is not as extensive. Here is brief overview of some work.

[Tang and Parsons, 2005] proposed a rather specific dialogue system for argumentation about means-end planning, not based on a formal model of argumentation.

[McBurney *et al.*, 2007] proposed a framework for multi-agent deliberation dialogues. The protocol is intended to allow for the open nature of deliberation, giving the agents much freedom for establishing goals, constraints, perspectives, facts, actions and evaluations. Accordingly, the dialogue cyclicly moves through various stages. After initial inform and propose stages, the agents evaluate and decide on actions in the consideration, revision, recommendation and confirmation stages. The framework does not assume a specific argumentation logic.

[Black and Atkinson, 2011] proposed a much more rigid system for two-agent deliberation based on Atkinson's [Atkinson, 2005] embedding of an argument scheme for practical reasoning in value-based argumentation frameworks. The rigidness of the system allows them to show that if the agents adhere to the dialogue protocol and construct their arguments on the basis of their own belief bases, then any agreed proposal is also acceptable to both agents individually.

Finally, [Kok *et al.*, 2011] proposed a dialogue system for multi-agent deliberation dialogues as part of an experimental setup for testing the usefulness of argumentation in such dialogues. The system is an instance of Prakken's [2005] framework for persuasion but adapted to deliberation. It incorporates $ASPIC^+$ as the underlying logic.

3.5.3 Inquiry

Only little work has been done on embedding argumentation in inquiry. Early work is McBurney and Parsons's [2001] model of scientific inquiry. More recently, [Black and Hunter, 2007; Black and Hunter, 2009] embedded Garcia and Simari's [2004] DeLP argumentation system in a protocol for inquiry dialogue. They combined the protocol with a strategy that selects exactly one of the legal moves to make. This allowed them to prove soundness and completeness properties with respect to the participants' belief bases, provided the agents construct their arguments from their own belief base.

3.6 Work on strategic aspects of argumentation

Dialogue systems for argumentation only cover the rules of the game, i.e., which moves are allowed; they do not cover principles for playing the game well, i.e., strategies, tactics and heuristics for the individual players. Above we already discussed some work that studies the combination of a protocol with strategies, such as [Black and Atkinson, 2011] for deliberation and [Black and Hunter, 2007; Black and Hunter, 2009] for inquiry. Moreover, as remarked above, the assertion and acceptance policies studied by [Parsons *et al.*, 2002; Parsons *et al.*, 2003] could be seen as heuristics for move selection (although Parsons, Wooldridge and Amgoud make them part of their protocols).

Other early work on strategic aspects of argumentation is [Amgoud and Maudet, 2002], who, building on the even earlier work of [Moore, 1993] on argumentation dialogues for intelligent tutoring, formulated move selection strategies and tactics based on human strategies in natural dialogues. One example is that agents have to choose between a *build* or *destroy* attitude, i.e., whether they want to support their own or to attack their opponent's position. This idea was later also used by [Kok, 2013] in his simulation experiments on whether argumentation is beneficial to deliberating agents.

In the context of dialogue games for abstract argumentation, Paul Dunne studied issues arising from the mismatch between the purpose of persuasion dialogues and the arguing agent's own objectives. In [Dunne, 2003] he studied the use of delay tactics and in [Dunne, 2006] he studied situations where agents have a 'hidden agenda'.

More recently, there is an emerging research strand on opponent modelling for strategic purposes, for example in terms of probability distributions or expected-utility distributions over the possible actions of the opponent [Matt and Toni, 2008; Thimm and Garcia, 2010; Oren and Norman, 2010; Hadjinikolis *et al.*, 2013; Rienstra *et al.*, 2013]. Somewhat earlier, [Riveret *et al.*, 2008] probabilistically modelled not an opponent but an impartial adjudicator who has the power to accept or reject premises of arguments put forward by the adversaries. In this work, probabilistic game theory can be used to determine optimal strategies.

Other recent work that uses game theory is that on mechanism design for argumentation [Rahwan and Larson, 2008; Rahwan *et al.*, 2009]. The goal here is to develop protocols that make unwanted behaviour (such as lying or withholding information) suboptimal.

All this recent work on strategic aspects of argumentation is still preliminary. The reader can consult [Thimm, 2014] for a recent overview. I confine myself to one concluding observation. On the one hand, the recent work on strategy, heuristics and tactics is a natural continuation of the earlier work on communication languages and protocols. However, in one respect it is a step backwards, since it generally assumes much simpler dialogue systems than were developed before, with, for example, much recent work assuming simple dialogue games for abstract argumentation semantics.

4 Application areas

Formal and computational models of argumentation have been applied in several areas. Although a comprehensive review is beyond the scope of this article, a brief overview is in order. For more detailed overviews the reader can consult [Modgil *et al.*, 2013] and some references given below. I will mainly focus on three main application areas, viz. medicine, law and debating technologies. In addition, in the literature many specific applications can be found, such as to recommender systems, trust and reputation management, robot soccer, waste management, licensing policy management, the internet of things, and so on.

Below I will only discuss applications of formal models of argumentation. In several areas there is much applied research based on informal or ad-hoc models of argumentation. For example, argumentation has been used in work on risk assessment and design rationale in software engineering for explaining why a design meets a design requirement or avoids a risk [Haley *et al.*, 2008; Franqueira *et al.*, 2011]

. Moreover, there is quite some work on support tools for argument visualisation [Reed *et al.*, 2007; ter Berg *et al.*, 2009] and collaborative argumentation and decision making [Conklin *et al.*, 2001; Scheuer *et al.*, 2010; Kirschner *et al.*, 2003], sometimes in educational contexts [Pinkwart and McLaren, 2012], and with applications for the social web [Schneider *et al.*, 2013]. Finally, recently research in argument mining [Palau and Moens, 2009; Lippi and Torroni, 2016] has become popular, which aims to recognise (elements of) arguments and their relations in natural-language texts.

As for the nature of the applications mentioned below, theoretical, user-oriented and fielded applications can be distinguished. *Theoretical applications* use a non-trivial domain example to demonstrate the adequacy or motivate design features of the model. In *user-oriented applications* (which usually are of computational architectures) the usefulness of the architecture for designated types of users or tasks is an essential aspect. *Fielded applications* have actually been used by the intended user group in a realistic context, either experimentally or in actual use.

4.1 Medical applications

Medicine has been an important application field of argumentation, with John Fox as a historically influential figure. Several systems developed by him and his colleagues have been experimentally tested or are even in actual use [Fox *et al.*, 2007], so these count as fielded applications. While their underlying argumentation model is rather simple, this group also studied formal foundations of their systems, e.g. in [Elvang-Göransson *et al.*, 1993] and [Krause *et al.*, 1995]. Moreover, [Fox and Parsons, 1997] proposed one of the first formal argumentation-based models of decision making, using arguments for expressing and comparing the positive and negative effects of medical treatments. This idea was combined with an argument-scheme approach by [Tolchinsky *et al.*, 2006; Tolchinsky *et al.*, 2012], who present a model for multi-agent deliberation about safety-critical medical actions, such as donor organ selection for patients. The intended system plays the role of a mediating agent whose task is to inform the participants about their valid move options, to decide whether an argument is relevant enough to be admitted into the process, and to evaluate the admitted arguments in order to assess whether the proposed action should be undertaken. Since this system was tested experimentally with medical doctors, it counts as a fielded application.

More recently, [Hunter and Williams, 2012] have applied argumentation in a user-oriented way to the problem of aggregating evidence-based arguments for and against treatment options from clinical trials. They use preference-based abstract argumentation frameworks instantiated with one-steps applications of domain-specific inference rules, and express argument preferences in terms of outcome indicators of the treatments. The approach was evaluated by comparison with recommendations made in published healthcare guidelines.

4.2 Legal applications

There has been much cross-pollination with the field of AI & Law [Prakken and Sartor, 2015]. This is understandable, given the inherently adversarial nature of the law and the importance of written justifications of legal decisions. Rule-based argumentation formalisms such as assumption-based argumentation and the system of [Prakken and Sartor, 1997] have been applied to preference-based reasoning with conflicting rules [Kowalski and Toni, 1996; Prakken and Sartor, 1996]. Prakken & Sartor also used their logic to formalise notions of burdens of proof [Prakken and Sartor, 2009], as was done by [Gordon and Walton, 2009] with their Carneades system. Work on applying dialogue systems to the formalisation of legal procedure was discussed above in Sections 3.1 and 3.3.3.

An important contribution of AI & Law to the formal study of argumentation is the study of the role of cases in argumentation; for a recent detailed overview see [Bench-Capon, 2017]. In Section 1 above the still influential HYPO system [Ashley, 1990] and its successor CATO [Aleven, 2003] were mentioned. Their underlying argumentation model is for 'factor'- or 'dimension'-based reasoning, where cases are collections of abstract fact patterns that favour or oppose a conclusion, either in an all-or nothing fashion (factors) or to varying degrees (dimensions). This work inspired subsequent formal work using the tools of formal argumentation, e.g. [Hage, 1993; Loui *et al.*, 1993; Prakken and Sartor, 1998; Bench-Capon and Sartor, 2003]. A key idea in this work is that case decisions give rise to conflicting rules (or conflicting sets of reasons) plus a preference expressing how the court resolved this conflict. In the notation of [Prakken and Sartor, 1998]:

r_1: *Pro-factors* $\Rightarrow$ *Decision*
r_2: *Con-factors* $\Rightarrow$ *Not D*ecision
$r_1 > r_2$

The rule preference expresses the court's decision that the pro factors in the body of rule r_1 together outweigh the con factors in the body of rule r_2. This approach allows for 'a fortiori' reasoning in that adding factors to a pro-decision rule or removing factors from a con-decision rule does not affect the rule priority. [Horty, 2011], using a non-argumentation-based nonmonotonic logic, formalises the conditions under which a decision is allowed or forced by body of precedents and then uses this to also formalise the concepts of following, distinguishing and overruling a precedent.

A related line of research is to compare cases not in terms of their factors but in terms of underlying legal and social values. [Berman and Hafner, 1993] argued that often a factor can be said to favour a decision by virtue of the purposes served or values promoted by taking that decision because of the factor. A choice in case of conflicting factors is then explained in terms of a preference ordering on the purposes, or values, promoted or demoted by the decisions suggested by the factors. Cases can then be compared in terms of the values at stake rather than on the factors they contain. [Bench-Capon, 2002] first computationally modelled this approach, leading to a series of papers culminating in [Prakken *et al.*, 2015] and using argument schemes for practical reasoning of the kinds also used in argumentation-based models of decision making (see Section 2.6.2 above).

All the AI & law applications mentioned so far are theoretical applications. User-oriented legal applications of argumentation are rare, with most applications in the field of e-democracy, e.g. [Cartwright and Atkinson, 2009; Gordon, 2011]. Finally, to my knowledge only one fielded application exists, namely, the CATO system, which was experimentally tested for teaching case-based argumentation skills to American law students.

4.3 Debating technologies

Most work on debating technologies is based on informal or ad-hoc models of argumentation; for overviews see the references given above. An exception is the work of the Arg-tech group at the University of Dundee, Scotland, led by Chris Reed. This group has developed various user-oriented web-based argumentation tools partly based on formal foundations [Bex *et al.*, 2013a]. For example, they have been using the so-called Argument Interchange Format [Chesñevar *et al.*, 2006], which was given a logical foundation in $ASPIC^+$ by [Bex *et al.*, 2013b] and they have an online implementation of an instance of $ASPIC^+$ called TOAST [Snaid and Reed, 2012]. Several tools developed by the Arg-tech group have been experimentally tested with intended users, so these count as fielded applications.

5 Conclusion

Looking back on the history of formal research on argumentation, there is a marked difference between the study of argumentation as inference and that of argumentation as dialogue. The theory of argumentation-based inference is mature, with an almost universally accepted formal foundation in Dung's theory of abstract argumentation frameworks and its extensions and with a converging study of structured argumentation, with just a small number of general frameworks and increasing knowledge about their relations. By contrast, the study of argumentation-based dialogue consists of a variety of different approaches and individual systems, all exciting work but with few unifying accounts or general frameworks. There are a few exceptions, such as a series of papers just after 2000 by Peter McBurney, Simon Parsons and others on principles for the design of dialogue systems e.g. [McBurney *et al.*, 2002; McBurney and Parsons, 2002], and my own formal framework for persuasion dialogue in [Prakken, 2005]. However, this work is still far from being foundational.

In my own personal opinion, the following are the four main main theoretical contributions of the field.

1. The idea that dialectical evaluation of arguments can be formalised. While logic textbooks routinely write that a valid argument does not dictate the acceptance of its conclusion since it can always be attacked on its premises, formal argumentation has shown that attack relations between arguments conform to patterns that can be formally studied. In its purest form this is captured in Dung's [1995] theory of abstract argumentation frameworks.

2. The idea of defeasible rules. Dogma has it that all arguments should be deductively valid, that is, the truth of their premises should guarantee the truth of their conclusion. The fields of informal logic, argumentation theory and epistemology have questioned this dogma and argued that arguments that fail to meet this standard of perfection can still be good, as long as they withstand critical scrutiny. The field of formal argumentation has shown that this idea can be formalised.

3. The idea that the principles for evaluating arguments in the context of a dialogue can be formalised. [Toulmin, 1958] first proposed that arguments should be evaluated not on their syntactic form but on whether they can be defended in a properly conducted dispute. He urged logicians of his day to study the principles of proper dispute. The formal study of dialogue studies has met this challenge and thus also opened the prospects for precise formal studies of strategy and tactics for persuasion.

4. The idea that reasoning under uncertainty can be formalised in a qualitative way. There is an increasing trend of advocating quantitative (especially Bayesian) models of uncertainty as the only way to reason about uncertainty. Likewise with quantitative models of decision making. However, for humans such quantitative theories are often hard to grasp, while they largely ignore the dialogical and procedural aspects of reasoning. This is especially a problem for applications with humans in the loop, such as support tools for human argu-

> mentation and decision making. Our field has shown that a natural qualitative theory of reasoning under uncertainty can be formalised.

However, there is, in my opinion, also an unfortunate recent development. While Dung's [1995] idea of abstract argumentation frameworks was a major breakthrough and is deservedly a key element in the formal study of argumentation-based inference, not all follow-up work is of the same generality. We have seen that several proposals for extending abstract argumentation frameworks with new elements implicitly make assumptions that are not in general satisfied. The same holds for work on the dynamics of abstract argumentation and for some work on probabilistic abstract argumentation. The resulting formalisms are thus abstract but not general in that they model special cases, such as the case in which all arguments, or all attacks, are independent of each other, or the special case in which all arguments are attackable.

It is worth noting that the word 'abstract' in Dung's [Dung, 1995] notion of abstract argumentation frameworks does not qualify 'argumentation' but 'frameworks'. In Dung's terminology, it is the framework that is abstract, not the argumentation. Strictly speaking there is no such thing as abstract argumentation, just as there is no such thing as structured argumentation. All there is is argumentation, which can be studied at various levels of abstraction. And in real argumentation not arguments but things like claims, reasons and grounds are the most basic elements. There is nothing wrong in principle with abstract studies of argumentation: abstraction is an indispensable tool in any kind of research. However, one should not forget that we all study the same phenomenon, so that the various levels of abstraction should be connected. I remind the reader of my (perhaps controversial) proposal in Section 2.5.3 of a methodological guideline that every new proposal for extending abstract argumentation frameworks with new elements should in the same paper be accompanied by at least one non-trivial instantiation, in order to demonstrate the significance of the new extension. In doing so, we would respect the historic roots of the abstract study of argumentation, since in his original 1995 paper Dung respected this guideline in a way that has since never been equalled.

It is time to conclude. The formal and computational study of argumentation has established itself as a mature field of research. Argumentation is a key word or topic in all main AI conferences, papers on argumentation are published in the major AI journals, and the field has its own COMMA conference plus several workshops (CNMA, ArgMas, TAFA). Theoretically, the field is in a healthy state with much exciting research. With respect to applications this is less so, but this holds for all theoretically interesting fields of research. There is every hope to be optimistic here too, as long as a too strong focus on abstract argumentation is avoided. Unlike, for example, constraint satisfaction or model checking, argumentation is not just a technique but an important aspect of human life. There will therefore always be the need for support tools for argumentation, and our field is arguably in an excellent position to provide these tools. In any case, it provides their formal foundations.

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Received 14 August 2017

Abstract Dialectical Frameworks. An Overview

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Abstract

This article describes abstract dialectical frameworks, or ADFs for short. ADFs are generalizations of the widely used Dung argumentation frameworks. Whereas the latter focus on a single relation among abstract arguments, namely attack, ADFs allow arbitrary relationships among arguments to be expressed. For instance, arguments may support each other, or a group of arguments may jointly attack another one while each single member of the group is not strong enough to do so. This additional expressiveness is achieved by handling acceptance conditions for each argument explicitly.

The semantics of ADFs are inspired by approximation fixpoint theory (AFT), a general algebraic theory for approximation based semantics developed by Denecker, Marek and Truszczyski. We briefly introduce AFT and discuss its role in argumentation. This puts us in a position to formally introduce ADFs and their semantics. In particular, we show how the most important Dung semantics can be generalized to ADFs. Furthermore, we illustrate the use of ADFs as semantical tool in various modelling scenarios, demonstrating how typical representations in argumentation can be equipped with precise semantics via translations to ADFs. We also present GRAPPA, a related approach where the semantics of arbitrary labelled argument graphs can be directly defined in an ADF-like manner, circumventing the need for explicit translations. Finally, we address various computational aspects of ADFs, like complexity, expressiveness and realizability, and present several implemented systems.

1 Introduction

This article is about abstract dialectical frameworks, or ADFs for short. ADFs are generalizations of Dung argumentation frameworks (AFs, see the chapter *Abstract Argumentation Frameworks and Their Semantics* of [Baroni *et al.*, in press]). AFs are very popular tools in argumentation. They abstract away from the content of particular arguments and focus on conflicts among arguments, where each argument is viewed as an atomic item. The only information AFs take into account is whether an argument attacks another one or not. Based on a set of arguments and an attack relation, different AF semantics single out coherent subsets of arguments which "fit" together, according to specific criteria. More formally, an AF semantics takes an argumentation framework as input and produces as output a collection of sets of arguments, called extensions.

AFs are typically not used directly for knowledge representation purposes, but as semantical tools: given a knowledge base KB in some knowledge representation formalism, the set of arguments induced by KB is formally defined and the attack relation on these arguments is identified. This defines an AF that can be evaluated according to a chosen semantics. The KB formulas supported by accepted arguments are then the ones which are accepted. This stepwise evaluation is often referred to as the argumentation process [Caminada and Amgoud, 2007].

Given that AFs are in wide use, a natural question to ask is why a generalization of AFs is useful in the first place. There are at least two possible answers to this question:

- the generalization is more expressive than AFs,
- the generalization allows for easier modelling.

In fact, it turns out that both answers apply to ADFs. We will discuss the issue of expressiveness in detail in Section 6.2. For the time being let us focus on the modelling issue. AFs restrict their attention to the attack relation, and the basic intuition is the following: assume an argument b is attacked by argument c, then whenever c is accepted b is defeated. But how about more fine-grained – or entirely different – relations which could be of potential interest? What if c alone is not strong enough and a second argument, say d, is needed to jointly defeat b? And, maybe even more importantly, aren't there situations where accepting an argument can be a reason for accepting another one, in other words, where arguments are in support rather than in attack relation? We do not claim here that examples like the ones just discussed cannot be modelled at all with AFs. However, additional nodes in the AF argument graph will be needed which have the sole purpose of modelling other relations indirectly, via attack. These nodes will often be entirely unrelated to the original knowledge base and thus meaningless from the perspective of the application.

Indeed, for these reasons many authors have felt the need to extend the functionality available in AFs in one way or another. Examples of extensions described in the literature are preference or value-based AFs [Simari and Loui, 1992; Amgoud and Cayrol, 2002; Amgoud and Vesic, 2011; Bench-Capon, 2003], AFs with support relations [Cayrol and Lagasquie-Schiex, 2005; Cayrol and Lagasquie-Schiex, 2013; Oren and Norman, 2008; Polberg and Oren, 2014], necessities [Nouioua, 2013], set attacks [Nielsen and Parsons, 2007], attacks on attacks [Modgil, 2009], recursive attacks [Baroni *et al.*, 2011] and AFs with weights [Martínez *et al.*, 2008; Dunne *et al.*, 2011; Coste-Marquis *et al.*, 2012] or probabilities [Hunter, 2013; Thimm, 2012]. We refer the reader to [Brewka *et al.*, 2014] for an overview of such extensions.

In a nutshell, ADFs are an attempt to unify several of these different approaches and to generalize AFs in a principled, systematic way. The basic idea is very simple. Consider again the conditions under which an argument, say b, with attackers c and d is accepted in an AF: b is accepted iff c is not accepted and d is not accepted. This condition can easily be expressed as the propositional formula $\neg c \wedge \neg d$. The acceptance condition for each argument in an AF is obtained in exactly the same way, by constructing the conjunction of the negations of its attackers. Once the implicit acceptance conditions which are at work in AFs are made explicit this way, the generalization ADFs build upon are pretty straightforward: rather than using implicit acceptance conditions of the form we just saw, ADFs use explicit acceptance conditions which can conveniently be expressed as arbitrary propositional formulas.

Let us see how explicit acceptance conditions allow us to handle some of the examples discussed above. We start with joint attack. If b can only be defeated jointly by c and d, then all we have to do is change the acceptance condition accordingly: rather than a conjunction, we have to use the disjunction $\neg c \vee \neg d$ as acceptance condition for b. The effect is that b is only defeated when both c and d are accepted, as intended. As soon as one of them is not accepted, b is no longer defeated.

Support can be handled in a similar manner. Assume g has two supporting arguments a and b, and one attacking argument c, as illustrated in Figure 1. We use $+$ and $-$ to indicate support and attack, respectively.

Note that the information about supporting and attacking links in the graph does not sufficiently specify under what conditions g should be accepted. Let us call a link active if its source node is accepted. There are various options we may want to choose, all of them expressible as a particular acceptance condition for g:

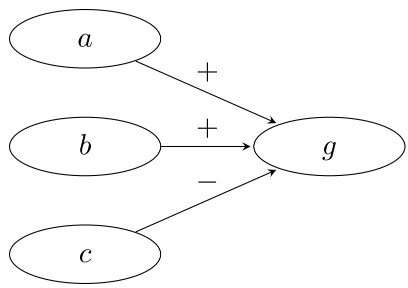


Figure 1: An argument with two supporters and one attacker.

- no negative and all positive links must be active: $\neg c \wedge (a \wedge b)$
- no negative and at least one positive link must be active: $\neg c \wedge (a \vee b)$
- no negative or both positive links must be active: $\neg c \vee (a \wedge b)$
- no negative or at least one positive link must be active: $\neg c \vee (a \vee b)$
- more positive than negative links must be active: $(\neg c \wedge (a \vee b)) \vee (a \wedge b)$

Note how it depends on the acceptance condition whether supporting links are "stronger than" attacking links (meaning that if all incoming links are active, the node is accepted), as in the last three items, or attacking links are "stronger than" supporting links (meaning that if all incoming links are active, the node is rejected), as for the first two items.

We hope these examples are sufficient to illustrate the additional modelling capabilities ADFs provide, and also the simplicity of the basic idea they rest upon. We will see, however, that generalizing the AF semantics to ADFs is far from being simple. This issue will be addressed in Section 3.

In spite of their additional expressiveness, we do not view ADFs primarily as a knowledge representation formalism. We rather consider them as "argumentation middleware", that is, as a framework which is particularly useful for providing semantics to other, maybe more user-friendly formalisms via translations [Brewka *et al.*, 2014]. We will further illustrate this in Section 4.

The rest of this article is organized as follows. In Section 2 we recall some relevant background and in particular discuss some relationships between approximation fixpoint theory and AFs which will be useful later. Section 3 introduces ADFs and their semantics formally. The presentation of this section is based on [Brewka *et* *al.*, 2013]. Section 4 illustrates the role of ADFs in argumentation, showing how they can be used for modelling. Section 5 describes GRAPPA (GRaph-based Argument Processing based on Patterns of Acceptance) along the lines of [Brewka and Woltran, 2014]; GRAPPA is an approach to graph-based argumentation which is closely related to ADFs and their underlying formal techniques. Section 6 discusses subclasses, computational aspects, and expressivity of ADFs. Section 6.1 focuses on an interesting special case of ADFs, so-called bipolar ADFs where each link in the ADF graph is attacking or supporting (or both). This rather expressive class is not only of practical interest, but also has nice computational properties. Expressiveness of ADFs and bipolar ADFs is investigated in Section 6.2, computational complexity in Section 6.3, and recent systems in Section 6.4. Section 7 concludes the article.

2 Approximation Fixpoint Theory in Abstract Argumentation

Denecker, Marek and Truszczyski [Denecker *et al.*, 2000] (henceforth shortened to DMT) introduced an algebraic framework for studying semantics of knowledge representation formalisms. In this framework – approximation fixpoint theory (AFT) – knowledge bases are associated with operators (functions) on algebraic structures (for example lattices). The fixpoints of those operators are then studied in order to analyse the semantics of knowledge bases. While this technique is standard to define semantics of programming languages and has indeed been used in early works on logic programming [van Emden and Kowalski, 1976], the major invention of DMT has been the important concept of an approximation of an operator. In the study of semantics of knowledge representation formalisms, elements of lattices represent objects of interest. Operators transform such objects into others according to the contents of a given knowledge base. Consequently, fixpoints of such operators are then objects that cannot be updated any more – informally speaking, the knowledge base can neither add information to a fixpoint nor remove information from it.

In classical approaches to fixpoint-based semantics, the underlying algebraic structure is the complete lattice of the set $\mathcal{V}_2 = \{v : A \to \{\mathbf{t}, \mathbf{f}\}\}$ of all two-valued interpretations over some vocabulary A ordered by the truth ordering $\leq_t$ with

$$v_1 \leq_t v_2 \text{ if and only if } \forall a \in A : v_1(a) = \mathbf{t} \implies v_2(a) = \mathbf{t}.^1$$

Consequently, an operator O on this lattice $(\mathcal{V}_2, \leq_t)$ takes as input a two-valued interpretation $v \in \mathcal{V}_2$ and returns a revised interpretation $O(v) \in \mathcal{V}_2$. The intuition

[1] $(\mathcal{V}_2, \leq_t)$ is isomorphic to $(2^A, \subseteq)$ via $v \mapsto v^{-1}(\mathbf{t}) = \{a \in A \mid v(a) = \mathbf{t}\}$.

of the operator is that the revised interpretation $O(v)$ incorporates additional knowledge that is induced by the knowledge base associated to O from interpretation v. Based on this intuition, fixpoints of O correspond to the models of the knowledge base.

To study fixpoints of operators O, DMT investigate fixpoints of their *approximating operators* $\mathcal{O}$. When O operates on two-valued interpretations $\mathcal{V}_2$, its approximation $\mathcal{O}$ operates on *three-valued* interpretations $\mathcal{V}_3 = \{v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}\}$. The three truth values $\mathbf{t}$ (true), $\mathbf{f}$ (false), and $\mathbf{u}$ (undefined) can be ordered by the information ordering $\leq_i$. This ordering intuitively assigns a greater information content to the classical truth values $\{\mathbf{t}, \mathbf{f}\}$ than to undefined $\mathbf{u}$; more formally, we have $\mathbf{u} <_i \mathbf{t}$ and $\mathbf{u} <_i \mathbf{f}$ and $\leq_i$ is the reflexive transitive closure of $<_i$. The partially ordered set $(\{\mathbf{t}, \mathbf{f}, \mathbf{u}\}, \leq_i)$ forms a complete meet-semilattice with the meet operation $\sqcap_i$.[2] This meet can be read as *consensus* and assigns $\mathbf{t} \sqcap_i \mathbf{t} = \mathbf{t}$, $\mathbf{f} \sqcap_i \mathbf{f} = \mathbf{f}$, and returns $\mathbf{u}$ otherwise. The ordering $\leq_i$ can be generalized to three-valued interpretations in a pointwise fashion:

$$v_1 \leq_i v_2 \text{ if and only if } \forall a \in A : v_1(a) \in \{\mathbf{t}, \mathbf{f}\} \implies v_1(a) = v_2(a).^3$$

Again, the resulting algebraic structure is a complete meet-semilattice; its $\leq_i$-maximal elements are exactly the two-valued interpretations $\mathcal{V}_2$, which form an $\leq_i$-antichain. Intuitively, in that complete meet-semilattice, a single three-valued interpretation

$$v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$$

serves to approximate a set $[v]_2 = \{w \in \mathcal{V}_2 \mid v \leq_i w\}$ of two-valued interpretations. For example, for the vocabulary $A = \{a, b, c\}$, the three-valued interpretation $v = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{f}\}$ approximates the set $\{w_1, w_2\}$ of two-valued interpretations where $w_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}\}$ and $w_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}\}$.

In a similar vein, a three-valued operator $\mathcal{O} : \mathcal{V}_3 \to \mathcal{V}_3$ *approximates* a two-valued operator $O : \mathcal{V}_2 \to \mathcal{V}_2$ if and only if

1. for all $v \in \mathcal{V}_2$, we have $\mathcal{O}(v) = O(v)$ ($\mathcal{O}$ agrees with O on two-valued v), and
2. for all $v_1, v_2 \in \mathcal{V}_3$, $v_1 \leq_i v_2 \implies \mathcal{O}(v_1) \leq_i \mathcal{O}(v_2)$ ($\mathcal{O}$ is $\leq_i$-monotone).

DMT [Denecker *et al.*, 2000] showed that in this case fixpoints of $\mathcal{O}$ approximate fixpoints of O. More specifically, for every fixpoint v_2 of O, there is a fixpoint v_3

[2] A complete meet-semilattice is such that every non-empty finite subset has a greatest lower bound, the meet; and every non-empty directed subset has a least upper bound. A subset is directed iff any two of its elements have an upper bound in the set.

[3] $(\mathcal{V}_3, \leq_i)$ is isomorphic to $(\{M \subseteq A \cup \{\neg a \mid a \in A\} \mid a \in M \implies \neg a \notin M\}, \subseteq)$ via the mapping $v \mapsto \{a \in A \mid v(a) = \mathbf{t}\} \cup \{\neg a \mid a \in A, v(a) = \mathbf{f}\}$.

of $\mathcal{O}$ such that $v_2 \in [v_3]_2$. Moreover, an approximating operator $\mathcal{O}$ always has a fixpoint, which need not be the case for two-valued operators O. In particular, $\mathcal{O}$ has an $\leq_i$-least fixpoint, which approximates *all* fixpoints of O.

In subsequent work, DMT [Denecker *et al.*, 2004] presented a general, abstract way to define the most precise approximation of a given operator $O : \mathcal{V}_2 \to \mathcal{V}_2$. Most precise here refers to a generalisation of $\leq_i$ to operators, where for $\mathcal{O}_1, \mathcal{O}_2 : \mathcal{V}_3 \to \mathcal{V}_3$, they define $\mathcal{O}_1 \leq_i \mathcal{O}_2$ iff for all $v \in \mathcal{V}_3$ it holds that $\mathcal{O}_1(v) \leq_i \mathcal{O}_2(v)$. Specifically, DMT then show that the most precise – called the *ultimate* – approximation of O is given by the operator $\mathcal{U}_O : \mathcal{V}_3 \to \mathcal{V}_3$ that maps a given $v \in \mathcal{V}_3$ to

$$\mathcal{U}_O(v) : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\} \quad \text{with} \quad a \mapsto \begin{cases} \mathbf{t} & \text{if } w(a) = \mathbf{t} \text{ for all } w \in \{O(x) \mid x \in [v]_2\} \\ \mathbf{f} & \text{if } w(a) = \mathbf{f} \text{ for all } w \in \{O(x) \mid x \in [v]_2\} \\ \mathbf{u} & \text{otherwise} \end{cases}$$

This definition is remarkable since previously, approximations of operators had to be devised by hand rather than automatically derived. DMT [Denecker *et al.*, 2004] give additional definitions introducing stable semantics that are only of minor interest here and will be introduced in a special form later.

AFT on AFs

AFT can be used for defining semantics of AFs as follows [Strass, 2013a]. The stable semantics for AFs can be understood as a two-valued semantics given by the fixpoints of an operator (going back to Pollock [1987]) on two-valued interpretations.

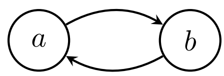
Definition 1. *For each* AF $F = (A, R)$*, the operator* $U_F : \mathcal{V}_2 \to \mathcal{V}_2$ *yields – for a given interpretation* $v : A \to \{\mathbf{t}, \mathbf{f}\}$ *– a new interpretation*

$$U_F(v) : A \to \{\mathbf{t}, \mathbf{f}\} \quad \textit{with} \quad a \mapsto \begin{cases} \mathbf{f} & \textit{if } \exists b \in A : v(b) = \mathbf{t}, (b, a) \in R \\ \mathbf{t} & \textit{otherwise} \end{cases}$$

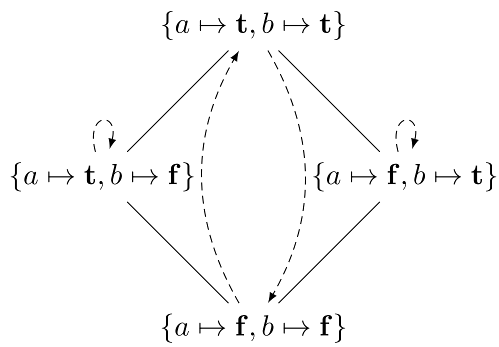
Intuitively, all arguments that are attacked in F by some argument that is true in v are set to false in U_F and set to true otherwise, that is, if unattacked by all $\mathbf{t}$ arguments of v. (So the U is for "unattacked".) It is easy to see that the fixpoints of this operator exactly correspond to stable extensions [Strass, 2013a, Proposition 4.4].

Proposition 2. *Let* $F = (A, R)$ *be an AF and* $v : A \to \{\mathbf{t}, \mathbf{f}\}$ *be an interpretation. Then* $v = U_F(v)$ *iff the set* $v^{-1}(\mathbf{t}) = \{a \in A \mid v(a) = \mathbf{t}\}$ *is a stable extension of* F*.*

Example 3. *Consider the* AF $F_1 = (A_1, R_1)$ *with* $A_1 = \{a, b\}$ *and* $R_1 = \{(a, b), (b, a)\}$*:*

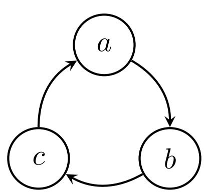


Below, we depict the complete lattice $(\{v : A_1 \to \{\mathbf{t}, \mathbf{f}\}\}, \leq_t)$ *of two-valued interpretations over* A_1 *ordered by the truth ordering as a Hasse diagram (i.e. straight lines show direct* $\leq_t$*-neighbours), and how the operator* U_{F_1} *assigns its points to others (dashed arrows).*

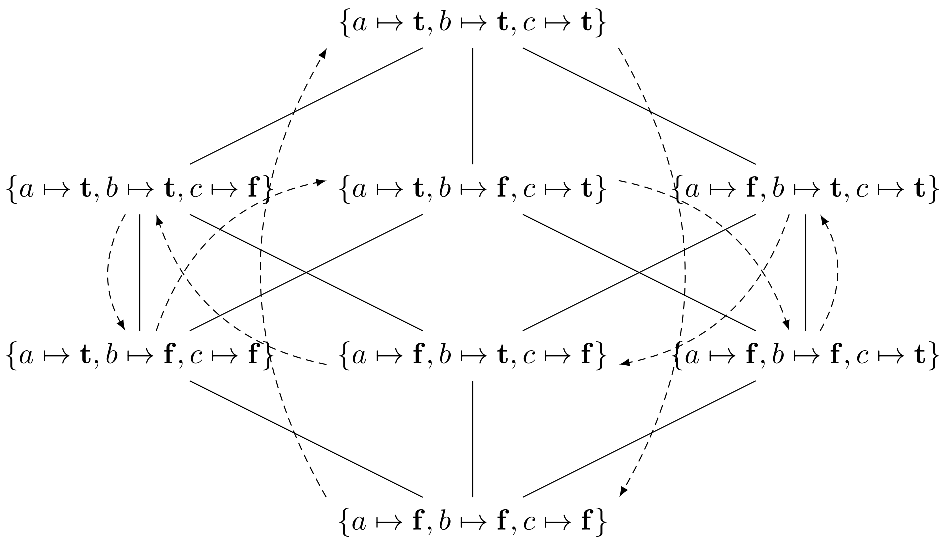


It can be seen from the diagram that the operator has two fixpoints, $\{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}\}$ *and* $\{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}\}$*. They correspond one-to-one to the stable extensions* $\{a\}$ *and* $\{b\}$ *of the AF* F_1*.*

Example 4. *In contrast, consider the* AF $F_2 = (A_2, R_2)$ *with* $A_2 = \{a, b, c\}$ *and* $R_2 = \{(a, b), (b, c), (c, a)\}$*:*



Again, we depict the complete lattice $(\{v : A_2 \to \{\mathbf{t}, \mathbf{f}\}\}, \leq_t)$ *and how the operator* U_{F_2} *assigns its points to others.*



The picture makes it obvious that U_{F_2} has no fixpoint, in accordance with the fact that F_2 has no stable extension.

Using the definitions of Denecker, Marek and Truszczyski, it is easy to obtain the ultimate approximation of U_F. (See also [Strass, 2013a, Proposition 4.1].)

Corollary 5. *Given an interpretation $v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$, the three-valued operator $\Upsilon_F : \mathcal{V}_3 \to \mathcal{V}_3$ yields a new interpretation*

$$\Upsilon_F(v) : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\} \quad \textit{with} \quad a \mapsto \begin{cases} \mathbf{f} & \textit{if } \exists b \in A : v(b) = \mathbf{t}, (b, a) \in R \\ \mathbf{t} & \textit{if } \forall b \in A : (b, a) \in R \implies v(b) = \mathbf{f} \\ \mathbf{u} & \textit{otherwise} \end{cases}$$

For any given AF F, the fixpoints of U_F constitute the stable semantics of F. The ultimate approximation Υ_F approximates U_F, thus the semantics induced by Υ_F then intuitively approximate AF stable semantics. More specifically, the following result is straightforward [Strass, 2013a, Section 4]:[4]

Proposition 6. *Let $F = (A, R)$ be an AF and $v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ be an interpretation.*

1. v is complete for F iff $v = \Upsilon_F(v)$.

[4]Given an AF $F = (A, R)$, an extension $E \subseteq A$ uniquely determines a three-valued interpretation v_E by letting $v_E(a) = \mathbf{t}$ if $a \in E$, $v_E(a) = \mathbf{f}$ if a is attacked by E in F, and $v_E(a) = \mathbf{u}$ otherwise. Similarly, a three-valued interpretation $v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ uniquely determines an extension $E_v = \{a \mid v(a) = \mathbf{t}\}$. This allows us to switch freely between extensions and interpretations.

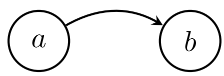
2. v is admissible for F iff $v \leq_i \Upsilon_F(v)$.

3. v is preferred for F iff v is $\leq_i$-maximal admissible.

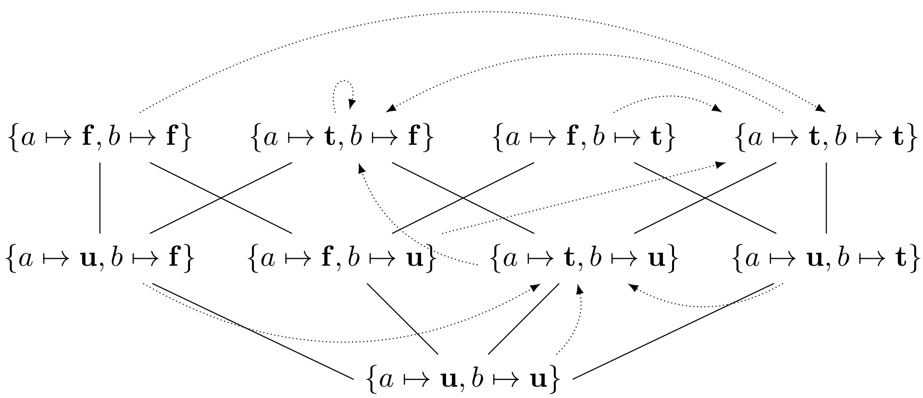
4. v is grounded for F iff v is the $\leq_i$-least fixpoint of Υ_F.

In the next section, we will use approximation fixpoint theory and this result to define the semantics of ADFs in a straightforward way.

Example 7. *Consider the AF $F_3 = (A_3, R_3)$ with $A_3 = \{a, b\}$ and $R_3 = \{(a, b)\}$:*

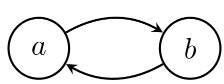


Below, we depict the associated meet-semilattice $(\{v : A_3 \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}\}, \leq_i)$ of the set of all three-valued interpretations over A_3 ordered by the information ordering, and how the operator Υ_{F_3} maps those interpretations to others.

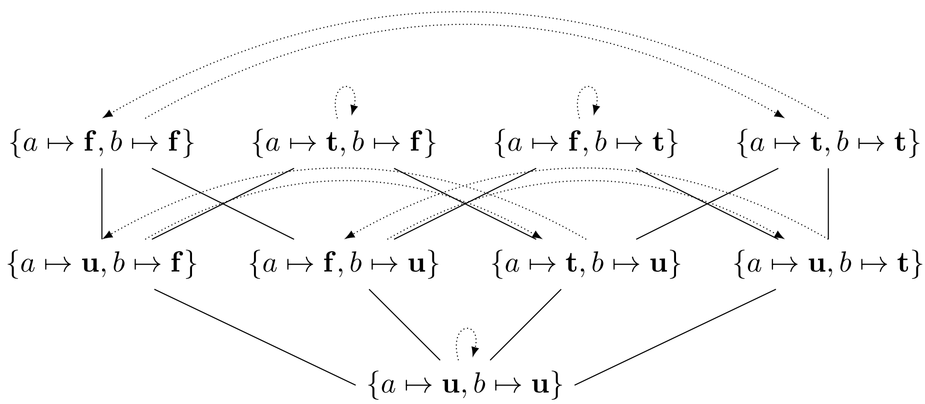


The picture shows how the grounded semantics can be obtained by following the dotted line starting in the $\leq_i$-least element up to the operator's single fixpoint. (In fact, it obviates that all (sufficiently long) sequences of operator applications lead to the fixpoint, showing that this interpretation really is the intended meaning of F_3.)

Example 8. *Reconsider the AF $F_1 = (A_1, R_1)$ from Example 3 with $A_1 = \{a, b\}$ and $R_1 = \{(a, b), (b, a)\}$:*



Again, we show the complete meet-semilattice $(\{v : A_1 \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}\}, \leq_i)$ *along with the mappings of the operator* Υ_{F_1} *(dotted arrows).*



In this picture, the operator U_{F_1} *re-appears in the top row of all two-valued interpretations. Those form a complete lattice with respect to* $\leq_t$*, but an antichain with respect to* $\leq_i$*. Likewise, the two fixpoints of* U_{F_1} *re-appear as fixpoints of* Υ_{F_1} *in the top row. The additional fixpoint of* Υ_{F_1} *consequently constitutes the grounded semantics of* F_1*.*

As we have seen, the operator Υ_F arises naturally from a straightforward application of ultimate approximation [Denecker *et al.*, 2004] to an operator proposed by Pollock [1987]. It is interesting to observe that the assignments of the operator correspond precisely to what has independently been defined as "legal argument labellings" [Caminada and Gabbay, 2009].

3 ADFs: Syntax and Semantics

Like an AF, an abstract dialectical framework (ADF) is a directed graph whose nodes represent arguments, statements or positions. One can think of the nodes as arbitrary items which can be accepted or not. The links represent dependencies. However, unlike a link in an AF, the meaning of an ADF link can vary. The status of a node s only depends on the status of its parents (denoted $par(s)$), that is, the nodes with a direct link to s. In addition, each node s has an associated acceptance condition C_s specifying the exact conditions under which s is accepted. C_s is a function assigning to each subset of $par(s)$ one of the truth values $\mathbf{t}$, $\mathbf{f}$.[5] Intuitively, if for some

[5]In the original paper *in* and *out* were used. We prefer truth values here as they allow us to apply standard logical terminology.

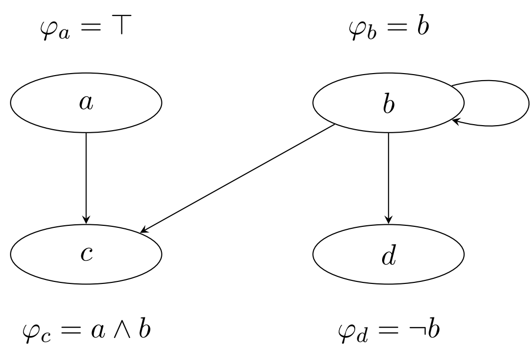
$R \subseteq par(s)$ we have $C_s(R) = \mathbf{t}$, then s will be accepted provided the nodes in R are accepted and those in $par(s) \setminus R$ are not accepted.

Definition 9. *An* abstract dialectical framework *is a tuple $D = (S, L, C)$ where*

- *S is a set of statements (positions, nodes),*
- *$L \subseteq S \times S$ is a set of links,*
- *$C = \{C_s\}_{s \in S}$ is a set of total functions $C_s : 2^{par(s)} \to \{\mathbf{t}, \mathbf{f}\}$, one for each statement s. C_s is called acceptance condition of s.*

In many cases it is convenient to represent acceptance conditions as propositional formulas. For this reason we will frequently use a logical representation of ADFs (S, L, C) where C is a collection $\{\varphi_s\}_{s \in S}$ of propositional formulas.[6]

Example 10. *In the following* ADF*, which will act as running example throughout the article, we use formulas to specify acceptance conditions.*



Intuitively, φ_a states that a should always be accepted. Condition φ_b expresses a kind of self-support, which can be utilized as a guess whether or not to accept b. Finally, c should be accepted if both a and b are, while d is attacked by statement b.

Unless specified differently we will tacitly assume that the acceptance formulas specify the parents a node depends on implicitly. It is then not necessary to give the links in the graph explicitly. We thus can represent an ADF D as a tuple (S, C) where S and C are as above and L is implicitly given as $(a, b) \in L$ iff a appears in φ_b.

[6] More precisely, each acceptance condition C_s will be represented as a propositional formula φ_s over the vocabulary $par(s)$.

The different semantics of ADFs over statements S are based (via approximation fixpoint theory) on the notion of a two-valued model. A two-valued interpretation $v : S \to \{\mathbf{t}, \mathbf{f}\}$ – a mapping from statements to the truth values true and false – is a *two-valued model* (*model*, if clear from the context) of an ADF (S, C) whenever for all statements $s \in S$ we have $v(s) = v(\varphi_s)$, that is, v maps exactly those statements to true whose acceptance conditions are satisfied under v.[7]

Approximation Fixpoint Theory on ADFs

We now come back to AFT and illustrate its role to define semantics for ADFs [Strass, 2013a; Brewka *et al.*, 2013]. As AFT deals with operator-based semantics and how to approximate them, the starting point is an operator for the two-valued semantics: the notion of an ADF model allows us to associate a two-valued operator to a given ADF.

Definition 11. *Let $D = (S, \{\varphi_s\}_{s\in S})$ be an ADF. The operator $G_D : \mathcal{V}_2 \to \mathcal{V}_2$ takes an input $v : S \to \{\mathbf{t}, \mathbf{f}\}$ and returns an updated interpretation*

$$G_D(v) : S \to \{\mathbf{t}, \mathbf{f}\} \quad \textit{with} \quad s \mapsto v(\varphi_s)$$

In words, the operator takes a two-valued interpretation v and returns a two-valued interpretation $G_D(v)$ mapping each $s \in S$ to the truth value that is obtained by evaluating φ_s with v. It is easy to see that this operator characterises the ADF model semantics [Strass, 2013a, Proposition 3.4].

Proposition 12. *Let $D = (S, L, C)$ be an ADF and $v : S \to \{\mathbf{t}, \mathbf{f}\}$ be a two-valued interpretation. Then v is a (two-valued) model of D iff $v = G_D(v)$.*

Example 13. *For the ADF D from Example 10, Figure 2 depicts the complete lattice $(\{v : S \to \{\mathbf{t}, \mathbf{f}\}\}, \leq_t)$ and how the operator G_D assigns its points to others.*

Using the general operator-based definitions of Denecker, Marek and Truszczyski [Denecker *et al.*, 2004], it is again straightforward to determine the ultimate approximation of G_D. Recall from the section on approximation fixpoint theory (Section 2) that the set $\mathcal{V}_3$ of all three-valued interpretations over S forms a complete meet-semilattice with respect to the information ordering $\leq_i$. The consensus meet operation $\sqcap_i$ of this semilattice is given by $(v_1 \sqcap_i v_2)(s) = v_1(s) \sqcap_i v_2(s)$ for all $s \in S$. The least element of this semilattice is the interpretation $v_{\mathbf{u}} : S \to \{\mathbf{u}\}$ mapping all

[7] In an earlier paper [Brewka *et al.*, 2013], there was the notion of a "three-valued model". The development and analysis of that concept has been discontinued.

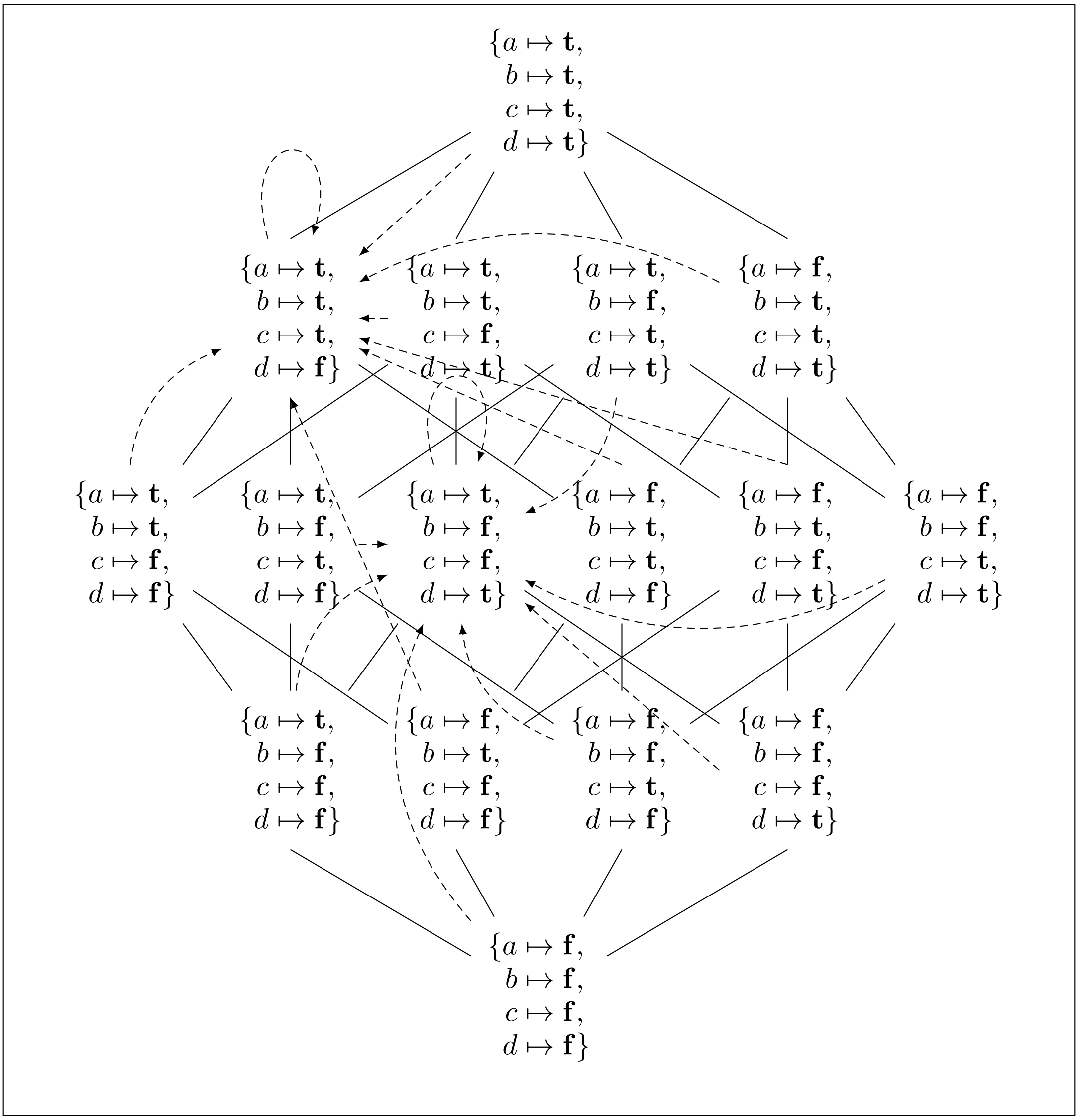


Figure 2: Complete lattice of two-valued interpretations for Example 10; dashed arrows visualise the assignments of the operator G_D. It can be readily seen that G_D has two fixpoints, whence D has two models (Proposition 12).

statements to undefined – the least informative interpretation. The ultimate approximation of the two-valued ADF operator G_D is now obtained as follows [Strass, 2013a, Lemma 3.12]:

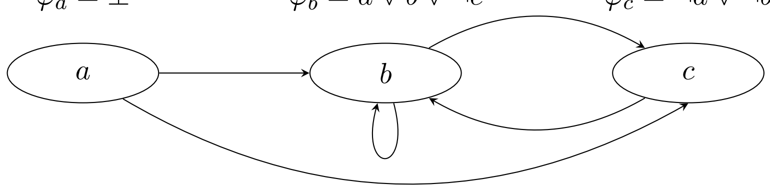
Corollary 14. *Let D be an* ADF*. The operator $\Gamma_D : \mathcal{V}_3 \to \mathcal{V}_3$ is the ultimate approximation of G_D and is defined as follows: for an* ADF *D and a three-valued interpretation v, the revised interpretation $\Gamma_D(v)$ is given by*

$$\Gamma_D(v) : S \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\} \quad \textit{with} \quad s \mapsto \textstyle\bigsqcap_i \{w(\varphi_s) \mid w \in [v]_2\}$$

That is, for each statement s, the operator returns the consensus truth value for its acceptance formula φ_s, where the consensus takes into account all possible two-valued interpretations w that extend the input valuation v. If this v is two-valued, then $[v]_2 = \{v\}$, thus $\Gamma_D(v)(s) = v(\varphi_s) = G_D(v)(s)$ and Γ_D indeed approximates G_D.

Example 15. *Consider the* ADF *$D_1 = (S_1, L_1, C_1)$ given by $S_1 = \{a, b, c\}$, and L_1 and C_1 given as follows:*

$\varphi_a = \bot$ $\qquad\qquad$ $\varphi_b = a \lor b \lor \neg c$ $\qquad\qquad$ $\varphi_c = \neg a \lor \neg b$



Roughly, a cannot be accepted. Statement b supports itself, and is furthermore supported by a and attacked by c – more precisely, b can be accepted if a can be accepted or b can be accepted or c can be rejected. In turn, c is jointly attacked by a and b – c can only be rejected if both a and b are accepted, otherwise c is accepted. Figure 3 shows the associated complete meet-semilattice $(\{v : S_1 \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}\}, \leq_i)$ along with the mappings of the operator Γ_{D_1}.

It is now an easy corollary of Proposition 6 to generalize the standard AF semantics to ADFs:

Definition 16. *Let $D = (S, L, C)$ be an* ADF *and $v : S \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ be an interpretation.*

1. *v is* complete *for D iff $v = \Gamma_D(v)$.*

2. *v is* admissible *for D iff $v \leq_i \Gamma_D(v)$.*

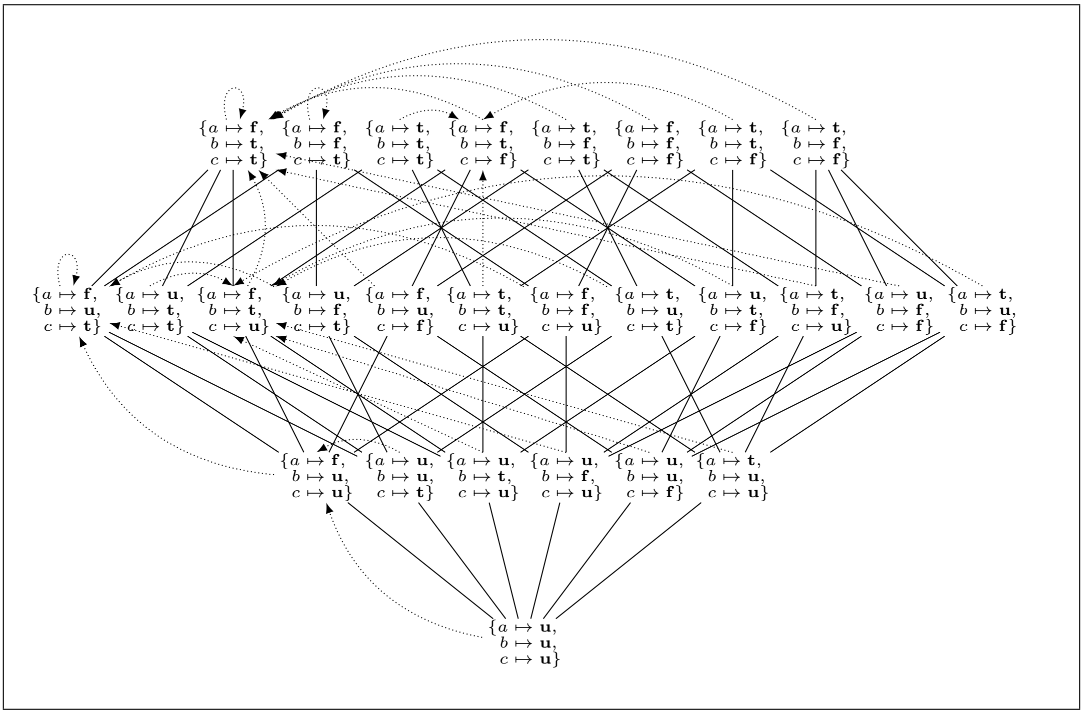


Figure 3: Complete meet-semilattice of three-valued interpretations over $S_1 = \{a, b, c\}$ under the information ordering for Example 15; dotted arrows visualise mappings of the operator Γ_{D_1}. It can be seen that Γ_{D_1} has a $\leq_i$-least fixpoint, which is situated right $\leq_i$-beneath its two-valued models, the other two fixpoints of Γ_{D_1}.

3. *v is* preferred *for D iff v is $\leq_i$-maximal admissible.*

4. *v is* grounded *for D iff v is the $\leq_i$-least fixpoint of Γ_D.*

Incidentally, Brewka and Woltran [2010] already defined the operator Γ_D (manually) and used it to define the grounded semantics. Thus the grounded semantics can be seen as the greatest possible consensus between all acceptable ways of interpreting the ADF at hand. A three-valued interpretation is admissible for an ADF D iff it does not make an unjustified commitment that the operator Γ_D will subsequently revoke.

There is an alternative and perhaps slightly more accessible way of introducing the operator Γ_D. We will briefly pursue this way for illustration, and start out with an additional definition. For a propositional formula φ over vocabulary S and a three-valued interpretation $v : S \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$, the *partial valuation of φ by v* is the formula

$$\varphi^v = \varphi[p/\top : v(p) = \mathbf{t}][p/\bot : v(p) = \mathbf{f}]$$

Intuitively, given a three-valued interpretation v and a formula φ, the partial evaluation of φ with v takes the two-valued part of v and replaces the evaluated variables with their truth values. For example, consider the propositional formula $\varphi = a \vee (b \wedge c)$ and the interpretation $v_1 = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}, c \mapsto \mathbf{u}\}$. Statement c with $v_1(c) = \mathbf{u}$ will remain in φ, while a and b are replaced, and we get $\varphi^{v_1} = \bot \vee (\top \wedge c)$. Now assume that an ADF $D = (S, \{\varphi_s\}_{s \in S})$ is given via acceptance formulas; for this D and a three-valued interpretation v, the revised interpretation $\Gamma_D(v)$ is given by

$$\Gamma_D(v) : S \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\} \quad \text{with} \quad s \mapsto \begin{cases} \mathbf{t} & \text{if } \varphi_s^v \text{ is irrefutable} \\ \mathbf{f} & \text{if } \varphi_s^v \text{ is unsatisfiable} \\ \mathbf{u} & \text{otherwise} \end{cases}$$

An irrefutable formula is a formula that is satisfied under any two-valued interpretation (i.e. the formula is a tautology).

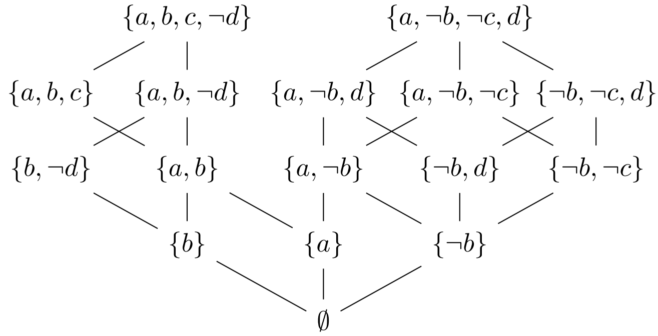
For reasons of brevity, we will sometimes shorten the notation of a three-valued interpretation $v = \{a_1 \mapsto t_1, \ldots, a_n \mapsto t_n,\}$ with statements $a_1, \ldots, a_n$ and truth values $t_1, \ldots, t_n$ to $v \mathrel{\hat{=}} \{a_i \mid v(a_i) = \mathbf{t}\} \cup \{\neg a_i \mid v(a_i) = \mathbf{f}\}$. For instance, $v = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{f}\} \mathrel{\hat{=}} \{a, \neg c\}$.

We now show some concrete interpretations and semantics for an example.

Example 17. *As we have seen before, for the* ADF *D from Example 10 we obtain the following two-valued models:*

- $v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{f}\} \mathrel{\hat{=}} \{a, b, c, \neg d\}$
- $v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\} \mathrel{\hat{=}} \{a, \neg b, \neg c, d\}$

Unfortunately, due to its sheer size ($3^4 = 81$ interpretations), we cannot depict the semi-lattice $(\{S \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}\}, \leq_i)$ and will henceforth resort to textual descriptions. The grounded interpretation of D is $v_3 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}, d \mapsto \mathbf{u}\} \mathrel{\hat{=}} \{a\}$. The admissible interpretations (ordered by $\leq_i$) of our example ADF *are as follows:*



We verify that $v_4 \hat{=} \{a, \neg b, \neg c\}$ is admissible in the example ADF. *Statement a's acceptance condition is a tautology. This means that under any three-valued interpretation v' it holds that $\Gamma_D(v')(a) = \mathbf{t}$, and, in particular, $\Gamma_D(v_4)(a) = v_4(a) = \mathbf{t}$. Acceptance condition of statement b is the formula b. Such an acceptance condition (a single unnegated variable) implies that for any three-valued interpretation v' that assigns a value to b, it holds that $\Gamma_D(v')(b) = v'(b)$. If b is assigned $\mathbf{t}$ by v', then $\varphi_b^{v'}$ is a tautology, if b is assigned $\mathbf{f}$, then $\varphi_b^{v'}$ is unsatisfiable, and if b is assigned $\mathbf{u}$ by v', then $\varphi_b^{v'} = b$ is neither a tautology nor unsatisfiable. The acceptance condition of statement c is $a \wedge b$. Evaluating φ_c under v_4 gives $\varphi_c^{v_4} = \top \wedge \bot \equiv \bot$, and $\Gamma_D(v_4)(c) = \mathbf{f} = v_4(c)$. Finally, $v_4(d) = \mathbf{u}$ and $\varphi_d = \neg b$. Since for the undefined truth value it holds that $\mathbf{u} \leq_i \mathbf{t}$ and $\mathbf{u} \leq_i \mathbf{f}$, if a three-valued interpretation v' assigns undefined to a statement, then applying the operator Γ_D under v' cannot return a truth value with less information than $\mathbf{u}$ for that statement. For our example interpretation, we have $v_4(d) \leq_i \Gamma_D(v_4)(d) = \mathbf{t}$.*

The complete interpretations of our example ADF *are*

$$v_3 \hat{=} \{a\}, \quad v_5 \hat{=} \{a, b, c, \neg d\}, \quad v_6 \hat{=} \{a, \neg b, \neg c, d\}.$$

The latter two, v_5 and v_6, are the preferred interpretations.

The definition of stable model semantics for ADFs [Brewka *et al.*, 2013] is based on ideas from Logic Programming (LP) where stable models strengthen the notion of minimal models by excluding self-justifying cycles of atoms. In LP, this is achieved by a test which picks a candidate model M, uses M to reduce the original logic program to a program without negative literals, and then checks whether M coincides with the (typically unique) least model of the reduced program. This way self-justifying cycles cannot appear. What we do for an ADF D is very similar: to check whether a two-valued model v of D is *stable* we do the following:

- we eliminate in D all nodes with v-value $\mathbf{f}$ and corresponding links,

- we replace eliminated nodes in acceptance conditions by $\bot$,
- we check whether nodes that are $\mathbf{t}$ in v coincide with those that are $\mathbf{t}$ in the grounded interpretation of the reduced ADF.

This is captured in the following definition [Brewka *et al.*, 2013, Definition 6]. (See also [Strass and Wallner, 2015, Proposition 2.4] for an alternative definition via AFT.)

Definition 18. *Let $D = (S, L, C)$ be an* ADF *with $C = \{\varphi_s\}_{s\in S}$ and $v : S \to \{\mathbf{t}, \mathbf{f}\}$ be a two-valued model of D. Define the reduced* ADF *D^v with $D^v = (S^v, L^v, C^v)$, where*

- $S^v = \{s \in S \mid v(s) = \mathbf{t}\}$
- $L^v = L \cap S^v \times S^v$
- $C^v = \{\varphi^v_s\}_{s\in S^v}$ *where for each $s \in S^v$, we set $\varphi^v_s = \varphi_s[b/\bot : v(b) = \mathbf{f}]$.*

Denote by w the unique grounded interpretation of D^v. Now the two-valued model v of D is a stable model *of D if and only if for all $s \in S$, we find that $v(s) = \mathbf{t}$ implies $w(s) = \mathbf{t}$.*

Note that a stable model of an ADF D is a model of D by definition (v is assumed to be a model). In the reduct for a model v, (i) only statements assigned to true by v are present, (ii) only links with both ends being statements assigned to true by v are considered, and (iii) in each acceptance formula of the remaining statements we replace statements $b \in S$ that v maps to false by their truth value, i.e., in these acceptance conditions variables assigned to false by v are replaced by $\bot$ (and the remaining statements/variables remain unmodified in the formulas). This definition straightforwardly expresses the intuition underlying stable models: if all statements the model v takes to be false are indeed false, we must find a constructive proof for all statements the model takes to be true.

Example 19. *Consider the* ADF *D given by*

$$\varphi_a = \top, \quad \varphi_b = \neg a \vee c, \quad \varphi_c = b.$$

It has two models: $v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}\}$ and $v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}\}$. Let us check whether they are stable models. For v_1, the reduct, D^{v_1}, is equal to D (every statement is assigned to true by v_1, thus all statements and links remain in the reduct and no statement is replaced by $\bot$ in an acceptance condition). The *grounded interpretation of D is $v_3 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}\}$, implying that v_1 is not stable in D, since the grounded interpretation of D^{v_1} is not equal to v_1.*[8]

For the other model of D, the reduct $D^{v_2} = (S^{v_2}, L^{v_2}, C^{v_2})$ with $S^{v_2} = \{a\}$, $L^{v_2} = \emptyset$, and $\varphi_a = \top$. The grounded interpretation of D^{v_2} is $v_4 = \{a \mapsto \mathbf{t}\}$. The final condition of Definition 18, $v_2(a) = \mathbf{t}$ implies $v_4(a) = \mathbf{t}$, is satisfied, and, therefore, v_2 is a stable model of D. Further, v_2 is the only stable model of D, since we considered all models of D, only one being stable, and any other interpretation cannot be stable for D, since being a model is a prerequisite for being stable.

Next, we illustrate that there are cases where an ADF has a model, but no stable model.

Example 20. *Consider the* ADF *D given by*

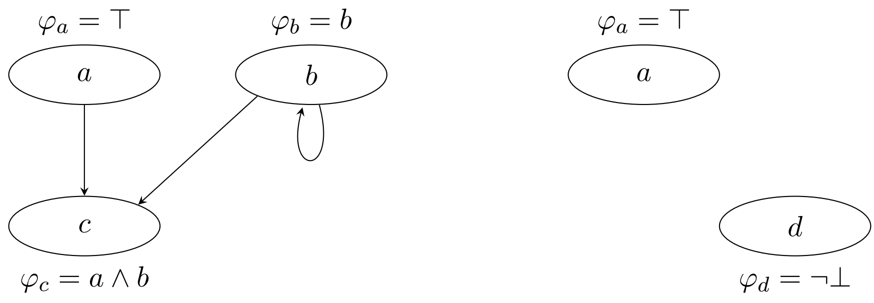
$$\varphi_a = c, \quad \varphi_b = c, \quad \varphi_c = a \leftrightarrow b.$$

The only two-valued model of D is $v = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}\}$. Since c is true because a and b are and vice versa, the model contains unintended cyclic support and thus should not be stable. Indeed, for the reduct we get $D^v = D$. Let us compute the grounded semantics of D. We start with interpretation $w = \{a \mapsto \mathbf{u}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}\}$. Since none of the acceptance formulas is a tautology or an unsatisfiable formula, w is already a fixpoint of Γ_D and thus the grounded interpretation of D. Hence v is not a stable model and D has no stable models, just as intended. Since v is a minimal model of D the example illustrates that in Definition 18 we actually need the grounded semantics; requiring v to be among the (subset-inclusion or information) minimal two-valued models of the reduct is insufficient, in contrast to, e.g., stable semantics of logic programs.

For our running example, the concept of reduct is applied as follows.

Example 21. *The ADF from Example 10 has two two-valued models, namely $v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{f}\}$ and $v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{f}, d \mapsto \mathbf{t}\}$. We obtain the reducts for each model of D as follows:*

[8] The definition of stable models in this article, taken from [Brewka *et al.*, 2013, Definition 6], supersedes the definition of stable models in the original paper on ADFs [Brewka and Woltran, 2010, Definition 6] in that the new definition corrects certain unintended results. For instance, v_1 in Example 19 is the only stable model according to the old definition, but this is not the case under the new definition. The model v_1 violates the basic intuition of stable semantics that all elements of a stable model should have a non-cyclic justification: in the model v_1 it holds that b is accepted because c is and vice versa (these two statements have supporting links to each other; see Section 6.1 for a formalization of attacking and supporting links between statements).



Reduct D^{v_1} *Reduct D^{v_2}*

The grounded interpretation of reduct D^{v_1} is $\{a\}$, v_1 is thus not a stable model of D. For v_2, the reduct D^{v_2} has the grounded interpretation $\{a \mapsto \mathbf{t}, d \mapsto \mathbf{t}\}$. The model v_2 of D is thus the single stable model of D.

Well-known relationships between semantics defined on Dung AFs carry over to ADFs. This is formalized in the next theorem [Brewka *et al.*, 2013, Theorem 3].

Theorem 3.1. *Let D be an* ADF.

- *Each stable model of D is a two-valued model of D;*
- *each two-valued model of D is a preferred interpretation of D;*
- *each preferred interpretation of D is complete;*
- *each complete interpretation of D is admissible;*
- *the grounded interpretation of D is complete.*

We illustrate the relationships in Figure 4 where an arrow from a σ-interpretation to a τ-interpretation denotes that every σ-interpretation is a τ-interpretation. Further, again similarly as in AFs, any ADF possesses at least one admissible, complete, preferred, and grounded interpretation, while this is not guaranteed for models and stable models.

In addition to the semantical relationships generalizing those known from AFs, semantics on ADFs also directly generalize semantics for AFs. We first define for a given AF its associated ADF.

Definition 22. *For an* AF *$F = (A, R)$, define the* ADF associated to *F as $D_F = (A, R, C)$ with $C = \{\varphi_a\}_{a \in A}$ such that for each $a \in A$, the acceptance condition is*

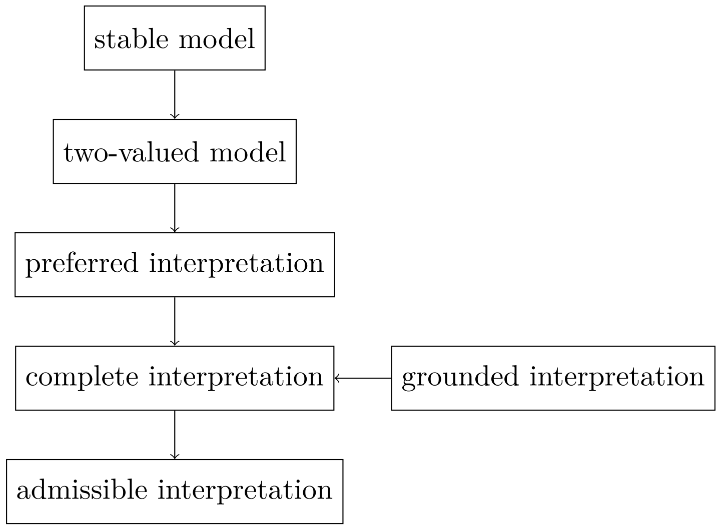


Figure 4: Relations between ADF *semantics*

given by

$$\varphi_a = \bigwedge_{\substack{b \in A, \\ (b,a) \in R}} \neg b$$

Now we can formalize the way ADFs, and their semantics, generalize AFs in the next two theorems [Brewka *et al.*, 2013].

Theorem 3.2. *Let $F = (A, R)$ be an* AF *and D_F its associated* ADF*. For any two-valued interpretation v for A, the following are equivalent:*

(A) the set $v^{-1}(\mathbf{t}) = \{a \in A \mid v(a) = \mathbf{t}\}$ is a stable extension of F,

(B) v is a stable model of D_F,

(C) v is a two-valued model of D_F.

Note that for AF-based ADFs, there is no distinction between models and stable models. The intuitive explanation for this is that stable semantics on ADFs breaks cyclic supports, which cannot arise in AFs because they cannot (directly) express support.

More generally, we can also show that our definitions are indeed proper generalizations of Dung's notions for AFs as given in Proposition 6. The result is due to [Brewka *et al.*, 2013].

Theorem 3.3. *Let F be an* AF *and D_F its associated* ADF*. An interpretation is admissible, complete, preferred, grounded for F iff it is admissible, complete, preferred, grounded for D_F.*

On AFs, if v is a preferred interpretation (a stable model) for an AF F it holds that there is no preferred interpretation (stable model) $v' \neq v$ such that the set of statements assigned to true by v is a subset of the statements assigned to true by v', i.e., $\{s \mid v(s) = \mathbf{t}\} \not\subseteq \{s \mid v'(s) = \mathbf{t}\}$. On general ADFs, this property does not hold for preferred interpretations and two-valued models, i.e., there are ADFs with two preferred interpretations (models) v and v' such that $\{s \mid v(s) = \mathbf{t}\} \subseteq \{s \mid v'(s) = \mathbf{t}\}$.

Example 23. *Consider* ADF $D = (\{a\}, \{(a, a)\}, \{\varphi_a = a\})$*. Both $v_1 = \{a \mapsto \mathbf{f}\}$ and $v_2 = \{a \mapsto \mathbf{t}\}$ are models and preferred interpretations of D. It holds that $\{s \mid v(s) = \mathbf{t}\} = \emptyset \subsetneq \{a\} = \{s \mid v'(s) = \mathbf{t}\}$.*

On the other hand, for any ADF D with stable models v_1 and v_2, it holds that $v_1 \leq_t v_2$ implies $v_1 = v_2$ [Strass, 2013a, Proposition 3.8], that is, such strict relationships cannot occur between *stable* models. (This follows easily from AFT.)

4 ADFs as Modelling Tools

In this section we will provide various examples illustrating why – as we believe – ADFs are useful tools in formal argumentation. We discussed the term argumentation middleware in the introduction already. We now want to give a clearer picture what we actually mean by this. More precisely, we will discuss various graphical representations of argumentation scenarios users may find useful. In each case we define the semantics of the chosen representation by providing a formal translation to ADFs. The representation is thus equipped – via the translation – with the whole range of Dung semantics we have defined for ADFs. We also discuss how ADFs can serve as a tool for providing semantics to systems based on strict and defeasible inference rules, again via a translation.

4.1 Weights and Preferences

In our informal discussion in the introduction we have already shown how graphical representations based on link types (+ for supporting, − for attacking) can be

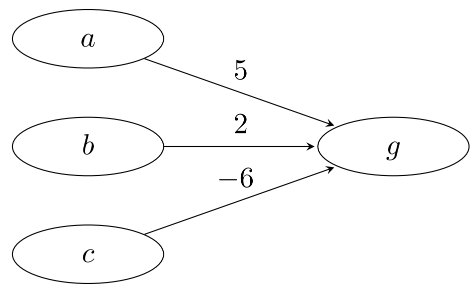


Figure 5: An argument graph with weighted links.

modeled using ADFs. The same is obviously true for links annotated with numerical weights. Throughout the paper we will assume a positive weight represents support, a negative weight attack, in both cases with a given strength. An example can be found in Figure 5.

The figure uses a weighted graph to represent a simple argumentation scenario. We will provide the graph with a formal semantics based on translating it to an ADF. There are various ways of interpreting the numbers and of actually deriving specific ADF acceptance conditions from representations like this one. We first have to specify how the numbers should actually be used to decide whether a node is accepted or not. Recall that a link is active if its source node is accepted. A straightforward idea is to accept a node whenever the sum of the weights of all active links pointing to the node is positive. We will call this strategy *sum-of-weights* (sow). For node g in Figure 5 this amounts, as we will see, to the following acceptance condition: $(\neg c \wedge (a \vee b)) \vee (a \wedge b)$.

Secondly, we need to take care of those nodes which do not depend on other nodes, that is, nodes without incoming links. We will call these nodes *input nodes* and denote the input nodes of a graph G as $\mathit{input}(G)$. It is often useful to consider input nodes as parameters whose truth values can be chosen freely, with the aim to explore the consequences of a particular choice. Consequently, our translation will depend on the assignment of truth values to the input nodes.

Definition 24. *Let $G = (N, E, I)$ be a labelled graph with nodes N, edges E and (integer) labelling function $I : E \to \mathbb{Z}$. Let $A \subseteq \mathit{input}(G)$ be the subset of input nodes considered true (the other input nodes are considered false). The sum-of-weights translation of G under A is the* ADF *$D = (S, L, C)$ with $S = N$, $L = E$, and the acceptance condition C_s (represented as a formula ϕ_s) is defined as follows:*

$$\phi_s = \begin{cases} \top, & \textit{if } s \in A \\ \bot, & \textit{if } s \in \textit{input}(G) \setminus A \\ \phi_{sow}(s), & \textit{otherwise} \end{cases}$$

where the formula $\phi_{sow}(s)$ is the disjunction of all conjunctions of literals built from parent nodes of s which represent truth value assignments under which the sum of weights of active links is positive.

Let us check how the acceptance condition for node g in Figure 5 is obtained. The following table shows 8 possible assignments of truth values to g's parent nodes, together with the sum of values of active links:

a	b	c	
t	**t**	**t**	1
t	**t**	**f**	7
t	**f**	**t**	-1
t	**f**	**f**	5
f	**t**	**t**	-4
f	**t**	**f**	2
f	**f**	**t**	-6
f	**f**	**f**	0

The sum of weights of active links is positive in 4 of the 8 lines, the acceptance condition of g is the disjunction of the conjunctions corresponding to these lines, that is:

$$(a \wedge b \wedge c) \vee (a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge b \wedge \neg c)$$

which can be simplified to $(\neg c \wedge (a \vee b)) \vee (a \wedge b)$, the formula presented earlier.

Of course, there are many more strategies how to evaluate the numbers. One possibility is to check whether the maximal positive weight of an active link is higher than the maximal negative weight of an active link. This leads to a different definition of acceptance conditions for non-input nodes. We leave the details to the reader and just mention that in Figure 5 the acceptance condition for g under this new strategy becomes $(\neg c \wedge (a \vee b))$.

Qualitative preferences can be handled in a similar manner. Let us first introduce prioritized argument graphs.

Definition 25. *A prioritized argument graph is a tuple $G = (S, L^+, L^-, >)$ where S is the set of nodes, L^+ and L^- are subsets of $S \times S$, the supporting and attacking links, and $>$ is a strict partial order (irreflexive, transitive, antisymmetric) on S representing preferences among the nodes.*

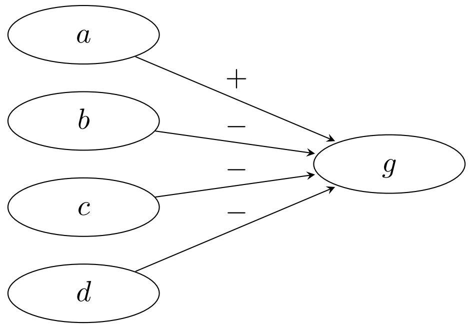


Figure 6: An argument graph with qualitative weights.

As before, we will translate prioritized argument graphs to ADFs. We illustrate the translation using an example. Assume we are given the graph in Figure 6.

Assume further the preference ordering is $a > c$ and $g > d$, that is a is strictly preferred to c, g to d. We want to capture the following intuition: an attacker (represented by label $-$ in the graph) does not succeed if the attacked node is more preferred than the attacker, or if there is a more preferred supporting node (represented by label $+$ in the graph).

We treat input nodes as in Definition 24. The general scheme for deriving formulas expressing the corresponding acceptance condition ϕ_s for a node s with a non-empty set of parents is the following: we create a conjunction of implications, one for each attacker t of s which is not less preferred than s. The left side of the implication (the precondition) consists of the attacker t, the right side (conclusion) is the disjunction of all supporting nodes of s which are more preferred than t.

In the example above, the only attackers which are not less preferred than g are b and c. For b we obtain the implication $b \to \mathbf{f}$ (as there is no supporting node more preferred than b and the empty disjunction is equivalent to $\mathbf{f}$). For attacker c we obtain the implication $c \to a$, as a is more preferred than c. This yields the following acceptance condition for g: $(b \to \mathbf{f}) \wedge (c \to a)$ or, equivalently $\neg b \wedge (c \to a)$.

As a matter of fact, preferences are often not given in advance, as assumed in the example, but an issue of debate themselves. One way to model situations where the preference relation $>$ is established dynamically in the course of argumentation is the following. Let us assume some nodes represent (possibly conflicting) preference information, that is information about which pairs of nodes belong to $>$. The idea is to guess a (stable, preferred, grounded) interpretation M and then to verify whether M can be generated in a way satisfying the preference relation it contains. To do so we extract the preference information from the relevant nodes in M. We then check whether M can be reconstructed under this (now static) preference information using the techniques described above. We thus verify whether the preferences represented in the model itself were taken into account adequately.

Definition 26. *An argument graph with dynamic preferences is a tuple*

$$G = (S, L^+, L^-, P)$$

where S is the set of nodes, L^+ and L^- are subsets of $S \times S$, the supporting and attacking links, and $P : S \to S \times S$ is a partial function.

The function P assigns preference information to some of the nodes in S. If $P(a) = (b, c)$ then node a carries the information that b is preferred over c. For a three-valued interpretation M we use $>_M$ to denote the smallest strict partial order on S containing the set $\{(b, c) \mid P(a) = (b, c), M(a) = \mathbf{t}\}$. Note that $>_M$ may be undefined, e.g. if M contains two nodes with conflicting preference information. The semantics of argument graphs with dynamic preferences is now defined as follows:

Definition 27. *Let $G = (S, L^+, L^-, P)$ be an argument graph with dynamic preferences, A a subset of its input nodes. E is a (stable, preferred, grounded) interpretation of G under A iff $>_E$ is a strict partial order and E is a (stable, preferred, grounded) interpretation of the prioritized argument graph $D_E = (S, L^+, L^-, >_E)$ under A.*

We thus guess an interpretation E of the intended type, extract from E the corresponding strict partial order on S, and check whether E is among the intended interpretations of the (non-dynamic) prioritized argument graph which is based on the extracted preference information. The evaluation of the prioritized graph is based on the translation to ADFs described earlier in this section. For further details see [Brewka *et al.*, 2013].

4.2 Proof Standards

Proof standards are well known and play an important role in legal reasoning. They are based on the intuitive idea that decisions or verdicts which have drastic consequences, say for a defendant, should be based on stronger, less doubtful criteria than decisions with limited consequences, say a small fine. Farley and Freeman [Farley and Freeman, 1995] introduced a model of legal argumentation which distinguishes four types of arguments (in decreasing order of strength):

- *valid* arguments based on deductive inference,

- *strong* arguments based on inference with defeasible rules,
- *credible* arguments where premises give some evidence,
- *weak* arguments based on abductive reasoning.

By using values $V = \{+v, +s, +c, +w, -v, -s, -c, -w\}$ we will distinguish pro and con links of the corresponding types in argument graphs, where the type of a link is inherited from the type of its source node.

Based on these argument types, Farley and Freeman define the following proof standards:

- *Scintilla of Evidence*: at least one pro-argument is accepted.
- *Preponderance of Evidence*: at least one pro-argument is accepted, all accepted con arguments are outweighed by stronger accepted pro arguments.
- *Dialectical Validity*: there is at least one credible accepted pro-argument, none of the other side's arguments is accepted.
- *Beyond Reasonable Doubt*: there is at least one strong accepted pro-argument, none of the other side's arguments is accepted.
- *Beyond Doubt*: there is at least one valid active pro-argument, none of the other side's arguments is accepted.

Again we will show how these notions can be formalized using ADFs.

Consider the labelled graph in Figure 7. Let us focus on the acceptance condition for g, represented as a propositional formula. The condition obviously depends on g's proof standard. For scintilla of evidence it is sufficient that at least one pro-argument is accepted. There are two such arguments, a and b, the acceptance condition thus is $a \lor b$. For preponderance of evidence at least one pro-argument must be accepted, and in addition each accepted con-argument must be outweighed by a stronger pro-argument. In our case this means that if c is accepted, then the stronger pro-argument b must also be accepted, and d cannot be accepted, as there is no stronger pro-argument than the valid argument d. Taken together this yields the formula $(a \lor b) \land (c \to b) \land \neg d$. In a similar manner we obtain the formulas for g for the remaining proof standards, as shown in the following table:

Scintilla of evidence:	$a \lor b$
Preponderance of evidence:	$(a \lor b) \land (c \to b) \land \neg d$
Dialectical validity:	$b \land \neg c \land \neg d$
Beyond reasonable doubt:	$b \land \neg c \land \neg d$
Beyond doubt:	$\bot$

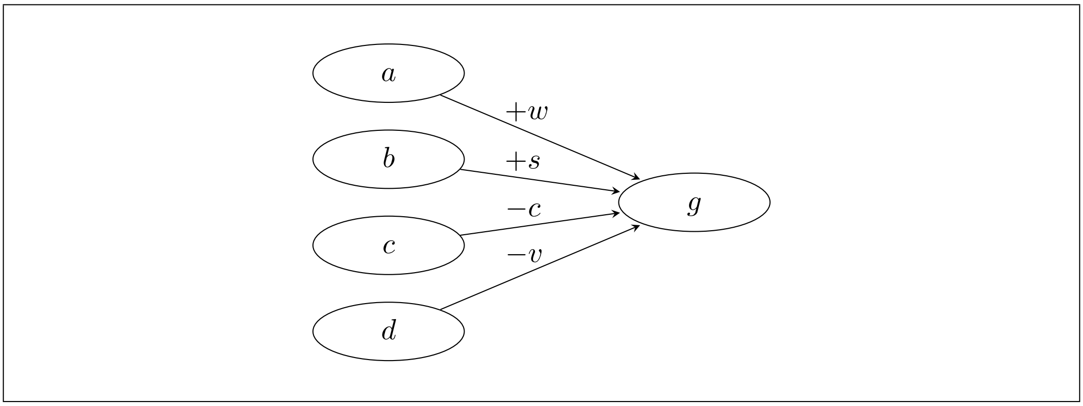


Figure 7: A Farley/Freeman argument graph.

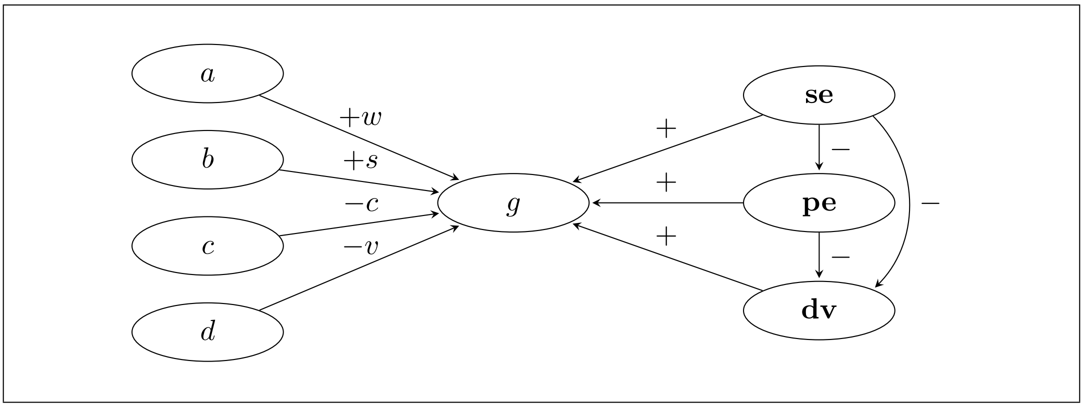


Figure 8: A graph with dynamic proof standards.

It is even possible to choose the proof standard dynamically. For ease of presentation let's focus on three proof standards, namely scintilla of evidence, preponderance of evidence and dialectical validity, represented as **se**, **pe** and **dv**, respectively.[9] Consider the graph in Figure 8 which should be viewed as part of a larger argument graph. The idea here is that scintilla of evidence is the default proof standard. If the corresponding node **se** is attacked from outside (e.g. since a crime was committed), then preponderance of evidence becomes the active proof standard. If also the corresponding node **pe** is attacked from outside (e.g. since the crime has serious consequences), then dialectical validity will be active. To model this intuition, the

[9]The type of these nodes is irrelevant and thus left out.

acceptance condition of node g becomes:

$$(\mathbf{se} \land (a \lor b)) \lor (\mathbf{pe} \land (a \lor b) \land (c \to b) \land \neg d) \lor (\mathbf{dv} \land b \land \neg c \land \neg d).$$

4.3 Carneades

Carneades [Gordon *et al.*, 2007] is an advanced model of argumentation based on a graphical representation of arguments and the propositions involved in them. Each proposition has an associated proof standard (scintilla of evidence, preponderance of evidence, clear and convincing evidence, beyond reasonable doubt, dialectical validity). There is some paraconsistency at work in the system as scintilla of evidence allows both a proposition and its negation to be accepted at the same time. The ADF graphs we will construct later will for this reason have separate nodes for each proposition p and its complement $\overline{p}$. A major restriction of Carneades is that cycles in the graph are not allowed (which means the system handles only cases where all Dung semantics coincide).

Let us start with some basic definitions underlying Carneades. Our presentation follows [Brewka and Gordon, 2010].

Definition 28. *An* argument *is a tuple $\langle P, E, c\rangle$ with premises P, exceptions E ($P \cap E = \emptyset$) and conclusion c. c and elements of P, E are literals.*

An argument evaluation structure *(CAES) is a tuple $\mathcal{S} = \langle args, as, weight, standard\rangle$, where*

- *args is a set of arguments generating an acyclic argument graph,*
- *as is a consistent set of literals,*
- *weight assigns a real number to each argument, and*
- *standard maps propositions to a proof standard.*

The argument graph generated by a CAES is obtained as follows: each literal occurring in an argument *arg* becomes a node; each argument *arg* becomes a node; each premise of an argument *arg* is linked to the corresponding argument node *arg* via a link labelled with +, each exception via a link labelled with -; an additional link, labelled with *weight*(*arg*), connects *arg* and the conclusion of *arg*.

The central notions in Carneades are *applicability* of arguments and *acceptability* of propositions. These notions are defined via mutual recursion. Note that for the recursion to bottom out it is essential that Carneades is acyclic.

Definition 29. *We say an argument $\langle P, E, c\rangle \in$ args is* applicable *in $\mathcal{S}$ iff*

- *$p \in P$ implies $p \in as$ or [$\overline{p} \notin as$ and p acceptable in $\mathcal{S}$], and*

- *$p \in E$ implies $p \notin as$ and [$\overline{p} \in as$ or p is not acceptable in $\mathcal{S}$].*

Based on the applicability of arguments, we can define what it means for a proposition p to be *acceptable* in $\mathcal{S}$. As expected, acceptability depends on p's proof standard. The Carneades proof standards differ form those of Farley and Freeman. In particular, they depend on numerical values:

- $standard(p) = se$: there is an applicable argument for p,

- $standard(p) = pe$: p satisfies se, and the maximum weight assigned to an applicable argument pro p is greater than the maximum weight of an applicable argument con p,

- $standard(p) = ce$: p satisfies pe, and the maximum weight of an applicable pro argument exceeds a threshold α, and difference between the maximum weight of applicable pro arguments and the maximum weight of applicable con arguments exceeds a threshold β,

- $standard(p) = bd$: p satisfies ce, and the maximum weight of the applicable con arguments is less than a threshold γ,

- $standard(p) = dv$: there is an applicable argument pro p and no applicable argument con p.

We now show how arguments and the generated argument graphs are represented using ADFs. The translation to ADFs is based on the techniques we have seen so far in this section. Consider the argument $a = \langle\{\mathrm{bird}\}, \{\mathrm{peng}, \mathrm{ostr}\}, \mathrm{flies}\rangle$ with $weight(a) = 0.8$. This argument is represented graphically as shown in Figure 9.

Apart from the duplication of propositions/complements the graphical representation corresponds to the original Carneades graph. Using techniques similar to the ones described earlier, we can properly define acceptance conditions such that an argument node is **t** in the ADF graph iff the argument is applicable, and a proposition node is **t** iff the proposition is acceptable. The acceptance condition of an argument node *arg* requires that all premises of *arg* are true, all exceptions false (assumptions can be handled by an easy preprocessing step). The acceptance condition of a proposition node depends on the proof standard and is modelled along the lines of what we have discussed earlier in this section. We leave the details to the reader. Note that we will resume our discussion of Carneades at the end of Section 5 where we show how the relevant acceptance conditions can be formalized in GRAPPA.

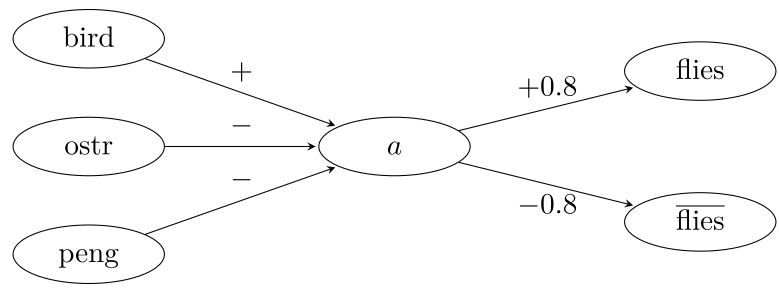


Figure 9: A Carneades argument represented graphically.

What has been gained by this reconstruction? Why is it useful? First of all, it shows the generality of ADFs. Secondly, it puts Carneades on safe formal ground. But in addition, and this is probably the main advantage, it allows us to give up the restriction of Carneades to acyclic argument graphs. Nothing in our translation rests on the assumption that Carneades is acyclic. The translation works perfectly well also for cyclic argument evaluation structures. The only difference is that the resulting ADF graph will have cycles as well. But handling cycles of this kind is part of the core functionality of ADFs, and they have a variety of different semantics to offer for this case, as we have seen in Section 3.

4.4 Rule-based Languages

A major strand of research in formal argumentation is concerned with using argumentation techniques to assign semantics to simple rule-based languages (see the paper *Abstract Rule-based Argumentation* by Modgil and Prakken in this issue). Those languages are simple logic-inspired formalisms working with inference rules on a set of propositional literals. Inference rules can be strict, in which case the conclusion of the inference (a literal) must necessarily hold whenever all antecedents (also literals) hold. Inference rules can also be defeasible, which means that the conclusion *usually* holds whenever the antecedents hold. Here, the word "usually" suggests that there could be exceptional cases where a defeasible rule has not been applied [Pollock, 1987] (for example to avoid an imminent inconsistency).

Most of the existing works in this area translate rule-based languages to AFs by constructing arguments and identifying attacks. But this approach is not always without problems, as Caminada and Amgoud [Caminada and Amgoud, 2007] observed. (They even devised a set of *rationality postulates* for capturing the intended behavior of semantics for rule-based languages.) While there exist AF-based solutions to those problems [Wyner *et al.*, 2013], we concentrate here on one approach using ADFs as target language [Strass, 2013b; Strass, 2015b]. Translating to ADFs instead of AFs has the additional benefit of tackling the problem of cyclic justifications amongst arguments on the semantic level instead of the syntactic one (like it is done in the ASPIC approach [Caminada and Amgoud, 2007] among others). We only give intuitions here and refer the reader to the original paper(s) for details [Strass, 2013b; Strass, 2015b].

Inspired by the approach of Wyner et al. [Wyner *et al.*, 2013], Strass [Strass, 2013b; Strass, 2015b] directly uses the literals from the theory base as statements that express whether the literal holds. He also uses rule names as statements indicating that the rule is applicable. Additionally, for each rule r he creates a statement -r indicating that the rule has not been applied. Not applying a rule is acceptable for defeasible rules, but unacceptable for strict rules since it would violate the closure postulate. This is enforced via integrity constraints saying that it may not be the case in any model that the rule body holds but the head does not hold: Technically, for a strict rule r, he introduces a conditional self-attack of -r; this self-attack becomes active if (and only if) the body of r is satisfied but the head of r is not satisfied, thereby preventing this undesirable state of affairs from getting included in a model. Defeasible rules offer some degree of choice, whence it is left to the semantics whether or not to apply them. This choice is modelled by a mutual attack cycle between r and -r. The remaining acceptance conditions are equally straightforward:

- Opposite literals attack each other.

- A literal is accepted whenever some rule deriving it is applicable, that is, all rules with head ψ support statement ψ.

- A strict rule is applicable whenever all of its body literals hold, that is, the body literals of r are exactly the supporters of r.

- Likewise, a defeasible rule is applicable whenever all of its body literals hold, and additionally the negation of its head literal must not hold.

Strass [2013b, 2015b] showed that the approach satisfies the rationality postulates of Caminada and Amgoud [2007]. Furthermore, this method has a mild computational complexity (with an at most quadratic blowup from rule-based theory to ADF formalization, while there can be exponential to infinite blowup in other approaches).

5 Graph-based Argument Processing

We have seen in Section 4 how ADFs can be used to provide graphical representations of argumentation scenarios with semantics. The different approaches were based on translations from some graphical representation to ADFs. In a nutshell, the GRAPPA approach [Brewka and Woltran, 2014] described in this section addresses the opposite question: is it possible to extend the formal techniques underlying ADFs in such a way that the semantics of various kinds of graphical representations can be defined directly for these representations, without the detour of a translation? More specifically, we will consider arbitrary (edge) labelled graphs. Such graphs are highly popular for visualizing argumentation scenarios, and indeed the literature on argumentation is full of such representations. The goal of this section is to define various semantics directly for such labelled graphs.

Another way of looking at the approach is the following: Dung AFs actually can be seen as graphs where all edges have the same label, which is left implicit for this reason. In addition, all nodes have the same type of acceptance condition. Dung's seminal contribution can thus be characterized as defining various semantics for specific graphs with a single label and uniform acceptance conditions. Our goal is to generalize this to arbitrary labelled graphs with flexible, user-defined acceptance conditions.

GRAPPA requires two major changes. First of all, the acceptance conditions can no longer be propositional formulas built from parent nodes, as in ADFs. We rather have to define them in terms of the labels of active links in the graph, that is links whose source nodes are accepted (true). More precisely, since it may be relevant whether there are multiple active links with the same label, we have to consider multisets of labels. An acceptance condition will thus be a function assigning a truth value to each multiset of labels. Secondly, we have to modify the operator Γ_D for ADFs D as defined in Section 3 in such a way that the new acceptance conditions are taken into account adequately.

In the following we describe multisets as functions into the natural numbers. Intuitively, the number assigned to an element describes the number of occurrences of the element in the multiset.

Definition 30. *An acceptance function over a set of labels L is a function $c : (L \to \mathbb{N}) \to \{\mathbf{t}, \mathbf{f}\}$.*

The set of all acceptance functions over L is denoted F^L.

Definition 31. *A labelled argument graph (LAG) is a tuple $G = (S, E, L, \lambda, \alpha)$ where*

- *S is a set of nodes (statements),*
- *E is a set of edges (dependencies),*
- *L is a set of labels,*
- *$\lambda : E \to L$ assigns labels to edges,*
- *$\alpha : S \to F^L$ assigns L-acceptance-functions to nodes.*

The characteristic operator Γ_G of a LAG G basically does what the corresponding operator does for ADFs: it takes a three-valued (or, equivalently, partial) interpretation v and produces a new one v'. In doing so, it checks which truth values of nodes in S can be justified by v. This is done by considering all possible completions of v, more precisely the multisets of active labels induced by completions of v. These multisets are obtained by including an occurrence of a particular label for each occurrence of that label in a link which is active in the completion. If the acceptance function of s yields $\mathbf{t}$ under all completions (more precisely, for all multisets induced by any completion), then v' assigns $\mathbf{t}$ to s. If the acceptance function of s yields $\mathbf{f}$ under all completions, then v' assigns $\mathbf{f}$ to s. In all other cases the value remains undefined.

Here are the formal details. Note that we represent here three-valued interpretations v as sets of literals: nodes true in v appear positively in the set, nodes assigned false appear negated, and undefined nodes are left out.

Definition 32. *Let $G = (S, E, L, \lambda, \alpha)$ be a LAG, v a three-valued interpretation of S. m_s^v, the multiset of active labels of $s \in S$ in G under v, is defined as*

$$m_s^v(l) = |\{(e, s) \in E \mid e \in v, \lambda((e, s)) = l\}|$$

for each $l \in L$.

The characteristic operator Γ_G of G takes a three-valued interpretation v of S and produces a revised three-valued interpretation $\Gamma_G(v)$ of S.

Definition 33. *Let $G = (S, E, L, \lambda, \alpha)$ be a LAG, v a three-valued interpretation of S. $\Gamma_G(v) = P_G(v) \cup N_G(v)$ with*

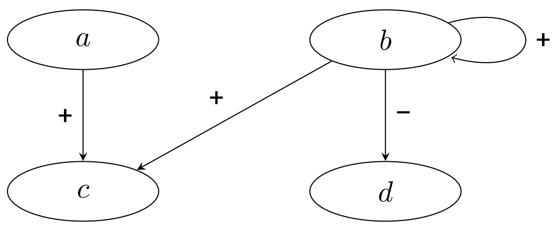
$$P_G(v) = \left\{ s \;\middle|\; \alpha(s)(m) = \mathbf{t} \text{ for each } m \in \{m_s^{v'} \mid v' \in [v]_c\} \right\}$$
$$N_G(v) = \left\{ \neg s \;\middle|\; \alpha(s)(m) = \mathbf{f} \text{ for each } m \in \{m_s^{v'} \mid v' \in [v]_c\} \right\}$$

With this new operator we can define the semantics of GRAPPA in exactly the same way as was done for ADFs:

Definition 34. *Let $G = (S, E, L, \lambda, \alpha)$ be a LAG, v a three-valued interpretation of S.*

- *v is a model of G iff v is total and $v = \Gamma_G(v)$,*
- *v is grounded in G iff v is the least fixed point of Γ_G,*
- *v is admissible in G iff $v \subseteq \Gamma_G(v)$,*
- *v is preferred in G iff v is $\subseteq$-maximal admissible in G,*
- *v is complete in G iff $v = \Gamma_G(v)$.*

Example 35. *This is a variation of Example 10. Consider the LAG with $S = \{a, b, c, d\}$ and $L = \{\mathsf{+}, \mathsf{-}\}$. The following graph shows the labels of each link.*



For simplicity, let us assume all nodes have the same acceptance condition requiring that all positive links must be active (that is the respective parents must be t) and no negative link is active.[10] *We obtain two models, namely $v_1 = \{a, b, c, \neg d\}$ and $v_2 = \{a, \neg b, \neg c, d\}$. The grounded interpretation is $v_3 = \{a\}$. The 16 admissible interpretations are exactly the same as for Example 17. Among the admissible interpretations $\{a, b, c, \neg d\}$ and $\{a, \neg b, \neg c, d\}$ are preferred. Complete interpretations are these two and in addition $\{a\}$.*

Now let us turn to stable semantics. The idea underlying stable semantics is to exclude self-justifying cycles. Again this semantics can be defined along the lines of the corresponding definition for ADFs in [Brewka *et al.*, 2013]: take a model v, reduce the LAG based on v and check whether the grounded extension of the reduced LAG coincides with the nodes true in v. Here is the definition:

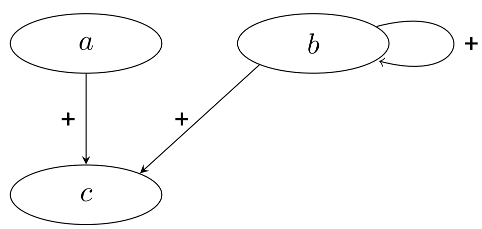
Definition 36. *Let $G = (S, E, L, \lambda, \alpha)$ be a LAG, v a model of G, $S^v = v \cap S$. v is a stable model of G iff v restricted to S^v is the grounded interpretation of $G^v = (S^v, E^v, L, \lambda^v, \alpha^v)$, the v-reduct of G, where*

[10] In the pattern language developed later in this section this can be expressed as $(\#_t(\mathsf{+}) - \#(\mathsf{+}) = 0) \wedge (\#(\mathsf{-}) = 0)$.

- $E^v = E \cap (S^v \times S^v)$,
- λ^v *is* λ *restricted to* S^v,[11]
- α^v *is* α *restricted to* S^v.

Observe that in α^v we did not have to alter the values of the function, i.e. the true and false multisets remain the same (although some of them might become "unused" since the number of parents shrinked). We will see later that this exactly matches the stable semantics for ADFs from [Brewka *et al.*, 2013]. For the moment, we continue our running example.

Example 37. *For Example 35 we obtained two models,* $v_1 = \{a, b, c, \neg d\}$ *and* $v_2 = \{a, \neg b, \neg c, d\}$. *In* v_1 *the justification for* b *is obviously based on a cycle. The* v_1*-reduct of our graph is*



It is easy to see that the grounded interpretation of the reduced graph is $\{a\}$, v_1 *is thus not a stable model, as intended. We leave it to the reader to verify that* v_2 *indeed is a stable model.*

Results about the semantics carry over from ADFs [Brewka *et al.*, 2013].

Proposition 38. *Let* G *be a LAG. The following inclusions hold:*

$$stb(G) \subseteq mod(G) \subseteq pref(G) \subseteq com(G) \subseteq adm(G),$$

where $stb(G)$,$mod(G)$, $pref(G)$, $com(G)$ *and* $adm(G)$ *denote the sets of stable models, models, preferred interpretations, complete interpretations and admissible interpretations of* G*, respectively. Moreover,* $pref(G) \neq \emptyset$*, whereas* $mod(G') = \emptyset$ *for some LAG* G'.

[11]Given a function $f : M \to N$ and $M' \subseteq M$, f restricted to M' is the function $f' : M' \to N$ such that $f'(m) = f(m)$ for all $m \in M'$.

A remaining question is how to actually specify acceptance functions for grappa. In [Brewka and Woltran, 2014] a specific pattern language has been developed for this purpose. This pattern language allows for the specification of conditions on multisets of labels. In the patterns one can refer to the number of total and active labels of specific types, to minimal/maximal numerical labels of active links. It is also possible to use simple arithmetics and relations.

More precisely, grappa acceptance functions are specified using *acceptance patterns* over a set of labels L defined as follows:

- A *term* over L is of the form $\#(l)$, $\#_t(l)$ (with $l \in L$), or min, min_t, max, max_t, sum, sum_t, $count$, $count_t$.
- A *basic acceptance pattern* (over L) is of the form $a_1 t_1 + \cdots + a_n t_n \, R \, a$, where the t_i are terms over L, the a_is and a are integers and $R \in \{<, \leq, =, \neq, \geq, >\}$.
- An *acceptance pattern* (over L) is a basic acceptance pattern or a Boolean combination of acceptance patterns.

A grappa instance then is a labelled argument graph with acceptance functions represented as acceptance patterns:

Definition 39. *A* grappa instance *is a tuple $G = (S, E, L, \lambda, \pi)$ where S is a set of statements, E a set of edges, L a set of labels, λ an assignment of labels to edges, and π an assignment of acceptance patterns over L to all elements of S.*

We still need to specify what the acceptance function represented by a particular pattern assigned to a node s is. Recall that an acceptance function assigns a truth value in $\{\mathbf{t}, \mathbf{f}\}$ to a multiset of labels. We will define this function by specifying a satisfaction relation $\models$ between multisets and patterns: the basic idea is that a multiset receives value $\mathbf{t}$ iff it satisfies the corresponding pattern. The actual definition is slightly more complicated, though, as some of the terms (actually those indexed with t) are actually independent of the multiset, but depend on the node s, more precisely on the labels of links – active or not – with target s. For this reason, satisfaction of a pattern depends on both a multiset of labels and the node the pattern is assigned to via π. For a multiset of labels $m : L \to \mathbb{N}$ and $s \in S$ the value function val_s^m is:

$$
\begin{array}{ll}
val_s^m(\#l) & = m(l) \\
val_s^m(\#_t l) & = |\{(e,s) \in E \mid \lambda((e,s)) = l\}| \\
val_s^m(min) & = \mathbf{min}\{l \in L \mid m(l) > 0\} \\
val_s^m(min_t) & = \mathbf{min}\{\lambda((e,s)) \mid (e,s) \in E\}
\end{array}
$$

$$\begin{array}{ll}
val_s^m(max) & = \mathbf{max}\{l \in L \mid m(l) > 0\} \\
val_s^m(max_t) & = \mathbf{max}\{\lambda((e,s)) \mid (e,s) \in E\} \\
val_s^m(sum) & = \sum_{l \in L} m(l) \\
val_s^m(sum_t) & = \sum_{(e,s) \in E} \lambda((e,s)) \\
val_s^m(count) & = |\{l \mid m(l) > 0\}| \\
val_s^m(count_t) & = |\{\lambda((e,s)) \mid (e,s) \in E\}|
\end{array}$$

$min_{(t)}$, $max_{(t)}$, $sum_{(t)}$ are undefined in case of non-numerical labels. For $\emptyset$ they yield the neutral element of the corresponding operation, i.e.

$$\begin{aligned}
val_s^m(sum) = val_s^m(sum_t) = 0, \\
val_s^m(min) = val_s^m(min_t) = \infty, \\
val_s^m(max) = val_s^m(max_t) = -\infty.
\end{aligned}$$

Let m and s be as before. For basic acceptance patterns the *satisfaction relation* $\models$ is defined by

$$(m,s) \models a_1 t_1 + \cdots + a_n t_n R\, a \quad \text{iff} \quad \sum_{i=1}^{n} \left(a_i\, val_s^m(t_i)\right) R\, a.$$

The extension to Boolean combinations is as usual. The acceptance function represented by pattern p at node s then is the function assigning $\mathbf{t}$ to multiset m iff $(m,s) \models p$.

Example 40. *Let $L = \{\texttt{++}, \texttt{+}, \texttt{-}, \texttt{--}\}$ be a set of labels representing strong support, support, attack and strong attack, respectively. Assume a node s is accepted if its (active) support is stronger than its attack, where we measure strength by counting the respective links, hereby multiplying strong support/attack with a factor of 2. This can be specified using the following pattern for s:*

$$2(\#\texttt{++}) + (\#\texttt{+}) - 2(\#\texttt{--}) - (\#\texttt{-}) > 0.$$

We conclude this section by showing how the necessary patterns for Carneades argument graphs, which we discussed in Section 4.3, can be defined in GRAPPA. Recall that these graphs have two kinds of nodes, argument nodes and propositions nodes. The pattern for all argument nodes is

$$((\#_t\texttt{+}) - (\#\texttt{+}) = 0) \wedge ((\#\texttt{-}) = 0).$$

which says that all premises and none of the exceptions must be accepted. The patterns for proposition nodes depend on their proof standard. Recall that some of these standards have additional numerical parameters α, β and γ. The terms max and min represent the maximal, respectively minimal, label of an active link:

- scintilla of evidence: $\max > 0$
- preponderance of evidence: $\max + \min > 0$
- clear and convincing evidence: $(\max > \alpha) \wedge (\max + \min > \beta)$
- beyond reasonable doubt: $(\max > \alpha) \wedge (\max + \min > \beta) \wedge (-\min < \gamma)$
- dialectical validity: $(\max > 0) \wedge (\min > 0)$

This representation of the acceptance conditions underlying Carneades is not only extremely simple. It has the big advantage that it is uniform: the patterns for all nodes with the same proof standard are actually the same. This is different from representations of proof standards and other notions we discussed in Section 4 in ADFs where the acceptance condition for each node depends on its specific parents.

6 Computational Aspects

In the introduction we discussed, in an informal manner, relationships between statements (arguments) that are supporting or attacking, in the sense that a statement can have a positive or negative influence on the acceptance of another statement. General ADFs have a generic notion of links (dependencies) between statements. However, such links can be formally categorized into 4 groups, depending on whether they have an attacking or supporting nature (or both or neither). This leads to the notion of so-called bipolar ADFs (BADFs for short) which contain only attacking or supporting dependencies. We will introduce them, based on the original definition of [Brewka and Woltran, 2010], in Section 6.1, together with the formalization of attacking and supporting links. Such BADFs are a subclass of general ADFs, yet have appealing computational properties. They generalize AFs in a direct manner, but are strictly "in-between" AFs and general ADFs w.r.t. their corresponding expressiveness. Results relating to expressiveness are presented in Section 6.2. Further, many frameworks arising in argumentation in AI, other than AFs, can be translated to BADFs [Polberg, 2016] (partially under semantics not discussed in this article).

From a computational perspective, BADFs have the following interesting properties: they have the same worst-time complexity as AFs for many semantics, while general ADFs typically exhibit higher computational complexity. We summarize these results in Section 6.3, followed by Section 6.4 that gives pointers to recent systems for computing reasoning tasks on ADFs and BADFs.

6.1 Bipolar ADFs

As we have seen in previous sections, the concept of acceptance condition is quite powerful. A natural question is to what extent different restrictions of acceptance conditions may form interesting subclasses of ADFs. One such subclass are bipolar ADFs, as already defined in [Brewka and Woltran, 2010]. This class relies on the concept of attacking and supporting links which are defined as follows.

Let $D = (S, L, C)$ be an ADF. Formally, a link $(r, s) \in L$ is

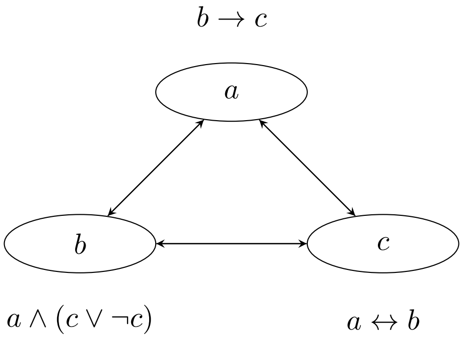
- supporting in D iff for all $R \subseteq par(s)$, we have $C_s(R) = \mathbf{t}$ implies $C_s(R \cup \{r\}) = \mathbf{t}$;
- attacking in D iff for all $R \subseteq par(s)$, we have $C_s(R \cup \{r\}) = \mathbf{t}$ implies $C_s(R) = \mathbf{t}$.

We use $L^+ \subseteq L$ to denote all supporting and $L^- \subseteq L$ to denote all attacking links of L in an ADF $D = (S, L, C)$.

Example 41. *In Figure 10 we see an example* ADF $D = (S, L, C)$ *with* $S = \{a, b, c\}$ *and acceptance conditions* $\varphi_a = b \to c$, $\varphi_b = a \land (c \lor \neg c)$, *and* $\varphi_c = a \leftrightarrow b$. *On the right of that figure the link types are shown. Let us investigate why some of the links are supporting or attacking. Looking at the acceptance condition of* a, φ_a, *and the parents of* a *then we have the following relevant sets of statements (shown as two-valued interpretations):*

$$\begin{array}{rrcl} v_1 \hat{=} & \{\neg b, \neg c\} & \models & \varphi_a \\ v_2 \hat{=} & \{b, \neg c\} & \not\models & \varphi_a \\ v_3 \hat{=} & \{\neg b, c\} & \models & \varphi_a \\ v_4 \hat{=} & \{b, c\} & \models & \varphi_a \end{array}$$

We see, e.g., that the link (c, a) *is supporting, because whenever* c *is added to a subset of parents that is mapped to* $\mathbf{t}$ *by* C_a *(switched to true in every model of* φ_a*) then the new set (interpretation) is again mapped to true by acceptance condition* C_a *(is a model of* φ_a*). More concretely,* v_1, v_3, *and* v_4 *are models of acceptance condition* φ_a. *Switching the truth value of* c *to true in each of them, results in* v_3 *and* v_4 *(assigning* c *to true in* v_1 *and* v_3 *results in both cases with* v_3, *and assigning* c *to true in* v_4 *is again equal to* v_4*). Both* v_3 *and* v_4 *are models of* φ_a. *This means* (c, a) *is a supporting link. Similarly, link* (b, a) *is attacking because whenever we*



L^+	L^-
(a, b)	(b, a)
(c, b)	(c, b)
(c, a)	

Figure 10: An ADF *with link types.*

remove b from a set of parents of a that is mapped to $\mathbf{t}$ *by C_a we get a set that is likewise mapped to* $\mathbf{t}$ *by C_a.*

Links (a, b) which are *both* attacking and supporting are so-called redundant links. The reason to call such a link redundant is that switching the truth value of a in any interpretation does not change the evaluation of acceptance condition φ_b w.r.t. the original interpretation and the modified interpretation. A link that is *neither* attacking nor supporting is called dependent.

Example 42. *Continuing Example 41, the link (c, b) is a redundant link. This link is both attacking and supporting. Redundancy means that the evaluation of φ_b is independent of the value of c (formula φ_b only depends on the truth value of a). In contrast, the links (b, c) and (a, c) are dependent links. For instance, $\{\neg a, \neg b\} \models \varphi_c$ and $\{a, \neg b\} \not\models \varphi_c$ taken together show that (a, c) is not supporting in this* ADF*. To see that (a, c) is not attacking, consider $\{a, b\} \models \varphi_c$ and $\{\neg a, b\} \not\models \varphi_c$.*

An ADF $D = (S, L, C)$ is bipolar (a BADF) if all links in L are supporting or attacking or both, i.e., $L = L^+ \cup L^-$. For example, our running example ADF from Example 10 is a BADF. Further, for any AF F its associated ADF D_F is bipolar, in fact each link in D_F is attacking.

Bipolar ADFs are still a quite expressible class; they allow acceptance conditions not only to express simply attack and support (for example $\neg a_1 \wedge \cdots \wedge \neg a_n \wedge s_1 \wedge \cdots \wedge s_m$ expressing that a statement is attacked by statements a_i and supported by statements s_j), but more advanced relations, like e.g. $((\neg a_1 \vee s_1) \wedge (\neg a_2 \vee s_2)) \vee \neg a_3$; in fact, all examples given in Section 4 are also bipolar ADFs. We would like to mention here that bipolar ADFs behave differently than the prominent class of bipolar AFs

[Cayrol and Lagasquie-Schiex, 2013]. Indeed, several concepts of support relations have been discussed in the literature (abstract, deductive, necessary, and evidential support), thus a detailed discussion is beyond the scope of this article, and we refer the reader to works relating ADFs to formalisms including support [Polberg and Oren, 2014; Polberg, 2016]. However, what is important to state is that bipolar ADFs treat support and attack as equally strong concepts. Given the generality of bipolar ADFs which allow to "mix" support and attack as exemplified above, a distinct handling of support and attack in ADFs, e.g. as separated concepts in the language instead of a property of links and acceptance conditions, would require a lot of additional machinery.

Acceptance conditions in BADFs are, in fact, not only interesting for defining ADFs. The study of the concept of *bipolar Boolean functions* has meanwhile found applications outside of ADFs. Baumann and Strass (2016) have analyzed the integer sequence that arises when considering for each positive integer n the number of bipolar Boolean functions in n arguments. The resulting sequence is novel and has been added to the Online Encyclopedia of Number Sequences[12]. In further related work, Alviano, Faber, and Strass [Alviano *et al.*, 2016] applied the concept of bipolar Boolean functions to aggregates in answer set programming and obtained a novel class of aggregates whose model checking problems (according to the semantics of Pelov *et al.* [Pelov *et al.*, 2007] and Son and Pontelli [Son and Pontelli, 2007]) can be decided in deterministic polynomial time. They even identify a class that goes beyond bipolar Boolean functions but still retains polynomial-time decidability; this might constitute an interesting avenue for research that extends the bipolarity concept of ADFs.

6.2 Expressiveness and Realizability

Expressiveness of a formalism $\mathcal{F}$ (i.e. the set of structures available in a formalism) with a semantics σ over a vocabulary A can be defined as the set of interpretation-sets over A that elements of $\mathcal{F}$ (the knowledge bases $\mathsf{kb} \in \mathcal{F}$ of that formalism) can produce. Formally, the *signature* of a formalism $\mathcal{F}$ w.r.t. semantics σ is the set

$$\Sigma^{\sigma}_{\mathcal{F}} = \{\sigma(\mathsf{kb}) \mid \mathsf{kb} \in \mathcal{F}\}$$

Intuitively, expressiveness is a basic measure of the capabilities of formalism $\mathcal{F}$ under σ, because it characterizes what "can and cannot be done" with $\mathcal{F}$ under semantics σ [Gogic *et al.*, 1995]. Whenever we have two formalisms, say $\mathcal{F}_1$ and $\mathcal{F}_2$, that share a semantics σ and we find that $\Sigma^{\sigma}_{\mathcal{F}_1} \subsetneq \Sigma^{\sigma}_{\mathcal{F}_2}$, then this intuitively means that $\mathcal{F}_2$ is

[12] `https://oeis.org/A245079`

strictly more expressive than $\mathcal{F}_1$: all sets $V \subseteq \mathcal{V}_3$ that can be realized with $\mathcal{F}_1$ can be realized with $\mathcal{F}_2$, and there is at least one set $V \subseteq \mathcal{V}_3$ that can be realized with $\mathcal{F}_2$ but not with $\mathcal{F}_1$.

For AFs, BADFs and ADFs under various semantics, their relative expressiveness is summarized in the following result [Strass, 2015c; Strass, 2015a; Linsbichler *et al.*, 2016b].

Theorem 6.1. *For $\sigma \in \{adm, com, prf, mod\}$, we find that*

$$\Sigma^{\sigma}_{AF} \subsetneq \Sigma^{\sigma}_{BADF} \subsetneq \Sigma^{\sigma}_{ADF}.$$

For the stable model semantics stb, we find that

$$\Sigma^{mod}_{AF} = \Sigma^{stb}_{AF} \subsetneq \Sigma^{stb}_{BADF} = \Sigma^{stb}_{ADF}.$$

Furthermore, for the model semantics we have

$$\Sigma^{mod}_{ADF} = \mathcal{V}_2 = \{v : A \to \{\mathbf{t}, \mathbf{f}\}\},$$

that is, ADFs under the model semantics are universally expressive.

Example 43. *We give example sets of interpretations that can be used to witness $\Sigma^{prf}_{AF} \subsetneq \Sigma^{prf}_{BADF} \subsetneq \Sigma^{prf}_{ADF}$. Consider $S = \{a, b, c\}$ and interpretations $v_1 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}\}$, $v_2 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}, c \mapsto \mathbf{t}\}$, and $v_3 = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}\}$. To see that $\{v_1, v_2, v_3\} \in \Sigma^{prf}_{BADF}$, consider the ADF over S with acceptance conditions $\varphi_a = \neg b \vee \neg c$, $\varphi_b = \neg a \vee \neg c$, and $\varphi_c = \neg a \vee \neg b$. It is easy to verify that this ADF is bipolar and that $\{v_1, v_2, v_3\}$ constitute its preferred interpretations. On the other hand, from results in [Dunne* et al.*, 2015] it follows that there is no AF with preferred extensions $\{a, b\}$, $\{a, c\}$, and $\{b, c\}$. In fact, this is quite easy to see: consider there would exist an AF F with those three preferred extensions. Then, there cannot be an attack in F between a and b, and moreover $\{a, b\}$ defends itself in F; the same holds for the pairs a, c, and b, c. But then, $\{a, b, c\}$ has to be conflict-free in F and defends itself, and thus $\{a, b\}$ (and likewise, $\{a, c\}$ and $\{b, c\}$) cannot be preferred in F; a contradiction.*

For $\Sigma^{prf}_{BADF} \subsetneq \Sigma^{prf}_{ADF}$, we use an example given in [Linsbichler et al.*, 2016a, Theorem 8]: consider $S' = \{a, b\}$ and interpretations $v_4 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{t}\}$, $v_5 = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}\}$, and $v_6 = \{a \mapsto \mathbf{f}, b \mapsto \mathbf{u}\}$. For $X' = \{v_4, v_5, v_6\}$ we have $X' \in \Sigma^{prf}_{ADF}$, but $X' \notin \Sigma^{prf}_{BADF}$. For general ADFs, one example ADF is $D' = (S', L', \{\varphi_a = a, \varphi_b = a \leftrightarrow b\})$. All three interpretations v_4, v_5, and v_6 are preferred interpretations of D'. This ADF D' is not bipolar (due to φ_b, see Example 42). There is no BADF that has X' exactly as its preferred interpretations.*[13]

[13] For an automated way to check whether for a given set of three-valued interpretations there is an ADF, BADF, or AF that has exactly this set as its σ-interpretations, one can use the system UNREAL [Linsbichler *et al.*, 2016a], available at `http://www.dbai.tuwien.ac.at/proj/adf/unreal/`.

While this shows that BADFs can do strictly more than AFs, and in turn ADFs can do strictly more than BADFs (with the exception of the stable model semantics), there is little information on *what exactly* these signatures look like. Work on precisely characterizing signatures has been carried out for AFs (see the paper *On the Nature of Argumentation Semantics* by Baumann in this issue. There has also been work on characterizing realizability for ADFs under two-valued [Strass, 2015a] and three-valued [Pührer, 2015; Linsbichler *et al.*, 2016b] semantics.

Finally, initial results on characterizing the representational *succinctness* of these formalisms have recently been obtained. Succinctness not only takes into account *what* formalisms can realize, but also *to what representational cost*, that is, what amount of space is needed to represent the smallest knowledge base realizing some desired set of interpretations. Again, the capabilities of different formalisms can be compared with respect to this measure [Gogic *et al.*, 1995]. As one promising result on ADFs, it turned out that even BADFs are exponentially more succinct than normal logic programs [Strass, 2015a].

6.3 Computational Complexity

The computational complexity of ADFs is well-studied [Strass and Wallner, 2014; Strass and Wallner, 2015; Gaggl *et al.*, 2015; Brewka *et al.*, 2013; Polberg and Wallner, 2017; Wallner, 2014]; for an overview we refer the reader to the paper *Computational Problems of Formal Argumentation* by Dvořák and Dunne in this issue. For the reader's convenience we repeat here the main results. For a specified semantics σ, the main reasoning tasks for ADFs to solve are:

- Credulous acceptance of a statement: is statement s assigned to true in at least one interpretation under semantics σ?
- Skeptical acceptance of a statement: is statement s assigned to true in all interpretations under semantics σ?
- Interpretation verification: is a given interpretation an interpretation under semantics σ?
- Interpretation existence: is there an interpretation under semantics σ?
- Non-trivial interpretation existence: is there an interpretation under semantics σ assigning true or false to some statement?

Briefly put, complexity of reasoning tasks on general ADFs is situated one level higher in the polynomial hierarchy compared to the corresponding tasks on AFs. For

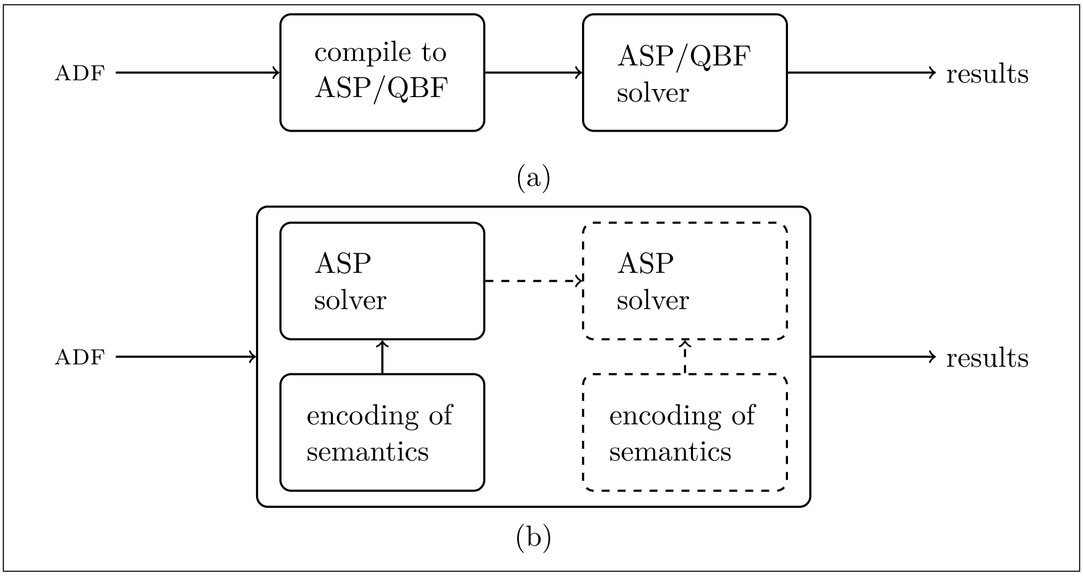


*Figure 11: Workflow for systems based on (a) instance-based compilation (*QADF*,* GrappaVis*, and* YADF*), and (b) static encodings (*diamond *and* GrappaVis*)*

badfs complexity of reasoning stays at the same level as reasoning on afs for most reasoning tasks, if the link type (attack or support) for each link is known (part of the input). Thus, badfs offer more modeling capabilities than afs while having the same (worst-case) computational cost as afs for many reasoning tasks.

6.4 Systems

Systems for implementing reasoning on adfs rely on declarative encodings in answer-set programming (ASP) [Brewka *et al.*, 2011] or quantified Boolean satisfiability, and utilize available solvers for these languages [Gebser *et al.*, 2011; Lonsing and Biere, 2010]. Most prominently, the diamond family[14] [Strass and Ellmauthaler, 2017; Ellmauthaler and Strass, 2016; Ellmauthaler and Strass, 2014; Ellmauthaler and Strass, 2013] consists of ASP-based systems for reasoning on adfs. In each diamond version an adf is encoded via ASP facts and, when augmented with static encodings for semantics, several reasoning tasks can be solved by computing answer-sets of the resulting ASP. Depending on the complexity of the reasoning task and used options in diamond one call (in some family members two calls) to an ASP-solver are carried out to solve the given problem instance. diamond includes dedicated badf-specific

[14] `http://diamond-adf.sourceforge.net/`

encodings that make use of BADFs' upper complexity bounds.

The system QADF[15] [Diller *et al.*, 2015] uses solvers for quantified Boolean formulas (QBFs) to perform reasoning on ADFs. In QADF, in contrast to DIAMOND, each ADF instance is compiled to a QBF incorporating both the input ADF and the chosen semantics, i.e., the encodings for the semantics are not static.

GrappaVis[16] [Heißenberger, 2016] is a system implementing GRAPPA (see Section 5) and incorporates both instance-based compilation of GRAPPA input into declarative ASP encodings and static encodings for the semantics utilizing in both cases one ASP solver call.

The system YADF[17] [Brewka *et al.*, 2017] is an ASP-based system for ADFs, based on the encodings for GRAPPA used in GrappaVis. This system compiles ADF instances into one program to call an ASP solver (once).

The basic workflows for DIAMOND, QADF, GrappaVis, and YADF are shown in Figure 11. With this figure we illustrate that QADF, GrappaVis, and YADF implement algorithms that take an instance of an ADF, compile this instance, together with the chosen semantics and reasoning task, to one instance of an ASP or a QBF. On the other hand, DIAMOND and GrappaVis implement algorithms that take an instance of an ADF, add to this instance a static encoding for the semantics and reasoning task, and give these to an ASP solver (with calling such a solver once or twice, depending on the task). The difference between (a) and (b) is that in (a) ADF and semantics have to be compiled together into one input for the solver, while for (b) semantics can be encoded separately (and modified separately).

A technique to cope with the high computational complexity of reasoning on ADFs was proposed by Linsbichler (2014). The technique is based on splitting the input ADF into partitions and solving one partition and transforming and solving the other partitions accordingly.

7 Conclusion

In this article, we have reviewed the argumentation formalism of abstract dialectical frameworks (ADFs). In contrast to Dung style frameworks, ADFs allow for a much more general specification of the interrelationship between the arguments. We have discussed how standard semantics like admissible, grounded, complete, preferred and stable can be generalized to ADFs by making use of the well known approximation fixpoint theory due to Denecker, Marek and Truszczyski [Denecker *et al.*, 2004].

[15] http://www.dbai.tuwien.ac.at/proj/adf/qadf/
[16] http://www.dbai.tuwien.ac.at/proj/adf/grappavis/
[17] http://www.dbai.tuwien.ac.at/proj/adf/yadf/

Alternative approaches to defining ADF semantics can be found in the works of Polberg and colleagues [Polberg *et al.*, 2013; Polberg, 2014a; Polberg, 2014b; Polberg, 2015]. Likewise, further well-known semantics for AFs have been generalized to ADFs, e.g. naive, stage, and the cf2 family of semantics [Gaggl and Strass, 2014] and an alternative, symmetric version of the naive semantics [Strass and Wallner, 2015].

A further subclass of ADFs, related to a certain notion of acyclicity and different from BADFs, is investigated in [Polberg, 2015; Polberg, 2016]. Other authors have analyzed the relationship of ADFs and logic programs [Strass, 2013a; Alviano and Faber, 2015] and in the course of that have defined new ADF semantics, like approximate stable models [Strass, 2013a], F-stable models [Alviano and Faber, 2015], and the grounded fixpoint semantics [Bogaerts *et al.*, 2015]. The whole ADF formalism has even been lifted to the probabilistic case [Polberg and Doder, 2014].

We also addressed the modelling capabilities of ADFs; for a thorough discussion on the relation between ADFs and other argumentations frameworks, see also [Polberg, 2017]. A further application of ADFs in the context of legal reasoning can be found in [Al-Abdulkarim *et al.*, 2014; Al-Abdulkarim *et al.*, 2016]. The use of ADFs in text exploration has been investigated in [Cabrio and Villata, 2016]. Finally, we discussed the GRAPPA approach which makes use of ADF-like semantics in a flexible graph-based formalism. GRAPPA is the formal system underlying a mobile argumentation app developed by Pührer [2017].

Acknowledgements

We thank the anonymous reviewers for various comments which helped to significantly improve the article. This research has been supported by the German Research Foundation (DFG) (project BR 1817/7-2), the Austrian Science Fund (FWF) (projects I2854, P25521, and P30168-N31), and by Academy of Finland under grants 251170 COIN and 284591.

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 Received 6 May 2016

Abstract Rule-Based Argumentation

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Abstract

This article reviews abstract rule-based approaches to argumentation, in particular the *ASPIC*$^+$ framework. In *ASPIC*$^+$ and its predecessors, going back to the seminal work of John Pollock, arguments can be formed by combining strict and defeasible inference rules and conflicts between arguments can be resolved in terms of a preference relation on arguments. This results in abstract argumentation frameworks (a set of arguments with a binary relation of defeat), so that arguments can be evaluated with the theory of abstract argumentation. First the basic *ASPIC*$^+$ framework is reviewed, possible ways to instantiate it are discussed and how these instantiations can satisfy closure and consistency properties. Then the relation between *ASPIC*$^+$ and other work in formal argumentation and nonmonotonic logic is discussed, including a review of how other approaches can be reconstructed as instantiations of *ASPIC*$^+$. Further developments and variants of the basic *ASPIC*$^+$ framework are also reviewed, including developments with alternative or generalised notions of attack and defeat and variants with further constraints on arguments. Finally, implementations and applications of *ASPIC*$^+$ are briefly reviewed and some open problems and avenues for further research are discussed.

1 Introduction

One of the oldest research strands in the logical study of argumentation is to allow for arguments that combine strict and defeasible inference rules. Strict inference rules are intended to capture deductively valid inferences, where the truth of the premises guarantees the truth of the conclusion. Defeasible inference rules are instead meant to capture presumptive inferences, where the premises only create a presumption in favour of the conclusion, which can be refuted by evidence to the contrary. This approach was introduced in AI by [Pollock, 1987; Pollock, 1990; Pollock, 1992; Pollock, 1994; Pollock, 1995], previously studied by e.g. [Lin and Shoham, 1989; Simari and Loui, 1992; Vreeswijk, 1997; Prakken and Sartor, 1997] and [Garcia and Simari, 2004] and currently studied by e.g. [Dung and Thang, 2014; Dung, 2014; Dung, 2016] and in work on the $ASPIC^+$ framework [Prakken, 2010; Modgil and Prakken, 2013; Modgil and Prakken, 2014; Caminada *et al.*, 2014; Li and Parsons, 2015; Grooters and Prakken, 2016].

While Dung's seminal theory of abstract argumentation frameworks [Dung, 1995] has proved to be extremely influential, it adopts a level of abstraction that precludes provision of guidelines for choosing how to define arguments and attacks from knowledge bases, and a study of how these choices should be made to ensure rational outcomes yielded by evaluation of the justified arguments under Dung's semantics. The above-mentioned work, which partly originates from before Dung's article, addresses these issues. This article presents the current consolidation of this research strand: the $ASPIC^+$ framework for structured argumentation. The *ASPIC* framework was initially developed as an output of a European Union project on argumentation [Amgoud *et al.*, 2006] and further developed into the $ASPIC^+$ framework, initially in [Prakken, 2010], and subsequently in [Modgil and Prakken, 2013]. The principal aims of $ASPIC^+$ were to: 1) generalise *ASPIC* so as to provide a natural knowledge representation framework in which to formalise a wide variety of existing and novel instantiations of abstract argumentation frameworks, while; 2) providing guidelines for instantiations that use features typically incorporated at the abstract level of these frameworks; in particular the use of preferences, which were introduced at the abstract level to determine the success of attacks as defeats [Amgoud and Cayrol, 2002], but may violate rationality postulates unless one carefully accounts for their use when instantiating abstract argumentation frameworks.

Importantly, the strict and defeasible inference rules in $ASPIC^+$are not part of the logical object language (in which the premises and conclusions of arguments are expressed), but are *metalevel* rules for encoding inference over well-formed formulas in some object level language. Also, the $ASPIC^+$ framework abstracts from the nature and origin of the inference rules and from the nature of the language over which they are defined. The resulting abstract nature[1] of $ASPIC^+$means that it provides a framework enabling the study of various logical instantiations of abstract argumentation frameworks, and conditions under which the extensions of these frameworks (and hence the defined inference relation over the instantiating knowledge base of

[1]The aforementioned features of $ASPIC^+$are shared by earlier work in this tradition, such as the work of Pollock and [Vreeswijk, 1997], and justifies the title of this article.

logical formulae, identified by the conclusions of justified arguments in extensions) satisfy the rationality postulates in [Caminada and Amgoud, 2007] (for example that the conclusions of arguments in an extension are mutually consistent). In fact, Assumption-Based Argumentation (ABA) ([Bondarenko *et al.*, 1997]), which only has strict rules, can also be regarded as abstract rule-based argumentation, since ABA also abstracts from the nature and origin of its inference rules. However, we will (except for some brief comparisons) not discuss ABA in this article, as it is reviewed elsewhere, e.g. in [Toni, 2014]. The same holds for a particular instantiation of the rule-based approach: Defeasible Logic Programming, which is reviewed in e.g. [Garcia and Simari, 2014].

In a rule-based approach, arguments are formed by chaining applications of inference rules into inference trees or graphs. This approach can be contrasted with approaches defined in terms of logical consequence notions, in which arguments are premises-conclusion pairs where the premises are consistent and imply the conclusion according to the consequence notion of some adopted 'base logic'. Examples of this approach are classical-logic argumentation [Cayrol, 1995; Besnard and Hunter, 2001; Besnard and Hunter, 2008; Gorogiannis and Hunter, 2011] and its generalisation into abstract Tarskian-logic argumentation [Amgoud and Besnard, 2013]. It is important to note that, unlike these logic-based approaches, rule-based approaches in general do not adopt a single base logic but two base logics, one for the strict and one for the defeasible rules. This issue will be discussed in detail in Section 3.1 of this article. Moreover, we will review how 'base logic' approaches [Hunter, 2010] can be formalised as instances of $ASPIC^+$ in which the logical language is a full propositional or first-order language and the only inference rules defined over this language are strict, and corresponds to the inference rules of the base logic.

This article is organised as follows. In Section 2 we incrementally introduce features of the $ASPIC^+$ framework. We first introduce the basic framework in which arguments are built from strict and or defeasible inference rules, and are grounded in fallible or infallible premises. Various notions of attacks as well as the use of preferences to determine defeats are defined. The basic framework can thus capture rule-based approaches to argumentation of the type dating back to John Pollock's work in formal epistemology, and formalisms for encoding the well-known schemes and critical questions approach to argumentation developed by the informal logic community (notably [Walton, 1996]), and widely used to accommodate more human orientated rather than formal logic based instantiations. We then define a version of $ASPIC^+$ that generalises the standard notion of negation used to identify when the claim of one argument is in conflict with an element in the attacked argument. In this way an asymmetric notion of conflict can be represented that allows for instantiations by logical languages with negation as failure, and the study of formalisms such as ABA as instances of $ASPIC^+$.

In Section 3 we provide guidance on how to choose and define the premises and strict and defeasible rules that comprise $ASPIC^+$ arguments, and the preference relations that are used to determine the success of attacks as defeats. We then specify formal guidelines as to how one should make the aforementioned choices to ensure satisfaction of the rationality postulates in [Caminada and Amgoud, 2007]. We also discuss the extent to which reasoning with defeasible rules and/or preferences can be reduced to reasoning in systems that do not distinguish between strict and defeasible rules, and/or do not use preferences. Finally, we discuss how argument schemes with critical questions can be reconstructed in $ASPIC^+$ as defeasible inference rules.

Section 4 then reviews the relation of $ASPIC^+$ with other works on argumentation and nonmonotonic logic. We show how some existing argumentation formalisms can be reconstructed in the $ASPIC^+$ framework; in particular, ABA as formulated in [Dung *et al.*, 2007], the Carneades system [Gordon *et al.*, 2007; Gordon and Walton, 2009a], and argumentation formalisms based on Tarskian abstract logics [Amgoud and Besnard, 2013] and in particular classical logic argumentation [Gorogiannis and Hunter, 2011]. We will also discuss how the inference relations of existing non-monotonic logics, in particular Preferred Subtheories [Brewka, 1989] and Prioritised Default Logic [Brewka, 1994a], can be endowed with argumentation semantics through instantiation of the $ASPIC^+$ framework. We conclude by reviewing how our structured approach to argumentation sheds light on developments of the theory of abstract argumentation frameworks, including the use of preferences and values, support relations, attacks on attacks, resolutions of attacks and the dynamics of abstract argumentation frameworks.

Further developments of the $ASPIC^+$ framework will be discussed in Section 5, in particular studies of alternative notions of attack, studies of generalised notions of attack and defeat, and studies of further consistency, minimality and chaining restrictions on arguments. Implementations and applications of $ASPIC^+$ are discussed in Section 6 and we conclude with a discussion of open problems and future research directions in Section 7.

2 $ASPIC^+$: Defining the Framework

2.1 The underlying ideas

People argue to remove doubt about a claim [Walton, 2006, p. 1], by giving reasons why one should accept the claim and by defending these reasons against criticism. The strongest way to remove doubt is to show that the claim deductively follows from indisputable grounds. A mathematical proof from the axioms of arithmetic is like this; its grounds are mathematical axioms, while its inferences are deductively sound. So such a proof cannot be attacked on its grounds or its inferences. However, in real life our grounds may not be indisputable and may provide less than conclusive support for their claim.

Suppose we believe that John was in Holland Park some morning and that Holland Park is in London. Then we can deductively reason from these beliefs, to conclude that John was in London that morning. While this reasoning cannot be attacked, the argument is still fallible since its grounds may turn out to be wrong. For instance, Jan may tell us that he met John in Amsterdam that morning around the same time, challenging our belief that John was in Holland Park that morning, since witnesses usually speak the truth. Maybe we have a supporting reason for our belief that John was in Holland Park; that we went jogging in Holland Park and saw John and that our senses are usually accurate. But given Jan's testimony, perhaps our senses betrayed us? But then we discover Jan has a reason to lie, since John is a suspect in a robbery in Holland Park that morning and Jan and John are friends. We then conclude that the basis for questioning our belief that John was in Holland Park that morning (namely, that witnesses usually speak the truth and Jan witnesses John in Amsterdam) does not apply to witnesses who have a reason to lie. So our reason in support of our belief is undefeated and we accept it.

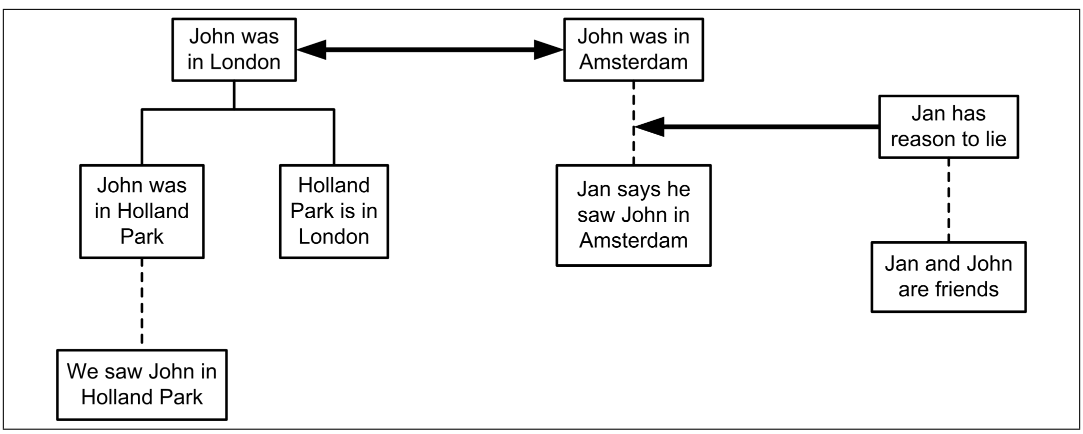

Figure 1: An informal example

This example is displayed in Figure 1, where the strict inference is visualised with solid lines, the defeasible inferences with dotted lines and the attack relations with arrow. The defeasible inferences within arguments are supposed to be licensed by the generalisations in the example.

If we want to formalise a logic for argumentation, then this simple example already suggests a number of issues to be addressed. First, the claims and beliefs in our example were supported in various ways: in the first case we appealed to the principles of deductive inference when concluding that John was in London. *ASPIC*$^+$ is therefore designed so that arguments can be constructed using deductive or *strict* inference rules that license deductive inferences from premises to conclusions. However, in the other two cases the reasoning from grounds to claim appealed to the reliability of, respectively, our senses and witnesses as sources of information. Should these kinds of support (inferences) from grounds to claims be modelled as deductive?

To help answer this question, consider that our informal example contains three ways of attacking an argument: 1) Our initial argument that John was in London was attacked by the witness argument on its ground, or *premise*, that John was in Holland Park that morning; 2) The initial argument was then extended with an additional argument for the attacked premise, but the extended argument was still attacked (by the witness argument) on the (now) intermediate conclusion that John was in Holland Park that morning; 3) Finally, we counterattacked the witness argument not on a premise or conclusion but on the reasoning from the grounds to the claim: namely, the inference step from the premise that Jan said he met John in Amsterdam that morning to the claim that John was in Amsterdam that morning (note that here we regard the principle that witnesses usually speak the truth as an inference rule).

Now, returning to the question whether all kinds of inference should be deductive, the second type of attack would not be possible on the deductively inferred intermediate conclusion since the nature of deductive support is that if all antecedents of a deductively valid inference rule are true, then its consequent must also be true. So if we have reason to believe that the conclusion of a deductive inference is not true, then there must be something wrong with its premises (which may in turn be the conclusions of subarguments). It is for this very same reason that the third type of attack on a deductive inferential step is also not possible.

ASPIC$^+$ is therefore designed to comply with the common-sense and philosophically argued position ([Pollock, 1995, p.41]; [Pollock, 2009, p. 173]) advocating the rationality of supporting claims with grounds that do not deductively entail them. In other words, the fallibility of an argument need not only be located in its premises, but can also be located in the inference steps from premises to conclusion. Thus, arguments in *ASPIC*$^+$ can be constructed using *defeasible* inference rules, and arguments can be attacked on both the conclusions, and application of, such defeasible inference rules, in keeping with the interpretation that the premises of such a rule presumptively rather than deductively support their conclusions.

As well as *fallible* premises that can be attacked, *ASPIC*$^+$ also allows to distinguish premises that are axiomatic and so cannot be attacked. We discuss the uses of such premises in Section 2.2.1, but for the moment we can summarise by saying that *ASPIC*$^+$ arguments can be constructed from fallible and infallible premises (respectively called *ordinary* and *axiom* premises in Section 2.2.1), and strict and defeasible inference rules, and that arguments can be attacked on their ordinary premises, the conclusions of defeasible inference rules, and the defeasible inference steps themselves. Finally, a key feature of the *ASPIC*$^+$ framework is that it accommodates the use of preferences over arguments, so that an attack from one argument to another only succeeds (as a defeat) if the attacked argument is not stronger than (strictly preferred to) the attacking argument, according to some given preference relation. The justified *ASPIC*$^+$ arguments are then evaluated with respect to the abstract argumentation framework relating *ASPIC*$^+$ arguments by the defeat relation. Since requirements for use of preferences in argumentation (and more generally for conflict resolution in non-monotonic logics) are well established in the literature, we will here not justify the need for preferences. However, examples are given in the remainder of the paper.

2.2 The basic framework with symmetric negation

2.2.1 Argumentation systems, knowledge bases, and arguments

ASPIC$^+$ is a general framework that allows one to choose a *logical language* $\mathcal{L}$ closed under negation $\neg$ (which we later replace with a more general notion of conflict) and two (possibly empty) sets of *strict* ($\mathcal{R}_s$) and *defeasible* ($\mathcal{R}_d$) inference rules. One also specifies well-formed formulas in $\mathcal{L}$ that correspond to (i.e., name) defeasible rules in $\mathcal{R}_d$ via a partial function n. These names can then be used when attacking arguments on defeasible inference steps. Informally, $n(r)$ is a well-formed formula (wff) in $\mathcal{L}$ which says that the defeasible rule $r \in \mathcal{R}$ is applicable, so that an argument claiming $\neg n(r)$ attacks the inference step in the corresponding rule[2].

Definition 2.1 (Argumentation systems)**.** *An* argumentation system *is a triple* $AS = (\mathcal{L}, \mathcal{R}, n)$ *where:*

- $\mathcal{L}$ *is a logical language with a unary negation symbol* $\neg$*.*
- $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ *is a set of strict* ($\mathcal{R}_s$) *and defeasible* ($\mathcal{R}_d$) *inference rules of the form* $\varphi_1, \ldots, \varphi_n \to \varphi$ *and* $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ *respectively (where* φ_i, φ *are meta-variables ranging over wff in* $\mathcal{L}$*), and* $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$*.*

[2] n is a partial function since you may want to enforce that some defeasible inference steps cannot be attacked.

- *n is a partial function such that* $n : \mathcal{R}_d \longrightarrow \mathcal{L}$.

We write $\psi = -\varphi$ *just in case* $\psi = \neg\varphi$ *or* $\varphi = \neg\psi$ *(we will sometimes informally say that formulas* φ *and* $-\varphi$ *are each other's negation).*

It is important to stress here that *ASPIC*$^+$'s strict and defeasible inference rules are *not* object-level formulae in the language $\mathcal{L}$, but are meta to the language, allowing one to deductively, respectively defeasibly, infer the rule's consequent from the rule's antecedents. Such inference rules may range over arbitrary formulae in the language, in which case they will, as usual in logic, be specified as *schemes*. For example, a scheme for strict inference rules capturing modus ponens for the material implication of classical logic can be written as $\alpha, \alpha \supset \beta \rightarrow \beta$[3], where α and β are metavariables for wff in $\mathcal{L}$. Alternatively, strict or defeasible inference rules may be domain-specific in that they reference specific formulae, as in the defeasible inference rule concluding that an individual flies if that individual is a bird: *Bird* $\Rightarrow$ *Flies*. We will further discuss these distinct uses of inference rules in Section 3.1.

ASPIC$^+$ also requires that one specify a knowledge base from which the premises of an argument can be taken, where one can distinguish between ordinary premises which are uncertain and so can be attacked, and axiom premises that are certain and so cannot be attacked.

Definition 2.2 (Knowledge bases). *A* knowledge base *in an* $AS = (\mathcal{L}, \mathcal{R}, n)$ *is a set* $\mathcal{K} \subseteq \mathcal{L}$ *consisting of two disjoint subsets* $\mathcal{K}_n$ *(the* axioms*) and* $\mathcal{K}_p$ *(the* ordinary premises*).*

An argumentation theory consists of an argumentation system and a knowledge base:

Definition 2.3 (Argumentation theory). *An* argumentation theory *is a tuple* $AT = (AS, \mathcal{K})$ *where* AS *is an argumentation system and* $\mathcal{K}$ *is a knowledge base in* AS.

ASPIC$^+$ arguments are now defined relative to an argumentation theory $AT = (AS, \mathcal{K})$, and chain applications of the inference rules from AS into inference graphs, starting with elements from the knowledge base $\mathcal{K}$. In what follows, for a given argument, the function `Prem` returns all the formulas of $\mathcal{K}$ (called *premises*) used to build the argument, `Conc` returns its conclusion, `Sub` returns all its sub-arguments, `DefRules` returns all the defeasible rules of the argument and `TopRule` returns the last inference rule used in the argument.

[3] In this article we use $\supset$ to denote the material implication connective of classical logic.

Definition 2.4 (Argument). *An* argument A *on the basis of an argumentation theory with a knowledge base* $\mathcal{K}$ *and an argumentation system* $(\mathcal{L}, \mathcal{R}, n)$ *is any structure obtainable by applying one or more of the following steps finitely many times:*

1. φ *is an argument if* $\varphi \in \mathcal{K}$ *with:* $\texttt{Prem}(A) = \{\varphi\}$, $\texttt{Conc}(A) = \varphi$, $\texttt{Sub}(A) = \{\varphi\}$, $\texttt{DefRules}(A) = \emptyset$, $\texttt{TopRule}(A) =$ *undefined.*

2. $A_1, \ldots, A_n \rightarrow \psi$ *is an argument if* $A_1, \ldots, A_n$ *are arguments such that there exists a strict rule*
   $\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \rightarrow \psi$ *in* $\mathcal{R}_s$.
   $\texttt{Prem}(A) = \texttt{Prem}(A_1) \cup \ldots \cup \texttt{Prem}(A_n)$,
   $\texttt{Conc}(A) = \psi$,
   $\texttt{Sub}(A) = \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n) \cup \{A\}$.
   $\texttt{DefRules}(A) = \texttt{DefRules}(A_1) \cup \ldots \cup \texttt{DefRules}(A_n)$,
   $\texttt{TopRule}(A) = \texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \rightarrow \psi$

3. $A_1, \ldots, A_n \Rightarrow \psi$ *is an argument if* $A_1, \ldots, A_n$ *are arguments such that there exists a defeasible rule* $\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \Rightarrow \psi$ *in* $\mathcal{R}_d$.
   $\texttt{Prem}(A) = \texttt{Prem}(A_1) \cup \ldots \cup \texttt{Prem}(A_n)$,
   $\texttt{Conc}(A) = \psi$,
   $\texttt{Sub}(A) = \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n) \cup \{A\}$,
   $\texttt{DefRules}(A) = \texttt{DefRules}(A_1) \cup \ldots \cup \texttt{DefRules}(A_n) \cup$
   $\{\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \Rightarrow \psi\}$,
   $\texttt{TopRule}(A) = \texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \Rightarrow \psi$.

Each of these functions $\texttt{Func}$ are also defined on sets of arguments $S = \{A_1, \ldots, A_n\}$ as follows: $\texttt{Func}(S) = \texttt{Func}(A_1) \cup \ldots \cup \texttt{Func}(A_n)$. Moreover, for any argument A we define $\texttt{Prem}_n(A) = \texttt{Prem}(A) \cap \mathcal{K}_n$ and $\texttt{Prem}_p(A) = \texttt{Prem}(A) \cap \mathcal{K}_p$.

Example 2.5. *Consider a knowledge base in an argumentation system with* $\mathcal{L}$ *consisting of* $p, q, r, s, t, u, v, x, d_1, d_2, d_3, d_4, d_5$ *and their negations, with* $\mathcal{R}_s = \{s_1, s_2\}$ *and* $\mathcal{R}_d = \{d_1, d_2, d_3, d_4, d_5, d_6\}$, *where*[4]

d_1:	$p \Rightarrow q$	d_4:	$u \Rightarrow v$	s_1:	$p, q \rightarrow r$
d_2:	$s \Rightarrow t$	d_5:	$v, x \Rightarrow \neg t$	s_2:	$v \rightarrow \neg s$
d_3:	$t \Rightarrow \neg d_1$				

[4]In the examples that follow we may use terms of the form s_i, d_i or f_i, to identify strict or defeasible inference rules or items from the knowledge base. We will assume that the d_i names are those assigned by the n function of Definition 2.1.

Let $\mathcal{K}_n = \{p\}$ and $\mathcal{K}_p = \{s, u, x\}$. Note that in presenting the example, we have informally used names d_i to refer to defeasible inference rules. We now define the n function that formally assigns wff d_i to such rules, i.e., for any rule informally referred to as d_i, we have that $n(d_i) = d_i$, so that '$n(d_1) = d_1$' is a shorthand for $n(p \Rightarrow q) = d_1$. In further examples we will often specify the n function in the same way.[5]

An argument for r (i.e., with conclusion r) is displayed in Figure 2, with the premises at the bottom and the conclusion at the top of the tree. In this and the next figure, the type of a premise is indicated with a superscript and defeasible inferences, underminable premises and rebuttable conclusions are displayed with dotted lines. The figure also displays the formal structure of the argument. We have that

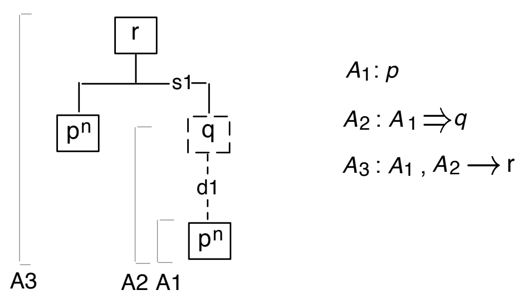

Figure 2: An argument

$\texttt{Prem}(A_3) =$	$\{p\}$	$\texttt{DefRules}(A_3) =$	$\{d_1\}$
$\texttt{Conc}(A_3) =$	r	$\texttt{TopRule}(A_3) =$	s_1
$\texttt{Sub}(A_3) =$	$\{A_1, A_2, A_3\}$		

The distinction between two kinds of inference rules and two kinds of premises motivates a distinction into four kinds of arguments.

Definition 2.6 (Argument properties)**.** *An argument A is* strict *if* $\texttt{DefRules}(A) = \emptyset$*;* defeasible *if* $\texttt{DefRules}(A) \neq \emptyset$*;* firm *if* $\texttt{Prem}(A) \subseteq \mathcal{K}_n$*;* plausible *if* $\texttt{Prem}(A) \cap \mathcal{K}_p \neq \emptyset$*. An argument is* fallible *if it is defeasible or plausible and* infallible *otherwise. We write $S \vdash \varphi$ if there exists a strict argument for φ with all premises taken from S, and $S \mid\!\sim \varphi$ if there exists a defeasible argument for φ with all premises taken from S.*

[5] In our further examples we will often leave the logical language $\mathcal{L}$ and the n function implicit, trusting that they will be obvious.

Example 2.7. *In Example 2.5 the argument A_1 is both strict and firm, while A_2 and A_3 are defeasible and firm. Furthermore, we have that $\mathcal{K} \vdash p$, $\mathcal{K} \mid\!\sim q$ and $\mathcal{K} \mid\!\sim r$.*

In logic-based approaches to argumentation [Besnard and Hunter, 2008; Amgoud and Besnard, 2013] arguments are often required to be minimal in that no proper subset of their premises should logically (according to the adopted base logic) imply the conclusion. In the *ASPIC*$^+$ context such a constraint would be fine for applications of strict rules and below we will review work that imposes such constraints on *ASPIC*$^+$ arguments (Sections 4.2 and 5.1). However, minimality cannot be required for application of defeasible inference rules, since defeasible rules that are based on more information may well make an argument stronger. For example, *Observations done in ideal circumstances are usually correct* is stronger than *Observations are usually correct.*

Another requirement of logic-based approaches, namely, that an argument's premises have to be consistent, can optionally be imposed in basic *ASPIC*$^+$, leading to two variants of the basic framework. We define a special class of arguments whose premises are 'c-consistent' (for 'contradictory-consistent'). In this way *ASPIC*$^+$ can be used as a framework for reconstructing logic-based argumentation formalisms, as we will further discuss in Section 4.2.

Definition 2.8 (c-consistency)**.** *A set $S \subseteq \mathcal{L}$ is* c-consistent *if for no ϕ is it the case that $S \vdash \phi$ and $S \vdash -\phi$. Otherwise S is said to be* c-inconsistent*. We say that $S \subseteq \mathcal{L}$ is* minimally c-inconsistent *iff S is c-inconsistent and $\forall S' \subset S$, S' is* c-consistent*.*

Definition 2.9 (c-consistent arguments)**.** *An argument A is* c-consistent *iff* $\texttt{Prem}(A)$ *is c-consistent.*

2.2.2 Attack and defeat

ASPIC$^+$ generates abstract argumentation frameworks consisting of arguments related by binary defeats. Having defined arguments above, we now define the attack relation and then apply preferences to determine the defeat relation (in fact [Dung, 1995] called his relation "attack" but we reserve this term for the basic notion of conflict, to which we then apply preferences).

Attack We first present the three ways in which *ASPIC*$^+$ arguments can be in conflict (i.e., attack). Arguments can be attacked on a conclusion of a defeasible inference (rebutting attack), on a defeasible inference step itself (undercutting attack), or on an ordinary premise (undermining attack). In Section 2.1 we argued that arguments cannot be attacked on their strict inferences. In Section 3.3 we will also show that attacks on conclusions of strict inferences may result in violation of rationality postulates. In Section 5.3 we will discuss to what extent alternative definitions of rebutting attack still make sense.

To define undercutting attack, the function n of an AS is used, which assigns to elements of $\mathcal{R}_d$ a well-formed formula in $\mathcal{L}$. Recall that informally, $n(r)$ (where $r \in R_d$) means that r is applicable. Then an argument using r is undercut by any argument with conclusion $-n(r)$.

Definition 2.10 (Attacks)**.** *A attacks B iff A undercuts, rebuts or undermines B, where:*

- *A undercuts argument B (on B') iff $\texttt{Conc}(A) = -n(r)$ for some $B' \in \texttt{Sub}(B)$ such that B''s top rule r is defeasible.*
- *A rebuts argument B (on B') iff $\texttt{Conc}(A) = -\varphi$ for some $B' \in \texttt{Sub}(B)$ of the form $B''_1, \ldots, B''_n \Rightarrow \varphi$.*
- *Argument A undermines B (on φ) iff $\texttt{Conc}(A) = -\varphi$ for an ordinary premise φ of B.*

This definition allows for a distinction between direct and indirect attack: an argument can be indirectly attacked by directly attacking one of its proper subarguments. This distinction will turn out to be crucial for a proper application of preferences when determining whether attacks succeed as defeats.

Example 2.11. *In our running example argument A_3 cannot be undermined, since all its premises are axioms. A_3 can potentially be rebutted on A_2, with an argument for $\neg q$. However, the argumentation theory of our example does not allow the construction of such a rebuttal. Likewise, A_3 can potentially be undercut on A_2, with an argument for $\neg d_1$. Our example does allow the construction of such an undercutter, namely:*

B_1: s
B_2: $B_1 \Rightarrow t$
B_3: $B_2 \Rightarrow \neg d_1$

B_3 has an ordinary premise s, and so can be undermined on B_1 with an argument for $\neg s$:

C_1: u
C_2: $C_1 \Rightarrow v$
C_3: $C_2 \rightarrow \neg s$

Note that since C_3 has a strict top rule, argument B_1 does not *in turn rebut C_3.*

Argument B_3 can potentially be rebut or undercut on either B_2 or B_3, since both of these subarguments of B_3 have a defeasible top rule. Our argumentation theory only allows for a rebutting attack on B_2:

C_1: u
C_2: $C_1 \Rightarrow v$
D_3: x
D_4: $C_2, D_3 \Rightarrow \neg t$

All arguments and attacks in the example are displayed in Figure 3.

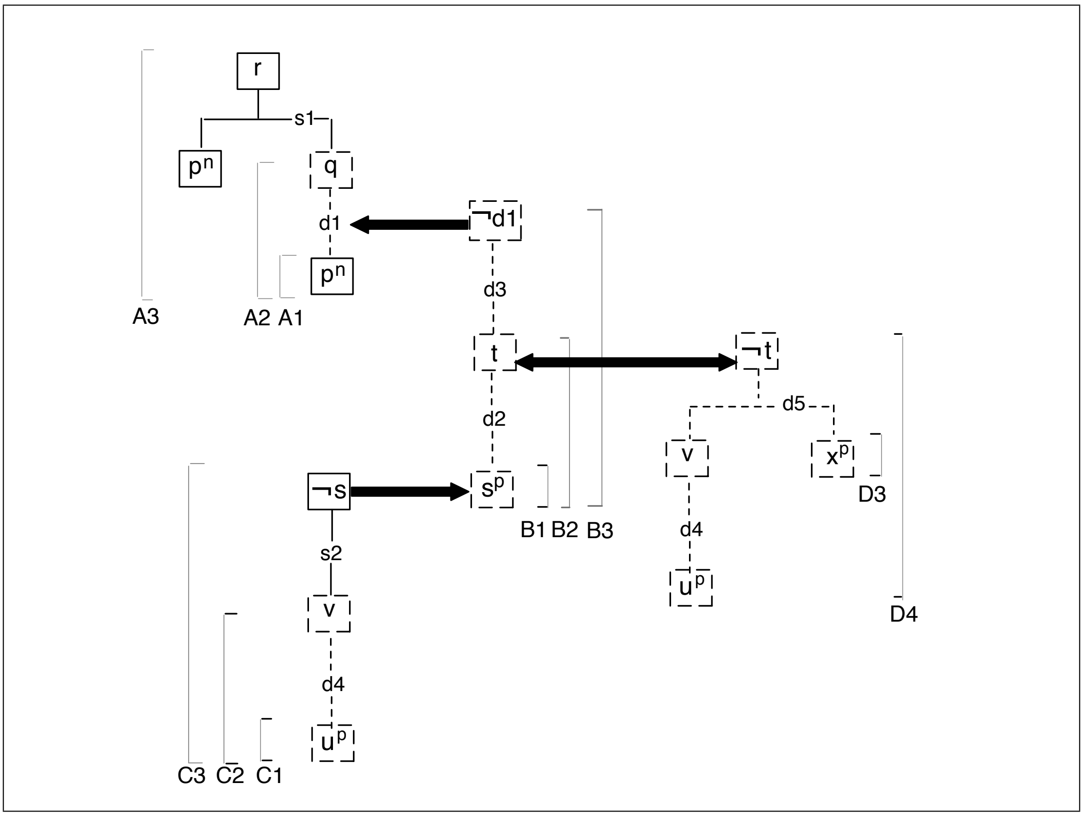

Figure 3: The arguments and attacks in the running example

Defeat The attack relation tells us which arguments are in conflict with each other. If an argument A *successfully attacks*, i.e., defeats, B, then A can be used as a counter-argument to B. Whether an attack from A to B (on its sub-argument B') succeeds as a defeat, may depend on the relative strength of A and B', i.e., whether B' is *strictly stronger than, or strictly preferred* to A. Only the success of undermining and rebutting attacks is contingent on preferences; undercutting attacks succeed as defeats independently of any preferences (see [Modgil and Prakken, 2013] for a discussion as to why this is the case). $ASPIC^+$ allows for any strict binary preference ordering $\prec$ on the set of all arguments that can be constructed on the basis of an argumentation theory. Note that in this article we formalise argument orderings not as they are defined in [Modgil and Prakken, 2013], but as they are defined in an erratum available online at `https://nms.kcl.ac.uk/sanjay.modgil/AIJfinalErratum`. The erratum essentially reverts to the directly defined strict partial ordering $\prec$ over arguments as employed in [Prakken, 2010]. Then (as illustrated in Section 3.2), the non-strict $\preceq$ is defined so that $A \preceq B$ iff $A \prec B$ or the fallible elements in A and B that are used in deciding preferences, are the same. Moreover, [Modgil and Prakken, 2013] identify conditions under which argument orderings are well-behaved in that they ensure satisfaction of the rationality postulates. The erratum modifies these conditions, which in [Modgil and Prakken, 2013] are stated by reference to non-strict orderings over sets of defeasible rules (ordinary premises), but in the erratum are stated with respect to strict orderings over sets of defeasible rules (ordinary premises). This has been done in order to address a counterexample to rationality pointed out by Sjur Dyrkolbotn (personal communication), assuming the conditions as stated in [Modgil and Prakken, 2013][6]. We will review these conditions later in this article.

Definition 2.12 (Successful rebuttal, undermining and defeat)**.**

- A successfully rebuts B *if* A *rebuts* B *on* B' *and* $A \nprec B'$*.*
- A successfully undermines B *if* A *undermines* B *on* φ *and* $A \nprec \varphi$*.*
- A defeats B *iff* A *undercuts or successfully rebuts or successfully undermines* B*. (In general, we say* A strictly *defeats* B *if* A *defeats* B *and* B *does not defeat* A*).*

The success of rebutting and undermining attacks thus involves comparing the conflicting arguments at the points where they conflict; that is, by comparing those arguments that are in a *direct* rebutting or undermining relation with each other. The definition of successful undermining exploits the fact that an argument premise is also a subargument, so the preference $A \nprec \varphi$ is well defined.

[6]Note that the erratum also addresses a counterexample to rationality in [Dung, 2016].

Example 2.13. *In our running example, the undercutting attack of B_3 on A_2 (and thereby on A_3) succeeds as a defeat irrespective of the argument ordering between B_3 and A_2. The undermining attack of C_3 on B_1 succeeds if $C_3 \not\prec B_1$. If B_2 and D_4 are incomparable, then these two arguments defeat each other, while D_4 strictly defeats B_3. If $D_4 \prec B_2$ then B_2 strictly defeats D_4 while if $B_2 \prec D_4$ then D_4 strictly defeats both B_2 and B_3.*

Let us now put all these elements together; that is the arguments and attacks defined on the basis of an argumentation theory, and a preference ordering over the arguments (here we write '*(c-)SAF*' as meaning '*SAF* or *c-SAF*'):

Definition 2.14 (*c-SAF*s)**.** *Let AT be an argumentation theory (AS, KB). A (c-)structured argumentation framework (*(c-)SAF*) defined by AT, is a triple $\langle \mathcal{A}, \mathcal{C}, \preceq \rangle$ where*

- *In a SAF, $\mathcal{A}$ is the set of all arguments constructed from KB in AS satisfying Definition 2.4;*
- *In a c-SAF, $\mathcal{A}$ is the set of all c-consistent arguments constructed from KB in AS satisfying Definition 2.4;*
- *$\preceq$ is a preference ordering on $\mathcal{A}$;*
- *$(X, Y) \in \mathcal{C}$ iff X attacks Y.*

Note that a *c-SAF* is a *SAF* in which all arguments are required to have a c-consistent set of premises.

Example 2.15. *In our running example $\mathcal{A} = \{A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3, D_3, D_4\}$, while $\mathcal{C}$ is such that B_3 attacks both A_2 and A_3, argument C_3 attacks all of B_1, B_2, B_3, argument D_4 attacks both B_2 and B_3 and, finally, B_2 attacks D_4.*

2.2.3 Generating abstract argumentation frameworks

We now instantiate abstract argumentation frameworks with *ASPIC*$^+$ arguments and defeats.

Definition 2.16 (Argumentation frameworks)**.** *An* abstract argumentation framework (*AF*) corresponding to a *(c-)SAF $= \langle \mathcal{A}, \mathcal{C}, \preceq \rangle$ is a pair $(\mathcal{A}, \mathcal{D})$ such that $\mathcal{D}$ is the defeat relation on $\mathcal{A}$ determined by $\langle \mathcal{A}, \mathcal{C}, \preceq \rangle$.*

The justified arguments of the above defined abstract argumentation frameworks are then defined under various semantics, as in [Dung, 1995]:

Definition 2.17 (Dung Semantics)**.** *Let $(\mathcal{A}, \mathcal{D})$ be an AF and $S \subseteq \mathcal{A}$. Then:*

- *S is* conflict free *iff $\forall X, Y \in S$: $(X, Y) \notin \mathcal{D}$*[7]*.*
- *$X \in \mathcal{A}$ is* acceptable with respect to *S iff $\forall Y \in \mathcal{A}$ such that $(Y, X) \in \mathcal{D}$: $\exists Z \in S$ such that $(Z, Y) \in \mathcal{D}$.*
- *S is an* admissible *set iff S is conflict free and $X \in S$ implies X is acceptable w.r.t. S.*
- *S is a* complete *extension iff S is admissible and if $X \in \mathcal{A}$ is acceptable w.r.t. S then $X \in S$;*
- *S is a* preferred *extension iff it is a set inclusion maximal complete extension;*
- *S is the* grounded *extension iff it is the set inclusion minimal complete extension;*
- *S is a* stable *extension iff S is conflict free and $\forall Y \notin S$, $\exists X \in S$ s.t. $(X, Y) \in \mathcal{D}$.*

For $T \in \{$complete, preferred, grounded, stable$\}$, X is sceptically*, respectively* credulously *justified on the basis of AF under the T semantics if X belongs to all, respectively at least one, T extension of AF.*

It is now also possible to define a consequence notion for well-formed formulas. Several definitions are possible. One is:

Definition 2.18 (Justified Formulae)**.** *A wff $\varphi \in \mathcal{L}$ is* sceptically justified *on the basis of a* (c-)SAF *under semantics T if φ is the conclusion of a sceptically justified argument on the basis of the AF corresponding to the* (c-)SAF *under semantics T, and* credulously justified *on the basis of a* (c-)SAF *under semantics T if φ is not sceptically justified and is the conclusion of a credulously justified argument on the basis of the AF corresponding to the* (c-)SAF *under semantics T.*

An alternative definition of skeptical justification is:

[7] Note that in [Modgil and Prakken, 2013] we motivate the use of the *ASPIC*$^+$ attack relation to define conflict-free sets (a set of arguments is conflict-free if there does not exist an attack between any of its contained arguments), and then only use the *ASPIC*$^+$ defeat relation to determine the acceptability of arguments. It turns out that under certain conditions, this way of evaluating the status of arguments is equivalent to Definition 2.17's use of the defeat relation for *both* determining conflict freeness and acceptability of arguments.

> A wff $\varphi \in \mathcal{L}$ is *sceptically justified* on the basis of the *(c-)SAF* under semantics T if all T-extensions of the AF corresponding to the *(c-)SAF* contain an argument with conclusion φ.

While the original definition of skeptical justification requires that there is one argument for φ that is in all extensions, the alternative definition allows that different extensions contain different arguments for φ. In multiple-extension semantics this can make a difference in, for example, cases with so-called floating conclusions; cf. Example 25 of [Prakken and Vreeswijk, 2002].

Example 2.19. *In our running example, if D_4 strictly defeats B_2, then we have a unique extension in all semantics, namely, $E = \{A_1, A_2, A_3, C_1, C_2, C_3, D_3, D_4\}$. If in addition C_3 does not defeat B_1, then the extension also contains B_1. In both cases this yields that wff r is sceptically justified.*

Alternatively, if B_2 strictly defeats D_4, then the status of r depends on whether C_3 defeats B_1. If it does, then we again have a unique extension in all semantics consisting of the set S, so r is sceptically justified. By contrast, if C_3 does not defeat B_1, we obtain a unique extension with A_1, B_1, B_2, B_3, C_1, C_2, C_3 and D_3, so r is neither sceptically nor credulously justified.

Finally, if B_2 and D_4 defeat each other, then the outcome again depends on whether C_3 defeats B_1. If it does, then the situation is as in the previous case – a unique extension E – but if C_3 does not defeat B_1, then the grounded extension consists of A_1, B_1, C_1, C_2, C_3, D_3. So in the latter case, in grounded semantics r is neither sceptically nor credulously justified. However, in preferred and stable semantics we then obtain two alternative extensions: the first contains D_4, A_2 and A_3, while the second instead contains B_2 and B_3 and so excludes A_2 and A_3. So in the latter case r is credulously, but not sceptically justified under stable and preferred semantics.

2.3 The basic framework with possibly non-symmetric negation

The notion of an argumentation system in Section 2.2.1, assumed a language $\mathcal{L}$ with a unary negation symbol $\neg$, which was used in the definition of conflict-based attack. The standard classical interpretation of $\neg$ licenses a symmetric notion of conflict-based attack, so that an argument consisting of an ordinary premise ϕ or with a defeasible top rule concluding ϕ, *symmetrically* attacks an argument consisting of an ordinary premise $\neg\phi$ or with a defeasible top rule concluding $\neg\phi$. However, the *ASPIC*$^+$ framework as presented in [Prakken, 2010; Modgil and Prakken, 2013], accommodates a more general notion of conflict, by defining an argumentation system to additionally include a function $^-$ that, for any wff $\psi \in \mathcal{L}$, specifies the set of wff's that are in conflict with ψ, so that one can define both an asymmetric and symmetric notion of conflict-based attack. Formally:

Definition 2.20 ($^-$ function)**.** $^-$ *is a function from $\mathcal{L}$ to $2^{\mathcal{L}}$, such that:*

- φ *is a* contrary *of* ψ *if* $\varphi \in \overline{\psi}$, $\psi \notin \overline{\varphi}$ *;*
- φ *is a* contradictory *of* ψ *(denoted by '$\varphi = -\psi$'), if* $\varphi \in \overline{\psi}$, $\psi \in \overline{\varphi}$.

Now $\texttt{Conc}(A) \in \overline{\varphi}$ ($\texttt{Conc}(A) \in \overline{n(r)}$) replaces $\texttt{Conc}(A) = -\varphi$ ($\texttt{Conc}(A) = -n(r)$) in Definition 2.10's definition of attacks. This induces a generalised notion of an argumentation system as a four-tuple $AS = (\mathcal{L}, ^-, \mathcal{R}, n)$ where $\mathcal{L}$, $\mathcal{R}$ and n are defined as in Definition 2.1 and $^-$ is as just defined. The special case of Definition 2.1 can then be reformulated as the case where $^-$ is defined in terms of classical negation as $\alpha \in \overline{\beta}$ iff α is of the form $\neg\beta$ or β is of the form $\neg\alpha$ (i.e., for any wff α, α and $\neg\alpha$ are contradictories). Below we will continue to refer to the special case with $\neg$ as a triple, leaving the $^-$ function implicit.

The rationale for these more general notions of conflict and attack is two-fold. Firstly, one can for pragmatic reasons state that two formulae are in conflict, rather than requiring that one implies the negation of another; for example, assuming a predicate language with the binary '$<$' relation, one can state that any two formulae of the form $\alpha < \beta$ and $\beta < \alpha$ are contradictories. Secondly, the $^-$ function allows for an asymmetric notion of negation. This enables reconstruction of assumption-based argumentation (ABA) in *ASPIC*$^+$ (indeed the idea of using a $^-$ function is taken from [Bondarenko *et al.*, 1997]). We briefly review this reconstruction in Section 4.1. Closely related to its use in reconstructing ABA, the contrary function allows for the modelling of negation as failure (as in logic programming). Using the negation as failure symbol $\sim$ (also called 'weak' negation, in contrast to the 'strong' negation symbol $\neg$), then $\sim \alpha$ denotes the negation of α under the assumption that α is not provable (i.e., the negation of α is assumed in the absence of evidence for α). Given this intended reading of $\sim$ it is not meaningful to assert that such an assumption brings into question (and so initiates an attack on) the evidence whose very absence is required to make the assumption in the first place. In other words, if A is an argument consisting of the premise $\sim \alpha$, and B concludes α (the contrary of $\sim \alpha$), then B attacks A, but not vice versa. Furthermore, since the very construction of A is invalidated by evidence to the contrary, i.e., B, then such attacks succeed as defeats *independently* of preferences.

To accommodate the notion of contrary, and attacks on contraries succeeding as defeats independently of preferences, we further modify Definition 2.10 to distinguish the special cases where $\texttt{Conc}(A)$ is a contrary of φ, in which case we say that A *contrary rebuts* B and A *contrary undermines* B, and then modify Definition 2.12 so that:

- A successfully rebuts B if A contrary rebuts B, or A rebuts B on B' and $A \not\prec B'$.
- A successfully undermines B if A contrary undermines B, or A undermines B on ϕ and $A \not\prec \phi$.

The definition of undercutting attack does not need to be changed.

To illustrate the use of negation as failure, suppose one wants arguments to be built from a propositional language that includes both $\neg$ and $\sim$. One could then define $\mathcal{L}$ as a language of propositional literals, composed from a set of propositional atoms $\{a, b, c, \dots\}$ and the symbols $\neg$ and $\sim$. Then:

- α is a *strong literal* if α is a propositional atom or of the form $\neg\beta$ where β is a propositional atom (strong negation cannot be nested).
- α is a wff of $\mathcal{L}$, if α is a strong literal or of the form $\sim \beta$ where β is a strong literal (weak negation cannot be nested).

Then $\alpha \in \overline{\beta}$ iff (1) α is of the form $\neg\beta$ or β is of the form $\neg\alpha$; or (2) β is of the form $\sim \alpha$ (i.e., for any wff α, α and $\neg\alpha$ are contradictories and α is a contrary of $\sim \alpha$). Finally, for any $\sim \alpha$ that is in the antecedent of a strict or defeasible inference rule, one is required to include $\sim \alpha$ in the ordinary premises.

Consider now Example 2.5, where we now have that $u \in \overline{\sim u}$, and we replace the rule $d_4 : u \Rightarrow v$ with d_4': $\sim u \Rightarrow v$, and add $\sim u$ to the ordinary premises: $\mathcal{K}_p = \{\sim u, s, u, x\}$. Then, the arguments C_3 and D_4 are now replaced by arguments C_3' and D_4' each of which contain the sub-argument $E : \sim u$ (instead of $C_1 : u$). Then $C_1 : u$ contrary undermines, and so defeats, C_3' and D_4' on $\sim u$.

3 Instantiating the *ASPIC*$^+$ Framework

ASPIC$^+$ is a framework for specifying systems, and so leaves one fully free to make choices as to the logical language, the strict and defeasible inference rules, the axioms and ordinary premises in a knowledge base, and the argument preference ordering. In this section we discuss various more or less principled ways to make these choices, and then show specific uses of *ASPIC*$^+$.

3.1 Choosing strict and defeasible rules

3.1.1 Domain specific strict inference rules

ASPIC$^+$ allows the specification of domain specific strict inference rules, as illustrated by the following example (based on Example 4 of [Caminada and Amgoud, 2007]) in which the strict inference rules capture definitional knowledge, namely, that bachelors are not married.

Example 3.1. *Let* $\mathcal{R}_d = \{d_1, d_2\}$ *and* $\mathcal{R}_s = \{s_1, s_2\}$*, where:*

$$\begin{array}{llll} d_1 = & WearsRing \Rightarrow Married & s_1 = & Married \rightarrow \neg Bachelor \\ d_2 = & PartyAnimal \Rightarrow Bachelor & s_2 = & Bachelor \rightarrow \neg Married \end{array}$$

Finally, let $\mathcal{K}_p = \{WearsRing, PartyAnimal\}$*. Consider the following arguments.*

$$\begin{array}{llll} A_1: & WearsRing & B_1: & PartyAnimal \\ A_2: & A_1 \Rightarrow Married & B_2: & B_1 \Rightarrow Bachelor \\ A_3: & A_2 \rightarrow \neg Bachelor & B_3: & B_2 \rightarrow \neg Married \end{array}$$

We have that A_3 *rebuts* B_3 *on its subargument* B_2 *while* B_3 *rebuts* A_3 *on its subargument* A_2*. Note that* A_2 *does not rebut* B_3*, since* B_3 *applies a strict rule; likewise for* B_2 *and* A_3*.*

In Example 3.1, the rules s_1 and s_2 are 'transpositions' of each other, and $\mathcal{R}_s$ is 'closed under transposition', in the sense that:

Definition 3.2 (Closure under Transposition)**.** *Let* $AT = (AS, \mathcal{K})$ *be an argumentation theory. Then* AT *is* closed under transposition *iff if* $\phi_1, \ldots, \phi_n \rightarrow \psi \in \mathcal{R}_s$*, then for* $i = 1 \ldots n$*,* $\phi_1, \ldots, \phi_{i-1}, -\psi, \phi_{i+1}, \ldots, \phi_n \rightarrow -\phi_i \in \mathcal{R}_s$*.*

In general it is a good idea to ensure that an argumentation theory is closed under transposition, since a strict (deductive) rule $q \rightarrow \neg s$ expresses that if q is true, then this guarantees the truth of $\neg s$, no matter what. Hence, if we have s, then q cannot hold, otherwise we would have $\neg s$. In general, if the negation of the consequent of a strict rule holds, then we cannot have all its antecedents, since if we had all of them, then its consequent would hold. This is the very meaning of a strict rule. So it is very reasonable to include in $\mathcal{R}_s$ the transposition of a strict rule that is in $\mathcal{R}_s$. A second reason for ensuring closure under transposition is that it ensures satisfaction of [Caminada and Amgoud, 2007]'s rationality postulates, as illustrated later in Section 3.3.

3.1.2 Strict inference rules and axioms based on deductive logics

Some find the use of domain-specific strict inference rules rather odd; why not instead express them as material implications in $\mathcal{L}$ and put them in the knowledge base as axiom premises? One then reserves the strict inference rules for general patterns of deductive inference, since one might argue that this is what inference rules are meant for in logic. $ASPIC^+$ therefore allows one to base your strict inference rules (and axioms) on a deductive logic of one's choice. One can do so by choosing a semantics for a particular choice of $\mathcal{L}$ with an associated monotonic notion of semantic consequence, and then letting $\mathcal{R}_s$ be rules that are sound with respect to that semantics. For example, suppose $\mathcal{R}_s$ should conform to classical logic, given a standard propositional (or first-order) language, such that arguments can contain any classically valid inference step over this language. This can be done in two ways: a crude way and a sophisticated way.

A crude way is to simply put all valid propositional (or first-order) inferences over your language of choice in $\mathcal{R}_s$. So if a propositional language has been chosen, then $\mathcal{R}_s$ can be defined as follows. (where $\vdash_{PL}$ denotes standard propositional-logic consequence). For any finite $S \subseteq \mathcal{L}$ and any $\varphi \in \mathcal{L}$:[8]

$$S \to \varphi \in \mathcal{R}_s \text{ if and only if } S \vdash_{PL} \varphi$$

In fact, with this choice of $\mathcal{R}_s$, strict parts of an argument don't need to be more than one step long. For example, if rules $S \to \varphi$ and $\varphi \to \psi$ are in $\mathcal{R}_s$, then $S \cup \{\varphi\} \to \psi$ will also be in $\mathcal{R}_s$. Note also that using this method, strict rules will be closed under transposition, because of the properties of classical logic.

It should be noted that this way of using a logic as the origin of the strict rule makes some implicit assumptions on the chosen logic, for example that it is compact (everything implied by an infinite set is implied by a finite subset) and satisfies the Cut rule (if S implies φ and $S \cup \{\varphi\}$ implies ψ then S implies ψ). In Section 5.1 we return to this issue.

Let us illustrate the crude approach with a variation of Example 3.1. We retain the defeasible rules d_1 and d_2 but we replace the domain-specific strict rules s_1 and s_2 with a single material implication $\mathit{Married} \supset \neg \mathit{Bachelor}$ in $\mathcal{K}_n$. Moreover, we put all propositionally valid inferences over our language in $\mathcal{R}_s$, including, for example, all inferences instantiating the modus ponens scheme $\varphi, \varphi \supset \psi \to \psi$. Then the arguments change as follows:

A_1:	$\mathit{WearsRing}$	B_1:	$\mathit{PartyAnimal}$
A_2:	$A_1 \Rightarrow \mathit{Married}$	B_2:	$B_1 \Rightarrow \mathit{Bachelor}$
A_3:	$\mathit{Married} \supset \neg \mathit{Bachelor}$	B_3:	$\mathit{Married} \supset \neg \mathit{Bachelor}$
A_4:	$A_2, A_3 \to \neg \mathit{Bachelor}$	B_4:	$B_2, B_3 \to \neg \mathit{Married}$

Now A_4 rebuts B_4 on B_2 while B_4 rebuts A_4 on A_2.

[8]Although antecedents of rules formally are sequences of formulas, we will sometimes abuse notation and write them as sets.

A sophisticated way to base the strict part of *ASPIC*$^+$ on a deductive logic of one's choice is to build an existing axiomatic system for the logic into *ASPIC*$^+$. Its axiom(s) (typically a handful) can be encoded in $\mathcal{K}_n$ and its inference rule(s) (typically just one or a few) in $\mathcal{R}_s$. For example, there are axiomatic systems for classical logic with just four axioms and just one inference rule, namely, modus ponens (i.e, $\varphi \supset \psi, \varphi \rightarrow \psi$)[9]. With this choice of $\mathcal{R}_s$, strict parts of an argument could be very long, since in logical axiomatic systems, proofs of even trivial validities might be long. However, this difference with the crude way is not very big, since if we want to be crude, we must, to know whether $S \rightarrow \varphi$ is in $\mathcal{R}_s$, first construct a propositional proof of φ from S.

With the sophisticated way of building classical logic into our argumentation system, argument A_4 in our example stays the same, since modus ponens is in $\mathcal{R}_s$. However, argument B_4 will change, since modus tollens is not in $\mathcal{R}_s$. In fact, B_4 will be replaced by a sequence of strict rule applications, together being an axiomatic proof of $\neg$*Married* from *Married* $\supset \neg$*Bachelor* and *Bachelor*.

Note that in the sophisticated method, closure under transposition may not hold; our example above does not contain modus tollens (that is, $\varphi \supset \psi, -\psi \rightarrow -\varphi$). However, this desirable form of reasoning can also be enforced without explicitly transposing rules. Recall that $S \vdash \varphi$ was defined as 'there exists a strict argument for φ with all premises taken from S'. Now it turns out that if $\vdash$ contraposes, then this is just as good as closure of the strict rules under transposition. Contraposition of $\vdash$ means that if $S \vdash \varphi$, then if we replace one element s of S with $-\varphi$, then $-s$ is strictly implied (if $\vdash$ corresponds to classical provability, as enforced by our choice of axioms and inference rules, then $\vdash$ does indeed contrapose).

Definition 3.3 (Closure under Contraposition)**.** *Let $AT = (AS, \mathcal{K})$ be an argumentation theory. We say that AT is* closed under contraposition *iff for all $S \subseteq \mathcal{L}$, $s \in S$ and ϕ, if $S \vdash \phi$, then $S \backslash \{s\} \cup \{-\phi\} \vdash -s$.*

Again, as will be discussed in Section 3.3, closure under contraposition also ensures satisfaction of rationality postulates.

3.1.3 Choosing defeasible inference rules

Regarding the choice of defeasible rules, the question as to whether these can be derived from a logic of our choice, just as with strict rules, is controversial. Some philosophers argue that all rule-like structures that we use in daily life are "inference licenses" and so cannot be expressed in the logical object language. In this view,

[9] As explained above, this strictly speaking is not a rule but a scheme, with meta variables ranging over $\mathcal{L}$.

all defeasible generalisations are inference rules, whether they are domain-specific or not, and are applied to formulas from $\mathcal{L}$ to support new formulas from $\mathcal{L}$.

Others (usually logicians) take a more standard-logic approach (e.g. [Kraus *et al.*, 1990; Pearl, 1992]) whereby all contingent knowledge should be expressed in the object language, and so they reject the idea of domain-specific defeasible inference rules (for the same reason they don't like domain-specific strict rules). They introduce a new connective, e.g., $\rightsquigarrow$, into $\mathcal{L}$ where (informally) $p \rightsquigarrow q$ is read as "If p then normally/typically/usually q". They then want to give a model-theoretic semantics for this connective just as logicians give a model-theoretic semantics for all connectives, except that semantics for these defeasible conditionals focus on a *preferred class* of models (e.g., all models where things are as normal as possible) instead of *all* models of a theory as in semantics for deductive logics. Hence, the model-theoretic interpretation of $p \supset q$ is that q is true in *all* models of p, whereas the model theoretic interpretation of $p \rightsquigarrow q$ is that q is true in all *preferred* models of p.

What inference rules for $\rightsquigarrow$ could result from such an approach? On two things there is consensus: modus ponens for $\rightsquigarrow$ is defeasibly but not deductively valid, so the rule $\varphi \rightsquigarrow \psi, \varphi \Rightarrow \psi$ should go into $\mathcal{R}_d$. There is also consensus that contraposition for $\rightsquigarrow$ is deductively invalid, so the rule $\varphi \rightsquigarrow \psi \rightarrow -\psi \rightsquigarrow -\varphi$ should *not* go into $\mathcal{R}_s$. However, here the consensus ends. Should the defeasible analogue of this rule go into $\mathcal{R}_d$ or not? Opinions differ at this point[10].

Let us illustrate the difference between the two approaches, by including defeasible modus ponens for $\rightsquigarrow$ in $\mathcal{R}_d$, and replacing the defeasible inference rules d_1 and d_2 (in Example 3.1) with object-level conditionals expressed in $\mathcal{L}$ and included in $\mathcal{K}_p$:

$$\begin{array}{l} \mathit{WearsRing} \rightsquigarrow \mathit{Married} \in \mathcal{K}_p \text{ and } \mathit{PartyAnimal} \rightsquigarrow \mathit{Bachelor} \in \mathcal{K}_p \\ \mathcal{R}_d = \{\varphi \rightsquigarrow \psi, \varphi \Rightarrow \psi\} \end{array}$$

The arguments then change as follows (assuming the crude incorporation of classical logic):

A_1:	$\mathit{WearsRing}$	B_1:	$\mathit{PartyAnimal}$
A_2:	$\mathit{WearsRing} \rightsquigarrow \mathit{Married}$	B_2:	$\mathit{PartyAnimal} \rightsquigarrow \mathit{Bachelor}$
A_3:	$A_1, A_2 \Rightarrow \mathit{Married}$	B_3:	$B_1, B_2 \Rightarrow \mathit{Bachelor}$
A_4:	$\mathit{Married} \supset \neg \mathit{Bachelor}$	B_4:	$\mathit{Married} \supset \neg \mathit{Bachelor}$
A_5:	$A_3, A_4 \rightarrow \neg \mathit{Bachelor}$	B_5:	$B_3, B_4 \rightarrow \neg \mathit{Married}$

[10]See Chapter 4 of [Caminada, 2004] for a very readable overview of the discussion.

Now A_5 rebuts B_5 on B_3 while B_5 rebuts A_5 on A_3.

Concluding, if desired, at least some of the choices concerning defeasible inference rules can be based on model-theoretic semantics for nonmonotonic logics. However, it is an open question whether a model-theoretic semantics is the *only* criterion by which we can choose our defeasible rules. Some have based their choice on other criteria, since they do not primarily see defeasible rules as logical inference rules but as principles of human cognition or rational action, so that they should be based on foundations other than semantics. For example, John Pollock based his defeasible reasons on his account of epistemology. Others have based their choice of defeasible reasons on the study of argument schemes in informal argumentation theory. We give examples of both these approaches in Section 3.5.

3.2 Choosing argument preference orderings

A well studied use of preferences in the non-monotonic logic literature is based on the use of preference orderings over formulae in the language or defeasible inference rules. If $ASPIC^+$ is to be used as a framework for giving argumentation-based characterisations of non-monotonic formalisms augmented with preferences, then it needs to provide an account of how these preference orderings can be 'lifted' to preferences over arguments. Since $ASPIC^+$ uses defeasible inference rules and ordinary premises, both may come equipped with preference orderings $\leq$ on $\mathcal{R}_d$ and $\leq'$ on $\mathcal{K}_p$, which in general may be distinct, in keeping with the ontologically distinct nature of rules and premises. For example, the ordinary premises may represent the content of percepts from sensors or of witness testimonies, whose preference ordering reflects the relative reliability of the sensors, respectively witnesses. The defeasible rules may, for example, be ordered based on probabilistic strength, on temporal precedence (defeasible rules acquired later are preferred to those acquired earlier), on the basis of principles of legal precedence, and so on. The challenge is to then define a preference over arguments A and B based on the preferences over their constituent ordinary premises *and* defeasible rules.

We now define two argument preference orderings, called the weakest-link and last-link orderings. These orderings are in turn based on partial preorders $\leq$ on $\mathcal{R}_d$ and $\leq'$ on $\mathcal{K}_p$, where as usual, $X <^{(')} Y$ iff $X \leq^{(')} Y$ and $Y \not\leq^{(')} X$ (note that we may represent these orderings in terms of the strict counterpart they define). However, these preferences relate individual defeasible rules, respectively ordinary premises, whereas when comparing two arguments, we want to compare them on the (possibly non-singleton) *sets of* rules/premises that these arguments are constructed from. So, to define these argument preferences, we need to first define a strict set ordering $\lhd_{\mathsf{s}}$ over sets of rules/premises, where for any sets of defeasible rules/ordinary premises

S and S', we intuitively want that:
1) if S is the empty set, it cannot be that $S \lhd_{\mathtt{s}} S'$;
2) if S' is the empty set, it must be that $S \lhd_{\mathtt{s}} S'$ for any non-empty S .
In other words, arguments that have no defeasible rules (ordinary premises) are, modulo the premises (rules), strictly stronger than (preferred to) arguments that have defeasible rules (ordinary premises). Hence the following definition explicitly imposes these constraints, and then gives two alternative ways of defining$\lhd_{\mathtt{s}}$; the so called `Elitist` and `Democratic` ways (i.e., $\mathtt{s} = \mathtt{Eli}$ and `Dem` respectively). `Eli` compares sets on their minimal and `Dem` on their maximal elements.

Definition 3.4 (Orderings $\lhd_{\mathtt{s}}$)**.** *Let Γ and Γ' be finite sets*[11]*. Then $\lhd_{\mathtt{s}}$ is defined as follows:*

1. *If $\Gamma = \emptyset$ then $\Gamma \not\lhd_{\mathtt{s}} \Gamma'$;*
2. *If $\Gamma' = \emptyset$ and $\Gamma \neq \emptyset$ then $\Gamma \lhd_{\mathtt{s}} \Gamma'$;*
   else, assuming a preordering $\leq$ over the elements in $\Gamma \cup \Gamma'$, then if :
3. $\mathtt{s} = \mathtt{Eli}$*:*
   $\Gamma \lhd_{\mathtt{Eli}} \Gamma'$ *if* $\exists X \in \Gamma$ *s.t.* $\forall Y \in \Gamma'$, $X < Y$.
   else, if:
4. $\mathtt{s} = \mathtt{Dem}$*:*
   $\Gamma \lhd_{\mathtt{Dem}} \Gamma'$ *if* $\forall X \in \Gamma$, $\exists Y \in \Gamma'$, $X < Y$.

For $\mathtt{s} = \mathtt{Eli}$ *or* $\mathtt{s} = \mathtt{Dem}$*:* $\Gamma \unlhd_{\mathtt{s}} \Gamma'$ *iff* $\Gamma = \Gamma'$ *or* $\Gamma \lhd_{\mathtt{s}} \Gamma'$

Now the **last-link principle** strictly prefers an argument A over another argument B if the last defeasible rules used in B are less preferred ($\lhd_{\mathtt{s}}$) than the last defeasible rules in A or, in case both arguments are strict, if the premises of B are less preferred than the premises of A. The concept of 'last defeasible rules' is defined as follows.

Definition 3.5 (Last defeasible rules)**.** *Let A be an argument.*

- $\mathtt{LastDefRules}(A) = \emptyset$ *iff* $\mathtt{DefRules}(A) = \emptyset$.
- *If* $A = A_1, \ldots, A_n \Rightarrow \phi$, *then* $\mathtt{LastDefRules}(A) = \{\mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \Rightarrow \phi\}$, *else* $\mathtt{LastDefRules}(A) = \mathtt{LastDefRules}(A_1) \cup \ldots \cup \mathtt{LastDefRules}(A_n)$.

[11]Notice that it suffices to restrict $\lhd$ to finite sets since *ASPIC*$^+$ arguments are assumed to be finite (in Definition 2.14) and so their sets of ordinary premises/defeasible rules must be finite.

For example, letting $\mathcal{K} = \{p, q\}$, $\mathcal{R}_s = \{r, s \rightarrow t\}$ and $\mathcal{R}_d = \{p \Rightarrow r;\ q \Rightarrow s\}$, then
$\mathtt{LastDefRules}(A) = \{p \Rightarrow r;\ q \Rightarrow s\}$ where A is the argument for t.

The above definition is now used to compare pairs of arguments as follows:

Definition 3.6 (Last link principle)**.** *Let A and B be two arguments. Then $A \prec B$ iff:*

1. $\mathtt{LastDefRules}(A) \triangleleft_{\mathsf{s}} \mathtt{LastDefRules}(B)$*; or*
2. $\mathtt{LastDefRules}(A)$ *and* $\mathtt{LastDefRules}(B)$ *are empty and* $\mathtt{Prem_p}(A) \triangleleft_{\mathsf{s}} \mathtt{Prem_p}(B)$.

Then $B \preceq A$ iff $B \prec A$ or, if $\mathtt{LastDefRules}(A) \neq \emptyset$ *then* $\mathtt{LastDefRules}(A) = \mathtt{LastDefRules}(B)$*, else* $\mathtt{Prem_p}(A) = \mathtt{Prem_p}(B)$.

Example 3.7. *Suppose in our running example that $u <' s$, $x <' s$, $d_2 < d_5$ and $d_4 < d_2$. Applying the last-link ordering to check whether C_3 defeats B_1, we compare* $\mathtt{LastDefRules}(C_3) = \{d_4\}$ *with* $\mathtt{LastDefRules}(B_1) = \emptyset$*. Clearly, $\{d_4\} \triangleleft_{\mathtt{Eli}} \emptyset$, so $C_3 \prec B_1$, so C_3 does not defeat B_1. Next, to check whether D_4 defeats B_2, we compare* $\mathtt{LastDefRules}(B_2) = \{d_2\}$ *with* $\mathtt{LastDefRules}(D_4) = \{d_5\}$*. Since $d_2 < d_5$ we have that* $\mathtt{LastDefRules}(B_2) \triangleleft_{\mathtt{Eli}} \mathtt{LastDefRules}(D_4)$*, so D_4 strictly defeats B_2.*

The **weakest-link principle** considers not the last but *all* uncertain elements in an argument.

Definition 3.8 (Weakest link principle)**.** *Let A and B be two arguments. Then $A \prec B$ iff*

1. *If both B and A are strict, then* $\mathtt{Prem_p}(A) \triangleleft_{\mathsf{s}} \mathtt{Prem_p}(B)$*, else;*
2. *If both B and A are firm, then* $\mathtt{DefRules}(A) \triangleleft_{\mathsf{s}} \mathtt{DefRules}(B)$*, else;*
3. $\mathtt{Prem_p}(A) \triangleleft_{\mathsf{s}} \mathtt{Prem_p}(B)$ *and* $\mathtt{DefRules}(A) \triangleleft_{\mathsf{s}} \mathtt{DefRules}(B)$

Then $B \preceq A$ iff $B \prec A$ or, $\mathtt{DefRules}(A) = \mathtt{DefRules}(B)$ *and* $\mathtt{Prem_p}(A) = \mathtt{Prem_p}(B)$.

Example 3.9. *In our running example to check whether C_3 defeats B_1 according to the weakest-link ordering, we first compare* $\mathtt{Prem}_p(C_3) = \{u\}$ *with* $\mathtt{Prem}_p(B_1) = \{s\}$*. Since $u <' s$ we have that* $\mathtt{Prem}_p(C_3) \triangleleft_{\mathtt{Eli}} \mathtt{Prem}_p(B_1)$*. Also,* $\mathtt{DefRules}(C_3) = \{d_4\} \triangleleft_{\mathtt{Eli}} \mathtt{DefRules}(B_1) = \emptyset$*, and so $C_3 \prec B_1$ and C_3 does not defeat B_1.*
For B_2 and D_4: $\mathtt{Prem}_p(D_4) = \{u, x\} \triangleleft_{\mathtt{Eli}} \mathtt{Prem}_p(B_2) = \{s\}$ *since $u <' s$ and $x < s'$. Then since $d_4 < d_2$,* $\mathtt{DefRules}(D_4) = \{d_4, d_5\} \triangleleft_{\mathtt{Eli}} \mathtt{DefRules}(B_2)\{d_2\}$*. So $D_4 \prec B_2$ and B_2 strictly defeats D_4.*

We next present two examples illustrating the suitability of the last-, respectively, weakest-link orderings. Consider an example relating to whether people misbehaving in a university library may be denied access to the library.[12]

Example 3.10. *Let* $\mathcal{K}_p = \{Snores,\ Professor\}$, $\mathcal{R}_d =$

$\{Snores \Rightarrow_{d_1} Misbehaves;$
$Misbehaves \Rightarrow_{d_2} AccessDenied;$
$Professor \Rightarrow_{d_3} \neg AccessDenied\}.$

Assume that $Snores <' Professor$ *and* $d_1 < d_2,\ d_1 < d_3,\ d_3 < d_2$*, and consider the following arguments.*

A_1:	$Snores$	B_1:	$Professor$
A_2:	$A_1 \Rightarrow Misbehaves$	B_2:	$B_1 \Rightarrow \neg AccessDenied$
A_3:	$A_2 \Rightarrow AccessDenied$		

Let us apply the ordering to the arguments A_3 *and* B_2*. The rule sets to be compared are* $\texttt{LastDefRules}(A_3) = \{d_2\}$ *and* $\texttt{LastDefRules}(B_2) = \{d_3\}$*. Since* $d_3 < d_2$ *we have that* $\texttt{LastDefRules}(B_2) \lhd_{\texttt{Eli}} \texttt{LastDefRules}(A_3)$*, hence* $B_2 \prec A_3$*. So* A_3 *strictly defeats* B_2*, hence* A_3 *is justified in any semantics, and we conclude AccessDenied.*

With the weakest-link principle the ordering between A_3 *and* B_2 *is different. Both A and B are plausible and defeasible so we are in case (3) of Definition 3.8. Since* $Snores <' Professor$*, we have that* $\texttt{Prem}_p(A_3) \lhd_{\texttt{Eli}} \texttt{Prem}_p(B_2)$*. Furthermore, the rule sets to be compared are now* $\texttt{DefRules}(A_3) = \{d_1, d_2\}$ *and* $\texttt{DefRules}(B_2) = \{d_3\}$*. Since* $d_1 < d_3$ *we have that* $\texttt{DefRules}(A_3) \lhd_{\texttt{Eli}} \texttt{DefRules}(B_2)$*. So now we have that* $A_3 \prec B_2$*. Hence* B_2 *now strictly defeats* A_3 *and we conclude instead that* $\neg AccessDenied$*.*

Which outcome is better? Some have argued that the last-link ordering gives the better outcome since the conflict really is between the two legal rules about whether someone may be denied access to the library, while d_1 just provides a sufficient condition for when a person can be said to misbehave. The existence of a conflict on whether someone may be denied access to the library is in no way relevant for the issue of whether a person misbehaves when snoring. More generally, it has been argued that for reasoning with legal (and other normative) rules the last-link ordering is appropriate. However, in an example of exactly the same form, with the legal rules replaced by empirical generalisations, intuitions seem to favour the weakest-link ordering:

[12] In all examples below, sets that are not specified are assumed to be empty. Moreover, sometimes we will attach the rule names to the $\Rightarrow$ symbol. Note that these attached indices have no formal meaning and are for ease of reference only.

Example 3.11. *Let* $\mathcal{K}_p = \{BornInScotland,\ FitnessLover\}$, $\mathcal{R}_d =$

$\{BornInScotland \Rightarrow_{d_1} Scottish;$
$Scottish \Rightarrow_{d_2} LikesWhisky;$
$FitnessLover \Rightarrow_{d_3} \neg LikesWhisky\}.$

Assume that $BornInScotland <' FitnessLover$ *and* $d_1 < d_2,\ d_1 < d_3,\ d_3 < d_2$, *and consider the following arguments.*

A_1:	$BornInScotland$	B_1:	$FitnessLover$
A_2:	$A_1 \Rightarrow Scottish$	B_2:	$B_1 \Rightarrow \neg LikesWhisky$
A_3:	$A_2 \Rightarrow LikesWhisky$		

This time it seems reasonable to conclude $\neg LikesWhisky$, *since the epistemic uncertainty of the premise and* d_1 *of* A_3 *should propagate to weaken* A_3. *And this is the outcome given by the weakest-link ordering. So it could be argued that for epistemic reasoning the weakest-link ordering is appropriate.*

3.3 The rationality postulates of Caminada and Amgoud (2007) and their satisfaction in *ASPIC*$^+$

ASPIC$^+$ leaves one fully free to choose a language, what is an axiom and what is an ordinary premise and how to specify strict and defeasible rules. However some care needs to be taken in making these choices, to ensure that the result of argumentation is guaranteed to be well-behaved in the sense that the desirable properties proposed by [Caminada and Amgoud, 2007] are satisfied. Before presenting these properties, we define required notions of direct and indirect consistency in terms of the contrary function (recall Definition 2.20).

Definition 3.12 (Direct and Indirect Consistency)**.** *For any* $S \subseteq \mathcal{L}$, *let the closure of* S *under strict rules, denoted* $Cl(S)$, *be the smallest set containing* S *and the consequent of any strict rule in* $\mathcal{R}_s$ *whose antecedents are in* $Cl(S)$. *Then a set* $S \subseteq \mathcal{L}$ *is*

- directly consistent *iff* $\nexists\ \psi, \varphi \in S$ *such that* $\psi \in \overline{\varphi}$
- indirectly consistent *iff* $Cl(S)$ *is directly consistent.*

Let E be any complete extension of an abstract argumentation framework corresponding to a (c)-*SAF* as defined in Section 2.2.3.

Sub-argument Closure: For any argument A in E, all sub-arguments of A are in E, i.e., for all $A \in E$: if $A' \in \mathtt{Sub}(A)$ then $A' \in E$.

Closure under Strict Rules: If E contains arguments with conclusions $\alpha_1, \ldots . \alpha_n$, then any arguments obtained by applying only strict inference rules to these conclusions, are in E, i.e., $\{\texttt{Conc}(A)|A \in E\} = Cl(\{\texttt{Conc}(A)|A \in E\})$.

Direct Consistency: The conclusions of arguments in E are directly consistent, i.e., $\{\texttt{Conc}(A)|A \in E\}$ is consistent.

Indirect Consistency: The conclusions of arguments in E are indirectly consistent, i.e., $Cl(\{\texttt{Conc}(A)|A \in E\})$ is consistent.

We next review the work done on identifying sufficient conditions for $ASPIC^+$ satisfying [Caminada and Amgoud, 2007]'s four rationality postulates.

3.3.1 The work of Caminada and Amgoud (2007), Prakken (2010) and Modgil and Prakken (2013)

The first relevant condition is that an argumentation theory is closed under transposition or contraposition. If neither is satisfied, then since strict rule applications cannot be attacked, direct consistency may be violated. Consider our first version of Example 3.1. Suppose we only have the strict rule s_1 so that B_3 cannot be constructed (given the absence of s_2). We still have that A_3 rebuts B_2. Suppose now that $d_1 < d_2$ and we apply the last-link argument ordering. Then A_3 does not defeat B_2. In fact, no argument in the example is defeated, so we end up with a single extension (under all semantics) which contains arguments for both *Bachelor* and $\neg$*Bachelor* and so violates direct and indirect consistency. However, with transposition we also have s_2. Then B_3 can be constructed, which rebuts A_3 on A_2. Under the last-link ordering (assuming again that $d_1 < d_2$) we still have that A_3 does not defeat B_2, but now B_3 strictly defeats A_2. We have a unique extension in all semantics, containing all arguments except A_2 and A_3. This extension does not violate consistency.

One might argue that the above violation of consistency, before inclusion of the transposed rule s_2, arises because $ASPIC^+$ forbids attacks on strictly derived conclusions. Consistency would not be violated if B_2 was allowed to attack A_3. However, apart from the reasons discussed in Section 2.2.2, another reason for prohibiting attacks on strictly derived conclusions is that if allowed, extensions may not be strictly closed or indirectly consistent, even if the strict rules are closed under transposition. To see why, suppose we allow attacks on strict conclusions, so that B_2 attacks A_3, A_2 attacks B_3, and A_3 and B_3 attack each other in Example 3.1. Suppose also that all knowledge-base items and defeasible rules are of equal preference, and we apply the weakest- or last-link argument ordering. Then all rebutting attacks in the example succeed. But then the set $\{A_1, A_2, B_1, B_2\}$ is admissible and is in fact both a stable and preferred extension. But this violates strict closure and indirect consistency. The extension contains an argument for *Bachelor* but not for $\neg$*Married*, which strictly follows from it by rule s_2. Likewise, the extension contains an argument for *Married* but not for $\neg$*Bachelor*, which strictly follows from it by rule s_1. So the extension is not closed under strict rule application. Moreover, the extension is indirectly inconsistent, since its strict closure contains both *Married* and $\neg$*Married*, and both *Bachelor* and $\neg$*Bachelor*.

Other requirements for satisfying the postulates are expressed in the following definition of a 'well-defined' structured argumentation framework (recall Definition 2.14), which references the notion of a 'reasonable' preference relation that is subsequently explained and defined:

Definition 3.13 (Well defined *(c-)SAF*s)**.** *A (c-)SAF* $(\mathcal{A}, \mathcal{C}, \preceq)$ *defined by an an argumentation theory* $AT = (AS, \mathcal{K})$*, where* $AS = (\mathcal{L}, ^-, \mathcal{R}, n)$ *and* $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$*, is said to be* well defined *iff:*

- AT *is* closed under transposition *or* closed under contraposition.
- $Cl_{\mathcal{R}_s}(\mathcal{K}_n)$ *is consistent (in which case* $\mathcal{K}$ *is said to be* axiom consistent*).*
- *If* $\mathcal{A}$ *is restricted to be the set of c-consistent arguments, then* $\mathcal{A}$ *is* c-classical*. That is to say, for any minimal c-inconsistent* $S \subseteq \mathcal{L}$ *and for any* $\varphi \in S$*, it holds that* $S \setminus \{\varphi\} \vdash -\varphi$ *(i.e., amongst all arguments defined there exists a strict argument with conclusion* $-\varphi$ *with all premises taken from* $S \setminus \{\varphi\}$*).*
- well formed *if whenever* φ *is a contrary of* ψ *then:*
  - $\psi \notin \mathcal{K}_n$*; and*
  - ψ *is not the consequent of a strict rule.*
- $\preceq$ *is reasonable.*

The property of transposition (and the alternative contraposition) has been discussed above. That the axiom premises are required to be consistent when closed under strict rules is self-evident given that axiom premises represent indisputable information or axioms of a deductive logic. The c-classicality condition is only required to hold when using $ASPIC^+$ to reconstruct Tarskian logic, and in particular classical logic approaches to argumentation, where $\mathcal{A}$ is restricted to arguments with consistent premises. Intuitively, c-classicality says that for every minimally c-inconsistent set of wff and any of its elements the remaining maximally c-consistent subset gives rise to an argument against the element. The intuition underlying the well-formed property should be apparent given the motivation for use of the contrary function and preference independent attacks on contraries, as discussed in Section 2.3. We now elaborate on the notion of reasonable preference orderings.

Before doing so, we define the following notion of strict continuations of arguments, which we define in a slightly different way than [Modgil and Prakken, 2013]. The new definition is arguably simpler but does not affect the proofs of Modgil and Prakken. It identifies arguments that are formed by extending a set of arguments with only strict inferences into a new argument, so that the new argument can only be attacked on the arguments that it extends.

Definition 3.14 (Strict continuations). *The set of* strict continuations *of a set of arguments from $\mathcal{A}$ is the smallest set satisfying the following conditions:*

1. *Any argument A is a strict continuation of $\{A\}$.*
2. *If $A_1, \dots, A_n$ and $S_1, \dots, S_n$ are such that for each $i \in \{1, \dots, n\}$, A_i is a strict continuation of S_i and $\{B_{n+1}, \dots, B_m\}$ is a (possibly empty) set of strict-and-firm arguments, and $\texttt{Conc}(A_1), \dots, \texttt{Conc}(A_n), \texttt{Conc}(B_{n+1}), \dots, \texttt{Conc}(B_m) \rightarrow \varphi$ is a strict rule in R_s, then $A_1, \dots, A_n, B_{n+1}, \dots, B_m \rightarrow \varphi$ is a strict continuation of $S_1 \cup \ldots \cup S_n$.*

If argument A is a strict continuation of arguments $\{A_1, \dots, A_n\}$, then A is a strict argument over $\{\texttt{Conc}(A_1), \dots, \texttt{Conc}(A_n)\}$.

Example 3.15. In Example 2.5 (see Figure 3) all arguments are strict continuations of the singleton set containing themselves while A_3 is a strict continuation of $\{A_1, A_2\}$ and C_3 is a strict continuation of $\{C_2\}$.

Definition 3.16 (Reasonable Argument Orderings). *An argument ordering $\preceq$ is* reasonable *iff:*

1. *i) $\forall A, B$, if A is strict and firm and B is plausible or defeasible, then $B \prec A$;*
   ii) $\forall A, B$, if B is strict and firm then $B \not\prec A$;
   iii) $\forall A, A', B$ such that A' is a strict continuation of $\{A\}$, if $A \not\prec B$ then $A' \not\prec B$, and if $B \not\prec A$ then $B \not\prec A'$ (i.e., applying strict rules to a single argument's conclusion and possibly adding new axiom premises does not weaken, respectively strengthen, arguments).
2. *Let $\{C_1, \dots, C_n\}$ be a finite subset of $\mathcal{A}$, and for $i = 1 \dots n$, let $C^{+\backslash i}$ be some strict continuation of $\{C_1, \dots, C_{i-1}, C_{i+1}, \dots, C_n\}$. Then it is not the case that: $\forall i$, $C^{+\backslash i} \prec C_i$.*

A reasonable argument ordering essentially amounts to requiring that: arguments that are both strict and firm are strictly preferred over all plausible or defeasible arguments, and no argument is strictly preferred to a strict and firm argument

(1i) and 1ii)); the strength (and implied relative preference) of an argument is determined exclusively by the defeasible rules and/or ordinary premises (1iii)); the preference ordering is acyclic (2).

Indeed, a strict relation $\lhd$ (on sets of ordinary premises or defeasible rules) results in a preference ordering (under either weakest- or last-link) that is reasonable, if $\lhd$ satisfies the following conditions:

Definition 3.17 (Inducing reasonable orderings)**.** $\lhd$ *is said to be* reasonable inducing *if* $\lhd$ *is a strict partial ordering (irreflexive and transitive) such that:*

for any $\mathtt{kr} \in \{\mathtt{LastDefRules}, \mathtt{DefRules}, \mathtt{Prem_p}\}$*, for all arguments* $B_1, \ldots,$ B_n, A *such that* $\bigcup_{i=1}^{n} \mathtt{kr}(B_i) \lhd \mathtt{kr}(A)$*, it holds that for some* $i = 1 \ldots n$*,* $\mathtt{kr}(B_i) \lhd \mathtt{kr}(A)$

It can be shown that both $\lhd_{\mathtt{Eli}}$ and $\lhd_{\mathtt{Dem}}$ (recall Definition 3.4) are reasonable inducing.

We are now in a position to state some important results proved in [Modgil and Prakken, 2013]. Any (c)-structured argumentation framework satisfies the rationality postulate of sub-argument closure. Moreover, if a (c-)structured argumentation framework is well-defined then the postulates of strict closure and direct and indirect consistency are also satisfied by the $ASPIC^+$ framework as defined with the contrary function in Section 2.3.

Theorem 3.18 (Sub-argument Closure)**.** *Let* $\Delta = (\mathcal{A}, \mathcal{C}, \preceq)$ *be a* (c-)SAF *and* E *a complete extension of the AF corresponding to* Δ*. Then for all* $A \in E$*: if* $A' \in \mathtt{Sub}(A)$ *then* $A' \in E$*.*

Theorem 3.19. *Let* $\Delta = (\mathcal{A}, \mathcal{C}, \preceq)$ *be a well-formed* (c-)SAF *and* E *a complete extension of the AF corresponding to* Δ*. Then*

Closure under Strict Rules $\{\mathtt{Conc}(A) | A \in E\} = Cl_{R_s}(\{\mathtt{Conc}(A) | A \in E\})$*;*

Direct consistency $\{\mathtt{Conc}(A) | A \in E\}$ *is consistent;*

Indirect consistency $Cl_{R_s}(\{\mathtt{Conc}(A) | A \in E\})$ *is consistent.*

Finally, note that if no strict rules or axiom premises are included in the argumentation theory, then the preference ordering need *not* be reasonable in order for all four rationality postulates to be satisfied (indeed no assumptions as to the properties of the preference ordering are required in this case). Thus the requirement that the defined (c-)SAF be well-defined does not apply.

3.3.2 The work of Dung and Thang (2014) and Grooters (2014)

For the case without preferences and knowledge bases, [Dung and Thang, 2014] identify weaker conditions for satisfying the rationality postulates than those discussed above. [Dung and Thang, 2014] formulate their results in terms of an adaptation of [Amgoud and Besnard, 2013] abstract-logic approach to abstract argumentation with abstract attack and support relations between arguments. After defining their adaptation they apply it to what they call "rule-based systems", which are a pair of sets of strict and defeasible rules defined over a propositional literal language. Since they adopt the $ASPIC^+$ definitions of an argument and of defeat (which they call 'attack') they thus effectively study a class of $ASPIC^+$ instantiations with an empty knowledge base and with no preferences. Below we summarise their definitions and results as holding for this class of $ASPIC^+$ instantiations, adapting fragments of [Grooters, 2014] and [Grooters and Prakken, 2016]. In doing so, we implicitly assume a given $ASPIC^+$ structured argumentation framework generated by a rule-based instantiation in the sense of [Dung and Thang, 2014], which we will call a 'rule-based' $ASPIC^+$ SAF.

First, an argument A is a *basic defeasible argument* iff $\texttt{TopRule}(A) \in \mathcal{R}_d$, and a set X of arguments is called *inconsistent* if $\texttt{Conc}(X)$ is indirectly inconsistent.

Definition 3.20 (Base of an argument)**.** *Let A be an argument and BA a finite set of subarguments of A. BA is a* base *of A if*

- $\texttt{Conc}(A) \in Cl_{\mathcal{R}_s}(\texttt{Conc}(BA))$*;*
- *For each argument C, C defeats A if and only if C defeats BA.*

The following example shows the intuitive idea of a base.

Example 3.21. Let $\mathcal{R}_s = \{c \to d\}$ and $\mathcal{R}_d = \{\Rightarrow a; \Rightarrow b; a, b \Rightarrow c\}$. Then the following arguments can be constructed: $A_1 :\Rightarrow a$, $A_2 :\Rightarrow b$, $A_3 : A_1, A_2 \Rightarrow c$ and $A_4 : A_3 \to d$. See Figure 4.

A_4 can only be attacked on its subarguments A_1, A_2, or A_3 because of the strict top rule. Every argument that attacks A_1 or A_2 also attacks A_3, so every argument that attacks A_4 also attacks A_3. It is easy to see that every argument that attacks A_3 also attacks A_4. $\texttt{Conc}(A_4) \subseteq Cl_{\mathcal{R}_s}(\texttt{Conc}(A_3))$, so $\{A_3\}$ is a base of A_4. The same kind of reasoning applies to the fact that the set $\{A_1, A_2, A_3\}$ is also a base of A_4. However note that the set $\{A_1, A_2\}$ is not a base of A_4, because A_4 can be rebutted (on A_3) without A_1 or A_2 being attacked.

Definition 3.22 (Generation of arguments)**.** *An argument A is said to be* generated *by a set of arguments S, if there is a base B of A such that $B \subseteq \texttt{Sub}(S)$. The set of all arguments generated by S is denoted by $GN(S)$.*

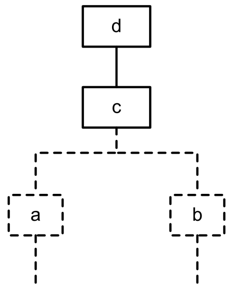

Figure 4: Arguments of Example 3.21

[Dung and Thang, 2014] show that for every set of arguments S, $\texttt{Sub}(S) \subseteq GN(S)$ and for every complete extension E, $GN(E) = E$. [Grooters, 2014] notes that these results immediately imply that each rule-based *ASPIC*$^+$ *SAF* satisfies the closure under subarguments postulate, since for every complete extension E: $\texttt{Sub}(E) \subseteq GN(E) = E$ ([Dung and Thang, 2014] do not consider the subargument-closure postulate).

Definition 3.23 (Compact)**.** *A rule-based* ASPIC$^+$ *SAF is* compact *if for each set of arguments S, $GN(S)$ is closed under strict rules.*

[Dung and Thang, 2014] show that each rule-based *ASPIC*$^+$ *SAF* is compact and that each compact rule-based *ASPIC*$^+$ *SAF* satisfies strict closure, so each rule-based *ASPIC*$^+$ *SAF* satisfies the closure under strict rules postulate.

Definition 3.24 (Cohesive)**.** *A rule-based* ASPIC$^+$ *SAF is* cohesive *if for each inconsistent set of arguments S, $GN(S)$ is conflicting (attacks itself).*

Definition 3.25 (Self-contradiction axiom)**.** *A rule-based* ASPIC$^+$ *SAF is said to satisfy the* self-contradiction axiom *if for each minimal inconsistent set $X \subseteq \mathcal{L}$: $\neg X \subseteq Cl_{\mathcal{R}_s}(X)$ (where $\neg X = \{\neg l \mid l \in L\}$).*

[Dung and Thang, 2014] then show that each cohesive rule-based *ASPIC*$^+$ *SAF* satisfies the indirect-consistency postulate and, moreover, that each rule-based *ASPIC*$^+$ *SAF* that satisfies the self-contradiction axiom is cohesive. Combining these two results, it follows that each rule-based *ASPIC*$^+$ *SAF* that satisfies the self-contradiction axiom, also satisfies indirect consistency. This result generalises the corresponding results discussed in the previous subsection, since satisfying the self-contradiction axiom is a weaker notion than closure under transposition. First, [Dung and Thang, 2014] prove that the latter implies the former in that each rule-based *ASPIC*$^+$ *SAF* that is closed under transposition satisfies the self-contradiction axiom. They then give the following counterexample to the converse implication.

Example 3.26. Let $\mathcal{L} = \{a, \neg a, b, \neg b\}$ and $\mathcal{R}_d = \emptyset$ and $\mathcal{R}_s = \{a \rightarrow b\} \cup \{x, \neg x \rightarrow y \mid x \in \{a, b\}$ and $y \in \mathcal{L}\}$. This system satisfies the self-contradiction axiom but is not closed under transposition.

It is worth noting that [Grooters, 2014] generalised all these results to the case with arbitrary logical languages with symmetric negation, c-consistent nonempty knowledge bases and reasonable argument orderings, and for both SAFs and for c-SAFs. Moreover, she did so alternatively for closure under transposition and closure under contraposition. In doing so, it was shown that the following weaker version of the self-contradiction axiom suffices:

Definition 3.27 (Weak self-contradiction axiom)**.** *A rule-based* ASPIC$^+$ *(c-)SAF is said to satisfy the* weak self-contradiction axiom *if for each minimal inconsistent set $X \subseteq \mathcal{L}$ there is a $\sigma \in X$ such that $\neg\sigma \in Cl_{\mathcal{R}_s}(X)$.*

3.4 On the need for the various elements of *ASPIC*$^+$

ASPIC$^+$ as a general framework is quite expressive. The question therefore arises whether all these elements are really needed.

3.4.1 The need for knowledge bases

The *ASPIC* system as presented in [Caminada and Amgoud, 2007] did not have knowledge bases. Instead, certain and uncertain premises were encoded as strict rules $\rightarrow \varphi$ and defeasible rules $\Rightarrow \varphi$. Others, such as [Dung and Thang, 2014], [Li and Parsons, 2015] and [Dung, 2016] also adopt this idea. Yet there are good reasons to retain knowledge bases. To start with, the distinction between knowledge (or beliefs) and inference rules is a natural one, widely adopted in logic. Furthermore, this distinction allows a systematic study of encodings of logical consequence notions in the set of strict rules, as we will see below. We therefore conclude that although dispensing with knowledge bases might have practical advantages in specific applications, a general theory of argumentation-based inference should retain the formal distinction between knowledge and inference rules.

3.4.2 The need for strict rules and axiom premises

[Li and Parsons, 2015] show that every *ASPIC*$^+$ *SAF* with a weakest-link ordering that satisfies the rationality postulates can be translated into a *SAF* with no strict rules and no axiom premises and that (for all of [Dung, 1995]'s semantics) validates exactly the same conclusions as the original SAF. Their basic idea is that each strict rule is translated to a corresponding defeasible rule and each axiom premise to an ordinary premise, and the argument ordering is then extended so as to give the new elements resulting from the translations of strict rules or axiom premises, precedence over all conflicting elements. While this result is theoretically interesting, we still believe that the distinction between strict and defeasible inference rules is a natural one and is philosophically grounded. For example, the observation that the inclusion of strict rules allows a systematic study of encodings of logical consequence notions also applies here. We also believe that the distinction between disputable (ordinary) and undisputable (axiom) premises is a practically useful one. For these reasons we claim that a general framework for structured argumentation should leave room for these distinctions.

3.4.3 The need for preferences

In the context of ABA, [Kowalski and Toni, 1996] proposed a way to encode preferences with a specific use of assumptions in strict rules with the effect that if a preferred rule applies, the assumption in a non-preferred conflicting rule is attacked. The same can in fact be done with defeasible rules. However, [Kowalski and Toni, 1996]'s proposal does not cover any of the argument orderings discussed in this article. Outside of argumentation, a systematic treatment for [Brewka, 1994b; Brewka, 1994a]'s prioritised default logic was given by [Delgrande and Schaub, 2000], who showed that prioritised default theories can be translated into equivalent ordinary default theories. In Section 4.5 we will discuss the relation between prioritised default logic and $ASPIC^+$.

In general, the question as to whether $ASPIC^+$ argument orderings can be encoded in $ASPIC^+$ rule sets or knowledge bases is still an open question. We conjecture that such translations may be very hard to give for argument orderings that depend on global properties of an argument, such as weakest-link orderings.

3.4.4 The need for defeasible rules

Perhaps the most controversial issue is whether defeasible inference rules are needed. In Section 2.1 we illustrated with an informal example that there are three ways to attack an argument: on its premises, on its defeasible inferences, and on the conclusions of its defeasible inferences. In Section 2.2.2 we saw that $ASPIC^+$ explicitly allows all three forms of attack. However, some would argue that the second and third type of attacks can be simulated using only deductive rules (specifically the deductive rules of classical logic) by augmenting the antecedents of these rules with normality premises. For example, with regard to the second type of attack, could we in our example of Section 2.1 not say that our argument claiming that John was in Holland Park that morning since we saw him there has an implicit premise *our senses functioned normally*, and that the argument that John was in Amsterdam that morning in fact attacks this implicit premise, rather than its claim, thus reducing attacks on conclusions to attacks on premises? With regard to the third type of attack, could we not say that instead of attacking the defeasible inference step from Jan's testimony to the claim that John was in Amsterdam, we could model this step as deductive, and then add the premise that normally witnesses speak the truth, and then direct the attack at this premise? In other words, can we reduce attacks on inferences to attacks on premises? These informal arguments have some formal backing since, as we will discuss in more detail in Section 5.2, [Dung and Thang, 2014] have shown that defeasible inference rules can in $ASPIC^+$ be reduced to strict rules.

In answer to these questions, we first claim that there is some merit in modelling the everyday practice of 'jumping to defeasible conclusions' and of considering arguments for contradictory conclusions. This is especially important given that one of the argumentation paradigm's key strengths is its characterisation of formal logical modes of reasoning in a way that corresponds with human modes of reasoning and debate.

We next note that some have argued that such deductive simulations are prone to yielding counterintuitive results. To illustrate, consider a instantiation of $ASPIC^+$ with no defeasible rules and in which the strict rules correspond to classical propositional logic as defined in Section 3.1.2, and assume that natural-language generalisations 'If P then normally Q' are formalised as material implications $P \supset Q$ in $\mathcal{K}_p$. The idea is that since $P \supset Q$ is an ordinary premise, its use as a premise can be undermined in exceptional cases. Observe that by classical reasoning we then have a strict argument for $\neg Q \supset \neg P$. Some say that this is problematic. Consider the following example: 'This alarm in this building usually does not give false alarms', so (strictly) 'false alarms in this building are usually not given by this alarm'. This strikes some as counterintuitive, since the first generalisation is consistent with the situation that this alarm is the only one in the building that gives false alarms, so the contraposition of 'If P then normally Q' cannot be deductively valid.

A more refined classical approach is to give the material implication an extra normality condition N, which informally reads as 'everything is normal as regards P implying Q', and which is also put in $\mathcal{K}_p$. The idea then is that exceptional cases give rise to underminers of N. However, $(P \wedge N) \supset Q$ also deductively contraposes, namely, as $(\neg Q \wedge N) \supset \neg P$, so we still have the controversial deductive validity of contraposition for generalisations. In the false-alarm example the contraposition of the rule with the added normality condition would read: 'any false alarm in this building which is usual with respect to false alarms in this building cannot be this alarm', which is clearly not deductively entailed by the initial generalisation given that it is consistent with the situation that this alarm is the only one in the building that gives false alarms.

One way to argue why classical simulations may give counter-intuitive results is to recall that a number of researchers provide statistical semantics for defeasible inference rules. These semantics regard a defeasible rule of the form $P \Rightarrow Q$ as a qualitative approximation of the statement that the conditional probability of Q, given P, is high. The laws of probability theory then tell us that this does not entail that the conditional probability of $\neg P$, given $\neg Q$, is high. The problem with the classical-logic approach is then that it conflates this distinction by turning the conditional probability of Q given P into the unconditional probability of $P \supset Q$, which then has to be equal to the unconditional probability of $\neg Q \supset \neg P$.

So far we have argued that contrapositive inferences with defeasible conditionals cannot be deductively valid (for a more detailed argument see [Modgil and Prakken, 2014, Section 4.5]). One way to respect this is to formalise defeasible natural-language conditionals as domain-specific defeasible inference rules in $ASPIC^+$ (see Section 3.1.3 above and in more detail [Modgil and Prakken, 2014]). However, this makes it hard to capture some logical properties of defeasible conditionals. For example, it might be argued that modus tollens and contraposition, although deductively invalid, are still defeasibly valid. For instance, in crime investigations the police often reason: if this person was at the crime, then we must be able to find his DNA at the crime scene; we have not been able to find his DNA at the crime scene, so presumably he was not at the crime scene. This seems a perfectly rational way of reasoning, provided that the modus-tollens inference is regarded as defeasible. Perhaps this can be captured by formalising generalisations with a defeasible object level connective $\leadsto$, as discussed above in Section 3.1.3 and by adding the appropriate strict and defeasible inference rules for $\leadsto$ to $\mathcal{R}_s$ and $\mathcal{R}_d$. For example, defeasible modus tollens could be added as follows:

$$\neg\psi, \varphi \leadsto \psi \Rightarrow \neg\varphi$$

However, doing so is not straightforward, since the above encoding of the defeasible modus pollens principle is in the form of an inference rule used in construction of $ASPIC^+$ arguments, while in contrast, the current nonmonotonic logics for defeasible conditionals model such principles at the level of the consequence relation (which in $ASPIC^+$ is defined in terms of the outcome of argument evaluation; cf. Definition 2.18 above). This suggests the following topic for future research: how to instantiate the sets of strict and defeasible rules in $ASPIC^+$ in such a way that the semantic insights on defeasible conditionals obtained in other areas of nonmonotonic logic are respected?

So far our discussion has focused on argumentation based reasoning as it applies to beliefs (i.e., reasoning about what is the case, often called *epistemic reasoning* by philosophers). However argumentation is often about what to do, prefer or value (what philosophers often call *practical reasoning*). Here too it has been argued on philosophical grounds that reasons for doing, preferring or valuing cannot be expressed in classical logic since they do not contrapose. This view can of course not be based on a statistical semantics, since statistics only applies to epistemic reasoning. Space limitations prevent us from giving more details about these philosophical arguments.

Finally, as further discussed in Section 4.1, [Dung and Thang, 2014] show for the case without preferences and knowledge bases that $ASPIC^+$ defeasible rules can be equivalently translated into theories of assumption-based argumentation (ABA). Since, as also discussed further in Section 4.1, ABA can be reconstructed as a special case of $ASPIC^+$ with no knowledge bases, defeasible rules or preferences, [Dung and Thang, 2014]'s result implies that the defeasible rules of $ASPIC^+$ SAFs with no knowledge bases or preferences can be translated into strict $ASPIC^+$ rules.

3.4.5 The value of translation results

Translation results like the ones of [Dung and Thang, 2014] and [Li and Parsons, 2015] on translating one type of rule into the other, and possible future results on encoding preferences in rules, are theoretically interesting and may have practical benefits. For example, [Dung and Thang, 2014]'s result makes it possible to use ABA tools for implementing fragments of $ASPIC^+$ without preferences. However, such translation results should be interpreted with care. Logic is full of such results and they do not necessarily mean that the translated system is less useful or less interesting. For example, nobody would say that the fact that all connectives of propositional logic can be translated into a single one means that presentations of propositional logic with the usual five or six connectives are unnecessarily complicated; on the contrary, versions with just one connective would lead to unnecessarily complex knowledge representations. Likewise, versions of $ASPIC^+$ with both strict and defeasible rules and with preferences may lead to more compact and more natural representations. Moreover, nobody would say that translations of modal logic into first-order predicate logic show that modal logic is superfluous. On the contrary, modal logics often provide systematic treatments of modalities in ways that their first-order translations do not. Likewise, $ASPIC^+$ provides a theory of reasoning with a combination of strict and defeasible rules and allows a general study of argumentation with preferences, something which formalisms with only strict or only defeasible rules or formalisms without preferences do not provide.

3.5 Argument schemes and critical questions

We concluded Section 3.1.3 by remarking on the use of defeasible inference rules as principles of cognition in John Pollock's work and as argument schemes in informal argumentation theory. We now illustrate how both approaches can be formalised in *ASPIC*$^+$ and how strict inference rules can also be accommodated when doing so.

John Pollock formalised defeasible rules for reasoning patterns involving perception, memory, induction, temporal persistence and the statistical syllogism, as well as undercutters for these reasons. In *ASPIC*$^+$ his principles of perception and memory can be written as follows:

$$d_p(x,\varphi): \quad \texttt{Sees}(x,\varphi) \Rightarrow \varphi$$
$$d_m(x,\varphi): \quad \texttt{Recalls}(x,\varphi) \Rightarrow \varphi$$

In fact, these defeasible inference rules are schemes for all their ground instances (that is, for any instance where x and φ are replaced by ground terms denoting a specific perceiving agent and a specific perceived state of affairs). Therefore, their names $d_p(x,\varphi)$ and $d_m(x,\varphi)$ as assigned by the n function are in fact also schemes for names. A proper name is obtained by instantiating these variables by the same ground terms as used to instantiate these variables in the scheme. Thus it becomes possible to formulate undercutters for one instance of the scheme (say for Jan who saw John in Amsterdam) while leaving another instance unattacked (say for Bob who saw John in Holland Park). Note, finally, that these schemes assume a naming convention for formulas in a first-order language, since φ is a term in the antecedent while it is a well-formed formula in the consequent. In the remainder we will leave this naming convention implicit.

Now undercutters for d_p state circumstances in which perceptions are unreliable, while undercutters of d_m state conditions under which memories may be flawed. For example, a well-known cause of false memories of events is that the memory is distorted by, for instance, seeing pictures in the newspaper or watching a TV programme about the remembered event. A general undercutter for distorted memories could be

$$u_m(x,\varphi): \quad \texttt{DistortedMemory}(x,\varphi) \Rightarrow \neg d_m(x,\varphi)$$

combined with information such as

$$\forall x,\varphi(\texttt{SeesPicturesAbout}(x,\varphi) \supset \texttt{DistortedMemory}(x,\varphi))$$

Pollock's epistemic inference schemes are in fact a subspecies of argument schemes. The notion of an argument scheme was developed in philosophy and is currently an important topic in the computational study of argumentation. Argument schemes are stereotypical non-deductive patterns of reasoning, consisting of a set of premises and a conclusion that is presumed to follow from them. Uses of argument schemes are evaluated in terms of critical questions specific to the scheme. An example of an epistemic argument scheme is the scheme from the position to know [Walton, 1996, pp. 61–63]:

$$\frac{\begin{array}{l} A \text{ is in the position to know whether } P \text{ is true} \\ A \text{ asserts that } P \text{ is true} \end{array}}{P \text{ is true}}$$

Walton gives this scheme three critical questions:

1. Is A in the position to know whether P is true?
2. Did A assert that P is true?
3. Is A an honest (trustworty, reliable) source?

A natural way to formalise reasoning with argument schemes is to regard them as defeasible inference rules and to regard critical questions as pointers to counterarguments. For example, in the scheme from the position to know, questions (1) and (2) point to underminers (of, respectively, the first and second premise) while question (3) points to undercutters (the exception that the person is for some reason not credible).

Accordingly, we formalise the position to know scheme and its undercutter as follows:

$$\begin{array}{ll} d_w(x,\varphi): & \texttt{PositionToKnow}(x,\varphi), \texttt{Says}(x,\varphi) \Rightarrow \varphi \\ u_w(x,\varphi): & \neg\texttt{Credible}(x) \Rightarrow \neg d_w(x,\varphi) \end{array}$$

We will now illustrate the modelling of both Pollock's defeasible reasons and Walton's argument schemes with our example from Section 2.1, focusing on a specific class of persons who are in the position to know, namely, witnesses. In fact, witnesses always report about what they observed in the past, so they will say something like "I remember that I saw that John was in Holland Park". Thus an appeal to a witness testimony involves the use of three schemes: first the position to know scheme is used to infer that the witness indeed remembers that he saw that John was in Holland Park, then the memory scheme is used to infer that he indeed saw that John was in Holland Park, and finally, the perception scheme is used to infer that

John was indeed in Holland Park. Now recall that John was a suspect in a robbery in Holland Park, that Jan testified that he saw John in Amsterdam on the same morning, and that Jan is a friend of John. Suppose now we also receive information that Bob read newspaper reports about the robbery in which a picture of John was shown. One way to model this in $ASPIC^+$ is as follows.

The knowledge base consists of the following facts (since we don't want to dispute them, we put them in $\mathcal{K}_n$):

f_1: $\texttt{PositionToKnow}(Bob, \texttt{Recalls}(Bob,$
$\texttt{Sees}(Bob, \texttt{InHollandPark}(John))))$
f_2: $\texttt{Says}(Bob, \texttt{Recalls}(Bob, \texttt{Sees}(Bob, \texttt{InHollandPark}(John))))$
f_3: $\texttt{SeesPicturesAbout}(Bob, \texttt{Sees}(Bob, \texttt{InHollandPark}(John)))$
f_4: $\forall x, \varphi.(\texttt{SeesPicturesAbout}(x, \varphi) \supset \texttt{DistortedMemory}(x, \varphi))$
f_5: $\forall x.\texttt{InHollandPark}(x) \supset \texttt{InLondon}(x)$
f_6: $\texttt{PositionToKnow}(Jan, \texttt{Recalls}(Jan, \texttt{Sees}(Jan, \texttt{InAmsterdam}(John))))$
f_7: $\texttt{Says}(Jan, \texttt{Recalls}(Jan, \texttt{Sees}(Jan, \texttt{InAmsterdam}(John))))$
f_8: $\texttt{Friends}(Jan, John)$
f_9: $\texttt{SuspectedRobber}(John)$
f_{10}: $\forall x, y, \varphi.\texttt{Friends}(x, y) \wedge \texttt{SuspectedRobber}(y) \wedge$
$\texttt{InvolvedIn}(y, \varphi) \supset \neg\texttt{Credible}(x)$
f_{11}: $\texttt{InvolvedIn}(John, \texttt{Recalls}(Jan, \texttt{Sees}(Jan, \texttt{InAmsterdam}(John))))$
f_{12}: $\forall x \neg(\texttt{InAmsterdam}(x) \wedge \texttt{InLondon}(x))$

Combining this with the schemes from perception, memory and position to know, we obtain the following arguments (for reasons of space we don't list separate lines for arguments that just take an item from $\mathcal{K}$).

A_3: $f_1, f_2 \Rightarrow_{dw} \texttt{Recalls}(Bob, \texttt{Sees}(Bob, \texttt{InHollandPark}(John)))$
A_4: $A_3 \Rightarrow_{dm} \texttt{Sees}(Bob, \texttt{InHollandPark}(John))$
A_5: $A_4 \Rightarrow_{dp} \texttt{InHollandPark}(John)$
A_7: $A_5, f_5 \rightarrow \texttt{InLondon}(John)$

This argument is undercut (on A_4) by the following argument applying the undercutter for the memory scheme:

B_3: $f_3, f_4 \rightarrow \texttt{DistortedMemory}(Bob, \texttt{Sees}(Bob, \texttt{InHollandPark}(John)))$
B_4: $B_3 \Rightarrow_{um} \neg d_m(Bob, \texttt{Sees}(Bob, \texttt{InHollandPark}(John)))$

Moreover, A_7 is rebutted (on A_5) by the following argument:

C_3: $f_6, f_7 \Rightarrow_{dw} \texttt{Recalls}(Jan, \texttt{Sees}(Jan, \texttt{InAmsterdam}(John)))$
C_4: $C_3 \Rightarrow_{dm} \texttt{Sees}(Jan, \texttt{InAmsterdam}(John))$
C_5: $C_4 \Rightarrow_{dp} \texttt{InAmsterdam}(John)$
C_8: $C_5, f_5, f_{12} \rightarrow \neg\texttt{InHollandPark}(John)$

This argument is also undercut, namely on C_3, based on the undercutter of the position to know scheme:

D_4: $f_8, f_9, f_{10}, f_{11} \rightarrow \neg\texttt{Credible}(Jan)$
D_5: $D_4 \Rightarrow_{uw} \neg d_w(Jan, \texttt{Recalls}(Jan, \texttt{Sees}(Jan, \texttt{InAmsterdam}(John))))$

Finally, C_8 is rebutted on C_5 by the following continuation of argument A_7:

A_8: $A_7, f_5, f_{12} \Rightarrow \neg\texttt{InAmsterdam}(John)$

A_8 is in turn undercut by B_4 (on A_4) and rebutted by C_8 (on A_5).

Because of the two undercutting arguments, neither of the testimony arguments are credulously or sceptically justified in any semantics. Let us now see what happens if we do not have the two undercutters. Then we must apply preferences to the rebutting attack of C_8 on A_5 and to the rebutting attack of A_8 on C_5. As it turns out, exactly the same preferences have to be applied in both cases, namely, those between the three defeasible-rule applications in the respective arguments. And this is what we intuitively want.

Finally, we note that counterarguments based on critical questions of argument schemes may themselves apply argument schemes. For example, we may believe that Jan and John are friends because another witness told us so. Or we may believe that Holland Park is in London because a London taxi driver told us so (an application of the so-called expert testimony scheme).

4 Relationship with other Argumentation Formalisms

As shown in various publications on $ASPIC^+$, its generality allows the reconstruction of various other systems and frameworks as special cases of $ASPIC^+$. In this section we review this work in some detail. We also discuss the relationship of $ASPIC^+$ with various developments of abstract argumentation frameworks.

4.1 Assumption-based argumentation

Assumption-based argumentation (ABA) emerged from attempts to give an argumentation-theoretic semantics to logic-programming's negation as failure, and has developed into a general framework for nonmonotonic logics [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997; Toni, 2014]. ABA assumes a 'deductive system', consisting of a set of inference rules defined over some logical language. Given a set of so-called 'assumptions' formulated in the logical language, arguments are then deductions of claims using rules and supported by sets of assumptions. In general, ABA leaves both the logical language and set of inference rules unspecified, so that like $ASPIC^+$, it is an abstract framework for structured argumentation. However, unlike $ASPIC^+$, ABA only allows attacks on an argument's assumptions, so that ABA's rules are effectively strict inference rules. In order to express conflicts between arguments, ABA makes a minimum assumption on the logical language, namely, that each assumption has a contrary. That b is a contrary of a, written as $b = \overline{a}$, informally means that b contradicts a. An argument using an assumption a is then attacked by any argument for conclusion $\overline{a}$. In [Bondarenko *et al.*, 1997] an argumentation-theoretic semantics is then given which is very much like [Dung, 1995]'s abstract approach, except that [Bondarenko *et al.*, 1997] considers sets of assumptions rather than sets of arguments. However, [Dung *et al.*, 2007] showed that an equivalent fully argument-based formulation can be given.

In this section we first discuss how ABA can be reconstructed in $ASPIC^+$ and then how some instantiations of $ASPIC^+$ can be reconstructed in ABA.

4.1.1 Reconstructing ABA in $ASPIC^+$

Above we remarked that [Bondarenko *et al.*, 1997]'s version of ABA is strictly speaking not an instantiation of [Dung, 1995]'s abstract argumentation frameworks but that [Dung *et al.*, 2007] gave an equivalent formulation of ABA in such frameworks. [Prakken, 2010] showed that this reconstructed version of ABA can in turn be reconstructed as a special case of $ASPIC^+$ extended with possibly non-symmetric negation (see Section 2.3 above). In $ASPIC^+$ as defined by [Prakken, 2010], the ordinary premises were further divided into 'really' ordinary premises and assumptions and the assumption premises were used to model ABA assumptions. However, as observed by [Modgil and Prakken, 2013, Section 3.1], one can do without such specialised premises and model assumptions as ordinary premises. ABA can then be reconstructed as the special case of $ASPIC^+$ with empty sets of defeasible rules and axiom premises and no preferences.

First the main definitions of ABA are recalled.

Definition 4.1 (Def. 2.3 of [Dung *et al.*, 2007])**.** *A* deductive system *is a pair* $(\mathcal{L}, \mathcal{R})$ *where*

- $\mathcal{L}$ *is a formal language consisting of countably many sentences, and*

- *$\mathcal{R}$ is a countable set of inference rules of the form $\alpha_1, \ldots, \alpha_n \to \alpha$.*[13] *$\alpha \in \mathcal{L}$ is called the* conclusion *of the inference rule, $\alpha_1, \ldots, \alpha_n \in \mathcal{L}$ are called the* premises *of the inference rule and $n \geq 0$.*

Definition 4.2 (Def. 2.5 of [Dung *et al.*, 2007])**.** *An* assumption-based argumentation framework *(ABF) is a tuple $(\mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-})$ where*

- *$(\mathcal{L}, \mathcal{R})$ is a deductive system.*
- *$\mathcal{A} \subseteq \mathcal{L}$, $\mathcal{A} \neq \emptyset$. $\mathcal{A}$ is the set of* assumptions*.*
- *If $\alpha \in \mathcal{A}$, then there is no inference rule of the form $\alpha_1, \ldots, \alpha_n \to \alpha \in \mathcal{R}$.*
- *$^{-}$ is a total mapping from $\mathcal{A}$ into $\mathcal{L}$. $\overline{\alpha}$ is the* contrary *of α.*

ABA arguments are then defined in terms of deductions. To remain as close as possible to $ASPIC^{+}$, we here give the tree-based definition of [Toni, 2014] (with some minor stylistic rephrasings). The proofs of [Prakken, 2010] instead use [Dung *et al.*, 2007]'s sequence-based definition, which essentially presents one particular order in which a tree-style argument can be constructed.

Definition 4.3 ([Toni, 2014])**.** *A* deduction *for a conclusion α supported by premises $S \subseteq \mathcal{L}$ is a finite tree with nodes labelled by sentences in $\mathcal{L}$ or by τ*[14]*. Each leaf is either τ or a sentence in S. each non-leave α' has, as children, the elements of the body of some rule in $\mathcal{R}$ with head α'.*

Then an assumption-based argument is defined as follows.

Definition 4.4 (Def. 2.6 of [Dung *et al.*, 2007])**.** *An* argument *for a conclusion on the basis of an ABF is a deduction of that conclusion whose premises are all assumptions (in $\mathcal{A}$).*

As for notation, the existence of an argument for a conclusion α supported by a set of assumptions A is denoted by $A \vdash \alpha$, or by $A \vdash_{ABF} \alpha$ if it has to be distinguished from the existence of a strict argument according to Definition 2.4 with the same premises and conclusion; the latter will below be denoted by $A \vdash_{AT} \alpha$.

Finally, Dung et al.'s notion of argument attack is defined as follows.

Definition 4.5 (Def. 2.7 of [Dung *et al.*, 2007])**.**

- *An argument $A \vdash \alpha$* attacks *an argument $B \vdash \beta$ if and only if $A \vdash \alpha$ attacks an assumption in B;*

[13] In [Dung *et al.*, 2007] the arrows are from right to left.
[14] τ represents 'true' and stands for the empty body of rules.

- *an argument* $A \vdash \alpha$ attacks an assumption β *if and only if* α *is the contrary* $\overline{\beta}$ *of* β.

The $ASPIC^+$ argumentation theory corresponding to an assumption-based argumentation framework is then in [Prakken, 2010] defined as follows.[15]

Definition 4.6 (Mapping ABFs to ATs)**.** *Given an assumption-based argumentation framework* $ABF = (\mathcal{L}_{ABF}, \mathcal{R}_{ABF}, \mathcal{A}, {}^{-}{}_{ABF})$, *the corresponding argumentation theory* $AT_{ABF} = (AS, \mathcal{K})$, *where* $AS = (\mathcal{L}_{AT}, {}^{-}{}_{AT}, \mathcal{R}_{AT}, n)$ *and* $\mathcal{K} = K_n \cup \mathcal{K}_p$, *is defined as follows:*

- $\mathcal{L}_{AT} = \mathcal{L}_{ABF}$
- $\varphi \in \overline{\psi}_{AT}$ *iff* $\varphi = \overline{\psi}_{ABF}$
- $\mathcal{R}_{AT} = \mathcal{R}_s = \mathcal{R}_{ABF}$
- $\mathcal{K}_n = \emptyset$
- $\mathcal{K}_p = \mathcal{A}$
- n *is undefined.*

Then it can be shown that for all ABFs: there exists an argument $A \vdash_{ABF} \alpha$ if and only if there exists an argument $A \vdash_{AT} \alpha$. From this it follows for all ABFs and for every argument $A \vdash_{ABF} \alpha$ and every argument $A \vdash_{AT} \alpha$: $A \vdash_{ABF} \alpha$ is attacked by an argument $B \vdash_{ABF} \beta$ if and only if $A \vdash_{AT} \alpha$ is defeated by an argument $B \vdash_{AT} \beta$. Then the main correspondence result can be proven:

Theorem 4.7 (Thm. 8.8 of [Prakken, 2010])**.** *For all ABFs, and for any semantics S subsumed by complete semantics and any set E:*

1. *if E is an S-extension of ABF then E_{AT} is an S-extension of AT, where* $E_{AT} = \{A \vdash_{AT} \alpha \mid A \vdash_{ABF} \alpha \in E\}$;
2. *if E is an S-extension of AT then E_{ABF} is an S-extension of ABF, where* $E_{ABF} = \{A \vdash_{ABF} \alpha \mid A \vdash_{AT} \alpha \in E\}$.

[15] In fact, in [Prakken, 2010] the ABA assumptions were translated into $ASPIC^+$ assumption-type premises, which in [Prakken, 2010] was an additional category of premises. However, as remarked by [Modgil and Prakken, 2013], the translation also succeeds when defined as below.

Theorem 4.7 says that there is a one-to-one correspondence between the extensions of an ABF and those of its corresponding AT. Note also that the above results carry over to [Verheij, 2003]'s DefLog argumentation system since, as observed by Verheij, DefLog can be translated into ABA.

One virtue of this reconstruction of ABA in $ASPIC^+$ is that one can then identify conditions under which ABA satisfies rationality postulates (by requiring, for instance, that the strict rules are closed under transposition).

4.1.2 Reconstructing instantiations of $ASPIC^+$ in ABA

[Dung and Thang, 2014] have shown that their rule-based systems, which are a special case of $ASPIC^+$ with no knowledge base and no preferences, can be translated into ABA instantiations. They do this by translating every defeasible rule $p_1, \ldots, p_n \Rightarrow q$ as a strict rule $d_i, p_1, \ldots, p_n, not\neg q \rightarrow q$, where

- $d_i = n(p_1, \ldots, p_n \Rightarrow q)$ in $ASPIC^+$;
- $d_i, not\neg q \in \mathcal{A}$ (i.e., they are ABA assumptions);
- $q = \overline{not\neg q}$ and for all φ: $\varphi = \overline{\neg\varphi}$ and $\neg\varphi = \overline{\varphi}$

[Dung and Thang, 2014] then show (on the assumption that $ASPIC^+$ rule names do not occur as antecedents or consequents in $ASPIC^+$ rules), that for each semantics subsumed by complete semantics the resulting ABA framework validates the same conclusions as the original $ASPIC^+$ SAF. Generalising this result to cases with preferences is still an open question.

4.2 Tarskian abstract logics and classical-logic argumentation

[Amgoud and Besnard, 2013] present an abstract approach to defining the structure of arguments and attacks, based on Tarski's notion of an abstract logic that only assumes some unspecified logical language $\mathcal{L}$, and a consequence operator over this language, which to each subset of $\mathcal{L}$ assigns a subset of $\mathcal{L}$ (its logical consequences). Tarski then assumed a number of constraints on Cn (see [Amgoud and Besnard, 2013] for a more detailed account of these constraints). Finally, Tarski defined a set $S \subseteq \mathcal{L}$ as *consistent* iff $Cn(S) \neq \mathcal{L}$. In [Amgoud and Besnard, 2013], an argument is a pair (S, p) where $S \subseteq \mathcal{L}$ is consistent, $p \in Cn(S)$ and S is a minimal (under set inclusion) set satisfying these conditions. Then (S, p) attacks (T, q) iff $\{p, q'\}$ is inconsistent for some $q' \in T$.

[Modgil and Prakken, 2013, Section 5.2] show that $ASPIC^+$ can be used to reconstruct, and extend with preferences, the Tarskian logic approach. For the strict rules, they choose (for any finite $S \subseteq \mathcal{L}$):

$S \rightarrow p \in \mathcal{R}_s$ iff $p \in Cn(S)$

Then given any $\Sigma \subseteq \mathcal{L}$, they let $\mathcal{K}_p = \Sigma$, $\mathcal{R}_d = \emptyset$. Also, $\forall \phi \in \mathcal{L}$, ϕ has a contradictory ψ, and if $\phi = -\psi$ then $Cn(\{\phi, \psi\}) = \mathcal{L}$ and if $Cn(\{\phi, \psi\}) = \mathcal{L}$ then $\exists \phi' \in Cn(\{\phi\})$ s.t. $\phi' = -\psi$. They then show that given a reasonable argument preference ordering $\preceq$ (possibly defined on the basis of an ordering $\leq'$ over Σ), the c-SAF is well defined. Hence one obtains an account of [Amgoud and Besnard, 2013]'s Tarskian logic abstract argumentation approach that is extended with preferences and is well behaved with respect to rationality postulates. Two issues to note are that the reconstruction employs $ASPIC^+$ undermining attacks, which differ from the abstract logic attacks defined above which rely on showing that the claim and attacked premises are inconsistent. However, [Modgil and Prakken, 2013] show that the use of $ASPIC^+$ attacks does not change the outcome in the sense that the complete (and hence grounded, preferred and stable) extensions remain the same irrespective of whether we use the abstract logic notion of an attack instead. Moreover, $ASPIC^+$ imposes no subset minimality conditions on the premises of arguments. However, [Modgil and Prakken, 2013] show that if subset minimal arguments are not strengthened by adding 'irrelevant' premises – i.e., if A is subset minimal and $A \not\prec B$ then $A' \not\prec B$ where $\texttt{Prem}(A') \supset \texttt{Prem}(A)$ – then the conclusions of arguments in complete extensions remains the same whether or not we exclude arguments that are not subset minimal.

[Modgil and Prakken, 2013] then applied this to a reconstruction of so-called classical argumentation [Cayrol, 1995; Besnard and Hunter, 2001; Besnard and Hunter, 2008; Gorogiannis and Hunter, 2011], which formalises arguments as minimal classical consequences from consistent and finite premise sets in standard propositional or first-order logic. In particular, [Gorogiannis and Hunter, 2011] study classical logic instantiations of abstract argumentation frameworks. [Modgil and Prakken, 2013] reconstruct this as a specific instance of the above formulation of the Tarskian abstract logic approach, with Cn the classical consequence operator (below denoted as $\models$). This yields the following instantiation of $ASPIC^+$:

Definition 4.8 (Classical argumentation with preferences reconstructed in $ASPIC^+$)**.** *Let $\mathcal{L}'$ be a classical-logic language, $\Sigma \subseteq \mathcal{L}'$ and $\leq'$ a partial preorder on Σ. A* classical-logic argumentation theory *based on $(\mathcal{L}', \Sigma, \leq')$ is a pair $(AS, \mathcal{K})$ such that AS is an argumentation system $(\mathcal{L}, {}^-, \mathcal{R}, n)$ where:*

1. *$\mathcal{L} = \mathcal{L}'$;*
2. *$\varphi \in \overline{\psi}$ iff $\varphi = \neg\psi$ or $\psi = \neg\varphi$;*
3. *$\mathcal{R}_d = \emptyset$, and for all finite $S \subseteq \mathcal{L}$ and $p \in \mathcal{L}$, $S \rightarrow p \in \mathcal{R}_s$ iff $S \models p$.*

$\mathcal{K}$ is a knowledge base such that $\mathcal{K}_n = \emptyset$ and $\mathcal{K}_p = \Sigma$.
$(\mathcal{A}, \mathcal{C}, \preceq)$ is the c-SAF *based on $(AS, \mathcal{K})$ as defined in Definition 2.14 and where $\preceq$ is defined in terms of $\leq'$ as in Section 3.2.*

[Gorogiannis and Hunter, 2011] define seven attack relations and prove that only the following two ensure satisfaction of the rationality postulate of indirect consistency:

- Y *directly undercuts* X if $\texttt{Conc}(Y) \equiv \neg p$ for some $p \in \texttt{Prem}(X)$
- Y *directly defeats* X if $\texttt{Conc}(Y) \vdash_c \neg p$ for some $p \in \texttt{Prem}(X)$

Since classical logic can be specified as a Tarskian abstract logic, [Modgil and Prakken, 2013] can prove via their reconstruction of abstract-logic argumentation, that the *ASPIC*$^+$ notion of undermining attacks is equivalent to direct undercuts and defeats in that the complete extensions generated are the same. Moreover, from the results described above in Section 3.2 it follows that their extension of classical-logic argumentation with preferences satisfies the rationality postulates. Indeed, [Modgil and Prakken, 2013] argue that the extension to include preferences is needed if classical-logic argumentation is to be effectively used in arbitrating amongst conflicts, since as shown in ([Cayrol, 1995; Gorogiannis and Hunter, 2011; Amgoud and Besnard, 2013]), there is a one-to-one correspondence between the (premises of arguments in in) preferred/stable extensions of abstract argumentation frameworks instantiated by a classical-logic knowledge base and the maximal consistent subsets of the knowledge base. This is to be expected, given the monotonicity of classical logic (and thus the absence of logical mechanisms to withdraw previously derivable contradictory inferences).

4.3 Carneades

As shown by [Van Gijzel and Prakken, 2011; Van Gijzel and Prakken, 2012], the Carneades system of [Gordon *et al.*, 2007; Gordon and Walton, 2009b] can be reconstructed as a special case of basic *ASPIC*$^+$ with a generalised contrariness relation. A Carneades argument is a triple $\langle P, E, c \rangle$ where P is a set of *premises*, E a set of *exceptions* and c the *conclusion*, which is either pro or con a *statement* s. Carneades does not assume that premises and conclusions are connected by inference rules. Also, all arguments are elementary, that is, they contain a single inference step; they are combined in recursive definitions of *applicability* of an argument and *acceptability* of its conclusion. In essence, an *argument* is *applicable* if (1) all its premises are given as facts or else are acceptable conclusions of other arguments, and (2) none of its exceptions are given as facts or as acceptable conclusions of other arguments. A *statement* is *acceptable* if it satisfies its *proof standard*. Facts are stated by an *audience*, which also provides numerical *weights* for each argument plus *thresholds* for argument weights and differences in argument weights. In the publications on Carneades five proof standards are defined. One is *preponderance of the evidence*:

> Statement p satisfies *preponderance of the evidence* iff there exists at least one applicable argument pro p for which the weight is greater than the weight of any applicable argument con p.

In the $ASPIC^+$ reconstruction of Carneades the facts are reconstructed as elements of $\mathcal{K}_n$, while the Carneades notions of applicability and acceptability are encoded in the $ASPIC^+$ defeasible inference rules. For every Carneades argument $a = \langle P, E, c \rangle$, a defeasible rule $P \Rightarrow_{app_a} arg_a$ is added, saying that if P then a is applicable[16]. Moreover, a defeasible rule $arg_a \Rightarrow_{acc_a} c$ is added, saying that if a is applicable, its conclusion is acceptable. Here, app_a and acc_a are the respective names of these rules in $\mathcal{L}$ according to the naming convention n. Thus a Carneades argument $\langle P, E, c \rangle$ pro statement s induces an $ASPIC^+$ argument:

A_1: P
A_2: $A_1 \Rightarrow_{app_a} arg_a$
A_3: $A_2 \Rightarrow_{acc_a} s$

It should be noted that effectively, a Carneades argument is analogous to a defeasible inference rule, since the representation (P, E, c) does not assume that the facts P are given as part of the argument; rather it is the *applicability* of the argument that depends on facts or arguments for P. This justifies the translation of Carneades arguments into $ASPIC^+$ defeasible rules.

Next, for each exception $e \in E$, a rule $e \Rightarrow \neg app_a$ is added to $\mathcal{R}_d$ and $\neg app_a = \overline{app_a}$ is added to the contrariness relation. So such rules can be used to undercut the $ASPIC^+$ version of an argument on its first step. Moreover, for each argument b with a conclusion c' that conflicts with s, we have that $arg_b = \overline{acc_a}$ if this is dictated by the proof standard for s. For example, if the standard for s is preponderance of the evidence, then $arg_b = \overline{acc_a}$ just in case $weight(a) \leq weight(b)$. Thus the Carneades proof standards and argument weights are not incorporated in the $ASPIC^+$ argument ordering but in the $ASPIC^+$ contrariness relation.

For example, a Carneades argument $b = \langle P', E', c' \rangle$ where c' is con s, induces an $ASPIC^+$ argument:

[16] The idea to make the applicability step explicit by means of an argument node was adapted from [Brewka and Gordon, 2010].

B_1: P'
B_2: $B_1 \Rightarrow_{app_b} arg_b$
B_3: $B_2 \Rightarrow_{acc_b} \neg s$

Then A_3 rebuts B_3 if $weight(b) < weight(a)$, B_3 rebuts A_3 if $weight(a) < weight(b)$ and both rebut each other if $weight(a) = weight(b)$. Since in the *ASPIC*$^+$ reconstruction all defeasible arguments are equally strong, all these rebutting attacks succeed as defeat.

[Van Gijzel and Prakken, 2011; Van Gijzel and Prakken, 2012] then prove that under this reconstruction, *ASPIC*$^+$ SAFs corresponding to a Carneades theory always have a unique extension, which is the same in all of [Dung, 1995]'s semantics. This perhaps surprising result is partly due to strong non-circularity assumptions made in Carneades on its 'inference graph', which contains all constructible arguments. [Van Gijzel and Prakken, 2011; Van Gijzel and Prakken, 2012] also prove that the conclusions of the justified arguments in *ASPIC*$^+$ correspond to the conclusions of the acceptable arguments in Carneades.

4.4 Defeasible Logic Programming

Defeasible logic programming (DeLP) is a logic-programming-based argumentation system originating from (but not equivalent to) [Simari and Loui, 1992]. The main publication on DeLP is [Garcia and Simari, 2004], which we will take as the basis for our discussion. Although DeLP is similar to *ASPIC*$^+$, it cannot be fully reconstructed as an instance. Elements of DeLP that instantiate *ASPIC*$^+$ are a predicate-logic literal language with ordinary negation, a set of indisputable facts, two sets of strict and defeasible rules, and a binary argument ordering. DeLP arguments can be reconstructed as *ASPIC*$^+$ arguments with the additional constraint that their sets of conclusions are consistent under application of strict rules in that for no φ it holds that $\texttt{Conc}(A) \vdash \varphi, \neg\varphi$.

DeLP's definition of attack is similar but not equivalent to *ASPIC*$^+$'s notion of rebutting attack. Instead (and translated to *ASPIC*$^+$ vocabulary), A attacks B at B's subargument B' if $\texttt{Conc}(A) \cup \texttt{Conc}(B') \vdash \varphi, \neg\varphi$ for some wff φ. Note that this allows an attack on a conclusion of a strict rule, but such an attack will never exist without an attack on a previous defeasible step in the argument as well. Apart from this difference, DeLP's notion of defeat is defined as in *ASPIC*$^+$: A defeats B if A attacks B on B' and $A \not\prec B'$. It remains to be investigated whether adopting DeLP's notion of rebutting attack in *ASPIC*$^+$ would lead to different outcomes.

A main difference with *ASPIC*$^+$ is that DeLP as defined in [Garcia and Simari, 2004] does not evaluate arguments by generating abstract argumentation frameworks. Instead, DeLP's notion of *warrant* is defined in a way that is similar to the argument game of grounded semantics [Prakken, 1999; Modgil and Caminada, 2009] but with some significant differences. Briefly, the argument game for grounded semantics is between a proponent and an opponent of an argument A, where the proponent begins with A and then the players take turns such that the opponent defeats or strictly defeats the proponent's previous argument while the proponent strictly defeats the opponent's previous argument; in addition, the proponent is not allowed to repeat his own arguments. An argument A is justified if the proponent has a winning strategy in a game starting with A. DeLP's notion of warrant is equivalent to this notion of justification but its game rules are different. First, if one player weakly defeats the previous argument then the next player must strictly defeat that argument, while if one player strictly defeats the previous argument then the next player may either weakly or strictly defeat it. Second, no player may reuse a subargument from one of its earlier moves.

It would be interesting to adopt the game rules of grounded semantics in DeLP's notion of warrant, which would then establish a clear link between DeLP and the theory of abstract argumentation. Among other things, this would facilitate the study of the satisfaction of rationality postulates in DeLP.

4.5 *ASPIC*$^+$ characterisations of non-monotonic reasoning formalisms

A key reason for the prominence of argumentation (in particular Dung's theory of abstract argumentation frameworks) in knowledge representation and reasoning, is its characterisation of non-monotonic reasoning in terms of the dialectical exchange of argument and counter-argument. Indeed, in [Dung, 1995], argumentation-based characterisations of logic programming, Reiter's [1980] Default Logic and Pollock's [1987] argumentation system are formalised. The theory thus provides foundations for reasoning by individual computational and human agents, and distributed non-monotonic reasoning ('dialogue') amongst agents.

ASPIC$^+$ continues in this tradition, formalising logic programming instantiations of abstract argumentation frameworks, whereby the defeasible rules are rules in a logic program, the strict rules and axiom premises are empty, the preference relation is empty, and (as described in Section 2.3) the ordinary premises are the negation as failure ($\sim$) assumptions in the antecedents of defeasible rules, and we define the contrary function $\forall\alpha \in \mathcal{L}$: $\alpha \in \overline{\sim\alpha}$.

Brewka's Preferred Subtheories [Brewka, 1989] can also be formalised as an instance of *ASPIC*$^+$'s formalisation of classical-logic argumentation (as outlined in Section 4.2). The arguments and attacks are defined by a base Σ of propositional classical wff equipped with a total ordering $\leq'$ which is used by the set comparison $\lhd_{\texttt{Eli}}$ to define weakest link preferences over arguments. One then obtains an argumentation-based characterisation of non-monotonic inference defined by Preferred Subtheories. The latter starts with a stratification $(\Sigma_1, \ldots, \Sigma_n)$ of the totally ordered Σ ($\alpha, \beta \in \Sigma_i$ iff $\alpha \equiv' \beta$ and $\alpha \in \Sigma_i, \beta \in \Sigma_j, i < j$ iff $\beta \in \Sigma <' \alpha \in \Sigma$). A 'preferred subtheory' (*ps*) is obtained by taking a maximal under set inclusion consistent subset of Σ_1, maximally extending this with a subset of Σ_2, and so on. Multiple individually consistent preferred subtheories may be constructed, and [Modgil and Prakken, 2013] show that each *ps* corresponds to the premises of arguments in a stable extension. Hence, α is classically entailed from a *ps* iff α is the conclusion of an argument in a stable extension. Then α is a sceptical (credulous) Preferred Subtheories inference iff α is entailed by all (respectively at least one) *ps*, iff α is sceptically (credulously) justified under the stable semantics (as defined in Definition 2.18).

More recently, *ASPIC*$^+$ has been used to provide an argumentative characterisation of Brewka's *Prioritised Default Logic* (*PDL*) [Brewka, 1994a]. *PDL* upgrades [Reiter, 1980]'s Default Logic to include a strict partial ordering $<_D$ on a finite set D of first order normal defaults of the form $\frac{\theta:\phi}{\phi}$. Then given a set W of first order formulae, and a linearisation $<^+$ of $<_D$, one iteratively applies the highest ordered default whose antecedent is in the first order closure of the result obtained in the previous iteration. Intuitively, one starts with the classical consequences E_0 of W, and then adds the consequent of the highest ordered default whose antecedent is contained in E_0. Then closure under classical consequence obtains E_1, to which one adds the consequent of the highest ordered default whose antecedent is contained in E_1, and so on, until $E_{n+1} = E_n$ is the unique extension of $(D, W, <)$. In [Young *et al.*, 2016], an *ASPIC*$^+$ *SAF* is defined in which the contrary function is defined so as to formalise classical negation, $\mathcal{R}_s$ characterises inference in first order classical logic, the axiom premises $\mathcal{K}_n$ is defined as W ($\mathcal{K}_p = \emptyset$), $\mathcal{R}_d = \{\theta \Rightarrow \phi | \frac{\theta:\phi}{\phi} \in D\}$ (with the naming function n undefined), and $<_D$ the ordering on $\mathcal{R}_d$. A linear 'structure preference' ordering $<_{SP}$ is defined, which modifies $<_D$ so as to account for the dependency amongst rules in $\mathcal{R}_d$ (i.e., for any set of rules applicable given all rules thus far applied, $<_{SP}$ picks out the $<_D$ maximal rule, and the process is repeated for the set of rules that are subsequently applicable). Then the *disjoint* elitist ordering – $\Gamma \lhd_{\texttt{DEli}} \Gamma'$ iff $\exists r \in \Gamma \setminus \Gamma', \forall r' \in \Gamma' \setminus \Gamma : r <_{SP} r'$ – is used to define an ordering over arguments according to the weakest link principle. [Young *et al.*, 2016] then show that the single extension E of $(D, W, <)$ corresponds to the conclusions of arguments in the (provably) unique stable extension of the corresponding *ASPIC*$^+$ *SAF*.

4.6 The relationship of *ASPIC*$^+$ with developments of the theory of abstract argumentation frameworks

ASPIC$^+$ is designed to generate abstract argumentation frameworks in the sense of [Dung, 1995]. Over the years, various extensions of abstract argumentation frameworks with further elements have been proposed, such as with preferences ([Amgoud and Cayrol, 1998]'s preference-based argumentation frameworks or $PAFs$), values ([Bench-Capon, 2003]'s value-based argumentation frameworks or $VAFs$), attacks on attacks ([Modgil, 2009]'s extended argumentation frameworks or $EAFs$) and abstract support relations between arguments (e.g. [Cayrol and Lagasquie-Schiex, 2009]'s bipolar argumentation frameworks or $BAFs$). The question arises as to what extent *ASPIC*$^+$ can be seen as instantiations of these frameworks. Moreover, work has recently been done on the dynamics of abstract argumentation frameworks, such as deleting or adding arguments or attacks; e.g. [Baroni and Giacomin, 2008; Baroni *et al.*, 2011b; Baumann and Brewka, 2010]. The question also arises as to what extent can the dynamics of argumentation, as studied in these works, be applied to *ASPIC*$^+$. These questions are answered in this section.

4.6.1 *E-ASPIC*$^+$: Structuring Extended Argumentation Frameworks

[Modgil, 2009] extended abstract argumentation frameworks to accommodate arguments that attack attacks, and in so doing enabled integration of arguments that express preferences over other arguments. The essential idea is that given an attack from A to B, then if the argument C expresses a strict preference for B over A, C attacks (and so invalidates the success of) the attack from A to B. A modified definition of the acceptability of arguments was defined for these *Extended Argumentation Frameworks* (*EAF*s), and [Modgil, 2009] showed that one can reconstruct [Prakken and Sartor, 1997]'s logic-programming-based argumentation system with defeasible preferences as an instance of EAFs. In this reconstruction, arguments built from rules expressing preferences over other 'object level' rules, constitute arguments expressing preferences over the arguments built from the object level rules.

However, as with Dung's original abstract argumentation frameworks, the abstract *EAF*s can in principle yield extensions that violate the rationality postulates. Hence [Modgil and Prakken, 2010] define a version of *ASPIC*$^+$ – E-*ASPIC*$^+$ – that generate a special class of *bounded hierarchical EAF*s in which the finite arguments $\mathcal{A}$ can be stratified into $\mathcal{A}_1, \ldots, \mathcal{A}_n$, such that if $C \in \mathcal{A}_i$ ($i \neq 1$) expresses a preference for B over A, then $A, B \in \mathcal{A}_{i-1}$. As in *ASPIC*$^+$ arguments are constructed from strict and defeasible rules, and axiom and ordinary premises, and in addition to the usual notions of attack, E-*ASPIC*$^+$ defines a function over sets of arguments

$\mathcal{A}' \subseteq \mathcal{A}$, that maps $\mathcal{A}'$ to a strict preference over some $B, A \in \mathcal{A}$. In this way, *EAF*s are conservatively modified to allow for attacks on attacks to originate from *sets of*, rather than *single*, arguments. As well as the notion of a well-defined *SAF*[17] [Modgil and Prakken, 2010] additionally identify a condition that if $\mathcal{A}' \subseteq \mathcal{A}$ expresses that $A \prec B$ and $\mathcal{A}'' \subseteq \mathcal{A}$ expresses that $B \prec A$, then $\mathcal{A}'$ and $\mathcal{A}''$ respectively contain arguments X and Y that have contradictory conclusions, or some X and Y such that X can be extended by strict rules to an argument X^+ such that X^+ and Y have contradictory conclusions. [Modgil and Prakken, 2010] then show that the generated *bounded hierarchical EAF*s satisfy [Caminada and Amgoud, 2007]'s rationality postulates.

4.6.2 Abstract support relations

There have been several recent proposals to extend abstract argumentation frameworks with abstract support relations, such as [Cayrol and Lagasquie-Schiex, 2005; Cayrol and Lagasquie-Schiex, 2009; Cayrol and Lagasquie-Schiex, 2013]'s Bipolar Argumentation Frameworks (*BAFs*), the work of [Martinez *et al.*, 2006] and [Oren and Norman, 2008]'s Evidential Argumentation Systems (*EASs*). Various semantics for such frameworks have been defined, claiming to capture different notions of support. For example, [Boella *et al.*, 2010a] study semantics of what they call "deductive" support, which satisfies the constraint that if A is acceptable and A is a deductive support of B, then B is acceptable. [Nouioua and Risch, 2011] consider "necessary support", which satisfies the constraint that if B is acceptable and A is a necessary support of B, then A is acceptable.

One question is whether the *ASPIC*$^+$ notion of a subargument instantiates any of these notions. Here we first discuss [Dung and Thang, 2014]'s simple way of formalising [Nouioua and Risch, 2011] intuitions concerning necessary support, namely, by adding a binary support relation $\mathcal{S}$ on $\mathcal{A}$ to *AFs* with the sole additional constraint that if B supports C and A defeats B then A also defeats C. The semantics of the resulting abstract argumentation frameworks is simply defined by choosing one of the semantics for the corresponding pair $(\mathcal{A}, \mathcal{D})$. Thus the support relation $\mathcal{S}$ is only used to constrain the defeat relation $\mathcal{D}$. [Prakken, 2014] calls the resulting frameworks *SuppAFs* and notes that *ASPIC*$^+$ can be reconstructed as an instance of *SuppAFs* as follows. Take $\mathcal{D}$ to be *ASPIC*$^+$'s defeat relation and $\mathcal{S}$ to be *ASPIC*$^+$'s subargument relation between arguments. It is then immediate from Definitions 2.10 and 2.12 that *ASPIC*$^+$'s notion of defeat satisfies [Dung and Thang, 2014]'s constraint on $\mathcal{D}$ in terms of $\mathcal{S}$.

[17] Where the requirement that an argument ordering is reasonable is adapted to the setting of *EAF*s.

An equivalent reformulation of *SuppAFs* does make use of support relations in its semantics. In [Prakken, 2013] $ASPIC^+$ as presented above was reformulated in terms of [Pollock, 1994]'s recursive labellings, and this reformulation was abstracted to *SuppAFs* in [Prakken, 2014]. First, [Prakken, 2013] defines a notion of p-defeat (for "Pollock-defeat"), which captures direct defeat between arguments:

Definition 4.9 (p-Attack)**.** *A p-attacks B iff A p-undercuts, p-rebuts or p-undermines B, where:*

- *A p-undercuts argument B iff $\texttt{Conc}(A) = -n(r)$ and B has a defeasible top rule r.*
- *A p-rebuts argument B iff $\texttt{Conc}(A) = -\texttt{Conc}(B)$ and B has a defeasible top rule.*
- *Argument A p-undermines B iff $\texttt{Conc}(A) = -\varphi$ and $B = \varphi$, $\varphi \notin \mathcal{K}_n$.*

Definition 4.10 (p-Defeat)**.** *A p-defeats B iff:A p-undercuts B, or; A p-rebuts/p-undermines B and $A \nprec B$.*

Then [Prakken, 2013] proves that A defeats B according to Definition 2.12 iff A p-defeats B or A p-defeats a proper subargument B' of B. Now if the support relation of a SuppAF is taken to be $ASPIC^+$'s notion of an 'immediate' subargument and the defeat relation of a *SuppAF* is taken to be p-defeat, then the following definition is equivalent to [Dung, 1995]'s semantics for AFs (and so for *SuppAFs*).

Definition 4.11 (p-labellings for *SuppAFs*)**.** *Let $(\mathcal{A}, \mathcal{D}, \mathcal{S})$ be a* SuppAF *corresponding to a (c-)$SAF = (\mathcal{A}, \mathcal{D})$ where $\mathcal{D}$ is defined as p-defeat and where $\mathcal{S}$ is defined as $(A, B) \in \mathcal{S}$ iff B is of the form $B_1, \ldots, B_n \rightarrow / \Rightarrow \varphi$ and $A = B_i$ for some $1 \leq i \leq n$. Then (In, Out) is a* p-labelling *of* SuppAF *iff $In \cap Out = \emptyset$ and for all $A \in \mathcal{A}$ it holds that:*

1. *A is labelled in iff:*
   - *(a) All arguments that p-defeat A are labelled out; and*
   - *(b) All B that support A are labelled in.*
2. *A is labelled out iff:*
   - *(a) A is p-defeated by an argument that is labelled in; or*
   - *(b) Some B that supports A is labelled out.*

Exploiting the well-known correspondences between labelling- and extension-based semantics [Caminada, 2006], [Prakken, 2014] shows that the complete extensions defined thus for *SuppAFs* generated from $ASPIC^+$ with p-defeat are exactly the complete extensions of *SuppAFs* as generated above from $ASPIC^+$ with defeat.

[Prakken, 2014] also showed for preferred semantics that $ASPIC^+$ instantiates [Oren and Norman, 2008]'s evidential argumentation systems. One might expect that classical-logic instantiations of $ASPIC^+$ instantiate [Boella *et al.*, 2010a]'s version of bipolar argumentation frameworks for "deductive support". However, [Prakken, 2014] showed that this is not the case. This raises the question as to how one might instantiate [Boella *et al.*, 2010a]'s notion of deductive support.

More generally, the question arises as to the relation of the various accounts of abstract support relations with formalisms for structured argumentation. To the best of our knowledge, the only papers studying this question are [Prakken, 2014] and [Modgil, 2014]. [Modgil, 2014] discusses this issue under the assumption that arguments and their relations are constructed from a $ASPIC^+$ argumentation theory. He discusses how examples in the literature used to motivate the need for support relations essentially amount to the supporting argument A concluding some ϕ that is: 1) either a premise in the supported argument B; 2) the conclusion of a defeasible rule in B, or; 3) A provides the missing sub-argument for the *enthymeme* B (i.e., B is an incomplete argument). For example, letting A be an argument constructed from α and $\alpha \Rightarrow_{r_1} \beta$ then illustrating the three cases, B consists of: 1) β and $\beta \Rightarrow_{r_2} \delta$; 2) γ, $\gamma \Rightarrow_{r_3} \beta$ and $\beta \Rightarrow_{r_2} \delta$; 3) $\beta \Rightarrow_{r_2} \delta$.

Given this analysis, the underlying premises and rules can then be seen to generate additional arguments without the need for support relations; for example, in case 1) the additional argument $B' : A \Rightarrow_{r_2} \delta$. Hence, one would expect that the justification status of arguments obtained by the modified definitions of acceptability in abstract argumentation frameworks augmented by support relations, corresponds to their evaluation in a standard abstract argumentation framework of arguments and attacks, instantiated by the additional arguments generated by the same premises and rules. In case 1), this would mean that the status of B in the augmented framework in which B is supported by A, is the same as the status of B in the original framework consisting of A, B and B'. However, [Modgil, 2014] shows that this correspondence does not always hold[18]. He concludes from this that only when examining abstract concepts in a structured approach can one gain some insight into the appropriate use of these abstract level concepts in evaluating arguments. Indeed, [Modgil, 2014] provides a similar analysis of collective attacks [Nielsen and Parsons, 2007] and recursive attacks on attacks [Baroni *et al.*, 2011a] that have

[18]Note that [Modgil, 2014] is careful to acknowledge that these observations apply to the case where arguments and their relations are generated by instantiating sets of formulae, rather than by human authoring of arguments. He argues that in the latter context additional relations between arguments incorporated in abstract argumentation frameworks may well be warranted by human oriented uses of argument, and goes on to argue the need for complementary empirical studies of human argumentation.

been incorporated at the abstract level and that have led to modified definitions of acceptability.

4.6.3 Preference- and value-based argumentation frameworks

[Amgoud and Cayrol, 1998] added to abstract argumentation frameworks (AFs) a preference relation on $\mathcal{A}$, resulting in *preference-based argumentation frameworks* ($PAFs$), which are triples of the form $\langle \mathcal{A}, attacks, \preceq \rangle$. An argument A then *defeats* an argument B if A attacks B and $A \not\prec B$. Thus each PAF generates an AF of the form $\langle \mathcal{A}, defeats \rangle$, to which Dung's theory of AFs can be applied. [Bench-Capon, 2003] proposed a variant called *value-based argumentation frameworks* ($VAFs$), in which each argument is said to promote some (legal, moral or societal) value. Attacks in an $VAFs$ succeed only if the value promoted by the attacked argument is strictly preferred to the value of the attacking argument, according to a given ordering on the values (an *audience*).

The question arises as to what happens if $ASPIC^+$ is reformulated so as to generate $PAFs$ instead of Dung's original AFs. This can be easily done, since $ASPIC^+$ instantiations already generate a set of arguments with an attack relation and define a binary argument ordering. However, this may lead to violation of rationality postulates, even in cases where $ASPIC^+$ satisfies them.

Consider the following example from [Prakken, 2012b; Modgil and Prakken, 2013].

$$A : p$$
$$B_1 : \neg p$$
$$B_2 : B_1 \Rightarrow q$$

Here p and $\neg p$ are ordinary premises. Note that B_1 is a subargument of B_2. In $ASPIC^+$ we then have that A and B_1 directly attack each other while, moreover, A indirectly attacks B_2, since it directly attacks B_2's subargument B_1. These attack relations are displayed in Figure 5(a).

Assume next that $B_1 \prec A$ and $A \prec B_2$ (such an ordering could be the result of a last-link ordering). The PAF modelling then generates the following single defeat relation: A defeats B_1; see Figure 5(b). Then we have a single extension (in whatever semantics), namely, $\{A, B_2\}$. So not only A but also B_2 is justified. However, this violates [Caminada and Amgoud, 2007]'s rationality postulate of subargument closure of extensions, since B_2 is in the extension while its subargument B_1 is not. This problem is not restricted to subargument closure; [Prakken, 2012b] also discusses examples in which the postulate of indirect consistency is violated.

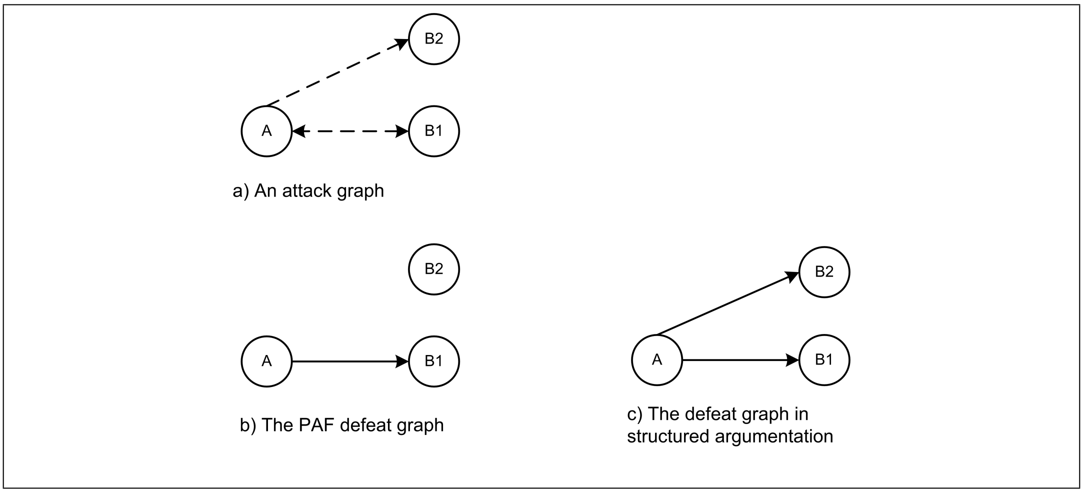

Figure 5: The attack graph

The cause of the problem is that the PAF modelling of this example cannot recognise that the reason why A attacks B_2 is that A directly attacks B_1, which is a subargument of B_2. So the PAF modelling fails to capture that in order to check whether A's attack on B_2 succeeds, we should compare A not with B_2 but with B_1. Now since $B_1 \prec A$, then in $ASPIC^+$ we also have that A defeats B_2; see Figure 5(c). So the single extension (in whatever semantics) is $\{A\}$, and so closure under subarguments is respected.

This shows that under the assumption that $PAFs$ (and also $VAFs$) are instantiated by logical formulae, then these only behave correctly with respect to the rationality postulates, if all attacks are direct. We can conclude that for a principled analysis of the use of preferences to resolve attacks, the structure of arguments must be made explicit, since the structure of arguments is crucial in determining how preferences must be applied to attacks.

A more general word of caution is in order here. Although it is tempting to extend abstract argumentation frameworks with additional elements, one should resist the temptation to think that for any given argumentation phenomenon the most principled analysis is at the level of abstract argumentation. In fact, it often is the other way around, since at the abstract level crucial notions like claims, reasons and grounds are abstracted away.

4.6.4 Dynamics of argumentation

Recently much work has been done on the nature and effects of change operations on a given argumentation state, e.g. [Modgil, 2006; Baroni and Giacomin, 2008; Rotstein *et al.*, 2008; Baumann and Brewka, 2010; Cayrol *et al.*, 2010; Boella *et al.*, 2010b; Baroni *et al.*, 2011b]. Among other things, enforcing and preservation properties are studied. Enforcement concerns the extent to which desirable outcomes can or will be obtained by changing an argumentation state, while preservation is about the extent to which the current status of arguments is preserved under change. Almost all this work is done for abstract argumentation frameworks. In particular, the following operations on abstract argumentation frameworks have been studied: addition or deletion of (sets of) arguments and addition or deletion of (sets of) attack relations. Deleting attacks can here be seen as an abstraction from the use of preferences to resolve attacks into defeats.

The question arises as to what extent this work is relevant for $ASPIC^+$. Here too our above word of caution applies. At first sight, it would seem that the most principled analysis of argumentation dynamics is at the level of abstract argumentation frameworks. However, upon closer inspection it turns out that such analyses, because they ignore the structure of arguments, often implicitly make assumptions that are not in general satisfied by $ASPIC^+$ instantiations (and neither by other formalisms for structured argumentation). For example, abstract models of argumentation dynamics do not recognise that some arguments are not attackable (such as deductive arguments with certain premises) or that some attacks cannot be deleted (for example between arguments that were determined to be equally strong), or that the deletion of one argument implies the deletion of other arguments (when the deleted argument is a subargument of another, as in Figure 5 above), or that the deletion or addition of one attack implies the deletion or addition of other attacks (for example, attacking an argument implies that all arguments of which the attacked argument is a subargument are also attacked; in Figure 5 above attacking B_1 implies attacking B_2). These considerations imply that formal results pertaining to the abstract model are only relevant for specific cases, and fail to cover many realistic situations in argumentation that can be expressed in $ASPIC^+$. To give a very simple example, in models that allow the addition of arguments and attacks, any non-selfattacking argument A can be made a member of every extension by simply adding non-attacked attackers of all A's attackers. However, this result at the abstract level does not carry over to instantiations in which not all arguments are attackable. Here too, we see the importance of being aware of what the model abstracts from.

For these reasons we have in [Modgil and Prakken, 2012] proposed a model of preference dynamics in *ASPIC*$^+$, that arguably overcomes several limitations of [Baroni *et al.*, 2011b]'s resolution-based semantics for abstract argumentation frameworks when applied to preference-based dynamics.[19] The latter allows that symmetric attacks are replaced by asymmetric attacks (i.e., the symmetric attacks are 'resolved'). We argued that from the perspective of instantiated abstract argumentation frameworks, it is the use of preferences that provides the clearest motivation for obtaining resolutions. But then studying the use of preferences at the structured *ASPIC*$^+$ level suggests that one must also account for the resolution of asymmetric attacks, that preferences may also result in removal of both attacks in a symmetric attack, and that certain resolutions may be impossible, because assuming a preference that removes one attack may necessarily imply removal of another attack, or because some attacks cannot be removed by preferences (e.g. undercut attacks and attacks on contraries). These subtleties can only be appreciated at the structured level, and are thus not addressed by the study of resolutions at the abstract level adopted by [Baroni *et al.*, 2011b], in which only resolutions of symmetric attacks are considered, and all possible resolutions are considered possible.

5 Further Developments of ASPIC+

In Section 2 we presented what we called the 'basic' *ASPIC*$^+$ framework in two stages, first with symmetric negation and then generalising it with possibly asymmetric negation. As a matter of fact, this basic framework is the result of various revisions and incremental extensions [Amgoud *et al.*, 2006; Prakken, 2010; Modgil and Prakken, 2013; Modgil and Prakken, 2014]. Also, in [Modgil and Prakken, 2013], the basic framework in fact comes in four variants, resulting from whether the premises of arguments are assumed to be c-consistent or not and whether conflict-freeness is defined with the attack or the defeat relation (recall footnote 7). So instead of a single *ASPIC*$^+$ framework there in fact exists a family of such frameworks. And this family is growing. In this section we discuss recent work that modifies the *ASPIC*$^+$ framework in some respects, especially with new constraints on arguments or with modified or generalised notions of attack. We consider this development of variants of *ASPIC*$^+$ a healthy situation, since it amounts to a systematic investigation of the effects of different design choices within a common approach, which may each be applicable to certain kinds of problems.

[19] We recognise that there may be other uses of resolution-based semantics to which our criticism does not apply.

5.1 Consistency and chaining restrictions motivated by contamination problems

Some recent work on $ASPIC^+$ has studied further constraints on arguments in an attempt to address the so-called contamination problem originally discussed by [Pollock, 1994; Pollock, 1995].[20] This problem arises if the strict inference rules are chosen to correspond to classical logic and if they are then combined with defeasible rules. The problem is how the trivialising effect of the classical Ex Falso principle can be avoided when two arguments that use defeasible rules have contradictory conclusions. The problem is especially hard since any solution should arguably preserve satisfaction of the rationality postulates of [Caminada and Amgoud, 2007]. In addition, [Caminada *et al.*, 2012] claim that any solution should also satisfy a new set of postulates that are meant to express the idea that information irrelevant to a part of the argumentation system should not affect the conclusions drawn from that part.

The following abstract example illustrates the problem. Assume that the strict rules of an argumentation system correspond to classical logic, i.e. $X \rightarrow \varphi \in \mathcal{R}_s$ if and only if $X \vdash \varphi$ and X is finite (where $\vdash$ denotes classical consequence).

Example 5.1. Let $\mathcal{R}_d = \{p \Rightarrow q;\ r \Rightarrow \neg q;\ t \Rightarrow s\}$, $\mathcal{K}_p = \emptyset$ and $\mathcal{K}_n = \{p, r, t\}$, while $\mathcal{R}_s$ corresponds to classical logic. Then the corresponding abstract argumentation framework includes the following arguments:

A_1: p $\quad A_2$: $A_1 \Rightarrow q$
B_1: r $\quad B_2$: $B_1 \Rightarrow \neg q$ $\quad C$: $A_2, B_2 \rightarrow \neg s$
D_1: t $\quad D_2$: $D_1 \Rightarrow s$

Figure 6 displays these arguments and their attack relations. Argument C attacks D_2. Whether C defeats D_2 depends on the argument ordering but plausible argument orderings are possible in which $C \not\prec D_2$ and so C defeats D_2. This is problematic, since s can be any formula, so any defeasible argument unrelated to A_2 or B_2, such as D_2, can, depending on the argument ordering, be defeated by C. Clearly, this is extremely harmful, since the existence of just a single case of mutual rebutting attack, which is very common, could trivialise the system. For instance, in this example neither of A_2 nor B_2 are in the grounded extension, since they defeat each other. But then the grounded extension does not defend D_2 against C and therefore does not contain D_2.

[20] Some parts of this section have been taken or adapted from [Grooters and Prakken, 2016].

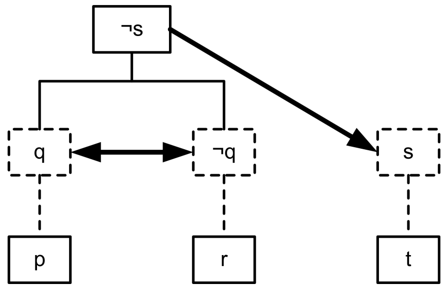

Figure 6: Illustrating trivialisation

It should be noted that simply disallowing application of strict rules to inconsistent sets of formulas does not help, since then an argument for $\neg s$ can still be constructed as follows:

A_3: $A_2 \to q \vee \neg s$
C': $A_3, B_2 \to \neg s$

Note that argument C' does not apply any strict inference rule to an inconsistent set of formulas.

[Grooters and Prakken, 2016] propose the following formalisation of the property of trivialisation.

Definition 5.2 (Trivialising argumentation systems)**.** *An argumentation system AS is* trivialising *iff for all $\varphi, \psi \in \mathcal{L}$ and all knowledge bases $\mathcal{K}$ such that $\{\varphi, \neg\varphi\} \subseteq \mathcal{K}$ a strict argument on the basis of $\mathcal{K}$ can be constructed in AS with conclusion ψ.*

The research problem then is identifying classes of non-trivialising argumentation systems. The argumentation system in our example is clearly trivialising since $\mathcal{R}_s$ contains strict rules $\varphi, \neg\varphi \to \psi$ for all $\varphi, \psi \in \mathcal{L}$.

Example 5.1 does not cause any problems for preferred or stable semantics, since A_2 and B_2 attack each other and at least one of these attacks will (with non-circular argument orderings) succeed as defeat. Therefore, all preferred or stable extensions contain either A_2 or B_2 but not both. Since both A_2 and B_2 attack C (by directly attacking one of its subarguments), C is for each preferred or stable extension defeated by at least one argument in the extension, so C is not in any of these extensions, so D_2 is in all these extensions. This is intuitively correct since there is no connection between D_2 and the arguments A_2 and B_2. [Pollock, 1994; Pollock, 1995] thought that this line of reasoning for preferred semantics suffices to show that his recursive-labelling approach (which was later in [Jakobovits and Vermeir, 1999] proved to be equivalent to preferred semantics) adequately deals with this problem. However, [Caminada, 2005] showed that the example can be extended in ways that also cause problems for preferred and stable semantics. Essentially, he replaced the facts p and r with defeasible arguments for p and r and let both these arguments be defeated by a self-defeating argument. On the one hand, such self-defeating arguments cannot be in any extension, since extensions are conflict free. However, if a self-defeating argument is not defeated by other arguments, it prevents any argument that it defeats from being acceptable with respect to an extension. In our example, if both A_2 and B_2 are defeated by a self-defeating argument that is otherwise undefeated, then neither A_2 not B_2 is in any extension, so no argument in an extension defends D_2 against C. To solve the problem, two approaches are possible. One is to change the definitions of the argumentation formalism, while the other is to derive the strict inference rules from a weaker logic than classical logic.

The first approach is taken by [Wu, 2012] and [Wu and Podlaszewski, 2015], who for the $ASPIC^+$ framework require that for each argument the set of conclusions of all its subarguments are classically consistent. They show that this solution partially works for a restricted version of $ASPIC^+$ without preferences, in that for complete semantics, both the original postulates of [Caminada and Amgoud, 2007] and the new ones of [Caminada *et al.*, 2012] are satisfied. However, their results do not cover stable, preferred or grounded semantics, while they give counterexamples to the consistency postulates for the case with preferences.

A second approach to solve the problem is to replace classical logic as the source for strict rules with a weaker, monotonic paraconsistent logic, in order to invalidate the Ex Falso principle as a valid strict inference rule. [Grooters and Prakken, 2016] explored this possibility. They first showed that two well-known paraconsistent logics, the system C_ω of [Da Costa, 1974] and the Logic of Paradox of [Priest, 1979; Priest, 1989], cannot be used for these purposes, since they induce violation of the postulate of indirect consistency. They then investigated Rescher and Manor's 1970 paraconsistent consequence notion of *weak consequence*. A set S of wff's weakly' implies a wff φ just in case at least one consistent subset of S classically implies φ. While thus initially taking the second approach, [Grooters and Prakken, 2016] had to combine it with the first approach (changing the definitions). Chaining strict rules in arguments has to be disallowed since the notion of weak consequence does not satisfy the Cut rule. For a counterexample, consider the set $\Gamma = \{a, \neg a \wedge b\}$. Then $\Gamma \vdash_W b$ and $\Gamma, b \vdash_W a \wedge b$, while it is not the case that $\Gamma \vdash_W a \wedge b$.

[Grooters and Prakken, 2016] proved that this solution avoids trivialisation and for well-behaved c-$SAFs$ satisfies all closure and consistency postulates (where the strict-closure postulate has to be changed to closure under one-step application of strict rules). Illustrating their solution with the above example, we see that the contaminating argument C cannot be constructed since its conclusion $\neg s$ follows from no consistent subset of $\{q, \neg q\}$, while the contaminating argument C' cannot be constructed since it chains two strict rules.

[Grooters and Prakken, 2016] also showed that with [Wu and Podlaszewski, 2015]'s stronger condition that the set of all conclusions of all subarguments of an argument must be consistent, consistency and strict closure are not satisfied. [Grooters and Prakken, 2016] did not attempt to prove Caminada *et al.*'s 2012 'contamination' postulates, for two reasons. First, they wanted to obtain results for all of [Dung, 1995]'s semantics and, second, they argued that Caminada *et al.*'s postulates in fact capture a stronger intuitive notion than the notion of trivialisation.

The work of [Grooters and Prakken, 2016] gives rise to some more general observations on [Caminada and Amgoud, 2007]'s original postulate of closure under strict rules. Above we suggested that $\mathcal{R}_s$ can be chosen to correspond to any monotonic logic with consequence notion $\vdash$ by letting $S \to \varphi \in \mathcal{R}_s$ if and only if $S \vdash \varphi$ and S is finite. However, the fact that the weak-consequence notion $\vdash_W$ does not satisfy the Cut rule illustrates that when $\mathcal{R}_s$ is thus defined, a system that is closed under $\mathcal{R}_s$ as defined in Section 3.1.2, could allow for inferences that are invalid according to $\vdash$. For these reasons, [Grooters and Prakken, 2016] not only reformulated their definition of strict closure but also proposed a new rationality postulate of *logical closure* and showed that their adapted version of $ASPIC^+$ also satisfies this postulate for well-behaved c-$SAFs$.

We also briefly note that [Grooters and Prakken, 2016] also studied minimality constraints on strict-rule applications and the exclusion of circular arguments. They show that if these two constraints are combined with their adoption of weak consequence as the source of the strict rules, then if both the knowledge base and the set of defeasible rules is finite, then each argument has at most a finite number of attackers, i.e., their framework generates so-called finitary argumentation frameworks in the sense of [Dung, 1995], which is computationally beneficial.

Finally, [D'Agostino and Modgil, 2016] provide a formalisation of classical argumentation with preferences in which arguments are triples (Δ, Γ, α) such that α is classically entailed by $\Delta \cup \Gamma$[21], and where Δ are the premises assumed true, and Γ the premises supposed true 'for the sake of argument'. The idea is that if a trivialising argument $(\{q, \neg q\}, \emptyset, s)$ defeats $(\{s\}, \emptyset, s) \in E$ (where E is an extension under any semantics), then $Y = (\emptyset, \{q, \neg q\}, \perp)$ defeats $X = (\{q, \neg q\}, \emptyset, s)$ (Y supposes for the sake of argument the premises of X). Moreover, since the premises whose truth Y commits to are empty, Y cannot be defeated and so can be included in any E

[21] [D'Agostino and Modgil, 2016] allow for arguments with inconsistent premises, as they argue that arguments with inconsistent premises, and hence the trivialising effect of such arguments, should be excluded dialectically (as in real-world reasoning and debate), rather than checking for consistency prior to inclusion of the argument in an abstract argumentation framework.

in order to defend $(\{s\}, \emptyset, s)$, thus negating the trivialising effect of X. [D'Agostino and Modgil, 2016] then show that under certain conditions, the consistency and closure postulates, as well as Caminada *et al.*'s additional contamination postulates are satisfied. As the authors note, an interesting direction for future research would be to see if their approach can be applied to the full *ASPIC*$^+$ framework.

5.2 Dung (2016) on rule-based argumentation systems

Recently, [Dung, 2016] has continued the formal study of [Dung and Thang, 2014]'s rule-based argumentation systems. Recall that these comprise of strict and defeasible inference rules over a propositional literal language, where axiom, respectively ordinary, premises p are simulated with rules $\rightarrow p$ and $\Rightarrow p$. [Dung, 2016] adds a transitive preference relation $\leq$ on $\mathcal{R}_d$, so that he defines rule-based systems as a triple $(\mathcal{R}_s, \mathcal{R}_d, \leq)$. In addition, he confines his study to knowledge bases with a consistent strict closure. Above we explained that [Dung and Thang, 2014] adopt the *ASPIC*$^+$ definitions of argument and defeat (which they call attack) and thus effectively study a class of *ASPIC*$^+$ instantiations. [Dung, 2016] also adopts the *ASPIC*$^+$ definition of an argument and still assumes that rule-based systems generate abstract argumentation frameworks in the sense of [Dung, 1995] (in our notation $(\mathcal{A}, \mathcal{D})$). However, Dung now abstracts from particular definitions of defeat ($\mathcal{D}$) and instead defines properties that defeat relations should have, thus effectively generalising *ASPIC*$^+$ on its notion of defeat. He then studies conditions under which defeat relations satisfy these properties.

Since this work is quite recent, we confine ourselves to a brief summary and discussion. In doing so, we will replace Dung's term 'attack' with 'defeat', in order to be consistent with the terminology in this article. This replacement is justified since in [Dung, 2016] it is the attack relation in terms of which arguments are evaluated, so it plays the role of *ASPIC*$^+$'s defeat relation.

Dung introduces two new rationality postulates. His postulate for *attack monotonicity* informally says that strengthening an argument cannot eliminate an attack of that argument on another. Let us illustrate this with Figure 2, interpreting the horizontal arrows as defeat relations. Then this postulate says, for instance, that if D_4's argument C_2 for v is replaced with a necessary premise v (or in [Dung, 2016]'s case a strict rule $\rightarrow v$) or with a strict and firm argument from u to v, then the new version of D_4 still defeats B_2. Next, Dung's postulate of *credulous cumulativity* informally means that changing a conclusion of an argument in some extension to a necessary fact cannot eliminate that extension.

Dung then identifies several sets of conditions under which one or both of these postulates and/or the original postulates of [Caminada and Amgoud, 2007] are satisfied. For the details of these very valuable results we refer the reader to his own publication. Dung then continues by investigating several definitions of defeat in terms of the preference relation $\leq$ on $\mathcal{R}_d$ on whether they satisfy these various postulates. Since he also assumes here that strict arguments cannot be defeated, this part of his study effectively concerns instantiations of $ASPIC^+$ as defined above in Section 2.2. Here Dung obtains both positive and negative results. For example, elitist orderings as defined in [Modgil and Prakken, 2013] are shown to satisfy attack monotonicity but not credulous cumulativity and indirect consistency, while democratic orderings as defined in [Modgil and Prakken, 2013] and Definition 3.4 above are shown to satisfy credulous cumulativity and indirect consistency but not attack monotonicity. As for Dung's results on consistency, these are a special case of [Modgil and Prakken, 2013]'s results for democratic orderings but they contain counterexamples to their results for the elitist orderings. However, these counterexamples do not apply to [Prakken, 2010]'s original way to define the elitist orderings, which has been incorporated in the above Definition 3.4, or to the erratum to [Modgil and Prakken, 2013] (which is available online at `https://nms.kcl.ac.uk/sanjay.modgil/AIJfinalErratum`).

The question arises as to whether Dung's two new postulates really are desirable in general. Our answer is positive for attack monotonicity but, following [Prakken and Vreeswijk, 2002, section 4.4], negative for credulous cumulativity. The point is that strengthening a defeasible conclusion to an indisputable fact may make arguments stronger than before, which can give them the power to defeat other arguments that they did not defeat before. This may in turn result in the loss of the extension from which the conclusion was promoted to an indisputable fact. We illustrate this with [Dung, 2016]'s own example. Informally: professors normally teach, administrators normally do not teach, deans are normally professors and all deans are administrators (so with transposition anyone who is not an administrator is not a dean). The question is whether some particular dean teaches. In rules:

$$
\begin{array}{lll}
Dean \Rightarrow_{d1} Professor & Professor \Rightarrow_{d2} Teach & Administrator \Rightarrow_{d3} \neg\, Teach \\
Dean \rightarrow Administrator & \neg\, Administrator \rightarrow \neg\, Dean \rightarrow Dean &
\end{array}
$$

Assume further that $d_1 < d_3 < d_2$. We have the following arguments on whether the dean teaches:

$$
\begin{array}{llll}
A_1: & \rightarrow Dean & B_1: & \rightarrow Dean \\
A_2: & A_1 \Rightarrow_{d1} Professor & B_2: & B_1 \rightarrow Administrator \\
A_3: & A_2 \Rightarrow_{d2} Teach & B_3: & B_2 \Rightarrow_{d3} \neg\, Teach
\end{array}
$$

(A_1 and B_1 are, of course, the same argument; B_3 is called A_3 by [Dung, 2016], while he does not explicitly name A_1/B_1 and B_2.) With the elitist or democratic weakest-link ordering as defined in Definition 3.4 above, argument B_3 strictly defeats A_3, so in all semantics a unique extension is obtained in which the dean is a professor but does not teach.

Suppose now the defeasibly justified conclusion *Professor* is added as a fact. This gives rise to a new argument:

C_1: $\rightarrow$ *Professor*
C_2: $C_1 \Rightarrow_{d2}$ *Teach*

Now the elitist ordering yields that C_2 strictly defeats B_3, so again in all semantics a unique extension is obtained but now it contains that the dean teaches. So we have lost the original extension, which illustrates violation of credulous cumulativity.

In our opinion, this outcome is the intuitive one, since by adding *Professor* as a fact, we have promoted its status from a defeasibly justified conclusion to an indisputable fact; as a consequence, argument A_3 can be strengthened by replacing its defeasible subargument A_2 with the strict-and-firm subargument C_1; no wonder then that the thus strengthened argument C_2 has, unlike its weaker version A_3, the power to defeat B_3.

Despite this minor criticism, we believe that Dung's latest investigations are a very valuable addition to the study of rule-based argumentation.

5.3 Variants of rebutting attack

Several papers have considered alternative definitions of rebutting attack in which an argument can under specific conditions also be rebutted on the conclusions of strict inferences.

5.3.1 Unrestricted rebuts

In $ASPIC^+$ as presented so far, arguments can only be rebutted on conclusions of defeasible-rule applications. [Caminada and Amgoud, 2007] call this *restricted rebut*. They also study *unrestricted rebut*, which allows rebuttals on the conclusion of a strict inference provided that at least one of the argument's subarguments is defeasible. Their replacement of restricted with unrestricted rebut leads to a variant of their simplified version of $ASPIC^+$ (which is in fact equivalent to [Dung and Thang, 2014]'s rule-based systems). They prove that for grounded semantics the rationality postulates are (under the usual conditions) satisfied but they provide a counterexample for stable and preferred semantics, presented above in Section 3.3 with a modification of Example 3.1.

[Caminada *et al.*, 2014] argue in favour of unrestricted rebut on the grounds that this would lead to more natural presentations of dialogues. They argue that when applying argumentation in dialogical settings, the notion of restricted rebuts sometimes forces agents to commit to statements they have insufficient reasons to believe. In abstract terms, suppose an agent Ag_1 submitting an argument A whose top rule is a strict rule $s_1 = \alpha_1, \ldots, \alpha_n \rightarrow \alpha$, where for $i = 1 \ldots n$, α_i is an ordinary premise in A or the head of a defeasible rule in A. Now suppose Ag_2 has an argument B that defeasibly concludes $\neg\alpha$. Since B does not rebut A on α, then to attack A requires that Ag_2 construct, for some $i = 1 \ldots n$, an argument B' that extends B *and the arguments concluding* α_j, $j \neq i$, with the transposition $s_1^i = \alpha_1, \ldots, \alpha_{i-1}, \neg\alpha, \alpha_{i+1}, \alpha_n \rightarrow \neg\alpha_i$. But then Ag_2 is forced to commit to her interlocutors' arguments concluding α_j, $j \neq i$, for which she has no reasons to believe.

[Caminada *et al.*, 2014] give the following concrete example.

> John: *"Bob will attend conferences AAMAS and IJCAI this year, as he has papers accepted at both conferences."*
> Mary: *"That won't be possible, as his budget of £1000 only allows for one foreign trip."*

Formally, this discussion could be modelled using an argumentation theory with $\mathcal{R}_d \supseteq \{\text{accA} \Rightarrow \text{attA};\ \text{accI} \Rightarrow \text{attI};\ \text{budget} \Rightarrow \neg(\text{attA} \wedge \text{attI})\}$ and $\mathcal{R}_s \supseteq \{\rightarrow \text{accA};\ \rightarrow \text{accI};\ \rightarrow \text{budget};\ \text{attA}, \text{attI} \rightarrow \text{attA} \wedge \text{attI}\}$.

A direct formalisation of the above arguments is then:

J_1:	$\rightarrow \text{accA}$	M_1:	$\rightarrow \text{budget}$
J_2:	$J_1 \Rightarrow \text{attA}$	M_2:	$M_1 \Rightarrow \neg(\text{attA} \wedge \text{attI})$
J_3:	$\rightarrow \text{accI}$		
J_4:	$J_3 \Rightarrow \text{attI}$		
J_5:	$J_3, J_4 \rightarrow \text{attA} \wedge \text{attI}$		

In *ASPIC*$^+$, Mary's argument does *not* attack John's argument, since the conclusion Mary wants to attack $(\text{attA} \wedge \text{attI})$ is the consequent of a strict rule. Mary can only attack John's argument by attacking the consequent of one of the defeasible rules, that is, by uttering one of the following two statements.

> Mary$'$: *"Bob can't attend AAMAS because he will attend IJCAI, and his budget does not allow him to attend both."*
> Mary$''$: *"Bob can't attend IJCAI because he will attend AAMAS, and his budget does not allow him to attend both."*

The associated formal counterarguments are as follows.[22]

M_1:	$\rightarrow$ budget		
M_2:	$M_1 \Rightarrow \neg(\text{attA} \wedge \text{attI})$		
J_3:	$\rightarrow$ accI	J_1:	$\rightarrow$ accA
J_4:	$J_3 \Rightarrow$ attI	J_2:	$J_1 \Rightarrow$ attA
M_5':	$M_2, J_4 \rightarrow \neg$attA	M_5'':	$M_2, J_2 \rightarrow \neg$attI

According to [Caminada *et al.*, 2014] the problem with this is that Mary does not know which of the two conferences Bob will attend, but $ASPIC^+$ with restricted rebut forces her to assert that Bob will attend one or the other. They argue that from the perspective of commitment in dialogue [Walton and Krabbe, 1995], this is unnatural.

[Caminada *et al.*, 2014] then define a restricted version of basic $ASPIC^+$ as presented above in Section 2.2 – which they call $ASPIC^-$ – that substitutes strict rules with empty antecedents for axiom premises, and defeasible rules with empty antecedents for ordinary premises. Moreover, $ASPIC^-$ allows unrestricted rebuts on the conclusions of strict rules. They then show that under the assumption of a *total* ordering on the defeasible rules, and assuming either the `Elitist` or `Democratic` set comparisons used in defining weakest- or last-link preferences, all of [Caminada and Amgoud, 2007]'s rationality postulates are satisfied for well-behaved $SAFs$, but only for the grounded semantics. They have thus generalised [Caminada and Amgoud, 2007]'s results for some specific cases with preferences.

5.3.2 Weak rebuts and an alternative view on the rationality postulates

[Prakken, 2016] studies a weaker version of unrestricted rebut, motivated by the general observation that deductive inferences may weaken an argument. His argument is that when a deductive inference is made from the conclusions of at least two 'fallible' (defeasible or plausible) subarguments, the deductive inference can be said to aggregate the degrees of fallibility of the individual arguments to which it is applied. This in turn means that the deductive inference may be less preferred than either of these subarguments, so that a successful attack on the deductive inference does not necessarily imply a successful attack on one of its fallible subarguments. And this in turn means that there can be cases where it is rational to accept a set of arguments that is not strictly closed and that violate indirect consistency. Note that this line of reasoning does not apply to cases where a deductive inference is applied to at

[22] Assuming $\mathcal{R}_s$ ito be closed under transposition, the fact that $\mathcal{R}_s$ contains $\text{attA}, \text{attI} \rightarrow \text{attA} \wedge \text{attI}$ implies that $\mathcal{R}_s$ also contains $\neg(\text{attA} \wedge \text{attI}), \text{attI} \rightarrow \neg\text{attA}$ and $\text{attA}, \neg(\text{attA} \wedge \text{attI}) \rightarrow \neg\text{attI}$.

most one fallible subargument: then the amount of fallibility of the new argument is exactly the same as the amount of fallibility of the single fallible argument to which the deductive inference is applied. Accordingly, [Prakken, 2016] defines *weak rebut* as allowing rebuttals on the conclusion of a strict inference, provided that the strict inference is applied to at least two fallible subarguments. Moreover, he argues that there are cases where argument orderings cannot be required to satisfy all properties of a reasonable argument ordering as defined in Definition 3.16.

[Prakken, 2016] illustrates this with the lottery paradox, a well-known paradox from epistemology, first discussed by [Kyburg, 1961]. Imagine a fair lottery with one million tickets and just one prize. If the principle is accepted that it is rational to accept a proposition if its truth is highly probable, then for each ticket T_i it is rational to accept that T_i will not win while at the same time it is rational to accept that exactly one ticket will win. If we also accept that everything that deductively follows from a set of rationally acceptable propositions is rationally acceptable, then we have two rationally acceptable propositions that contradict each other: we can join all individual propositions $\neg T_i$ into a big conjunction $\neg T_1 \wedge \ldots \wedge \neg T_{1,000,000}$ with one million conjuncts, which contradicts the certain fact that exactly one ticket will win.

Many views on this paradox exist. [Prakken, 2016] wants to formalise the view that for each individual ticket it is rational to accept that it will not win while at the same time it is not rational to accept the conjunction of these acceptable beliefs. He considers the following modelling of the lottery paradox in $ASPIC^+$. Let $\mathcal{L}$ be a propositional language built from the set of atoms $\{T_i \mid 1 \leq i \leq 1,000,000\}$. Then let X denote a well-formed formula $X_1 \veebar \ldots \veebar X_{1,000,000}$ where $\veebar$ is exclusive or and where each X_i is of one of the following forms:

- If $i = 1$ then $X_i = T_1 \wedge \neg T_2 \wedge \ldots \wedge \neg T_n$
- If $i = n$ then $X_i = \neg T_1 \wedge \neg T_2 \wedge \ldots \wedge \neg T_{n-1} \wedge T_n$
- Otherwise $X_i = \neg T_1 \wedge \ldots \wedge \neg T_{i-1} \wedge T_i \wedge \neg T_{i+1} \wedge \ldots \wedge \neg T_n$

Next we choose $\mathcal{K}_p = \{\neg T_i \mid 1 \leq i \leq 1,000,000\}$, $\mathcal{K}_n = \{X\}$, $\mathcal{R}_s$ as consisting of all propositionally valid inferences from finite sets and $\mathcal{R}_d = \emptyset$.

The following arguments are relevant for any i such that $1 \leq i \leq 1,000,000$.

$$\neg T_i \qquad \text{and} \qquad \neg T_1, \ldots, \neg T_{i-1}, \neg T_{i+1}, \ldots, \neg T_{1,000,000}, X \rightarrow T_i \text{ (call it } A_i\text{)}$$

[Prakken, 2016] then equates rational acceptability with sceptical justification (see Definition 2.18 above). Making $\neg T_i$ sceptically justified for all i requires for all i that

$A_i \prec \neg T_i$, to prevent A_i from defeating $\neg T_i$. Then we have a single extension in all semantics containing arguments for all conclusions $\neg T_i$ but not for their conjunction.

Note that adopting the above argument ordering requires that Condition (2) of Definition 3.16 of reasonable argument orderings is dropped, since it excludes such an argument ordering. On the other hand, Condition (1) of Definition 3.16 can be retained. In particular, Condition (1.iii) captures that applying a strict rule to the conclusion of a single argument A to obtain an argument A' does not change the 'preferedness' of A' compared to A. This is reasonable in general, since A and A' have exactly the same set of fallible elements (ordinary premises and/or defeasible inferences). [Prakken, 2016] calls argument orderings that satisfy Condition (1) of Definition 3.16 *weakly reasonable* argument orderings. Finally, he proposes weakened versions of the postulates of strict closure and indirect consistency, according to which these properties are only required to hold for subsets of extensions with at most one fallible argument. He then proves that if weak rebut is allowed in addition to restricted rebut and argument orderings are required to be weakly reasonable, then the original postulate of direct consistency plus the weakened postulates of strict closure and indirect consistency are satisfied if AT is closed under contraposition or transposition and $\texttt{Prem}(A) \cup \mathcal{K}_n$ is indirectly consistent.

[Prakken, 2016] concludes with some general observations on the relation between deduction and justification. He argues to have shown that preservation of truth (the definition of deductively valid arguments) does not imply preservation of rational acceptance, since truth and rational acceptance are different things. However, he also argues that deduction still plays an important role in argumentation. Deductive inference rules are still available as argument construction rules and if an argument with a strict top rule has no attackers or all its attackers are less preferred, then the argument may still be sceptically justified. The specifics of the adopted argument ordering are essential here. For instance, in the lottery paradox the argument ordering might allow that application of the conjunction rule to a small number of conclusions $\neg T_i$ is still sceptically justified.

5.4 Attacks from sets of arguments to arguments

[Baroni *et al.*, 2015] consider a variant of $ASPIC^+$ by adapting an idea originally proposed by [Vreeswijk, 1997] in the context of his 'abstract argumentation systems', which are a predecessor of $ASPIC^+$. In Vreeswijk's systems a counterargument is in fact a *set* of arguments: a set Σ of arguments is *incompatible* with an argument τ iff the conclusions of $\Sigma \cup \{\tau\}$ give rise to a strict argument for $\perp$. [Baroni *et al.*, 2015] adapt this idea to $ASPIC^+$, where the 'nodes' of the abstract argumentation frameworks generated by the modification are sets of arguments instead of individual arguments. They then prove satisfaction of [Caminada and Amgoud, 2007]'s rationality postulates under similar conditions as in [Modgil and Prakken, 2013].

[Baroni *et al.*, 2015]'s proposal is motivated by criticism of the $ASPIC^+$ treatment of generalised contrariness relations. However, we believe that they just criticise specific uses of this generalised contrariness relation and that the problems they discuss can be avoided by proper definitions of contrariness. Nevertheless, their ideas are very interesting and also apply to basic $ASPIC^+$ with ordinary negation. For example, it would be interesting to see if their variant of $ASPIC^+$ provides an alternative way to model the examples discussed by [Caminada *et al.*, 2014]. More generally, it would be interesting to see if their variant of $ASPIC^+$ can be reconstructed as generating AFs that allow attacks from sets of arguments to arguments as in e.g. [Bochman, 2003].

6 Implementations and applications

6.1 Implementations

Various implementations of instantiations of $ASPIC^+$are available online, all with domain-specific inference rules defined over literal-like languages, and with argument orderings based on rule preferences.

The original ASPIC inference engine The original inference engine from the ASPIC project (designed by Matthew South on the basis of a prototype of Gerard Vreeswijk) is available online at `http://aspic.cossac.org/`, with a demonstrator with example inputs available at `http://aspic.cossac.org/ArgumentationSystem/`. Rules can be formulated over a language with predicate-logic literals with ordinary negation. The implementation allows for choosing between restricted and unrestricted rebut. The implementation of restricted rebut deviates from its formal definition in that it also allows rebuttals between two arguments that both have a strict top rule. Arguments can be evaluated alternatively with a last- and a weakest-link argument ordering and with sceptical grounded or credulous preferred semantics.

Visser's Epistemic and Practical Reasoner Wietske Visser took the ASPIC deliverable ([Amgoud *et al.*, 2006]) as the basis for her Epistemic and Practical Reasoner (EPR), available at `http://www.wietskevisser.nl/research/epr/`. Rules can be formulated over a language of propositional literals with ordinary negation, optionally augmented with a 'desirable' modality for modelling practical reasoning. EPR implements argument games for sceptical grounded and credulous preferred semantics, as well as [Prakken, 2006]'s game for combined epistemic and practical reasoning. It also implements as an option [Prakken, 2005]'s mechanism for accrual of arguments.

ArgTech's TOAST Mark Snaith of ArgTech at the University of Dundee, Scotland, developed an implementation called TOAST ([Snaid and Reed, 2012]) based on [Prakken, 2010], available at `www.arg-tech.org/index.php/toast-an-aspic-implementation/`. Rules can be formulated over a language of propositional literals with ordinary negation plus optionally a user-specified contrariness relation. TOAST allows for argument evaluation with an elitist weakest- or last-link ordering and in grounded, preferred, stable and semi-stable semantics. Interestingly, TOAST can receive input specified in the AIF format, so that it can be connected to argumentation tools that can export to AIF ([Bex *et al.*, 2013a]). More on this will be said in the following subsection.

6.2 Logical specifications of the Argument Interchange Format

There is substantial interest in the development of argumentation support tools enabling the structuring of individual arguments and the dialogical exchange of argument in offline and online tools supporting human reasoning and debate (for example see `www.arg-tech.org`). A key aim is to then organise human authored arguments into abstract argumentation frameworks, so ensuring that the assessment of arguments is formally and rationally grounded and enabling 'mixed initiative' argumentation integrating both machine and human authored arguments [Modgil *et al.*, 2013]. These developments, as well as the burgeoning interest in logic-based models of argument, have motivated formulation of a standardised format – the *Argument Interchange Format* (*AIF*) [Chesñevar *et al.*, 2006] – for representation of human authored arguments and arguments constructed in logic.

The *AIF* is an ontology that broadly speaking distinguishes between information (propositions and sentences) and schemes which are general patterns of reasoning such as applications of inference rules, or conflict or preferences between information. Instances of these information and schemes classes constitute nodes that can be organised into *AIF* graphs representing argumentation knowledge. In [Bex *et al.*, 2013b], two-way translations are defined between *AIF* graphs and both $ASPIC^+$ and E-$ASPIC^+$ argumentation theories, and a number of information preserving properties are proved in both cases. The latter essentially prove that given certain assumptions on the given *AIF* graphs, the translation functions are identity-preserving (i.e. translating from the *AIF* graph to (E-)$ASPIC^+$ and back again yields the same graph as we started out with).

One can then translate *AIF* representations of human authored arguments and their interactions defined in the above-mentioned argumentation support tools, and translate these to instantiations of (E-)$ASPIC^+$ so enabling evaluation under Dung's semantics. This is explored in [Bex *et al.*, 2013b], in which arguments and their interactions authored in the *Rationale* tool [ter Berg *et al.*, 2009] are translated to the *AIF* and then to $ASPIC^+$ arguments, attacks and defeats. In this way, $ASPIC^+$ is placed in the wider spectrum of not just formal but also philosophical and linguistic approaches to argumentation.

6.3 Other applications of $ASPIC^+$

$ASPIC^+$ has been applied both in purely theoretical models and in implemented architectures.

6.3.1 Theoretical applications

Some theoretical applications of $ASPIC^+$ amount to the formulation of sets of argument schemes for specific forms of reasoning in $ASPIC^+$. [van der Weide *et al.*, 2011] and [van der Weide, 2011] use a combination of $ASPIC^+$ and [Wooldridge *et al.*, 2006]'s system for meta-argumentation for specifying argument schemes for reasoning about preferences in argumentation-based decision making. [Bench-Capon and Prakken, 2010] and [Bench-Capon *et al.*, 2011] formulate argument schemes for policy debates in E-$ASPIC^+$. [Prakken *et al.*, 2015] and [Bench-Capon *et al.*, 2013], inspired by earlier AI & Law work of e.g. [Ashley, 1990] and [Aleven, 2003], model factor-based legal reasoning with precedents in $ASPIC^+$, with argument schemes formalised as defeasible rules and auxiliary definitions concerning (sets of) factors, their origins, their relations and their preferences as first-order axioms. This allows the formalisation of arguments like the following:

> **Plaintiff** The current case and precedent *Bryce* share pro-plaintiff factors $\{f_1, f_2\}$ and pro-defendant factors $\{f_3\}$, the pro-plaintiff factors outweigh the pro-defendant factors since *Bryce* was decided for the plaintiff; therefore, the current case should be decided for me.
>
> **Defendant** But unlike the current case, Bryce also contained pro-plaintiff factor f_4, so it is relevantly different from the current case, so the outcome of *Bryce* does not control the current case.
>
> **Plaintiff** But the current case contains factor f_5 and both f_4 and f_5 are a special case of the more abstract factor f_6, so this difference between *Bryce* and the current case is not relevant.

Other theoretical applications of *ASPIC*$^+$ concern case studies. [Prakken, 2012a] modelled the legal and evidential reasoning in the American *Popov v. Hayashi* case, an ownerships dispute between two baseball fans about a baseball hit in the 500th homerun of a famous American baseball player. [Prakken, 2015] modelled a legislative debate and an American labour law dispute as argumentation-based decision making involving goals, values and preferences.

Finally, some theoretical applications use *ASPIC*$^+$ as a component of a more general reasoning model. [Müller and Hunter, 2012] used a simple instantiation of *ASPIC*$^+$ with no knowledge base, only defeasible rules and no preferences as a reasoning component in a formal model of decision making. [Prakken *et al.*, 2013] applied *ASPIC*$^+$ in a dialogue model of collaborative IT security risk assessment. Finally, [Timmer *et al.*, 2017] used *ASPIC*$^+$ for generating explanations of forensic Bayesian networks.

6.3.2 Applications in implemented architectures

Some implemented architectures proposed in the literature have used implementations of *ASPIC*$^+$ as a component. [Kok, 2013] used *ASPIC*$^+$ as the agent reasoning mechanism in a testbed for inter-agent deliberation dialogue, meant for testing whether the use of argumentation is beneficial to the individual agents or to the group to which they belong. This testbed is available online at `https://bitbucket.org/erickok/baidd`. [Toniolo *et al.*, 2015] used *ASPIC*$^+$ as a reasoning component in their *CISpaces* sensemaking tool for intelligence analysis. [Yun and Croitoru, 2016] used the original ASPIC inference engine for reasoning with possibly inconsistent ontologies in ontology-based data access. Finally, [van Zee *et al.*, 2016] used the TOAST implementation of *ASPIC*$^+$ as a component of a framework for rationalising goal models using argument diagrams.

7 Open problems and avenues for future research

The study of abstract rule-based argumentation with both strict and defeasible rules has a long history, ultimately going back to the seminal work of [Pollock, 1987], passing through intermediate stages [Simari and Loui, 1992; Pollock, 1995; Vreeswijk, 1997; Prakken and Sartor, 1997; Garcia and Simari, 2004] and currently consolidated in the work on *ASPIC*$^+$. As this article has shown, the approach is a fruitful one, a mature metatheory is developing and there is a growing number of implementations and applications. Yet many open questions and avenues for future research remain. Here we list some of the (in our opinion) most important ones.

- The study of argument preference relations and their properties is relatively underdeveloped. More can be done here, for example, relating argument orderings to work in decision theory or to probability theory (see also the next point), or combining different preference criteria for different kinds of problems, such as for epistemic versus practical reasoning.

- A recent research trend in formal argumentation is the combination of argumentation-based inference with probability theory. This is not surprising, since argumentation has from the early days been proposed as a model for reasoning under uncertainty. One question that arises here is how characterisations of the strength or relative preference of arguments relate to probability theory. Much recent work on probabilistic argumentation assigns probabilities to arguments in abstract argumentation frameworks, as in [Li *et al.*, 2012; Hunter and Thimm, 2014]. However, assigning probabilities to arguments is problematic, since in probability theory probabilities are assigned to the truth of statements or to outcomes of events, and an argument is neither a statement nor an event. What is required here is a precise specification of what the probability of an argument means in terms of its elements. How to do this in the context of abstract rule-based argumentation is still largely an open question. A preliminary answer is given by [Hunter, 2013] but only for the case of classical-logic argumentation.

- The contamination problems referred to in Section 5.1 remain to be solved for the fully general $ASPIC^+$framework. As briefly discussed at the end of Section 5.1, the work of [D'Agostino and Modgil, 2016] suggests directions for future development of the $ASPIC^+$framework such that one can establish conditions under which the additional rationality postulates of [Caminada *et al.*, 2012] are satisfied.

- In contrast to abstract argumentation, the study of computational aspects of rule-based argumentation and the various ways it can be instantiated is seriously underdeveloped. Much work can still be done on algorithms and complexity results for rule-based argumentation involving defeasible rules and preferences.

- While there is a growing body of work on the dynamics of abstract argumentation, the work of [Modgil and Prakken, 2012] in $ASPIC^+$ is to our knowledge still the only account of the dynamics of structured argumentation. Much remains to be done here.

- Another important research topic is implementation of more expressive instantiations than those existing today. It would, for example, be interesting to integrate state-of-the art propositional, first-order or modal-logic theorem provers in $ASPIC^+$ implementations.

- Finally, with an eye to practical applications it is important to conduct comparative case studies involving various formalisms, such as $ASPIC^+$, assumption-based argumentation, Carneades or [Brewka and Woltran, 2010]'s abstract dialectical frameworks. It would be especially interesting to study issues like naturalness and conciseness of representations.

Acknowledgments

We would like to express our thanks to those who have worked on the precursors of the $ASPIC^+$ framework (reviewed at the beginning of this article), as well as those who have provided valuable feedback (including Peter Young and Leon van der Torre), and those who have taken the time and effort to critically and constructively study $ASPIC^+$ and as a consequence have helped us improve $ASPIC^+$; notably, Phan Minh Dung in [Dung, 2016] and especially Sjur Dyrkolbotn (personal communication), whose constructive observations motivated us to write the erratum (`https://nms.kcl.ac.uk/sanjay.modgil/AIJfinalErratum`) to [Modgil and Prakken, 2013].

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Received 4 July 2017

Assumption-Based Argumentation: Disputes, Explanations, Preferences

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Abstract

Assumption-Based Argumentation (ABA) is a form of structured argumentation with roots in non-monotonic reasoning. As in other forms of structured argumentation, notions of argument and attack are not primitive in ABA, but are instead defined in terms of other notions. In the case of ABA these other notions are those of rules in a deductive system, assumptions, and contraries.

ABA is equipped with a range of computational tools, based on dispute trees and amounting to dispute derivations, and benefiting from equivalent views of the semantics of argumentation in ABA, in terms of sets of arguments and, equivalently, sets of assumptions. These computational tools can also provide the foundation for multi-agent argumentative dialogues and explanation of reasoning outputs, in various settings and senses.

ABA is a flexible modelling formalism, despite its simplicity, allowing to support, in particular, various forms of non-monotonic reasoning, and reasoning with some forms of preferences and defeasible rules without requiring any additional machinery. ABA can also be naturally extended to accommodate further reasoning with preferences.

1 Introduction

Assumption-Based Argumentation (ABA) [Bondarenko *et al.*, 1993; 1997; Dung *et al.*, 2009; Toni, 2014] is a form of *structured argumentation* [Besnard *et al.*, 2014] with roots in non-monotonic reasoning [Brewka *et al.*, 1997]. Differently from abstract argumentation [Dung, 1995] but as in other forms of structured argumentation, e.g. DeLP [García and Simari, 2014] and deductive arguments [Besnard and Hunter, 2014], notions of argument and attack are not primitive in ABA, but are instead defined in terms of other notions. In the case of ABA these notions are those of *rules* in an underlying *deductive system*, *assumptions* and their *contraries*: arguments are supported by rules and assumptions and attacks are directed against (assumptions deducible from) assumptions supporting arguments, by building arguments for the contrary of these assumptions. Semantics of ABA frameworks can be characterised in terms of sets of assumptions (or *extensions*) [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997; Dung *et al.*, 2007] meeting desirable requirements, including, but not limited to, the two core requirements of *closedness* (where a set of assumptions is *closed* iff it consists of all the assumptions deducible from it) and *conflict-freeness* (where a set of assumptions is *conflict-free* iff it does not attack itself). The closedness requirement is guaranteed to be fulfilled automatically for all sets of assumptions for restricted kinds of *ABA frameworks*, referred to as *flat* [Bondarenko *et al.*, 1997]. The ABA semantics of admissible, preferred, complete, well-founded, stable and ideal extensions [Bondarenko *et al.*, 1997; Dung *et al.*, 2007] differ in which additional desirable requirements they impose upon sets of assumptions, but can all be seen as providing argumentative counterparts of semantics that had previously been defined for non-monotonic reasoning, by appropriately instantiating (flat and non-flat) ABA frameworks [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997] to "match" existing frameworks for non-monotonic reasoning.

Flat ABA is equipped with a range of computational tools, based on dispute trees [Dung *et al.*, 2006; Dung *et al.*, 2007] and amounting to dispute derivations [Dung *et al.*, 2006; Dung *et al.*, 2007; Toni, 2013], and benefiting from equivalent views of the semantics of argumentation in flat ABA, in terms of sets of arguments and, equivalently, sets of assumptions [Dung *et al.*, 2007]. These computational tools can also provide the foundation for inter-agent *ABA dialogues* in various settings and senses [Fan and Toni, 2011b; Fan and Toni, 2011a; Fan and Toni, 2011c; Fan and Toni, 2012b; Fan and Toni, 2012a; Fan and Toni, 2012c; Fan *et al.*, 2014; Fan and Toni, 2014b; Fan and Toni, 2016] and explanations of reasoning outputs, in various settings and senses, e.g. to explain (non-)membership in answer sets of logic programs [Schulz and Toni, 2016], to explain "goodness" of decisions [Fan and Toni, 2014a; Fan *et al.*, 2013; Zhong *et al.*, 2014] and, more generically, to explain admissibility of sentences in any flat instance of ABA [Fan and Toni, 2015c].

ABA is a flexible modelling formalism, despite its simplicity, allowing to support, in particular, reasoning with some forms of preferences and defeasible rules without requiring any additional machinery [Kowalski and Toni, 1996; Toni, 2008b; Thang and Luong, 2013; Fan *et al.*, 2013], but accommodating preferences at the "object-level". ABA can also be naturally extended to accommodate further reasoning with preferences, e.g. as in [Wakaki, 2014] or as ABA^+ in [Čyras and Toni, 2016a; Čyras and Toni, 2016b].

This paper is organised as follows. In Section 2 we recap the basic definitions of ABA frameworks and semantics, focusing on semantics that have been inspired by semantics for non-monotonic reasoning, and summarising properties of semantics, distinguishing amongst generic and flat ABA frameworks. In Section 3 we illustrate two instances of ABA, capturing autoepistemic logic and logic programming, and respectively requiring non-flat and flat ABA frameworks. From Section 4 to Section 7 we focus on flat ABA frameworks. In particular, in Section 4 we summarise how flat ABA frameworks can be equivalently understood, for all semantics considered in this paper, as abstract argumentation frameworks [Dung, 1995], following the results in [Dung *et al.*, 2009], and, vice versa, abstract argumentation frameworks can be equivalently understood, for all semantics considered in this paper, as flat ABA frameworks, following the results in [Toni, 2012]. In Section 5 we provide an overview and illustration of the basis of all computational machinery for ABA, in the flat case, namely dispute trees [Dung *et al.*, 2006; Dung *et al.*, 2007] and dispute derivations [Dung *et al.*, 2006; Dung *et al.*, 2007; Toni, 2013]. In this section we also illustrate how this machinery can be adapted to provide a foundation for inter-agent ABA dialogues [Fan and Toni, 2014b]. In Section 6 we overview various uses of (flat) ABA to provide explanations of reasoning outputs [Schulz and Toni, 2016; Fan and Toni, 2014a; Fan *et al.*, 2013; Zhong *et al.*, 2014; Fan *et al.*, 2014; Fan and Toni, 2015c]. In Section 7 we overview various existing approaches to accommodating preferences in (flat) ABA [Kowalski and Toni, 1996; Toni, 2008a; Toni, 2008b; Thang and Luong, 2013; Fan *et al.*, 2013] or extending ABA to accommodate reasoning with preferences [Wakaki, 2014; Čyras and Toni, 2016a; Čyras and Toni, 2016b]. Finally, in Section 8 we conclude, emphasising, in particular, omissions and future work.

This paper complements other earlier surveys of ABA [Dung *et al.*, 2009; Toni, 2012; Toni, 2014]. In particular, all earlier surveys focused exclusively on flat ABA frameworks. These are powerful knowledge representation mechanisms, as, for example, they fully capture logic programming (see Section 3) and default logic [Reiter, 1980] (see [Bondarenko *et al.*, 1997]), both widely used formalisms for non-monotonic reasoning and knowledge representation and reasoning, as well as, for instance, some forms of decision-making (see Section 7). However, non-flat frameworks allow to capture additional forms of reasoning, including the kind of non-monotonic reasoning encapsulated by autoepistemic logic (see Section 3), as well as circumscription [McCarthy, 1980], amongst others (see [Bondarenko *et al.*, 1997]). For example, in non-flat ABA one can represent beliefs as assumptions that can be deduced via rules from other assumptions.

Moreover, differently from earlier surveys, this paper summarises uses of ABA for non-monotonic reasoning (Section 3) and defeasible reasoning (Section 7) as well as the explanatory power of ABA (Section 6) afforded by its computational machinery. At the same time, this paper ignores other aspects of ABA, emphasised instead in the earlier surveys, such as the equivalence between different presentations of ABA in the literature, e.g. alternative views of arguments (as trees [Dung *et al.*, 2009] rather than as forward [Bondarenko *et al.*, 1997] or backward [Dung *et al.*, 2006] deductions).

2 ABA frameworks and semantics

In this section we introduce ABA frameworks [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997; Dung *et al.*, 2009; Toni, 2014] and their standard semantics of admissible, preferred, complete, well-founded (called *grounded* in the specific case of flat ABA frameworks), stable and ideal extensions [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997; Dung *et al.*, 2007] as sets of assumptions. All the semantics considered have counterparts in logic programming, in the sense that they correspond to semantics of logic programs in the logic programming instance of ABA (see Section 3).

Definition 2.1. *An* ABA framework *is a tuple* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *where*

- $\langle \mathcal{L}, \mathcal{R} \rangle$ *is a* deductive system*, with* $\mathcal{L}$ *a* language *(a set of sentences) and* $\mathcal{R}$ *a set of* (inference) rules*, each with a* head *and a* body*, where the head is a sentence in* $\mathcal{L}$*, and the body consists of* $m \geq 0$ *sentences in* $\mathcal{L}$*;*
- $\mathcal{A} \subseteq \mathcal{L}$ *is a (non-empty) set, with elements referred to as* assumptions*;*
- $\overline{}$ *is a total mapping from* $\mathcal{A}$ *into* $\mathcal{L}$*;* $\overline{a}$ *is referred to as the* contrary *of* a*, for* $a \in \mathcal{A}$*.*

Rules in $\mathcal{R}$ can be written in different formats, e.g. a rule with head σ_0 and body $\sigma_1, \ldots, \sigma_m$ may be written as

$$\sigma_0 \leftarrow \sigma_1, \ldots, \sigma_m \qquad \text{or} \qquad \frac{\sigma_1, \ldots, \sigma_m}{\sigma_0}.$$

Note that $\leftarrow$ is not to be interpreted as logical implication, when used to represent rules in ABA as above. In the remainder of this paper, we will use these two syntactic conventions for writing rules interchangeably. Moreover, unless specified otherwise, we will assume as given a generic ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$. Note also that sentences have a contrary if, and only if, they are assumptions. This contrary is not to be confused with negation, which may or may not occur in $\mathcal{L}$.

Rules in ABA frameworks can be chained to form *deductions*. These can be defined in several ways, notably in a forward [Bondarenko *et al.*, 1997], a backward [Dung *et al.*, 2006] or a tree-style manner [Dung *et al.*, 2009]. We use here the latter style, as follows:

Definition 2.2. *A* deduction for $\sigma \in \mathcal{L}$ supported by $S \subseteq \mathcal{L}$ and $R \subseteq \mathcal{R}$, *denoted* $S \overset{R}{\vdash} \sigma$ *(or simply* $S \vdash \sigma$*), is a (finite) tree with*

- *nodes labelled by sentences in* $\mathcal{L}$ *or by* τ,[1]
- *the root labelled by* σ,
- *leaves either* τ *or sentences in* S,
- *non-leaves* σ' *with, as children, the elements of the body of some rule in* $\mathcal{R}$ *with head* σ'*, and* R *the set of all such rules.*

Example 2.3. *Consider an ABA framework* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *with* $\mathcal{R} = \{x \leftarrow c, z \leftarrow y, b, y \leftarrow, a \leftarrow b\}$ *and* $\mathcal{A} = \{a, b, c\}$.[2] *The following are examples of deductions, denoted as indicated (first with the supporting rules and then without):*

$$\begin{array}{ccccc}
a & \begin{array}{c} a \\ | \\ b \end{array} & \begin{array}{ccc} & z & \\ y & & b \end{array} & \begin{array}{c} y \\ | \\ \tau \end{array} & \begin{array}{ccc} & z & \\ y & & b \\ | & & \\ \tau & & \end{array} \\[1em]
\{a\} \overset{\{\}}{\vdash} a & \{b\} \overset{\{a \leftarrow b\}}{\vdash} a & \{y, b\} \overset{\{z \leftarrow y, b\}}{\vdash} z & \{\} \overset{\{y \leftarrow\}}{\vdash} y & \{b\} \overset{\{z \leftarrow y, b,\ y \leftarrow\}}{\vdash} z \\[1em]
\{a\} \vdash a & \{b\} \vdash a & \{y, b\} \vdash z & \{\} \vdash y & \{b\} \vdash z
\end{array}$$

Note that deductions for assumptions have a non-empty rule support only if they occur as head of rules, and sentences occurring as head of rules with an empty body are always supported by an empty set of sentences (and a singleton set of rules).

Semantics of ABA frameworks are defined in terms of sets of assumptions meeting desirable requirements. One such requirement is being *closed* under deduction, defined as follows:

[1] $\tau \notin \mathcal{L}$ represents "true" and stands for the empty body of rules. In other words, each rule with empty body can be interpreted as a rule with body τ for the purpose of presenting deductions as trees.

[2] Throughout, we often omit to specify the language $\mathcal{L}$, as it is implicit from the rules and assumptions. Also, if the contraries of assumptions are not explicitly defined, then they are assumed to be different from each other and any other explicitly mentioned sentences.

Definition 2.4. *The* closure *of a set of sentences $S \subseteq \mathcal{L}$ is*

$$Cl(S) = \{\sigma \in \mathcal{A} \mid \exists\ S' \overset{R}{\vdash} \sigma,\ S' \subseteq S,\ R \subseteq \mathcal{R}\}.$$

A set of assumptions $A \subseteq \mathcal{A}$ is closed *iff $A = Cl(A)$.*

In Example 2.3, $\{a, b\}$ is closed whereas $\{b\}$ is not.

Note that, in some ABA frameworks, sets of assumptions are guaranteed to be closed. These ABA frameworks are referred to as *flat* and, as we will see later, exhibit additional properties than generic ABA frameworks.

Definition 2.5. *An ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ is* flat *iff for every $A \subseteq \mathcal{A}$, A is closed.*

The ABA framework in Example 2.3 is not flat, whereas the following is an example of a flat ABA framework.

Example 2.6. *An ABA framework with $\mathcal{R} = \{r \leftarrow b, c, \quad q \leftarrow, \quad p \leftarrow q, a\}$ and $\mathcal{A} = \{a, b, c\}$ is guaranteed to be flat. Here, as in all flat ABA frameworks, deductions for assumptions can only be supported by an empty set of rules, e.g. there is a single deduction for a:*

$$\{a\} \overset{\{\}}{\vdash} a\ .$$

It is easy to see that if no assumption is the head of a rule, then an ABA framework is flat [Dung *et al.*, 2006]. However, an ABA framework can be flat even if some assumptions are heads of rules. For instance, in an ABA framework with $\mathcal{R} = \{a \leftarrow x\}$ and $\mathcal{A} = \{a\}$, the assumption a appears as the head of the rule $a \leftarrow x$, but since x is not deducible from any set of assumptions, all sets of assumptions in this ABA framework are guaranteed to be closed, and so the framework is flat. Note, however, that "dummy" rules such as $a \leftarrow x$ above, whose body is not deducible from any set of assumptions, could without loss of generality be deleted from ABA frameworks, as they generate no conclusions. On the other hand, the ABA framework in Example 2.3 has no such "dummy" rules and is not flat (as, indeed, $\{b\}$ is not closed).

The remaining desirable requirements met by sets of assumptions, as semantics for ABA frameworks, are given in terms of a notion of *attack* between sets of assumptions, defined as follows:

Definition 2.7. *A set of assumptions $A \subseteq \mathcal{A}$* attacks *a set of assumptions $B \subseteq \mathcal{A}$ iff there are $A' \subseteq A$ and $b \in B$ such that $A' \vdash \overline{b}$.*

The following definitions of semantics for ABA are adapted from [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997; Dung *et al.*, 2007].

Definition 2.8. *A set of assumptions (or* extension*) is* conflict-free *iff it does not attack itself. A set of assumptions/extension $A \subseteq \mathcal{A}$ is*

- admissible *iff it is closed, conflict-free and, for every $B \subseteq \mathcal{A}$, if B is closed and attacks A, then A attacks B;*
- preferred *iff it is maximally (w.r.t. $\subseteq$) admissible;*
- complete *iff it is admissible and contains all assumptions it* defends*, where A defends a iff for every $B \subseteq \mathcal{A}$, if B is closed and attacks $\{a\}$, then A attacks B;*
- stable *iff it is closed, conflict-free and, for every $a \notin A$, A attacks $\{a\}$;*
- well-founded *iff it is the intersection of all complete extensions;*
- ideal *iff A is maximal (w.r.t. $\subseteq$) such that*
  (i) it is admissible, and
  (ii) for all preferred extensions $P \subseteq \mathcal{A}$, $A \subseteq P$.

Note that ideal sets of assumptions were originally defined, in [Dung *et al.*, 2007], in the context of flat ABA frameworks only. The original definition naturally generalises to general, possibly non-flat, ABA frameworks as given above. Note also that, in the case of flat ABA frameworks, the term *grounded* is conventionally used instead of *well-founded* (e.g. in [Dung *et al.*, 2007]): we will adopt this convention too later in the chapter.

Example 2.9. *Consider a non-flat ABA framework with rules $\mathcal{R} = \{x \leftarrow c, z \leftarrow b, \quad a \leftarrow b\}$, $\mathcal{A} = \{a, b, c\}$ and $\overline{a} = x$, $\overline{b} = y$, $\overline{c} = z$. Then, $\{c\}$ is closed and conflict-free. It is attacked by $\{b\}$, which cannot be counter-attacked but is not closed and thus can be disregarded; it is also attacked by the closed $\{a, b\}$, which is counter-attacked by $\{c\}$. Thus, $\{c\}$ is admissible, as well as preferred and complete. $\{\}$ is also admissible and complete, and thus well-founded, but not preferred. $\{b\}$ is not admissible, because it is not closed. Moreover, the closed $\{a, b\}$ is admissible because it is conflict-free and $\{b\}$ counter-attacks the closed $\{c\}$ which attacks $\{a, b\}$. Finally, $\{a, b\}$ is preferred and complete, and thus $\{\}$ is ideal.*

Note that a set of assumptions/extension can be seen as characterising the set of all sentences in the given ABA framework for which deductions exist supported by (subsets of) the extension:

Definition 2.10. *The* consequences *of an extension* $A \subseteq \mathcal{A}$ *is*

$$Cn(A) = \{\sigma \in \mathcal{L} \mid \exists\ A' \vdash \sigma,\ A' \subseteq A\}.$$

As an illustration, in Example 2.9, $Cn(\{c\}) = \{c, x\}$.

In the remainder of the paper, when a sentence belongs to the consequences of an admissible / preferred / stable / complete / well-founded / ideal extension we will say that it is admissible / preferred / stable / complete / well-founded / ideal, respectively. Thus, in Example 2.9, x is admissible.

The following properties on relationships amongst extensions according to various semantics hold for generic (possibly non-flat) ABA frameworks:

Theorem 2.11. *Let* $A \subseteq \mathcal{A}$ *be a set of assumptions.*

(i) If A *is stable, then it is preferred.*

(ii) If A *is admissible, then there is some* $P \subseteq \mathcal{A}$ *such that* P *is preferred and* $A \subseteq P$.

(iii) If A *is stable, then it is complete.*

(iv) If A *is ideal and* $S \subseteq \mathcal{A}$ *is the intersection of all preferred extensions, then* $A \subseteq S$.

(v) If A *is the intersection of all preferred extensions and admissible, then it is ideal.*

(vi) If A *is ideal, then for each set of assumptions* B *attacking* A *there exists no admissible set of assumptions* $B' \subseteq \mathcal{A}$ *such that* $B' \supseteq B$.

(vii) If A *is well-founded, then for every* $S \subseteq \mathcal{A}$*, if* S *is stable, then* $A \subseteq S$.

Proof.

(i) See proof of Theorem 4.6 in [Bondarenko *et al.*, 1997].

(ii) See proof of Theorem 4.9 in [Bondarenko *et al.*, 1997].

(iii) See proof of Theorem 5.5 in [Bondarenko *et al.*, 1997].

(iv) By definition, $A \subseteq P$ for every preferred $P \subseteq \mathcal{A}$, so $A \subseteq S$.

(v) The intersection of all preferred extensions A is a $\subseteq$-maximal set of assumptions that is contained in every preferred extension, so if A is in addition admissible, then it is by definition ideal.

(vi) Assume A is ideal and let B attack A. By contradiction, assume there exists an admissible $B' \supseteq B$. Then, by (ii) above, there is a preferred set of assumptions P such that $B' \subseteq P$. By definition of ideal extension, $A \subseteq P$, hence P is not conflict-free, contradicting its admissibility.

(vii) By definition, the well-founded extension is contained in every complete extension. Also, by (iii) above, every stable extension is complete. Therefore, the well-founded extension must be contained in every stable extension.

□

Note that item (v) was given and proven in [Dung *et al.*, 2007] (as Theorem 2.1(iv)) in the case of abstract argumentation frameworks [Dung, 1995].

The following properties on existence of extensions according to various semantics hold for generic (possibly non-flat) ABA frameworks.

Theorem 2.12.

(i) If there is an admissible extension, then there is at least one preferred extension.

(ii) If the empty set of assumptions is closed, then there is at least one preferred extension.

(iii) If the empty set of assumptions is closed, then there exists an ideal extension.

Proof.

(i) Directly from Theorem 2.11(ii) (see also comments after the proof of Theorem 4.9 in [Bondarenko *et al.*, 1997]).

(ii) Directly from (i) above, as the empty set, if closed, is necessarily admissible (see also comments after the proof of Theorem 4.9 in [Bondarenko *et al.*, 1997]).

(iii) If $\{\}$ is closed, then it is admissible. So by (i) above, there is a preferred extension. Hence, the intersection S of preferred extensions exists too. Given that $\{\}$ is admissible, there must then be a $\subseteq$-maximal admissible subset of S, i.e. an ideal extension.

□

For a simple example of a (necessarily non-flat) ABA framework in which the empty set is not closed, consider $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with $\mathcal{R} = \{a \leftarrow, \quad x \leftarrow a\}$, $\mathcal{A} = \{a\}$ and $\overline{a} = x$: here, $\{\} \vdash a$, so that $\{\}$ is not closed; note also that no set is admissible, because any admissible set needs to be a closed superset of the empty set, and since there are deductions $\{\} \vdash a$ as well as $\{\} \vdash x$, where x is the contrary of a, no closed superset of $\{\}$ is conflict-free.

Flat ABA frameworks fulfil the following property, often referred to as the *Fundamental Lemma* (see e.g. [Dung, 1995; Bondarenko *et al.*, 1997]):

Theorem 2.13. *Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ be a* flat *ABA framework, and let $A \subseteq \mathcal{A}$ be an admissible set of assumptions that defends assumptions $a, a' \in \mathcal{A}$. Then $A \cup \{a\}$ is admissible and defends a'.*

Proof. See proof of Theorem 5.7 in [Bondarenko *et al.*, 1997]. □

Note that non-flat ABA frameworks need not in general fulfil the Fundamental Lemma: consider $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with $\mathcal{R} = \{c \leftarrow a, b\}$, $\mathcal{A} = \{a, b, c\}$ and $\overline{a} = x$, $\overline{b} = y$, $\overline{c} = z$; it is non-flat, because $\{a, b\} \vdash c$; observe that both $\{a\}$ and $\{b\}$ are closed and unattacked, so, for instance, $\{a\}$ is admissible and defends b; however, $\{a, b\}$ is not closed, and so not admissible.

Flat ABA frameworks also fulfil additional properties concerning relationships between semantics, as follows:

Theorem 2.14. *Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ be a* flat *ABA framework, and let $A \subseteq \mathcal{A}$ be a set of assumptions.*

(i) If A is preferred, then it is complete.

(ii) If A is grounded, then it is minimally (w.r.t. $\subseteq$) complete.

(iii) If A is grounded, then for every $P \subseteq \mathcal{A}$, if P is preferred, then $A \subseteq P$.

(iv) If A is ideal, then it is complete.

(v) If A is ideal and $G \subseteq \mathcal{A}$ is grounded, then $A \supseteq G$.

(vi) If A is maximally (w.r.t. $\subseteq$) complete, then it is preferred.

(vii) If A is admissible, then it is ideal iff for each set of assumptions B attacking A there exists no admissible set of assumptions $B' \subseteq \mathcal{A}$ such that $B' \supseteq B$.

Proof.

(i) Directly from Theorem 5.7 in [Bondarenko *et al.*, 1997], see Corollary 5.8 in [Bondarenko *et al.*, 1997].

(ii) See proof of Theorem 6.2 in [Bondarenko *et al.*, 1997].

(iii) See proof of Theorem 6.4 in [Bondarenko *et al.*, 1997].

(iv) Let I be ideal and suppose it defends $a \in \mathcal{A}$. Due to flatness, $I \cup \{a\}$ is admissible, and hence contained in every preferred extension. So $a \in I$ by $\subseteq$-maximality of I.

(v) Directly from (iv) and (ii) above.

(vi) If A was $\subseteq$-maximally complete but not preferred, then, by Theorem 1(ii), there would be some preferred yet not complete P such that $A \subsetneq P \subseteq \mathcal{A}$, contrary to (i) above.

(vii) See Theorem 3.3 in [Dung *et al.*, 2007].

$\square$

Note that items (iv) and (v) were given and proven in [Dung *et al.*, 2007] (as items (ii) and (iii) respectively in Theorem 2.1), in the case of abstract argumentation frameworks. Also, (vii) was given and proven as Lemma 4(a) in [Dunne, 2009].

The following examples show that the properties in Theorem 2.14 may not hold, in general, for non-flat ABA frameworks.

Example 2.15. *Consider an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with $\mathcal{R} = \{d \leftarrow c\}$, $\mathcal{A} = \{a, b, c, d\}$ and $\overline{a} = p$, $\overline{b} = a$, $\overline{c} = b$, $\overline{d} = d$. Then $\{a\}$ is preferred and ideal, but not complete, as it defends c. (Cf. Theorem 2.14(i), (iv).) Note that $\{a, c\}$ is not admissible, as it is not closed whereas $\{a, c, d\}$ is closed, but not admissible as not conflict-free.*

In this example, there is no complete extension and thus no well-founded extension, and thus the ideal extension is not a superset of the well-founded extension.

Example 2.16. *Consider an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with $\mathcal{R} = \{p \leftarrow a,\ p \leftarrow b,\ \ c \leftarrow\}$, $\mathcal{A} = \{a, b, c, d\}$ and $\overline{a} = b$, $\overline{b} = a$, $\overline{c} = d$, $\overline{d} = p$. Here, the complete extensions are $\{a, c\}$ and $\{b, c\}$, and thus $\{c\}$ is well-founded, but it is not (minimally) complete, as it does not defend itself against (the attacking) $\{d\}$. (Cf. Theorem 2.14(ii).) This also shows that even if there is an admissible extension, there need not be an ideal extension.*

Example 2.17. *Consider an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with $\mathcal{R} = \{d \leftarrow c,\ f \leftarrow e,\ \ p \leftarrow d,\ \ p \leftarrow e\}$, $\mathcal{A} = \{a, b, c, d, e, f\}$ and $\overline{a} = f$, $\overline{b} = a$, $\overline{c} = b$, $\overline{d} = p$, $\overline{e} = q$, $\overline{f} = a$. Then $\{e, f, b\}$ is the only complete extension, and thus the well-founded extension. Moreover $\{a\}$ and $\{e, f, b\}$ are (the only) preferred extensions, and $\{e, f, b\} \not\subseteq \{a\}$. Therefore, there exists a preferred extension that does not contain the well-founded extension. (Cf. Theorem 2.14(iii).)*

Example 2.18. *Consider an ABA framework* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *with* $\mathcal{R} = \{q \leftarrow a,\ r \leftarrow b,\ \ c \leftarrow q, r,\ \ z \leftarrow a,\ \ z \leftarrow b,\ \ z \leftarrow c\}$, $\mathcal{A} = \{a, b, c\}$ *and* $\overline{a} = c$, $\overline{b} = c$, $\overline{c} = z$. *Here, every* $A \subseteq \mathcal{A}$ *containing* c *is not conflict-free, so not admissible. Also,* $\{a, b\}$ *is not closed, so not admissible. However,* $\{a\}$ *is admissible, but not complete, as it defends* b. *Likewise* $\{b\}$ *is admissible, but not complete. Indeed, both* $\{a\}$ *and* $\{b\}$ *are preferred, yet not complete. Therefore,* $\{\}$ *is* $\subseteq$*-maximally complete, yet not preferred. (Cf. Theorem 2.14(vi).)*

Example 2.19. *Consider an ABA framework* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *with* $\mathcal{R} = \{z \leftarrow c,\ c \leftarrow a, b\}$, $\mathcal{A} = \{a, b, c\}$ *and* $\overline{a} = x$, $\overline{b} = y$, $\overline{c} = z$. *Then* $\{a\}$ *is admissible (and preferred) and unattacked. Observe that* $\{a, x\}$ *is not closed, and* $\{a, c\}$, $\{x, c\}$, $\{a, x, c\}$ *are not conflict-free. So* $\{b\}$ *is preferred, yet* $\{a\} \nsubseteq \{b\}$, *so that* A *is not ideal. (Cf. Theorem 2.14(vii).)*

Flat ABA frameworks fulfil additional properties concerning existence of extensions w.r.t. various semantics, as follows:

Theorem 2.20. *Let* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *be a* flat *ABA framework.*

(i) There is at least one preferred extension.

(ii) There is a unique ideal extension.

(iii) There is at least one complete extension.

(iv) There is a unique grounded extension and it is the least fixed point of Def, where, for $A \subseteq \mathcal{A}$, *Def(A)* $= \{a \in \mathcal{A} \mid A$ *defends* $a\}$.

Proof.

(i) Directly from the second item of Theorem 2.12, as, in the case of flat ABA frameworks, the empty set (like any other set of assumptions) is guaranteed to be closed (see also [Bondarenko *et al.*, 1997]).

(ii) Follows from Theorem 2.12(iii).

(iii) Directly from (i) above and Theorem 2.14(i).

(iv) See proof of Theorem 6.2 in [Bondarenko *et al.*, 1997].

□

Note that (ii) above was given and proven in [Dung *et al.*, 2007] in the case of abstract argumentation frameworks.

The following examples show that the properties in Theorem 2.20 may not hold, in general, for non-flat ABA frameworks.

Example 2.21. *Consider an ABA framework* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *with* $\mathcal{R} = \{a \leftarrow\}$, $\mathcal{A} = \{a\}$ *and* $\overline{a} = a$*. Here,* $\{\}$ *is not closed and* $\{a\}$ *is not conflict-free. Thus, no set of assumptions is admissible. Hence, there is no preferred, complete, ideal or well-founded extension.*

Finally, consider an example which shows that, differently from flat ABA frameworks, in general, an ideal extension need not be unique for non-flat ABA frameworks.

Example 2.22. *Consider an ABA framework* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *with assumptions* $\mathcal{A} = \{a, a', b, b', c, d\}$*, rules* $\mathcal{R} = \{c \leftarrow a, a', \quad \overline{c} \leftarrow d, \quad \overline{d} \leftarrow a, b, \quad \overline{d} \leftarrow a', b', \quad a' \leftarrow a, b, c, \quad a \leftarrow a', b, c, \quad a' \leftarrow a, b', c, \quad a \leftarrow a', b', c\}$*, and contraries* $\overline{b} = b'$, $\overline{b'} = b$.[3] *Here,* $\{\}$ *is closed, so admissible. There are two preferred extensions:* $\{a, a', b, c\}$ *and* $\{a, a', b', c\}$*. Their intersection* $\{a, a', c\}$ *is not admissible, because it cannot defend against (the closed attacking)* $\{d\}$*. Likewise,* $\{a, c\}$ *and* $\{a', c\}$ *are not admissible. Also,* $\{a, a'\}$ *is not closed. However, both* $\{a\}$ *and* $\{a'\}$ *are admissible, and hence ideal extensions.*

Note that additional properties hold for (generic and/or flat) ABA frameworks of restricted kinds, for instance, where "cycles" are not allowed (e.g. *stratified* and *order-consistent* ABA frameworks, see [Bondarenko *et al.*, 1997] for details). Moreover, additional properties hold for other special classes of ABA frameworks, in addition to flat ABA frameworks, namely *normal* [Bondarenko *et al.*, 1997] and *simple* [Dimopoulos *et al.*, 2002] ABA frameworks (see [Bondarenko *et al.*, 1997; Dimopoulos *et al.*, 2002] for details).

3 ABA and non-monotonic reasoning

In this section we illustrate two instances of ABA for Non-Monotonic Reasoning, namely Autoepistemic Logic (AEL) [Moore, 1985] and Logic Programming (LP). The formal definitions of these instances, as well as correspondence results between the semantics for ABA as given in Definition 2.8 and their original semantics, can be found in [Bondarenko *et al.*, 1997; Schulz and Toni, 2015]

For illustration, as well as a running example throughout the chapter, we will use the following extract from the Nationwide[4] building society's 2016 policy for UK/EU Breakdown Assistance:

[3] For readability, with an abuse of notation we may sometimes assume that the contraries (in this case, $\overline{c}$ and $\overline{d}$) of assumptions (in this case, c and d) are actually symbols in the language (different from other explicitly mentioned sentences).

[4] `www.nationwide.co.uk`

> COVERED FOR: UK/EU Breakdown Assistance for account holder(s) in any private car they are travelling in
>
> NOT COVERED FOR: private cars not registered to the account holder(s) unless the account holder(s) are in the vehicle at the time of the breakdown

We consider a person, Mary (denoted simply as m), who is an account holder travelling in a friend's car (denoted as c) when the car breaks down somewhere in the EU. In the remainder of this section we show how the application of the policy above to Mary's case can be represented in the AEL and LP instances of ABA, as given in general in [Bondarenko *et al.*, 1997]. In giving the concrete instantiations below we will use the following abbreviations: ah stands for "account holder"; tr stands for "travelling"; pr stands for "private vehicle"; cov stands for "covered"; reg stands for "registered"; cov' stands for "there is an exception to being not covered".

3.1 Breakdown Assistance policy in the AEL instance of ABA

The application of the Breakdown Assistance policy to Mary's case can be represented in the AEL instance of ABA as follows:

$\mathcal{L} =$ a modal language containing a modal operator L (where $L\sigma$ stands for "σ is believed") as well as atoms $ah(m),\ tr(m,c),\ pr(c),\ cov(m,c), reg(c,m),\ cov'(m,c), in(m,c)$

$\mathcal{R} =$ a complete set of inference rules of classical logic for $\mathcal{L}$ together with the following inference rules (all with an empty body):

$$\frac{}{ah(m) \wedge tr(m,c) \wedge pr(c) \wedge \neg L \neg cov(m,c) \rightarrow cov(m,c)}$$

$$\frac{}{\neg reg(c,m) \wedge \neg L cov'(m,c) \rightarrow \neg cov(m,c)}$$

$$\frac{}{in(m,c) \rightarrow cov'(m,c)} \quad \frac{}{ah(m)} \quad \frac{}{tr(m,c)}$$

$$\frac{}{pr(c)} \quad \frac{}{\neg reg(c,m)} \quad \frac{}{in(m,c)}$$

$\mathcal{A} = \{L\sigma, \neg L\sigma \mid \sigma \in \mathcal{L}\}$

$\overline{L\sigma} = \neg L\sigma$ and $\overline{\neg L\sigma} = \sigma$ for any $\sigma \in \mathcal{L}$

Note that, in this ABA framework, $\mathcal{R}$ includes domain-independent rules, e.g.

$$\frac{\sigma_1 \wedge \sigma_2}{\sigma_1} \ \text{ for any } \sigma_1, \sigma_2 \in \mathcal{L},$$

as well as domain-specific rules, e.g.

$$\frac{}{in(m,c)}.$$

Note also that this ABA framework (as well as any other AEL instance of ABA) is not flat [Bondarenko *et al.*, 1997], as, for instance, the set of assumptions $\{Lcov(m), \neg Lcov(m)\}$ is not closed, because it is classically inconsistent. Nonetheless, for this instance, the empty set of assumptions is closed.

Given this representation in ABA, the problem of determining whether Mary should be covered or not amounts to determining whether $cov(m)$ is stable (following the conventional AEL approach of determining whether $cov(m)$ belongs to a consistent stable expansion [Moore, 1985] of the theory consisting of the heads of the domain-specific part of $\mathcal{R}$), or preferred, or well-founded etc. (by adopting any of the other ABA semantics). In this particular example, all ABA semantics agree that Mary should be covered, by assuming $\neg L\neg cov(m,c)$, in agreement with the original semantics of AEL, as predicted by the general correspondence Theorem 3.18 in [Bondarenko *et al.*, 1997] and the fact that, in this example, all ABA semantics agree with the semantics of stable extensions. As an illustration, $\{\neg L\neg cov(m,c)\}$ is admissible, since it is conflict-free, closed and the (closed) set of assumptions $\{\neg Lcov'(m,c)\}$ attacking it, as well as all its (closed) supersets, are attacked by the (closed) empty set of assumptions.

Note that, in the AEL instance of ABA, beliefs, of the form $L\,\sigma$ or $\neg L\,\sigma$, are assumptions that may occur as heads of rules. For example, the earlier AEL instance of ABA may be extended so that $\mathcal{R}$ includes also

$$\frac{}{L\,ah(m)}$$

to represent that Mary is believed to be an account holder. This kind of knowledge cannot be directly represented in flat ABA.

3.2 Breakdown Assistance policy in the LP instance of ABA

The application of the Breakdown Assistance policy to Mary's case can be represented in the LP instance of ABA as follows:

$$\begin{aligned}
\mathcal{R} = \quad & \{cov(m,c) \leftarrow ah(m), tr(m,c), pr(c), not\ \neg cov(m,c),\\
& \neg cov(m,c) \leftarrow \neg reg(c,m), not\ cov'(m,c),\\
& cov'(m,c) \leftarrow in(m,c),\\
& ah(m) \leftarrow,\quad tr(m,c) \leftarrow,\quad pr(c) \leftarrow,\quad \neg reg(c,m) \leftarrow,\quad in(m,c) \leftarrow\}\\
\mathcal{A} = \quad & \{not\ p(t_1,t_2),\ not\ q(t) \mid p \in \{cov, tr, \neg cov, \neg reg, cov', in\},\\
& \qquad\qquad\qquad\qquad\qquad q \in \{ah, pr\},\\
& \qquad\qquad\qquad\qquad\qquad t_1, t_2, t \in \{m, c\}\}\\
\mathcal{L} = \quad & \mathcal{A} \cup \{x \mid not\ x \in \mathcal{A}\}\\
\overline{not\, x} = \quad & x \text{ for any } not\ x \in \mathcal{A}
\end{aligned}$$

So, $\mathcal{L}$ is the Herbrand base of (the logic program) $\mathcal{R}$ together with all negation as failure (NAF) literals that can be built from this Herbrand base, and $\mathcal{A}$ is the set of all these NAF literals. Note that in this illustration we treat $\neg cov$ and $\neg reg$ as predicate symbols.

Given this representation in ABA, the problem of determining whether Mary should be covered or not amounts to determining, for instance, whether $cov(m, c)$ is stable (following the stable model semantics [Gelfond and Lifschitz, 1988] for $\mathcal{R}$, seen as a logic program, by virtue of the correspondence Theorem 3.13 in [Bondarenko *et al.*, 1997]), or admissible/preferred (following the preferred extension semantics [Dung, 1991] for $\mathcal{R}$, by virtue of the correspondence Theorem 4.5 in [Bondarenko *et al.*, 1997]), or grounded (following the well-founded model semantics [Gelder *et al.*, 1991] for $\mathcal{R}$, by virtue of the correspondence Theorem 3.13 in [Bondarenko *et al.*, 1997]), or ideal (following the scenario semantics [Alferes *et al.*, 1993] for $\mathcal{R}$ In this particular example, all ABA semantics agree that Mary should be covered, by assuming $not\ cov(m, c)$. As an illustration, $\{not\ cov(m, c)\}$ is admissible, since it is conflict-free and the set of assumptions $\{not\ cov'(m, c)\}$ attacking it, as well as all its supersets, are attacked by the empty set of assumptions.

4 ABA versus abstract argumentation

In this section we focus on the relationship between flat ABA frameworks and Abstract Argumentation (AA) frameworks [Dung, 1995]. In particular, flat ABA is an instance of AA, under all semantics considered in this paper and, conversely, AA is an instance of (flat) ABA.

Flat ABA frameworks are instances of AA frameworks where arguments are deductions supported by sets of assumptions and attacks are defined by appropriately lifting the notion of attack between sets of assumptions to a notion of attack between arguments [Dung *et al.*, 2007; Toni, 2012].

Definition 4.1. *Let $\mathcal{ABA} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ be a flat ABA framework.*

- *An* argument *for $\sigma \in \mathcal{L}$ supported by $A \subseteq \mathcal{A}$ and $R \subseteq \mathcal{R}$, denoted $A \overset{R}{\vdash}_{arg} \sigma$ (or simply $A \vdash_{arg} \sigma$), is such that there is a deduction $A \overset{R}{\vdash} \sigma$.*
- *An argument $A \vdash_{arg} \sigma$* attacks *an argument $B \vdash_{arg} \pi$ iff there is $b \in B$ such that $\sigma = \overline{b}$.*

Then $\mathcal{AA}(\mathcal{ABA}) = (Args, attack)$ is the corresponding AA framework *of $\mathcal{ABA}$ with $Args$ the set of all arguments (as in the first bullet) and attack the set of all pairs $(\mathtt{a}, \mathtt{b})$ such that $\mathtt{a}, \mathtt{b} \in Args$ and $\mathtt{a}$ attacks $\mathtt{b}$ (as in the second bullet).*

Note that $Args$ contains an argument for every assumption in $\mathcal{A}$ as illustrated by the following example.

Example 4.2. *Consider an ABA framework $\mathcal{ABA}$ with rules and assumptions as in Example 2.6 and $\overline{a} = r$, $\overline{b} = q$, $\overline{c} = p$. Then $\mathcal{AA}(\mathcal{ABA})$ is $(Args, attack)$ with $Args = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{p}, \mathtt{q}, \mathtt{r}\}$ where $\mathtt{a} = \{a\} \vdash_{arg} a$, $\mathtt{b} = \{b\} \vdash_{arg} b$, $\mathtt{c} = \{c\} \vdash_{arg} c$, $\mathtt{p} = \{a\} \vdash_{arg} p$, $\mathtt{q} = \{\} \vdash_{arg} q$, $\mathtt{r} = \{b, c\} \vdash_{arg} r$, and $attack = \{(\mathtt{p}, \mathtt{c}), (\mathtt{p}, \mathtt{r}), (\mathtt{q}, \mathtt{b}), (\mathtt{q}, \mathtt{r}), (\mathtt{r}, \mathtt{a}), (\mathtt{r}, \mathtt{p})\}$.*

The semantics of an AA framework corresponding to a flat ABA framework can be determined using the standard AA semantics [Dung, 1995; Dung *et al.*, 2007]. For all ABA semantics considered in this paper, the semantics of a flat ABA framework corresponds to the semantics of its corresponding AA framework, as follows:

Theorem 4.3. *Let $\mathcal{ABA} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ be a flat ABA framework and let $\mathcal{AA}(\mathcal{ABA})$ be its corresponding AA framework.*

(i) If a set of assumptions $A \subseteq \mathcal{A}$ is admissible / preferred / stable / complete / grounded / ideal in $\mathcal{ABA}$, then the union of all arguments supported by any $A' \subseteq A$ is admissible / preferred / stable / complete / grounded / ideal, respectively, in $\mathcal{AA}(\mathcal{ABA})$.

(ii) The union of all sets of assumptions supporting the arguments in an admissible / preferred / stable / complete / grounded / ideal set of arguments in $\mathcal{AA}(\mathcal{ABA})$ *is admissible / preferred / stable / complete / grounded / ideal, respectively, in* $\mathcal{ABA}$*.*

Proof. See the proof of Theorem 2.2 in [Dung *et al.*, 2007] for admissible, grounded and ideal extensions, the proof of Theorem 1 in [Toni, 2012]for stable extensions[5], and the proof of Theorem 6.1 and 6.3 [Caminada *et al.*, 2015] for complete and preferred extensions respectively. □

Note that for the preferred, stable, complete, grounded, and ideal semantics the correspondence between the extensions of a flat ABA framework and the extensions of the corresponding AA framework is one-to-one. For the admissible semantics, instead, the correspondence is one-to-many, i.e. the union of all sets of assumptions supporting the arguments in an admissible extension may be the same for various admissible extensions of the corresponding AA framework, as illustrated in the following example.

Example 4.4. *The ABA framework from Example 4.2 has two admissible extensions:* $\{\}$ *and* $\{a\}$*. In contrast, the corresponding AA framework has five admissible extensions:* $\mathtt{A}_1 = \{\}$, $\mathtt{A}_2 = \{\mathtt{q}\}$, $\mathtt{A}_3 = \{\mathtt{p}\}$, $\mathtt{A}_4 = \{\mathtt{p},\mathtt{q}\}$, $\mathtt{A}_5 = \{\mathtt{p},\mathtt{a}\}$, $\mathtt{A}_6 = \{\mathtt{q},\mathtt{a}\}$, $\mathtt{A}_7 = \{\mathtt{p},\mathtt{q},\mathtt{a}\}$*. However, the union of all sets of assumptions supporting the arguments in* $\mathtt{A}_1$ *and* $\mathtt{A}_2$ *is* $\{\}$*, so both correspond to the first admissible extension of the ABA framework. Similarly, the union of all sets of assumptions supporting arguments in the other admissible extensions (*$\mathtt{A}_3 - -\mathtt{A}_7$*) of the AA framework is* $\{a\}$*, so they all correspond to the second admissible extension of the ABA framework.*

Theorem 4.3 shows that, under the semantics considered therein, flat ABA frameworks are an instance of AA frameworks and the semantics of ABA can alternatively be defined in terms of extensions as sets of arguments, as in [Dung *et al.*, 2007], rather than in terms of extensions as sets of assumptions, as in [Bondarenko *et al.*, 1993; Bondarenko *et al.*, 1997]. This implies, for example, that existing machinery for computing extensions of AA frameworks can be used to compute extensions of ABA frameworks whose corresponding AA frameworks are finite. Conversely, as shown below, AA frameworks are an instance of flat ABA frameworks, that is any AA framework can be translated into a corresponding flat ABA framework such that their respective extensions correspond [Toni, 2012]. This implies, in particular, that existing machinery for determining whether sentences are admissible / preferred

[5]The proof of Theorem 1 in [Toni, 2012] actually considers a different notion of stable extension, but can naturally be modified to prove the result indicated here.

/ complete / grounded / ideal in flat ABA (see Section 5) can be used to determine whether arguments in an AA framework belong to an admissible / preferred / complete / grounded / ideal extension. The ABA framework corresponding to an AA framework has the arguments in the AA framework as (the only) assumptions and appropriate notions of contraries of these assumptions and rules to encode the attacks between the arguments in the original AA framework, as follows:

Definition 4.5. *Let $\mathcal{AA} = (Args, attack)$ be an AA framework. The* corresponding ABA framework *of $\mathcal{AA}$ is $\mathcal{ABA}(\mathcal{AA}) = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with*

- $\mathcal{A} = Args$;
- $\mathcal{L} = \mathcal{A} \cup \{\mathtt{a}^c \mid \mathtt{a} \in \mathcal{A}\}$;
- *for all* $\mathtt{a} \in \mathcal{A}$: $\overline{\mathtt{a}} = \mathtt{a}^c$;
- $\mathcal{R} = \{\mathtt{b}^c \leftarrow \mathtt{a} \mid (\mathtt{a}, \mathtt{b}) \in attack\}$.

Note that clearly the corresponding ABA framework of any AA framework is flat since assumptions never occur in the head of a rule, by construction.

Since the set of arguments in a given AA framework coincides with the set of assumptions of the corresponding ABA framework, there is a straight-forward one-to-one correspondence between all semantics of the AA and ABA framework.

Theorem 4.6. *Let $\mathcal{AA} = (Args, attack)$ be an AA framework and let $\mathcal{ABA}(\mathcal{AA})$ be its corresponding ABA framework.*

(i) If $\mathtt{A} \subseteq Args$ is admissible / preferred / stable / complete / grounded / ideal in $\mathcal{AA}$, then $\mathtt{A}$ is admissible / preferred / stable / complete / grounded / ideal, respectively, in $\mathcal{ABA}(\mathcal{AA})$.

(ii) If $A \subseteq \mathcal{A}$ is admissible / preferred / stable / complete / grounded / ideal set of arguments in $\mathcal{ABA}(\mathcal{AA})$, then A is admissible / preferred / stable / complete / grounded / ideal, respectively, in $\mathcal{AA}$.

Proof. See proof of Theorem 2 in [Toni, 2012] for admissible. As noted in [Toni, 2012], the proof for other semantics is similar. □

Example 4.7. *Consider the AA framework $\mathcal{AA}$ with $Args = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and $attack = \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{b}, \mathtt{c})\}$. The corresponding ABA framework is $\mathcal{ABA}(\mathcal{AA})$ with $\mathcal{A} = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, $\overline{\mathtt{a}} = \mathtt{a}^c$, $\overline{\mathtt{b}} = \mathtt{b}^c$, $\overline{\mathtt{c}} = \mathtt{c}^c$, and $\mathcal{R} = \{\mathtt{b}^c \leftarrow \mathtt{a},\ \mathtt{a}^c \leftarrow \mathtt{b},\ \mathtt{c}^c \leftarrow \mathtt{b}\}$. The admissible extensions of $\mathcal{AA}$ are $\{\}$, $\{\mathtt{a}\}$, $\{\mathtt{b}\}$, and $\{\mathtt{a}, \mathtt{c}\}$, which are exactly the admissible extensions of $\mathcal{ABA}(\mathcal{AA})$. Correspondence as dictated by Theorem 4.6 hold for the other semantics considered therein too.*

5 Dispute trees, dispute derivations and ABA dialogues

In this section we overview the main existing computational machinery for flat ABA frameworks, allowing to determine whether sentences are admissible (and therefore preferred, by Theorem 2.11 (ii), and complete, by Theorem 2.11 (ii) and Theorem 2.14 (i)), grounded, or ideal.[6] This machinery is based on the computation of *dispute trees* (overviewed in Section 5.1), using *dispute derivations* (illustrated in Section 5.2) that, in particular, can be executed amongst agents to form *ABA dialogues* (illustrated in Section 5.3).

5.1 Dispute trees

Dispute trees [Dung *et al.*, 2006; Dung *et al.*, 2007] provide an abstraction of the problem of determining whether arguments in AA frameworks belong to an admissible / grounded / ideal extension. Since flat ABA frameworks correspond to special instances of AA frameworks (see Section 4), dispute trees can be used to determine whether sentences are admissible / grounded / ideal, respectively, as well as identifying assumptions in admissible / grounded / ideal extensions for ABA, respectively, supporting arguments for these sentences. Dispute trees can be defined abstractly for any abstract argumentation framework as follows:

Definition 5.1. *Let* $(Args, attack)$ *be any abstract argumentation framework. A* dispute tree *for* $\mathtt{a} \in Args$ *is a tree* $\mathcal{T}$ *such that:*

(i) *every node of* $\mathcal{T}$ *is of the form* $[\mathtt{L} : \mathtt{x}]$*, with* $\mathtt{L} \in \{\mathtt{P}, \mathtt{O}\}$*,* $\mathtt{x} \in Args$*: the node is* labelled *by argument* $\mathtt{x}$ *and assigned the* status *of either* proponent ($\mathtt{P}$) *or* opponent ($\mathtt{O}$)*;*

(ii) *the root of* $\mathcal{T}$ *is a* $\mathtt{P}$ *node labelled by* $\mathtt{a}$*;*

(iii) *for every* $\mathtt{P}$ *node* n*, labelled by some* $\mathtt{b} \in Args$*, and for every* $\mathtt{c} \in Args$ *such that* $\mathtt{c}$ *attacks* $\mathtt{b}$*, there exists a child of* n*, which is an* $\mathtt{O}$ *node labelled by* $\mathtt{c}$*;*

(iv) *for every* $\mathtt{O}$ *node* n*, labelled by some* $\mathtt{b} \in Args$*, there exists exactly one child of* n *which is a* $\mathtt{P}$ *node labelled by some* $\mathtt{c} \in Args$ *such that* $\mathtt{c}$ *attacks* $\mathtt{b}$*;*

(v) *there are no other nodes in* $\mathcal{T}$ *except those given by 1–4.*

[6] In general, this machinery cannot be used to determine whether a sentence is stable, as this requires the computation of a full extension, as discussed in [Dung *et al.*, 2002]. However, for restricted types of flat ABA frameworks whose preferred extensions are guaranteed to be stable, determining whether a sentence is admissible amounts to determining whether it is stable, too.

The defence set *of a dispute tree $\mathcal{T}$, denoted by $\mathcal{D}(\mathcal{T})$, is the set of all arguments labelling* P *nodes in $\mathcal{T}$.*

Example 5.2. *Given the AA framework with $Args = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}, \mathtt{g}\}$ and $attack = \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{c}), (\mathtt{d}, \mathtt{e}), (\mathtt{d}, \mathtt{f}), (\mathtt{e}, \mathtt{d}), (\mathtt{e}, \mathtt{f}), (\mathtt{f}, \mathtt{g}), (\mathtt{g}, \mathtt{f})\}$, consider the trees in the figure below. The tree on the left is not a dispute tree since an opponent node is a leaf node, thus violating condition (iv) in Definition 5.1. In contrast, the trees in the middle and on the right satisfy all conditions and are thus dispute trees for* c *and* d*, respectively.*

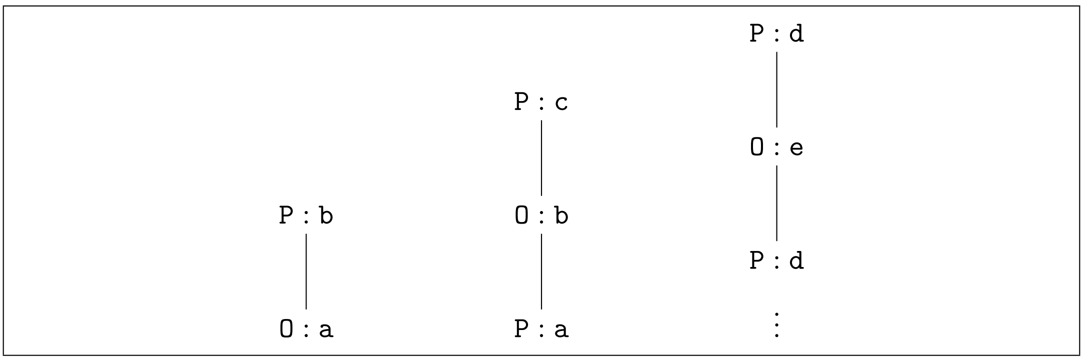


Figure 1: Only the middle and right of the three trees are dispute trees.

In order to help determine membership of arguments in admissible / grounded / ideal extensions of AA frameworks, dispute trees need to fulfil special requirements, as follows:

Definition 5.3. *Let $(Args, attack)$ be any abstract argumentation framework. A dispute tree $\mathcal{T}$ (for some argument in Args) is*

- admissible *iff no argument in $\mathcal{T}$ labels both* P *and* O *nodes;*
- grounded *iff it is finite;*
- ideal *iff for no argument* a *in $\mathcal{T}$ labelling an* O *node there exists an admissible dispute tree for* a*.*

Example 5.4. *Consider again the AA framework from Example 5.2. The dispute tree shown in the middle of Figure 1 is admissible since no argument labels both a proponent and an opponent node, as well as grounded since it is finite. Furthermore, the dispute tree is ideal since its only opponent node is labelled with* b *and there are no dispute trees for* b*, and thus there are no admissible dispute trees for* b*. The* *dispute tree for* d *on the right of Figure 1 is admissible, but not grounded since it is infinite. It is furthermore not ideal since there is an admissible dispute tree for* e *(obtained by exchanging* d *and* e *in the dispute tree for* d *on the right of Figure 1).*

The left of Figure 2 gives an example of a dispute tree which is ideal but not grounded. The opponent nodes of this dispute tree are all labelled by argument f*. Since the only dispute tree for* f *is the one displayed in the middle of Figure 2, which is not an admissible dispute tree since argument* d *(as well as* e*) labels both an opponent and a proponent node, the dispute tree for* g *on the left of Figure 2 is ideal. Note that there are other admissible dispute trees for* g *which are not ideal. For example the one on the right of Figure 2 is not ideal since there exists an admissible dispute tree for* e*.*

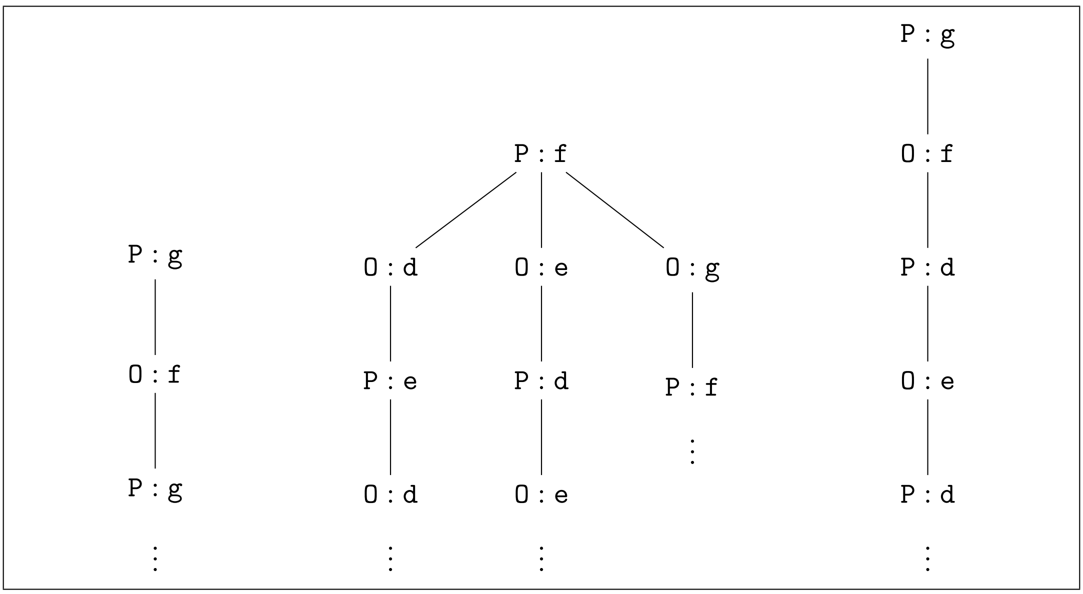


Figure 2: Three dispute trees constructed from the AA framework in Example 5.2.

Theorem 5.5. *Let* $(Args, attack)$ *be any abstract argumentation framework.*

(i) If $\mathcal{T}$ *is an admissible dispute tree for an argument* a *then the defence set of* $\mathcal{T}$ *is admissible.*

If $\mathtt{a} \in \mathtt{A}$ *for some admissible set of arguments* $\mathtt{A} \subseteq Args$ *then there exists an admissible dispute tree for* a *with defence set* $\mathtt{A}'$ *such that* $\mathtt{A}' \subseteq \mathtt{A}$ *and* $\mathtt{A}'$ *is admissible.*

(ii) If $\mathcal{T}$ is an ideal dispute tree for an argument a *then the defence set* A *of $\mathcal{T}$ is such that* A *is admissible and* A $\subseteq I$ *with I the ideal extension of $(Args, attack)$.*

If a $\in I$ *with I the ideal extension of $(Args, attack)$, then there exists an ideal dispute tree for* a *with defence set* A *and* A $\subseteq I$*.*

(iii) If $\mathcal{T}$ is a grounded dispute tree for an argument a *then the defence set* A *of $\mathcal{T}$ is such that* A *is admissible and* A $\subseteq G$ *with G the grounded extension of $(Args, attack)$.*

If a $\in G$ *with G the grounded extension of $(Args, attack)$, then there exists a grounded dispute tree for* a *with defence set* A *and* A $\subseteq G$*.*

Proof. (i) See proof of Theorem 3.2 in [Dung *et al.*, 2007].

(ii) See proof of Theorem 3.4 in [Dung *et al.*, 2007].

(iii) Follows directly from Theorem 3.7 in [Kakas and Toni, 1999].

□

Example 5.6. *As discussed in Example 5.4, the dispute tree in the middle of Figure 1 is admissible and grounded. As stated in Theorem 5.5 the defence set,* {a, c}*, is admissible and is a subset of the grounded extension of the AA framework from Example 5.2, in fact in this case it coincides with the grounded extension. The ideal extension of the AA framework is* {a, c, g} *and we saw that there exists an ideal dispute tree for* g *(on the left of Figure 2) whose defence set is* {g}*, which is a subset of the ideal extension.*

In order to determine whether a sentence is admissible / grounded / ideal, given a flat ABA framework, a dispute tree for an argument for that sentence can be used, by virtue of the correspondence results overviewed in Section 4 and Theorem 5.5 above. For example, given the ABA framework in Section 3.2, the dispute tree in Figure 3 for argument $\{not\,\neg cov(m,c)\} \vdash_{arg} cov(m,c)$ can be used to determine that $cov(m,c)$ is admissible, grounded and ideal. Indeed, this is a dispute tree since the leaf node cannot be attacked and no other opponent node can attack the root. Moreover, it is trivially admissible and, since it is finite, it is grounded. Finally, it is ideal as no admissible dispute tree for its only opponent node exists (as $\{\} \vdash_{arg} cov'(m,c)$ cannot be attacked).

5.2 Dispute derivations

Dispute derivations [Dung *et al.*, 2006; Dung *et al.*, 2007; Toni, 2013; Craven and Toni, 2016] are algorithms for determining whether a given sentence, in the language

$$\texttt{P} : \{not\,\neg cov(m,c)\} \vdash_{arg} cov(m,c)$$

$$\texttt{O} : \{not\; cov'(m,c)\} \vdash_{arg} \neg cov(m,c)$$

$$\texttt{P} : \{\} \vdash_{arg} cov'(m,c)$$

Figure 3: A dispute tree for $\{not\,\neg cov(m,c)\} \vdash_{arg} cov(m,c)$ for the flat ABA framework in Section 3.2.

of a flat ABA framework, is admissible, grounded or ideal. Different kinds of dispute derivations can be defined for the different semantics, as in [Dung *et al.*, 2006; Dung *et al.*, 2007], or the same template of dispute derivations can be instantiated differently for the different semantics, as in [Toni, 2013; Craven and Toni, 2016] and, for the LP instance of ABA, in [Kakas and Toni, 1999]. Dispute derivations for determining whether a sentence is admissible can also be used to determine whether the sentence is complete or preferred [Toni, 2013]. All given notions of dispute derivations are defined as games between (fictional) *proponent* (P) and *opponent* (O) players, as for dispute trees. All given notions are sound and, for restricted types of flat ABA frameworks (referred to as *p-acyclic* [Dung *et al.*, 2006]), complete [Dung *et al.*, 2006; Dung *et al.*, 2007; Toni, 2013]. The most recently defined types of dispute derivations are complete in general [Craven and Toni, 2016], for the admissible and grounded semantics. Different types of dispute derivations also differ in the data structures they deploy as well as their outputs:

- the dispute derivations of [Dung *et al.*, 2006; Dung *et al.*, 2007] deploy sets of assumptions and output admissible sets of assumptions in all cases, and, in the case of grounded/ideal semantics, these sets of assumptions are contained in the grounded/ideal extension, respectively;

- the dispute derivations of [Toni, 2013; Craven and Toni, 2016] deploy a mixture of sets of assumptions and sets of *potential arguments*, i.e. deductions supported by any sets of sentences (rather than assumptions) and with sentences in the support possibly marked as “seen”, and output admissible sets of assumptions in all cases, as for the previous types of dispute derivations, as well as dialectical structures from which admissible / grounded / ideal dispute trees can be obtained.

We illustrate dispute derivations for the LP instance of ABA representation of the Breakdown Assistance policy, in Section 3.2, and refer to the original papers for formal definitions and results. In the illustration, we focus on the dispute derivations of [Toni, 2013], since they are generalisations of the earlier dispute derivations of [Dung *et al.*, 2006; Dung *et al.*, 2007] but still in the same spirit. Instead, the dispute derivations of [Craven and Toni, 2016] are based on a different conceptual model for ABA, where arguments and sets of arguments are defined as graphs instead (see [Craven and Toni, 2016] for details).

The (flat) ABA framework of Section 3.2 can be used to determine whether Mary should be covered, by determining whether $cov(m,c)$ is admissible (and thus, for this particular ABA framework, grounded, ideal etc.), i.e. if it belongs to an admissible extension. This can be determined in turn by means of a dispute derivation for $cov(m,c)$. This dispute derivation starts with a *potential argument* by `P`:[7]

$$\{\} \vdash^{p}_{\{cov(m,c)\}} cov(m,c),$$

namely a deduction $\{cov(m,c)\} \vdash cov(m,c)$ with no sentence in the support $\{cov(m,c)\}$ marked as "seen" (and the sentence $cov(m,c)$ in the support still "unseen"). In the first step of the derivation, then, `P` needs to "expand" its potential argument, while `O` is watching and can only put forward new potential arguments when `P` has sufficiently expanded its own potential arguments so as to have identified assumptions in their "unseen" support that `O` can attack (automatically rendering them "seen"). In this simple illustration, `P` will necessarily expand the initial potential argument to

$$\{\} \vdash^{p}_{\{ah(m),tr(m,c),pr(c),not\,\neg cov(m,c)\}} cov(m,c)$$

and identify the assumption $not\,\neg cov(m,c)$ as an element of the *defence set* of the dispute tree that the dispute derivation will output (if successful). At this stage `O` may opt to *eagerly* attack this assumption or *patiently* wait for `P` to carry on "expanding" its potential argument until it becomes an *actual argument*. This choice for `O` (and, in an analogous situation, for `P`) is dictated by the *selection function*, a parameter in the definition of dispute derivations. Whichever this selection function, at some later stage in the derivation the initial potential argument by `P` will become the *actual argument*

$$\{not\,\neg cov(m,c)\} \vdash^{p}_{\{\}} cov(m,c) \qquad (\mathtt{P}_{cov})$$

[7] In general, a potential argument is of the form $A \vdash^{p}_{S} \sigma$, for $A \subseteq \mathcal{A}$, $S \subseteq \mathcal{L}$, and $\sigma \in \mathcal{L}$, where the superscript p stands for "*potential*". Given $A \vdash^{p}_{S} \sigma$, there is a deduction for σ supported by $A \cup S$ (and some set of rules), with S the set of "unseen" sentences in this support and A the set of "seen" assumptions, as illustrated later.

attacked by a potential argument by $\mathtt{O}$

$$\{not\,cov'(m,c)\} \vdash^{p}_{U} \neg cov(m,c) \qquad\qquad (\mathtt{O}_{\neg cov}(U))$$

where, depending on the selection function, U may be as follows:

- $U = \{\neg reg(c,m)\}$, or
- $U = \{\}$.

In both cases, at some earlier stage, $\mathtt{P}$ will have chosen $not\,cov'(m,c)$, in the "unseen" support of a potential argument by $\mathtt{O}$, as a *culprit*, causing that assumption to be marked as "seen" from that stage onwards. Note that $\mathtt{O}$'s potential argument $\mathtt{O}_{\neg cov}(U)$, whichever U, is necessarily obtained by "expanding" the potential argument

$$\{\} \vdash^{p}_{\{\neg cov(m,c)\}} \neg cov(m,c)$$

put forward earlier by $\mathtt{O}$ to attack $\mathtt{P}$'s defence set element $not\,\neg cov(m,c)$. Also, when $\mathtt{P}$ "sees" $not\,cov'(m,c)$ and chooses it as a culprit in $\mathtt{O}_{\neg cov}(U)$, it creates a potential argument

$$\{\} \vdash^{p}_{\{cov'(m,c)\}} cov'(m,c)$$

which is later "expanded" to

$$\{\} \vdash^{p}_{\{\}} cov'(m,c). \qquad\qquad (\mathtt{P}_{cov'})$$

Since $\mathtt{O}$ cannot possibly attack this argument, the derivation terminates successfully, returning, as output, the defence set $\{not\,\neg cov(m,c)\}$ as well as the dialectical structure

$$\begin{tikzcd}
\mathtt{P}_{cov} \\
\mathtt{O}_{\neg cov'}(U) \arrow[u] \\
\mathtt{P}_{cov'} \arrow[u]
\end{tikzcd}$$

from which the dispute tree in Figure 3 is obtained.

In general, the defence set and the set of culprits are used to perform various kinds of *filtering* to save computation (to prevent players from attacking assumptions they have already attacked) as well as to guarantee that the computed defence set is conflict-free. Different semantics require different combinations of these filtering mechanisms. Moreover, the ideal semantics requires additional subcomputation to guarantee that the dispute tree is indeed ideal (namely that there exists no admissible dispute tree for the argument held at any opponent node).

5.3 ABA dialogues

ABA dialogues, as given in [Fan and Toni, 2014b; Fan and Toni, 2012a; Fan and Toni, 2011a], can be viewed as a distributed computation of dispute trees amongst agents, holding different ABA frameworks, but with the same underlying language $\mathcal{L}$.[8] An ABA dialogue is a sequence of *utterances*. The *content* of utterances may be a rule, an assumption, a contrary, or a claim whose "acceptability" (under admissible / grounded / ideal semantics) needs to be ascertained. The dialogue model can be used to support several dialogue types, e.g. information seeking and persuasion [Fan and Toni, 2011c; Fan and Toni, 2012a; Fan and Toni, 2012c; Fan *et al.*, 2014].

Syntactically, given two agents a_i and a_j, let $\mathcal{ID}$ be a (non-empty, possibly infinite) set that is totally ordered, with the ordering given by $<$, and contains a special element ID_0 which is the least element w.r.t. $<$. Then, utterances are denoted as tuples:

$$\langle a_i, a_j, T, C, \mathrm{ID}\rangle,$$

where

- a_i is the agent making this utterance;
- a_j is the recipient;
- C (the *content*) is of one of the following forms:
  - $\mathit{claim}(\chi)$ for some $\chi \in \mathcal{L}$ (a *claim*),
  - $\mathit{rl}(\sigma_0 \leftarrow \sigma_1, \ldots, \sigma_m)$ for some $\sigma_0, \ldots, \sigma_m \in \mathcal{L}$ with $m \geq 0$ (a *rule*),
  - $\mathit{asm}(\alpha)$ for some $\alpha \in \mathcal{L}$ (an *assumption*),
  - $\mathit{ctr}(\alpha, \sigma)$ for some $\alpha, \sigma \in \mathcal{L}$ (a *contrary*),
  - a *pass* sentence π, such that $\pi \notin \mathcal{L}$.
- $\mathrm{ID} \in \mathcal{ID} \setminus \{\mathrm{ID}_0\}$ (the *identifier*).
- $T \in \mathcal{ID}$ (the *target*); we impose that $T < \mathrm{ID}$.

Through a dialogue δ, the participating agents construct a joint *ABA framework* $\mathcal{F}_\delta$ *drawn from* δ. This $\mathcal{F}_\delta$ contains all information that the two agents have uttered

[8] Here, as in [Gaertner and Toni, 2008], we (equivalently) define the contrary of an assumption as a total mapping from an assumption to a (non-empty) set of sentences, instead of a mapping from an assumption to a sentence as in the original ABA. This lends itself better to a dialogical setting, as agents may hold different sentences as contrary to the same assumption.

in the dialogue and gives the context for examining the "acceptability" of the claim of the dialogue. Conceptually, a dialogue is "successful" if its claim is "acceptable" in $\mathcal{F}_\delta$. Note that the claim of a dialogue may be a belief, and acceptability thereof an indication that the agents may legitimately uphold the belief, or a course of actions, and acceptability thereof an indication that the agents may legitimately choose to adhere to it. Indeed, "acceptability" has so far shown to be an important criterion for assessing the outcome of various types of dialogues [Fan and Toni, 2011c; Fan and Toni, 2012a; Fan and Toni, 2012c; Fan *et al.*, 2014], and thus "successful" dialogues can be seen as building blocks of a widely deployable framework for distributed interactions in multi-agent systems.

Rather than checking "success" retrospectively, this can be guaranteed constructively by means of *legal-move functions* (see [Fan and Toni, 2011a; Fan and Toni, 2014b] for details) guaranteed to generate "successful" dialogues if a limited form of retrospective checking by means of *outcome functions* succeeds [Fan and Toni, 2011a; Fan and Toni, 2014b]. Dialogue goals, e.g. information-seeking, inquiry or persuasion, can be modelled with *strategy-move functions* [Fan and Toni, 2012a]. Given a dialogue, a legal-move function returns a set of allowed utterances that can be uttered to extend the dialogue. Legal-move functions can thus be viewed as dialogue protocols. Outcome functions are mappings from dialogues to true / false. Given a dialogue, an outcome function returns true if a certain property holds for that dialogue. From utterances allowed by legal-move functions, strategy-move functions further select the ones advancing dialogues towards their goals.

We illustrate ABA dialogues for information seeking, persuasion and inquiry for the flat ABA framework in Section 3.2 again, and refer to the original papers for formal definitions and results.

Informally, information seeking dialogues are dialogues with the inquirer agent seeking some specific information from the inquiree agent. In an information seeking dialogue, the inquirer agent does nothing but posing its query, whereas the inquiree agent puts forward information it possesses in answering the query. With the breakdown assistance policy example, suppose that the inquirer agent a_1 asks the inquiree agent a_2 about the existence of argument for the sentence $cov'(m,c)$, as follows:

$$\begin{array}{l}\langle a_1, a_2, 0, claim(cov'(m,c)), 1\rangle \\ \langle a_2, a_1, 1, rl(cov'(m,c) \leftarrow in(m,c)), 2\rangle \\ \langle a_2, a_1, 2, rl(in(m,c) \leftarrow), 3\rangle\end{array}$$

We can see that with a_1 and a_2 each using suitable strategy-move functions [Fan and Toni, 2012a], a_1 puts forward $cov'(m,c)$ as the claim of this dialogue and a_2 puts forward utterances 2 and 3 establishing the argument (in the ABA framework $\mathcal{F}_\delta$

drawn from the dialogue) for $cov'(m, c)$ supported by the empty set of assumptions and the two rules:

$$cov'(m, c) \leftarrow in(m, c) \text{ and } in(m, c) \leftarrow.$$

Persuasion dialogues are dialogues between two agents posing "incompatible" views towards some topic with the persuader trying to "prove" the topic and the persuadee trying to "disprove" it. Illustrating with the running example, we may have (for a_1 the persuader and a_2 the persuadee):

$$\begin{array}{l}
\langle a_1, a_2, 0, claim(not\ cov'(m, c)), 1\rangle \\
\langle a_1, a_2, 1, asm(not\ cov'(m, c)), 2\rangle \\
\langle a_2, a_1, 2, ctr(not\ cov'(m, c), cov'(m, c)), 3\rangle \\
\langle a_2, a_1, 3, rl(cov'(m, c) \leftarrow in(m, c)), 4\rangle \\
\langle a_2, a_1, 4, rl(in(m, c) \leftarrow), 5\rangle
\end{array}$$

Here, a_1 tries to establish the acceptability of $not\ cov'(m, c)$ by claiming it as an assumption, thus forming the argument $\{not\ cov'(m, c)\} \vdash not\ cov'(m, c)$, whereas a_2 puts forward the attacking argument $\{\} \vdash cov'(m, c)$ with utterances 3, 4 and 5. The presented persuasion behaviours of both agents are formally defined with strategy-move functions in [Fan and Toni, 2012c].

Inquiry dialogues are about two agents jointly "proving" or "disproving" the acceptability of some claim. Both agents put forward information supporting or attacking the claim. Again illustrated with the breakdown assistance policy example, we may have:

$$\begin{array}{l}
\langle a_1, a_2, 0, claim(cov(m, c)), 1\rangle \\
\langle a_1, a_2, 1, rl(cov(m, c) \leftarrow ah(m), tr(m, c), pr(c), not\ \neg cov(m, c)), 2\rangle \\
\langle a_1, a_2, 2, rl(ah(m) \leftarrow), 3\rangle \\
\langle a_1, a_2, 2, rl(tr(m, c) \leftarrow), 4\rangle \\
\langle a_1, a_2, 2, rl(pr(c) \leftarrow), 5\rangle \\
\langle a_1, a_2, 2, asm(not\ \neg cov(m, c)), 6\rangle \\
\langle a_2, a_1, 6, ctr(not\ \neg cov(m, c), \neg cov(m, c)), 7\rangle \\
\langle a_2, a_1, 7, rl(\neg cov(m, c) \leftarrow \neg reg(c, m), not\ cov'(m, c)), 8\rangle \\
\langle a_2, a_1, 8, rl(\neg reg(c, m) \leftarrow), 9\rangle \\
\langle a_2, a_1, 8, asm(not\ cov'(m, c)), 10\rangle \\
\langle a_2, a_1, 10, ctr(not\ cov'(m, c), cov'(m, c)), 11\rangle \\
\langle a_2, a_1, 11, rl(cov'(m, c) \leftarrow in(m, c)), 12\rangle \\
\langle a_2, a_1, 12, rl(in(m, c) \leftarrow), 13\rangle
\end{array}$$

With utterances 1-6, the argument $\{not\ \neg cov(m,c)\} \vdash cov(m,c)$ is formed. Utterances 7-10 form an attacking argument $\{not\ cov'(m,c)\} \vdash \neg cov(m,c)$, which is attacked by $\{\} \vdash cov'(m,c)$. The inquiry behaviour of agents is formally defined in [Fan and Toni, 2012a].

6 ABA and explanation

It is widely acknowledged that there is a strong interplay between argumentation and explanation, as for example discussed in [Seselja and Straßer, 2013]. In this section we overview existing proposals [Fan and Toni, 2015c; Schulz and Toni, 2016] using dispute trees in ABA (see Section 5) to provide (argumentative) explanations for why sentences should be concluded. Dispute trees for (flat) ABA can also serve as the basis for explanations in other settings, including various forms of decision-making [Fan and Toni, 2014a; Fan *et al.*, 2014; Zhong *et al.*, 2014; Fan *et al.*, 2013] and case-based reasoning [Čyras *et al.*, 2016] (see the original papers for details). In particular, natural language explanations can be drawn automatically from the dispute trees (see [Zhong *et al.*, 2014; Mocanu *et al.*, 2016] for details).

6.1 Dispute trees as explanations in flat ABA

We have seen (in Section 5) that dispute trees can be used to determine whether a sentence is admissible / grounded / ideal (and, as a consequence, preferred / complete). These dispute trees can also provide a computational counterpart for providing explanations for these sentences (being consequences of admissible / grounded / ideal / preferred / complete extensions, respectively). For example, the dispute tree in Figure 3 can be seen as providing an explanation for $cov(m,c)$, in the spirit of [Newton-Smith, 1981]:

> *... if I am asked to explain why I hold some general belief that p, I answer by giving my justification for the claim that p is true.*

Hence, if a belief q does not contribute to the justification of p, q should not be in the explanation of p. Dispute trees are explanations for (the argument in their root supporting) a sentence in that everything in them contribute to justifying the sentence. This informal notion can be formalised in terms of a notion of *related admissibility* of ABA arguments [Fan and Toni, 2015c],[9] in turn defined using a notion of *r-defence* [Fan and Toni, 2015c], given as follows:

[9]The notions defined in this section can be defined trivially for any AA framework too, as in [Fan and Toni, 2015c]. The notions for AA frameworks corresponding to ABA frameworks, given below, are an instantiation of the notions for any AA frameworks.

Definition 6.1. *Given an ABA framework* $\mathcal{ABA} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$*, let* $\mathcal{AA}(\mathcal{ABA}) = (Args, attack)$ *be the corresponding AA framework of* $\mathcal{ABA}$*.*

- *Given* $\mathtt{a}, \mathtt{b} \in Args$*,* $\mathtt{a}$ r-defends $\mathtt{b}$ *iff:*
  - *(i)* $\mathtt{a} = \mathtt{b}$*, or*
  - *(ii) there exists* $\mathtt{c} \in Args$ *such that* $\mathtt{a}$ *attacks* $\mathtt{c}$ *and* $\mathtt{c}$ *attacks* $\mathtt{b}$*, or*
  - *(iii) there exists* $\mathtt{c} \in Args$ *such that* $\mathtt{a}$ *r-defends* $\mathtt{c}$ *and* $\mathtt{c}$ *r-defends* $\mathtt{b}$*.*
- *Given* $\mathtt{a} \in Args$ *and* $\sigma \in \mathcal{L}$*,* $\mathtt{a}$ r-defends σ *iff there exists* $\mathtt{b} \in Args$ *such that* $\mathtt{b}$ *supports* σ *and* $\mathtt{a}$ *r-defends* $\mathtt{b}$*.*

As an illustration, given the ABA framework in Section 3.2:

$\{\} \vdash_{arg} cov'(m,c)$ r-defends $\{\} \vdash_{arg} cov'(m,c)$,
$\{\} \vdash_{arg} cov'(m,c)$ r-defends $\{not\ \neg cov(m,c)\} \vdash_{arg} cov(m,c)$,
$\{\} \vdash_{arg} cov'(m,c)$ r-defends $cov'(m,c)$,
$\{not\ \neg cov(m,c)\} \vdash_{arg} cov(m,c)$ r-defends $cov(m,c)$,
$\{\} \vdash_{arg} cov'(m,c)$ r-defends $cov(m,c)$.

The notion of related admissibility is obtained by combining the r-defence relation and standard admissibility as follows:

Definition 6.2. *Given an ABA framework* $\mathcal{ABA}$*, let* $\mathcal{AA}(\mathcal{ABA}) = (Args, attack)$ *be the corresponding AA framework of* $\mathcal{ABA}$*. A set of arguments* $\mathtt{A} \subseteq Args$ *is* related admissible *iff:*

(i) $\mathtt{A}$ *is admissible, and*

(ii) there exists a topic *sentence* σ *(of* $\mathtt{A}$*) such that* σ *is supported by some argument in* $\mathtt{A}$ *and for all* $\mathtt{b} \in \mathtt{A}$*,* $\mathtt{b}$ *defends* σ*.*

Intuitively, for a related admissible set of arguments $\mathtt{A}$ with topic sentence σ, no argument in $\mathtt{A}$ is "unrelated" to σ as all arguments in $\mathtt{A}$ r-defend σ.

As an illustration, given the ABA framework in Section 3.2,

$$\{\{\} \vdash_{arg} cov'(m,c)\}$$

is related admissible, with topic sentence $cov'(m,c)$, and

$$\{\{\} \vdash_{arg} cov'(m,c), \{not\ \neg cov(m,c)\} \vdash_{arg} cov(m,c)\}$$

is related admissible, with topic sentence $cov(m, c)$. Instead,

$$\{\{\} \vdash_{arg} cov'(m, c), \{not\ cov'(m, c)\} \vdash_{arg} \neg cov(m, c)\}$$

is not related admissible as it is not admissible; and

$$\{\{\} \vdash_{arg} ah(m), \{\} \vdash_{arg} pr(c)\}$$

is not related admissible as there does not exists a topic sentence σ such that it is defended by both $\{\} \vdash_{arg} ah(m)$ and $\{\} \vdash_{arg} pr(c)$.

Dispute trees correspond to explanations in that their defence sets are related admissible:

Theorem 6.3. *Given an ABA framework $\mathcal{ABA} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$, let $\mathcal{AA}(\mathcal{ABA}) = (Args, attack)$ be the corresponding AA framework of $\mathcal{ABA}$. Let $\sigma \in \mathcal{L}$.*

(i) Let $\mathtt{a} = A \vdash_{arg} \sigma \in Args$ and $\mathcal{T}$ be a dispute tree for $\mathtt{a}$. If $\mathcal{T}$ is admissible / grounded / ideal, then $\mathcal{D}(\mathcal{T})$ is related admissible.

(ii) If $\mathtt{A} \subseteq Args$ is related admissible, with topic sentence σ, then there is an admissible dispute tree $\mathcal{T}$ such that $\mathtt{A}' = \mathcal{D}(\mathcal{T})$ and $\mathtt{A}' \subseteq \mathtt{A}$.

Proof. (i) By definition 6.1, all arguments labelling $\mathtt{P}$ nodes ($\mathcal{D}(\mathcal{T})$) in a dispute tree r-defend the argument labelling the root note. By Theorem 5.5, all arguments labelling $\mathtt{P}$ nodes in an admissible / grounded / ideal dispute tree are admissible. Thus, by Definition 6.2, $\mathcal{D}(\mathcal{T})$ is related admissible.

(ii) If $\mathtt{A}$ is related admissible, by Definition 6.2, $\mathtt{A}$ is also admissible. By Theorem 5.5, there exists an admissible dispute tree $\mathcal{T}$ such that $\mathtt{A}' = \mathcal{D}(\mathcal{T})$ and $\mathtt{A}' \subseteq \mathtt{A}$.

□

6.2 Explanations for answer set programming

We have seen in Section 3.2 that a logic program can be encoded as an (equivalent) ABA framework such that the semantics of the ABA framework coincide with the semantics of the underlying logic program [Bondarenko *et al.*, 1997], for a wide range of semantics including the stable model (or *answer set*) semantics [Schulz and Toni, 2016; Schulz and Toni, 2015; Caminada and Schulz, 2015]. Logic programs under the answer set semantics (or *answer set programming*) can be applied in a wide range of scenarios [Baral and Uyan, 2001; Lifschitz, 2002; Eiter *et al.*, 2008; Delgrande *et al.*, 2009; Ricca *et al.*, 2010; Gebser *et al.*, 2011b; Boenn *et al.*, 2011;

Erdem, 2011; Ricca *et al.*, 2012; Terracina *et al.*, 2013], thanks also to the availability of efficient solvers for the computation of answer sets [Leone *et al.*, 2006; Gebser *et al.*, 2011a; Alviano *et al.*, 2015; Calimeri *et al.*, 2016]. These however do not provide any explanation of the answer sets computed. In particular, given one such answer set, there is no indication as to why a literal is or is not part of an answer set: this would instead be beneficial in human-computer interaction scenarios where logic programming is used for example to support human decision making.

As seen in Section 6.1, dispute trees do not only provide a way of determining whether or not a sentence is, for instance, admissible, but also an explanation as to *why* this is so.

Given that answer sets of a logic program correspond to stable extensions of the ABA framework encoding this logic program [Bondarenko *et al.*, 1997] and that if an answer set is guaranteed to exist then it is preferred (See Theorem 2.11 (i)), dispute trees can be used to determine, for a computed answer set and sentence in it, an explanation (in the form of a dispute tree) for why this is so. However, for the purpose of extracting explanations for literals in terms of other literals (rather than arguments, see [Schulz and Toni, 2016]), it is useful to single out, from the set of rules supporting ABA arguments, the rules with an empty body (referred to as *facts* in LP):

Definition 6.4. *Given a flat ABA framework* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$*, we say that* $(A, F) \vdash_{arg} \sigma$ *is a* fact-based-argument *for* $\sigma \in \mathcal{L}$ *supported by* $A \subseteq \mathcal{A}$ *and* $F \subseteq \{\pi \leftarrow \mid \pi \leftarrow \in \mathcal{R}\}$*, if there is an argument* $A \overset{R}{\vdash}_{arg} \sigma$ *such that* $F = R \cap \{\pi \leftarrow \mid \pi \leftarrow \in \mathcal{R}\}$*.*

A generalisation of dispute trees, which we call *explanation trees* [Schulz and Toni, 2016], where nodes are labelled by fact-based-arguments[10] can be used to explain why a literal is contained in a given answer set.

As an example, consider the ABA framework in Section 3.2, and the logic program amounting to its rules. This logic program has only one answer set: $\{ah(m)$, $tr(m, c)$, $pr(c)$, $\neg reg(c, m)$, $in(m, c)$, $cov(m, c)$, $cov'(m, c)\}$.

The explanation tree in Figure 4 justifies why Mary is covered, i.e. why $cov(m, c)$ is contained in the answer set. It expresses that there is evidence that Mary is covered (given by the argument with conclusion $cov(m, c)$ in the root proponent node) since Mary is the account holder and she is travelling in a car which is a private vehicle (facts supporting the argument), and since it can be assumed that there is no evidence that Mary is not covered ($not\ \neg cov(m, c)$ is an assumption). Even though there is evidence against this assumption, i.e. there is evidence that that Mary is not covered (given by the argument with conclusion $\neg cov(m, c)$ in

[10]For better readability we will omit the symbol $\leftarrow$ for all facts in the set F.

$$\texttt{P} : (\{not\ \neg cov(m,c)\}, \{ah(m), tr(m,c), pr(c)\}) \vdash_{arg} cov(m,c)$$
$$|$$
$$\texttt{O} : (\{not\ cov'(m,c)\}, \{\neg reg(c,m)\}) \vdash_{arg} \neg cov(m,c)$$
$$|$$
$$\texttt{P} : (\{\}, \{in(m,c)\}) \vdash_{arg} cov'(m,c)$$

Figure 4: An explanation tree justifying why Mary is covered in the running example

the opponent node) because she is not registered on the car, this evidence can be disregarded since Mary was in the car at the time of the breakdown (given by the proponent argument with conclusion $cov'(m,c)$, which attacks the assumptions $not\,cov'(m,c)$ of the opponent node). Note that this explanation tree is the same as the dispute tree in Figure 3 except that it uses fact-based-arguments.

In contrast to dispute trees which are used to justify only the containment of an argument *in* an extension, explanation trees can also explain why a literal is *not in* an answer set. In that case, explanation trees have an opponent node as their root, as illustrated by the explanation tree below which justifies why it is not the case that Mary is not covered (why $\neg cov(m,c)$ is not part of the answer set)

$$\texttt{O} : (\{not\ cov'(m,c)\}, \{\neg reg(c,m)\}) \vdash_{arg} \neg cov(m,c)$$
$$|$$
$$\texttt{P} : (\{\}, \{in(m,c)\}) \vdash_{arg} cov'(m,c)$$

Note that this explanation tree is a sub-tree of the previous explanation tree in Figure 4 justifying why $cov(m,c)$ is contained in the answer set.

Since explanation trees whose root node is a proponent node are dispute trees and since arguments which are in a stable extension are also in an admissible extension (Theorem 2.11 (i)), it follows from the relationhip between admissible extensions and admissible dispute trees given in Theorem 5.5 (i) that explanation trees starting with proponent nodes are admissible dispute trees. Thus, for literals contained in the answer set, explanation trees illustrate that the literal is supported by an admissible subset of this answer set.

Explanation trees whose root node is an opponent node have an explanation tree for a literal contained in the answer set as its direct sub-tree. Thus, this direct sub-tree is an admissible dispute tree. This means that literals not contained in the answer set are justified by illustrating that they are attacked by an admissible subset of the answer set.

In summary, explanation trees provide justifications of literals with respect to an answer set in terms of *admissible subsets* of this answer set [Schulz and Toni, 2016].

7 ABA and reasoning with preferences

Argumentation and preferences come a long way, see e.g. [Simari and Loui, 1992]. In general, preferences can be used to express, for instance, agents' degrees of belief, imperatives (moral, legal, etc.), aims, wishes. There are numerous methods in knowledge representation and reasoning to account for preference information, see e.g. [Prakken and Sartor, 1999; Kakas and Moraitis, 2003; Delgrande *et al.*, 2004; Brewka *et al.*, 2010; Domshlak *et al.*, 2011], and, in particular, several argumentation formalisms handling preferences, see e.g. [Bench-Capon, 2003; Modgil, 2009; Modgil and Prakken, 2014; García and Simari, 2014; Besnard and Hunter, 2014; Amgoud and Vesic, 2014; Baroni *et al.*, 2011], where preferences help to discriminate amongst information such as extensions, arguments, assumptions, rules, decisions and goals [Wakaki, 2014; Besnard and Hunter, 2014; Čyras and Toni, 2016a; Modgil and Prakken, 2014; Fan *et al.*, 2013]. There are various ways to deal with preferences in ABA too [Kowalski and Toni, 1996; Toni, 2008b; Thang and Luong, 2013; Fan *et al.*, 2013; Wakaki, 2014; Čyras and Toni, 2016a; Čyras and Toni, 2016b]. In this section we illustrate (by way of examples) these latter approaches. At a high-level, they can be divided in two groups: *meta level* approaches ([Wakaki, 2014; Čyras and Toni, 2016a; Čyras and Toni, 2016b], see Section 7.1), which, roughly, account for preferences at the semantic level, and *object level* approaches ([Kowalski and Toni, 1996; Toni, 2008b; Thang and Luong, 2013; Fan *et al.*, 2013], see Section 7.2), which, roughly, encode preferences within the existing ABA components (e.g. rules and assumptions) and avoid the need to modify the semantics of ABA frameworks.

Note that the examples chosen for the illustrations in this section have been selected for their simplicity, to give a high-level idea of the various approaches overviewed, and may not convey the full sophistication and usefulness of these approaches: the interested reader can find details as well as formal results in the original papers.

7.1 Handling preferences in ABA at the meta-level

[Wakaki, 2014] follows the ideas of prioritized logic programming [Sakama and Inoue, 2000] and equips ABA with explicit preferences by introducing a binary preference relation $\leqslant$ over the language $\mathcal{L}$. (For $a, b \in \mathcal{L}$, $a \leqslant b$ expresses that 'a is less or equally preferred than b'.) This ordering $\leqslant$ is then used to compute, by comparing consequences of extensions, a preference ordering $\sqsubseteq$ over extensions so as to select the most "preferable" extensions (i.e. the $\sqsubseteq$-maximal ones) of the underlying ABA framework. Such meta-level preference treatment can be well illustrated via scenarios of decision making with preferences, as in the following example.

Example 7.1. *Mary needs to decide what insurance policy to buy. Following the approach of [Fan* et al.*, 2013], information relevant to the decision making is represented via two tables,* $\mathtt{T_{DA}}$ *and* $\mathtt{T_{GA}}$*, as illustrated in Table 1, where*

- $\mathtt{T_{DA}}$ *describes relations between* decision candidates *(Policy 1 (P_1), Policy 2 (P_2)) and* attributes *(£50, £70, no exceptions (*no_ex*));*
- $\mathtt{T_{GA}}$ *describes relations between* goals *(*cheap *and full coverage (*full*)) and* attributes*.*

	£50	£70	no_ex
P_1	0	1	1
P_2	1	0	0

	£50	£70	no_ex
cheap	1	0	0
full	0	0	1

Table 1: $\mathtt{T_{DA}}$(left) and $\mathtt{T_{GA}}$(right), for Example 7.1.

Intuitively, each decision candidate has *certain attributes (P_1 has £70 and no_ex; P_2 has £50); and each goal can be* met *by certain attributes (*cheap *is met by £50;* full *is met by no_ex).*

In addition, suppose that the goal full *is preferred over* cheap*. In p_ABA, we can represent this information as a framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{}, \leqslant \rangle$, with the underlying ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with*

$$
\begin{aligned}
\mathcal{R} = \{ & \textit{£}70 \leftarrow P_1, \quad \textit{no\_ex} \leftarrow P_1, \quad \textit{£}50 \leftarrow P_2, \quad \textit{cheap} \leftarrow \textit{£}50, \\
& \textit{full} \leftarrow \textit{no\_ex}, \quad \overline{P_2} \leftarrow P_1, \quad \overline{P_1} \leftarrow P_2 \}, \\
\mathcal{A} = \{ & P_1, P_2 \}, \ \textit{and} \\
& \textit{cheap} \leqslant \textit{full}, \ \textit{cheap} \leqslant \textit{cheap}, \ \textit{full} \leqslant \textit{full}.
\end{aligned}
$$

$\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *has two preferred / stable extensions* $\{P_1\}$ *and* $\{P_2\}$*, with conclusions* $Cn(\{P_1\}) = \{P_1, \pounds 70,\ no\_ex,\ full\ \}$ *and* $Cn(\{P_2\}) = \{P_2, \pounds 50,\ cheap\ \}$*. We then find* $\{P_2\} \sqsubseteq \{P_1\}$ *and* $\{P_1\} \not\sqsubseteq \{P_2\}$*, so that* $\{P_1\}$ *is a* $\sqsubseteq$*-maximal extension, and is hence selected as the "preferable" one. Buying Policy 1 is thus deemed the better decision to take.*

Preferences in ABA can also be utilized to modify the attack relation between sets of assumptions, akin to approaches to argumentation with preferences such as [Bench-Capon, 2003; Modgil and Prakken, 2014; Amgoud and Vesic, 2014; Besnard and Hunter, 2014]. For instance, ABA$^+$ [Čyras and Toni, 2016a; Čyras and Toni, 2016b] equips ABA with a binary preference relation $\leqslant$ over assumptions, and incorporates preferences directly into the attack relation so as to *reverse* attacks that stem from sets containing assumptions less preferred than the one whose contrary is deduced, as illustrated next.

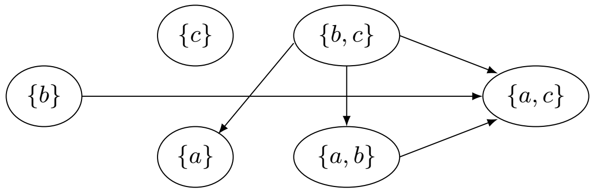
Example 7.2. *Suppose that Mary has decided to buy Policy 1, as suggested in Example 7.1. However, Mary has also found some information on the Internet about the policy: source C says that under certain circumstances (c), the policy applies only to citizens of certain specified countries; source B says that sometimes (say, assuming c), the policy applies only to UK residents (*$UK \leftarrow b, c$*); source A says that sometimes (assuming c) the policy applies only to non-UK residents (*$non\_UK \leftarrow a, c$*). Mary trusts the source A the least (i.e.* $a < b$*,* $a < c$*). What is Mary justified believing in about the applicability of the policy, given certain circumstances?*

We can formalize this in ABA$^+$ *as follows: consider* $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{}, \leqslant \rangle$ *with*

$$
\begin{aligned}
&\mathcal{A} = \{a, b, c\},\\
&\mathcal{R} = \{non\_UK \leftarrow a, c, \quad UK \leftarrow b, c\},\\
&\overline{a} = UK, \quad \overline{b} = non\_UK,\\
&a < b, \quad a < c,
\end{aligned}
$$

where the assumptions stand for the possibility to trust the sources and preferences indicate their relative credibility, rules are drawn given that information from sources A and B is applicable under certain circumstances (c), also given that sources A and B are in conflict.

The underlying ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ *admits both* $\{a, c\}$ *and* $\{b, c\}$ *as stable / preferred extensions. In ABA*$^+$*, attacks from* $\{a, c\}$ *to (any set of assumptions containing) b are reversed, due to the a's lower credibility in comparison with b. Hence,* $\{b, c\}$ *is a unique stable / preferred extension, arguably the desirable outcome. This can be seen clearly given the graph depicted below, omitting, for readability, assumption sets* $\{\}$ *and* $\{a, b, c\}$*, as well as attacks to and from them:*



7.2 Handling preferences in ABA at the object-level

Instead of equipping ABA frameworks with explicit preference relations as in Section 7.1, and then modifying the semantics of ABA (by either comparing extensions or modifying the attack relation), preferences can be encoded within the existing components (rules, assumptions and contraries) without modifying the semantics.

For instance, [Kowalski and Toni, 1996; Toni, 2008b] deal with preferences between rules by adding conditions (i.e. assumptions) to the body of rules expressing that the rules are not attacked by other higher preference rules, by appropriately defining contraries of these assumptions. For illustration, consider the following example:

Example 7.3. *In our breakdown policy example of Section 3, the rules in the ABA instance for LP of section 3.2 can be modified by adding assumptions as follows:*

$$cov(m, c) \leftarrow ah(m), tr(m, c), pr(c), not\ \neg cov(m, c), a_{cov(m,c)},$$
$$\neg cov(m, c) \leftarrow \neg reg(c, m), not\ cov'(m, c), a_{\neg cov(m,c)}.$$

If a preference of the second rule over the first one is to be expressed, then one could assign contraries

$$\overline{a_{cov(m,c)}} = \neg cov(m, c), \quad \overline{a_{\neg cov(m,c)}} = a^c,$$

where a^c is new to $\mathcal{L}$.

More generally (as in [Toni, 2008b]), one can assume a naming *function assigning distinguished names to elements (e.g. rules) of a given domain, and given preferences over the elements of the domain, consider a language that includes sentences expressing those preferences. For example, the two rules above can be given names r and r' respectively, and the language $\mathcal{L}$ would contain a "preference sentence" $r < r'$ expressing that the second rule is preferred over the first one. Then, when mapping the domain into an ABA framework, a rule*

$$\neg cov(m, c) \leftarrow r < r', a_{\neg cov(m,c)},$$

could be added, so as to account for preferences, which could be stated e.g. via a rule $r < r' \leftarrow$. This way, ABA can also account for dynamic preferences *(see e.g. [Prakken and Sartor, 1999]), i.e. preferences that are themselves deducible using rules, possibly from other assumptions.*

Yet another way to deal with preferences in ABA on the object level is used in [Thang and Luong, 2013] when translating Brewka's preferred subtheories [Brewka, 1989] into ABA. To capture the interplay between classically inconsistent sentences and partial preference information among them, [Thang and Luong, 2013] introduce assumptions for representing sentences in the domain language as well as for determining their acceptance status in the construction of preferred subtheories, and further introduce rules for: deriving sentences from their corresponding assumptions; deriving contraries of the least preferred elements of minimally inconsistent subsets; enforcing (non-)acceptance of an assumption iff the statuses more preferred assumptions are determined. This is illustrated next.

Example 7.4. *Let us rewrite the rules from Example 7.2 as*

$$\alpha, \gamma \rightarrow \neg UK, \quad \beta, \gamma \rightarrow UK$$

(where $\rightarrow$ is material implication) to constitute the facts (world knowledge), and let $T = \{\alpha, \beta, \gamma\}$ be the theory representing the defeasible knowledge, with preferences $\alpha < \beta$ and $\alpha < \gamma$. This partial order $<$ admits two extensions to total orders, namely $\alpha < \beta < \gamma$ and $\alpha < \gamma < \beta$, both of which result in the same preferred subtheory of T, namely $\{\beta, \gamma\}$.

The domain can be mapped into an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ with (for readability treating contraries of assumptions as symbols in the language)

$$\begin{aligned}
\mathcal{A} =& \{a_\alpha, a_\beta, a_\gamma\} \cup \{b_\alpha, b_\beta, b_\gamma\},\\
\mathcal{R} =& \{\alpha \leftarrow a_\alpha, \quad \beta \leftarrow a_\beta, \quad \gamma \leftarrow a_\gamma\} \cup \{\overline{a_\alpha} \leftarrow a_\beta, \overline{b_\beta}, a_\gamma, \overline{b_\gamma}\} \cup \\
& \{\overline{a_\alpha} \leftarrow b_\alpha, \quad \overline{a_\beta} \leftarrow b_\beta, \quad \overline{a_\gamma} \leftarrow b_\gamma\} \cup \\
& \{\overline{b_\beta} \leftarrow, \quad \overline{b_\gamma} \leftarrow, \quad \overline{b_\alpha} \leftarrow a_\beta, \overline{b_\beta}, a_\gamma, \overline{b_\gamma}, \\
& \overline{b_\alpha} \leftarrow \overline{a_\beta}, \overline{b_\beta}, a_\gamma, \overline{b_\gamma}, \quad \overline{b_\alpha} \leftarrow a_\beta, \overline{b_\beta}, \overline{a_\gamma}, \overline{b_\gamma}, \quad \overline{b_\alpha} \leftarrow \overline{a_\beta}, \overline{b_\beta}, \overline{a_\gamma}, \overline{b_\gamma}\}.
\end{aligned}$$

This $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ has a unique stable extension $\{a_\beta, a_\gamma\}$, corresponding to the unique preferred subtheory of T.

Another example of preferences dealt with in ABA within the object-level is to support decision making with preferences over goals. Differently from the other approaches overviewed in this section, this method is specific to decision making settings, and uses preferences over sentences (the goals) within decision criteria (e.g. various kinds of "dominance", see [Fan *et al.*, 2013]) for choosing "best" decisions. This can be illustrated in the context of the same decision making setting of Example 7.1:

Example 7.5. *Given the two tables,* $\mathtt{T_{DA}}$ *and* $\mathtt{T_{GA}}$*, in Table 1, as well as the preference* $full > cheap$*, the problem of identifying the "best" decisions, namely those "meeting the more preferred goals that no other decisions meet", can be represented in ABA with*

$$\begin{array}{rll}
\mathcal{R} = & \{ & has(P_1, £70) \leftarrow, \quad has(P_1,\ no_ex) \leftarrow, \quad has(P_2, 50) \leftarrow, \\
& & satBy(cheap, £50) \leftarrow, \quad satBy(full) \leftarrow no_ex, \\
& & prefer(full, cheap) \leftarrow\} \cup \\
& \{ & met(X,Y) \leftarrow has(X,Z), satBy(Y,Z) \mid X \in \{P_1,P_2\}, \\
& & Y \in \{cheap, full\},\ Z \in \{£50, £70,\ no_ex\ \}\ \} \cup \\
& \{ & sel(X) \leftarrow met(X,Y), noBetterThan(X,Y) \mid X \in \{P_1,P_2\}, \\
& & Y \in \{cheap, full\}\ \} \cup \\
& \{ & better(X,Y) \leftarrow met(X',Y'), prefer(Y',Y), X \neq X' \mid \\
& & X, X' \in \{P_1,P_2\},\ Y, Y' \in \{cheap, full\}\ \} \\
\mathcal{A} = & \{ & noBetterThan(X,Y) \mid X \in \{P_1,P_2\},\ Y \in \{cheap, full\}\ \} \\
\overline{not\ x} = & & better(X,Y)\ \ \textit{for any}\ \ x = noBetterThan(X,Y) \in \mathcal{A}
\end{array}$$

Then

$$\{\{noBetterThan(P_1, full)\} \vdash_{arg} sel(P_1)\}$$

is admissible whereas

$$\{\{noBetterThan(P_2, cheap)\} \vdash_{arg} sel(P_2)\}$$

is not, as the latter is attacked by $\{\} \vdash_{arg} better(P_2, cheap)$*. Indeed, Policy 1 is the "best" decision in this simple setting.*

8 Conclusion

This paper overviews research, spanning over more than two decades (from [Bondarenko *et al.*, 1993] onwards), on Assumption-Based Argumentation (ABA), a framework for structured argumentation motivated by and emerging from non-monotonic reasoning. We have focused on the semantic foundations of ABA, in the general case as well as for the special case of flat ABA frameworks, while also providing an overview of the computational machinery (flat) ABA is equipped with as well as its uses for explaining argumentative conclusions. Finally, we have overviewed, with the aid of examples, uses and generalisations of ABA to support reasoning with preferences.

This paper is meant as a taster of ABA rather than a comprehensive technical presentation, and complements other earlier overviews [Dung *et al.*, 2009; Toni, 2012; Toni, 2014]. In particular, it focuses on the case of general (possibly non-flat) frameworks rather than flat frameworks as in the earlier overviews, and provides a taster of explanation and the treatment of preferences in ABA.

We omitted to mention several aspects of ABA. For instance, there are several other instances of ABA for non-monotonic reasoning (see [Bondarenko *et al.*, 1997]), and ABA has also been shown to admit Adaptive Logic and ASPIC+ without preferences as instances [Heyninck and Straßer, 2016]. Other ABA semantics have been presented in the literature, e.g. the semi-stable semantics [Caminada *et al.*, 2015]. Moreover, formulation of (some) ABA semantics in terms of labellings, in the spirit of those proposed for abstract argumentation [Caminada and Gabbay, 2009], have been proposed [Schulz and Toni, 2014; Schulz and Toni, 2015; Schulz and Toni, 2017]. Further, the computational complexity of several reasoning problems in several instances of ABA is known [Dimopoulos *et al.*, 2002; Dunne, 2009], and several systems for (flat) ABA are publicly available (see `robertcraven.org/proarg/` and `www-abaplus.doc.ic.ac.uk`). Recent work also shows that (sets of) arguments in ABA can be re-interpreted as graphs, with conceptual and computational advantages [Craven and Toni, 2016]. We have seen in Section 7 that ABA has been extended to accommodate reasoning with preferences: other extensions of ABA also exist, notably the probabilistic ABA of [Dung and Thang, 2010]. Finally, we have not delved into applications of ABA: these are overviewed in earlier surveys [Dung *et al.*, 2009; Toni, 2012; Toni, 2014] or other papers [Gao *et al.*, 2016; Fan and Toni, 2016]. In particular, [Gao *et al.*, 2016] uses related admissibility in ABA (see Section 6.1) to coordinate and resolve conflicts amongst agents, while also guaranteeing that privacy is preserved, in some sense, whereas [Fan and Toni, 2016] reinterprets the problem of determining solutions in games in normal form in ABA, using ABA dialogues (as summarised in Section 5.3) to determine these solutions in a distributed fashion, without agents fully disclosing their preferences.

There are several open issues in ABA as well as several directions for future work. We have seen, in Section 6.2, that explanations as to why sentences are not "acceptable" may be useful [Schulz and Toni, 2016]. The concept of "not-explanations" can be defined, in general, in abstract argumentation [Fan and Toni, 2015b]: it would be useful to define this notion also for ABA. Other forms of explanations have been defined, notably for explaining inconsistencies in LP [Schulz *et al.*, 2015]: it would be interesting to define a notion of explanation for the lack of (e.g. stable) extensions in generic ABA. Some preliminary work [Zhong *et al.*, 2014; Mocanu *et al.*, 2016] indicates that natural language explanations can be naturally drawn from dispute trees computed by dispute derivations: it would be interesting to develop this work further and test the usefulness of the generated explanations in practice. Further, in multi-agent settings, it would be interesting to further study strategic behaviour of agents using ABA as their language of interaction [Fan and Toni, 2012c; Fan and Toni, 2015a; Gao *et al.*, 2016; Fan and Toni, 2016]. From a computational viewpoint, (flat) ABA is equipped with several (sound and complete) algorithms for determining the "acceptability" of sentences (and compute extensions "supporting" them): it would be interesting to see how these algorithms can be generalised to the case of any, possibly non-flat, ABA frameworks and/or deployed when preferences are given, e.g. in the spirit of Gorgias (see `gorgiasb.tuc.gr/index.html`) and dealt with at the meta-level (as in Section 7.1). Moreover, it would be interesting to identify (sound and complete) computational machinery for determining extensions of ABA, without having to resort to implementations of abstract argumentation by using the mapping described in Section 4.

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Received 17 July 2017

Argumentation Semantics as Formal Discussion

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Abstract

In the current review paper, we provide an overview of how mainstream argumentation semantics can be interpreted in terms of structured discussion. The idea is that an argument is justified according to a particular argumentation semantics iff it is possible to win a discussion of a particular type. Hence, different argumentation semantics correspond to different types of discussion. Our aim is to provide an overview of what these discussions look like, and their formal correspondence to argumentation semantics.

1 Introduction

The term "argumentation", when used in an informal way, calls upon intuitions of arguments being exchanged in some kind of interactive discussion. Yet, the notion of discussion plays a relatively limited role in abstract argumentation theory, which mainly focuses on various principles (called "argumentation semantics") for selecting nodes from a graph. As such, there seems to be quite a gap between (abstract) argumentation theory as described in much of the literature, and as it occurs in everyday life.

In order to address this gap, attempts have been made to express argumentation semantics in terms of structured discussion. More precisely, the idea is that an argument is accepted w.r.t. a particular argumentation semantics iff it is possible to successfully defend the argument using a particular kind of discussion. In the current paper we provide an overview of what the different kinds of discussion are, and how they formally relate to their associated argumentation semantics.

Although the discussion protocols (which we will often refer to as "discussion games") can serve as proof procedures of their associated argumentation semantics,

The current paper will also be published as a chapter in the Handbook of Formal Argumentation.

their potential application is much wider than that. One could for instance use the discussion games for the purpose of human computer interaction. Suppose a knowledge-based system has determined that a particular argument (say, about how to treat a patient) should be accepted, and communicates this to its user (say, a doctor). When the user asks why this is the case, what should probably be avoided is a highly technical answer of the form "because the argument is in the minimal fixpoint of monotonic function F".[1] Instead, one would like the user to critically question the answer, and be able to utter counter arguments to see whether these are properly addressed (by the system providing counter counter arguments). As an example of such a human-computer discussion, consider the following dialogue:
System: The patient is best off with medicine X, because this is the most effective.
User: But the patient is diabetic, for which medicine X could have side effects.
System: Recent studies have shown that these side effects are relatively minor.
So instead of the system immediately providing the full justification for its answer (say, by providing the entire grounded extension) in engages in a discussion with its user. Ideally, such a discussion should be "natural" in the sense that the human-computer interaction looks as much as possible as human-human interaction (say, if the doctor were to discuss the case with a more senior colleague).

Apart from being natural, the discussion should also be sound and complete. That is, the ability to win the discussion for a particular argument (that is, to have a winning strategy for the argument in the discussion game) should coincide with the argument being justified according to a pre-defined argumentation semantics. Soundness and completeness imply that if the system provides an answer ("argument A is (or is not) justified according to a particular argumentation semantics") the system can successfully defend itself in the discussion with the user. When this discussion is also perceived as natural by the user, this will hopefully increase the user's confidence in the system's answer.

Soundness and completeness also imply that what we are looking for are essentially proof procedures for particular argumentation semantics. Several of these have been stated in the literature. Inclusion in the current review paper is done based on two criteria:

(1) does the discussion game have any link with natural discussion concepts, like described in philosophy or linguistics?

(2) is the discussion game such that it guarantees the absence of any exponential blowups, in either time or space?

[1] Which basically says the argument is in the grounded extension.

Criterion (1) is the reason why for instance we have not included any discussion games for sceptical preferred semantics (like those of Doutre and Mengin [2004] and Dung and Thang [2007]). Criterion (2) is the reason why we did not include a detailed treatment of tree-based discussion games (like those of Prakken and Sartor [1997], Caminada [2004], Modgil and Caminada [2009] and Dung *et al.* [2007].[2]

The remaining part of this paper is structured as follows. First, in Section 2 we briefly recall some basic definitions and results from abstract argumentation theory. Then, in Section 3 we describe a discussion game for (credulous) preferred semantics [Caminada *et al.*, 2014a], and explain that it contains aspects of Socratic discussion. Then, in Section 4 we briefly state how this discussion game can be reapplied in the context of ideal semantics [Caminada *et al.*, 2014a]. In Section 5 we subsequently describe a discussion game for stable semantics [Caminada and Wu, 2009], basically by making minor modifications to the earlier described discussion game for (credulous) preferred semantics. In Section 6 we then describe a different discussion game in the context of grounded semantics [Caminada, 2015a] and explain its relationship with persuasion dialogue. Then, in Section 7 we briefly examine tree-based discussion games and explain one of their main disadvantages: the possibility of an exponential blowup in time or space. We round off with a discussion in Section 8.

2 Formal Preliminaries

In the current section, we briefly recall some basic definitions from abstract argumentation theory. For current purposes, we restrict ourselves to finite argumentation frameworks.

Definition 1 (argumentation framework)**.** *An* argumentation framework *is a pair* (Ar, att) *where* Ar *is a finite set of entities called* arguments *and* att *is a binary relation on* Ar*.*

Given an argumentation framework (Ar, att), $A, A' \in Ar$ and $\mathcal{A}rgs$, $\mathcal{A}rgs' \subseteq Ar$, we say that (1) A attacks A' iff $(A, A') \in att$, (2) A attacks $\mathcal{A}rgs$ iff A attacks some argument in $\mathcal{A}rgs$, (3) $\mathcal{A}rgs$ attacks A iff some argument in $\mathcal{A}rgs$ attacks A, and (4) $\mathcal{A}rgs$ attacks $\mathcal{A}rgs'$ iff some argument in $\mathcal{A}rgs$ attacks some argument in $\mathcal{A}rgs'$.

Definition 2 (preliminaries, extension-based)**.** *Let* (Ar, att) *be an argumentation framework. A set* $\mathcal{A}rgs \subseteq Ar$ *is* conflict-free *iff* $\mathcal{A}rgs$ *does not attack itself. A set* $\mathcal{A}rgs \subseteq Ar$ defends $A \in Ar$ *iff for each* $B \in Ar$ *that attacks* A*, it holds that* $\mathcal{A}rgs$ *attacks* B*.*

[2] How tree-based discussion games can lead to an exponential blowup is explained in Section 7.

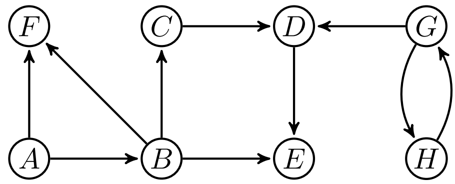


Figure 1: An argumentation framework to illustrate strong admissibility.

Definition 3 (admissibility, extension-based)**.** *Let* (Ar, att) *be an argumentation framework. A set* $\mathcal{A}rgs \subseteq Ar$ *is* admissible *iff* $\mathcal{A}rgs$ *is conflict-free and each* $A \in \mathcal{A}rgs$ *is defended by* $\mathcal{A}rgs$*.*

Definition 4 (strong admissibility, extension-based)**.** *Let* (Ar, att) *be an argumentation framework. A set* $\mathcal{A}rgs \subseteq Ar$ *is* strongly admissible *iff each* $A \in \mathcal{A}rgs$ *is defended by some* $\mathcal{A}rgs' \subseteq \mathcal{A}rgs \setminus \{A\}$ *which in its turn is again strongly admissible.*

It has been proved that each strongly admissible set is conflict-free as well as admissible [Baroni and Giacomin, 2007; Caminada, 2014].

As an example, consider the argumentation framework of Figure 1. Here, the set $\{A, C\}$ is strongly admissible as A is defended by $\emptyset \subseteq \{A, C\} \setminus \{A\}$ which is trivially strongly admissible, and C is defended by $\{A\} \subseteq \{A, C\} \setminus \{C\}$ which is strongly admissible (as A is defended by $\emptyset \subseteq \{A\} \setminus \{A\}$). The set $\{G\}$, however, is admissible but not strongly admissible as G is not defended by any subset of $\{G\} \setminus \{G\}$.

Definition 5 (completeness, extension-based)**.** *Let* (Ar, att) *be an argumentation framework. A set* $\mathcal{A}rgs \subseteq Ar$ *is a* complete extension *iff* $\mathcal{A}rgs$ *is conflict-free and the set of arguments defended by* $\mathcal{A}rgs$ *is equal to* $\mathcal{A}rgs$*.*

Definition 6 (semantics, extension-based)**.** *Let* (Ar, att) *be an argumentation framework. A set* $\mathcal{A}rgs \subseteq Ar$ *is called*

1. *a* grounded extension *iff* $\mathcal{A}rgs$ *is the minimal (w.r.t.* $\subseteq$*) complete extension*
2. *a* preferred extension *iff* $\mathcal{A}rgs$ *is a maximal (w.r.t.* $\subseteq$*) complete extension*
3. *a* stable extension *iff* $\mathcal{A}rgs$ *is a complete extension that attacks each argument in* $Ar \setminus \mathcal{A}rgs$
4. *an* ideal extension *iff* $\mathcal{A}rgs$ *is the maximal (w.r.t.* $\subseteq$*) complete extension that is not attacked by any complete extension*

We recall that each argumentation framework has precisely one grounded extension, precisely one ideal extension, one or more preferred extensions and zero or more stable extensions.

The above definition describes grounded, preferred, stable and ideal semantics uniformly in terms of complete semantics. However, for our purposes it is sometimes useful to describe these semantics in terms of (strong) admissibility.

Theorem 1 (semantics, extension-based)**.** *Let (Ar, att) be an argumentation framework. A set $\mathcal{A}rgs \subseteq Ar$ is*

1. *a preferred extension iff $\mathcal{A}rgs$ is a maximal (w.r.t. $\subseteq$) admissible set*
2. *a grounded extension iff $\mathcal{A}rgs$ is the maximal (w.r.t. $\subseteq$) strongly admissible set*
3. *a stable extension iff $\mathcal{A}rgs$ is an admissible set that attacks each argument in $Ar \setminus \mathcal{A}rgs$*
4. *an ideal extension iff $\mathcal{A}rgs$ is the maximal (w.r.t. $\subseteq$) admissible set that is not attacked by any admissible set*

Apart from the extension-based view on argumentation semantics, there is also the labelling-based view [Caminada, 2006; Caminada and Gabbay, 2009; Caminada, 2011; Baroni *et al.*, 2011] of which we now provide a brief overview.

Definition 7 (preliminaries, labelling-based)**.** *Let (Ar, att) be an argumentation framework. An* argument labelling *is a function $\mathcal{L}ab : Ar \to \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$. We define $\mathtt{in}(\mathcal{L}ab)$ as $\{A \in Ar \mid \mathcal{L}ab(A) = \mathtt{in}\}$, $\mathtt{out}(\mathcal{L}ab)$ as $\{A \in Ar \mid \mathcal{L}ab(A) = \mathtt{out}\}$ and $\mathtt{undec}(\mathcal{L}ab)$ as $\{A \in Ar \mid \mathcal{L}ab(A) = \mathtt{undec}\}$. We sometimes write a labelling as a triple $(\mathtt{in}(\mathcal{L}ab), \mathtt{out}(\mathcal{L}ab), \mathtt{undec}(\mathcal{L}ab))$. If $\mathcal{L}ab_1$ and $\mathcal{L}ab_2$ are labellings, we write $\mathcal{L}ab_1 \sqsubseteq \mathcal{L}ab_2$ when $\mathtt{in}(\mathcal{L}ab_1) \subseteq \mathtt{in}(\mathcal{L}ab_2)$ and $\mathtt{out}(\mathcal{L}ab_1) \subseteq \mathtt{out}(\mathcal{L}ab_2)$. Moreover, we write $\mathcal{L}ab_1 \approx \mathcal{L}ab_2$ when $\mathtt{in}(\mathcal{L}ab_1) \cap \mathtt{out}(\mathcal{L}ab_2) = \emptyset$ and $\mathtt{out}(\mathcal{L}ab_1) \cap \mathtt{in}(\mathcal{L}ab_2) = \emptyset$.*

Definition 8 (admissibility, labelling-based)**.** *Let $\mathcal{L}ab$ be a labelling of argumentation framework (Ar, att). $\mathcal{L}ab$ is called an* admissible labelling *iff for each $A \in Ar$ it holds that*

1. *if $\mathcal{L}ab(A) = \mathtt{in}$ then for each $B \in Ar$ that attacks A it holds that $\mathcal{L}ab(B) = \mathtt{out}$*
2. *if $\mathcal{L}ab(A) = \mathtt{out}$ then there exists a $B \in Ar$ that attacks A such that $\mathcal{L}ab(B) = \mathtt{in}$*

In order to define strong admissibility in the context of argument labellings, we first need to introduce the concept of a min-max numbering.

Definition 9 (min-max numbering)**.** *Given an admissible labelling $\mathcal{L}ab$ of argumentation framework (Ar, att), a min-max numbering is a function $\mathcal{MM}_{\mathcal{L}ab} : \mathtt{in}(\mathcal{L}ab) \cup \mathtt{out}(\mathcal{L}ab) \rightarrow \mathbb{N} \cup \{\infty\}$ such that for each $A \in \mathtt{in}(\mathcal{L}ab) \cup \mathtt{out}(\mathcal{L}ab)$ it holds that*

- *if $\mathcal{L}ab(A) = \mathtt{in}$ then $\mathcal{MM}_{\mathcal{L}ab}(A) = \max(\{\mathcal{MM}_{\mathcal{L}ab}(B) \mid B$ attacks A and $\mathcal{L}ab(B) = \mathtt{out}\}) + 1$ (with $\max(\emptyset)$ defined as 0)*
- *if $\mathcal{L}ab(A) = \mathtt{out}$ then $\mathcal{MM}_{\mathcal{L}ab}(A) = \min(\{\mathcal{MM}_{\mathcal{L}ab}(B) \mid B$ attacks A and $\mathcal{L}ab(B) = \mathtt{in}\}) + 1$ (with $\min(\emptyset)$ defined as ∞)*

It can be proved that each admissible labelling has a unique min-max numbering [Caminada, 2014].[3]

Definition 10 (strong admissibility, labelling-based)**.** *Let $\mathcal{L}ab$ be a labelling of argumentation framework (Ar, att). $\mathcal{L}ab$ is called a* strongly admissible labelling *iff it is an admissible labelling whose associated min-max numbering yields natural numbers only (so no argument is numbered ∞).*

From Definition 10 it trivially follows that each strongly admissible labelling is also an admissible labelling.

As an example, consider the argumentation framework shown in Figure 1. Here $\mathcal{L}ab_1 = (\{A, C, E, G\}, \{B, D, H\}, \{F\})$ is an admissible labelling with associated min-max numbering $\mathcal{MM}_{\mathcal{L}ab_1} = \{(A\colon 1), (B\colon 2), (C\colon 3), (D\colon 4), (E\colon 5), (G\colon \infty), (H\colon \infty)\}$, which implies that $\mathcal{L}ab_1$ is not strongly admissible. Furthermore, $\mathcal{L}ab_2 = (\{A, C, E\}, \{B, D, F\}, \{G, H\})$ is an admissible labelling with associated min-max numbering $\mathcal{MM}_{\mathcal{L}ab_2} = \{(A\colon 1), (B\colon 2), (C\colon 3), (D\colon 4), (E\colon 5), (F\colon 2)\}$, which implies that $\mathcal{L}ab_2$ is indeed a strongly admissible labelling.

Definition 11 (completeness, labelling-based)**.** *Let $\mathcal{L}ab$ be a labelling of argumentation framework (Ar, att). $\mathcal{L}ab$ is called a* complete labelling *iff for each $A \in Ar$ it holds that*

1. *if $\mathcal{L}ab(A) = \mathtt{in}$ then for each $B \in Ar$ that attacks A it holds that $\mathcal{L}ab(B) = \mathtt{out}$*

[3] The min-max numbering can be constructed in an iterative way, starting from the unnumbered `in`-labelled arguments without attackers (these are numbered 1), then the unnumbered `out`-labelled arguments that are attacked by these (these are numbered 2), etc. When a particular iteration provides no new argument numbers, the remaining unnumbered `in` and `out`-labelled arguments are numbered ∞. See the work of Caminada [2014] for details.

2. *if* $\mathcal{L}ab(A) = \texttt{out}$ *then there exists a* $B \in Ar$ *that attacks* A *such that* $\mathcal{L}ab(B) = \texttt{in}$

3. *if* $\mathcal{L}ab(A) = \texttt{undec}$ *then not for each* $B \in Ar$ *that attacks* A *it holds that* $\mathcal{L}ab(B) = \texttt{out}$ *and there does not exist a* $B \in Ar$ *that attacks* A *such that* $\mathcal{L}ab(B) = \texttt{in}$

Definition 12 (semantics, labelling-based)**.** *Let* (Ar, att) *be an argumentation framework. A labelling* $\mathcal{L}ab$ *is called*

1. *a* grounded labelling *iff it is the minimal (w.r.t.* $\sqsubseteq$*) complete labelling*

2. *a* preferred labelling *iff it is a maximal (w.r.t.* $\sqsubseteq$*) complete labelling*

3. *a* stable labelling *iff it is a complete labelling with* $\texttt{undec}(\mathcal{L}ab) = \emptyset$

4. *an* ideal labelling *iff it is the maximal (w.r.t.* $\sqsubseteq$*) complete labelling that is compatible (*$\approx$*) with every complete labelling*

We recall that each argumentation framework has precisely one grounded labelling, precisely one ideal labelling, one or more preferred labellings and zero or more stable labellings.

The above definition describes grounded, preferred, stable and ideal semantics in terms of complete labellings. However, it is sometimes useful to be able to describe these semantics in terms of (strong) admissibility, similar to what was done earlier for the extension-based semantics.

Theorem 2 (semantics, labelling-based)**.** *Let* (Ar, att) *be an argumentation framework. A labelling* $\mathcal{L}ab$ *is called*

1. *a* preferred labelling *iff it is a maximal (w.r.t.* $\sqsubseteq$*) admissible labelling*

2. *a* grounded labelling *iff it is the maximal (w.r.t.* $\sqsubseteq$*) strongly admissible labelling*

3. *a* stable labelling *iff it is an admissible labelling with* $\texttt{undec}(\mathcal{L}ab) = \emptyset$

4. *an* ideal labelling *iff it is the maximal (w.r.t.* $\sqsubseteq$*) admissible labelling that is compatible (*$\approx$*) with every admissible labelling*

To be able to easily switch between the labelling-based approach and the extension-based approach, we introduce two functions `Lab2Ext` and `Ext2Lab`, such that for an admissible labelling $\mathcal{L}ab$, $\texttt{Lab2Ext}(\mathcal{L}ab)$ is defined as $\texttt{in}(\mathcal{L}ab)$, and for an admissible set $\mathcal{A}rgs$, $\texttt{Ext2Lab}(\mathcal{A}rgs)$ is defined as $(\mathcal{A}rgs, \{A \in Ar \mid A \text{ is attacked by } \mathcal{A}rgs\}, \{A \in Ar \mid A \notin \mathcal{A}rgs \text{ and } A \text{ is not attacked by } \mathcal{A}rgs\})$ where Ar is the set of all arguments in the argumentation framework. It holds that if $\mathcal{L}ab$ is a (strongly) admissible labelling (resp. a complete, grounded, preferred, stable or ideal labelling) then $\texttt{Lab2Ext}(\mathcal{L}ab)$ is a (strongly) admissible set (resp. a complete, grounded, preferred, stable or ideal extension). It also holds that if $\mathcal{A}rgs$ is a (strongly) admissible set (resp. a complete, grounded, preferred, stable or ideal extension) then $\texttt{Ext2Lab}(\mathcal{A}rgs)$ is a (strongly) admissible labelling (resp. complete, grounded, preferred, stable or ideal labelling). Moreover, when restricted to complete (or resp. grounded, preferred, stable or ideal) extensions and labellings, the functions `Lab2Ext` and `Ext2Lab` become bijections that are each other's inverses [Caminada, 2006; Caminada and Gabbay, 2009].

The above results imply that:

- in order to determine whether an argument is in a preferred extension, it suffices to determine whether the argument is labelled `in` by an admissible labelling
- in order to determine whether an argument is in the grounded extension, it suffices to determine whether the argument is labelled `in` by a strongly admissible labelling
- in order to determine whether an argument is in a stable extension, it suffices to determine whether the argument is labelled `in` by an admissible labelling without `undec`
- in order to determine whether an argument is in the ideal extension, it suffices to determine whether the argument is labelled `in` by an admissible labelling that is compatible with every admissible labelling

In the sections that follow, we will apply the above observations to provide discussion games for preferred, grounded, stable and ideal semantics.

3 Preferred Semantics

In the current section, we describe the discussion game for preferred semantics as stated by Caminada *et al.* [2014a].[4] The idea of the preferred discussion game is to

[4] The discussion game of Caminada *et al.* [2014a] consists of a labelling-based reinterpretation of the work of Vreeswijk and Prakken [2000].

show membership of a preferred extension by constructing an admissible labelling where the argument in question is labelled `in`.

The preferred discussion game has two players which we will refer to as M and S. Player M starts; his task is to defend the fact that he has a reasonable position (admissible labelling) in which a particular argument is accepted (labelled `in`). Player S then tries to confront M with the consequences of M's own position, and asks for these consequences to be resolved. Player M is successful if he is able to address all the issues pointed out by player S, without being led to a contradiction.

As an example of how such a discussion can take place, consider the argumentation framework of Figure 2.

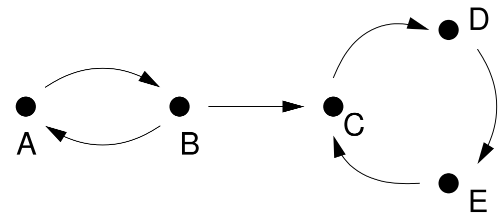


Figure 2: An argumentation framework

Here, the player M can win the discussion game for argument D in the following way.

Example 1.

M: $\texttt{in}(D)$
"I have an admissible labelling in which D is labelled `in`."

S: $\texttt{out}(C)$
"But then in your labelling it must also be the case that D's attacker C is labelled `out`. Based on which grounds?"

M: $\texttt{in}(B)$
"C is labelled `out` because B is labelled `in`."

S: $\texttt{out}(A)$
"But then in your labelling it must also be the case that B's attacker A is labelled `out`. Based on which grounds?"

M: $\texttt{in}(B)$
"A is labelled `out` because B is labelled `in`."

As is shown in the above example, the discussion moves of player M are statements that particular arguments are labelled `in` in M's labelling. The moves of player S, on the other hand, are meant to confront M with the consequences of his own position: "if you think that argument X is labelled `in` then you must also hold that X's attacker Y is labelled `out` in your labelling." That is, by uttering $\texttt{out}(Y)$, player S points out that player M is implicitly committed to the fact that Y should be rejected. This means that player M has to explain *why* Y should be rejected. That is, the moves of player S can be seen as *questions* about why a particular argument Y should be labelled `out`. The moves of player M (except his first move) can then be interpreted as the *answers* to the questions of player S. Each answer follows directly to the question raised by player S. That is:

Each move of M (except the first) contains an attacker of the argument in the directly preceding move of S. (1)

Every time player M claims that an argument is labelled `in`, player S should be given the opportunity to state that as a consequence of this, player M is implicitly committed that *all* attackers of the argument are labelled `out`. The problem, however, is that each move of player S is a statement about just *one* argument. In order to deal with this problem, player S should be given the opportunity to react on the same `in`-labelled argument several times, each time confronting player M with a different `out`-labelled argument. This means that player S should be allowed to react not just on the immediately preceding move of player M, but on *any* previous move of player M.

Each move of player S contains an attacker of an argument contained in some (not necessarily the directly preceding) move of player M. (2)

Another issue is whether player S should be allowed to repeat his own moves. Recall that each move essentially contains a question ("Based on which grounds is argument Y labelled `out`?"). At the moment player S repeats one of his moves, this question has already been answered by player M, so there is no good reason to ask again. In order to avoid the discussion from going round in circles, it does not make sense to allow player S to repeat his moves.

Player S is not allowed to repeat his moves. (3)

On the other hand, Example 1 does illustrate the need for player M to be able to repeat his moves (like $\texttt{in}(B)$). This is because some of the questions of S (like "why is argument C `out`" and "why is argument A `out`") can have the same answer ("because argument B is `in`").

Player M is allowed to repeat his moves. (4)

The argumentation framework of Figure 2 can also be used for an example of a game won by player S:

Example 2.

M: in(E)
"I have an admissible labelling in which E is labelled in."

S: out(D)
"But then in your labelling it must be the case that E's attacker D is labelled out. Based on which grounds?"

M: in(C)
"D is labelled out because C is labelled in."

S: out(E)
"But then in your labelling it must be the case that C's attacker E is labelled out. This contradicts your earlier claim that E is labelled in."

The above example illustrates that when player S manages to use an argument uttered previously by player M, player S has won the game. After all, if player M claims an argument to be in and player S subsequently manages to confront player M with the fact that in M's own position, the same argument should be labelled out, then player S has successfully pointed out a contradiction in M's position.

If player S uses an argument previously used by player M, then player S wins the discussion game. (5)

One can ask a similar question regarding what happens when player M uses one of the arguments previously used by player S. The fact that player S performed an out move means that the argument must be labelled out in the labelling of player M. If player M then subsequently claims that the same argument is labelled in, then he has directly contradicted himself.

If player M uses an argument previously used by player S, then player S wins the discussion game. (6)

There also exists a third condition under which player S wins the game. This is when player M is unable to answer one of the questions of S. This can be the case when there exists no attacker against an argument uttered by player S. Hence, player S asks why a particular argument is labelled out but player M is unable to come up with any attacker to be labelled in. In that case, player M has lost the game, for not being able to answer the critical questions of player S.

If player M cannot make a move any more, player S wins the discussion game. (7)

Similarly, one might examine what happens when it is player S who cannot make a move any more. This essentially means that player S has run out of questions. All possible relevant questions have already been asked; all relevant issues have already been raised. Moreover, player M has managed to answer all questions in a satisfactory way. Therefore, player M has survived the process of critical questioning, hence winning the discussion.

If player S cannot make a move any more, player M wins the discussion game. (8)

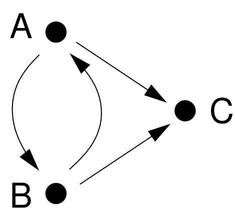


Figure 3: An argumentation framework with floating attack

As a last illustration of how the discussion game functions, consider the argumentation framework of Figure 3. Argument C is not in any admissible set. It is illustrative to see what happens if player M tries to defend C.

Example 3.

M: `in`(C)
"I have an admissible labelling in which C is labelled `in`."

S: `out`(A)
"But then in your labelling C's attacker A must be labelled `out`. Based on which grounds?"

M: `in`(B)
"A is labelled `out` because B is labelled `in`."

S: `out`(B)

"But from the fact that you hold C to be `in`, it follows that C's attacker B must be labelled `out`. This contradicts with your earlier claim that B is labelled `in`."

The above example illustrates the need for player S to be able to respond not only to the immediately preceding move, but to any past move of player M; in the example, $\texttt{out}(B)$ is a response to $\texttt{in}(C)$. This is because, as we have mentioned before, for an argument to be labelled `in`, *all* its attackers have to be `out`, so player S may need to respond to a move of player M with more than one countermove.

When putting observations (1) to (8) together, we obtain the following description of the discussion game

Definition 13. *Let (Ar, att) be an argumentation framework. A preferred discussion is a sequence of moves $[\Delta_1, \Delta_2, \ldots, \Delta_n]$ ($n \geq 0$) such that:*

- *each move Δ_i ($1 \leq i \leq n$) where i is odd is called an M-move and is of the form $\texttt{in}(A)$, where $A \in Ar$*
- *each move Δ_i ($1 \leq i \leq n$) where i is even is called an S-move and is of the form $\texttt{out}(A)$, where $A \in Ar$*
- *for each S-move $\Delta_i = \texttt{out}(A)$ ($2 \leq i \leq n$) there exists an M-move $\Delta_j = \texttt{in}(B)$ ($j < i$) such that A attacks B*
- *for each M-move $\Delta_i = \texttt{in}(A)$ ($3 \leq i \leq n$) it holds that Δ_{i-1} is of the form $\texttt{out}(B)$, where A attacks B*
- *there exist no two S-moves Δ_i and Δ_j with $i \neq j$ and $\Delta_i = \Delta_j$*

A preferred discussion $[\Delta_1, \Delta_2, \ldots, \Delta_n]$ is said to be finished *iff (1) there exists no Δ_{n+1} such that $[\Delta_1, \Delta_2, \ldots, \Delta_n, \Delta_{n+1}]$ is a preferred discussion, or there exists an M-move and an S-move containing the same argument, and (2) no subsequence $[\Delta_1, \ldots, \Delta_m]$ ($m < n$) is finished. A finished preferred discussion is won by player S if there exist an M-move and an S-move containing the same argument. Otherwise, it is won by the player making the last move (Δ_n).*

The soundness and completeness of the game described above is stated in the following theorem.

Theorem 3 (Caminada and Wu [2009]; Caminada *et al.* [2014a])**.** *Let (Ar, att) be an argumentation framework and $A \in Ar$.*

1. *If there exists a preferred discussion for A that is won by player M, then there exists a preferred extension that contains A.*

2. *If there exists a preferred extension that contains A then player M has a winning strategy*[5] *for the preferred discussion game.*

The correctness of Theorem 3 can be seen as follows. As for point 1, it has to be observed that what the game essentially does is to build an admissible labelling of which the `in`-labelled arguments coincide with the M-moves and the `out`-labelled arguments coincide with the S-moves (all the other arguments are labelled `undec`). The resulting labelling is well-defined in the sense that no argument is labelled both `in` and `out` (otherwise there would be an argument that is subject to both an M-move and an S-move, in which case player S would have won the discussion). Moreover, the fact that player M wins the discussion also means that he made the last move, which implies that (i) each `out`-labelled argument has an `in`-labelled attacker. Also, the fact that player S cannot move anymore implies that (ii) each `in`-labelled argument has all its attackers labelled `out`. From (i) and (ii) it follows that the labelling yielded by the game is indeed an admissible one, satisfying the conditions of Definition 8. In this admissible labelling, argument A is labelled `in` (since A was the subject of the first M-move). This implies that A is element of an admissible set, and therefore also element of a preferred extension.

As for point 2, it should be mentioned that the fact that A is in a preferred extension by definition implies that A is in an admissible set ($\mathcal{A}rgs$), which then implies that A is labelled `in` by an admissible labelling $\mathcal{L}ab = \texttt{Ext2Lab}(\mathcal{A}rgs)$. This makes it possible for player M to win the game simply by staying within the borders of admissible labelling $\mathcal{L}ab$. That is, as long as player M only plays arguments that are labelled `in` by $\mathcal{L}ab$, each move of player S has to be an argument that is labelled `out` by $\mathcal{L}ab$, which then implies that player M can always react with an argument that is labelled `in` by $\mathcal{L}ab$, etc. If player M follows such a strategy, there will never be an M-move and an S-move for the same argument (this is because $\mathcal{L}ab$ is a well-defined labelling, meaning that no argument is labelled both `in` and `out`). Moreover, the fact that player S cannot repeat himself means that the game has to finish in a finite number of moves. As player M can *always* react on a move of player S, this means that the last move has to be an M-move. Hence, player M wins the game.

From points 1 and 2 together, it follows that if there is at least one preferred discussion that is won by player M, then M has a winning strategy for the preferred discussion game. This is not the case in alternative discussion games for preferred semantics, like the one described by Modgil and Caminada [2009]. In their approach, a single discussion game does not prove membership (for this, the presence of a

[5] Winning strategy in the sense of [Caminada *et al.*, 2014a, Definition 5.6]. Informally this means that player M has a way of winning the discussion, regardless of what moves player S decides to play.

winning strategy is really necessary). From informal perspective, this is rather odd, as in everyday life the aim of a (persuasion) discussion is to convince the other party in a single discussion. This means that at the end of the discussion, the other party has to have heard sufficient evidence to accept the main claim. This is the case in the above described preferred discussion game, but not in the alternative discussion game of Modgil and Caminada [2009].

As we have observed, an admissible labelling can serve as a "roadmap" for winning the preferred discussion game.[6] However, an argument can be labelled `in` by more than one admissible labelling, which raises the question of which admissible labelling to choose as a basis to play the game. It can be verified that given an admissible labelling $\mathcal{L}ab$ (with $\mathcal{L}ab(A) = \mathtt{in}$ and $\mathtt{out}(\mathcal{L}ab)$ being minimal w.r.t. set inclusion) the number of moves required in the game for main argument A is $2 \cdot |\mathtt{out}(\mathcal{L}ab)| + 1$ (see [Caminada *et al.*, 2014a] for details). Hence, in order to be able to finish the game in as few moves as possible (which could be desirable from the perspective of human-computer interaction if the aim of the game is to convince a human user) one should try to find an admissible labelling $\mathcal{L}ab$ where $|\mathtt{out}(\mathcal{L}ab)|$ is minimal. This is a computationally hard problem, as even verifying whether a particular admissible labelling has this property is coNP complete [Caminada *et al.*, 2014a].

The essential nature of the preferred discussion game is that of critically questioning a particular position, and to see whether the proponent of this position (player M) can avoid being led to a contradiction (by player S). As such, the preferred discussion game bears a close resemblance to the concept of Socratic discussion, as well as to its modern variants like critical interviews or cross-examinations in court.[7] The general idea is to have somebody take a position and then iteratively confront him (through questioning) with what appears to be the consequences of this position, in the hope of ultimately leading him to a contradiction. We refer to the work of Caminada *et al.* [2014a] for a details.

4 Ideal Semantics

An ideal set of arguments, as was originally defined by Dung *et al.* [2007], is an admissible set that is a subset of each preferred extension. It can be proved that the maximal ideal set (commonly known as the *ideal extension*) is unique and is a complete extension as well.

[6]For details, we refer to the work of Caminada *et al.* [2014a].

[7]In fact, in the work of Caminada *et al.* [2014a] player S stands for Socrates and player M stands for Menexenus, which is one of Socrates's historic discussion partners.

An alternative but equivalent way of characterising the ideal extension is as the maximal admissible set that is not attacked by any admissible set (like is done in Theorem 1) or as the maximal complete extension that is not attacked by any complete extension (like is done in Definition 6). It can be proved that for each admissible sets $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ it holds that $\mathcal{A}rgs_1$ attacks $\mathcal{A}rgs_2$ iff $\mathcal{A}rgs_2$ attacks $\mathcal{A}rgs_1$. This gives rise to the labelling-based descriptions of ideal semantics of Theorem 2 and Definition 12.[8]

For current purposes, our characterisation of the ideal extension is as the maximal admissible set that is not attacked by any admissible set. To determine membership of the ideal extension, one then needs to find an admissible set (although not necessarily the maximal one) that contains the argument in question and is not attacked by any admissible set. This makes it possible to express ideal semantics using the preferred discussion game. Basically, the discussion whether an argument is in an ideal extension consists of two phases. In the first phase, one runs the preferred discussion game, as is described in the previous section. This is to determine whether the argument is in an admissible set. Then, in the second phase of the discussion, one needs to determine whether this set is attacked by another admissible set. This is done by again running the preferred discussion game for each of the arguments that were rejected (labelled `out`) during the first phase of the discussion, this time trying to defend (label `in`) the argument.

As an example, consider again the argumentation framework of Figure 2. Now consider the question of whether argument D is in an ideal set. The first phase of the discussion would be like Example 1 (page 9). Then, in the second phase of the discussion, one has to try to find an argument that was labelled `out` during the first phase[9] (say A) and can be defended in a new preferred discussion game. Such a game would be as follows.

M: $\texttt{in}(A)$
"I have a reasonable position (admissible labelling) in which A is accepted (labelled `in`*)."*

S: $\texttt{out}(B)$
"Then in your position, argument B must be rejected (labelled `out`*). Based on which grounds?"*

M: $\texttt{in}(A)$
"B is rejected (labelled `out`*) because A is accepted (labelled* `in`*)."*

Hence, we have an admissible set $\{A\}$ that attacks the admissible set $\{B, D\}$ found during the first phase, so the admissible set $\{B, D\}$ of the first phase is not

[8] Recall that each complete extension (labelling) is also an admissible set (labelling).

[9] Recall that the preferred game is such that the `out`-labelled arguments are the attackers of the `in`-labelled arguments (which is not necessarily the case for admissible labellings in general).

an ideal set.[10]

The overall procedure for ideal semantics puts an extra burden on the proponent of the argument. Not only does he have to win the preferred discussion game in the first phase, but he has to win it in such a way[11] that the resulting position (labelling) cannot be argued against in the second phase.

5 Stable Semantics

In the current section, we describe a discussion game for credulous stable semantics based on the work of Caminada and Wu [2009]. Before doing so, it may be illustrative to see why the preferred discussion game does not work for stable semantics. Consider again the argumentation framework of Figure 2. Even though A is in an admissible set and in a preferred extension ($\{A\}$), A is not in a stable extension. To see why A is in an admissible set, consider the following discussion:

M: `in`(A) "I have an admissible labelling where A is labelled `in`."

S: `out`(B) "Then in your labelling, argument B must be labelled `out`. Based on which grounds?"

M: `in`(A) "B is labelled `out` because A is labelled `in`."

The point is, however, that once it has been decided that A is labelled `in` and B is labelled `out`, it is not possible anymore to label the remaining arguments such that final result will be a stable labelling. This can be seen as follows. Suppose C is labelled `in`. Then E must be labelled `out`, so D should be labelled `in`, which means that C would be labelled `out`. Contradiction. Similarly, suppose that C is labelled `out`. Then E must be labelled `in`, so D should be labelled `out`, so C should be labelled `in`. Again, contradiction.

There exist many ways to characterize a stable extension [Caminada and Gabbay, 2009]. For our purposes, the most useful characterization is that of an admissible set which attacks every argument that is not in it (Theorem 1). When one translates this to labellings, one obtains an admissible labelling where each argument is labelled either `in` or `out` (that is, no argument is labelled `undec`, Theorem 2).

It appears that a discussion game for stable semantics requires an additional type of move: `question`. To illustrate the role of this new move, imagine a politician being interviewed for TV. At first the discussion may be about financial matters (say, whether the banking system should be nationalized). Then, the discussion may be about the consequences of the politician's opinion ("If you accept to nationalize the

[10] In fact, for the argumentation framework of Figure 2, the only ideal set is the empty set.

[11] Since an argument can be element of more than one admissible set, there can be different ways to win the preferred discussion game.

banks, then you must reject the possibility to improve healthcare, because there will not be enough money left to do so."). However, at some moment, the interviewer could choose to totally change topic ("By the way, what are your opinions about abortion?"). It is this change of topic that is enabled by the `question` move.[12]

For the discussion game for stable semantics, we use the `question` move to involve those arguments that have never been uttered before so that we are able to label all the arguments in Ar. By questioning an argument ($\texttt{question}(A)$), player S (the opponent) asks player M (the proponent) to give an explicit opinion on whether A should be labelled `in` or `out`. If player M thinks that A should be labelled `in` then he should respond with $\texttt{in}(A)$. If, on the other hand, player M thinks that A should be labelled `out` then he should respond with $\texttt{in}(B)$ where B is a attacker of A. The discussion game for stable semantics can thus be described as follows:

- Player M (the proponent) and player S (the opponent) take turns. Player M starts.
- Each move of player S is either of the form $\texttt{out}(A)$, where A is a attacker of some (not necessarily the directly preceding) move of player M, or of the form $\texttt{question}(A)$, where A is an argument that has not been uttered in the discussion before (by either player M or player S).
- The first move of player M is of the form $\texttt{in}(A)$, where A is the main argument of the discussion. The following moves of player M are also of the form $\texttt{in}(A)$ although A no longer needs to be the main claim. If the directly preceding move of player S is of the form $\texttt{out}(B)$ then A is a attacker of B. If the directly preceding move of player S is of the form $\texttt{question}(B)$ then A is either equal to B or a attacker of B.
- Player S is not allowed to repeat any of his `out` moves.
- Player M is allowed to repeat his own `in` moves.

Player S wins if there is an argument A that has been subject to both an `in` move (by player M) and an `out` move (by player S). Otherwise, the discussion continues until one of the players cannot move anymore, in which case the discussion is won by the player making the last move.

[12] One of the reasons the `question` move is needed is because stable semantics does not satisfy the property of *directionality* [Baroni and Giacomin, 2007]. This means that for determining the status of an argument, not just the "ancestors" (the attackers, the attackers of these attackers, etc) are relevant but also the "offspring" (the attacked, the attacked of the attacked, etc) as well as arguments from unconnected parts of the graph.

To illustrate the use of the discussion game, consider the argumentation framework depicted in Figure 4.

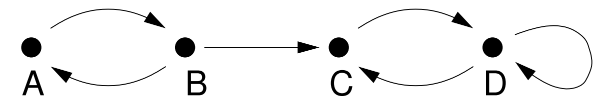


Figure 4: Another argumentation framework

Suppose player M would like to start a discussion about A.
M: `in`(A) "I have a stable labelling in which A is labelled `in`."
S: `out`(B) "Then in your labelling, A's attacker B must be labelled `out`. Based on which grounds?"
M: `in`(A) "B is labelled `out` because A is labelled `in`."
S: `question`(C) "What about C?"
M: `in`(C) "C is labelled `in`."
S: `out`(D) "Then C's attacker D must be labelled `out`. Based on which grounds?"
M: `in`(C) "D is labelled `out` because C is labelled `in`."
Player M wins the discussion, since player S cannot move anymore.

The above example also shows that the outcome of a discussion may depend on player M's response to a question move. For instance, if player M would have replied to `question`(C) with `in`(D), then he would have lost the discussion, since player S would then move `out`(D).

As an example of a discussion that cannot be won by player M, consider the discussion for argument B. This discussion has to be lost by player M since the argumentation framework of Figure 4 has only one stable extension: $\{A, C\}$, which does not include B.
M: `in`(B) "I have a stable labelling in which B is labelled `in`."
S: `out`(A) "Then in your labelling, B's attacker A must be labelled `out`. Based on which grounds?"
M: `in`(B) "A is labelled `out` because B is labelled `in`."
S: `question`(C) "What about C?"
M: `in`(D) "C is labelled `out` because its attacker D is labelled `in`."
S: `out`(D) "Then D's attacker D (itself) must be labelled `out`. Contradiction."
Player M would still have lost the discussion if he had responded to `question`(C) with `in`(C) instead of with `in`(D). This is because then player S would have reacted with `out`(B) and would therefore still have won the discussion.

Formally, the stable discussion game can be described as follows.

Definition 14. *Let* (Ar, att) *be an argumentation framework. A* stable discussion *is a sequence of moves* $[\Delta_1, \Delta_2, \dots, \Delta_n]$ $(n \geq 0)$ *such that:*

- *each* Δ_i $(1 \leq i \leq n)$ *where* i *is odd (which is called an* M-move*) is of the form* $\mathtt{in}(A)$*, where* $A \in Ar$*.*
- *each* Δ_i $(1 \leq i \leq n)$ *where* i *is even (which is called an* S-move*) is of the form* $\mathtt{out}(A)$ *where* $A \in Ar$*, or of the form* $\mathtt{question}(A)$ *where* $A \in Ar$*.*
- *For each S-move* $\Delta_i = \mathtt{out}(A)$ $(2 \leq i \leq n)$ *there exists an M-move* $\Delta_j = \mathtt{in}(B)$ $(j < i)$ *where* A *attacks* B*.*
- *For each M-move* $\Delta_i = \mathtt{in}(A)$ $(3 \leq i \leq n)$ *it either holds that (1)* $\Delta_{i-1} = \mathtt{out}(B)$ *where* A *attacks* B*, or (2)* $\Delta_{i-1} = \mathtt{question}(B)$ *where either* $A = B$ *or* A *attacks* B*.*
- *For each S-move* $\Delta_i = \mathtt{out}(A)$ $(1 \leq i \leq n)$ *there does not exist an S-move* $\Delta_j = \mathtt{out}(A)$ *with* $j < i$*.*
- *For each S-move* $\Delta_i = \mathtt{question}(A)$ $(1 \leq i \leq n)$ *there does not exist any move* Δ_j $(j < i)$ *of the form* $\mathtt{in}(A)$*,* $\mathtt{out}(A)$ *or* $\mathtt{question}(A)$*.*
- *For each M-move* $\Delta_i = \mathtt{in}(A)$ $(1 \leq i \leq n)$ *there does not exist an S-move* $\Delta_j = \mathtt{out}(A)$ *with* $j < i$*.*

A stable discussion $[\Delta_1, \Delta_2, \dots, \Delta_n]$ *is said to be* finished *iff (1) there exists no* Δ_{n+1} *such that* $[\Delta_1, \Delta_2, \dots, \Delta_n, M_{n+1}]$ *is a stable discussion, or there exists an M-move* $\mathtt{in}(A)$ *and an S-move* $\mathtt{out}(A)$ *for the same argument* A*, and (2) no subsequence* $[\Delta_1, \dots, \Delta_m]$ $(m < n)$ *is finished. A finished stable discussion is won by player S if there exists an M-move* $\mathtt{in}(A)$ *and an S-move* $\mathtt{out}(A)$ *for the same argument* A*. Otherwise it is won by the payer making the last move* Δ_n*.*

It turns out that an argument is in at least one stable extension iff the proponent can win the stable discussion game for it.

Theorem 4. *Let* (Ar, att) *be an argumentation framework and* $A \in Ar$*.*

1. *If there exists a stable discussion for* A *that is won by player M, then* A *is in a stable extension.*
2. *If* A *is in a stable extension, then player M has a winning strategy for the stable discussion game.*

As for point 1, it can be observed that what the discussion game essentially does is to build a stable labelling $\mathcal{L}ab$ with $\texttt{in}(\mathcal{L}ab) = \{A \mid$ there exists an M-move $\texttt{in}(A)\}$ and $\texttt{out}(\mathcal{L}ab) = \{A \mid$ there exists an S-move $\texttt{out}(A)\} \cup \{A \mid$ there exists an S-move $\texttt{question}(A)$ that was responded to with $\texttt{in}(B)$ where B attacks $A\}$. It can be verified that $\mathcal{L}ab$ is an admissible labelling without any argument being labelled `undec`. Hence, $\mathcal{L}ab$ is a stable labelling in the sense of Theorem 2. As A is labelled `in` by $\mathcal{L}ab$ (since A is the subject of the first M-move) it holds that A is in $\texttt{Lab2Ext}(\mathcal{L}ab)$. Hence, A is in a stable extension.

As for point 2, it should be mentioned that player M can win the game simply by staying within the borders of the stable labelling $\mathcal{L}ab = \texttt{Ext2Lab}(\mathcal{A}rgs)$ (with $\mathcal{A}rgs$ being the stable extension that contains A, the argument that the discussion will start with). That is, as long as player M only plays arguments that are labelled `in` by $\mathcal{L}ab$, each `out` move of player S will be labelled `out` by $\mathcal{L}ab$, which then implies that player M can always react with an argument that is labelled `in` by $\mathcal{L}ab$, etc. Moreover, when player S does a $\texttt{question}(A)$ move, either A itself or an attacker of A is labelled `in` by $\mathcal{L}ab$, which again means that player M can always respond with an argument that is labelled `in` by $\mathcal{L}ab$. As the argumentation framework is finite and player S cannot repeat himself, it follows that the game will finish in a finite number of moves. As player M can *always* react to the moves of player S, this means that the last move has to be an M-move. Hence, player M wins the game.[13]

Definition 14 describes the discussion game for credulous stable semantics (that is, it can used to determine whether an argument is in at least one stable extension). It is, however, relatively straightforward to re-apply this game in the context of sceptical stable semantics (that is, to determine whether an argument is in every stable extension). The idea is that an argument A is in each stable extension iff no attacker of A is in any stable extension. So in order to determine whether A is in every stable extension, one could try to play the stable discussion game for each attacker of A. If for none of these attackers the discussion game can be won, argument A is in each stable extension.

6 Grounded Semantics

So far, we have mainly focussed on the preferred discussion game and its slightly modified variants for ideal and stable semantics. In the current section we will focus on a fundamentally different type of discussion game, in the context of grounded semantics.

One of the main differences between the preferred discussion game and the

[13] A more elaborate proof can be found in [Caminada and Wu, 2009].

grounded discussion game to be introduced in the current section is a conceptual one. To properly understand this difference, it is useful to take the perspective of complete labellings. We recall that a complete labelling (Definition 11) is a labelling where one has reasons for each argument one accepts (because all its attackers are rejected), reasons for each argument one rejects (because it has an attacker that is accepted), and reasons for each argument one abstains from having an explicit opinion about (because there are insufficient grounds to accept it and insufficient grounds to reject it). As such, a complete labelling can be seen as a reasonable position on how to evaluate the conflicting information represented in the argumentation framework. The preferred discussion game determines whether an argument is accepted (labelled `in`) by *at least one* such reasonable position.[14] The grounded discussion game, to be introduced in the current section, determines whether an argument is accepted (labelled `in`) by *every* such reasonable position.[15] That is, from the perspective of complete labellings, the preferred discussion game is about whether an argument *can be* accepted, whereas the grounded discussion game is about whether an argument *has to be* accepted.

The difference between determining whether an argument can be accepted and whether an argument has to be accepted is reflected in the nature of the associated discussion game. If the discussion is merely about whether an argument can be accepted (that is, about whether there exists a reasonable position in which the argument is accepted) then arguing against this means pointing out that any position in which the argument is accepted is somehow not reasonable. That is, the opponent tries to lead the proponent of such a position towards a contradiction.[16] Hence, the admissible discussion game has at least some properties of Socratic discussion [Caminada, 2008; Caminada *et al.*, 2014a]. If, on the other hand, the discussion is about whether an argument has to be accepted (that is, about whether the argument is accepted in each reasonable position) then the discussion gets a totally different nature. If an argument is accepted in each reasonable position, then in particular one's discussion partner, by being reasonable, should accept the argument. So the discussion becomes one of trying to *convince* the discussion partner that he has to accept a particular argument. That is, the discussion partner should be shown that by being reasonable, he cannot avoid having to accept the argument in question. As such, the nature of the discussion becomes that of persuasion dialogue [Walton and

[14]This is because an argument is labelled `in` by some admissible labelling iff it is labelled `in` by some complete labelling.

[15]This is because an argument is labelled `in` by the grounded labelling iff it is labelled `in` by every complete labelling.

[16]like saying, "if you think that argument X is labelled `in`, then it follows that X's attacker Y should be labelled `out`, but previously you claimed that Y should be labelled `in`."

Krabbe, 1995].

Now that the conceptual difference between the preferred discussion game and the grounded discussion game has been explained, we will take a closer look at the technical differences. Although the preferred discussion game is used to determine membership of a preferred extension, it does so by determining membership of an admissible set (labelling).[17] This has the advantage of not having to construct the entire preferred extension (labelling), as constructing an admissible set (labelling) will be sufficient. Similarly, although the grounded discussion game is used to determine membership of the grounded extension, it does so by determining membership of a strongly admissible set (labelling) [Baroni and Giacomin, 2007; Caminada, 2014].[18] This has the advantage of not having to construct the entire grounded extension (labelling) as constructing a strongly admissible set (labelling) will be sufficient.

The grounded discussion game [Caminada, 2015a,b] that we will described in the current section has two players (proponent and opponent) and is based on four different moves, each of which has an argument as a parameter.

HTB(A) ("A has to be the case")
: With this move, the proponent claims that A has to be labelled `in` by every complete labelling, and hence also has to be labelled `in` by the grounded labelling.

CB(B) ("B can be the case, or at least cannot be ruled out")
: With this move, the opponent claims that B does not have to be labelled `out` by every complete labelling. That is, the opponent claims there exists a complete labelling where B is labelled `in` or `undec`, and that B is therefore not labelled `out` by the grounded labelling.

CONCEDE(A) ("I agree that A has to be the case")
: With this move, the opponent indicates that he now agrees with the proponent (who previously did an *HTB*(A) move) that A has to be the case (labelled `in` by every complete labelling, including the grounded).

RETRACT(B) ("I give up that B can be the case")
: With this move, the opponent indicates that he no longer believes that B can

[17] Recall that an admissible set (labelling) can always be extended to a preferred extension (labelling), as a preferred extension (labelling) is a maximal admissible set (labelling).

[18] Recall that a strongly admissible set (labelling) can always be extended to the grounded extension (labelling), as the grounded extension (labelling) is the maximal strongly admissible set (labelling) (see Theorem 2 and the work of Baroni and Giacomin [2007] and Caminada [2014].

be `in` or `undec`. That is, the opponent acknowledges that B has to be labelled `out` by every complete labelling, including the grounded.

One of the key ideas of the discussion game is that the proponent has burden of proof. He has to establish the acceptance of the main argument and make sure the discussion does not go around in circles. The opponent merely has to cast sufficient doubts.

The game starts with the proponent uttering an *HTB* statement. After each *HTB* statement (either the first one or a subsequent one) the opponent utters a sequence of one or more *CB*, *CONCEDE* and *RETRACT* statements, after which the proponent again utters an *HTB* statement, etc. In the argumentation framework of Figure 1 the discussion could go as follows.

(1) P: $HTB(C)$	(4) O: $CONCEDE(A)$
(2) O: $CB(B)$	(5) O: $RETRACT(B)$
(3) P: $HTB(A)$	(6) O: $CONCEDE(C)$

In the above discussion, C is called *the main argument* (the argument the discussion starts with). The discussion above ends with the main argument being conceded by the opponent, so we say that the proponent wins the discussion.

As an example of a discussion that is lost by the proponent, it can be illustrative to examine what happens if the proponent claims that B has to be the case.

(1) P: $HTB(B)$ (2) O: $CB(A)$

After the second move, the discussion is terminated, as the proponent cannot make any further move, since A does not have any attackers. This brings us to the precise preconditions of the discussion moves.

$HTB(A)$ Either this is the first move, or the previous move was $CB(B)$, where A attacks B, and no *CONCEDE* or *RETRACT* move is applicable.

$CB(A)$ A is an attacker of the last $HTB(B)$ statement that is not yet conceded, the directly preceding move was not a *CB* statement, argument A has not yet been retracted, and no *CONCEDE* or *RETRACT* move is applicable.

$CONCEDE(A)$ There has been an $HTB(A)$ statement in the past, of which every attacker has been retracted, and $CONCEDE(A)$ has not yet been moved.

$RETRACT(A)$ There has been a $CB(A)$ statement in the past, of which there exists an attacker that has been conceded, and $RETRACT(A)$ has not yet been moved.

Apart from the preconditions mentioned above, all four statements also have the additional precondition that no *HTB-CB* repeats have occurred. That is, there should be no argument for which *HTB* has been uttered more than once, *CB* has been uttered more than once, or both *HTB* and *CB* have been uttered. In the first and second case, the discussion is going around in circles, which the proponent has to prevent as he has burden of proof. In the third case, the proponent has been contradicting himself, as his statements are not conflict-free. In each of these three cases, the discussion comes to an end with no move being applicable anymore. The above conditions are made formal as follows.

Definition 15. *Let $AF = (Ar, att)$ be an argumentation framework. A* grounded discussion *is a sequence of discussion moves constructed by applying the following principles.*

BASIS (*HTB*) *If $A \in Ar$ then $[HTB(A)]$ is a grounded discussion.*

STEP (*HTB*) *If $[M_1, \dots, M_n]$ $(n \geq 1)$ is a grounded discussion without HTB-CB repeats,*[19] *and no CONCEDE or RETRACT move is applicable,*[20] *and $M_n = CB(A)$ and B is an attacker of A then $[M_1, \dots, M_n, HTB(B)]$ is also a grounded discussion.*

STEP (*CB*) *If $[M_1, \dots, M_n]$ $(n \geq 1)$ is a grounded discussion without HTB-CB repeats, and no CONCEDE or RETRACT move is applicable, and M_n is not a CB move, and there is a move $M_i = HTB(A)$ $(i \in \{1 \dots n\})$ such that the discussion does not contain $CONCEDE(A)$, and for each move $M_j = HTB(A')$ $(j > i)$ the discussion contains a move $CONCEDE(A')$, and B is an attacker of A such that the discussion does not contain a move $RETRACT(B)$, then $[M_1, \dots, M_n, CB(B)]$ is a grounded discussion.*

STEP (*CONCEDE*) *If $[M_1, \dots, M_n]$ $(n \geq 1)$ is a grounded discussion without HTB-CB repeats, and $CONCEDE(B)$ is applicable then $[M_1, \dots, M_n,\ CONCEDE(B)]$ is a grounded discussion.*

STEP (*RETRACT*) *If $[M_1, \dots, M_n]$ $(n \geq 1)$ is a grounded discussion without HTB-CB repeats, and $RETRACT(B)$ is applicable then $[M_1, \dots, M_n,\ RETRACT(B)]$ is a grounded discussion.*

[19] We say that there is a *HTB-CB* repeat iff $\exists i, j \in \{1 \dots n\} \exists A \in Ar : (M_i = HTB(A) \vee M_i = CB(A)) \wedge (M_j = HTB(A) \vee M_j = CB(A)) \wedge i \neq j$.

[20] A move $CONCEDE(B)$ is applicable iff the discussion contains a move $HTB(A)$ and for every attacker A of B the discussion contains a move $RETRACT(B)$, and the discussion does not already contain a move $CONCEDE(B)$. A move $RETRACT(B)$ is applicable iff the discussion contains a move $CB(B)$ and there is an attacker A of B such that the discussion contains a move $CONCEDE(A)$, and the discussion does not already contain a move $RETRACT(B)$.

It can be observed that the preconditions of the moves are such that a proponent move (HTB) can never be applicable at the same moment as an opponent move (CB, $CONCEDE$ or $RETRACT$). That is, proponent and opponent essentially take turns in which each proponent turn consists of a single HTB statement, and every opponent turn consists of a sequence of $CONCEDE$, $RETRACT$ and CB moves.

Definition 16. *A grounded discussion $[M_1, \ldots, M_n]$ is called* terminated *iff there exists no move M_{n+1} such that $[M_1, \ldots, M_n, M_{n+1}]$ is a grounded discussion. A terminated grounded discussion (with A being the main argument) is won by the proponent iff the discussion contains $CONCEDE(A)$, otherwise it is won by the opponent.*

To illustrate why the discussion has to be terminated after the occurrence of an HTB-CB repeat, consider the following discussion in the argumentation framework of Figure 1.

(1) P: $HTB(G)$	(3) P: $HTB(G)$
(2) O: $CB(H)$	

At the third move, an HTB-CB repeat occurs and the discussion is terminated (opponent wins). Hence, termination after an HTB-CB repeat is necessary to prevent the discussion from going on perpetually.

Theorem 5. *Every discussion will terminate after a finite number of steps.*

From the fact that a discussion terminates after an HTB-CB repeat, the following result follows.

Lemma 1. *No discussion can contain a $CONCEDE$ and $RETRACT$ move for the same argument.*

The soundness and completeness of the game described above is stated in the following theorem.

Theorem 6 (Caminada [2015a])**.** *Let (Ar, att) be an argumentation framework and let $A \in Ar$.*

1. *If there exists a grounded discussion for A that is won by player P, then A is labelled* `in` *by the grounded labelling.*
2. *If A is labelled* `in` *by the grounded labelling, then player P has a winning strategy for A in the grounded discussion game.*

The correctness of Theorem 6 can be seen as follows. As for point 1, it can be observed that what the discussion game actually does is to construct a strongly admissible labelling of which the `in`-labelled arguments coincide with the *CONCEDE* moves, and the `out`-labelled arguments coincide with the *RETRACT* moves. In fact, it can be proved by induction that at each state of the discussion, the labelling where each *CONCEDE* move is labelled `in` and each retract move is labelled `out` is strongly admissible [Caminada, 2015b]. The fact that the discussion is won by player P implies that the main argument (A) has been conceded. So at the end of the discussion, we have a strongly admissible labelling where argument A is labelled `in`. Hence, by Theorem 2, A is labelled `in` by the grounded labelling.

As for point 2, it should be mentioned that a strongly admissible labelling (for instance the grounded labelling) with its associated min-max numbering can serve as a roadmap for winning the discussion. The proponent will be able to win if, whenever he has to do an *HTB* move, he prefers to use an `in` argument with the lowest min-max number that attacks the directly preceding *CB* move. We refer to this as a *lowest number strategy.*[21]

It turns out that when applying such a strategy, the game stays within the boundaries of the strongly admissible labelling (that is, within its `in` and `out` labelled part). As long as each *HTB* move of the proponent is related to an `in`-labelled argument, it follows that all the attackers are labelled `out` (Definition 8, first bullet) so each *CB* move the opponent utters in response will be related to an `out`-labelled argument. This `out`-labelled argument will then have at least one `in`-labelled attacker (Definition 8, second bullet) as a candidate for the proponent's subsequent *HTB* move.

The next thing to be observed is that when the proponent applies a lowest number strategy, the game will not terminate due to any *HTB*-*CB* repeats. This is due to the facts that (1) after a move $HTB(A)$ is played (for some argument A) all subsequent *CB* and *HTB* moves will be related to arguments with lower min-max numbers than A until a move $CONCEDE(A)$ is played, and (2) after a move $CB(A)$ is played (for some argument A), all subsequent *HTB* and *CB* moves will be related to arguments with lower min-max numbers than A, until a move $RETRACT(A)$ is played. We refer to [Caminada, 2015b] for details.

[21] We write "*a* lowest number strategy" instead of "*the* lowest number strategy" as a lowest number strategy might not be unique due to different lowest numbered `in`-labelled arguments being applicable at a specific point. In that case it is sufficient to pick an arbitrary one.

7 Tree-Based Discussion Games

The discussion games that were described in the previous sections are not the only ones that have been stated for preferred, stable, ideal and grounded semantics. In fact, various alternative dialectical proof procedures can be found in the literature, many of them are based on the concept of dialectical trees [Dung *et al.*, 2007; Modgil and Caminada, 2009; Thang *et al.*, 2009]. In the current section, we aim to provide an impression of these tree-based discussion games, and explain some of their disadvantages compared to the discussion games described in the previous sections. Rather than giving an overview of all tree-based discussion games that have been stated in the literature, we will focus our attention on one of them: the Standard Grounded Game [Prakken and Sartor, 1997; Caminada, 2004; Modgil and Caminada, 2009].

The Standard Grounded Game (SGG) [Prakken and Sartor, 1997; Caminada, 2004; Modgil and Caminada, 2009] is one of the earliest dialectical proof procedures for grounded semantics. Each game[22] consists of a sequence $[A_1, \ldots, A_n]$ $(n \geq 1)$ of arguments, moved by the proponent and opponent taking turns, with the proponent starting. That is, a move A_i $(i \in \{1 \ldots n\})$ is a proponent move iff i is odd, and an opponent move iff i is even. Each move, except the first one, is an attacker of the previous move. In order to ensure termination even in the presence of cycles, the proponent is not allowed to repeat any of his moves. A game is terminated iff no next move is possible; the player making the last move wins. Formally, the Standard Grounded Game can be defined as follows.

Definition 17. *A* discussion *in the Standard Grounded Game is a finite sequence* $[A_1, \ldots, A_n]$ $(n \geq 1)$ *of arguments (sometimes called* moves*), of which the odd moves are called P-moves (Proponent moves) and the even moves are called O-moves (Opponent moves), such that:*

1. *every O-move is an attacker of the preceding P-move (that is, every A_i where i is even and $2 \leq i \leq n$ attacks A_{i-1})*
2. *every P-move except the first one is an attacker of the preceding O-move (that is, every A_i where i is odd and $3 \leq i \leq n$ attacks A_{i-1})*
3. *P-moves are not repeated (that is, for every odd $i, j \in \{1, \ldots, n\}$ it holds that if $i \neq j$ then $A_i \neq A_j$)*

[22] What we call an SGG game is called a "line of dispute" in [Modgil and Caminada, 2009].

A discussion is called terminated *iff there is no* A_{n+1} *such that* $[A_1, \ldots, A_n, A_{n+1}]$ *is a discussion. A terminated discussion is said to be* won *by the player making the last move.*

As an example, in the argumentation framework of Figure 1 $[C, B, A]$ is terminated and won by the proponent (as A has no attackers, the opponent cannot move anymore) whereas $[G, H]$ is terminated and won by the opponent (as the only attacker of H is G, which the proponent is not allowed to repeat). It is sometimes possible for the proponent to win a game even if the main argument is not in the grounded extension. An example would be $[F, B, A]$. This illustrates that in order to show that an argument is in the grounded extension, a single game won by the proponent is not sufficient. Instead, what is needed is a *winning strategy.* This is essentially a tree in which each node is associated with an argument such that (1) each path from the root to a leaf constitutes a terminated discussion won by the proponent, (2) the children of each proponent node (a node corresponding with a proponent move) coincide with all attackers of the associated argument, and (3) each opponent node (a node corresponding with an opponent move) has precisely one child, whose argument attacks the argument of the opponent node.

Formally, *argument tree* is a tree of which each node ($\mathtt{n}$) is labelled with an argument ($Arg(\mathtt{n})$). The *level* of a node is the number of nodes in the path to the root. This leads to the following formal definition of a winning strategy in the context of the Standard Grounded Game.

Definition 18. *A* winning strategy *of the Standard Grounded Game for argument A is an argument tree, where the root is labelled with A, such that*

1. *for each path from the root* ($\mathtt{n}_{root}$) *to a leaf node* ($\mathtt{n}_{leaf}$) *it holds that the arguments on this path form a terminated discussion won by P*
2. *for each node at odd level* $\mathtt{n}_P$ *it holds that* $\{Arg(\mathtt{n}_{child}) \mid \mathtt{n}_{child}$ *is a child of* $\mathtt{n}_P\}$ $= \{B \mid B$ *attacks* $Arg(\mathtt{n}_P)\}$ *and the number of children of* $\mathtt{n}_P$ *is equal to the number of attackers of* $Arg(\mathtt{n}_P)$
3. *each node of even level* $\mathtt{n}_O$ *has precisely one child* $\mathtt{n}_{child}$*, and* $Arg(\mathtt{n}_{child})$ *attacks* $Arg(\mathtt{n}_O)$

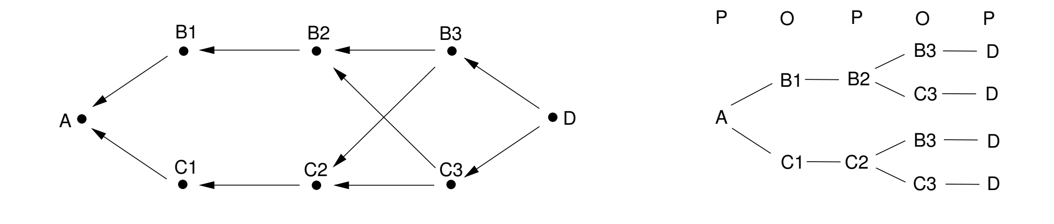
It has been proved that an argument is in the grounded extension iff the proponent has a winning strategy for it in the SGG [Prakken and Sartor, 1997; Caminada, 2004]. Moreover, it has also been shown that an SGG winning strategy defines a strongly admissible labelling, when each argument of a proponent node is labelled `in`, each argument of an opponent node is labelled `out` and all remaining arguments are labelled `undec` [Caminada, 2014].

As an example, in the argumentation framework of Figure 1 the winning strategy for argument E would be the tree consisting of the two branches $E - B - A$ and $E - D - C - B - A$, thus proving its membership of the grounded extension by yielding the strongly admissible labelling $(\{A, C, E\}, \{B, D\}, \{F, G, H\})$.

As can be observed from this example, a winning strategy of the SGG can contain some redundancy when it comes to multiple occurrences of the same arguments in different branches. In the current example, the redundancy is relatively mild (consisting of just the two arguments A and B) but other cases exist where the SGG requires a number of moves in the winning strategy that is *exponential* w.r.t. the size of the strongly admissible labelling the winning strategy is defining. As an example, consider the argumentation framework of Figure 5 (top left). The winning strategy of the SGG is in the same figure (top right). Now consider what would happen if one would start to extend the argumentation framework by duplicating the middle part. That is, suppose we have arguments $B_1, \ldots, B_n$ and $C_1, \ldots, C_n$ (with n being an odd number), as well as arguments A and D. Suppose that for every $i \in \{1, \ldots, n-1\}$ B_{i+1} attacks B_i, and C_{i+1} attacks C_i, and that for each *even* $i \in \{2, \ldots n-1\}$ B_{i+1} attacks C_i, and C_{i+1} attacks B_i, and that B_1 and C_1 attack A, and that D attacks B_n and C_n. In that case, the branches in the SGG winning strategy would split at every O-move. So for $n = 3$ (as is the case in Figure 5) the number of branches is four, for $n = 5$ it is eight, etc. In general, the number of branches in the SGG winning strategy is $2^{(n+1)/2}$, with the number of nodes in the SGG winning strategy being $1 + 2\Sigma_{i=1}^{(n+1)/2} 2^i$. Hence, the number of steps needed in a winning strategy of the SGG can be *exponential* in relation to the size (number of `in` and `out` labelled arguments) of the strongly admissible labelling that the SGG winning strategy is constructing.[23]

As for the Grounded Discussion Game (GDG) as described in Section 6, the situation is different. As was mentioned in Section 6, what the GDG essentially does is to construct a strongly admissible labelling of which the `in` labelled arguments coincide with the *CONCEDE* moves and the `out` labelled arguments coincide with the *RETRACT* moves. It can be observed that no argument occurs in both a *CONCEDE* and *RETRACT* move (otherwise the argument would also have occurred in both an *HTB* and *CB* move, and the discussion would have terminated before reaching the *CONCEDE* and *RETRACT* moves) and that for each argument there exists at most one *CONCEDE* move and at most one *RETRACT* move. As we assume the game is won by the proponent, who is playing a lowest number strategy, there will be no *HTB*-*CB* repeats. This implies that for each *CONCEDE* move, there exists precisely one *HTB* move, and for each *RETRACT* move, there exists

[23] We thank Mikołaj Podlaszewski for this example.



(1) P: $HTB(A)$	(5) P: $HTB(D)$	(9) O: $RETRACT(C_3)$	(13) P: $HTB(C_2)$
(2) O: $CB(B_1)$	(6) O: $CONCEDE(D)$	(10) O: $CONCEDE(B_2)$	(14) O: $CONCEDE(C_2)$
(3) P: $HTB(B_2)$	(7) O: $RETRACT(B_3)$	(11) O: $RETRACT(B_1)$	(15) O: $RETRACT(C_1)$
(4) O: $CB(B_3)$	(8) O: $CB(C_3)$	(12) O: $CB(C_1)$	(16) O: $CONCEDE(A)$

Figure 5: The Standard Grounded Game (SGG) versus the Grounded Discussion Game (GDG).

precisely one *CB* move. This means that the total number of moves (in a game won by the proponent, who is applying a lowest number strategy) is two times the number of `in` labelled arguments (which accounts for the *HTB* and *CONCEDE* moves) plus two times the number of `out` labelled arguments (which accounts for the *CB* and *RETRACT* moves). Hence, the number of moves in the game is *linear* in relation to the size (number of `in` and `out` labelled arguments) of the strongly admissible labelling the GDG is constructing.[24]

Hence, whereas for the Grounded Discussion Game, constructing a strongly admissible labelling (which is needed to show membership of the grounded extension) requires a linear number of moves, for the Standard Grounded Game this requires a potentially exponential number of moves. This makes the GDG a better choice for purposes of human-computer interaction, assuming that the human user's time is precious.

It should be mentioned that the possibility of an exponential blowup in the number of moves is not restricted to the SGG, but is a feature of tree-based discussion games in general. For instance, the above sketched example also leads to an exponential number of moves in the preferred semantics game of Modgil and Caminada [2009] and in the ideal semantics game of Dung *et al.* [2007]. The key feature of these approaches is that they require a winning strategy to show membership of a (grounded, preferred or ideal) extension. It is this winning strategy that is responsible for the exponential blowup. In the discussion games described in sections 3, 5 and 6, however, no winning strategy is required, as just a single game won by the proponent is sufficient to prove membership of a (preferred, stable or grounded)

[24]See [Caminada, 2015a] for details.

extension.[25]

8 Discussion

What the above described discussion games for preferred semantics (Section 3), stable semantics (Section 5) and grounded semantics (Section 6) have in common is that (1) a single game won by the proponent is sufficient to prove membership of a (preferred, stable or grounded) extension, and (2) if an argument is member of a (preferred, stable or grounded) extension then the proponent has a winning strategy for it. This is evidenced by theorems 3, 4 and 6. In tree-based discussion games, like those of Dung *et al.* [2007], Modgil and Caminada [2009] and Thang *et al.* [2009] point (1) is altered such that a single game won by the proponent is *not* sufficient to prove membership of an extension; for this a winning strategy is needed. Having to provide such a winning strategy in a dialectical way can be troublesome for two reasons. First of all, the tree of the winning strategy would need to be "linearized" as discussions take place not in branching time but in linear time. But even if linearization takes place, one still has to deal with the fact that the original (tree-based) winning strategy could have a size that is exponentially related to the (strongly) admissible labelling it is based on. The discussion games presented in sections 3, 5 and 6 have the advantage that they are not tree-based and hence do not have these problems.

One can ask the question whether it is always possible (for any argumentation semantics) to define a discussion game that satisfies the points (1) and (2) mentioned above. For instance, the procedure sketched in Section 4 (ideal semantics) does not satisfy point (1). This is because in the second phase of the discussion, when trying to find an admissible set that attacks the admissible set obtained in the first phase of the discussion, not finding such a set could be due to the proponent making the "wrong" choices during the second phase, rather than due to the actual absence of such a set. It would be a challenge to change the discussion procedure for ideal semantics such that both points (1) and (2) are satisfied. An even greater challenge would be to formulate discussion games (still satisfying points (1) and (2)) for semi-stable, stage or even CF2 semantics.

As the tree-based discussion games of Dung *et al.* [2007], Modgil and Caminada [2009] and Thang *et al.* [2009] violate point (1) but satisfies point (2), one can ask the question of whether there also exists a discussion game that satisfies point (1)

[25] It can be proved that the preferred discussion game (Section 3) is linear in the number of moves required. See [Caminada *et al.*, 2014a] for details. Using similar techniques one can also prove that the stable discussion game (Section 5) requires only a linear number of moves.

but violates point (2). The answer is affirmative, as is evidenced by the work of Caminada and Podlaszewski [2012]. Here, the ability to win the discussion game might depend on cooperation of the opponent. So even though an argument being in the grounded extension implies the existence of a discussion for it that is won by the proponent, it does not imply that the proponent also has a winning strategy.[26] For the purpose of human-computer interaction, this property is undesirable, as the computer should be able to win the discussion (for an argument that is actually in the grounded extension) regardless of how the human user choses to utter the possible counterarguments.

The discussion games presented in the current paper have been stated in the context of *abstract* argumentation theory. This raises the question of whether these discussion games are also suitable in the context of *instantiated* argumentation, like ASPIC+ [Modgil and Prakken, 2014] ABA [Toni, 2014] or logic-based argumentation [Gorogiannis and Hunter, 2011] Technically, this should not be a problem, as each of these formalisms provides an instantiation of Dung's abstract argumentation theory. That is, each of these formalisms specifies what arguments can be constructed and how these attack each other, starting from a particular knowledge base. Although applying the discussion games in the context of instantiated argumentation is technically straightforward, there is a catch. The question is whether the notion of attack of the instantiated argumentation formalism is defined in such a way that it allows for moves that can be considered as intuitive during the course of the discussion. For instance, in ASPIC+ it can be the case that a discussion partner utters an argument with conclusion c, which cannot be replied to with an argument for conclusion $\neg c$ (even though such an argument is well-formed and perhaps even justified) because the definition of attack is such that it does not attack the argument with conclusion c. This is like having your discussion partner uttering an argument for a claim (c) which you know is not the case, but you're not allowed to reply with an argument that directly rebuts this claim. We refer to the work of Caminada *et al.* [2014b] for details.

As mentioned in the introduction, one of the possible applications of the discussion games is for the purpose of human-computer interaction. The context here is that of a shared knowledge base[27] (say, of medical research and clinical evidence) that allows for the construction of arguments (say, regarding to how to treat a particular

[26] We refer to [Caminada, 2015a] for a specific example.

[27] A particularly interesting situation is where such a shared knowledge base is absent, that is, where proponent and opponent each have their own private knowledge base and associated argumentation framework. In that case, both proponent and opponent learn new information from each other during the course of the discussion. This puts additional constraints on the discussion protocol. We refer to [Caminada and Sakama, 2015] for details.

patient). As the knowledge base can be complex and huge, it is not always directly obvious what the justified arguments are. Although a software implementation of (instantiated) argumentation theory can help to provide an answer, the correctness of this answer might need to be explained to a human user. Our hypothesis is that human-computer discussion can contribute to acceptance of argument-based entailment. In order to test this hypothesis, one would need to perform experiments in which the user's confidence in the argument-based entailment is tested, before and after performing the discussion game. Experiments like these is what we would like to perform in the near future.

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Received

Argumentation Schemes. History, Classifications, and Computational Applications

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Abstract

Argumentation schemes can be described as abstract structures representing the most generic types of argument, constituting the building blocks of the ones used in everyday reasoning. This paper investigates the structure, classification, and uses of such schemes. Three goals are pursued: 1) to describe the schemes, showing how they evolved and how they have been classified in the traditional and the modern theories; 2) to propose a method for classifying them based on ancient and modern developments; and 3) to outline and show how schemes can be used to describe and analyze or produce real arguments. To this purpose, we will build on the traditional distinctions for building a dichotomic classification of schemes, and we will advance a modular approach to argument analysis, in which different argumentation schemes are combined together in order to represent each step of reasoning on which a complex argument relies. Finally, we will show how schemes are applied to formal systems, focusing on their applications to Artificial Intelligence, AI & Law, argument mining, and formal ontologies.

1 Introduction

The purpose of this paper is threefold: 1) to describe the schemes, showing how they evolved and how they have been classified in the traditional and the modern theories; 2) to propose a method for classifying them based on ancient and modern developments; and 3) to outline and show how schemes are interrelated and can be organized in a modular way to describe natural arguments or produce complex arguments. Historically, the schemes evolved from the Aristotelian topics, the so-called places to find arguments. But looking over the descriptions Aristotle presented of them in the *Topics*, for the most part they do not appear to very much resemble the argumentation schemes in the contemporary list of Walton, Reed and Macagno [Walton *et al.*, 2008]. Of course there are exceptions, such as the topic for argument from analogy described in Aristotle, which is recognizable as standing for the same kind of argument as the current scheme for argument from analogy, even though the detailed description of it is quite different.

Argumentation schemes are instruments for argumentation, involving the activity of critically evaluating a viewpoint and the reasons given in its support. For this reason, every scheme has a corresponding set of critical questions, representing its defeasibility conditions and the possible weak points that the interlocutor can use to question the argument and evaluate its strength. A critic who has no counter-arguments ready to hand can search through the list of critical questions matching the argument he is confronted with in order to look for clues on how the argument can be attacked that might suggest sources of evidence that could be used to build up a whole line of argumentation that furnishes a way of refuting the argument.

The fundamental challenge that a theory of argumentation schemes needs to face is the problem of finding a useful and sound classification system. The schemes need to be usable, easily identifiable, and at the same time they need to allow the user to detect the most specific pattern of argument that can fit the text or that can be employed for producing an argument suitable to the circumstances and the purpose. In any classification system, entities can be classified in many different ways, depending on the purpose of the classification. The purpose of the classification system will determine the criteria for classification that are adopted in that system. For example, a much more detailed classification of animals may be useful in biology than the kind of classification that might be useful for law, or for classifying animals as they are spoken and written about in everyday conversational English. We need to begin by specifying the purpose of the classification, so that some guidance can be given on how to identify the criteria used in the classification system. From this perspective it is useful to examine how the study of argumentation schemes evolved.

2 Introducing argumentation schemes

Argumentation schemes represent forms of argument that are widely used in everyday conversational argumentation, and in other contexts such as legal and scientific argumentation. But for the most part these arguments are not adequately modeled by deductive forms of reasoning of the kind familiar in classical logic or as statistical inferences based on the standard Bayesian account of probability. They represent the premise-conclusion structure of an argument, and they are defeasible. Their defeasibility conditions are shown as a set of critical questions, dialectical instruments to help begin the procedure of testing the strength and acceptability of an argument by weighing the pro and con arguments.

2.1 Nature of the schemes

Argumentation schemes are stereotypical patterns of inference, combining semantic-ontological relations with types of reasoning and logical axioms and representing the abstract structure of the most common types of natural arguments [Macagno and Walton, 2015]. The argumentation schemes provided in [Walton *et al.*, 2008] describe the patterns of the most typical arguments, without drawing distinctions between material relations (namely relations between concepts expressed by the warrant of an argument), types of reasoning (such as induction, deduction, abduction), and logical rules of inference characterizing the various types of reasoning (such as *modus ponens*, *modus tollens*, etc.). For this reason, argumentation schemes fall into distinct patterns of reasoning such as abductive, analogical, or inductive ones, and ones from classification or cause to effect.

In order to design a system for classifying the schemes, it is useful to understand their limits, and investigate how the dimensions of an argument (material relation and logical form) are merged. For example, consider argument from cause to effect [Walton *et al.*, 2008, p.328]:

Major premise	Generally, if A occurs, then B will (might) occur.
Minor premise	In this case, A occurs (might occur).
Conclusion	Therefore, in this case B will (might) occur).

Table 1: Argument from cause to effect

This argumentation scheme is based on a defeasible *modus ponens* scheme [Verheij, 2003a] which is combined with a semantic causal relation between two events. The material (semantic) relation is merged with the logical one. However, this combination represents only one of the possible types of inferences that can be drawn from the same semantic-ontological connection. The actual relationship between the material and the logical relation is much more complex. For example, consider the classic Aristotelian causal link between "having fever" and "breathing fast," and see how this cause-effect relation can be used to draw a conclusion based on different logical rules [Macagno and Walton, 2015; Macagno, 2015])

1. He had fever. (Fever causes breathing fast). Therefore, he (must have) breathed fast.
2. He did not breathe fast. (Fever causes breathing fast). Therefore, he had no fever.
3. He is breathing fast. (Fever causes breathing fast). Therefore, he might have fever.
4. He has no fever. (Fever causes breathing fast). Therefore, he may be not breathing fast.
5. You may have fever. When I had fever, I was breathing fast, and you are breathing fast.

Cases (1) and (2) proceed logically from defeasible deductive axioms, i.e. the defeasible *modus ponens* (in 1), and the defeasible *modus tollens* (in 2). Cases 3 and 4 proceed from abductive reasoning. In (3) the conclusion is drawn by affirming the consequent, while in (4) the denial of the antecedent can be rephrased by contraposition as "not breathing fast is caused by having no fever," leading to a conclusion drawn abductively [Walton *et al.*, 2008, pp.169–173]. In (5) the conclusion is based on an inductive generalization from one single case.

Schemes represent only the prototypical matching between semantic relations and logical rules (types of reasoning and axioms). This matching is, however, only the most common one. The material and the logical relations can combine in several different ways. Hence this distinction needs to be taken into account order to classify the schemes.

2.2 Why schemes are important

Critics often ask how these schemes can be justified, given that they resisted analysis as deductive or inductive forms of argument of the kind recognized as valid in the dominant 20th-century logic tradition [Walton and Sartor, 2013].

Schemes are becoming extremely important for practical reasons. First, argumentation schemes are instruments for analyzing and recognizing natural arguments occurring in ordinary and specialized discourse. For example, arguments from political discourse have been analyzed using the schemes, and the argumentative profiles of the candidates have been brought to light considering their preferences of the types of arguments used [Hansen & Walton, 2013]. Thousands of real examples of these forms of argument have been analyzed in the argumentation literature, such as the considerable literature on fallacies, with the aid of tools like argument mapping [Reed *et al.*, 2007; Rowe *et al.*, 2006]. On this basis, the structure, use, and importance of schemes for argumentation studies have been justified inductively. This method consists in the following steps:

1. The structure of a scheme is outlined considering the literature on the topic.

2. A significant mass of examples of arguments is analyzed using the scheme, adapting and modifying the scheme so that it can best describe the specific natural arguments.

3. It is shown that the form of argument represented by the scheme under analysis is significantly important for the study of argumentation as it occurs in natural language discourse (and other specialized contexts such as legal discourse).

4. Empirical justification is given that this form of argument needs to be recognized as a basic scheme for argumentation.

Second, schemes are instruments that can be used for the purpose of teaching critical thinking. Informal logic is a field is known for having grown from its origins in textbooks that departed from formal logic and instead proceeded on the basis of analyzing numerous examples of arguments from ordinary discourse, such as those taken from magazines and newspapers. There is an abundance of such textbooks full of examples of everyday arguments related to topics such as the informal fallacy of appeal to authority, false cause, and so forth. During its growth stage and subsequent theoretical flowering, the field followed this trend by stressing the importance of analyzing real arguments "on the hoof". For example, the handbook *Informal Logic* [Walton, 1989] was based on hundred 150 key examples, many of them illustrating forms of argument now identified with argumentation schemes, including personal attack, uses and abuses of expert opinion, arguments from analogy, arguments from correlation to cause, and so forth. These textbooks and continued academic writings on informal logic contained a very large number of such examples, often analyzed in minute detail. Argumentation schemes, such as argument from expert opinion, are tested against the real examples, to discuss the respects in which the abstract scheme fits or does not fit the vagaries of the real-life example. This body of data confirms that certain types of arguments, mainly the ones subsequently identified as argumentation schemes, are not only extremely common, but are also highly influential in daily practices of argumentation.

Third, schemes can be used in education both for teaching students how to argue and for learning through argumentation [Erduran and Jimenez-Aleixandre, 2007; Erduran and Jiménez Aleixandre, 2012; Rapanta and Walton, 2016]. The interest in argumentation and the patterns for representing natural arguments is growing [Rapanta *et al.*, 2013]. The argumentation schemes illustrated in [Walton, 1995; Walton *et al.*, 2008] have been applied to science education in order to represent students' arguments and improve the quality thereof [Rapanta and Macagno, 2016], retrieve the implicit premises, and assess and rebut their reasoning in a systematic fashion [Macagno and Konstantinidou, 2013], or to assess the quality of argumentation [Duschl *et al.*, 1999; Ozdem *et al.*, 2013]. However, a crucial problem arising out of the use of schemes in education is their differentiation [Kim *et al.*, 2010; Nussbaum and Edwards, 2011]. Students often fail to understand the differences between various types of arguments, and the recent developments in education tend to conflate the schemes instead of providing criteria for classifying or distinguishing between them.

Fourth, schemes have now been recognized as important for argument mining, and it has also been recognized that there are too many schemes for handy use [Mochales Palau and Moens, 2009; Mochales Palau and Moens, 2011]. Configuring the relationships between clusters of them, and the internal structure of each cluster, would help in the research efforts to apply the schemes as working tools to a broader range of problems as the field of computational linguistics has moved forward.

From a theoretical point of view, schemes fit into current formal argumentation models such as ASPIC+ [Prakken *et al.*, 2015], DefLog [Verheij, 2003a] and the Carneades Argumentation System [Walton and Gordon, 2012]. Among the basic schemes presented in the list of 60+ schemes in chapter 9 of [Walton *et al.*, 2008] are argument from expert opinion, argument from sign, argument from example, argument from commitment, argument from position to know, argument from lack of knowledge, practical reasoning (argument from goal to action), argument from cause to effect, the sunk costs argument, argument from analogy, *ad hominem* argument, and the slippery slope argument. These schemes are at this point well enough recognized in the argumentation literature that no detailed account of them needs to be given in this paper, except for the ones that we will focus on to illustrate general characteristics of schemes discussed in detail in the paper.

Moreover, Walton and Sartor [Walton and Sartor, 2013] have shown that the basic defeasible schemes can be justified by the teleological argument. According to this reasoning, the use of a specific scheme is warranted by the fact that it can serve an agent's goals better than using nothing, and better than other alternative schemata the agent has at its disposal. This kind of justification of basic schemes is essentially a practical one saying that these schemes, even at their current state of development, are proving to be useful in such areas as artificial intelligence and multiagent computing. Defeasible schemes allow agents to arrive at a presumptive conclusion on how to proceed in a situation where continuing to collect evidence may cause delay, taking time and costing money.

This form of justification of schemes applies both to goals of epistemic cognition (getting to the truth of a matter) and goals of practical cognition (making the best choice in given circumstances). The importance of the schemes has also been acknowledged in the history of dialectics. The forms of argument, their critical and defeasible dimension, and their structure was long ago acknowledged in the earlier concerns of the Sophists, who pointed out forms of argument useful for persuasion and deliberation [Schiappa, 1999; Tindale, 2010]. In the *Topics* [Aristotle, 1991b] and in the *Rhetoric* [Aristotle, 1991a], Aristotle set out a list of topics that, providing the abstract and general hypothetical premises of dialectical syllogisms, can be considered to be the predecessors of the argument patterns developed in modern times [Macagno *et al.*, 2014; Rubinelli, 2009; Macagno *et al.*, 2014].

The tradition of the topics was continued through the Middle Ages, with various theories aimed at providing a classification and an analysis of the nature of the schemes [Bird, 1962; Gabbay and Woods, 2008; Green-Pedersen, 1984; Green-Pedersen, 1987; Stump, 1982; Stump, 1989]. Study of the kinds of schemes that are the focus of this paper was eclipsed during the Enlightenment, as the dominant view became firmly entrenched that the only forms of reasoning that can be identified with rational thinking are those of deductive logic, and inductive reasoning of the kind used in games of chance. But the study of these schemes made a comeback in the 20th century at the beginning of, and after the rise of argumentation studies as a respectable discipline, once the basic schemes were identified by Hastings [1963], Perelman and Olbrechts-Tyteca [1969], Kienpointner [1992], Walton [1995], Grennan [1997], and Walton, Reed and Macagno [2008]. From that point onwards, the study of schemes has been recognized as important for building computational models of argumentation, and especially for applying these models to argumentation in natural language discourse.

2.3 Classification of the schemes: how to proceed

In this paper, it is shown how the complex project of classifying schemes needs to proceed by matching a top-down approach with a bottom-up approach, and in particular that this bottom-up approach needs to begin by studying relationships between clusters of nested schemes. From a top-down approach, dichotomic criteria of classification need to be found, allowing the user to decide the scheme needed, both by direct identification and by exclusion. For this purpose, an overview of the existing classification systems developed in the tradition and in the recent theories can provide useful criteria. From a bottom-up approach, relationships within groups of schemes need to be studied, and then how one group fits with another can be studied. Walton [2012] took a bottom-up approach that began with some examples at the ground level of cases where two schemes seem to apply to the same real example of an argument found in a text, leading to a difficulty of determining which scheme fits the argument. Working from there, we identify clusters of schemes that fit together, and then at the next step, we examine how these clusters can be fitted together. Once clusters of schemes are fitted together into larger groups, we can gradually learn how they fit into an overarching system.

3 The topics in the dialectical and rhetorical tradition

Argumentation schemes describe patterns from which specific arguments can be drawn. In this sense, they can be seen as the modern development of the traditional concept of *topos*, the conditional expressing a generic principle from which some of the specific premises warranting the conclusion in an argument can be drawn. The purpose of this section is to show how the ancient account of *topoi* and *loci* can be considered as the ground and the predecessor of the modern theory of schemes.

3.1 Aristotle

The idea of providing general principles of inference from which various arguments can be drawn was the ground of Aristotle's *Topics* and *Rhetoric*. The Aristotelian *topoi* can be conceived as principles [De Pater, 1965, pp.150–159] having often the form of "P, then Q". The various semantic (material) relations between P and Q, or the "nature of the things which the terms of the argument represent or stand for" [Green-Pedersen, 1987, p.413], constitute the differences between the various *topoi*. For example, P and Q can be related by a relation of genus-species, *definiens-definiendum*, contraries, similarity, etc. The function of the *topoi* in the mechanism of argument production can be explained as follows [Slomkowski, 1997, p.45]:

> The enthymemes seem to be instances of *topoi*; or, expressed differently, enthymemes are arguments which are warranted by the principle expressed in the *topos*. Thus hypothetical syllogism would fall under a *topos* insofar as it falls under its major premiss in which the essence of the hypothetical syllogism is expressed.

Topoi can be considered as the external general rules of reasoning of an enthymeme, or the genera of the major premises of dialectical and rhetorical syllogisms. *Topoi* can work as rules, namely as the principle of inference guaranteeing the passage from an enthymematic premise to the conclusion. For example, we consider the following enthymeme [Slomkowski, 1997, p.51]: âĂć

âĂć Doing greater injustice is a greater evil.

âĂć From "what is more A is more B", you may infer: "A is B".

âĂć Doing injustice is an evil.

The *topos* can be also used as a general principle from which it is possible to draw the specific premises of a hypothetical syllogism [Bird, 1960; Bird, 1962; Macagno *et al.*, 2014]. For example, the same argument mentioned above can be completed by adding the major premise that is an instantiation (an axiom-instance) of the *topos* from the more [Slomkowski, 1997, p.53] (Table 2):

General principle	If being more A is more B, then A is B.
Specific instantiation of the *topos* as a premise	If doing greater injustice (A) is a greater evil (B), then doing injustice (A) is an evil (B).
Minor premise	Doing greater injustice (A) is a greater evil (B).
Conclusion	Doing injustice (A) is an evil (B)

Table 2: *Topoi* as general principles of inference

The aforementioned mechanism of specification (or instantiation) of the *topoi* brings to light a fundamental distinction that Aristotle draws between generic *topoi* and the *idia* (the specific topics) [Rubinelli, 2009, pp.59–70]. While generic *topoi* are abstract and commonly shared conditionals under which specific premises can be found, the specific *topoi* represent premises warranting the conclusion ([De Pater, 1965, p.134]; [Stump, 1989, p.29]) that are accepted within specific disciplines, such as ethics, law, or medicine. For example, consider the following specific topic [Lawson, 1885, p.262]:

> Where a person does an act, he is presumed in so doing to have intended that the natural and legal consequences of his act shall result.

In specific domains of knowledge, specific *topoi* can be listed as instruments of invention, premises that can be used to construct arguments in support of typical conclusions.

Generic topics can be considered as abstractions from the specific ones, or more correctly, an abstraction from a large number of specific topics. They provide classes of both necessary and defeasible inferences [Bird, 1960; Bird, 1962; Christensen, 1988; Drehe, 2011; Stump, 2004]. In the first class fall some maxims setting out definitional properties of meta-semantic concepts, i.e. concepts representing semantic relations between concepts, such as definition, genus, and property. For example the *locus* from definition, which establishes the convertibility between definition and *definiendum*, represents also the essential logical characteristic that a predicate needs to have in order be considered as a "discourse signifying what a thing is." Other *loci*, such as the ones based on analogy or the more and the less, are only defeasible, as they represent only commonly accepted relationships. In the *Topics* [Aristotle, 1991b], Aristotle focuses most of his analysis on the topics governing the meta-semantic relations between concepts, i.e. genus, property, definition, and accident. The Aristotelian account was developed in the Latin and medieval dialectical tradition, which developed classifications of the topics (called *loci*) based on the type of material relation they represent.

3.2 Cicero

Cicero [Cicero, 2003] reduced the Aristotelian list of *topoi* to 20 *loci* or maxims, grouping them in generic categories (differences) and dividing them in two broad classes, the intrinsic and the extrinsic topics [Stump, 1989]. While the first ones proceed directly from the subject matter at issue (for instance, its semantic properties), the external topics (the Aristotelian arguments from authority) support the conclusion through contextual elements (for instance, the source of the speech act expressing the claim) (Cicero, *Topica*, 8, 3–4). In between there are the topics that concern the relationship between a predicate and the other predicates of a linguistic system (for instance, its relations with its contraries or alternatives). We represent the topics of Cicero in Table 3 below.

Cicero pointed out some *loci* that, on his view, are principally used by dialecticians. Such topics, named *loci* from antecedents, consequents, and incompatibles (no. 8, 9, and 10 in Table 1), represent patterns of reasoning based only on the meaning of the connector of the hypothetical premise (if...then). For instance, if such a premise holds, and the antecedent is affirmed, the consequent follows necessarily (topic from antecedents) (Cicero, *Topica*, 53, 1–25). These *loci* seem to be aimed at establishing commitments based on previous commitments. In other words, instead of increasing the acceptability of a viewpoint based on the acceptability of the content of the premises on which it is grounded, such topics lead the interlocutor to the acceptance of a conclusion because of his previous acceptance of

Intrinsic		**Extrinsic**
Directly from the subject matter	***From things somehow related to the subject matter***	
1. *definitio* • By material parts (whole-part definition) • By essential parts (genus-species definition) 2. *notatio* (etymological relation)	1. *Coniugata* (inflectional relations) 2. *Genus* (genus-species relation) 3. *Forma* (species-genus relation) 4. *Similitudo* (similarity relation) 5. *Differentia* (difference relation) 6. *Contraria* (4 types of opposite relation) 7. *Adiuncta* (relation of concomitance) 8. *Antecedentia* 9. *Consequentia* 10. *Repugnantia* (incompatibles) 11. *Efficentia* (cause-effect relation) 12. *Effecta* (effect-cause relation) 13. *Ex comparatione maiorum, minorum, parium* (comparison)	Authority

Table 3: Cicero - Classification of generic topics

other propositions [Green-Pedersen, 1984, p.256].

Cicero connected the theory of topics to the division of discourse according to the Hermagoras stasis, the issue of the discussion, formulating the proposition to be proved or disputed [Kennedy, 1963, p.303]. He provided a classification of the topics according to their function for addressing a specific type of issue, namely conjecture, definition, and qualification (Cicero, *Topica*, 87) (Table 4).

Conjecture	**Definition**	**Qualification**
Cause, effect, circumstances	Definition, description, notation, division, partition, consequent, antecedent, inconsistencies, cause and effect, *adiuncta*.	Comparison

Table 4: Cicero - Division of topics by issue

Cicero's classification of topics became the ground for Boethius' works, which are the basis of the medieval dialectical tradition ([Stump, 1982]; [Stump, 1989]; [Stump, 2004]).

3.3 Boethius

Boethius commented on and organized Cicero's *loci* in his *In Ciceronis Topica* and *De Topicis Differentiis*, distinguishing between necessary and plausible connections and between dialectical and rhetorical *loci*. The treatise on *De Topicis Differentiis* includes *loci* that in Cicero and previously in Aristotle were distinguished as dialectical and rhetorical topics.

Boethius underscored how while dialectical *loci* stem from the rules of prediction and the logic-semantic properties of the predicates, rhetorical *topoi* represent the possible connections between things having different qualities (*De Topicis Differentiis*,1215C).[1] Some dialectical topics, such as topics from definition or genus and species, are necessary [Macagno and Walton, 2014, Ch.3], while others (for instance, from *adiuncta*) represent only frequent connections. This relation between probable and necessary consequence was studied in the Middle Ages. Garlandus Compotista classified topics according to their logical (demonstrative) role. Topics from whole (which includes definition and genus), along with part and equal became the foundations of categorical syllogism [Stump, 1982, p.277].

In Boethius the Aristotelian *topoi* are interpreted as *maximae propositiones* falling under *differentiae*, genera of these maxims. *Maximae propositiones* are general principles, also called axioms. They are general (indefinite in respect to particulars) and generic propositions that several arguments can instantiate, and they have warranting the conclusion in an argument as a primary role. The relationship between the terms of the premises and the conclusion, namely the respect under which they are regarded, is called *differentia*, representing the criterion of appropriateness or the genus of maxims. The maxim is found from the genus of the *maximae propositiones* and the relationship between the terms of the first premise [Stump, 1989, p.6]. The structure of a topic are illustrated in Table 5.

First term:	Every virtue is advantageous.
Middle term:	Justice is a virtue.
Second term:	Therefore justice is advantageous.
Maxim:	What belongs to the genus, belongs to the species.
Differentia:	From the whole, i.e. the genus

Table 5: Argument and maxim in Boethius

[1] Rhetorical *loci* are similar in form to the dialectical ones, but they proceed from frequent connections between things, from stereotypes and not from semantic properties of concepts (for instance, usually people addicted to alcohol are dissolute, this person is alcoholic, therefore he is dissolute. See Boethius *De Topicis Differentiis*1215b).

Topoi are divided into three main categories: intrinsic, extrinsic and intermediate. While the first two categories are similar to Cicero's organization, the third is based on different principles. *Loci medii* represent semantic connections of grammatical relations, such as from words stemming from the same root, or semantic relations of division underlying the definition of the word (Table 6).

Intrinsic Loci		
From substance	From things accompanying the substance	
•From the definition •From the description •From the explanation of the name	•From the whole (genus) •From the integral whole •From a part (species) •From the parts of an integral whole •From efficient cause •From the matter	•From the end •From the form •From the generation (effects) •From the corruption •From uses •From associated accidents
Intermediate Loci	**Extrinsic Loci**	
•From inflections •From coordinates •From division	•From estimation about a thing •From similar •From what is more •From things that are less •From proportion	•From contraries •From opposites with reference to privation and possession •From relative opposites •From opposites with reference to affirmation and negation •From transumption

Table 6: Boethius - Division of the dialectical *loci*

Boethius distinguishes the dialectical *loci* from the rhetorical ones. Rhetorical topics are drawn from not from the concepts (representing the abstract relations between concepts), but from the things and how things usually are. For example, while the dialectical topic from genus proceeds from the definition of a concept (if a person is drunk, he is also intoxicated), the rhetorical one concerns how a more generic concept is usually related to a more specific one (usually if someone is not dissipated, he does not get drunk). Boethius takes from Cicero the rhetorical topics, not dealing with the abstract principles of inference concerning concepts, but with the circumstances concerning the specific cases[2]. For instance, reasoning from place,

[2]They are different from the preceding topics, because the preceding topics either contained deeds or adhered to deeds in such a way that they could not be separated, as place, time, and the rest, which do not desert the action performed. But those things that are associated with the

name, time depends on the fact, stem from the factors of the event and not from the logic-semantic relations between concepts. The rhetorical topics are organized into the four classes pointed out by Cicero (*De Topicis Differentiis*,1212A-1214A) (Table 7).

<table>
<tr><th colspan="4">Intrinsic Loci</th></tr>
<tr><th colspan="2">Person</th><th colspan="2">Action</th></tr>
<tr>
<td>• Name (Verres)
• Natura (Barbar)
• Mode of life (Friend of nobles)
• Fortune (Rich)
• Studies (Architect)</td>
<td>• Luck (Exiled)
• Feelings (Lover)
• Disposition (Wise)
• Purpose
• Deeds
• Words</td>
<td>• Gist of the deed (Murder of a relative)
• Before the deed (He stole a sword)
• While the deed occurs (He struck violently)
• After the deed (He hid him in a secret place)</td>
<td>• When: Time (night) and opportunity (people were sleeping)
• Where: Place (bedroom)
• How: Method (secretly)
• With the aid: Means (with many men)</td>
</tr>
<tr><th>Comparing circumstances</th><th colspan="3">Extrinsic Loci</th></tr>
<tr>
<td>• Species
• Genus
• Contrary
• Result
• Greater
• Lesser
• Equal</td>
<td colspan="3">• By what name to call what has been done
• Who are the doers of the deed
• Who approve of its having been thought up
• What is the law, custom, agreement, judgment, opinion, and theory for the thing.
• Whether the thing is contrary to custom
• Whether men generally agree to these things.</td>
</tr>
</table>

Table 7: Boethius - Division of the rhetorical *loci*

3.4 Abaelardus

During the Middle Ages, the focal point of the study of argument was the connection between dialectics and demonstration. Beginning with the XI century, Garlandus Compotista analysed the categorical syllogisms as proceeding from topics from whole, part, and equal. On the other hand, he conceived all the topics under the logical forms of topics from antecedent and consequent, whose *differentiae* (the *genera* of *maximae propositiones*) are the syllogistic rules [Stump, 1982, p.277]. In the XII

action do not adhere to the action itself but are accidents of the circumstances, and they provide an argument only when they enter into comparison. The arguments, however, are taken not from contrariety but from a contrary, and not from similarity but from a similar, so that the argument seems to be taken not from a relationship [such as contrariety] but from things associated with the action [such as contraries]. Those things are associated with the action which are related to the very action at issue (*De Topicis Differentiis*,1214B 6-1214C 19).

century, Abelard in his *Dialectica* examined the structure of dialectical consequence in its components for the first time [Kienpointner, 1987, p.283]).

Abelard described topics as imperfect inferences, different from valid categorical syllogisms. In this work, the *maxima propositio*, expressing a principle of inference, is related to the function of invention. The *maxima* is the general principle that is useful for finding the propositions accepted by everybody or the by the wise (the *endoxa*) relative to the subject dealt with in the argument. From this perspective, the structure of an argument is similar to a syllogism. The main difference lies in the nature of the assumptions, the propositions connecting the general principles to the subject of the reasoning. While dialectical inferences depend on the content of the propositions (or, rather, on the terms and their connections), syllogisms depend only on the form. The difference between form and content can be explained with the following cases. A syllogism such as:

> Every man is an animal
> But every animal is animate
> Therefore, every man is animate

depends on a rule of inference, that is [Abaelardus, 1970, p.262]:

> *posito antecedenti ponitur consequens* (if the antecedent is affirmed, the consequent is affirmed as well))

The connection between the terms of the inference depends only on their position in the propositions. On the other hand, dialectical inferences cannot be resolved only by considering the positions of the terms. These inferences are imperfect, since assumptions are needed for the conclusion to follow from the premises. For instance, the consequence

> If he is a man, he is an animate being

is necessarily valid since it is known that "animate being" is the genus of man and "whatever is predicated of the species is predicated of the genus as well." The inference depends on the local connection between the terms, on the *habitudo*. The *habitudo* is the topical relation, the semantic-ontological respect under which the terms are connected to each other in a (dialectical) syllogism ([Green-Pedersen, 1984, p.185]; [Green-Pedersen, 1987, p.415]), and on which the strength of the inference depends [Abaelardus, 1970, pp.254-257]. The mechanism of an argument scheme can be shown by the ancient model of Abelard [Abaelardus, 1970, p.315], in which the assumptions were connected to the axioms, to the maxims the *locus* proceeded from [Stump, 1989, p.36] (Table 8).

Consequence	If Socrates is a man, he is an animate being.
Maxim	What the species is said of, the genus is said of as well.
Assumption	But "man," which is the species of "animate being" is said of Socrates; also therefore "animate being," which is clearly its genus.
Assumption 1	"Man" is a species of "animate being."
Syllogism 1	• What the species is said of, the genus is said of as well. • Man is species of "animate being". • Therefore, if man is said of anything, "animate being" is said of it as well.
Syllogism 2	• If "man" is said of anything, "animate being" is said of it as well. • Socrates is a man. • Therefore Socrates is an animate being.

Table 8: Rules of inference and the material structure of arguments in Abelard

In the example above, the passage from the predicate "to be a man" attributed to the subject to the different predicate "to be an animate being" is grounded on a relation of semantic inclusion between these two predicates, i.e. a genus-species relation [Bird, 1962]. This relationship guarantees the inference based on a rule (the maxim) that expresses a necessary consequence of the concept of genus itself. The genus expresses the generic fundamental features of a concept, answering to the question "what is it?" and is attributed to all the concepts different in kind (Aristotle, *Topics* 102a 31-32). For this reason, it is predicated of what the species is predicated of.

After Abelard, in the 12th century, the notion of form of inference was developed into a reduction of all topical inferences to syllogisms. Later on, in the 13th century analytical consequences were analysed as following from topics "*dici de omni*" and "*dici de nullo*" (Every A is B, Every B is C, therefore every A is C). Demonstration is for this reason based on a topical relation (from the whole)[Green-Pedersen, 1984, p.256].

4 Modern Theories of Schemes

In the modern and contemporary theories on argumentation (or argument) schemes, several types of classification have been advanced [Walton *et al.*, 2008]. In this section, the most relevant theories on schemes and the classification thereof will be summarized.

4.1 Perelman and the New Rhetoric

Perelman and Olbrechts-Tyteca divided their system of *topoi* into two broad categories, defined based on the two purposes that they considered to be the basic ones, finding associations and dissociations between concepts [Perelman and Olbrechts-Tyteca, 1969, p.190]. According to the *New Rhetoric*, arguments from association are divided in three main classes: Quasi-logical Arguments, Relations Establishing the Structure of Reality, and Arguments based on the Structure of Reality, while dissociation constitutes a distinct class. This classification can be represented in Table 9.

<table>
<tr><th colspan="2">Quasi-Logical Arguments</th><th colspan="2">The Relations Establishing the Structure of Reality</th></tr>
<tr><td colspan="2" rowspan="2">▪ Contradiction and Incompatibility
▪ Identity and Definition
▪ Analyticity, Analysis and Tautology
▪ The Rule of Justice
▪ Arguments of Reciprocity
▪ Arguments of Transitivity
▪ Inclusion of the Part in the Whole
▪ Division of the Whole into its Parts
▪ Arguments by Comparison
▪ Argumentation by Sacrifice</td><td>Establishment through Particular Case</td><td>Reasoning by Analogy</td></tr>
<tr><td>▪ Example
▪ Illustration
▪ Model and Anti-model</td><td>▪ Analogy
▪ Metaphor</td></tr>
<tr><th colspan="4">Arguments based on the Structure of Reality</th></tr>
<tr><td>Sequential Relations</td><td>The Relations of Coexistence</td><td>Double Hierarchy Argument</td><td>Differences of Degree and Order</td></tr>
<tr><td>▪ Causal Link
▪ Pragmatic Argument
▪ Ends and Means
▪ Argument of Waste
▪ Argument of Direction
▪ Unlimited Development</td><td>▪ Analogy
▪ The person and His Acts
▪ Argument from Authority
▪ The Speech as an Act of the Speaker
▪ The Group and its Members
▪ Act and Essence
▪ Symbolic Relation</td><td></td><td></td></tr>
</table>

Table 9: Classification of the arguments in the New Rhetoric

This classification is based several criteria, namely on the conceptual/ontological structure (association-dissociation; the reference to the structure of reality), the logical structure (quasi-logical vs. non-logical arguments), and the type of relations between concepts (sequential vs. coexistence). However, the interrelation between all these criteria is not specified, and there is not a unique rationale linking all such different arguments.

4.2 Toulmin

A different approach is provided by Toulmin, Rieke and Janik (1984), in which they classified arguments based on the basic functions of the warrants on which the arguments are grounded. Nine general classes of arguments were distinguished, subdivided into subclasses [Toulmin *et al.*, 1984], shown in Figure 1.

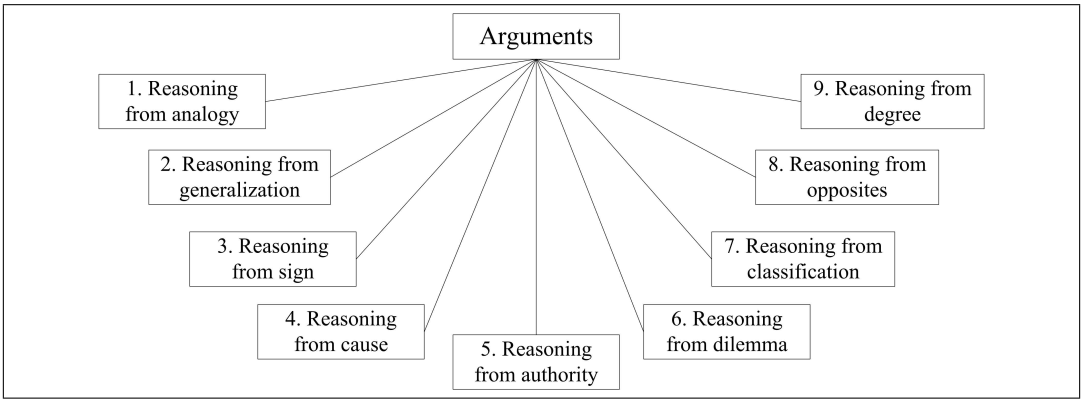

Figure 1: Classification of the arguments in Toulmin

Also in this case, different criteria are used in the classification. Some schemes represent types of reasoning (such as generalization, sign, or analogy); others are characterized by logical rules of inference (dilemma, opposites); others refer to the content of the argument (authority, classification, cause, degree). The relationship between the various criteria is not given.

4.3 Kienpointner

Kienpointner in *Alltagslogik* provides a complex and fine-grained classification, based on four criteria: 1) the type of inference; 2) the epistemic nature of the premises; 3) the dialectical function of the conclusion; and 4) the pragmatic function of the conclusion. On his view, every scheme 1) can proceed from different logical rules; 2) must be real (namely based upon the truth or likeliness of the premises), or fictive (grounded upon the mere possibility) (epistemic nature of the premises); 3) it must be pro or contra a certain thesis (dialectical function); and 4) it must have either a descriptive or a normative conclusion (pragmatic function) [Kienpointner, 1992, p.241]. In this sense, all the schemes can have descriptive or normative, pro or contra, real or fictive variants. The classification provided in *Alltagslogik* groups 21 schemes in three abstract classes characterized by the typology of the inferential rule: argument schemes using a rule; argument schemes establishing a rule by means of induction; argument schemes both using and establishing a rule (Figure 2).

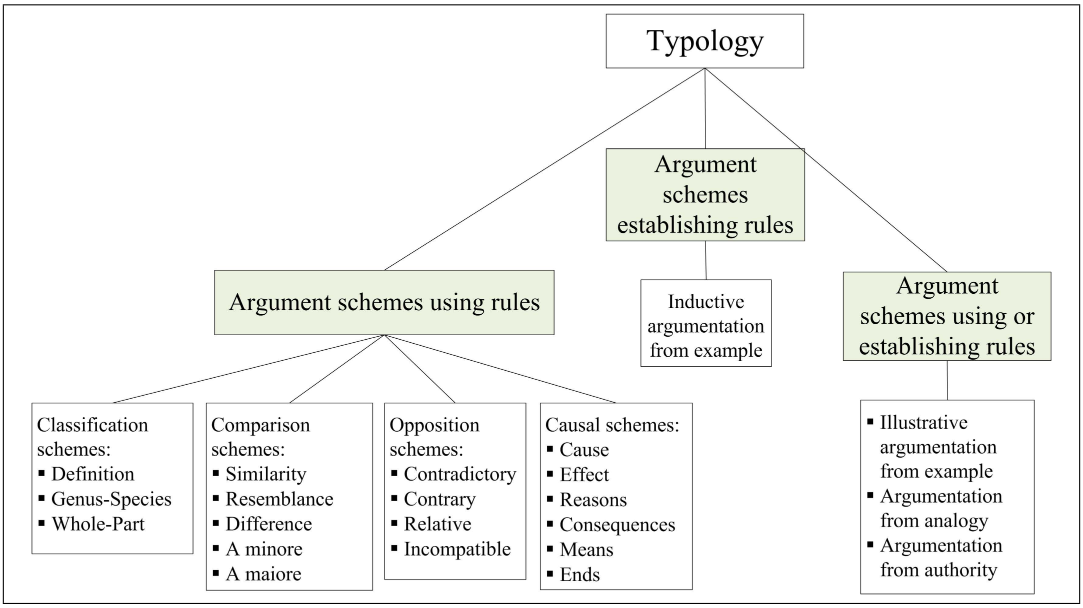

Figure 2: Classification of the arguments in Kienpointner

The first class, as shown in Figure 2, is subdivided in its turn in four content-based categories: classification, comparison, opposition, and causal schemes [Kienpointner, 1992, p.246]. Based on the aforementioned criteria, all the argument schemes may in turn have descriptive or normative variants, different logical forms (*Modus Ponens*, *Modus Tollens*, Disjunctive Syllogism, etc.), different dialectical purposes (establishing or countering a viewpoint), and different word-world relation (fictive – real).

This system of classification is aimed at distinguishing first the type of reasoning (induction, deduction), and then differentiating between the various material relations. The possible limitation of this system is that while the material relation of many deductive schemes is specified and distinguished, the content dimension of the inductive schemes is not pointed out.

4.4 Pragma-Dialectics

The pragma-dialectical system of classification of schemes consists of three basic schemes [Van Eemeren and Grootendorst, 1992]: 1) symptomatic argumentation; 2) argumentation based on similarities; and 3) the instrumental argumentation. The first scheme represents type of argumentation in which the speaker tries to convince his interlocutor "by pointing out that something is symptomatic of something else." In this type of pattern, what is stated in the argument premise is a sign or symptom of what is stated in the conclusion. The second scheme is grounded on a relation of analogy between what is stated in the argument premise and what is stated in the conclusion. In the third type of scheme the argument and the conclusion are linked by a very broad relation of causality. Other arguments are classified under these categories [Van Eemeren and Grootendorst, 1992]. For instance, arguments based on inherent qualities or a characteristic part of an entity or from authority are regarded as belonging to the symptomatic argumentation; arguments pointing out the consequences of an action or based on the means-end relationship are considered as subclasses of causal arguments [Garssen, 2001].

This system of classification is grounded on a twofold criterion. While causal argumentation is characterized by a material relation, analogical argumentation represents a type of reasoning independent from the specific content of the premises and conclusion. Symptomatic argumentation is a combination of these two criteria, as a sign or a symptom presupposes an abductive pattern and a material causal relation.

4.5 Grennan

In Grennan's [Grennan, 1997, pp.163-165] typology, the structurally valid inductive[3] inference patterns are classified according to 9 warrant types, derived from Ehninger and Brockreide's typology [Brockriede and Ehninger, 1963]. The warrant types include possible reasons for inferring conclusions from premises, all belonging to the "logical mode" (and not to other types of motivations, such as emotions). The argument patterns can be summarized as follows:

1. **Cause to Effect**: The phenomenon mentioned in P produces the one in C.
2. **Effect to Cause**: The phenomenon mentioned in P is best explained by C.
3. **Sign**: The phenomenon mentioned in P is symptomatic (naturally or conventionally) of one reported in C.
4. **Sample to Population**: What is true of sample of X is also true of other Xs.
5. **Parallel Case**: What is true of the referent of P is also true of other Xs.
6. **Analogy**: $B1$ is to $B2$ in C as $A1$ is to $A2$ in P.
7. **Population to Sample**: What is true of Known Xs is also true of this X.

[3]Inferences, in an informal logic perspective, are considered inductive, since argumentation does not deal with deductive validity. The criterion for discriminating between acceptable and unacceptable patterns is provided by a logical intuition.

8. **Authority**: S (the assertor of C) is a reliable source.

9. **Ends-Means**: The action mentioned in C generally achieves the end mentioned in P.

The patterns mentioned above are individuated on the basis of the warrant type. Together with this criterion of argument classification, Grennan presents a typology of claims. Each argument can be analysed relative to the type of warrant and to the kind of conclusion to be supported. The types of claim identified by Grennan [Grennan, 1997, p.162] can be represented in Table 10.

Type of Claim	Example
1. *Obligation Claims:* X must do A.	"Sam must apologize."
2. *Supererogatory Actuative Claims:* X ought to do A (they express a judgment that is in the interests of someone other than X for X to do A)	"I ought to help the needy in this area."
3. *Prudential Actuative Claims:* X ought to do A.	"Canadians ought to avoid heart diseases."
4. *Evaluative Claims*, of which there are three kinds: grading, rating, and comparison.	"This is a good cantaloupe." "Steffi Graf is the best female tennis player at this time." "Gretzky is a better hockey player than Howe was."
5. *Physical-World Claims*, which include both physical brute facts and institutional facts.	"The sun is setting." "The Dodgers beat the Giants three to two in eleven innings."
6. *Mental-World Claims*, which ascribe mental phenomena.	"He is upset."
7. *Constitutive-Rule Claims*, which are based on definitions and other necessary truths and falsehoods.	"In this election, majority should be defined as a majority of members present and voting." "Solid iron does not float in water."
8. *Regulative-Rule Claims*, which express obligations and prohibitions.	"Driving on the right is obligatory."

Table 10: Grennan: Classification of schemes

The types of warrant and the types of claim are the two criteria underlying Grennan's typology of argument patterns, each characterized by a premise, a warrant, and a conclusion. In the diagram below are represented the valid and useful patterns of arguments for obligation claims resulting from this classification [Grennan, 1997, p.162] (Figure 3).

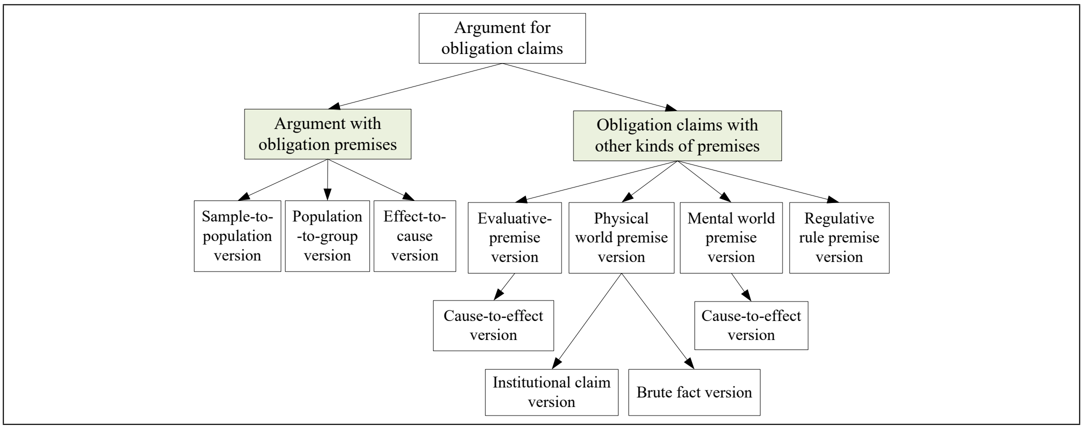

Figure 3: Classification of the arguments for obligation claims in Grennan

Grennan's typology develops the distinction between the warrant type and the kind of conclusion. The typology is extremely deep as regards the relation between speech acts and argument, but is limited to 8 warrant types.

4.6 Katzav and Reed

Rooted in the schemes presented by Walton [Walton, 1995], the classification system of Katzav and Reed [Katzav and Reed, 2004b] aims to classify an argument by virtue of the "relation of conveyance" that the complex proposition constituting the argument represents. These relations of conveyance describe how it is that one fact necessitates another, such as in the following example [Katzav and Reed, 2004a, p.2]:

> Consider, by way of illustration, a case in which the causal relation is operative: in the circumstances, the fact that the US military attacked Iraq caused the fall of Saddam′s regime. Thus, in the circumstances, and via or in virtue of the obtaining of a causal relation, the fact that the US military attacked Iraq necessitated, or made it liable that, Saddam′s regime fell.
>
> Using the causal relation and the above statements about Saddam′s regime, we can construct the following simple argument:
>
> (1) Saddam′s regime fell, because the US military attacked Iraq and if the US military were to attack Iraq, Saddam′s regime would fall.
>
> In (1), the fact that the US military attacked Iraq is represented as conveying, via the causal relation, the fact that Saddam′s regime fell. That the relation of conveyance represented is the causal relation is implicit in the subjunctive conditional "if the US military were to attack Iraq, Saddam′s regime would fall."

In [Katzav and Reed, 2004b] the nature of such relations of conveyance is unpacked and connected to the concepts of warrant and scheme and to the work of Kienpointner [Kienpointner, 1992] and Walton [Walton, 1995]in particular. In [Katzav and Reed, 2004a], they sketch a high-level classification of relations of conveyance. At the topmost level, they distinguish between "internal" and "external" relations, whereby the former depend solely upon intrinsic features (and therefore encompass definitional, cladistic, mereological and normative relations, amongst others), whilst external relations depend upon extrinsic features (thereby covering such as spatiotemporal and casual relations, amongst others). Beneath this, the classification is further broken down into groups of schemes: those of specification, constitution, analyticity and identity under intrinsic relations and causal and non-causal under extrinsic (due largely to the fact that so many schemes rely upon causal relations). The full top-level classification tree (which identifies the main branches but does not give an exhaustive specification) is given in the scheme below:

Internal relation of conveyance
Relation of specification

- Relation of species to genus
- Relation of species to genus
- Relation of genus to species
- Determinable-determinate
- Etc.

Relation of constitution

- Abstract fact constitution
- Constitution of normative facts
- Constitution of positive normative facts
- Constitution of negative normative facts
- Constitution of non-normative abstract facts
- Constitution of necessary conditions
- Constitution of causal law
- Constitution of singular causal conditionals
- Constitution of constitution facts
- Constitution of Possibility
- Constitution of Impossibility
- Etc.

Concrete fact constitution

- Species/kind instance constitution
- Property instance constitution
- Property constitution by properties
- Property constitution by particulars
- Etc.
- Constitution of singular causal facts
- Relation of a part to a whole
- Relation of whole to one of its parts
- Etc.

Relation of analyticity

- Relation of sameness of meaning
- Relation of stipulative definition
- Relation of implication

Relation of identity

- Relation of qualitative identity
- Relation of numerical identity
- Etc.

External relation of conveyance
Non-causal dependence

- Non-causal law
- Conservation
- Conserved quantity
- Conserved quality
- Etc.
- Symmetry
- Spatial symmetry
- Etc.
- Nomological incompatibility
- Thing location incompatibility
- Thing type incompatibility
- Etc.
- Topological structure conveyance

Causal dependence

- Efficient cause conveyance
- Causal law
- Singular cause to effect
- Singular effect to cause
- Common cause
- Final cause conveyance

Though the mapping from individual relations of conveyance in this classification to the argumentation schemes in [Walton, 1995] and particularly [Walton *et al.*, 2008]is not a trivial 1-to-1 correspondence, those schemes have been slotted in successfully in later work with a computational focus such as [Bex and Reed, 2011].

4.7 Lumer and Dove

The last system of classification that we consider was provided by Lumer and Dove (Lumer & Dove, 2011), using three general classes, each including subclasses:

1. Deductive argument schemes
    - Elementary deductive argument schemes;
    - Analytical arguments:
    - Definitoric arguments
    - Subsuming legal arguments:
2. Probabilistic argument schemes
    - Pure probabilistic argument schemes (statistics, signs);
    - Impure probabilistic argument schemes (best explanation);
3. Practical argument schemes
    - Pure practical argument for pure evaluations;
    - Impure practical argument schemes (for justification of actions; justification of instruments);
    - Arguments for evaluations based on adequacy conditions;
    - Arguments for welfare-ethical value judgements;
    - Practical arguments for theoretical theses.

This system consists of a mix of two distinct criteria, logical and pragmatic. While the first two classes are characterized by the type of reasoning, the last one is a type of argument with a specific pragmatic purpose, recommending a course of action. Moreover, the subclasses are defined based on both logic-based and content-based criteria, where together with distinctions based on the logical form (analytic schemes; probabilistic schemes) there are subclasses based on the nature of the premises (definitoric; subsuming).

All these types of classification show how a sole criterion is not sufficient for providing a clear and comprehensive classification of schemes. In order to understand what criteria can be used and in what abstract categories can be considered as the most basic ones, it is necessary to analyze the structure of the schemes. Once the common components of these heterogeneous combinations of premises and conclusions are brought to light, it is possible to find criteria for organizing them for specific purposes.

5 Using the schemes: A classification system

Argumentation schemes can be conceived as the prototypical combination of semantic (or topical) relations with logical rules of inference [Macagno and Walton, 2015; Macagno *et al.*, 2016; Walton and Macagno, 2015]. A classification based on the semantic link can provide an instrument for bringing to light the material relation between premises and conclusion, but the same semantic relation can be combined with types and rules of reasoning, and lead to various types of conclusion. For instance, causal relations are the ground of the argument from cause to effect, but also of arguments from sign and practical reasoning. Argumentation schemes merge the most common combinations between types of reasoning and material relations. For this reason, we need first to distinguish between these two levels, distinguishing between the various types of reasoning in Table 11.

Type of reasoning	Deductive axioms	Induction	Abduction
Type of argument	Argument from definition, genus...	Argument from example	Argument from (improper) signs
	Argument from cause to effect	...	Practical reasoning
	Argument from consequences	...	Argument from best explanation
	Argument from commitment	...	...

Table 11: Types of argument and types of reasoning

A multi-logical perspective needs to be taken into account as a classification criterion, in which the logical form can be described using distinct types of reasoning, which in turn can include various logical rules of inference (MP, MT). However, in the Latin and Medieval tradition, the formal rules of inference are treated as maxims and not as distinct levels of abstraction. For this reason, the two levels of the general, semantic topics and of the logical rules are not distinguished, and the possible interconnections between them are not taken into account. The modern theories of argumentation schemes propose classifications essentially mirroring the ancient approach. The logical rules are treated at the same level as the semantic-ontological topics, and not as distinct levels of abstraction. A possible solution is to acknowledge the discrepancy between logical form and semantic content as a divergence in kind, and try to show how these two levels can be interconnected.

A possible overarching principle can be found in the pragmatic function of the schemes, namely what they have been intended for. Argumentation schemes can be thought of as instruments for reconstructing and building arguments (intended as discourse moves), i.e. analytical or invention tools. For this reason, in order to provide a classificatory system to retrieve and detect the needed scheme it can be useful to start from the intended purpose of an argumentation scheme. From an analytical point of view, the analysis of an argument in a discourse, a text, or dialogue presupposes a previous understanding of the communicative goal (and, therefore, the "pragmatic" meaning) of the argument and the components thereof. For example, an argument can be aimed at classifying a state of affairs, supporting the existence of a state of affairs, or influencing a decision-making process.

This teleological classification needs to be combined with a practical one. The generic purposes of a move need to be achieved by means of an inferential passage. In this sense, the classificatory system needs to account for the possible (argumentative) means to achieve the pragmatic purpose of an argument. Not all the semantic relations underlying the schemes can support all the possible conclusions or purposes of an argument. Definitional schemes are aimed at supporting the classification of a state of affairs; they cannot lead to the prediction or retrodiction of an event. Similarly, a pattern of reasoning based on the evaluation of the consequences of an action or an event can be used to establish the desirability of a course of action brining it about. However, it cannot be reasonably used to establish the truth or falsity (or acceptability) of a proposition. For this reason, the analysis of the pragmatic meaning (i.e. the purpose) of an argument provides a criterion for restricting the paradigm of the possible means to achieve it. The crucial problem is to find categories of argument purposes that can establish criteria for distinguishing among classes of semantic relations, which in turn can be specified further according to the means to achieve such goals.

The first distinction to be made is based on the nature of the subject matter, which can be 1) a course of action or 2) a state of affairs. In the first case, the goal is to support the desirability or non-desirability of an action; in the second case, the schemes are aimed at providing grounds for the acceptability of a judgment on a state of affairs. The ancient dialectical accounts (Cicero, *Topica*; Boethius, *De Topicis Differentiis*) distinguished between two types of argumentative "means" to support a conclusion, namely the "internal" and the "external" arguments. The first ones are based on the characteristics of the subject matter (such as arguments from definition or cause). The latter derive their force from the source of the statement, namely from the authority of who advances the judgment or the proposal (arguments from authority). This first distinction can be represented as shown in Figure 4.

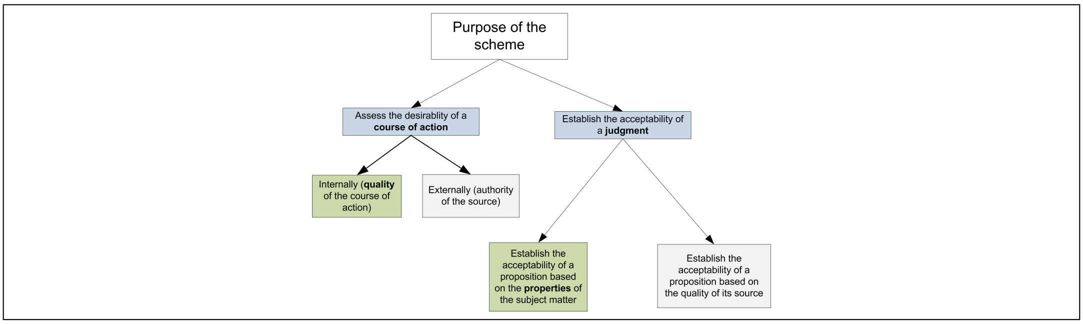

Figure 4: Purposes of an argument

The acceptability of a conclusion can be supported externally in two ways. If the argument is aimed at establishing the desirability of a course of action, the authority can correspond to the role of the source ("You should do it because he told you that!"). Otherwise, the popular practice can be a reason for pursuing a course of action ("We should buy a bigger car. Everyone drives big cars here!"). External arguments can be represented in Figure 5.

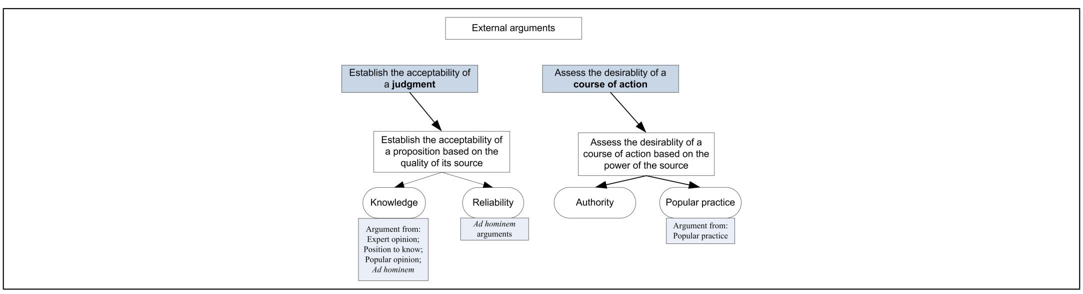

Figure 5: External arguments

When external arguments are used to support also a judgment on a state of affairs, the relevant quality of the source is not the speaker's authority (connected with the consequences of not complying with the orders/conforming to common behavior) but rather with the source's superior knowledge. The quality of the source can be also used negatively to show that a source is not reliable (it is not a good source), and that consequently the conclusion itself should be considered as doubtful (*ad hominem* arguments).

Internal arguments can be divided into the two categories of arguments aimed at assessing the desirability of a course of action, and the ones supporting the acceptability of a judgment. Courses of action can be classified as desirable or not depending on the quality of their consequences (the course of action is a condition of a resulting positive or negative state of affairs) or their function in bringing about a desired goal (an action is productive of a pursued state of affairs) (Figure 6).

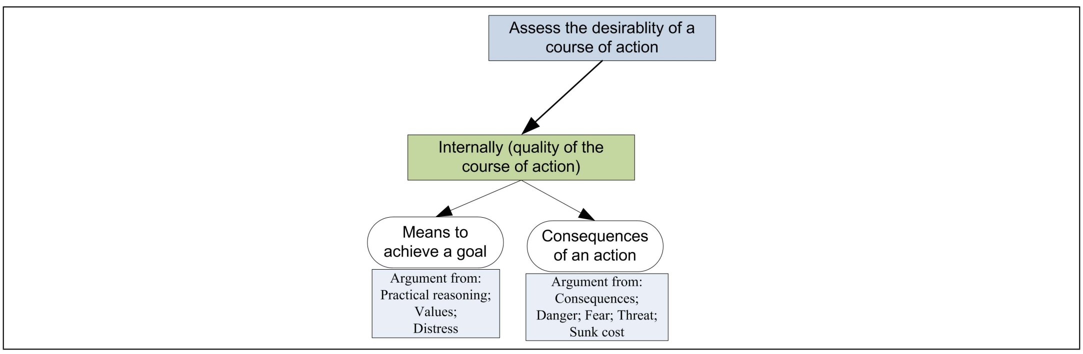

Figure 6: Internal practical arguments

The arguments used to provide grounds for a judgment on a state of affairs can be divided according to the nature of the predicate that is to be attributed. The most basic differentiation can be traced between the predicates that attribute the existence of a state of affairs (the occurrence of an event or the existence of an entity in the present, the past, or the future), and the ones representing factual or evaluative properties.

The arguments supporting a prediction or a retrodiction are aimed at establishing whether or not an event has occurred or will occur, or whether an entity was or will be present (existent). The arguments proceeding from casual relations (in particular from material and efficient causes) bear out this type of conclusion. The other type of predicates can be divided in two categories: factual judgments and value judgments. The first type of predicates can be attributed by means of reasoning from classification, grounded on descriptive (definitional) features and supporting the attribution of a categorization to an entity or an event (Bob is a man; Tom is a cat). Value judgments are classifications that are not based on definitions of categorical concepts (to be a cat) but rather on values, or rather hierarchies of values. Such judgments proceed from criteria (or more specifically, criteria of importance to the audience to whom the argument is presented) for classifying what is commonly considered to be "good" or "bad." Also the reasoning underlying the attribution of evaluative predicates, such as "to be a criminal," can be considered as belonging to this group of arguments. These latter patterns are grounded on signs of an internal disposition of character, which in its turn is evaluated. The distinctions discussed above are summarized in Figure 7 below.

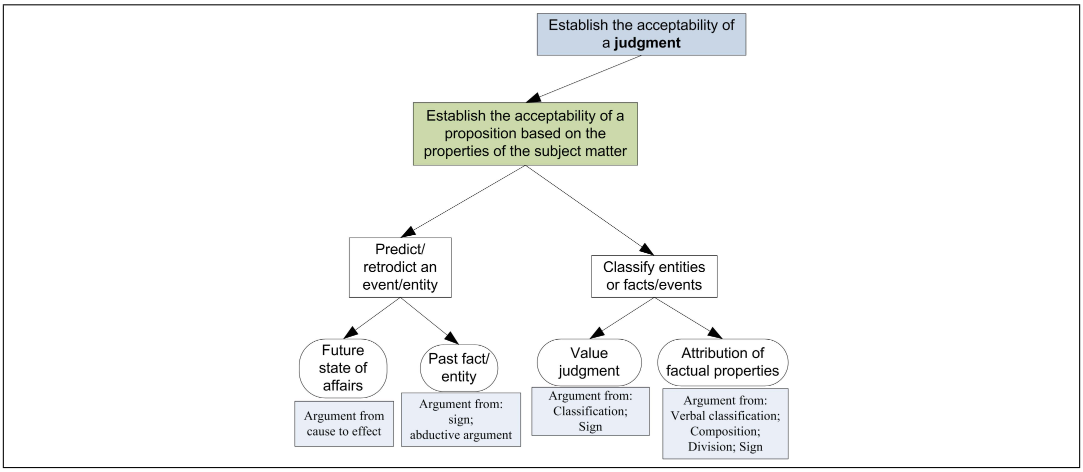

Figure 7: Establishing the acceptability of a judgment (SoA)

This system of classification of argumentation schemes is based on the interaction between two criteria, the (pragmatic) purpose of an argument and the means to achieve it. This tree model can be used both for analytical and production purposes. In the first case, the speaker's intention is reconstructed by examining the generic purpose of his move, and then the possible choices that he made to support it, based on the linguistic elements of the text. Depending on the desired level of preciseness, the analysis can be narrowed down until detecting the specific scheme, namely the precise combination of the semantic principle and the logical rule supporting the conclusion. In this fashion, the analyst can decide where to stop his reconstruction. This analytical model can be of help also for educational purposes, as it can be adapted to various teaching needs and levels. For production purposes, the nature of the viewpoint to be supported can be analyzed using the most generic criteria set out above (What is under discussion, a decision or a fact? The occurrence of an event or its classification? The naming of a state of affairs or its qualification?). Such questions closely resemble the ones that were at the basis of the rhetorical theory of stasis, namely the issues that can be discussed [Heath, 1994]. These distinctions are then combined with the specific alternative strategies to support the defended viewpoint.

The aforementioned system of classification can also account for the interrelation between the semantic relation and the different types of reasoning, namely logical forms. For example, the desirability of a course of action can be assessed internally by taking into consideration the means to achieve a goal. This pattern of reasoning can be stronger or weaker depending on whether there is only one or several alternatives. The paradigm of the possible means will determine whether the reasoning is abductive or deductive, resulting in a more or less defeasible conclusion. The same principle applies to the other semantic relations, such as the ones proceeding from cause or classification, which can be shaped logically according to inductive, analogical, deductive, or abductive types of reasoning.

6 A bottom-up approach to classification: Clusters of decision-making schemes

Argumentation schemes are characterized by both "family" resemblances and actual interconnections [Walton and Macagno, 2015]. Practical reasoning, value-based reasoning, value-based practical reasoning, argument from positive consequences, argument from negative consequences, and the slippery slope argument are related by the same similar structure based on value judgments and practical outcome. Such schemes are often also interconnected when we analyze the structure of actual arguments. However, in order to understand and choose between similar and interrelated schemes, it is necessary to examine their relations and their differences. The simplest and most intuitive version of the scheme for practical reasoning (Table 13) uses the first-person pronoun "I" to represents a rational agent, an entity that has goals, some knowledge of its circumstances, and the capability of taking action to change those circumstances. It also has sensors to perceive its circumstances, and to perceive at least some of the consequences of its actions when it acts to change its circumstances. Such a rational agent also therefore has the capability for feedback. When it perceives changes in its circumstances due to its own actions, it can modify its actions or goals accordingly, depending on whether the consequences of its actions are deemed to contribute to its goals or not. This simplest form of practical reasoning [Walton *et al.*, 2008, p.95] can be described as a fast and frugal heuristic for jumping to a quick conclusion that may later need to be retracted in the light of further considerations (Table 12).

Major Premise:	I have a goal G.
Minor Premise:	Carrying out this action A is a means to realize G.
Conclusion:	Therefore, I ought (practically speaking) to carry out this action A.

Table 12: Argument from Practical reasoning

The defeasible nature of this simple form of practical reasoning is brought out by the observation that it typically provides a starting point for action that needs to be challenged by the asking of critical questions as the agent moves ahead. Below is the standard set of critical questions matching this scheme.

CQ1 What other goals do I have that should be considered that might conflict with G?

CQ2 What alternative actions to my bringing about A that would also bring about G should be considered?

CQ3 Among bringing about A and these alternative actions, which is arguably the most efficient?

CQ4 What grounds are there for arguing that it is practically possible for me to bring about A?

CQ5 What consequences of my bringing about A should also be taken into account?

The last critical question, CQ5, often called the side effects question, concerns assessment of the potential negative consequences of carrying out the action described in the conclusion of the scheme. If negative consequences of this course of action are identified, that is a reason for withdrawing the conclusion and considering an alternative course of action that might avoid the negative consequences. Use of the term "negative" implies that values are involved, and that a rational agent is assumed to have values as well as goals that it bases its practical reasoning on.

A complication is that there is another closely related argumentation scheme associated with this critical question, Argument from negative consequences. This scheme, widely recognized in the literature, cites known or estimated consequences of a proposed course of action as presenting a reason, or set of reasons, against taking the course of action initially indicated by the practical reasoning scheme. Argument from negative consequences also has a positive form. According to the scheme for argument from positive consequences, known or estimated consequences that have a positive value for the agent are cited as a reason, or set of reasons, supporting the carrying out of the action initially considered. Below the versions of the two basic argumentation schemes for arguments from consequences are formulated as they were in [Walton *et al.*, 2008, p.101]. The first one is called argument from positive consequences (Table 13).

Premise:	If A is brought about, good consequences will plausibly occur.
Conclusion:	Therefore A should be brought about.

Table 13: Argument from positive consequences

Premise:	If A is brought about, bad consequences will plausibly occur.
Conclusion:	Therefore A should not be brought about.

Table 14: Argument from negative consequences

The second one is called argument from negative consequences (Table 14).

In both instances, an implicit premise could be made explicit in the scheme stating that if good (bad) consequences will plausibly occur, A should (not) be brought about. As with the basic form of practical reasoning, arguments from positive or negative consequences are defeasible. The premise offers a reason to accept a proposal for action tentatively, subject to exceptions as new circumstances come to be known by the agent. In these formulations, the expression “good consequences” refers to consequences taken by the agent to have positive value, and the expression “bad consequences” refers to actions taken to have negative value. These observations bring us to another pair of schemes closely related to the ones for argument from positive consequences and argument from negative consequences.

The relationship between a state of affairs, its classification according to a value, and the commitment to an action is represented in terms of value. Values (differently from [Atkinson *et al.*, 2005];[Bench-Capon, 2003]) are regarded as grounds for a type of reasoning independent from and related to (or rather, presupposed by) practical reasoning. This reasoning guarantees the so-called “practical classification” [Westberg, 2002, p.163] of a state of affairs and the commitment thereto. The scheme for argument from positive value is formulated in Table 15 as in [Walton *et al.*, 2008, p.321]:

Premise 1:	Value V is positive as judged by agent A.
Premise 2:	If V is positive, it is a reason for A to commit to goal G.
Conclusion:	V is a reason for A to commit to goal G.

Table 15: Argument from positive value

The corresponding scheme representing argument for argument from negative value is formulated in Table 16.

Premise 1:	Value V is negative as judged by agent A.
Premise 2:	If V is negative, it is a reason for retracting commitment to goal G.
Conclusion:	V is a reason for retracting commitment to goal G.

Table 16: Argument from negative value

Argument from positive consequences typically supports an argument taking the form of basic practical reasoning by giving justification for going ahead with the contemplated action. Argument from negative consequences presents a reason against taking the action being considered by citing consequences of it that would contravene the values of the agent.

Another more complex argumentation scheme has also been recognized in the literature [Bench-Capon, 2003] that combines all the schemes mentioned above. This scheme describes a form of argument called goal-based practical reasoning that combines basic practical reasoning with value-based reasoning. The version of this scheme (Table 17) is from [Walton *et al.*, 2008, p.324].

The scheme for value-based practical reasoning can also be formulated in a more explicit way that brings out an important aspect of practical reasoning, namely the circumstances of the case that can be observed by the agent and used by as a basis for reaching a decision on what to do. According to the version of the scheme formulated in [Atkinson *et al.*, 2005], any action the agent takes can be seen as a transition from the current set of circumstances to a new set of circumstances, as the agent moves forward to attempt to realize its goal.

Premise 1:	I have a goal G.
Premise 2:	G is supported by my set of values, V.
Premise 3:	Bringing about A is necessary (or sufficient) for me to bring about G.
Conclusion:	Therefore, I should (practically ought to) bring about A.

Table 17: Argument from goal-based practical reasoning

The last decision-making argument is the slippery slope argument, sometimes also called the wedge argument. Different varieties of slippery slope argument have been recognized, such as the causal slippery slope argument, the precedent slippery slope argument, the linguistic slippery slope argument, which depends on the vagueness of terms or concepts, and a more complex (all-in) form of slippery slope argument that combines the simpler variants. A good place to start is a simple version of the slippery slope type of argument formulated as the basic scheme in [Walton *et al.*, 2008, p.340] (Table 18).

First Step Premise:	A_0 is up for consideration as a proposal that seems initially like something that should be brought about.
Recursive Premise:	Bringing up A_0 would plausibly lead (in the given circumstances) to A_1, which would in turn plausibly lead to A_2, and so forth, through the sequence $A_2, \ldots A_n$.
Bad Outcome Premise:	A_n is a horrible (disastrous, bad) outcome.
Conclusion:	A_0 should not be brought about.

Table 18: Argument from goal-based practical reasoning

According to [Walton *et al.*, 2008, p.340], the following three critical questions match this basic scheme.

CQ1 What intervening propositions in the sequence linking up A_0 with $_n$ are actually given?

CQ2 What other steps are required to fill in the sequence of events, to make it plausible?

CQ3 What are the weakest links in the sequence, where specific critical questions should be asked on whether one event will really lead to another?

So here we have a cluster of schemes all closely related to each other. The argument from negative consequences is one of the critical questions matching the basic scheme, but the scheme for argument from negative consequences is itself based on the closely related scheme for argument from negative values.

Clarifying the relationships among this cluster of schemes enables us to draw an important distinction widely discussed in the philosophical literature on practical reasoning between two distinct types of practical reasoning: instrumental practical reasoning and value-based practical reasoning. When it comes to classifying the arguments within this cluster of schemes, it would seem reasonable to venture as a hypothesis that the basic scheme for practical reasoning is the simplest form of it, while the scheme for value-based practical reasoning is a more complex variant of the scheme. It combines the basic scheme with the schemes for argument from values. On this approach to drawing distinctions within the cluster, arguments from positive consequences can be taken as species of arguments from positive value, and arguments from negative consequences can be taken as species of arguments from negative value. Practical experience in using assistants to use argumentation schemes to identify types of arguments in natural language text suggests that the assistants sometimes find it difficult to classify a particular argument identified in a text as fitting one or more of these schemes. It can be helpful for this purpose is to give the assistants identification conditions that attempt to formulate key essential requirements of the type of argument represented by a particular scheme.

The following is a set of three identification conditions for the type of argument matching the scheme for instrumental practical reasoning: (1) An agent (or group of agents in the case of multiagent reasoning) is attempting to arrive at a reasoned decision on what course of action to take in a given set of circumstances requiring some action, (2) the circumstances provide evidence on which to build pro and con arguments, arguments for and against the course of action being considered, (3) the agent is basing its decision on its goals, as well as its perception of the circumstances of the case, (4) arguments need to be weighed against each other as stronger or weaker reasons for taking this action or not, and (5) the agent purports to be using this evaluation of the stronger or weaker reasons as its basis for taking the action or not. Here the four conditions describe an agent deciding whether to take a particular course of action or not. But it needs to be recognized that in some situations there may be several alternative courses of action to be considered, and the agent is trying to decide which of them would be the best course of action, based on the reasons provided by its goals and the circumstances of the case.

The identification conditions for the value-based species of practical reasoning are the same as the five identification conditions for instrumental practical reasoning, except that another condition needs to be added: (6) the agent is justifying its decision based on its values, as well as on its goals and its perception of the circumstances of the case. The aforementioned cluster of arguments is characterized by several types of relations, which can be of help in distinguishing them and detecting their possible nets. For example, argument from negative consequences is one of the questions matching the scheme for argument from practical reasoning. So this relationship could be described by saying that argument from negative consequences is a counterargument, a rebuttal or undercutter that can defeat an argument from practical reasoning in a given case, provided that the negative consequences can be specified, and provided that it can be shown that these consequences are indeed negative.

Already from these remarks one relationship emerges. Argument from negative consequences is based on argument from values, and is a species of argument from values. Another relationship already shown above, is that value-based practical reasoning is a more complex form of argument than instrumental practical reasoning. Value-based practical reasoning is a species of instrumental practical reasoning with argument from values added on to it.

Another relationship that emerges is that the slippery slope type of argument is clearly a subtype and special instance of argument from negative consequences. It is less evident that the slippery slope argument is also a species of value-based practical reasoning. However, it can be seen that it is. In the case of the slippery slope argument, the agent doing the decision-making must be assumed to have some goals and values in mind that the other party, the agent attacking its argument, can appeal to when mounting a slippery slope argument. Let's call the two parties the agent and critic. The slippery slope type of argument is inherently negative. The critic is using the argument to warn the agent that if he takes a first step, or continues a series of steps that he has already started, these steps will lead to a loss of control that cannot be anticipated in advance so that the sequence of actions will ultimately result in a catastrophic outcome. The critic has to assume that the agent has some values that both of them share, so that they can both agree that the outcome warned of by the critic is catastrophic, that is highly negative and worth avoiding. The critic has to assume that the agent has some goals and is acting in a rational manner so that it is trying to either achieve or at least be consistent with these goals as it carries out an action supposedly designed to fulfill them. Otherwise the critic's argument is not going to have much force and will be unlikely to deter the agent from moving ahead.

What especially distinguishes the slippery slope as a distinctive type of argument are three premises, the recursive premise, the grey zone premise and the loss of control premise. Given these observations, we can see how the value-based practical reasoning argument is embedded into the basic slippery slope argument and is a part of it. As shown in Figure 8, the basic slippery slope type of argument, represented by the scheme formulated above, is at the center of a cluster of other related schemes.

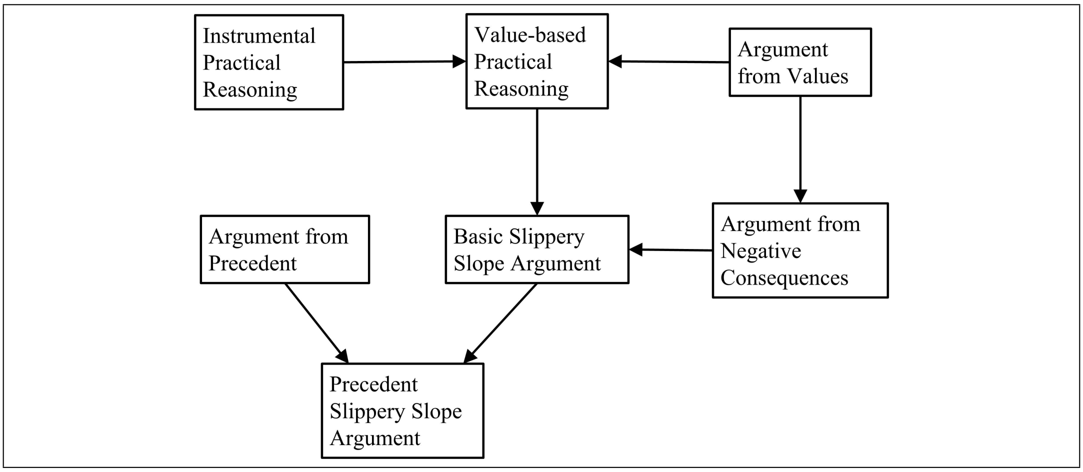

Figure 8: Cluster of Schemes

The basic slippery slope argument is derived from value-based practical reasoning as its core argument structure, where value-based practical reasoning is a combination of instrumental practical reasoning and argument from values. So here it is shown how these schemes are structured together into a cluster. It is also shown that the basic slippery slope argument is a species of argument from negative consequences, as scheme that is in turn built partly from the scheme for argument from values. So these five schemes form a cluster. But the basic slippery slope argument also has several subtypes, including the precedent slippery slope argument, the causal slippery slope argument, and the variety of slippery slope argument deriving from vagueness of a verbal criterion. According to the analysis of the slippery slope argument given in [Walton, 1992] these four species of the slippery slope argument are subtypes of a more general form of argument called the all-in slippery slope argument.

Here we put forward the hypothesis that there is a basic, minimal type of slippery slope argument from which these other more specialized variants are derived. To indicate the existence of such connections in Figure 8, we have inserted the name of the scheme for the precedent slippery slope argument underneath the schemes for argument from precedent and the basic slippery slope argument. This classification indicates another aspect of the cluster of schemes surrounding the category of slippery slope arguments.

7 Using argumentation schemes: Nets of Argumentation Schemes

Argumentation schemes are imperfect bridges between the logical (or quasi-logical) level and the conceptual one [Macagno and Walton, 2015; Macagno, 2015]. From a conceptual (material) point of view, schemes usually represent an inferential step from a specific type of premise to a specific type of conclusion. However, there is a crucial gap between the complexity of natural argumentation, characterized by several conceptual passages leading to a conclusion, and the schemes. In order to reason from consequences, we need to classify a state of affairs, evaluate it positively or negatively, and then suggest a suitable course of action, which can lead to further reasoning steps, for example from commitment. A single argumentation scheme cannot capture the complexity of such real argumentation. For this reason, we need to conceive the relationship between arguments and schemes in a modular way, in terms of nets of schemes.

A real argument can be described through interconnected and interdependent argumentation schemes, each of them bringing to light a single argumentative step that can be explicit, presupposed, or simply implied. In order to explain the idea of nets of schemes, we consider the following example taken from the debates during the conflict between Russia and Ukraine in 2014. In this case, the British Foreign secretary William Hague commented on Russia's intervention in Crimea and Ukraine as follows[4]:

Example 7.1 (The Hague Speech)**.** *Be in no doubt, there will be consequences. The world cannot say it is OK to violate the sovereignty of other nations. This clearly is a violation of the sovereignty independence and territorial integrity of Ukraine. If Russia continues on this course we have to be clear this is not an acceptable way to conduct international relations.*

This example is apparently an easy case of argument from consequences, in which Russia's continuation of its military operations is depicted by the British Foreign Secretary as leading to undesirable consequences. However, this reasoning involves also a classification of Russia's behavior as a"violation of the sovereignty independence and territorial integrity of Ukraine," and a qualification of this behavior as unacceptable by the UK and the "world." By pointing out the shared values to which the world countries are committed (the sovereignty of other nations cannot be violated), the speaker makes explicit the commitment against Russia's behav-

[4]Ukraine crisis: William Hague warns Russia of economic fallout. *The Guardian*, 3 March 2014. Retrieved from: https://is.gd/Kw8Vax. (Accessed on 15 May 2017)

ior, which is represented by the vague notion of "consequences." We represent this structure in Figure 9.

Figure 9: Net of arguments in the Hague Example

In Figure 9 the dotted boxes represent the tacit premises and the tacit ultimate conclusion, which are taken for granted by the speaker but are needed for reconstructing his reasoning. The classification, the reasoning from commitment, and the argument from consequences are deeply interconnected. The alleged world's commitment to consequences against Russia depends on the classification of the state of affairs [Macagno and Walton, 2014; Walton and Macagno, 2009], which fits into the value of "protecting nations' sovereignty." This commitment leads to an implicit threat, namely a consequence that is presupposed to be negatively evaluated by Russia.

This analysis can be applied to the structure of a slippery slope argument, such as the one advanced by the Russian defense analysts in reply to the help provided by the United States to Ukraine (which includes weapons and hardware)[5]:

Example 7.2 (The Global Escalation)**.** *U.S. provision of military aid to Ukraine would be seen by Moscow as a declaration of war and spark a global escalation of Ukraine's separatist conflict, Russian defense analysts said.*

This argument stems from a classification (US provision of military help is a declaration of war), and leads to a chain of negative consequences (global escalation)

[5]Russia Would See U.S. Moves to Arm Ukraine as Declaration of War. *The Moscow Times*, 9 February 2015. Retrieved from: https://is.gd/hxO6MW (Accessed on 15 May 2017)

that ultimately are going to affect the Western countries. Also in this case, the central argument (the slippery slope) is associated with other arguments (argument from classification and from values), resulting in the net shown in the graph in Figure 10.

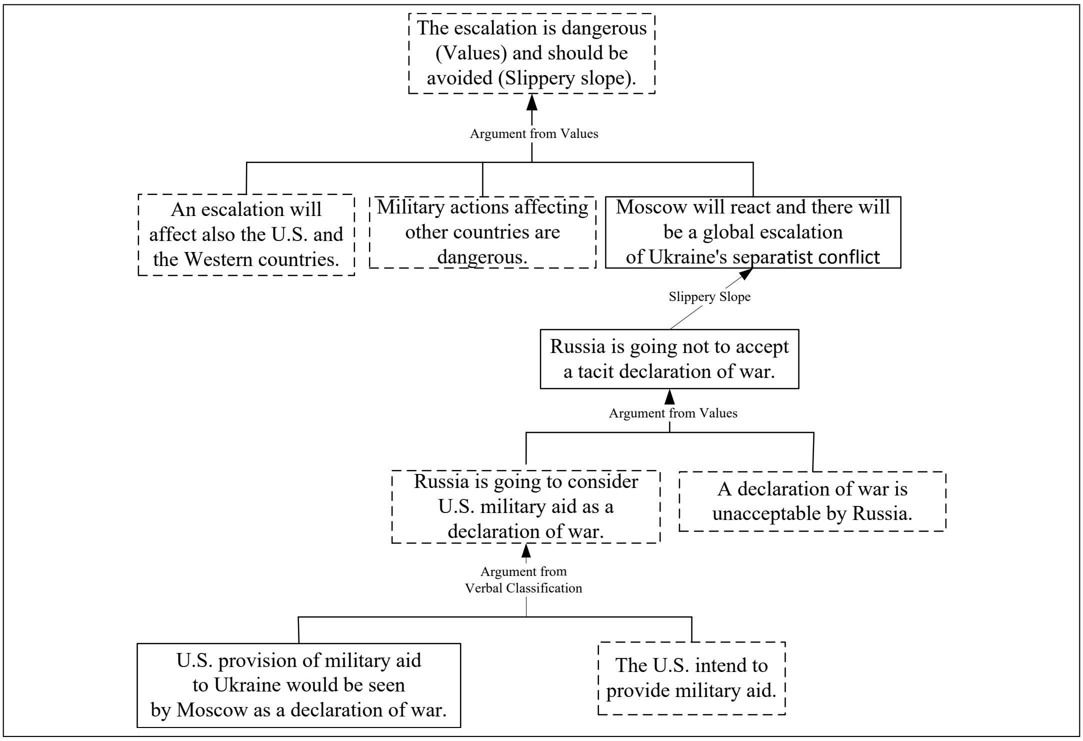

Figure 10: Net of arguments in the global escalation example

In this case, the classification justifies the slippery slope, whose force partially depends on the fact that the escalation is claimed to be global, affecting also other countries. The evaluation of this consequence therefore combines with the chain of events claimed by the analysts, and leads to the practical conclusion of avoiding the provision of military aid.

A special feature of this example is its compressed style of presentation. Slippery slope is a complex form of argument built around a connected sequence of actions and consequences starting from an initial action or policy and then proceeding through a sequence to an eventual outcome. However in many examples, the intervening sequence is left implicit, concealing a chain of intervening propositions that have to be filled in as implicit assumptions of the argument. These implicit assumptions are needed to make it fit the scheme for the slippery slope type of argument. The example really is a slippery slope argument, but in order to prove that it is, several implicit premises or conclusions have to be filled in that are essential. These intervening links are basically filled in by common knowledge concerning the normal way we expect military inventions to take place and to have consequences. By using an argument map that reveals the network of argumentation into which the given slippery slope argument fits, the puzzle of unraveling the network of argumentation using a cluster can be solved in any given case of argument interpretation.

On the perspective presented in this section, we notice that argumentation schemes appear in nets instead of in clear and independent occurrences. A scheme can capture only one passage of reasoning, while the nets can map a more complex argumentative strategy, involving distinct and interdependent steps.

8 Using Argumentation Schemes in AI and law

In the sections above we have shown how argumentation schemes have been developed theoretically, providing a system of classification and representation thereof. One of the most important areas of application of the schemes is computing, and in particular artificial intelligence. In this section, we will show very briefly how argumentation schemes have been used in AI and AI and Law, and in particular the principles guiding the formalization thereof. It is far from a complete survey, but merely attempts to show how schemes are currently being applied and modeled. It also tries to convey very briefly how schemes have evolved as they have been used for different purposes in different AI systems and areas. The discussion includes the problem of how to model critical questions matching each scheme, and how schemes are being used in AI and Law in argument mining, case-based reasoning and statutory interpretation.

The paper that introduced argumentation schemes to the AI and law community was [Verheij, 2003b]. This paper proposed the use of argumentation schemes, as a main tool for analysis in AI and law, stating [Verheij, 2003b, p.168] that the argumentation scheme is "a concept borrowed from the field of argumentation theory." Verheij investigated how argumentation schemes could be formalized for use in computational settings. He proposed [Verheij, 2003b, p.176] that any argumentation scheme can be expressed in the following format: Premise 1, Premise 2,. . . , Premise n, therefore Conclusion. Verheij visually represented the graph structure of an argumentation scheme by building an argument mapping software tool called ArguMed.

A formal analysis of argumentation schemes of Reed and Walton [Reed and Walton, 2005] defined a set of attributes, T, associated with propositions by a typing relation that associates every proposition to a set of attributes called a type. On this analysis a scheme is comprised of a set of tuples <SName, SConclusion, SPremises> where SName is some arbitrary token [Reed and Walton, 2005, p.179]. The gist of the analysis is that a particular scheme is given a unique name which is associated with a conclusion type and a set of premise types. An instantiation of a scheme of a type represented by a unique name must have a conclusion of the right type, and each premise must also be of the right type.

Prakken [Prakken, 2005, p.34] remarked that schemes act very much like the rules used in rule-based computer systems. The problem was that AI systems, as well as argument mapping tools of the kind used in argumentation theory, including the software systems developed in AI to assist with the building of argument diagrams, use a model of argument where the premises and conclusions are propositions. Along these lines, the structure is basically a graph with arcs joining the various points representing the propositions that can be identified as premises or conclusions. So far then it seemed that schemes were amenable to being fitted into AI systems without undue difficulty, but the central problem posed at that point was how to model the distinctive set of critical questions matching each scheme. One proposal, commented on below, is to model the critical questions as additional premises of an argument fitting a scheme.

But there was a big problem with this way of proceeding because different critical questions act in different ways in this regard. Sometimes merely asking a critical question is enough to defeat the target argument, whereas in other instances the asking of the question does not defeat the target argument unless some evidence is offered. The issue turned out to be one of burden of proof [Gordon *et al.*, 2007]. In some instances, merely asking a critical question is enough to shift the burden of proof onto the proponent who put forward the argument. In other instances, the burden of proof does not shift unless the questioner can provide some backup evidence to support the question.

Verheij [Verheij, 2003b] noted that there were variations on how the critical questions work in this regard. He noted that critical questions that point to exceptions to a general rule only undercut an argument while others could be seen refuting the argument in one of two different ways. One way is to deny an implicit assumption on which the argument depends. Another is to point to counter-arguments that can be used to attack the given argument. Verheij [Verheij, 2003b, p.180] showed that critical questions can perform four distinctively different kinds of roles:

1. They can be used to question whether a premise of a scheme holds.

2. They can point to exceptional situations in which a scheme defaults.

3. They can frame conditions for the proper use of a scheme.

4. They can indicate other arguments that might be used to attack the scheme.

It is currently widely assumed in AI that there are three ways you can attack an argument. You can attack one or more of the premises (premise attack), you can attack the conclusion (conclusion attack), or you can attack the inferential link joining the premises to the conclusion (for example by arguing that an exception applies). The last mode of attack is called undercutting [Pollock, 1995]. The first role would be that of a premise attack. The second and third roles would be undercutting attacks. The fourth role might refer to an undercutter but could also perhaps be taken to refer to a conclusion attack. So here the problem is posed of how to model critical questions given that critical questions can perform more than one function.

ASPIC+ [Prakken, 2010] is a formal argumentation system that consists of a logical language L with a binary contrariness relation that operates like negation along with two kinds of inference rules, strict and defeasible, defined over L. ASPIC+ is based on the abstract argumentation framework (Dung, 1995) which can be defined as a pair (Args, R), where Args is a set of arguments and a binary relation R on Args is called the attack relation. The underlying idea of the formalism is that each argument in a sequence of argumentation forming a directed graph structure can be defeated by other arguments so that a_2 defeats a_1, a_3 defeats $a_2, \ldots,$ and defeats a_{n-1}. Arguments in the graph can be labeled as "in" or "out". An argument is rejected (out) if it is attacked by any other argument that is in. An argument is accepted if it is not attacked by any other argument that is "in". Note that the notions of argument and argument attack are taken as primitive in an abstract argumentation system, so that such a system by itself provides no way of modeling the premises and the conclusion.

In the system developed in [Prakken *et al.*, 2015] for case-based reasoning, preferences among factors are established in the present case, and then these preferences can be applied to the current case. One of the argumentation schemes (CS1) of can be used to briefly explain how such schemes are meant to be used in legal arguments from precedent. In all these schemes, for purposes of presentation, it is assumed that the arguer is putting forward the current argument (curr) to support the side of the plaintiff.

$$\frac{\begin{array}{l}\text{commonPfactors(curr; prec)} = p,\\ \text{commonDfactors(curr; prec)} = d,\\ \text{preferred}(p; d)\end{array}}{\text{outcome(curr)} = \text{Plaintiff}}$$

According to this scheme, the current argument should be decided for the plaintiff because the common p factors were preferred to the common d factors in the precedent argument. [Prakken *et al.*, 2015] uses a running example to illustrate how an argument fitting a scheme can be attacked by other arguments in the formal system representing the argumentation in a legal case.

Argument schemes are being used in AI and Law for argument mining. Moens, Mochales Palau, Boiy and Reed devised techniques for automatically classifying arguments in legal texts by using indicators of rhetorical structure expressed by conjunctions and adverbial groupings [Moens *et al.*, 2007, p.226]. They identify words, pairs of successive words, sequences of three successive words, adverbs, verbs and modal auxiliary verbs. This work has been applied to legal argumentative texts [Mochales Palau and Moens, 2009; Mochales Palau and Moens, 2011]. By classifying types of arguments using argumentation schemes they built a system for searching for arguments in legal cases [Mochales Palau and Moens, 2008]. The project used human annotators supervised by legally trained personnel to identify arguments in texts of the European Court of Human Rights [Mochales Palau and Ieven, 2009]. Their results suggested that it would help to have additional criteria that can be applied to judge whether a given argument fits a particular scheme.

Rahwan et al. [Rahwan *et al.*, 2011] carried forward research on the automated identification of particular schemes by developing an OWL-based ontology of argumentation schemes in description logic that showed how description logic inference techniques can be used to reason about automatic argument classification. Their method of identifying schemes has been implemented in a web-based system called Avicenna [Rahwan *et al.*, 2011, pp. 11–13]. A user can search arguments by using schemes along with other tools.

Gordon and Walton [Gordon and Walton, 2006] proposed a solution to the problem of how to model critical questions by using three kinds of premises (ordinary premises, assumptions and exceptions) in the Carneades Argumentation System (CAS). This solution used information about the dialectical status of statements (undisputed, at issue, accepted or rejected) to model critical questions in such a way as to allow the burden of proof to be allocated to the proponent of the argument or the critical questioner as appropriate for the case in point. On this way of proceeding, ordinary premises need to be supported by further arguments even if they have not been questioned. In the case of exceptions, however, the critical questioner is the one who has to offer evidential support to make his criticism defeat the argument.

Version 4 is the current implemented formal and computational system of CAS, based on the formal model of argument [Gordon and Walton, 2016] called CAS2. CAS2 provides support for cumulative arguments, cyclic argument graphs, practical reasoning, and multi-criteria decision analysis. The source code of all four versions can be accessed on the Internet[6]. Carneades 4 is now online [7]. CAS2, as implemented in version 4 of Carneades, provides a formal model that uses argumentation schemes.

In the CAS2 model [Gordon and Walton, 2016] an argumentation scheme is defined as a tuple (e, v, g), where e is a function for weighing arguments which instantiate a scheme, v is a function for validating arguments, to test whether they properly instantiate an argumentation scheme, and g is a function for generating arguments by instantiating the scheme. The validation function tells us whether the argument instantiates a particular scheme, but then, once a set of schemes has been specified, the system can apply their validation functions to given argument to whether that scheme is instantiated, or not, by the given argument.

An argument is defined as a tuple (S, P, C, U), where S is the scheme instantiated by the argument; P, a finite subset of L, is the set of premises of the argument; c, a member of L, is the conclusion of the argument. U is an undercutter of the argument [Pollock, 1995]. In version 4 of CAS an issue is defined as a tuple (O, F), where O represents the options (called the alternative positions) of the issue, and F is the proof standard of the issue. Argument graphs in CAS version 4 are tripartite, rather than bipartite, as in the previous versions, with separate nodes for statements, arguments and issues. Argument diagrams in version 4 are extended with a new node type, diamonds, for representing issues. There can be any number of issues you like in a single diagram. Argument evaluation is carried out by labeling statements *in, out* or *undecided*. A statement is *in* if and only if it has been assumed to be acceptable to a rational audience, or has been derived from such assumptions via the application of the arguments, argument weighing functions and proof standards used in CAS. A statement is *out* if and only if it is neither assumed nor supported by arguments and would therefore be rejected by a rational audience. A statement is *undecided* if it is neither *in* nor *out*.

Carneades 3 uses backwards-chaining, in a goal-directed way, whereas Carneades 4 uses forwards-reasoning to derive arguments from argumentation schemes and assumptions. Both strategies, forwards and backwards reasoning, have their advantages. Forwards reasoning allows CAS to invent arguments using argumentation schemes, such as the scheme for argument from expert opinion, where the conclusion is a second-order variable ranging over propositions. Only Carneades 4 can construct arguments using formalizations of all of the twenty or so schemes currently built into the system.

Case-based reasoning (CBR) is vitally important for AI and Law and for understanding legal reasoning generally. CBR evaluates an argument in a given case

[6] Retrieved from: https://github.com/carneades

[7] Retrieved from: http://carneades.fokus.fraunhofer.de/carneades

by comparing and contrasting its features to those of prior cases that have already been evaluated [Aleven, 1997]. These prior cases are stored in a knowledge base which supplies similar precedent cases that can be pro or con the evaluation being considered in the given case. In some systems widely known in AI and law [Ashley, 1990], judgments of similarity between a pair of cases are decided by the factors that they share. Special argumentation schemes have been built to model arguments from precedent using factors in case-based reasoning [Gordon and Walton, 2009; Wyner and Bench-Capon, 2007; Wyner *et al.*, 2011]. Prakken et al. [Prakken *et al.*, 2015] offered a formal version of these legal case-based argumentation schemes using ASPIC+.

Walton, Sartor and Macagno [Walton *et al.*, 2016] showed how canons of interpretation can be translated into argumentation schemes. This project was carried out by by analyzing the most common types of statutory arguments found in legal examples and certain key forms of interpretive legal argumentation found in the work of Tarello [Tarello, 1980] and McCormick and Summers [MacCormick and Summers, 1991]. Steps were carries out to show how these legally recognizable forms of argument can be formulated as argumentation schemes. Among the schemes modeled are argument from ordinary meaning, argument from technical meaning, argument from precedent, argument from purpose, *a contrario* argument, historical argument and the non-redundancy argument. It was shown using classical examples of statutory interpretation in law how these schemes (and others) can be incorporated into computational argumentation systems such as CAS and APSIC+ and applied to displaying the pro-contra structure argumentation in legal cases using argument mapping tools.

In the following sections we will illustrate shortly two other computational applications of argumentation schemes, namely their role in argument mining and formal ontologies.

9 Using Schemes for Argument Mining

Argumentation schemes also have an important role to play in a major new area of computational research into argumentation: argument mining. Argument mining focuses on the development of algorithms and techniques for the automatic extraction of argument structure from natural language text. Though it has connections to areas such as sentiment analysis and opinion mining, it represents a substantially more demanding task. There are two features that make argument mining so difficult. The first concerns the availability of data and the second, the limits of statistical approaches to language understanding.

Many approaches to mining syntactic and semantic structure from unrestricted natural language have, since the late 1990s, been based heavily in statistical analysis: essentially, modelling the regularities in language by examining and comparing many, many different examples. The most robust syntactic parsers, for example, are based not on theoretical linguistic analysis — which proved on the whole to be too limited and too brittle — but on statistical models based on corpora typically comprising millions of examples [Koehn *et al.*, 2003]. Though the machine learning mechanisms upon which such techniques depend vary, one feature that they share is the need for such large datasets from which to draw regularities. If, therefore, argument mining is to be able to deploy the same techniques, it requires large datasets, and datasets not just of argumentation *per se*, by argumentation that has been analysed for its structure. As anyone involved in the teaching of critical thinking skills will attest, such analysis of argument structure is both demanding and extremely time consuming. Until very recently there were few datasets, and those that did exist were available in idiosyncratic representation languages, with little re-use between research teams and projects — so what effort was invested in data collection and analysis was regularly lost. Two approaches have started to change this.

First, there have been attempts to collect datasets specifically for community use. The first example is the Internet Argument Corpus, IAC [Walker *et al.*, 2012], which collects 390,000 examples. The problem facing the IAC is that it is designed primarily from a text-processing viewpoint, with little argumentation theory sitting behind it. As a result, the conception of argument that it embodies is very thin and more or less unrecognisable to researchers from argumentation theory and computational models of argument, viz., quote-response pairs with associated polarity (additional features including sarcasm and nastiness are marked for subsets). A second example is more directly rooted in informed models of argumentation. The Potsdam Microtext Corpus [Peldszus and Stede, 2016] provides artificially constrained — but completely human-generated, natural language — arguments that are structured according to the work of Freeman [Freeman, 1991] with explicit distinction between, for example, linked and convergent arguments, undercutting and rebutting attacks and so on. Another unique advantage of the Microtext Corpus is that it has been professionally translated so that both English and German versions exist: to our knowledge this is the first parallel corpus of argumentation. On the other hand, the fact that every argument is required to contain a total of five components (premises and conclusions), whilst providing a vitally useful “laboratory” for testing techniques, risks placing a severe limitation on the subsequent generalizability of those techniques to unrestricted arguments in the wild. The limited size of the corpus — just 130 examples — also places limitations on what can be accomplished using traditional statistical machine learning techniques.

The second approach has been to provide infrastructure specifically for collecting, publishing, sharing and re-using corpora. Whilst there are now several platforms for online analysis of argument (argunet[8], debategraph[9], AGORA-net[10], Rationale-Online[11], etc.) none provide open access to the data in machine processable ways, except, as far as we are aware, for the infrastructure offered by the Argument Web [Rahwan *et al.*, 2011; Bex *et al.*, 2013]. The Argument Web is a vision for an interconnected web of arguments and debates, regardless of the software used to create them, analyse them or extract them, and regardless, too, of the uses — academic, social or commercial — to which they might be put. The vision supports, for example, the academic analysis of an argument presented in a political broadcast; the automated analysis of responses to it on social media; the deployment of automated dialogue games for online users to interact with both original and responses; the automated summary of the status of the debate to a government policy department; and the delivery of a corpus comprising the debate to researchers in argument mining. Argumentation schemes in the style of [Walton *et al.*, 2008] form a cornerstone of the Argument Web, as a way of providing a rich ontology of reasoning forms. Further details of this ontology occur in the next section; here we focus on the tools and the ways in which they can be used to develop corpora.

Though the first publicly available corpus of argumentation was developed using Araucaria (viz. AraucariaDB, see [Reed and Walton, 2005]), the software itself is now very old and virtually obsolete. Though it remains the only software to handle large analyses, such as the ones developed by Wigmore for mapping cases, and the only to interchange between Wigmore, Toulmin and Freeman styles of analysis, it has been superseded in its core functionality by the Online Visualisation of Argument tool, OVA [Janier *et al.*, 2014]. OVA provides a simple-to-use interface for analysing existing argumentation in both monologue and, in the extended OVA+, also dialogue. It supports enthymeme reconstruction; argumentation scheme analysis; critical question processing; serial, linked, convergent and divergent structures; undercutting, rebutting and undermining attacks; and in OVA+, locution analysis; dialogue game rule analysis; illocutionary force identification; the role of *ethos* [Duthie *et al.*, 2016] and personal attacks; and ultimately, full Inference Anchoring Theory analysis [Budzysnka and Reed, 2011]. Analyses from OVA can be stored in AIFdb, a database infrastructure fabric for storing and accessing argument data [Lawrence *et al.*, 2012].

[8]Retrieved from: http://www.argunet.org/ (Accessed on 10 May 2016)

[9]Retrieved from: http://debategraph.org/Stream.aspx?nid=61932&vt=ngraph& dc=focus (Accessed on 10 May 2016)

[10]http://agora.gatech.edu/ (Accessed on 10 May 2016)

[11]https://www.rationaleonline.com/ (Accessed on 10 May 2016)

One side effect of using AIFdb is that the data is easily transportable to other forms, both representational (in being able to convert to formats required for Carneades [Walton and Gordon, 2012] and Rationale [van Gelder, 2007], for example), and processable — in being able to convert via ASPIC+ [Modgil and Prakken, 2013] to abstract frameworks [Dung, 1995] via formal equivalences established in [Bex *et al.*, 2013]. More importantly for our current purposes, sets of analyses in AIFdb can be configured to constitute a corpus using the AIFdb corpus management tools [Lawrence and Reed, 2015] available online at corpora.aifdb.org, and AIFdb current constitutes the largest publicly available dataset of analysed argumentation. These tools enable research teams to define corpora comprising both analysed argumentation and raw text; both argumentative and non-argumentative source material; both raw data and metadata. Corpora themselves are aggregable providing flexible structuring options to manage dependencies between teams, projects, and objectives. The original AraucariaDB corpus is available on this infrastructure, but so too are smaller datasets focusing specifically on argumentation schemes, such as the Argument Schemes in the Moral Maze, comprising excerpts from the BBC Moral Maze radio programme that involve 35 instances of argumentation schemes and the ExpertOpinion-PositiveConsequences corpus comprising 71 examples of just these two schemes.

With the availability of appropriate datasets becoming less of an impediment, various approaches to automatically recognising argument structure have been developed. The majority have been focused specifically on statistical models, which brings us to the second major challenge facing argument mining: the limits of such models. Whilst it is certainly the case that statistical approaches are starting to deliver results for argument mining, and will undoubtedly continue to do so, it is also the case that the more sophisticated conceptions of argument developed in argumentation theory remain extraordinarily demanding. The reason for this lies precisely in their sophistication. With so many patterns of argumentation, so many structures, so many ways in which components can be left implicit, so many types of reasoning, the amount of data required to train statistical models becomes not just unwieldy but unreasonable and, quite probably, unattainable.

Consider a comparison with syntactic analysis, where statistical models have been so successful. The number of rules governing how different parts of speech can be legally combined run in theoretical linguistics to tens of examples. In statistical models, it is hundreds (which is why they are so successful). The number of rules governing how argument components can be assembled (and left implicit) runs, by combination across argumentation schemes, to thousands or more. So whilst we might expect statistically oriented techniques to deliver us good results on simple and strongly generalizable aspects of argument recognition, for the type of analysis that is typically taught to students of critical thinking classes, more is required. It is looking increasingly likely that having strong, well defined conceptions of argument, dialogue and argument schemes provide exactly the sort of additional information required to guide machine learning processes by acting, in essence, as priors to that process: defining expectations about what is likely to be seen. This combination of statistical and structural approaches is looking very promising. In particular, we provide examples here that tap in specifically to structure provided by argument schemes.

Feng and Hirst [Wei Feng and Hirst, 2011] aimed to classify arguments into the type of scheme employed. Like some of the earliest work in argument mining, such as [Moens and Mochales Palau, 2007], they also used the AraucariaDB corpus as a starting point, because it was the only dataset at that time with annotated examples of argumentation schemes. They used the 65 argumentation schemes from [Walton *et al.*, 2008], but emphasized the importance of the five schemes they found to be the most commonly used ones in their corpus: argument from example, argument from cause to effect, practical reasoning, argument from consequences and argument from verbal classification [Wei Feng and Hirst, 2011]. The number of occurrences of these most common five schemes constituted 61% of the kinds of arguments identified in their database [Wei Feng and Hirst, 2011, p.998]. They used a variety of features with which to train the machine learning classifiers including key words and phrases as textual indicators of argumentation schemes. They identified, for example, twenty-eight keywords and phrases associated with the scheme for practical reasoning, including “want”, “aim”, “objective”, and modal verbs like “should”, “must” and “need” [Wei Feng and Hirst, 2011, p.991]. Their results were extremely promising, providing classification accuracies ranging from 0.64 to 0.98.

Building on this approach, Lawrence and Reed ([Lawrence and Reed, 2015] extended the model to use argumentation schemes not just as a target for machine learning but to aid the very process of identifying argumentative structure (rather than presupposing it as input, as in Feng and Hirst). The intuition is that argumentation schemes do not connect propositions that are all alike, but rather are associated with particular types of propositions. In this way, arguments from positive consequence will typically conclude with a normative statement in the subjunctive mood; arguments from expert opinion will typically have a premise which reports, either directly or indirectly, the speech of another; arguments from analogy will include a premise which attributes some property to some individual; and so on. If it is possible to identify instances of some of these types, it will constrain the potential argument structures that can be reassembled. If, for example, an automatic algorithm can spot the lexeme said, there is a reasonable chance that we have reported speech, which in turn increases the chance that it is part of an expert opinion argument. If we can find the lexeme expert in a sentence close by, we can be even more sure we have argument from expert opinion and can start looking nearby for a conclusion — and that conclusion is likely to be a sentence which has strong semantic similarity with the clause that follows "said". In this way, knowing *a priori* about argumentation scheme structure helps to constrain the problem of automatically recognising the argument structure. It turns out that this hypothesis is borne out by results.

Lawrence and Reed report (ibid.) results ranging from an F1 performance of 0.59 to 0.91 for detecting scheme components and of 0.62 to 0.88 for identifying scheme instances. Operationalising argumentation scheme structure in this way depends, however, upon "knowledge engineering", or, more specifically, "ontology engineering" — the construction of explicit computational models that capture scheme structure and the commonalities, similarities and classificatory relationships between schemes. It is to this question that we turn next.

10 Schemes in Formal Ontologies

The Argument Interchange Format, AIF, is not just a representation language for argument structure; it also has a formal definition rooted in description logic; that is to say, it provides a core ontology for describing argument (though that core is rather compact, it admits of extension using "adjunct ontologies" that extend it to handle features such as dialogical interaction, user- and social-oriented features, and so on). The AIF was laid out initially in [Chesñevar *et al.*, 2006] and extended in its description logic specification in [Rahwan *et al.*, 2007]. Given this basis, it is then rather straightforward to extend it to further specify not just that two propositions might be linked by an application of a rule of inference (or "RA"), but to also specify the different types of such rules of inference, that is to define an ontology of argumentation schemes. This ontology not only describes the structure of argumentation schemes in machine-processable form, but also defines relationships between schemes (such as generalisation-specification relationships) and relationships between scheme components (such as that knowledge assertions occur as premises in several different schemes). By way of example, snippets of the ontology concerned with the argumentation scheme from expert opinion are shown in Figure 11 below.

```
<Class IRI="#ExpertOpinion_Inference"/>
    <ObjectIntersectionOf>
      <Class IRI="#Presumptive_Inference"/>
      <ObjectSomeValuesFrom>
        <ObjectProperty IRI="#hasConclusion"/>
        <Class IRI="#KnowledgePosition_Statement"/>
      </ObjectSomeValuesFrom>
      <ObjectSomeValuesFrom>
        <ObjectProperty IRI="#hasFieldExpertise_Premise"/>
        <Class IRI="#FieldExpertise_Statement"/>
      </ObjectSomeValuesFrom>
      <ObjectSomeValuesFrom>
        <ObjectProperty IRI="#hasKnowledgeAssertion_Premise"/>
        <Class IRI="#KnowledgeAssertion_Statement"/>
      </ObjectSomeValuesFrom>
    </ObjectIntersectionOf>

<Class IRI="#ExpertOpinion_Inference"/>
    <ObjectIntersectionOf>
      <ObjectSomeValuesFrom>
        <ObjectProperty IRI="#hasCredibilityOfSource_Presumption"/>
        <Class IRI="#CredibilityOfSource_Statement"/>
      </ObjectSomeValuesFrom>
      <ObjectSomeValuesFrom>
        <ObjectProperty IRI="#hasExpertiseBackUpEvidence_Presumption"/>
        <Class IRI="#ExpertiseBackUpEvidence_Statement"/>
      </ObjectSomeValuesFrom>
      <ObjectSomeValuesFrom>
        <ObjectProperty IRI="#hasExpertiseInconsistency_Exception"/>
        <Class IRI="#ExpertiseInconsistency_Statement"/>
      </ObjectSomeValuesFrom>
      <ObjectSomeValuesFrom>
        <ObjectProperty IRI="#hasLackOfReliability_Exception"/>
        <Class IRI="#LackOfReliability_Statement"/>
      </ObjectSomeValuesFrom>
    </ObjectIntersectionOf>

<Class IRI="#ExpertiseInconsistency_Conflict"/>
    <ObjectIntersectionOf>
      <Class IRI="#Exception_Conflict"/>
      <ObjectExactCardinality cardinality="1">
        <ObjectProperty IRI="#hasConflictedElement"/>
        <Class IRI="#ExpertOpinion_Inference"/>
      </ObjectExactCardinality>
      <ObjectExactCardinality cardinality="1">
        <ObjectProperty IRI="#hasExpertiseInconsistency_Exception"/>
        <Class IRI="#ExpertiseInconsistency_Statement"/>
      </ObjectExactCardinality>
                          </ObjectIntersectionOf>
```

Figure 11: Snippet of Argumentation Scheme Ontology

In the first stanza, the conclusion and premises concerning the knowledge assertion (that the expert said something) and the field expertise (that the speaker is indeed an expert) are set up. In the second stanza, the remaining premises (those captured as presumptions and exceptions) are added in, covering credibility, backup evidence, consistency between experts and expert reliability. The third stanza shows how one of these, consistency between experts, can be used to drive a stereotypical way of attacking this inference — i.e. the posing of a critical question (see [Reed and Walton, 2005] for the mechanics of operationalizing critical questions in this way). The aim here is just to give a flavour of how all the important components of argumentation schemes — structure, description and critical questions — can be captured in a formal ontology. The full ontology is available online at http://arg.tech/aif.owl, and is used by many of the Argument Web online services.

Two benefits of this approach are demonstrated in [Rahwan *et al.*, 2011]. The first is an economy in specification, that allows more specific schemes to be defined in terms of minor additions to more general ones. The second, much more importantly, is that these structures support automated reasoning, in three distinct ways. First, it becomes possible to reason across argument structures, identifying, for example, transitivity of inferences, so that if X is used to infer Y, and Y to infer Z, the dependence of X on Z can be inferred automatically. Of course such reasoning is not at all unique to ontologically based systems, but is a convenient side benefit. An ontologically more interesting way of performing automated reasoning is to perform automatic classification. This is where formal ontologies, and the reasoning systems constructed on top of them, excel. Rahwan et al., exemplify this technique by showing how fear appeal arguments are naturally classifiable as a subset of negative consequence arguments. The third and final way of performing automated reasoning is also of use in designing and implementing dialogue systems. By virtue of hierarchical relationships between schemes that are represented in, or inferable from, the ontology, it also becomes possible to infer appropriate critical questions that might be asked of a given argument. Thus, for example, all of the critical questions of a superclass can be asked of an instance of a sub-class of argumentation schemes. In these ways, formal representation of argumentation schemes in an explicit ontology can contribute to the computational techniques for analysing, processing and interacting with arguments.

11 Conclusions

Argumentation schemes represent the abstract structures of the most common and stereotypical arguments used in everyday conversation and specific fields, such as law, science and politics. They appear as a set of premises having an abstract form with variables and constants, leading to an abstract conclusion. They are abstract in the sense that they provide a form for structuring inferential relation between the premises and the conclusion. Some schemes are based on the most abstract relations (classification, cause, authority), while others specify the most abstract premises including some further detail (negative consequences; expert opinion *ad populum* argument) [Walton *et al.*, 2008].

The abstract nature of the schemes allows the analyst to detect the structure of natural arguments, and recognize patterns occurring in everyday reasoning. This paper has shown how they can be applied to real arguments in natural language discourse, and in technical discourse as well, for that matter. In this paper, it has been shown how these schemes, at their current state of development can be used as tools to identify kinds of arguments in a text, and beyond that how they can be an important part of argument evaluation. Throughout the history of logic and rhetoric there has always been some uncertainty about the role of the topics [Bird, 1962]. Some have seen them as forms of logical inference that can be used to show that arguments are valid, where the term "valid" is used in a wider sense that can include not only deductively valid arguments but also defeasible arguments that have an identifiable structure as fitting a particular topic. Others have seen the topics of as having a search function that can be used to find arguments to prove a designated conclusion. The search function is supposed to help an arguer select arguments that have premises accepted by the audience to whom the argument is directed [Kienpointner and Kindt, 1997].

Schemes can also be used for argument construction. As we saw in this paper, an argumentation scheme is taken to have a warranting function that enables an inference to be drawn from a set of premises to a conclusion. This practical way of justifying schemes indicates their usefulness not only for argument evaluation, but also for argument construction, also called argument invention in the long history of the subject tracing back to the Sophists and Aristotle. An argument invention device would enable an arguer to search for an argument that could be used to support a claim s/he wants to prove [Kienpointner, 1987]. When viewed in this way, topics can be seen to have a use as components of an argument construction function, for use in a system for finding arguments. The schemes can be used as instruments for producing arguments, allowing the user to decide the type of argument he considers the most applicable to his purpose, and then develop a specific line of reasoning from the premises or evidential facts he has to the conclusion he needs to prove. In this guise, the schemes are dialectical instruments for use in the task of argument construction.

The advent of IBM's new Watson Debater tool [Aharoni *et al.*, 2014] is a leap forward for argument invention because it enables a user to quickly search through a database such as Wikpedia and find useful pro and con arguments supporting or attacking a designated claim. Once this tool comes onto the market, it will greatly stimulate research on argument invention in argumentation studies. The Debater tool does not (so far) use argumentation schemes, but there is a formal and computational argumentation system, the Carneades Argumentation System (CAS)[12]. By inputting information into the CAS find arguments assistant, a user who has a

[12]https://carneades.github.io/ (Accessed on 10 May 2016)

database containing propositions recording the commitments of the audience, the automated assistant constructs a chain of argumentation where the conclusion of the chain is the proposition is the goal proposition that the speaker wants to persuade the audience to accept, called the arguer's ultimate claim, or ultimate *probandum*, the proposition to be proved, in the language of the ancient *stasis* theory [Walton and Gordon, 2012]. The argument assistant searches through the commitments of the audience and uses a repository of argumentation schemes in its knowledge base to collect a set of arguments moving from these premises to the ultimate claim. If there are such arguments available the assistant gives that information, but may suggest a partial way forward.

Argumentation schemes are instruments that can be used in different ways to many disciplines addressing the analysis of discourse in general, and reasoned discourse in particular. The current research on schemes can improve noticeably the field of application and make this tool crucial for a deeper analysis of argumentative exchanges. To this purpose, argumentation schemes need first to be integrated within a theory of discourse interpretation. Schemes can be powerful instruments for representing arguments and relations between sentences. However, at present they presuppose an interpretation of discourse. This line of research could show how schemes can represent interpretation, and how they can be used to assess what interpretation is the best one [Macagno, 2012]. A second challenge in this area is to link the theory of dialogue types and discourse moves (utterances) to argumentation schemes [Macagno and Bigi, 2017]. By showing how certain schemes are the most adequate to pursue specific dialogical ends, it is possible to map not only a set of useful tools for argument production, but also a set of presumptions for interpreting and classifying arguments based on the type of dialogue.

Acknowledgments

Fabrizio Macagno would like to thank the FundaÃğÃčo para a CiÃłncia e a Tecnologia for the research grants no. IF/00945/2013, PTDC/IVC-HFC/1817/2014, and PTDC/MHC-FIL/0521/2014.

Douglas Walton would like to thank the Social Sciences and Humanities Research Council of Canada for Insight Grant 435-2012-0104.

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Received 2 February 2017

Computational Problems in Formal Argumentation and their Complexity

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Abstract

In this paper we give an overview of the core computational problems arising in formal argumentation together with a complexity analysis highlighting different sources of computational complexity. To this end we consider three prominent argumentation formalisms from the literature, that are Dung's abstract argumentation frameworks, assumption-based argumentation, and abstract dialectical frameworks, each of which allows to highlight different sources of computational complexity in formal argumentation. As most of these problems turn out to be of high complexity we also consider properties of instances, like being in a specific graph class, that reduce the complexity and thus allow for more efficient algorithms. Finally, we also show how to apply techniques from parametrized complexity that allow for a more fine-grained complexity classification.

1 Introduction

In the literature several models for formal argumentation were introduced and discussed. They propose different ways to construct arguments, draw conclusions and each of these models comes with several proposals for semantics as to how coherent sets of arguments or statements should be selected. In this paper we address the computational issues appearing in argumentation formalisms and that have to be tackled when implementing argumentation systems. That is, we will identify core problems of abstract argumentation and present basic procedures to solve them together with hardness results, based on computational complexity theory, that show some problems to have inherent complexity that cannot be circumvented by any algorithm.

The computational problems of formal argumentation occur at several places in the argumentation process. (A1) First, when instantiating argumentation frameworks from knowledge bases one has to deal with the task of constructing arguments and identifying conflicts (or even more complex) relations between arguments. (A2) Second, given the arguments and conflicts between them one has to find coherent sets of arguments that can be simultaneously accepted (w.r.t. a selected semantics). (A3) Finally, given the coherent sets of arguments one has to draw conclusions based on this selection. The computations in the first and third item often correspond to problems that are purely located in the underlying logic, e.g., to evaluating an inference operator. The second item is at the core of formal argumentation. That is, we are given arguments and relations (e.g. attacks) between them, and have to evaluate them w.r.t. to an argumentation semantics. This may require computing all extensions of a given semantics, the acceptance status of some argument w.r.t. some semantics, or finding some witness or counter example for a claim.

In this work we consider computational problems in three argumentation formalisms, i.e. Dung's abstract argument frameworks [Dung, 1995; Baroni *et al.*, 2011a], assumption-based argumentation [Toni, 2014] and abstract dialectical frameworks [Brewka *et al.*, 2013]. Dung's abstract argument frameworks are a model for (A2) which only consists of abstract entities called arguments and a binary attack relation between them. There is no instantiation process or computation of conclusions. Thus, it is perfectly suited for studying the computational issues involved in (A2). Assumption-based argumentation models the whole process (A1), (A2) and (A3), starting from a knowledge base, constructing arguments and conflicts, and finally returning conclusions. By comparing assumption-based argumentation with the results for Dung's abstract argumentation we are able to highlight the computational costs for (A1) and (A3). Finally, we consider abstract dialectical frameworks (ADFs) which are a richer model for (A2). As for Dung's abstract argument frameworks, here only abstract entities are considered but instead of just a binary attack relation ADFs allow for more complex relations between these entities. On the basis of ADFs we will highlight the impact of the allowed relations between arguments on the computational complexity of the reasoning tasks.

Notice by the term "*computational problem*" we mean the task of when presented with a *description* of some *input*, e.g., the vertices and edges in a graph, a collection of numeric values, producing an output related in a specified way to this input. For example: reporting the set of vertices forming the end-points of at least two edges, returning the collection of numerical values *sorted* in increasing order. One special type of computational problem is of particular interest: the class of so-called *decision* problems. These concern determining whether the given input structure has a particular property of interest, e.g. given a graph as before does it contain a cycle?, given a list of numbers, does the largest exceed 100?

The formal study of computational problems has two principal foci:

A. The construction of ("efficient") *algorithms* to *solve* the problem. That is methods which when presented with an input instance *always* report the *correct* output.

B. To categorise collections of computational problems that are "similar" in terms of their "best" algorithms, and thence provide a formal proof that *every* algorithmic approach must take some number of steps.

Thus (A) is concerned with positive constructive demonstration of an *upper* bound on a problem's *computational complexity* while (B) is a (more negative) statement prescribing *lower* bounds on computational complexity.

Why are these focal points of importance? To gain some insight to this consider the well-known computational problem of *sorting*: given a collection of N numbers $< a_1, a_2, \ldots, a_n >$ return this collection in increasing order of its members. Here are three informally presented "sorting algorithms":

S1 Generate each possible ordering, π, of $< a_1, a_2, \ldots, a_N >$ in turn: return the first ordering found that is correct.

S2 Form a new ordering by comparing for each $i > 1$ the (current) a_{i-1} and a_i: if $a_i > a_{i-1}$ exchange the pair. Repeat with the new ordering produced until the collection is sorted.

S3 If $N = 1$ the list is already sorted. Otherwise (recursively) sort the two list $< a_1, \ldots, a_{N/2} >$ and $< a_{N/2+1}, \ldots, a_N >$ and "merge" the two sorted lists to give the final output.

On the surface, in the sense that all three methods are correct there appears to be little to choose between these three methods. If, however, we examine their performance a very different picture emerges.

Method (S1) in the worst case (no matter how the successive ordering are produced) requires $N!$ steps: if $N = 100$ this is roughly 10^{200}

Method (S2) in the worst case needs N^2 steps: for $N = 100$ this is 10^4.

Method (S3) takes of the order of $N \log_2 N$ steps: with $N = 100$ this is about $10^{2.5}$.

Now (S1) is unusable as a *realistic* algorithm: even with a high-performance computer implementation capable of executing 10^{12} operations per second, in the worst case (S1) will require 10^{150} *years.* On a much slower machine (say 100 operations per second) even a “naive” implementation of (S2) will have finished in about 2 *minutes* and (S3) in just over 1 second.

Although a quite extreme case is being considered, this overview of one particular range of algorithmic methods for a computational problem does highlight two significant issues:

H1. The efficiency of an algorithm is a crucial factor in determining its practical usability: if (S1) were the *only known* sorting method, tasks such as organising records in a database would not be possible.

H2. Developments in technology – the platforms on which algorithms are realised – have minimal impact: a reasonable algorithm (S2 or S3) even running on an antiquated very slow machine (100 ops/sec) will easily outperform a very inefficient approach (such as S1) even if this is run on a machine with significant computational power (10^{12} ops/sec)

The study of algorithms for computational problems in argumentation has made notable advances over the last twenty years. There is, however, a significant issue that besets many of its computational concerns: that within the technical classifications of problem difficulty presented in the field of *computational complexity theory* there is powerful evidence that the prospects for identifying efficient solution methods are extremely limited: that is to say, in terms of the sorting method example given, the status of best known worst-case methods for important computational problems in argumentation is more likely to be characterised by (S1) than (S2) or (S3).

Our intention in this work is to present a survey of computational complexity results that have been obtained within formal argumentation.

Prior to embarking on this overview, in order to provide some necessarily technical background, we give an very informal basic introduction to the ideas and techniques used in this study.

From a practical point, complexity classification is in particular crucial when one considers implementing argumentation reasoning tasks by a *reduction approach.* That is, instead of designing and implementing complex algorithms and systems from scratch, one might reduce the new reasoning tasks to related formalisms where sophisticated solvers already exist. For instance, for a broad range of argumentation semantics one can reduce the task of computing a set of coherent arguments of an argumentation framework to computing a model of a propositional formula that can be efficiently constructed from the argumentation framework [Besnard and Doutre, 2004]. Now one can exploit the sophisticated systems to deal with propositional formulae to get an efficient system for the encoded argumentation semantics with relatively small effort. In the reduction approach the complexity of the actual problem and the corresponding problem in the target formalism are crucial for the following reasons: given that an actual problem has higher complexity than the designated target problem we know that there is no efficient encoding of our problem and we might consider a different target formalism. On the other hand if the target problem is of higher complexity we may end up with unnecessarily high computational costs. In such a case it might be a good idea to encode the problem within a restriction of the target formalism, providing lower complexity.

The remainder of the paper is organised as follows. In Section 2 we give a brief introduction to computational complexity. That is we introduce the techniques and complexity classes we will use in the later parts of the paper. In Section 3 we consider Dung's abstract argumentation frameworks and the main computational problems thereof. In Section 4 we consider computational problems in assumption-based argumentation. In Section 5 we consider computational problems in abstract dialectical frameworks. Finally, in Section 6 we summarise and discuss the presented results as well as related results not covered by this paper.

2 A brief Introduction to Computational Complexity Theory

In very informal terms, computational complexity theory is the field of computer science concerned with grouping computational problems (in the sense we introduced above) into so-called "complexity classes". Such classes are captured by different resource requirements, typically measured by quantities such as Time (number of steps taken by an algorithm) or Space (amount of "memory" needed). Thus a *complexity class* $\mathcal{C}$ is a *set* of computational problems, and when we say that "problem P is in the complexity class $\mathcal{C}$" (or P has complexity $\mathcal{C}$) this indicates that there *exists* an algorithm that solves P and meets the resource criteria prescribed by $\mathcal{C}$. For example, as illustrated by the methods discussed in the introduction, the computational problem of "sorting n numbers" is in the (function) complexity class of problems solvable in time $n \log n$ (evidenced by method S3).

Now already this basic description raises many issues, among which we have:

a. How do we avoid proliferating "complexity classes" because of different technological capabilities, i.e., having to formulate a "complexity theory for Apple Mac machines", another for IBM hardware, and yet another for Windows O/S, etc. etc.?

b. How do we formalise notions of "input size" and relate such to the computational complexity of a problem?

c. How do we, in a precise sense, group distinct computational problems into collections of similar behaviour?

Before developing these questions further, we observe that it is convenient to focus on *decision problems*. That is to say, problems that separate input *instances* into two disjoint sets:

- **Positive** instances x of problem P: those on which P reports the answer **true** (equivalently, 1 or **yes**).
- **Negative** instances x of problem P: those on which P reports the answer **false** (equivalently, 0 or **no**).

In order to abstract away from the trivialities of platform specifics, algorithms are considered as realised on some standard "*model of computation*". While a huge number of such models have appeared in the technical literature[1] those adopted in computational complexity, ultimately, derive from *Turing machine (*TM*) programs*. The exact specification of these is unimportant for the purposes of this overview. The interested reader is referred to any standard textbook for further details (e.g., [Papadimitriou, 1994; Arora and Barak, 2009]).

By fixing a standard basis for specifying algorithms (that is, TM programs) we obtain methods for addressing questions (b) and (c). At the most rarefied abstract level of TM operation, "input size" is simply the total number of *characters* (symbols) appearing in the input data. Usually (although not invariably) this will take the form of a sequence of *binary* "digits". The important feature is that the input sequence uses only characters from a *fixed finite* set or *alphabet* no matter whether these characters are digits, letters, or any other type of characters. [2]

While "length of the input string" offers a common basis for comparison, it can be somewhat cumbersome for practical analysis. Fortunately (and certainly in the case of abstract argumentation problems which are our principal interest) there is, usually, some supporting structure to a problem instance which can serve as a size parameter. For example, returning to the example of "sorting", instead of considering the total number of *bits* to represent instances (which could be $n \log_2 k$ when

[1] In one form or another the abstraction "model of computation" can be traced back almost a hundred years: its first appearance being with respect to capturing the notion of "computational problems that *can* be solved".

[2] In complexity matters, if such a set contains at least 2 distinct symbols, it makes little difference whether the alphabet has 2 or 1000 or more symbols. In contrast, however, *unary* (single symbol) encodings may lead to notably different algorithmic behaviour.

non-negative integers of value $< 2^k$ are involved), since most sorting methods work at the level of numeric comparisons (as opposed to individual bit-level manipulation), the size of an instance can reasonably be viewed as the number of values (N) to be sorted. In the consideration of decision problems arising in Dung's formalism a typical instance will specify an argumentation framework, that is to say a *directed graph*, (A, R) and a subset S of arguments: hence the "obvious" input size parameter is simply "the number of arguments in A". Notice that, total input size is bounded polynomially in n, as the size of each part of input, i.e., of A, R and S, is bounded polynomially in n.

We can now deal with the second part of (b): relating such notions of "size" to problem complexity, in particular precise interpretations of "problem P has lower (time) complexity than problem Q". Notice that such statements combine *two separate* claims:

C1. That there *exists* an algorithm A_P solving P that runs in time $T_P(n)$ on instances of size n.

C2. That *every* algorithm A_Q solving Q takes times at least $T_Q(n)$ on instances of size n and $T_Q(n)$ is "larger" than $T_P(n)$.[3]

Let us focus now on problems concerning AFs in which the dominant input component is a directed graph, (A, R). Considering the character of the algorithm, A_P, associated with this there is an infinite sequence,

$$\{< A_P, R_P >_{(1)}, < A_P, R_P >_{(2)}, \ldots, < A_P, R_P >_{(k)}, \ldots\}$$

for which $< A_P, R_P >_k$ is an AF having exactly k arguments. In addition, the number of steps (run-time) of A_P on the instance $< A_P, R_P >_k$ is not exceeded by any other instance (A, R) in which A has exactly k arguments. Such an instance, $< A_P, R_P >_k$ is called a "*worst-case* input for A_P". In this way the run-time function, T_P, is just the

$$T_P(n) =_{\text{def}} \text{The number of steps } A_P \text{ takes when given the input } < A_P, R_P >_{(n)}$$

Now, unless we are dealing with a highly artificial and contrived problem, P, one will typically have $T_P(n+1) > T_P(n)$.[4] This clarifies the precise meaning of (C1), and

[3]Typically, one is interested in the asymptotic behaviour with growing input size n, i.e., whether there is an there exists n_0 such that $T_Q(n) >= T_P(n)$ for every $n >= n_0$.

[4]We are, of course, ignoring minor issues whereby P requires (A, R) to have a particular structure rendering frameworks with some numbers of arguments unsuitable, e.g., problems in which A must have an even number of arguments and are ill-defined when the size of A is an odd number.

by a similar analysis we can associate run-time functions, T_Q with every algorithm solving Q. The statement "P has smaller complexity than Q" is thus a *positive* (upper bound) claim about algorithms for problem P and a *negative* lower bound claim about all algorithms for Q: as the number of arguments in A increases we will see a growing disparity between the worst-case time that P requires to deliver an answer (using A_P) compared to the worst-case time that *any algorithm*, A_Q takes to deliver its answer.

Much of the focus of computational complexity theory is in grouping problems into classes where this disparity is at its most extreme: these extremes and the techniques for placing problems at either end of the spectrum of difficulty are the subject of the next subsection.

2.1 Basic Complexity Classes

Here we briefly review the *complexity classes* used in this work and their relations. As discussed above the high-level idea of complexity theory is to group problems with similar resource requirements in complexity classes and also put these classes into an order so that we can distinguish between "easier" and "harder" problems.

2.1.1 Polynomial-Time

By convention, a problem is viewed as having an efficient algorithmic solution if it can be placed into the class P (*polynomial-time*) of all problems that have a polynomial-time algorithm, i.e., an algorithm that for each instance x (of size $|x|$) produces its answer after at most $|x|^k$ steps, for a fixed constant k. It is noted that this a rather coarse-grained classification: problems whose fastest algorithm runs in time n^{100} are considered to be "efficiently solvable". This may seem rather arbitrary, however, there is a very noticeable performance difference between methods whose run-time is bounded by n^k and those that cannot be so bounded.

An important subclass we will consider is L (*logarithmic space*), which consists of the problems that can be solved in logarithmic space (not counting input and output) and polynomial-time. Just as P is seen as the class of computational problems with efficient "sequential" algorithms, so L is the class having efficient "parallel" algorithms (see, e.g., [Greenlaw *et al.*, 1995]).

We consider problems in the classes L, P to be computationally tractable, while we will consider problems in all the other classes to be intractable or computationally hard.

2.1.2 The classes NP, coNP and DP

Often a decision question can be solved by finding a witness for the instance satisfying the questioned property. For instance if we ask whether an AF has a stable extension, a way to answer that positively would be to actually compute a stable extension as witness.

Taking this view, we may associate with any instance x of a decision problem Q, a set $W(x)$ of potential *witnesses* that x has the property of interest. For example, for admissibility semantics if we are interested in whether a specified argument p is credulously accepted with respect to admissibility the instances have the form $((A,R),p)$ (with $p \in A$) and potential witnesses are all subsets of $A \setminus \{p\}$. A witness S in this set is valid for the instance if and only if the set $S \cup \{p\}$ is admissible.

The class NP. The complexity class NP (*non-deterministic polynomial-time*) can be characterised by such witnesses. A decision problem is in the class NP if (i) for each instance x there is a set $W(x)$ of potential witnesses, which are of polynomial size in $|x|$, such that (ii) one can verify that a $y \in W(x)$ is actually a witness for x in polynomial time and (iii) x is a "yes" instance if and only if at least one $y \in W(x)$ is a witness for x.

In the above example for an AF F the potential witnesses $W(F)$ would be all the subsets of arguments. Verifying whether a set is admissible is in polynomial time and F is a positive instance iff [5] at least one of these sets is a stable extension.

Formally the specification of a decision problem in terms of witness sets can be seen in the following way. Let x be an instance of a (decision) problem Q we write, $Q(x) = 1$ if x is a "yes" instance of Q, and $Q(x) = 2$ if x is a "no" instance of Q. We have a *binary* relation $W_Q(x,y)$ for which $< x,y > \in W_Q$ iff y is a valid witness that x is a positive instance of the (decision) problem Q. This yields,

$$Q(x) = 1 \quad \Leftrightarrow \quad \exists\ y \in W(x) : \ < x,y > \in W_Q$$

Thus the class NP can be interpreted as those decision problems, Q, for which the membership problem $< x,y > \in W_Q$ can be decided in time polynomial in the size of x. Notice that this constraint immediately forces y (a valid witness) also to have size polynomial in $|x|$.

The class coNP. The quantifier in our formalisation of NP is an existential one. If we modify this to

$$\forall\ y \in W(x) \ \ < x,y > \notin W_Q$$

[5] We will frequently use "iff" as short form for "if and only if".

then we obtain the important class coNP capturing instances that *do not* have the property of interest. For example if we wish to demonstrate that an argument, x is inadmissible then it suffices to show "for every subset S of A the set $S \cup \{x\}$ either is not conflict-free or has an undefended argument".

We, now, briefly summarise some developments of this view of "decision problems as witness testing".

The first of these is the concept of "oracles": in an *oracle* computation we are provided with a "black-box" for witness testing which given a problem instance x provides the answer for "$Q(x) = 1$?" in a single computational step. Now such oracle machines may be considered with respect to arbitrary complexity classes, so P^A describes the class of "decision problems that have a polynomial-time algorithm *that makes use of an oracle for a decision problem in the complexity class A*".

For example, for an NP oracle we might use "existence of a stable extension". In exploiting such an oracle to solve another decision problem B "in polynomial-time" we might use an algorithm which, given an instances p of B, constructs one or more (but at most polynomial in $|p|$) frameworks F_1^p, F_2^p, etc., using the answer to "does F_k^p have a stable extension?" to determine if p should be accepted as an instance of B.

The class DP. A number of important classes have been found to occur in complexity analysis of argumentation via such oracles. Among them we have DP, the so-called "difference class" of decision problems whose members are captured by the intersection of instances x accepted by a problem L_1 (with $L_1 \in \mathsf{NP}$) and x accepted by a problem L_2 (with $L_2 \in$ coNP). For example the set of pairs of propositional formulae $< \varphi_1, \varphi_2 >$ in which φ_1 is satisfiable and φ_2 is not so (the SAT-UNSAT problem) is in DP since its positive instances are the intersection of

$$\begin{aligned} L_1 &= \{ < \varphi, \psi > \ : \ \varphi \text{ is satisfiable } \} \\ L_2 &= \{ < \varphi, \psi > \ : \ \psi \text{ is unsatisfiable } \} \end{aligned}$$

2.1.3 The Polynomial-time Hierarchy

The notion of "oracle" can also be used in defining the important "Polynomial-time Hierarchy" (PH). Consider the quantifier formulation of NP and coNP

$$\begin{array}{l} \exists\ y \ < x, y > \in W_Q \\ \forall\ y \ < x, y > \notin W_Q \end{array}$$

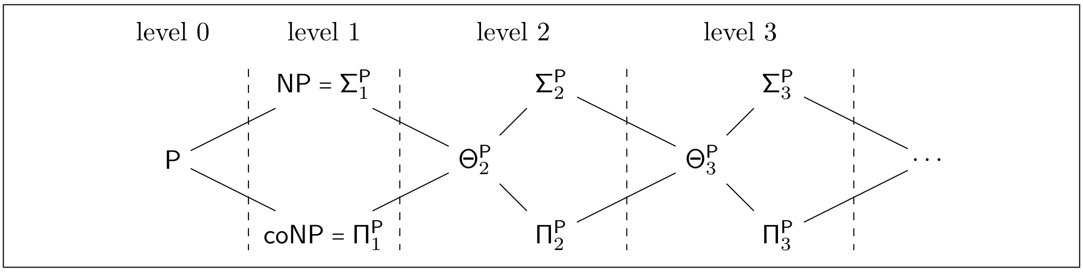

Figure 1: Levels of the polynomial-hierarchy. An edge denotes that all problems in the class on the left side are also contained in the class on the right side. Notice that only classes relevant for this paper are shown.

This uses a (polynomial) time decidable *binary* relation and a *single* quantifier. We could, however, extend this further, e.g.

$$\exists\ y_1\ \forall\ y_2\ W_Q^2(x, y_1, y_2)$$
$$\forall\ y_1\ \exists\ y_2\ \neg W_Q^2(x, y_1, y_2)$$

or even

$$\exists\ y_1\ \forall\ y_2\ \exists\ y_3\ W_Q^3(x, y_1, y_2, y_3)$$
$$\forall\ y_1\ \exists\ y_2\ \forall\ y_3\ \neg W_Q^3(x, y_1, y_2,\ y_3)$$

and, generally

$$Q_1\ y_1\ Q_2\ y_2\ \ldots\ Q_k\ y_k\ W_Q^k(x, y_1, y_2, \ldots, y_k)$$

In the last case we have k *alternating* quantifiers (that is $\exists$ is followed by $\forall$ and vice-versa) and the predicate $W_Q^k(x, y_1, y_2, \ldots, y_k)$ is decidable in time polynomial in $|x|$. When the opening (Q_1) quantifier is $\exists$ this defines the class of languages Σ_k^{P}; when this quantifier is $\forall$ we have Π_k^{P}. The *polynomial-hierarchy* (PH) is

$$\mathsf{PH} \quad = \quad \bigcup_{k=0}^{\infty} \Sigma_k^{\mathsf{P}} \quad = \quad \bigcup_{k=0}^{\infty} \Pi_k^{\mathsf{P}}$$

We sometimes refer to *levels of the polynomial-hierarchy*, where the k-th level is formed by the classes Σ_k^{P} and Π_k^{P}. For instance on the first level there are the classes NP and coNP while on the second level there are the classes Σ_2^{P} and Π_2^{P}. Moreover, we will later introduce a further family of complexity classes Θ_k^{P}, and will consider the class Θ_k^{P} to be in the k-th level of the polynomial hierarchy (cf. Figure 1).

How does this relate to the concept of "oracle machines"? The answer to this is given by examining the quantifier structure in more depth. We have required the inner most $(k+1)$-ary predicate W_Q^k to be (deterministic) polynomial-time computable. If we have an oracle for the decision problem implied by removing the *first* quantifier then this class of languages (when $Q_1 = \exists$) is formed by languages which belong to NP given access to a $\Sigma_{k-1}^{\mathsf{P}}$ oracle, conventionally denoted $\mathsf{NP}^{\Sigma_{k-1}^{\mathsf{P}}}$, while Π_k^{P} (first quantifier is $\forall$) are those problems computable in coNP with a $\Sigma_{k-1}^{\mathsf{P}}$ oracle.

For example, consider the "quantified SAT problem" one version of which involves two disjoint sets of propositional variables, X and Y, and asks of a given formula $\varphi(X, Y)$ whether $\exists\ \alpha_X \forall\ \beta_Y\ \varphi(\alpha_X, \beta_Y)$, that is, "can we find an assignment of values to the X variables (α_X) which renders the formula $\varphi(\alpha_X, Y)$ a tautology?". Given an oracle for satisfiability we can test $\varphi(\alpha_X, Y) \equiv \top$ to be a "single step", by testing the negated formula for satisfiability. The implied NP question ("can we find ...") is handled by a "polynomial" algorithm with access to this oracle so that $\Sigma_2^{\mathsf{P}} = \mathsf{NP}^{\mathsf{NP}}$. Notice that, in our example we can also directly use an oracle for the coNP problem of tautology which gives us $\mathsf{NP}^{\mathsf{NP}} = \mathsf{NP}^{\mathsf{coNP}}$. That is, for an oracle machine it does not matter whether it has access to a NP or coNP oracle (or more generally to a $\Sigma_{k-1}^{\mathsf{P}}$ or Π_{k-1}^{P} oracle) as it can easily switch "yes" and "no" answers after an oracle call.

In total we can treat PH as groups of problems described via alternation of a fixed number (k) of quantifiers or in terms of polynomial-time oracle machines exploiting oracles to the immediately lower level, i.e. both Σ_k^{P} and Π_k^{P} use access to a $\Sigma_{k-1}^{\mathsf{P}}$ oracle.

Moreover, we consider related oracle complexity classes that have only restricted access to their oracle. Concretely, the class $\Theta_k^{\mathsf{P}} = \mathsf{P}^{\Sigma_{k-1}^{\mathsf{P}}[\log(|x|)]}$ contains problems decidable by a deterministic polynomial-time algorithm that is allowed to make a logarithmic number (w.r.t. input size) of $\Sigma_{k-1}^{\mathsf{P}}$-oracle calls. An alternative characterisation for Θ_k^{P} is that the deterministic algorithm is allowed to make linearly (in the input size) so called non-adaptive calls to the $\Sigma_{k-1}^{\mathsf{P}}$-oracle, that is all oracle calls are evaluated in parallel. When using this alternative characterisation the class Θ_k^{P} is sometimes also denoted as $\mathsf{P}_{\parallel}^{\Sigma_{k-1}^{\mathsf{P}}}$.

Notice that all complexity classes we consider can be solved by in worst-case exponential time algorithms that only require polynomial space. However, problems on different levels of the polynomial hierarchy behave quite differently, and methods that work reasonable for problems at the NP, coNP level might not work as well for Σ_2^{P} or Π_2^{P}-hard problems.[6]

[6] In the context of formal argumentation such a behaviour can be observed at the results of the First International Competition on Computational Models of Argumentation [Thimm and Villata, 2015; Thimm *et al.*, 2016].

2.2 Reductions, Hardness and Completeness

At the conclusion of the preceding sub-section we referred to particular problems as "among the hardest Π_2^{P} problems". This (at the time of writing) does *not* mean we can formally demonstrate that *every* problem that can be classified as belonging to Π_2^{P} may be solved by a (deterministic) algorithm whose run-time is no worse than that of the best algorithm for, e.g., semi-stable skeptical reasoning. It does, however, mean the following: *if* we can find an NP (or even P) algorithm for skeptical semi-stable reasoning *then* we can construct NP (resp. P) algorithms *for every problem in the class* Π_2^{P}, i.e. it would follow that the classes Π_2^{P} and NP (resp. P) contained *exactly* the same decision problems. Despite this, throughout this work we will follow the standard assumptions in computational complexity theory and consider problems in higher levels of the polynomial hierarchy to be harder than problems in the lower levels of the polynomial-hierarchy.[7]

2.2.1 Polynomial Reducibility

The key idea used to support this claim is that of *polynomial reducibility.* Suppose we have two decision problems – F and G say. These have sets of instances I_F and and I_G. Now, while we may not be able to formally prove that either problem is intractable we can argue, using the following approach, that if G is decidable in polynomial time then F is also.

Build an *efficient* procedure, τ, transforming any instance of F into an instance of G, i.e., $\tau : I_F \to I_G$ and with the property that $x \in I_F$ is a positive instance of F iff $\tau(x) \in I_G$ is a positive instance G.

With such a transformation procedure any algorithm for G can be used as a sub-routine to give an algorithm for F. So were it the case that $G \in \mathsf{P}$, as τ is efficient, it follows that $F \in \mathsf{P}$ also. By contraposition, it can be shown that if $F \notin \mathsf{P}$ it must be the case that $G \notin \mathsf{P}$. When such a transformation can be found between decision problems F and G as above, we say that "F is *polynomially-reducible* to G" using the notation $F \leq_p G$ to describe this relationship.

Notice that the form of instances for F and G do not have to be identical: G could, for example, be a decision problem concerning propositional formulae and F one whose instances are AFs: a transformation between the two would define how a formula is constructed from a given AF.

[7] This relates to two famous open problems in complexity theory, namely to show that $\mathsf{P} \neq \mathsf{NP}$ and to show that the polynomial hierarchy is an infinite hierarchy and does not collapse at a certain level, i.e, $\Sigma_k^{\mathsf{P}} \neq \Sigma_{k+1}^{\mathsf{P}}$ for all $k > 0$. Both statement are widely believed but (at the time of writing) there are no formal proofs.

2.2.2 Hardness and Completeness

The concept of reducibility offers a means to argue that the class NP differs from the class P and formalise the notion of "hardest" problem of a complexity class. Intuitively we consider a problem to be among the "hardest" problems of a complexity class if an efficient method for the problem would yield efficient methods for *all* problems in the class. That is, an efficient method for just *one* of the "hardest" problems would yield efficient methods for *all* problems in the class. Formally, for any complexity class, $\mathcal{C}$, a decision problem G is said to be $\mathcal{C}$*-hard* if

$$\forall\ F \in \mathcal{C} \quad F \leq_p G$$

If, in addition $G \in \mathcal{C}$ then G is said to be $\mathcal{C}$*-complete*.

So the class of NP-complete problems are those problems in NP to which any other problem in NP can be polynomially reduced. The class of known NP-complete problems includes many well-studied combinatorial, logic, and graph problems for which no efficient algorithm has been discovered, in some cases after several centuries of study. Among these are: deciding if a propositional formula has a model (SAT); deciding if a graph has a path that contains every vertex exactly once (a variant of the so-called Travelling Salesperson Problem), deciding if a given argument is acceptable w.r.t. Dung's stable semantics.

It is considered highly unlikely that every single one of these problems can be solved efficiently. In order to prove that no NP-complete problem can be solved in polynomial time it would suffice to show that just *one* could not be.

Thus, a proof that a problem G is NP-complete is seen as very strong evidence that F is intractable. Given the transitivity of $\leq_p$ all that is required to proof NP-hardness is a known NP–hard problem (F say) and a transformation, τ, to witness $F \leq_p G$. In order to obtain NP-completeness one has to additionally give a procedure that decides G and fits the definition of NP, we sometimes call such a procedure a NP-algorithm (more generally $\mathcal{C}$-algorithm for complexity class $\mathcal{C}$).

Next let us briefly reconsider our restrictions on reductions. All the complexity classes $\mathcal{C}$ considered in this paper, except L, are *closed under polynomial reductions*, that is whenever a problem A can be polynomial-time reduced to a problem $B \in \mathcal{C}$ then also A belongs to $\mathcal{C}$. Notice that any problem in the class P and in particular those in the class L would be complete for P with respect to polynomially-reducibility. Thus when differentiating between problems in L and P one uses the concept of *logspace-reducibility* where the transforming procedure is required to work in logarithmic space. In particular, P-completeness results are stated w.r.t. logspace-reducibility.

2.2.3 Complete Problems for the Polynomial Hierarchy.

To show that a problem A is hard for a specific complexity class $\mathcal{C}$ one typically starts from a problem B that is complete for the class $\mathcal{C}$ and provides a reduction from B to A. In the following we briefly introduce some canonical complete problems for the complexity classes in the polynomial-hierarchy.

As already mentioned a famous NP-complete problem is deciding if a propositional formula has a model (SAT). On the other side standard coNP-complete problems are verifying that a propositional formula is a tautology (TAUT) or that a propositional formula has no model (UNSAT). The canonical DP-complete problem is the earlier mentioned $\mathsf{SAT}-\mathsf{UNSAT}$ problem.

The complete problems for classes Σ_k^{P} and Π_k^{P} are given by quantified SAT problems (cf. Section 2.1.3). That is, one is given a propositional formula φ whose variables are split up in k disjoint sets $X_1, \dots X_k$ and the possible assignments for these sets X_1 are quantified with alternating existential and universal quantifiers. A quantified boolean formula (QBF) is then of the form

$$Q_1 X_1 \, Q_2 X_2 \, \dots Q_k X_k \, \varphi(X_1, \dots X_k)$$

with Q_i being alternating $\exists, \forall$ quantifiers (i.e., $\exists$ is followed by $\forall$ and vice versa). Deciding whether a QBF with k quantifiers and $Q_1 = \exists$ is valid is the canonical Σ_k^{P}-complete problem while deciding whether a QBF with k quantifiers and $Q_1 = \forall$ is valid is the canonical Π_k^{P}-complete problem.

As the second level of the polynomial-hierarchy is of special interest in the setting of formal argumentation we next introduce minimal model satisfiability (MINSAT) as another problem that is Σ_2^{P}-complete [Eiter and Gottlob, 1993]. In the MINSAT problem one is given a propositional formula φ over variables X and a variable x thereof and has to decide whether the variable is true in some minimal model of φ.

2.3 Parametrized Complexity

Classical complexity theory deals with the complexity of problems w.r.t. the size of the instance. However, often the complexity of a problem does not mainly depend on the size of an instance but on some (structural) properties of the instance. That is, we can solve huge instances efficiently as long as some property is satisfied or the obstacles in the structure are bounded independent of the size. The field of parametrized complexity theory[8] deals with this observation. The idea is to

[8]We just briefly introduce the concepts relevant for this paper; for comprehensive introductions to parametrized complexity the reader is referred to [Flum and Grohe, 2006; Niedermeier, 2006; Cygan *et al.*, 2015].

consider parametrized problems, i.e., the problem description contains a designated parameter (typically an integer) which is instantiated by each problem instance. An example for a parametrized problem is given a graph G and an integer parameter k deciding whether G has a clique of size k.

Definition 2.1. *A parametrized (decision) problem is called* fixed-parameter tract able *(or in* FPT*) if it can be determined in time $f(k) \cdot |x|^{O(1)}$ for a computable function f.*

Now given that a problem is in FPT and just consider those instances where the parameter is bounded by some constant then we can decide an instance with a polynomial-time algorithm. Only the constants in the polynomial-time bound are affected by the parameter, but not the order of the polynomial.

Beside FPT there is also a weaker form of tractability w.r.t. a parameter allowing the order of the polynomial to depend on the parameter.

Definition 2.2. *A parametrized (decision) problem is* slice-wise polynomial *(or in* XP*) if it can be determined in time $f(k) \cdot |x|^{g(k)}$ for computable functions f, g.*

A problem in XP can be solved in polynomial time if we bound the parameter, but distinguishing it from FPT the order of the polynomial may highly depend on the bound of the parameter.

Let us briefly present the relations between the classes FPT, XP and P:

$$\mathsf{P} \subseteq \mathsf{FPT} \subseteq \mathsf{XP}$$

When considering unparametrized problems and talking about FPT we have to mention the used parameter explicitly. Thus we say a problem P is fixed-parameter tractable w.r.t. the parameter k iff the corresponding parametrized problem (P, k) is fixed-parameter tractable.

3 Complexity of Dung's Abstract Argumentation

We start our analysis with Dung's Abstract Argumentation Frameworks. These frameworks consist of a set of abstract arguments and a relation representing directed conflicts or attacks between these arguments. Then rules, so called semantics, are defined to select coherent sets of arguments that can be accepted simultaneously. That is, abstract argument frameworks focus on the core issue of argumentation, i.e., resolving conflicts between arguments.

This part of the paper is organised as follows: In Section 3.1 we recall the basic definitions of Dung's Abstract Argumentation Frameworks and the most popular semantics for it. That is, beside the semantics introduced by Dung [Dung, 1995], we consider ideal [Dung *et al.*, 2007], semi-stable [Verheij, 1996; Caminada *et al.*, 2012], stage [Verheij, 1996] and cf2 [Baroni *et al.*, 2005] semantics. Then in Section 3.2 we discuss the core computational Problems of Abstract Argumentation and define formal variants that serve as basis for the complexity analysis in Section 3.3. In Section 3.4 we consider potential computational advantages when the argumentation frameworks fall into some specific graph class. The potential of techniques from parametrized complexity theory is discussed in Section 3.5. In Section 3.6 we discuss some computational issues specific to labelling-based argumentation semantics. Finally, in Section 3.7 we summarise and discuss the presented results and give additional pointers to literature.

3.1 Dung's Abstract Argumentation Frameworks

In this section we introduce (abstract) argumentation frameworks [Dung, 1995] and recall the semantics we study (for a comprehensive introduction the reader is referred to [Baroni *et al.*, 2011a]).

Definition 3.1. *An* argumentation framework (AF) *is a pair $F = (A, R)$ where A is a (finite) set of arguments and $R \subseteq A \times A$ is the attack relation. The pair $(a, b) \in R$ means that a attacks b. We say that an argument $a \in A$ is* defended *(in F) by a set $S \subseteq A$ if, for each $b \in A$ such that $(b, a) \in R$, there exists $c \in S$ such that $(c, b) \in R$.*

Indeed when studying computational complexity we are only interested in AFs where the set A is finite.

Semantics for argumentation frameworks are defined as functions σ which assign to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for σ the functions *na*, *gr*, *st*, *ad*, *co*, *cf2*, *id*, *pr*, *sst* and *stg* which stand for naive, grounded, stable, admissible, complete, cf2, ideal, preferred, semi-stable and stage semantics, respectively. Towards the definition of these semantics we have to introduce a few more formal concepts.

Definition 3.2. *Given an AF $F = (A, R)$, the* characteristic function $\mathrm{F}_F : 2^A \to 2^A$ *of F is defined as $\mathrm{F}_F(S) = \{x \in A \mid x \text{ is defended by } S\}$.*

Definition 3.3. *For a set $S \subseteq A$ and an argument $a \in A$, we say S* attacks *a (resp. a attacks S) in case there is an argument $b \in S$, such that $(b, a) \in R$ (resp. $(a, b) \in R$). Moreover, for a set $S \subseteq A$, we denote the set of arguments attacked by (resp. attacking) S as $S^+_R = \{x \mid S \text{ attacks } x\}$ (resp. $S^-_R = \{x \mid x \text{ attacks } S\}$), and define the* range of S *as $S^\oplus_R = S \cup S^+_R$.*

We are now prepared to give the formal definitions of the abstract argumentation semantics we will consider. Notice that we restrict ourselves to extension-based semantics, but some aspects of labelling-based semantics are discussed in Section 3.6).

Definition 3.4. *Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is* conflict-free (in F), *if there are no $a, b \in S$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of conflict-free sets of F. For a conflict-free set $S \in cf(F)$, it holds that*

- *$S \in na(F)$, if there is no $T \in cf(F)$ with $T \supset S$;*
- *$S \in st(F)$, if $S^+_R = A \setminus S$;*
- *$S \in ad(F)$, if $S \subseteq \mathrm{F}_F(S)$;*
- *$S \in co(F)$, if $S = \mathrm{F}_F(S)$;*
- *$S \in gr(F)$, if $S \in co(F)$ and there is no $T \in co(F)$ with $T \subset S$;*
- *$S \in pr(F)$, if $S \in ad(F)$ and there is no $T \in ad(F)$ with $S \subset T$;*
- *$S \in id(F)$ if S is $\subseteq$-maximal among $\{S' \mid S' \in ad(F)$ and $S' \subseteq E$ for each $E \in pr(F)\}$.*
- *$S \in sst(F)$, if $S \in ad(F)$ and there is no $T \in ad(F)$ with $S^\oplus_R \subset T^\oplus_R$;*
- *$S \in stg(F)$, if there is no $T \in cf(F)$, with $S^\oplus_R \subset T^\oplus_R$.*

We recall that for each AF F, the grounded semantics yields a unique extension, the grounded extension, which is the least fixed-point of the characteristic function F_F.

Finally, we give the recursive definition of cf2 semantics (see [Baroni *et al.*, 2005; Gaggl and Woltran, 2013] for further reference).

Definition 3.5. *Given an argumentation framework $F = (A, R)$, then $E \in cf2(F)$, if*

- *$E \in na(F)$ if $|SCCs_F| = 1$, and*
- *$\forall S \in SCCs_F\ (E \cap S) \in cf2(F{\downarrow}_{UP_F(S,E)})$ otherwise.*

Here $SCCs_F$ denotes the set of strongly connected components of F, and for any $E, S \subseteq A$, $UP_F(S, E) = \{a \in S \mid \nexists b \in E \setminus S : (b, a) \in R\}$. Moreover, for $S \subseteq A$ we use $F{\downarrow}_S$ to denote the AF $(A \cap S, R \cap S \times S)$, i.e., the AF that one obtains when restricting F to the arguments in S.

We recall some basic properties of these semantics. For each AF F we have the following subset relations:

$$st(F) \subseteq stg(F) \subseteq na(F) \subseteq cf(F),$$

$$st(F) \subseteq sst(F) \subseteq pr(F) \subseteq co(F) \subseteq ad(F) \subseteq cf(F),$$

and $st(F) \subseteq cf2(F) \subseteq na(F)$. Furthermore, for any of the considered semantics σ except stable semantics we have that $\sigma(F) \neq \emptyset$ holds, i.e., these semantics always propose at least one extension. Grounded and ideal semantics always yield exactly one extension, thus we also say that they are unique status semantics, and the ideal extension is always a complete extension. With slight abuse of notation we sometimes use $gr(F)$, resp. $id(F)$, to refer to the unique grounded, resp. ideal, extension of F. Moreover, stable, semi-stable, and stage semantics coincide for AFs with at least one stable extension.

3.2 Computational Problems

In general an argumentation semantics assigns several extensions to a single framework, but at the end of the day we want to make a conclusion about arguments. There are different ways to aggregate the acceptance status of an argument from the set of extensions, which mirrors different levels of scepticism. First it is quite clear that an argument which is in no extension at all should not be accepted, but in certain situations it might be fine to accept an argument that appears in just one extension, this is what we will call *credulous reasoning*. On the other hand in situations where one has to be cautious one might demand that an argument is in all extensions, we refer to this as *skeptical reasoning*.

These reasoning modes give rise to the following computational problems for argumentation semantics σ.

- *Credulous Acceptance* $Cred_\sigma$: Given AF $F = (A, R)$ and an argument $a \in A$. Is a contained in some $S \in \sigma(F)$?

- *Skeptical Acceptance* $Skept_\sigma$: Given AF $F = (A, R)$ and an argument $a \in A$. Is a contained in each $S \in \sigma(F)$?

If an AF has no stable extensions, according to our definition of skeptical acceptance, all arguments are skeptically accepted. This may be unwanted and hence one might consider a variation of the skeptical acceptance problem asking whether an argument is contained in all extensions and there exists at least one extension [Dunne and Wooldridge, 2009].

In practice, one often is interested in computing all extensions or a certain number of extensions. However, complexity theory provides much better tools for decision problems than for function problems and thus one usually sticks to decision problems when analysing the computational problems, in our case credulous and skeptical acceptance. Nevertheless, the complexities of credulous and skeptical acceptance together give a good impression of the complexity to actually compute the extensions.

Beside these reasoning problems there are also several other computational problems in the field of abstract argumentation. In this work we consider the most prominent ones of them. First of all one might be interested in verifying given extensions, which may come from another agent or potentially corrupted file, or simply as part of a reasoning algorithm.

- *Verification of an extension* Ver_σ: Given AF $F = (A, R)$ and a set of arguments $S \subseteq A$. Is $S \in \sigma(F)$?

Another task is deciding whether an AF provides any coherent conclusion. That can be deciding whether it has at least one extension, in the case of stable semantics, or whether it has an extension different from the empty set, for all the other semantics under our consideration.

- *Existence of an extension* $Exists_\sigma$: Given AF $F = (A, R)$. Is $\sigma(F) \neq \emptyset$?
- *Existence of a non-empty extension* $Exists_\sigma^{\neg\emptyset}$: Given AF $F = (A, R)$. Does there exist a set $S \neq \emptyset$ such that $S \in \sigma(F)$?

Finally, we will also consider the problem of deciding whether a semantics yields a unique extension for a given an AF.

- *Uniqueness of the solution* $Unique_\sigma$: Given AF $F = (A, R)$. Is there a unique set $S \in \sigma(F)$, i.e., is $\sigma(F) = \{S\}$?

3.3 Computational Complexity

A typical complexity analysis of a problem consists of two parts. First, we have to give an upper bound for the complexity of the problem. That is, we have to either give an algorithm showing the problem can be solved within a class $\mathcal{C}$ or we reduce the problem to another problem already shown to be in the class $\mathcal{C}$. Second, we want to prove lower bounds for the complexity of the problem. That is, we consider a problem that was shown to be hard for some complexity class $\mathcal{C}'$ and reduce it to the current problem. That is, we show the problem to be $\mathcal{C}'$-hard. In case that

σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ	$Exists_\sigma$	$Exists_\sigma^{\neg\emptyset}$	$Unique_\sigma$
cf	in L	trivial	in L	trivial	in L	in L
na	in L	in L	in L	trivial	in L	in L
gr	P-c	P-c	P-c	trivial	in L	trivial
st	NP-c	coNP-c	in L	NP-c	NP-c	DP-c
ad	NP-c	trivial	in L	trivial	NP-c	coNP-c
co	NP-c	P-c	in L	trivial	NP-c	coNP-c
cf2	NP-c	coNP-c	in P	trivial	in L	in P
id	Θ_2^P-c	Θ_2^P-c	Θ_2^P-c	trivial	Θ_2^P-c	trivial
pr	NP-c	Π_2^P-c	coNP-c	trivial	NP-c	coNP-c
sst	Σ_2^P-c	Π_2^P-c	coNP-c	trivial	NP-c	in Θ_2^P
stg	Σ_2^P-c	Π_2^P-c	coNP-c	trivial	in L	in Θ_2^P

Table 1: Complexity of Dung's abstract argumentation ($\mathcal{C}$-c denotes completeness for class $\mathcal{C}$).

the classes $\mathcal{C}$ and $\mathcal{C}'$ coincide we obtain that the studied problem is $\mathcal{C}$-complete, and have an exact classification of the complexity of the problem.

The complexity landscape of abstract argumentation semantics is given in Table 1 and discussed below. For Dung's semantics the "in P" and "trivial" are immediately by properties of the corresponding semantics [Dung, 1995]; results for naive semantics are due to Coste-Marquis *et al.* [2005]; results for stable, admissible and preferred semantics follow from results on logic programs by Dimopoulos and Torres [1996], except for the Π_2^P-completeness of $Skept_{pr}$ which is due to Dunne [2009]; the complexity of ideal semantics is due to Dunne [2009]; results for complete semantics are due to Coste-Marquis *et al.* [2005]; results for semi-stable and stage semantics are due to Caminada *et al.* [2012] and Dvořák and Woltran [2010]; the results for *cf2* semantics are due to Gaggl and Woltran [2013] and the analysis of polynomial-time problems that distinguishes problems that can be solved in L from problems that are P-complete is due to Dvořák and Woltran [2011] [Dvořák, 2012a].

In accordance with the above we will first consider upper bounds for the introduced reasoning problems and then discuss hardness results for them.

3.3.1 Upper Bounds for the Computational Complexity

Most of the problems we introduced in the previous section will fall into one of the complexity classes based on non-deterministic algorithms, e.g. NP and coNP, and thus most of the upper bound are by guess and check algorithms that first non-deterministically guess a potential extension and then verify that it is indeed an extension and satisfies the desired properties.

Standard Reasoning Procedures. The standard algorithm for credulous acceptance first non-deterministically guesses a set of arguments, and then verifies that the set is an extension for the considered semantics and contains the argument under question. The answer to the credulous acceptance query is yes if at least one of the possible guesses evaluates to true. Now let V be the complexity for verifying an extension then the above gives use a NP^{V} algorithm for credulous acceptance. We next consider skeptical acceptance and show that it has a $\mathsf{coNP}^{\mathsf{V}}$ algorithm. To show that a problem falls into a $\mathsf{coNP}^{\mathcal{C}}$ class one could follow the definition of coNP and give an algorithm that first guesses a potential witness and then tests whether the potential witness satisfies certain conditions that can be tested in P^{V}. This conditions have to be such that an instance is positive iff all possible guesses evaluate to true. However, often it is more convenient to consider the complementary problem and provide a $\mathsf{NP}^{\mathcal{C}}$ algorithm for that problem. That is, instead of skeptical acceptance we consider the problem of showing that an argument is not skeptically accepted. The standard algorithm non-deterministically guesses a set of arguments, and then verifies that it is an extension and does not contain the argument under question. The answer to the skeptical acceptance query is yes only if each possible guess evaluates to false. Let V be again the complexity for verifying an extension then the above gives use a $\mathsf{coNP}^{\mathsf{V}}$ algorithm for skeptical acceptance. Towards upper bounds for *Cred* and *Skept* we next consider upper bounds for the verification problems.

Verifying Extensions. For conflict-free, naive, stable, admissible and complete semantics we only have to check whether for the given set certain attacks exist, respectively do not exist. For instance to verify a stable extension we have to verify that (a) between arguments in the extension there is no attack and (b) that all arguments not in the extension are attacked by at least one argument in the extension. This can clearly be done in polynomial time and as it only needs two pointers to arguments also in logarithmic space.[9] Since polynomial time oracles do not add any computational power they can be neglected, i.e., $\mathsf{NP}^{\mathsf{P}} = \mathsf{NP}$ and

[9]For the corresponding result for cf2 semantics see [Nieves *et al.*, 2009; Gaggl and Woltran, 2013].

$\mathsf{coNP}^{\mathsf{P}} = \mathsf{coNP}$. This gives NP, resp. coNP, upper bounds for credulous and skeptical acceptance under these semantics.

Next consider semantics that require maximisation, that are *pr*, *sst*, *stg* (we will see later that for ideal semantics no maximisation is required). Again the basic criterion of being admissible or conflict-free can be easily checked in polynomial time but the maximality criterion adds some complexity. To show that checking whether a set S is an extension is coNP we again give a non-deterministic algorithm for the complimentary problem, of falsifying the set S to be an extension. This is done by first testing whether S is not admissible (for *pr*, *sst*) or not conflict-free (for *stg*) and then guessing a set $T \supset S$ and testing whether it is admissible (for *pr*, *sst*) or not conflict-free (for *stg*). The algorithm successfully falsifies the set S to be an extension iff the first test succeeds or the second test succeeds for at least on guess. In other words, the set S is an extension only if the first and the second tests fails for all possible guessed sets T. Combined with the NP^{V}, $\mathsf{coNP}^{\mathsf{V}}$ resp., algorithm for credulous, skeptical resp., acceptance the above coNP-algorithms for verification give Σ_2^{P}, resp. Π_2^{P} algorithms, for the credulous, resp. skeptical, acceptance problems.

Improved Procedures. For many semantics the above upper bounds are already optimal, but there are some cases where we can improve over them.

First, consider *conflict-free* and naive sets. If an argument is not self-attacking then it certainly will appear in a conflict-free set and thus also in a naive extension. Thus credulous acceptance can be decided by just testing whether the argument under question is self-attacking. Considering skeptical acceptance we have that the empty set is always conflict-free and admissible. Thus for conflict-free and admissible semantics we can reply "no" to each skeptical acceptance query without looking at the actual framework.

For *naive sets* we know that an argument is in a naive set iff it has no self-attack and none of its neighbours is in the set. Thus for skeptical acceptance we just have to test whether the argument is not self-attacking and all of its neighbours are not credulously accepted, i.e., they are self-attacking.

The *grounded* semantics can be computed by iterating the characteristic function until the least fixed-point is reached [Dung, 1995]. The characteristic function can be computed in polynomial time and, as the least fixed-point is reached after at most linearly many iterations. That is, the grounded extension can be computed in polynomial time and the decision problems can then be easily answered. Moreover, as the grounded extension is the unique minimal complete extension skeptical acceptance for complete semantics is exactly the problem of testing whether an argument is contained in the grounded extension and thus in polynomial time.

As each admissible set can be extended to a preferred extension and each preferred extension is admissible we have $Cred_{ad} = Cred_{pr}$. That is, for *credulous acceptance* under *preferred* semantics it suffices to consider admissible sets and thus an NP algorithm suffices.

Finally, for *ideal* semantics there is an alternative characterisation that allows for a Θ_2^P algorithm [Dunne, 2009]. That is, the ideal extension is the maximal admissible set that is not attacked by any other admissible set. The algorithm first computes the credulously accepted arguments (w.r.t. preferred semantics) via an NP-oracle and then considers the set of arguments that are credulously accepted but not attacked by any credulous accepted argument. Within this set one then computes the ideal extension by a polynomial-time algorithm that iteratively removes arguments which are not defended.

3.3.2 Hardness results

Given the complexity upper bounds from above we are now going for hardness results that show that these upper bounds are optimal. We start with what we call the *standard translation* from propositional formulae to argumentation frameworks and then discuss some prototypical hardness results that extend the standard translation.

Standard Translation. On the one hand the standard translation will give us our first hardness results and on the other hand it is part of almost all reductions in abstract argumentation. To show hardness one typically starts from the standard translation and adds modifications to match the actual problem and semantics.

Reduction 3.6. *Given a propositional formula φ in CNF given by a set of clauses C over the atoms Y, we define the* standard translation *from φ as $F_\varphi = (A, R)$, where*

$$\begin{aligned} A &= \{\varphi\} \cup C \cup Y \cup \bar{Y} \\ R &= \{(c, \varphi) \mid c \in C\} \cup \\ &\quad \{(x, c) \mid x \in c, c \in C\} \cup \{(\bar{x}, c) \mid \bar{x} \in c, c \in C\} \cup \\ &\quad \{(x, \bar{x}), (\bar{x}, x) \mid x \in Y\} \end{aligned}$$

The AF F_φ from Reduction 3.6 is illustrated in Figure 2. The intuition behind the construction is as follows. Assume we are arguing whether the formula φ is true. For an atom y_i we have two arguments, y_i claiming the atom is true, $\bar{y}_i$ claiming the atom is false and thus $\neg y_i$ is true. As exactly one of y_i and $\bar{y}_i$ is true they are mutually attacking. Now consider the argument φ, which can be interpreted as "the

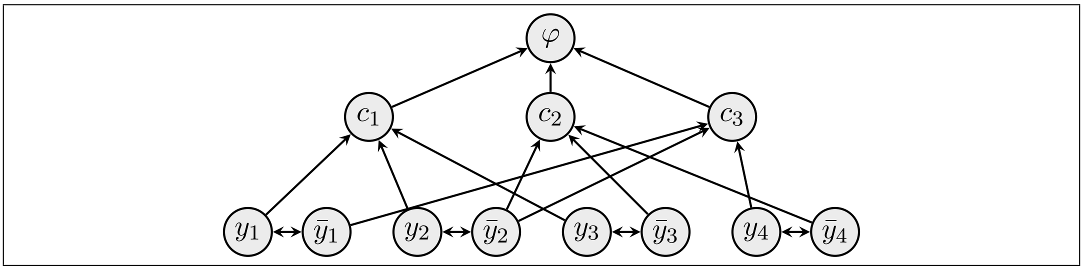

Figure 2: Illustration of the standard translation F_φ, for the propositional formula φ with clauses $\{\{y_1, y_2, y_3\}, \{\bar{y}_2, \bar{y}_3, \bar{y}_4)\}, \{\bar{y}_1, \bar{y}_2, y_4\}\}$.

formula φ is true". This argument is attacked by the arguments c_i which can be read as "clause c_i is not satisfied". Clearly if one clause is false the whole formula is not satisfied. Finally, if one of the literals in a clause is true the whole clause is true and thus an argument c_i is attacked by all arguments corresponding to literals in c_i.

Credulous Acceptance. For the NP-hardness of credulous acceptance consider the AF F_φ constructed by the above reduction. It is not to hard to show that each model I_Y[10] of φ corresponds to a stable extension of F_φ that consists of the argument φ, the arguments y_i for $y_i \in I_Y$, and the arguments $\bar{y}_i$ for $y_i \in Y \setminus I_Y$. Moreover also the converse holds, i.e., each stable extension of F_φ containing the argument φ corresponds to a model of φ. Thus, F_φ has a stable extension containing φ iff φ has a model. The same holds for admissible sets, complete, preferred and cf2 extensions. Thus Reduction 3.6 is a reduction from SAT to credulous reasoning under these semantics and as it can be clearly performed in polynomial time we obtain that credulous reasoning under these semantics is NP-hard.

Skeptical Acceptance. To show coNP-hardness of skeptical acceptance for stable, preferred and cf2 semantics we extend the standard translation F_φ by an additional argument $\bar{\varphi}$ that is attacked by φ. In the resulting AF G_φ the argument $\bar{\varphi}$ is skeptically accepted w.r.t. the mentioned semantics iff φ is not credulously accepted iff φ is unsatisfiable [Dimopoulos and Torres, 1996; Gaggl and Woltran, 2013]. Thus we have a reduction from UNSAT to skeptical acceptance showing coNP-hardness. Notice that this reduction does not work for admissible and complete semantics as for both the empty set is an extension neither containing φ nor $\bar{\varphi}$.

[10]A model I_Y is a subset of the variables Y such that if we set all variables in I_Y to true and all arguments in $Y \setminus I_Y$ to false the formula φ evaluates to true.

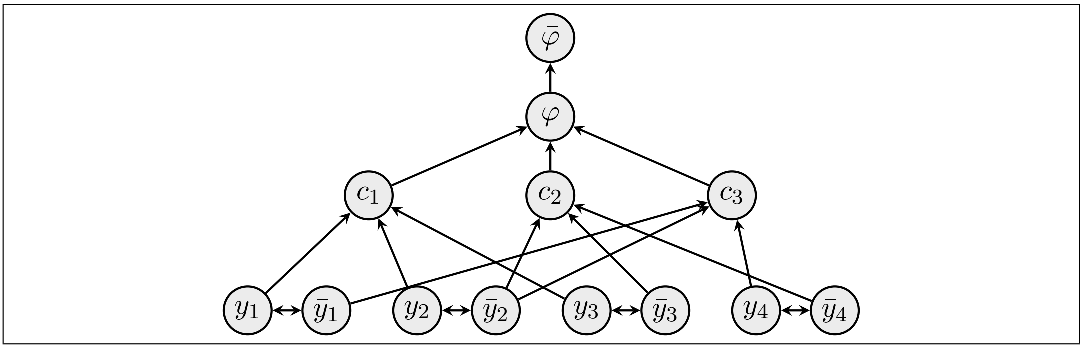

Figure 3: Illustration of the reduction G_φ, for the propositional formula φ with clauses $\{\{y_1, y_2, y_3\}, \{\bar{y}_2, \bar{y}_3, \bar{y}_4)\}, \{\bar{y}_1, \bar{y}_2, y_4\}\}$.

Skeptical Acceptance with Preferred Semantics. Skeptical acceptance with preferred semantics is a prototypical problem for the second level of the polynomial-hierarchy. The hardness proof is reported in [Dunne and Bench-Capon, 2002] and we next discuss a slight variation of the reduction presented there. That is, we give a reduction from the Π_2^{P}-complete problem $QSAT_\forall^2$ of deciding whether a $QBF_\forall^2$ formula is valid to skeptical acceptance with preferred semantics. That is, given a $QBF_\forall^2$ formula $\forall Y \exists Z\, \varphi(Y, Z)$ with φ being a CNF formula we construct an AF as follows. We first apply the standard reduction from propositional CNF formulae to AFs and then add an additional argument $\bar{\varphi}$ which is attacked by φ, attacks itself and attacks all arguments z, $\bar{z}$ for $z \in Z$ (but not the arguments y, $\bar{y}$ for $y \in Y$). The full reduction is given below and illustrated in Figure 4.

Reduction 3.7. *Given a $QBF_\forall^2$ formula $\Phi = \forall Y \exists Z\, \varphi(Y, Z)$ with φ being a CNF formula given by a set of clauses C over atoms $X = Y \cup Z$, we define the following translation from Φ to $H_\Phi = (A, R)$, where*

$$
\begin{aligned}
A &= \{\varphi, \bar{\varphi}\} \cup C \cup X \cup \bar{X} \\
R &= \{(c, \varphi) \mid c \in C\} \cup \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \\
&\qquad \{(x, c) \mid x \in c, c \in C\} \cup \{(\bar{x}, c) \mid \bar{x} \in c, c \in C\} \cup \\
&\qquad \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \bar{\varphi})\} \cup \{(\bar{\varphi}, z), (\bar{\varphi}, \bar{z}) \mid z \in Z\}
\end{aligned}
$$

In the Reduction 3.7 we have that each interpretation $I_Y \subseteq Y$ corresponds to an admissible set $\{y \mid y \in I_Y\} \cup \{\bar{y} \mid y \in Y \setminus I_Y\}$ in H_Φ while arguments z and $\bar{z}$ are attacked by φ and thus can only be in an admissible set if also φ is in that set. To make φ admissible we have to find $I_Y \subseteq Y$ and $I_Z \subseteq Y$ that together satisfy φ. Moreover, as each $c \in C$ is in conflict with φ and attacked by either z and $\bar{z}$

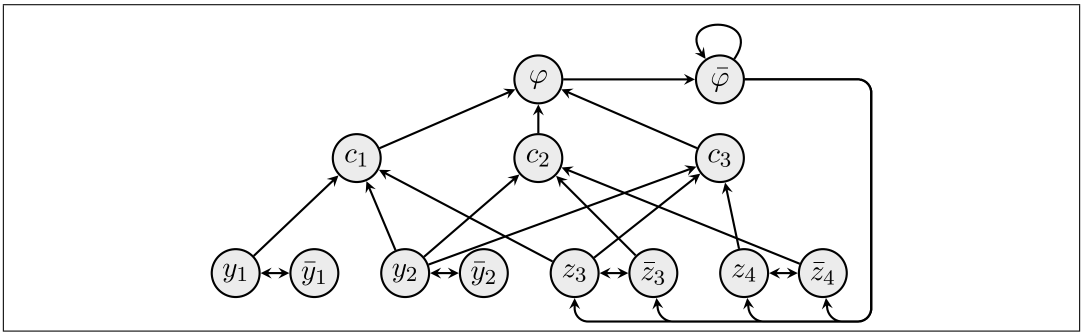

Figure 4: Illustration of the reduction for Π_2^{P}-hardness of $Skept_{pr}$. The AF H_Φ, for $\Phi = \forall y_1 y_2 \exists z_3 z_4 \; ((y_1 \vee y_2 \vee z_3) \wedge (y_2 \vee \neg z_3 \vee \neg z_4) \wedge (y_2 \vee z_3 \vee z_4))$.

for some $z \in Z$ none of them can be in an admissible set. We then have that a set $\{y \mid y \in I_Y\} \cup \{\bar{y} \mid y \in Y \setminus I_Y\}$ is a preferred extension, i.e., a subset maximal admissible set, iff there is no I_Z such $I_Y \cup I_Z$ satisfies φ. That is, there is a preferred extension in H_Φ not containing the argument φ iff the $QBF_\forall^2$ formula $\forall Y \exists Z \, \varphi(Y, Z)$ is false. Thus, we have a polynomial reduction from the Π_2^{P}-complete problem of $QSAT_\forall^2$ to skeptical acceptance with preferred semantics which proves the Π_2^{P}-hardness of the latter.

Acceptance with Grounded Semantics. Here we consider the problem of deciding whether an argument is in the grounded extension and show that it is P-hard. To this end we first have to introduce the P-complete problem HORNSAT. A *definite Horn-clause* c is a disjunction over literals from a countable domain U such that c contains exactly one positive literal. A definite Horn-formula is the conjunction over definite Horn-clauses. For example consider the definite Horn-formula $\varphi = x \wedge (\neg x \vee \neg y \vee z) \wedge (\neg y \vee \neg z \vee x)$. A more convincing way to denote definite Horn-formulae is as set of clauses and moreover denoting clauses as (logically equivalent) rules. Thus, our example formula φ can be denoted as $\varphi = \{\rightarrow x, x \wedge y \rightarrow z, y \wedge z \rightarrow x\}$. It is well-known that a definite Horn-formula has a unique minimal model which can be computed in polynomial time. Moreover, the problem HORNSAT of deciding whether an atom is in the minimal model of a definite Horn formula is P-complete [Kasif, 1986].

Next, in order to show P-hardness of $Cred_{gr}$, we give a logspace-reduction from the P-complete problem HORNSAT, to $Cred_{gr}$ (see Figure 5). Starting from a definite Horn formula one constructs an AF with one argument for each Horn clause; one argument $\bar{x}$ for each variable x; and an additional argument z for the variable that we are asking for being in the minimal model. All the arguments $\bar{x}$ are self attacking.

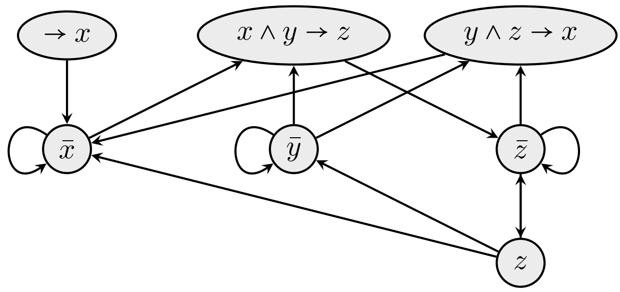

Figure 5: Illustration of the reduction for P-hardness of $Cred_{gr}$, that is $F_{\varphi,z}$ for $\varphi = \{\to x, x \wedge y \to z, y \wedge z \to x\}$.

Each argument corresponding to a rule is attacked by all arguments $\bar{x}$ for which the variable x is in the body of the rule and the argument attacks the argument $\bar{h}$ where h is the head of the rule. Finally, the argument z is only attacked by $\bar{z}$ but attacks all $\bar{x}$ arguments. The reduction is formally stated below.

Reduction 3.8. *Let $\varphi = \{r_l : b_{l,1} \wedge \cdots \wedge b_{l,i_l} \to h_l \mid 1 \leq l \leq n\}$ be a definite Horn theory over atoms X. We construct the AF $F_{\varphi,z} = (A, R)$ as follows:*

$$
\begin{aligned}
A =& \varphi \cup \bar{X} \cup \{z\} \\
R =& \{(\bar{x}, \bar{x}), (z, \bar{x}) \mid x \in X\} \cup \{(\bar{z}, z)\} \cup \\
& \{(r_l, h_l), (b_{l,j}, r_l) \mid r_l \in \varphi, 1 \leq j \leq i_l)\}
\end{aligned}
$$

The intuition behind the above reduction is that an argument corresponding to a rule is in the grounded extension only if all atoms in the rule body are in the minimal model of φ and an argument $\bar{x}$ is attacked by the grounded extension only if x is in the minimal model. That is, when computing the grounded extension via iteratively applying the characteristic function we simulate the following algorithm for deciding whether z is in the minimal model of φ. The algorithm starts with the rules with empty body and adds their rule heads to the minimal model. Then it iteratively considers all rules with the body already being part of the minimal model and adds their heads to the minimal model until either z is added or a fixed-point is reached. Notice that as soon as z is added to the grounded extension all arguments corresponding to rules are defended and thus also added to the grounded extension. We then have that z is in the minimal model of the Horn-formula φ iff z is in the grounded extension of $F_{\varphi,z}$ iff $\{r_l \mid 1 \leq l \leq n\} \cup \{z\}$ is the grounded extension of $F_{\varphi,z}$. This shows the P-hardness of credulous acceptance as well as of verifying the grounded extension.

3.3.3 Existence and Uniqueness of Extensions

Next, let us consider the results for the existence of (non-trivial) extensions and the uniqueness of solutions in Table 1.

Existence problems. First notice that the Exists_σ problem is only relevant for stable semantics as all the other semantics always lead at least one extensions. Moreover, for AFs with at least one argument the problems Exists_{st} and $\mathit{Exists}_{st}^{\neg\emptyset}$ coincide. The standard (non-deterministic) algorithm for $\mathit{Exists}_\sigma^{\neg\emptyset}$ first guesses a non-empty set and then verifies that it is an extension. Now let be V be the complexity for verifying an extension then the above gives use a NP^V algorithm for $\mathit{Exists}_\sigma^{\neg\emptyset}$. However, the algorithm is only adequate for *st*, *ad*, *co*, and *id* semantics, for the other semantics the problem can be solved more efficiently: for *cf* and *na* semantics it suffices to find one argument that is no self-attacking; for *gr* semantics one tests whether there is an argument that is not attacked by other arguments; for *cf2* and *stg* semantics the problem reduces to test whether there is a non-empty conflict-free set; and finally for *pr* and *sst* semantics the problem reduces to check whether there is a non-empty admissible set.

Uniqueness. When testing for the uniqueness of extensions again stable semantics has a special behaviour. While for all the other semantics we are guaranteed that there is at least one extension for stable semantics we have to perform an additional check that there exists an extension. To check that there are not two (or more) extensions we use the following NP^V procedure that shows that an AF has at least two extensions. It first non-deterministically guesses two sets and then verifies that they are different from each other and both are extensions (for the latter the V oracle is used). That is we have an coNP^V for testing that an AF has at most one extension, which for all semantics, except stable, is equivalent to Unique_σ.

Let us now briefly discuss the results for the specific semantics listed in Table 1 (cf. [Dvořák, 2017]). First, *cf* semantics yield a unique extension iff all arguments in the AF are self-attacking, and naive semantics yield a unique extensions if there is no conflict between non-self-attacking arguments. Both criteria can be easily tested in L. Second, *gr* and *id* always yield a unique extension and thus an algorithm can answer “yes” without any computation. For *st*, *ad*, and *co* we can use the coNP^V algorithm. However, for stable we have to use an additional NP-algorithm to test whether there exists an extension, resulting in a DP algorithm for Unique_{st}. The situation of *cf2* is different. As shown in [Kröll *et al.*, 2017] one can enumerate *cf2* extensions with polynomial delay and thus can also test uniqueness in polynomial time (by computing the first two *cf2* extensions). For *pr* semantics the coNP^V algorithm can be improved by the observations that an AF has two (or more) preferred extensions iff it has two admissible sets that are in conflict with each other. Thus it suffices to guess two sets, and verify that both sets are admissible and there is a conflict between the two sets. For *sst*, and *stg* the exact complexity is still open but one can also do better than the standard algorithm. That is, the standard algorithm would give a Π_2^{P}-algorithm but one can actually decide uniqueness with a Θ_2^{P}-algorithm [Dvořák, 2017].

3.4 Computational Advantages of Specific Graph-Classes

As most of the reasoning tasks are hard for most of the semantics, one is interested in criteria that make concrete instances tractable. Here we consider special graph classes such that abstract argumentation frameworks within this graph class can be evaluated efficiently. However, these tractability results often only hold for specific semantics and not for the others. This section is based on [Dunne, 2007] and follow up work. In the following we omit semantics where the reasoning tasks are already in L in the general case.

3.4.1 Acyclic AFs

For acyclic AFs we have that each argument is either contained in the grounded extension or attacked by an argument in the grounded extension. Thus the grounded extension is the only stable extension and all the semantics under our consideration coincide. Thus, for all semantics reasoning reduces to computing the grounded extension, but which itself remains P-hard even for acyclic bipartite AFs [Dvořák, 2012a]. The results are summarised in Table 2. Notice that $Skept_{ad}$ is trivially false even in the general case.

σ	*gr*	*st*	*ad*	*co*	*pr*	*sst*	*stg*	*cf2*	*id*
$Cred_\sigma$	P-c	P-c	P-c	P-c	P-c	P-c	P-c	P-c	P-c
$Skept_\sigma$	P-c	P-c	trivial	P-c	P-c	P-c	P-c	P-c	P-c

Table 2: Complexity for acyclic AFs.

For admissibility based semantics there is a conceptual difference how they deal with even (length) and odd (length) cycles. In an even-cycle there are three admissible sets, the empty set, the set of odd numbered arguments and the set of even numbered arguments, while for an odd-cycle the only admissible set is the empty set. Due to different treatments even and odd-cycles have a quite different impact on the computational complexity.

Even-cycle free AFs. Let us first consider the impact of even-cycles for admissibility based semantics. By an observation in [Dunne and Bench-Capon, 2001] each AF with at least two preferred extensions has an even-cycle. This even holds for complete extensions, i.e., each AF with two complete extensions has an even-cycle [Dvořák, 2012a]. The number of even-cycles in an AF bounds the number of complete and thus also preferred extensions. Thus if an AF has no even-cycles the grounded extension is as well the unique preferred extension and therefore the only candidate for being a stable extension. Again the reasoning tasks for the admissibility based semantics reduce to computing the grounded extension.

σ	*gr*	*st*	*ad*	*co*	*pr*	*sst*	*stg*	*cf2*	*id*
$Cred_\sigma$	P-c	P-c	P-c	P-c	P-c	P-c	Σ_2^{P}-c	NP-c	P-c
$Skept_\sigma$	P-c	P-c	trivial	P-c	P-c	P-c	Π_2^{P}-c	coNP-c	P-c

Table 3: Complexity results for even-cycle free AFs.

The picture is different for stage and cf2 semantics which are not based on admissibility and handle odd and even-cycles in a similar way. Both maintain their full complexity for even-cycle free AFs [Dvořák and Gaggl, 2016]. The results for even-cycle free AFs are summarised in Table 3.

Odd-cycle free AFs. Odd-cycles are of interest as they distinguish stable from preferred semantics. By a result from [Dung, 1995] in the absence of odd-cycles stable and preferred semantics coincide, i.e., the AF is coherent. As this implies that there is at least one stable extension also semi-stable and stage semantics coincide with stable and preferred semantics in odd-cycle free AFs. But then the complexity of preferred, semi-stable and stage drops down to the complexity of stable, which however stays the same as in the general case. Also admissible, complete and cf2 are not profiting from the absence of odd-cycles, which is proven by the fact that both the standard translation and the modification for skeptical acceptance do not make use of odd-cycles [Dimopoulos and Torres, 1996; Gaggl and Woltran, 2013]. An overview is given in Table 4.[11]

[11] The result for ideal has not been stated before, but is immediate by a generic result in [Dunne *et al.*, 2013] stating that $Cred_{id}$ belongs to $\mathsf{coNP}^{\mathsf{V}}$ where V is the complexity of Ver_{pr} and the fact that the reduction for coNP hardness in [Dunne, 2008] constructs an odd-cycle free AF.

σ	*gr*	*st*	*ad*	*co*	*pr*	*sst*	*stg*	*cf2*	*id*
$Cred_\sigma$	P-c	NP-c	NP-c	NP-c	NP-c	NP-c	NP-c	NP-c	coNP-c
$Skept_\sigma$	P-c	NP-c	trivial	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c

Table 4: Complexity results for odd-cycle free AFs.

3.4.2 Bipartite AFs

Bipartite AFs, AFs where the arguments can be partitioned on two conflict-free sets, are a special case of odd-cycle free AFs and thus again stable, preferred, semi-stable, and stage semantics coincide. There is a polynomial time algorithm for computing the credulously accepted arguments [Dunne, 2007], which is based on the following observation. Let the arguments be partitioned in two conflict-free sets A, B. Arguments in A are only attacked by arguments in B and can only be defended by arguments in A. Now the algorithms starts with the set A and tests if it is admissible. If yes then all arguments in the set A are credulously accepted, otherwise all arguments in A which are not defended can not be in any admissible set, i.e., they are not credulously accepted. In the latter case the algorithm removes the undefended arguments and tests the new set for being admissible. It proceeds until it reaches an admissible set (which might be empty). At the end we have that all arguments which are in the computed admissible set are credulously accepted and the remaining arguments in A are not. We then apply the same algorithm to the set B to compute the remaining credulously accepted arguments.

To decide skeptical acceptance we can use that for stable semantics an argument is skeptically accepted iff none of its attackers is credulously accepted.[12] Hence given that we can compute all credulously accepted arguments in polynomial time we can also decide skeptical acceptance in P.

In [Dvořák and Gaggl, 2016] it was shown that the above algorithm also works for cf2 semantics. The result for ideal semantics follows from the result for preferred semantics and the polynomial-time algorithm in [Dunne *et al.*, 2013] that computes the ideal extension given the skeptically accepted arguments w.r.t. preferred semantics.

The results are summarised in Table 5. Notice that while in bipartite AFs we can efficiently compute credulous and skeptical acceptance, in contrast to previous tractable fragments, we cannot compute all extensions nor have a good handle on them. This is mirrored by the fact that deciding whether two arguments appear together in one extension is NP-hard [Dunne, 2007]. One can also imagine to consider

[12] Each stable extension has to either contain the argument or one of its attackers.

generalisations of bipartite graphs, so called k-partite graphs, where the arguments can be divided into k conflict-free sets. However, for $k \geq 3$ there are no computational advances from k-partite graphs [Dunne, 2007].

σ	*gr*	*st*	*ad*	*co*	*pr*	*sst*	*stg*	*cf2*	*id*
$Cred_\sigma$	P-c	P-c	P-c	P-c	P-c	P-c	P-c	P-c	P-c
$Skept_\sigma$	P-c	P-c	trivial	P-c	P-c	P-c	P-c	P-c	P-c

Table 5: Complexity results for bipartite AFs.

3.4.3 Symmetric AFs

Here we consider AFs where each attack is symmetric. As each attacker is immediately defended by the symmetric attack the notion of admissibility reduces to conflict-freeness. Considering grounded semantics, in each non-trivial connected component of arguments all arguments are attacked by at least one other argument and thus none of them can be in the grounded extension. Thus computing the grounded extension reduces to find all isolated arguments which can be done in L. Hence all reasoning tasks for grounded, admissible, complete, and preferred semantics are in L. Moreover, also cf2 coincides with naive semantics [Dvořák and Gaggl, 2016]. Finally for symmetric AFs the set of skeptically accepted arguments is always admissible and thus the skeptically accepted arguments coincide with the ideal extension.

σ	*gr*	*st*	*ad*	*co*	*pr*	*sst*	*stg*	*cf2*	*id*
$Cred_\sigma$	L	L/NP-c	L	L	L	L/Σ_2^{P}-c	L/Σ_2^{P}-c	L	L
$Skept_\sigma$	L	L/coNP-c	trivial	L	L	L/Π_2^{P}-c	L/Π_2^{P}-c	L	L

Table 6: Complexity results for symmetric AFs.

For symmetric AFs one often also requires [Coste-Marquis *et al.*, 2005] that the AF is irreflexive, i.e., it has no self attacks. In that case each naive extension is also a stable extension and thus also semi-stable, and stage coincides with naive. That is, the corresponding reasoning tasks become tractable. However, if we allow self-attacks in the framework then these three semantics maintain their full complexity [Dvořák, 2012a].

graph class	*ad*	*co*	*pr*	*sst*	*st*	*stg*	*cf2*	*id*
acyclic	FPT	FPT	FPT	FPT	FPT	hard	hard	FPT
noeven	XP	XP	XP	XP	XP	hard	hard	XP
bipartite	hard	hard	hard	hard	hard	hard	hard	hard
symmetric	hard	hard	hard	hard	hard	hard	hard	hard

Table 7: Complexity of acceptance problems, parametrized by the distance from graph classes that allows for efficient algorithms.

3.5 Fixed-Parameter Tractable Fragments

In the previous section we considered properties of the graph structure that make argumentation problems tractable. In this section we study parameters that are quantitative measures for some kind of structure in the graph, with the goal to find parameters such that the complexity of the problem rather scales with the parameter than with input size. That is, we are looking for parameters that allow for fixed-parameter tractable algorithms.

Backdoors for abstract argumentation. One approach to fixed-parameter tractable algorithms is the so called backdoor approach [Dvořák *et al.*, 2012a]. The idea is to start from a tractable fragment, define some kind of distance to the tractable fragment, and then use the distance as a parameter for the reasoning problem. The hope behind this approach is that the running time will scale with the distance to the tractable fragment instead of jumping instantly to the full problem complexity when leaving the fragment. In argumentation one can use the graph classes discussed above as tractable fragments and as distance one considers the number of arguments that have to be deleted from an AF to fall into the graph class.

Definition 3.9. *Let $\mathcal{G}$ be a graph class and $F = (A, R)$ an AF. We define* $\mathrm{dist}_{\mathcal{G}}(F)$ *as the minimal number k such that there exists a set $S \subseteq A$ (the* backdoor set*) with $|S| = k$ and $(A \smallsetminus S, R \cap ((A \smallsetminus S) \times (A \smallsetminus S)) \in \mathcal{G}$. If there is no such set S we define* $\mathrm{dist}_{\mathcal{G}}(F) = \infty$.

We will see that this parametrization only works for certain fragments and semantics, while for other fragments and semantics we have the full complexity even for AFs with constant distance to the fragment. Table 7 summarises the results for the semantics under our considerations (again we omit semantics already tractable in the general case). All results are due to [Ordyniak and Szeider, 2011;

Dvořák *et al.*, 2012a] [13], except the results for cf2 semantics which are due to Dvořák and Gaggl, 2016]. The entries in Table 7 are to be read as follows. FPT: all reasoning tasks are in FPT; XP: all reasoning tasks are in XP; hard: all problems are as hard as for general graphs even for instances with a fixed distance to the fragment (and are at least NP/coNP-hard for distance 1).

In the remainder of the section we will first present the algorithm that underlies the FPT and XP results and then exemplify some hardness results.

FPT backdoor-algorithms. The positive results are all based on the fact that the number of complete extensions is small and we can compute them efficiently. As soon as we have the complete extensions all the reasoning tasks for admissibility based semantics can be answered efficiently. The algorithms for complete semantics consist of two parts: first one has to compute the backdoor set; second given the backdoor set one has to compute the complete extensions.

Let us first consider computing a backdoor. The detection of *acyc*-backdoors for AFs is equivalent to the so-called *directed feedback vertex set* problem in graph theory, which is known to be fixed-parameter tractable [Chen *et al.*, 2008]. For detecting *noeven*-backdoors in AFs the following algorithm is known, which only shows the problem to be in XP. By a result of Robertson *et al.* [1999] one can test in polynomial time whether a graph is in *noeven* or not. Now, to find a backdoor of size k one can simply iterate over all sets of size k and test whether removing these arguments break all even-cycles. As there are $\Theta(n^k)$ many such sets the algorithm is not fixed-parameter tractable and thus only shows the problem to be in XP.

Now let us assume we already have a backdoor set. We consider labels for arguments that correspond to their status in the extension. An argument is labelled `in` if it is in the extension, `out` if it is not in the extension but attacked by an argument in the extension, and `undec` otherwise. The algorithm tests all possible assignments of labels to the arguments in the backdoor set (these are 3^k many) and for each of them propagates the labels to the remaining AF (which is acyclic or noeven) according to the characteristic function. That is, a node gets label `out` as soon as one attacker is labelled `in`, label `in` if all attackers are labelled `out` and label `undec` if all attackers are labelled and none of the above applies. Finally, one considers the set of arguments labelled `in` and keeps the set if it is a complete extension or withdraws them otherwise. As we do this for each possible labelling of the backdoor set we finally get the set of all complete extensions, which is of size at most 3^k. As one can propagate the labels in polynomial time the, total running

[13]Notice, that ideal semantics is not explicitly mentioned in [Dvořák *et al.*, 2012a], but the results follow immediately from the results presented for preferred semantics.

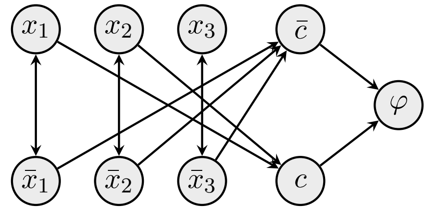

Figure 6: Hardness reduction for $Cred_{ad}$ and backdoors to bipartite graphs, illustrated for the propositional formula φ, with clauses $c = \{x_1, x_2\}$, and $\bar{c} = \{\bar{x}_1, \bar{x}_2, \bar{x}_3\}$.

time of the algorithm is 3^k multiplied by some polynomial and thus in FPT.

Finally combining the results for computing a backdoor set and for evaluating an AF given a backdoor we have an FPT algorithm for acyclic AFs and an XP algorithm for noeven AFs. Notice that the XP complexity for noeven AFs comes solely from the algorithm for computing the backdoor, the evaluation itself is in FPT.

Hardness Results. The hardness proofs work very much like for the general case, one has to give a reduction from a hard problem but additionally take into account the graph structure [Dvořák *et al.*, 2012; Dvořák 2012c; Dvořák *et al.*, 2014a; Dvořák and Gaggl, 2016]. We exemplify such a reduction for credulous reasoning under admissible, complete, preferred and cf2 semantics and backdoors for bipartite graphs. To this end consider the standard translation from proposition logic and the NP-hard problem monotone SAT, of deciding whether a formula in CNF where each clause either contains solely positive or solely negative literals is satisfiable. As each clause either contains solely positive literals or solely negative literals the graph constructed by the standard translation is almost bipartite (cf. Figure 6). That is, there are no edges between the arguments corresponding to positive literals and negative clauses and no edges between the arguments corresponding to negative literals and positive clauses. Thus, when deleting φ from the graph the graph becomes bipartite with two independent sets, one containing the positive literals and the negative clauses, and one containing the negative literals and the positive clauses. We obtain that credulous reasoning is NP-hard even for graphs with distance 1 to bipartite graphs.

Further FPT Results. Besides backdoors to tractable fragments several other approaches for parametrizations can be found in the literature. One approach is to consider graph parameters that measure structural properties, most prominently tree-width, a parameter that, roughly speaking, measures how tree-like a graph is. Results for tree-width (and the related parameter clique-width) can be either obtained by dynamic programming algorithms that exploit the structural properties or by powerful meta-theorems. These meta-theorems basically say that every property which can be characterised by a formula from monadic-second order logic (MSO) over a graph structure can be tested in FPT w.r.t. tree-width and clique-width. Results via the MSO meta-theorems are given in [Dunne, 2007; Dvořák *et al.*, 2012c] concrete dynamic programming algorithms are given in [Dvořák *et al.*, 2012b; Charwat, 2012] for tree-width and in [Dvořák *et al.*, 2010] for clique-width. Moreover, in [Dvořák *et al.*, 2012b] a lot of parameters specific to directed graphs, e.g. directed tree-width, are shown to be not applicable for FPT algorithms in abstract argumentation.

Finally for semantics harder than NP one can also think about backdoors to graph classes that allow to solve problems in NP or coNP [Dvořák *et al.*, 2014a]. While this does not give FPT results it still reduces complexity, with notable effects on the practical resolvability.

3.6 Computational Problems related to Labelling-Based Semantics

So far our complexity analysis was in terms of extension-based semantics (which is in accordance with the literature), in this section we discuss some computational aspects related to labelling-based semantics.

Labelling-based semantics. Beside the so-called extension-based semantics we have considered so far, there are several approaches defining argumentation semantics via certain kinds of argument labellings. As an example we consider the popular approach of 3-valued labellings by Caminada and Gabbay [2009] and in particular their complete labellings. Basically, such a labelling is a three-valued function $\mathcal{L}ab$ that assigns one of the labels `in`, `out` and `undec` to each argument, with the intuition behind these labels being the following. An argument is labelled with: `in` if it is accepted, i.e., it is defended by the `in` labelled arguments; `out` if there are strong reasons to reject it, i.e., it is attacked by an accepted argument; `undec` if the argument is undecided, i.e., neither accepted nor attacked by accepted arguments. Complete labellings can be one-to-one mapped to complete extensions by considering the set of `in` labelled arguments and vice versa, by labelling all arguments in the extension with `in` all arguments attacked by the extension with `out` and the remaining arguments with `undec` [Caminada and Gabbay, 2009]. Notice that this is not only a property of complete semantics but this one-to-one correspondence holds for most argumentation semantics.

Computational problems. Given the above correspondence between labellings and extensions, the tasks of computing all labellings and all extensions are, from

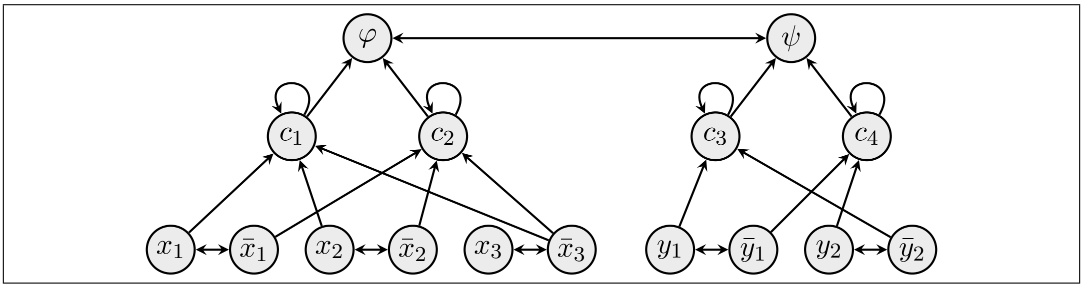

Figure 7: Reduction for showing the DP-hardness of weak acceptance. AF $F_{\varphi,\psi}$ for the propositional formulae φ, with clauses $\{x_1, x_2, \bar{x}_3\}$,$\{\bar{x}_1, \bar{x}_2, \bar{x}_3\}$, and ψ, with clauses $\{y_1, \bar{y}_2\}$, $\{\bar{y}_1, y_2\}$.

a computational point of view, equivalent and the same holds for credulous and skeptical reasoning. However, three-valued labellings allow for more fine-grained acceptance statuses of arguments. Wu and Caminada 2010 introduced the notion of justification status of an argument w.r.t. a semantics which is given by the set of labels that are assigned by at least one labelling of the semantics. That is, given an AF F and a labelling-based semantics $\sigma_{\mathcal{L}ab}$ the *justification status* $\mathcal{JS}_{\sigma_{\mathcal{L}ab}}(F, a)$ of an argument a in F is given by $\mathcal{JS}_{\sigma_{\mathcal{L}ab}}(F, a) = \{\mathcal{L}ab(a) \mid \mathcal{L}ab \in \sigma_{\mathcal{L}ab}(F)\}$. The above definition gives rise to eight different justification statuses, most prominently the set $\{\texttt{in}\}$ called strong accept, which corresponds to skeptical acceptance, and the set $\{\texttt{in}, \texttt{undec}\}$ called weak accept.[14] We are now faced with the computational problem of *verifying the justification status* of an argument, which was studied in [Dvořák, 2012b].

Algorithms. Compared with credulous and skeptical acceptance, where we either search for an extension containing a specific argument or for an extension not containing a specific argument, the problem of testing whether an argument has a specific justification status, e.g., whether it is a weakly accepted, has two sources of complexity. First, we have to search for labellings that assign the labels appearing in the justification status, e.g., `in` and `undec` for weak acceptance, and second we have to make sure that no labelling assigns one of the labels not in the in the justification status, e.g., `out` for weak acceptance, to a. For complete semantics this means we have to perform both an NP search for the good labels and a coNP search for the bad labels, which together gives a DP algorithm.

[14]Notice that credulous acceptance of argument a corresponds to the query $\texttt{in} \in \mathcal{JS}_{\sigma}(F, a)$.

Hardness. To prove DP-hardness of weak acceptance w.r.t. complete semantics one starts from an instance (φ, ψ) of the DP-complete SAT−UNSAT problem and constructs the AF $F_{\varphi,\psi}$ (see Figure 7) as follows. First one applies the standard translation to each of the two formulae and then makes the arguments c corresponding to clauses unacceptable, by adding self-attacks. Finally the arguments φ and ψ are connected by a mutual attack. As in the standard translation we have that φ, respectively ψ, is credulously accepted iff φ, respectively ψ, is satisfiable. Moreover, (a) φ is labelled `out` iff ψ is labelled `in` by some labelling, i.e., if ψ is credulously accepted, and (b) the grounded labelling maps all arguments to `undec` and thus `undec` $\in \mathcal{JS}_{\sigma_{\mathcal{L}ab}}(F_{\varphi,\psi}, \varphi)$. We then have that the argument φ is weakly accepted iff φ is satisfiable and ψ is unsatisfiable, that is iff (φ, ψ) is a "yes" instance of SAT−UNSAT. Thus we have a reduction from SAT−UNSAT to weak acceptance and can conclude that also the latter is DP-hard.

3.7 Discussion

As illustrated in Table 1 there is a significant difference in the computational complexity between the different semantics. Let us first consider the polynomial-time computable semantics. Grounded semantics distinguishes itself from the remaining semantics by the fact that it has a unique extension which can be efficiently computed in an iterative fashion by applying the characteristic function. For conflict-free and naive sets the good complexity comes from the fact that we can decide the reasoning problems without computing the actual conflict-free, respectively naive, sets. However, there are AFs with exponentially many conflict-free, respectively naive, sets and there are non-standard problems the are computationally hard, for instance counting the number of conflict-free, respectively naive, sets [Baroni *et al.*, 2010].

On the NP, coNP layer of the polynomial-hierarchy we have semantics with potentially exponentially many extensions but where each set itself can be easily tested to be an extension. That is, the source of the computational hardness is the fact that one, in the worst case, has to check many sets to find a witness for credulous acceptance, respectively to find a counter-example for skeptical acceptance. However, these problems can be efficiently encoded in formalisms where the corresponding problems are NP- and coNP-hard, like propositional logic, and then can be evaluated with corresponding systems for these formalisms [Besnard and Doutre, 2004].

Finally, we have semantics that require some sort of subset maximisation which adds an additional source of complexity. Thus, these semantics are harder than NP and located at the second level of the polynomial-hierarchy. For reduction-based approaches this implies that one cannot efficiently translate them to a single instance of propositional logic but has either to consider richer formalisms like QBFs [Egly and Woltran, 2006; Arieli and Caminada, 2012] or ASP [Egly *et al.*, 2010] or consider iterative approaches [Cerutti *et al.*, 2014; Dvořák *et al.*, 2014a] that make several calls to a SAT-Solver. The different levels of hardness of different semantics are also mirrored by the results of the First International Competition on Computational Models of Argumentation [Thimm and Villata, 2015; Thimm *et al.*, 2016], where the computational tasks for preferred semantics appear significantly harder than the corresponding tasks for stable or complete semantics.

Notice that there are several established semantics which are beyond the scope of this work. First there is the scheme of resolution-based semantics [Baroni *et al.*, 2011c], with resolution-based grounded semantics being the most prominent instantiation. A comprehensive complexity analysis for resolution-based grounded semantics can be found in [Baroni *et al.*, 2011c], which is complemented by results in [Dvořák *et al.*, 2012c; Dvořák *et al.*, 2014b]. Another semantics we neglected is eager semantics [Caminada, 2007], whose complexity was studied in the generalised setting of parametrized ideal semantics [Dunne *et al.*, 2013].

4 Complexity of Assumption-based Argumentation

With Dung's abstract argumentation frameworks we focused on the issue of finding coherent sets of simultaneously acceptable arguments, but neglected the effort for constructing these frameworks and for drawing conclusions from the accepted arguments. With Assumption-based Argumentation [Bondarenko *et al.*, 1997] we now switch to a formalism that covers the whole argumentation process. That is, arguments and conflicts are constructed from a knowledge base, then acceptable sets, i.e., extensions, are identified, and finally one draws conclusions from the extensions. We are in particular interested in how these additional steps affect the overall computational complexity.

In this section will discuss complexity results for Assumption-based Argumentation which are due to the work of Dimopoulos *et al.* [1999; 2000; 2002] and the later work on ideal semantics [Dunne, 2009].[15] We first briefly introduce assumption-based frameworks and the different semantics thereof and define the core reasoning problems in assumption-based argumentation. We then discuss procedures to solve the reasoning problems, which give us upper bounds for the computational complexity. As most of these procedures are of high complexity we also discuss the special case of flat ABFs which allows for a milder complexity. Finally, we discuss some

[15]The complexity of Assumption-based Argumentation was also briefly discussed in the earlier survey on the complexity of argumentation [Dunne and Wooldridge, 2009].

hardness results showing that the presented procedures are essentially optimal.

4.1 Assumption-based Argumentation

We first briefly recall the definitions of assumption-based argumentation, for a comprehensive introduction the reader is referred to [Toni, 2014].

For an assumption-based framework we assume a *deductive system* $(\mathcal{L}, \mathcal{R})$, where $\mathcal{L}$ is a formal language and $\mathcal{R}$ a set of inference rules that induces a derivability relation $\vdash$. Given a theory $T \subseteq \mathcal{L}$ the *deductive closure* $Th(T)$ of T is defined as $Th(T) = \{\alpha \in \mathcal{L} \mid T \vdash \alpha\}$.

Definition 4.1. *An abstract* assumption-based framework *(ABF) is a tuple* $\left\langle \mathcal{L}, \mathcal{R}, A, \overline{} \right\rangle$ *with* $(\mathcal{L}, \mathcal{R})$ *a deductive system,* $A \subseteq \mathcal{L}$ *is a (non-empty finite) set, with elements referred to as assumptions; and the* contrary *function* $\overline{}$*, a total mapping from* A *into* $\mathcal{L}$*.*

An extension of an ABF is a set of assumptions $\Delta \subseteq A$ meeting some requirements.

Definition 4.2. *Given an ABF and an assumption set* $\Delta \subseteq A$ *we say that* Δ attacks *an assumption* $\alpha \in A$ *if* $\bar{\alpha} \in Th(\Delta)$*. Further we say that an assumption set* Δ *attacks an assumption set* Δ' *if* Δ *attacks at least one* $\alpha \in \Delta'$

We will further require that assumptions sets are closed, i.e., we can not derive additional assumptions.

Definition 4.3. *We call an assumption set* Δ closed *if* $Th(\Delta) \cap A = \Delta$*.*

It is often the case that the derivability relation is such that all assumption sets are closed, in that case we call the ABF *flat*.

We are now prepared to define the standard semantics for ABFs.

Definition 4.4. *Given an ABF* F *and an assumption set* $\Delta \subseteq A$*.* Δ *is called*

- *stable extension* ($\Delta \in st(F)$)*, if* Δ *is closed,* Δ *does not attack itself, and* Δ *attacks each assumption* $\alpha \in A \setminus \Delta$*.*
- *admissible set* ($\Delta \in ad(F)$)*, if* Δ *is closed,* Δ *does not attack itself, and for all closed assumption sets* $\Delta' \subseteq A$*, if* Δ' *attacks* Δ *then also* Δ *attacks* Δ'*.*
- *preferred extension* ($\Delta \in pr(F)$)*, if* Δ *is a subset-maximal admissible assumption set.*

Moreover, for flat frameworks also *ideal semantics* can be defined [Dung *et al.*, 2006; 2007]. The unique ideal extensions $id(F)$ is the maximal admissible set Δ that is contained in all preferred extensions.[16]

We have that every stable assumption set is also a preferred assumption set, and every preferred assumption set is an admissible assumptions set, but not vice versa. However, each admissible assumption set is a subset of some preferred assumption set. Moreover, if the ABF is flat the empty assumption set is always admissible.

4.2 Reasoning Problems

As for abstract argumentation we are mainly interested in computing acceptance statuses of statements instead of extensions. However, the reasoning tasks we consider will give us a good impression of the complexity of computing extensions. That is, we again consider credulous and skeptical acceptance but now of a sentence $\varphi \in \mathcal{L}$ instead of an argument. More concretely we either want to decide whether there is at least one extension that entails φ (credulous reasoning) or whether φ is entailed by each extension. This gives rise to the following computational problems for an assumption-based argumentation semantics σ.

- *Credulous Acceptance $Cred_\sigma$*: Given ABF F and a sentence $\varphi \in \mathcal{L}$. Is $\varphi \in Th(\Delta)$ for some assumption set $\Delta \in \sigma(F)$?
- *Skeptical Acceptance $Skept_\sigma$*: Given ABF F and a sentence $\varphi \in \mathcal{L}$. Is $\varphi \in Th(\Delta)$ for all assumption sets $\Delta \in \sigma(F)$?

Beside the above reasoning problems we again consider the task of verifying extensions, i.e., one is given an assumption set and has to verify that it is an extension of a given semantics σ.

- *Verification of an Extension Ver_σ*: Given ABF $F = \langle T, A, \bar{\ } \rangle$ and an assumption set $\Delta \subseteq A$. Is $\Delta \in \sigma(F)$?

4.3 Procedures to solve ABA Reasoning Problems

In ABA, new computational challenges come up when compared with Dung's abstract argumentation. While in Dung's abstract argumentation arguments and attacks are given explicitly, they are only given implicitly in ABFs and depend on the set of assumptions and the derivability relation $\vdash$. That is, we get two additional sources of complexity: (1) the construction of arguments, and (2) the identification

[16]Notice that uniqueness and other properties of the ideal extension are only guaranteed for flat ABFs.

	General ABFs			*Flat ABFs*		
σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ
st	$\mathsf{NP}^{\mathcal{C}}$	$\mathsf{coNP}^{\mathcal{C}}$	$\mathsf{P}^{\mathcal{C}}$	$\mathsf{NP}^{\mathcal{C}}$	$\mathsf{coNP}^{\mathcal{C}}$	$\mathsf{P}^{\mathcal{C}}$
ad	$\mathsf{NP}^{\mathsf{NP}^{\mathcal{C}}}$	$\mathsf{coNP}^{\mathsf{NP}^{\mathcal{C}}}$	$\mathsf{coNP}^{\mathcal{C}}$	$\mathsf{NP}^{\mathcal{C}}$	$\mathcal{C}$	$\mathsf{P}^{\mathcal{C}}$
id	–	–	–	$\mathsf{P}_{\parallel}^{\mathsf{NP}^{\mathcal{C}}}$	$\mathsf{P}_{\parallel}^{\mathsf{NP}^{\mathcal{C}}}$	$\mathsf{P}_{\parallel}^{\mathsf{NP}^{\mathcal{C}}}$
pr	$\mathsf{NP}^{\mathsf{NP}^{\mathcal{C}}}$	$\mathsf{coNP}^{\mathsf{NP}^{\mathsf{NP}^{\mathcal{C}}}}$	$\mathsf{NP}^{\mathsf{NP}^{\mathcal{C}}}$	$\mathsf{NP}^{\mathcal{C}}$	$\mathsf{coNP}^{\mathsf{NP}^{\mathcal{C}}}$	$\mathsf{NP}^{\mathcal{C}}$

Table 8: Complexity upper bounds for different types of ABFs. $\mathcal{C}$ denotes the complexity of deciding the $\vdash$ relation.

of conflicts between them. Both highly depend on the complexity of deciding the derivability relation $\vdash$. Thus, upper bounds for the complexity in assumption-based argumentation usually assume that the derivability relation $\vdash$ can be decided in some complexity class $\mathcal{C}$ and the actual complexity results are then given in terms of some $\mathcal{C}$-oracle complexity classes.

Verifying an Assumption Set. First, we consider the problem of verifying an assumption set Δ as an extension and start with stable semantics. We have to check that (i) Δ is closed, (ii) Δ is conflict-free, and (iii) Δ attacks every assumption $\alpha \in A \setminus \Delta$. Each of these checks can be done in $\mathsf{P}^{\mathcal{C}}$ as follows: For (i) one has to check whether $\Delta \vdash \alpha$ for $\alpha \in A \setminus \Delta$ which just requires a linear number of $\vdash$ computations. For (ii) one has to check whether $\Delta \nvdash \bar{\alpha}$ for $\alpha \in \Delta$ which again just requires a linear number of $\vdash$ computations. Finally, (iii) can also be checked by a linear number of $\vdash$ computations and thus a stable set can be verified in $\mathsf{P}^{\mathcal{C}}$. For admissible semantics verification is a bit harder. Here instead of condition (iii) we have to verify that for all closed assumption sets $\Delta' \subseteq A$, if Δ' attacks Δ then also Δ attacks Δ'. This can be done with a $\mathsf{coNP}^{\mathcal{C}}$-algorithm that guesses a counter-example Δ' and then verifies via the $\mathcal{C}$ oracle that Δ' is closed, Δ' attacks Δ, and Δ' is not attacked by Δ. In total we have that verifying an admissible extension is in $\mathsf{coNP}^{\mathcal{C}}$. For preferred semantics we additionally have to take into account the maximality check which leads to a $\mathsf{coNP}^{\mathsf{NP}^{\mathcal{C}}}$-algorithm.

Reasoning. The complexity upper bounds for skeptical and credulous reasoning are immediate by the algorithms for verifying extensions. We can decide the acceptance of a sentence by first guessing an assumption set, second verifying that the guessed set is an extension and finally deciding via a $\mathcal{C}$ oracle whether the extension entails the queried sentence. The corresponding complexity results are given in the left part of Table 8 (recall that ideal semantics was only introduced for flat ABFs). As for Dung's AFs, credulous reasoning with preferred semantics reduces to credulous reasoning with admissible semantics and thus has a lower complexity than skeptical reasoning. Moreover, in contrast to Dung's AFs, we have a complexity gap between stable and admissible semantics, which is due to the fact that for admissible extensions for each attacking assumption set we have to test whether it is closed or not.

Flat ABFs. Flat ABFs as a special class of ABFs that provide milder complexity. Recall that in flat ABFs each assumption set is already closed and we thus do not have to check this in the algorithms. Let us now reconsider the problem of verifying an admissible extension Δ. As Δ is closed we only have to check whether (i) Δ is conflict-free, and (ii) for all assumption sets $\Delta' \subseteq A$, if Δ' attacks Δ then also Δ attacks Δ'. The latter simplifies to checking whether $\{\alpha \in A \mid \Delta \nvdash \bar{\alpha}\}$ does not attack Δ, which can be decided in $\mathsf{P}^{\mathcal{C}}$. Thus, verifying admissible extensions in flat ABFs is in $\mathsf{P}^{\mathcal{C}}$ and hence also verifying preferred extensions is in $\mathsf{coNP}^{\mathcal{C}}$. This gives improved complexity bounds for credulous and skeptical acceptance listed in in Table 8 in the column *flat*. Finally, notice that in flat ABFs the empty set is always admissible and thus only the assumptions contained in $Th(\varnothing)$ are skeptically accepted. That is, skeptical reasoning reduces to testing whether $\varphi \in Th(\varnothing)$, which is in $\mathcal{C}$.

The *ideal extension* can be computed by the same algorithm as for Dung AFs [Dunne, 2009]. That is, one first determines the credulously accepted assumptions w.r.t. admissible semantics that are not attacked by other credulously accepted assumptions. Given those arguments one iteratively removes assumptions that can not be defended until an admissible set, the ideal extension, is reached. Overall, this gives an $\mathsf{P}_{\parallel}^{\mathsf{NP}^{\mathcal{C}}}$ algorithm for credulous and skeptical reasoning as well as for the verification problem.

4.4 Complexity lower bounds

While the upper bounds can be given in a generic fashion, which immediately gives upper bounds/algorithms for each instantiation, hardness results only exist for concrete formalisms. However, the complexity results for the concrete instantiations [Dimopoulos *et al.*, 2002] show that the generic upper bounds are tight in the sense that there are formalisms where the lower bounds match the generic upper bounds. In Table 9 we list the complexity results for Autoepistemic Logic (AEL) [Moore, 1985], Logic Programming (LP) [Gelfond and Lifschitz, 1988], and Default Logic (DL) [Reiter, 1980] all the results are due to Dimopoulos *et al.* [2002] and Dunne [2009]. For

Autoepistemic Logic we have that the ABF is not flat and deciding the $\vdash$ relation is coNP-complete. Thus the complexity results in Table 9 exactly match the generic upper bounds for general ABFs. In contrast, Logic Programming and Default Logic result flat ABFs and for the former the $\vdash$ relation is in P and for the latter the $\vdash$ relation is coNP-complete. In both cases the complexity results in Table 9 exactly match the generic upper bounds for flat ABFs.

	type	stability		Admissibility		Preferability		Ideal
		cred.	skept.	cred.	skept.	cred.	skept.	cred.
AEL	general	Σ_2^{P}-c	Π_2^{P}-c	Σ_3^{P}-c	Π_3^{P}-c	Σ_3^{P}-c	Π_4^{P}-c	–
LP	flat	NP-c	coNP-c	NP-c	P-c	NP-c	Π_2^{P}-c	Θ_2^{P}-c
DL	flat	Σ_2^{P}-c	Π_2^{P}-c	Σ_2^{P}-c	coNP-c	Σ_2^{P}-c	Π_3^{P}-c	Θ_3^{P}-c

Table 9: Completeness results for instantiations of ABA.

For a hardness proof in ABA one has to construct a certain knowledge base in the considered formalism instead of arguments interlinked with conflicts. Thus hardness proofs in the context of ABA are of a different nature than in Dung's abstract argumentation. To exemplify such an hardness proof we next present the hardness result for credulous admissible reasoning in Default Logic which is Σ_2^{P}-complete [Dimopoulos *et al.*, 2002].

Default logic as ABA. A Default theory (W, D) that consists of a set W of propositional formulae[17], called background theory, and a set D of default rules of the form $\frac{\alpha: M\beta_1 \ldots M\beta_n}{\gamma}$, where α, β_i, γ are sentences in propositional logic, can be interpreted as assumption-based framework $\langle \mathcal{L}, \mathcal{R}, A, \overline{} \rangle$ [Bondarenko *et al.*, 1997]. As deductive system one uses the deductive system of propositional logic extended by the set D of default rules, where the intuitive meaning of a default rule is that if we know α is the case and have no basis on which to suppose any $\neg\beta_i$ holds it is reasonable to assume γ. The ABF is now built as follows: the set of assumptions A consists of the expressions of the form $M\beta$, the contrary $\overline{M\beta}$ of an assumption $M\beta$ is $\neg\beta$ and the derivability relation $\vdash$ is given by $\Delta \vdash \phi$ iff $\phi \in Th_{DL}(W \cup \Delta)$ where Th_{DL} is the deductive closure of the deductive system described above.

Hardness of credulous admissible reasoning in DL. To show hardness for credulous admissible reasoning in Default Logic, we give a reduction from the Σ_2^{P}-

[17]Notice that instead of using propositional logic one could also define Default logic on top of first-order logic or any other formal logic.

hard problem $QSAT_{\exists}^2$ of deciding whether a $QBF_{\exists}^2$ is valid. That is, we start with a QBF $\exists Y \forall Z\, \varphi(Y,Z)$ and construct a DL theory $(\emptyset, D)$ and thus the corresponding ABF F as follows. To construct the set of default rules D, we add the two default rules (i) $\frac{My}{y}$ and (ii) $\frac{M\neg y}{\neg y}$ for each variable $y \in Y$. This corresponds to the ABF F with $A = \{My, M\neg y \mid y \in Y\}$ and $\overline{My} = \neg y$, $\overline{M\neg y} = y$ for all $y \in Y$. By that we have that an admissible set can only contain either My or $M\neg y$ but not both, and thus that the admissible sets correspond to the partial truth assignments of Y. Moreover for an admissible set E we have that $E \vdash \varphi$ iff $\varphi(Y,Z)$ is true for all assignments Z under the partial assignment given for Y. That is, φ is credulously accepted iff there is a partial assignment of Y such that for each assignment to Z the formula $\varphi(Y,Z)$ evaluates to true, that is iff $\exists Y \forall Z \varphi(Y,Z)$ is valid.

Example 4.5. *Consider the QBF* $\Phi = \exists y_1, y_2 \forall z_1, z_2\, (y_1 \vee z_2 \vee \neg z_3) \wedge (\neg y_2 \vee z_3)$. *The above reduction would construct*

- *the default rules* $\frac{My_1}{y_1}$, $\frac{My_2}{y_2}$, $\frac{M\neg y_1}{\neg y_1}$ *and* $\frac{M\neg y_1}{\neg y_1}$;
- *the assumption set* $A = \{My_1, My_2, M\neg y_1, M\neg y_2\}$; *and*
- *the contrary function* $\overline{}$ *with* $\overline{My_1} = \neg y_1$, $\overline{My_2} = \neg y_2$, $\overline{M\neg y_1} = y_1$, *and* $\overline{M\neg y_2} = \neg y_2$.

Now, by the above, it must be that Φ *is valid if and only if there is an admissible set* $E \subseteq A$ *such that* $E \vdash (y_1 \vee z_2 \vee \neg z_3) \wedge (\neg y_2 \vee z_3)$. *The formula* Φ *is valid as setting* y_1 *to true and* y_2 *to false makes both clauses true no matter which truth value is assigned to the variables* z_1 *and* z_2. *On the other hand also the set* $E = \{My_1, M\neg y_2\}$ *is admissible and, by our default rules, we have* $E \vdash (y_1 \vee z_2 \vee \neg z_3) \wedge (\neg y_2 \vee z_3)$. $\Diamond$

4.5 Discussion

The upper bounds for the complexity of the reasoning problems in Table 8 indicate that assumption-based argumentation indeed has a higher complexity than just Dung style argumentation. However, by the discussed results for flat argumentation in Table 8 and the concrete instantiations of ABA in Table 9 one can see that the actual complexity heavily depends on the complexity of derivability relation $\vdash$ and the type of the assumption-based framework. For instance for LP we have a flat assumption-based framework and a tractable derivability relation and end up with the same complexity bounds as for Dung's abstract argumentation frameworks. The complexity of deciding the derivability relation directly corresponds to the costs of constructing an argument, or drawing some conclusion when already given an extension. Thus the parameter $\mathcal{C}$ in Table 8 can be interpreted as the costs of these two steps, i.e., constructing arguments and drawing conclusions, in the argumentation process.

For general ABFs the complexity of assumption-based argumentation is quite high and thus it is promising to consider some restrictions of the formalism to get better algorithms. In this work we considered flat ABFs which reduced the complexity significantly. In [Dimopoulos *et al.*, 2002] also two other classes, namely so called simple and normal ABFs, are studied and shown to have computational advantages for certain problems.

5 Computational Problems in Abstract Dialectical Frameworks

In this section we consider abstract dialectical frameworks, a generalisation of Dung style abstract argumentation frameworks. While arguments are still abstract entities abstract dialectical frameworks allow for more complex relations between the arguments. That is, each abstract dialectical framework has a link relation between the arguments, which is not necessarily an attack relation. The semantics of the links is given by acceptance conditions for each argument that define the acceptance status of an argument in dependence on the acceptance status of the predecessor arguments. This allows for classical binary attacks between arguments but also for joint attacks, support and more complex dependencies.

This section is based on the works of Brewka *et al.* [2013], Wallner [2014, Chapter 4], and Strass and Wallner [2015] and organised as follows. We first define abstract dialectical frameworks and semantics thereof. We then discuss and formally define the core computational problems and consider the general computational complexity of abstract dialectical frameworks. Moreover, we discuss a restricted class of ADFs, so called bipolar ADFs, that only allow for links that are attacking or supporting (but might be both), and their computational advantages.

5.1 Abstract Dialectical Frameworks (ADFs)

Here we give a very brief discussion of Abstract Dialectical Frameworks.Notice that in the literature there are several proposals how to define semantics for ADFs, here we will follow the lines of Brewka et al. [Brewka *et al.*, 2013].

Definition 5.1 ([Brewka *et al.*, 2013]). *An abstract dialectical framework is a tuple $D = (S, L, C)$ where*

- *S is a (finite) set of abstract arguments / statements,*

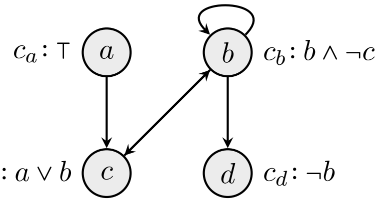

Figure 8: Illustration of an ADF $D = (S, L, C)$ with $S = \{a, b, c, d\}$, $L = \{(a, c), (b, b), (b, c), (c, b), (b, d)\}$, and $C = \{c_a : \top, c_b : b \wedge \neg c,\ c_c : a \vee b, c_d : \neg b\})$.

- *$L \subseteq S \times S$ is a set of links,*
- *$C = \{C_s\}_{s \in S}$ is a set of total functions $C_s : 2^{par(s)} \to \{\mathbf{t}, \mathbf{f}\}$, one for each statement $s \in S$. C_s is called acceptance condition of s.*

Here, we will assume that each acceptance condition C_s is given by a propositional formula φ_s over the predecessors of s. An example is provided in Figure 8.

As first semantics we define (two-valued) *models* of ADFs. To this end we consider two-valued interpretations I that to each $s \in S$ assign either $\mathbf{t}$ or $\mathbf{f}$. Given an interpretation I we will use $I^{\mathbf{t}}$ to denote the set $\{s \in S \mid I(s) = \mathbf{t}\}$ and $I^{\mathbf{f}}$ to denote the set $\{s \in S \mid I(s) = \mathbf{f}\}$.

Definition 5.2. *Let $D = (S, L, C)$ be an ADF, a two-valued interpretation I defined over S is a two-valued model of D if $I \models \varphi_s$ for each $s \in I^{\mathbf{t}}$ and $I \not\models \varphi_s$ for each $s \in I^{\mathbf{f}}$.*

Most of the ADF semantics are based on *3-valued interpretations* [Kleene, 1952] that map each argument in S to one of the values $\mathbf{t}$, $\mathbf{f}$ and $\mathbf{u}$. The three values $\mathbf{t}$, $\mathbf{f}$, $\mathbf{u}$ are ordered, by $<_i$, such that $\mathbf{u} <_i \mathbf{t}$, $\mathbf{u} <_i \mathbf{f}$, and $\mathbf{t}$,$\mathbf{f}$ are incomparable. This ordering is then extended to interpretations such that for 3-valued interpretations I, J we have $I \leq_i J$ iff $I(s) \leq_i J(s)$ for all $s \in S$. We say that a two-valued interpretation I extends a 3-valued interpretation J iff $I \leq_i J$. That is, all arguments mapped to $\mathbf{f}$ or $\mathbf{t}$ by J are mapped to the same by I and all arguments that are mapped to $\mathbf{u}$ by J are mapped to either $\mathbf{t}$ or $\mathbf{f}$ by I. Given a 3-valued interpretation J, by $[J]_2$ we denote the set of all two-valued interpretations that extend J.

In Dung's abstract argumentation frameworks the characteristic function and its fixed-points are central in the definition of the semantics. We next define the operator Γ_D that will be central in our definitions of ADF semantics. Γ_D generalises the characteristic function in two directions: (i) it gives a three valued assignment on arguments, i.e., beside marking arguments as accepted it also explicitly marks arguments as rejected; and (ii) it allows for the more general acceptance conditions of ADFs.Given an interpretation I, the operator Γ_D computes the arguments that should be set to $\mathbf{t}$ or $\mathbf{f}$ under the current interpretation I.

Definition 5.3. *For an ADF D and a three-valued interpretation I, the interpretation $\Gamma_D(I)$ is given by*

$$\Gamma_D(I)(s) = \bigsqcap\{w(\varphi_s) \mid w \in [I]_2\}$$

where $\sqcap$ is the consensus operation that assigns $\mathbf{t} \sqcap \mathbf{t} = \mathbf{t}$, $\mathbf{f} \sqcap \mathbf{f} = \mathbf{f}$, and assigns $\mathbf{u}$ otherwise.

We are now prepared to define admissibility based semantics.

Definition 5.4 ([Brewka *et al.*, 2013])**.** *A three-valued interpretation I for an ADF D is*

- *the grounded interpretation iff it is the least fixed point of Γ_D.*
- *admissible iff $I \leq_i \Gamma_D(I)$;*
- *complete iff $I = \Gamma_D(I)$.*
- *preferred iff it is $\leq_i$-maximal admissible.*

Finally, one can define stable semantics which, in order to avoid cyclic support, makes use of a reduced ADF in the definition.

Definition 5.5 ([Brewka *et al.*, 2013])**.** *Let $D = (S, L, C)$ be an ADF with $C = \{\varphi_s\}_{s\in S}$. A two-valued model I of D is a stable model of D iff $E_I = \{s \in S : I(s) = \mathbf{t}\}$ equals the set of statements that are $\mathbf{t}$ in the grounded interpretation of the reduced ADF $D^I = (E_I, L^I, C^I)$, where $L^I = L \cap (E_I \times E_I)$ and for $s \in E_I$ we set $\varphi^I_s = \varphi_s[b/\mathbf{f} : I(b) = \mathbf{f}]$.*

5.2 Computational Problems

As the nature of abstract dialectical frameworks is quite similar to the nature of Dung's abstract argumentation frameworks also the core computational problems coincide. That is, we first have *credulous reasoning*, i.e., an argument is accepted if it is mapped to $\mathbf{t}$ by at least one interpretation, and *skeptical reasoning*, i.e., an argument is accepted only if it is mapped to $\mathbf{t}$ by all interpretations. These two reasoning modes again give rise to the following computational problems for argumentation semantics σ.

- *Credulous Acceptance* $Cred_\sigma$: Given ADF $D = (S, L, C)$ and an argument $a \in S$. Is there an interpretation $I \in \sigma(D)$ with $I(a) = \mathbf{t}$?
- *Skeptical Acceptance* $Skept_\sigma$: Given ADF $D = (S, L, C)$ and an argument $a \in S$. Is $I(a) = \mathbf{t}$ for each interpretation $I \in \sigma(D)$?

Beside these reasoning problems we also consider the problem of *verifying* a given interpretation, and deciding whether an ADF provides any *coherent conclusion*. Depending on the actual semantics the latter can corresponds to deciding whether the ADF has at least one interpretation, or whether the ADF has an interpretation that maps at least one statement to either $\mathbf{t}$ or $\mathbf{f}$.

- *Verification of an interpretation* Ver_σ: Given an ADF $D = (S, L, C)$ and an interpretation I. Is $I \in \sigma(F)$?
- *Existence of an interpretation* $Exists_\sigma$: Given an ADF $D = (S, L, C)$. Is $\sigma(F) \neq \emptyset$?
- *Existence of a non-trivial interpretation* $Exists_\sigma^{\neg\emptyset}$: Given an ADF $D = (S, L, C)$. Does there exist an interpretation I with $I(a) \in \{\mathbf{t}, \mathbf{f}\}$ for some argument $a \in S$.

5.3 Complexity Results for ADFs

The ability of ADFs to express more complex relations between arguments resulted in more evolved definitions of the semantics and as we will discuss next also increases the complexity of the core reasoning tasks. As the complexity results for ADFs in Table 10 show, all the non-trivial reasoning tasks are one level higher in the polynomial-hierarchy than for Dung's AFs. The main reason for that is the complexity of the Γ_D operator which replaces the characteristic function. While the characteristic function can be evaluated in polynomial time (and even logarithmic space) deciding problems associated with the Γ_D operator are in general NP/coNP-hard.[18]

In this section we discuss the complexity of admissible and grounded semantics in more detail.

[18]For instance testing whether an argument is mapped to $\mathbf{t}$ is basically the validity problem of propositional logic.

σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ	$Exists_\sigma$	$Exists_\sigma^{\neg\emptyset}$
gr	coNP-c	coNP-c	DP-c	trivial	coNP-c
model	NP-c	coNP-c	in P	NP-c	NP-c
st	Σ_2^P-c	Π_2^P-c	coNP-c	Σ_2^P-c	Σ_2^P-c
ad	Σ_2^P-c	trivial	coNP-c	trivial	Σ_2^P-c
co	Σ_2^P-c	coNP-c	DP-c	trivial	Σ_2^P-c
pr	Σ_2^P-c	Π_3^P-c	Π_2^P-c	trivial	Σ_2^P-c

Table 10: Complexity of ADFs ($\mathcal{C}$-c denotes completeness for class $\mathcal{C}$).

Complexity of admissible semantics. Again the most fundamental problem is to verify that an interpretation is admissible. To show that the problem is in coNP we give an NP algorithm [Wallner, 2014] to falsify the admissibility of an interpretation I. Such an algorithm would guess an argument s, such that (i) $I(s) = \mathbf{t}$ or (ii) $I(s) = \mathbf{f}$, and a 2-valued interpretation $J \in [I]_2$ extending I such that either, in case (i), $J(\varphi_s) = \mathbf{f}$ or, in case (ii), $J(\varphi_s) = \mathbf{t}$. As I is admissible iff no such pair s, J exists this is a NP algorithm that falsifies I being admissible and thus the complementary problem of verifying an admissible interpretation is in coNP.

The coNP-hardness is by reduction from the coNP-complete UNSAT problem [Wallner, 2014] of testing whether a propositional formula is unsatisfiable. To this end consider a propositional formula φ over variables X and construct an ADF $D = (S, L, C)$ as follows: The set of arguments S consists of X and an additional argument a, each $x \in X$ is linked towards a, and the acceptance conditions are given by $c_x = x$ for $x \in X$ and $c_a = \varphi$. Now we consider the interpretation I mapping all $x \in X$ to $\mathbf{u}$ and a to $\mathbf{f}$. We have that the two-valued interpretations $J \in [I]_2$ correspond to the two-valued interpretations of φ and thus $\Gamma_D(I)(a) = \mathbf{f}$ iff φ has no model. That is, I is admissible iff φ is unsatisfiable and, as the ADF D can be constructed in polynomial time, coNP-hardness follows.

Combining the coNP verification algorithm for admissible semantics with the standard guess and check algorithms gives a Σ_2^P upper bound for credulous reasoning with admissible, complete and preferred semantics, and a Π_2^P upper bound for verifying a preferred interpretation. The latter then gives a Π_3^P algorithm for skeptical reasoning with preferred semantics.

Complexity of grounded semantics. The computational properties of grounded semantics in ADFs are quite in contrast to the computational proper-

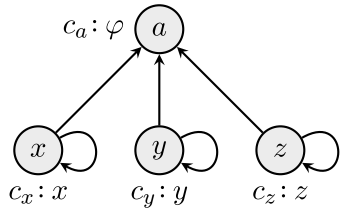

Figure 9: Illustration of the ADF constructed in the reduction from UNSAT to the problem of verifying an admissible interpretation in an ADF, for a propositional formula φ over atoms x, y, z.

ties of grounded semantics in AFs. When considering grounded semantics in ADFs, a straight forward algorithm is, starting from the three-valued model mapping all arguments to $\mathbf{u}$, and then iteratively apply the operator Γ_D until a fixed-point is reached. The straight forward algorithm is only a P^{NP}-algorithm, because of the costly evaluation of Γ_D. However, due to a sophisticated characterisation of grounded semantics [Wallner, 2014] there is a more efficient way to test whether an argument is mapped to $\mathbf{t}$ in the grounded interpretation of an ADF. Also notice that verifying the grounded interpretation is in DP as we have to do verify both the $\mathbf{t}$, $\mathbf{f}$ assignments and the $\mathbf{u}$ assignments.

The DP-hardness of verifying the grounded interpretation is by a reduction from the DP-complete SAT–UNSAT problem [Brewka and Woltran, 2010; Wallner, 2014]. To this end consider an instance (φ, ψ) of SAT–UNSAT where φ is a propositional formula over atoms X and ψ is a propositional formula over different atoms Y. In polynomial time we construct the ADF $D = (S, L, C)$ with $S = X \cup Y \cup \{d, s, v\}$, $L = \{(x, s) \mid x \in X\} \cup \{(y, v) \mid y \in Y\} \cup \{(d, s)\}$ and the acceptance conditions $c_x = x$ for $x \in X \cup Y$, $c_d = d$, $c_s = \varphi \wedge d$ and $c_v = \psi$. Now we consider the interpretation I with $I(v) = \mathbf{f}$ and $I(a) = \mathbf{u}$ for all the other arguments $a \in S \setminus \{v\}$. We next argue that I is the grounded model iff (φ, ψ) is a "yes" instance of SAT–UNSAT. Let G be the grounded model. First notice that the arguments $a \in X \cup Y \cup \{d\}$ do not have incoming edges from other arguments. Whenever $J(a) = \mathbf{u}$ then there are both an $I_1 \in [J]_2$ with $I_1(a) = \mathbf{t}$ and an $I_2 \in [J]_2$ with $I_2(a) = \mathbf{f}$, and thus also $\Gamma_D(J)(a) = \mathbf{u}$. That is, the grounded model G maps all arguments in $X \cup Y \cup \{d\}$ to $\mathbf{u}$. Now consider the argument s and $c_s = \varphi \wedge d$. The two-valued interpretations $J \in [G]_2$ correspond to the two-valued interpretations over $X \cup Y \cup \{d\}$. That is, either (a) φ has a model and we can satisfy $\varphi \wedge d$ by setting d to $\mathbf{t}$ as well as falsify $\varphi \wedge d$ by setting d to $\mathbf{f}$ and thus $\Gamma_D(G)(s) = G(s) = \mathbf{u}$, or (b) φ is unsatisfiable and thus $\Gamma_D(J)(s) = G(s) = \mathbf{f}$. One the other hand, for v and $c_v = \psi$, we have that either ψ is unsatisfiable and $\Gamma_D(G)(v) = G(v) = \mathbf{f}$, ψ is satisfiable but not valid and

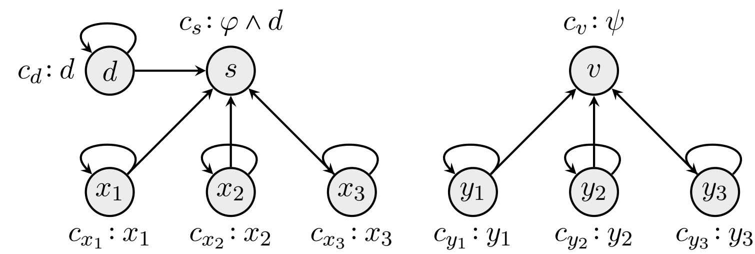

Figure 10: Illustration of the ADF constructed in the reduction from SAT−UNSAT to the problem of verifying the grounded model of an ADF, for an propositional formulae φ, ψ over atoms $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2 y_3\}$ respectively.

$\Gamma_D(G)(v) = G(v) = \mathbf{u}$, or ψ is valid and $\Gamma_D(G)(v) = G(v) = \mathbf{t}$. Hence, we have that $G = I$ iff φ is satisfiable and ψ is unsatisfiable.

5.4 Complexity of Bipolar ADFs with Known Link Types

Again there are certain instances of ADFs that do not have the worst-case complexity, but can be processed with milder complexity. Here we discuss so called *Bipolar ADFs* which put some restriction on the link structure, i.e., each link has to be supporting or attacking (but might be both). For a given set $X \subseteq S$ let I_X be the two-valued interpretation with $I^{\mathbf{t}} = X$ and $I^{\mathbf{f}} = S \setminus X$. A link (a, b) is called *supporting* if there is no $X \subseteq S$ such that $I_X \models \varphi_b$ and $I_{X \cup \{a\}} \not\models \varphi_b$; whereas it is called *attacking* if there is no $X \subseteq S$ such that $I_X \not\models \varphi_b$ and $I_{X \cup \{a\}} \models \varphi_b$. While in general testing the link type is itself coNP-complete [Brewka and Woltran, 2010; Ellmauthaler, 2012] there are certain applications of ADFs where the link type is known beforehand [Brewka and Gordon, 2010; Strass, 2013]. This motivates the research on *bipolar AFs with know link types* which we discuss in the remainder of this section.

The main observation that leads to the better complexity results for bipolar AFs (see Table 11) is that the operator Γ_D can be efficiently computed when all the links are attacking or supporting. The matching hardness results are then by the lower bounds for Dung's abstract argumentation (cf. Table 1) and the observation that AFs can be interpreted as bipolar ADFs with known link types [Brewka *et al.*, 2013] as follows.[19] Given an AF (A, R) the equivalent ADF is given by (A, R, C) with $C = \{c_a : \bigwedge_{(b,a) \in R} \neg b \mid a \in A\}$. Notice that all the links are indeed attacking.

[19]Notice that both ADF semantics models and stable models are generalisations of Dung's stable semantics.

σ	$Cred_\sigma$	$Skept_\sigma$	Ver_σ	$Exists_\sigma$	$Exists_\sigma^{\neg\emptyset}$
gr	P-c	P-c	P-c	trivial	in P
model	NP-c	coNP-c	in P	NP-c	NP-c
st	NP-c	coNP-c	in P	NP-c	NP-c
ad	NP-c	trivial	in P	trivial	NP-c
co	NP-c	P-c	P-c	trivial	NP-c
pr	NP-c	Π_2^P-c	coNP-c	trivial	NP-c

Table 11: Complexity of Bipolar ADFs with know link types ($\mathcal{C}$-c denotes completeness for class $\mathcal{C}$).

To compute $\Gamma_D(I)$ in general ADFs, we have to consider all 2-valued interpretations that extend I, which is coNP-hard, but given the link type of each link we only have to check two 2-valued interpretations for each argument as follows [Wallner, 2014; Strass and Wallner, 2014]. Let *Supp* be the set of supporting links and *Att* the set of attacking links, but there might be links that are both supporting and attacking (such links are called redundant links). For $s \in S$ consider the the formula φ_s and the interpretations J_1, J_2 as follows.

1. $J_1(s) = \begin{cases} \mathbf{t} & \text{if } I(s) = \mathbf{t}, \text{ or } I(s) = \mathbf{u} \text{ and } (s,a) \in Supp \\ \mathbf{f} & \text{if } I(s) = \mathbf{f}, \text{ or } I(s) = \mathbf{u} \text{ and } (s,a) \in Att \setminus Supp \end{cases}$

2. $J_2(s) = \begin{cases} \mathbf{t} & \text{if } I(s) = \mathbf{t}, \text{ or } I(s) = \mathbf{u} \text{ and } (s,a) \in Att \\ \mathbf{f} & \text{if } I(s) = \mathbf{f}, \text{ or } I(s) = \mathbf{u} \text{ and } (s,a) \in Supp \setminus Att \end{cases}$

The interpretation J_1 sets all yet undecided supporters to true and all yet undecided (non-redundant) attackers to false. If $J_1(\varphi_s) = \mathbf{f}$ then no 2-valued interpretation extending I satisfies φ_s, and thus $\Gamma_D(I)(s) = \mathbf{f}$. Otherwise if $J_1(\varphi_s) = \mathbf{t}$ then clearly $\Gamma_D(I)(s) \neq \mathbf{f}$. The interpretation J_2 sets all yet undecided (non-redundant) supporters to false and all yet undecided attackers to true. Now, whenever $J_2(\varphi_s) = \mathbf{t}$ then all 2-valued interpretations extending I satisfy φ_s, and thus $\Gamma_D(I)(s) = \mathbf{t}$. Otherwise if $J_2(\varphi_s) = \mathbf{f}$ then clearly $\Gamma_D(I)(s) \neq \mathbf{t}$. Hence, we can compute $\Gamma_D(I)$ by just considering J_1 and J_2 and set $\Gamma_D(I)(s) = \mathbf{t}$ if $J_2(\varphi_s) = \mathbf{t}$; $\Gamma_D(I)(s) = \mathbf{f}$ if $J_1(\varphi_s) = \mathbf{f}$; and $\Gamma_D(I)(s) = \mathbf{u}$ otherwise.

Now, as $\Gamma_D(I)$ can be computed in polynomial time, we can also (i) efficiently compute the grounded model by iteratively applying $\Gamma_D(I)$. Moreover, (ii) verifying an admissible or complete interpretation just requires to apply the Γ_D operator once and thus can be done in polynomial time. The remaining results in Table 11 are by the combination of the polynomial-time verification algorithms with the standard guess and check algorithms as used for Dung's AFs.

6 Discussion

In this work we presented complexity results for the three argumentation formalisms of Dung's Abstract Argumentation Frameworks, Assumption-based Argumentation and Abstract Dialectical Frameworks. We have identified several sources of computational complexity: (i) the construction of arguments and the interlinking structure, e.g., the attack relation, (ii) the search for coherent sets of arguments, and (iii) the decision about certain conclusions. Points (i) and (iii) are present in the complexity results for Assumption-based Argumentation where the complexity of algorithms heavily depends on the complexity of the derivability relation, which is the essential ingredient to build arguments, identify conflicts, and draw conclusions. Point (ii) is present in all three formalisms. The discussed results show that the actual computational complexity in this step may highly depend on the chosen semantics and reasoning task. Moreover, faced with the typically high complexity we discussed approaches to identify instances with lower complexity and solve them more efficiently.

Implications for the Design of Systems and Algorithms. First given the upper bounds of the complexity analysis we have first guidelines how to implement argumentation semantics, and on the computational resources required for that. An efficient system should process any instance within the resources given by the upper bound but moreover also should perform better on easier instances. In particular an efficient system should also be able to process instances that fall into one of the tractable fragments with milder complexity more efficiently. A system for abstract argumentation that is explicitly built around this idea is CEGARTIX [Dvořák *et al.*, 2014a], that is based on easier fragments for semantics on the second level of the polynomial-hierarchy.

The complexity classification of a semantics is also crucial for reduction-based implementations. To get an appropriate reduction the target formalism should have a similar complexity as the argumentation semantics, or one should only use a fragment of the target formalism with similar complexity. One example is the ASPARTIX [Egly *et al.*, 2010] system that encodes abstract argumentation problems in logic-programming in a query-based fashion. That is, the system provides fixed encodings for the supported argumentation semantics (the queries) that are then evaluated on the encoding of considered AF (the input data).[20] The polynomial-time computable grounded semantics is encoded as stratified logic program, a fragment whose data-complexity is in polynomial time (even P-complete), the semantics at the NP, coNP level are encoded as programs without disjunction in the rule heads, the data-complexity of this kind of logic programs is on the NP / coNP level, and the full expressiveness of disjunctive logic programs is only used for the argumentation semantics whose complexity is at the second level of the polynomial-hierarchy.[21]

Another example is the work on intertranslatability of abstract argumentation semantics where one aims to efficiently translate one argumentation semantics to another, by modifying the argumentation framework [Dvořák and Woltran, 2011; Dvořák and Spanring, 2016]. Here a gap in the complexities of the semantics immediately gives a negative result.

Function Complexity. In this paper we restricted ourselves to what we consider to be the core computational problems and in particular to decision problems. In terms of computational complexity function problems, problems where one wants to compute a number, extensions or the set of extensions, are only rarely studied, notable exceptions are the research line on ideal semantics [Dunne, 2009; Dunne *et al.*, 2013], the work on counting the number of extensions [Baroni *et al.*, 2010, and the work on computing an admissible set that results in a minimal socratic discussion [Caminada *et al.*, 2016]. Recently, Kröll *et al.* started the research on enumeration complexity in abstract argumentation [Kröll *et al.*, 2017], where one is interested in the computational cost per extension.

Fine-Grained Lower Bounds. Lower bounds from classical computational complexity theory like NP-hardness indicate that there are no polynomial-time algorithms. However, they neither indicate lower bounds for the constants in the exponent of exponential running times nor rule out subexponential algorithms at all. That is, there is still some gap between the best known algorithms for the hard problems, they are exponential-time (see, e.g., [Nofal *et al.*, 2014]), and the existing lower bounds. To overcome this gap, in the field of combinatorial algorithms, so called conditional lower bounds are studied (see, e.g., [Abboud and Williams, 2014]). That is, one uses conjectures about lower bounds for well studied algorithmic problems. To obtain a lower bound for a new problem one then reduces the problem from the conjecture to the problem under question such that a faster algorithm for the new

[20] The specific encoding of an AF as logic program has became popular beyond the logic programming setting as the so-called ASPARTIX-format for encoding AFs.

[21] For a survey on the complexity of logic-programs see [Dantsin *et al.*, 2001].

problem would imply a faster algorithm for the original problem and thus would contradict the conjecture. By that one gets an algorithmic lower bound for the new problem conditioned on the original conjecture. Probably the most prominent such conjecture is the (strong) exponential-time hypothesis (S)ETH [Impagliazzo and Paturi, 1999], with ETH conjecturing that there is no subexponential algorithm for 3-SAT, and SETH conjecturing that there is no algorithm for CNF-SAT that runs in time $2^{(1-\epsilon)n} \cdot poly(n,m)$, for every constant $\epsilon > 0$ and polynomial $poly(n,m)$. As many of the existing reductions in formal argumentation are based on propositional logic (S)ETH is also a promising starting point for closing these complexity gaps in formal argumentation.

Complexity Analysis of further Argumentation Formalisms. In this paper we only cover three argumentation formalisms, while there are many more around and many of them come with a complexity analysis. Below we give a brief overview and pointers to the relevant literature. First, there are formalisms, e.g. AFRAs [Baroni *et al.*, 2011b], that extend Dung's Abstract argumentation frameworks and can be efficiently reduced to them. For such formalisms the complexity results for AFs directly extend to the new formalism. Second, there are extensions of AFs that can not be reduced in such a direct way and thus need their own complexity analysis. Most prominently: The complexity of Extended argumentation frameworks was studied in [Dunne *et al.*, 2010] and later complemented by a result in [Dvořák *et al.*, 2015]; Valued-based argumentation has been discussed in an earlier survey [Dunne and Wooldridge, 2009] on the complexity of abstract argumentation and more recent results can be found in, e.g., [Dunne, 2010; Kim *et al.*, 2011]; weighted argumentation systems and their complexity have been studied in [Dunne *et al.*, 2011]; and Constrained Argumentation Frameworks [Coste-Marquis *et al.*, 2006]. Finally, there are complexity results for logic-based argumentation formal isms. Complexity aspects of Deductive Argumentation were for instance considered in [Besnard *et al.*, 2009; Creignou *et al.*, 2011], while the complexity of Defeasible Logic Programming (DeLP) was studied in [Cecchi *et al.*, 2006].

Complexity Analysis in Formal Argumentation. At the current state of the field for most argumentation formalisms we already have a good understanding of the computational complexity of the fundamental problems and the important semantics. However, this by no means says that all research questions in that direction are solved. Indeed the field of formal argumentation is very active and with almost every new research topic there come associated computational problems that should be analysed w.r.t. their computational complexity. Let us exemplify three such occasion where a complexity analysis can deepen our understanding: (a) For a newly proposed semantics the complexity of the fundamental reasoning problems should be analysed in order to compare it with existing semantics and identified computational benefits/drawbacks. (b) When expanding existing argumentation formalisms with additional (syntactic) concepts one is interested in the (additional) computational costs of these concepts. That is by how much the complexity increases or whether one can add these concepts without any computational drawbacks. (c) When considering novel tasks for argumentation systems a complexity classification gives a first impression on the feasibility of the new task and guides the way to efficient implementations. For instance, recently the field of dynamics of argumentation received some attention [Diller *et al.*, 2015; Snaith and Reed, 2016; Wallner *et al.*, 2016; Kim *et al.*, 2013] and raised a couple of computational problems, e.g., the so-called extension enforcement problem [Baumann and Brewka, 2010] where one aims to modify an AF such that a certain set of arguments becomes acceptable. The work of [Wallner *et al.*, 2016] first gives a comprehensive complexity analysis of the enforcement problem and then turns these results into algorithms and the prototype system Pakota [22].

Acknowledgments

The authors are grateful to Hannes Strass, Johannes P. Wallner and an anonymous reviewer for their thoughtful comments on earlier versions of this paper which helped to improve the quality of presentation.

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Received 23 November 2016

Foundations of Implementations for Formal Argumentation

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Abstract

We survey the current state of the art of general techniques, as well as specific software systems for solving tasks in abstract argumentation frameworks, structured argumentation frameworks, and approaches for visualizing and analysing argumentation. Furthermore, we discuss challenges and promising techniques such as parallel processing and approximation approaches. Finally, we address the issue of evaluating software systems empirically with links to the International Competition on Computational Models of Argumentation.

1 Introduction

Compared to related areas such as argumentation theory [van Eemeren *et al.*, 2014], research conducted in the formal argumentation community seeks *formal* accounts of argumentation with explicit links to knowledge representation and reasoning, and artificial intelligence [Brachman and Levesque, 2004; Russell and Norvig, 2003]. An important feature for these accounts is *computability*, i. e., the possibility to provide algorithmic methods to solve problems.

In this paper, we survey general computational techniques and concrete implementations for solving problems related to formal argumentation. We distinguish between: (1) Approaches to abstract argumentation frameworks, (2) Approaches to structured argumentation frameworks (such as ASPIC+ and DeLP), and (3) Other approaches, including semi-formal systems related to visualization of argumentation processes or exchange of arguments on the web.

Between them, the most active research direction within the formal argumentation community[1] is devoted to the first category—algorithms and systems for abstract argumentation frameworks—reviewed in Section 2. The relevant computational problems and their (high) computational complexity have been studied in e. g. [Dunne and Wooldridge, 2009]. Here, we focus on the algorithmic issues and techniques to handle the high computational complexity of some of those problems. The development of implementations has accelerated recently, also due to the foundation of the *International Competition on Computational Models of Argumentation* (ICCMA):[2] besides discussing general techniques we will also survey concrete systems.

We will also look at techniques and systems solving problems for structured approaches to formal argumentation. Due to the multitude of different approaches to structured argumentation, computational techniques and algorithms are usually tailored towards specific approaches. We will discuss them in Section 3.

In order to complement our survey we will also have a brief look at other systems that incorporate some kind of (semi-)formal argumentation such as argument schemes and argumentation technologies (or *debating technologies*) which are popular in many other fields besides the formal argumentation community. In contrast to the perspective of artificial intelligence and knowledge representation usually taken by researchers in the formal argumentation community, the focus of the systems in this third category is on human-computer interaction and supporting critical thinking. We will discuss these systems in Section 4, concluding the survey part of this paper.

In Section 5 we will look beyond the current state of the art of algorithms and systems and current challenges for the development of systems, such as parallelization and approximation algorithms, focusing on abstract and structured argumentation approaches. A recent effort to promote the development of systems for solving argumentation tasks is the ICCMA: the first instance of the competition took place

[1] Approaches in the third category are also addressed by other research communities such as human-computer-interaction and web science.

[2] `http://argumentationcompetition.org` (on 27/04/2017).

in 2015 [Thimm *et al.*, 2016]. We will discuss this competition and general methods for empirically evaluating systems in Section 6.

2 Abstract Argumentation Implementations

In this section we will give an overview of implementations for abstract Argumentation Frameworks (AFs) following the approach from Dung [Dung, 1995] and give an overview of existing systems for Dung's framework as well as for some related formalisms.

One can divide the implementations for abstract AFs into two categories: the *reduction-based approach* and the *direct approach.* The former one reduces the problem at hand into another formalism to exploit existing solvers from the other formalism. We will discuss this method and the dedicated implementations in the following subsection. The other possibility is to design algorithms to directly solve the problem. This implementation method will be presented in Subsection 2.2. For a more detailed discussion on implementation methods for AFs we refer to [Charwat *et al.*, 2015].

Before we go into details on the different approaches we briefly introduce the background on abstract argumentation [Dung, 1995] and the notation we will use in this section. For comprehensive surveys on argumentation semantics the interested reader is referred to [Baroni *et al.*, 2011a].

Definition 2.1. *An* argumentation framework *(AF) is a pair $AF = \langle Ar, att\rangle$, where Ar is a finite set of arguments and $att \subseteq Ar \times Ar$ is the attack relation. The pair $\langle a, b\rangle \in Ar$ means that a attacks b. A set $S \subseteq Ar$ of arguments attacks b (in AF), if there is an $a \in S$, such that $\langle a, b\rangle \in att$. An argument $a \in Ar$ is* defended *by $S \subseteq Ar$ (in AF) iff, for each $b \in Ar$, it holds that, if $\langle b, a\rangle \in att$, then S attacks b (in AF). Given a set $S \subseteq Ar$, $S^+ = \{a \in Ar \mid \langle b, a\rangle \in att, b \in S\}$, and $S^- = \{a \in Ar \mid \langle a, b\rangle \in att, b \in S\}$.*

The inherent conflicts between the arguments are solved by selecting subsets of arguments, where a semantics σ assigns a collection of sets of arguments to an argumentation framework AF. The basic requirement for all semantics is that none of the selected arguments attack each other[3].

[3]We concentrate here on the basic Dung-style argumentation framework, and do not consider approaches like value-based argumentation frameworks (VAFs) [Bench-Capon, 2003] or inconsistency tolerant semantics [Dunne *et al.*, 2009] (where this requirement does not hold), as our main focus is on implementation methods.

Definition 2.2. *Let $AF = \langle Ar, att\rangle$ be an AF. A set $S \subseteq Ar$ is said to be* conflict-free *(in AF), if there are no $a, b \in S$, such that $\langle a, b\rangle \in att$. We denote the collection of sets which are conflict-free (in AF) by $cf(F)$.*

Definition 2.3. *Let $AF = \langle Ar, att\rangle$ be an AF, then $S \in cf(AF)$ is*

- *a* stable *extension, i. e. $S \in \mathcal{E}_{\mathcal{ST}}(AF)$, if each $a \in Ar \setminus S$ is attacked by S in AF;*
- *an* admissible *extension, i. e. $S \in \mathcal{E}_{\mathcal{AD}}(AF)$, if each $a \in S$ is defended by S;*
- *a* preferred *extension, i. e. $S \in \mathcal{E}_{\mathcal{PR}}(AF)$, if $S \in \mathcal{E}_{\mathcal{AD}}(AF)$ and for each $T \in \mathcal{E}_{\mathcal{AD}}(AF)$, $S \not\subset T$;*
- *a* complete *extension, i. e. $S \in \mathcal{E}_{\mathcal{CO}}(AF)$, if $S \in \mathcal{E}_{\mathcal{AD}}(AF)$ and for each $a \in Ar$ defended by S it holds that $a \in S$;*
- *the* grounded *extension (of AF), i. e. the unique set $S = \mathcal{E}_{\mathcal{GR}}(AF)$, if $S \in \mathcal{E}_{\mathcal{CO}}(AF)$ and for each $T \in \mathcal{E}_{\mathcal{CO}}(AF)$, $T \not\subset S$.*

The typical problems of interest in abstract argumentation are the following decision problems for given $AF = \langle Ar, att\rangle$, a semantics σ, $a \in Ar$ and $S \subseteq Ar$:

- Verification Ver_σ: is $S \in \mathcal{E}_\sigma(AF)$?
- Credulous acceptance $Cred_\sigma$: is a contained in at least one σ extension of AF?
- Skeptical acceptance $Skept_\sigma$: is a contained in every σ extension of AF?
- Non-emptiness $Exists_\sigma^{\neg\emptyset}$: is there any $S \in \mathcal{E}_\sigma(AF)$ for which $S \neq \emptyset$?

Computational complexity of decision problems on AFs is well-studied. For an overview see e. g. [Dunne and Wooldridge, 2009].

2.1 Reduction-based Implementations

Reduction-based implementations are a very common approach as one benefits from very sophisticated solvers developed and improved by several communities. The underlying idea is to exploit existing efficient software which has originally been developed for other purposes. To this end, one has to formalize the reasoning problems within other formalisms such as constraint-satisfaction problems (CSP) [Rossi *et al.*, 2006], propositional logic [Biere *et al.*, 2009] or answer-set programming (ASP) [Brewka *et al.*, 2011]. The general methodology of the reduction-based approach is to reduce the problem at hand to the target formalism, run the solver (of the target formalism) and interpret the output as the solutions of the original problem, as depicted in Figure 1.

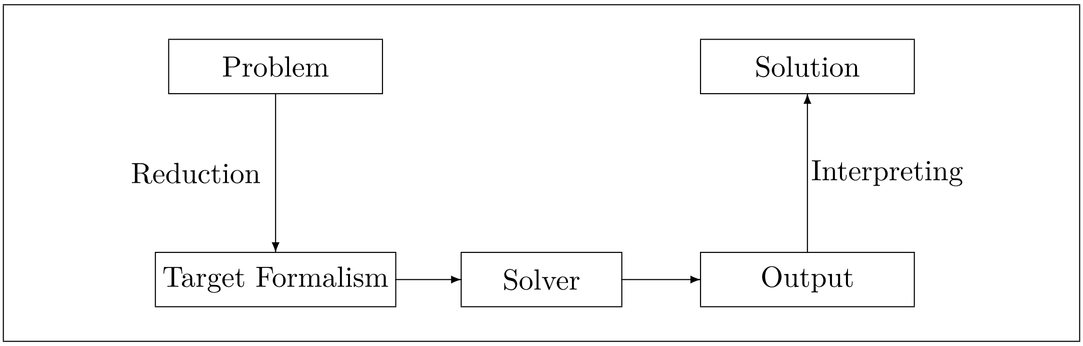


Figure 1: Reduction-based approach.

2.1.1 SAT-based Approach

Reductions to SAT have been first advocated in [Dunne and Bench-Capon, 2002] and [Dunne and Bench-Capon, 2003] and then further developed by Besnard and Doutre [Besnard and Doutre, 2004], and later extended by means of quantified propositional logic [Arieli and Caminada, 2013; Egly and Woltran, 2006]. Several prominent systems use reductions to SAT, such as **Cegartix** [Dvořák *et al.*, 2014] and **{j}ArgSemSAT** [Cerutti *et al.*, 2014c; Cerutti *et al.*, 2016b; Cerutti *et al.*, 2017] that both rely on iterative calls to SAT solvers for argumentation semantics of high complexity (i. e. being located on the second level of the polynomial hierarchy). Further SAT-based systems include **prefMaxSAT** [Vallati *et al.*, 2015; Faber *et al.*, 2016], which uses the MaxSAT approach for the computation of preferred semantics; the **LabSATSolver** [Beierle *et al.*, 2015], which uses propositional formulas based on labellings and, for the subset maximization task, the PrefSat Algorithm [Cerutti *et al.*, 2014a] that then become **{j}ArgSemSAT**. The system **CoQuiAAS** [Lagniez *et al.*, 2015], which also uses SAT encodings for some semantics, will be explained in Subsection 2.1.2, as the maximization task necessary for instance for preferred semantics is performed by means of constraint programming.

Background. Let us consider a set of propositional variables (or atoms) $\mathcal{P}$ and the connectives $\wedge, \vee, \rightarrow$ and $\neg$, denoting respectively the logical conjunction, disjunction, material implication and negation. The constants $\top$ and $\bot$ denote respectively *true* and *false*. In addition, we consider quantified Boolean formulae (QBF) with the universal quantifier $\forall$ and the existential quantifier $\exists$ (both over atoms), that is, given a formula ϕ, then $Qp\phi$ is a QBF, with $Q \in \{\forall, \exists\}$ and $p \in \mathcal{P}$. $Q\{p_1, \ldots, p_n\}\phi$ is a shorthand for $Qp_1 \cdots Qp_n\phi$. A propositional variable p in a QBF ϕ is free if it does not occur within the scope of a quantifier Qp and bound otherwise. If ϕ

contains no free variable, then ϕ is said to be closed and otherwise open. We will write $\phi[p/\psi]$ to denote the result of uniformly substituting each free occurrence of p with ψ in formula ϕ.

An interpretation $I \subseteq \mathcal{P}$ defines for each propositional variable a truth assignment where $p \in I$ indicates that p evaluates to true while $p \notin I$ indicates that p evaluates to false. This generalizes to arbitrary formulae in the standard way: Given a formula ϕ and an interpretation I, then ϕ evaluates to true under I (i. e., I satisfies ϕ) if one of the following holds (with $p \in \mathcal{P}$).

- $\phi = p$ and $p \in I$
- $\phi = \neg p$ and $p \notin I$
- $\phi = \psi_1 \wedge \psi_2$ and both ψ_1 and ψ_2 evaluate to true under I
- $\phi = \psi_1 \vee \psi_2$ and one of ψ_1 and ψ_2 evaluates to true under I
- $\phi = \psi_1 \rightarrow \psi_2$ and ψ_1 evaluates to false or ψ_2 evaluates to true under I
- $\phi = \exists p \psi$ and one of $\psi[p/\top]$ and $\psi[p/\bot]$ evaluates to true under I
- $\phi = \forall p \psi$ and both $\psi[p/\top]$ and $\psi[p/\bot]$ evaluate to true under I.

If an interpretation I satisfies a formula ϕ, denoted by $I \models \phi$, we say that I is a model of ϕ.

Reductions to propositional logic. The first reduction-based approach [Besnard and Doutre, 2004; Egly and Woltran, 2006] we consider here uses propositional logic formulae (without quantifiers) to encode the problem of finding admissible sets. Given an AF $AF = \langle Ar, att \rangle$, for each argument $a \in Ar$ a propositional variable v_a is used. Then, $S \subseteq Ar$ is an extension under semantics σ iff $\{v_a \mid a \in S\} \models \phi$, with ϕ being a propositional formula that evaluates AF AF under semantics σ (below we will present in detail how to translate AFs into formulae). Formally, the correspondence between sets of extensions and models of a propositional formula can be defined as follows.

Definition 2.4. *Let $\mathcal{T} \subseteq 2^{Ar}$ be a collection of sets of arguments and let $\mathcal{I} \subseteq 2^{\mathcal{P}}$ be a collection of interpretations. We say that $\mathcal{T}$ and $\mathcal{I}$ correspond to each other, in symbols $\mathcal{T} \cong \mathcal{I}$, if*

1. *for each $S \in \mathcal{T}$, there exists an $I \in \mathcal{I}$, such that $\{a \mid v_a \in I, a \in Ar\} = S$;*
2. *for each $I \in \mathcal{I}$, there exists an $S \in \mathcal{T}$, such that $\{a \mid v_a \in I, a \in Ar\} = S$; and*

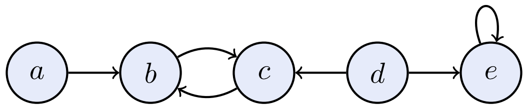


Figure 2: Example argumentation framework.

3. $|\mathcal{T}| = |\mathcal{I}|$.

Given an AF $AF = \langle Ar, att \rangle$, the following formula can be used to solve the enumeration problem of admissible semantics.

$$adm_{Ar,att} := \bigwedge_{a \in Ar} \Big((v_a \rightarrow \bigwedge_{\langle b,a \rangle \in att} \neg v_b) \wedge (v_a \rightarrow \bigwedge_{\langle b,a \rangle \in att} (\bigvee_{\langle c,b \rangle \in att} v_c)) \Big) \tag{1}$$

Note that an empty conjunction is treated as $\top$, whereas the empty disjunction is treated as $\bot$.

The models of $adm_{Ar,att}$ now correspond to the admissible sets of AF, i. e., we have $\mathcal{E}_{\mathcal{AD}}(AF) \cong \{M \mid M \models adm_{Ar,att}\}$. The first conjunction in (1) ensures that the resulting set of arguments is conflict-free, that is, whenever we accept an argument a (i. e., v_a evaluates to true under a model), all its attackers cannot be accepted. The second conjunct expresses the defense of arguments by stating that, if we accept a, then for each attacker b, some defender c must be accepted as well.

Example 2.5. *Let $AF = \langle Ar, att \rangle$ be an AF with $Ar = \{a, b, c, d, e\}$ and $att = \{\langle a, b \rangle, \langle b, c \rangle, \langle c, b \rangle, \langle d, c \rangle, \langle d, e \rangle, \langle e, e \rangle\}$ as depicted in Figure 2. The corresponding propositional formula $adm_{Ar,att}$ is as follows.*

$$\begin{aligned} adm_{Ar,att} \equiv & (v_a \rightarrow \top) \wedge \\ & (v_b \rightarrow (\neg v_a \wedge \neg v_c)) \wedge \\ & (v_c \rightarrow (\neg v_b \wedge \neg v_d)) \wedge \\ & (v_d \rightarrow \top) \wedge \\ & (v_e \rightarrow (\neg v_d \wedge \neg v_e)) \wedge \\ & (v_a \rightarrow \top) \wedge \\ & (v_b \rightarrow (\bot \wedge (v_b \vee v_d))) \wedge \\ & (v_c \rightarrow ((v_a \vee v_c) \wedge \bot)) \wedge \\ & (v_d \rightarrow \top) \wedge \\ & (v_e \rightarrow (\bot \wedge v_d)) \end{aligned}$$

It is easy to see that $\mathcal{I} = \{I_1, I_2, I_3, I_4\}$ *represents the set of models of* $adm_{Ar,att}$*, where*

$$\begin{aligned}
I_1 &= \{v_a \mapsto \bot, v_b \mapsto \bot, v_c \mapsto \bot, v_d \mapsto \bot, v_e \mapsto \bot\},\\
I_2 &= \{v_a \mapsto \top, v_b \mapsto \bot, v_c \mapsto \bot, v_d \mapsto \bot, v_e \mapsto \bot\},\\
I_3 &= \{v_a \mapsto \bot, v_b \mapsto \bot, v_c \mapsto \bot, v_d \mapsto \top, v_e \mapsto \bot\},\\
I_4 &= \{v_a \mapsto \top, v_b \mapsto \bot, v_c \mapsto \bot, v_d \mapsto \top, v_e \mapsto \bot\}.
\end{aligned}$$

As $\mathcal{T} = \{S_1, S_2, S_3, S_4\}$*, with* $S_1 = \{\}$*,* $S_2 = \{a\}$*,* $S_3 = \{d\}$ *and* $S_4 = \{a, d\}$*, is the set of all admissible sets of AF we clearly have the correspondence* $\mathcal{I} \cong \mathcal{T}$ *as desired.*

Reductions to quantified Boolean formulas. For problems beyond NP we require a more expressive formalism than propositional logic. For this purpose we consider QBFs. In the following we will show how to reduce a given AF into a QBF such that the models of the QBF correspond to the preferred extensions of the AF [Egly and Woltran, 2006].

In order to realize the maximality check for preferred semantics we need to be able to compare two sets of atoms w.r.t. set inclusion. Consider the formula

$$Ar < Ar' := \bigwedge_{a \in Ar} (v_a \to v_{a'}) \land \neg \bigwedge_{a' \in Ar'} (v_{a'} \to v_a),$$

where $Ar' = \{a' \mid a \in Ar\}$. This formula ensures that any model $M \models (Ar < Ar')$ satisfies $\{a \in Ar \mid v_a \in M\} \subset \{a \in Ar \mid v_{a'} \in M\}$. Now we can state the QBF $prf_{Ar,att}$ for preferred extensions. Let the quantified variables be $Ar'_v = \{v_{a'} \mid a' \in Ar'\}$ and $att' = \{\langle a', b' \rangle \mid \langle a, b \rangle \in att\}$. Then

$$prf_{Ar,att} := adm_{Ar,att} \land \neg \exists Ar'_v((Ar < Ar') \land adm_{Ar',att'})\ . \tag{2}$$

Thus, for any AF $AF = \langle Ar, att \rangle$ an interpretation I is a model of $prf_{Ar,att}$ iff it satisfies the formula for admissible sets and there exists no "bigger" interpretation I' that also satisfies the the corresponding formula for admissible sets.

Example 2.5 (continued) *There,* I_4 *is the only interpretation which satisfies the QBF* $prf_{Ar,att}$ *and the corresponding set* S_4 *is the only preferred extension of AF.*

Similar approaches have been proposed by Arieli and Caminada in [Arieli and Caminada, 2013] and for Abstract Dialectical Frameworks by Diller *et al.* in **QADF** [Diller *et al.*, 2015].

Iterative application of SAT solvers. The final approach we outline here is based on the idea of iteratively searching for models of propositional formulae and has been instantiated in the systems **{j}ArgSemSAT** [Cerutti *et al.*, 2014a; Cerutti *et al.*, 2014c; Cerutti *et al.*, 2016b] and **Cegartix** [Dvořák *et al.*, 2014]. The idea is to use an algorithm which iteratively constructs formulae and searches for models of these formulae. A new formula is generated based on the model of the previous one (or based on the fact that the previous formula is unsatisfiable). At some point the algorithm reaches a final decision and terminates.

The iterative approach is suitable when the problem to be solved cannot be decided in general—under standard complexity theoretic assumptions—by the satisfiability of a single propositional formula, constructible in polynomial time without quantifiers. This is, for instance, the case with skeptical acceptance under preferred semantics, where the corresponding decision problem is Π_2^P-complete. Instead of reducing the problem to a single QBF formula, the solving task is delegated to the iterative scheme of an algorithm querying a SAT solver multiple times.

The algorithms for preferred semantics work roughly as follows. To compute preferred extensions we traverse the search space of a computationally simpler semantics. For instance, we can iteratively search for admissible sets or complete extensions and iteratively extend them until we reach a maximal set, which is a preferred extension. By generating a new candidate for an admissible set or a complete extension, which is not contained in an already visited preferred extension, we can enumerate all preferred extensions in this manner. This allows answering both credulous and skeptical reasoning problems as well.

For deciding e. g. skeptical acceptance of an argument under preferred semantics one requires, in the worst case, an exponential number of calls to the SAT solver—under standard complexity-theoretic assumptions. However, the actual number of SAT calls in the iterative SAT scheme depends on the number of preferred extensions of the given AF, see [Dvořák *et al.*, 2014].

In the following, we sketch the **Cegartix** approach from [Dvořák *et al.*, 2014] for skeptical acceptance of an argument under preferred semantics. The algorithm returns YES if a is skeptically accepted, NO otherwise. To do so we try to construct a preferred extension which does not contain a. If this is possible we know that a is not skeptically accepted under preferred semantics, otherwise the algorithm returns YES.

1) Check if there is an interpretation I satisfying the formula ϕ (initially $\phi = adm_{Ar,att} \wedge \neg v_a$). If such an interpretation I exists, go to Step 2. Otherwise there is no admissible set which does not contain a, and the algorithm returns YES.

2) Try to add new arguments to I by updating it (as long as possible) with interpretations satisfying the formula

$$adm_{Ar,att} \wedge \neg v_a \wedge (\bigwedge_{a \in Ar, v_a \in I} v_a) \wedge (\bigvee_{a \in Ar, v_a \notin I} v_a).$$

3) For the maximized interpretation I, check if it is possible to add the argument a to it by checking for models of the formula

$$\phi' = adm_{Ar,att} \wedge (\bigwedge_{a \in Ar, v_a \in I} v_a) \wedge (\bigvee_{a \in Ar, v_a \notin I} v_a).$$

If there is an interpretation I' satisfying ϕ', there is a preferred extension which contains a. Otherwise, there is a preferred extension, namely the one represented by the interpretation I, which does not contain the argument a. In this case the algorithm outputs NO and terminates.

4) The algorithm continues with the search for a different preferred extension which does not contain the arguments of I by modifying the formula ϕ as follows:

$$\phi' = \phi \wedge (\bigvee_{a \in Ar, v_a \notin I} v_a).$$

Go to Step 1.

Example 2.5 (continued) *Let us exemplify the algorithm of* ***Cegartix*** *on our AF from Example 2.5, where we want to decide skeptical acceptance of the argument d. We know that there are four interpretations satisfying the formula for admissible sets and only I_1 and I_2 satisfy the formula $\phi = adm_{Ar,att} \wedge \neg v_d$ of Step 1. Let us continue with $I = I_1$ which represents the admissible set $S_1 = \{\}$. In Step 2, we update I by setting v_a to $\top$. Remember, we cannot set v_d to $\top$ as ϕ contains the clause $\neg v_d$. In Step 3 we check if there is an I' satisfying the formula $\phi' = adm_{Ar,att} \wedge v_a \wedge (v_b \vee v_c \vee v_d \vee v_e)$. Indeed $I' = \{v_a \mapsto \top, v_b \mapsto \bot, v_c \mapsto \bot, v_d \mapsto \top, v_e \mapsto \bot\}$ is a model of ϕ', thus we constructed a preferred extensions, namely $S = \{a, d\}$ containing the argument a. In Step 4 we update our formula to $\phi = adm_{Ar,att} \wedge \neg v_d \wedge (v_b \vee v_c \vee v_d \vee v_e)$ and go to Step 1. In the next iteration, we check the new formula ϕ for models, but as ϕ is not satisfiable the algorithm outputs YES and terminates.*

One can use a modified version of the above algorithm to enumerate all preferred extensions. More concretely, one can add the obtained preferred extension from Step 2 to the output-set and then update the formula as in Step 4, while omitting Step 3. Further, the conjunct containing a negated variable for the queried argument must be removed. The PrefSat approach [Cerutti *et al.*, 2014a] as implemented in the system **{j}ArgSemSAT** [Cerutti *et al.*, 2014c; Cerutti *et al.*, 2016b] uses this method to compute all preferred labellings.

2.1.2 Reductions to Constraint Satisfaction Problems

In the following we introduce reductions to another target formalism, namely Constraint Satisfaction Problems (CSPs) [Rossi *et al.*, 2006], which allow to solve combinatorial search problems. Reductions to CSP have been addressed by Amgoud and Devred [Amgoud and Devred, 2011] and Bistarelli, Pirolandi, and Santini [Bistarelli *et al.*, 2009; Bistarelli and Santini, 2010; Bistarelli and Santini, 2011; Bistarelli and Santini, 2012b; Bistarelli and Santini, 2012a]; the latter works led to the development of the **ConArg** system. Further systems based on CSP are **CoQuiAAS** [Lagniez *et al.*, 2015] and **ASGL** [Sprotte, 2015]. The approach of CSP is inherently related to propositional logic reductions as introduced in Subsection 2.1.1, see also [Walsh, 2000] for a formal analysis of the relation between the two approaches.

A CSP can generally be described by a triple (X, D, C), where $X = \{x_1, \ldots, x_n\}$ is the set of variables, $D = \{D_1, \ldots, D_n\}$ is a set of finite domains for the variables and $C = \{c_1, \ldots, c_m\}$ a set of constraints. Each constraint c_i is a pair (h_i, H_i) where $h_i = (x_{i1}, \ldots, x_{ik})$ is a k-tuple of variables and H_i is a k-ary relation over D. In particular, H_i is a subset of all possible variable values representing the allowed combinations of simultaneous values for the variables in h_i. An assignment v is a mapping that assigns to every variable $x_i \in X$ an element $v(x_i) \in D_i$. An assignment v satisfies a constraint $((x_{i1}, \ldots, x_{ik}), H_i) \in C$ iff $(v(x_{i1}), \ldots, v(x_{ik})) \in H_i$. Finally, a solution is an assignment v to all variables such that all constraints are satisfied, denoted by $(v(x_1), \ldots, v(x_n))$.

Finding a valid assignment of a CSP is in general NP-complete. Nevertheless, several programming libraries support constraint programming, like ECLiPSe,[4] SWI Prolog,[5] Gecode,[6] JaCoP,[7] Choco,[8] Turtle[9] (just to mention some of them) and allow for efficient implementations of CSPs. These constraint programming solvers make use of techniques like backtracking and local search.

Given an AF $AF = \langle Ar, att \rangle$, the associated CSP (X, D, C) is specified as $X = Ar$ and for each $a_i \in X$, $D_i = \{0, 1\}$. The constraints are formulated depending on the specific semantics σ. For example, solutions that correspond to conflict-free sets

[4] `http://eclipseclp.org/` (on 27/04/2017).
[5] `http://www.swi-prolog.org/` (on 27/04/2017).
[6] `http://www.gecode.org/` (on 27/04/2017).
[7] `https://github.com/radsz/jacop` (on 27/04/2017).
[8] `http://www.choco-solver.org/` (on 27/04/2017).
[9] `https://github.com/timfel/turtle` (on 27/04/2017).

can be obtained by defining a constraint for each pair of arguments a and b with $\langle a, b\rangle \in att$, where the two variables may not be set to 1 at the same time. Here, the constraint is of the form $((a, b), ((0, 0), (0, 1), (1, 0)))$ which is equivalent to the cases when the propositional formula $(a \rightarrow \neg b)$ evaluates to true.

In the following, we will use the notation from [Amgoud and Devred, 2011], because it reflects the similarities between the CSP approach and the reductions to propositional logic as outlined above.

For admissible semantics we get the following constraints.

$$C_{\mathcal{AD}} = \Big\{(a \rightarrow \bigwedge_{b:\langle b,a\rangle \in att} \neg b) \wedge (a \rightarrow \bigwedge_{b:\langle b,a\rangle \in att} (\bigvee_{c:\langle c,b\rangle \in att} c)) \Bigm| a \in Ar\Big\} \tag{3}$$

The first part ensures conflict-free sets and the second part encodes the defense of arguments. Then, for an AF $AF = \langle Ar, att\rangle$ and its associated admissible CSP $(X, D, C_{\mathcal{AD}})$, $(v(x_1), \ldots, v(x_n))$ is a solution of the CSP iff the set $\{x_j, \ldots, x_k\}$ s.t. $v(x_i) = 1$ is an admissible set in AF.

Example 2.5 (continued) *For our AF we obtain the following admissible CSP* $(X, D, C_{\mathcal{AD}})$*.* $X = A$*, for each* $a_i \in X$ *we have* $D_i = \{0, 1\}$ *and*

$$\begin{aligned} C_{\mathcal{AD}} = \{&(a \rightarrow \top) \wedge (a \rightarrow \top), (b \rightarrow \neg a \wedge \neg c) \wedge (b \rightarrow \bot \wedge d), \\ &(c \rightarrow \neg b \wedge \neg d) \wedge (c \rightarrow (a \vee c) \wedge \bot), (d \rightarrow \top) \wedge (d \rightarrow \top), \\ &(e \rightarrow \neg d \wedge \neg e) \wedge (e \rightarrow \bot \vee d)\}. \end{aligned}$$

This CSP has the following solutions: $(0, 0, 0, 0, 0)$*,* $(1, 0, 0, 0, 0)$*,* $(0, 0, 0, 1, 0)$*,* $(1, 0, 0, 1, 0)$ *which correspond to the admissible sets of AF, namely* $\{\}, \{a\}, \{d\}$ *and* $\{a, d\}$*.*

Most CSP solvers do not support subset maximization. Thus, for preferred semantics, Bistarelli and Santini [2012a] propose an approach that iteratively computes admissible/complete extensions and adds constraints to exclude certain sets, such that one finally obtains the preferred extensions.

Reductions to Weighted Partial Max-SAT. This approach has been implemented in **CoQuiAAS** [Lagniez *et al.*, 2015] and in **prefMaxSAT** [Vallati *et al.*, 2015; Faber *et al.*, 2016] and is particularly tailored to maximization problems as needed to compute preferred semantics. A *Weighted Partial Max-SAT* problem is a problem which maximizes the sum of weights associated to constraints, where the term *partial* means that some constraints have an infinite weight, which means they need to be satisfied. The system **CoQuiAAS** uses a SAT-Solver but the problem of Weighted Partial Max-SAT is more related to Constraint Programming, therefore we discuss this approach in this section, but of course it is also closely related to the previous section.

The computation of preferred extensions in [Lagniez *et al.*, 2015] is based on complete extensions which are obtained as follows. For an AF $AF = \langle Ar, att\rangle$ and for each $a \in Ar$ we use a boolean variable v_a.

$$comp_{Ar,att} :=$$
$$\bigwedge_{a\in Ar} \Big(v_a \to (\bigwedge_{b\in Ar:\langle b,a\rangle\in att} \neg v_b) \wedge (v_a \leftrightarrow (\bigwedge_{b\in Ar:\langle b,a\rangle\in att} \bigvee_{c\in Ar:\langle c,b\rangle\in att} v_c))\Big)$$

The models of $comp_{Ar,att}$ correspond to the complete extensions of AF, i. e., we have $\mathcal{E}_{\mathcal{CO}}(F) \cong \{M | M \models comp_{Ar,att}\}$. Then, the maximal models of $comp_{Ar,att}$ correspond to the preferred extensions of AF. To obtain these one uses the concept of a *maximal satisfiable subset* (MSS). For a set of formulas $\mathcal{F}$ the set of formulas $\mathcal{S} \subseteq \mathcal{F}$ is a MSS iff $\mathcal{S}$ is satisfiable and for each $c \in \mathcal{F} \setminus \mathcal{S}$, $\mathcal{S} \cup \{c\}$ is unsatisfiable.

Now, the computation of preferred extension reduces to the computation of MSSs of the sets of weighted formulas

$$prf_{Ar,att} = \{(comp_{Ar,att}, +\infty), (a_1, 1), \ldots, (a_n, 1)\}$$

where $a_1, \ldots, a_n \in Ar$.

2.1.3 Reductions to Answer Set Programming

The use of logic programming to solve abstract argumentation problems has been initiated by several authors (the survey article by Toni and Sergot [Toni and Sergot, 2011] provides a good overview), including the approach proposed by Nieves *et al.* [Nieves *et al.*, 2008], where the program is re-computed for every input instance; Wakaki and Nitta [Wakaki and Nitta, 2008], who use labelling-based semantics; and the approach by Egly *et al.* [Egly *et al.*, 2010a], which follows extension-based semantics. Here, we focus on the latter—the ASPARTIX approach—[Egly *et al.*, 2010a; Dvořák *et al.*, 2013a; Gaggl *et al.*, 2015], which relies on a query-based implementation where the argumentation framework to be evaluated is provided as an input database. From this point of view, the SAT or CSP methods can be seen as a compiler-like approach to abstract argumentation, while the ASP method acts like an interpreter.

A large collection of such ASP queries is provided by the **ASPARTIX-D** and **ASPARTIX-V** systems. Furthermore, the **DIAMOND** system [Ellmauthaler and Strass, 2014] for *Abstract Dialectical Frameworks* (ADFs), as well as the **GERD** system [Dvořák *et al.*, 2015] for *extended argumentation frameworks* (EAFs) are based on ASP. In the following, we first give a brief introduction to ASP. We then present how the computation of admissible sets can be encoded in ASP. In order to obtain preferred extensions, it is necessary to check for subset-maximality of admissible sets. We will give pointers to the literature on several approaches for the subset-maximality check and refer to [Charwat *et al.*, 2015] for a detailed discussion.

Background. Let us consider disjunctive logic program under the answer-set semantics [Gelfond and Lifschitz, 1991].[10] We fix a countable set $\mathcal{U}$ of *(domain) elements*, also called *constants*, and suppose a total order $<$ over the domain elements. An *atom* is an expression $p(t_1, \ldots, t_n)$, where p is a *predicate* of arity $n \geq 0$ and each t_i is either a variable or an element from $\mathcal{U}$. An atom is *ground* if it is free of variables. $B_{\mathcal{U}}$ denotes the set of all ground atoms over $\mathcal{U}$.

A *(disjunctive) rule* r with $n \geq 0$, $m \geq k \geq 0$, $n + m > 0$ is of the form

$$a_1 \vee \cdots \vee a_n \leftarrow b_1, \ldots, b_k, \ \mathit{not}\, b_{k+1}, \ldots, \ \mathit{not}\, b_m$$

where $a_1, \ldots, a_n, b_1, \ldots, b_m$ are atoms, and "*not*" stands for *default negation*. An atom a is a positive literal, while $\mathit{not}\, a$ is a default-negated literal. The *head* of r is the set $H(r) = \{a_1, \ldots, a_n\}$ and the *body* of r is $B(r) = B^+(r) \cup B^-(r)$ with $B^+(r) = \{b_1, \ldots, b_k\}$ and $B^-(r) = \{b_{k+1}, \ldots, b_m\}$. A rule r is *normal* if $n \leq 1$ and a *constraint* if $n = 0$. A rule r is *safe* if each variable in $H(r)$ occurs in $B^+(r)$. A rule r is *ground* if no variable occurs in r. A *fact* is a ground rule with a single literal in the head and with an empty body. An *(input) database* is a set of facts. A program is a finite set of safe disjunctive rules. For a program π and an input database D, we often write $\pi(D)$ instead of $D \cup \pi$. If each rule in a program is normal (resp. ground), we call the program normal (resp. ground).

For any program π, let U_π be the set of all constants appearing in π. $Gr(\pi)$ is the set of rules $r\tau$ obtained by applying, to each rule $r \in \pi$, all possible substitutions τ from the variables in r to elements of U_π. An *interpretation* $I \subseteq B_{\mathcal{U}}$ *satisfies* a ground rule r iff $H(r) \cap I \neq \emptyset$ whenever $B^+(r) \subseteq I$ and $B^-(r) \cap I = \emptyset$. I satisfies a ground program π, if each $r \in \pi$ is satisfied by I. A non-ground rule r (resp. a program π) is satisfied by an interpretation I iff I satisfies all groundings of r (resp. $Gr(\pi)$). $I \subseteq B_{\mathcal{U}}$ is an *answer set* of π iff it is a subset-minimal set satisfying the *Gelfond-Lifschitz reduct* $\pi^I = \{H(r) \leftarrow B^+(r) \mid I \cap B^-(r) = \emptyset, r \in Gr(\pi)\}$. For a program π, we denote the set of its answer sets by $\mathcal{AS}(\pi)$.

Reduction to ASP. We now provide fixed queries for admissible sets in such a way that an argumentation framework AF is given as an input database $\widehat{F}$ and the

[10] For further background, see [Eiter *et al.*, 1997; Brewka *et al.*, 2011].

answer sets of the program $\pi_e(\widehat{F})$ are in a certain one-to-one correspondence with the respective extensions, where $e \in \{\mathcal{AD}, \mathcal{PR}\}$. For an AF $AF = \langle Ar, att\rangle$, we define

$$\widehat{F} = \{\ \mathrm{arg}(a) \mid a \in Ar\} \cup \{\mathrm{att}(a,b) \mid \langle a,b\rangle \in att\ \}.$$

We have to guess candidates for the selected type of extensions and then check whether a guessed candidate satisfies the corresponding conditions, where default negation is an appropriate concept to formulate such a guess within a query. In what follows, we use unary predicates $\mathrm{in}(\cdot)$ and $\mathrm{out}(\cdot)$ to perform a guess for a set $S \subseteq Ar$, where $\mathrm{in}(a)$ means $a \in S$.

Similar to Definition 2.4, we define the subsequent notion of correspondence which is relevant for our purposes.

Definition 2.6. *Let $\mathcal{T} \subseteq 2^{\mathcal{U}}$ be a collection of sets of domain elements and let $\mathcal{I} \subseteq 2^{B_{\mathcal{U}}}$ be a collection of sets of ground atoms. We say that $\mathcal{T}$ and $\mathcal{I}$ correspond to each other, in symbols $\mathcal{T} \cong \mathcal{I}$, iff*

1. *for each $S \in \mathcal{T}$, there exists an $I \in \mathcal{I}$, such that $\{a \mid \mathrm{in}(a) \in I\} = S$;*
2. *for each $I \in \mathcal{I}$, there exists an $S \in \mathcal{T}$, such that $\{a \mid \mathrm{in}(a) \in I\} = S$; and*
3. *$|\mathcal{T}| = |\mathcal{I}|$.*

Let $AF = \langle Ar, att\rangle$ be an argumentation framework. The following program fragment guesses, when augmented by $\widehat{F}$, any subset $S \subseteq A$ and then checks whether the guess is conflict-free in AF:

$$\begin{aligned}\pi_{cf} = \{\ &\mathrm{in}(X) \leftarrow \mathit{not}\,\mathrm{out}(X), \mathrm{arg}(X);\\ &\mathrm{out}(X) \leftarrow \mathit{not}\,\mathrm{in}(X), \mathrm{arg}(X);\\ &\leftarrow \mathrm{in}(X), \mathrm{in}(Y), \mathrm{att}(X,Y)\ \}.\end{aligned}$$

The program module $\pi_{\mathcal{AD}}$ for the admissibility test is as follows:

$$\begin{aligned}\pi_{\mathcal{AD}} =\ \pi_{cf} \cup \{\ &\mathrm{defeated}(X) \leftarrow \mathrm{in}(Y), \mathrm{att}(Y,X);\\ &\leftarrow \mathrm{in}(X), \mathrm{att}(Y,X), \mathit{not}\,\mathrm{defeated}(Y)\ \}.\end{aligned}$$

For each conflict-free set one computes the arguments defeated by the set via the predicate defeated/1. The constraint then rules out those sets where an argument in the guessed set is attacked by an argument which is not defeated by the set, thus there is an argument in the conflict-free set which is not defended.

For any AF $AF = \langle Ar, att\rangle$, the admissible sets of AF correspond to the answer sets of $\pi_{\mathcal{AD}}$ augmented by $\widehat{F}$, i. e. $\mathcal{E}_{\mathcal{AD}}(AF) \cong \mathcal{AS}(\pi_{\mathcal{AD}}(\widehat{F}))$.

For semantics beyond NP we need to make use of *disjunction* in the logic program. There are several different ways how to encode these semantics. The first approach was to use the so called *saturation encodings* as pointed out in [Egly *et al.*, 2010a] which are part of **ASPARTIX**. Other encodings also incorporated in **ASPARTIX** are the *metasp encodings* [Dvořák *et al.*, 2013a], and the recently proposed encodings based on *conditional disjunction* which make use of a particular property of preferred semantics as shown in [Gaggl *et al.*, 2015].

2.2 Direct Implementations

A direct implementation refers to a dedicated algorithm for a reasoning problem of a specific semantics. The advantage is that direct implementations directly incorporate some problem-specific shortcuts, which is often not possible—or it leads to limited improvement—in the case of reduction-based implementations.

2.2.1 Labelling-based Algorithms

Many direct implementations are based on an alternative characterization for semantics using certain labelling functions for arguments [Verheij, 1996b; Doutre and Mengin, 2001; Modgil and Caminada, 2009; Nofal *et al.*, 2014b; Nofal *et al.*, 2014a; Verheij, 2007]. A labelling usually assigns each argument one of the following labels $\Lambda = \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$, which stand for accepted, rejected and undecided arguments. A labelling is a total function $\mathcal{L}ab : Ar \to \Lambda$. In the following we write $\mathtt{x}(\mathcal{L}ab)$ for $\{a \in Ar \mid \mathcal{L}ab(a) = \mathtt{x}\}$. For instance, $\mathtt{in}(\mathcal{L}ab)$ is the set of all in-labeled arguments. Sometimes we will also represent a labelling $\mathcal{L}ab$ as the triple $\langle \mathtt{in}(\mathcal{L}ab), \mathtt{out}(\mathcal{L}ab), \mathtt{undec}(\mathcal{L}ab)\rangle$.

One advantage of labellings is that the label of one argument has an immediate consequence to its neighbours. For example, if an argument a is labeled with `in`, all arguments attacked by a will be labeled with `out`. Such labelling-based algorithms have been materialized in several systems, see Table 1.

Enumeration. Several labelling-based algorithms to enumerate all extensions for various semantics have been proposed. For instance, the algorithm in [Nofal *et al.*, 2014a] makes use of five labels, namely $\Lambda = \{\mathtt{in}, \mathtt{out}, \mathtt{must\_out}, \mathtt{blank}, \mathtt{undec}\}$, where the additional label `blank` denotes the not yet labeled arguments and `must_out` is assigned to arguments that attack `in`-labeled arguments. Initially all arguments are labeled with `blank`. Then, the algorithm selects an $a \in \mathtt{blank}(\mathcal{L}ab)$ which is labeled with `in` in the left branch and `undec` in the right branch of the search tree. Every time an argument a is labeled with `in` all arguments attacked by it are labeled `out` and all remaining arguments which attack a are labeled with `must_out`. These steps are repeated until there are no arguments left to be labeled. The algorithm stores a preferred extension in one branch if each argument has one of the labels `in`, `out` and `undec` and the `in`-labeled arguments are not a subset of a previously stored preferred extension. Then, the algorithm backtracks to try to find all preferred extensions.

For the selection of the next argument to be labeled out from $\texttt{blank}(\mathcal{L}ab)$ the following heuristics are used.

- Don't pick an argument a to label it `in` iff there is a $b \in \{a\}^-$ such that $\mathcal{L}ab(b) \neq \texttt{out}$ and there is no $c \in \{b\}^-$ with $\mathcal{L}ab(c) = \texttt{blank}$.
- Don't pick an argument a to label it `undec` iff each $b \in \{a\}^-$ is either labeled with `out` or `must_out`.
- First select those `blank`-labeled argument to be labeled `in` which are not attacked at all or all its attacker are labeled with `out` or `must_out`.
- Otherwise, select a `blank`-labeled argument to be labeled `in` which attacks the most not `out`-labeled arguments.

Here we have only considered the case of preferred semantics, but for most of the semantics labelling-based algorithms have been proposed in the literature: algorithms for grounded and stable semantics are given in [Modgil and Caminada, 2009]; algorithms for semi-stable and stage semantics can be found in [Caminada, 2007; Caminada, 2010; Modgil and Caminada, 2009]. Recently [Nofal, 2013] studied improved algorithms for enumerating grounded, complete, stable, semi-stable, stage and ideal semantics. Labelling-based Algorithms are implemented in the **ArguLab** [Podlaszewski *et al.*, 2011] system as well as in the **ArgTools** [Nofal *et al.*, 2012].

Decision Procedures. In the following we will exemplify the use of labellings in an algorithm dedicated to credulous reasoning with preferred semantics, following the work of [Verheij, 2007], which is implemented in the **CompArg** system. In credulous reasoning one is only interested if a particular argument is accepted in at least one extension, thus we try to produce a witness (or counter-example) for this argument, instead of computing all extensions.

The algorithm starts with labelling the queried argument with `in` and all the other arguments with `undec`. Then, it iterates the following two steps. Firstly, it checks whether the set of `in`-labeled arguments is conflict-free and if so label all arguments attacking them with `out`. Otherwise terminate the branch of the algorithm. Secondly, for each argument a which is labeled `out` but not attacked by an argument labelled `in`, it picks an `undec` labeled attacker b of a and label it with `in`. In case there are several such arguments, it starts a new branch of the algorithm for each choice. If no such argument exists it terminates the branch. It stops a branch as soon as no more changes to labellings are made. In that case, it has reached an admissible labelling acting as proof for the credulous acceptance of the queried argument.

Consider the AF of Example 2.5 and the argument c. In the first step we obtain the following intermediate labelling

$$\mathcal{L}ab_1 = \langle \{c\}, \{\}, \{a, b, d, e\} \rangle.$$

As $\texttt{in}(\mathcal{L}ab_1)$ is conflict-free, we label all arguments attacking c with `out`:

$$\mathcal{L}ab_2 = \langle \{c\}, \{b, d\}, \{a, e\} \rangle.$$

Next we need to make arguments b and d *legally out* by labelling at least one of their attacker with `in`. In case of b this is already fulfilled as c is labeled with `in`. However, the argument d has no attacker, so the algorithm stops. We could not construct an admissible labelling for accepting the argument c, thus it is not credulously accepted under preferred semantics.

2.2.2 Dynamic Programming-based Approaches

We briefly mention the dynamic programming-based approach, which is defined on tree decompositions of argumentation frameworks. Many argumentation problems have been shown to be solvable in linear time for AFs of bounded tree-width [Dunne, 2007; Dvořák *et al.*, 2012c; Courcelle, 1989].

First introduced in [Dvořák *et al.*, 2012b], this approach especially aims at the development of efficient algorithms that turn complexity-theoretic results into practice. The algorithms from [Dvořák *et al.*, 2012b] are capable of solving credulous and skeptical reasoning problems under admissible and preferred semantics. Later, this approach was extended to work with stable and complete semantics [Charwat, 2012]. Further fixed-parameter tractability results were obtained for AFs with bounded clique-width [Dvořák *et al.*, 2010] and in the work on backdoor sets for argumentation [Dvořák *et al.*, 2012a]. Negative results for other graph parameters like bounded cycle-rank, directed path-width, and Kelly-width can be found in [Dvořák *et al.*, 2012b].

Systems implemented towards this approach are **dynPARTIX** [Charwat, 2012; Dvořák *et al.*, 2013b] as well as **D-FLAT** [Bliem, 2012; Bliem *et al.*, 2012]. **D-FLAT**

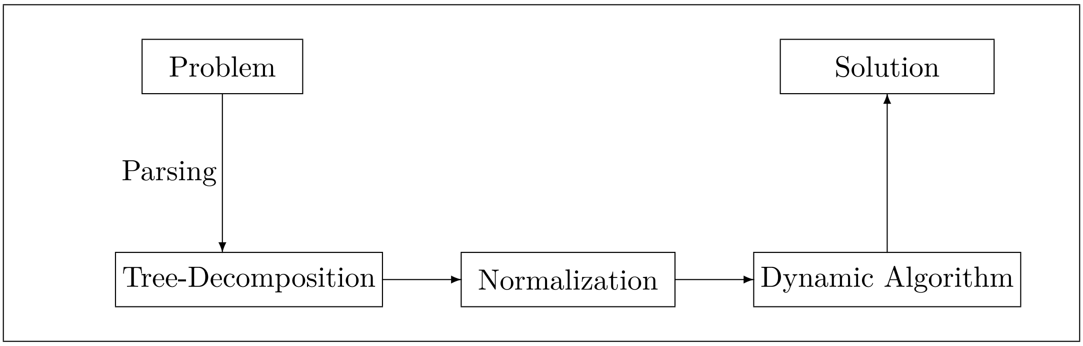


Figure 3: Dynamic-programming approach based on tree-decompositions.

is a general-purpose system that is capable of solving problems from multiple domains. The methodology underlying both of these systems is to build a tree-decomposition of a framework and then run a dynamic programming algorithm on the tree-decomposition to obtain the extensions of the desired semantics, as depicted in Figure 3. For an extensive discussion of the approach we refer the reader to [Charwat *et al.*, 2015].

2.3 Summary

In this section we discussed the two main approaches to implement abstract argumentation frameworks, namely the reduction-based and the direct implementation approach. Systems which implement the reduction-based approach are very popular, as they benefit from highly sophisticated solvers. One can say that they delegate the difficult part of the design of an efficient algorithm to the solvers of the target formalism. This might be the reason why so many solvers make use of this approach (see Table 1). On the other side the direct implementations can incorporate shortcuts if specific properties for certain structures in AFs are known, and in particular when it comes to the reasoning problems of skeptical and credulous acceptance, these algorithms can benefit from them. Many direct implementation algorithms make use of labellings. Table 1 summarizes all systems.

3 Structured Argumentation Implementations

This section gives an overview of algorithmic approaches to structured argumentation [Besnard *et al.*, 2014] and their respective systems. In contrast to abstract argumentation where arguments are interpreted as abstract entities and only logical relationships between arguments are taken into account, structured argumentation

	Direct	Reduction-Based	Type	Reference
{j}ArgSemSAT		Yes	SAT	[Cerutti *et al.*, 2014c; Cerutti *et al.*, 2016b; Cerutti *et al.*, 2017]
ArgTools	Yes		Labellings	[Nofal *et al.*, 2014b]
ArguLab	Yes		Labellings	[Podlaszewski *et al.*, 2011]
ASGL		Yes	CSP	[Sprotte, 2015]
ASPARTIX-D		Yes	ASP, SAT	[Egly *et al.*, 2010a; Gaggl and Manthey, 2015]
ASPARTIX-V		Yes	ASP	[Gaggl *et al.*, 2015]
ASSA	Yes		Matrices	[Hadjisoteriou, 2015]
Carneades	Yes		Labellings	[Gordon *et al.*, 2007]
Cegartix		Yes	SAT	[Dvořák *et al.*, 2014]
CompArg	Yes		Labellings	[Verheij, 2007]
ConArg		Yes	CSP	[Bistarelli *et al.*, 2015]
CoQuiAAS		Yes	SAT	[Lagniez *et al.*, 2015]
DIAMOND		Yes	ASP	[Ellmauthaler and Strass, 2014]
Dungell	Yes		Haskell	[van Gijzel and Nilsson, 2013]
EqArgSolver		Yes	Equations, Labellings	[Rodrigues, 2016]
GERD		Yes	ASP	[Dvořák *et al.*, 2015]
GRIS		Yes	Equations, Labellings	[Gabbay and Rodrigues, 2015]
LabSATSolver		Yes	SAT, Labellings	[Beierle *et al.*, 2015]
LamatzSolver	Yes			[Lamatz, 2015]
prefMaxSAT		Yes	SAT	[Vallati *et al.*, 2015; Faber *et al.*, 2016]
ProGraph	Yes			[Groza and Groza, 2015]
QADF		Yes	QBF, Labellings	[Diller *et al.*, 2015]
ZJU-ARG	Yes		Labellings	[Liao *et al.*, 2013]

Table 1: Summary of abstract argumentation implementations.

considers an argument's internal structure for several aspects including evaluation. Within formal argumentation, formalisms for structured argumentation assume a formalized knowledge base, often in a logical or rule-based form, from which arguments and their relations are constructed. Conceptually, formalisms for structured argumentation often follow the steps of the so-called argumentation process or argumentation pipeline (see e. g. [Dung, 1995, Sections 4 and 5] and [Caminada and Amgoud, 2007, Section 2]):

1. argument construction;
2. determining conflicts among arguments;
3. evaluation of acceptability of arguments; and
4. drawing conclusions.

Argument construction typically refers to the task of building arguments composed of a claim and a derivation of that claim (e. g. a proof tree) from the given knowledge base. Moreover, conflicts need to be recorded, e. g., when claims of two arguments are contradictory, or when the derivation of an argument's claim contradicts with the claim of another argument. Evaluation of acceptability refers to formal means of finding acceptable arguments, and finally conclusions can be drawn from the acceptable arguments.

From a computational point of view, all of the steps of the process taken individually can be quite computationally expensive: for instance even construction of single arguments may be computationally complex (NP-hard in cases); a large number of arguments may be constructed; finding conflicts can be non-trivial; and evaluation of acceptability has in general a high complexity, as in the case of abstract argumentation.

Several algorithmic approaches have been proposed, which result in a quite heterogeneous and evolving field comprising of many different solutions. In the following we highlight properties that distinguish algorithms for structured argumentation from each other.

Reasoning on structural or abstract representation. The first aspect that distinguishes algorithms and systems for structured argumentation is that they may deviate from the conceptual argumentation process. In particular, the approaches can be roughly categorized whether they perform

- (query-based) structural reasoning; or
- reasoning on an abstract representation.

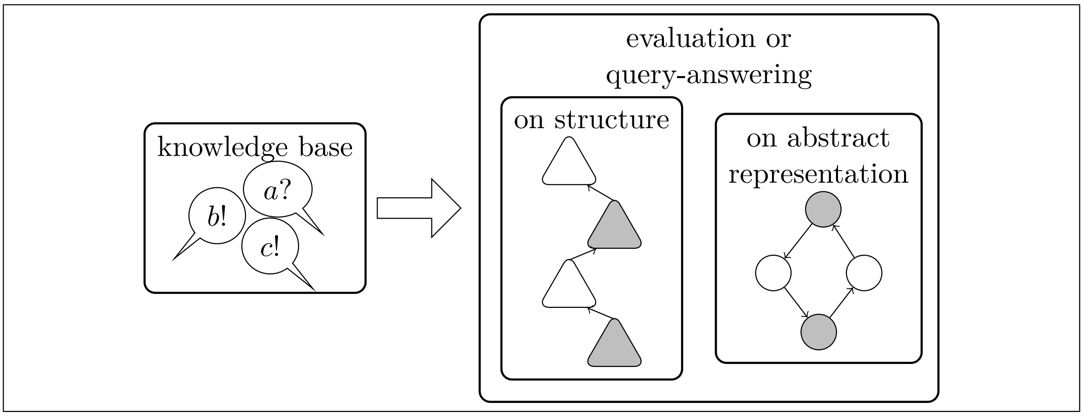


Figure 4: Argumentation process from a computational point of view

The latter classification encompasses algorithms that explicitly construct an abstract representation, e. g. an AF, and perform reasoning solely on that representation. Algorithms following the other approach construct no such representation, but combine argument construction, conflict discovery, and argument evaluation in possibly interleaving steps and take structured information from the input knowledge base into consideration in possibly every step.

Algorithms that perform structural reasoning are typically query-based, i. e., decide acceptability of a certain claim, and construct arguments for and counterarguments against the queried claim from the knowledge base. A structural approach can restrict argument construction more easily than the abstract approach, in particular for query-based reasoning, since structural information can be used to determine which arguments have an effect on the query or the currently processed argument.

On the other hand, the abstract approach first "compiles" the structured knowledge base and subsequently all reasoning can be performed on the abstraction. In some cases "full" knowledge of all arguments occurring in the abstract representation is required to perform reasoning, e. g. for stable semantics. Conceptually, the abstract approach follows more closely the argumentation process. We illustrate structural and abstract approaches to algorithms for structured argumentation in Figure 4. In this figure triangles are arguments with internal structure and round vertices are abstract arguments.

Dedicated and reduction-based approaches. Similarly as for approaches to implement abstract argumentation, we can distinguish between direct or dedicated approaches and reduction-based approaches to implement structured argumentation. An approach is reduction-based if the input is translated to a problem of another target formalism with available solvers for that problem. Direct algorithms solve the problem at hand with a domain-specific dedicated algorithm. Direct algorithms have the benefit of incorporating domain-specific properties and optimizations more easily. On the other hand, reduction approaches can re-use off-the-shelf solvers. Reduction-based approaches for structured argumentation typically incorporate all involved tasks, i.e., argument construction, conflict evaluation, and deciding acceptability of arguments. When constructing an abstract representation, approaches to structured argumentation can also be hybrid systems, i.e., providing a direct or reduction-based approach for constructing the abstraction, and providing another for abstract reasoning. Usual target systems for reduction-based approaches are Prolog systems, solvers for Boolean satisfiability (SAT) and related formalisms, and solvers for answer-set programming (ASP) [Brewka *et al.*, 2011]. We also call an algorithm or system reduction-based if it incorporates a translation of subproblems to a target language with available solvers.

Considered Approaches. In the following we overview concrete algorithmic approaches to structured argumentation, introducing them with examples and discussing the main computational problems, properties of interest from a computational point of view, and algorithms and systems proposed to solve the problem.[11] We focus on implemented algorithms for abstract rule-based argumentation (in particular concrete instantiations of the general ASPIC+ formalism) [Prakken, 2010; Modgil and Prakken, 2014], assumption-based argumentation (ABA) [Bondarenko *et al.*, 1997; Toni, 2014], argumentation based on logic programming, in particular based on defeasible logic programs (DeLPs) [García and Simari, 2004; García and Simari, 2014], argumentation based on classical logic [Besnard and Hunter, 2008], and Carneades [Gordon *et al.*, 2007]. Complementing information can be found in a review of implementations for defeasible reasoning [Bryant and Krause, 2008], in particular sections 4.2.7, 4.3.1, 4.3.2, 4.3.3, and 4.3.4; in the review for argumentation for the social web [Schneider *et al.*, 2013]; and in the overview on research in argumentation systems given by [Simari, 2011].

3.1 Abstract Rule-Based Argumentation

In this section we focus on systems for abstract rule-based argumentation, in particular concrete instantiations of the ASPIC+ [Prakken, 2010; Modgil and Prakken, 2014] formalism. We begin with a brief introduction to a concrete instantiation

[11]Tools presented and referenced within the following subsections sometimes do not solve the same reasoning tasks proposed for a formalism. We refer the reader to the references for each algorithm and tool for the exact problem definitions that are solved.

rules	arguments
$p \rightarrow b$	$A_1 : p$
$b \Rightarrow f$	$A_2 : A_1 \rightarrow b$
$p \rightarrow \overline{f}$	$A_3 : A_1 \rightarrow \overline{f}$
	$A_4 : A_2 \Rightarrow f$

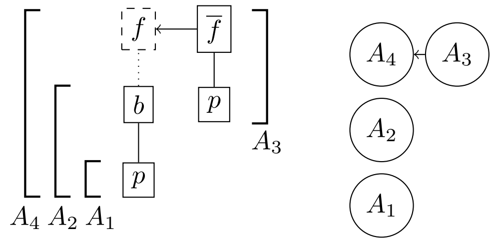


Figure 5: Tweety example knowledge base in ASPIC+ (left) with axiom p, structure of corresponding arguments (middle), and AF (right).

of ASPIC+ following notation of [Modgil and Prakken, 2014]. Input in this formalism is a knowledge base consisting of several components, central among them are (ordinary) premises and axioms, defeasible and strict rules, and preferential information. Semantics are specified via a translation to an abstract argumentation framework. Arguments are constructed by chaining premises or axioms with rules. Conflicts among arguments are defined via so-called undercuts, rebuts, and undermining among arguments, all respecting the preferential information.

We illustrate the concepts in a toy example knowledge base in Figure 5.

Example 3.1. *Figure 5 shows two strict rules (with a simple arrow $\rightarrow$) and one defeasible rule (using a double-lined arrow $\Rightarrow$), and assuming p (Tweety is a penguin) to be an axiom, one can infer the four arguments shown in the figure, namely by a strict rule that Tweety is a bird (b), that birds normally fly (via a defeasible rule inferring f), and that penguins do not fly (via a strict rule inferring $\overline{f}$; note that overlining indicates contrariness). The structure of the arguments is visualized in the middle of Figure 5 where we also see the only conflict in this example, namely that argument A_3 attacks A_4 via rebut (contradictory conclusions). On the right of Figure 5 the abstract AF is shown.*

Computational problems for abstract rule-based argumentation include argument construction, conflict discovery, and semantic evaluation. These problems may be tackled in an intertwined way, for instance interleaving construction and evaluation or following more closely the argumentation process step-by-step and thus firstly constructing the abstract argumentation framework and then proceeding by semantical evaluation.

As a rough and general outline for algorithms based on structural reasoning, given a potential conclusion (e. g. Tweety can fly in example Figure 5), arguments can be constructed via backward chaining using rules until premises or axioms are found. For instance, argument A_4 can be constructed from conclusion f and back-chaining of two rules until axiom p is reached. Counterarguments can be found in a similar manner by back-chaining from conclusions of arguments that would attack the arguments constructed so far. The so constructed arguments, i. e., arguments in favor of the queried claim and the counterarguments, corresponds to a game-theoretic approach to compute acceptability of the given query (and one of its argument in favor) under the specified semantics. For instance, one can conclude that A_3 is contained in an admissible set $\{A_3\}$.

We begin our survey of systems for abstract rule-based argumentation with the **TOAST** system[12] [Snaith and Reed, 2012]. **TOAST** directly follows the steps of the argumentation pipeline by constructing an abstract AF from given input knowledge base and delegates the reasoning tasks to a dedicated AF reasoner, namely the Dung-O-Matic web service [Snaith *et al.*, 2010]. As an example, given the input in Figure 5 (left) the system would return a semantical evaluation of the AF shown on the right of that figure. The **TOAST** and Dung-O-Matic system together provide a system supporting axioms, premises, assumptions, and preferential information (last link and weakest link principles, see also [Modgil and Prakken, 2014]), rules, and a user-specified contrariness relation. The system further supports reasoning on the resulting AF under grounded, preferred, semi-stable, and stable semantics. **TOAST** is available as both a Java-based web service and web form.

Next we overview contributions to systems for abstract rule-based argumentation by Vreeswijk, which influenced subsequent successor systems. These systems follow query-based structural reasoning. Vreeswijk's works for argumentation systems are well summarized in the survey of [Bryant and Krause, 2008, Sections 4.3.1, 4.3.2, 4.3.3, and 4.3.4]. A system that resulted from Vreeswijk's PhD thesis [Vreeswijk, 1993], **IACAS** (InterActive Argumentation System), was written in LISP and is one of the earliest implementations of structured argumentation that is capable of handling input with strict and defeasible rules. This system allows for argument generation for or against a queried claim, and concluding its acceptability taking all the arguments into consideration. Vreeswijk's argumentation system (**AS**) is a Ruby-based implementation that handles strict and defeasible rules and tries to construct an admissible set containing an argument that concludes the queried claim. Two systems based on Vreeswijk's **AS** have been developed, namely the **ASPIC Inference Engine** and **Argue tuProlog** [Bryant *et al.*, 2006].

The **ASPIC Inference Engine** is available from the ASPIC resources at the Cancer Research UK's Advanced Computation Laboratory.[13] It provides both a web-based front-end and a Java-based system that implement query-based structural

[12] `http://www.arg.dundee.ac.uk/toast/` (on 27/04/2017).

[13] `http://aspic.cossac.org` (on 27/04/2017).

reasoning under grounded and (credulous) admissible semantics. The Java-based implementation offers a graphical user-interface.

A reduction approach to the language of Prolog is used in **Argue tuProlog** and the system is presented in [Bryant *et al.*, 2006]. The reduction utilizes a game-theoretic approach for implementing ASPIC, similarly as the previous approaches. In contrast to reduction approaches for other formalisms, **Argue tuProlog** reduces the input to several Prolog queries, i. e., every query for an argument for each player is instantiated as a separate Prolog call and thus the dialogue can be terminated at any time.

We conclude this section with Wietske Visser's Epistemic and Practical Reasoner (**EPR**)[14] [Visser, 2008] which is a direct Java-based implementation that implements query-based reasoning under grounded semantics, (credulous) admissible semantics, and e-p semantics [Prakken, 2006]. The system provides a graphical user-interface, and is documented in detail in Wietske Visser's master's thesis [Visser, 2008].

3.2 Assumption based argumentation

In assumption-based argumentation (ABA) [Bondarenko *et al.*, 1997; Toni, 2014], arguments and conflicts are drawn from three main components: a knowledge base, a set of assumptions, and a contrariness relation. We illustrate these concepts in Figure 6. On the left of Figure 6 we see an ABA framework, with four rules, the set of assumptions A containing a and e, and the contrariness relation relating the two assumptions to be contrary to f and d respectively (denoted via $\overline{a} = f$ and $\overline{e} = d$). Arguments (in squares) and conflicts (with solid arrows) that can be drawn from this framework are shown on the right of the figure. These arguments correspond to proof trees of claims. More concretely, the arguments' structure is based on the rules with the conclusion shown on the top of the squares and attacks take place based on assumptions and their contraries. For instance, the argument with f as the conclusion attacks the argument with conclusion b, since this argument requires the assumption a which is the contrary of f ($\overline{a} = f$). Arguments without assumptions are not attacked, e. g. argument with conclusion c.

Semantics of ABA can be defined via extensions as sets of arguments or, equivalently, as sets of assumptions. For instance, in the example in Figure 6 the set of arguments with claims for c, f, and e (that in this instance uniquely determine the corresponding arguments) is an admissible extension of the ABA framework (no attacks between these arguments are present and all attackers from outside are counterattacked). The corresponding set of assumptions is $\{e\}$.

[14] `http://www.wietskevisser.nl/research/epr/` (on 27/04/2017).

$$\begin{array}{ll} c \leftarrow & A = \{a, e\} \\ b \leftarrow a & \overline{a} = f \\ d \leftarrow b, c & \overline{e} = d \\ f \leftarrow e & \end{array}$$

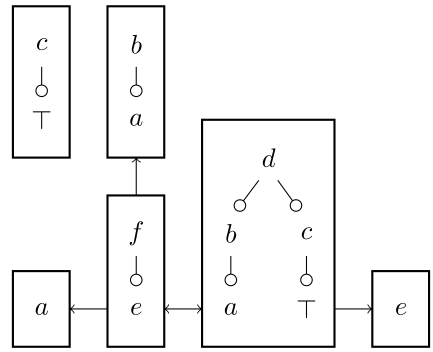


Figure 6: ABA framework (left) and its corresponding arguments and attacks (right)

A typical reasoning task for ABA frameworks is to check whether an argument for a given claim is contained in an extension under a specified semantics. The computational complexity for reasoning with an abstract ABA formalism has been investigated in [Bondarenko *et al.*, 1997]. In [Bondarenko *et al.*, 1997] decision problems for credulous and skeptical acceptance are studied and the complexity ranges from polynomial-time decidable to completeness for Σ_4^P, a class on the fourth level of the polynomial hierarchy.

Common to several algorithms for computing acceptability of a given claim under a specified semantics in a given ABA framework are so-called *dispute derivations* [Craven and Toni, 2016; Dung *et al.*, 2006; Dung *et al.*, 2007; Gaertner and Toni, 2007b; Gaertner and Toni, 2008; Toni, 2013]. Intuitively, dispute derivations can be seen as a game-theoretic constructive proof of acceptability of the given claim by constructing (part of) the argument in favor of the claim as well as constructing (parts of) its counterarguments and their counterarguments. Dispute derivations were proposed for grounded, admissible, and ideal semantics, called respectively GB, AB, and IB[15] dispute derivations [Dung *et al.*, 2007], which are an advancement of the proof trees proposed in [Dung *et al.*, 2006]. In [Gaertner and Toni, 2007b; Gaertner and Toni, 2008] *structured* dispute derivations were proposed that explicitly compute the dialectical structure hidden in dispute derivations, e. g., computing the attack structure explicitly. A parametrized version of dispute derivations was proposed in [Toni, 2013] that have a richer output incorporating both equivalent views of semantics of ABA, namely the view of extensions as sets of arguments and sets of assumptions.

In this paper we illustrate concepts of dispute derivations by showing GB-dispute

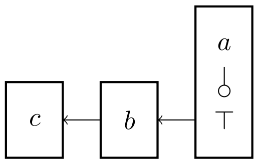
[15]Here, the "B" stands for belief.

derivations [Dung *et al.*, 2007]. In Figure 7 we see on the left a representation of a simple ABA framework with assumptions $A = \{b, c\}$ and a rule that infers a without assumptions. The grounded extension of this ABA framework contains the arguments for c and a, which are uniquely determined in this particular framework. A GB-dispute derivation is a sequence of quadruples (P_i, O_i, A_i, C_i) with integer i denoting the sequence or step. The ingredients for a step are the sentences or nodes for proponent (P_i) and opponent (O_i), the assumptions for defense of the queried claim (A_i) and assumptions for the opponent, so-called culprits (C_i). The component P_i is a set of sentences and both A_i and C_i are sets of assumptions. The second component of the quadruple, O_i, is a set of sets containing sentences. For querying acceptability for a claim α we initialize with $P_0 = \{\alpha\}$, $A_0 = \alpha \cap A$, and empty O_0 and C_0, where A is the set of assumptions in the ABA framework. We next illustrate the basics of GB-dispute derivations by recalling the corresponding sequences from [Dung *et al.*, 2007], where we assume a selection function f that selects at each step either an element in P_i or in O_i and in the latter case an element of the set selected. For a given ABA framework and a selection function f, a GB-dispute derivation of a defense set D for sentence α is a finite sequence of quadruples

$$(P_0, O_0, A_0, C_0), \ldots, (P_i, O_i, A_i, C_i), \ldots, (P_n, O_n, A_n, C_n)$$

with $P_0 = \{\alpha\}$, $A_0 = \alpha \cap A$, and empty O_0 and C_0; $P_n = O_n = \emptyset$ and $A_n = D$; and for every $0 \leq i < n$ and $X = f(P_i, O_i, A_i, C_i)$ the selected element s. t.

1. if $X \in P_i$ then
   (a) if $X \in A$ then
   $$\begin{array}{lllllll} P_{i+1} & = & P_i \setminus X, & A_{i+1} & = & A_i, \\ C_{i+1} & = & C_i, & O_{i+1} & = & O_i \cup \{\{\overline{X}\}\} \end{array}$$
   (b) else (there exists a rule $X \leftarrow R$ with body R s.t. $C_i \cap R = \emptyset$)
   $$\begin{array}{lllllll} P_{i+1} & = & (P_i \setminus X) \cup R, & A_{i+1} & = & A_i \cup (A \cap R), \\ C_{i+1} & = & C_i, & O_{i+1} & = & O_i \end{array}$$
2. else ($T \in O_i$ is selected with $X \in T$)
   (a) if $X \in A$ then
   $$\begin{array}{lllllll} P_{i+1} & = & P_i \cup \{\overline{X}\}, & A_{i+1} & = & A_i \cup (\{\overline{X}\} \cap A), \\ C_{i+1} & = & C_i \cup \{X\}, & O_{i+1} & = & O_i \setminus \{T\} \end{array}$$
   (b) else
   $$\begin{array}{lllllll} P_{i+1} & = & P_i, & A_{i+1} & = & A_i, \\ C_{i+1} & = & C_i, & O_{i+1} & = & (O_i \setminus \{T\}) \cup \\ & & & & & \{T \setminus \{X\} \cup R \mid X \leftarrow R \in \mathcal{R}\} \end{array}$$



i	P_i	O_i	A_i	C_i	selected
0	$\{c\}$	$\emptyset$	$\{c\}$	$\emptyset$	–
1	$\emptyset$	$\{\{b\}\}$	$\{c\}$	$\emptyset$	c
2	$\{a\}$	$\emptyset$	$\{c\}$	$\{b\}$	$\{b\}$, b
3	$\{\top\}$	$\emptyset$	$\{c\}$	$\{b\}$	a

Figure 7: ABA with $A = \{b, c\}$, $\overline{b} = a$, $\overline{c} = b$, and rule $a \leftarrow$ (left); GB-dispute derivation for c (right)

with $\mathcal{R}$ the set of rules of the given ABA framework. In Figure 7 we see on the right a sequence of a GB-dispute derivation. Briefly put, in each step in the sequence we select either an element of proponent or opponent, which in turn can either be assumptions or non-assumptions. Depending on the choice, different updates to the step have to be applied. For instance, if we choose an assumption of the proponent, then we remove that assumption from the sentence the proponent holds and add the contrary to the opponent who may construct an argument in favor of the contrary. We can note that each step in the sequence individually is straightforward to compute, however computation relies heavily on the selection function (also on selecting a rule in one case), which is discussed in more detail e. g. in [Gaertner and Toni, 2007b; Craven and Toni, 2016], which also highlights design choices for an algorithm based in dispute derivations.

Several systems have been developed implementing algorithms based on variants of dispute derivations. Current state of the art of dispute-derivation-based algorithms and systems for ABA are query-based and reason on the structural level and generally do not construct the full abstract representation to perform reasoning. Interestingly, most implementations, that build upon dispute derivations, rely on a reduction to Prolog with one exceptions **sxdd** [Craven *et al.*, 2012], which is an implementation in C++.

The system **CaSAPI**,[16] which stands for "Credulous and Sceptical Argumentation: Prolog Implementation", is, as the name suggests, an implementation for ABA in Prolog. In version 2.0 [Gaertner and Toni, 2007a], **CaSAPI** implements GB, AB, and IB dispute derivations to perform query-based structural reasoning. Further, in versions 3.0 [Gaertner and Toni, 2007b] and 4.3 [Gaertner and Toni, 2008; Dung *et al.*, 2007] structured dispute derivations are employed. Nowadays, **CaSAPI** acts as a precursor system for more recent systems.

Several tools with refined dispute derivations and reduction to Prolog have been

[16] `http://www.doc.ic.ac.uk/~ft/CaSAPI/` (on 27/04/2017).

proposed and implemented to perform query-based structural reasoning for ABA.[17] In the tool **proxdd** [Toni, 2013] the parametrized versions of dispute derivations are used. Graph-based versions of dispute derivations have been applied in the systems **grapharg** [Craven *et al.*, 2013] and its follow-up system **abagraph** [Craven and Toni, 2016]. These tools include graphical visualization.

Recently, two systems for ABA were developed which are not based on dispute derivations: **ABAplus**[18] and the system from [Lehtonen *et al.*, 2017], which we call here **ABAToAF**. Both of these systems compute semantics of ABA frameworks via an AF reasoner, ASPARTIX [Egly *et al.*, 2010a], on an abstract representation of the ABA framework.

The system **ABAplus** implements ABA^+ [Cyras and Toni, 2016a], an extension of ABA with preferences. More concretely, this system provides computations for flat ABA^+ frameworks satisfying the axiom of weak contraposition [Cyras and Toni, 2016b] (this class subsumes flat ABA frameworks). The system **ABAplus** is capable of enumeration of extensions (as sets of assumptions together with their conclusions) under grounded, complete, preferred, stable, and ideal semantics. In contrast to systems described above, **ABAplus** constructs an abstract AF to reason on the ABA, with arguments being sets of assumptions, with the AF being solved via encodings of ASPARTIX. The system **ABAplus** generates arguments, using Python, based on (i) sets of assumptions that deduce contraries of assumptions and (ii) singleton sets of assumptions. Both the ABA^+ framework and the enumerated extensions are visualized in a web frontend.

The other system for ABA that relies on an AF reasoner, **ABAToAF**, constructs arguments and attacks, similarly to **ABAplus**, based on sets of assumptions and derived sentences. Argument construction, implemented in Java 8, approximates here the restriction to generate arguments only for those sets of assumptions where at least one sentence can be derived from such a set, but not any proper subset. The system **ABAToAF** solves credulous (under admissible and stable semantics) and skeptical (under stable semantics) acceptance queries via calling an ASP solver on modified ASPARTIX encodings on the constructed AF.

Empirical evaluations of systems for ABA have been carried out for **sxdd** [Craven *et al.*, 2012], **grapharg** [Craven *et al.*, 2013], **abagraph** [Craven and Toni, 2016], and **ABAToAF** [Lehtonen *et al.*, 2017].

The work of [Craven and Toni, 2016], based on preliminary research of [Craven *et al.*, 2013], improves on several computational aspects of dispute derivations by altering the arguments' tree-structure to general graphs and introducing graphical

[17] Available at `http://www.doc.ic.ac.uk/~rac101/proarg/` (on 27/04/2017).

[18] Web front end available at `http://www-abaplus.doc.ic.ac.uk/` (on 27/04/2017) and stand-alone version at `https://github.com/zb95/2016-ABAPlus/` (on 27/04/2017).

dispute derivations (graph-DDs). In addition to tackle certain circularity questions for computation, in [Craven and Toni, 2016] an improvement for the problems of so-called flabbiness and bloatedness is provided. Briefly put, flabbiness refers to the potential shortcoming that the same sentence or claim is proved in several different ways, and bloatedness talks about deriving a claim in multiple ways in different arguments in an extension. That is, the former talks about computation of claims for individual arguments and the latter talks about computation of extension-based acceptability questions incorporating redundancy. In [Craven and Toni, 2016] graph-DDs are proposed for admissible and grounded semantics.

3.3 Argumentation based on logic programming

In this section we focus on algorithms and systems for argumentation based on logic programming, in particular defeasible logic programming [García and Simari, 2004; García and Simari, 2014].A defeasible logic program (DeLP) consists of strict ($\leftarrow$) and defeasible ($\leftarrow\!\!\prec$) rules as illustrated in Figure 8. Arguments in a DeLP are composed of a claim (a literal) and a set of defeasible rules. Acceptance of arguments is decided via a dialectical tree, see Figure 8 (right) for an example which includes an argument (A, a) that argues for literal a with set of rules A, arguments $(B_1, {\sim}b)$ and $(B_2, {\sim}b)$ that argue for (strongly) negated b, and argument $(E, {\sim}e)$ that argues for (strongly) negated e. Argument $(B_2, {\sim}b)$ defeats (A, a) because the former contradicts a subargument of the latter (arguing for b). Such a dialectical tree is then marked conceptually in a bottom-up manner with undefeated U and defeated D, i. e., leaves are undefeated and arguments are defeated if at least one child node is undefeated. Arguments are undefeated if all its children are defeated. Important for determining conflicts are preference relations which can either be given as input or derived via specificity, see [García and Simari, 2004; Stolzenburg *et al.*, 2003] for details. In our example, the argument (A, a) is not warranted, simply because it is defeated by $(B_1, {\sim}b)$. If the rules used in argument $(B_1, {\sim}b)$ would be removed from the input DeLP, then argument (A, a) would be warranted.

Complexity of decision problems in DeLP has been studied in [Cecchi *et al.*, 2006], showing complexity results for problems of deciding whether a given structure is an argument in a given DeLP (polynomial-time decidable), existence of arguments (a problem in NP), and further results regarding data complexity.

Algorithms for DeLP, which are based on dialectical trees, inherently solve query-based structural reasoning and check whether the queried claim is acceptable or warranted in a dialectical tree. Regarding enhancements for algorithms for computing acceptance of DeLPs, as stated in the survey of [Bryant and Krause, 2008],

A	B_1	B_2	E
$a \leftarrowtail b$	$\sim b \leftarrowtail d$	$\sim b \leftarrowtail e$	$\sim e \leftarrow g$
$b \leftarrowtail c$	$d \leftarrow$	$e \leftarrowtail f$	$g \leftarrow$
$c \leftarrow$		$f \leftarrow$	

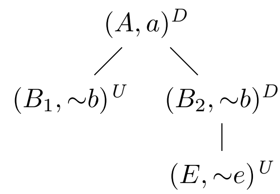


Figure 8: DeLP knowledge base (left) and dialectical tree (right)

three concepts have been proposed to optimize efficiency for deciding acceptance in DeLPs: (i) pruning of dialectical trees [Chesñevar *et al.*, 2000], (ii) using pre-compiled arguments in a dialectical database [Capobianco *et al.*, 2004], and (iii) using parallelism [García and Simari, 2000]. We briefly illustrate these concepts and also refer the reader to the survey [Bryant and Krause, 2008] which includes a section on DeLP (Section 4.2.7).

For pruning of dialectical trees, as can be seen in the example dialectical tree of Figure 8, we do not need to consider all arguments in the tree to determine the dialectical status of the root argument. In particular, since argument $(B_1, \sim b)$ is undefeated, it is immediate that the top argument in this case is defeated. Therefore the right subtree is not relevant for concluding the overall result. Details on general pruning procedures for DeLP can be found in [Chesñevar *et al.*, 2000], in particular how to "choose" the most promising argumentation line (path from root to a leaf in a dialectical tree) that determines an answer to the acceptability question as soon as possible.

In [Capobianco *et al.*, 2004] for speeding up algorithms for ODeLP, a pre-compiled so-called dialectical database is suggested. Briefly put, potential arguments and defeats from the initial knowledge base are pre-compiled. In this way queries can incorporate first look-ups in the pre-compiled dialectical database.

For exploiting parallelism, in [García and Simari, 2000] it is suggested to parallelize computation for (i) finding several arguments for the same conclusion, (ii) discovering several defeaters for an argument, and (iii) finding several argumentation lines.

For concrete systems, DeLP reasoning has been implemented in Prolog accessible via the **DeLP client**,[19] and in the general-purpose libraries of **Tweety**[20] [Thimm, 2014]. In **Tweety** both the algorithm outlined in [García and Simari, 2004] for marking a dialectial tree and a translation to an AF have been implemented (the latter does not preserve the dialectical semantics of DeLP and only interprets the ar-

[19]Web interface available at `http://lidia.cs.uns.edu.ar/delp_client/` (on 27/04/2017).
[20]`http://tweetyproject.org` (on 27/04/2017).

guments and counterargument relationship within an abstract framework). **Tweety** also provides a web-interface for DeLP. Also, an abstract machine called JAM (justification abstract machine) [García, 1997] has been designed for DeLP. Furthermore, a reduction to ASP is given in [Thimm and Kern-Isberner, 2008].

Two further notable reduction-based approaches for extensions of DeLP have been proposed and implemented.[21] Possibilistic DeLP (P-DeLP) extends DeLP rules by attaching levels of strength. In [Alsinet *et al.*, 2010] a recursive semantics for P-DeLP has been proposed, the corresponding framework is called RP-DeLP. An ASP-based approach to compute queries for RP-DeLP, i. e., to decide if a literal is warranted in the framework, is presented and experimentally evaluated in [Alsinet *et al.*, 2012], which is based on results and complexity bounds of [Alsinet *et al.*, 2011]. We call the corresponding system **ASP-RP-DeLP**. A reduction-based approach to SAT for multiple outputs of R-DeLP, we call the system **SAT-R-DeLP**, has been presented in [Alsinet *et al.*, 2013] and also experimentally evaluated in that paper. The SAT approach is based on results of [Alsinet *et al.*, 2011].

3.4 Argumentation based on classical logic

In argumentation based on classical logic, or deductive argumentation, arguments and conflicts are generated from a (classical) logic knowledge base [Besnard and Hunter, 2008]. A knowledge base is here a set of formulas and arguments are pairs (S, C) of support S and claim C. The first component is a consistent, minimal (w.r.t. $\subseteq$) subset of the knowledge base that entails the claim, which in turn is a formula. Arguments can be compared w.r.t. conservativeness, i. e., (S, C) is more conservative than (S', C') iff $S \subseteq S'$ and $C' \models C$. Several notions of conflicts among arguments have been studied [Gorogiannis and Hunter, 2011]. We illustrate here the notion of (canonical) undercuts. Argument (S, C) undercuts (S', C') if $C = \neg(\phi_1 \wedge \cdots \wedge \phi_n)$ with $\{\phi_1, \ldots, \phi_n\} \subseteq S'$. Canonical undercuts incorporate notions of maximal conservativeness and canonical enumeration of formulas, i. e., the sequence of formulas ϕ_i in the conjunction C does not matter. In Figure 9 we see on the left (a) a knowledge base and on the right (c) three arguments where the middle one is a canonical undercut of the top one and the bottom one a canonical undercut of the middle one. Note that in contrast to other structured approaches to argumentation, the arrows in formulas in this section denote logical (material) implication, i. e., within formulas $a \rightarrow b$ is logically equivalent to $\neg a \leftarrow \neg b$ and $\neg a \vee b$. A further important notion is that of (complete) argument trees. A given argument is the root of an argument tree, for each node its children are its canonical

[21] Available via web-front-end at `http://arinf.udl.cat/rp-delp` (on 27/04/2017).

knowledge base		
a	$\{a, a \to b, \neg b\}$	$(\{a, a \to b\}, b)$
$a \to b$	$\vert$	$\uparrow$
$\neg b$	$\{\neg b, \neg b \to \neg a, a\}$	$(\{\neg b\}, \neg(a \wedge (a \to b)))$
$\neg b \to \neg a$		$\uparrow$
		$(\{\neg b \to \neg a, a\}, b)$
(a)	(b)	(c)

Figure 9: Knowledge base for deductive argumentation (a), inconsistent subsets of that knowledge base (b), and argument tree based on the inconsistent subsets as constructed by compilation-based approach (c)

undercuts, and the support of no node is a subset of the union of supports of all its ancestor nodes.

Computational complexity is in general very high for deductive argumentation [Parsons *et al.*, 2003; Hirsch and Gorogiannis, 2010; Wooldridge *et al.*, 2006; Creignou *et al.*, 2011], as can be intuitively explained from the definitions which incorporate both minimality and entailment properties.[22] Complexity of finding individual arguments has been analyzed in [Parsons *et al.*, 2003], decisions problems concerning instantiation of argument graphs with classical logic in [Wooldridge *et al.*, 2006], and finding argument trees in [Hirsch and Gorogiannis, 2010]. Complexity for problems for deductive argumentation based on propositional logic can reach up to PSPACE.

Proposed algorithms and systems for deductive argumentation are based on minimal unsatisfiable subsets (MUSes) of formulas [Besnard and Hunter, 2006; Besnard *et al.*, 2010], connection graphs [Efstathiou and Hunter, 2011; Efstathiou and Hunter, 2008], reductions to QBF [Besnard *et al.*, 2009] and ASP [Charwat *et al.*, 2012], so-called "contours" [Hunter, 2006b] and approximate arguments [Hunter, 2006a]. Algorithms that utilize contours, approximate arguments, and one MUS-based approach [Besnard and Hunter, 2006] are also discussed in detail in the book [Besnard and Hunter, 2008].

We begin with our algorithmic overview with two MUS-based approaches. The first one [Besnard and Hunter, 2006] falls into the general scheme of knowledge compilation [Darwiche and Marquis, 2002] where a given input is compiled into a

[22] Another explanation for complexity of deductive argumentation is to consider its connection to (propositional) abduction, see [Besnard and Hunter, 2014, Section 7.4]. Complexity of propositional abduction is analyzed in [Eiter and Gottlob, 1995], with problems complete for Σ_2^P a class that is presumably more complex than the class NP.

structure to which one can pose queries that are computationally easier to compute on that structure compared to the original input. For deductive argumentation, the input knowledge base is compiled into a graph consisting of minimal inconsistent subsets of the knowledge base as the vertices and edges between non-disjoint subsets.

In Figure 9 we see in the middle (b) the compiled graph from knowledge base in the left (a). Given an argument, say $(\{a, a \to b\}, b)$ (top right of Figure 9) one can construct an argument tree for this argument using the inconsistent subsets. Note that the support $\{a, a \to b\}$ of this argument is contained in a MUS. The remainder of that MUS ($\neg b$) then is the support for a canonical undercut of the argument, since both parts of the MUS, $\{a, a \to b\}$ and $\{\neg b\}$, each entail a negated conjoined subset of the other, e. g. $\{\neg b\}$ entails $\neg(a \wedge (a \to b))$. Using this line of reasoning recursively, one can construct all counterarguments and in turn the argument tree (shown on the right of Figure 9). For details on the algorithm see [Besnard and Hunter, 2006]. The compilation-based approach has been implemented in the **Tweety** libraries [Thimm, 2014] which can be configured to use different MUS solvers, for instance MARCO [Liffiton *et al.*, 2016] or MIMUS [McAreavey *et al.*, 2014].

Another approach using MUSes [Besnard *et al.*, 2010] directly constructs arguments and counterarguments with a MUS solver, without an "offline" compilation beforehand. The idea underlying argument construction of [Besnard *et al.*, 2010] is that (S, C) is an argument iff $S \cup \{\neg C\}$ is a MUS of the knowledge base together with $\neg C$. Conditions of minimality and entailment for argument (S, C) follow from the fact that if $S \cup \{\neg C\}$ is a MUS, then S is consistent and entails C and S' with $S' \subset S$ does not entail C. The algorithms for argument construction and argument tree generation proposed in [Besnard *et al.*, 2010], **BA** and **BT**, follow this line of reasoning and directly incorporate algorithmic issues like construction of formulas in conjunctive normal form. Algorithm **BA** has been implemented with the MUS solver HYCAM [Grégoire *et al.*, 2009] and experimentally evaluated in [Besnard *et al.*, 2010].

A different approach for generating argument trees for a given claim is proposed in [Efstathiou and Hunter, 2011], building on earlier work in [Efstathiou and Hunter, 2008] which utilizes connection graphs. Connection graphs consist of clauses as vertices and edges between clauses with complementary literals. Briefly put, for a given claim one can reduce the connection graph in such a way that, if non-empty, a support for the claim is contained in the reduced connection graph. In [Efstathiou and Hunter, 2011] this idea is used to construct argument trees. The approach has been implemented in Java in the tool **JArgue** and experimentally evaluated.

Reduction-based approaches are given in [Besnard *et al.*, 2009; Charwat *et al.*, 2012]. The former is a reduction to QBF and the latter to ASP. The latter has been implemented in the system called **vispartix**[23] within the tool ARVis [Ambroz *et al.*, 2013] for visualizing relations between answer-sets of an ASP encoding. In **vispartix** an AF is generated from a given knowledge base and pre-specified set of claims, and conflicts are constructed as specified in [Gorogiannis and Hunter, 2011], thus partially deviating from other works in this section. The construction process is done via two ASP calls, the first constructing the arguments and the second constructing the attacks. In a final step the AF is visualized. Semantics can be computed via tools developed for AFs.

Algorithms following the concept of contours [Hunter, 2006b] are based on the idea of providing boundaries of what is provable in a knowledge base. Briefly put, an upper (lower) contour stores for a given formula which subsets of the knowledge base entail (do not entail) the formula. Finally, algorithms for approximate arguments [Hunter, 2006a] are based on the idea of relaxing one of the conditions for arguments (consistency, entailment, or minimality).

3.5 Carneades

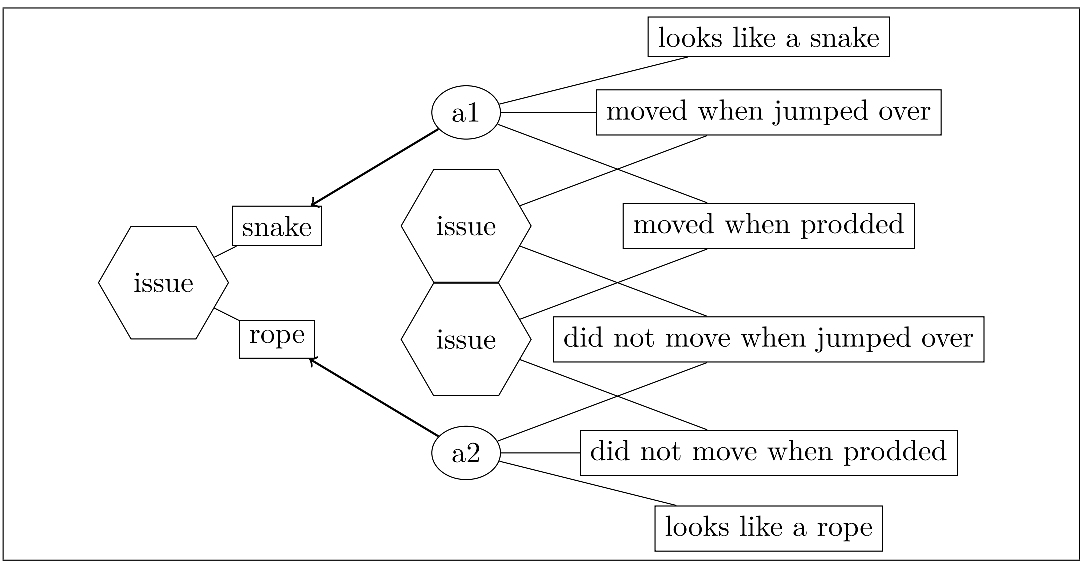


Figure 10: Example Carneades argument graph

Carneades [Gordon and Walton, 2016; Gordon *et al.*, 2007] is both a formal model of argument structure and evaluation, and a system[24] implementing the

[23] `http://www.dbai.tuwien.ac.at/proj/argumentation/vispartix/` (on 27/04/2017).
[24] `https://carneades.github.io/` (on 27/04/2017).

model. Evaluation of acceptance incorporates proof standards [Freeman and Farley, 1996], argument strength, and several ingredients available to a user. We illustrate briefly some of the capabilities of Carneades in a simple example[25] in Figure 10 and refer the reader for more details on the language and acceptability definitions to the literature [Gordon and Walton, 2016]. On the right part of Figure 10 there are six statements, i. e., that an object looks like a snake or a rope, and whether the object moved when jumped over or prodded. *Issue nodes* connect contradictory statements. Two arguments are formed (a1 and a2), which build on their premises (right of the figure) to conclude (left of the figure) that the seen object is indeed a snake or a rope. Let us assume that the object indeed looks like a snake and a rope (e.g. due to poor illumination), but neither did the object move when prodded with a stick nor when jumped over (e. g. by an adventurous person). In this case we conclude that the object is indeed a rope and not a snake (all premises of argument a2 are given but only one for a1).

The system Carneades (currently in version 4.2), features collaborative argument construction, argument visualization, and argument evaluation both for the structured arguments like we have seen in Figure 10 and also for Dung's AFs under grounded, complete, preferred, and stable semantics. Construction of structured arguments relies partially on internal calls to Prolog, and evaluation in the Carneades system can be classified as structural reasoning, since explicit abstract representation in the form of an AF is not utilized. Carneades is also available as a web-service and front-end [Gordon, 2012; Gordon, 2013], and includes a detailed manual.

3.6 Further implementations

Here we give pointers to related algorithms and implementations for structured argumentation that fall outside the previous sections.

In addition to other approaches to structured argumentation, **Tweety** [Thimm, 2014] features an implementation to structured argumentation as proposed in [Thimm and García, 2010]. Further, Wyner et al's [Wyner *et al.*, 2013] approach to instantiate rule-based knowledge bases with strict and defeasible rules as AFs has been encoded in ASP[26] [Strass, 2014].

A translational approach[27] to implement structured argumentation formalisms has been proposed in [van Gijzel and Nilsson, 2014] using Haskell as the programming

[25]Example taken from `http://carneades.github.io/` (on 27/04/2017). Variants of this example are discussed in [Walton *et al.*, 2014].

[26]Main ASP encoding available under `http://sourceforge.net/p/diamond-adf/code/ci/master/tree/lib/theorybase.lp` (on 27/04/2017).

[27]`http://www.cs.nott.ac.uk/~bmv/COMMA/` (on 27/04/2017).

language to capture definitions of these formalisms as directly as possible inside the programming language. For instance, in [van Gijzel and Nilsson, 2014] it is shown how to utilize this approach to translate Carneades to AFs: we call the corresponding system **CarneadesToDung**.

3.7 Summary

In this section we have given an overview of several algorithmic approaches to structured argumentation and their respective systems. Formalisms developed for structured argumentation and their implementations draw a quite heterogeneous picture. In particular, algorithms and systems range from query evaluation on the given structure to reasoning on an abstract representation where structural information is abstracted away. In Table 2 we see a summary of the presented approaches that have implementations and how they can be classified. Systems implementing structural reasoning typically solve queries in the form of deciding acceptance of a given claim and constructing arguments for this claim and counterarguments against the claim in a recursive fashion. Abstract reasoning involves construction of an abstract representation, i. e., an AF, and performing reasoning on this representation resulting typically in sets of extensions. For reduction-based approaches, the column "language" refers to the target formalism of the approach. These systems typically also include parsers or compilers written in an imperative language that translate or reduce the given input to the formalism. In this table, ASP stands for answer-set programming, SAT for satisfiability solvers, and MUS for solvers capable of solving problems related to minimal unsatisfiable subsets of formulas.

The **Tweety** libraries [Thimm, 2014] implement several reasoning tasks from multiple formalisms for structured argumentation. We name the respective approaches in parenthesis for **Tweety**. We note that not all tools mentioned in Table 2 provide reasoning support themselves, i. e., some tools focus on argument construction and delegate evaluation to other systems. The tools **BA** [Besnard *et al.*, 2010] and **vispartix** [Charwat *et al.*, 2012] handle argument construction for deductive argumentation without evaluation, in particular, **BA** generates arguments and **vispartix** an AF. One of **Tweety**'s algorithms translates a given DeLP to an AF and leaves the choice for an AF reasoner to the user. **CarneadesToDung** [van Gijzel and Nilsson, 2014] translates input as specified in the Carneades model to a Dung AF. **TOAST** [Snaith and Reed, 2012] incorporates Dung-O-Matic [Snaith *et al.*, 2010] for evaluation.

	Direct	Reduction	Language	(Query-based) Structural Reasoning	Reasoning on Abstract Representation
ASPIC+					
TOAST	Yes		Java		Yes
ASPIC Inference Engine	Yes		Java	Yes	
EPR	Yes		Java	Yes	
Argue tuProlog		Yes	Prolog	Yes	
ABA					
CaSAPI		Yes	Prolog	Yes	
proxdd		Yes	Prolog	Yes	
abagraph		Yes	Prolog	Yes	
grapharg		Yes	Prolog	Yes	
ABAplus		Yes	ASP		Yes
ABAToAF		Yes	ASP		Yes
DeLP					
DeLP client		Yes	Prolog	Yes	
Tweety (DeLP)	Yes		Java	Yes	
Tweety (DeLP to AF)	Yes		Java		Yes
ASP-RP-DeLP		Yes	ASP	Yes	
SAT-R-DeLP		Yes	SAT	Yes	
Deductive					
JArgue	Yes		Java	Yes	
Tweety (deductive)		Yes	Java/MUS	Yes	
vispartix		Yes	ASP		Yes
BA		Yes	MUS		
Carneades					
Carneades		Yes	Prolog	Yes	
CarneadesToDung			Haskell		Yes

Table 2: Summary table for structured implementations.

4 Other Implementation Approaches

This paper would not be complete without a description of implemented systems that provide a general purpose gateway to formal structures of argumentation. They are, for instance, systems supporting text annotation for producing corpora that can be exploited by argument mining algorithms as well as systems for supporting critical thinking by the means of formal models of argumentation thus reusing elements discussed in previous sections. Our aim here is to summarize the most notable examples with some guidance for the reader interested in using—or reusing—existing implementations.

In particular, we analyse 34 promising implementations chosen among those that are active projects. Since it is beyond the scope of this paper to provide a comprehensive description for each of those, we briefly review them in Section 4.1. Moreover, there are four additional projects that, although they appear to have been discontinued, have been relevant from an academic perspective, and we believe they should be mentioned in order to provide the reader with a complete background. Those are reviewed in Section 4.2, while in Section 4.3 we provide a comparative analysis of the active projects. Finally, the excellent review of Schneider et al. [Schneider *et al.*, 2013] mentions other interesting projects—mostly online platforms—that are briefly discussed in Section 4.4, even if they do not implement any evident formal model of argumentation.

4.1 Active Projects

The following 34 systems are representative among active projects incorporating some argumentation techniques.

AGORA [Hoffmann, 2005; Hoffmann, 2007] is a Computer-Supported Collaborative Argument Visualization (CSCAV) tool. An argument is defined here as a set of statements—claim and one or more reasons—where the reasons jointly provide support for the claim, or are at least meant to support the claim.

AIFdb [Lawrence *et al.*, 2012b] is a database solution for the Argument Web thus implementing the AIF model of arguments [Bex *et al.*, 2013; Rahwan *et al.*, 2011; Chesñevar *et al.*, 2006]. AIFdb offers an array of web service interfaces allowing a wide range of software to interact with the same argument data. Various dataset are available as part of the Argument Corpora [Reed, 2013].

AnalysisWall [Bex *et al.*, 2013] is a collaborative workspace, a touchscreen measuring 11 feet by 7 feet, located at the University of Dundee.

Arg&Dec [Aurisicchio *et al.*, 2015] is a web application for collaborative decision-making, encompassing the quantitative argumentation-based framework QuAD, and its decision matrix model, assisting their comparison through automated transformation.

ArgTeach [Dauphin and Schulz, 2014] is an interactive tutor that facilitates the learning of different labelling semantics in abstract argumentation. It now exists both as a standalone desktop application and as a web application.[28]

ArgTrust [Tang *et al.*, 2012] relates the grounds of an argument to the agent that supplied the information, and can be used as the basis to compute acceptability statuses of arguments that take trust into account.

ArgueApply [Pührer, 2017] is a Java app for mobile phones, with a graphical interface, that lets users put forward arguments, and positive or negative links between arguments, in a fragment of the GRAPPA [Brewka and Woltran, 2014] language.[29]

ArgMed [Hunter and Williams, 2012; Williams *et al.*, 2015] is a project investigating the use of computational argumentation for analysing and aggregating clinical evidence for making recommendations. In addition to the theoretical framework, it also has a public website.[30]

ArguMed [Verheij, 1998] introduces **ARGUE!**, based on the logical system CUMULA that abstractly models defeasible argumentation [Verheij, 1996a]. The development of **ARGUE!** was soon followed by the **ArguMed** family [Verheij, 2003a] based on the DefLog system [Verheij, 2003b], where dialectical arguments consist of statements that can have two types of connections between them: a statement can support another, or a statement can attack another. Dialectical arguments can be evaluated with respect to a set of prima facie justified assumptions.

[28] `http://www-argteach.doc.ic.ac.uk/` (on 27/04/2017).
[29] `http://www.informatik.uni-leipzig.de/~puehrer/ArgueApply/` (on 27/04/2017).
[30] `http://www0.cs.ucl.ac.uk/staff/a.hunter/projects/argmed/` (on 27/04/2017).

Argument Blogging [Bex *et al.*, 2014] allows users to construct debate and discussions across blogs, linking existing and new online resources to form distributed, structured conversations. Arguments and counterarguments can be posed by giving opinions on one's own blog and replying to other bloggers' posts. The resulting argument structure is connected to the Argument Web [Bex *et al.*, 2013], in which argumentative structures are made semantically explicit and machine-processable.

Argunet [Schneider *et al.*, 2007] is a desktop tool coupled with an open source federation system for sharing argument maps.

Arvina [Bex and Reed, 2012; Lawrence *et al.*, 2012a] is a dialogical support system that allows for the structured execution of a reasoning process by implementing dialogue protocols and then allowing users to play the dialogue game against virtual agents and against each other in an instant-messaging environment.

ASPARTIXWeb [Egly *et al.*, 2010b] is a web-based interface to the ASPARTIX system for computing extensions for various semantics of abstract argumentation.[31]

bCisive is a professional argument mapping and critical thinking support system.[32]

CISpaces [Toniolo *et al.*, 2014; Toniolo *et al.*, 2015] is an agent-based tool to help intelligence analysts in acquiring, evaluating, and interpreting information in collaboration. Agents assist analysts in reasoning with different types of evidence to identify what happened and why, what is credible, and how to obtain further evidence. Argument schemes lie at the heart of the tool, and sensemaking agents assist analysts in structuring evidence and identifying plausible hypotheses. A crowdsourcing agent is used to reason about structured information explicitly obtained from groups of contributors, and provenance is used to assess the credibility of hypotheses based on the origin of the supporting information.

Cohere/Compendium [De Liddo and Buckingham Shum, 2010; Shum, 2008] is an open source software for sensemaking using argumentation maps and annotation.

[31] `http://rull.dbai.tuwien.ac.at:8080/ASPARTIX/index.faces` (on 27/04/2017).
[32] `https://www.bcisiveonline.com/` (on 27/04/2017).

ConargWeb is a web-based interface to the Conarg system for computing extensions of Dung's argumentation frameworks.[33]

CoPe_it! [Tzagarakis *et al.*, 2009] is a tool to support synchronous and asynchronous argumentative collaboration in a Web environment. It introduces the notion of incremental formalization of argumentative collaboration. The tool permits a stepwise evolution of the argumentation space, through which formalization is not imposed by the system but is at the user's control. By permitting the users to formalize the discussion as the collaboration proceeds, more advanced services can be made available. Once the collaboration has been formalized to a certain point, CoPe_it! can exhibit an active behavior facilitating the decision making process.

D-BAS [Krauthoff *et al.*, 2016] is a web and dialogue-based system to facilitate online argumentation, with the aim to guide users through statements, their pro-arguments and counterarguments, and adding new arguments as well as conflicts between these arguments.[34]

Debategraph [Macintosh, 2009] is a collaborative debate visualisation tool.

GERD [Dvořák *et al.*, 2015] is a web-based interface of an ASP-based system for enumerating extensions of various semantics of the framework from [Modgil, 2009], which extends Dung's abstract argumentation framework with preferences among arguments.[35]

Gorgias [Kakas and Moraitis, 2003] is a general argumentation framework that combines preference reasoning and abduction. It can form the basis for reasoning about adaptable preference policies in the face of incomplete information from dynamic and evolving environments [Kakas *et al.*, 1994].

Gorgias-B [Spanoudakis *et al.*, 2016] supports the development of applications of argumentation under **Gorgias**. **Gorgias-B** guides the developer to structure their knowledge at several levels. The first level serves for enumerating the possible decisions and arguments that can support these options under some conditions, while each higher level serves for resolving conflicts at the previous level by taking into account default or contextual knowledge.

[33] `http://www.dmi.unipg.it/conarg/` (on 27/04/2017).
[34] `https://dbas.cs.uni-duesseldorf.de/` (on 27/04/2017).
[35] `http://gerd.dbai.tuwien.ac.at/index.php` (on 27/04/2017).

Grafix [Cayrol *et al.*, 2014] is a graphical tool for handling abstract argumentation frameworks and bipolar frameworks. Grafix allows editing and drawing of argumentation graphs (or sets of graphs), and the execution of some "predefined treatments" (called "server treatments") on the current graph(s), such as, e. g., computing various acceptability semantics, or computing the strength of arguments.

GrappaVis is a Java graphical tool to specify GRAPPA [Brewka and Woltran, 2014] and ADF [Brewka *et al.*, 2013] frameworks, evaluate them, and visualize the results of the evaluation. In particular, GRAPPA is a general semantical framework for assigning a precise meaning to graphical models of arguments or labelled argument graphs, which makes them suitable for automatic evaluation. GRAPPA rests on the notion of explicit acceptance conditions, as discussed in ADF [Brewka *et al.*, 2013].[36]

MARFs (Markov Argumentation Random Fields) [Tang *et al.*, 2016] is a system combining elements of formal argumentation theory and probabilistic graphical models. In doing so it provides a principled technique for the merger of probabilistic graphical models and non-monotonic reasoning.

Opinion Space [Faridani *et al.*, 2010] is an online interface incorporating ideas from deliberative polling, dimensionality reduction, and collaborative filtering that allows participants to visualize and navigate through a diversity of comments.

OVA+ [Janier *et al.*, 2014] provides a drag-and-drop interface for analysing textual arguments. It is designed to work with web pages It is available as a web interface and does not require a local installation. It also natively handles AIF structures, and supports real-time collaborative analysis.

Parmenides [Cartwright and Atkinson, 2008; Cartwright *et al.*, 2009; Cartwright and Atikinson, 2009] is primarily a forum by which government bodies can present policy proposals to the public so that users can submit their opinions on the justification presented for a particular policy. Within Parmenides, the justification for action is structured to exploit a specific representation of persuasive argument based on the use of argumentation schemes and critical questions.

[36] `http://www.dbai.tuwien.ac.at/proj/adf/grappavis/` (on 27/04/2017).

PIRIKA (PIlot for the RIght Knowledge and Argument) [Oomidou *et al.*, 2014] is an argument-based communication tool for humans and agents, which supplements current communication systems such as Twitter. It allows for asynchronous argumentation for anyone, anytime, anywhere on any issues, as well as synchronous argumentation and stand-alone argumentation.

Quaestio-it [Evripidou and Toni, 2014] is based on a framework for modelling and analysing social discussions. It offers debating infrastructure for opinion exchanges between users and providing support for extracting intelligent answers to user-posed questions.

Rationale is a professional argument mapping and critical thinking support system.[37]

Reason [Introne, 2009] is a platform for supporting group decisions by leveraging the argumentative structure of deliberative conversation to drive a decision support algorithm. The platform uses argument visualization to mediate the collaborators' conversation.

Truthmapping is a professional, collaborative argument mapping tool.[38]

4.2 Discontinued Projects

In addition to the 34 systems discussed in Section 4.1, we briefly mention the following four as well. Although discontinued at the time of writing, those works have significantly impacted the research field and are still inspirational.

Avicenna [Rahwan *et al.*, 2011] is an OWL-based argumentation system that consists of three main tiers: the data tier, the middle tier, and the client tier. The argumentation ontology is stored in the form of RDF statements (triples) in the back-end database, which constitutes the data tier. The middle tier is responsible for reasoning based on description logics and the interface to the web, through which applications in the client tier connect.

[37] `http://rationale.austhink.com/` (on 27/04/2017).
[38] `https://www.truthmapping.com/` (on 27/04/2017).

Dispute Finder [Ennals *et al.*, 2010] is a browser extension that alerts a user when information they read online is disputed by a source that they might trust. Dispute Finder examines the text on the page that the user is browsing and highlights any phrases that resemble known disputed claims. If a user clicks on a highlighted phrase then Dispute Finder shows her a list of articles that support other points of view.

SEAS [Lowrance *et al.*, 2008] is a collaborative, semi-automatic approach to evidential reasoning that uses template-based structured argumentation. Graphical depictions of arguments readily convey lines of reasoning, from evidence through to conclusions, making it easy to compare and contrast alternative lines of reasoning.

Trellis [Chklovski *et al.*, 2003] allows users to add their observations, viewpoints, and conclusions as they analyze information by making semantic annotations to documents and other on-line resources. Users can associate specific claims with particular locations in documents used as "sources" for analysis, and then structure these statements into an argument detailing pros and cons on a certain issue.

4.3 Comparative Analysis

To provide a concise overview over the *active* systems discussed in Section 4.1, we identified seven features that characterize the commonalities and differences among those systems, namely whether a system

(F1) is able to handle some form of structured argumentation;

(F2) gives the ability to manipulate arguments;

(F3) is collaborative;

(F4) enables a dialogue between different parties involved in its usage; and, in particular, if it

(F5) enables a dialogue based on speech acts;

(F6) includes a reasoner based on Dung's theory of abstract argumentation; or if it

(F7) includes a reasoner not based on Dung's theory of abstract argumentation.

It is evident that F5 is a specific case of F4: if a system offers speech acts, by definition it also offers a dialogue system. Moreover, F6 and F7 only apparently are mutually exclusive: indeed, a system can offer multiple choices of reasoners—the case of **CISpaces**—or it can encompass Dung's theory of abstract argumentation as a special case—e. g. **MARFs**.

Table 3 provides a comparative overview of the 34 active projects from Section 4.1 with respect to the seven features identified. This list of features is clearly far from being complete or unquestionable. However, it is sufficient for describing a large variety of possible usages of the systems.

Indeed, if a system supports F1 and F6, it is evident that it can be used in the *conventional* meaning of structured argumentation and perhaps it implements a specific approach for structured argumentation [Besnard *et al.*, 2014]. This is, for instance, the case of **OVA+**, which allows to represent and reason about ASPIC+ knowledge bases. Moreover, since **OVA+** also possesses the feature F2, it is evident that it can be used interactively; and since it possesses F3 as well, it can used in a distributed fashion.

It is worth noticing that there is only one system exhibiting all the seven features, CISpaces, which is unfortunately not (yet) available as an open-source implementation. Differently from OVA, CISpaces implements a subset of ASPIC, notably the ability to express only defeasible rules, and it follows a customised methodology for handling preferences, similar to ASPIC+ but using AFRA [Baroni *et al.*, 2011b] as the meta-representation system. However, it also encompasses both the ability to use an evolution of **ArgTrust** as a web-service, as well as models of probabilistic reasoning based on [Li *et al.*, 2012].

To conclude this analysis, it is worth showing the chronological evolution of all 38 systems reviewed in this survey, depicted in Figure 11. It is evident that 2014 has been the most prolific year, as also testified by the significant number (19) of demo submissions to COMMA 2014.

4.4 Projects for Informal Argumentation

Following the review of Schneider et al. [2013], there are further systems worth mentioning that make use of "informal" argumentation techniques. Indeed, they tend to be closer to user experience and they generally have a low entry barrier. At the same time, they do not offer much support for structuring arguments in a formal fashion, nor automated reasoning capabilities.

There is a large number of social networking debating systems such as Arguehow,[39] Climate CoLab [Gürkan *et al.*, 2010], ConsiderIt [Kriplean *et al.*, 2011],

[39] `http://arguehow.com/` (on 27/04/2017).

	F1	F2	F3	F4	F5	F6	F7
AGORA	Yes	Yes	Yes				
AIFdb	Yes		Yes			Yes	
AnalysisWall	Yes	Yes	Yes			Yes	
Arg&Dec		Yes	Yes	Yes			Yes
ArgTeach						Yes	
ArgTrust	Yes	Yes					Yes
ArgueApply		Yes	Yes	Yes		Yes	Yes
ArgMed	Yes	Yes				Yes	
ArguMed	Yes	Yes					Yes
Argument Blogging	Yes	Yes	Yes				
Argunet	Yes	Yes	Yes				
Arvina	Yes			Yes	Yes	Yes	
ASPARTIXWeb						Yes	
bCisive	Yes	Yes					
CISpaces	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Cohere/Compendium	Yes	Yes	Yes				
ConargWeb		Yes				Yes	
CoPe_it!	Yes	Yes	Yes				
D-BAS	Yes		Yes	Yes			
Debategraph	Yes	Yes	Yes	Yes			
GERD						Yes	Yes
Grafix						Yes	Yes
GrappaVis	Yes					Yes	Yes
Gorgias	Yes	Yes					Yes
Gorgias-B	Yes	Yes					
MARFs	Yes					Yes	Yes
Opinion Space			Yes				
OVA+	Yes	Yes	Yes	Yes		Yes	
Parmenides	Yes			Yes			
PIRIKA		Yes	Yes			Yes	
Quaestio-it	Yes	Yes	Yes	Yes			Yes
Rationale	Yes	Yes					
Reason	Yes	Yes	Yes				
Truthmapping	Yes	Yes	Yes	Yes			

Table 3: Comparative overview of systems (discontinued systems are omitted) using some form of formal argumentation. F1: structured argumentation; F2: argument manipulation; F3: collaborative; F4: enables dialogues, F5: based on speech acts; F6: Dung's reasoner, or F7: non-Dung's reasoner.

2017	**ArgueApply**
2016	**D-BAS**
	Gorgias-B
	GrappaVis
	MARFs
2015	**Arg&Dec**
	GERD
2014	**ArgTeach**
	Argument Blogging
	CISpaces
	ConargWeb
	Grafix
	OVA+
	PIRIKA
	Quaestio-it
2013	**AnalysisWall**
	bCisive
	Rationale
2012	**AIFdb**
	ArgMed
	Arvina
2011	**ArgTrust**
	Avicenna (discontinued)
2010	**ASPARTIXWeb**
	Opinion Space
	Dispute Finder (discontinued)
2009	**CoPe_it!**
	Debategraph
	Reason
2008	**Cohere/Compendium**
	Parmenides
	SEAS (discontinued)
2007	**Argunet**
2005	**AGORA**
2004	**Truthmapping**
2003	**ArguMed**
	Gorgias
	Trellis (discontinued)
1998	**ARGUE!** (then **ArguMed** in 2003)

Figure 11: History of systems from Section 4, both active and discontinued. The year refers to the first tracked publication or to the first time the system appears online.

ConvinceMe,[40], CreateDebate,[41] Debate.org,[42] Debatepidia,[43] Debatewise,[44] Hypernews,[45] and LivingVote.[46] Further systems worth mentioning are, e. g., Belvedere,[47] an open-source critical thinking support system; the Cabanac's annotation system[48] for investigating social validation of arguments in comments; and DiscourseDB,[49] that is used to collaboratively collect policy-related commentary.

5 Challenges

In this section we discuss current challenges in devising and implementing algorithms for solving problems related to formal argumentation. In particular, for abstract argumentation problems we discuss parallel algorithms (Section 5.1), approximation algorithms (Section 5.2), and dynamic selection of algorithms depending on graph features (Section 5.3). We also have a brief look at advanced techniques and the related challenges for some structured argumentation approaches (Section 5.4).

5.1 Parallelization

Reasoning tasks related to computational models of argumentation in general, and abstract argumentation in particular, are usually hard from the perspective of computational complexity, cf. e. g. [Dunne and Wooldridge, 2009]. In order to make systems applicable to real-world scenarios, specific measures have to be taken in order to overcome the NP-complexity barrier—or even higher. One such measure is to use *parallelization*. Modern computing systems usually provide many CPU cores that allow for multiple threads to be executed in parallel. Moreover, grid- or cluster-based systems collect the computational capacity of many single machines and provide an abstraction with access to many computing cores. In order to exploit the computational power of such parallel systems, algorithms have to be devised that allow for the decomposition of complex problems, independent solving of the individual sub-problems, and an effective aggregation of the partial results into a global solution. While not every computational problem allows for such a parallelization—or

[40]`http://hamschank.com/convinceme/index.html` (on 27/04/2017).
[41]`http://www.createdebate.com/` (on 27/04/2017).
[42]`http://debate.org` (on 27/04/2017).
[43]`http://www.debatepedia.com/` (on 27/04/2017).
[44]`http://debatewise.org/` (on 27/04/2017).
[45]`http://sourceforge.net/projects/hypernews/` (on 27/04/2017).
[46]`http://www.livingvote.org/` (on 27/04/2017).
[47]`http://belvedere.sourceforge.net/` (on 27/04/2017).
[48]`http://www.irit.fr/~Guillaume.Cabanac/expe/` (on 27/04/2017).
[49]`http://www.discoursedb.org/` (on 27/04/2017).

at least does not allow for parallelization with a significant gain in performance—parallelization has been applied to many NP-complete (or harder) problems in the past with some success, most notably to the problem SAT [Hölldobler *et al.*, 2011] allowing for considerable speed-ups on certain subclasses of instances.

For abstract argumentation, a natural feature to exploit for devising parallel algorithms is SCC-recursiveness [Baroni *et al.*, 2005]. A semantics is SCC-recursive if the problem of enumerating the extensions for the graph as a whole can be be decomposed in computing the extensions of its *strongly connected components*[50] (SCC). Once SCCs have been identified, extensions can be computed on each SCC separately and the resulting sub-extensions can be joined in order to obtain the extensions of the whole graph paying attention to the inter-dependencies among different SCCs.[51] This basic approach is followed by the algorithm presented in [Cerutti *et al.*, 2015], which itself is an enhancement to the previously published algorithm from [Cerutti *et al.*, 2014e].

The approach for parallelizing the computation of extensions in abstract argumentation outlined in [Cerutti *et al.*, 2015] is effective as long as the number of SCCs is "relatively" large in comparison to the size of the argumentation framework. Computing the SCCs of a graph can be done in polynomial time (see e. g. Tarjan's algorithm [Tarjan, 1972]) and, thus, the computational overhead of decomposing the problem is negligible in comparison to the computational effort of computing extensions, which is, as discussed before, often NP-hard or harder, depending on the chosen semantics. The computational effort required for the aggregation step is highly dependent on the actual instance of the problem and may be exponential in the worst case, as a sub-graph may possess an exponential number of extensions [Baumann and Strass, 2014] that need to be aggregated. However, for "reasonable" instances, this step is also negligible in comparison to the effort of computing extensions. As the empirical evaluation in [Cerutti *et al.*, 2015] suggests, exploiting SCC-recursiveness for parallelization may yield a speedup (up to 280%) when increasing the number of cores from 1 to 4.

Another approach to parallelization is not based on decomposing a problem into sub-problems, but on parallel execution of different algorithms for the whole problem. For many computationally hard problems there is usually a limited number of algorithms that can solve "most" of the instances in reasonable time, and the core problem is to determine *which* algorithm should be selected to solve a particular instance. This problem is called the *Algorithm selection* problem and will be dis-

[50]A subgraph of a directed graph is a strongly connected component, if there is a directed path from every vertex to each vertex and the subgraph is maximal.

[51]Other decomposition methods might take advantages of I/O-multipoles [Baroni *et al.*, 2014], but no approaches have been yet proposed.

cussed in more detail in Section 5.3. It is worth noticing that [Vallati *et al.*, 2017] proposes a first parallel algorithm selection approach. A straightforward solution to this problem is to devise a meta-algorithm that runs several algorithms on the original problem in parallel. As soon as the first algorithm terminates, the meta-algorithm terminates as well and the result of the meta-algorithm is the result of this algorithm. This approach, also called *variant-based parallel computation*, has been implemented in [Craven *et al.*, 2012] for the problem of deciding acceptance of arguments in *assumption-based argumentation* (ABA)[52] and has been applied in the medical domain. More specifically, the approach of [Craven *et al.*, 2012] is based on discussion games and different algorithms for solving acceptance use different expansion strategies in advancing the game.

The two approaches from above are complementary in the way how parallelization is realized. While the first approach uses a single algorithm and decomposes the problem instance into a parallel execution, the second approach uses multiple algorithms on the whole problem. Of course, combinations of the paradigms are imaginable.

5.2 Approximation Techniques

Parallelization offers an approach to overcome complexity barriers while maintaining soundness and completeness. A different and also often applied approach is to give up soundness and/or completeness and devise *approximation algorithms*, see e. g. [Vazirani, 2002; Cormen *et al.*, 2009]. Roughly, an approximation algorithm is not expected to solve the problem correctly but only within a certain margin of error. On the other hand, an approximation algorithm is expected to be more efficient than a correct algorithm.

In general, an algorithm A is said to be an ϵ-approximation algorithm for an optimization problem P (with $\epsilon > 0$), if for every instance the output of A is in the interval $[(1-\epsilon)C, (1+\epsilon)C]$, where C is the optimal solution, and ϵ thus represents the relative error in the approximation. Usually, one is interested in polynomial-time ϵ-approximation algorithms with ϵ being as small as possible. In case the algorithm returns more refined solutions—i. e. it decreases the ϵ-approximation further—if provided with additional runtime, it belongs to the class of *anytime algorithms*.

Approximation techniques for problems of abstract argumentation have not been investigated in-depth yet, with only very few exceptions. For example, the equational approach to abstract argumentation (see also [Gabbay, 2012; Gabbay and Rodriguez, 2014]) views an argumentation framework as a generator of equations for value

[52] While ABA is actually an approach to structured argumentation, we discuss it here as it is the only known parallel approach to structured argumentation.

assignments V such that $V(X) = 1$ indicates that X is in; $V(X) = 0$ indicates that X is out; and $V(X) \in (0,1)$ that X is undecided. In [Gabbay and Rodriguez, 2014] the authors introduce an iteration schema for computing complete extensions, starting from an arbitrary assignment V_0 and then, by use of a specific update rule, generating a sequence of assignments $V_0, V_1, \ldots$. In [Gabbay and Rodriguez, 2014] it is shown that this sequence will eventually converge and form a complete extension. This algorithm can therefore be interpreted as an anytime algorithm for computing complete extensions, but a thorough analysis of this algorithm in terms of approximation quality has not been done yet.

In the area of *probabilistic abstract argumentation* [Li *et al.*, 2012; Thimm, 2012; Hunter, 2014], which is concerned with combining abstract argumentation frameworks with probabilistic reasoning, approximation techniques from probabilistic reasoning have been applied to overcome the additional complexity necessary to deal with quantitative uncertainty [Hadoux *et al.*, 2015; Li *et al.*, 2012]. As probabilistic abstract argumentation is a topic that will be covered in later volumes of this handbook, we omit discussing these techniques here.

In summary, approximation techniques for computational models of arguments are still underdeveloped, but may gain attention in the near future.

5.3 Algorithm Selection

In Section 5.1 we already discussed the variant-based parallel computation approach of [Craven *et al.*, 2012] which is a specific solution for solving the *Algorithm Selection* problem by running different algorithms for the same problem in parallel. If parallelization is not possible for devising an algorithm, another solution is given by the *algorithm portfolio* approach [Rice, 1976; Leyton-Brown *et al.*, 2003; Xu *et al.*, 2008]. A portfolio is a meta-algorithm that has access to several specific algorithms for solving the same problem. When presented with a problem instance, the meta-algorithm selects one of those specific algorithms. In the case of *dynamic portfolios*, the meta-algorithm first extracts some *features* of the problem instance and then selects an algorithm that has, in a preprocessing step, proven to be the best algorithm for instances with the given features. This approach has been proven quite successful in solving many hard problems, such as SAT [Xu *et al.*, 2008].

The crucial step in developing a dynamic portfolio algorithm is to define which features are relevant both to assess the quality of the algorithms in the preprocessing step and to select the appropriate algorithm during runtime. Furthermore, it is important that the overhead introduced for computing features of the problem instance during runtime is "reasonably" small. In [Vallati *et al.*, 2014b; Cerutti *et al.*, 2014b] the authors presented 50 features of abstract argumentation frameworks and derived empirical performance models (EPMs) to determine the "best" implementation for enumerating preferred extensions, given CPU-time as evaluation criterion and a limited set of solvers. The features considered there were basic graph theory-based measures such as size of the graph, average degree of arguments, flow hierarchy, and so on. The two EPMs presented in [Cerutti *et al.*, 2014b] show an overall accuracy of 80% (classification) and, depending on the implementation, the ability to predict quite accurately the runtime required by a solver to enumerate the preferred extensions (regression). Unsurprisingly, the set of most informative features—according to a greedy forward search-based on the Correlation-based Feature Selection attribute evaluator [Hall, 1998] and with respect to the experimental setting used by the authors—includes the density of the argumentation graph, as well as number of SCCs and the size of the maximum SCC. When the computed EPMs have been applied to the problem of algorithm selection, both of them perform significantly well: in 78% of cases (resp. 75%) the classification-based EPM (the regression-based EPM) selects the best implementation. In most of the cases, 83%, both EPMS select the same algorithm, which is the correct one in 82% of cases.

Complete static and dynamic portfolios have been proposed in [Cerutti *et al.*, 2016d], and parallel portfolios are proposed and discussed in [Vallati *et al.*, 2017]. However, it is still unclear whether there may be better features to use for the selection problem or whether a combination of different techniques discussed in this section may yield improved performance. In [Brochenin *et al.*, 2015], *abstract solvers* [Nieuwenhuis *et al.*, 2006] are used as a formal machinery to formally specify different algorithms addressing extension-enumeration problems. By using these formalizations, algorithms could be combined and extended to more effective algorithms. Hence, using this machinery to also include the concepts discussed in this section may be a fruitful endeavor.

5.4 Advanced Techniques for Structured Argumentation

In structured argumentation, further computational problems than argument evaluation may occur. Many approaches to structured argumentation consider a knowledge base formalized in some logical formalism, and then derive arguments and conflicts between them on top of that, cf. Figure 4. Therefore, additional computational effort is required to construct arguments and to discover the conflict relationship between them. In general, computational approaches to structured argumentation can be categorized in two classes: those that use abstract argumentation frameworks as the underlying argument evaluation mechanism and those that provide proprietary evaluation mechanisms.

For the class of approaches providing proprietary evaluation mechanisms—such as Defeasible Logic Programming and earlier versions of Deductive Argumentation—the processes of argument construction, defeat discovery, and argument evaluation are usually intertwined, but each step still imposes some challenges.[53]

For argument construction, an important issue is relevance of arguments. In particular, for approaches building on classical logics—such as Deductive Argumentation—the number of arguments that can be derived from knowledge base may be potentially infinite. Given a specific query to the knowledge base, usually only those arguments are constructed that are relevant to the query and possess a certain normal form (in Deductive Argumentation these are the *maximally conservative undercuts*). In [Besnard and Hunter, 2006] an effective method for constructing both arguments and the defeat relation for a certain query is presented. This method relies on a preprocessing step that generates a so-called *compilation* from a knowledge base, which is an undirected graph with vertices being the minimal inconsistent subsets of the knowledge base and two vertices are connected if they have a non-empty intersection. Given a specific query, a traversal algorithm allows the complete construction of an argument tree from this compilation. Considering only *approximate arguments* [Hunter, 2006a]—e. g. arguments which are not necessarily minimal—also allows to gain efficiency by trading-off completeness or soundness (to some extent).

Another advanced technique for structured argumentation is *pruning of dialectical trees* in, e. g., Defeasible Logic Programming [Chesñevar *et al.*, 2000; Chesñevar and Simari, 2007; Rotstein *et al.*, 2011]. This technique also offers a solution to refrain from considering all arguments for evaluating a query. This is realized by only expanding the dialectical tree so far until the evaluation status of the query is decided. For example, if an argument possesses multiple attackers, and it can already be decided that the first attacker is ultimately accepted and defeats the argument, then there is no need to evaluate the acceptance status of the remaining attackers as it can already be decided that the argument under consideration is not acceptable. Yet another approach to address the very same issue is to evaluate different argumentation lines in a dialectical tree in parallel [García and Simari, 2000].

[53]For those approaches relying on abstract argumentation for argument evaluation, similar sophisticated techniques as outlined in this and the previous sections apply, but will not be discussed separately.

6 Evaluation of Implementations

While theoretical approaches to computational models of argumentation are usually analytically evaluated using rationality postulates or comparison of behavior on toy examples—see e. g. [Gorogiannis and Hunter, 2011; Caminada and Amgoud, 2005; Amgoud, 2014]—the evaluation of algorithms and implementations focuses on the three aspects of *correctness*, *performance*, and *usability*. The correctness of algorithms and implementations is usually shown in an analytical way and involves showing that the algorithmic representation corresponds to the formal definition, e. g. that the result of performing an algorithm indeed returns the grounded extension of a given abstract argumentation framework. In order to evaluate an *algorithm* with respect to performance, one usually conducts an analytical runtime or complexity analysis. For the performance evaluation of *implementations* an empirical evaluation on either artificial or real-world benchmarks and runtime measurement on the corresponding computational problems is essential for obtaining a comparative analysis of different approaches. Finally, in order to evaluate the usability of implementations, user studies have to be performed.

For the remainder of this section, we will focus on the problem of empirical performance evaluation of implementations of computational models of argumentation. In particular, we will focus on evaluations of implementations that solve problems for abstract argumentation frameworks, cf. Section 2. Those problems are an important aspect of any evaluation of implementations as well, as they provide clear formalizations of what are the expected outcomes of computational tasks. Another important aspect of such evaluations is the identification of suitable benchmarks, i. e. abstract argumentation graphs, that can be used to compare the performance of different implementations, which we discuss in Section 6.1. We discuss existing comparative analyses, in particular the *International Competition on Computational Models of Argumentation* (ICCMA),[54] in Section 6.2.

6.1 Benchmark Examples

A crucial issue in setting up an evaluation of an implementation of abstract argumentation problems is the identification of argument graphs that are used as benchmark examples. Ideally, real-world applications would provide these kind of benchmark graphs in order to test implementations on actually existing problems. Unfortunately, the availability of real-world benchmarks for argumentation problems is quite limited, some few exceptions are [Cabrio *et al.*, 2013; Cabrio and Villata, 2014b;

[54] `http://argumentationcompetition.org` (on 27/04/2017).

Cabrio and Villata, 2014a] and AIFdb.[55] Moreover, these benchmarks are tailored towards problems of argument mining [Wells, 2014] and their representation as abstract argumentation frameworks usually lead to topologically simple graphs, such as cycle-free graphs, which are unsuitable for comparing abstract argumentation solvers: all classical semantics coincide with grounded semantics on cycle-free graphs [Dung, 1995]. In order to compare solvers for—among others—preferred and stable semantics, artificially-generated argumentation graphs have been used so far.

Generating graphs for testing computational approaches or hypotheses on physical or social phenomena has already some tradition in network theory [Erdös and Rényi, 1959; Albert and Barabási, 2002; Pfeiffer *et al.*, 2012; Tabourier *et al.*, 2011; Barabasi and Albert, 1999]. However, it is questionable whether these graph models are suitable to model argumentation problems. For instance, the Barabási-Albert model [Barabasi and Albert, 1999] generates networks based on *preferential attachment.* The concept *preferential attachment* refers to the tendency of nodes that have already many connections to other nodes, to receive even more connections in the evolution of the network: an example of this phenomenon is the saying "the rich get richer, while the poor get poorer." To the best of our knowledge, there is no evidence that real-world argumentation adheres to this concept. Another concept from network theory often (indirectly) implemented in graph models is that of *triangle closure*, i. e., the tendency of nodes directly connecting to the neighbors of its neighbors (as in the saying "the friend of my friend is also my friend"). This concept is hardly applicable to argumentation graphs as this would imply that *defense* (an argument attacking the attacker of another argument) tends also to be a *direct attack* (the first argument attacking the argument it also defends).

Graph models from network theory also usually generate undirected graphs. Adapting a model to generate directed edges is of course trivial, but it is questionable whether the resulting graphs have any interpretation with respect to the original intention of the model.

Finally, from the perspective of challenging benchmarks for abstract argumentation, the graphs generated by such models are usually also not adequate. Initial experiments for ICCMA'15 [Thimm *et al.*, 2016] (see also below and the next section) suggest that those generated graphs usually contain an empty or a very small grounded extensions, usually no stable extensions (also due to the triangle closure property), and very few and small complete and preferred extensions. The latter observation is due to the fact that these graph models aim at modeling the "small world" property of many real-world graphs.[56] This leads to many arguments di-

[55]`http://corpora.aifdb.org` (on 27/04/2017).

[56]This property basically states that there are always "relatively short" paths from any node to every other node [Watts and Strogatz, 1998], provided that the network is connected and not too

rectly or indirectly being in conflict with each other. However, these models have been used for benchmark generation in earlier evaluations of implementations of abstract argumentation solvers [Bistarelli *et al.*, 2013; Bistarelli *et al.*, 2014].

In order to provide challenging benchmarks, ICCMA'15 used proprietary graph generators, each addressing different aspects of computationally hard graphs for specific semantics. For example, the `StableGenerator` aims at generating graphs with many stable extensions, and thus also many complete and preferred extensions. Graphs generated by this generator pose substantial combinatorial challenges for solvers addressing the computational tasks of determining (skeptical or credulous) acceptance of arguments and of enumerating extensions. For a given number of arguments, this generator first identifies a subset of these arguments to form an acyclic subgraph which will contain the grounded extension. Afterwards, another subset of arguments is randomly selected and attacks are randomly added from some arguments within this set to all arguments outside the set (except to the arguments identified in the first step). This process is repeated until a number of desired stable extensions is reached. The source code for this and other generators can be found in the source code repository[57] of *probo* [Cerutti *et al.*, 2014f], the benchmark suite used to run the competition. Another general tool for generating argumentation frameworks from a set given graph features is given by AFBenchGen[58] [Cerutti *et al.*, 2014d; Cerutti *et al.*, 2016a].

6.2 Comparative Analysis

The first systematic evaluations of implementations of abstract argumentation solvers have been conducted in [Bistarelli *et al.*, 2013; Bistarelli *et al.*, 2014]. In these evaluations a small number of implementations have been evaluated with respect to runtime on graphs generated by different graph models from social networking theory such as the Barabási-Albert model (see above). A similar performance evaluation is provided in [Vallati *et al.*, 2014a; Cerutti *et al.*, 2016d]. In addition, in [Cerutti *et al.*, 2016c] the authors discuss the effect of solver and instances configuration on performance.

A large-scale and systematic comparison of different implementations of computational models of argumentation is offered by the *International Competition on Computational Models of Argumentation* (ICCMA)[59], which has already been men-

complete. For example the theory of "six degrees of separation" suggests that in the social network of the known world the longest shortest path between any two persons is six.

[57] `http://sourceforge.net/p/probo/code/HEAD/tree/trunk/src/net/sf/probo/generators/` (on 27/04/2017).

[58] `https://sourceforge.net/projects/afbenchgen/` (on 27/04/2017).

[59] `http://argumentationcompetition.org` (on 27/04/2017).

tioned before and is an international event established in 2014. The first instance of the competition took place in 2015 and focused on comparing implementations for various decision and enumeration problems in abstract argumentation.

The competition in 2015 received 18 solvers from research groups in Austria, China, Cyprus, Finland, France, Germany, Italy, Romania, and UK. It was conducted using the benchmark framework *probo* [Cerutti *et al.*, 2014f], which provides the possibility to run the instances on the individual solvers, verify the results, measure the runtime, and log the results accordingly. The software *probo* is written in Java and requires the implementation of a simple command line interface from the participating solvers.[60] All benchmark graphs—generated using proprietary generation algorithms, see previous section—were made available in two file formats. The *trivial graph format*[61] (TGF) is a simple representation of a directed graph which simply lists all appearing vertices and edges. The *Aspartix format* (APX) [Egly *et al.*, 2008] is an abstract argumentation-specific format which represents an argumentation framework as facts in a logic programming-like way. In order to verify the answers of solvers, the solutions for all instances were computed in advance using the *Tweety libraries for logical aspects of artificial intelligence and knowledge representation*[62] [Thimm, 2014]. Tweety contains naïve algorithms for all considered semantics that implement the formal definitions of all semantics in a straightforward manner and thus provides verified reference implementations for all considered problems. Besides serving as the benchmark framework for executing the competition, *probo* also contains several abstract classes and interfaces for solver specification that can be used by participants in order to easily comply with the solver interface specification.

The competition in 2015 evaluated the runtime performance of the solvers for four different semantics and four different computational tasks, yielding a total of 16 tracks. Among the best solvers throughout all tracks were CoQuiAAS, ArgSemSAT, and LabSATSolver (see also Section 2). For detailed performance comparisons and current competitions see the webpage of ICCMA.[63]

7 Discussion

In this paper we discussed (1) approaches for addressing reasoning problems in abstract argumentation frameworks; (2) approaches for handling structured argu-

[60]See `http://argumentationcompetition.org/2015/iccma15notes_v3.pdf` (on 27/04/2017) for the formal interface description.

[61]`http://en.wikipedia.org/wiki/Trivial_Graph_Format` (on 27/04/2017).

[62]`http://tweetyproject.org` (on 27/04/2017).

[63]`http://argumentationcompetition.org` (on 27/04/2017).

mentation frameworks; and (3) other approaches that might be relevant to the argumentation community although they do not belong to the previous two classes.

As per approaches for abstract argumentation frameworks, it is beyond doubt that currently the majority of proposals adopt a reduction-based approach (Section 2.1), thus relying on SAT-solvers, or CSP-solvers, or ASP-solvers. However, we have covered the few direct implementations as discussed in Section 2.2.

Coming to approaches for structured argumentation frameworks, we considered the four large families developed in some 20 years of studies, viz. (in alphabetical order) ABA, ASPIC+, Deductive argumentation, and DeLP. We also considered the case of Carneades, which is both a formal model of argument structure and evaluation, and a system implementing the model.

Then, we reviewed 34 implemented systems that provide a general purpose gateway to formal structures of argumentation. They can be systems for producing corpora that can be exploited by argument mining algorithms as well as system for supporting critical thinking by the means of formal models of argumentation.

This touches one of the main topic of discussion still open in the community, namely applying machine learning techniques for automatic argument elicitation from natural language text, or *argument mining*, see [Budzynska *et al.*, 2014; Wells, 2014]. This is a fast growing research field, but at the same time, it encompasses a large variety of topics, from mining legal arguments, to mining tweets, and it is unlikely to have a *one-size-fits-all* approach. At the same time, this is an extremely young research field and best practices did not yet emerge in the community.

While we did not devote space to argument mining techniques, we instead discussed what are the main challenges we envisage for implementation of formal argumentation, as well as what are sensible ways for comparing different implementations. In particular, we reviewed (Section 5) the few approaches for making systems applicable to real-world scenarios, and thus overcoming the NP-complexity barrier, namely parallelization and approximation techniques. Moreover, machine learning techniques might also play an important role in selecting the right solver for a specific problem. There are, indeed, some embryonic approaches for automatic algorithm selection on the basis of abstract argumentation frameworks features. However, most—if not all—of the reviewed approaches consider abstract argumentation frameworks only.

This leads us to the last element of discussion we touched in this paper (Section 6), namely how to compare different systems by the means of benchmarks and competitions. Although the community already made a move in the context of abstract argumentation, with the first edition of the International Competition of Computational Models of Argumentation, we still have a long way ahead for addressing questions related to structured argumentation. Comparative studies on different formalisms, i. e. [Schulz and Caminada, 2015] and [Heyninck and Straßer, 2016], might shed some light on common grounds, thus allowing for a fair comparison.

Acknowledgements

This work has been funded by the Austrian Science Fund (FWF): P25521, I2854 and P30168, and by Academy of Finland under grants 251170 COIN and 284591.

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Received 31 May 2016

Rationality Postulates: applying argumentation theory for non-monotonic reasoning

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Abstract

The current review paper examines how to apply Dung's theory of abstract argumentation to define meaningful forms of non-monotonic inference. The idea is that arguments are constructed using strict and defeasible inference rules, and that it is then examined how these arguments attack (or defeat) each other. The thus defined argumentation framework provides the basis for applying Dung-style semantics, yielding a number of extensions of arguments. As each of the constructed arguments has a conclusion, an extension of arguments has an associated extension of conclusions. It are these extensions of conclusions that we are interested in. In particular, we ask ourselves whether each of these extensions is (1) consistent, (2) closed under the strict inference rules and (3) free from undesired interference. We examine the current generation of techniques to satisfy these properties, and identify some research issues that are yet to be dealt with.

1 Introduction

Argumentation, as it takes place in everyday life, is never completely abstract. Commonly, arguments are exchanged in order to determine what to do or what to believe. These arguments tend to be composed of reasons, some of which are strict and some of which are defeasible. Strict reasons (like rules of logic) provide conclusive evidence for a claim (like "Socrates is a man. All men are mortal. Therefore, Socrates is mortal.") whereas defeasible reasons (like rules of thumb) provide evidence for their claim that is only valid in the absence of counter evidence (like "Tux is a bird. Therefore Tux can fly."). The existence of defeasible reasons illustrates that for

The current paper will also be published as a chapter in the Handbook of Formal Argumentation.

commonsense reasoning, classical logic is often not sufficient, and that some form of nonmonotonic reasoning (as for instance provided by formal argumentation theory) is necessary.

Whereas defeasible reasons (formally represented as defeasible rules) provide a basis for nonmonotonic reasoning, strict reasons (formally represented as strict rules) provide the ability to model hard constraints (like "given our budget, if we acquire both product X and Y, then we cannot acquire product Z anymore"). By doing so, strict rules provide an important aspect of commonsense reasoning: the ability to reason about an outside world that has particular constraints (for instance of physical or financial nature) that are not subject to discussion.[1]

Suppose one would like to apply Dung's theory in the presence of strict and defeasible rules. That is, the idea is to apply the strict and defeasible rules to construct the arguments of the argumentation framework.[2] How can one be sure that the outcome makes sense from a logical perspective? Suppose there exists a rule representing the reason "given the current budget, if we acquire both product X and Y, then we cannot acquire product Z anymore", together with various other rules. In that case, what one would like to avoid is arguments for buying product X, Y and Z becoming justified (perhaps even in the same extension) because this would mean the constraint is violated. In principle, we could of course look inside of the arguments to check that what we select does not violate any constraint. However, the whole idea of Dung's abstract argumentation theory[3] is *not* to look at the internal structure of the arguments, and to select them based purely on their position in the graph. However, if one cannot look inside of the arguments when selecting them, then how does one make sure that the overall outcome (regarding conclusions on, say, what to do or what to believe) makes any sense?

In the current paper, we examine the question of how to apply Dung's theory

[1] Some argumentation researchers have claimed (personal communication) that if one digs deep enough, even strict rules start to have exceptions, and that therefore only defeasible rules exist. While this may be true from a philosophical perspective, one often wants to restrict the domain of reasoning and not take the more esoteric exceptions into account. The rule "given the current budget, if we acquire both product X and Y, we cannot acquire product Z anymore" may have exceptions if one is willing to steal, but this exception is of little relevance when the setting is a meeting at work. Also, the very idea of modelling information (be it by means of rules or by any other means) is that one limits oneself to a particular Universe of Discourse. Hence, strict rules can be seen as defeasible rules whose exceptions are beyond our current Universe of Discourse.

[2] Basically, this is done by chaining the rules together into inference trees, like is for instance done in [Modgil and Prakken, 2014; Toni, 2014; Caminada *et al.*, 2014b; Caminada *et al.*, 2015].

[3] Keep in mind that in Dung's theory, arguments are abstract, not atomic. Atomic would mean that arguments have no internal structure at all. Abstract means that arguments do have an internal structure, but that one does not take this structure into account (that is, one has *abstracted* from the internal structure).

of abstract argumentation for the purpose of non-monotonic reasoning with strict and defeasible rules. That is, we examine how to apply abstract argumentation semantics while making sure the overall outcome (in terms of justified conclusions) still makes sense. The remaining part of this paper is structured as follows. First, we will state some formal preliminaries on rule-based argumentation in Section 2. Then, in Section 3 we examine three desirable properties of the overall outcome (direct consistency, indirect consistency and closure) and examine various ways of satisfying these properties. Then, in Section 4 we examine two additional desirable properties (non-interference and crash resistance) that are particularly relevant when the strict rules are derived from classical logic, and again examine various ways of satisfying these properties. We round off with a summary and discussion in Section 5.

2 Formal Preliminaries

In the current section, we outline the process of constructing an argumentation framework from a set of strict and defeasible rules. For current purposes, we base our approach on the work of Caminada *et al.* [2014b].[4]

Definition 1. *Given a logical language that is closed under negation ($\neg$), an* argumentation system *is a tuple $AS = (\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ where:*

- *$\mathcal{R}_s$ is a finite set of strict inference rules of the form $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ (where φ_i, φ are meta-variables ranging over $\mathcal{L}$ and $n \geq 0$)*
- *$\mathcal{R}_d$ is a finite set of defeasible inference rules of the form $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ (where φ_i, φ are meta-variables ranging over $\mathcal{L}$ and $n \geq 0$)*
- *$\mathtt{n}$ is a partial function such that $\mathtt{n} : \mathcal{R}_d \longrightarrow \mathcal{L}$*
- *$\leq$ is a partial pre-order on $\mathcal{R}_d$*

We write $\psi = -\varphi$ in case $\psi = \neg\varphi$ or $\varphi = \neg\psi$ (we will sometimes informally say that formulas φ and $-\varphi$ are each other's negation).

To keep things simple, we assume that the logical language $\mathcal{L}$ consists of literals only.[5]

[4] As such, we will for instance not consider the notion of contraries [Modgil and Prakken, 2014] or any other notions in ASPIC+ that are not relevant for current purposes.

[5] In Section 4 we generalise things by having $\mathcal{L}$ be the language of propositional logic.

In the following definition, arguments are constructed from strict and defeasible rules in an inductive way. This process starts from the strict and defeasible rules with empty antecedents (so where $n = 0$).

Definition 2. *An* argument *A on the basis of an argumentation system $AS = (\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ is defined as:*

1. $A_1, \ldots, A_n \rightarrow \psi$ *if* $A_1 \ldots A_n$ *($n \geq 0$) are arguments, and there is a strict rule* $Conc(A_1), \ldots, Conc(A_n) \rightarrow \psi$ *in* $\mathcal{R}_s$. *In that case we define*
   $Conc(A) = \psi$,
   $Sub(A) = Sub(A_1) \cup \ldots \cup Sub(A_n) \cup \{A\}$.
   $DefRules(A) = DefRules(A_1) \cup \ldots \cup DefRules(A_n)$,
   $TopRule(A) = Conc(A_1), \ldots, Conc(A_n) \rightarrow \psi$

2. $A_1, \ldots, A_n \Rightarrow \psi$ *if* $A_1 \ldots A_n$ *($n \geq 0$) are arguments, and there is a defeasible rule* $Conc(A_1), \ldots, Conc(A_n) \Rightarrow \psi$ *in* $\mathcal{R}_d$. *In that case we define*
   $Conc(A) = \psi$,
   $Sub(A) = Sub(A_1) \cup \ldots \cup Sub(A_n) \cup \{A\}$,
   $DefRules(A) = DefRules(A_1) \cup \ldots \cup DefRules(A_n) \cup \{Conc(A_1), \ldots, Conc(A_n) \Rightarrow \psi\}$,
   $TopRule(A) = Conc(A_1), \ldots, Conc(A_n) \Rightarrow \psi$.

Furthermore, for any argument A and set of arguments E:

- *A is* strict *iff $DefRules(A) = \emptyset$;* defeasible *iff $DefRules(A) \neq \emptyset$;*
- *If $DefRules(A) = \emptyset$, then $LastDefRules(A) = \emptyset$, else; if $A = A_1, \ldots, A_n \Rightarrow \phi$ then $LastDefRules(A) = \{Conc(A_1), \ldots, Conc(A_n) \Rightarrow \phi\}$, otherwise $LastDefRules(A) = LastDefRules(A_1) \cup \ldots \cup LastDefRules(A_n)$.*
- $Concs(E) = \{Conc(A) \mid A \in E\}$
- *The closure under strict rules of E, denoted $Cl_S(E)$ is the smallest set containing $Concs(E)$ and the consequent of any strict rule in $\mathcal{R}_s$ whose antecedent is contained in $Cl_S(E)$.*

For current purposes (as well as is done in [Caminada and Amgoud, 2007; Prakken, 2010; Caminada *et al.*, 2014b]) we assume that the set of strict rules is consistent in the following way.

Definition 3. *Let $AS = (\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ be an argumentation system. We say that AS and $\mathcal{R}_s$ are consistent iff no strict arguments A and B exist such that* $\mathtt{Conc}(A) = -\mathtt{Conc}(B)$

Definition 4. *Let A and B be arguments. We say that*

- *A undercuts B (on B') iff $Conc(A) = -\mathtt{n}(r)$ for some $B' \in Sub(B)$ with $TopRule(B') = r$ and $r \in \mathcal{R}_d$*
- *A restrictively rebuts B (on B') iff $Conc(A) = -Conc(B')$ for some $B' \in Sub(B)$ with $TopRule(B') \in \mathcal{R}_d$*
- *A unrestrictively rebuts B (on B') iff $Conc(A) = -Conc(B')$ for some $B' \in Sub(B)$ with B' being a defeasible argument*

To illustrate the difference between restricted rebut and unrestricted rebut, first consider the example of an argumentation system AS_1 with $\mathcal{R}_s = \emptyset$ and $\mathcal{R}_d = \{\Rightarrow a;\ a \Rightarrow b;\ \Rightarrow c;\ c \Rightarrow \neg b\}$. Here, the argument $(\Rightarrow a) \Rightarrow b$ restrictively and unrestrictively rebuts the argument $(\Rightarrow c) \Rightarrow \neg b$, and vice versa. In the argumentation system AS_2 with $\mathcal{R}_s = \{\rightarrow a;\ a \rightarrow b\}$ and $\mathcal{R}_d = \{\Rightarrow c;\ c \Rightarrow \neg b\}$, the argument $(\rightarrow a) \rightarrow b$ restrictively and unrestrictively rebuts the argument $(\Rightarrow c) \Rightarrow \neg b$, but the argument $(\Rightarrow c) \Rightarrow \neg b$ does not restrictively or unrestrictively rebut the argument $(\rightarrow a) \rightarrow b$. In the argumentation system AS_3 with $\mathcal{R}_s = \{a \rightarrow b;\ \rightarrow c\}$ and $\mathcal{R}_d = \{\Rightarrow a;\ c \Rightarrow \neg b\}$ the argument $(\Rightarrow a) \rightarrow b$ restrictively and unrestrictively rebuts the argument $(\rightarrow c) \Rightarrow \neg b$, and the argument $(\rightarrow c) \Rightarrow \neg b$ unrestrictively (but not restrictively) rebuts the argument $(\Rightarrow a) \rightarrow b$. To sum up, with restrictive rebut one needs to check whether the *last* rule of the attacked conclusion[6] is defeasible whereas with unrestricted rebut one needs to check whether *any previous* rule of the attacked conclusion is defeasible.

The intuition behind unrestricted rebut is that a conclusion is defeasible iff it has been derived using at least one defeasible rule. If the conclusion has been derived using strict rules only, then the conclusion is strict and cannot be argued against. The intuition behind restricted rebut, on the other hand, is that (like in classical logic) in order to argue against a particular derivation, one has to argue against its premises. So instead of attacking the consequent of a strict rule, one has to attack its antecedent, unless this antecedent itself consists of the consequents of strict rules, in which case one has to keep on going backwards until finding a defeasible rule. It holds that if A restrictively rebuts B, then A also unrestrictively rebuts B, but not vice versa.

One last subtle aspect of the definition of restricted and unrestricted rebut (Definition 4) is that one only looks at the subargument B' that yields the conclusion that one is arguing against. So in the argumentation system AS_4 with $\mathcal{R}_s = \{\rightarrow c;\ c \rightarrow \neg b\}$ and $\mathcal{R}_d = \{\Rightarrow a;\ a \Rightarrow b;\ \neg b \Rightarrow d\}$ the argument $(\Rightarrow a) \Rightarrow b$

[6] meaning: of the conclusion one argues against by providing an argument for its contrary

does *not* (restrictively or unrestrictively) rebut the argument $((\to c) \to \neg b) \Rightarrow d$, even though the latter argument is defeasible, because the subargument that yields the attacked conclusion $\neg b$ is strict.

The difference between restricted and unrestricted rebut is relevant not just because they are based on different intuitions, but also because choosing to implement either restricted or unrestricted rebut has consequences for how one should define the rest of the argumentation formalism if the aim is to yield some kind of reasonable output in terms of justified conclusions. Details will follow further on in the current paper.

Apart from (restrictive and unrestrictive) rebutting, Definition 4 also introduces the concept of undercutting. Whereas with rebutting, one argues against the conclusion of an argument (or against the conclusion of a subargument), with undercutting one argues against the applicability of a particular defeasible rule. A classical example of undercutting has been given by Pollock [1995]: "If an object looks red, then it actually is red, unless it is illuminated by a red light". Formally, this can be modelled using argumentation system AS_5 with $\mathcal{R}_s = \{\to \mathit{looksred};\ \to \mathit{redlight}\}$, $\mathcal{R}_d = \{\mathit{looksred} \Rightarrow \mathit{isred};\ \mathit{redlight} \Rightarrow \neg \mathit{lris}\}$ and $\mathtt{n}(\mathit{looksred} \Rightarrow \mathit{isred}) = \mathit{lris}$. Here, the argument $(\to \mathit{looksred}) \Rightarrow \mathit{isred}$ is undercut by the argument $(\to \mathit{redlight}) \Rightarrow \neg \mathit{lris}$. Although undercutting does not play a major role in the remaining part of the current paper, we have still chosen to introduce it, as it is a piece of functionality that can be implemented while still warranting an overall reasonable outcome regarding the justified conclusions.

Another piece of functionality that some formalisms have implemented is that of argument strength.[7] Argument strength is often defined based on an ordering of the defeasible rules. However, as arguments can be constructed using more than one defeasible rule, one needs a way of applying the strength ordering between *individual* rules to determine a strength ordering between *sets* of rules. Two principles for doing so have been defined in the literature: the elitist and the democratic set ordering [Modgil and Prakken, 2014; Caminada *et al.*, 2014b].

Definition 5. *Let* $\leq \subseteq (\mathcal{R}_d \times \mathcal{R}_d)$ *be a total pre-ordering on the defeasible inference rules, where as usual,* $r < r'$ *iff* $r \leq r'$ *and* $r \not\leq r'$*, and* $r \equiv r'$ *iff* $r \leq r'$ *and* $r' \leq r$*. Then for any* $\mathcal{E}, \mathcal{E}' \subseteq \mathcal{R}_d$ $\trianglelefteq_{\mathtt{s}}$ *(*$\mathtt{s} \in \{\mathtt{Eli}, \mathtt{Dem}\}$*) is defined as follows:*

1. *If* $\mathcal{E} = \emptyset$ *then* $\mathcal{E} \ntrianglelefteq_{\mathtt{s}} \mathcal{E}'$ *;*
2. *If* $\mathcal{E}' = \emptyset$ *and* $\mathcal{E} \neq \emptyset$ *then* $\mathcal{E} \trianglelefteq_{\mathtt{s}} \mathcal{E}'$*; else:*

[7]Argument strength is sometimes referred to as *argument preferences* in the work of Prakken [2010], Modgil and Prakken [2014] and of Caminada *et al.* [2014b].

3. if $\mathtt{s} = \mathtt{Eli}$*:* $\mathcal{E} \trianglelefteq_{\mathtt{Eli}} \mathcal{E}'$ *if* $\exists r_1 \in \mathcal{E}$ *s.t.* $\forall r_2 \in \mathcal{E}'$*,* $r_1 \leq r_2$*; else:*

4. if $\mathtt{s} = \mathtt{Dem}$*:* $\mathcal{E} \trianglelefteq_{\mathtt{Dem}} \mathcal{E}'$ *if* $\forall r_1 \in \mathcal{E}$*,* $\exists r_2 \in \mathcal{E}'$*,* $r_1 \leq r_2$*.*

As usual $\mathcal{E} \triangleleft_{\mathtt{s}} \mathcal{E}'$ *iff* $\mathcal{E} \trianglelefteq_{\mathtt{s}} \mathcal{E}'$ *and* $\mathcal{E}' \ntrianglelefteq_{\mathtt{s}} \mathcal{E}$

The elitist and democratic set ordering principles assume the presence of sets of defeasible rules. This leads to the question of how to determine the relevant sets of defeasible rules when one argument rebuts another. Again, two principles have been formulated in the literature, called *weakest link* and *last link*. With weakest link, one takes into account *all* defeasible rules (of both the rebutting argument and the rebutted (sub)argument), whereas with last link, one takes into account only the *last* defeasible rule(s). Given the weakest link and the last link principles for determining the sets of relevant defeasible rules, as well as the elitist and democratic set ordering principles for evaluating these sets of defeasible rules, one can identify four different principles for determining argument strength.

Definition 6. *Let Ar be the set of arguments that can be constructed using argumentation system* $(\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$*. Then* $\forall A, B \in Ar$*:*

1. $A \preceq_{\mathtt{Ewl}} B$ *iff* $\mathtt{DefRules}(A) \trianglelefteq_{\mathtt{Eli}} \mathtt{DefRules}(B)$

2. $A \preceq_{\mathtt{Ell}} B$ *iff* $\mathtt{LastDefRules}(A) \trianglelefteq_{\mathtt{Eli}} \mathtt{LastDefRules}(B)$

3. $A \preceq_{\mathtt{Dwl}} B$ *iff* $\mathtt{DefRules}(A) \trianglelefteq_{\mathtt{Dem}} \mathtt{DefRules}(B)$

4. $A \preceq_{\mathtt{Dll}} B$ *iff* $\mathtt{LastDefRules}(A) \trianglelefteq_{\mathtt{Dem}} \mathtt{LastDefRules}(B)$

where `Ewl`*,* `Ell`*,* `Dwl` *and* `Dll` *respectively denote '*`Elitist` *weakest link',*
'`Elitist` *last link', '*`Democratic` *weakest link' and '*`Democratic` *last link'.*
We may write $A \prec_p B$ *iff* $A \preceq_p B$ *and* $B \npreceq_p A$*, and write* $A \approx_p B$ *iff* $A \preceq_p B, B \preceq_p A$ *(where* $p \in \{\mathtt{Ewl}, \mathtt{Ell}, \mathtt{Dwl}, \mathtt{Dll}\}$*). It is straightforward to show that* $\prec_p$ *is a strict partial ordering (irreflexive, transitive and asymmetric).*

We are now ready to define the overall notion of defeat. For this, we follow the approach of formalisms like ASPIC+ [Modgil and Prakken, 2014] and ASPIC- [Caminada *et al.*, 2014b], where the notion of defeat stands for attack after argument strength has been taken into account. It is defeat, not attack, that is then used to define the argumentation framework.

Definition 7. *Let Ar be the set of arguments that can be constructed using argumentation system* $AS = (\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$*. Let* $\preceq_p$ *be the associated argument strength order on Ar as defined in Definition 6. Then* $def_{ur} \subseteq Ar \times Ar$ *is defined as*

$(A, B) \in def_{ur}$ iff A undercuts B or A unrestrictively rebuts B on B' and $A \not\prec_p B'$, and $def_{rr} \subseteq Ar \times Ar$ is defined as $(A, B) \in def_{rr}$ iff A undercuts B or A restrictively rebuts B on B' and $A \not\prec_p B'$.

We observe that the set of arguments Ar, together with the associated defeat relation (either def_{ur} or def_{rr}) defines a Dung-style argumentation framework. On this argumentation framework, one can then apply the standard argumentation semantics.

3 Direct Consistency, Indirect Consistency and Closure

To illustrate the issue of rationality postulates, consider the following example.

Example 1 ([Caminada and Amgoud, 2007]). *Consider an argumentation system $AS = (\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ with $\mathcal{R}_s = \{\rightarrow r;\ \rightarrow n;\ m \rightarrow hs;\ b \rightarrow \neg hs\}$, $\mathcal{R}_d = \{r \Rightarrow m;\ n \Rightarrow b\}$, $\mathtt{n} = \emptyset$ and $\leq = \emptyset$.*

An intuitive interpretation of this example is the following:
"John wears a ring (r) on his finger. John is also a regular nightclubber (n). Someone who wears a ring on his finger is usually married (m). Someone who is a regular nightclubber is usually bachelor (b). Someone who's married by definition has a spouse (hs). Someone who's bachelor by definition does not have a spouse ($\neg hs$)."

We can construct the following arguments.

$A_1 : \rightarrow r \quad A_3 : A_1 \Rightarrow m \quad A_5 : A_3 \rightarrow hs$
$A_2 : \rightarrow n \quad A_4 : A_2 \Rightarrow b \quad A_6 : A_4 \rightarrow \neg hs$

If one were to apply unrestricted rebut, the only defeat would be between A_5 and A_6. That is, $def_{ur} = \{(A_5, A_6), (A_6, A_5)\}$. This then implies that for instance the grounded extension is $\{A_1, A_2, A_3, A_4\}$, yielding the associated set of (grounded) justified conclusions $\{r, n, m, b\}$. The problem with these conclusions, however, is that they do not take into account the meaning of the strict rules of the argumentation system: that if one holds the antecedent of a strict rule to be the case, one must also hold what deductively follows from it (the consequent of the rule). For instance, from the fact that we obtain m, together with the strict rule $m \rightarrow hs$ we should also have obtained hs, as a married person by definition has a spouse, so by John being married we cannot escape the conclusion that he has a spouse. Yet, the fact that John has a spouse is not represented in the set of justified conclusions (that is, $hs \notin \{r, n, m, b\}$). This brings us to the first problem: the set of justified conclusions is not closed under the strict rules.

Another problem appears when also applying the strict rule $b \rightarrow \neg hs$. After all, John is also considered to be a bachelor, so we cannot escape the conclusion that he *does not have a spouse ($\neg hs$). However, when we also apply the rule $m \rightarrow hs$, as we did earlier, then we derive that John both has a spouse and does not have a spouse. So not only is our set $\{r, n, m, b\}$ of justified conclusions not closed under the strict rules, if we do try to compute its closure, this closure turns out to be inconsistent!*

So far, we examined what happens regarding the justified conclusions in case we apply unrestricted rebut. However, if we were to base the defeat relation on restricted rebut instead, then the outcome would even be worse, as the defeat relation would become empty (that is, $def_{rr} = \emptyset$) which means that (when still applying grounded semantics) one obtains $\{A_1, A_2, A_3, A_4, A_5, A_6\}$ as the grounded extension and $\{r, n, m, b, hs, \neg hs\}$ as the associated justified conclusions. So here, we don't even need to close the justified conclusions under the strict rules in order to obtain an inconsistent outcome, as the set of justified conclusions is already inconsistent by itself.

From Example 1 we observe that there are at least three desirable properties a set of conclusions should satisfy.

Postulate 1. *Let $S \subseteq \mathcal{L}$ be a set of justified conclusions yielded by an argumentation system. S should satisfy:*

- direct consistency*, meaning that* $\neg\exists x : x, -x \in S$
- closure*, meaning that* $Cl_{\mathcal{R}_s}(S) = S$
- indirect consistency*, meaning that* $\neg\exists x : x, -x \in Cl_{\mathcal{R}_s}(S)$

Early formalisations of argumentation theory tried to avoid problems like those illustrated in Example 1 by tinkering with the definition of defeat. However, as explained by Caminada and Amgoud [2007], this does not actually lead to the properties of Postulate 1 being satisfied. Clearly, some more fundamental solutions are needed. In the following two subsections, we examine some of the solutions that have been described in the literature, distinguishing between solutions that have been obtained for restricted rebut and solutions that have been obtained for unrestricted rebut.

3.1 Restricted Rebut Solutions

In the current section, we examine some of the solutions that have been described in the literature for satisfying direct consistency, indirect consistency and closure when the defeat relation is based on restricted rebut.

We recall that, when applying restricted rebut to Example 1 this results in the empty defeat relation, that is $def_{rr} = \emptyset$. One could argue that this is because something is wrong with the information encoded in the argumentation system AS, in particular with the set of strict rule $\mathcal{R}_s$. If one were for instance to add the additional strict rules $\neg hs \rightarrow \neg m$ and $hs \rightarrow \neg b$ then the problem would be solved. This is because one could then construct additional arguments $A_7 : A_5 \rightarrow \neg b$ and $A_8 : A_6 \rightarrow \neg m$. It holds that A_7 restrictively rebuts A_4 (as well as each argument that contains A_4, so also A_6 and A_8) and that A_8 restrictively rebuts A_3 (as well as each argument that contains A_3, so also A_5 and A_7). So overall we obtain the argumentation framework shown in Figure 1. This argumentation framework yields the grounded extension $\{A_1, A_2\}$ (with associated conclusions $\{r, n\}$) and preferred extensions $\{A_1, A_2, A_3, A_5, A_7\}$ (with associated conclusions $\{r, n, m, hs, \neg b\}$) and $\{A_1, A_2, A_4, A_6, A_8\}$ (with associated conclusions $\{r, n, b, \neg hs, \neg m\}$). As we can see, each set of conclusions yielded under grounded or preferred semantics satisfies the postulates of direct consistency, closure and indirect consistency.

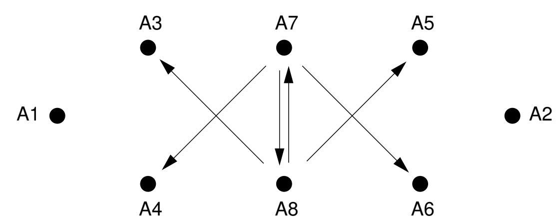

Figure 1: Argumentation framework of Example 1 after adding the rules $\neg hs \rightarrow \neg m$ and $hs \rightarrow \neg b$.

Adding the rules $\neg hs \rightarrow \neg m$ and $hs \rightarrow \neg b$ can be seen as a reasonable thing to do. After all, $\mathcal{R}_s$ already contains a rule $m \rightarrow hs$, meaning that without possible exception, someone who is married by definition has a spouse. This implies that someone who does not have a spouse cannot be married. Hence, $\neg hs \rightarrow \neg m$. Using similar reasoning, one can use the rule $b \rightarrow \neg hs$ to derive $hs \rightarrow \neg b$. Hence, the rules $\neg hs \rightarrow \neg m$ and $hs \rightarrow \neg b$ were already "implicitly" contained in $\mathcal{R}_s$. Adding them explicitly can therefore be seen as doing justice to $\mathcal{R}_s$, and has as a side effect that the postulates of direct consistency, closure and indirect consistency become satisfied.

Adding the "contraposed" version of a strict rule is relatively straightforward when the antecedent of the rule consists just of a single formula (as is for instance the case for $m \rightarrow hs$ and $b \rightarrow \neg hs$) but gets more complicated when the antecedent consists of multiple formulas. For this, a generalised version of contraposition is needed, which is referred to as *transposition* [Caminada and Amgoud, 2007].

Definition 8 ([Caminada and Amgoud, 2007]). *Let $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ ($n \geq 0$) be a strict rule. A transposed version of this rule is of the form $\varphi_1, \ldots, \varphi_{i-1}, -\varphi, \varphi_{i+1}, \ldots, \varphi_n \rightarrow -\varphi$ (for some $i \in \{1 \ldots n\}$). We say that a set of strict rules $\mathcal{R}_s$ is closed under transposition when for each strict rule in $\mathcal{R}_s$, each of its transposed versions is also in $\mathcal{R}_s$.*

As an example, the strict rule $a, \neg b, c \rightarrow d$ has three transposed versions: $\neg d, \neg b, c \rightarrow \neg a$; $a, \neg d, c \rightarrow b$ and $a, \neg b, \neg d \rightarrow \neg c$.

An example of an argumentation formalism that applies transposition to satisfy direct consistency, closure and indirect consistency is ASPIC+ [Modgil and Prakken, 2014]. In ASPIC+ the following design choices have been made:

- the set of strict rules $\mathcal{R}_s$ is consistent and closed under transposition
- restricted rebut is applied
- argument strength is based on a partial pre-order on the defeasible rules, together with either the last-link or weakest link selection principle and either the elitist or democratic set ordering principle[8]
- the argumentation semantics is complete-based, meaning that it selects one or more complete extensions (examples of complete-based semantics are grounded, preferred, complete, semi-stable, ideal and eager semantics)

It is shown that under these choices, the overall outcome of the formalism satisfies direct consistency, closure and indirect consistency.

To understand why transposition plays an important role in satisfying the properties of direct consistency, closure and indirect consistency, it can be useful to give a sketch of proof. We start with the property of direct consistency. Suppose, towards a contradiction, that there exists a complete extension yielding conclusions that are directly inconsistent. This means there exists an argument A for conclusion c and an argument B for conclusion $-c$ (see Figure 2). As the set of strict rules $\mathcal{R}_s$ is consistent, at least one of these arguments must be defeasible. Assume without loss of generality that argument A is defeasible. Then A must contain at least one defeasible rule. Now, identify a defeasible rule r that is "as high as possible" in A (that is, whose distance to the conclusion c is minimal). Let e be the consequent of r and let A_i be the subargument of A that has r as its top rule (so $Conc(A_i) = e$). Let $A_1, \ldots, A_n$ be the subarguments of A that have the same "depth" as A_i (that

[8]More precisely, argument strength has to be based on a *reasonable argument ordering* [Modgil and Prakken, 2014], which is satisfied by applying either the weakest link or the last link selection principle, in combination with applying either the democratic or the elitist set ordering principle.

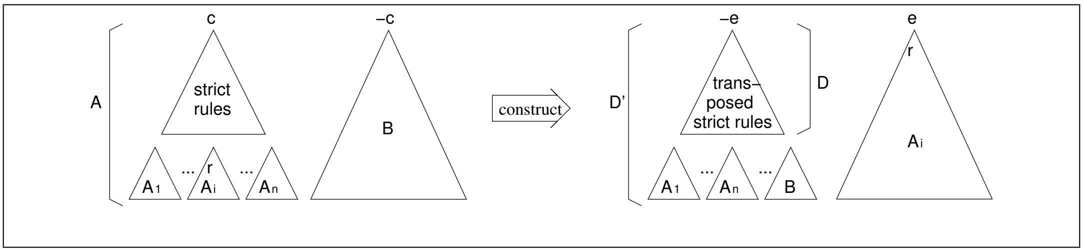

Figure 2: Sketch of proof direct consistency (restricted rebut)

is, whose respective top-rules have the same distance to conclusion c). It turns out to be possible to build an argument D' that defeats A_i by deriving conclusion $-e$. Recall that "above" each A_i there are only strict rules in A (after all, r was the "highest" defeasible rule in A). In case these strict rules consist of only one layer, there exists a single strict rule $Conc(A_1), \ldots, Conc(A_n) \rightarrow c)$ with transposed version $Conc(A_1), \ldots, Conc(A_{i-1}), -c, Conc(A_{i+1}), \ldots, Conc(A_n) \rightarrow -Conc(A_i)$, so $Conc(A_1), \ldots, Conc(A_{i-1}), Conc(B), Conc(A_{i+1}), \ldots, Conc(A_n) \rightarrow -c$, which implies we can use $A_1, \ldots A_{i-1}$, B and $A_{i+1}, \ldots, A_n$ to construct an argument that restrictively rebuts A_i. In case the strict rules above each A_i consist of more than one layer, then one can still use transposition to construct an argument that restrictively rebuts A_i (basically by induction over the number of layers of strict rules). Let D' be the thus constructed argument that restrictively rebuts A_i. As A_i is a subargument of A, it follows that D' also restrictively rebuts A. From the fact that we are considering a complete extension, it follows that the extension has to contain an argument (say C) that defeats D'. However, as each defeasible rule of D' also occurs in A or B, it follows that C also defeats A or B.[9] Hence, the complete extension is not conflict-free. Contradiction.

It is important to observe that the above sketch of proof uses the facts that (1) $\mathcal{R}_s$ is consistent, (2) $\mathcal{R}_s$ is closed under transposition, (3) restricted rebut is being applied, and (4) we are considering a complete extension (or at least an admissible set).[10]

As for the property of closure, suppose there exists a strict rule $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ and that the conclusions $\varphi_1, \ldots, \varphi_n$ are yielded by our complete extension. We need to show that conclusion φ is also yielded by the complete extension. From the fact

[9] This is straightforward to see when the strength ordering between the rules is empty, but also holds when the strength ordering is non-empty. See the work of Modgil and Prakken [2013] for details.

[10] There are also some requirements regarding argument strength. These are such that $\preceq_{\texttt{Ewl}}$, $\preceq_{\texttt{Ell}}$, $\preceq_{\texttt{Dwl}}$, and $\preceq_{\texttt{Dll}}$ (Definition 6) satisfy them. We refer to the work of Modgil and Prakken [2013; 2014] for details.

that conclusions $\varphi_1, \ldots, \varphi_n$ are yielded, it follows that the complete extension contains arguments $A_1, \ldots, A_n$ with conclusions $\varphi_1, \ldots, \varphi_n$ respectively. Now consider the argument $A : A_1, \ldots, A_n \rightarrow \varphi$. Let B be an arbitrary argument that defeats A. Then from the definition of defeat, it follows that B also defeats at least one of $A_1, \ldots, A_n$. From the fact that our extension is complete (and therefore also admissible) it follows that it contains an argument (say C) that defeats B. This means that A is defended by the complete extension, and must therefore also be contained in the complete extension.[11] This then implies that the complete extension also yields conclusion $Conc(A) = \varphi$.

Given that we have obtained both direct consistency and closure, the property of indirect consistency is trivially satisfied.

As was mentioned above, the property of transposition plays an important role for satisfying direct consistency, closure and indirect consistency. However, if one takes a closer look at the above sketch of proof, what is actually applied is a property that is more general than transposition. Going back to Figure 2 then what is actually needed is that if from $Conc(A_1), \ldots, Conc(A_n)$ one can apply strict rules to derive c, then from $Conc(A_1), \ldots, Conc(A_{i-1}), -c, Conc(A_{i+1}), \ldots,\ Conc(A_n)$ one can also apply strict rules to derive $-Conc(A_i)$. This property is called *contraposition* by Modgil and Prakken [2013; 2014], who show that direct consistency, closure and indirect consistency are satisfied when the set of strict rules is closed under contraposition.

One can ask the question of whether it is possible to derive even more general conditions than transposition and contraposition, under which direct consistency, closure and indirect consistency are still satisfied. This question is answered positively by Dung and Thang [2014] who present a semi-abstract approach that abstracts away from most aspects of argument structure (making explicit only the notions of a conclusion and that of a subargument). However, their approach does rely on particular constraints on the defeat relation, and it can be observed that these constraints can only be satisfied under restricted (and not unrestricted) rebut.[12]

3.2 Unrestricted Rebut Solutions

Although restricted rebut has become the most popular principle for defining the overall defeat relationship (as is for instance evidenced by the various versions of the

[11]Notice that for this reasoning step, a complete extension is really needed; an admissible set is not sufficient.

[12]More precisely, unrestricted rebut trivialises the notion of a *base* [Dung and Thang, 2014], which prevents the results of Dung and Thang [2014] from being applied in the context of unrestricted rebut.

ASPIC+ formalism [Prakken, 2010; Modgil and Prakken, 2013; Modgil and Prakken, 2014]) it does have some disadvantages, especially when applied in a dialectical context. Consider for instance the following discussion taken from [Caminada *et al.*, 2014b].

John: *"Bob will attend both AAMAS and IJCAI this year, as he has papers accepted at each of these conferences."*

Mary: *"That won't be possible, as his budget of £1000 only allows for one foreign trip."*

Formally, this discussion can be modelled using the argumentation system $(\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ with $\mathcal{R}_d = \{accA \Rightarrow attA;\ accI \Rightarrow attI;\ budget \Rightarrow \neg attboth\}$ and $\mathcal{R}_s = \{\rightarrow accA;\ \rightarrow accI;\ \rightarrow budget;\ attA, attI \rightarrow attboth;\ \neg attboth, attI \rightarrow \neg attA;\ attA, \neg attboth \rightarrow \neg attI\}$.[13]

John: $((\rightarrow accA) \Rightarrow attA), ((\rightarrow accI) \Rightarrow attI) \rightarrow attboth$

Mary: $(\rightarrow budget) \Rightarrow \neg attboth$

The problem is that when applying restricted rebut, Mary's argument does not defeat John's argument. This is because the conclusion that Mary wants to attack ($attboth$) is the consequent of a strict rule. If Mary wants to restrictively rebut John's argument, she can only do so by attacking the consequent of a defeasible rule. That is, she would be forced to choose to defeat either $attA$ or $attI$, meaning that she essentially has to utter one of the following statements.

Mary$'$: *Bob won't attend AAMAS because he will already attend IJCAI, and his budget doesn't allow him to attend both.*

Mary$''$: *Bob won't attend IJCAI because he will already attend AAMAS, and his budget doesn't allow him to attend both.*

The associated formal counterarguments are as follows.

Mary$'$: $((\rightarrow budget) \Rightarrow \neg attboth), ((\rightarrow accI) \Rightarrow attI) \rightarrow \neg attA$

Mary$''$: $((\rightarrow accA) \Rightarrow attA), ((\rightarrow budget) \Rightarrow \neg attboth) \rightarrow \neg attI$

Critically, Mary does not *know* which of the two conferences Bob will attend, yet the principle of restricted rebut *forces* her to make concrete statements on this. From the perspective of commitment in dialogue [Walton and Krabbe, 1995], this is unnatural. One should not be forced to commit to things one has insufficient reasons to believe in.

It should be stressed that the problem outlined above is particularly relevant in dialectical contexts, where different agents make commitments during the exchange of arguments. This contrasts with a formalism like ASPIC+, which is more monolithic in nature, in that from the given rules and premises, one constructs a graph of each other defeating arguments and simply *computes* which arguments (and associated

[13] We observe that $\mathcal{R}_s$ is consistent and closed under transposition.

conclusions) are justified. Concepts like different agents, communication steps or commitment stores do not play a role in ASPIC+, and hence restricted rebut *seems* acceptable. However, if one wants to add dialectical aspects to formal argumentation (c.f., [Caminada and Wu, 2009; Caminada and Podlaszewski, 2012; Caminada *et al.*, 2014a]) then one is forced to take the limitations of restricted rebut seriously.

The obvious way to deal with problems like sketched above would be to simply replace restricted rebut by unrestricted rebut (thus replacing def_{rr} by def_{ur}). Unfortunately, doing so also has far reaching consequences regarding the ability to satisfy the postulates of indirect consistency and closure. This is illustrated by the following example, taken from [Caminada and Wu, 2011].

Example 2. *Consider the argumentation system $(\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ with $\mathcal{R}_s = \{\rightarrow jw;\ \rightarrow mw;\ \rightarrow sw;\ mt, st \rightarrow \neg jt;\ jt, st \rightarrow \neg mt;\ jt, mt \rightarrow \neg st\}$ and $\mathcal{R}_d = \{jw \Rightarrow jt;\ mw \Rightarrow mt;\ sw \Rightarrow st\}$. This example can be interpreted as follows. John, Mary and Suzy want to go cycling in the countryside ($\rightarrow jw;\ \rightarrow mw;\ \rightarrow sw$). They have a tandem bicycle that each of them would like to be on ($jw \Rightarrow jt;\ mw \Rightarrow mt;\ sw \Rightarrow st$). However, as the tandem only has two seats, if two of them are on it, the third one cannot be on it ($mt, st \rightarrow \neg jt;\ jt, st \rightarrow \neg mt;\ jt, mt \rightarrow \neg st$). Using this argumentation system, we can construct the following arguments.*

$A_1 : \rightarrow jw \quad A_4 :\ A_1 \Rightarrow jt \quad A_7 :\ A_5, A_6 \rightarrow \neg jt$
$A_2 : \rightarrow mw \quad A_5 :\ A_2 \Rightarrow mt \quad A_8 :\ A_4, A_6 \rightarrow \neg mt$
$A_3 : \rightarrow sw \quad A_6 :\ A_3 \Rightarrow st \quad A_9 :\ A_4, A_5 \rightarrow \neg st$

When applying restricted rebut (and assuming the empty rule strength ordering) argument A_7 defeats A_4 (as well as A_8 and A_9, which contain A_4), argument A_8 defeats A_5 (as well as A_7 and A_9, which contain A_5) and argument A_9 defeats A_6 (as well as A_7 and A_8, which contain A_6). This yields the argumentation framework at the left hand side of Figure 3, which we will refer to as AF_{rr}.

AF_{rr} has four complete extensions: $\{A_1, A_2, A_3, A_5, A_6, A_7\}$ (yielding conclusions $\{jw, mw, sw, \neg jt, mt, st\}$), $\{A_1, A_2, A_3, A_4, A_6, A_8\}$ (yielding conclusions $\{jw, mw, sw, jt, \neg mt, st\}$), $\{A_1, A_2, A_3, A_4, A_5, A_9\}$ (yielding conclusions $\{jw, mw, sw, jt, mt, \neg st\}$), and $\{A_1, A_2, A_3\}$ (yielding conclusions $\{jw, mw, sw\}$). The first three complete extensions are also preferred (as well as stable and semi-stable). The last one is also grounded. We observe that the conclusions of each complete extension satisfy direct consistency, closure and indirectly consistency.

Now, let us consider what happens if we were to replace restricted rebut by unrestricted rebut. In that case, A_7 would still defeat A_4 (as well as A_8 and A_9), A_8 would still defeat A_5 (as well as A_7 and A_9) and A_9 would still defeat A_6 (as well as A_7 and A_8). However, additionally A_4 would defeat A_7, A_5 would defeat A_8 and A_6 would defeat A_9. This is because A_7, A_8 and A_9 are defeasible arguments, as

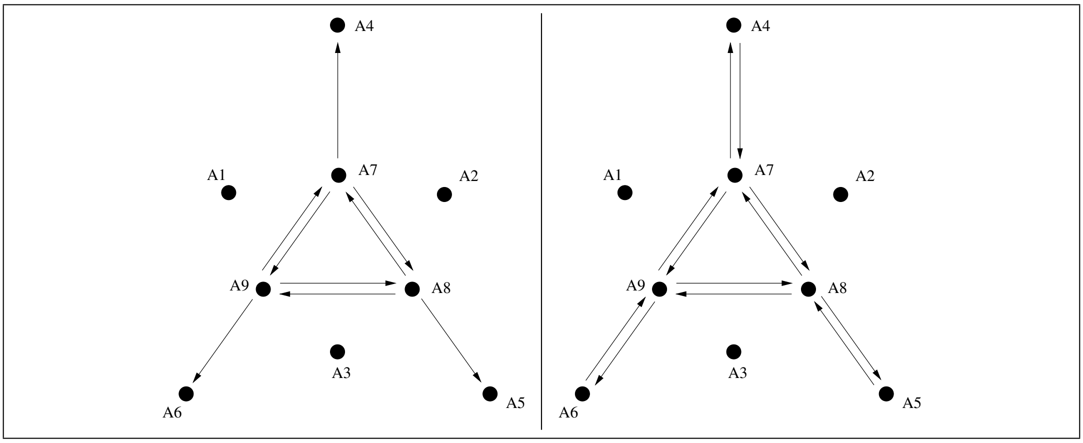

Figure 3: restricted rebut versus unrestricted rebut

their subarguments contain defeasible rules. So with unrestricted rebut, the arguments A_4, A_5 and A_6 are able to "strike back" against their respective defeaters. This yields the argumentation framework at the right hand side of Figure 3, which we will refer to as AF_{ur}. AF_{ur} has five complete extensions. The first four are the same as those of AF_{rr}. The fifth one is $\{A_1, A_2, A_3, A_4, A_5, A_6\}$ yielding conclusions $\{jw, mw, sw, jt, mt, st\}$, hence violating closure and indirect consistency. As this fifth complete extension is also preferred, stable and semi-stable, we have a counterexample against applying unrestricted rebut under each of these semantics.

Example 2 illustrates a fundamental difference between restricted and unrestricted rebut. Whereas under restricted rebut (in combination with $\mathcal{R}_s$ being consistent and closed under transposition or contraposition) any admissible set of arguments will yield conclusions that are indirectly consistent, under unrestricted rebut admissibility alone is not sufficient (the set $\{A_1, A_2, A_3, A_4, A_5, A_6\}$ being the counter example). It turns out that what is needed is a property that is stronger than admissibility: strong admissibility [Baroni and Giacomin, 2009; Caminada, 2014].[14] We observe that although the set $\{A_1, A_2, A_3, A_4, A_5, A_6\}$ is admissible, it is not strongly admissible. Furthermore, we observe that the set $\{A_1, A_2, A_3\}$ is both admissible and strongly admissible and yields conclusions $\{jw, mw, sw\}$ that are closed and indirectly consistent.

As the grounded extension is the unique biggest strongly admissible set [Baroni and Giacomin, 2009; Caminada, 2014], grounded semantics is a natural starting

[14]We recall that a set of arguments $\mathcal{A}rgs$ is strongly admissible iff each $A \in \mathcal{A}rgs$ is defended by some $\mathcal{A}rgs' \subseteq \mathcal{A}rgs \setminus \{A\}$ which in its turn is again strongly admissible. Informally, the idea of strong admissibility is that each argument should be defended without going around in circles.

point for proving the properties of direct consistency, indirect consistency and closure when applying unrestricted rebut. Proving the property of direct consistency is relatively straightforward. After all, if the grounded extension was to yield conclusions that are directly inconsistent, it would have to contain two arguments A and B with opposite conclusions. As $\mathcal{R}_s$ is consistent, at least one of them has to be defeasible, which means that one would defeat (unrestrictedly rebut) the other, which would implies that the grounded extension is not conflict-free. Contradiction.

Proving the property of closure is a bit more complex, as it is done by induction using the inductive definition of the grounded extension. We refer to the work of Caminada and Amgoud [2007] and of Caminada *et al.* [2014b] for details. Indirect consistency then follows trivially from direct consistency and closure.

As for argument strength, two possibilities have been observed when it comes to satisfying closure and indirect consistency under unrestricted rebut. The first approach, of Caminada and Amgoud [2007], is to essentially have the empty ordering on the defeasible rules. A later approach, by Caminada *et al.* [2014b] is to have a total (!) pre-order among the defeasible rules.

An overall overview of approaches to satisfy direct consistency, closure and indirect consistency is provided in Table 1.

4 Non-Interference and Crash Resistance

One of the issues to decide when formulating an argumentation system is whether the (strict and defeasible) rules should be domain dependent or domain independent. An example of a domain dependent strict rule would be $cow \rightarrow mammal$. An example of a domain independent strict rule would be modus ponens, so $cow,\ cow \supset mammal \rightarrow mammal$. When the aim is to implement domain independent reasoning, the most obvious thing to do would be to base the strict rules on some form of classical logic. For current purposes, we examine what happens if one were to base the set of strict rules on propositional logic.

Definition 9. *Given the language $\mathcal{L}$ of propositional logic, a* defeasible theory *is a tuple $(P, \mathcal{R}_d, \mathtt{n}, \leq)$ where*

- *P is a consistent set of propositions (called* premises*)*
- *$\mathcal{R}_d$ is a set of defeasible rules of the form $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ (where φ_i, φ are meta-variables ranging over $\mathcal{L}$)*
- *$\mathtt{n}$ is a function such that $\mathtt{n} : \mathcal{R}_d \longrightarrow \mathcal{L}$*

defeat based on	argument strength	semantics	other conditions	example formalism
restricted rebut	empty	any complete-based semantics	$\mathcal{R}_s$ consistent and closed under transposition	ASPIC [Caminada and Amgoud, 2007]
unrestricted rebut	empty	grounded semantics	$\mathcal{R}_s$ consistent and closed under transposition	ASPIC [Caminada and Amgoud, 2007]
restricted rebut	partial pre-order $\mathcal{R}_d$, last link or weakest link, elitist or democratic	any complete-based semantics	$\mathcal{R}_s$ consistent and closed under transposition/ contraposition	ASPIC+ [Modgil and Prakken, 2014]
unrestricted rebut	total pre-order $\mathcal{R}_d$, last link or weakest link, elitist or democratic	grounded semantics	$\mathcal{R}_s$ consistent and closed under transposition	ASPIC− [Caminada *et al.*, 2014b]

Table 1: Approaches for satisfying closure and direct/indirect consistency

Given a defeasible theory $(P, \mathcal{R}_d, \mathtt{n}, \leq)$, *we define the associated argumentation system as* $(\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ *with* $\mathcal{R}_s = \{\rightarrow \varphi \mid \varphi \in P\} \cup \{\varphi_1, \ldots, \varphi_n \rightarrow \varphi \mid \varphi_1, \ldots, \varphi_n \vdash \varphi\}$

As P is a consistent set of formulas, $\mathcal{R}_s$ will be consistent. Moreover, $\mathcal{R}_s$ is also closed under transposition. This is because the set $\{\rightarrow \varphi \mid \varphi \in P\}$ is trivially closed under transposition (as a rule with an empty antecedent does not have any transposed versions) and the set $\{\varphi_1, \ldots, \varphi_n \rightarrow \varphi \mid \varphi_1, \ldots, \varphi_n \vdash \varphi\}$ is closed under transposition as $\varphi_1, \ldots, \varphi_n \vdash \varphi$ implies $\varphi_1, \ldots, \varphi_{i-1}, -\varphi, \varphi_{i+1}, \ldots, \varphi_n \vdash -\varphi$. However, basing strict rules on classical logic also brings an additional type of problems. Consider the following example.

Example 3. *Consider the defeasible theory* $(P, \mathcal{R}_d, \mathtt{n}, \leq)$ *with* $P = \{js, mns\}$, $\mathcal{R}_d = \{js \Rightarrow s;\ mns \Rightarrow \neg s;\ wfr \Rightarrow r\}$ *and* n *and* $\leq$ *being the empty ordering. This example can be interpreted as follows. John says the cup of coffee contains sugar,*

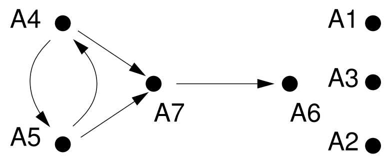

Figure 4: Strict rules as classical logic can have side effects (simple example)

so it probably contains sugar ($\rightarrow js$; $js \Rightarrow s$). Mary says the cup of coffee does not contain sugar ($\rightarrow mns$; $mns \Rightarrow \neg s$). The weather forecaster predicts rain tomorrow, so it will rain tomorrow ($\rightarrow wfr$; $wfr \Rightarrow r$). Hence, although we're not sure about whether the cup of coffee contains sugar, at least we should believe that it will rain tomorrow. Using this argumentation system, at least the following arguments can be constructed.

$A_1 : \rightarrow js \qquad A_4 : \ A_1 \Rightarrow s$

$A_2 : \rightarrow mns \quad A_5 : \ A_2 \Rightarrow \neg s$

$A_3 : \rightarrow wfr \quad A_6 : \ A_3 \Rightarrow r$

*However, classical logic also yields the strict rule $s, \neg s \rightarrow \neg r$, as $s, \neg s \vdash \neg r$ (*ex falso quodlibet*). With this rule, we can construct the following argument.*

$A_7 : \ A_4, A_5 \rightarrow \neg r$

This yields the argumentation framework of Figure 4.[15]

If one were to apply for instance grounded semantics, the grounded extension $\{A_1, A_2, A_3\}$ would yield conclusions $\{j, m, wf\}$. Thus, the weather forecast is not believed because John and Mary are having a disagreement about a cup of coffee.

The first thing to observe about Example 3 is that the underlying problem cannot be solved simply by removing rules with an inconsistent antecedent. This is because the effects of the rule $s, \neg s \rightarrow \neg r$ can be simulated by the rules $s \rightarrow s \vee \neg r$ and $s \vee \neg r, \neg s \rightarrow \neg r$, which still allow us to construct an argument for $\neg r$ from A_4 and A_5.

One approach that has been proposed in the literature [Prakken, 2010] is to change the semantics. If one were to apply for instance not grounded but preferred semantics to the argumentation framework of Figure 4, then two extensions would result: $\{A_1, A_2, A_3, A_4, A_6\}$ (yielding conclusions $\{j, m, wf, s, r\}$) and $\{A_1, A_2, A_3, A_5, A_6\}$ (yielding conclusions $\{j, m, wf, \neg s, r\}$). We observe that each set of conclusions contains r, so r is a justified conclusion under preferred semantics.

Although changing grounded semantics to preferred semantics seems to yield the

[15]Notice that we are applying restricted rebut, but similar problems also occur when applying unrestricted rebut.

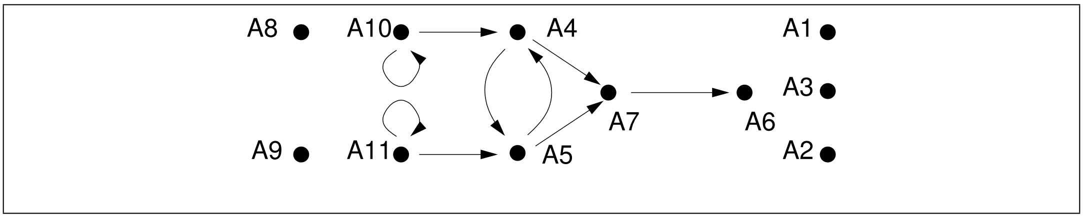

Figure 5: Strict rules as classical logic can have side effects (complex example)

desired outcome in Example 3, there exists a slightly more complex example where preferred semantics does *not* yield the desired outcome.

Example 4. *Consider the defeasible theory* $(P, \mathcal{R}_d, \mathtt{n}, \leq)$ *with* $P = \{js, mns, junrel, munrel, wfr\}$, $\mathcal{R}_d = \{js \Rightarrow s;\ mns \Rightarrow \neg s;\ wfr \Rightarrow r;\ junrel \Rightarrow \neg jrel;\ munrel \rightarrow \neg mrel\}$, $\mathtt{n}(js \Rightarrow s) = \mathtt{n}(junrel \Rightarrow \neg jrel) = jrel$, $\mathtt{n}(mns \Rightarrow \neg s) = \mathtt{n}(munrel \Rightarrow \neg mrel) = mrel$ *and* $\leq$ *being the empty ordering. So now, in addition to John saying that the cup of coffee contains sugar, he also says that he is unreliable, so John is probably unreliable* ($junrel \Rightarrow \neg jrel$)*. However, if John is unreliable, then the fact that he says something is no longer a reason to believe it. Hence the rule* ($js \Rightarrow s$) *is undercut, just like the rule* ($junrel \Rightarrow \neg jrel$)*. Similarly, in addition to Mary saying that the cup of coffee does not contain sugar, she also says that she is unreliable, so Mary is probably unreliable* ($munrel \Rightarrow \neg mrel$)*. However, if Mary is unreliable, then the fact that she says something is no longer a reason to believe it. Hence the rule* ($mns \Rightarrow \neg s$) *is undercut, just like the rule* ($munrel \Rightarrow \neg mrel$)*. Overall, we can construct at least the following arguments.*

$A_1 : \rightarrow js$	$A_4 :\ A_1 \Rightarrow s$
$A_2 : \rightarrow mns$	$A_5 :\ A_2 \Rightarrow \neg s$
$A_3 : \rightarrow wfr$	$A_6 :\ A_3 \Rightarrow r$
$A_8 : \rightarrow junrel$	$A_{10} :\ A_8 \Rightarrow \neg jrel$
$A_9 : \rightarrow munrel$	$A_{11} :\ A_9 \Rightarrow \neg mrel$

Classical logic again yields the strict rule $s, \neg s \rightarrow \neg r$*, which allows the construction of the following argument.*

$A_7 :\ A_4, A_5 \rightarrow \neg r$

This yields the argumentation framework of Figure 5.[16]

In the argumentation framework of Figure 5 there exists just a single complete extension (that is also grounded, preferred, ideal and semi-stable): $\{A_1, A_2, A_3, A_8, A_9\}$ *yielding conclusions* $\{js, mns, wfr, junrel, munrel\}$*. So again, we have that the weather forecast is not believed (under* any *admissibility-based semantics) because*

[16]Notice that we are again applying restricted rebut, although similar problems also occur when applying unrestricted rebut.

John and Mary are having a disagreement about a cup of coffee.

Before continuing to discuss some solutions that have been proposed in the literature, it can be useful to first define what precisely is it that we are trying to satisfy. Or, to put it in other words, what is the property that is actually being violated in Example 3 and Example 4? For this, we follow the approach of Caminada *et al.* [2012].

First of all, if $DT = (P, \mathcal{R}_d, \mathtt{n}, \leq)$ is a defeasible theory, then we write $\mathtt{Atoms}(DT)$ for the set of all propositional atoms occurring in DT. We say that defeasible theories DT_1 and DT_2 are syntactically disjoint iff $\mathtt{Atoms}(DT_1) \cap \mathtt{Atoms}(DT_2) = \emptyset$. For syntactically disjoint defeasible theories $DT_1 = (P_1, \mathcal{R}_{d1}, \mathtt{n}_1, \leq_1)$ and $DT_2 = (P_2, \mathcal{R}_{d2}, \mathtt{n}_2, \leq_2)$ we define the union $DT_1 \cup DT_2$ as $(P_1 \cup P_2, \mathcal{R}_{d1} \cup \mathcal{R}_{d2}, \mathtt{n}_1 \cup \mathtt{n}_2, \leq_1 \cup \leq_2)$. Also, given a defeasible theory DT, we define its consequences $Cn_\sigma(DT)$ as $\{Concs(\mathcal{A}rgs_1\}, \ldots, Concs(\mathcal{A}rgs_n)\}$ where $\mathcal{A}rgs_1, \ldots, \mathcal{A}rgs_n$ are the extensions of arguments (under semantics σ) of the argumentation framework yielded by defeasible theory DT. Given a set of propositions S and a set of propositional atoms $\mathcal{A}$, we define $S_{|\mathcal{A}}$ as $\{\varphi \in S \mid \text{each atom in } \varphi \text{ is an element of } \mathcal{A}\}$. Similarly, given a set $\mathcal{S} = \{S_1, \ldots, S_n\}$ where each S_i $(i \in \{1 \ldots n\})$ is a set of propositions, we define $\mathcal{S}_{|\mathcal{A}}$ as $\{S_{1|\mathcal{A}}, \ldots, S_{n|\mathcal{A}}\}$.

Definition 10. *An argumentation formalism (applying semantics σ) satisfies non-interference iff for every pair of syntactically disjoint defeasible theories DT_1 and DT_2 it holds that $Cn_\sigma(DT_1)_{|\mathtt{Atoms}(DT_1)} = Cn_\sigma(DT_1 \cup DT_2)_{|\mathtt{Atoms}(DT_1)}$.*

To see how non-interference can be violated, consider again Example 3. In essence, the defeasible theory of this example can be seen as the union of two syntactically disjoint defeasible theories $DT_1 = (P_1, \mathcal{R}_{d1}, \mathtt{n}_1, \leq_1)$ and $DT_2 = (P_2, \mathcal{R}_{d2}, \mathtt{n}_2, \leq_2)$ with $P_1 = \{wfr\}$, $\mathcal{R}_{d1} = \{wfr \Rightarrow r\}$, $P_2 = \{js, mns\}$, $\mathcal{R}_{d2} = \{js \Rightarrow s;\ mns \Rightarrow \neg s\}$, $\mathtt{n}_1 = \mathtt{n}_2 = \emptyset$ and $\leq_1 = \leq_2 = \emptyset$. When applying grounded semantics, it holds that $Cn_{gr}(DT_1)_{|\mathtt{Atoms}(DT_1)} = \{\{wfr, r\}\}$ whereas $Cn_{gr}(DT_1 \cup DT_2)_{|\mathtt{Atoms}(DT_1)} = \{\{wfr\}\}$. So merging DT_1 with the completely unrelated defeasible theory DT_2 affects the outcome that is relevant w.r.t. DT_1. Hence, non-interference is violated.

An even stronger property is that of crash resistance.

Definition 11. *A defeasible theory $DT_1 = (P_1, \mathcal{R}_{d1}, \mathtt{n}_1, \leq_1)$ (with $\mathtt{Atoms}(DT_1) \subsetneq \mathtt{Atoms}(\mathcal{L})$) is called contaminating (under semantics σ) iff for each syntactically disjoint defeasible theory DT_2 it holds that $Cn_\sigma(DT_1) = Cn_\sigma(DT_1 \cup DT_2)$. An argumentation formalism satisfies crash resistance iff there exists no defeasible theory that is contaminating.*

To see how crash resistance can be violated, consider Example 4. Again, the defeasible theory of this example can be seen as the union of two syntactically disjoint defeasible theories $DT_1 = (P_1, \mathcal{R}_{d1}, \mathtt{n}_1, \leq_1)$ and $DT_2 = (P_2, \mathcal{R}_{d2}, \mathtt{n}_2, \leq_2)$ with $P_1 = \{js, mns, junrel, munrel\}$, $\mathcal{R}_{d1} = \{js \Rightarrow s;\ mns \Rightarrow \neg s;\ junrel \Rightarrow \neg jrel;\ munrel \Rightarrow \neg mrel\}$, $\mathtt{n}_1(js \Rightarrow s) = \mathtt{n}_1(junrel \Rightarrow \neg jrel) = jrel$, $\mathtt{n}_1(mns \Rightarrow \neg s) = \mathtt{n}_1(munrel \Rightarrow \neg mrel) = mrel$, $\leq_1 = \emptyset$, $P_2 = \{wfr\}$, $\mathcal{R}_{d2} = \{wfr \Rightarrow r\}$, $\mathtt{n}_2 = \emptyset$ and $\leq_2 = \emptyset$. When applying stable semantics, it holds that $Cn_{st}(DT_1) = \emptyset$, just like $Cn_{st}(DT_1 \cup DT_2) = \emptyset$. Moreover, it can be verified that for *any* DT'_2 that is syntactically disjoint with DT_1, it holds that $Cn_{st}(DT_1 \cup DT'_2) = \emptyset$, hence violating crash resistance under stable semantics.

Conceptually, the difference between non-interference and crash resistance is as follows. A violation of non-interference means that a defeasible theory somehow influences the entailment of a completely unrelated (syntactically disjoint) defeasible theory when being merged to it. A violation of crash resistance is more severe, as this means that a defeasible theory influences the entailment of a completely unrelated (syntactically disjoint) defeasible theory to such an extent that the actual contents of this other defeasible theory become totally irrelevant. An argumentation formalism that satisfies non-interference also satisfies crash resistance.[17]

Now that the relevant properties have been identified, we proceed to examine some of the approaches in the literature for satisfying these. The first approach to be discussed is that of Wu and Podlaszewski [2015]. Their main idea is simply to erase inconsistent arguments[18] from the argumentation framework before applying argumentation semantics.

Definition 12. *Let (Ar, def) be the argumentation framework constructed from defeasible theory DT (by applying restricted rebut). Let Ar_c be $\{A \in Ar \mid A$ is consistent $\}$ and let def_c be $def \cap (Ar_c \times Ar_c)$. (Ar_c, def_c) is defined as the inconsistency cleaned argumentation framework of DT.*

As an example of how Definition 12 is used, in Example 3 and Example 4 argument A_7 would be removed, as well as all attacks from and to A_7. The resulting inconsistency cleaned argumentation framework is such that r is a conclusion of each complete extension.

One of the main results proved by Wu and Podlaszewski [2015] is that removing inconsistent arguments from the argumentation framework does not lead to any violations of direct consistency, closure and indirect consistency.[19] They also prove

[17] That is, as long as the argumentation formalism is *non-trivial* in the sense of [Caminada *et al.*, 2012].

[18] An argument A is called inconsistent iff $\{Conc(A') \mid A' \in Sub(A)\}$ is inconsistent.

[19] This is unlike what for instance would happen when removing self-defeating (self-undercutting) arguments, which can lead to violations of closure. As an example (free after [Pollock, 1995]) take the argumentation system $(\mathcal{R}_s, \mathcal{R}_d, \mathtt{n}, \leq)$ with $\mathcal{R}_s = \{\rightarrow a;\ b \rightarrow \neg c;\ c \rightarrow \neg b\}$, $\mathcal{R}_d = \{a \Rightarrow b\}$, $\mathtt{n}(a \Rightarrow$

that the properties of non-interference and crash resistance are satisfied. However, the work of Wu and Podlaszewski [2015] assumes that the strength ordering among the defeasible rules is the empty one, and they provide an example of how their approach of erasing inconsistent arguments violates consistency and closure when applying non-empty rule strengths in combination with the last link principle.

The second approach to be discussed is that of Grooters and Prakken [2016]. Here, one of the basic ideas is to change the way strict rules are generated from propositional logic. Instead of generating a strict rule $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ whenever $\varphi_1, \ldots, \varphi_n \vdash \varphi$, they are generating such a strict rule only when from some consistent set $\Phi \subseteq \{\varphi_1, \ldots, \varphi_n\}$ it holds that $\Phi \vdash \varphi$. So instead of the strict rules coinciding with *all* propositional entailment, the idea is to have the strict rules coinciding with *consistent* propositional entailment.

However, ruling out inconsistent inferences alone is not sufficient, as the problem of *ex falso quodlibet* can also occur when successively applying several strict inference steps, as was for instance observed earlier, using the rules $s \rightarrow s \vee r$ and $s \vee r, \neg s \rightarrow \neg r$. The solution proposed by Grooters and Prakken [2016] is simple: when constructing arguments, disallow the application of a strict rule after the application of another strict rule.

It has to be mentioned that the approach of Grooters and Prakken [2016] has not been proven to satisfy any of the properties of direct consistency, closure, indirect consistency, non-interference and crash-resistance. Weaker properties have been proven instead. We refer to [Grooters and Prakken, 2016] for details.

5 Discussion

It is important to observe that the properties examined in the current paper (sometimes called "rationality postulates" in the literature) are not specific to argumentation theory. In fact, they are general properties that can be applied to each formalism for non-monotonic reasoning that aims to encapsulate some form of strict reasoning. This is why the notion of an argument is not mentioned in the postulates of direct consistency, closure, indirect consistency, non-interference and crash-resistance.

$b) = c$ and $\leq = \emptyset$. Here, we can construct arguments $A_1 :\rightarrow a$, $A_2 : A_1 \Rightarrow b$ and $A_3 : A_2 \rightarrow \neg c$. It holds that A_3 defeats (undercuts) both itself and A_2. This yields a unique complete extension $\{A_1\}$ whose set of conclusions $\{a\}$ satisfies direct consistency, closure and indirect consistency. However, if one were to remove the self-defeating argument A_3, then this would yield a unique complete extension $\{A_1, A_2\}$, whose set of conclusions $\{a, b\}$ violates closure, as it contains b but not $\neg c$. The key point is that whenever one removes a particular class of arguments from the argumentation framework (be it inconsistent or self-attacking arguments) one has to examine whether this results in any violations of direct consistency, indirect consistency and closure.

Instead, these postulates are defined purely based on the *outcome* (in terms of conclusions) of the argumentation formalism. That is, the postulates abstract from the notion of an argument.

This is not to say that no postulates have been formulated specifically about the arguments yielded (instead of about the conclusions yielded). An example of such a postulate would be subargument closure [Caminada and Amgoud, 2007]. This postulate says that if a particular extension contains argument A, then it should also contain all subarguments of A (so each $A' \in Sub(A)$). Satisfying subargument closure is not difficult. From the definition of defeat (under either restricted or unrestricted rebut) it follows that each argument that attacks A' also attacks A. So from A being in, say, a complete extension it follows that A is defended against these attackers, so A' is also being defended. Therefore, A' is also part of the complete extension (which contains everything it defends).

In the current paper, we have mainly focused on rule-based argumentation formalisms, like ASPIC+. However, similar issues also play a role in classical logic based argumentation [Gorogiannis and Hunter, 2011]. Here, the idea is, given a set of propositions Δ (called the *knowledge base*), to construct arguments as pairs $\langle \Phi, \varphi \rangle$ where φ is a proposition (called the *conclusion*) and Φ is a set of propositions (called the *assumptions*) such that $\Phi \vdash \varphi$, $\Phi \not\vdash \perp$ and $\neg \exists \phi \in \Phi$: $\Phi \setminus \{\phi\} \vdash \varphi$. Given this argument form, various ways of defining the notion of defeat (or *attack*, as no strength order is taken into account) are examined, especially for their ability to yield a consistent outcome. We refer to the work of Gorogiannis and Hunter [2011] for details. While Gorogiannis and Hunter [2011] do not consider use of preferences, a recent alternative formalisation of classical logic argumentation of D'Agostino and Modgil [2016] satisfies the consistency and non-contamination postulates while supporting the use of preferences. Moreover, this is done without the requirement that an argument's premises need to be checked for consistency and subset minimality, and with the resulting argumentation frameworks only including finite subsets of the arguments defined by a set of classical well-formed formulas. As such, their theory provides a rational account that is suitable for resource bounded agents.

One key point that we want to emphasise is that the satisfaction of rationality postulates is *not* just a matter of theoretical elegance. If we were to apply argumentation theory for practical purposes, to determine what should be the actions to take, and our formalism tells us to put three people on a tandem bicycle, then this advice will be of little use, as the actions to implement it will fail. If we believe the world to be such that there exist some hard (inviolable) constraints, then it makes sense to model these using nondefeasible (strict) rules and expect the argumentation formalism to deal with them in a proper way. Similarly, if one were for instance to build a robot that uses argumentation theory for its internal reasoning, what we would like to avoid is the situation where after being fed some specific snippets of input (like John whispering in its ear “The cup of coffee contains sugar, and I’m unreliable”, and Mary whispering in its ear “The cup of coffee contains no sugar, and I’m unreliable”) all inference will come to a grinding halt, and the robot essentially stops functioning. Hence, satisfaction of the rationality postulates is important not just for theoretical elegance, but also to make the theory suitable for actual applications.

Given the important role of rationality postulates when it comes to applications of argumentation theory, we observe that the current state of affairs (at the time of writing) is somewhat unsatisfying. As for the postulates of direct consistency, closure and indirect consistency, there seems to be a dilemma. If, on one hand, one chooses to implement restricted rebut then these postulates can be satisfied under any complete-based semantics. The disadvantage, however, is that restricted rebut can be seen as unintuitive, especially in a dialectical context. If, on the other hand, one chooses to implement unrestricted rebut, then the notion of defeat becomes more in line with natural discussion. The disadvantage, however, is that one can only apply grounded semantics, which tends to yield a very sceptical result. Moreover, satisfaction of the rationality postulates is only guaranteed if the strength order on the defeasible rules is either empty or total (hence ruling out a proper partial oder).

As for the postulates of non-inferference and crash resistance, the situation is even more troublesome. First of all, all the approaches that we are aware of [Wu, 2012; Wu and Podlaszewski, 2015; Podlaszewski, 2015; Grooters and Prakken, 2016] work only with restricted rebut. Moreover, the approach of Wu and Podlaszewski [2015] requires the empty ordering regarding rule strength, whereas in many application domains different rules can have different strengths. The work of Grooters and Prakken [2016], does allow for a non-empty rule strength ordering, but fails to prove any of the forementioned postulates, opting to prove much weaker properties instead.

Overall, when it comes to the development of formal argumentation theory, one can observe that the topic of pure abstract argumentation tends to receive quite some more research attention than the topic of instantiated argumentation. Much work has for instance been done on how to select nodes from a graph. However, the real challenge is how to select nodes from a graph *in a meaningful way*, that is, such that the overall outcome makes sense from a logical perspective so the conclusions could be relied upon regarding what to do or what to believe. If formal argumentation is to be applied in situations that matter, some proper solutions to the issue of rationality postulates would be highly desirable.

Afterword

At the time the current paper was submitted, a paper of Heyninck and Straßer [2017] has just been accepted to be presented at IJCAI 2017. The authors' main idea is to allow for arguments to be attacked on several of its (sub)conclusions (that is, on the conclusions of one or more of its subarguments). This is done by an attacker with a disjunctive conclusion, such that each disjunct is the contrary (negation) of one of the (sub)conclusions of the attacked argument. As far as we know, this yields the first ever instantiation of Dung's argumentation theory that (1) works with a combination of classical logic and defeasible inference rules, (2) satisfies *all* the rationality postulates and (3) implements argument preferences.

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 Received 26 May 2017

The Principle-Based Approach to Abstract Argumentation Semantics

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Abstract

The principle-based or axiomatic approach is a methodology to choose an argumentation semantics for a particular application, and to guide the search for new argumentation semantics. This article gives a complete classification of the fifteen main alternatives for argumentation semantics using the twenty-seven main principles discussed in the literature on abstract argumentation, extending Baroni and Giacomin's original classification with other semantics and principles proposed in the literature. It also lays the foundations for a study of representation and (im)possibility results for abstract argumentation, and for a principle-based approach for extended argumentation such as bipolar frameworks, preference-based frameworks, abstract dialectical frameworks, weighted frameworks, and input/output frameworks.

1 The principle-based approach

A considerable number of semantics exists in the argumentation literature. Whereas examining the behaviour of semantics on examples can certainly be insightful, a need for more systematic study and comparison of semantics has arisen. Baroni and Giacomin [2007] present a classification of argumentation semantics based on a set of principles. In this article, we extend their analysis with other principles and semantics proposed in the literature over the past decade.

The principle-based approach is a methodology that is also successfully applied in many other scientific disciplines. It can be used once a unique universal method is replaced by a variety of alternative methods, for example, once a variety of modal logics is used to represent knowledge instead of unique first order logic. The principle-based approach is also called the axiomatic approach, or the postulate based approach (for example in AGM theory change by Alchourrón *et al.* [1985]).

Maybe the best known example of the principle-based approach is concerned with the variety of voting rules, a core challenge in democratic societies, see, e.g., Tennenholtz and Zohar [2016]. It is difficult to find two countries that elect their governments in the same way, or two committees that decide using exactly the same procedure. Over the past two centuries many voting rules have been proposed, and researchers were wondering how we can know that the currently considered set of voting rules is sufficient or complete, and whether there is no better voting rule that has not been discovered yet. Voting theory addresses what we call the choice and search problems inherent to diversity:

Choice problem: If there are many voting rules, then how to choose one voting rule from this set of alternatives in a particular situation?

Search problem: How to guide the search for new and hopefully better voting rules?

In voting theory, the principle-based approach was introduced by Nobel prize winner Kenneth Arrow. The principle-based approach classifies existing approaches based on axiomatic principles, such that we can select a voting rule based on the set of requirements in an area. Moreover, there may be sets of principles for which no voting rule exist yet. Beyond voting theory, the principle-based approach has been applied in a large variety of domains, including abstract argumentation.

Formal argumentation theory, following the methodology in non-monotonic logic, logic programming and belief revision, defines a diversity of semantics. This immediately raises the same questions that were raised before for voting rules, and in many other areas. How do we know that the currently considered set of semantics is sufficient or complete? May there be a better semantics that has not been discovered yet? Moreover, the same choice and search problems of voting theory can be identified for argumentation theory as well:

Choice problem: If there are many semantics, then how to choose one semantics from this set of alternatives in a particular application?

Search problem: How to guide the search for new and hopefully better argumentation semantics?

The principle-based approach again addresses both problems. For example, if one needs to exclude the possibility of multiple extensions, one may choose the grounded or ideal semantics. If it is important that at least some extension is available, then stable semantics should not be used. As another common example, consider the admissibility principle that if an argument in an extension is attacked, then it is defended against this attack by another argument in the extension. If one needs a semantics that is admissible, then for example CF2 or stage2 cannot be chosen.

Principles have also been used to guide the search for new semantics. For example, the principle of resolution was defined by Baroni and Giacomin [2007], well before resolution based semantics were defined and studied by Baroni *et al.* [2011b]. Likewise it may be expected that the existing and new principles will guide the further search for suitable argumentation semantics. For example, consider the conflict-freeness principle that says that an extension does not contain arguments attacking each other. All semantics studied in this article satisfy this property. If one needs to define new argumentation semantics that are para-consistent in the sense that its extensions are not necessarily conflict free [Arieli, 2015], then one can still adopt other principles such as admissibility in the search for such para-consistent semantics.

The principle-based approach consists of three steps.

The first step in the principle-based approach is to define a general function, which will be the object of study. Kenneth Arrow defined social welfare functions from preference profiles to aggregated preference orders. For abstract argumentation, the obvious candidate is a function from graphs to sets of sets of nodes of the graph. Following Dung's terminology, we call the nodes of the graph arguments, we call sets of nodes extensions, we call the edges attacks, and we call the graphs themselves argumentation frameworks. Moreover, we call the function an argumentation semantics. Obviously nothing hinges on this terminology, and in principle the developed theory could be used for other applications of graph theory as well.

We call this function from argumentation frameworks to sets of extensions a two valued function, as a node is either in the extension, or not. Also multi valued functions are commonly used, in particular three valued functions conventionally called labelings. For three valued labelings, the values are usually called in, out and undecided. Other more general functions have been considered in abstract argumentation, for example in value based argumentation, bipolar argumentation, abstract dialectical frameworks, input/output frameworks, ranked semantics, and more. The principle-based approach can be applied to all of them, but in this article we will not consider such generalisations.

The second step of the principle-based approach is to define the principles. The central relation of the principle-based approach is the relation between semantics and principles. In abstract argumentation a two valued relation is used, such that every semantics either satisfies a given property or not. In this case, principles can be defined also as sets of semantics, and they can be represented by a constraint on the function from argumentation frameworks to sets of extensions. An alternative approach used in some other areas gives a numerical value to represent to which degree a semantics satisfies a principle.

The third step of the principle-based approach is to classify and study sets of principles. For example, a set of principles may imply another one, or a set of principles may be satisfiable in the sense that there is a semantics that satisfies all of them. A particular useful challenge is to find a set of principles that characterises a semantics, in the sense that the semantics is the only one that satisfies all the principles. Such characterisations are sometimes called representation theorems.

The principles used in a search problem are typically desirable, and desirable properties are sometimes called postulates. For the mathematical development of a principle-based theory, it may be less relevant whether principles are desirable or not.

Before we continue, we address two common misunderstandings about the principle-based approach, which are sometimes put forward as objections against it.

The first point is that not every function from argumentation frameworks to sets of extensions is an argumentation semantics. In other words, the objection is sometimes raised against the axiomatic approach that it allows for counterintuitive or even absurd argumentation semantics, just like the objection may be raised that not every function from preference profile to candidates is a voting rule. However, in the principle-based approach, such counterintuitive alternatives are excluded by the principles, they are not excluded a priori.

It may be observed that in formal argumentation, this objection is not restricted to principle-based abstract argumentation. A general framework for structured argumentation like ASPIC+ also allows for many counterintuitive or even absurd argumentation theories. However, from the perspective of the principle-based approach, the generality of the ASPIC+ approach can be used to study which combinations of definitions lead to argumentation theories satisfying desired principles [Caminada, forthcoming].

The second point is that a semantics is fundamentally different from a principle. In general a semantics is a function from argumentation frameworks to sets of extensions, and principles can be defined as sets of such functions and represented by a constraint on such functions. This misunderstanding arises because there are examples where a property can be represented as a semantics. For example, the completeness principle may be defined to state that each extension is complete, and the complete semantics may be defined such that the set of extensions of an argumentation framework are *all* its complete extensions. Likewise, some authors transform the admissibility principle into a “semantics” that associates with a framework all the admissible extensions. In this article we do not consider an admissibility semantics defined in this sense, only the admissibility principle.

Finally, we end this introduction with two methodological observations. First, we note that both argumentation semantics and argumentation principles can be organised and clustered in various ways. For example, sometimes a distinction is made between the set of admissibility based semantics and the set of naive based semantics, which are semantics satisfying the admissibility principle and the maximal conflict free principle respectively. In this article we have organised the semantics and principles in a way that seemed reasonable to us, but we did not use a systematic approach and we expect that some readers might have preferred an alternative organisation.

Second, while writing the article, several readers and reviewers have suggested additional semantics and principles to us. For example, we did not systematically study all resolution based semantics. The reason is pragmatic: this article has been growing while we were writing, and at some moment we needed to finish it. Moreover, we excluded several semantics proposed in the literature, such as AD1, AD2, CF1 introduced by Baroni *et al.* [2005], because they have not been further discussed or applied in the formal argumentation literature. However, if some of them will become more popular in the future, then the principle-based study in this article has to be extended to them as well. Finally, dynamic principles are studied by Baroni *et al.* [2014], Rienstra *et al.* [2015] and Baumann [forthcoming].

The layout of this article is as follows. Section 2 introduces the setting and notation, Section 3 introduces the argumentation semantics we study in the rest of the article, and Section 4 introduces the principles and presents the table detailing which principles are satisfied by each semantics.

2 Setting and notations

The current section introduces the setting and notations.

Definition 2.1 (Argumentation framework, [Dung, 1995])**.** *An* argumentation framework *is a couple $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ where $\mathcal{A}$ is a finite set and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. The elements of $\mathcal{A}$ are called* arguments *and $\mathcal{R}$ is called attack relation. We say that a* attacks *b if $(a, b) \in \mathcal{R}$; in that case we also write $a\mathcal{R}b$. For a set $S \subseteq \mathcal{A}$ and an argument $a \in \mathcal{A}$, we say that S attacks a if there exists $b \in S$ such that $b\mathcal{R}a$; we say that a attacks S if there exists $b \in S$ such that $a\mathcal{R}b$. We say that S attacks a set P if there exist $a \in S$, $b \in P$ such that a attacks b.*

We define $S^+ = \{a \in \mathcal{A} \mid S \text{ attacks } a\}$ and $S^- = \{a \in \mathcal{A} \mid a \text{ attacks } S\}$. Moreover, for an argument a, we define $a^+ = \{b \in \mathcal{A} \mid a \text{ attacks } b\}$ and

$a^- = \{b \in \mathcal{A} \mid b$ *attacks* $a\}$. *We define* $S^-_{out} = \{a \in \mathcal{A} \mid a \notin S$ *and* a *attacks* $S\}$. *The set of all argumentation frameworks is denoted by* $\mathcal{AF}$.

We can observe that an argumentation framework is just a finite graph. In the rest of the article, $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ stands for an argumentation framework.

Definition 2.2 (Projection, union, subset)**.** *For an argumentation framework* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ *and a set* $S \subseteq \mathcal{A}$, *we define* $\mathcal{F}\downarrow_S = (S, \mathcal{R} \cap (S \times S))$. *Let* $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ *and* $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$ *be two argumentation frameworks. We define their union by* $\mathcal{F}_1 \cup \mathcal{F}_2 = (\mathcal{A}_1 \cup \mathcal{A}_2, \mathcal{R}_1 \cup \mathcal{R}_2)$. *We write* $\mathcal{F}_1 \subseteq \mathcal{F}_2$ *if and only if* $\mathcal{A}_1 \subseteq \mathcal{A}_2$ *and* $\mathcal{R}_1 \subseteq \mathcal{R}_2$.

For a set S, we denote its powerset by 2^S. Now we define the notion of semantics. It is a function that, given an argumentation framework $(\mathcal{A}, \mathcal{R})$, returns a set of subsets of $\mathcal{A}$.

Definition 2.3 (Semantics)**.** *An extension-based semantics is a function* σ *such that for every argumentation framework* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, *we have* $\sigma(\mathcal{F}) \in 2^{2^{\mathcal{A}}}$. *The elements of* $\sigma(\mathcal{F})$ *are called* extensions.

Our definition requires a semantics to satisfy universal domain, i.e. to be defined for every argumentation framework. We could give a more general definition, thus allowing a semantics to be defined only for some argumentation frameworks. We do not do that in order to simplify the setting, since all the semantics of interest for our study are defined for all argumentation frameworks.

3 Semantics

This section introduces different argumentation semantics we study in the rest of the article. Note that most of the properties from the literature, which we study in Section 4, can appear in two variants: extension-based and labelling-based. In this article, we present their versions for extension-based approach.

We start by introducing the notions of conflict-freeness and admissibility.

Definition 3.1 (Conflict-freeness, admissibility, strong admissibility)**.** *Let* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ *and* $S \subseteq \mathcal{A}$. *Set* S *is* conflict-free *in* $\mathcal{F}$ *if and only if for every* $a, b \in S$, $(a, b) \notin \mathcal{R}$.

Argument $a \in \mathcal{A}$ *is* defended *by set* S *if and only if for every* $b \in \mathcal{A}$ *such that* $b\mathcal{R}a$ *there exists* $c \in S$ *such that* $c\mathcal{R}b$. *Argument* $a \in \mathcal{A}$ *is* strongly defended *by set* S*if and only if for every* $b \in \mathcal{A}$ *such that* $b\mathcal{R}a$ *there exists* $c \in S \setminus \{a\}$ *such that* $c\mathcal{R}b$ *and* c *is strongly defended by* $S \setminus \{a\}$. S *is* admissible *in* $\mathcal{F}$ *if and only if it is* *conflict-free and it defends all its arguments. S is* strongly admissible *in $\mathcal{F}$ if and only if it is conflict-free and it strongly defends all its arguments.*

Stable, complete, preferred and grounded semantics were introduced by Dung [1995]:

Definition 3.2 (Complete, stable, grounded, preferred semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$.*

- *Set S is a* complete extension *of $\mathcal{F}$ if and only if it is conflict-free, it defends all its arguments and it contains all the arguments it defends.*
- *Set S is a* stable extension *of $\mathcal{F}$ if and only if it is conflict-free and it attacks all the arguments of $\mathcal{A} \setminus S$.*
- *S is the* grounded extension *of $\mathcal{F}$ if and only if it is a minimal with respect to set inclusion complete extension of $\mathcal{F}$.*
- *S is a* preferred extension *of $\mathcal{F}$ if and only if it is a maximal with respect to set inclusion admissible set of $\mathcal{F}$.*

Dung [1995] shows that each argumentation framework has a unique grounded extension. Stable extensions do not always exist, i.e. there exist argumentation frameworks whose set of stable extensions is empty. Semi-stable semantics [Verheij, 1996; Caminada, 2006b] guarantees that every argumentation framework has an extension. Furthermore, semi-stable semantics coincides with stable semantics on argumentation frameworks that have at least one stable extension.

Definition 3.3 (Semi-stable semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a semi-stable extension of $\mathcal{F}$ if and only if it is a complete extension and $S \cup S^+$ is maximal with respect to set inclusion among complete extensions, i.e. there exists no complete extension S_1 such that $S \cup S^+ \subset S_1 \cup S_1^+$.*

Ideal semantics [Dung *et al.*, 2007] is an alternative to grounded semantics. Like grounded semantics, ideal semantics always returns a unique extension, which is also a complete extension [Dung *et al.*, 2007]. From the definition of the grounded semantics, we conclude that the ideal extension is a superset of the grounded extension. Ideal semantics is thus less sceptical than grounded semantics.

Definition 3.4 (Ideal semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is the ideal extension of $\mathcal{F}$ if and only if it is a maximal with respect to set inclusion admissible subset of every preferred extension.*

We now introduce eager semantics [Caminada, 2007].

Definition 3.5 (Eager semantics). *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is the eager extension of $\mathcal{F}$ if and only if it is the maximal with respect to set inclusion admissible subset of every semi-stable extension.*

Caminada [2007] shows that each argumentation framework has a unique eager extension and that the eager extension is also a complete extension. Note that eager semantics is similar to ideal semantics: the ideal extension is the unique biggest admissible subset of every preferred extension; the eager extension is the unique biggest admissible subset of each semi-stable extension. Since each semi-stable extension is a preferred extension [Caminada, 2006], the eager extension is a superset of the ideal extension.

In our article, we want to conduct an exhaustive investigation of properties of extension-based semantics. Thus, for the sake of completeness, we introduce even the semantics that are not very commonly used or studied in the literature, like stage semantics, naive semantics and prudent variants of grounded, complete, stable and preferred semantics.

Stage semantics [Verheij, 1996] was defined in a slightly different setting than ours; we provide an alternative but equivalent definition [Verheij, 1996; Baroni *et al.*, 2011a].

Definition 3.6 (Stage semantics). *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a stage extension of $\mathcal{F}$ if and only if S is a conflict-free set and $S \cup S^+$ is maximal with respect to set inclusion, i.e. S is conflict-free, and there exists no conflict-free set S_1 such that $S \cup S^+ \subset S_1 \cup S_1^+$.*

Note the difference between semi-stable and stage semantics: semi-stable extension is a complete extension whereas stage extension is a conflict-free set; stage extension is not necessarily an admissible set.

Definition 3.7 (Naive semantics). *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a naive extension of $\mathcal{F}$ if and only if S is a maximal conflict-free set.*

Prudent semantics [Coste-Marquis *et al.*, 2005] is based on the idea that an extension should not contain arguments a and b if a indirectly attacks b. An indirect attack is an odd length attack chain.

Definition 3.8 (Indirect conflict). *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, $S \subseteq \mathcal{A}$ and $a, b \in \mathcal{A}$. We say that a indirectly attacks b if and only if there is an odd-length path from a to b with respect to the attack relation. We say that S is without indirect conflicts and we write $\texttt{wic}(S)$ if and only if there exist no $x, y \in S$ such that x indirectly attacks y.*

The semantics introduced by Dung (grounded, complete, stable, preferred) is based on admissibility; prudent semantics is based on p-admissibility. Prudent semantics is called grounded prudent, complete prudent, stable prudent and preferred prudent by Coste-Marquis *et al.* [2005]. In order to make the names shorter, we call them p-grounded, p-complete, p-stable and p-preferred.

Definition 3.9 (p-admissible sets)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-admissible set in $\mathcal{F}$ if and only if every $a \in A$ is defended by S and S is without indirect conflicts.*

Definition 3.10 (p-complete semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-complete extension in $\mathcal{F}$ if and only if S is a p-admissible set and for every argument $a \in \mathcal{A}$ we have: if a defended by S and $S \cup \{a\}$ is without indirect conflicts, then $a \in S$.*

We now introduce p-characteristic function, which is needed to define p-grounded semantics. Note that grounded semantics can be defined using characteristic function, but we preferred to provide an alternative equivalent definition.

Definition 3.11 (p-characteristic function)**.** *The p-characteristic function of an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ is defined as follows:*

- $\mathcal{CF}^p_{\mathcal{F}} : 2^{\mathcal{A}} \to 2^{\mathcal{A}}$
- $\mathcal{CF}^p_{\mathcal{F}}(S) = \{a \in \mathcal{A} \mid S \text{ defends } a \text{ and } \mathtt{wic}(S \cup \{a\})\}$

Definition 3.12 (p-grounded semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. Let j be the lowest integer such that*

$$\underbrace{\mathcal{CF}^p_{\mathcal{F}}(\mathcal{CF}^p_{\mathcal{F}}(\dots \mathcal{CF}^p_{\mathcal{F}}}_{j \text{ times}}(\emptyset)\dots) = \underbrace{\mathcal{CF}^p_{\mathcal{F}}(\mathcal{CF}^p_{\mathcal{F}}(\dots \mathcal{CF}^p_{\mathcal{F}}}_{j+1 \text{ times}}(\emptyset)\dots) = S.$$

The p-grounded extension is the set S.

The p-grounded extension is a p-complete extension [Coste-Marquis *et al.*, 2005]. Note that it is not the case in general that the p-grounded extension is included into every p-preferred extension [Coste-Marquis *et al.*, 2005].

Definition 3.13 (p-stable semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-stable extension in $\mathcal{F}$ if and only if S is without indirect conflicts and S attacks (in a direct way) each argument in $\mathcal{A} \setminus S$.*

Definition 3.14 (p-preferred semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-preferred extension if and only if S is a maximal for set inclusion p-admissible set.*

Evert p-stable extension is a p-preferred extension [Coste-Marquis *et al.*, 2005].

We now introduce CF2 semantics [Baroni *et al.*, 2005]. For more explanations and examples, the reader is referred to the original paper. The definition of this semantics is complicated; we must introduce several auxiliary definitions in order to present it.

Let us first introduce the notion of strongly connected component (SCC) introduced by Baroni *et al.* [2005].

Definition 3.15 (Strongly Connected Component)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. The binary relation of path-equivalence between nodes, denoted as $PE_{\mathcal{F}} \subseteq \mathcal{A} \times \mathcal{A}$, is defined as follows:*

- *for every $a \in \mathcal{A}$, $(a, a) \in PE_{\mathcal{F}}$*
- *given two distinct arguments $a, b \in \mathcal{A}$, we say that $(a, b) \in PE_{\mathcal{F}}$ if and only if and only if there is a path from a to b and a path from b to a.*

The strongly connected components of $\mathcal{F}$ are the equivalence classes of arguments under the relation of path-equivalence. The set of strongly connected components is denoted by $SCCS_{\mathcal{F}}$. Given an argument $a \in \mathcal{A}$, notation $SCC_{\mathcal{F}}(a)$ stands for the strongly connected component that contains a.

In the particular case when the argumentation framework is empty, i.e. $\mathcal{F} = (\emptyset, \emptyset)$, we assume that $SCCS_{\mathcal{F}} = \{\emptyset\}$. The choices in the antecedent strongly connected components determine a partition of the nodes of S into three subsets: defeated, provisionally defeated and undefeated. D stands for defeated, P for provisionally defeated and U for undefeated.

Definition 3.16 (D, P, U [Baroni *et al.*, 2005])**.** *Given an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, a set $\mathcal{E} \subseteq \mathcal{A}$ and a strongly connected component $S \in SCCS_{\mathcal{F}}$, we define:*

- $D_{\mathcal{F}}(S, \mathcal{E}) = \{a \in S \mid (\mathcal{E} \cap S^-_{out})$ *attacks* $a\}$
- $P_{\mathcal{F}}(S, \mathcal{E}) = \{a \in S \mid (\mathcal{E} \cap S^-_{out})$ *does not attack* a *and* $\exists b \in (S^-_{out} \cap a^-)$ *such that* $\mathcal{E}$ *does not attack* $b\}$
- $U_{\mathcal{F}}(S, \mathcal{E}) = S \setminus (D_{\mathcal{F}}(S, \mathcal{E}) \cup D_{\mathcal{F}}(S, \mathcal{E}))$

We define $UP_{\mathcal{F}}(S, \mathcal{E}) = U_{\mathcal{F}}(S, \mathcal{E}) \cup P_{\mathcal{F}}(S, \mathcal{E})$.

Definition 3.17 (CF2 semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{E} \subseteq \mathcal{A}$. Set $\mathcal{E}$ is an extension of CF2 semantics if and only if*

- *$\mathcal{E}$ is a naive extension of $\mathcal{F}$ if $|SCCS_{\mathcal{F}}| = 1$*
- *for every $S \in SCCS_{\mathcal{F}}$, $(\mathcal{E} \cap S)$ is a CF2 extension of $\mathcal{F} \downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})}$ otherwise*

Observe that $\mathcal{F} \downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})} = \{a \in S \mid \text{ there exists no } b \in \mathcal{E} \setminus S \text{ s.t. } (b, a) \in \mathcal{R}\}$.

We now introduce stage2 semantics [Dvorák and Gaggl, 2016].

Definition 3.18. *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{E} \subseteq \mathcal{A}$. Set $\mathcal{E}$ is a stage2 extension if and only if*

- *$\mathcal{E}$ is a stage extension of $\mathcal{F}$ if $|SCCS_{\mathcal{F}}| = 1$*
- *for every $S \in SCCS_{\mathcal{F}}$, $(\mathcal{E} \cap S)$ is a stage2 extension of $\mathcal{F} \downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})}$ otherwise*

Dvorák and Gaggl [2016] showed that every stage2 extension is a CF2 extension and that every stable extension is a stage2 extension.

This ends the discussion on extension based semantics of abstract argumentaton. There exist additional proposals for argumentation semantics in the literature, such as for example resolution based semantics of Baroni *et al.* [2011b], but we do not consider them in this article.

In this article, we focus on the extension-based approach, which means that each semantics is defined by specifying the extensions it returns for a given argumentation framework. There exists an alternative, labelling-based approach. Instead of calculating extensions, it provides labellings, one labelling being a function that attaches to every argument a label *in*, *out* or *undec* (which stands for "undecided").

Definition 3.19 (Labelling-based semantics)**.** *Let $\Lambda = \{in, out, undec\}$. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. A labelling on $\mathcal{F}$ is a total function $\mathcal{L}ab : \mathcal{A} \to \Lambda$. A labelling-based semantics is a function λ defined for every element of $\mathcal{AF}$ such that for every argumentation framework $\mathcal{F}$, we have that $\lambda(\mathcal{F})$ is a set of labellings on $\mathcal{F}$.*

To illustrate, let us provide a labelling-based definition of complete semantics.

Definition 3.20 (Complete labelling)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{L}ab$ a labelling on $\mathcal{F}$. We say that $\mathcal{L}ab$ is a complete labelling if and only if for every $a \in \mathcal{A}$:*

- *if a is labelled in then all its attackers are labelled out*
- *if a is labelled out then none of its attackers is labelled in*
- *if a is labelled undec then not all its attackers are labelled out and none of its attackers is labelled in.*

We denote by in($\mathcal{L}ab$) (resp. out($\mathcal{L}ab$), und($\mathcal{L}ab$)) the set of arguments labelled in (resp. out, und).

For every $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, the set of complete extensions under σ is exactly the set $\{in(\mathcal{L}ab) \mid \mathcal{L}ab$ is a complete labelling$\}$.

Moreover, there exists a general way that allows to obtain a labelling-based definition of a semantics given its extension-based definition, under the condition that the semantics returns conflict-free sets.

Definition 3.21 (Extension to labelling)**.** *Given an extension $\mathcal{E}$, labelling $\mathcal{L}ab_{\mathcal{E}}$ is defined as follows: $\mathcal{L}ab_{\mathcal{E}}(a) = in$ if $a \in \mathcal{E}$, $\mathcal{L}ab_{\mathcal{E}}(a) = out$ if $a \in \mathcal{E}^+$, $\mathcal{L}ab_{\mathcal{E}}(a) = und$ otherwise. Then, given a semantics σ, we say that $\mathcal{L}ab$ is a σ labelling of $\mathcal{F}$ if and only if there exists $\mathcal{E} \in \sigma(\mathcal{F})$ such that $\mathcal{L}ab = \mathcal{L}ab_{\mathcal{E}}$.*

Other ways to obtain a labelling from an extension are possible, for example we could say that an argument is *out* if it is attacked by an argument in the extension, or it attacks an argument in the extension. This would make the definition of *out* more symmetric and more in line with naive based semantics. However, it seems such alternatives have not been explored systematically in the literature. Moreover, even if extension and labelling based semantics are inter-translatable, it may affect other definitions such as equivalence of frameworks. Finally, using Definition 3.21, every principle defined in terms of extension based semantics can be translated into labelings and vice versa, though one of the definitions may be more compact or intuitive than the other.

We saw an intuitive way to define complete labellings in Definition 3.20. Intuitive labelling-based definitions of other semantics also exist in the literature. For example: a grounded labelling is a complete labelling such that the set of arguments labelled *in* is minimal with respect to set inclusion among all complete labellings; a stable labelling is a complete labelling such that the set of undecided arguments is empty; a preferred labelling is a complete labelling such that the set of arguments labelled *in* is maximal with respect to set inclusion among all complete labellings. The reader interested in more details about the labelling-based approach is referred to the paper by Baroni *et al.* [2011a].

4 List of Principles

This section presents the properties from the literature and reviews all the semantics with respect to the properties.

Definition 4.1 (Isomorphic argumentation frameworks)**.** *Two argumentation frameworks $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ and $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$ are* isomorphic *if and only if there*

	Defence	Admiss.	Strong adm.	Naivety	Ind. CF	Reinst.	Weak reinst.	CF--reinst.
complete	✓	✓	×	×	×	✓	✓	✓
grounded	✓	✓	✓	×	×	✓	✓	✓
preferred	✓	✓	×	×	×	✓	✓	✓
stable	✓	✓	×	✓	×	✓	✓	✓
semi-stable	✓	✓	×	×	×	✓	✓	✓
ideal	✓	✓	×	×	×	✓	✓	✓
eager	✓	✓	×	×	×	✓	✓	✓
p-complete	✓	✓	×	×	✓	×	×	×
p-grounded	✓	✓	✓	×	✓	×	×	×
p-preferred	✓	✓	×	×	✓	×	×	×
p-stable	✓	✓	×	✓	✓	✓	✓	✓
naive	×	×	×	✓	×	×	×	✓
CF2	×	×	×	✓	×	×	✓	✓
stage	×	×	×	✓	×	×	×	✓
stage2	×	×	×	✓	×	×	✓	✓

Table 1: Properties of semantics: basic properties, admissibility and reinstatement

exists a bijective function $m : \mathcal{A}_1 \to \mathcal{A}_2$*, such that* $(a, b) \in \mathcal{R}_1$ *if and only if* $(m(a), m(b)) \in \mathcal{R}_2$*. This is denoted by* $\mathcal{F}_1 \doteq_m \mathcal{F}_2$*.*

The first property, called "language independence" by Baroni and Giacomin [2007] is an obvious requirement for argumentation semantics. It is sometimes called abstraction [Amgoud and Besnard, 2013; Bonzon *et al.*, 2016a] or anonymity [Amgoud *et al.*, 2016].

Principle 1 (Language independence)**.** *A semantics* σ *satisfies the* language independence *principle if and only if for every two argumentation frameworks* $\mathcal{F}_1$ *and* $\mathcal{F}_2$*, if* $\mathcal{F}_1 \doteq_m \mathcal{F}_2$ *then* $\sigma(\mathcal{F}_2) = \{m(\mathcal{E}) \mid \mathcal{E} \in \sigma(\mathcal{F}_1)\}$*.*

It is immediate to see that all the semantics satisfy language independence, since the definitions of semantics take into account only the topology of the graph, and not the arguments' names.

Conflict-freeness is one of the basic principles. Introduced by Dung [1995] and stated as a principle by Baroni and Giacomin [2007], it is satisfied by all argumentation semantics studied in this article. Note that one can define a non conflict-free semantics [Arieli, 2015]. As another example of relaxing conflict-freeness consider the work by Dunne *et al.* [2011], who introduce a framework where each attack is associated a weight; given an inconsistency budget β, they accept to disregard the set of attacks up to total weight of β.

Principle 2 (Conflict-freeness)**.** *A semantics σ satisfies the* conflict-freeness *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, $\mathcal{E}$ is conflict-free set in $\mathcal{F}$.*

Defence is a well-known property introduced by Dung [1995].

Principle 3 (Defence)**.** *A semantics σ satisfies the* defence *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, for every $a \in \mathcal{E}$, $\mathcal{E}$ defends a.*

Baroni and Giacomin [2007] show that complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable satisfy defence and that CF2 does not satisfy defence. Let us consider the four remaining semantics: stage, stage2, eager and naive. The argumentation framework from Figure 1 shows that stage, stage2 and naive semantics violate defence since they all return three extensions: $\{a\}$, $\{b\}$ and $\{c\}$. Eager semantics satisfies defence (this follows directly from its definition).

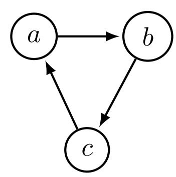

Figure 1: Stage, stage2, naive and CF2 semantics violate admissibility, defence and reinstatement, since they return three extensions: $\{a\}$, $\{b\}$ and $\{c\}$.

Baroni and Giacomin [2007] suppose that every extension is conflict-free. Thus an extension defends all it arguments if and only if it is admissible. However, if conflict-freeness is seen as an optional criterion, we can distinguish between the principles of admissibility and defence.

Principle 4 (Admissibility)**.** *A semantics σ satisfies the* admissibility *principle if and only if for every argumentation framework $\mathcal{F}$, every $\mathcal{E} \in \sigma(\mathcal{F})$ is admissible in $\mathcal{F}$.*

Observation 1. *If a semantics σ satisfies admissibility it also satisfies conflict-freeness and defence.*

We now study the notion of strong admissibility [Baroni and Giacomin, 2007].

Principle 5 (Strong admissibility)**.** *A semantics σ satisfies the* strong admissibility *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$ it holds that $a \in \mathcal{E}$ implies that $\mathcal{E}$ strongly defends a.*

Observation 2. *If a semantics σ satisfies strong admissibility then it satisfies admissibility.*

To understand the notion of strong admissibility, consider the example from Figure 2. Set $\{a, d\}$ is admissible but is not strongly admissible. Informally speaking, this is because a is defended by d whereas d is defended by a. The intuition behind strong admissibility is that this kind of defence is not strong enough because it is cyclic, i.e. arguments defend each other. However, argument e is not attacked, thus $\{e\}$ is strongly admissible. Furthermore, $\{e\}$ strongly defends a, so $\{a, e\}$ is strongly admissible. Also, $\{a, e\}$ strongly defends d. Thus $\{a, d, e\}$ is strongly admissible.

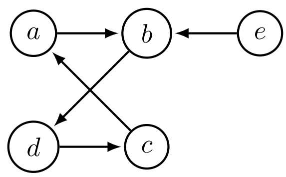

Figure 2: Set $\{a, d\}$ is admissible but is not strongly admissible. Set $\{a, d, e\}$ is admissible and strongly admissible.

Baroni and Giacomin [2007] show that grounded and p-grounded semantics satisfy strong admissibility and that complete, preferred, stable, semi-stable, ideal, p-complete, p-preferred, p-stable and CF2 do not satisfy this principle. Let us consider stage, stage2, eager and naive semantics. Since stage, stage2 and naive semantics violate admissibility, they also violate strong admissibility. To see that eager semantics violates strong admissibility too, consider the example from Figure 3, suggested by Caminada [2007]. The eager extension is $\{b, d\}$; this set is not strongly admissible since it does not strongly defend b.

Another principle, which we call *naivety*, says that every extension under semantics σ is a naive extension.

Principle 6 (Naivety)**.** *A semantics σ satisfies the* naivety *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, $\mathcal{E}$ is maximal for set inclusion conflict-free set in $\mathcal{F}$.*

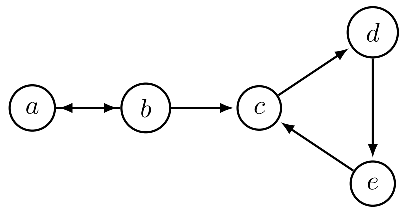

Figure 3: Eager semantics violates strong admissibility because eager extension $\{b, d\}$ does not strongly defend b. The same example shows that eager semantics violates directionality. Observe that $U = \{a, b\}$ is an unattacked set. Denote the whole framework by $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. The eager extension of $\mathcal{F}$ is the set $\{b, d\}$ whereas the eager extension of $\mathcal{F} \downarrow_U$ is the empty set.

We see directly from the definitions of stable, stage, naive, p-stable and CF2 semantics that they satisfy naivety. Since every stage2 extension is also a CF2 extension [Dvorák and Gaggl, 2016], naivety is also satisfied by stage2 semantics. It is easy to see that the other semantics violate this principle.

Coste-Marquis *et al.* [2005] introduced prudent semantics, which are based on the notion of indirect conflict-freeness.

Principle 7 (Indirect conflict-freeness)**.** *A semantics σ satisfies the* indirect conflict-freeness *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, $\mathcal{E}$ is without indirect conflicts in $\mathcal{F}$.*

Observation 3. *If a semantics σ satisfies indirect conflict-freeness then it satisfies conflict-freeness.*

By examining the definitions of prudent semantics, we see that they all satisfy indirect conflict-freeness, since this concept is built in through the use of p-admissibility and p-characteristic function.

The other semantics do not satisfy indirect conflict-freeness. To show this, consider the argumentation framework depicted in Figure 4, suggested by [Coste-Marquis *et al.*, 2005]. All the semantics except prudent ones have an extension containing both a and e. Hence, they violate indirect conflict-freeness since e indirectly attacks a.

Defence says that an extension must defend all the arguments it contains. Reinstatement can be seen as its counterpart, since it says that an extension must contain all the arguments it defends. This principle was first studied in a systematic way by Baroni and Giacomin [2007].

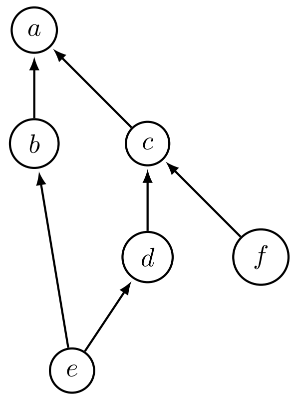

Figure 4: All semantics except prudent semantics violate indirect conflict-freeness. They all yield an extension containing both a and e, even if e indirectly attacks a.

Principle 8 (Reinstatement)**.** *A semantics σ satisfies the* reinstatement *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, for every $a \in \mathcal{A}$ it holds that if $\mathcal{E}$ defends a then $a \in \mathcal{E}$.*

The results in Table 1 concerning complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable and CF2 semantics were proved by Baroni and Giacomin [2007]. To summarise, all the semantics they study satisfy reinstatement except p-grounded, p-complete, p-preferred and CF2. Let us consider eager, stage, stage2 and naive semantics.

Regarding eager semantics, suppose that $\mathcal{E}$ is an eager extension and that a is defended by $\mathcal{E}$. The eager extension is a complete extension [Caminada, 2007], and complete semantics satisfies reinstatement. Thus, $a \in \mathcal{E}$, which means that eager semantics satisfies reinstatement.

Stage, stage2 and naive semantics violate reinstatement, as proved by [Dvorák and Gaggl, 2016]. Another way to see this is to consider the counter-example from Figure 1.

Baroni and Giacomin [2007] study another property called weak reinstatement.

Principle 9 (Weak reinstatement)**.** *A semantics σ satisfies the* weak reinstatement *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$ it holds that*

$$\mathcal{E} \text{ strongly defends } a \text{ implies } a \in \mathcal{E}.$$

Observation 4. *If a semantics σ satisfies reinstatement then it satisfies weak reinstatement.*

The results in Table 1 concerning complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable and CF2 semantics were proved by Baroni and Giacomin [2007]. From Observation 4 we conclude that eager semantics satisfies weak reinstatement.

Stage and naive semantics violate weak reinstatement as can be seen from Figure 5. This was also shown by Dvorák and Gaggl [2016]. Namely, $\{b\}$ is a stage and a naive extension that strongly defends a but does not contain it. Stage2 semantics does satisfy weak reinstatement [Dvorák and Gaggl, 2016].

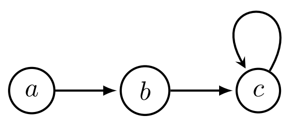

Figure 5: Stage and naive semantics violate weak reinstatement, since $\mathcal{E} = \{b\}$ is an extension that strongly defends a, but $\mathcal{E}$ does not contain a.

The reinstatement principle makes sure that as soon as an argument a is defended by an extension $\mathcal{E}$, a should belong to $\mathcal{E}$—without specifying that a is not in conflict with arguments of $\mathcal{E}$. To take this into account, another principle was defined by Baroni and Giacomin [2007].

Principle 10 ($\mathcal{CF}$-reinstatement)**.** *A semantics σ satisfies the* $\mathcal{CF}$-reinstatement *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, for every $a \in \mathcal{A}$ it holds that if $\mathcal{E}$ defends a and $\mathcal{E} \cup \{a\}$ is conflict-free then $a \in \mathcal{E}$.*

Observation 5. *If a semantics σ satisfies reinstatement then it satisfies $\mathcal{CF}$-reinstatement.*

The results in Table 1 concerning complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable and CF2 semantics were proved by Baroni and Giacomin [2007].

If $\mathcal{E}$ is a naive extension and a an argument such that $\mathcal{E}$ defends a and $\mathcal{E} \cup \{a\}$ is conflict-free, then $a \in \mathcal{E}$ since $\mathcal{E}$ is a maximal conflict-free set. This means that naive semantics satisfies $\mathcal{CF}$-reinstatement.

Observation 5 implies that eager semantics satisfies $\mathcal{CF}$-reinstatement.

	I-max.	Allowing abstention	Crash resistance	Non--interference	Direct.	Weak--direct.	Semi--direct.
complete	×	✓	✓	✓	✓	✓	✓
grounded	✓	✓	✓	✓	✓	✓	✓
preferred	✓	×	✓	✓	✓	✓	✓
stable	✓	×	×	×	×	✓	×
semi-stable	✓	×	✓	✓	×	×	×
ideal	✓	✓	✓	✓	✓	✓	✓
eager	✓	✓	✓	✓	✓	✓	✓
p-complete	×	✓	✓	✓	×	×	✓
p-grounded	✓	✓	✓	✓	✓	✓	✓
p-preferred	✓	×	✓	✓	×	×	✓
p-stable	✓	×	×	×	×	✓	×
naive	✓	×	✓	✓	×	×	✓
CF2	✓	×	✓	✓	✓	✓	✓
stage	✓	×	✓	✓	×	×	×
stage2	✓	×	✓	✓	✓	✓	✓

Table 2: Properties of semantics, part 2

Stage and stage2 semantics satisfy $\mathcal{CF}$-reinstatement, as shown by Dvorák and Gaggl [2016].

The next principle was first considered by Baroni and Giacomin [2007]. It says that an extension cannot contain another extension.

Principle 11 (I-maximality)**.** *A semantics σ satisfies the* I-maximality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E}_1, \mathcal{E}_2 \in \sigma(\mathcal{F})$, if $\mathcal{E}_1 \subseteq \mathcal{E}_2$ then $\mathcal{E}_1 = \mathcal{E}_2$.*

I-maximality is trivially satisfied by single extension semantics. It is thus satisfied by eager semantics. We see directly from the definitions of naive and stage semantics that they satisfy I-maximality. Dvorák and Gaggl [2016] show that stage2 semantics satisfies I-maximality. Baroni and Giacomin [2007] show that I-maximality is satisfied by all other semantics except complete and p-complete semantics.

Baroni *et al.* [2011a] define a principle called rejection, which says that if an argument a is labelled in and a attacks b, then b should be labelled out. If we use the translation from extension to a labelling we mentioned in Definition 3.21, we see that all the labellings satisfy this property. However, it would be possible to be more general by defining a labelling-based semantics that does not satisfy this property. Let us define a semantics σ that always returns a unique labelling such that an argument is labelled in if it is not attacked, it is labelled undec if it is attacked by exactly one argument and it is labelled out otherwise. Consider the example from

Figure 5: argument a will be labelled in, argument b undec and argument c out, which violates the rejection principle.

We next consider the allowing abstention principle [Baroni *et al.*, 2011a].

Principle 12 (Allowing abstention)**.** *A semantics σ satisfies the* allowing abstention *principle if and only if for every argumentation framework $\mathcal{F}$, for every $a \in \mathcal{A}$, if there exist two extensions $\mathcal{E}_1, \mathcal{E}_2 \in \sigma(\mathcal{F})$ such that $a \in \mathcal{E}_1$ and $a \in \mathcal{E}_2^+$ then there exists an extension $\mathcal{E}_3 \in \sigma(\mathcal{F})$ such that $a \notin (\mathcal{E}_3 \cup \mathcal{E}_3^+)$.*

Baroni *et al.* [2011a] show that complete semantics satisfies the previous principle and that preferred, stable, semi-stable, stage and CF2 semantics falsify it. Observe that unique status semantics trivially satisfy this principle. Allowing abstention is thus satisfied by grounded, ideal, eager and p-grounded semantics.[1]

Let us now consider the remaining semantics, namely: naive, p-stable, p-preferred, p-complete and stage2 semantics.

We first prove that p-complete semantics satisfies allowing abstention. We start with a lemma.

Lemma 4.2. *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework, $\mathtt{GE}_p$ its p-grounded extension and $\mathcal{E} \subseteq \mathcal{A}$ be a set that defends all its arguments. Then, $\mathcal{E}$ does not attack $\mathtt{GE}_p$.*

Proof. Let $\mathcal{CF}^p$ be the p-characteristic function. Denote ${\mathtt{GE}_p}^0 = \emptyset$, ${\mathtt{GE}_p}^1 = \mathcal{CF}^p(\emptyset)$, ${\mathtt{GE}_p}^2 = \mathcal{CF}^p(\mathcal{CF}^p(\emptyset))$, ... and denote by $\mathtt{GE}_p$ the p-grounded extension of $\mathcal{F}$. Let $\mathcal{E}$ be a set that defends all its arguments. By means of contradiction, suppose that there exist $x \in \mathcal{E}$, $y \in \mathtt{GE}_p$ such that $x\mathcal{R}y$. Let $k \in \mathbb{N}$ be the minimal number such that $y \in {\mathtt{GE}_p}^k$. From the definition of function $\mathcal{CF}^p$, there exists $l < k$ such that there exists $y_1 \in {\mathtt{GE}_p}^l$ such that $y_1\mathcal{R}x$. Since $\mathcal{E}$ defends all its arguments, there exists $x_1 \in \mathcal{E}$ such that $x_1\mathcal{R}y_1$. Again, there exists $m < l$ such that there exists $y_2 \in {\mathtt{GE}_p}^m$ such that $y_2\mathcal{R}x_1$. By continuing this process, we conclude that there exists $y_s \in {\mathtt{GE}_p}^1$ such that there exists $x_s \in \mathcal{E}$ such that $x_S\mathcal{R}y_s$. This is impossible, since the arguments of ${\mathtt{GE}_p}^1$ are not attacked. Contradiction. □

Proposition 4.3. *p-complete semantics satisfies allowing abstention.*

Proof. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, let $a, b \in \mathcal{A}$, let $b\mathcal{R}a$ and let $\mathcal{E}_1$ and $\mathcal{E}_2$ be p-complete extensions such that $a \in \mathcal{E}_1$ and $b \in \mathcal{E}_2$. Denote by $\mathtt{GE}_p$ the p-grounded extension of $\mathcal{F}$. Let us prove that $a \notin \mathtt{GE}_p$ and that $\mathtt{GE}_p$ does not attack a. First, since $b\mathcal{R}a$

[1] Note that Table 2 by Baroni *et al.* [2011a] specifies that grounded semantics does not satisfy dilemma abstaining. The reason is that Baroni et al. consider the property as being “non-applicable” to unique status semantics (personal communication, 2016).

and b belongs to a p-complete extension (and every p-complete extension defends all its arguments), Lemma 4.2 implies that $a \notin \mathtt{GE}_p$. Let us now show that $\mathtt{GE}_p$ does not attack a. By means of contradiction, suppose the contrary. Let $b \in \mathtt{GE}_p$ be an argument such that $b\mathcal{R}a$. Since $a \in \mathcal{E}_1$, and $\mathcal{E}_1$ defends all its arguments, then there exists $c \in \mathcal{E}_1$ such that $c\mathcal{R}b$. Contradiction with Lemma 4.2. Thus, it must be that $\mathtt{GE}_p$ does not attack a. It is known that the p-grounded extension is a p-complete extension [Coste-Marquis *et al.*, 2005]. Thus, we showed that there exists a p-complete extension that neither contains nor attacks argument a. □

To see why naive, p-stable, p-preferred and stage2 semantics violate allowing abstention, consider the argumentation framework depicted in Figure 6. The principle is violated since all those semantics return two extensions, $\{a\}$ and $\{b\}$.

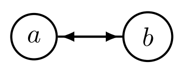

Figure 6: Several semantics violate allowing abstention principle.

To define crash resistance [Caminada *et al.*, 2012], we first need to introduce the following two definitions.

Definition 4.4 (Disjoint argumentation frameworks)**.** *Two argumentation frameworks $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ and $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$ are disjoint if and only if $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$.*

A framework $\mathcal{F}^\star$ is contaminating if joining $\mathcal{F}^\star$ with an arbitrary disjoint framework $\mathcal{F}$ results in a framework $\mathcal{F} \cup \mathcal{F}^\star$ having the same extensions as $\mathcal{F}^\star$. The intuition behind this definition is that $\mathcal{F}^\star$ contaminates every framework.

Definition 4.5 (Contaminating)**.** *An argumentation framework $\mathcal{F}^\star$ is contaminating for a semantics σ if and only if for every argumentation framework $\mathcal{F}$ disjoint from $\mathcal{F}^\star$ it holds that $\sigma(\mathcal{F}^\star \cup \mathcal{F}) = \sigma(\mathcal{F}^\star)$.*

A semantics is crash resistant if and only if there are no contaminating frameworks. The intuition behind this name is that a contaminating framework causes the system to crash.

Principle 13 (Crash resistance)**.** *A semantics σ satisfies the* crash resistance *principle if and only if there are no contaminating argumentation frameworks for σ.*

Crash resistance forbids only the most extreme form of interferences between disjoint subgraphs. A stronger property, non-interference, was defined by Caminada *et al.* [2012]. We first need to define a notion of isolated set, i.e. a set that neither attacks outside arguments nor is attacked by them.

Definition 4.6 (Isolated set of arguments)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. A set $S \subseteq \mathcal{A}$ is isolated in $\mathcal{F}$ if and only if*

$$((S \times (\mathcal{A} \setminus S)) \cup ((\mathcal{A} \setminus S) \times S)) \cap \mathcal{R} = \emptyset.$$

A semantics satisfies non-interference principle if for every isolated set S, the intersections of the extensions with set S coincide with the extensions of the restriction of the framework on S.

Principle 14 (Non-interference)**.** *A semantics σ satisfies the* non-interference *principle if and only if for every argumentation framework $\mathcal{F}$, for every set of arguments S isolated in $\mathcal{F}$ it holds that $\sigma(\mathcal{F}\downarrow_S) = \{\mathcal{E} \cap S \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

The previous principle can be made even stronger by considering the case when the set S is not attacked by the rest of the framework, but can attack the rest of the framework. Let us formalize the notion of an unattacked set.

Definition 4.7 (Unattacked arguments)**.** *Given an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, a set U is* unattacked *if and only if there exists no $a \in \mathcal{A} \setminus U$ such that a attacks U. The set of unattacked sets in $\mathcal{F}$ is denoted $\mathcal{US}(\mathcal{F})$.*

We can now define the principle of directionality, introduced by Baroni and Giacomin [2007].

Principle 15 (Directionality)**.** *A semantics σ satisfies the* directionality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $U \in \mathcal{US}(\mathcal{F})$, it holds that $\sigma(\mathcal{F}\downarrow_U) = \{\mathcal{E} \cap U \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

Baroni *et al.* [2011a] show the following dependencies between directionality, interference and crash resistance.

Observation 6. *Directionality implies non interference, and non interference implies crash resistance.*

Let us see which semantics satisfy directionality. Baroni and Giacomin [2007] proved that complete, grounded, preferred, ideal, p-grounded and CF2 semantics satisfy directionality. They also showed that stable, semi-stable, p-complete, p-stable and p-preferred semantics violate this principle. Baroni *et al.* [2011a] show that stage semantics does not satisfy directionality; however, Dvorák and Gaggl [2016] show that stage2 semantics does satisfy directionality.

The only remaining semantics are eager and naive. The argumentation framework from Figure 7 shows that naive semantics does not satisfy directionality. The

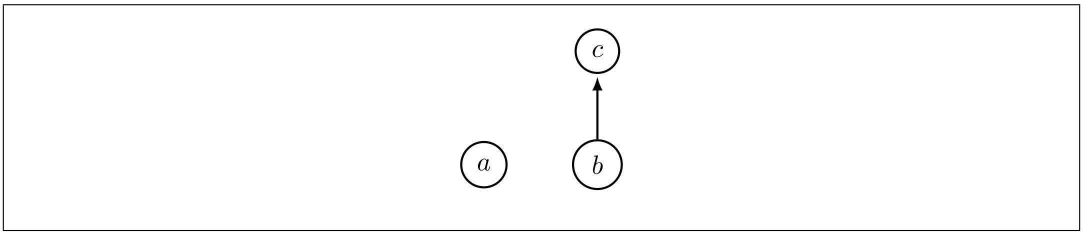

Figure 7: Naive semantics violates directionality and weak directionality. Denote the whole framework by $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. Let $U = \{a, b\}$. Observe that $\{a, c\}$ is a naive extension of $\mathcal{F}$ but that $\{a\}$ is not a naive extension of $\mathcal{F}\downarrow_U$.

argumentation framework from Figure 3 shows that eager semantics does not satisfy directionality.

Let us now consider non-interference. Baroni *et al.* [2011a] showed that non-interference is satisfied by complete, grounded, preferred, semi-stable, ideal, stage and CF2 semantics. Eager semantics satisfies non-interference since it satisfies directionality. From the definition of non-interference we see that this principle is satisfied by naive semantics. Since p-grounded semantics satisfies directionality, it also satisfies non-interference.

Proposition 4.8. *p-complete, p-preferred semantics satisfy non-interference.*

Proof. We present the proof for p-complete semantics, the one for p-preferred semantics is similar. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{A}' \subseteq \mathcal{A}$ be an isolated set in $\mathcal{F}$. Denote by $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ the restriction of $\mathcal{F}$ on $\mathcal{A}'$. Let us first suppose that $\mathcal{E}$ is a complete prudent extension of $\mathcal{F}$. Denote $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}'$. We have $icf(\mathcal{E}')$. It is easy to see that every $\alpha \in \mathcal{E}'$ is defended by $\mathcal{E}'$ from all attacks from $\mathcal{A}'$. Also, for an $\alpha \in \mathcal{A}' \setminus \mathcal{E}'$, we can easily see that either $\mathcal{E}' \cup \{\alpha\}$ is not without indirect conflicts or α is attacked by some argument and not defended by $\mathcal{E}'$. Suppose now that $\mathcal{E}'$ is a complete prudent extension of $\mathcal{F}'$. Then $\mathcal{E}'$ is p-admissible in $\mathcal{F}$, so there must be a complete prudent extension $\mathcal{E}''$ of $\mathcal{F}$ such that $\mathcal{E}' \subseteq \mathcal{E}''$. □

Stage2 semantics satisfies non-interference since it satisfies directionality. Finally, p-stable semantics violates non-interference. Indeed, as we will soon see, p-stable semantics violates crash resistance. Since non-interference implies crash resistance, we conclude that p-stable semantics violates non-interference.

Let us now consider crash resistance. Baroni *et al.* [2011a] showed that non-interference is satisfied by complete, grounded, preferred, semi-stable, ideal, stage and CF2 semantics. Eager, naive, p-grounded, p-complete, p-preferred and stage2 semantics satisfy crash resistance since they satisfy non-interference. To see that stable semantics and p-stable semantics violate crash resistance, consider the framework $\mathcal{F}^* = (\{a\}, \{(a, a)\})$. We see that $\mathcal{F}^*$ is contaminating for stable and p-stable semantics. Thus, they both violate crash resistance.

Let us now consider two variants of directionality, called weak directionality and semi-directionality suggested by M. Giacomin (personal communication, 2016).

Principle 16 (Weak directionality)**.** *A semantics σ satisfies the* weak directionality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $U \in \mathcal{US}(\mathcal{F})$, it holds that $\sigma(\mathcal{F}\downarrow_U) \supseteq \{\mathcal{E} \cap U \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

Principle 17 (Semi-directionality)**.** *A semantics σ satisfies the* semi-directionality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $U \in \mathcal{US}(\mathcal{F})$, it holds that $\sigma(\mathcal{F}\downarrow_U) \subseteq \{\mathcal{E} \cap U \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

Observation 7. *A semantics σ satisfies directionality if and only if σ satisfies both weak directionality and semi-directionality.*

Thus, grounded, complete, preferred, ideal, eager, p-grounded, stage2 and CF2 semantics satisfy both weak directionality and semi-directionality. It is immediate from the definition that stable semantics satisfies weak directionality. Since stable semantics does not satisfy directionality, it does not satisfy semi-directionality.

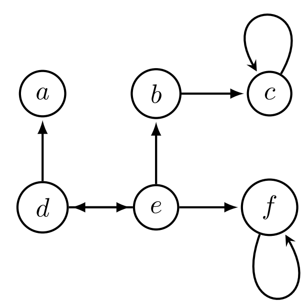

Figure 8: Semi-stable and stage semantics violate weak directionality. Let $U = \{d, e, f\}$. Set $\{b, d\}$ is an extension of this argumentation framework, but $\{b\}$ is not an extension of the restriction of this framework on U.

Example from Figure 8 shows that semi-stable semantics does not satisfy weak directionality. To see that semi-stable semantics does not satisfy semi-directionality, consider the example from Figure 9, suggested by M. Giacomin. Stage semantics violates weak directionality, the same counter-example as for semi-stable semantics

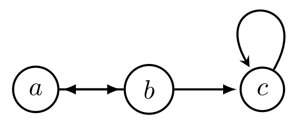

Figure 9: Semi-stable and stage semantics violate semi-directionality. Let $U = \{a, b\}$. Set $\{a\}$ is an extension of the restriction of the framework on U, but there is no extension $\mathcal{E}$ of the whole framework such that $\mathcal{E} \cap U = \{a\}$.

(Figure 8) can be used. Stage semantics also violates semi-directionality, and we can again use the same counter-example as for semi-stable semantics (Figure 9).

Directly from the definition of naive semantics we see that it satisfies semi-directionality. Since it does not satisfy directionality, we conclude from Observation 7 that it does not satisfy weak directionality.

Proposition 4.9. *p-complete and p-preferred semantics satisfy semi-directionality.*

Proof. We present the proof for p-complete semantics, the proof for p-preferred semantics is similar. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework, $U \subseteq \mathcal{A}$ an unattacked set and $\mathcal{F}' = \mathcal{F} \downarrow_U$ the restriction of $\mathcal{F}$ on U. Let $\mathcal{E}'$ be a p-complete extension of $\mathcal{F}'$. Then $\mathcal{E}'$ is without indirect conflicts and is p-admissible in $\mathcal{F}'$. It is immediate to see that $\mathcal{E}'$ is also p-admissible in $\mathcal{F}$. It is clear that there exists no $x \in U \setminus \mathcal{E}'$ such that x is defended by $\mathcal{E}'$ and $\mathcal{E}' \cup \{x\}$ is without indirect conflicts. Thus, there exists a (possibly empty) set $\mathcal{E} \subset (\mathcal{A} \setminus U)$ such that $\mathcal{E} \cup \mathcal{E}'$ is a p-complete extension. □

Since both p-complete and p-preferred semantics violate directionality, the previous proposition and Observation 7 imply that they both violate weak directionality.

Directly from the definition of p-stable semantics, we see that this semantics satisfies weak directionality. From Observation 7 we conclude that it does not satisfy semi-directionality.

We now consider the six properties related to skepticism and resolution adequacy [Baroni and Giacomin, 2007].

The first definition says that a set of extensions $\texttt{Ext}_1$ is more skeptical than $\texttt{Ext}_2$ if the set of skeptically accepted arguments with respect to $\texttt{Ext}_1$ is a subset of the set of skeptically accepted arguments with respect to $\texttt{Ext}_2$.

Definition 4.10 ($\preceq^E_\cap$)**.** *Let* $\mathtt{Ext}_1$ *and* $\mathtt{Ext}_2$ *be two sets of sets of arguments. We say that* $\mathtt{Ext}_1 \preceq^E_\cap \mathtt{Ext}_2$ *if and only if*

$$\bigcap_{\mathcal{E}_1 \in \mathtt{Ext}_1} \mathcal{E}_1 \subseteq \bigcap_{\mathcal{E}_2 \in \mathtt{Ext}_2} \mathcal{E}_2.$$

The previous definition compares only the intersections of extensions. A finer criterion was introduced by Baroni *et al.* [2004].

Definition 4.11 ($\preceq^E_W$)**.** *Let* $\mathtt{Ext}_1$ *and* $\mathtt{Ext}_2$ *be two sets of sets of arguments. We say that* $\mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2$ *if and only if*

$$\textit{for every } \mathcal{E}_2 \in \mathtt{Ext}_2, \textit{ there exists } \mathcal{E}_1 \in \mathtt{Ext}_1 \textit{ such that } \mathcal{E}_1 \subseteq \mathcal{E}_2.$$

Baroni and Giacomin [2007] refine the previous relation by introducing the following definition.

Definition 4.12 ($\preceq^E_S$)**.** *Let* $\mathtt{Ext}_1$ *and* $\mathtt{Ext}_2$ *be two sets of sets of arguments. We say that* $\mathtt{Ext}_1 \preceq^E_S \mathtt{Ext}_2$ *if and only if* $\mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2$ *and*

$$\textit{for every } \mathcal{E}_1 \in \mathtt{Ext}_1, \textit{ there exists } \mathcal{E}_2 \in \mathtt{Ext}_2 \textit{ such that } \mathcal{E}_1 \subseteq \mathcal{E}_2.$$

Letters W and S in the previous definitions stand for *weak* and *strong.* Baroni and Giacomin [2007] showed that the three relations are reflexive and transitive and that they are also in strict order of implication. Namely, given two sets of sets of arguments $\mathtt{Ext}_1$ and $\mathtt{Ext}_2$, we have

Observation 8.

$$\mathtt{Ext}_1 \preceq^E_S \mathtt{Ext}_2 \textit{ implies } \mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2$$

$$\mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2 \textit{ implies } \mathtt{Ext}_1 \preceq^E_\cap \mathtt{Ext}_2$$

We now define a skepticism relation $\preceq^A$ between argumentation frameworks. It says that $\mathcal{F}_1 \preceq^A \mathcal{F}_2$ if $\mathcal{F}_1$ may have some symmetric attacks where $\mathcal{F}_2$ has a directed attack.

Definition 4.13 ($\preceq^A$)**.** *Given an argumentation framework* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$*, the conflict set is defined as* $\mathtt{CONF}(\mathcal{F}) = \{(a, b) \in \mathcal{A} \times \mathcal{A} \mid (a, b) \in \mathcal{R} \textit{ or } (b, a) \in \mathcal{R}\}$*. Given two argumentation frameworks* $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ *and* $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$*, we say that* $\mathcal{F}_1 \preceq^A \mathcal{F}_2$ *if and only if* $\mathtt{CONF}(\mathcal{F}_1) = \mathtt{CONF}(\mathcal{F}_2)$ *and* $\mathcal{R}_2 \subseteq \mathcal{R}_1$*.*

Observe that $\preceq^A$ is a partial order, as it consists of an equality and a set inclusion relation [Baroni and Giacomin, 2007]. Note that within the set of argumentation frameworks comparable with a given argumentation framework $\mathcal{F}$, there might be several maximal elements with respect to $\preceq^A$, since there might be several ways to replace all symmetric attacks by asymmetric ones.

We can now introduce the skepticism adequacy principle. Its idea is that if $\mathcal{F}$ is more skeptical than $\mathcal{F}'$ then the set of extensions of $\mathcal{F}$ is more skeptical than that of $\mathcal{F}'$.

Principle 18 (Skepticism adequacy)**.** *Given a skepticism relation $\prec^E$ between sets of sets of arguments, a semantics σ satisfies the $\preceq^E$-*skepticism adequacy *principle if and only if for every two argumentation frameworks $\mathcal{F}$ and $\mathcal{F}'$ such that $\mathcal{F} \preceq^A \mathcal{F}'$ it holds that $\sigma(\mathcal{F}) \preceq^E \sigma(\mathcal{F}')$.*

For example if $\mathcal{F}$ consists of two arguments a and b attacking each other and $\mathcal{F}'$ has only an attack from a to b, then the intersection of the extensions of $\mathcal{F}$ ($\emptyset$ for all semantics) is a subset of extensions of $\mathcal{F}'$, typically $\{a\}$. Roughly speaking: the more symmetric attacks we replace, the more we know, but we do not loose any accepted arguments.

Observation 9.

- *If σ satisfies $\preceq^E_S$-sk. adequacy then it satisfies $\preceq^E_W$-sk. adequacy*
- *If σ satisfies $\preceq^E_W$-sk. adequacy then it satisfies $\preceq^E_\cap$-sk. adequacy*

Let us see which semantics satisfy skepticism adequacy. Baroni and Giacomin [2007] proved all the results for grounded, complete, stable, preferred, semi-stable, ideal, all four prudent and CF2 semantics.

Eager semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy, as illustrated by the example depicted in Figure 10. From Observation 9, we conclude that eager semantics violates $\preceq^E_W$-skepticism adequacy and $\preceq^E_S$-skepticism adequacy.

Naive semantics satisfies all three variants of skepticism adequacy since $\texttt{CONF}(\mathcal{F}_1) = \texttt{CONF}(\mathcal{F}_2)$ implies $\sigma(\mathcal{F}_1) = \sigma(\mathcal{F}_2)$.

Stage semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy, as illustrated by the example from Figure 11. From Observation 9, we conclude that stage semantics violates $\preceq^E_W$-skepticism adequacy and $\preceq^E_S$-skepticism adequacy.

Finally, stage2 semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy, as illustrated by the example from Figure 12. From Observation 9, we conclude that stage2 semantics violates $\preceq^E_W$-skepticism adequacy and $\preceq^E_S$-skepticism adequacy.

Let us now consider resolution adequacy [Baroni and Giacomin, 2007].

	$\preceq^E_\cap$-sk. ad.	$\preceq^E_W$-sk. ad.	$\preceq^E_S$-sk. ad.	$\preceq^E_\cap$-res. ad.	$\preceq^E_W$-res. ad.	$\preceq^E_S$-res. ad.
complete	✓	✓	×	×	×	×
grounded	✓	✓	✓	×	×	×
preferred	×	×	×	✓	✓	✓
stable	✓	✓	×	✓	✓	✓
semi-stable	×	×	×	✓	✓	×
ideal	×	×	×	×	×	×
eager	×	×	×	×	×	×
p-complete	×	×	×	×	×	×
p-grounded	×	×	×	✓	×	×
p-preferred	×	×	×	×	×	×
p-stable	×	×	×	✓	✓	×
naive	✓	✓	✓	✓	✓	✓
CF2	✓	✓	×	×	×	×
stage	×	×	×	✓	✓	×
stage2	×	×	×	×	×	×

Table 3: Properties of semantics, skepticism and resolution adequacy

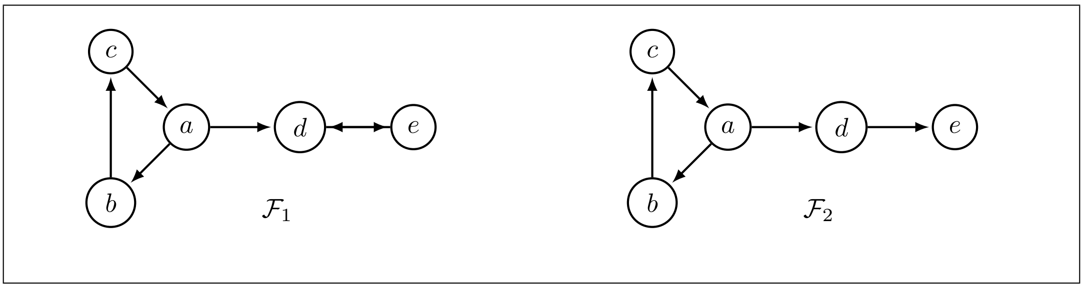

Figure 10: Eager semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy. We have $\mathcal{F}_1 \preceq^A \mathcal{F}_2$. The eager extension of $\mathcal{F}_1$ is $\{e\}$ and the eager extension of $\mathcal{F}_2$ is $\emptyset$. Thus the set of skeptically accepted arguments of $\mathcal{F}_1$ equals $\{e\}$ is not a subset of the set of skeptically accepted arguments of $\mathcal{F}_2$.

Definition 4.14 (RES)**.** *We denote by $RES(\mathcal{F})$ the set of all argumentation frameworks comparable with $\mathcal{F}$ and maximal with respect to $\preceq^A$.*

Definition 4.15 (UR)**.** *Given an argumentation framework $\mathcal{F}$ and a semantics σ, we define $UR(\mathcal{F}, \sigma) = \bigcup_{\mathcal{F}' \in RES(\mathcal{F})} \sigma(\mathcal{F}')$.*

Principle 19 (Resolution adequacy, [Baroni and Giacomin, 2007])**.**
*Given a skepticism relation $\preceq^E$ between sets of sets of arguments, a semantics σ satisfies the $\preceq^E$-*resolution adequacy *principle if and only if for every argumentation framework $\mathcal{F}$ we have $UR(\mathcal{F}, \sigma) \preceq^E \sigma(\mathcal{F})$.*

Figure 11: Stage semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy. We have $\mathcal{F}_1 \preceq^A \mathcal{F}_2$. Framework $\mathcal{F}_1$ has a unique stage extension $\{a\}$ and framework $\mathcal{F}_2$ has two stage extensions $\{a\}$ and $\{b\}$. Thus the set of skeptically accepted arguments of $\mathcal{F}_1$ equals $\{a\}$ is not a subset of the set of skeptically accepted arguments of $\mathcal{F}_2$, which is the empty set.

Figure 12: Stage2 semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy. We have $\mathcal{F}_1 \preceq^A \mathcal{F}_2$. Framework $\mathcal{F}_1$ has a unique stage2 extension $\{a\}$ and framework $\mathcal{F}_2$ has three stage2 extensions $\{a\}$, $\{b\}$ and $\{c\}$. Thus the set of skeptically accepted arguments of $\mathcal{F}_1$ equals $\{a\}$ is not a subset of the set of skeptically accepted arguments of $\mathcal{F}_2$, which is the empty set.

We consider three variants of the resolution adequacy principle: $\preceq^E_\cap$-resolution adequacy, $\preceq^E_W$-resolution adequacy and $\preceq^E_S$-resolution adequacy.

Observation 10.

- *If σ satisfies $\preceq^E_S$-res. adequacy then it satisfies $\preceq^E_W$-res. adequacy*
- *If σ satisfies $\preceq^E_W$-res. adequacy then it satisfies $\preceq^E_\cap$-res. adequacy*

The results regarding grounded, complete, stable, preferred, semi-stable, ideal, all four prudent and CF2 semantics were shown by Baroni and Giacomin [2007].

Eager semantics violates $\preceq_{\cap}^{E}$-resolution adequacy, as illustrated by the example from Figure 13. Consequently, it does not satisfy the other two forms of resolution adequacy. Consider naive semantics; from its definition we see that for every argu-

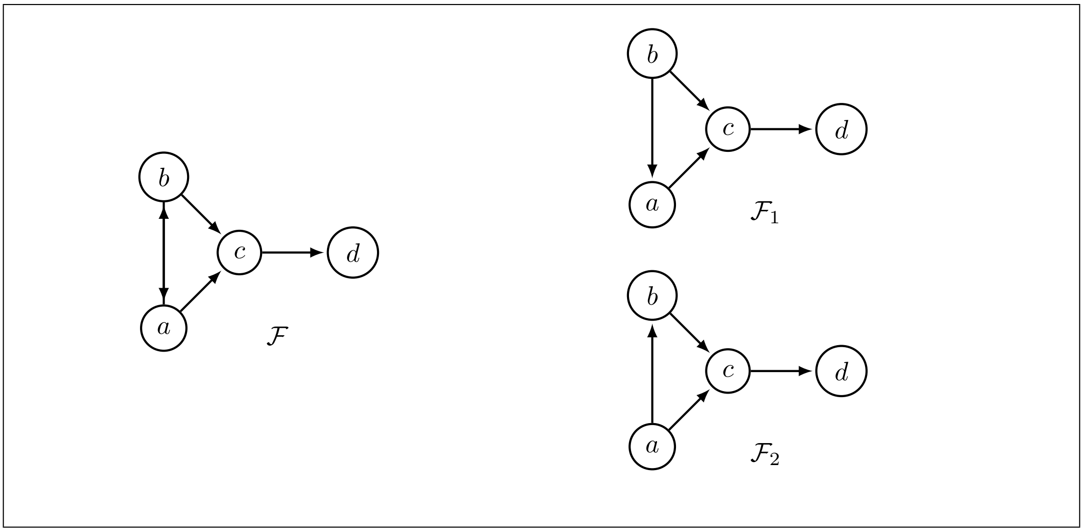

Figure 13: Eager semantics does not satisfy $\preceq_{\cap}^{E}$-resolution adequacy. We have $RES(\mathcal{F}) = \{\mathcal{F}_1, \mathcal{F}_2\}$. Namely, the eager extension of $\mathcal{F}_1$ is $\{b, d\}$ and the eager extension of $\mathcal{F}_2$ is $\{a, d\}$. Since the eager extension of $\mathcal{F}$ is the empty set, and $\{a, d\} \cap \{b, d\} = \{d\} \not\subseteq \emptyset$, the criterion is not satisfied.

mentation framework $\mathcal{F}$, for every $\mathcal{F}' \in RES(\mathcal{F})$, we have $\sigma(\mathcal{F}) = \sigma(\mathcal{F}')$. Thus, naive semantics satisfies all three forms of resolution adequacy.

Proposition 4.16. *Stage semantics satisfies $\preceq_{W}^{E}$-resolution adequacy.*

Proof. To show this, it is sufficient to show the following claim: for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, for every stage extension $\mathcal{E}$ of $\mathcal{F}$, there exists $\mathcal{F}' \in RES(\mathcal{F})$ such that $\mathcal{E}$ is a stage extension of $\mathcal{F}'$. Let $\mathcal{E}$ be a stage extension of $\mathcal{F}$. Let $\mathcal{F}' = (\mathcal{A}, \mathcal{R}') \in RES(\mathcal{F})$ be such that for every $a, b \in \mathcal{A}$ if $a \in \mathcal{E}$ then $(a, b) \in \mathcal{R}'$. (In other words, all attacks from $\mathcal{E}$ are preserved.) $\mathcal{E}$ is conflict-free in $\mathcal{F}'$, and all the attacks from $\mathcal{E}$ are preserved. Observe that the set of conflict-free sets of $\mathcal{F}$ and the set of conflict-free sets of $\mathcal{F}'$ coincide. Also, no conflict-free set attacks more arguments in $\mathcal{F}'$ than it attacks in $\mathcal{F}$. Thus, since $\mathcal{E}$ is a stage extension in $\mathcal{F}$, it is also a stage extension in $\mathcal{F}'$.

From the fact that for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, for every stage extension $\mathcal{E}$ of $\mathcal{F}$, there exists $\mathcal{F}' \in RES(\mathcal{F})$ such that $\mathcal{E}$ is a stage extension of $\mathcal{F}'$, we conclude that stage semantics satisfies $\preceq_{W}^{E}$-resolution adequacy. □

Since stage semantics satisfies $\preceq^E_W$-resolution adequacy, then it satisfies $\preceq^E_\cap$-resolution adequacy. The example from Figure 14 shows that stage semantics does not satisfy $\preceq^E_S$-resolution adequacy.

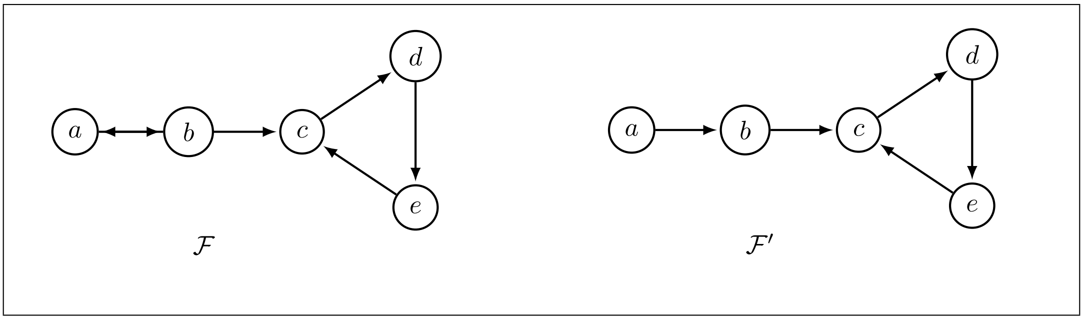

Figure 14: Stage semantics does not satisfy $\preceq^E_S$-resolution adequacy. We have $\mathcal{F}' \in RES(\mathcal{F})$, set $\mathcal{E}' = \{a, c\}$ is a stage extension of $\mathcal{F}'$, but there exists no stage extension $\mathcal{E}$ of $\mathcal{F}$ such that $\mathcal{E}' \subseteq \mathcal{E}$.

Stage2 semantics violates $\preceq^E_\cap$-resolution adequacy, as illustrated by the example from Figure 15. Consequently, it does not satisfy the other two forms of resolution adequacy.

	Succinctness	Tightness	Conflict- -sensitiveness	Com- -closure	SCC- -recursiveness	Cardinality
complete	×	×	×	✓	✓	1+
grounded	×	✓	✓	✓	✓	1
preferred	×	×	✓	✓	✓	1+
stable	×	✓	✓	✓	✓	0+
semi-stable	×	×	✓	✓	×	1+
ideal	×	✓	✓	✓	×	1
eager	×	✓	✓	✓	×	1+
p-complete	×	×	×	×	×	1+
p-grounded	×	✓	✓	✓	×	1
p-preferred	×	✓	✓	✓	×	1+
p-stable	×	✓	✓	✓	×	0+
naive	×	✓	✓	✓	×	1+
CF2	✓	✓	✓	✓	✓	1+
stage	×	✓	✓	✓	×	1+
stage2	✓	✓	✓	✓	✓	1+

Table 4: Properties of semantics, part 4

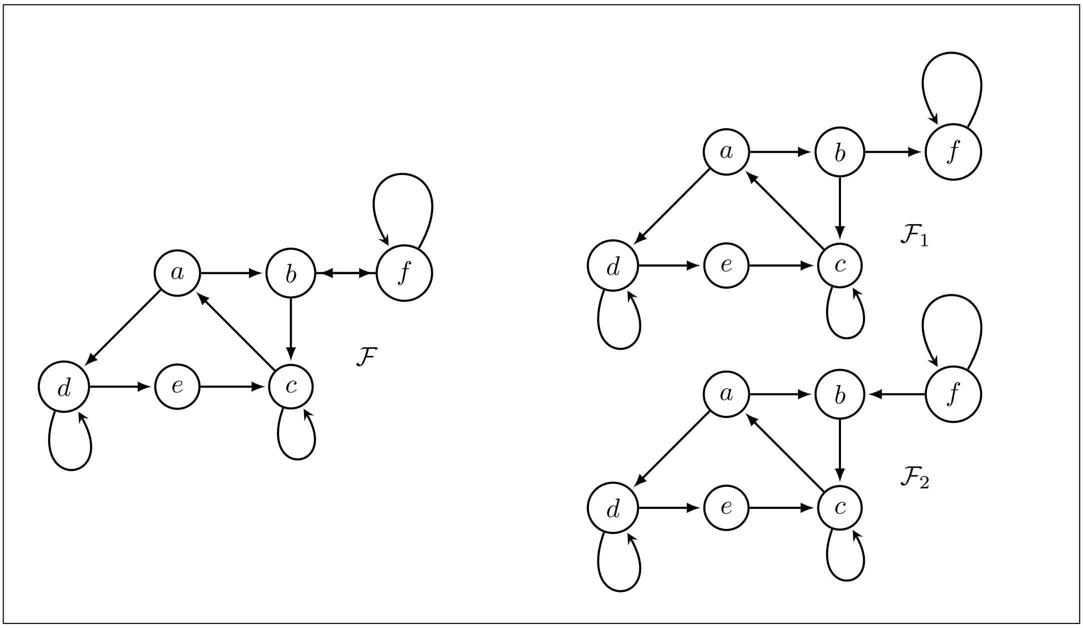

Figure 15: (Example provided by Wolfgang Dvorak, personal communication) Stage2 semantics does not satisfy $\preceq^E_\cap$-resolution adequacy. We have $RES(\mathcal{F}) = \{\mathcal{F}_1, \mathcal{F}_2\}$. Namely, the stage2 extensions of $\mathcal{F}$ are $\{a, e\}$ and $\{b, e\}$, and the stage2 extension of $\mathcal{F}_1$ and $\mathcal{F}_2$ is $\{a, e\}$. Since $\{a, e\} \not\subseteq \{a, e\} \cap \{b, e\} = \{e\}$, the criterion is not satisfied. The intuitive reason for the different behaviour from stage is that resolutions can break up a SCC into several SCCS and arguments that are not in the same SCC are not considered for range maximality.

Baroni *et al.* [2011b] introduce resolution-based family of semantics, which are developed to satisfy the resolution properties.

Let us now consider the last group of properties listed in Table 4. We first need to define the notion of strong equivalence [Oikarinen and Woltran, 2010]. Two frameworks $\mathcal{F}_1$ and $\mathcal{F}_2$ are strongly equivalent if for every argumentation framework $\mathcal{F}_3$, we have that $\mathcal{F}_1 \cup \mathcal{F}_3$ has the same extensions as $\mathcal{F}_2 \cup \mathcal{F}_3$.

Definition 4.17 (Strong equivalence)**.** *Two argumentation frameworks $\mathcal{F}_1$ and $\mathcal{F}_2$ are strongly equivalent with respect to semantics σ, in symbols $\mathcal{F}_1 \equiv^\sigma_s \mathcal{F}_2$ if and only if for each argumentation framework $\mathcal{F}_3$, $\sigma(\mathcal{F}_1 \cup \mathcal{F}_3) = \sigma(\mathcal{F}_2 \cup \mathcal{F}_3)$.*

An attack is redundant in $\mathcal{F}$ if removing it does not change the extensions of any $\mathcal{F}'$ that contains $\mathcal{F}$.

Definition 4.18 (Redundant attack)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework and σ and semantics. Attack $(a, b) \in \mathcal{R}$ is said to be redundant in $\mathcal{F}$ with respect to σ if and only if for all argumentation frameworks $\mathcal{F}'$ such that $\mathcal{F} \subseteq \mathcal{F}'$ we have $\sigma(\mathcal{F}') = \sigma(\mathcal{F}' \setminus (a, b))$.*

We can now define the succinctness principle [Gaggl and Woltran, 2013].

Principle 20 (Succinctness)**.** *A semantics σ satisfies the* succinctness *principle if and only if no argumentation framework contains a redundant attack with respect to σ.*

Gaggl and Woltran [2013] show that a semantics σ satisfies succinctness if and only if for every two argumentation frameworks $\mathcal{F}_1$ and $\mathcal{F}_2$ strong equivalence under σ coincides with $\mathcal{F}_1 = \mathcal{F}_2$.

Only CF2 and stage2 semantics satisfy succinctness. Namely, Oikarinen and Woltran [2010] showed that the notions of strong equivalence and syntactic equivalence do not coincide under complete, grounded, preferred, stable, semi-stable and ideal semantics. Gaggl and Woltran [2013] show that strong equivalence and syntactic equivalence do not coincide under stage and naive semantics. They also show that strong equivalence coincides with syntactic equivalence under CF2 semantics. Dvorák and Gaggl [2016] show that the same is true under stage2 semantics, which means that it also satisfies succinctness.

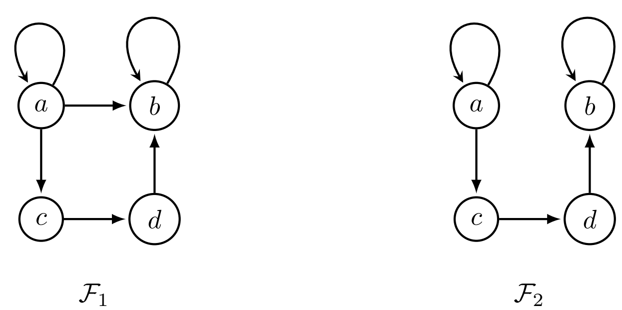

Figure 16: Several semantics violate succinctness

Consider eager semantics. Using Theorem 2 by Oikarinen and Woltran [2010], we can see that $\mathcal{F}_1$ and $\mathcal{F}_2$ from Figure 16 are strongly equivalent under semi-stable semantics. Since the eager semantics is uniquely determined by the set of semi-stable extensions, this means that $\mathcal{F}_1$ and $\mathcal{F}_2$ are strongly equivalent under eager semantics. Hence, eager semantics does not satisfy succinctness. Let us now show that all four prudent semantics violate succinctness.

Let $\mathcal{F}_1 = (\mathcal{A}, \mathcal{R}_1)$ and $\mathcal{F}_2 = (\mathcal{A}, \mathcal{R}_2)$ be the two argumentation frameworks from Figure 16. Let $\mathcal{F} = (\mathcal{A}', \mathcal{R}')$ be an arbitrary argumentation framework. Denote $\mathcal{F}_1' = \mathcal{F}_1 \cup \mathcal{F}$ and $\mathcal{F}_2' = \mathcal{F}_2 \cup \mathcal{F}$. Let us prove that the sets without indirect conflicts of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. It is immediate that if $\mathcal{E} \subseteq \mathcal{A} \cup \mathcal{A}'$ is not without indirect conflicts in $\mathcal{F}_2'$, it is also not without indirect conflicts in $\mathcal{F}_1'$, since $\mathcal{R}_2 \subseteq \mathcal{R}_1$. Let $\mathcal{E} \subseteq \mathcal{A} \cup \mathcal{A}'$ and let us prove that if $\mathcal{E}$ is not without indirect conflicts in $\mathcal{F}_1'$ then it is not without indirect conflicts in $\mathcal{F}_2'$. Let $\{(x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n)\} \subseteq \mathcal{R}_1 \cup \mathcal{R}'$ with n being even and $x_1, x_n \in \mathcal{E}$. If $\{(x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n)\} \subseteq \mathcal{R}_2 \cup \mathcal{R}'$ then $\mathcal{E}$ clearly has an indirect conflict in $\mathcal{F}_2'$. Otherwise, it must be that for some $i \in \{1, \ldots, n-1\}$ we have $x_i = a$ and $x_{i+1} = b$. Then $\{(x_1, x_2), \ldots, (x_i, c), (c, d), (d, x_{i+1}), \ldots, (x_{n-1}, x_n)\} \subseteq \mathcal{R}_2 \cup \mathcal{R}'$, thus $\mathcal{E}$ is not without indirect conflicts in $\mathcal{F}_2'$. Hence, the sets without indirect conflicts of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. It is immediate to see that $\mathcal{E} \subseteq \mathcal{A} \cup \mathcal{A}'$ defends all it arguments in $\mathcal{F}_1'$ if and only if it defends all its arguments in $\mathcal{F}_2'$. Thus, the sets of p-complete extensions of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. Also, the p-grounded extension of $\mathcal{F}_1'$ is exactly the p-grounded extension of $\mathcal{F}_2'$. Since every $\mathcal{E}$ without indirect conflicts attacks an argument x in $\mathcal{F}_1'$ if and only if $\mathcal{E}$ attacks x in $\mathcal{F}_2'$, p-stable extensions of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. Since the sets without indirect conflicts coincide, then maximal sets without indirect conflict coincide. Thus, p-preferred extensions of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. We conclude that all variants of prudent semantics violate succinctness.

The next principle we consider is tightness. Let us first define the notion of pairs. A couple (a, b) is in $\mathcal{P}airs$ if there is an extension containing both a and b.

Definition 4.19 ($\mathcal{P}airs$)**.** *Given a set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$, we define*

$$\mathcal{P}airs(\mathcal{S}) = \{(a, b) \mid \text{ there exists } \mathcal{E}_i \in \mathcal{S} \text{ such that } \{a, b\} \subseteq \mathcal{E}_i\}.$$

Tightness was introduced by Dunne *et al.* [2015]. Roughly speaking, it says that if argument a does not belong to extension $\mathcal{E}$, then there must be argument $b \in \mathcal{E}$ which is somehow incompatible with a.

Principle 21 (Tightness)**.** *A set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ is tight if and only if for every extension $\mathcal{E}_i$ and for every $a \in \mathcal{A}$ that appears in at least one extension from $\mathcal{S}$ it holds that if $\mathcal{E}_i \cup \{a\} \notin \mathcal{S}$ then there exists $b \in \mathcal{E}_i$ such that $(a, b) \notin \mathcal{P}airs(\mathcal{S})$.*

A semantics σ satisfies the tightness *principle if and only if for every argumentation framework $\mathcal{F}$, $\sigma(\mathcal{F})$ is tight.*

Dunne *et al.* [2015] show that stable, stage and naive semantics satisfy tightness. Example 4 from their paper shows an argumentation framework $\mathcal{F}$ such that

$\sigma(\mathcal{F}) = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$ with $\mathcal{E}_1 = \{a, b\}$, $\mathcal{E}_2 = \{a, d, e\}$, $\mathcal{E}_3 = \{b, c, e\}$, under preferred and semi-stable semantics. This example shows that those two semantics violate tightness since $\{a, b, e\}$ is not an extension.

Directly from the definition of tightness, we conclude that unique status semantics satisfy this principle.

Observation 11. *If σ is a semantics that returns exactly one extension for every argumentation framework then σ satisfies tightness.*

Hence, grounded, p-grounded, ideal and eager semantics satisfy tightness. The example from Figure 17 shows that complete and p-complete semantics violate tightness.

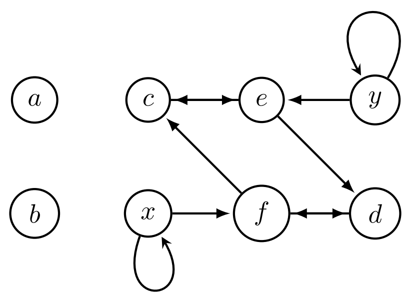

Figure 17: Complete and p-complete semantics violate tightness. There are two extensions $\mathcal{E}_1 = \{a, b\}$, $\mathcal{E}_2 = \{a, b, c, d\}$. Tightness is not satisfied since set $\mathcal{E}_1 \cup \{c\}$ is not an extension.

From Proposition 1 by Dunne *et al.* [2015], we have that the set of naive extensions is tight for every argumentation framework. Note that when σ is naive semantics and $\mathcal{F}$ an argumentation framework, all the elements of $\sigma(\mathcal{F})$ are pairwise incomparable with respect to $\subseteq$ (i.e. for each S, S', $S \subseteq S'$ implies $S = S'$). Hence, we can apply Lemma 2 by Dunne *et al.* [2015] and obtain

Observation 12. *If every extension under σ is a maximal conflict-free set, σ satisfies tightness.*

As an immediate consequence, p-stable, CF2 and stage2 semantics satisfy tightness. We now show that p-preferred semantics also satisfies this principle.

Proposition 4.20. *p-preferred semantics satisfies tightness.*

Proof. We use the proof by reductio ad absurdum. Let $\mathcal{E}$ be a p-preferred extension and let a be a credulously accepted argument such that

$$\text{for every } b \in \mathcal{E} \text{ there is a preferred p-extension } \mathcal{E}'' \text{ s.t. } \{a, b\} \subseteq \mathcal{E}'' \tag{1}$$

By means of contradiction, let us suppose that $\mathcal{E}' = \mathcal{E} \cup \{a\}$ is not a p-preferred extension. From (1), we conclude that $\mathcal{E}'$ is without indirect conflicts. Set $\mathcal{E}'$ is not p-admissible, since that would mean that $\mathcal{E}$ is not a maximal p-admissible set. Since $\mathcal{E}'$ is without indirect conflicts and $\mathcal{E}$ is p-admissible, there exists an argument b_1 such that $b_1 \mathcal{R} a$ and there is no $b' \in \mathcal{E}'$ such that $b' \mathcal{R} b_1$. Denote $B_1 = \{b \mid b \mathcal{R} a\}$.

Note that $\mathcal{E} \neq \emptyset$, since $\mathcal{E} = \emptyset$ would imply that there are no other p-preferred extensions and, consequently, a would not be credulously accepted. Thus, $\mathcal{E} \neq \emptyset$. Let $b \in \mathcal{E}$. From (1), there exists a p-preferred extension $\mathcal{E}_1$ such that $b \in \mathcal{E}_1$ and $a \in \mathcal{E}_1$. Since $a \in \mathcal{E}_1$ then for every $b_1^i \in B_1$ there exists $b_2^i \in \mathcal{E}_1$ such that $b_2^i \mathcal{R}$. Let $B_2 = \{b' \in \mathcal{E}_1 \mid \text{ there exists } b'' \in B_1 \text{ s.t. } b' \mathcal{R} b''\}$. In words, B_2 is the set of arguments from $\mathcal{E}_1$ that attack B_1 (they defend a from B_1).

Let us show that $\mathcal{E} \cup B_2$ is without indirect conflicts. By means of contradiction, suppose $\mathcal{E}$ indirectly attacks B_2. Then $\mathcal{E}$ indirectly attacks a, contradiction. Suppose now that B_2 indirectly attacks $\mathcal{E}$. Since $\mathcal{E}$ is p-admissible, then $\mathcal{E}$ attacks B_2, and thus (like in the previous case) $\mathcal{E}$ indirectly attacks a. Contradiction. So it must be that $\mathcal{E} \cup B_2$ is without indirect conflicts. Note also that since $B_2 \subseteq \mathcal{E}_1$ and $a \in \mathcal{E}_1$, we have that $\mathcal{E}_2 = \mathcal{E} \cup \{a\} \cup B_2$ is without indirect conflicts.

Note that $\mathcal{E}_2$ is not p-admissible, since it is a strict superset of a p-preferred extension. Set $\mathcal{E}$ is p-admissible and B_2 defends a so it must be that some argument(s) of B_2 are not defended by $\mathcal{E}_2$.

Let $B_3 = \{b \mid b \mathcal{R} B_2\}$. It must be that $B_3 \setminus B_2 \neq \emptyset$. Since $B_2 \subseteq \mathcal{E}_1$, and $\mathcal{E}_1$ is p-admissible, there exists $B_4 \subseteq \mathcal{E}_1$ such that B_4 defends B_2. Let $B_4 = \{b' \in \mathcal{E}_1 \mid \text{ there exists } b'' \in B_3 \text{ such that } b' \mathcal{R} b''\}$.

Note that $\mathcal{E}_4 = \mathcal{E} \cup \{a\} \cup B_2 \cup B_4$ is without indirect conflicts. By using the similar reasoning as in the case of $\mathcal{E}_2$, we conclude that $\mathcal{E}_4$ is not p-admissible. Let $B_5 = \{b \mid b \mathcal{R} B_4\}$. We have $B_5 \setminus (B_1 \cup B_3) \neq \emptyset$. By continuing this process, we construct an infinite sequence of different arguments $(b_1, b_3, \ldots, b_{i+1}, \ldots)$ such that $b_1 \in B_1$, $b_3 \in B_3 \setminus B_1$, $\ldots$, $b_{i+1} \in B_{i+1} \setminus (B_1 \cup \ldots \cup B_{i-1})$, $\ldots$, which is impossible, since the set of arguments is finite. □

We now study the notion of conflict-sensitiveness [Dunne *et al.*, 2015]. Note that an equivalent principle was called adm-closure in some papers.

Principle 22 (Conflict-sensitiveness)**.** *A set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ is conflict-sensitive if and only if for every two extensions $\mathcal{E}_i$, $\mathcal{E}_j$ such that $\mathcal{E}_i \cup \mathcal{E}_j \notin \mathcal{S}$ it holds that there exist $a, b \in \mathcal{E}_i \cup \mathcal{E}_j$ such that $(a, b) \notin \mathcal{P}airs_{\mathcal{S}}$.*

A semantics σ satisfies the conflict-sensitiveness *principle if and only if for every argumentation framework $\mathcal{F}$, $\sigma(\mathcal{F})$ is conflict-sensitive.*

This principle checks whether the fact that $\mathcal{E}_i \cup \mathcal{E}_j$ is not an extension is justified by existence of $a \in \mathcal{E}_i$ and $b \in \mathcal{E}_j$ that cannot be taken together. Dunne *et al.* [2015] show that every tight set is also conflict-sensitive. Thus, grounded, stable, ideal, stage, eager, naive, p-grounded, p-stable, p-preferred, stage2 and CF2 semantics satisfy conflict-sensitiveness. Proposition 2 by Dunne *et al.* [2015] shows that preferred and semi-stable semantics satisfy conflict-sensitiveness. Our example from Figure 18 shows that complete and p-complete semantics violate this principle. As for tightness, it does not seem that violating this principle is a necessarily a bad thing. It can be rational to ask for both a and b in order to defend e. There is no conflict between a and e, it is just that e needs to be defended from both c and d.

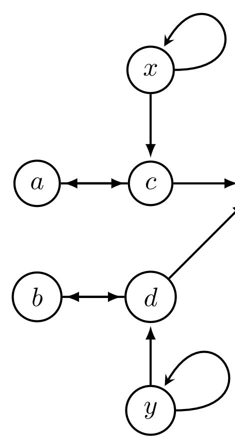

Figure 18: Complete and p-complete semantics violate conflict-sensitiveness. There are four extensions $\mathcal{E}_1 = \emptyset$, $\mathcal{E}_2 = \{a\}$, $\mathcal{E}_3 = \{b\}$, $\mathcal{E}_4 = \{a, b, e\}$. Conflict-sensitiveness is not satisfied since set $\{a, b\}$ is not an extension.

Let us now turn to com-closure [Dunne *et al.*, 2015]. To define this principle, we first need to introduce the notion of completion set. Completion sets are the smallest extensions that contain a given set.

Definition 4.21 (Completion set)**.** *Given a set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ and a set of arguments $\mathcal{E}$, set $\mathcal{E}'$ is a completion set of $\mathcal{E}$ in $\mathcal{S}$ if and only if $\mathcal{E}'$ is a minimal for $\subseteq$ set such that $\mathcal{E}' \in \mathcal{S}$ and $\mathcal{E} \subseteq \mathcal{E}'$.*

Roughly speaking, com-closure says that, given a set of extensions $\mathcal{S}$, if for every $\mathcal{T} \subseteq \mathcal{S}$ each two arguments from sets of $\mathcal{T}$ appear in some extension of $\mathcal{S}$, then $\mathcal{T}$ can be extended to an extension in a unique way.

Principle 23 (Com-closure)**.** *A set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ is com-closed if and only if for every $\mathcal{T} \subseteq \mathcal{S}$ the following holds: if $(a, b) \in \mathcal{P}airs_{\mathcal{S}}$ for each $a, b \in \cup_{\mathcal{E}_i \in \mathcal{T}} \mathcal{E}_i$, then $\cup_{\mathcal{E}_i \in \mathcal{T}} \mathcal{E}_i$ has a unique completion set in $\mathcal{S}$.*

A semantics σ satisfies the com-closure *principle if and only if for every argumentation framework $\mathcal{F}$, $\sigma(\mathcal{F})$ is com-closed.*

Dunne *et al.* [2015] show that each conflict-sensitive set of extensions is com-closed. Thus, all the semantics that satisfy conflict-sensitiveness also satisfy com-closure. Their Proposition 4 shows that complete semantics is com-closed. To see that p-complete semantics does not satisfy com-closure, consider the graph from Figure 19.

We now study the notion of SCC-recursiveness, which was introduced by Baroni *et al.* [2005].

Principle 24 (SCC-recursiveness)**.** *A semantics σ satisfies the* SCC-recursiveness *principle if and only if for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ we have $\sigma(\mathcal{F}) = \mathcal{GF}(\mathcal{F}, \mathcal{A})$, where for every $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and for every set $C \subseteq \mathcal{A}$, the function $\mathcal{GF}(\mathcal{F}, C) \subseteq 2^{\mathcal{A}}$ is defined as follows: for every $\mathcal{E} \subseteq \mathcal{A}$, $\mathcal{E} \in \mathcal{GF}(\mathcal{F}, C)$ if and only if*

- *in case $|SCCS_{\mathcal{F}}| = 1$, $\mathcal{E} \in \mathcal{BF}_S(\mathcal{F}, C)$,*
- *otherwise, $\forall S \in SCCS_{\mathcal{F}}, (\mathcal{E} \cap S) \in \mathcal{GF}(\mathcal{F} \downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})}, U_{\mathcal{F}}(S, \mathcal{E}) \cap C)$,*

where $\mathcal{BF}_S(\mathcal{F}, C)$ is a function, called base function*, that, given an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, such that $|SCCS(\mathcal{F})| = 1$ and a set $C \subseteq \mathcal{A}$, gives a subset of $2^{\mathcal{A}}$.*

Baroni *et al.* [2005] proved that grounded, complete, stable and preferred semantics satisfy SCC-recursiveness. CF2 and stage2 semantics also satisfy this principle, since they are defined by using SCC recursive schema. None of the remaining semantics satisfies SCC-recursiveness. To show that ideal, semi-stable, stage and eager semantics does not satisfy SCC-recursiveness, consider the examples from Figures 20 and 21, which are both due to M. Giacomin (personal communication, 2016). Naive semantics does not satisfy SCC-recursiveness since it ignores the direction of attacks. Consider the example from Figure 22. All four prudent semantics violate SCC-recursiveness. Consider the argumentation framework from Figure 4. Let σ be any of the four prudent semantics. In this example, every argument forms an SCC.

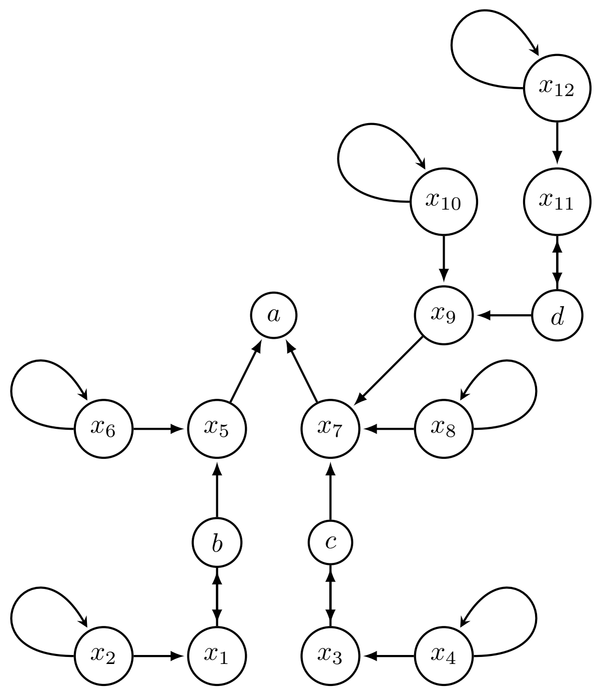

Figure 19: p-complete semantics is not com-closed. There are eight p-complete extensions: $\mathcal{E}_1 = \emptyset$, $\mathcal{E}_2 = \{b\}$, $\mathcal{E}_3 = \{c\}$, $\mathcal{E}_4 = \{d\}$, $\mathcal{E}_5 = \{b, d\}$, $\mathcal{E}_6 = \{c, d\}$, $\mathcal{E}_7 = \{b, c, d\}$, $\mathcal{E}_8 = \{b, c, a\}$. Let $\mathcal{T} = \{\mathcal{E}_2, \mathcal{E}_3\}$. Com-closure is not satisfied since set $\{b, c\}$ has two competition sets, namely $\mathcal{E}_7$ and $\mathcal{E}_8$.

Thus, each extension must contain both e and f. Furthermore, no extension can contain neither of b, c, d, since they are all attacked by e of f. Finally, if σ satisfied SCC-recursiveness, each extension would contain a, which is not the case.

The results considering cardinality are easy to obtain.

We do not include several properties that are not satisfied by any of the studied semantics. Let us mention three such properties. Downward closure [Dunne *et al.*, 2015] basically says that each subset of each extension is an extension. Non-triviality [Dunne *et al.*, 2012] says that it is not the case that $\sigma(\mathcal{F}) = \{\emptyset\}$; in words, the empty set is not the only extension. Decisiveness [Dunne *et al.*, 2012] is a stronger principle that asks that every framework has exactly one extension $\mathcal{E}$ and that $\mathcal{E}$ is not empty.

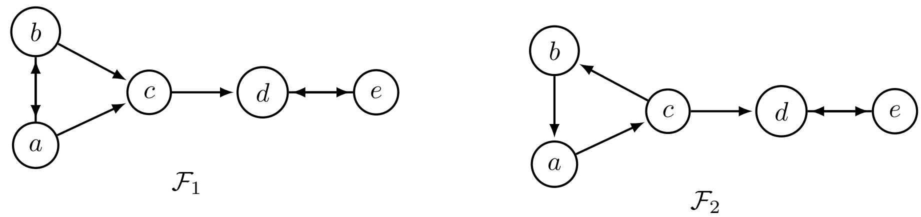

Figure 20: Ideal semantics is not SCC-recursive. Both in $\mathcal{F}_1$ and in $\mathcal{F}_2$, there are two SCCs: $S_1 = \{a, b, c\}$ and $S_2 = \{d, e\}$. Suppose ideal semantics is SCC-recursive. Then, we can calculate the ideal extension of an argumentation framework by starting from S_1 and then continuing to S_2. Denote by $\mathcal{F}_1^1$ the restriction of $\mathcal{F}_1$ on S_1 and by $\mathcal{F}_2^1$ the restriction of $\mathcal{F}_2$ on S_1. The ideal extension of $\mathcal{F}_1^1$ is the empty set. The ideal extension of $\mathcal{F}_2^1$ is also the empty set. So the exact same information is transferred to the next SCC, S_2. The second SCC, S_2 is the same for both frameworks, so given the same information from S_1, both frameworks should have the same ideal extension. However, $\sigma(\mathcal{F}_1) = \emptyset$ whereas $\sigma(\mathcal{F}_2) = \{e\}$. Thus, ideal semantics does not satisfy SCC-recursiveness.

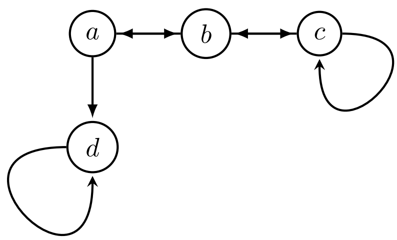

Figure 21: Semi-stable, stage and eager semantics violate SCC-recursiveness. Let σ be stage, semi-stable or eager semantics. Consider the first SCC, $S_1 = \{a, b, c\}$. If we restrict the argumentation framework to S_1, the only extension under σ is $\{b\}$. If σ satisfied SCC-recursiveness, each extension of this framework would contain b, which is not the case, since $\{a\}$ is an extension of this framework under σ.

5 Summary and outlook

The principle-based approach has developed over the past ten years into a cornerstone of formal argumentation theory, because it allows for a more systematic study and comparison of argumentation semantics. In this article we give a com-

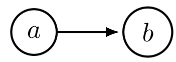

Figure 22: Naive semantics does not satisfy SCC-recursiveness. Note that the first SCC is $S_1 = \{a\}$. If naive semantics satisfied SCC-recursiveness, every naive extension of the whole framework would contain a, which is not the case since $\{b\}$ is a naive extension of this framework too.

plete analysis of the fifteen main alternatives for argumentation semantics using the twenty-seven main principles discussed in the literature on abstract argumentation. Moreover, Caminada [forthcoming] discusses the principles used in structured argumentation, which he calls rationality postulates, and Dung [2016] analyses *prioritised* argumentation using a principle-based or axiomatic approach.

The principle-based approach has also been used to provide a more systematic study and analysis of the semantics of extended argumentation frameworks, of the aggregation of argumentation frameworks, and of the dynamics of argumentation frameworks. For example, principles of ranking-based semantics have been proposed [Amgoud and Ben-Naim, 2016; Amgoud *et al.*, 2017; Bonzon *et al.*, 2016b], where the output is not a set of extensions but a ranking on the set of arguments, and principles have been developed for bipolar argumentation [Cayrol and Lagasquie-Schiex, 2015]. Likewise we expect a further systematic study of weighted argumentation frameworks, preference-based argumentation frameworks, input/output frameworks, abstract dialectical frameworks, and so on.

It may be expected that the principle-based approach will play an even more prominent role in the future of formal argumentation, as the number of alternatives for argumentation semantics increases, new argumentation principles are introduced, and more requirements of actual applications are expressed in terms of such principles. Moreover, in the future applications and principles concerned with infinite frameworks may become more prominent. For example, when the set of arguments becomes infinite, it may be that there are no semi-stable extensions. However, Baumann [forthcoming] illustrates how a meaningful version of eager semantics can be defined, which no longer has the property that it always returns exactly one extension.

Finally, the principle-based approach to formal argumentation may lead to the study of impossibility and possibility results, as well as the development of representation theorems characterising sets of argumentation semantics. The use of the principle-based approach in other areas of reasoning, such as voting theory or AGM theory change, may inspire such further formal investigations.

Acknowledgements

The authors thank Ringo Baumann, Pietro Baroni, Liao Beishui, Martin Caminada, Wolfgang Dvořák and Massimiliano Giacomin for their useful comments and suggestions.

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Received 8 June 2017

On the Nature of Argumentation Semantics: Existence and Uniqueness, Expressibility, and Replaceability

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Abstract

This article is devoted to argumentation semantics which play the flagship role in Dung's abstract argumentation theory. Almost all of them are motivated by an easily understandable intuition of what should be acceptable in the light of conflicts. However, although these intuitions equip us with short and comprehensible formal definitions it turned out that their intrinsic properties such as *existence and uniqueness*, *expressibility*, and *replaceability* are not that easily accessible. The article reviews the mentioned properties for almost all semantics available in the literature. In doing so we include two main axes: namely first, the distinction between extension-based and labelling-based versions and secondly, the distinction of different kind of argumentation frameworks such as finite or unrestricted ones.

1 Introduction

Given the large variety of existing logical formalisms it is of utmost importance to select the most adequate one for a specific purpose, e.g. for representing the knowledge relevant for a particular application or for using the formalism as a modeling tool for problem solving. Awareness of the nature of a logical formalism, in other words, of its fundamental intrinsic properties, is indispensable and provides the basis of an informed choice. Apart from the deeper understanding of the considered formalism, the study of such intrinsic properties can help to identify interesting fragments or to develop useful extensions of a formalism. Moreover, the obtained insights can be used to refine existing algorithms, or even give rise to new ones.

Presumably, the best-known intrinsic property of logics is *monotonicity*. *Monotonic logics* like first order logic are perfectly suitable for the formalization of universal truths since in these logics, whenever a formula ϕ is a logical consequence of a set of axioms Σ, it remains true forever and without exception even if we add new axioms to Σ. Formalisms which do not satisfy monotonicity, commonly referred to as *nonmonotonic logics*, allow for defeasible reasoning, i.e. it is possible to withdraw former conclusions (cf. [Brewka, 1992; Gabbay *et al.*, 1994] for excellent overviews). Both kinds of logics have their traditional application domains and apart from this fundamental choice there are many other comparison criteria influencing the decision which logic or which specific semantics of a logic to use in a certain context.

One of the first intrinsic properties which comes to mind is *computational complexity*, i.e. how expensive is it to solve typical decision problems in the candidate formalism. A further related issue is *modularity* which is, among other things, engaged with the question whether it is possible to divide a given theory in subtheories, s.t. the formal semantics of the entire theory can be obtained by constructing the semantics of the subtheories. Both topics were studied in-depth for mainstream nonmonotonic formalisms like default logic [Gottlob, 1992; Turner, 1996], logic programming under certain semantics [Lifschitz and Turner, 1994; Dantsin *et al.*, 1997] as well as abstract argumentation frameworks under various argumentation semantics [Baroni *et al.*, 2005; Baumann, 2011; Liao *et al.*, 2011; Dvořák, 2012].

In this article we give an overview of three further intrinsic properties of abstract argumentation semantics.

1. *existence and uniqueness* Is it possible, and if so how, to *guarantee* the existence of at least one or exactly one extension/labelling by considering the structure of a given AF F only? (cf. Section 2)

2. *expressibility* Is it possible, and if so how, to *realize* a given candidate set of extensions/labellings within a single AF F? (cf. Section 3)

3. *replaceability* Is it possible, and if so how, to *simplify* parts of a given AF F, s.t. the modified version F' and F cannot be semantically distinguished by further information which might be added later to both simultaneously? (cf. Section 4)

The question whether a certain formalism always provides one with a formal meaning or even with a uniquely determined semantical answer is a crucial factor for its suitability for the application in mind. For instance, in contrast to problem solving where a plurality of solutions may possibly be desired, in decision making one might be interested in guaranteeing a single answer provided by a logical formalism. It is well-known that a given theory in propositional logic neither has to possess a model nor, in case of existence, has there to be exactly one. The same applies to logic programs under stable model semantics. In contrast, a propositional theory of positive formulae is always satisfiable and definite logic programs constitute a subclass of logic programs where even uniqueness is guaranteed. In Section 2 we will see that Dung's abstract argumentation semantics behave in a similar way, i.e. the existence or uniqueness of extensions/labellings depend on structural restrictions of argumentation frameworks.

Expressibility is concerned with the expressive power of logical formalisms. The question here is which kinds of model sets are realizable, that is, can be the set of models of a single knowledge base of the formalism. This is a decisive property from an application angle since potential necessary or sufficient properties of model sets may rule out a logic or make it perfectly appropriate for representing certain solutions. For instance, it is well-known that in case of propositional logic any finite set of two-valued interpretations is realizable. This means, given such a finite set $\mathcal{I}$, we always find a set of formulae T, s.t. $Mod(T) = \mathcal{I}$. In case of normal logic programs it is obvious that not all model sets can be expressed, since any set of stable models forms a $\subseteq$-antichain. Remarkably, being such an antichain is not only necessary but even sufficient for realizability w.r.t. stable model semantics [Eiter *et al.*, 2013; Strass, 2015]. In case of abstract argumentation we are equipped with a high number of semantics and in Section 3 we will see that characterizing properties are not that easy. Moreover, as expected, representational limits highly depend on the chosen semantics.

In case of propositional logic we have that – in contrast to all non-monotonic logics available in the literature – standard equivalence, i.e. sharing the same models, even guarantees intersubstitutability in any logical context without loss of information. As an aside, it is not the monotonicity of a certain logic but rather the so-called *intersection property* which guarantees this behavior (cf. [Baumann and Strass, 2016]). scenarios since it allows to simplify parts of a theory without looking at the rest. For this reason, much effort has been devoted to characterizing *strong equivalence* for nonmonotonic formalisms, such as logic programs [Lifschitz *et al.*, 2001], causal theories [Turner, 2004], default logic [Turner, 2001] and nonmonotonic logics in general [Truszczynski, 2006; Baumann and Strass, 2016]. In Section 4 we will see that characterization theorems in case of abstract argumentation are quite different from those for the aforementioned formalisms since being strongly equivalent can be decided by looking at the syntax only.

2 Existence and Uniqueness

Given a certain logical formalism $\mathcal{L}$ together with its semantics $\sigma_{\mathcal{L}}$. One central question is whether the semantics provides any $\mathcal{L}$-theory T with a formal meaning, i.e. $|\sigma_{\mathcal{L}}(T)| \geq 1$. A more demanding property than *existence* is *uniqueness*, i.e. $|\sigma_{\mathcal{L}}(T)| = 1$ for any $\mathcal{L}$-theory T. Clearly, these properties are interesting from several perspectives. For instance, in case of uniqueness, we observe a coincidence of sceptical and credulous reasoning modes. More precisely, if $\sigma_{\mathcal{L}}(T) = \{E\}$, then $\bigcap \sigma_{\mathcal{L}}(T) = \bigcup \sigma_{\mathcal{L}}(T) = E$. Furthermore, if a theory T is interpreted as *meaningful* if and only if $\sigma_{\mathcal{L}}(T) \neq \emptyset$, then existence might be a desired property. If the latter has to be neglected in the general case, then one further challenge is to identify sufficient properties of $\mathcal{L}$-theories guaranteeing their meaningfulness.

Let us turn to abstract argumentation frameworks [Dung, 1995]. Due to the practical nature of argumentation most work in the literature restricts itself to the case of finite AFs, i.e. any considered AF consists of finitely many arguments and attacks only. For this class of AFs a proof or disproof of existence or uniqueness is mostly straightforward. In the general infinite case however conducting such proofs is more intricate. It usually involves the proper use of set theoretic axioms, like the *axiom of choice* or equivalent statements. Dung already proposed the existence of preferred extensions in the case of infinite argumentation frameworks. It has later on (e.g. [Caminada and Verheij, 2010]) been pointed out that Dung has not been precise with respect to the use of principles. The existence of semi-stable extensions for finitary[1] argumentation frameworks was first shown by Weydert with the use of model-theoretic techniques [Weydert, 2011]. Later on, Baumann and Spanring presented a first comprehensive overview of results regarding existence and uniqueness for a whole bunch of semantics considered in the literature [Baumann and Spanring, 2015]. They provided complete or alternative proofs of already known results and contributed missing results for the infinite or finitary case. We mention two interesting results: Firstly, eager semantics is exceptional among the universally defined semantics since either there is exactly one or there are infinitely many eager extensions. Secondly, stage semantics behaves similarly to semi-stable in the sense that extensions are guaranteed as long as finitary AFs are considered. A further step forward in the systematic analysis of argumentation semantics in the infinite case was presented in [Spanring, 2015]. Spanring studied the relation between non-existence of extensions and the number of non-finitary arguments. It was shown that there are AFs where one single non-finitary argument causes a collapse[2] of semi-stable semantics. Interestingly, all known AFs which do not provide any stage extension possesses infinitely many non-finitary arguments. It is an open question whether this observation applies in general [Spanring, 2015, Conjecture 14].

[1] An argument is called *finitary* if it receives finitely many attacks only. Moreover, an AF is said to be *finitary* if and only if it consists of finitary arguments only (cf. Definition 2.1).

[2] The term *collapse* was firstly introduced in [Spanring, 2015] and it refers to a semantics not providing any extension/labelling for a given AF.

2.1 Basic Definitions in Dung's Abstract Argumentation Theory

For the sake of self-containedness we review all relevant definitions (for more introductory comments we refer the reader to [Baroni *et al.*, 2011]). The standard way of defining argumentation frameworks is to introduce a certain reference set $\mathcal{U}$, so-called *universe of arguments* and to require, that all arguments used in AFs are elements of this set. More formally, for any AF $F = (A, R)$ we have $A \subseteq \mathcal{U}$ and $R \subseteq A \times A$. In order to be able to consider AFs possessing an arbitrary finite number of arguments or even infinitely many we have to request that $|\mathcal{U}| \geq \aleph_0 = |\mathbb{N}|$. No further conditions are imposed. In the following we use $\mathcal{F}$ as an abbreviation for the set of all AFs (induced by $\mathcal{U}$). An AF F is called *finite* if it possesses finitely many arguments only. Furthermore, we say that F is *finitary* if every argument has only finitely many attackers.

Definition 2.1. *An AF $F = (A, R)$ is called*

1. *finite if $|A| \in \mathbb{N}$,*
2. *finitary if for any $a \in A$, $|\{b \in A \mid (b, a) \in R\}| \in \mathbb{N}$ and*
3. *arbitrary or unrestricted if $F \in \mathcal{F}$.*

In order to formalize the notions of *existence* and *uniqueness* in the context of abstract argumentation theory we have to clarify what we precisely mean by a *semantics*. In the literature two main approaches to argumentation semantics can be found, namely so-called *extension-based* and *labelling-based* versions. The main difference is that extension-based versions return a set of sets of arguments (so-called *extensions*) for any given AF in contrast to a set of sets of n-tupels (so-called *labellings*) as in case of labelling-based approaches. However, from a mathematical point of view both kinds of semantics are instances of Definition 2.2. More precisely, extension-based versions are covered by $n = 1$ and labelling-based approaches can be obtained by setting $n \geq 2$. We use $\left(2^{\mathcal{U}}\right)^n$ to denote the n-ary cartesian power of $2^{\mathcal{U}}$, i.e. $\left(2^{\mathcal{U}}\right)^n = \underbrace{2^{\mathcal{U}} \times \cdots \times 2^{\mathcal{U}}}_{n-\text{times}}$.

Definition 2.2. *A semantics is a function $\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^n}$ for some $n \in \mathbb{N}$, s.t. $F = (A, R) \mapsto \sigma(F) \subseteq \left(2^A\right)^n$.*

We now introduce the two different definedness statuses of argumentation semantics which capture the notions of existence and uniqueness, namely so-called *universal* and *unique definedness*. Both versions are relativized to a certain set of

AFs. If clear from context, unimportant or if $\mathcal{C} = \mathcal{F}$ we will not mention explicitly the considered set of AFs.

Definition 2.3. *Given a semantics σ and a set $\mathcal{C}$ of AFs. We say that σ is*

1. *universally defined w.r.t. $\mathcal{C}$ if $\forall F \in \mathcal{C}$, $|\sigma(F)| \geq 1$ and*
2. *uniquely defined w.r.t. $\mathcal{C}$ if $\forall F \in \mathcal{C}$, $|\sigma(F)| = 1$.*

In this section we are interested in definedness statuses w.r.t. finite, finitary and arbitrary frameworks. Besides conflict-free and admissible sets (abbreviated by *cf* and *ad*) we consider a large number of mature semantics, namely naive, stage, stable, semi-stable, complete, preferred, grounded, ideal, eager semantics as well as the more exotic cf2 and stage2 semantics (abbreviated by *na*, *stg*, *stb*, *ss*, *co*, *pr*, *gr*, *il*, *eg*, *cf2* and *stg2* respectively). In the following we introduce the extension-based versions of these semantics (indicated by $\mathcal{E}_\sigma$). Any considered semantics possesses a 3-valued labelling-based version (denoted as $\mathcal{L}_\sigma$). It is important to note that for all considered semantics we do not observe any differences between the definedness statuses of their labelling-based and extension-based versions. For the mature semantics this is due the fact that there is a one-to-one correspondence between σ-extensions and σ-labellings implying that $|\mathcal{E}_\sigma(F)| = |\mathcal{L}_\sigma(F)|$ for any AF F (for more details confer Paragraph *Basic Properties and a Fundamental Relation* in Section 4).

Before presenting the definitions we have to introduce some notational conventions. Given an AF $F = (A, R)$ and a set $E \subseteq A$. We use E^+_F or simply, E^+ for $\{b \mid (a, b) \in R, a \in E\}$. Moreover, $E^\oplus_F$ or simply, $E^\oplus$ is called the *range* of E and stands for $E^+ \cup E$. We say *a attacks b* (in F) if $(a, b) \in R$. An argument a is *defended* by E (in F) if for each $b \in A$ with $(b, a) \in R$, b is attacked by some $c \in E$. Finally, $\Gamma_F : 2^A \to 2^A$ with $I \mapsto \{a \in A \mid a \text{ is defended by } I\}$ denotes the so-called *characteristic function* (of F) [Dung, 1995].

Definition 2.4. *Let $F = (A, R)$ be an AF and $E \subseteq A$.*

1. *$E \in \mathcal{E}_{cf}(F)$ iff for no $a, b \in E$, $(a, b) \in R$,*
2. *$E \in \mathcal{E}_{na}(F)$ iff $E \in \mathcal{E}_{cf}(F)$ and for no $I \in \mathcal{E}_{cf}(F)$, $E \subset I$,*
3. *$E \in \mathcal{E}_{stg}(F)$ iff $E \in \mathcal{E}_{cf}(F)$ and there is no $I \in \mathcal{E}_{cf}(F)$, s.t. $E^\oplus \subset I^\oplus$,*
4. *$E \in \mathcal{E}_{stb}(F)$ iff $E \in \mathcal{E}_{cf}(F)$ and $E^\oplus = A$,*
5. *$E \in \mathcal{E}_{ad}(F)$ iff $E \in \mathcal{E}_{cf}(F)$ and E defends all its elements,*
6. *$E \in \mathcal{E}_{ss}(F)$ iff $E \in \mathcal{E}_{ad}(F)$ and there is no $I \in \mathcal{E}_{ad}(F)$, s.t. $E^\oplus \subset I^\oplus$,*

7. $E \in \mathcal{E}_{co}(F)$ *iff* $E \in \mathcal{E}_{ad}(F)$ *and for any* $a \in A$ *defended by* E *in* F*,* $a \in E$*,*

8. $E \in \mathcal{E}_{pr}(F)$ *iff* $E \in \mathcal{E}_{ad}(F)$ *and for no* $I \in \mathcal{E}_{co}(F)$*,* $E \subset I$*,*

9. $E \in \mathcal{E}_{gr}(F)$ *iff* E *is the* $\subseteq$*-least fixpoint of* Γ_F*,*

10. $E \in \mathcal{E}_{il}(F)$ *iff* $E \in \mathcal{E}_{ad}(F)$*,* $E \subseteq \bigcap \mathcal{E}_{pr}(F)$ *and there is no* $I \in \mathcal{E}_{co}(F)$ *satisfying* $E \subset I \subseteq \bigcap \mathcal{E}_{pr}(F)$*,*

11. $E \in \mathcal{E}_{eg}(F)$ *iff* $E \in \mathcal{E}_{ad}(F)$*,* $E \subseteq \bigcap \mathcal{E}_{ss}(F)$ *and there is no* $I \in \mathcal{E}_{co}(F)$ *satisfying* $E \subset I \subseteq \bigcap \mathcal{E}_{ss}(F)$*.*

Finally, we introduce the recursively defined cf2 and stage2 semantics [Baroni *et al.*, 2005; Dvořák and Gaggl, 2012].

Definition 2.5. *Let* $F = (A, R)$ *be an AF and* $E \subseteq A$*.*

1. $E \in \mathcal{E}_{cf2}(F)$ *iff*

- $E \in \mathcal{E}_{na}(F)$ *if* $|SCCs_F = 1|$ *and*
- $\forall S \in SCCs_F (E \cap S) \in \mathcal{E}_{cf2}\left(F|_{UP_F(S,E)}\right)$,

2. $E \in \mathcal{E}_{stg2}(F)$ *iff*

- $E \in \mathcal{E}_{stg}(F)$ *if* $|SCCs_F = 1|$ *and*
- $\forall S \in SCCs_F (E \cap S) \in \mathcal{E}_{stg2}\left(F|_{UP_F(S,E)}\right)$.

Here $SCCs_F$ *denotes the set of all strongly connected components of* F*, and for any* $E, S \subseteq A$*,* $UP_F(S, E) = \{a \in S \mid \nexists b \in E \setminus S : (b, a) \in R\}$*.*

The following proposition summarizes well-known subset relations between the considered semantics. For two semantics σ, τ and a certain set of AFs $\mathcal{C}$ we use $\sigma \subseteq_{\mathcal{C}} \tau$ as a shorthand for $\sigma(F) \subseteq \tau(F)$ for any AF $F \in \mathcal{C}$. The presented relations hold for both extension-based as well as labelling-based versions of the considered semantics. In the interest of readability we present the relations graphically.

Proposition 2.6. *For semantics* σ *and* τ*,* $\sigma \subseteq_{\mathcal{F}} \tau$ *iff there is a path of solid arrows from* σ *to* τ *in Figure 1. A dotted arrow indicates that the corresponding subset relation is guaranteed for finite frameworks only.*

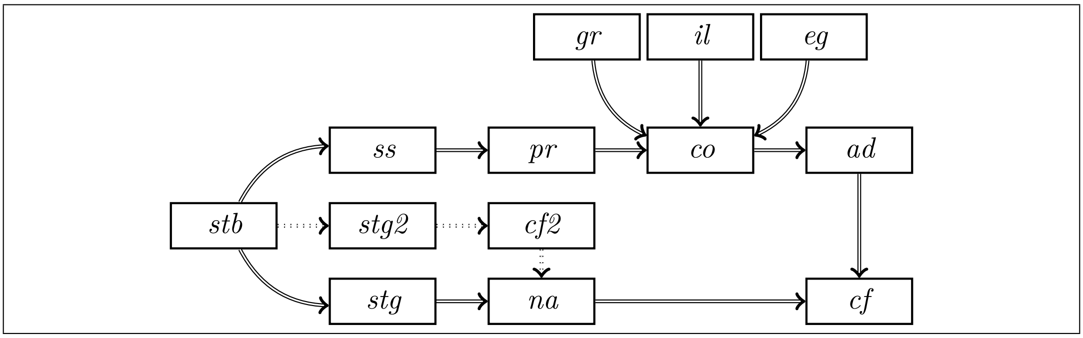
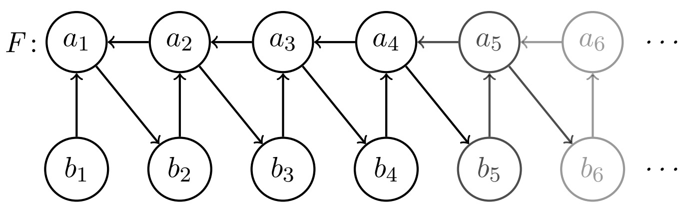


Figure 1: Subset Relations between Semantics

Detailed proofs can be found in [Baumann, 2014b, Proposition 2.7] as well as [Gaggl and Dvořák, 2016, Section 3.1]. Note that the shorthand $\sigma \subseteq_{\mathcal{C}} \tau$ requires that both semantics are total functions on $\mathcal{C}$ since a framework to which one of these semantics is undefined renders the subset shorthand undefined itself. The following simple example shows that Definition 2.5 does not always provide a definite answer on whether a certain candidate set is an *cf2*-extension or *stg2*-extension, respectively. This is due to the fact that the defined recursion does not terminate necessarily in case of non-finite AFs.[3] Consequently, *stg2* and *cf2* are not total functions regarding arbitrary frameworks.

Example 2.7 (Infinite Recursion [Baumann and Spanring, 2017])**.** *Consider the following AF $F = (A \cup B, R)$ where*

- $A = \{a_i \mid i \in \mathbb{N}\}$, $B = \{b_i \mid i \in \mathbb{N}\}$ *and*
- $R = \{(b_i, a_i), (a_{i+1}, a_i), (a_i, b_{i+1}) \mid i \in \mathbb{N}\}$



Let $\sigma \in \{cf2, stg2\}$. We want to check whether the candidate set $E = \{b_i \mid i \in \mathbb{N}\}$ is a σ-extension. Observe that the AF F possesses two SCCs, namely one consisting of the single argument b_1 and the other containing the remaining arguments, i.e. $S_1 = \{b_1\}$ and $S_2 = (A \cup B) \setminus \{b_1\}$. For S_1 we end up with the

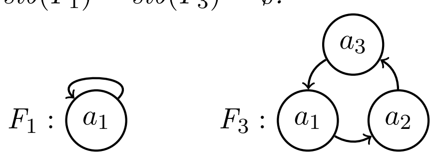
[3] We mention that the inventors of both semantics considered finite AFs only [Baroni *et al.*, 2005; Dvořák and Gaggl, 2012]. In case of finite AFs any recursion will terminate no matter which candidate set is considered.

base case returning a positive answer. For S_2 we have to consider the AF $F' = F|_{UP_F(S_2,E)} = F|_{(A\cup B)\setminus\{a_1,b_1\}}$ (since a_1 is attacked by $b_1 \in E \setminus S_2$) and the set $S' = E \cap S_2 = \{b_i \mid i \in \mathbb{N}, i \geq 2\}$. Obviously, determining whether S' is an σ-extension w.r.t. F' is equivalent to decide whether S is an σ-extension w.r.t. F. This means, the consideration of the candidate set E leads to infinite recursion.

2.2 Finite AFs

As a matter of fact, in order to show that a certain semantics σ is not universally defined w.r.t. a certain set $\mathcal{C}$ it suffices to present an AF $F \in \mathcal{C}$, s.t. $\sigma(F) = \emptyset$. Contrastingly, an affirmative answer w.r.t. universal definedness requires a proof involving all AFs in $\mathcal{C}$. Let us consider finite AFs first. It is well-known that stable semantics does not warrant the existence of extensions/labellings even in the case of finite AFs. Witnessing examples are given by odd-cycles (cf. Example 2.8). Interestingly, in case of finite AFs we have that being odd-cycle free is sufficient for warranting at least one stable extension/labelling.[4]

Example 2.8. *The following minimalistic AFs cause a collapse of stable semantics, i.e. $stb(F_1) = stb(F_3) = \emptyset$.*



Observe that both frameworks do possess semi-stable, stage2 as well as stage extensions/labellings. The extensions are as follows: For any $\sigma \in \{ss, stg2, stg\}$, $\tau \in \{stg2, stg\}$, $\mathcal{E}_\sigma(F_1) = \{\emptyset\} = \mathcal{E}_{ss}(F_3)$ and $\mathcal{E}_\tau(F_3) = \{\{a_1\},\{a_2\},\{a_3\}\}$.

Let us consider now semi-stable semantics. Example 2.8 shows that AFs may possess semi-stable extensions even in the absence of stable extensions. Are semi-stable extensions possibly guaranteed in case of finite AFs? Consider the following explanations about the existence of semi-stable extensions taken from [Caminada, 2006]:

> For every argumentation framework there exists at least one semi-stable extension. This is because there exists at least one complete extension, and a semi-stable extension is simply a complete extension in which some property (the union of itself and the arguments it defeats) is maximal.

[4] This is due to the fact that firstly, in case of finite AFs, being odd-cycle free coincides with being *limited controversial* [Dung, 1995, Definition 32] and secondly, any limited controversial AFs warrants the existence of at least one stable extensions [Dung, 1995, Corollary 36].

We would like to point out two issues. Firstly, the presented explanation should not be understood as: Since any semi-stable extension is a complete one and complete semantics is universally defined we conclude that semi-stable semantics is universally defined. Accepting this kind of (false) argumentation would imply the universal definedness of stable semantics since also any stable extension is a complete one. The second issue is that the presented explanation is not precise about *why* it is guaranteed that the non-empty set of complete extensions possesses at least one range-maximal member. The following statement gives a more precise explanation [Caminada *et al.*, 2012]:

> For every (finite) argumentation framework, there exists at least one semi-stable extension. This is because there exists at least one complete extension (the grounded) and the fact that the argumentation framework is finite implies that there exist at most a finite number of complete extensions. The semi-stable extensions are then simply those complete extensions in which some property (its range) is maximal.

This means, the additional argument that we have to compare finitely many complete extensions only justifies the universal definedness of semi-stable extensions in case of finite AFs. Obviously, in case of infinite AFs we cannot expect to have finitely many complete extensions implying that this kind of argumentation is no longer valid for finitary as well as infinite AFs in general.

In the rest of this subsection we want to argue why all considered semantics except the stable one are universally defined in case of finite AFs.[5] Remember that many semantics are looking for certain $\subseteq$-maximal elements. The main advantage in case of finiteness is that it is simply impossible to have infinite $\subseteq$-chains which guarantees the existence of $\subseteq$-maximal elements. Consider the following more detailed explanations. Given a finite AF $F = (A, R)$, i.e. $|A| = n \in \mathbb{N}$. Consequently, $1 \leq \left|2^A\right| = 2^n \in \mathbb{N}$. By definition of any extension-based semantics σ we derive $0 \leq |\mathcal{E}_\sigma(F)| \leq 2^n$ since $\mathcal{E}_\sigma(F) \subseteq 2^A$ (cf. Definition 2.2). This means, for any finite F and any semantics σ we have at least one candidate set for being a σ-extension (namely, the empty set) and at most finitely many σ-extensions. In any case, the empty set is conflict-free as well as admissible, i.e. $|\mathcal{E}_{cf}(F)|, |\mathcal{E}_{ad}(F)| \geq 1$. Furthermore, naive and preferred semantics are looking for $\subseteq$-maximal conflict-free or admissible sets, respectively. Since we have finitely many conflict-free as well as admissible sets only we derive the universal definedness of naive and preferred semantics in case of finite AFs. Combining $\mathcal{E}_{pr} \subseteq \mathcal{E}_{co}$ and $|\mathcal{E}_{pr}(F)| \geq 1$ yields the

[5] We mention that grounded, ideal and eager semantics are even uniquely defined w.r.t. finite AFs. This will be a by-product of Theorem 2.23, Corollary 2.22 as well as Theorem 2.25.

universal definedness of complete semantics in case of finite AFs. Moreover, since $1 \leq |\mathcal{E}_{cf}(F)|, |\mathcal{E}_{ad}(F)| \leq 2^n$ is given we obtain the universal definedness of stage and semi-stable semantics in case of finite AFs because the existence of $\subseteq$-range-maximal is guaranteed. Let us consider ideal and eager semantics. Candidate sets of both semantics are admissible sets being in the intersection of all preferred or semi-stable extensions, respectively. Note that there is at least one admissible set satisfying this property, namely the empty one since definitely $\emptyset \subseteq \bigcap \mathcal{E}_{pr}(F) \subseteq \mathcal{U}$ as well as $\emptyset \subseteq \bigcap \mathcal{E}_{ss}(F) \subseteq \mathcal{U}$. This means, the sets of candidates are non-empty and finite which guarantees the existence of $\subseteq$-maximal elements implying the universal definedness of ideal and eager semantics in case of finite AFs. The grounded extension, i.e. the $\subseteq$-least fixpoint of the characteristic function Γ_F, is guaranteed due to the monotonicity of Γ_F and the famous Knaster-Tarski theorem [Tarski, 1955]. Finally, even the more exotic stage2 as well as cf2 semantics are universally defined w.r.t. finite AFs. This can be seen as follows: Obviously, finitely many as well as initial SCCs are guaranteed due to finiteness. Consequently, one may start with computing stage/naive extension on these initial components and "propagate" the resulting extensions to the subsequent SCCs and so on. This procedure will definitely terminate and ends up with stage2/cf2 extensions. Apart from stable semantics we have argued that the extension-based versions of all considered semantics are universally defined w.r.t. finite AFs. In case of mature semantics, the result carry over to their labelling-based versions since any of these semantics possesses a one-to-one-correspondence between extensions and labellings. This property does not hold in case of admissible as well as conflict-free sets. However, since any admissible/conflict-free set induce at least one admissible/conflict-free labelling the result applies to their labelling versions too.

2.3 Arbitrary AFs

2.3.1 Non-well-defined Semantics

In contrast to all other semantics available in the literature, cf2 as well as stage2 semantics were originally defined recursively. The recursive schema is based on the decomposition of AFs along their strongly connected components (SCCs). Roughly speaking, the schema takes a base semantics σ and proceeds along the induced partial ordering and evaluates the SCCs according to σ while propagating relevant results to subsequent SCCs. This procedure defines a $\sigma 2$ semantics.[6] Given so-called *SCC-recursiveness* (cf. [Baroni *et al.*, 2005]) we have to face some difficulties

[6]Following this terminology we have to rename *cf2* semantics to *na2* semantics since its base semantics is the naive semantics and not conflict-free sets.

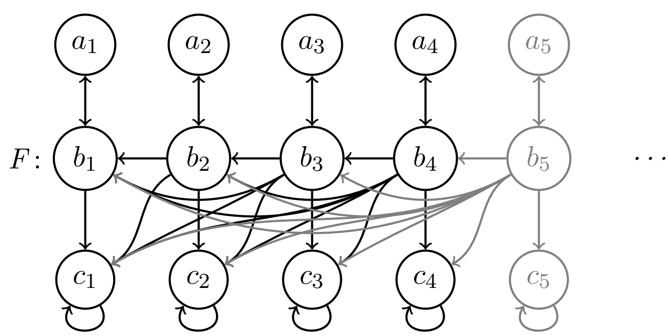
in drawing conclusions with respect to infinite AFs. Firstly, arbitrary AFs need not to possess initial SCCs which is granted for finite AFs. This makes checking whether a certain set is an $\sigma 2$-extension more complicated and in particular, especially due to the recursive definitions not that easy to handle. Secondly, even worse, even if an AF as well as subsequent subframeworks of it possess initial SCCs there is no guarantee that any recursion will stop in finitely many steps. More precisely, as shown in Example 2.7 there might be candidate sets which lead to infinite recursion, i.e. the base case will never be considered. In [Gaggl and Dvořák, 2016, Propositions 2.12 and 3.2] the authors considered alternative non-recursive definitions of cf2 as well as stage2 semantics in case of finite AFs. It is an open question whether these definitions overcome the problem of undefinedness for arbitrary frameworks.

2.3.2 Collapsing Semantics

Dealing with finite AFs is a common as well as attractive and reasonable restriction, due to their computational nature. In the subsection before we have argued that apart from stable semantics all considered semantics are universally defined w.r.t. finite AFs. It is an important observation that warranting the existence of σ-extensions/labellings in case of finite AFs does not necessarily carry over to the infinite case, i.e. the semantics σ does not need to be universally defined w.r.t. arbitrary AFs. Take for instance semi-stable and stage semantics. To the best of our knowledge the first example showing that semi-stable as well as stage semantics does not guarantee extensions/labellings in case of non-finite AFs was given in [Verheij, 2003, Example 5.8.] and is picked up in the following example.

Example 2.9 (Collapse of Stage and Semi-stable Semantics). *Consider the following AF $F = (A \cup B \cup C, R)$ where*

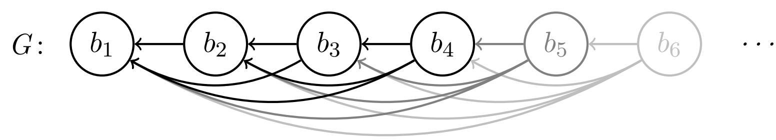
- $A = \{a_i \mid i \in \mathbb{N}\}$, $B = \{b_i \mid i \in \mathbb{N}\}$, $C = \{c_i \mid i \in \mathbb{N}\}$ *and*
- $R = \{(a_i, b_i), (b_i, a_i), (b_i, c_i), (c_i, c_i) \mid i \in \mathbb{N}\} \cup \{(b_i, b_j), (b_i, c_j) \mid i, j \in \mathbb{N}, j < i\}$



The set of preferred and naive extensions coincide, in particular $\mathcal{E}_{pr}(F) = \mathcal{E}_{na}(F) = \{A\} \cup \{E_i \mid i \in \mathbb{N}\}$ *where* $E_i = (A \setminus \{a_i\}) \cup \{b_i\}$*. Furthermore, none of these extensions is* $\subseteq$*-range-maximal since* $A^{\oplus} \subsetneq E_i^{\oplus} \subsetneq E_{i+1}^{\oplus}$ *for any* $i \in \mathbb{N}$*. In consideration of* $ss \subseteq pr$ *and* $stg \subseteq na$ *(cf. Figure 1) we conclude that this framework possesses neither semi-stable nor stage extensions/labellings.*

In Example 2.7 we have seen that cf2 as well as stage2 semantics are not well-defined in general. This means, there are infinite AFs and candidate sets leading to an infinite recursion implying that there is no definite answer on whether such a set is an extension. However, the following example shows that even if for any candidate set a definitive decision is possible there need not to be an extension in contrast to finite AFs.

Example 2.10 (Collapse of Cf2 and Stage2 Semantics)**.** *Taking into account the AF* $F = (A \cup B \cup C, R)$ *from Example 2.9. Consider the AF* $G = F|_B$*, i.e. the restriction of* F *to* B*.*



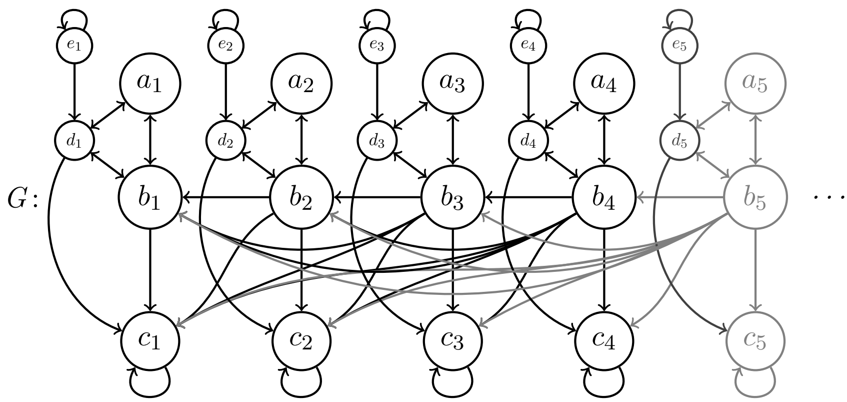
Let $\sigma \in \{cf2, stg2\}$*. Obviously, any argument* b_i *constitutes a SCC* $\{b_i\}$ *which is evaluated as* $\{b_i\}$ *by the base semantics of* σ*. Consequently,* $\emptyset$ *cannot be a* σ*-extension. Furthermore, a singleton* $\{b_j\}$ *cannot be a* σ*-extension either. The* b_i*'s for* $i > j$ *are not affected by* $\{b_j\}$ *and thus, the evaluation of* $G|_{UP_G(\{b_i\},\{b_j\})} = G|_{\{b_i\}} = (\{b_i\}, \emptyset)$ *do not return* $\emptyset$ *as required. Finally, any set containing more than two arguments would rule out at least one of them and thus, cannot be a* σ*-extension. Hence,* $|\mathcal{E}_\sigma(G)| = |\mathcal{L}_\sigma(G)| = 0$*.*

In Example 2.9 we have seen an AF F without any semi-stable and stage extensions/labellings. In [Baumann and Spanring, 2015] the authors studied the question of existence-dependency between both semantics in case of infinite AFs. More precisely, they studied whether it is possible that some AF does have semi-stable but no stage extensions or vice versa, there are stage but no semi-stable extensions. The following Example 2.11 shows that stage extensions might exist even if semi-stable semantics collapses.[7]

[7]The AF $G = F|_B$ depicted in Example 2.10 witnesses the reverse case. It can be checked that $\mathcal{E}_{ss}(G) = \{\emptyset\}$ and $\mathcal{E}_{stg}(G) = \emptyset$ (cf. [Baumann and Spanring, 2015, Example 2] for further explanations).

Example 2.11 (No Semi-Stable but Stage Extensions/Labellings).
Consider again the AF F depicted in Example 2.9. Using the components of F we define $G = (A \cup B \cup C \cup D \cup E, R \cup R')$ where

- $D = \{d_i \mid i \in \mathbb{N}\}$ *and* $E = \{e_i \mid i \in \mathbb{N}\}$ *and*
- $R' = \{(a_i, d_i), (d_i, a_i), (b_i, d_i), (d_i, b_i), (d_i, c_i), (e_i, d_i), (e_i, e_i) \mid i \in \mathbb{N}\}$



In comparison to Example 2.9 we do not observe any changes as far as preferred and semi-stable semantics are concerned. In particular, $\mathcal{E}_{pr}(G) = \{A\} \cup \{E_i \mid i \in \mathbb{N}\}$ where $E_i = (A \setminus \{a_i\}) \cup \{b_i\}$ and again, none of these extensions is $\subseteq$-range-maximal. Hence, $\mathcal{E}_{ss}(G) = \emptyset$. Observe that we do have additional conflict-free as well as naive sets, especially the set D. Since any $e \in E$ is self-defeating and unattacked and furthermore, $D^{\oplus} = A \cup B \cup C \cup D$ we conclude, $\mathcal{E}_{stg}(G) = \{D\}$. Due to the one-to-one correspondence the collapse or non-collapse transfer to their labelling-based versions.

2.3.3 Universally Defined Semantics

We now turn to semantics which are universally defined w.r.t. the whole class of AFs. The first non-trivial result in this line was already proven by Dung himself, namely the universal definedness of the extension-based version of preferred semantics [Dung, 1995, Corollary 12]. He argued that the Fundamental Lemma (cf. [Dung, 1995, Lemma 10]) immediately implies that the set of all admissible sets is a complete partial order which means that any $\subseteq$-chain possesses a least upper bound. Then (and this was not explicitly stated in [Dung, 1995]), due to the famous *Zorn's lemma* [Zorn, 1935] the existence of $\subseteq$-maximal admissible sets, i.e. preferred extensions, is guaranteed.

In order to get an idea how things work in the general case we illustrate some proofs in more detail. We will see that a proof of universal definedness w.r.t. arbitrary AFs is completely different to the argumentation in case of finite ones. In order to keep this section self-contained we start with Zorn's lemma and an equivalent version of it.

Lemma 2.12 ([Zorn, 1935])**.** *Given a partially ordered set $(P, \leq)$. If any $\leq$-chain possesses an upper bound, then $(P, \leq)$ has a maximal element.*

Lemma 2.13. *Given a partially ordered set $(P, \leq)$. If any $\leq$-chain possesses an upper bound, then for any $p \in P$ there exists a maximal element $m \in P$, s.t. $p \leq m$.*

Having Lemma 2.13 at hand we may easily argue that any conflict-free/admissible set is bounded by a naive/preferred extension.

Lemma 2.14. *Given $F = (A, R)$ and $E \subseteq A$,*

1. *if $E \in \mathcal{E}_{cf}(F)$, then there exists $E' \in \mathcal{E}_{na}(F)$ s.t. $E \subseteq E'$ and*
2. *if $E \in \mathcal{E}_{ad}(F)$, then there exists $E' \in \mathcal{E}_{pr}(F)$ s.t. $E \subseteq E'$.*

Proof. For $F = (A, R)$ we have the associated power set lattice $(2^A, \subseteq)$. Consider now the partially ordered fragments $\mathcal{C} = (\mathcal{E}_{cf}(F), \subseteq)$ as well as $\mathcal{A} = (\mathcal{E}_{ad}(F), \subseteq)$. In accordance with Lemma 2.13 the existence of naive and preferred supersets is guaranteed if any $\subseteq$-chain possesses an upper bound in $\mathcal{C}$ or $\mathcal{A}$, respectively. Given a $\subseteq$-chain $\mathcal{E} \subseteq \mathcal{E}_{cf}(F)$ or $\mathcal{E} \subseteq \mathcal{E}_{ad}(F)$, respectively. Consider now $\bar{E} = \bigcup \mathcal{E}$. Obviously, $\bar{E}$ is an upper bound of $\mathcal{E}$, i.e. for any $E \in \mathcal{E}$, $E \subseteq \bar{E}$. It remains to show that $\bar{E}$ is conflict-free or admissible, respectively. Conflict-freeness is a finite condition. This means, if there were conflicting arguments $a, b \in \bar{E}$ there would have to be some conflict-free sets $E_a, E_b \in \bar{E}$, s.t. $a \in E_a$ and $b \in E_b$. Since $\mathcal{E}$ is a $\subseteq$-chain we have $E_a \subseteq E_b$ or $E_b \subseteq E_a$ which contradicts the conflict-freeness of at least one of them. Assume now $\bar{E}$ is not admissible. Consequently, there is some $a \in \bar{E}$ that is not defended by $\bar{E}$. Furthermore, there has to be an $E_a \in \mathcal{E}$, s.t. $a \in E_a$ contradicting the admissibility of $E_a \in \mathcal{E}_{ad}(F)$. □

According to the last lemma, we may deduce the universal definedness of the extension-based versions of preferred as well as naive semantics as long as, for any AF F, the existence of at least one conflict-free or admissible set is guaranteed. This is an easy task since the empty set is conflict-free as well as admissible even in the case of arbitrary AFs. Consequently, universal definedness of both extension-based semantics is given and the same applies to their labelling-based versions due to their one-to-one correspondence.

Theorem 2.15. *Let $\sigma \in \{pr, na\}$. The semantics σ is universally defined.*

Remember that no matter which cardinality a considered AF possesses, we have that any preferred extension/labelling is a complete extension/labelling (Proposition 2.6). Thus, having the universal definedness of preferred semantics at hand we deduce that even complete semantics is universally defined w.r.t. the whole class of AFs .

Theorem 2.16. *The semantics co is universally defined.*

Let us consider now eager and ideal semantics. An eager extension is defined as the $\subseteq$-maximal admissible set that is a subset of each semi-stable extension. This is very similar to the definition of an ideal extension where the role of semi-stable extensions is taken over by preferred ones. On a more abstract level, both semantics are instantiations of the following schema.

Definition 2.17. *Let σ be a semantics (so-called base semantics). We define the σ-parametrized semantics ad^σ as follows. For any AF F,*

$$\mathcal{E}_{ad^\sigma} = \max_{\subseteq} \left\{ E \in \mathcal{E}_{ad}(F) \;\middle|\; E \subseteq \bigcap_{S \in \mathcal{E}_\sigma(F)} S \right\}.$$

These kind of semantics were firstly introduced in [Dvorák *et al.*, 2011]. The authors studied general properties of these semantics in case of finite AFs with the additional restriction that the base semantics σ has to be universally defined. The following general theorem requires neither finiteness of AFs, nor any assumption on the base semantics.

Theorem 2.18. *Any σ-parametrized semantics is universally defined.*

Proof. Given an AF $F = (A, R)$ and a σ-parametrized semantics ad^σ. Consider the set $\Sigma = \left\{ E \in \mathcal{E}_{ad}(F) \mid E \subseteq \bigcap_{S \in \mathcal{E}_\sigma(F)} S \right\}$. Note that in the collapsing case, i.e. $\mathcal{E}_\sigma(F) = \emptyset$, we have: $\bigcap_{S \in \mathcal{E}_\sigma(F)} S = \{x \in \mathcal{U} \mid \forall S \in \mathcal{E}_\sigma(F) : x \in S\} = \mathcal{U}$. However, in any case $\Sigma \neq \emptyset$ since for any F, $\emptyset \in \mathcal{E}_{ad}(F)$ and obviously, $\emptyset \subseteq \bigcap_{S \in \mathcal{E}_\sigma(F)} S \subseteq \mathcal{U}$. In order to show that $\mathcal{E}_{ad^\sigma}(F) \neq \emptyset$ it suffices to prove that $(\Sigma, \subseteq)$ possesses maximal elements. We will use Zorn's lemma. Given a $\subseteq$-chain $\mathcal{E} \in 2^\Sigma$. Consider now $\bar{E} = \bigcup \mathcal{E}$. Analogously to the proof of Lemma 2.14 we may easily show that $\bar{E}$ is conflict-free and even admissible. Moreover, since for any $E \in \mathcal{E}$, $E \subseteq \bigcap_{S \in \sigma(F)} S$ we deduce $\bar{E} \subseteq \bigcap_{S \in \sigma(F)} S$ guaranteeing $\bar{E} \in \Sigma$. Now, applying Lemma 2.12, we deduce the existence of $\subseteq$-maximal elements in Σ, i.e. $|\mathcal{E}_{ad^\sigma}(F)| \geq 1$ concluding the proof. $\square$

In particular, we obtain the result for the extension-based versions of eager and ideal semantics and thus, due to the one-to-one correspondence for both labelling-based versions too.

Corollary 2.19. *Let $\sigma \in \{eg, il\}$. The semantics σ is universally defined.*

One obvious question is whether the statement above can be strengthened in the sense that both semantics are even uniquely defined w.r.t. the whole class of AFs. The following proposition, in particular the second item, shows that the unique definedness of eager semantics w.r.t. finite frameworks does not carry over to the general unrestricted case.

Proposition 2.20. *For any F we have:*

1. $ss(F) = \emptyset \Rightarrow eg(F) = pr(F)$ *and*
2. $ss(F) = \emptyset \Rightarrow |eg(F)| \geq \aleph_0 = |\mathbb{N}|$.

Proof. We show both assertions for the extension-based versions.
1.) Given $F = (A, R)$ and let $\mathcal{E}_{ss}(F) = \emptyset$. Hence, $\bigcap_{S \in \mathcal{E}_{ss}(F)} S = \mathcal{U}$. Consequently, $\mathcal{E}_{ss}(F) = \max_{\subseteq} \{E \in \mathcal{E}_{ad}(F) \,|\, E \subseteq \mathcal{U}\}$. This means, $\mathcal{E}_{ss}(F) = \mathcal{E}_{pr}(F)$.
2.) We show the contrapositive. Assume $|\mathcal{E}_{eg}(F)| = n$ for some finite cardinal $n \in \mathbb{N}$. Due to the first statement we derive, $|\mathcal{E}_{pr}(F)| = n$. Since $ss \subseteq pr$ (cf. Proposition 2.6) we have finitely many candidates only. Furthermore, among these preferred extensions has to be at least one $\subseteq$-range-maximal set implying $\mathcal{E}_{ss}(F) \neq \emptyset$. □

In a nutshell, if we observe a collapse of semi-stable semantics, then eager and preferred semantics coincide and moreover, we necessarily have infinitely many eager extensions/labellings. An AF witnessing such a behaviour can be found in Example 2.9.

2.3.4 Uniquely Defined Semantics

Although eager and ideal semantics are instances of σ-parametrized semantics we have shown the non-unique definedness (Proposition 2.20) for eager semantics only. This is no coincidence since preferred semantics, the base semantics of ideal semantics is universally defined in contrast to semi-stable semantics, the base semantics of the eager semantics. Moreover, the following theorem shows that any σ-parametrized semantics warrants the existence of exactly one extension if σ-extensions are conflict-free as well as guaranteed ([Dvorák *et al.*, 2011, Proposition 1]).

Theorem 2.21. *Given a σ-parametrized semantics ad^σ, s.t. $\sigma \subseteq cf$ and σ is universally defined w.r.t. a class $\mathcal{C}$, then ad^σ is uniquely defined w.r.t. $\mathcal{C}$.*

Proof. Given an AF $F = (A, R)$. We already know $|\mathcal{E}_{ad^\sigma}(F)| \geq 1$ (Theorem 2.18). Hence, it suffices to show $|\mathcal{E}_{ad^\sigma}(F)| \leq 1$. Suppose, to derive a contradiction, that for some $I_1 \neq I_2$ we have $I_1, I_2 \in \mathcal{E}_{ad^\sigma}(F)$. Consequently, by Definition 2.17, $I_1, I_2 \in \mathcal{E}_{ad}(F)$ and $I_1, I_2 \subseteq \bigcap_{S \in \mathcal{E}_\sigma(F)} S$ as well as neither $I_1 \subseteq I_2$, nor $I_2 \subseteq I_1$. Obviously, $I_1 \cup I_2 \subseteq \bigcap_{S \in \mathcal{E}_\sigma(F)} S$. Since $\mathcal{E}_\sigma(F) \neq \emptyset$ and I_1 as well as I_2 has to be subsets of any σ-extension (which are conflict-free by assumption) we deduce $I_1, I_2 \in \mathcal{E}_{cf}(F)$ and thus, $I_1 \cup I_2 \in \mathcal{E}_{cf}(F)$. Furthermore, since both sets are admissible in F we derive $I_1 \cup I_2 \in \mathcal{E}_{ad}(F)$ contradicting the $\subseteq$-maximality of at least one of the sets I_1 and I_2. □

Corollary 2.22. *The semantics il is uniquely defined.*

A further prominent representative of uniquely defined semantics w.r.t. the whole class of AFs is the grounded semantics. Its unique definedness was already implicitly given in [Dung, 1995]. Unfortunately, this result was not explicitly stated in the paper. Nevertheless, in [Dung, 1995, Theorem 25] it was shown that firstly, the set of all complete extensions form a complete semi-lattice w.r.t. subset relation, i.e. the existence of a $\subseteq$-greatest lower bound for any non-empty subset S is implied. Secondly, it was proven that the grounded extension is the $\subseteq$-least complete extension. Consequently, the existence of such a $\subseteq$-least extension is justified via setting $S = \mathcal{E}_{co}(F)$ for any given F. Alternatively, one may stick to the original definition of the grounded extension, namely as $\subseteq$-least fixpoint of the characteristic function Γ_F and argue that the monotonicity of Γ_F as well as the Knaster-Tarski theorem [Tarski, 1955] imply its existence.

Theorem 2.23. *The semantics gr is uniquely defined.*

2.4 Finitary AFs

Let us consider now finitary AFs, i.e. AFs where each argument receives finitely many attacks only. It was already observed by Dung itself that finitary AFs possess useful properties. More precisely, if an AF is finitary, then the characteristic function Γ is not only monotonic, but even ω-continuous [Dung, 1995, Lemma 28] (which does not hold in case of arbitrary AFs [Baumann and Spanring, 2017, Example 1]). This implies that the least fixed point of Γ, i.e. the unique grounded extension, can be "computed" in at most ω steps by iterating Γ on the empty set (cf. [Rudin, 1976] for more details). A further advantage of finitary AFs is that for some semantics σ, the existence or even uniqueness of σ-extension is guaranteed which cannot be shown in general.

Consider again the AF F depicted in Example 2.9. In contrast to finite AFs where the existence of semi-stable as well stage extensions is guaranteed we observed a collapse of both semantics. Not that F is not finitary since, for example, the argument b_1 receives infinitely many attacks. A positive answer in case of semi-stable semantics, i.e. universal definedness w.r.t. finitary AFs was conjectured in [Caminada and Verheij, 2010, Conjecture 1] and firstly proven by Emil Weydert in [Weydert, 2011, Theorem 5.1]. Weydert proved his result in a first order logic setup using generalized argumentation frameworks. Later on, Baumann and Spanring provided an alternative proof using transfinite induction. Moreover, they showed that even stage semantics warrants the existence of at least one extension in case of finitary AFs [Baumann and Spanring, 2015, Theorem 14]. For detailed proofs we refer the reader to the mentioned scientific papers.

Theorem 2.24. *Let $\sigma \in \{ss, stg\}$. The semantics σ is universally defined w.r.t. finitary AFs.*

Applying Theorem 2.21 we derive that exactly one eager extension/labelling is guaranteed as long as the AF in question is finitary.

Theorem 2.25. *The semantics eg is uniquely defined w.r.t. finitary AFs.*

2.5 Summary of Results and Conclusion

In this section we gave an overview on the question whether certain semantics guarantee the existence or even unique determination of extensions/labellings. We have seen that these properties may vary from subclass to subclass. The following table gives a comprehensive overview over results presented in this section. The entry "$\exists$" ("$\exists!$") in row *certain* and column σ indicates that the semantics σ is universally (uniquely) defined w.r.t. the class of *certain* frameworks. No entry reflects the situation that a *certain* AF can be found which do not provide any σ-extension/labelling, i.e. σ collapses. The two question marks represent open problems. Note that we already observed that cf2 as well as stage2 semantics are not well-defined in case of finitary as well arbitrary AFs. This means, there are infinite AFs and candidate sets leading to an infinite recursion implying that there is no definite answer on whether such a set is an extension (Example 2.7). Nevertheless, even if for any candidate set a definitive decision is possible there are infinite (but non-finitary) AFs where both semantics collapse (Example 2.10). In [Baumann and Spanring, 2015, Conjecture 1] it is conjectured that this is impossible in case of finitary frameworks.

	stb	*ss*	*stg*	*cf2*	*stg2*	*pr*	*ad*	*co*	*gr*	*il*	*eg*	*na*	*cf*
finite		$\exists$	$\exists$	$\exists$	$\exists$	$\exists$	$\exists$	$\exists$	$\exists!$	$\exists!$	$\exists!$	$\exists$	$\exists$
finitary		$\exists$	$\exists$	?	?	$\exists$	$\exists$	$\exists$	$\exists!$	$\exists!$	$\exists!$	$\exists$	$\exists$
arbitrary						$\exists$	$\exists$	$\exists$	$\exists!$	$\exists!$	$\exists$	$\exists$	$\exists$

Table 1: Definedness Statuses of Semantics

For a detailed complexity analysis of the associated decision problems, i.e. *Given an AF F. Is $|\sigma(F)| \geq 1$ or even, $|\sigma(F)| = 1$?* we refer the reader to [Dvořák, 2012]. The mentioned decisions problems are considered for finite AFs only since the input-length, i.e. the length of the formal encoding of an AF has to be finite (for finite representations of infinite AFs we refer the reader to [Baroni *et al.*, 2013]). Due to the table above some complexity results are immediately clear. For instance, the existence problem is trivial for all considered semantics except the stable one. An upper bound for the complexity of the uniqueness problem can be obtained via the complexity of the corresponding verification problem, i.e. *Given an AF F and a set E. Is $E \in \mathcal{E}_\sigma(F)$?*. More precisely, an algorithm which decides the uniqueness problem is the following two-step procedure: first, guessing a certain set E non-deterministically and second, verifying whether this set is an σ-extension.

As already mentioned, most of the literature concentrate on finite AFs for several reasons, especially due to their computational nature. However, allowing an infinite number of arguments is essential in applications where upper bounds on the number of available arguments cannot be established a priori, such as for example in dialogues [Belardinelli *et al.*, 2015] or modeling approaches including time or action sequences [Baumann and Strass, 2012]. Moreover, even actual infinite AFs frequently occur in the instantiation-based context. More precisely, the semantics of so-called *rule-based argumentation formalisms* (cf. [Besnard and Hunter, 2008; Prakken, 2010]) is given via the evaluation of induced Dung-style AFs. In this context, even a finite set of rules may lead to an infinite set of arguments as observed in (cf. [Caminada and Oren, 2014; Strass, 2015]).

In 2011, Baroni et al. wrote "As a matter of fact, we are not aware of any systematic literature analysis of argumentation semantics properties in the infinite case." [Baroni *et al.*, 2011, Section 4.4]. Since then only few works have contributed to a better understanding of infinite AFs. In [Baroni *et al.*, 2013] the authors studied to which extent infinite AFs can be finitely represented via formal languages and considered several decision problems within this context. In [Baumann and Spanring, 2015] a detailed study of the central properties of existence and uniqueness as presented in this section was given. Recently, the same authors addressed several central issues like *expressibility*, *intertranslatability* or *replaceability* (cf. Sections 3 and 4) in the general unrestricted case [Baumann and Spanring, 2017].

3 Expressibility

Given a certain logical formalism $\mathcal{L}$ used as knowledge representation language or modelling tool in general. Depending on the application in mind, it might be interesting to know which kinds of model sets are actually expressible in $\mathcal{L}$? More formally, if $\sigma_{\mathcal{L}}$ denotes the semantics of $\mathcal{L}$, we are interested in determining the set $\mathcal{R}_{\mathcal{L}} = \{\sigma_{\mathcal{L}}(T) \mid T \text{ is an } \mathcal{L}\text{-theory}\}$. This task, also known as *realizability* or *defineability*, highly depends on the considered formalism $\mathcal{L}$. Clearly, potential necessary or sufficient properties for being in $\mathcal{R}_{\mathcal{L}}$, i.e. being $\sigma_{\mathcal{L}}$*-realizable*, may rule out a logic or make it perfectly appropriate for a certain application. For instance, it is well-known that in case of propositional logic any finite set of two-valued interpretations is realizable. This means, given such a finite set $\mathcal{I}$, we always find a set of formulae T, s.t. $Mod(T) = \mathcal{I}$. Differently, in case of normal logic programs under stable model semantics we have that any finite candidate set is realizable if and only if it forms a $\subseteq$-antichain, i.e. any two sets of the candidate set have to be incomparable with respect to the subset relation. Remarkably, being such an $\subseteq$-antichain is not only necessary but even sufficient for realizability w.r.t. stable model semantics [Eiter *et al.*, 2013; Strass, 2015]. One major application of realizability issues are dynamic evolvements of $\mathcal{L}$-theories like in case of belief revision (cf. [Alchourrón *et al.*, 1985; Williams and Antoniou, 1998; Qi an dYang, 2008; Delgrande and Peppas, 2015; Delgrande *et al.*, 2008; Delgrande *et al.*, 2013, Baumann and Brewka, 2015; Diller *et al.*, 2015] for several knowledge representation formalisms). Roughly speaking, belief revision deals with the problem of integrating new pieces of information to a current knowledge base which is represented by a certain $\mathcal{L}$-theory T. To this end, you are typically faced with the problem of modifying the given theory T in such a way that the revised version S satisfies $\sigma_{\mathcal{L}}(S) = M$ for some model set M. Now, before trying to do this revision in a certain minimal way it is essential to know whether M is realizable at all, i.e. $M \in \mathcal{R}_{\mathcal{L}}$.

The first formal treatment of realizability issues w.r.t. extension-based argumentation semantics was recently given by Dunne *et al.* [2013; 2015]. They coined the term *signature* for the set of all realizable sets of extensions. The authors provided simple criteria for several mature semantics deciding whether a set of extensions is contained in the corresponding signature. For instance, two obvious necessary conditions in case of preferred semantics (as well as many other semantics) is that a candidate set $\mathbb{S}$ has to be non-empty, due to universal definedness of preferred semantics and second, $\mathbb{S}$ has to be a $\subseteq$-antichain, also known as *I-maximality criterion* [Baroni and Giacomin, 2007]. However, these conditions are not sufficient implying that further requirements has to hold. In case of preferred semantics it turned out that adding the requirement of so-called *conflict-sensitivity* indeed yield a set of characterizing properties. A $\subseteq$-antichain $\mathbb{S}$ is conflict-sensitive if for each pair of distinct sets A and B from $\mathbb{S}$ there are at least one $a \in A$ and one $b \in B$, s.t. a and b do not occur together in any set of $\mathbb{S}$. This implies that there exists an AF F in which the set of its preferred extension coincides with $\mathbb{S} = \{\{a,b\},\{a,c\},\{b,d\},\{c,d\}\}$. Furthermore, since $\{a,b\}$ and $\{b,d\}$ are already contained in $\mathbb{S}$ it is impossible to realize the set $\mathbb{T} = \mathbb{S} \cup \{\{a,d\}\}$ under preferred semantics. From a practical point of view, such realizability insights can be used to limit the search space when enumerating preferred extensions. More precisely, applying the mentioned characterization result we obtain that not only $\{a,d\}$, but also any other set $A \subseteq \{a,b,c,d\}$ can not be a further preferred extension of a certain AF given that we already computed all sets contained in $\mathbb{S}$. As a matter of fact, knowing that a certain set is realizable does not provide one automatically with a witnessing AF. Fortunately, there exist *canonical frameworks* showing realizability in a constructive fashion as shown in [Dunne *et al.*, 2013; Dunne *et al.*, 2015].

Later on, restricted versions of realizability were considered, namely *compact* as well as *analytic realizability* in case of extension-based semantics [Baumann *et al.*, 2014a; Baumann *et al.*, 2014b; Linsbichler *et al.*, 2015; Baumann *et al.*, 2016a]. Both versions are motivated by typical phenomena that can be observed for several semantics. First, there potentially exist arguments in a given AF that do not appear in any extension, so-called *rejected* arguments. Second, most of the argumentation semantics possess the feature of allowing *implicit conflicts*. An implicit conflict arises when two arguments are never jointly accepted although they do not attack each other. In order to understand in which way rejected arguments and implicit conflicts contribute to the expressive power of a certain semantics the notions of compact AFs as well as analytic AFs were introduced. The former kind disallows rejected arguments whereas the latter is free of implicit conflicts. It turned out that for many universally defined semantics the full range of expressiveness indeed relies on the use rejected arguments and implicit conflicts. This means, there are plenty of AFs which do not possess an equivalent AF which is in addition compact or analytic, respectively.

Recently, a first study of extension-based realizability w.r.t. arbitrary frameworks was presented in [Baumann and Spanring, 2017]. The authors compared the expressive power of several mature semantics in the unrestricted setting. Interestingly, the results reveal an intimate connection between arbitrary and finitely compact AFs in terms of expressiveness. Nevertheless, an in-depth analysis of realizability in the unrestricted setting is still missing. For instance, necessary and sufficient properties for being realizable are not considered so far.

There are only few works which have dealt with labelling-based realizability in the context of Dung-style argumentation frameworks. Dyrkolbotn showed that, as long as additional arguments are allowed any finite set of labellings is *realizable under projection* in case of preferred or semi-stable semantics [Dyrkolbotn, 2014]. In order to realize a set of labellings $\mathbb{S}$ under projection it suffices to come up with an AF F, s.t. its set of labellings modulo additional arguments coincide with $\mathbb{S}$. The second work by Linsbichler et al. deals with the standard notion of realizability adapted to labelling-based semantics [Linsbichler *et al.*, 2016]. The authors presented an algorithm which returns either "No" in case of non-realizability or a witnessing AF F in the positive case. Remarkably, the algorithm is not restricted to the formalism of abstract argumentation frameworks only. In fact, it can also be used to decide realizability in case of the more general abstract dialectical frameworks as well as various of its sub-classes [Brewka and Woltran, 2010; Brewka *et al.*, 2013].

3.1 Realizability and Signatures

Let us start with the two central concepts of this section, namely *realizability* as well as *signature*. In a nutshell, we say that a certain set $\mathbb{S}$ is realizable under the semantics σ, if there is an AF F such that its set of σ-labellings/σ-extensions coincides with $\mathbb{S}$. Collecting all realizable sets defines the concept of a signature. In accordance with the existing literature the main part of this section is devoted to finite realizability for extension-based semantics, i.e. signatures which contain set of σ-extensions of finite AFs only. Realizability w.r.t. labelling-based semantics as well as the consideration of infinite AFs will be briefly outlined only. Consider the following general definition of realizability in the context of abstract argumentation.

Definition 3.1. *Given a semantics $\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^n}$ and a set $\mathcal{C} \subseteq \mathcal{F}$. A set $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^n$ is σ-realizable w.r.t. $\mathcal{C}$ if there is an AF $F \in \mathcal{C}$, s.t. $\sigma(F) = \mathbb{S}$.*

Definition 3.2. *Given a semantics σ and a set $\mathcal{C} \subseteq \mathcal{F}$. The σ-signature w.r.t. $\mathcal{C}$ is defined as $\Sigma_\sigma^{\mathcal{C}} = \{\sigma(F) \mid F \in \mathcal{C}\}$.*

If clear from context or unimportant we simply speak of *signatures* and write Σ without mentioning a semantics σ or set of AFs $\mathcal{C}$. Similarly, we say that a certain set is *realizable* instead of σ-realizable w.r.t. $\mathcal{C}$. Please observe that both concepts are intimately connected via the following relation: for any set $\mathbb{S}$ we have, $\mathbb{S}$ is realizable if and only if $\mathbb{S} \in \Sigma$. Consequently, if $\mathbb{S}$ is not contained in Σ, then there is no framework whose extensions/labellings are exactly $\mathbb{S}$. Hence, instead of searching for witnessing AFs (which might not exist) it is very attractive to find necessary as well as sufficient properties for the containment of a set $\mathbb{S}$ to a certain signature locally, i.e. by properties of $\mathbb{S}$ itself.

3.2 Signatures w.r.t. Finite AFs

We start with finite realizability. Instantiating Definitions 3.1 and 3.2 with $\mathcal{C} = \{F \in \mathcal{F} \mid F \text{ finite}\}$ formally capture the notions of realizability as well as signatures relativised to finite AFs. Consider the following definitions.

Definition 3.3. *Given a semantics $\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^n}$. A set $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^n$ is finitely σ-realizable if there is an AF $F \in \{F \in \mathcal{F} \mid F \text{ finite}\}$, s.t. $\sigma(F) = \mathbb{S}$.*

Definition 3.4. *Given a semantics σ. The finite σ-signature is defined as $\{\sigma(F) \mid F \in \mathcal{F}, F \text{ finite}\}$ abbreviated by Σ^f_σ.*

We proceed with further notational shorthands (adjusted to the extension-based approach) which will be used throughout the whole section.

Definition 3.5 ([Dunne *et al.*, 2015])**.** *Given $\mathbb{S} \subseteq 2^{\mathcal{U}}$, we use*

- *$Args_{\mathbb{S}}$ to denote $\bigcup_{S \in \mathbb{S}} S$ and $\|\mathbb{S}\|$ for $|Args_{\mathbb{S}}|$,*
- *$Pairs_{\mathbb{S}}$ to denote $\{(a,b) \mid \exists S \in \mathbb{S} : \{a,b\} \subseteq S\}$ and*
- *$dcl(\mathbb{S})$ to denote (the so-called downward-closure) $\{S' \subseteq S \mid S \in \mathbb{S}\}$*

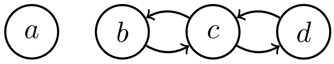
Furthermore, we say that $\mathbb{S}$ is an extension-set if $\|\mathbb{S}\|$ is a finite cardinal.

In order to familiarize the reader with the introduced definitions we give the following example.

Example 3.6. *Let $\mathbb{S} = \{\{a\},\{a,c\},\{a,b,d\}\}$. Then*

- *$Args_{\mathbb{S}} = \{a,b,c,d\}$ and $\|\mathbb{S}\| = 4$. This means, $\mathbb{S}$ is an extension-set.*
- *$Pairs_{\mathbb{S}} = \{(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(a,d),(b,d)\} \cup \{(b,a),(c,a),(d,a),(d,b)\}$*
- *$dcl(\mathbb{S}) = \{\emptyset,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{a,b,d\}\}$*

Furthermore, since naive extensions are defined as $\subseteq$-maximal sets and obviously, $\{a\} \subset \{a, c\}$ we deduce that $\mathbb{S}$ is not na-realizable, i.e. $\mathbb{S} \notin \Sigma^f_{\mathcal{E}_{na}}$. Regarding complete semantics we obtain $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{co}}$ witnessed by the following AF F.

F :


In the following we consider the signatures of the extension-based versions of stable, semi-stable, stage, naive, preferred, complete as well as grounded semantics [Dunne *et al.*, 2013; Dunne *et al.*, 2015]. We provide a bunch of properties where certain subsets of them exactly matches the containment conditions for certain signatures. All properties can be decided by looking on the set in question only.

3.2.1 Semantics based on Conflict-freeness

Our starting point are semantics based on conflict-free sets. Conflict-free sets by themselves inherited their conflict-freeness to any subset of them. More formally, the downward-closure does not vary the set of conflict-free sets for a given AF. A set possessing this property is called *downward-closed*. Clearly, downward-closedness does not hold in case of admissible sets as well as any other reasonable semantics σ where conflict-freeness is just one requirement among others for being a σ-extension. Take for instance naive semantics. Naive extension are defined as $\subseteq$-maximal conflict-free sets. Consequently, the set of all naive extensions is a $\subseteq$-antichain, i.e. any two naive extensions are *incomparable* w.r.t. subset relation. This property also applies to many other semantics, such as stable and stage semantics as well as any uniquely defined semantics. However, although incomparability is a necessary condition for many considered semantics it is certainly not sufficient. Consider therefore the following example taken from [Dunne *et al.*, 2015, Example 1].

Example 3.7. *Consider the $\subseteq$-antichain $\mathbb{S} = \{\{a, b\}, \{a, c\}, \{b, c\}\}$ and a semantics σ which selects its reasonable positions among the conflict-free sets, i.e. $\mathcal{E}_\sigma(F) \subseteq \mathcal{E}_{cf}(F)$ for any AF F. Now suppose there exists an AF F with $\mathcal{E}_\sigma(F) = \mathbb{S}$. Then F must not contain attacks between a and b, a and c, and respectively b and c. This means, $\{a, b, c\} \in \mathcal{E}_{cf}(F)$. But then $\mathcal{E}_\sigma(F)$ typically contains $\{a, b, c\}$.*

There are several ways to define the required property which excludes sets like $\mathbb{S}$ from above. It turned out that in order to characterize conflict-free based semantics like stable, stage and naive semantics a rather strong condition is required, so-called *tightness*. Roughly speaking, if an incomparable set is not tight, then there is a set $S \in \mathbb{S}$ and an argument a not belonging to S, s.t. for any $s \in S$ we find an other $S' \in \mathbb{S}$ with a and s being members of it. The idea behind the notion of being tight is simply that if an argument a does not occur in some extension S there must be a reason for that. The most simple reason one can think of is that there is a conflict between a and some $s \in S$, i.e. a and s do not occur jointly in any extension-set of $\mathbb{S}$ or, in other words, $(a, s) \notin Pairs_{\mathbb{S}}$. In a way, this limits the multitude of incomparable elements of an extension-set.

We proceed with the formal definitions.

Definition 3.8 ([Dunne *et al.*, 2013]). *Given $\mathbb{S} \subseteq 2^{\mathcal{U}}$. We call $\mathbb{S}$*

- *downward-closed if $\mathbb{S} = dcl(\mathbb{S})$,*
- *incomparable if $\mathbb{S}$ is a $\subseteq$-antichain and*
- *tight if for all $S \in \mathbb{S}$ and $a \in Args_{\mathbb{S}}$ it holds that if $S \cup \{a\} \notin \mathbb{S}$ then there exists an $s \in S$ such that $(a, s) \notin Pairs_{\mathbb{S}}$.*

Please observe that for incomparable $\mathbb{S}$, the premise of the tightness condition, i.e. $S \cup \{a\} \notin \mathbb{S}$, is always fulfilled. However, tightness and incomparability are independent of each other, i.e. neither tightness implies incomparability or comparability, nor incomparability implies tightness or non-tightness.

Example 3.9. *Consider again the extension-set $\mathbb{S} = \{\{a, b\}, \{a, c\}, \{b, c\}\}$ from Example 3.7. The set $\mathbb{S}$ is incomparable but not tight which can be seen as follows. If setting $S = \{a, b\}$ we observe $S \cup \{c\} \notin \mathbb{S}$. Moreover, for any $s \in S$ we find an $S' \in \mathbb{S}$, s.t. $\{s, c\} = S'$ implying that $(s, c) \in Pairs_{\mathbb{S}}$. More precisely, if $s = a$, then we have $S' = \{a, c\}$ and similarly, if $s = b$ we find $S' = \{b, c\}$.*

Furthermore, it can be checked that $\mathbb{S}' = \{\{a, b\}, \{a, c\}, \{b, d\}, \{c, d\}\}$ or $\mathbb{S}'' = \mathbb{S} \cup \{\{a, b, c\}\}$ are witnessing examples for incomparability and tightness or tightness and comparability, respectively.

Clearly, subsets of incomparable sets are incomparable. Such a kind of inheritance does not hold in case of tight sets (cf. $\mathbb{S}$ and $\mathbb{S}''$ as defined in Example 3.9). Nevertheless, there are non-trivial tight subsets of any tight set. For instance, in any case the set of all $\subseteq$-maximal elements is tight. Furthermore, if a tight set is even incomparable, then any subset of it is tight too.

In the following we present the main statements only. However, in many cases we provide some short comments indicating how to prove the statement in question. For full proofs we refer the reader to the referenced papers.

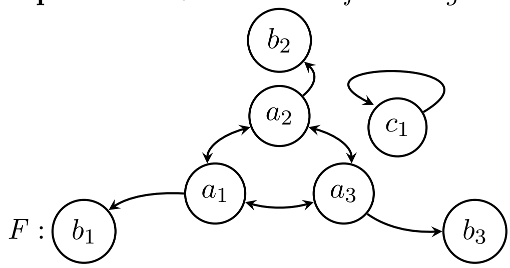
Lemma 3.10 ([Dunne *et al.*, 2015]). *For a tight extension-set $\mathbb{S} \subseteq 2^{\mathcal{U}}$ we have:*

1. *the $\subseteq$-maximal elements in $\mathbb{S}$ form a tight set, and*
2. *if $\mathbb{S}$ is incomparable then each $\mathbb{S}' \subseteq \mathbb{S}$ is tight.*

Note that the second statement of Lemma 3.10 implies that if the downward-closure of an incomparable extension-set $\mathbb{S}$ is tight, then $\mathbb{S}$ itself has to be tight too.

We proceed with a specific AF and check which properties apply to its different sets of extensions.

Example 3.11. *Consider the following AF F.*



Since c_1 is self-defeating as well as unattacked we obtain $\mathcal{E}_{stb}(F) = \emptyset$. Furthermore, $\mathcal{E}_{stg}(F) = \{\{a_1, b_2, b_3\}, \{a_2, b_1, b_3\}, \{a_3, b_1, b_2\}\}$ and $\mathcal{E}_{na}(F) = \mathcal{E}_{stg}(F) \cup \{\{b_1, b_2, b_3\}\}$. We observe,

1. *$\mathcal{E}_{stb}(F), \mathcal{E}_{stg}(F)$ as well as $\mathcal{E}_{na}(F)$ are incomparable,*
2. *$\mathcal{E}_{stb}(F), \mathcal{E}_{stg}(F)$ as well as $\mathcal{E}_{na}(F)$ are tight and additionally,*
3. *$dcl(\mathcal{E}_{na}(F))$ and $dcl(\mathcal{E}_{stb}(F))$ are tight and obviously,*
4. *$\mathcal{E}_{stg}(F)$ and $\mathcal{E}_{na}(F)$ are non-empty.*

The first and the last items are not surprising since firstly, all considered semantics satisfy the I-maximality criterion which is just another name for incomparability and secondly, in Section 2 we have already seen that stage extensions are guaranteed for finitary (hence, for finite) frameworks and naive semantics is even universally defined w.r.t. the whole class of AFs. This means, incomparability or non-emptiness of the mentioned sets of σ-extensions do not depend on the specific AF F, but rather apply to any finite AF. Consequently, these properties represent necessary properties regarding realizability. The tightness statements of the second and third items can be checked in a straightforward manner. We now examine that $dcl(\mathcal{E}_{stg}(F))$ is non-tight. This can be seen as follows: Firstly, $\{b_2, b_3\} \in dcl(\mathcal{E}_{stg}(F))$. Now, for b_1 the *premise of Definition 3.8 is satisfied, i.e.* $\{b_1, b_2, b_3\} \notin dcl(\mathcal{E}_{stg}(F))$*. Consequently, since* $\{b_1, b_2\}, \{b_1, b_3\} \in dcl(\mathcal{E}_{stg}(F))$ *and therefore,* $(b_1, b_2), (b_1, b_3) \in Pairs_{dcl(\mathcal{E}_{stg}(F))}$ *we deduce the non-tightness of* $dcl(\mathcal{E}_{stg}(F))$*. This means, tightness of the downward-closure of a given set can not be a necessary criterion for belonging to the stage signature.*

We now present the characterization theorems for conflict-free, naive, stable as well as stage signatures. It is somehow surprising that only a few simple properties are sufficient to characterize these different signatures.

Theorem 3.12 ([Dunne *et al.*, 2015])**.** *Given a set* $\mathbb{S} \subseteq 2^{\mathcal{U}}$*, then*

1. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{cf}} \Leftrightarrow \mathbb{S}$ *is a non-empty, downward-closed, and tight extension-set,*
2. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{na}} \Leftrightarrow \mathbb{S}$ *is a non-empty, incomparable extension-set and* $dcl(\mathbb{S})$ *is tight,*
3. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{stb}} \Leftrightarrow \mathbb{S}$ *is a incomparable and tight extension-set,*
4. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{stg}} \Leftrightarrow \mathbb{S}$ *is a non-empty, incomparable and tight extension-set.*

We mention that a proof of the characterization theorem above requires two directions. Let us fix a certain semantics $\sigma \in \{cf, na, stb, stg\}$. The first part is to show that for any finite AF F, $\mathcal{E}_\sigma(F)$ satisfies the mentioned properties. Now, for the second part, if a certain extension-set $\mathbb{S}$ satisfies the properties in question, then we have to find a finite AF F, s.t. $\mathcal{E}_\sigma(F) = \mathbb{S}$.

Let us start with the first part. It suffices to consider tightness only since downward-closedness, non-emptiness and incomparability are clear (cf. some explanations given in Example 3.11). It is easy to see that $\mathcal{E}_{cf}(F)$ is tight because if augmenting a conflict-free set S with a non-conflicting argument a yields a conflicting set, then obviously there has to be at least one element in $s \in S$, s.t. $\{a, s\}$ is conflicting. In order to prove that $dcl(\mathcal{E}_{na}(F))$ is tight, it suffice to see that $dcl(\mathcal{E}_{na}(F)) = \mathcal{E}_{cf}(F)$. Consequently, applying Lemma 3.10 we obtain the tightness of $\mathcal{E}_{na}(F)$. Furthermore, with the same lemma, we get that every $\mathbb{S} \subseteq \mathcal{E}_{na}(F)$ is tight. In consideration of $stb \subseteq stg \subseteq na$ (Proposition 2.6) it follows that $\mathcal{E}_{stb}(F)$ as well as $\mathcal{E}_{stg}(F)$ are tight.

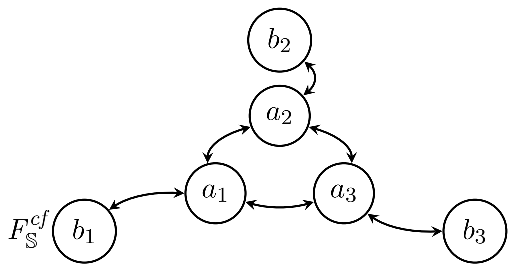
In order to show that the mentioned properties are not only necessary but even sufficient we have to come up with witnessing AFs. Consider therefore the following prototype.

Definition 3.13 ([Dunne *et al.*, 2015])**.** *Given an extension-set* $\mathbb{S}$*, we define the canonical argumentation framework for* $\mathbb{S}$ *as*

$$F^{cf}_{\mathbb{S}} = (Args_{\mathbb{S}}, (Args_{\mathbb{S}} \times Args_{\mathbb{S}}) \setminus Pairs_{\mathbb{S}}).$$

The idea behind the framework is simple: we draw a relation between two arguments iff they do not occur jointly in any set $S \in \mathbb{S}$. Consequently, for any $\mathbb{S}$, $F_{\mathbb{S}}^{cf}$ is symmetric. Moreover, in any case, it is self-loop-free since $a \in Args_{\mathbb{S}}$ implies $(a, a) \in Pairs_{\mathbb{S}}$. Let us consider the following example.

Example 3.14. *Let* $\mathbb{S} = \{\{a_1, b_2, b_3\}, \{a_2, b_1, b_3\}, \{a_3, b_1, b_2\}, \{b_1, b_2, b_3\}\}$ *and consider the corresponding canonical framework* $F_{\mathbb{S}}^{cf}$.



Please note that $\mathbb{S}$ *is non-empty, incomparable as well as possesses a tight downward-closure (cf. Example 3.11). Furthermore,* $F_{\mathbb{S}}^{cf}$ *realizes* $\mathbb{S}$ *under the naive semantics, i.e.* $\mathcal{E}_{na}\left(F_{\mathbb{S}}^{cf}\right) = \mathbb{S}$.

The following proposition shows that this is no coincidence.

Proposition 3.15 ([Dunne *et al.*, 2015])**.** *For each non-empty, incomparable extension-set* $\mathbb{S}$, *where* $dcl(\mathbb{S})$ *is tight,* $\mathcal{E}_{na}\left(F_{\mathbb{S}}^{cf}\right) = \mathbb{S}$.

Moreover, the canonical framework can also be used as witnessing framework in case of conflict-free sets as stated in the following proposition.

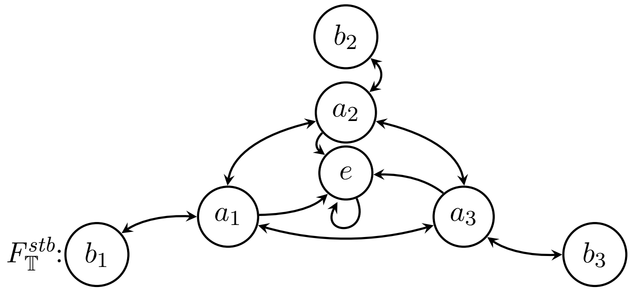
Proposition 3.16 ([Dunne *et al.*, 2015])**.** *For each non-empty, downward-closed and tight extension-set* $\mathbb{S}$, $\mathcal{E}_{cf}\left(F_{\mathbb{S}}^{cf}\right) = \mathbb{S}$.

We proceed with stable and stage semantics. In Theorem 3.12 the only difference between the characterizations of stable and stage signatures is the non-empty requirement for stage semantics. Remember that we are dealing with finite AFs and indeed in case of this restriction stable semantics is the only semantics which does not warrant the existence of extensions (cf. Table 1).[8] This means, stable semantics is the only semantics which may realize the empty extension-set (which is incomparable and tight too). The final step towards concluding Theorem 3.12 is to find witnessing frameworks for any non-empty, incomparable and tight extension-sets. At first we will show that the canonical framework does not do the job in case of

[8]For instance, $F = (\{a\}, \{(a, a)\})$ yields $\mathcal{E}_{stb}(F) = \emptyset$.

these semantics. More precisely, given a non-empty, incomparable as well as tight extension-set $\mathbb{S}$, then the sets of stable as well as stage extensions of the canonical framework $F_{\mathbb{S}}^{cf}$ do not necessarily coincide with $\mathbb{S}$.

Example 3.17. *Consider again Example 3.14. We define $\mathbb{T} = \mathbb{S} \setminus \{\{b_1, b_2, b_3\}\}$. Please note that $F_{\mathbb{T}}^{cf}$ and $F_{\mathbb{S}}^{cf}$ are identical since $Args_{\mathbb{S}} = Args_{\mathbb{T}}$ and $Pairs_{\mathbb{S}} = Pairs_{\mathbb{T}}$. Furthermore, according to Example 3.11 we have that $\mathbb{T}$ is non-empty, incomparable and tight, but $\mathcal{E}_{stb}\left(F_{\mathbb{T}}^{cf}\right) = \mathcal{E}_{stg}\left(F_{\mathbb{T}}^{cf}\right) = \mathcal{E}_{na}\left(F_{\mathbb{S}}^{cf}\right) = \mathbb{S} \neq \mathbb{T}$. In order to get rid of the undesired stable as well as stage extension $E = \{b_1, b_2, b_3\}$ we may simply add a new self-defeating argument e to $F_{\mathbb{S}}^{cf}$, s.t. e is attacked by all other arguments excepting those stemming from E. The following framework $F_{\mathbb{T}}^{stb}$ illustrates this idea. Convince yourself that $\mathcal{E}_{stb}\left(F_{\mathbb{T}}^{stb}\right) = \mathcal{E}_{stg}\left(F_{\mathbb{T}}^{stb}\right) = \mathbb{T}$.*



The following definition generalizes the construction idea from above to arbitrary many undesired sets. The subsequent proposition states that we have indeed found witnessing examples for non-empty, incomparable and tight extension-sets as required for Theorem 3.12.

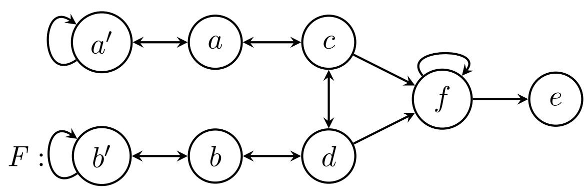
Definition 3.18 ([Dunne *et al.*, 2015])**.** *Given an extension-set $\mathbb{S}$ and its canonical framework $F_{\mathbb{S}}^{cf} = (A_{\mathbb{S}}^{cf}, R_{\mathbb{S}}^{cf})$. Let $\mathbb{X} = \mathcal{E}_{stb}\left(F_{\mathbb{S}}^{cf}\right) \setminus \mathbb{S}$ we define*
$F_{\mathbb{S}}^{stb} = \left(A_{\mathbb{S}}^{cf} \cup \{\bar{E} \mid E \in \mathbb{X}\}, R_{\mathbb{S}}^{cf} \cup \{(\bar{E}, \bar{E}), (a, \bar{E}) \mid E \in \mathbb{X}, a \in Args_{\mathbb{S}} \setminus E\}\right)$.

Proposition 3.19 ([Dunne *et al.*, 2015])**.** *For each non-empty, incomparable and tight extension-set $\mathbb{S}$, $\mathcal{E}_{stb}\left(F_{\mathbb{S}}^{stb}\right) = \mathcal{E}_{stg}\left(F_{\mathbb{S}}^{stb}\right) = \mathbb{S}$.*

3.2.2 Semantics based on Admissibility

Let us turn now to semantics based on admissible sets. In particular, we provide characterization theorems for the finite signatures w.r.t. admissible sets as well as preferred and semi-stable semantics. In contrast to semantics based on conflict-free sets where the notion of tightness played a decisive role (cf. Theorem 3.12) we have to introduce a new concept, so-called *conflict-sensitivity.* Conflict-sensitivity is a very basic property in the sense that it is fulfilled by almost all semantics σ (or rather, their corresponding sets of σ-extensions) available in the literature. Furthermore, it is strictly weaker than tightness, i.e. tight extension-sets are always conflict-sensitive, but not necessarily vice versa. To explain the difference between these two notions let us consider the following example taken from [Dunne *et al.*, 2015].

Example 3.20. *Consider the following framework F.*



We have $\mathcal{E}_{pr}(F) = \mathcal{E}_{ss}(F) = \mathbb{S} = \{A, B, C\} = \{\{a, b\}, \{a, d, e\}, \{b, c, e\}\}$. First, observe that $\mathbb{S}$ is not tight. This can be seen as follows: Obviously, $A \cup \{e\} \notin \mathbb{S}$, but both (a, e) and (b, e) are contained in $Pairs_{\mathbb{S}}$ since $\{a, e\} \subseteq B$ and $\{b, e\} \subseteq C$. This means, although $A \cup \{e\}$ is not a reasonable position w.r.t. preferred and semi-stable semantics we find witnessing extensions, namely B and C, showing that any argument in A is compatible with e, i.e. they can be accepted together. Please observe that this is not true for any two arguments in A and B or A and C, respectively. For instance, $b, d \in A \cup B$, but $(b, d) \notin Pairs_{\mathbb{S}}$ as well as $a, c \in A \cup C$, but $(a, c) \notin Pairs_{\mathbb{S}}$. Furthermore, the same applies to B and C, since $c, d \in B \cup C$ and $(c, d) \notin Pairs_{\mathbb{S}}$.

The following definition precisely formalizes the observed property of the AF F presented in the example above.

Definition 3.21 ([Dunne *et al.*, 2015])**.** *A set $\mathbb{S} \subseteq 2^{\mathcal{U}}$ is called conflict-sensitive if for each $A, B \in \mathbb{S}$ such that $A \cup B \notin \mathbb{S}$ it holds that $\exists a, b \in A \cup B : (a, b) \notin Pairs_{\mathbb{S}}$.*

As the name suggests, the property checks whether the absence of the union of any pair of extensions in an extension-set $\mathbb{S}$ is justified by a conflict indicated by $\mathbb{S}$. Note that for $a, b \in A$ (likewise $a, b \in B$), $(a, b) \in Pairs_{\mathbb{S}}$ holds by definition. Thus the property of conflict-sensitivity is determined by arguments $a \in A \setminus B$, $b \in B \setminus A$, for $A, B \in \mathbb{S}$. As already indicated tightness implies conflict-sensitivity as stated in the following lemma.

Lemma 3.22 ([Dunne *et al.*, 2015])**.** *Every tight extension-set is also conflict-sensitive.*

Similarly to Lemma 3.10 one may show that the set of all $\subseteq$-maximal elements of a conflict-sensitive set is conflict-sensitive too. Moreover, if the initial set is incomparable in addition, then even any subset of it is conflict-sensitive. Furthermore, in contrast to tight extension-sets it is possible to add the empty set to a conflict-sensitive set without loosing conflict-sensitivity.[9]

Lemma 3.23 ([Dunne *et al.*, 2015]). *For a conflict-sensitive ext.-set* $\mathbb{S} \subseteq 2^{\mathcal{U}}$*,*

1. *the* $\subseteq$*-maximal elements in* $\mathbb{S}$ *form a conflict-sensitive set,*
2. *if* $\mathbb{S}$ *is incomparable then each* $\mathbb{S}' \subseteq \mathbb{S}$ *is conflict-sensitive, and*
3. $\mathbb{S} \cup \{\emptyset\}$ *is conflict-sensitive.*

Having conflict-sensitivity at hand, we are now ready to present characterization theorems for the signatures w.r.t. admissible sets as well as preferred and semi-stable semantics. Interestingly, it turns out that preferred and semi-stable semantics are equally expressive in case of finite AFs, i.e. $\Sigma^f_{\mathcal{E}_{pr}} = \Sigma^f_{\mathcal{E}_{ss}}$.

Theorem 3.24 ([Dunne *et al.*, 2015]). *Given a set* $\mathbb{S} \subseteq 2^{\mathcal{U}}$*, then*

1. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{ad}} \Leftrightarrow \mathbb{S}$ *is a conflict-sensitive ext.-set containing* $\emptyset$*,*
2. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{pr}} \Leftrightarrow \mathbb{S}$ *is a non-empty, incomparable and conflict-sensitive ext.-set,*
3. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{ss}} \Leftrightarrow \mathbb{S}$ *is a non-empty, incomparable and conflict-sensitive ext.-set.*

Let us first argue that the mentioned properties are necessary conditions for being in the corresponding signature. For admissible sets it suffices to recall the following two facts: First, the empty set is admissible by definition; and second, if the union of two admissible sets is conflict-free, then the union is admissible too. In other words, if the union fails to be admissible, then there has to be a conflict proving the conflict-sensitivity of any set of admissible sets. Now, for preferred and semi-stable semantics. Non-emptiness is due to the already shown universal definedness of both semantics in case of finite AFs (cf. Table 1). Moreover, incomparability is clear since both semantics satisfy the I-maximality criterion [Baroni and Giacomin, 2007]. Finally, conflict-sensitivity of sets of admissible sets transfer to sets of preferred extensions via statement 1 of Lemma 3.23 and therefore also to sets of semi-stable extensions via statement 2 of Lemma 3.23 and the fact that $ss \subseteq pr$ (Proposition 2.6).

In order to show that the mentioned properties are not only necessary but even sufficient we have to come up with witnessing AFs. In contrast to conflict-free based semantics we have to find AFs which encode the central notion of admissibility.

[9]Note that any one-element extension-set $\mathbb{S} \neq \{\emptyset\}$ is tight, whereas $\mathbb{S} \cup \{\emptyset\}$ is not.

Please note that the already introduced canonical frameworks $F^{cf}_{\mathbb{S}}$ as well as $F^{stb}_{\mathbb{S}}$ (cf. Definitions 3.13 and 3.18) do not comply with the requirements. Consider therefore the following example.

Example 3.25. *Let us consider again the non-empty, incomparable as well as tight set $\mathbb{T} = \{\{a_1, b_2, b_3\}, \{a_2, b_1, b_3\}, \{a_3, b_1, b_2\}\}$ together with its corresponding canonical framework $F^{stb}_{\mathbb{T}}$ as presented in Example 3.17. Due to Lemma 3.22 we have that any tight extension-set is even conflict-sensitive and thus, $\mathbb{T}$ satisfies the necessary requirements of Theorem 3.24. Inspecting the canonical framework reveals that $\mathcal{E}_{pr}\left(F^{stb}_{\mathbb{T}}\right) = \mathbb{T} \cup \{\{b_1, b_2, b_3\}\} \neq \mathbb{T}$. Although, $\mathcal{E}_{ss}\left(F^{stb}_{\mathbb{T}}\right) = \mathbb{T}$ one may easily check that non-empty, incomparable as well as conflict-sensitive set $\mathbb{S} = \{\{a, b\}, \{a, d, e\}, \{b, c, e\}\}$ mentioned in Example 3.20 shows that this equality does not hold in general. Likewise, one may prove that the framework $F^{cf}_{\mathbb{S}}$ is not appropriated as a witnessing prototype for semi-stable as well as preferred semantics.*

It turned out that suitable canonical AFs can be built by means of so-called *defense-formulae* as introduced in the following definition.

Definition 3.26 ([Dunne *et al.*, 2015])**.** *Given an extension-set $\mathbb{S}$, the defense-formula $\mathcal{D}^{\mathbb{S}}_a$ of an argument $a \in Args_{\mathbb{S}}$ in $\mathbb{S}$ is defined as:*

$$\mathcal{D}^{\mathbb{S}}_a = \bigvee_{\substack{S \in \mathbb{S},\\ a \in S}} \ \bigwedge_{s \in S \setminus \{a\}} s.$$

$\mathcal{D}^{\mathbb{S}}_a$ given as (a logically equivalent) CNF is called CNF-defense-formula $\mathcal{CD}^{\mathbb{S}}_a$ of a in $\mathbb{S}$.

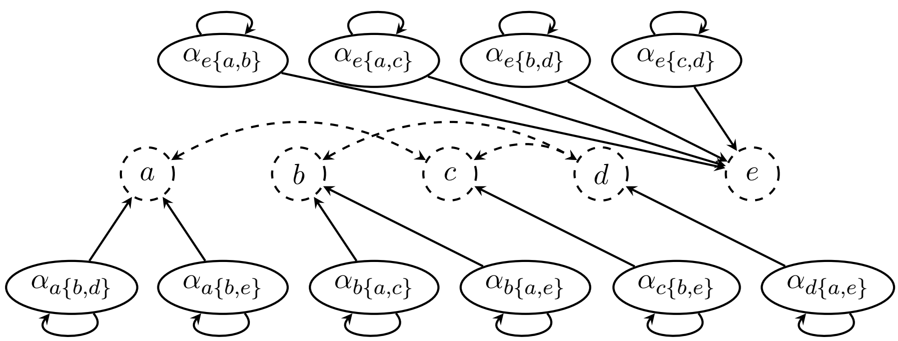
The main idea of the formula $\mathcal{D}^{\mathbb{S}}_a$ is to describe the conditions for the argument a being in an extension. Note that the variables coincide with the arguments. If $\mathbb{S}$ amounts to a set of admissible extensions, then each disjunct represents a set of arguments A which allows a to join in the sense that $A \cup \{a\}$ is a reasonable position w.r.t. admissible semantics. Put it differently, propositional models of $\mathcal{D}^{\mathbb{S}}_a \wedge a$ represent (if considered as set of atoms) supersets of certain reasonable position. Please not that a defense-formula $\mathcal{D}^{\mathbb{S}}_a$ is tautological if and only if $\{a\} \in \mathbb{S}$. We proceed with an example.

Example 3.27. *Consider again the non-empty, incomparable as well as conflict-sensitive set $\mathbb{S} = \{\{a, b\}, \{a, d, e\}, \{b, c, e\}\}$ stemming from Example 3.20. We obtain the following defense-formulae together with their corresponding CNF-defense-formulae (written in clause form).*

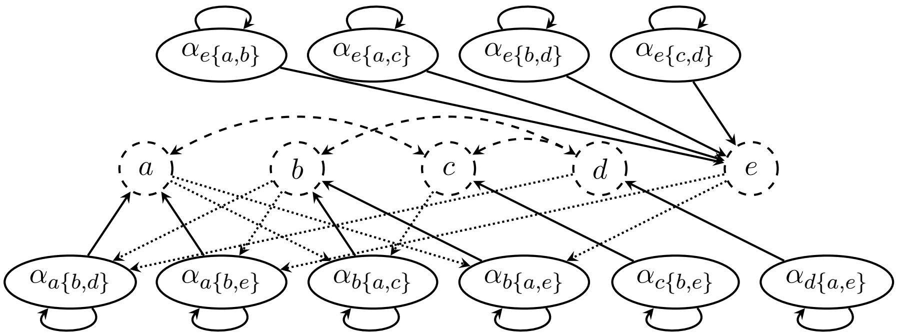
- $\mathcal{D}_a^{\mathbb{S}} = b \vee (d \wedge e) \equiv (b \vee d) \wedge (b \vee e)$ *and* $\mathcal{CD}_a^{\mathbb{S}} = \{\{b,d\},\{b,e\}\}$
- $\mathcal{D}_b^{\mathbb{S}} = a \vee (c \wedge e) \equiv (a \vee c) \wedge (a \vee e)$ *and* $\mathcal{CD}_b^{\mathbb{S}} = \{\{a,c\},\{a,e\}\}$
- $\mathcal{D}_c^{\mathbb{S}} = b \wedge e$ *and* $\mathcal{CD}_c^{\mathbb{S}} = \{\{b,e\}\}$
- $\mathcal{D}_d^{\mathbb{S}} = a \wedge e$ *and* $\mathcal{CD}_d^{\mathbb{S}} = \{\{a,e\}\}$
- $\mathcal{D}_e^{\mathbb{S}} = (a \wedge d) \vee (b \wedge c) \equiv (a \vee b) \wedge (d \vee b) \wedge (a \vee c) \wedge (d \vee c)$ *and* $\mathcal{CD}_d^{\mathbb{S}} = \{\{a,b\},\{a,c\},\{b,d\},\{c,d\}\}$

One simple idea for the realization of a certain set $\mathbb{S}$ under admissible semantics is the following two-step procedure. In the first step, we construct a framework F which maintains all elements of $\mathbb{S}$ as conflict-free sets. This can be done via the the canonical framework $F_{\mathbb{S}}^{cf}$. In the second step, we augment the initial framework $F_{\mathbb{S}}^{cf}$, s.t. only elements in $\mathbb{S}$ become admissible. The second step can be realized via adding a certain amount of additional arguments. More precisely, for any argument $a \in Args_{\mathbb{S}}$ we add n self-conflicting arguments $\alpha_{aC_1}, ..., \alpha_{aC_n}$ if $\left|\mathcal{CD}_a^{\mathbb{S}}\right| = |\{C_1, ..., C_n\}| = n$. Then, for any $i \in \{1,..,n\}$, α_{aC_i} attacks a and is in turn attacked by any argument in C_i. Consider therefore the following example.

Example 3.28. *Again consider the extension-set* $\mathbb{S} = \{\{a,b\},\{a,d,e\},\{b,c,e\}\}$ *and its corresponding CNF-defense-formulae as presented in Example 3.27. In accordance with the above mentioned two-step procedure we obtain the dashed AF* $F_{\mathbb{S}}^{cf}$ *first. Then, in view of the CNF-defense-formulae we have to add 10 additional self-defeating arguments which attacks their corresponding argument. This intermediate step is depicted below.*



Let us consider the set $\{a,b\} \in \mathbb{S}$. *In order for* $\{a,b\}$ *to be admissible we have to add counter-attacks for the arguments* $\alpha_{a\{b,d\}}$, $\alpha_{a\{b,e\}}$, $\alpha_{b\{a,c\}}$ *and* $\alpha_{b\{a,e\}}$. *For instance,* $\alpha_{a\{b,d\}}$ *is attacked by* b *and* d *and so forth. The following figure (built on top of the previous one) depicts resulting counter-attacks for the mentioned 4* *arguments highlighted as densely dotted edges. For the sake of clarity we do not perform this construction for the remaining arguments.*



The following definition precisely formalizes the mentioned two-step procedure.

Definition 3.29 ([Dunne *et al.*, 2015])**.** *Given an extension-set $\mathbb{S}$, the canonical defense-argumentation-framework $F^{def}_{\mathbb{S}} = (A^{def}_{\mathbb{S}}, R^{def}_{\mathbb{S}})$ extends the canonical AF $F^{cf}_{\mathbb{S}} = (Args_{\mathbb{S}}, R^{cf}_{\mathbb{S}})$ as follows:*

$$A^{def}_{\mathbb{S}} = Args_{\mathbb{S}} \cup \bigcup_{a \in Args_{\mathbb{S}}} \left\{ \alpha_{a\gamma} \mid \gamma \in \mathcal{CD}^{\mathbb{S}}_{a} \right\}, \text{ and}$$

$$R^{def}_{\mathbb{S}} = R^{cf}_{\mathbb{S}} \cup \bigcup_{a \in Args_{\mathbb{S}}} \left\{ (b, \alpha_{a\gamma}), (\alpha_{a\gamma}, \alpha_{a\gamma}), (\alpha_{a\gamma}, a) \mid \gamma \in \mathcal{CD}^{\mathbb{S}}_{a}, b \in \gamma \right\}.$$

The subsequent proposition shows that not only all elements in $\mathbb{S}$ become admissible in the constructed AF $F^{def}_{\mathbb{S}}$, but rather that the set of admissible sets of $F^{def}_{\mathbb{S}}$ exactly coincides with $\mathbb{S}$ given that $\mathbb{S}$ is conflict-sensitive as well as contains the empty set.

Proposition 3.30 ([Dunne *et al.*, 2015])**.** *For each conflict-sensitive ext.-set $\mathbb{S}$ where $\emptyset \in \mathbb{S}$, it holds that $\mathcal{E}_{ad}\left(F^{def}_{\mathbb{S}}\right) = \mathbb{S}$.*

Interestingly, we may even use the canonical defense-AF to show that any non-empty, incomparable and conflict-sensitive extension-set $\mathbb{S}$ can be realized under the preferred semantics. This can be seen as follows: First, via Lemma 3.23 we obtain the conflict-sensitivity of $\mathbb{S}\cup\{\emptyset\}$ since $\mathbb{S}$ is assumed to be conflict-sensitive. Consequently, using Proposition 3.31 we obtain $\mathcal{E}_{ad}\left(F^{def}_{\mathbb{S}\cup\{\emptyset\}}\right) = \mathbb{S} \cup \{\emptyset\}$. Since $F^{def}_{\mathbb{S}} = F^{def}_{\mathbb{S}\cup\{\emptyset\}}$ and due to the incomparability of $\mathbb{S}$, we have $\mathcal{E}_{pr}\left(F^{def}_{\mathbb{S}}\right) = \mathbb{S}$ as stated in the following proposition.

Proposition 3.31 ([Dunne *et al.*, 2015]). *For each non-empty, incomparable and conflict-sensitive extension-set* $\mathbb{S}$*, it holds that* $\mathcal{E}_{pr}\left(F^{def}_{\mathbb{S}}\right) = \mathbb{S}$.

Furthermore, due to a translation result by Dvořák and Woltran we obtain that any non-empty, incomparable and conflict-sensitive extension-set $\mathbb{S}$ can be realized under semi-stable semantics too. More precisely, in [Dvořák and Woltran, 2011] it is shown that for any AF F exists an AF F', s.t. $\mathcal{E}_{pr}(F) = \mathcal{E}_{ss}(F')$.

Proposition 3.32 ([Dunne *et al.*, 2015]). *Each non-empty, incomparable and conflict-sensitive extension-set* $\mathbb{S}$ *is ss-realizable.*

3.2.3 Uniquely Defined Semantics

Let us finally turn to grounded, ideal and eager semantics. Remember that all mentioned semantics warrants the existence of exactly one extension given that the frameworks in question are finite (cf. Table 1). Furthermore, it is hardly surprising that this property is even sufficient for being in the corresponding signature, since any one-element extension-set $\mathbb{S} = \{E\}$ can be realized via $F_E = (E, \emptyset)$. In particular, we obtain that all three semantics are equally expressive.

Theorem 3.33 ([Dunne *et al.*, 2016]). *Given a set* $\mathbb{S} \subseteq 2^{\mathcal{U}}$*, then*

1. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{gr}} \Leftrightarrow \mathbb{S}$ *is an extension-set with* $|\mathbb{S}| = 1$,
2. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{il}} \Leftrightarrow \mathbb{S}$ *is an extension-set with* $|\mathbb{S}| = 1$ *and*
3. $\mathbb{S} \in \Sigma^f_{\mathcal{E}_{eg}} \Leftrightarrow \mathbb{S}$ *is an extension-set with* $|\mathbb{S}| = 1$.

3.2.4 Summary of Results and Further Remarks

In this subsection we provide a comprehensive overview of characterization results w.r.t. extension-based realizability in case of finite AFs. The following table collect and combine the results of the previous three subsections. The table has to be interpreted as follows: Consider a certain column σ. Then, the entries "$\times$" in rows r_1,...,r_n indicate that for any extension-set $\mathbb{S}$, $\mathbb{S} \in \Sigma^f_{\mathcal{E}_\sigma} \Leftrightarrow r_1, ..., r_n$. Moreover, an entry "$\rightarrow$" in row r reflects the fact that the collection of the properties $r_1, ..., r_n$ imply property r.

	cf	na	stb	stg	ad	pr	ss	gr	il	eg
$\mathbb{S} \neq \emptyset$	$\times$	$\times$		$\times$	$\rightarrow$	$\times$	$\times$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$\emptyset \in \mathbb{S}$	$\rightarrow$				$\times$					
$\lvert\mathbb{S}\rvert = 1$								$\times$	$\times$	$\times$
$dcl(\mathbb{S})$ is tight		$\times$						$\rightarrow$	$\rightarrow$	$\rightarrow$
$\mathbb{S}$ is incomparable		$\times$	$\times$	$\times$		$\times$	$\times$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$\mathbb{S}$ is tight	$\times$	$\rightarrow$	$\times$	$\times$				$\rightarrow$	$\rightarrow$	$\rightarrow$
$\mathbb{S}$ is conflict-sensitive	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\times$	$\times$	$\times$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$dcl(\mathbb{S}) = \mathbb{S}$	$\times$									

Table 2: Characterizing Properties for Realizable Extension-sets

Remember that the decision whether a certain extension-set $\mathbb{S}$ is realizable can not be done via brute force (i.e., enumerating AFs and checking whether their extensions coincide with $\mathbb{S}$) since there are no a priori bounds on the number of required arguments. Consequently, the results depicted in Table 2 put us in a very good position since now, the question of realizability can be decided locally, i.e. by inspecting the set in question itself. Moreover, all mentioned properties can checked in polynomial time as shown in [Dunne *et al.*, 2015, Theorem 6]. For the majority of the properties tractability is immediately apparent. The only exception is tightness of the downward-closure of a given extension-set $\mathbb{S}$ since its size is not polynomially bounded in the size of $\mathbb{S}$ (cf. [Dunne *et al.*, 2015, Proposition 12] for a way out of this problem).

By inspecting the respective properties as depicted in Table 2, we can immediately put the signatures of different semantics in relation to each other. The following theorem includes the signature w.r.t. complete semantics in addition. The reason why we did not included complete semantics in our considerations is simply that a precise characterization of the complete signature is still an open problem. Nevertheless, certain necessary properties are already found [Dunne *et al.*, 2015, Proposition 4] justifying items 3 and 4 of the following theorem.

Theorem 3.34 ([Dunne *et al.*, 2015]). *The following relations hold*

1. $\Sigma^f_{\mathcal{E}_{na}} \subset \Sigma^f_{\mathcal{E}_{stg}} \subset \Sigma^f_{\mathcal{E}_{ss}} = \Sigma^f_{\mathcal{E}_{pr}}$,
2. $\Sigma^f_{\mathcal{E}_{stb}} = \Sigma^f_{\mathcal{E}_{stg}} \cup \{\emptyset\}$,
3. $\Sigma^f_{\mathcal{E}_{cf}} \subset \Sigma^f_{\mathcal{E}_{ad}} \subset \Sigma^f_{\mathcal{E}_{co}}$,
4. $\Sigma^f_{\mathcal{E}_{\sigma}} \subset \Sigma^f_{\mathcal{E}_{\tau}}$ *where* $\sigma \in \{gr, il, eg\}$, $\tau \in \{na, stb, stg, pr, ss, co\}$ *and*
5. $\left\{\mathbb{S} \cup \{\emptyset\} \mid \mathbb{S} \in \Sigma^f_{\mathcal{E}_{pr}}\right\} \subset \Sigma^f_{\mathcal{E}_{ad}}$.

The following Venn-diagram provides a compact overview of subset relations between the considered signatures. A bordered area represents a set of extension-sets. The outer ellipse $\mathcal{ES} = \{\mathbb{S} \subseteq 2^{\mathcal{U}} \mid \mathbb{S} \text{ is an ext.-set}\}$ stands for the set of all extension-sets over $\mathcal{U}$. Clearly, all other signatures are subsets of $\mathcal{ES}$ by definition. Furthermore, we use $\{\{\emptyset\}\}$ or $\{\emptyset\}$ the set consisting of the single extension-set $\{\emptyset\}$ (realizable by all considered semantics) or the set containing the empty extension-set (realizable by stable semantics only), respectively. The right side of Figure 2 shows signatures of semantics providing only incomparable extension-sets. The intersection of these signatures with $\Sigma^f_{\mathcal{E}_{co}}$ exactly coincides with $\Sigma^f_{\mathcal{E}_{gr}}$ as well as $\Sigma^f_{\mathcal{E}_{il}}$ and $\Sigma^f_{\mathcal{E}_{eg}}$ which contain all extension-sets $\mathbb{S}$ with $|\mathbb{S}| = 1$. Moreover, the only extension-set they have in common with the signatures of conflict-free and admissible sets is the extension-set containing the empty extension. This fact causes the "missing" intersection in the middle of Figure 2.

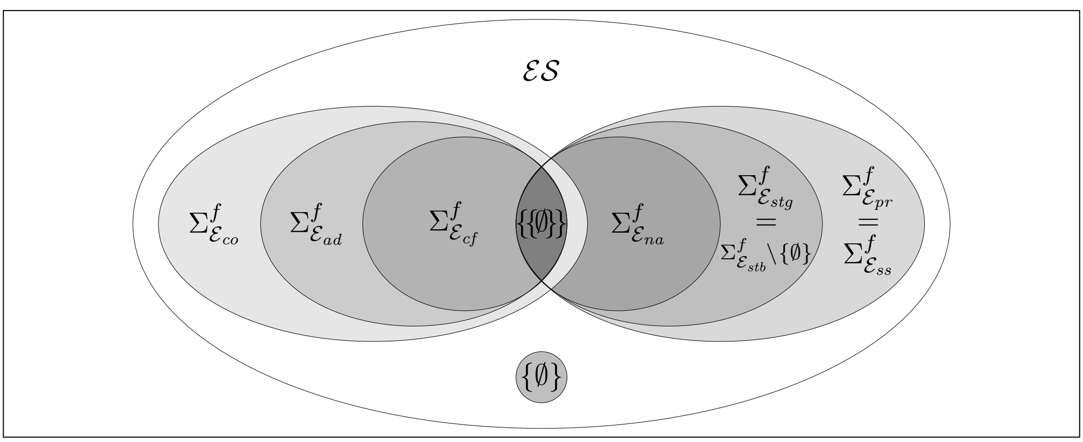


Figure 2: Subset Relations between Finite Signatures

Finally, we want to mention that all considered finite signatures, apart from the complete signature, are closed under non-empty intersections. More precisely, if two finitely σ-realizable sets $\mathbb{S}$ and $\mathbb{T}$ possess a non-empty intersection, then $\mathbb{S} \cap \mathbb{T}$ is finitely σ-realizable too. This feature is mainly due to the fact that subsets of incomparable and tight as well as incomparable and conflict-sensitive sets maintain these properties (cf. Lemmas 3.10 and 3.23).

Theorem 3.35 ([Dunne *et al.*, 2015])**.** *Let $\sigma \in \{cf, ad, na, stb, stg, pr, ss\}$. For any two finite AFs F_1, F_2 exists an finite AF F, s.t. $\mathcal{E}_\sigma(F) = \mathcal{E}_\sigma(F_1) \cap \mathcal{E}_\sigma(F_2)$ given that $\mathcal{E}_\sigma(F_1) \cap \mathcal{E}_\sigma(F_2) \neq \emptyset$.*[10]

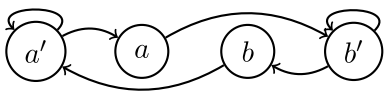
3.3 Signatures w.r.t. Finite, Compact AFs

So far we considered realizibility without any restriction (apart from finiteness) for witnessing AFs. This means, realizing AFs may contain *rejected* arguments, i.e. arguments which do not appear in any extension. Rejected arguments are natural ingredients in typical argumentation scenarios and it is a priori completely unclear in which ways rejected arguments contribute to the expressibility of a particular semantics. In order to have a handle for analyzing the effect of rejected arguments, the class of *compact* AFs and its induced signatures were introduced and studied [Baumann *et al.*, 2014a; Baumann *et al.*, 2014b; Baumann *et al.*, 2016a]. An AF is compact with respect to a semantics σ, if it does not contain rejected arguments, i.e. each of its arguments appears in at least one σ-extension. Now, the main question is whether it is possible to get rid of rejected arguments without changing the outcome? or, in other words: Under which circumstances can AFs be transformed into equivalent compact ones? Note that studying compactness is far from being an academic exercise since there is a fundamental computational significance: When searching for extensions, arguments span the search space, since extensions are to be found among the subsets of the set of all arguments. Hence the more arguments, the larger the search space. Compact AFs are argument-minimal since none of the arguments can be removed without changing the outcome, thus leading to a minimal search space.

Let us first have a brief look on the naive semantics, which is defined as $\subseteq$-maximal conflict-free sets: Here, it is rather easy to see that any AF can be transformed into an equivalent compact AF by just removing all self-defeating arguments. In other words, the same outcome (in terms of the naive extensions) can be achieved by a simplified AF without rejected arguments. This means, naive semantics does not lose expressive power if we stick to compact AFs. However, it is not hard to

[10] The prerequisite of a non-empty intersection can be dropped in case of stable semantics.

find semantics where this coincidence does not hold implying that for such semantics the full range of expressiveness indeed relies on the concepts of rejected arguments. Consider therefore the following non-compact AF F.

F : 

Let us consider admissible sets. We obtain $\mathbb{S} = \mathcal{E}_{ad}(F) = \{\emptyset, \{a, b\}\}$. Obviously, any attempt of realizing $\mathbb{S}$ with a compact AF $G = (\{a, b\}, R)$ is doomed to failure since if $\{a, b\}$ is admissible in G we necessarily obtain the admissibility of $\{a\}$ as well as $\{b\}$ proving $\mathbb{S} \neq \mathcal{E}_{ad}(G)$. It was one main result in [Baumann *et al.*, 2014a] to show that the finite, compact signatures w.r.t. stable, preferred, semi-stable, and stage semantics are strict subsets of their corresponding finite signatures. This means, in case of those semantics, sticking to finite, compact AFs implies a loss of expressive power.

3.3.1 Central Definitions and Preliminary Observations

In the following we formally introduce the central notions of *compact argumentation frameworks*, *compact realizibility* as well as *compact signatures.* As already stated, the main idea behind compact AFs is the absence of rejected arguments. For labelling-based semantics σ (i.e., a semantics returning n-tuples) we assume that the first component of their associated σ-labellings are interpreted as *acceptable sets of arguments* in analogy to σ-extensions in case of extension-based semantics. This means, if a certain argument occur in no first component of given σ-labellings we classify it as *rejected.* For a given labelling L we use L^{I} to refer to its first component.

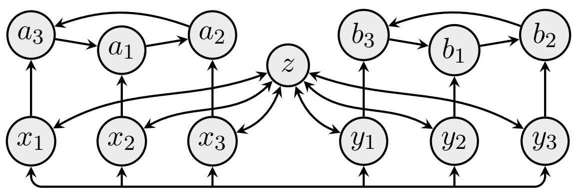
Definition 3.36. *Given a semantics $\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^n}$. An AF $F = (A, R)$ is compact for σ (or simply, σ-compact) if $Args_{\mathcal{E}_\sigma(F)} = A$ (in case of $n = 1$) or $Args_{\{L^I \mid L \in \mathcal{L}_\sigma(F)\}} = A$ (for $n \geq 2$), respectively.*

Although extension-based and labelling-based semantics are formally different semantics (according to Definition 2.2) we often speak of the extension-based version or labelling-based version of a certain semantics. This can be formally justified for the considered semantics since there is a close relationship between both versions (cf. Facts 4.38 and 4.39 for some formal relations). The following fact shows that for all considered semantics σ there is no need to distinguish between σ-compactness w.r.t. the extension-based version of σ and σ-compactness w.r.t. the labelling-based version of σ. As an aside, such a coincidence does not require a one-to-one correspondence between the extension-based and labelling-based version of a semantics σ. It suffices that any σ-extension induces a σ-labelling and vice versa in such a way that accepted arguments are preserved (cf. statements 1 and 2 of Fact 4.38).

Fact 3.37. *For any $\sigma \in \{stb, ss, stg, cf2, stg2, pr, ad, co, gr, il, eg, na, cf\}$ and any*[11] *AF F we have: F is compact for $\mathcal{E}_\sigma$ iff F is compact for $\mathcal{L}_\sigma$.*

In the following we use CAF_σ for AFs compact for σ. Moreover, we use CAF^f_σ to indicate that the considered frameworks are finite in addition. It is intuitively clear that there are AFs F being σ-compact without being τ-compact for two different semantics σ and τ. The following example firstly presented in [Baumann *et al.*, 2014a, Figure 1] provides us with a witnessing framework.

Example 3.38. *Consider the following AF F.*[12]

$F :$


The preferred extensions of F are $\mathcal{E}_{pr}(F) = \{\{z\}, \{x_1, a_1\}, \{x_2, a_2\}, \{x_3, a_3\}, \{y_1, b_1\}, \{y_2, b_2\}, \{y_3, b_3\}\}$, meaning that F is pr-compact ($F \in CAF^f_{pr}$) since each argument occurs in at least one preferred extension. On the other hand observe that $\mathcal{E}_{ss}(F) = \mathcal{E}_{pr}(F)\backslash\{\{z\}\}$ and $\mathcal{E}_{stg}(F) = \{\{x_i, a_i, b_j\}, \{y_i, b_i, a_j\} \mid 1 \leq i, j \leq 3\}$, i.e. z is not contained in any semi-stable or stage extension. Therefore F is neither compact for semi-stable nor compact for stage semantics (i.e. $F \notin CAF^f_{ss}$ and $F \notin CAF^f_{stg}$).

How are the different sets of compact AFs related? We start with an easy observation.

Lemma 3.39 ([Baumann *et al.*, 2016a])**.** *For any two semantics σ and τ such that for each AF F, for every $S \in \mathcal{E}_\sigma(F)$ there is some $S' \in \mathcal{E}_\tau(F)$ with $S \subseteq S'$, we have $CAF_\sigma \subseteq CAF_\tau$.*

Note that $\sigma \subseteq \tau$ is a special case of the premise of Lemma 3.39. Thus, $CAF_\sigma \subseteq CAF_\tau$, whenever $\sigma \subseteq \tau$ (see Figure 1 for an overview). Strict subset relations have to be proven by providing a witnessing AF as presented in Example 3.38. Moreover, $CAF_{pr} = CAF_{co} = CAF_{ad}$ as well as $CAF_{na} = CAF_{cf}$ is justified by Lemma 2.14 and the fact that $pr \subseteq co \subseteq ad$ and $na \subseteq cf$. Finally, in case of the

[11] Indeed, no finiteness restriction is required here.

[12] The construct in the lower part of the figure represents symmetric attacks between each pair of distinct arguments. We will make use of this style in illustrations throughout the whole section.

uniquely defined grounded and ideal semantics we have, $F = (A, R)$ is compact if and only if $R = \emptyset$. This in turn implies that F is compact for stable semantics. This means, $CAF_{gr} = CAF_{il} \subset CAF_{stb}$. Remember that eager semantics is uniquely defined w.r.t. finitary AFs only (Theorem 2.25, Example 2.9). Consequently, we may conclude $CAF^f_{gr} = CAF^f_{eg}$ only. Although, the majority of the results do not require the finiteness restriction we present the following theorem in terms of finite AFs. Detailed proofs for the relations between stable, semi-stable, preferred, stage and naive semantics can be found in [Baumann *et al.*, 2016a, Theorem 2].

Theorem 3.40. *The following relations hold:*

1. $CAF^f_{gr} = CAF^f_{il} = CAF^f_{eg}$,
2. $CAF^f_{pr} = CAF^f_{co} = CAF^f_{ad}$,
3. $CAF^f_{na} = CAF^f_{cf}$,
4. $CAF^f_{gr} \subset CAF^f_{stb} \subset CAF^f_{ss} \subset CAF^f_{pr} \subset CAF^f_{na}$,
5. $CAF^f_{stb} \subset CAF^f_{stg} \subset CAF^f_{na}$ *and*
6. $CAF^f_{stg} \not\subseteq CAF^f_{\sigma}$ *as well as* $CAF^f_{\sigma} \not\subseteq CAF^f_{stg}$ *for any* $\sigma \in \{pr, ss\}$.

The following figure concisely summarizes all relations mentioned in the theorem above. Directed arrows between two boxes have to be interpreted as strict subset relations between the mentioned sets of compact AFs in these boxes.

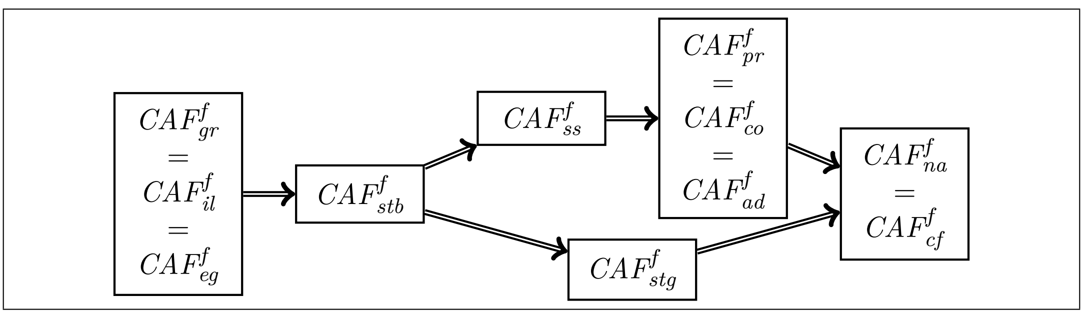


Figure 3: Subset Relations between Finite, Compact AFs

Instantiating Definitions 3.1 and 3.2 with $\mathcal{C} = CAF^f_{\sigma}$ formalize the notions of realizibility as well as signatures relativised to finite, compact AFs. Consider the following definitions.

Definition 3.41. *Given a semantics* $\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^n}$. *A set* $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^n$ *is finitely, compactly* σ*-realizable if there is an AF* $F \in CAF_\sigma^f$, *s.t.* $\sigma(F) = \mathbb{S}$.

Definition 3.42. *Given a semantics* σ. *The finite, compact* σ*-signature is defined as* $\left\{\sigma(F) \mid F \in CAF_\sigma^f\right\}$ *abbreviated by* $\Sigma_\sigma^{f,c}$.

It is clear that $\Sigma_\sigma^{c,f} \subseteq \Sigma_\sigma^f$ holds for any semantics σ, i.e. finite, compact realizibility implies finite realizibility. In the following we shed light on the question whether the mentioned subset relation is strict for a given semantics? In other words, we answer the question whether we indeed lose expressive power if sticking to compact AFs.

3.3.2 The Loss or Stability of Expressive Power

Let us consider the uniquely defined grounded, ideal and eager semantics first. We already stated that a set $\mathbb{S}$ is realizable w.r.t. these semantics if and if only if $\mathbb{S}$ is an one-element extension-set if considering finite AFs (Theorem 3.33). Furthermore, it is immediate that an extension-set $\mathbb{S} = \{E\}$ can be compactly realized via $F_E = (E, \emptyset)$. This means, these semantics do not lose expressive power if we restrict ourselves to compact AFs. Furthermore, the attentive reader may have noticed that the canonical argumentation framework $F_\mathbb{S}^{cf}$, which was used as a witnessing framework for conflict-free sets and naive semantics (cf. Definition 3.13 as well as Propositions 3.15 and 3.16), does not involve further artificial arguments. Thus, it verifies finite, compact realizibility and shows that there is no expressive loss in case of conflict-free sets and naive semantics. For the other considered semantics, namely admissible, stable, stage, semi-stable, preferred as well as complete semantics we have to accept a strict weaker expressibility if we stick to compact AFs. In order to prove that in case of these semantics the full range of expressiveness indeed relies on the concept of rejected arguments we have to come up with witnessing extension-sets. Consider therefore the following example.

Example 3.43. *The extension-set* $\mathbb{S} = \{\{a, b\}, \{a, d, e\}, \{b, c, e\}\}$ *is realizable under preferred as well as semi-stable semantics (cf. Example 3.20 for a realizing non-compact framework). Let* $\sigma \in \{pr, ss\}$. *Now suppose there exists an AF* $F = (\{a, b, c, d, e\}, R)$, *s.t.* $\mathcal{E}_\sigma(F) = \mathbb{S}$. *Since* $\{a, d, e\}, \{b, c, e\} \in \mathbb{S}$ *and* $\sigma \subseteq cf$ *we conclude that there is no attack in* R *involving* e, *i.e.* e *is an isolated argument in* F. *But then,* e *is contained in each* σ*-extension of* F *contradicting* $\{a, b\} \in \mathbb{S}$. *In Summary,* $\mathbb{S} \in \Sigma_{\mathcal{E}_\sigma}^f \setminus \Sigma_{\mathcal{E}_\sigma}^{f,c}$.

For further witnessing extension-sets we refer the reader to [Baumann *et al.*, 2016a, Propositions 35 and 57] and proceed with the main theorem.

Theorem 3.44. *It holds that*

1. $\Sigma^{f,c}_{\mathcal{E}_\sigma} = \Sigma^{f}_{\mathcal{E}_\sigma}$ *for* $\sigma \in \{cf, na, gr, il, eg\}$*, and*
2. $\Sigma^{f,c}_{\mathcal{E}_\sigma} \subset \Sigma^{f}_{\mathcal{E}_\sigma}$ *for* $\sigma \in \{ad, stb, stg, ss, pr, co\}$.

In both cases we may benefit of characterization theorems for finite signatures (cf. Theorems 3.12, 3.24 and 3.33). If both signatures are identical (first item), then necessary and sufficient properties for being finitely σ-realizable immediately carry over to finite, compact σ-realizibility. If we observe a strict subset relation (second item), then we obtain at least necessary properties for being in the finite, compact σ-signature.

Theorem 3.45. *Given a set* $\mathbb{S} \subseteq 2^{\mathcal{U}}$*, then*

1. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{cf}} \Leftrightarrow \mathbb{S}$ *is a non-empty, downward-closed and tight ext.-set,*
2. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{na}} \Leftrightarrow \mathbb{S}$ *is a non-empty, incomparable ext.-set and* $dcl(\mathbb{S})$ *is tight,*
3. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{gr}} \Leftrightarrow \mathbb{S}$ *is an ext.-set with* $|\mathbb{S}| = 1$,
4. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{il}} \Leftrightarrow \mathbb{S}$ *is an ext.-set with* $|\mathbb{S}| = 1$,
5. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{eg}} \Leftrightarrow \mathbb{S}$ *is an ext.-set with* $|\mathbb{S}| = 1$ *and*
6. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{stb}} \Rightarrow \mathbb{S}$ *is an incomparable and tight ext.-set,*
7. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{stg}} \Rightarrow \mathbb{S}$ *is a non-empty, incomparable and tight ext.-set,*
8. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{ad}} \Rightarrow \mathbb{S}$ *is a conflict-sensitive ext.-set containing* $\emptyset$,
9. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{pr}} \Rightarrow \mathbb{S}$ *is a non-empty, incomparable and conflict-sensitive ext.-set,*
10. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{E}_{ss}} \Rightarrow \mathbb{S}$ *is a non-empty, incomparable and conflict-sensitive ext.-set.*

3.3.3 Comparing Finite, Compact Signatures and Final Remarks

In the following we relate the finite, compact signatures of the semantics under consideration to each other. Recall that for finite signatures it holds that $\Sigma^{f}_{\mathcal{E}_{na}} \subset \Sigma^{f}_{\mathcal{E}_{stg}} = \left(\Sigma^{f}_{\mathcal{E}_{stb}} \setminus \{\emptyset\}\right) \subset \Sigma^{f}_{\mathcal{E}_{ss}} = \Sigma^{f}_{\mathcal{E}_{pr}}$ (cf. Figure 2). This picture changes dramatically when considering the relationships between finite, compact signatures as depicted in

Figure 4 (incomparable semantics only) and formally stated in Theorem 3.46. The dashed areas represent particular intersections for which the question of existence of extension-sets is still an open question.

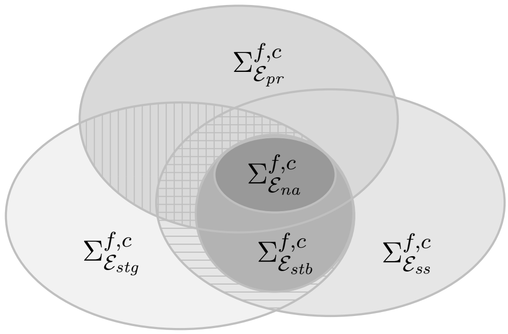


Figure 4: Subset Relations between Finite, Compact Signatures

We proceed with an enumeration of relationships between finite, compact signature including further semantics like conflict-free and admissible sets as well as grounded, ideal, eager and complete semantics. For formal proofs we refer the interested reader to [Baumann *et al.*, 2016a, Theorem 36, Proposition 58].

Theorem 3.46. *The following relations hold:*

1. $\Sigma^{f,c}_{\mathcal{E}_\sigma} \subset \Sigma^{f,c}_{\mathcal{E}_{na}} \subset \Sigma^{f,c}_{\mathcal{E}_\tau}$ *for* $\sigma \in \{gr, il, eg\}$ *and* $\tau \in \{stb, stg, ss, pr\}$,
2. $\Sigma^{f,c}_{\mathcal{E}_{stb}} \subset \Sigma^{f,c}_{\mathcal{E}_\sigma}$ *for* $\sigma \in \{stg, ss\}$,
3. $\Sigma^{f,c}_{\mathcal{E}_{cf}} \subset \Sigma^{f,c}_{\mathcal{E}_{ad}}$,
4. $\Sigma^{f,c}_{\mathcal{E}_{co}} \setminus \Sigma^{f,c}_{\mathcal{E}_\sigma} \neq \emptyset$ *and* $\Sigma^{f,c}_{\mathcal{E}_\sigma} \setminus \Sigma^{f,c}_{\mathcal{E}_{co}} \neq \emptyset$ *for* $\sigma \in \{cf, ad\}$,
5. $\Sigma^{f,c}_{\mathcal{E}_{pr}} \setminus \left(\Sigma^{f,c}_{\mathcal{E}_{stb}} \cup \Sigma^{f,c}_{\mathcal{E}_{ss}} \cup \Sigma^{f,c}_{\mathcal{E}_{stg}}\right) \neq \emptyset$,
6. $\Sigma^{f,c}_{\mathcal{E}_{stg}} \setminus \left(\Sigma^{f,c}_{\mathcal{E}_{stb}} \cup \Sigma^{f,c}_{\mathcal{E}_{pr}} \cup \Sigma^{f,c}_{\mathcal{E}_{ss}}\right) \neq \emptyset$,
7. $\Sigma^{f,c}_{\mathcal{E}_{stb}} \setminus \Sigma^{f,c}_{\mathcal{E}_{pr}} \neq \emptyset$,
8. $\left(\Sigma^{f,c}_{\mathcal{E}_{pr}} \cap \Sigma^{f,c}_{\mathcal{E}_{ss}}\right) \setminus \left(\Sigma^{f,c}_{\mathcal{E}_{stb}} \cup \Sigma^{f,c}_{\mathcal{E}_{stg}}\right) \neq \emptyset$ *and*
9. $\Sigma^{f,c}_{\mathcal{E}_{ss}} \setminus \left(\Sigma^{f,c}_{\mathcal{E}_{stb}} \cup \Sigma^{f,c}_{\mathcal{E}_{pr}} \cup \Sigma^{f,c}_{\mathcal{E}_{stg}}\right) \neq \emptyset$.

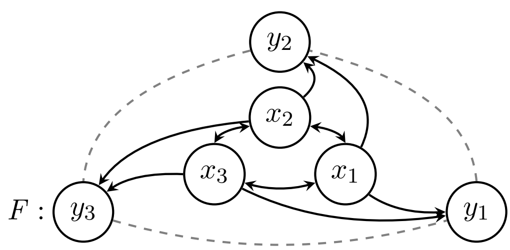
Comparing the results on expressiveness of the considered semantics as stated in Theorems 3.34 and 3.46 we observe notable differences. When allowing rejected arguments, preferred and semi-stable semantics are equally expressive and at the same time strictly more expressive than stable and stage semantics. As we have seen, this property does not carry over to the compact setting (with the exceptions $\Sigma^{f,c}_{\mathcal{E}_{stb}} \subset \Sigma^{f,c}_{\mathcal{E}_{ss}}$ and $\Sigma^{f,c}_{\mathcal{E}_{stb}} \subset \Sigma^{f,c}_{\mathcal{E}_{stg}}$) where signatures become incomparable.

Finally, regarding the open issues represented as dashed areas in Figure 4. More precisely, it is an open problem whether there are extension-sets lying in the intersection between $\Sigma^{f,c}_{\mathcal{E}_{pr}}$ (resp. $\Sigma^{f,c}_{\mathcal{E}_{ss}}$) and $\Sigma^{f,c}_{\mathcal{E}_{stg}}$ but outside of $\Sigma^{f,c}_{\mathcal{E}_{stb}}$. In [Baumann *et al.*, 2016a] it is conjectured that such extension-sets do not exist.

Conjecture 3.47 ([Baumann *et al.*, 2016a]). *It holds that* $\Sigma^{f,c}_{\mathcal{E}_{pr}} \cap \Sigma^{f,c}_{\mathcal{E}_{stg}} \subset \Sigma^{f,c}_{stb}$ *and* $\Sigma^{f,c}_{\mathcal{E}_{ss}} \cap \Sigma^{f,c}_{\mathcal{E}_{stg}} = \Sigma^{f,c}_{\mathcal{E}_{stb}}$.

3.4 Signatures w.r.t. Finite, Analytic AFs

We now turn to a further phenomenon, so-called *implicit conflicts* which can be frequently observed in typical argumentation scenarios. Consider therefore the following AF F.



Let us consider stable semantics. Please note that any x_i is jointly acceptable with one specific y_j. More precisely, $\mathcal{E}_{stb}(F) = \{\{x_1, y_3\}, \{x_2, y_1\}, \{x_3, y_2\}\}$ implying that we do not have any rejected arguments, i.e. F is stable compact. What can be said about the two pairs of arguments x_1 and x_2 as well as y_1 and y_2? First of all, both pairs represent a semantical conflict in F since neither of those pairs occur together in any stable extension. In case of x_1 and x_2, the conflict is even a syntactical one since both arguments attack each other in contrast to the pair consisting of y_1 and y_2. This difference leads to the distinction between syntactically underlined *explicit conflicts* and syntactically unfounded *implicit* ones (indicated by dashed lines). In order to understand how implicit conflicts contribute to the expressiveness of a certain semantics, the set of *analytic* AFs and its induced signatures were introduced and studied [Linsbichler *et al.*, 2015; Baumann *et al.*, 2016a]. An analytic framework, i.e. a framework which is free of implicit conflicts maximizes the information on conflicts. One main question is: under which circumstances an arbitrary framework can be transformed into an equivalent analytic one? This question is interesting from a theoretical as well as practical point of view. On the one hand, analytic frameworks are natural candidates for normal forms of AFs, and on the other maximizing the number of explicit conflicts might help argumentation systems to evaluate AFs more efficiently.

Let us consider again the extension-set $\mathbb{S} = \{\{x_1, y_3\}, \{x_2, y_1\}, \{x_3, y_2\}\}$ stemming from the AF F depicted above. Replacing the dashed arrows with symmetric attacks in F shows that $\mathbb{S}$ can be analytically realized under stable semantics. Interestingly, this is no coincidence, since it was shown that in case of stable semantics any AF can be transformed into an equivalent analytical one. However, in general it is not that easy to make implicit conflicts explicit since there are frameworks where any suitable transformation requires the use of additional arguments as shown in [Linsbichler *et al.*, 2015].

3.4.1 Central Definitions and Preliminary Observations

In this section we consider the central notions of *analytic argumentation frameworks*, *analytic realizability* as well as *analytic signatures*. In order to define analytic AF we have to differentiate between the concept of an attack (as a syntactical element) and the concept of a conflict (with respect to the evaluation under a given semantics). More precisely, if two arguments cannot be accepted together, i.e. no reasonable position contain them jointly as elements, we say that these arguments are in conflict. If this conflict is syntactically underlined by an attack between them, we call this conflict explicit, otherwise implicit. Now, an analytic framework is an AF which simply does not contain any implicit conflicts. Consider the following definition.

Definition 3.48. *Given a semantics $\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^n}$, an AF $F = (A, R)$ and two arguments $a, b \in A$. We say that*

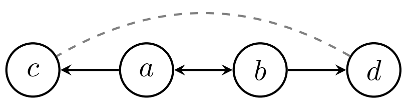
1. *a and b are in conflict for σ if $(a, b) \notin Pairs_{\mathcal{E}_\sigma(F)}$ (in case of $n = 1$) or $(a, b) \notin Pairs_{\{L^I \mid L \in \mathcal{L}_\sigma(F)\}}$ (for $n \geq 2$), respectively,*
2. *the conflict is explicit w.r.t. σ if $(a, b) \in R$ or $(b, a) \in R$, otherwise implicit,*
3. *the AF F is analytic for σ (or σ-analytic) if all conflicts are explicit.*

Please notice that Definition 3.48 does not require a and b to be different arguments. In particular, an argument that is not contained in any reasonable position is in conflict with itself. This conflict is explicit if the argument is self-attacking and implicit otherwise. Furthermore, for all considered semantics σ we observe there is no need to distinguish between σ-analyticality w.r.t. the extension-based version of σ and σ-analyticality w.r.t. the labelling-based version of σ (similarly as in case of σ-compactness as stated in Fact 3.37). Please note that this coincidence is justified for any semantics σ whenever $\mathcal{E}_\sigma(F) = \{L^{\mathrm{I}} \mid L \in \mathcal{L}_\sigma(F)\}$ is guaranteed.

Fact 3.49. *For any $\sigma \in \{stb, ss, stg, cf2, stg2, pr, ad, co, gr, il, eg, na, cf\}$ and any AF F we have: F is analytic for $\mathcal{E}_\sigma$ iff F is analytic for $\mathcal{L}_\sigma$.*

In the following we denote the set of all σ-analytic AFs as XAF_σ. To indicate that the frameworks under consideration are finite we use XAF^f_σ. We proceed with an example illustrating the new definitions.

Example 3.50. *As a simple example consider the following AF F depicted below.*

F : 

For $\sigma \in \{stb, pr, ss, stg\}$ we have $\mathcal{E}_\sigma(F) = \{\{a, d\}, \{b, c\}\}$. Observe that there is only one implicit conflict, namely the conflict between the arguments c and d, denoted by a dashed line. Hence, F is not σ-analytic, i.e. $F \notin XAF^f_\sigma$. However, since $\mathcal{E}_{na}(F) = \mathcal{E}_\sigma(F) \cup \{\{c, d\}\}$ we have that F is na-analytic, i.e. $F \in XAF^f_{na}$.

As indicated in Example 3.50 the sets of analytic AFs can differ for different semantics. Just like in case of compact AFs (cf. Lemma 3.39) one may easily verify the following lemma which allows to obtain a plenty of subset relations between sets of analytic AFs.

Lemma 3.51 ([Baumann *et al.*, 2016a])**.** *For any two semantics σ and τ such that for each AF F, for every $S \in \mathcal{E}_\sigma(F)$ there is some $S' \in \mathcal{E}_\tau(F)$ with $S \subseteq S'$, we have $XAF_\sigma \subseteq XAF_\tau$.*

In line with the existing literature we restrict our considerations to finite AFs. Regarding universal (but not uniquely) defined semantics we obtain the same relations as in case of compact AFs (see explanations below Lemma 3.39). In any case we have $XAF^f_{gr} \subseteq XAF^f_{il} \subseteq XAF^f_{eg}$ since ideal semantics accepts more arguments than grounded semantics and eager semantics is even more credulous than ideal semantics. Furthermore, $XAF^f_{eg} \subseteq XAF^f_{ss}$ because the unique eager extension is contained in all semi-stable extension by definition and moreover, semi-stable semantics guarantees reasonable positions in case of finite AFs. Now, let $F = (A, R)$ be analytic w.r.t. eager semantics and $\mathcal{E}_{eg}(F) = \{E\}$. We deduce that all arguments in $A \setminus E$ have to be self-defeating. Consequently, its corresponding (conflict-free) base semantics (cf.

Definition 2.17) warrants exactly one extension for F. More precisely, $\mathcal{E}_{ss}(F) = \{E\}$. Finally, due to the self-conflicting arguments and the admissibility of E we obtain $\mathcal{E}_{pr}(F) = \{E\}$ and thus, $\mathcal{E}_{il}(F) = \{E\}$ showing that F is even analytic w.r.t. ideal semantics, i.e. $XAF^f_{il} = XAF^f_{eg}$. The AF $F = (\{a, b\}, \{(a, b), (b, a), (b, b)\})$ proves that a similar result in case of grounded and ideal semantics does not hold. Detailed proofs for the relations between stable, semi-stable, preferred, stage and naive semantics can be found in [Baumann *et al.*, 2016a, Theorem 4].

Theorem 3.52. *The following relations hold:*

1. $XAF^f_{gr} \subset XAF^f_{il} = XAF^f_{eg} \subset XAF^f_{ss}$,
2. $XAF^f_{pr} = XAF^f_{co} = XAF^f_{ad}$,
3. $XAF^f_{na} = XAF^f_{cf}$,
4. $XAF^f_{stb} \subset XAF^f_{ss} \subset XAF^f_{pr} \subset XAF^f_{na}$,
5. $XAF^f_{stb} \subset XAF^f_{stg} \subset XAF^f_{na}$,
6. $XAF^f_{stg} \not\subseteq XAF^f_{\sigma}$ *and* $XAF^f_{\sigma} \not\subseteq XAF^f_{stg}$ *for any* $\sigma \in \{pr, ss\}$,
7. $XAF^f_{\sigma} \not\subseteq XAF^f_{\tau}$ *and* $XAF^f_{\tau} \not\subseteq XAF^f_{\sigma}$ *for any* $\sigma \in \{gr, il, eg\}$, $\tau \in \{stb, stg\}$.

The following figure summarizes all relation in a compact way. Similarly to Figure 3, a directed arrow between two boxes has to be interpreted as strict subset relation between the mentioned sets of analytic AFs therein.

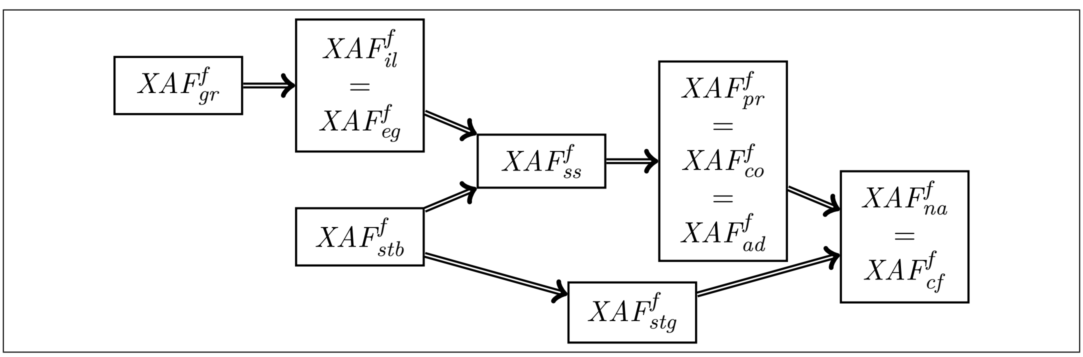


Figure 5: Subset Relations between Finite, Analytic AFs

At this point we want to mention that although Figures 3 and 5 look very similar we have that compactness and analyticality are sufficiently distinct properties.

More precisely, apart from the uniquely defined semantics as well as naive semantics and conflict-free sets no subset relations between the sets of compact and analytic frameworks can be stated in general. Sticking to self-loop-free AFs allows one to draw further relations such as analyticality implies compactness for any considered semantics. The main reason for this general relation is that rejected arguments has to be self-defeating in case of analytic frameworks. A selection of proofs of relations listed below can be found in [Baumann *et al.*, 2016a, Proposition 5-8].

Proposition 3.53. *Given an AF F, then*

1. $CAF^f_\sigma \subset XAF^f_\sigma$ *for* $\sigma \in \{gr, il, eg, na, cf\}$,
2. $CAF^f_\sigma \not\subseteq XAF^f_\sigma$ *and* $XAF^f_\sigma \not\subseteq CAF^f_\sigma$ *for* $\sigma \in \{ad, stb, ss, pr, stg, co\}$.

If F is self-loop-free in addition, then

3. $F \in XAF^f_\sigma$ *and* $F \in CAF^f_\sigma$ *for* $\sigma \in \{na, cf\}$,
4. $F \in XAF^f_\sigma \Leftrightarrow F \in CAF^f_\sigma$ *for* $\sigma \in \{gr, il, eg\}$ *and*
5. $F \in XAF^f_\sigma \Rightarrow F \in CAF^f_\sigma$ *for* $\sigma \in \{ad, stb, ss, pr, stg, co\}$.

We now precisely formalize the notions of realizibility as well as signatures relativised to finite, analytic AFs. This can be formally done via instantiating Definitions 3.1 and 3.2 with $\mathcal{C} = XAF^f_\sigma$.

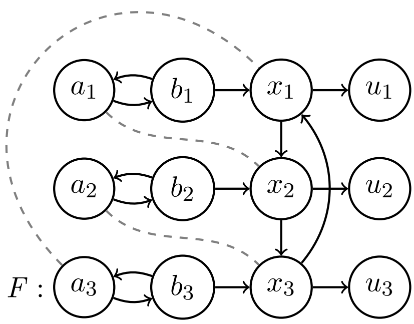
Definition 3.54. *Given a semantics* $\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^n}$. *A set* $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^n$ *is finitely, analytically σ-realizable if there is an AF* $F \in XAF^f_\sigma$, *s.t.* $\sigma(F) = \mathbb{S}$.

Definition 3.55. *Given a semantics σ. The finite, analytic σ-signature is defined as* $\left\{\sigma(F) \mid F \in XAF^f_\sigma\right\}$ *abbreviated by* $\Sigma^{f,x}_\sigma$.

3.4.2 The Loss or Stability of Expressive Power

Clearly, every set in the finite, analytic signature of a semantics is also contained in the finite signature. Remember that in case of compact AFs we do not lose any expressive power if considering the uniquely defined grounded, ideal and eager semantics as well as naive semantics and conflict-free sets (Theorem 3.44). These equal expressiveness results carry over to analytic AFs and moreover, even stable and stage semantics may realize the same sets. For instance, consider again the non-analytic AF F as introduced in Example 3.50. One may easily verify that adding an attack from c to d or vice versa yields an AF F' analytic for stable semantics which does not change the set of stable extensions. However, in general it is not that easy to make implicit conflicts explicit but it was shown that the use of additional arguments indeed allows one to turn any finite framework in an analytical one without changing the set of stable or stage extensions, respectively [Baumann *et al.*, 2016a, Proposition 28, Theorem 29]. For the sake of completeness, we mention that it was an open question for a while, known as *Explicit Conflict Conjecture* [Baumann *et al.*, 2014a], whether it is possible, under stable semantics, to translate a given AF into an equivalent analytic one without adding further arguments. In [Baumann *et al.*, 2016a] the conjecture was refuted for stable and even stage semantics. For the remaining semantics, i.e. admissible, semi-stable, preferred and complete semantics the conjecture does not hold either since in case of these semantics we even have that the finite, analytic signature is a strict subset of the corresponding finite one. This means, the full range of expressiveness indeed relies on the use of implicit conflicts. Consider the following example firstly presented in [Baumann *et al.*, 2016a, Example 6].

Example 3.56. *Take into account the AF $F = (A, R)$ as depicted below.*



Formally, we have

$$A = \{a_i, b_i, x_i, u_i \mid i \in \{1,2,3\}\} \text{ and}$$
$$R = \{(a_i, b_i), (b_i, a_i), (b_i, x_i), (x_i, u_i) \mid i \in \{1,2,3\}\} \cup \{(x_1, x_2), (x_2, x_3), (x_3, x_1)\}.$$

Regarding the extension-based version of preferred semantics we obtain the set $\mathbb{S} = \mathcal{E}_{pr}(F) = \{S_a, S_b, A_1, A_2, A_3, B_1, B_2, B_3\}$ *with*

$$S_a = \{a_1, a_2, a_3\} \qquad S_b = \{b_1, b_2, b_3, u_1, u_2, u_3\}$$
$$A_1 = \{a_2, a_3, b_1, x_2, u_1, u_3\} \qquad B_1 = \{a_1, b_2, b_3, x_1, u_2, u_3\}$$
$$A_2 = \{a_1, a_3, b_2, x_3, u_1, u_2\} \qquad B_2 = \{a_2, b_1, b_3, x_2, u_1, u_3\}$$
$$A_3 = \{a_1, a_2, b_3, x_1, u_2, u_3\} \qquad B_3 = \{a_3, b_1, b_2, x_3, u_1, u_2\}$$

We observe three implicit conflicts indicated by dashed lines. Consequently, F is not analytic w.r.t. preferred semantics. Moreover, we claim that $\mathbb{S}$ is not analytically pr-realizable at all. For a contradiction we assume that there exists an AF $G \in XAF^{f}_{pr}$, s.t. $\mathcal{E}_{pr}(G) = \mathbb{S}$. We now investigate this hypothetical AF G. The main idea is to show that if the conflict between a_1 and x_2 is made explicit, then $\mathbb{S} \neq \mathcal{E}_{pr}(G)$. First, note that G contains at least all arguments in A since $Args_{\mathbb{S}} = A$. Due to A_3 and B_3 we deduce that $S_a \cup \{u_2\}$ is conflict-free in G. Furthermore, due to A_1, the admissibility of S_a in G and the assumption that all conflicts has to be explicit, we infer that a_1 attacks x_2. Moreover, in consideration of $\mathbb{S}$, it is easy to see that x_2 is the only possible attacker of u_2 among $Args_{\mathbb{S}}$. This implies that S_a defends u_2 against all arguments in $Args_{\mathbb{S}}$. Finally, any additional argument $z \notin Args_{\mathbb{S}}$ in G must be attacked by S_a since G is analytic w.r.t. preferred semantics and S_a must be admissible. This causes $S_a \cup \{u_2\}$ to be admissible in G and hence, S_a cannot be preferred in G. In summary, any AF realizing $\mathbb{S}$ has to be non-analytic for preferred semantics, i.e. $\mathbb{S} \in \Sigma^{f}_{\mathcal{E}_{pr}} \setminus \Sigma^{f,x}_{\mathcal{E}_{pr}}$.

We proceed with the main theorem comparing finite signatures with their corresponding analytical ones.

Theorem 3.57 ([Baumann *et al.*, 2016a])**.** *It holds that*

1. $\Sigma^{f,x}_{\mathcal{E}_\sigma} = \Sigma^{f}_{\mathcal{E}_\sigma}$ *for* $\sigma \in \{cf, na, gr, il, eg, stb, stg\}$, *and*
2. $\Sigma^{f,x}_{\mathcal{E}_\sigma} \subset \Sigma^{f}_{\mathcal{E}_\sigma}$ *for* $\sigma \in \{ad, ss, pr, co\}$.

In the following we present characterization theorems for finite, analytic signatures or at least necessary properties for being finitely, analytically realizable. All results can be verified via combining the main theorem above as well as the already presented characterization theorems for finite signatures, namely Theorems 3.12, 3.24 and 3.33.

Theorem 3.58. *Given a set* $\mathbb{S} \subseteq 2^{\mathcal{U}}$, *then*

1. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{cf}} \Leftrightarrow \mathbb{S}$ *is a non-empty, downward-closed and tight ext.-set,*
2. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{na}} \Leftrightarrow \mathbb{S}$ *is a non-empty, incomparable ext.-set and* $dcl(\mathbb{S})$ *is tight,*
3. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{gr}} \Leftrightarrow \mathbb{S}$ *is an ext.-set with* $|\mathbb{S}| = 1$,
4. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{il}} \Leftrightarrow \mathbb{S}$ *is an ext.-set with* $|\mathbb{S}| = 1$,
5. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{eg}} \Leftrightarrow \mathbb{S}$ *is an ext.-set with* $|\mathbb{S}| = 1$,

6. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{stb}} \Leftrightarrow \mathbb{S}$ *is a incomparable and tight ext.-set,*

7. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{stg}} \Leftrightarrow \mathbb{S}$ *is a non-empty, incomparable and tight ext.-set and*

8. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{ad}} \Rightarrow \mathbb{S}$ *is a conflict-sensitive ext.-set containing* $\emptyset$*,*

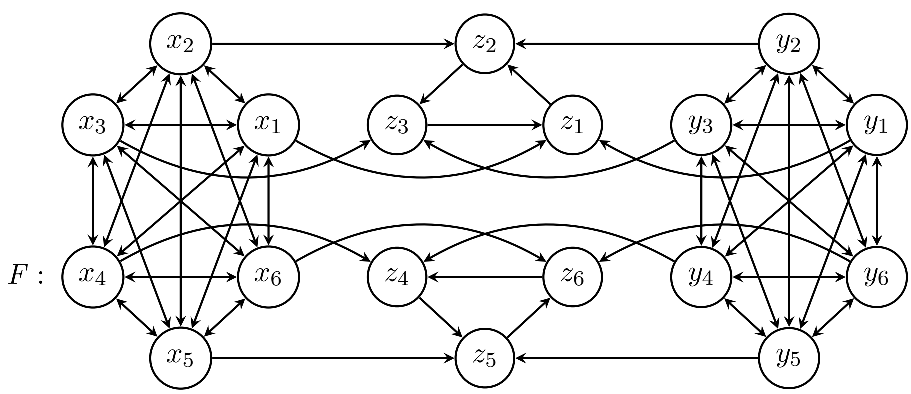
9. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{pr}} \Rightarrow \mathbb{S}$ *is a non-empty, incomparable and conflict-sensitive ext.-set,*

10. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{ss}} \Rightarrow \mathbb{S}$ *is a non-empty, incomparable and conflict-sensitive ext.-set.*

3.4.3 Comparing Finite, Analytic Signatures and Final Remarks

So far we have compared finite signatures and finite, analytic signatures for the semantics under consideration. We have seen, for example, that preferred and semi-stable semantics can realize strictly more when allowing the use of implicit conflicts, while this is not the case for stable and stage semantics. In the following we relate the finite, analytic signatures of all considered semantics. Remember that we observed a considerable variety in the relations between incomparable semantics if sticking from finite to finite, compact signatures (cf. Figures 2 and 4). However, in the analytic case we have slight differences only as illustrated in Figure 8 (for incomparable semantics) and formally stated in Theorem 3.60. For instance, preferred and semi-stable signatures do not coincide anymore as shown by the following example taken from [Baumann *et al.*, 2016a, Figure 9, Proof of Theorem 34].

Example 3.59. *Consider the following AF F as depicted below.*



The preferred extension of F can be compactly presented via a cyclic successor functions. More precisely, if $\mathrm{s}(1) = 2, \mathrm{s}(2) = 3, \mathrm{s}(3) = 1$ *and* $\mathrm{s}(4) = 5, \mathrm{s}(5) =$

$6, \mathrm{s}(6) = 4$, *then* $\mathcal{E}_{pr}(F) = \mathbb{S} = \mathbb{S}_0 \cup \mathbb{S}_1 \cup \mathbb{S}_2$ *with*

$$\mathbb{S}_0 = \Big\{\Big\{x_i, y_j, z_{\mathrm{s}(i)}, z_{\mathrm{s}(j)}\Big\} \mid i \in \{1,2,3\}, j \in \{4,5,6\} \textit{ or } i \in \{4,5,6\}, j \in \{1,2,3\}\Big\},$$
$$\mathbb{S}_1 = \Big\{\Big\{x_i, y_i, z_{\mathrm{s}(i)}\Big\} \mid i \in \{1,2,3,4,5,6\}\Big\} \textit{ and}$$
$$\mathbb{S}_2 = \Big\{\Big\{x_i, y_{\mathrm{s}(i)}, z_{\mathrm{s}(\mathrm{s}(i))}\Big\}, \Big\{x_{\mathrm{s}(i)}, y_i, z_{\mathrm{s}(\mathrm{s}(i))}\Big\} \mid i \in \{1,2,3,4,5,6\}\Big\}.$$

This means, F is pr-analytic and therefore, $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{E}_{pr}}$. We show now that $\mathbb{S} \notin \Sigma^{f,x}_{\mathcal{E}_{ss}}$. Assume that there is some $G = (B,S) \in XAF^f_{ss}$ with $\mathcal{E}_{pr}(G) = \mathbb{S}$. We take a look at $\mathbb{S}_1$ and more specifically $\{x_1, y_1, z_2\} \in \mathbb{S}_1$. Now we need an explicit conflict between x_1 and x_4, but in the selected set only x_1 can possibly defend against this attack, hence $(x_1, x_4) \in S$. The same argument works for x_1 and x_3 as well as z_2 and z_3, meaning that also $(x_1, x_3), (z_2, z_3) \in S$. For symmetry reasons $\{(x_i, x_j), (x_j, x_i), (y_i, y_j), (y_j, y_i) \mid i \in \{1,2,3\}, j \in \{4,5,6\}\} \subseteq S$ and $\{(x_{\mathrm{s}(i)}, x_i), (z_i, z_{\mathrm{s}(i)}) \mid i \in \{1,2\ldots 6\}\} \subseteq S$.

We take a look at $\mathbb{S}_2$ and more specifically $\{x_1, y_2, z_3\} \in \mathbb{S}_2$. As there should be an explicit conflict between x_1 and x_2 with only x_1 possibly defending this extension against x_2 we need $(x_1, x_2) \in S$. Further as in this set only y_2 and z_3 can possibly attack z_2 we have the set $\{y_2, z_3\}$ attacking z_2. For symmetry reasons $\{(x_i, x_{\mathrm{s}(i)}), (y_i, y_{\mathrm{s}(i)}) \mid i \in \{1,2\ldots 6\}\} \subseteq S$ and each set $\{x_i, z_{\mathrm{s}(i)}\}, \{y_i, z_{\mathrm{s}(i)}\}$ for $i \in \{1,2\ldots 6\}$ attacks z_i.

Finally we take a look at $\mathbb{S}_0$ and specifically the set $I = \{x_1, y_4, z_2, z_5\} \in \mathbb{S}_0$. Since I necessarily is an admissible extension in an analytic AF we have that I attacks all rejected arguments. By the above observations we now have that I even attacks all arguments not being member of I in G, which means that I is a stable extension and stable semantics and semi-stable semantics thus coincide on G. But then, with $J = \{x_1, y_1, z_2\} \in \mathbb{S}_1$ not being in conflict with for instance z_4 we have that J can not be a stable or semi-stable extension in G concluding $\mathbb{S} \notin \Sigma^{f,x}_{\mathcal{E}_{ss}}$.

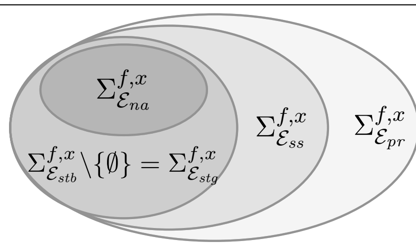


Figure 6: Subset Relations between Finite, Analytic Signatures

Theorem 3.60 ([Dunne *et al.*, 2015]). *The following relations hold:*

1. $\Sigma^{f,x}_{\mathcal{E}_\sigma} \subset \Sigma^{f,x}_{\mathcal{E}_{na}} \subset \Sigma^{f,x}_{\mathcal{E}_{stg}} \subset \Sigma^{f,x}_{\mathcal{E}_{ss}} \subset \Sigma^{f,x}_{\mathcal{E}_{pr}}$ *for* $\sigma \in \{gr, il, eg\}$,

2. $\Sigma^{f,x}_{\mathcal{E}_{stb}} = \Sigma^{f,x}_{\mathcal{E}_{stg}} \cup \{\emptyset\}$,

3. $\Sigma^{f,x}_{\mathcal{E}_{cf}} \subset \Sigma^{f,x}_{\mathcal{E}_{ad}}$ *and*

4. $\Sigma^{f,x}_{\mathcal{E}_{co}} \setminus \Sigma^{f,x}_{\mathcal{E}_\sigma} \neq \emptyset$ *and* $\Sigma^{f,x}_{\mathcal{E}_\sigma} \setminus \Sigma^{f,x}_{\mathcal{E}_{co}} \neq \emptyset$ *for* $\sigma \in \{cf, ad\}$.

3.5 Remarks on Unrestricted AFs and Intertranslatability

Recently, some first results regarding expressibility w.r.t. unrestricted frameworks were presented in [Baumann and Spanring, 2017]. Remember that the set of unrestricted frameworks, abbreviated by $\mathcal{F}$, contains all AFs $F = (A, R)$, s.t. $A \subseteq \mathcal{U}$ (cf. Section 2.1 for further information). This means, $\mathcal{F}$ contains finite as well as infinite AFs and especially, AFs possessing all available arguments. It is obvious that signatures w.r.t. unrestricted frameworks contain more realizable sets then their finite counterparts since finite AFs may realize finite as well as finitely many extensions only. The following definition formally captures all considered types of signatures (cf. Definitions 3.4, 3.42 and 3.55) without any finite assumption.

Definition 3.61. *Given a semantics σ. We call the set S the*

1. *(unrestricted) σ-signature if $S = \{\sigma(F) \mid F \in \mathcal{F}\}$ abbreviated by Σ_σ,*

2. *compact σ-signature if $S = \{\sigma(F) \mid F \in CAF\}$ abbreviated by Σ^c_σ and*

3. *analytic σ-signature if $S = \{\sigma(F) \mid F \in XAF\}$ abbreviated by Σ^x_σ.*

In [Baumann and Spanring, 2017] the authors were interested in a comparison of the expressive power of several mature semantics in the unrestricted setting. The following result shows that the relation between unrestricted signatures is intimately connected to their relation in case of finite, compact signatures. More precisely, non-empty relative complements in case of finite, compact signatures between two semantics carry over to their unrestricted versions. The main reason for this relation is the fact that unrestricted frameworks may contain any available argument of the universe $\mathcal{U}$.

Theorem 3.62 ([Baumann and Spanring, 2017]). *Given two semantics $\sigma, \tau \in \{na, stb, stg, ss, pr, co, gr, il, eg, cf, ad\}$ we have:*

1. *If $\Sigma^{f,c}_{\mathcal{E}_\sigma} \setminus \Sigma^{f,c}_{\mathcal{E}_\tau} \neq \emptyset$, then $\Sigma^{c}_{\mathcal{E}_\sigma} \setminus \Sigma^{c}_{\mathcal{E}_\tau} \neq \emptyset$ and*
2. *If $\Sigma^{c}_{\mathcal{E}_\sigma} \setminus \Sigma^{c}_{\mathcal{E}_\tau} \neq \emptyset$, then $\Sigma_{\mathcal{E}_\sigma} \setminus \Sigma_{\mathcal{E}_\tau} \neq \emptyset$.*

The following example illustrates the main proof idea.

Example 3.63. *Let $\mathcal{E} \in \Sigma^{f,c}_{\mathcal{E}_{pr}} \setminus \Sigma^{f,c}_{\mathcal{E}_{stb}}$ (cf. Figure 4) and $F = (A, R)$ a witnessing framework. This means, F is finite, $\mathcal{E}_{pr}(F) = \mathcal{E}$ and pr-compact, i.e. $\bigcup \mathcal{E} = A$. Consider now $H = (\mathcal{U}, R)$. Obviously, $\mathcal{E}' = \mathcal{E}_{pr}(H) = \{E \cup (\mathcal{U} \setminus A) : E \in \mathcal{E}\}$ and $\bigcup \mathcal{E}' = \mathcal{U}$ showing the σ-compactness of H. In particular, $\mathcal{E}' \in \Sigma^{c}_{\mathcal{E}_{pr}}$. Note that any stb-realization of $\mathcal{E}'$ has to be compact too since there are no additional arguments available. Assume $\mathcal{E}' \in \Sigma^{c}_{\mathcal{E}_{stb}}$, i.e. there is an AF $G' = (\mathcal{U}, R')$, s.t. $\mathcal{E}_{stb}(G') = \mathcal{E}'$. We observe that due to conflict-freeness there can not be attacks in G' between arguments from A and $\mathcal{U} \setminus A$ nor between any of the arguments from $\mathcal{U} \setminus A$. Consequently, $G = (A, R')$ is finite, $\mathcal{E}_{stb}(G) = \mathcal{E}$ and stb-compact implying that $\mathcal{E} \in \Sigma^{f,c}_{\mathcal{E}_{stb}}$ in contradiction to the initial assumption.*

Now we are prepared for a comparison in case of unrestricted frameworks. Ignoring the superscripts in Figure 4 provides you with a graphical representation for selected semantics.

Theorem 3.64. *For unrestricted signatures the following hold:*

1. $\{\{E\} \mid E \subseteq \mathcal{U}\} = \Sigma_{\mathcal{E}_\sigma} \subset \Sigma_{\mathcal{E}_{na}} \subset \Sigma_{\mathcal{E}_\tau}$ *for* $\sigma \in \{gr, il\}$, $\tau \in \{stb, stg, ss, pr\}$,
2. $\Sigma_{\mathcal{E}_{eg}} \subset \Sigma_{\mathcal{E}_{pr}}$,
3. $\Sigma_{\mathcal{E}_{stb}} \subset \Sigma_{\mathcal{E}_\sigma}$ *for* $\sigma \in \{stg, ss\}$,
4. $\Sigma_{\mathcal{E}_{pr}} \setminus \left(\Sigma_{\mathcal{E}_{stb}} \cup \Sigma_{\mathcal{E}_{ss}} \cup \Sigma_{\mathcal{E}_{stg}}\right) \neq \emptyset$,
5. $\Sigma_{\mathcal{E}_{stg}} \setminus (\Sigma_{\mathcal{E}_{stb}} \cup \Sigma_{\mathcal{E}_{pr}} \cup \Sigma_{\mathcal{E}_{ss}}) \neq \emptyset$,
6. $\Sigma_{\mathcal{E}_{stb}} \setminus \Sigma_{\mathcal{E}_{pr}} \neq \emptyset$,
7. $\Sigma_{\mathcal{E}_{ss}} \setminus \left(\Sigma_{\mathcal{E}_{stb}} \cup \Sigma_{\mathcal{E}_{pr}} \cup \Sigma_{\mathcal{E}_{stg}}\right) \neq \emptyset$,
8. $\Sigma_{\mathcal{E}_{co}} \setminus \Sigma_{\mathcal{E}_\sigma} \neq \emptyset$ *and* $\Sigma_{\mathcal{E}_\sigma} \setminus \Sigma_{\mathcal{E}_{co}} \neq \emptyset$ *for* $\sigma \in \{cf, ad\}$,
9. $\Sigma_{\mathcal{E}_{cf}} \subset \Sigma_{\mathcal{E}_{ad}}$.

Finally, we briefly consider the closely related topic of *intertranslatability*. Intertranslatability revolves around the idea of mapping one semantics to another. One main motivation for studying this issue is the possibility to reuse a solver for one semantics for another [Dvořák and Woltran, 2011]. The main tool for this endeavour are functions mapping AFs to AFs, so-called *translations* formally defined as follows.

Definition 3.65. *[Dvořák and Woltran, 2011] Given two semantics σ, τ. A function $f : \mathcal{F} \to \mathcal{F}$ is called an exact translation for $\sigma \to \tau$, if $\sigma(F) = \tau(f(F))$ for any AF F. It is called a faithful translation if for any AF F first $|\sigma(F)| = |\tau(f(F))|$ and second $\sigma(F) = \{S \cap A(F) \mid S \in \tau(f(F))\}$.*

Please note that for some semantics there are no exact translations available due to reasons inherent to those semantics. For instance, preferred semantics satisfies *I-maximality*, i.e. for any AF F, $\mathcal{E}_{pr}(F)$ forms a $\subseteq$-antichain [Baroni and Giacomin, 2007]. This implies that an exact translation $\mathcal{E}_{ad} \to \mathcal{E}_{pr}$ can not exist since for $F = (\{a\}, \emptyset)$ we observe $\{\emptyset, \{a\}\} = \mathcal{E}_{ad}(F)$. Sticking to faithful translations provides us with a positive answer if we consider finite AFs only [Spanring, 2012, Translation 3.1.85]. Interestingly, the considered translation does not serve in the general unrestricted case and interestingly, it was shown that a search for a suitable translation will never succeed (cf. [Baumann and Spanring, 2017, Example 6]).

The following theorem (a generalization of the finite version from [Dvořák and Spanring 2016, Section 6.1]) establishes a close relation between realizability and intertranslatability as promised, namely: if τ is not less expressive than σ, then σ can be exactly translated to τ and vice versa.

Theorem 3.66 ([Baumann and Spanring, 2017])**.** *Given two semantics σ and τ. We have: $\Sigma_\sigma \subseteq \Sigma_\tau$ if and only if there is an exact translation for $\sigma \to \tau$.*

The following Figure 7 illustrates translational (im)possibilities in an eye-catching way. Figure 7b summarizes known results regarding faithful translations in the finite case [Dvořák and Woltran, 2011; Spanring, 2012; Dvořák and Spanring, 2016], augmented with obvious insights for unique status semantics *il* and *eg*. For semantics σ, τ, encirclement in the same component indicates bidirectional translations. An arrow from σ to τ means directional translations. If there is no directed path (for instance for *na* to *cf*, or for *cf* to *gr*), then there is no translation. Figure 7a features the same visualization for unrestricted AFs. Dropping the finiteness restriction has some further consequences for the considered semantics, namely exact and faithful intertranslatability coincide. It is an open question whether both forms of translations are essentially the same in the general unrestricted setting. In consideration of Theorem 3.66 we may interpret Figure 7a as a comparison of the expressiveness of the considered semantics. That is, $\Sigma_\sigma \subset \Sigma_\tau$ if and only if there is a directed path from σ to τ.

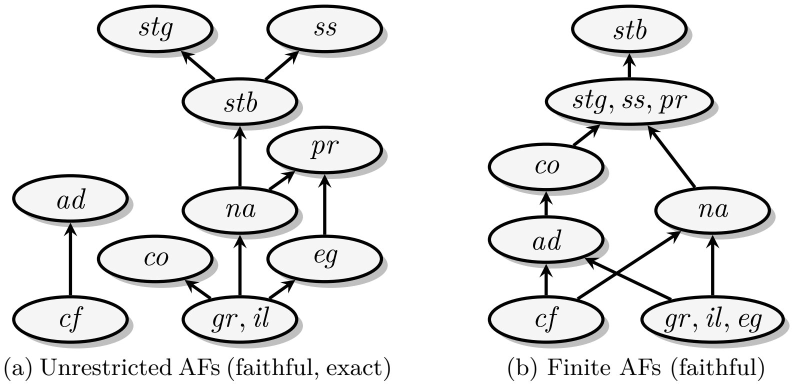


Figure 7: Translational (Im)Possibilities

As a final note, in contrast to the unrestricted setting Baumann and Spanring observed that for slightly restricted AFs $F = (A, R)$, s.t. $|A| \leq |\mathcal{U} \setminus A|$ it is possible to provide exact and efficiently computable translations from preferred to semi-stable semantics via $f(F) = F' = (A', R')$ with $A' = A \cup \{a' \mid a \in A\}$ and $R' = R \cup \{(a, a'), (a', a') \mid a \in A\}$. It is an interesting question whether this restriction allows for similar translational possibilities as in case of finite AFs.

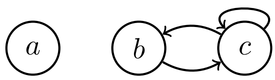
3.6 Realizability and Signatures for Labelling-based Versions

Although any considered semantics σ possesses an extension-based version (indicated by $\mathcal{E}_\sigma$) as well as a closely related 3-valued labelling-based version (indicated by $\mathcal{L}_\sigma$) we formally have that both versions are different semantics (or more precisely, functions) in the sense of Definition 2.2. This formal difference has some impact on realizability as well as signatures. Let us consider realizability in the realm of finiteness. As a matter of fact, for any considered 3-valued labelling-based version $\mathcal{L}_\sigma$ we have: if $F = (A, R)$ and $L = (L^{\mathrm{I}}, L^{\mathrm{O}}, L^{\mathrm{U}}) \in \mathcal{L}_\sigma(F)$, then $A = L^{\mathrm{I}} \cup L^{\mathrm{O}} \cup L^{\mathrm{U}}$. This means, σ-labellings assign a status to any argument in F. Now, in case of finite AFs we know that potentially realizable sets of labellings have to involve finitely many arguments only. Moreover, these finitely many arguments precisely determine the set A of witnessing AFs $F = (A, R)$.[13] Consider therefore the following example.

[13] This is exactly the point which does not carry over to finite realizability in case of extension-based semantics (cf. statement 2 of Theorem 3.44).

Example 3.67. *Consider the following set of 3-valued labellings* $\mathbb{S} = \{(\{a\}, \emptyset, \{b, c\}), (\{a, b\}, \{c\}, \emptyset)\}$. *Is* $\mathbb{S}$ *co-realizable? Since* $\{a\} \cup \emptyset \cup \{b, c\} = \{a, b, c\}$ *we deduce that candidates have to be members of* $\mathcal{C} = \{F = (A, R) \mid A = \{a, b, c\}\}$. *Note that* $|\mathcal{C}| = 2^{|\{a,b,c\}|^2} = 2^9 = 512$. *Clearly, this is a huge number, but it is a finite one. Consequently, the question of realizability can be decided by computing the* σ*-labellings of all AFs in* $\mathcal{C}$. *Of course, any intelligent search algorithm would involve further information like* $\{a, b\}$ *has to be conflict-free in a witnessing AF. Such an observation would decrease the number of candidates to* $2^5 = 32$. *However, in both cases one would find the unique witnessing framework* F, *i.e.* $\mathcal{L}_{co}(F) = \mathbb{S}$, *as depicted below.*

F:



The example above shows that the search space can be very large even in case of small numbers of arguments. Consequently, locally verifiable necessary as well as sufficient properties for realizability just like in case of extension-based semantics are of high interest too. To the best of our knowledge only two papers have dealt with labelling-based realizability in the context of AFs. The first study was presented by Dyrkolbotn [Dyrkolbotn, 2014]. The author showed that, as long as additional arguments are allowed any finite set of labellings is realizable under preferred and semi-stable semantics. It is important to emphasize that Dyrkolbotn uses a more relaxed notion of realizibility, namely *realizibility under projection* (cf. Definition 3.72). The other work [Linsbichler *et al.*, 2016] deals with the standard notion of finite realizability (Definition 3.3). The authors presented an algorithm which returns either "No" in case of non-realizibility or a witnessing AF F in the positive case. The algorithm is not purely a guess-and-check method since it also includes a propagation step where certain necessary properties of witnessing AFs are processed. Remarkably, the algorithm is not restricted to the formalism of abstract argumentation frameworks only. In fact, it can also be used to decide realizability in case of the more general abstract dialectical frameworks as well as various of its sub-classes [Brewka and Woltran, 2010; Brewka *et al.*, 2013].

3.6.1 Preliminary Results for Labelling-based Signatures

In the following we shed light on general relations between the labelling-based and extension-based signatures of the considered semantics. Fortunately, due to former characterization results we will even achieve characterizing or at least necessary properties for finite realizability regarding labelling-based versions. We proceed with the definition of an *labelling-set* which is the n-valued analogon (for $n \geq 2$) to an extension-set as introduced in Definition 3.5. A labelling-set is a finite set of n-tuples which are dealing with the same set of arguments and moreover, any n-tuple assigns exactly one status to each argument in question.

Definition 3.68. *Given $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^n$. $Args_{\mathbb{S}}$ denotes $\bigcup_{L=(L_1,\ldots,L_n)\in\mathbb{S}} \bigcup_{i=1}^{n} L_i$ and $\|\mathbb{S}\|$ stands for $|Args_{\mathbb{S}}|$. We say that $\mathbb{S}$ is a labelling-set if*

1. *$\|\mathbb{S}\|$ is a finite cardinal,*
2. *for any $L = (L_1, \ldots, L_n) \in \mathbb{S}$, $Args_{\mathbb{S}} = \bigcup_{i=1}^{n} L_i$ and*
3. *for any $L = (L_1, \ldots, L_n) \in \mathbb{S}$, $L_1, \ldots, L_n$ are pairwise disjoint.*

The following proposition establishes a connection between extension-based and labelling-based realizibility for any considered semantics. Roughly speaking, it states that labelling-based realizability requires extension-based realizability of the corresponding sets of in-labelled arguments. For a 3-tuple $L = (L_1, L_2, L_3)$ we also write $(L^{\mathrm{I}}, L^{\mathrm{O}}, L^{\mathrm{U}})$ as usual.

Proposition 3.69. *Given a set of 3-tuples $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^3$. For any semantics $\sigma \in \{stb, ss, stg, cf2, stg2, pr, ad, co, gr, il, eg, na, cf\}$ we have,*

1. $\mathbb{S} \in \Sigma_{\mathcal{L}_\sigma} \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma_{\mathcal{E}_\sigma}$ *(unrestricted realizability)*
2. $\mathbb{S} \in \Sigma^c_{\mathcal{L}_\sigma} \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma^c_{\mathcal{E}_\sigma}$ *(compact realizability)*
3. $\mathbb{S} \in \Sigma^x_{\mathcal{L}_\sigma} \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma^x_{\mathcal{E}_\sigma}$ *(analytic realizability)*
4. $\mathbb{S} \in \Sigma^f_{\mathcal{L}_\sigma} \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma^f_{\mathcal{E}_\sigma}$ *(finite realizability)*
5. $\mathbb{S} \in \Sigma^{f,c}_{\mathcal{L}_\sigma} \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma^{f,c}_{\mathcal{E}_\sigma}$ *(finite, compact realizability)*
6. $\mathbb{S} \in \Sigma^{f,x}_{\mathcal{L}_\sigma} \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma^{f,x}_{\mathcal{E}_\sigma}$ *(finite, analytic realizability)*

Please note that the implications above are justified for any semantics σ whenever the different versions of it satisfy $\mathcal{E}_\sigma(F) = \{L^{\mathrm{I}} \mid L \in \mathcal{L}_\sigma(F)\}$ for any relevant AF F. In the former sections we already presented characterization theorems or at least necessary properties for being finitely realizable regarding extension-based versions (cf. Theorems 3.12, 3.24 and 3.33). Combining these results with the proposition above yields the following necessary properties for finite realizability in the labelling-based case. Note that the mentioned implications apply to finite, compact as well as finite, analytic signatures too since $\Sigma^{f,c}_{\mathcal{L}_\sigma} \subseteq \Sigma^{f}_{\mathcal{L}_\sigma}$ as well as $\Sigma^{f,x}_{\mathcal{L}_\sigma} \subseteq \Sigma^{f}_{\mathcal{L}_\sigma}$ by definition. In case of grounded, ideal and eager semantics we have that being an one-element labelling-set is necessary and even sufficient for being finitely realizable. One may easily verify that the only-if-directions of these semantics are justified by the witnessing framework $F_L = (L^{\mathrm{I}} \cup L^{\mathrm{O}} \cup L^{\mathrm{U}}, \{(i,o) \mid i \in L^{\mathrm{I}}, o \in L^{\mathrm{O}}\} \cup \{(u,u) \mid u \in L^{\mathrm{U}}\})$ given that $\mathbb{S} = \{L\}$.

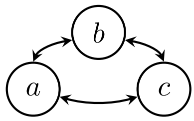
Theorem 3.70. *Given a set of 3-tuples $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^3$, then*

1. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{cf}} \Rightarrow \{L^{I} \mid L \in \mathbb{S}\}$ *is a non-empty, downward-closed and tight ext.-set,*
2. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{na}} \Rightarrow \{L^{I} \mid L \in \mathbb{S}\}$ *is a non-empty, incomparable ext.-set and* $dcl(\mathbb{S})$ *is tight,*
3. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{gr}} \Leftrightarrow \mathbb{S}$ *is a labelling-set with* $|\mathbb{S}| = 1$,
4. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{il}} \Leftrightarrow \mathbb{S}$ *is a labelling-set with* $|\mathbb{S}| = 1$,
5. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{eg}} \Leftrightarrow \mathbb{S}$ *is a labelling-set with* $|\mathbb{S}| = 1$,
6. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{stb}} \Rightarrow \{L^{I} \mid L \in \mathbb{S}\}$ *is a incomparable and tight extension-set,*
7. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{stg}} \Rightarrow \{L^{I} \mid L \in \mathbb{S}\}$ *is a non-empty, incomparable and tight ext.-set,*
8. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{ad}} \Rightarrow \{L^{I} \mid L \in \mathbb{S}\}$ *is a conflict-sensitive ext.-set containing* $\emptyset$,
9. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{pr}} \Rightarrow \{L^{I} \mid L \in \mathbb{S}\}$ *is a non-empty, incomparable and conflict-sensitive ext.-set,*
10. $\mathbb{S} \in \Sigma^{f}_{\mathcal{L}_{ss}} \Rightarrow \{L^{I} \mid L \in \mathbb{S}\}$ *is a non-empty, incomparable and conflict-sensitive ext.-set.*

3.6.2 Realizibility under Projection

We turn now to *realizability under projection* which was first considered in [Dyrkolbotn, 2014]. In order to realize a set of labellings $\mathbb{S}$ under projection it suffices to come up with an AF F, s.t. its set of labellings restricted to the relevant arguments coincide with $\mathbb{S}$. Consider therefore the following illustrating example.

Example 3.71. *Given* $\mathbb{S} = \{(\{a\}, \{b\}, \emptyset), (\{b\}, \{a\}, \emptyset), (\emptyset, \{a, b\}, \emptyset)\}$. *We observe that the corresponding set of sets of in-labelled arguments* $\mathbb{S}^I = \{\emptyset, \{a\}, \{b\}\}$ *violates incomparability. Thus, applying statement 9 of Theorem 3.70 we derive that* $\mathbb{S}$ *is not finitely pr-realizable. Consider now the following AF* F.

$F:$


We obtain $\mathcal{L}_{pr}(F) = \{(\{a\}, \{b, c\}, \emptyset), (\{b\}, \{a, c\}, \emptyset), (\{c\}, \{a, b\}, \emptyset)\}$. *Now, if we restrict any labelling* $L = (L^I, L^O, L^U) \in \mathcal{L}_{pr}(F)$ *to the arguments* a *and* b, *i.e.* $L|_{\{a,b\}} = (L^I \cap \{a, b\}, L^O \cap \{a, b\}, L^U \cap \{a, b\})$ *we obtain exactly all labellings in* $\mathbb{S}$. *In this sense,* $\mathbb{S}$ *is pr-realizable under projection.*

We proceed with the formal definitions. For the sake of completeness we introduce realizability under projection and its corresponding signatures w.r.t. any kind of semantics as defined in Definition 2.2.

Definition 3.72. *Given a semantics* $\sigma : \mathcal{F} \to 2^{(2^{\mathcal{U}})^n}$. *A set* $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^n$ *is* σ*-realizable under projection if there is an AF* F, *s.t.* $\sigma(F)|_{Args_{\mathbb{S}}} = \{E|_{Args_{\mathbb{S}}} \mid E \in \mathcal{E}_\sigma(F)\} = \mathbb{S}$ *(in case of* $n = 1$*) or* $\sigma(F)|_{Args_{\mathbb{S}}} = \{L|_{Args_{\mathbb{S}}} \mid L \in \mathcal{L}_\sigma(F)\} = \mathbb{S}$ *(for* $n \geq 2$*), respectively.*

Definition 3.73. *Given a semantics* σ. *The unrestricted as well as finite* σ*-projection-signatures are defined as follows:*

1. $\Sigma_\sigma^p = \{\sigma(F)|_B \mid F = (A, R) \in \mathcal{F}, B \subseteq A\}$ *and*
2. $\Sigma_\sigma^{f,p} = \{\sigma(F)|_B \mid F = (A, R) \in \mathcal{F}, F$ *is finite,* $B \subseteq A\}$

Analogously to Proposition 3.69 we state the following relation between labelling-based and extension-based versions of the considered semantics.

Proposition 3.74. *Given a set of 3-tuples* $\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^3$. *For any semantics* $\sigma \in \{stb, ss, stg, cf2, stg2, pr, ad, co, gr, il, eg, na, cf\}$ *we have,*

1. $\mathbb{S} \in \Sigma_{\mathcal{L}_\sigma}^p \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma_{\mathcal{E}_\sigma}^p$ *(unrestricted realizability under projection)*
2. $\mathbb{S} \in \Sigma_{\mathcal{L}_\sigma}^{f,p} \Rightarrow \{L^I \mid L \in \mathbb{S}\} \in \Sigma_{\mathcal{E}_\sigma}^{f,p}$ *(finite realizability under projection)*

As a matter of fact, any projection signature is a superset of the corresponding signature. The following question then arises naturally: how much more sets can be generated if we stick to realizability under projection? For instance, we have already seen that even comparable sets are realizable under projection by semantics satisfying incomparability (Example 3.71). It was the main result in [Dyrkolbotn, 2014, Theorem 3.1] that in case of semi-stable and preferred semantics indeed any 3-valued labelling-set is finitely realizable under projection. The proof relies on two basic constructions. The first step *generates* an AF, consisting of so-called *circuits*, s.t. its set of preferred as well as semi-stable labellings restricted to the relevant arguments contains any possible labelling. The second construction *eliminates* undesired labellings step by step. Combining this realizability result with statement 2 of Proposition 3.74 yields the following theorem.

Theorem 3.75. *Let $\sigma \in \{pr, ss\}$. We have,*

1. $\Sigma^{f,p}_{\mathcal{L}_\sigma} = \left\{\mathbb{S} \subseteq \left(2^{\mathcal{U}}\right)^3 \mid \mathbb{S} \textit{ is a labelling-set}\right\}$ *and*
2. $\Sigma^{f,p}_{\mathcal{E}_\sigma} = \left\{\mathbb{S} \subseteq 2^{\mathcal{U}} \mid \mathbb{S} \textit{ is an extension-set}\right\}$.

3.7 Final Remarks and Conclusion

We have dealt with different forms of realizability in the context of abstract argumentation frameworks. In accordance with the existing literature the main part of this section was devoted to finite realizability for extension-based semantics. However, for any semantics σ we may state the following general subset relations depicted as Venn-diagram.

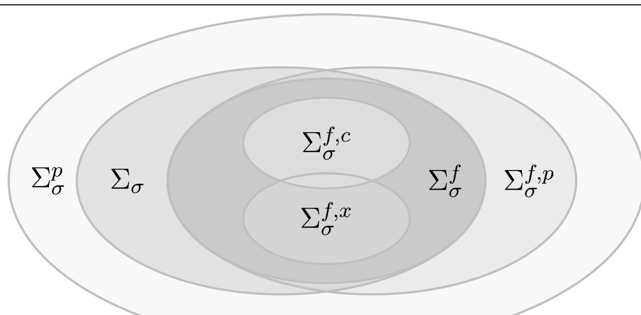


Figure 8: Subset Relations between Different Kinds of Signatures

In case of the extension-based versions of naive, grounded, ideal, eager, stable, stage, preferred and semi-stable semantics as well as conflict-free and admissible sets we provided exact characterizations for their corresponding general signatures. We have seen that for some semantics we do not lose any expressive power if sticking to compact or analytic AFs, i.e. $\Sigma_\sigma^f = \Sigma_\sigma^{f,c}$ or $\Sigma_\sigma^f = \Sigma_\sigma^{f,x}$, respectively. However, for certain prominent semantics, e.g. preferred semantics we have that the expressive power indeed relies on the use of rejected arguments or implicit conflicts. For such semantics, it remains an open problem to present exact characterizations for finite, compact or finite, analytic realizability, respectively. In case of labelling-based versions of semi-stable and preferred semantics we have seen that any labelling-set is realizable under projection. In [Dyrkolbotn, 2014] it was already noted that this equality does not hold for any semantics. For instance, the empty labelling is admissible for any AF F. Hence, in case of admissible semantics, no labelling-set is realizable under projection if it fails to include the empty labelling.

Finally, let us mention some computational issues not considered so far. It can be said that on the one hand, the classes of finite, compact and finite, analytic provide computational benefits both in practice and in terms of theoretical worst-case analysis. On the other hand testing for membership in one of the classes is, for most of the semantics, of rather high complexity and thus these classes cannot be directly used to improve systems. We refer the interested reader to [Baumann *et al.*, 2016a] for more details. Moreover, in general, given an extension-set $\mathbb{S}$, deciding whether $\mathbb{S}$ is compactly realizable is a hard problem, that is, by definition of the decision problem there are no good reasons to believe that we can do any better than guessing a compact AF and checking whether its extension-set coincides with $\mathbb{S}$. Nevertheless, for some semantics we have seen that finite, compact realizability can be characterized locally, i.e. by properties of $\mathbb{S}$ itself (as shown in Theorem 3.45). In this case, finite, compact realizability can be checked in polynomial time as shown in [Dunne *et al.*, 2015, Theorem 6]. Moreover, in [Baumann *et al.*, 2016a] a huge number of shortcuts to detect non-compactness are provided. By shortcut we mean a property of the given extension-set $\mathbb{S}$ that is easily computable (preferably in polynomial time) which (sometimes) provides us with a definitive answer to the decision problem. These shortcuts are related to numerical aspects of argumentation frameworks like results concerning maximal number of extensions [Baumann and Strass, 2013].

4 Replaceability

Given a certain logical formalism $\mathcal{L}$ and two syntactically different $\mathcal{L}$-theories T_1 and T_2. One central question is whether, and if so, how to decide that these $\mathcal{L}$-theories represent the same information? Of course, in order to answer this question we have to clarify what we exactly mean by sharing the same information first. Note that there is neither a uniquely determined, nor a certain preferred interpretation by the formalism $\mathcal{L}$ itself. For instance, equating information with possessing the same semantics yields to the well-known notion of *ordinary* or *standard equivalence*. This means, assuming that $\sigma_{\mathcal{L}}$ is the semantics of $\mathcal{L}$ we might answer that T_1 and T_2 are equivalent if and only if $\sigma_{\mathcal{L}}(T_1) = \sigma_{\mathcal{L}}(T_2)$. A more demanding interpretation of sharing the same information is to require that T_1 and T_2 are semantically indistinguishable even if further $\mathcal{L}$-theories T are added to both simultaneously. More formally, we may state: T_1 and T_2 are considered to be equivalent if and only if $\sigma_{\mathcal{L}}(T_1 \cup T) = \sigma_{\mathcal{L}}(T_2 \cup T)$ for any theory T. This notion is known as *strong equivalence* and is of high interest for any logical formalism since it allows one to locally replace, and thus give rise for simplification, parts of a given theory without changing the semantics of the latter. In contrast to classical (monotone) logics where standard and strong equivalence coincide (cf. [Baumann and Strass, 2016] for more detailed information on this issue), it is possible to find ordinary but not strongly equivalent objects for any nonmonotonic formalism available in the literature. Consequently, much effort has been devoted to characterizing strong equivalence for nonmonotonic formalisms such as logic programs [Lifschitz *et al.*, 2001], causal theories [Turner, 2004], default logic [Turner, 2001] as well as nonmonotonic logics in general [Truszczynski, 2006].

In [Oikarinen and Woltran, 2011] the authors introduced the notion of strong equivalence for abstract AFs. They provided a series of characterization theorems for deciding strong equivalence of two AFs with respect to several semantics. In view of the fact that strong equivalence is defined semantically it is the main and quite surprisingly insight that being strongly equivalent can be decided syntactically. More precisely, they introduced the notion of a *kernel* of an AF F which is (informally speaking) a subgraph of F where certain attacks are deleted and showed that syntactical identity of suitably chosen kernels characterizes strong equivalence w.r.t. the considered semantics. Strong equivalence is, as its name suggests, a very (and often unnecessarily to) strong notion of equivalence if dynamic evolvements are considered. In many argumentation scenarios the type of modification which may potentially occur can be anticipated and furthermore, more importantly, does not range over *arbitrary expansions* as required for strong equivalence. Let us consider the instantiation-based context where AFs are built from an underlying knowledge base. Here, we typically observe that older arguments and their corresponding attacks survive and only new arguments which may interact with the previous ones arise given that a new piece of information is added to the underlying knowledge base. This type of dynamic evolvement is a so-called *normal expansion* and its corresponding equivalence notion were firstly studied in [Baumann, 2012a]. Over the last five years several equivalence notions taking into account specific types of evolvements reflecting the nature of various argumentation scenarios were defined and characterized. The considered dynamic scenarios range from the most general form, so-called *updates* [Baumann, 2014a] where arguments and attacks can be deleted and added to different types of *expansions* [Oikarinen and Woltran, 2011; Baumann, 2012a; Baumann and Brewka, 2010] and *deletions* [Baumann, 2014a] where arguments and/or attacks are allowed to be added or deleted in a certain way only.

Into the year 2015 all characterization theorems were stated in terms of extension-based semantics. Recently, Baumann presents their labelling-based counterparts and showed that, although labelling-based semantics contain more information then there extension-based counterpart, there is a majority of equivalence relations where labelling-based and extension-based versions coincide [Baumann, 2016]. Even more recently, a first consideration of strong equivalence regarding unrestricted frameworks were presented in [Baumann and Spanring, 2017]. It turned out that there are no characterizational differences compared to the finite case as long as the AFs in question are *jointly expandable*, i.e. that the existence of fresh arguments is guaranteed.

Another approach somehow complementary to the ones mentioned before is presented in [Baroni *et al.*, 2014] where sharing the same information is interpreted as possessing the same Input/Output behavior. Roughly speaking, the main idea is to consider an argumentation framework as a kind of black box which receives some input from the external world (i.e, a set of external arguments) via incoming attacks and produces an output to the external world via outgoing attacks. Such an interacting module is called an *argumentation multipole*. Two multipoles connected with the same external world are considered as *Input/Output equivalent* if the effects, i.e. the produced labellings for external arguments are the same for any reasonable input-labelling. This notion yields the possibility of replacing a multipole with another one embedded in a larger framework without affecting the labellings of the unmodified part of the initial framework. In the following we shed light on equivalence notions induced by certain dynamic scenarios.

4.1 Dynamic Scenarios and Corresponding Equivalence Notions

There are two main classes of dynamic scenarios, namely *expansions* and *deletions*. Both of them can be further divided in *normal* and *local* versions. These scenarios are motivated by real-world argumentation as well as instantiation-based argumentation [Caminada and Amgoud, 2007]. For instance, let us consider the dynamics of a discussion or dispute illustrated by the following citation [Besnard and Hunter, 2009]:

> How does argumentation usually take place? Argumentation starts when an initial argument is put forward, making some claim. An objection is raised, in the form of a counterargument. The latter is addressed in turn, eventually giving rise to a counter-counterargument, if any. And so on.

This means, in order to strengthen the own point of view or to rebut the opponents arguments it is natural that one tries to come up with *stronger* arguments, i.e. new arguments which are not attacked by the former arguments. This type of dynamics is formally captured by so-called *strong expansions* [Baumann and Brewka, 2010]. The formal counterpart of it, so-called *weak expansions* [Baumann and Brewka, 2010], where the new arguments do not attack (but may be attacked by) the old ones seem to be more an academic exercise than a task with practical relevance with regard to real-world argumentation.[14] Let us turn to instantiation-based argumentation where arguments and attacks stem from an underlying knowledge base (cf. [Caminada and Amgoud, 2007; Besnard and Hunter, 2008]). What happens on the abstract level if a new piece of information is added? It turns out that in almost all deductive argumentation systems older arguments and their corresponding attacks survive and only new arguments which may interact with the previous ones arise. This type of dynamic evolvement is formally captured by so-called *normal expansions*. *Local expansions* in contrast, i.e. expansions where new attacks are added only correspond to re-instantiations if we change to a less restrictive notion of attack (cf. [Besnard and Hunter, 2001] for different attack notions).

We start with the definition of the different types of expansions together with some introducing examples.

Definition 4.1 ([Baumann and Brewka, 2010])**.** *An AF G is an expansion of AF $F = (A, R)$ (for short, $F \preceq_E G$) iff $G = (A \mathbin{\dot\cup} B, R \mathbin{\dot\cup} S)$ for some (maybe empty) sets B and S. An expansion is called*

1. *normal ($F \preceq_N G$) iff $\forall ab \;\; ((a,b) \in S \rightarrow a \in B \vee b \in B)$,*
2. *strong ($\mathcal{F} \preceq_S G$) iff $\mathcal{F} \preceq_N G$ and $\forall ab \;\; ((a,b) \in S \rightarrow \neg(a \in A \wedge b \in B))$,*
3. *weak ($\mathcal{F} \preceq_W G$) iff $\mathcal{F} \preceq_N G$ and $\forall ab \;\; ((a,b) \in S \rightarrow \neg(a \in B \wedge b \in A))$,*
4. *local ($F \preceq_L G$) iff $B = \emptyset$.*

For short, being a normal expansion means that new attacks must involve at least one new argument in contrast to local expansions where new attacks involve

[14] We mention that they do play a decisive role w.r.t. computational issues, so-called *splitting methods* (cf. [Baumann, 2011; Baumann *et al.*, 2011; Baumann *et al.*, 2012]).

old arguments only. Moreover, strong and weak expansions are normal and their names refer to properties of the additional arguments, namely arguments which are never attacked by former arguments (so-called *strong* arguments) and arguments which do not attack former arguments (so-called *weak* arguments).

Observe that any arbitrary expansion can be splitted up in a normal and a local part. This can be nicely seen in the following example.

Example 4.2. *The AF F is the initial framework. An arbitrary, normal, strong, weak or local expansion of it are F_E, F_N, F_S, F_W and F_L, respectively. Grey-highlighted arguments or attacks represent added information.*

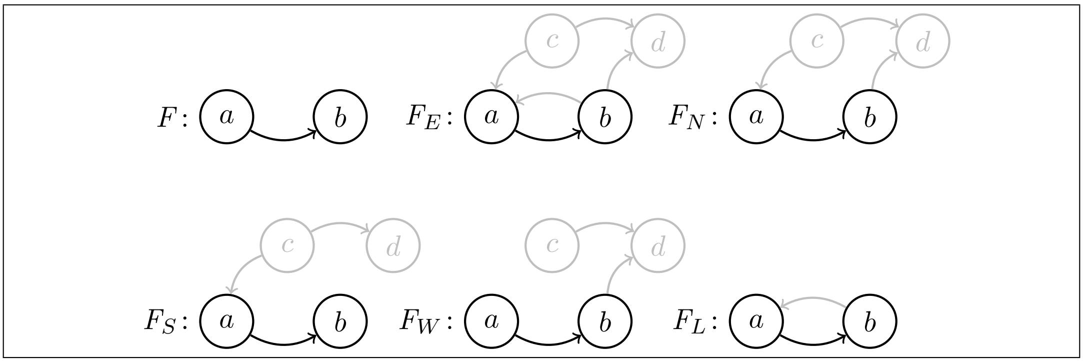


Figure 9: Different Kinds of Expansions

In 2014 the natural counter-parts (or more precisely, inverse operations) to arbitrary, normal and local expansions, so-called *deletions* were introduced [Baumann, 2014a]. Furthermore, the most general form of a dynamic scenario (where expansion and deletion can be combined) a so-called *update* were considered too. Analogously to expansions, any arbitrary deletion can be splitted in a normal and a local part. This means, a *normal deletion* retract arguments and their corresponding attacks. *Local deletions* in contrast delete attacks only.[15] The main motivation behind these notions stems from instantiation-based context. More precisely, a normal deletion on the abstract level correspond to deleting information of a given knowledge base. Changing to a more restrictive notion of attack correspond to a local deletion and a combination of both of them give rise to an arbitrary deletion on the abstract level. We proceed with the formal definitions as well as introductory examples.

[15]We mention that *strong* as well as *weak deletions* are not introduced/considered so far. They could be easily defined as inverse operations of their expansion counterparts. Before doing so, it would be interesting to identify real-world situations or instantiation-based dynamics were such kind of evolvements naturally occur.

Definition 4.3 ([Baumann, 2014a]). *Given an AF $F = (A, R)$, a set of arguments B and a set of attacks S as well as a further AF H. The AF*

$$G = (F \setminus [B, S]) \cup H := \Big((A, R \setminus S)|_{A \setminus B}\Big) \cup H$$

is called an update of F (for short, $F \asymp_U G$). An update is called a

1. *deletion ($F \succeq_D G$) iff $H = (\emptyset, \emptyset)$,*
2. *normal deletion ($F \succeq_{ND} G$) iff ($F \succeq_D G$) and $S = \emptyset$,*
3. *local deletion ($F \succeq_{LD} G$) iff $F \succeq_D G$ and $B = \emptyset$.*

Let us take a closer look at the definition of $G = (F \setminus [B, S]) \cup H$. The AF H plays the role of added information, i.e. it contains new arguments and attacks. Consequently, for all kind of deletions we have $H = (\emptyset, \emptyset)$ which leaves us with $G = F \setminus [B, S]$. The set B contains arguments which have to deleted. Since attacks depend on arguments we have to delete the attacks which involve arguments from B too. This operation is formally captured by the restriction of F to $A \setminus B$. Furthermore, the set S contains particular attacks which have to be deleted. This means, the pair $[B, S]$ does not necessarily have to be an AF. Therefore we use $[B, S]$ instead of (B, S). If clear from context we use B and S instead of $[B, \emptyset]$ or $[\emptyset, S]$, i.e. we simply write $F \setminus B$ as well as $F \setminus S$ for normal or local deletions, respectively.

Example 4.4. *The AF F represents the initial situation. An update as well as arbitrary, normal or local deletion of it are given by F_U, F_D, F_{ND} and F_{LD}. Grey-highlighted arguments or attacks represent added information in contrast to dotted arguments and attacks which represent deleted objects.*[16] *More formally, in accordance with Definition 4.3 we have that $F_U = (F \setminus [B, S]) \cup H$, $F_D = F \setminus [B, S]$, $F_{ND} = F \setminus B$, $F_{LD} = F \setminus S$ where the set of arguments $B = \{c\}$, the set of attacks $S = \{(b, a)\}$ and the AF $H = (\{b, d, e, f\}, \{(d, b), (e, f), (f, d)\})$.*

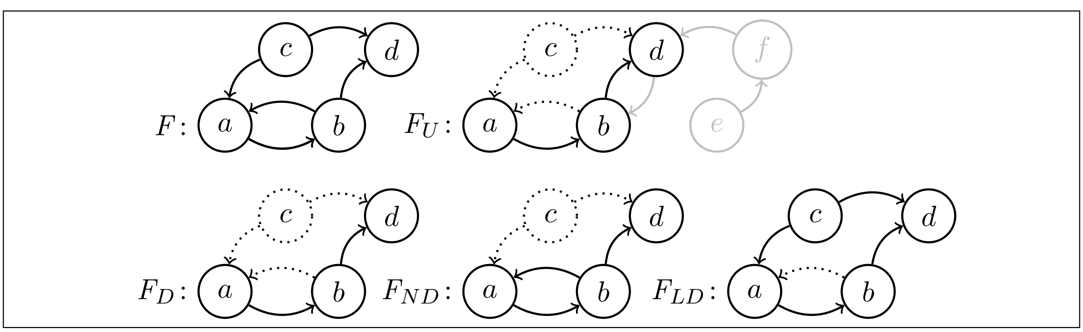
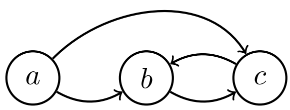


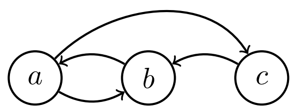
Figure 10: An Update and Different Kinds of Deletions

[16]This convention will be used throughout the whole section.

We now turn to the corresponding equivalence notions (cf. [Baumann and Strass, 2015, Section 3.8] for chronological order). Two AFs F and G are said to be *ordinarily equivalent* w.r.t. a semantics σ if they possess the same σ-extensions/labellings. In this case, we say that F and G possess the same *explicit* information. In contrast, sharing the same *implicit* information, i.e. being semantically indistinguishable w.r.t. any suitable future scenario is a much more demanding property which allows to replace F and G by each other without loss of semantical information.

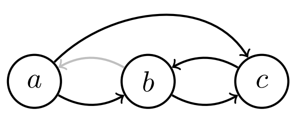
Example 4.5. *Consider the following AFs F and G. We have $\mathcal{E}_{pr}(F) = \mathcal{E}_{pr}(G) = \{\{a\}\}$. This means, F and G possess the same explicit information w.r.t. preferred semantics or in other words, they are ordinarily equivalent.*

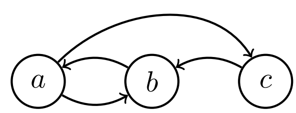
F:


G:


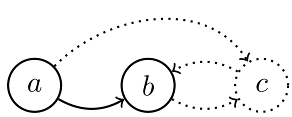
Assume that expansions as well deletions are the dynamic scenarios of interest. This means, we ask whether the AFs F and G even possess the same implicit information w.r.t. expansions or deletions, respectively? In order to give a negative answer one has to come up with one single dynamic scenario were the revised versions possess different preferred extensions. A positive answer in contrast is a statement about infinitely many dynamic scenarios (even in case of finite AFs). In this example, we give a negative answer for both modification types.

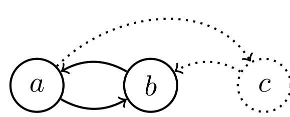
In case of expansions, we conjoin to both the AF $H = (\{a, b\}, \{(b, a)\})$. Consider the resulting frameworks below. We have $\mathcal{E}_{pr}(F \cup H) = \{\{a\}, \{b\}\}$ and since $G \cup H = G$ we obtain $\mathcal{E}_{pr}(G \cup H) = \{\{a\}\}$ without re-computing.

$F \cup H$:


$G \cup H$:


To reveal the inherent difference between F and G in case of deletions we may retract with the argument c. Consider the resulting (normal) deletions $F \setminus \{c\}$ and $G \setminus \{c\}$ of F or G, respectively. Now, $\{b\}$ becomes a preferred extension in $F \setminus \{c\}$ but still not in $G \setminus \{c\}$.

$F \setminus \{c\}$:


$G \setminus \{c\}$:


We now formally define what we precisely mean by possessing the same implicit information. As already stated, the first paper in this line of work was [Oikarinen and Woltran, 2011] engaged with characterizing *strong equivalence*. For the sake of clarity and comprehensibility we use the term *expansion equivalence* since strong equivalence [Oikarinen and Woltran, 2011, Definition 2] corresponds to semantical indistinguishability w.r.t. arbitrary expansions.

Definition 4.6. *Given a semantics σ. Two AFs F and G are*

1. *ordinarily equivalent w.r.t. σ (for short, $F \equiv^\sigma G$) iff $\sigma(F) = \sigma(G)$,*
2. *expansion equivalent w.r.t. σ (for short, $F \equiv^\sigma_E G$) iff for each AF H we have, $F \cup H \equiv^\sigma G \cup H$,*
3. *normal expansion equivalent w.r.t. σ (for short, $F \equiv^\sigma_N G$) iff for each AF H, such that $F \preceq_N F \cup H$ and $G \preceq_N G \cup H$ we have, $F \cup H \equiv^\sigma G \cup H$,*
4. *strong expansion equivalent w.r.t. σ (for short, $F \equiv^\sigma_S G$) iff for each AF H, such that $F \preceq_S F \cup H$ and $G \preceq_S G \cup H$ we have, $F \cup H \equiv^\sigma G \cup H$,*
5. *weak expansion equivalent w.r.t. σ (for short, $F \equiv^\sigma_W G$) iff for each AF H, such that $F \preceq_W F \cup H$ and $G \preceq_W G \cup H$ we have, $F \cup H \equiv^\sigma G \cup H$,*
6. *local expansion equivalent*[17] *w.r.t. σ (for short, $F \equiv^\sigma_L G$) iff for each AF H, such that $A(H) \subseteq A(F \cup G)$ we have, $F \cup H \equiv^\sigma G \cup H$.*
7. *update equivalent w.r.t. σ (for short, $F \equiv^\sigma_U G$) iff for any pair $[B, S]$ and any AF H we have, $(F \setminus [B, S]) \cup H \equiv^\sigma (G \setminus [B, S]) \cup H$,*
8. *deletion equivalent w.r.t. σ (for short, $F \equiv^\sigma_D G$) iff for any pair $[B, S]$ we have, $F \setminus [B, S] \equiv^\sigma G \setminus [B, S]$,*
9. *normal deletion equivalent w.r.t. σ (for short, $F \equiv^\sigma_{ND} G$) iff for any set of arguments B we have, $F \setminus B \equiv^\sigma G \setminus B$,*
10. *local deletion equivalent w.r.t. σ (for short, $F \equiv^\sigma_{LD} G$) iff for any set of attacks S we have, $F \setminus S \equiv^\sigma G \setminus S$,*

Remember that there are several relations between the considered dynamic scenarios. For instance, in accordance with Definitions 4.1 and 4.3, any normal expansion (deletion) is an arbitrary expansion (deletion). Furthermore, in the light of Definition 4.6, we certainly affirm that expansion equivalence is much more demanding then local expansion equivalence. In other words, local expansion equivalence of

[17] Note that a suitable AF H is not necessarily a local expansion of F and G in the sense of Definition 4.1. Nevertheless, we may loosely speak about local expansions.

two AFs is an immediate and unavoidable consequence of being expansion equivalent. Finally, any considered equivalence notion is at least as demanding then ordinary equivalence.[18] Please note that these relations do not depend on certain properties of a considered semantics. Consequently, Figure 11 gives a preliminary overview for such interrelations (arising from the definitions) between the introduced equivalence notions for any possible semantics. For reasons, which will become clearer later, we also consider the identity relation. For two equivalence notion Φ and Ψ we have $\Phi \subseteq \Psi$ iff there is a link from Φ to Ψ.

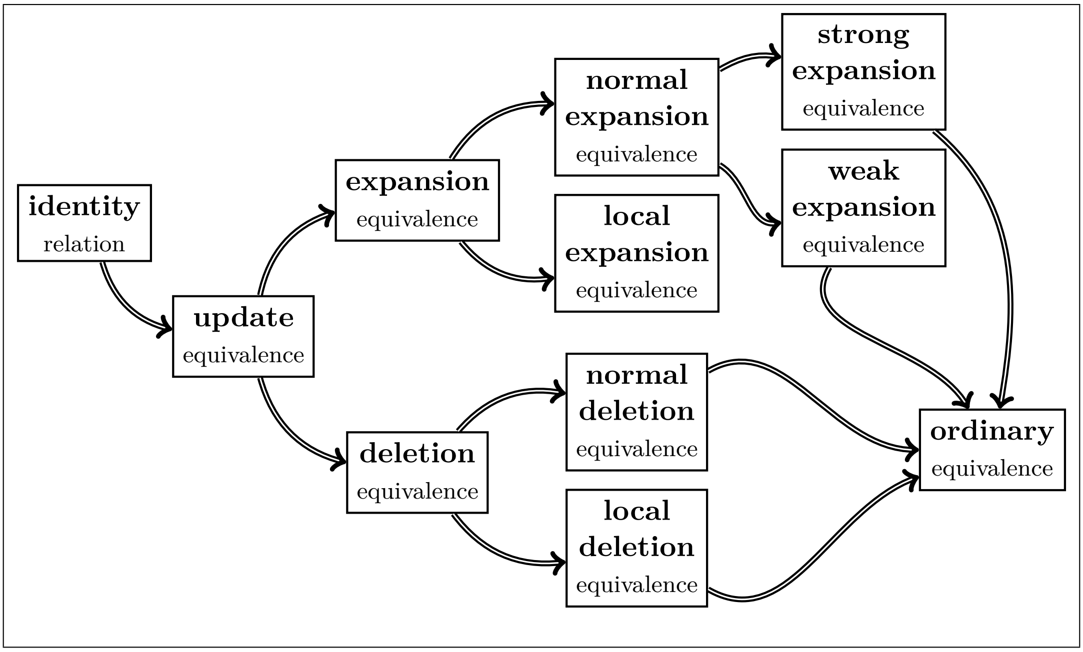


Figure 11: Preliminary Subset Relations between Equivalence Notions

In the remainder of this section we shed light on the question of *how* to determine whether two AFs are equivalent w.r.t. certain scenarios? As a by-product of these characterization results we will see that for many semantics the preliminary relations between the introduced equivalence notions depicted above can be delineated in a much more compact way. The majority of the presented characterization results is devoted to finite AFs as well as extension-based semantics. We will see that there are some differences if sticking to unrestricted frameworks or the corresponding labelling-based versions.

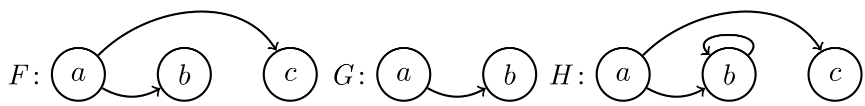
[18]The empty framework $(\emptyset, \emptyset)$ as well as the empty pair $[\emptyset, \emptyset]$ justifies this assertion for any type of expansions or deletions, respectively.

4.2 Characterization Theorems for Extension-based Semantics

4.2.1 The Central Notion of Expansion Equivalence

In order to get an idea of how to find a characterization we start with some reflections. For this purpose we consider the most restrictive semantics, namely the stable one as well as the most prominent type of equivalence, namely expansion equivalence. What are necessary features of expansion equivalence w.r.t. stable semantics, i.e. which properties are implied if two AFs F and G are expansion equivalent? In consideration of Figure 11 we deduce their ordinary equivalence, i.e. $\mathcal{E}_{stb}(F) = \mathcal{E}_{stb}(G)$. Note that possessing the same set of extensions neither imply sharing the same arguments nor sharing the same self-loops as shown in the following example.

Example 4.7. *Consider the AFs F, G and H. Each two of them are ordinarily equivalent since $\mathcal{E}_{stb}(F) = \mathcal{E}_{stb}(G) = \mathcal{E}_{stb}(H) = \{\{a\}\}$.*



The AFs $I_1 = (\{c\}, \emptyset)$ and $I_2 = (\{a,b,c\}, \{(b,a),(b,c)\})$ witness that neither F and G, nor F and H are expansion equivalent w.r.t. stable semantics. Convince yourself that $\mathcal{E}_{stb}(F \cup I_1) = \{\{a\}\} \neq \{\{a,c\}\} = \mathcal{E}_{stb}(G \cup I_1)$ and $\mathcal{E}_{stb}(F \cup I_2) = \{\{a\},\{b\}\} \neq \{\{a\}\} = \mathcal{E}_{stb}(G \cup I_2)$.

Restricting ourselves to finite AFs, it is not difficult to see that in case of expansion equivalence w.r.t. stable semantics the observed relation between non-sharing the same arguments/loops and non-equivalence does hold in general. In other words, possessing the same arguments as well as possessing the same loops are indeed necessary conditions for being expansion equivalent in the finite setting.

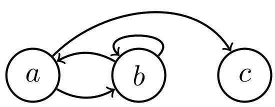
Let us summarize our observations in the following fact.

Fact 4.8. *Given two finite AFs F and G. If $F \equiv_E^{\mathcal{E}_{stb}} G$, then*

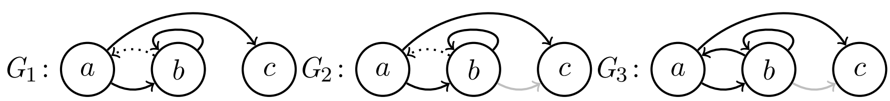
1. $\mathcal{E}_{stb}(F) = \mathcal{E}_{stb}(G)$,
2. $A(F) = A(G)$ *and*
3. $L(F) = L(G)$.

As already stated in Figure 11, being identical (i.e. $A(F) = A(G)$ and $R(F) = R(G)$) is sufficient for being expansion equivalent. Combining this undeniable fact together with the second and third items of Fact 4.8 encourages one to search for syntactical properties sufficient as well as necessary for being expansion equivalent. In order to guarantee the first item of Fact 4.8 we have to identify attacks which do not contribute anything when computing stable extensions. Moreover, these attacks which do not affect the evaluation of a given AF F have to be *redundant*, no matter how F is extended. Remember that being a stable extension can be simply verified by checking whether the set in question is conflict-free and possesses a full range.[19] This means, good candidates for "useless" attacks w.r.t. stable semantics should fulfill the following two properties: firstly, having or not having such an attack does not change the status of a set from being conflict-free to conflicting or vice versa and secondly, having or not having such an attack does not affect the range of a conflict-free set. Certainly, an attack (a, b) stemming from a self-defeating argument a does not change the conflict status of a certain set E. This can be seen as follows: If $a \in E$, then E was conflicting as well as remains conflicting after deleting or adding (a, b). Furthermore, if $a \notin E$, then E might be conflicting or not. In either case the conflict status of E does not change if (a,b) is added or removed since $\{a, b\} \not\subseteq E$. Finally, such an attack (a, b) might have an influence on the range of conflicting sets but it definitely has not in case of conflict-free sets since $a \notin E$ can not be questioned.

Example 4.9. *Consider the following AF F. We have, $\mathcal{E}_{stb}(F) = \{\{a\}\}$.*

F:


According to our considerations above adding or deleting an attack stemming from the self-defeating argument b does not change the semantics. Consider therefore the following three possible "manipulations".



Indeed, $\mathcal{E}_{stb}(F) = \mathcal{E}_{stb}(G_1) = \mathcal{E}_{stb}(G_2) = \mathcal{E}_{stb}(G_3) = \{\{a\}\}$ support our claims for the static case. We encourage the reader to try to do the impossible, namely semantically distinguish the AFs F and its manipulations by an arbitrary expansion.

It was the main result in [Oikarinen and Woltran, 2011] that expansion equivalence can be indeed decided by looking at the syntax only. The authors introduced

[19] The topic of *verifiability* of argumentation semantics σ was studied in [Baumann *et al.*, 2016b]. The main question is which (minimal amount of) information on top of conflict-free sets is exactly needed to determine whether a certain set is a σ-extension.

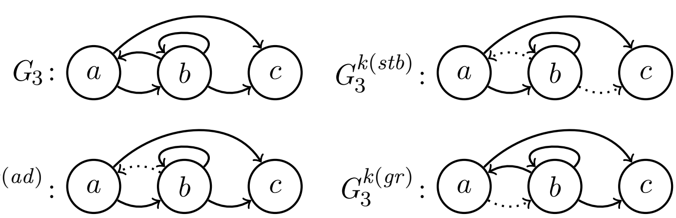
so-called *kernels* which are simply functions mapping each AF F to its redundancy-free version. This means, the kernel of an AF F does not possess any redundant attack. Put it differently, for any surviving attack exist at least one dynamic scenario were deleting this attack would cause a semantical difference. We proceed with the formal definition of the very first kernels already introduced in [Oikarinen and Woltran, 2011]. We sometimes call them *classical*.

Definition 4.10. *Let $\sigma \in \{stb, ad, gr, co\}$. The σ-kernel $k(\sigma) : \mathcal{F} \to \mathcal{F}$ with $k(\sigma)(F) = F^{k(\sigma)} = \left(A, R^{k(\sigma)}\right)$ for a given AF $F = (A, R)$ is defined as:*

$$\begin{aligned}
R^{k(stb)} &= R \setminus \{(a,b) \mid a \neq b, (a,a) \in R\},\\
R^{k(ad)} &= R \setminus \{(a,b) \mid a \neq b, (a,a) \in R, \{(b,a),(b,b)\} \cap R \neq \emptyset\},\\
R^{k(gr)} &= R \setminus \{(a,b) \mid a \neq b, (b,b) \in R, \{(a,a),(b,a)\} \cap R \neq \emptyset\},\\
R^{k(co)} &= R \setminus \{(a,b) \mid a \neq b, (a,a),(b,b) \in R\}.
\end{aligned}$$

In order to get an idea of how the classical kernels work we proceed with an example.

Example 4.11. *Consider again the AF G_3 depicted in Example 4.9. We apply now all classical kernels.*



The stable kernel deletes all attacks (a,b) stemming from a self-defeating argument a. A deletion of (a,b) in case of the grounded kernel additionally requires that a is counter-attacked by b or b is self-defeating or both. Interchanging a and b yields the condition for deletion in case of the grounded kernel. Finally, $G_3^{k(co)} = G_3$ since deleting an attack (a,b) w.r.t. the complete kernel requires that both arguments a and b are self-defeating.

Before turning to characterization theorems, we collect some useful properties of the introduced kernels. The following fact contains intrinsic properties of the classical kernels.[20] More precisely, any classical kernel k is *node-preserving* and *loop-preserving*, i.e. the sets of arguments and self-defeating arguments do not change

[20] Although most of the properties are immediately clear even in case of unrestricted frameworks we will state all of them for finite AFs only as done in the existing literature. The same applies to Fact 4.18. Some results regarding unrestricted frameworks can be found in Section 4.3.

if applying k. Moreover, in the absence of self-loops, each AF coincides with its classical kernels. Furthermore, the decision whether an attack (a, b) has to be deleted does not depend on further arguments than a and b. Put differently, the reason of being redundant is *context-free*, i.e. it stems from the arguments themselves. The last two properties claim that equality of kernels is *robust* w.r.t. further compositions as well as deleting arguments and corresponding attacks. For a given AF $F = (B, S)$ we use $A(F), R(F)$ and $L(F)$ to refer to its arguments, attacks and self-defeating arguments, i.e. $A(F) = B$, $R(F) = S$ and $L(F) = \{a \in A(F) \mid (a, a) \in R(F)\}$.

Fact 4.12 (cf. [Oikarinen and Woltran, 2011; Baumann, 2014a]). *Given $k \in \{k(stb), k(ad), k(gr), k(co)\}$. For any finite AF F we have:*

1. $A(F) = A\left(F^k\right)$, *(node-preserving)*
2. $L(F) = L\left(F^k\right)$, *(loop-preserving)*
3. $L(F) = \emptyset \;\Rightarrow\; F = F^k$ *and* *(sufficient condition for identity)*
4. $(a, b) \in R\left(F^k\right) \Leftrightarrow (a, b) \in R\left((F|_{\{a,b\}})^k\right)$. *(context-freeness)*

Furthermore, for finite AFs F and G we have:

4. *If $F^k = G^k$, then $(F \cup H)^k = (G \cup H)^k$ for any finite AF H* *($\cup$-robustness)*
5. *If $F^k = G^k$, then $(F \setminus B)^k = (G \setminus B)^k$ for any finite set of args B.* *($\setminus$-robustness)*

We proceed with extrinsic properties, i.e. features of kernels in presence of semantics. More precisely, stable, admissible, grounded and complete semantics are insensitive w.r.t. the application of their corresponding classical σ-kernel, i.e. the set of σ-extensions remains unchanged. Furthermore, the admissible kernel neither effects semi-stable, eager, preferred and ideal semantics. Similarly in case of stable kernel and stage semantics.

Fact 4.13 ([Oikarinen and Woltran, 2011; Gaggl and Woltran, 2013]). *For any finite AF F we have:*

1. $\mathcal{E}_\sigma(F) = \mathcal{E}_\sigma\left(F^{k(\sigma)}\right)$ *for $\sigma \in \{stb, ad, gr, co\}$,*
2. $\mathcal{E}_\sigma(F) = \mathcal{E}_\sigma\left(F^{k(ad)}\right)$ *for $\sigma \in \{ss, eg, pr, il\}$ and*
3. $\mathcal{E}_{stg}(F) = \mathcal{E}_{stg}\left(F^{k(stb)}\right)$.

As already mentioned, kernels play a decisive role in deciding expansion equivalence. In general, we say that an equivalence notion $\equiv$ *is characterizable through* k or simply, k *is a characterizing kernel (of* $\equiv$*)* if for any two AFs F and G, $F \equiv G$ iff $F^k = G^k$. This means, proving whether two frameworks are equivalent can be done by simply checking whether the corresponding kernels are identical. Note that all classical kernels can be efficiently constructed from a given AF. The following main theorem states that for all nine considered semantics σ there is a certain classical kernel k, s.t. expansion equivalence w.r.t. σ is characterizable through k in the finite setting. This is a very remarkable result since expansion equivalence is defined semantically. For instance, two finite AFs F and G are expansion equivalent w.r.t. stable semantics if and only if the associated stable kernels $F^{k(stb)}$ and $G^{k(stb)}$ are syntactically equal. Observe that there is no need to introduce further kernels since one single kernel may serve for different semantics.

Theorem 4.14. *[Oikarinen and Woltran, 2011; Gaggl and Woltran, 2013] For finite AFs F and G we have:*

1. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F^{k(\sigma)} = G^{k(\sigma)}$ *for any* $\sigma \in \{stb, ad, co, gr\}$,
2. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F^{k(ad)} = G^{k(ad)}$ *for any* $\sigma \in \{pr, il, ss, eg\}$ *and*
3. $F \equiv_E^{\mathcal{E}_{stg}} G \Leftrightarrow F^{k(stb)} = G^{k(stb)}$.

Having Theorem 4.14 at hand we can now formally verify that all AFs depicted in Example 4.9 are expansion equivalent w.r.t. stable semantics. This means, the recommended search for arbitrary expansions revealing semantical difference between them will never succeed. As an aside, one might get the impression that the syntactical characterization presented in Theorem 4.14 is somehow unique. This is not true. Consider therefore the equivalence class $[F]_E^{\mathcal{E}_{stb}} = \{G \mid F \equiv_E^{\mathcal{E}_{stb}} G\}$ induced by F. Mathematically speaking, the stable kernel $F^{k(stb)}$ represents the least (w.r.t. subgraph-relation) element in $[F]_E^{\mathcal{E}_{stb}}$. It is not difficult to prove that $[F]_E^{\mathcal{E}_{stb}}$ even possesses a greatest element, namely $F^{k'(stb)} = (A(F), R(F) \cup \{(a,b) \mid a \neq b, (a,a) \in R(F)\})$, i.e. the framework resulting from F by adding (instead of deleting) all redundant attacks. In case of finite AFs it can be shown with reasonable effort that expansion equivalence w.r.t. stable semantics is characterizable through $k'(stb)$ too. In the same manner, all other semantics considered in Theorem 4.14 possess alternative "greatest elements" characterizations. We will see that the so-called *naive kernel* (compare Definition 4.17) provides such a kind of characterization for naive semantics. The reason for this "choice" is simply that the induced equivalence classes do not necessarily possess a least element in case of naive semantics.

Finally, let us turn to the more exotic cf2 as well as stage2 semantics which are defined via a recursive schema based on the decomposition of AFs along their strongly connected components (SCCs). These semantics are exceptional regarding expansion equivalence since in contrast to all other semantics considered in this section we have that even attacks between two self-attacking arguments are *meaningful*. This means, the presence or absence of such attacks may change the outcome of an AF. Moreover, it turned out that any attack is non-redundant. In summary, expansion equivalence coincides with syntactical identity or more formally, for any finite AF F, $\left|[F]_E^{\mathcal{E}_{cf2}}\right| = \left|[F]_E^{\mathcal{E}_{stg2}}\right| = |\{F\}| = 1$.

Theorem 4.15. *[Gaggl and Woltran, 2013; Gaggl and Dvořák, 2016] Given semantics $\sigma \in \{cf2, stg2\}$. For finite AFs F and G we have,*

$$F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F = G.$$

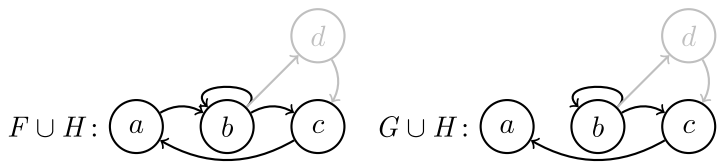
4.2.2 Further Equivalence Notions Characterizable through Kernels

Let us turn to the remaining equivalence notions? Are there similar syntax-based characterization results?

Weaker Notions of Expansion Equivalence Let us consider less demanding notions than expansion equivalence, e.g. normal and local expansion equivalence. In consideration of Definition 4.1 we do not have good reasons to believe that two AFs could be semantically distinguished by normal or local expansions, given that we only have a witnessing arbitrary expansion showing their non-equivalence. It was one surprising result in this line of research, that for many semantics, expansion equivalence coincide with definitorially weaker notions of it. This implies that weaker notions than expansion equivalence can be characterized by classical kernels too. The first results in this respect were already given in [Oikarinen and Woltran, 2011, Theorem 8]. The authors showed that for some semantics expansion equivalence and local expansion equivalence coincide if considering finite AFs. It is worthwhile to gain a thorough understanding of this relation since it actually means that if there is an arbitrary expansion which semantically distinguish two finite AFs, than there has to be a local expansion doing likewise. Later it was shown that even normal expansion equivalence coincides with expansion equivalence for a whole bunch of semantics [Baumann, 2012a]. Interestingly, in contrast to local expansion equivalence, there are (to the best of our knowledge) no semantics together with witnessing AFs known which show that this coincidence does not hold in general.

Example 4.16. *Consider the following AFs F and G. According to Theorem 4.14 they are not expansion equivalent w.r.t. preferred semantics since $F^{k(ad)} = F \neq G = G^{k(ad)}$.*

As already stated (up to now) normal expansion equivalence coincides with expansion equivalence for any considered semantics. One possible scenario which makes the predicted different behaviour explicit is the following.



Formally, we define $H = (\{b,c,d\},\{(b,d),(d,c)\})$ and we obtain $\{\{a,d\}\} = \mathcal{E}_{pr}(F \cup H) \neq \{\emptyset\} = \mathcal{E}_{pr}(G \cup H)$. We encourage the reader to try to find a witnessing example showing that F and G are not local expansion equivalent w.r.t. preferred semantics. Due to Theorem 4.20 there has to be at least one distinguishing local expansion.

How do the semantics behave in case of strong expansion equivalence? Remember, a special feature of strong expansions is that a former attack between old arguments will never become a counterattack to an added attack. In this sense, former attacks do not play a role with respect to being a potential defender of an added argument. Hence, in contrast to arbitrary expansions where such attacks might be relevant, we may delete them without changing the behavior with respect to further evaluations. To make this point clearer consider again the AF $F \cup H$ depicted in Example 4.16. Note that the already existing attack (a,b) in F becomes a defending attack of the newly added argument d. This means, such attacks in fact play an important role with respect to further evaluation in case of arbitrary expansions. It was one main result in [Baumann, 2012a] that for some semantics attacks like (a,b) in F are indeed redundant w.r.t. strong expansions. Even more surprising, strong expansion equivalence is characterizable through kernels. Therefore, more involved kernel definitions, so-called *σ-*-kernels* had to be introduced. These kernels allow more deletions than their classical counterparts for expansion equivalence. In contrast to them, σ-*-kernels are *context-sensitive*, i.e. the question whether an attack (a,b) is redundant can not be answered by considering the arguments a and b only [Baumann, 2014a].

The first three kernels presented in the definition below were firstly introduced in [Baumann, 2012a] with the objective to characterize strong expansion equivalence with respect to certain semantics. For the sake of completeness we also present the so-called *stg-*-kernel* as well as *na-kernel* [Baumann and Woltran, 2016; Baumann *et al.*, 2016b].[21]

Definition 4.17. *Let* $\sigma \in \{ad, gr, co, stg\}$. *The* σ-**-kernel* $k^*(\sigma) : \mathcal{F} \to \mathcal{F}$ *with* $k^*(\sigma)(F) = F^{k^*(\sigma)} = \left(A, R^{k^*(\sigma)}\right)$ *for a given AF* $F = (A, R)$ *is defined as:*

$$
\begin{aligned}
R^{k^*(ad)} &= R \setminus \{(a,b) \mid a \neq b, ((a,a) \in R \wedge \{(b,a),(b,b)\} \cap R \neq \emptyset) \\
&\qquad \vee ((b,b) \in R \wedge \forall c\ ((b,c) \in R \to \{(a,c),(c,a),(c,c),(c,b)\} \cap R \neq \emptyset))\}, \\
R^{k^*(gr)} &= R \setminus \{(a,b) \mid a \neq b, ((b,b) \in R \wedge \{(a,a),(b,a)\} \cap R \neq \emptyset) \\
&\qquad \vee ((b,b) \in R \wedge \forall c\ ((b,c) \in R \to \{(a,c),(c,a),(c,c)\} \cap R \neq \emptyset))\}, \\
R^{k^*(co)} &= R \setminus \{(a,b) \mid a \neq b, ((a,a),(b,b) \in R) \vee ((b,b) \in R \wedge (b,a) \notin R \\
&\qquad \wedge \forall c\ ((b,c) \in R \to \{(a,c),(c,a),(c,c),(c,b)\} \cap R \neq \emptyset))\}, \\
R^{k^*(stg)} &= R \setminus \{(a,b) \mid a \neq b, (a,a) \in R \vee \forall c\ (c \neq a \to (c,c) \in R)\} \\
R^{k(na)} &= R \cup \{(a,b) \mid a \neq b, \{(a,a),(b,a),(b,b)\} \cap R \neq \emptyset\}.
\end{aligned}
$$

The latter represents the so-called na-kernel $F^{k(na)} = \left(A, R^{k(na)}\right)$.

For an illustrating example we refer the reader to Example 4.19. Analogously to Fact 4.12 we collect some properties of the newly introduced kernels. The first three properties are immediately clear by definition.[22] The robustness w.r.t. deletions and corresponding attacks is less obvious but it is already shown for all considered kernels (except the *stg*-*-kernel) in case of finite AFs (cf. [Baumann, 2014a, Theorems 6,14]).

Fact 4.18. *Given kernels* $k \in \{k^*(ad), k^*(gr), k^*(co), k^*(stg), k(na)\}$ *as well as* $k^* \in \{k^*(ad), k^*(gr), k^*(co), k^*(stg)\}$. *For two finite AFs F and G we have:*

1. $A(F) = A\left(F^k\right)$, *(node-preserving)*
2. $L(F) = L\left(F^k\right)$, *(loop-preserving)*
3. $L(F) = \emptyset \Rightarrow F = F^{k^*}$ *and* *(sufficient condition for identity)*
4. *If* $F^k = G^k$, *then* $(F \setminus B)^k = (G \setminus B)^k$ *for any finite set of arguments B.* *($\setminus$-robustness)*

[21] As an aside, we use the supplement "*", whenever the kernel in question is non-classical and expansion equivalence is already characterized by another kernel.

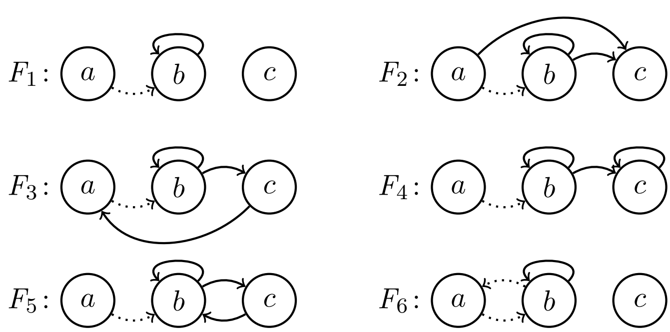
[22] The AF $F = (\{a,b\}, \{(a,b)\})$ shows that the naive kernel has to be excluded from item 3 of Fact 4.18 since $F^{k(na)} = (\{a,b\}, \{(a,b),(b,a)\}) \neq F$.

Let us consider the *ad*-*-kernel (which, as we shall see, characterizes strong expansion equivalence for preferred semantics) in more detail. Consider the first disjunct. This first condition is exactly the same as in case of the *ad*-kernel (compare Definition 4.10), i.e. an attack (a, b) has to be deleted if a is self-attacking and at least one of the attacks (b, a) or (b, b) exist. The second disjunct provides one with further options to delete an attack (a, b), namely if b is self-defeating and furthermore, for all arguments c which are attacked by b at least one of the following conditions has to be fulfilled:

1. a attacks c,
2. c attacks a,
3. c attacks c,
4. c attacks b.

The motivation for the second disjunct is the following: At first observe that b cannot be an element of any conflict-free set. Consequently, in case of strong expansions the attack (a, b) may only be relevant with respect to the defense of c. In the first three cases this relevance becomes unimportant since $\{a, c\}$ is conflicting. In the fourth case the redundancy of (a, b) with respect to the defense of c is given by the fact that c already defends itself against b. Please note that the consideration of $c = a$ or $c = b$ is not excluded by Definition 4.17. The following frameworks exemplify different cases.

Example 4.19. *The following graphs show six frameworks and their corresponding ad-*-kernels. The dotted attacks represent initial attacks which have to be deleted if applying the ad-*-kernel.*



Consider the formal description of $R^{k^*(ad)}$ as given in Definition 4.17. The AF F_1 is somehow the base case since the only argument c, s.t. $(b, c) \in R(F_1)$ is b itself.

Since $(b,b) \in R(F_1)$ we deduce that the considered intersection is non-empty and thus, the deletion of (a,b) is justified. The subsequent four frameworks F_2, F_3, F_4 and F_5 are the base case plus one further argument c different from a and b, s.t. for any $i \in \{2,3,4,5\}$, $(b,c) \in R(F_i)$. The last framework F_6 illustrates the case b counterattacks a. Note that the reason to delete (a,b) is somehow self-referential since (additionally to the base case) it is justified by $(a,b) \in R(F_6)$. Due to the first disjunct (i.e. just like in case of the classical ad-kernel) even the attack (b,a) has to be deleted.

We proceed with further characterization theorems.[23] An comprehensive overview of equivalence notion and their characterizing kernels in case of finite AFs and extension-based semantics is presented in Figure 12.

Theorem 4.20. *[Oikarinen and Woltran, 2011; Baumann„ 2012a; Baumann and Woltran, 2016] For finite AFs F and G we have the following coincidences.*

1. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F \equiv_N^{\mathcal{E}_\sigma} G$ *for* $\sigma \in \{stg, stb, ss, eg, ad, pr, il, gr, co, na, cf2, stg2\}$,
2. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F \equiv_L^{\mathcal{E}_\sigma} G$ *for* $\sigma \in \{ss, eg, ad, pr, il, na\}$ *and*
3. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F \equiv_S^{\mathcal{E}_\sigma} G$ *for* $\sigma \in \{stg, stb, ss, eg, na\}$.

Furthermore, for any two finite AFs F and G we have the following non-classical characterizations.

4. $F \equiv_L^{\mathcal{E}_{stg}} G \Leftrightarrow F^{k^*(stg)} = G^{k^*(stg)}$,
5. $F \equiv_S^{\mathcal{E}_\sigma} G \Leftrightarrow F^{k^*(ad)} = G^{k^*(ad)}$ *for* $\sigma \in \{ad, pr, il\}$,
6. $F \equiv_S^{\mathcal{E}_\sigma} G \Leftrightarrow F^{k^*(\sigma)} = G^{k^*(\sigma)}$ *for* $\sigma \in \{co, gr\}$ *and*
7. $F \equiv_E^{\mathcal{E}_{na}} G \Leftrightarrow F^{k(na)} = G^{k(na)}$.

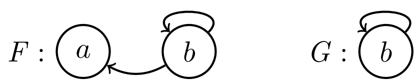
At this point we want to highlight a very surprising relation. Remember that normal expansion equivalence and normal deletion equivalence are completely unrelated in the general picture (cf. Figure 11). The observation that the characterizing kernels (including the identity map in case of cf2 and stage2 semantics) of normal expansion equivalence w.r.t. all considered semantics in this section satisfy \-robustness (cf. Facts 4.12 and 4.18) reveals that normal expansion equivalence implies normal deletion equivalence for these semantics.

[23] Please note that the results in case of cf2 and stage2 semantics have never been published before.

Corollary 4.21. *Given* $\sigma \in \{stg, stb, ss, eg, ad, pr, il, gr, co, na, cf2, stg2\}$ *and two finite AFs* F*and* G*. We have:* $F \equiv_N^{\mathcal{E}_\sigma} G \Rightarrow F \equiv_{ND}^{\mathcal{E}_\sigma} G$.

The attentive reader may have noticed that we do not have characterized local expansion equivalence w.r.t. stable, complete as well as grounded extension-based semantics. We mention that all three equivalence notions are already characterized but the characterization theorems are not purely kernel-based (cf. [Oikarinen and Woltran, 2011, Theorems 9,10,11]). Furthermore, it can be checked that none of the kernels presented in Definitions 4.10 and 4.17 serve as a characterizing kernel. Consider therefore the following example [Oikarinen and Woltran, 2011, Example 15].

Example 4.22. *The AFs* F *and* G *are local expansion equivalent w.r.t. stable semantics. This can be seen as follows. Given an AF* H*, s.t.* $A(H) \subseteq \{a, b\}$*. If* $(a, b) \in R(H)$ *and* $(a, a) \notin R(H)$*, we obtain* $\mathcal{E}_{stb}(F \cup H) = \mathcal{E}_{stb}(G \cup H) = \{\{a\}\}$*. Otherwise,* $\mathcal{E}_{stb}(F \cup H) = \mathcal{E}_{stb}(G \cup H) = \emptyset$*.*



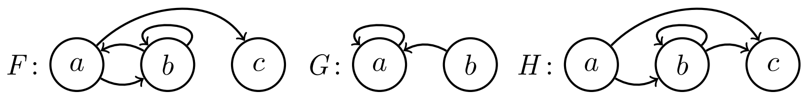
Remember that all introduced kernels are node-preserving (Facts 4.12 and 4.18). Consequently, none of them may serve as a characterizing kernel for local expansion equivalence w.r.t. stable semantics.

We mention that weak expansion equivalence is already characterized in case of stable semantics [Baumann, 2012a, Proposition 3] as well as admissible, preferred and complete semantics [Baumann and Brewka, 2015, Theorem 1]. All characterization results are not kernel-based. For instance, two AFs are weak expansion equivalent w.r.t. stable semantics iff both do not possess stable extensions at all or if they share the same arguments and at the same time possess the same stable extensions. Consequently, $F = (\{a\}, \{(a, a)\})$ and $G = (\{a, b, c\}, \{(a, b), (b, c), (c, a)\})$ are weak expansion equivalent w.r.t. stable semantics. Both frameworks witness that any potential characterizing kernel k is necessarily neither node- nor loop-preserving.

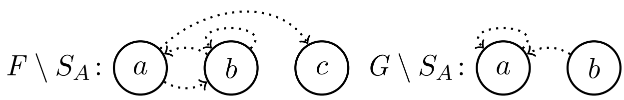
As a final note, we are not aware of any study of weaker notions of expansion equivalence in case of cf2 as well as stage2 semantics.

Notions of Deletion Equivalence and Update Equivalence We start with local deletion equivalence. Remember that local deletion equivalent AFs cannot be semantically distinguished by deleting a certain set of attacks in both simultaneously. How "strong" is this notion? Are there redundant attacks or even redundant arguments?

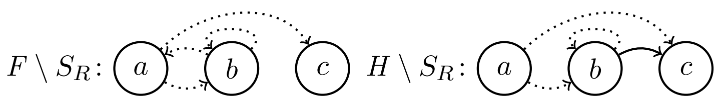
Example 4.23. *Consider the following AFs F, G and H.*



The AFs F and G do not possess the same arguments. Let us delete all occurring attacks, i.e. $S_A = R(F) \cup R(G)$. We obtain the following local deletions where $\{a,b,c\} \in \mathcal{E}_\sigma(F \setminus S_A) \setminus \mathcal{E}_\sigma(G \setminus S_A)$ for all semantics σ considered in this section.



The AFs F and H possess the same arguments but differ in their attack-relation, e.g. $(b,c) \in R(H) \setminus R(F)$. This difference can be made more explicit if defining $S_R = (R(F) \cup R(H)) \setminus \{(b,c)\}$. Consider the resulting local deletions.



Once again we have $\{a,b,c\} \in \mathcal{E}_\sigma(F \setminus S_R)$ for all known semantics σ and $\{a,b,c\} \notin \mathcal{E}_\sigma(H \setminus S_R)$ if assuming conflict-freeness of the considered semantics.

The observations above indicate that there is not much space for redundancy in case of local expansion equivalence and indeed, it was one main result in [Baumann, 2014a] that local expansion equivalence collapse to identity for all semantics considered in this section. Moreover, instead of proving this one by one for any semantics the author followed the line in [Baroni and Giacomin, 2007] and provide abstract criteria guaranteeing the coincidence with syntactical identity. These criteria are very weak requirements, namely *conflict-freeness* ($\mathcal{CF}$) and the principle of *isolate-inclusion* ($\mathcal{II}$). The latter is fulfilled by a semantics σ iff for any AF F, the set of all isolated arguments is contained in at least one σ-extension. Observe that any considered semantics apart from stable semantics satisfy $\mathcal{II}$.[24]

Theorem 4.24 ([Baumann, 2014a])**.** *Given a semantics σ satisfying $\mathcal{CF}$ and $\mathcal{II}$. For two finite AFs F and G we have:*

$$F \equiv^{\mathcal{E}_\sigma}_{LD} G \Leftrightarrow F = G.$$

Since being identical implies local deletion equivalence we deduce that all equivalence notion "inbetween" them collapse to identity too (cf. Figure 11).

[24] Note that only universally defined semantics σ, i.e. semantics which warrants the existence of at least one σ-extension (cf. Definition 2.3), may satisfy isolate-inclusion. A counter-example in case of stable semantics is given by $F = (\{a,b\}, \{(b,b)\})$. Obviously, a is isolated but $\mathcal{E}_{stb}(F) = \emptyset$. Nevertheless, local expansion equivalence in case of stable semantics collapse to identity too.

Proposition 4.25. *Given a semantics σ satisfying $\mathcal{CF}$ and $\mathcal{II}$. For any two finite AFs F and G we have:*

$$F \equiv_U^{\mathcal{E}_\sigma} G \Leftrightarrow F \equiv_D^{\mathcal{E}_\sigma} G \Leftrightarrow F = G.$$

This means, for semantics satisfying conflict-freeness and isolate-inclusion any argument/attack may play a crucial role with respect to further evaluations if updates, deletions or local deletions are considered. Note that the results may apply to future semantics. In order to refine the general picture (as depicted in Figure 11) for the semantics considered in this section we state the following relations.[25]

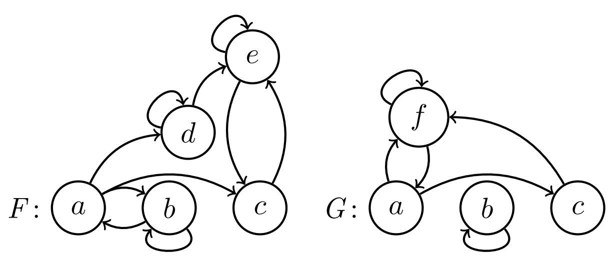
Corollary 4.26. *Let $\sigma \in \{stg, stb, ss, eg, ad, pr, il, gr, co, na, cf2, stg2\}$. For any two finite AFs F and G we have:*

1. $F \equiv_U^{\mathcal{E}_\sigma} G \Leftrightarrow F \equiv_D^{\mathcal{E}_\sigma} G \Leftrightarrow F \equiv_{LD}^{\mathcal{E}_\sigma} G \Leftrightarrow F = G,$ *(k = id)*
2. $F \equiv_D^{\mathcal{E}_\sigma} G \Rightarrow F \equiv_E^{\mathcal{E}_\sigma} G,$ *(deletion vs. expansion)*
3. $F \equiv_{LD}^{\mathcal{E}_\sigma} G \Rightarrow F \equiv_L^{\mathcal{E}_\sigma} G.$ *(local versions)*

4.2.3 The Exceptional Case of Normal Deletion Equivalence

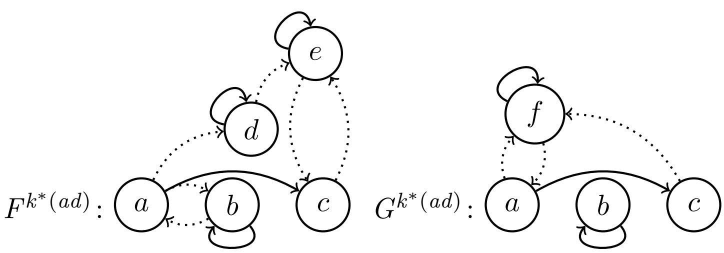
Normal deletion equivalence, where the retraction of arguments and corresponding attacks is considered, is exceptional in several regards. Firstly, the characterization theorems for admissible, complete and grounded semantics partially rely on σ-*-kernels. Remember that these kernels were originally introduced to characterize strong expansion (cf. Theorem 4.20). Secondly, normal deletion equivalent AFs do not even have to share the same arguments and thus give space for simplifications.

Example 4.27. *Consider the following AFs F and G. We have $\mathcal{E}_{ad}(F) = \mathcal{E}_{ad}(G) = \{\emptyset, \{a\}\}$. Even more, for any set of arguments B, $\mathcal{E}_{ad}(F \setminus B) = \mathcal{E}_{ad}(G \setminus B)$ showing their normal deletion equivalence, i.e. $F \equiv_{ND}^{\mathcal{E}_{ad}} G$.*



[25]The results in case of cf2 and stage2 semantics have never been published before. It can be checked that both semantics satisfy the preconditions of Theorem 4.25.

Observe that the non-shared arguments d, e and f do not play a role for the evaluation w.r.t. admissible semantics since firstly, they are self-defeating and thus cannot be part of an admissible set; and secondly, if they attack a non-looping argument shared by both arguments, e.g. e attacks c in F or f attacks a in G, then they are counter-attacked by the same argument, i.e. c attacks e in F and a attacks f in G. Consequently, they cannot influence potential admissible sets being a subset of $\{a, b\}$. *Finally, let us consider the ad-*-kernel of both frameworks (cf. Example 4.19 and the comments above for more details).*



Obviously, F and G do not possess the same kernels but note that their restrictions to the shared arguments do, i.e. $\left(F|_{\{a,b,c\}}\right)^{k^*(ad)} = \left(G|_{\{a,b,c\}}\right)^{k^*(ad)}$.

It turned out that the issues raised in Example 4.27 are essential to characterize normal deletion equivalence w.r.t. admissible semantics. In case of complete and grounded semantics slightly different conditions have to be fulfilled, namely w.r.t. the non-shared arguments we have "it is forbidden to be attacked" instead of "counter-attack if attacked" like in case of admissible semantics and furthermore, instead of the ad-*-kernel the corresponding σ-*-kernels are used. Consider therefore the following definition and the characterization theorem. We use Δ to denote the symmetric difference, i.e. $A\Delta A' = A \setminus A' \cup A' \setminus A$. Moreover, $NL(F) = A(F) \setminus L(F)$, i.e. $NL(F)$ contains all arguments of F which are not self-defeating.

Definition 4.28. *Given* $F = (A, R)$ *and* $G = (A', R')$ *and let* $\sigma \in \{co, gr\}$.

1. $Loop(F,G) \Leftrightarrow_{def} L\left(F \cup G|_{A\Delta A'}\right) = A\Delta A'$,
   ("non-shared args are self-defeating")
2. $Att^{ad}(F,G) \Leftrightarrow_{def} \forall b \in A \setminus A' \; \forall a \in NL(F|_{A\cap A'}) : ((b,a) \in R \rightarrow (a,b) \in R)$
   $\land \forall b \in A' \setminus A \; \forall a \in NL(G|_{A\cap A'}) : ((b,a) \in R' \rightarrow (a,b) \in R')$,
   ("counter-attack if attacked")
3. $Att^{\sigma}(F,G) \Leftrightarrow_{def} \forall b \in A \setminus A' \; \forall a \in NL(F|_{A\cap A'}) : (b,a) \notin R$
   $\land \forall b \in A' \setminus A \; \forall a \in NL(G|_{A\cap A'}) : (b,a) \notin R'$.
   ("it is forbidden to be attacked")

Theorem 4.29 ([Baumann, 2014a]). *Let $\sigma \in \{ad, co, gr\}$. Given two finite AFs $F = (A, R)$ and $G = (A', R')$ and let $I = A \cap A'$,*

$$F \equiv^{\mathcal{E}_\sigma}_{ND} G \Leftrightarrow Loop(F,G),\ \ Att^{\sigma}(F,G), (F|_I)^{k^*(\sigma)} = (G|_I)^{k^*(\sigma)}.$$

In contrast to admissible, complete and grounded semantics where normal deletion equivalence is indeed weaker than normal expansion equivalence we observe that these notions coincide in case of stable semantics. This means, normal deletion equivalence w.r.t. stable semantics is characterized by the classical stable kernel too.

The following theorem corrects the corresponding result in [Baumann, 2014a, Theorem 10] which did not take into account that an empty framework possess a stable extension, namely the empty one.[26]

Theorem 4.30. *For finite AFs F and G we have:*

$$F \equiv^{\mathcal{E}_{stb}}_{ND} G \Leftrightarrow F^{k(stb)} = G^{k(stb)}.$$

Proof. ($\Rightarrow$) We show the contrapositive, i.e. $F^{k(stb)} \neq G^{k(stb)} \Rightarrow F \not\equiv^{\mathcal{E}_{stb}}_{ND} G$.
1^{st} case: Assume $A\left(F^{k(stb)}\right) \neq A\left(G^{k(stb)}\right)$ and w.l.o.g. let $a \in A\left(F^{k(stb)}\right) \setminus A\left(G^{k(stb)}\right)$. Since the stable kernel is node-preserving (Fact 4.12) we obtain $G \setminus B = (\emptyset, \emptyset)$ and $F \setminus B \in \{(\{a\}, \emptyset), (\{a\}, \{(a,a)\})\}$ if $B = (A(F) \cup A(G)) \setminus \{a\}$. In either case, $\emptyset \in \mathcal{E}_{stb}(G) \setminus \mathcal{E}_{stb}(F)$ since $\mathcal{E}_{stb}(F) \in \{\emptyset, \{\{a\}\}\}$. From now on we assume $A\left(F^{k(stb)}\right) = A\left(G^{k(stb)}\right)$.
2^{nd} case: Consider $R\left(F^{k(stb)}\right) \neq R\left(G^{k(stb)}\right)$ and w.l.o.g. let $(a, b) \in R\left(F^{k(stb)}\right) \setminus R\left(G^{k(stb)}\right)$. Let $a = b$. Remember that the stable kernel is loop-preserving (Fact 4.12). Therefore, $(a, a) \in R(F) \setminus R(G)$. We obtain $G \setminus B = (\{a\}, \emptyset)$ and $F \setminus B = (\{a\}, \{(a, a)\})$ if $B = (A(F) \cup A(G)) \setminus \{a\}$. Hence, $\emptyset = \mathcal{E}_{stb}(F) \neq \mathcal{E}_{stb}(G) = \{\{a\}\}$. From now on we assume $L\left(F^{k(stb)}\right) = L\left(G^{k(stb)}\right)$. Consider now $a \neq b$. Consequently, $(a, b) \in R(F)$ and $(a, a) \notin R(F)$. Hence, $(a, a) \notin R(G)$ and furthermore, $(a, b) \notin R(G)$. Define $B = (A(F) \cup A(G)) \setminus \{a, b\}$. In any case, $\{a\} \in \mathcal{E}_{stb}(F \setminus B) \setminus \mathcal{E}_{stb}(G \setminus B)$ concluding the if-direction.
($\Leftarrow$) Given $F^{k(stb)} = G^{k(stb)}$. Applying Theorems 4.14 and 4.20 one after the other yields $F \equiv^{\mathcal{E}_{stb}}_{E} G$ and then $F \equiv^{\mathcal{E}_{stb}}_{N} G$. Finally, Corollary 4.21 justifies $F \equiv^{\mathcal{E}_{stb}}_{ND} G$ concluding the proof. $\square$

[26]We mention that Theorem 10 in [Baumann, 2014a] hold, given that resulting AFs have to be non-empty. The claimed normal deletion equivalence of the AFs F and G depicted in [Baumann, 2014a, Example 4] can be disproved by setting $B = \{a, b, c, f\}$.

4.2.4 Characterization Theorems in Case of Self-loop-free AFs

We already observed that apart from naive kernel any mentioned kernel k does not change anything if the considered AF F is self-loop-free, i.e. $F = F^k$ (cf. Facts 4.12 and 4.18). Consequently, any equivalence relation characterizable through such a kernel collapses to identity if we restrict ourselves to self-loop-free AFs. This is stated in the following theorem.

Theorem 4.31. *Given a relation $\equiv \subseteq \mathcal{F} \times \mathcal{F}$ characterizable through k where $k \in \{k(stb), k(ad), k(gr), k(co), k^*(ad), k^*(gr), k^*(co), k^*(stg)\}$. For any self-loop-free AFs F and G,*

$$F \equiv G \Leftrightarrow F = G.$$

We will refrain from listing all combinations of semantics and equivalence notions characterizable through a kernel mentioned in the theorem above. Please confer Figures 12 and 15 for compact overviews. For all such combinations, self-loop-free AFs are redundancy-free, i.e. all attacks as well as arguments may play a crucial role w.r.t. further evaluations and thus, there is no space for simplification. In the introductory part of this section we noted that many equivalence notions, e.g. normal and local expansion equivalence are motivated by the instantiation-based context where AFs are built from an underlying knowledge base. However, we want to mention that there are some formalisms like classical logic-based argumentation where self-attacking arguments do not occur [Besnard and Hunter, 2001, Theorem 4.13], while for other systems, e.g. ASPIC self-defeating arguments indeed may arise [Prakken, 2010, Section 7].

4.2.5 Summary of Results and Conclusion

In the presented results the notion of a kernel played a crucial role. Indeed, kernels are interesting from several perspectives: First, they allow to decide the corresponding notion of equivalence by a simple check for topological (i.e. syntactical) equality. Moreover, all kernels we have obtained so far can be efficiently constructed from a given argumentation framework. This means, if a certain equivalence notion is characterizable through such a kernel, then we have tractability of the associated decision problem.

The following Figure 12 provides a comprehensive overview of the state of the art in case of extension-based semantics. The entry "k" in row M and column σ indicates that $\equiv^{\mathcal{E}_\sigma}_M$ is characterizable through k. The abbreviation "id" stands for identity map and the question mark represents an open problem. Further abbreviations like "L" and "Att^σ" refer to additional conditions relevant in case of normal deletion equivalence (cf. Theorem 4.29). The entry "$[m, n]$" indicates three facts. First, the characterization problem is already solved in Theorem/Proposition n in m.[27] Second, the characterization result is not (purely) kernel-based and third, it can be checked that none of the introduced kernels serve as a characterization.

	stg	*stb*	*ss*	*eg*	*ad*	*pr*	*il*	*gr*	*co*	*na*	*cf2*	*stg2*
W	?	[1,3]	?	?	[2,1]	[3,1]	?	?	[2,1]	?	?	?
L	$k^*(stg)$	[4,9]	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	[4,10]	[4,11]	$k(na)$	?	?
E	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	$k(na)$	id	id
N	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	$k(na)$	id	id
S	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k^*(ad)$	$k^*(ad)$	$k^*(ad)$	$k^*(gr)$	$k^*(co)$	$k(na)$	?	?
ND	?	$k(stb)$	?	?	$k^*(ad)$ L, Att^{ad}	?	?	$k^*(gr)$ L, Att^{gr}	$k^*(co)$ L, Att^{co}	?	?	?
D	id	id	id	id	id	id	id	id	id	id	id	id
LD	id	id	id	id	id	id	id	id	id	id	id	id
U	id	id	id	id	id	id	id	id	id	id	id	id

Figure 12: Extension-based Characterizations for Finite AFs

Remember that any arbitrary expansion (deletion) can be split into a normal and local part. So one natural conjecture is that normal and local expansion (deletion) equivalence jointly imply expansion (deletion) equivalence. Using the results presented in this section we can not only verify the addressed conjecture but even give a significantly stronger result. In fact, the main and quite surprisingly relations for the considered semantics can be briefly and concisely stated in the following two equations, namely "normal expansion equivalence = expansion equivalence" and "local deletion equivalence = deletion equivalence".

[27]For m we use the following assignments: 1 = [Baumann, 2011], 2 =[Baumann and Brewka, 2015], 3 =[Baumann and Brewka, 2013] and 4 =[Oikarinen and Woltran, 2011]

The fact that different notions of equivalence might or might not coincide is interesting from a conceptual point of view. To illustrate this let us have a look at normal and strong expansion equivalence. Recall that normal expansions add new arguments and possibly new attacks which involve at least one of the fresh arguments, while strong expansions (a subclass of normal expansions) restrict the possible attacks between the new arguments and the old ones to a single direction. In dynamic settings, both concepts can be justified in the sense that new arguments might be raised but this will not influence the relation between already existing arguments. For strong expansions, only strong arguments will be raised, i.e. arguments which cannot be attacked by existing ones. The corresponding equivalence notions now check whether two AFs are "equally robust" to such new arguments, and indeed, normal expansion equivalence always implies strong expansion equivalence but the other direction is only true for some of the semantics, namely stage, stable, semi-stable, eager and naive semantics. One interpretation is that when two AFs are not normal expansion equivalent, then this can be made explicit by only posing strong arguments (not attacked by existing ones), while for the other semantics this is not the case. For this particular example, it seems that the notion of admissibility which is more "explicit" in the admissible, preferred, ideal, grounded and complete semantics is responsible for the fact that frameworks might be strong expansion equivalent but not normal expansion equivalent.

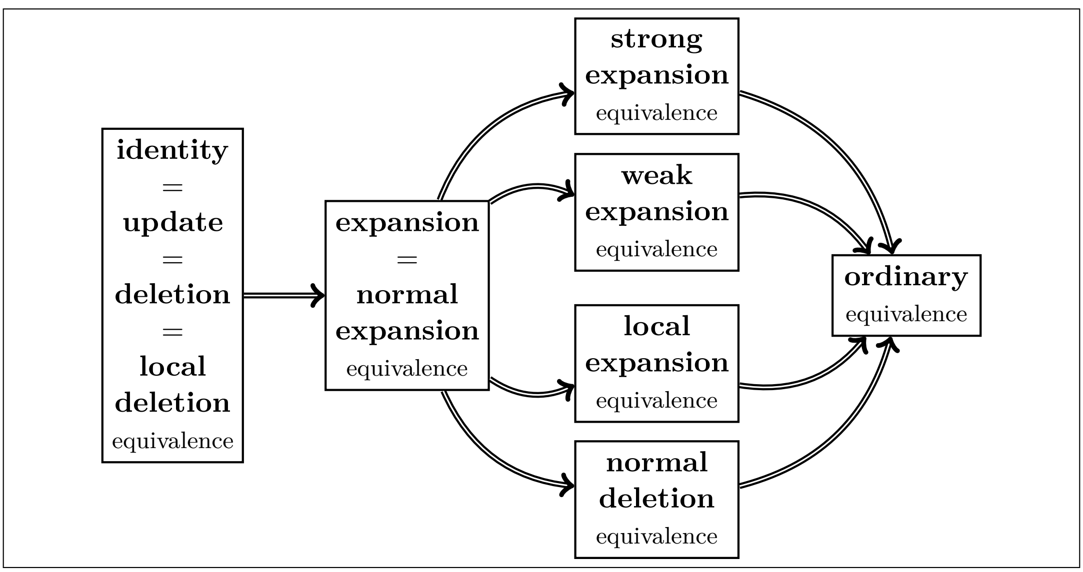


Figure 13: Relations for $\sigma \in \{stg, stb, ss, eg, ad, pr, il, gr, co, na, cf2, stg2\}$ - Extension-based Versions and Finite AFs

In Figure 11 we presented preliminary relations between several notions of equivalence which hold for any semantics. The refinement depicted in Figure 13 applies to any extension-based semantics considered in this section.

Finally, we present the overall picture for the most prominent semantics, namely the stable one. Interestingly, in contrast to Figure 13 all equivalence notions are comparable, i.e. they are totally ordered w.r.t. $\subseteq$. Comprehensive overviews for single semantics can be found in [Baumann, 2014b, Section 5.5.2] or [Baumann and Brewka, 2015b]. The latter also contains a comparison to different notions of *minimal change equivalence* firstly introduced in [Baumann, 2012b]. As an aside, very recently the authors of [Baumann *et al.*, 2017] introduced so-called *C-relativized equivalence* that subsumes ordinary and expansions equivalence as its extreme corner cases. The set C represents so-called *core* arguments which will not be directly touched by the possible expansions. This means, for any set C we obtain a further intermediate notion between expansion and ordinary equivalence. However, due to its recency further relations are not studied so far.

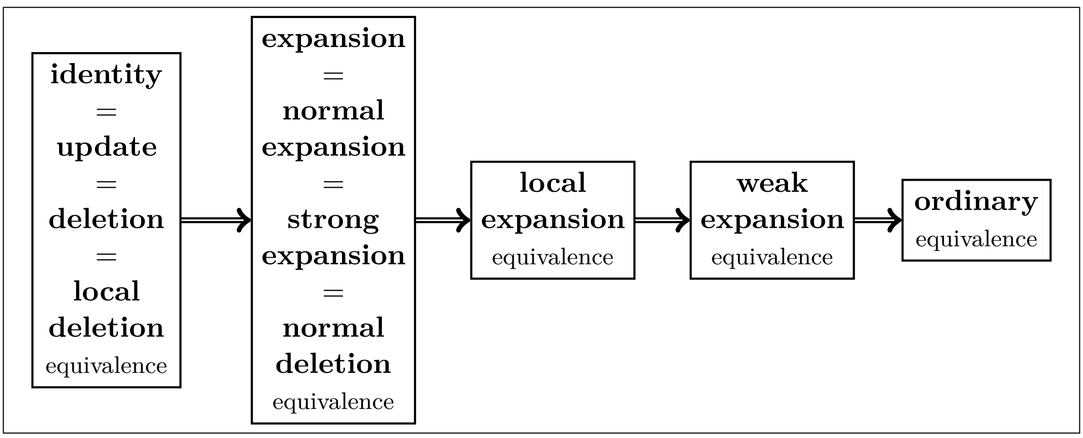


Figure 14: Stable Semantics - Extension-based Version and Finite AFs

4.3 Equivalence in the Light of Unrestricted Frameworks

Recently, a first study of several abstract properties in the unrestricted setting were presented in [Baumann and Spanring, 2017]. The main result regarding expansion equivalence can be summarized as follows: All characterization results carry over to the unrestricted setting as long as the AFs in question are *jointly expandable* (w.r.t. $\mathcal{U}$). Consider therefore the following definition and the corresponding characterization theorem.

Definition 4.32. *F and G are jointly expandable if $\mathcal{U} \setminus (A(F) \cup A(G)) \neq \emptyset$.*

Theorem 4.33. *[Baumann and Spanring, 2017] For jointly expandable AFs F and G we have:*

1. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F^{k(\sigma)} = G^{k(\sigma)}$ *for any* $\sigma \in \{stb, ad, co, gr, na\}$,
2. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F^{k(ad)} = G^{k(ad)}$ *for any* $\sigma \in \{pr, il, ss, eg\}$ *and*
3. $F \equiv_E^{\mathcal{E}_{stg}} G \Leftrightarrow F^{k(stb)} = G^{k(stb)}$.

The main proof strategies are straightforward extensions of those presented in [Oikarinen and Woltran, 2011]. However, finiteness assumptions are often used implicitly and one has to pay attention whether a certain reasoning step (e.g. subset relation between semantics, definedness statuses of semantics, finitely many extensions etc.) carry over to the infinite setting.

Interestingly, in case of the admissible as well as naive kernel we may even drop the restriction of joint expandability as stated in the following theorem.

Theorem 4.34. *[Baumann and Spanring, 2017] For unrestricted AFs F,G we have:*

1. $F \equiv_E^{\mathcal{E}_{na}} G \Leftrightarrow F^{k(na)} = G^{k(na)}$ *and*
2. $F \equiv_E^{\mathcal{E}_\sigma} G \Leftrightarrow F^{k(ad)} = G^{k(ad)}$ *for any* $\sigma \in \{ad, pr, il, ss, eg\}$.

The following two examples taken from [Baumann and Spanring, 2017] show that this assertion does not hold for all kernels considered in this section. The main reason for this different behaviour is that for some semantics it plays a decisive role whether AFs can be expanded by "fresh" arguments which is not given for unrestricted frameworks in general but guaranteed for jointly expandable AFs (cf. Definition 4.32).

Example 4.35. *Given $c \in \mathcal{U}$ and define $F = (\mathcal{U} \setminus \{c\}, \{(a,a) \mid a \in \mathcal{U} \setminus \{c\}\})$ and $G = (\mathcal{U}, \{(a,a) \mid a \in \mathcal{U} \setminus \{c\}\})$. For any H we observe $\mathcal{E}_{stb}(F \cup H) = \mathcal{E}_{stb}(G \cup H)$. In particular,*

$$\mathcal{E}_{stb}(F \cup H) = \begin{cases} \{\{c\}\}, & \text{if } \{(c,a) \mid a \in \mathcal{U} \setminus \{c\}\} \subseteq R(H) \text{and } (c,c) \notin R(H) \\ \emptyset, & \text{otherwise} \end{cases}$$

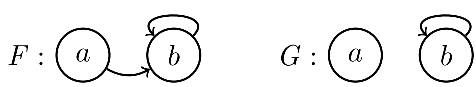
Consequently, $F \equiv_E^{\mathcal{E}_{stb}} G$ although $A(F) \neq A(G)$ (and thus, $F^{k(stb)} \neq G^{k(stb)}$).

Example 4.36. *Consider the AFs $F = (\mathcal{U}, \{(a,a) \mid a \in \mathcal{U}\})$ as well as $G = (\mathcal{U}, \{(a,b) \mid a,b \in \mathcal{U}, a \neq b)$. Applying the grounded kernel does not change anything for either framework, i.e. $F^{k(gr)} = F$ and $G = G^{k(gr)}$. Due to the absence of unattacked arguments we deduce $\mathcal{E}_{gr}(F \cup H) = \mathcal{E}_{gr}(G \cup H) = \{\emptyset\}$ for any AF H. Consequently, $F \equiv_E^{\mathcal{E}_{gr}} G$ although $F^{k(gr)} \neq G^{k(gr)}$.*

4.4 Characterization Theorems for Labelling-Based Semantics

We now return to the finite setting and consider the second main approach used for evaluating argumentation scenarios, namely labelling-based semantics. As a matter of fact, the labelling-based versions of all considered semantics provides one with more information than their extension-based counter-parts. More precisely, the defined 3-valued labellings assign a status to any argument of the considered AF F, i.e. in addition to the information which arguments are *accepted* we also have labels for the remaining arguments indicating that they are either *rejected* or *undecided* with respect to F (cf. [Baroni *et al.*, 2011] for more details). It is well known that many semantics establish a one-to-one correspondence between their extension-based and labelling-based versions. This means, any labelling is associated with exactly one extension and vice versa. It is not immediately apparent whether this property guarantees that there is a coincidence of the extension-based and labelling-based equivalence notions. In [Baumann, 2016] a negative answer was given. The main reason for the invalidity is that AFs may possess the same extensions without sharing the same arguments which is impossible in case of labellings since any argument has to be labelled. Furthermore, even sharing the same arguments does not ensure the validity of the converse direction. Consider therefore the following example.

Example 4.37. *Consider the AFs F and G as depicted below. Although both frameworks possess the same unique preferred extension, they do not share the same preferred labellings. More precisely,* $\mathcal{E}_{pr}(F) = \mathcal{E}_{pr}(G) = \{\{a\}\}$ *but* $\{(\{a\}, \{b\}, \emptyset)\} = \mathcal{L}_{pr}(F) \neq \mathcal{L}_{pr}(G) = \{(\{a\}, \emptyset, \{b\})\}$.



Moreover, observe that $F^{k^*(ad)} = G = G^{k^*(ad)}$. *Consequently, both frameworks are even strong expansion equivalent w.r.t. preferred extension-based semantics (Theorem 4.20). This means, equivalence notions may differ considerably if considered under the extension-based or labelling-based approach.*

In contrast to extension-based semantics where characterization results are spread over a high number of publications there is only one reference, namely [Baumann, 2016] concerned with labelling-based semantics. The author considered 8 different equivalence notions w.r.t. 8 prominent labelling-based semantics in the finite setting. In effect, similarly to extension-based semantics, almost all labelling-based equivalence notions can be decided syntactically. Differently from the extension-based approach we observe a much more homogeneous picture. For instance, there is no need for the more sophisticated σ-*-kernels as we will see.

4.4.1 Basic Properties and a Fundamental Relation

Before turning to the main results we start with some preliminary facts relating σ-extensions and σ-labellings. In the following we restrict ourselves to the semantics considered in [Baumann, 2016]. For any 3-valued labelling $L = (L_1, L_2, L_3)$ we use $L = (L^{\mathrm{I}}, L^{\mathrm{O}}, L^{\mathrm{U}})$ as usual.

Fact 4.38. *Given a finite AF $F = (A, R)$ and a set $E \subseteq A$. We write $E^{\mathcal{L}}$ for $(E, E^{+}, A \setminus E^{\oplus}))$. For all $\sigma \in \{stb, ss, eg, ad, pr, il, gr, co\}$ we have,*

1. *If $L \in \mathcal{L}_\sigma(F)$, then $L^I \in \mathcal{E}_\sigma(F)$,* *(extension induced by labelling)*
2. *If $E \in \mathcal{E}_\sigma(F)$, then $E^{\mathcal{L}} \in \mathcal{L}_\sigma(F)$ and* *(labelling induced by extension)*
3. *Obviously, $(E^{\mathcal{L}})^I = E$.* *($I \circ \mathcal{L} = id$)*

We point out that the first two properties mentioned in Fact 4.38 do not ensure that there is a one-to-one correspondence between σ-labellings and σ-extensions. This desirable feature (which would indeed justify the terms σ-labellings and σ-extensions) is given if additionally, labellings are uniquely determined by their in-labelled arguments.

Fact 4.39. *Given a finite AF $F = (A, R)$ and a set $E \subseteq A$. For all semantics $\sigma \in \{stb, ss, eg, pr, il, gr, co\}$ we have,*

1. *For any $L, M \in \mathcal{L}_\sigma(F), L^I = M^I$ iff $L = M$,* *(uniquely determined by in-labels)*
2. *Given $L \in \mathcal{L}_\sigma(F)$, then $(L^I)^{\mathcal{L}} = L$ and* *($\mathcal{L} \circ I = id$)*
3. *$|\mathcal{L}_\sigma(F)| = |\mathcal{E}_\sigma(F)|$.* *(same cardinality)*

As an aside, we mention that (although not immediately apparent) the first two items of Fact 4.39 are equivalent independently of any semantics definition. Please note that admissible labellings are excluded from Fact 4.39. The AF F depicted in Example 4.37 shows that this is no coincidence. It possesses two admissible labellings associated with one admissible extension. More precisely, the admissible labellings $(\{a\}, \{b\}, \emptyset)$ as well as $(\{a\}, \emptyset, \{b\})$ refer to the same admissible extension $\{a\}$.

We proceed with a general relation between labelling-based and extension-based versions of certain equivalence notion. More precisely, for any considered semantics and any equivalence notion presented in Definition 4.6 we have that being equivalent w.r.t. labellings implies being equivalent w.r.t. extensions. The main reason for this fundamental relation is the following lemma stating that possessing the same labellings implies sharing the same extensions. We mention that this property is already guaranteed if the semantics σ in question satisfies that any σ-extension induces an σ-labelling and vice versa (cf. statements 1 and 2 of Fact 4.38).

Lemma 4.40 ([Baumann, 2016]). *Given two finite AFs F and G. For any semantics $\sigma \in \{stb, ss, eg, ad, pr, il, gr, co\}$ we have,*

$$\mathcal{L}_\sigma(F) = \mathcal{L}_\sigma(G) \Rightarrow \mathcal{E}_\sigma(F) = \mathcal{E}_\sigma(G).$$

Proof. Reductio ad absurdum. Assume $\mathcal{E}_\sigma(F) \neq \mathcal{E}_\sigma(G)$. Then, w.l.o.g. exists $E \in \mathcal{E}_\sigma(F) \setminus \mathcal{E}_\sigma(G)$. Consequently, $E^{\mathcal{L}} \in \mathcal{L}_\sigma(F)$ (item 2 of Fact 4.38). Thus, $E^{\mathcal{L}} \in \mathcal{L}_\sigma(G)$ (assumption). Hence, $(E^{\mathcal{L}})^I \in \mathcal{E}_\sigma(G)$ (item 1 of Fact 4.38). Furthermore, $(E^{\mathcal{L}})^I = E \in \mathcal{E}_\sigma(G)$ (item 3 of Fact 4.38). Contradiction! $\square$

We now present the fundamental relation between labelling-based and extension-based equivalence notion.

Theorem 4.41 ([Baumann, 2016]). *Given two finite AFs F and G. For any $\sigma \in \{stb, ss, eg, ad, pr, il, gr, co\}$ and any $M \in \{W, L, E, N, S, ND, D, LD, U\}$ we have,*

$$F \equiv^{\mathcal{L}_\sigma}_M G \Rightarrow F \equiv^{\mathcal{E}_\sigma}_M G.$$

Proof. We show the contrapositive. Assume $F \not\equiv^{\mathcal{E}_\sigma}_M G$. This means, there is a certain scenario S according to M, s.t. $\mathcal{E}_\sigma(S(F)) \neq \mathcal{E}_\sigma(S(G))$.[28] Consequently, $\mathcal{L}_\sigma(S(F)) \neq \mathcal{L}_\sigma(S(G))$ (Lemma 4.40) proving $F \not\equiv^{\mathcal{L}_\sigma}_M G$. $\square$

In Example 4.37 we have seen that the converse direction does not hold in general. Nevertheless, there is huge number of equivalence notions where labelling-based and extension-based versions do indeed coincide (cf. Figure 15 for an overview).

4.4.2 Coincidences of Extension-based and Labelling-based Versions

Remember that the identity relation is the finest equivalence relation. Furthermore, it is already shown that deletion, local deletion as well as update equivalence w.r.t. $\mathcal{E}_\sigma$ collapse to identity (see Figure 13). Consequently, applying the fundamental relation stated in Theorem 4.41 we obtain the identical characterization results w.r.t. labelling-based semantics.

Theorem 4.42 ([Baumann, 2016]). *For finite AFs F and G, a scenario $M \in \{D, LD, U\}$ and a semantics $\sigma \in \{stb, ss, eg, ad, pr, il, gr, co\}$ we have,*

$$F \equiv^{\mathcal{L}_\sigma}_M G \Leftrightarrow F = G.$$

[28]For instance, in case of expansion equivalence (i.e. M = E) a scenario S is simply the union with a further AF H, i.e. $S(F) = F \cup H$ and $S(G) = G \cup H$.

Analogously to extension-based semantics (cf. Fact 4.13) we have that there are combinations of kernels and semantics σ, s.t. the application of a kernel does not vary the set of σ-labellings.

Fact 4.43. *For any finite AF F,*

1. $\mathcal{L}_\sigma(F) = \mathcal{L}_\sigma\left(F^{k(\sigma)}\right)$ *for* $\sigma \in \{co, stb, gr\}$ *and*

2. $\mathcal{L}_\tau(F) = \mathcal{L}_\tau\left(F^{k(ad)}\right)$ *for* $\tau \in \{ss, eg, pr, il\}$.

The fact above is the decisive property which allows one to carry over further kernel-based characterization results for extension-based semantics to their labelling-based version. In order to show this result it was necessary to find a condition for equality of two complete labellings of different AFs. Remember that two complete labellings of the same framework are identical if and only if they possess the same in-labelled arguments (Fact 4.39). In case of different AFs we have to require additionally that both frameworks share the same arguments and the same range w.r.t. the set of in-labelled arguments.

Fact 4.44. *Given two finite AFs F and G as well as $L \in \mathcal{L}_{co}(F)$ and $M \in \mathcal{L}_{co}(G)$. We have $L = M$ iff simultaneously $A(F) = A(G)$, $L^I = M^I$ and $R^+_F(L^I) = R^+_G(M^I)$.*

Please observe that admissible labellings do not fulfill Fact 4.44. Consider for instance again the AF F depicted in Example 4.37 and its two admissible labellings $(\{a\}, \{b\}, \emptyset)$ and $(\{a\}, \emptyset, \{b\})$.

We proceed with the main coincidence theorem. It stipulates that several expansion equivalence relations as well as weaker notions do not distinguish between their labelling-based and extension-based version. This means, kernel-based characterization results (depicted in Figure 11) carry over to labelling-based semantics. Similarly to extension-based semantics we present an overview of characterizing kernels at the end of this section (cf. Figure 15).

Theorem 4.45 ([Baumann, 2016])**.** *Given finite AFs F and G. We have,*

1. $F \equiv^{\mathcal{E}_\sigma}_M G \Leftrightarrow F \equiv^{\mathcal{L}_\sigma}_M G$ *for* $\sigma \in \{stb, ss, eg, pr, il, gr, co\}, M \in \{E, N\}$,

2. $F \equiv^{\mathcal{E}_\sigma}_L G \Leftrightarrow F \equiv^{\mathcal{L}_\sigma}_L G$ *for* $\sigma \in \{ss, eg, pr, il\}$ *and*

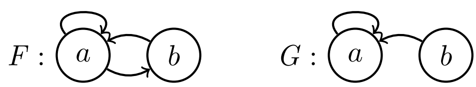
3. $F \equiv^{\mathcal{E}_\sigma}_S G \Leftrightarrow F \equiv^{\mathcal{L}_\sigma}_S G$ *for* $\sigma \in \{stb, ss, eg\}$.

4.4.3 Non-Coincidence of Extension-based and Labelling-based Versions

We now leave the realm of uniformity of extension-based and labelling-based characterizations. This section is divided into three parts. We start with characterization theorems for admissible labellings. In particular, we will see that the admissible kernel (originally introduced to characterize equivalence notions w.r.t. admissible extension-based semantics) does not serve as characterizing kernel for admissible labellings. We then proceed with strong expansion equivalence w.r.t. labellings. We will see that the remaining notions are characterizable via traditional kernels instead of σ-*-kernels. In the third part we consider normal deletion equivalence w.r.t. labelling-based semantics. In contrast to their extension-based versions where many notions has defied any attempt of solving, we present characterization theorems based on traditional kernels for all eight considered semantics.

Expansion Equivalence w.r.t. Admissible Labellings Expansion equivalence as well as its local, normal and strong versions w.r.t. admissible extensions are characterizable through the admissible kernel. The following example shows that this assertion does not hold in case of admissible labellings.

Example 4.46. *The following two AFs possess the same admissible kernels, namely* $F^{k(ad)} = G^{k(ad)} = F$. *Consequently, applying characterization theorems for extension-based semantics we obtain* $F \equiv^{\mathcal{E}_{ad}}_{M} G$ *for* $M \in \{L, E, N\}$ *(cf. Figure 12).*



Observe that $(\{b\}, \emptyset, \{a\}) \in \mathcal{L}_{ad}(G) \setminus \mathcal{L}_{ad}(F)$ *because the argument* a *cannot be undecided in* F *since it attacks the in-labelled argument* b. *Thus* $F \not\equiv^{\mathcal{L}_{ad}}_{M} G$ *for* $M \in \{L, E, N, S\}$.

Let us assume that the equivalence notions considered in the example above are characterizable through a certain kernel k. Due to the fundamental relation (Theorem 4.41) and the characterization results w.r.t. admissible extensions (Figure 12), we already know that the kernel k has to satisfy the following implication: $F^k = G^k \Rightarrow F^{k(ad)} = G^{k(ad)}$ for any two AFs F and G. This means, we are looking for a weaker kernel than the admissible one in the sense that first, everything which is redundant w.r.t. k has to be redundant w.r.t. the admissible kernel too; and second, an attack from a to b has to survive even if a is self-defeating and b counterattacks a. One candidate for k is the complete kernel since redundancy w.r.t. the complete kernel implies redundancy w.r.t. to the admissible one, and furthermore, it deletes an attack between two arguments if and only if both are self-defeating.

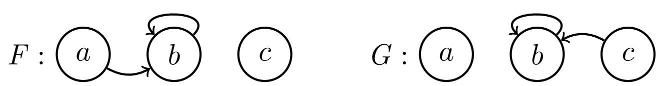
And indeed, it was shown that expansion equivalence as well as its local, normal and strong variant w.r.t. admissible labellings are characterizable through the complete kernel as stated by the following theorem.

Theorem 4.47 ([Baumann, 2016])**.** *Given finite AFs F and G. We have,*

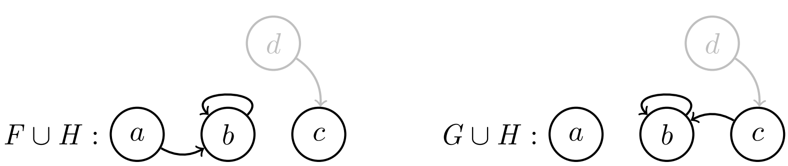
$$F \equiv_M^{\mathcal{L}_{ad}} G \Leftrightarrow F^{k(co)} = G^{k(co)} \text{ with } M \in \{L, E, N, S\}.$$

Strong Expansion Equivalence for Preferred, Ideal, Grounded and Complete Labellings In this subsection we present characterization theorems for strong expansion equivalence w.r.t. labelling-based preferred, ideal, grounded and complete semantics. Remember that in case of strong expansions a former attack between old arguments will never become a counterattack to an added attack. Consequently, in contrast to arbitrary expansions former attacks do not play a role with respect to being a potential defender of an added argument. The context-sensitive σ-*-kernels took these considerations into account and allow for more deletions than their classical counterparts.

Example 4.48. *According to Definition 4.17 we have, $F^{k^*(\sigma)} = G^{k^*(\sigma)}$ for any semantics $\sigma \in \{ad, gr, co\}$. More precisely, the attacks (a, b) in F as well as (c, b) in G are redundant w.r.t. all three σ-*-kernels. This means, in consideration of Figure 12 both frameworks are strong expansion equivalent w.r.t. the extension-based versions of preferred, ideal, grounded and complete semantics.*



Consider the following dynamic scenario where a stronger argument than the former ones is added. Formally, we conjoin the AF $H = (\{c, d\}, \{(d, c)\})$ to both frameworks F and G.



Note that both frameworks has to possess the same σ-extension since $G \equiv_S^{\mathcal{E}_\sigma} H$ for $\sigma \in \{pr, il, gr, co\}$ is already ensured. Furthermore, we observe $(\{a, d\}, \{b, c\}, \emptyset) \in \mathcal{L}_\sigma(F \cup H) \setminus \mathcal{L}_\sigma(G \cup H)$ since b cannot be out-labelled in $G \cup H$ because there is no in-labelled attacker. This means, $F \not\equiv_S^{\mathcal{L}_\sigma} G$ for $\sigma \in \{pr, il, gr, co\}$.

Analogously to the previous section let us assume that strong expansion equivalence w.r.t. the considered labelling-based semantics are characterizable through a certain kernel k. We immediately obtain, $F^k = G^k \Rightarrow F^{k^*(\sigma)} = G^{k^*(\sigma)}$ for any two AFs F and G. Possible candidates are the classical counterparts of the σ-*-kernels and indeed it was shown that these kernels guarantee the desired outcome. This means, in case of strong expansion equivalence w.r.t. preferred, ideal, grounded and complete semantics we have that the labelling-based version is characterizable through a classical σ-kernel if and only if the extension-based version is characterizable through the corresponding σ-*-kernel.

Theorem 4.49 ([Baumann, 2016]). *Given finite AFs F and G. We have,*

1. $F \equiv_S^{\mathcal{L}_\sigma} G \Leftrightarrow F^{k(ad)} = G^{k(ad)}$ *for* $\sigma \in \{pr, il\}$,
2. $F \equiv_S^{\mathcal{L}_{gr}} G \Leftrightarrow F^{k(gr)} = G^{k(gr)}$ *and*
3. $F \equiv_S^{\mathcal{L}_{co}} G \Leftrightarrow F^{k(co)} = G^{k(co)}$.

Normal Deletion Equivalence Characterizing normal deletion equivalence in case of extension-based semantics is exceptional in several regards. Remember that normal deletions retract arguments and their corresponding attacks. Firstly, only a few characterization results are achieved (cf. Figure 12). Furthermore, apart from stable semantics, none of the characterization results is purely kernel-based, i.e. beside the equality of kernels on certain parts of the frameworks further loop- as well as attack-conditions have to be satisfied. Finally, quite surprisingly, normal deletion equivalent AFs do not even have to share the same arguments enabling equivalence classes with an infinite number of elements. Being equivalent w.r.t. labellings and possessing different arguments at the same time is impossible in case of labellings since any argument has to be labelled. It turned out that any considered labelling-based semantics is characterizable through traditional kernels and thus, do not share any of the features mentioned above. Consider the following main theorem.

Theorem 4.50 ([Baumann, 2016]). *Given finite AFs F and G. We have,*

1. $F \equiv_{ND}^{\mathcal{L}_{stb}} G \Leftrightarrow F^{k(stb)} = G^{k(stb)}$,
2. $F \equiv_{ND}^{\mathcal{L}_\sigma} G \Leftrightarrow F^{k(ad)} = G^{k(ad)}$ *for* $\sigma \in \{ss, eg, pr, il\}$,
3. $F \equiv_{ND}^{\mathcal{L}_\sigma} G \Leftrightarrow F^{k(co)} = G^{k(co)}$ *for* $\sigma \in \{ad, co\}$ *and*
4. $F \equiv_{ND}^{\mathcal{L}_{gr}} G \Leftrightarrow F^{k(gr)} = G^{k(gr)}$.

4.4.4 Summary of Results and Conclusion

The following Figure 15 presents a comprehensive overview of the state of the art in case of labelling-based semantics. Analogously to Figure 12 the entry "k" in row M and column σ indicates that $\equiv_M^{\mathcal{L}_\sigma}$ is characterizable through k given the finiteness restriction. The abbreviation "id" stands for identity map and the question mark represents an open problem.[29] A grey-highlighted entry reflects the situation that extension-based and labelling-based version do not coincide.

	stb	ss	eg	ad	pr	il	gr	co
L	?	$k(ad)$	$k(ad)$	$k(co)$	$k(ad)$	$k(ad)$	?	?
E	$k(stb)$	$k(ad)$	$k(ad)$	$k(co)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$
N	$k(stb)$	$k(ad)$	$k(ad)$	$k(co)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$
S	$k(stb)$	$k(ad)$	$k(ad)$	$k(co)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$
ND	$k(stb)$	$k(ad)$	$k(ad)$	$k(co)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$
D	id	id	id	id	id	id	id	id
LD	id	id	id	id	id	id	id	id
U	id	id	id	id	id	id	id	id

Figure 15: Labelling-based Characterizations for Finite AFs

In contrast to extension-based semantics we observe a much more homogeneous

[29] In contrast to extension-based semantics the labelling versions of conflict-free-based semantics like stage, naive, cf2 as well as stage2 semantics (cf. [Caminada, 2011; Gaggl and Dvořák, 2016]) as well as weak expansion equivalence at all were not considered so far and thus, represent open problems too.

picture. Firstly, there is no need for the more sophisticated σ-*-kernels. Secondly, normal deletion equivalence w.r.t. labelling-based semantics is naturally incorporated in the overall picture in the sense that it coincides with its corresponding expansion, normal expansion and strong expansion equivalence notions.

The following Figure 16 applies to each one of the eight labelling-based semantics considered in this section. In comparison to Figure 11 where preliminary relations are depicted it illustrates (to a certain extent) a collapse of the diversity of the introduced equivalence notions in case of labelling-based semantics.

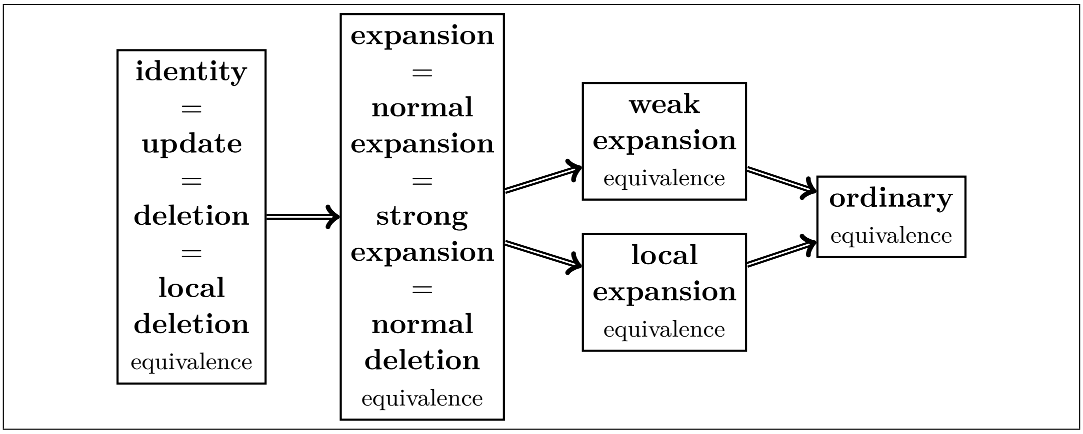


Figure 16: Relations for $\sigma \in \{stb, ss, eg, ad, pr, il, gr, co\}$ - Labelling-based Versions and Finite AFs

4.5 Final Remarks

In this section we motivated and discussed several notions of equivalence in the context of abstract argumentation and provided an exhaustive number of characterization theorems for extension-based as well as labelling-based semantics. In general we may state that Dung's abstract argumentation frameworks are a very compact formalism since the majority of the considered equivalence notion possess only little space for redundancy. Moreover, most of these notions collapse to identity if self-loop-free AFs are considered. This means, in this case any subframework of the AF in question may play a decisive role w.r.t. further evaluations and thus, cannot be locally replaced by another. This insight is sometimes used as an argument against the usefulness of the study of equivalence notions in the context of abstract argumentation. Obviously, we agree that if you are expecting much space for simplification, then the results are somehow disappointing but let us not lose sight of the fact that this is only clear *after* it has been proved. Furthermore, as already stated, the results underline that in case of abstract argumentation (almost) everything is meaningful similar to other non-monotonic formalisms available in the literature (cf. [Lifschitz *et al.*, 2001] for logic programs, [Turner, 2004] for causal theories, [Turner, 2001] for default logic and [Truszczynski, 2006] for nonmonotonic logics in general). However, one decisive difference to these formalisms is that equivalence notions in case of abstract argumentation can be decided syntactically. Indeed, kernels are interesting from several perspectives: First, they allow to decide the corresponding notion of equivalence by a simple check for topological equality and second, all kernels we have obtained so far can be efficiently constructed from a given argumentation framework. This means, if a certain equivalence notion is characterizable through such a kernel, then we have tractability of the associated decision problem.

Acknowledgements

We would like to thank Wolfgang Dvořák, Thomas Linsbichler and Christoph Spanring for fruitful discussions which helped to improve the article.

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Received 19 June 2017

Argumentation, Nonmonotonic Reasoning and Logic

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Abstract

In this study, we will explore the respective roles of logic and nonmonotonic reasoning in argumentation. As a first step, we introduce the notion of collective argumentation as a logical basis of argumentation frameworks, and provide it with a natural (four-valued) logical semantics. This will allows us, in particular, to augment the underlying language with appropriate logical connectives that will transform abstract argumentation frameworks into a reasoning system with full-fledged logical capabilities. On the way, we will show not only that argumentation and logic are important for nonmonotonic reasoning, but also the other way round, namely that the main nonmonotonic formalisms and argumentation systems constitute actually primary instantiations of Dung's abstract argumentation in appropriately extended logical languages.

1 The new stage of argumentation theory

Dung's argumentation frameworks are viewed today as a general formal basis of the argumentation theory, but they have deep roots in nonmonotonic reasoning, and it is due to these roots that they constitute a new stage in the development of the theory of argumentation.

Traditional formal argumentation theory (see, e.g., [Hamblin, 1971]) has been based, implicitly or explicitly, on the standard deductive paradigm, according to which our corpus of knowledge and beliefs comprises a set of propositional (factual or epistemic) assertions, coupled with a set of strict, universal deductive rules (= the Logic) that govern their acceptance. These rules allow us, in particular, to derive (support) further propositional claims, as well as to reveal possible inconsistencies among them. This underlying logic can also be given a precise argumentative (dialectical) formulation in the form of allowable attack and defense moves in argumentation games ([Lorenzen and Lorenz, 1978]). In this way, the traditional formal argumentation theory could be largely viewed as a 'human-friendly' instantiation of standard deductive reasoning.

The above deductive paradigm has been challenged, however, with the advent of nonmonotonic reasoning in AI. Studies in the latter (including related areas such as the AGM theory of belief revision), as well as contemporaneous studies of defeasible reasoning in philosophical logic (see [Pollock, 1987]), have shown that epistemic states underlying our reasoning are much more complex and structured than plain sets of beliefs governed by logic. They have shown, in particular, an important role of *default assumptions* in our reasoning, assumptions that we normally accept in the absence of evidence to the contrary. These assumptions usually appear in the conditional form *"If A, then normally B"*, and it can even be argued that such normality conditionals constitute one of the central ingredients of our commonsense epistemic states.

Despite their presumptive acceptability status, default assumptions are *defeasible*, that is, they can be attacked, and even eventually refuted, due to other assumptions and available evidence. The corresponding adjudication process, however, already cannot be represented as a deductive inference or proof in some logic, primarily because it is in general *non-local* and *non-monotonic*. The eventual acceptability of such assumptions depends on other assumptions present, and it can change from acceptance to rejection and vice versa with addition of new assumptions or facts. This reasoning process displays, however, distinctive features of genuine argumentation.

As a matter of fact, the intimate connections between argumentation and nonmonotonic reasoning has been noticed at the very beginning of the studies in nonmonotonic reasoning. The starting point of this understanding can be found already in the Truth Maintenance System (TMS) of [Doyle, 1979] and assumption-based truth maintenance (ATMS) of [de Kleer, 1986]. This understanding has even led to a general view of NMR as a theory of the reasoned use of assumptions in [Doyle, 1994]. In fact, even before Dung, a significant argumentation-based representation of default logic and other nonmonotonic formalisms has been suggested in [Lin and Shoham, 1989].

Developing further this line of research, Dung has shown a fundamental and unifying role of argumentation in logic programming and general nonmonotonic formalisms such as default and modal nonmonotonic logics. More precisely, he has shown that all these formalisms can be viewed as particular instantiations of a uniform argumentation scheme that implements the principle of default acceptability for arguments in an abstract framework based solely on a single relation of attack among them.

Dung's argumentation frameworks have had two crucial novel features. First, they implemented the basic principle of default acceptance for arguments[1]. On Dung's vision, however, this principle admits a number of different interpretations, which lead to different possible *nonmonotonic semantics* for the argumentation frameworks.

The second, more formal, novel feature of the Dung's formalism was the asymmetric (directional) character of the attack relation; it was this 'degree of freedom' that has allowed to provide an adequate representation of the above-mentioned nonmonotonic formalisms. This feature has also marked an important formal difference with the traditional, deductive argumentation that has been based primarily on symmetric inconsistency relations.

It could even be argued that the main contribution of Dung's theory has consisted in incorporating these two novel features as central conceptual ingredients of argumentation. It is this conceptual advancement that has given the argumentation theory its current impetus.

2 Logic in argumentation

One of the fundamental questions that have been re-opened, however, with the advent of Dung's argumentation frameworks was the question of the relation between argumentation and logic. Indeed, on the face of it, Dung's abstract frameworks do not include, or even require, any explicit logical components. On the other hand, it has been shown in subsequent argumentation literature that arbitrary, unrestricted combinations of argumentation frameworks with deductive rules may lead to patently inappropriate results, so such compositions should be constrained by some (more or less) reasonable 'rationality postulates', see, e.g., Caminada and Amgoud, 2007; Amgoud and Besnard, 2013; Dung and Thang, 2014].

As we are going to show in this study, though the relations between argumentation and logic have irrevocably changed, logic still plays (or, better, should play) an important role in argumentation. However, we contend that a crucial prerequisite for a proper understanding of this role amounts to a clear separation of the logical and non-monotonic aspects of argumentation.[2] In fact, the latter objective is not specific to argumentation theory, but pertains to all nonmonotonic formalisms.

In nonmonotonic formalisms, logic no longer 'pervades the world' (using the famous Wittgenstein's phrase). Namely, the logic, taken by itself, cannot provide

[1] Dung himself has called it the basic principle of argumentation and described it in [Dung, 1995b] as the principle *"The one who has the last word laughs best"*.

[2] In this respect, our construction below will be distinct from a large number of other suggested ways of combining logic with argumentation - see, e.g., [Boella *et al.*, 2005; Caminada and Gabbay, 2009; Gabbay, 2011; Strasser and Seselja, 2010].

the final 'output' of these formalisms; this latter task is relegated to the associated nonmonotonic semantics.

Nevertheless, logic still plays a distinctive and even crucial role in these formalisms. First of all, logic and its associated (monotonic) semantics should still provide a formal interpretation and meaning for the very syntax of a nonmonotonic formalism. Note that a nonmonotonic semantics is usually defined as a distinguished *subset* of the corresponding logical semantics, so it cannot be used for interpreting the source language. In addition, the logic provides deductive inferences that are 'safe' with respect to the nonmonotonic semantics, so it can be used to facilitate proofs and computations of the latter.

However, an even more profound benefit of the separation between logical and nonmonotonic aspects of a reasoning formalism emerges from the fact that, once the separation is made, many of these formalisms can be reconstructed as instantiations of the same nonmonotonic semantics in different logical languages.

The field of formal argumentation is abundant with different formalisms, which creates a fertile ground for extensive and rapid development. But there is also a lot of conceptual affinity among these argumentation formalisms, as well as between the latter and the major knowledge representation languages in AI. It is this affinity that allows us to use many of them for basically the same reasoning tasks. This situation creates, however, an obvious incentive for unification, namely for constructing a general theory of argumentation and reasoning where these formalisms could find their proper and hospitable place.

As we are going to show in this study, the logic appropriate for Dung's argumentation frameworks can be constructed on the basis of a four-valued logical semantics that can be found, in effect, already in [Jakobovits and Vermeir, 1999]. In that paper, the authors described a general semantic framework based on acceptance and rejection of arguments. This semantics was essentially four-valued, because assignments of acceptance and rejection to arguments were primarily viewed as mutually independent, which permitted valuations in which arguments can be both accepted and rejected, or neither accepted, nor rejected. The semantics suggested in [Jakobovits and Vermeir, 1999] were designed, however, to be generalizations of existing *nonmonotonic* argumentation semantics, so they incorporated also some non-logical, nonmonotonic features (see below). Still, we will show that a (properly generalized) four-valued semantics can be used as a *logical* basis of Dung's argumentation frameworks. It will allows us, in particular, to augment the underlying language with appropriate logical connectives that will transform this abstract argumentation to real argumentation reasoning with full-fledged logical capabilities.

As we are going to see in what follows, the assumption-based frameworks of [Bondarenko *et al.*, 1997] could be viewed as a 'focal point' of this logical development. On our reconstruction of the latter, assumption-based frameworks can be obtained from abstract Dung's frameworks just by adding a particular negation connective to the underlying language of arguments. This connective will also allow us to establish straightforward relations between attack and inference, as well as between (non-propositional) arguments and (propositional) assumptions that can be viewed as their reified counterparts in the object language.

Further stages of this logical development will allow us to provide a more systematic description of many other argumentation and general nonmonotonic formalisms, such as logic programming, default logic, abstract dialectical frameworks and the causal calculus.

The general picture that will emerge from this formal development is not only that argumentation is important for nonmonotonic reasoning, but also the other way round, namely that the main nonmonotonic formalisms and argumentation systems constitute actually primary instantiations of Dung's abstract argumentation in appropriately extended logical languages.

3 Abstract Collective Argumentation

As a general formal basis of argumentation theory, we will use the formalism of collective argumentation suggested in [Bochman, 2003a] as a 'disjunctive' generalization of Dung's argumentation theory. In this formalism, a primitive attack relation holds between *sets* of arguments[3]: in the notation introduced below, $a \hookrightarrow b$ says that a set a of arguments attacks a set of arguments b. This fact implies, of course, that these two sets arguments are incompatible. $a \hookrightarrow b$ says, however, more than that, namely that the set a of arguments, being accepted, provides a reason, or explanation, for rejection of the set of arguments b. Accordingly, the attack relation will not in general be symmetric, since in this situation acceptance of b need not give reasons for rejection of a. In addition, the attack relation is not reducible to attacks between individual arguments. For instance, we can disprove some conclusion jointly supported by a disputed set of arguments, though no particular argument in the set, taken alone, could be held responsible for this.

In what follows, $a, b, c, \dots$ will denote finite sets of arguments, while $u, v, w, \dots$ will denote arbitrary such sets. We will use the same agreements for the attack relation as for usual consequence relations. Thus, $a, a_1 \hookrightarrow b, B$ will have the same meaning as $a \cup a_1 \hookrightarrow b \cup \{B\}$, *etc.*

In what follows, proofs will be provided only for the main theorems. Proofs

[3]A similar idea has been suggested in [Nielsen S.H., 2007], though only for attacking sets, not attacked ones.

of all the other claims mentioned in this and the next section can be found in [Bochman, 2005].

Definition 1. *Let $\mathcal{A}$ be a set of* arguments. *A (collective)* attack relation *is a relation $\hookrightarrow$ on finite sets of arguments satisfying the following postulate:*

Monotonicity *If $a \hookrightarrow b$, then $a, a_1 \hookrightarrow b, b_1$.*

As we will see below, the above Monotonicity postulate turns out to be sufficient to characterize the primary logic behind the attack relation.

Though defined initially on finite sets of arguments, the attack relation can be extended to arbitrary such sets by imposing the compactness requirement: for any $u, v \subseteq \mathcal{A}$,

(**Compactness**) $u \hookrightarrow v$ iff there exist finite $a \subseteq u$ and $b \subseteq v$ such that $a \hookrightarrow b$.

The original Dung's argumentation frameworks can be seen as a special case of collective argumentation that satisfies additional properties (cf. [Kakas and Toni, 1999]):

Definition 2. *An attack relation will be called*

- affirmative *if no argument set attacks the empty set $\emptyset$;*
- local *if it satisfies the following condition:*

  (Locality) *If $a \hookrightarrow b, b_1$, then either $a \hookrightarrow b$, or $a \hookrightarrow b_1$.*

- normal *if it is both affirmative and local.*

Then the following facts can be easily verified:

Lemma 3.
- *If $\hookrightarrow$ is a normal attack relation, then $a \hookrightarrow b$ holds if and only if $a \hookrightarrow A$, for some $A \in b$.*
- *If $\hookrightarrow$ is a local attack relation, then $a \hookrightarrow b$ holds iff either $a \hookrightarrow \emptyset$, or $a \hookrightarrow A$, for some $A \in b$.*

Thus, the normal attack relation is reducible to the relation $a \hookrightarrow A$ between argument sets and single arguments, and the resulting theory will coincide, in effect, with that given in [Dung, 1995a]. A slightly more general local attack relation admits also constraints of the form $a \hookrightarrow$. Such a constraint says that the argument set a is *unacceptable* (due to Monotonicity, it attacks any argument whatsoever).

By an *argument theory* we will mean an arbitrary set of attacks $a \hookrightarrow b$ between finite argument sets. Now, since the Monotonicity condition is a 'Horn' one, any argument theory Δ generates a unique least attack relation that we will denote by $\hookrightarrow_\Delta$. The latter is obtained from Δ just by closing it with respect to the Monotonicity rule. Accordingly, $\hookrightarrow_\Delta$ can be described directly as follows:

$$u \hookrightarrow_\Delta v \quad \text{iff} \quad a \hookrightarrow b \in \Delta, \text{ for some } a \subseteq u, b \subseteq v.$$

An argument theory will be called *definite*, if it consists of attack rules of the form $a \hookrightarrow A$, where A is a single argument, and *singular*, if it has only attacks of the form $a \hookrightarrow b$, where b contains no more than one argument. Then the preceding lemma, coupled with the above representation, immediately implies the following simple observation:

Lemma 4. *An attack relation is local (respectively, normal) if and only if it is generated by a singular (resp., definite) argument theory.*

Thus, the differences between general, local and normal argumentation are reducible to the differences between corresponding generating argument theories.

3.1 Four-valued logical semantics

Collective argumentation can be given a four-valued semantics that can be seen as describing the (abstract) *meaning* of the attack relation. This formal meaning stems from the following understanding of an attack $a \hookrightarrow b$:

> *If all arguments in a are accepted, then at least one of the arguments in b should be rejected.*

The argumentation theory does not impose, however, the classical constraints on acceptance and rejection of arguments, so an argument can be both accepted and rejected, or neither accepted, nor rejected. Such an understanding can be captured formally by assigning any argument A a *subset* $\nu(A) \subseteq \{t, f\}$, where t denotes acceptance (truth), while f denotes rejection (falsity). This is nothing other than the well-known *Belnap's interpretation* of four-valued logic (see [Belnap, 1977]). On this understanding, $t \in \nu(A)$ means that an argument A is accepted, while $f \in \nu(A)$ means that A is rejected. In accordance with this, collective argumentation acquires a four-valued logical semantics described below.

Definition 5. *An attack $a \hookrightarrow b$ will be said to* hold *in a four-valued interpretation ν of arguments, if either $t \notin \nu(A)$, for some $A \in a$, or $f \in \nu(B)$, for some $B \in b$.*

An interpretation ν will be called a model *of an argument theory Δ if every attack from Δ holds in ν.*

Since an attack relation can be seen as a special kind of an argument theory, the above definition determines also the notion of a model for an attack relation.

Any pair (u, v) of argument sets determines a four-valued interpretation in which u is the set of true (i.e., accepted) arguments, while v is the set of arguments that are not false (non-rejected), and vice versa, any four-valued interpretation corresponds to such a pair of argument sets. In fact, we will often identify in what follows four-valued interpretations with their associated pairs of argument sets.

In this sense canonical models of an attack relation or an argument theory can be identified with bitheories described in the next definition.

Definition 6. *A pair* (u, v) *of arguments will be called a* bitheory *of an argument theory* Δ*, if* $u \not\hookrightarrow_\Delta v$[4]*.*

It can be easily verified that any bitheory of Δ corresponds to a four-valued model of the latter.

Definition 7.
- *A bitheory* (u, v) *of an argument theory will be called* consistent*, if* $u \subseteq v$*, and* complete*, if* $v \subseteq u$*.*
- *An argument set* u *will be called* consistent*, if* $u \not\hookrightarrow u$*.*

Consistent bitheories correspond to consistent four-valued interpretations, namely to interpretations in which no argument is both accepted and rejected. Similarly, complete bitheories correspond to complete four-valued interpretations in which every argument is either accepted, or rejected. Such constrained interpretations will play an important role in what follows. Finally, consistent argument sets correspond exactly to bitheories that are both consistent and complete. This notion of consistency provides an appropriate generalization of the notion of a conflict-free argument set in Dung's argumentation theory.

For a set I of four-valued interpretations, we will denote by $\hookrightarrow_I$ the set of all attacks that hold in each interpretation from I. Then the following result is actually a basic representation theorem showing that the four-valued semantics is adequate for collective argumentation.

Theorem 8. $\hookrightarrow$ *is an attack relation iff it coincides with* $\hookrightarrow_I$*, for some set of four-valued interpretations* I*.*

3.2 Logical kinds of argumentation

We will describe now three special kinds of collective argumentation called, respectively, classical, negative and positive argumentation. On the semantic level, these

[4]where $\not\hookrightarrow$ means that the attack relation does not hold.

kinds of argumentation will correspond to restrictions of four-valued reasoning to two- and three-valued reasoning. For all these kinds of argumentation, the attack relation will be defined by 'borrowing' arguments of the opposite side in order to disprove the latter. Classical argumentation will give an abstract description of classical consistency-based reasoning. Negative argumentation will be shown to be especially appropriate for describing the stable nonmonotonic semantics.

In ordinary disputation and argumentation the parties can provisionally accept some of the arguments defended by their adversaries in order to disprove the latter. Three basic cases of such an 'argument sharing' in attacking the opponents are described in the following definition (see also [Bondarenko *et al.*, 1997]).

Definition 9. *Given an attack relation* $\hookrightarrow$*, we will say that*

- a classically attacks b *(notation* $a \hookrightarrow^{\circ} b$*) if* $a, b \hookrightarrow a, b$*;*
- a negatively attacks b *(notation* $a \hookrightarrow^{-} b$*) if* $a \hookrightarrow a, b$*;*
- a positively attacks b *(notation* $a \hookrightarrow^{+} b$*) if* $a, b \hookrightarrow b$*.*

In a classical attack, the proponent shows, in effect, that her arguments are incompatible with that of the opponent. In a positive attack, the proponent temporarily accepts opponent's arguments in order to disprove the latter, while in a negative attack she shows that her arguments are sufficient for challenging an addition of the opponent's arguments. Clearly, if a attacks b directly, then it attacks the latter classically, positively and negatively, though not vice versa.

As can be seen, $\hookrightarrow^{\circ}$, $\hookrightarrow^{-}$ and $\hookrightarrow^{+}$ are also attack relations. Moreover, it turns out that all of them can be given an invariant structural characterization in terms of additional rules imposed on the attack relation. For explanatory reasons, we will begin below with a simplest such kind, namely the classical argumentation.

3.2.1 Classical argumentation

Classical argumentation can be seen as an 'upper bound' of collective argumentation; it is a simplest kind of argumentation which amounts, in effect, to classical consistency reasoning.

Definition 10. *An attack relation will be called* classical *if* $a, b \hookrightarrow a, b$ *always implies* $a \hookrightarrow b$*.*

It can be easily verified that an argument theory based on a classical attack $\hookrightarrow^{\circ}$ will be classical. Moreover, the latter determines a least classical 'closure' of the source attack relation:

Lemma 11. $\hookrightarrow^{\circ}$ *is a least classical attack relation containing* $\hookrightarrow$.

An immediate consequence of the above lemma is that classical attack relations are precisely attack relations of the form $\hookrightarrow^{\circ}$. In other words, classical attack relations provide a canonical description of argumentation based on a classical attack.

An attack relation is classical if and only if it satisfies:

(**Symmetry**) $\quad a \hookrightarrow b, c$ iff $a, b \hookrightarrow c$.

As a special case of Symmetry, we have

$$u \hookrightarrow v \ \text{ iff } \ \emptyset \hookrightarrow u, v \ \text{ iff } \ u, v \hookrightarrow \emptyset.$$

This shows that a classical attack amounts to inconsistency in the full classical sense. It should be noted in this respect that the classical closure $\hookrightarrow^{\circ}$ of an attack relation $\hookrightarrow$ preserves consistent argument sets. Namely, the definition of $\hookrightarrow^{\circ}$ immediately implies that it has the same consistent argument sets as $\hookrightarrow$.

As could be anticipated, classical argumentation can be characterized semantically by restricting the set of four-valued interpretations to classical two-valued ones, namely to interpretations that assign only **t** or only **f** to the arguments. This means that any argument is either accepted or rejected in an interpretation, but not both.

Theorem 12. *An attack relation is classical if and only if it is determined by a set of classical interpretations.*

Finally, classical argumentation can be seen as a combination of positive and negative argumentation. The proof follows immediately from the definitions of the respective attack relations.

Lemma 13. *For any attack relation* $\hookrightarrow$, $(\hookrightarrow^{-})^{+} = (\hookrightarrow^{+})^{-} = \hookrightarrow^{\circ}$.

The above result says, in particular, that positive and negative argumentation are incompatible on pain of collapsing to classical reasoning.

3.2.2 Negative argumentation

The definition below provides a general description of collective argumentation based on a negative attack.

Definition 14. *An attack relation will be called* negative *if* $a \hookrightarrow a, b$ *always implies* $a \hookrightarrow b$.

To begin with, it can be easily verified that any attack relation of the form $\hookrightarrow^-$ will be negative. Moreover, the latter determines a least negative closure of the source attack relation:

Lemma 15. $\hookrightarrow^-$ *is a least negative attack relation containing* $\hookrightarrow$*.*

An immediate consequence of the above lemma is that negative attack relations are precisely relations of the form $\hookrightarrow^-$. In other words, negative attack relations provide a canonical description of argumentation based on negative attacks.

The following result gives an important alternative characterization of negative argumentation.

Lemma 16. *An attack relation is negative iff it satisfies:*

(Import) *If* $a \hookrightarrow b, c$*, then* $a, b \hookrightarrow c$*.*

As a special case of Import, we have that if $a \hookrightarrow b$, then $a, b \hookrightarrow \emptyset$. Thus, any negative attack relation is bound to be non-affirmative. Furthermore, this implies that inconsistent argument sets attack any argument:

$$\text{If } v \hookrightarrow v, \text{ then } v \hookrightarrow u.$$

This feature is responsible for the fact that only stable sets constitute a reasonable nonmonotonic semantics for negative argumentation (see below).

Negative argumentation can be characterized semantically by restricting the set of possible four-valued interpretations to *consistent* ones that do not assign the set $\{t, f\}$ to arguments. This means that no argument can be both accepted and rejected in an interpretation.

Theorem 17. *An attack relation is negative if and only if it is determined by a set of consistent interpretations.*

3.2.3 Positive argumentation

The definition below provides a structural description of positive argumentation.

Definition 18. *An attack relation will be called* positive *if* $a, b \hookrightarrow b$ *always implies* $a \hookrightarrow b$*.*

Any attack relation $\hookrightarrow^+$ will be positive. Moreover, the latter determines a least positive extension of the source attack relation.

Lemma 19. $\hookrightarrow^+$ *is a least positive argument theory containing* $\hookrightarrow$*.*

The lemma implies that positive attack relations are precisely relations of the form $\hookrightarrow^+$, and hence they give a canonical description of argumentation based on positive attacks.

Similarly to negative argumentation, positive argumentation can be characterized by the 'exportation' property described in the lemma below:

Lemma 20. *An attack relation is positive iff it satisfies:*

(Export) *If $a, b \hookrightarrow c$, then $a \hookrightarrow b, c$.*

Positive argumentation can also be characterized semantically by restricting the set of possible four-valued interpretations to *complete* ones, namely to interpretations that do not assign $\emptyset$ to arguments. This means that every argument is either accepted or rejected in an interpretation (or both).

Theorem 21. *An attack relation is positive if and only if it is determined by a set of complete interpretations.*

The proof of the above theorem is perfectly similar to the case of negative argumentation. It turns out, however, that the positive argumentation, taken in its full generality, is not appropriate for the main nonmonotonic semantics, namely the stable semantics. Still, the reader can find in the literature a number of weaker argumentation systems that incorporate some of the features of positive argumentation. We will mention below only one important logical principle of this kind.

Consistent argumentation. A number of argumentation systems suggested in the literature (see, e.g., [Kakas *et al.*, 1994]) are based on the idea that inconsistent arguments should not form a legitimate attack on other arguments. A simplest way to incorporate this idea into an argumentation theory consists in using the following modification of an attack relation:

Definition 22. *Given an attack relation $\hookrightarrow$, we will say that a* consistently attacks *b (notation $a \hookrightarrow^c b$) if either $a \hookrightarrow b$, or $b \hookrightarrow b$.*

It turns out that this kind of an attack relation can also be given a logical description.

Definition 23. *An attack relation will be called* consistent *if $b \hookrightarrow b$ implies $\emptyset \hookrightarrow b$.*

As can be seen, consistent attack relations embody the most significant feature of positive argumentation, namely that inconsistent arguments are attacked by any argument. Still, consistency in the above sense is a weaker property than positivity (Export).

As before, it can be shown that consistent attack relations are precisely relations of the form $\hookrightarrow^c$. Finally, a semantic characterization of consistent argumentation can be obtained by requiring that any inconsistent argument set is rejected in at least one four-valued interpretation. This requirement is met by restricting the set of possible four-valued interpretations to *quasi-reflexive* ones, namely to sets of interpretations I such that if $(u, v) \in I$, then the corresponding classical interpretation (v, v) also belongs to I. As a matter of fact, this semantic constraint plays a prominent role in describing the logics appropriate for the stable semantics of logic programs (see [Bochman and Lifschitz, 2011]).

4 Nonmonotonic semantics

In the preceding section we have described a structural logical basis of argumentation. As we have argued in the introduction, however, the argumentation theory should be viewed as a two-layered formalism which has both logical and nonmonotonic components. This means that, in addition to the logical semantics, an argumentation formalism should be assigned also a *nonmonotonic* semantics that will determine the actual acceptance and rejection of arguments in each reasoning context. As one of its main objectives, the latter semantics should incorporate and thoroughly implement the basic principle of default acceptance for arguments.

Partly due to historical reasons (primarily, the logic programming origins), there is a bewildering number of nonmonotonic semantics that are actively investigated in the current argumentation literature. There have been a number of attempts to systematize these semantics (see, e.g., [Baroni and Giacomin, 2007]), though no uniform picture has been emerged.

In some sense, an attempt to systematize the various nonmonotonic semantics of argumentation is similar to an attempt to systematize logics in general, and as for the latter, it appears to be doomed from the very beginning. Worse still, the modern formal argumentation theory is still too young to provide substantive evidence for (or against) specific semantics, and thereby implicitly preserves hopes that they could be found useful in the future.

We will attempt to provide below a rough sketch of the basic principles and desiderata for constructing the nonmonotonic semantics of argumentation, which will also implicitly single out certain preferences, or priorities, between them. Our main underlying idea is that we should always try to apply the best (rather than the most general) semantics that is consistent with the constraints of the application in question. Of course, our position on this issue is not uncontroversial, but we contend that it is a reasonable and defensible position.

As a starting point, we will formulate the main *principle of argumentation* as the claim that arguments (in sharp distinction with factual assertions) bear with them the presumption of acceptance:

An argument is accepted unless there is a reason for its rejection.

One of the important ways of interpreting the above principle amounts to viewing arguments as *abducibles* in the framework of an argumentation theory. This understanding can serve as a guidance in determining the associated nonmonotonic semantics.

Now, in the framework of the formal argumentation theory, the reasons for rejection of arguments come only in the form of attacks by other arguments. Thus, our logical interpretation of the attack relation immediately sanctions that if an argument A attacks an argument B, and A is a accepted, then B should be rejected. In what follows, we will say that an argument is *refuted*, if it is attacked by an accepted argument set. Then our main principle of argumentation implies that an argument should be accepted whenever all its attacking arguments are not accepted. In other words, it evolves to

An argument is accepted if and only if it is not refuted.

Now, if we combine the above principle with the natural 'classical' requirement that any argument should be either accepted, or rejected, but not both, we will immediately obtain the primary nonmonotonic semantics of argumentation, the *stable semantics*[5]. According to this semantics, acceptable sets of arguments are conflict-free sets that attack any argument outside them.

In the general correspondence between Dung's argumentation theory and other nonmonotonic formalisms, the stable semantics corresponds to the main nonmonotonic semantics of the latter. This, as well as many other facts (some of which will be detailed later in this study), make the stable semantics a proper candidate on the role of the *standard nonmonotonic semantics* for argumentation, much in the same sense as the classical logic can be viewed as the standard logic for our reasoning (whatever the objections one could possibly have against this logic).

Despite its naturalness and simplicity, however, there are also quite simple argumentation frameworks where the stable semantics fails to determine an acceptable set of arguments[6]. Such situations create an obvious incentive for trying alternative,

[5] see also [Pollock, 1987].

[6] A simplest such framework comprises a single argument that attacks itself.

more tolerant, nonmonotonic semantics[7].

It turns out that the general four-valued logical semantics of acceptance and rejection of arguments provides all the necessary 'degrees of freedom' for defining such alternative nonmonotonic semantics, and the way to do this amounts to adopting different 'partial' generalizations of the main argumentation principle in the four-valued setting.

Retaining our earlier definition of refutation, a first such relaxed argumentation principle can be formulated as follows:

An argument is rejected if and only if it is refuted.

Note that the above principle is not equivalent to our original main argumentation principle, since the assignments of acceptance and rejection are logically independent. Instead, combined with our logical characterization of the attack relation, this principle will give us precisely the notion of labeling from [Jakobovits and Vermeir, 1999].

An even stronger general constraint on nonmonotonic semantics can be obtained by adding the following alternative generalization of the main argumentation principle:

An argument is accepted if and only if all its attackers are rejected.

Now, if we will restrict the set of valuations to consistent ones, we will obtain exactly the Caminada labellings (see [Caminada and Gabbay, 2009]). These labellings have been shown to encompass the main nonmonotonic semantics of Dung's argumentation frameworks.

In the rest of this section we are going to provide a more detailed description of the nonmonotonic semantics of argumentation.

4.1 Normal (Dung) argumentation

As a convenient starting point, we will describe now a range of nonmonotonic semantics for normal argument theories suggested in [Dung, 1995b]. All these semantics can be defined in terms of the following two notions:

[7] As a side remark, it is important to bear in mind, however, that the nonmonotonic semantics is not intended to replace the logical semantics in all its functions and capacities. In particular, the nonmonotonic semantics *should not* be required to deliver a consistent extension in any situation (just as a definite description in classical first-order logic cannot be required to always determine a unique referent).

Definition 24. *Given an attack relation $\hookrightarrow$, an argument A will be called* allowable *for a set of arguments u, if $u \not\hookrightarrow A$, and* acceptable *for u, if u attacks any argument set that attacks A.*

$[u]$ will denote the set of all assumptions allowable by u:

$$[u] \;=\; \{A \mid u \not\hookrightarrow A\}.$$

The origins of this operator can be found already in [Pollock, 1987], and it has been extensively used in [Dung, 1995b].

Note that $[\ ]$ is an anti-monotonic operator on argument sets, that is, $u \subseteq v$ implies $[v] \subseteq [u]$. Moreover, the set of arguments that are acceptable for u coincides with $[[u]]$, where $[[\ ]]$ is obviously a monotonic operator.

Using the above notions, we can give a rather simple characterization of the basic nonmonotonic models of a normal argumentation.

Definition 25. *An argument set u is*

- conflict-free *if $u \subseteq [u]$;*
- admissible *if it is conflict-free and $u \subseteq [[u]]$;*
- *a* complete extension *if it is conflict-free and $u = [[u]]$;*
- *a* preferred extension *if it is a maximal complete extension;*
- *a* stable extension *if $u = [u]$.*

As has been shown in [Dung, 1995a], the above models correspond to well-known semantics for normal logic programs.

4.2 Stable and partial stable semantics

Though the above notions and models of Dung's argumentation theory have been defined for arbitrary collective attack relations, it should be clear that they are adequate only for normal argumentation, since they are based only on singular attacks. Still, we will see that in some important cases the more general models defined below will coincide with their normal counterparts. Unfortunately, it will turn out that only a small part of the nice and well-organized structure of nonmonotonic models for normal argumentation can be transferred into a general framework of collective argumentation.

If (u, v) is a bitheory of an attack relation (that is, $u \not\hookrightarrow v$), then v is always included in a maximal set v_1 such (u, v_1) is a bitheory. The corresponding four-valued model contains a minimal set of rejected arguments for a given set u of accepted ones.

For an argument set u, we will denote by $\langle u \rangle$ the set of all maximal argument sets v such that $u \not\hookrightarrow v$. The operator $\langle u \rangle$ will play in what follows the same role as the operator $[u]$ in normal argumentation. In particular, using this operator, we can give the following rather simple description of stable and partial stable models of collective argumentation.

Definition 26.

- *An argument set u will be called* stable *if $u \in \langle u \rangle$.*
- *A bitheory (u, v) will be called* partial stable *if $u \in \langle v \rangle$, and $v \in \langle u \rangle$.*

As has been shown in [Bochman, 2003a], under a general correspondence between collective argumentation and *disjunctive* logic programs, the above models correspond precisely to the well-known semantics for such logic programs, given in the literature.

Stable argument sets correspond precisely to partial stable bitheories of the form (u, u). Note also that if (u, v) is a partial stable bitheory, then (v, u) will also be partial stable, and vice versa. A usual additional condition imposed on partial stable models amounts, however, to requiring that (u, v) should be a consistent bitheory (that is, $u \subseteq v$). The corresponding models will be called *consistent* partial stable bitheories. Note, however, that the bitheories (u, v) and (v, u) provide the same information about 'classical' acceptance and rejection of assumptions, namely they single out the same assumptions that are accepted without being rejected, and same rejected assumptions that are not also accepted.

The following lemmas give more direct, and often more convenient, descriptions of the above models.

Lemma 27. *An argument set u is stable iff* $u = \{A \mid u \not\hookrightarrow u, A\}$.

The above equation says that a stable argument u consists of all arguments A such that u does not attack $u \cup \{A\}$. A similar description can be given for partial stable models.

Lemma 28. (u, v) *is a partial stable bitheory if and only if*

$$v = \{A \mid u \not\hookrightarrow v, A\} \quad \textit{and} \quad u = \{A \mid v \not\hookrightarrow u, A\}.$$

The next result shows that stable argument sets and consistent partial stable bitheories of a normal attack relation coincide, respectively, with stable and complete extensions.

Lemma 29. *If $\hookrightarrow$ is a normal attack relation, then*

- *stable argument sets coincide with stable extensions;*
- (u, v) *is a consistent partial stable bitheory iff* u *is a complete extension, and* $v = [u]$.

Furthermore, it can be shown that partial stable bitheories are representable as stable argument sets of a certain 'doubled' attack relation.

Let $\hookrightarrow$ be an attack relation on a set $\mathcal{A}$ of arguments. For each $A \in \mathcal{A}$, we introduce a new argument A'. For any subset u of $\mathcal{A}$, we will denote by u' the set $\{A' \mid A \in u\}$. Now we define a new attack relation $\hookrightarrow_\circ$ on $\mathcal{A} \cup \mathcal{A}'$ as follows:

$$a, b' \hookrightarrow_\circ c, d' \;\equiv\; a \hookrightarrow d \text{ or } b \hookrightarrow c.$$

Then we have

Theorem 30. *A bitheory* (u, v) *is partial stable in* $\hookrightarrow$ *if and only if* $u \cup v'$ *is a stable argument set in* $\hookrightarrow_\circ$.

The above theorem shows, in effect, that partial stable models are essentially stable argument sets 'in disguise'. Unfortunately, in the case of collective argumentation they lack most of the structural properties they had in the normal case of Dung's theory. Most importantly, they do not form a lower semilattice (under the standard information order over four-valued models -see, e.g., [Fitting, 1991]) and, in particular, there may be no least partial stable model. In fact, unlike the normal case, collective argument theories may have no partial stable models at all.

Example 31. *(1) It can be verified that the argument theory*

$$\{\hookrightarrow A, B, C;\; A \hookrightarrow B;\; B \hookrightarrow C;\; C \hookrightarrow A\}$$

does not have any partial stable bitheories.

(2) the argument theory $\{\hookrightarrow A, B;\; A \hookrightarrow A;\; B \hookrightarrow B\}$ *does not have consistent partial stable models, though* $(\{A\}, \{B\})$ *and* $(\{B\}, \{A\})$ *are its partial stable bitheories.*

4.3 Admissibility semantics

Finally we will briefly consider collective counterparts of admissible argument sets in normal argumentation. Recall that the latter have been defined as conflict-free argument sets that counterattack any argument against them. This definition can be naturally generalized to collective argumentation as follows:

Definition 32. *A consistent argument set u will be called* admissible *if, for any v, if $v \hookrightarrow u$, then $u \hookrightarrow v$.*

Admissible argument sets can also be described in terms of the $\langle\ \rangle$ operator. Namely, u is admissible if no argument set from $\langle u \rangle$ attacks u. Clearly, admissibility reduces to Dung-admissibility for normal attack relations. Unfortunately, in the context of collective argumentation the notion of admissibility behaves in a much less ordered fashion than in the Dung's theory. Note, in particular, that even stable argument sets need not be admissible in this sense:

Example 33. *Let us consider an argument theory $\{A \hookrightarrow B \hookrightarrow A, B\}$. As can be seen, $\{B\}$ is a stable argument set of this theory, but it is not admissible: we have $A \hookrightarrow B$, though $B \not\hookrightarrow A$.*

It turns out, however, that consistent stable extensions are always both admissible and stable.

Lemma 34. *Any consistent stable extension of an attack relation is both an admissible and stable argument set.*

4.4 Underlying argumentation logics

Any nonmonotonic semantics implicitly determines an appropriate underlying logic, a logic that preserves this semantics under all expansions of the associated nonmonotonic theory. In this section, we describe the effects of imposing logical constraints, described earlier, on the nonmonotonic argumentation semantics.

As an 'upper' limiting case, the classical argumentation theory drastically simplifies the whole range of nonmonotonic semantics:

Lemma 35. *If $\hookrightarrow$ is a classical attack relation, then*

- *Admissible sets coincide with consistent sets;*
- *Stable sets coincide with maximal consistent sets;*
- *Partial stable bitheories coincide with stable ones.*

A more discriminate look reveals that it is the 'positive ingredient' of classical argumentation that could be held responsible for this trivializing effect. More precisely, already the consistency property produces the same result:

Lemma 36. *If $\hookrightarrow$ is a consistent attack relation, then*

- *stable argument sets coincide with maximal consistent sets;*

- *consistent partial stable bitheories are bitheories of the form* (u, u)*, where* u *is a stable argument set.*

On the other hand, the results below will show that negative argumentation constitutes an adequate and very convenient framework for studying the stable semantics.

As a first step, the next result shows that stable argument sets of an attack relation are precisely stable extensions of its negative closure.

Lemma 37. *Stable argument sets of an attack relation* $\hookrightarrow$ *coincide with the stable extensions of* $\hookrightarrow^-$.

The above result shows that Dung's stable extensions and stable argument sets of collective argumentation are indeed close relatives. As a consequence, we immediately obtain

Corollary 38.
- *Stable argument sets of a negative attack relation coincide with its stable extensions.*
- *Any attack relation* $\hookrightarrow$ *has the same stable argument sets as* $\hookrightarrow^-$.

The second claim above implies that Import is an argumentation rule that preserves stable argument sets, and hence negative argumentation turns out to be appropriate for the stable nonmonotonic semantics. An additional consequence of the above results is the eventual reduction of stable argument sets to stable extensions of Dung's argumentation theory. Lemma 37 says, in effect, that, after extending a given attack relation to a negative one (by closing it with respect to Import), we can restrict ourselves to its normal sub-relation; stable extensions of the resulting normal attack relation will coincide with stable argument sets of the original attack relation.

Our next result shows, however, that negative argumentation trivializes partial stable semantics.

Lemma 39. *Partial stable bitheories of a negative attack relation are bitheories of the form* (u, u)*, where* u *is a stable argument set.*

Thus, negative argumentation reduces partial stable models to stable ones. In fact, negative argumentation seems to exclude all nonmonotonic semantics other than the stable one.

Our final result shows that admissible argument sets still play an important role in negative argumentation.

Theorem 40. *Let* $\hookrightarrow$ *be a negative attack relation.*

- *If u is an admissible argument set in $\hookrightarrow$, and v a consistent argument set that includes u, then v is also admissible in $\hookrightarrow$.*
- *Stable argument sets of $\hookrightarrow$ coincide with maximal admissible argument sets.*

The above theorem demonstrates that the structure of admissible and stable argument sets in negative argumentation is very simple. Namely, they behave much like logically consistent sets. There is, however, a crucial difference: the empty set $\emptyset$ is not, in general, admissible. Moreover, a negative attack relation may have no admissible arguments at all; this happens precisely when it has no stable argument sets.

5 Negation, deduction and assumptions

The notion of an argument is often taken as primitive in argumentation theory, which, among other theoretical advantages, allows for a possibility of considering arguments that are non-propositional in character (e.g., arguments as inference rules, or derivations). Still, there exists a natural, direct connection between abstract argumentation frameworks and traditional deductive argumentation; it has been established, in effect, already in [Bondarenko *et al.*, 1997][8]. In this formalism of *assumption-based argumentation* arguments were constructed as plain deductive arguments that may involve, however, auxiliary propositional *assumptions*. Moreover, the attack relation can already be defined in this framework, so the assumption-based argumentation can be viewed as a special case of Dung's abstract argumentation. Nevertheless, it has been shown in [Bondarenko *et al.*, 1997] that this special kind of argumentation still provides a natural and powerful generalization of the main nonmonotonic formalisms and various semantics for logic programming.

As we are going to show in this section, the entire formalism of assumption-based argumentation can be obtained just by adding a single negation connective to the logical system of abstract argumentation, a connective that is actually implicit in the formalism of [Bondarenko *et al.*, 1997] in the form of the contrary mapping on assumptions. This move will also constitute a first, and most important, step towards a full-fledged theory of *propositional argumentation* that will be described subsequently.

Let us extend our underlying language with a negation connective $\sim$ having the

[8] See also [Kowalski and Toni, 1996].

following precise (four-valued) semantic definition[9]:

$$\sim A \text{ is accepted iff } A \text{ is rejected}$$
$$\sim A \text{ is rejected iff } A \text{ is accepted.}$$

The above definition makes $\sim$ a particular four-valued connective; it will be called a *global negation*, since it switches the evaluation contexts between acceptance and rejection.

An axiomatization of this negation in abstract argumentation theory can be obtained by imposing the following rules on the attack relation:

$$\begin{gathered} A \hookrightarrow \sim A \qquad \sim A \hookrightarrow A \\ \text{If } a \hookrightarrow A, b \text{ and } a, \sim A \hookrightarrow b, \text{ then } a \hookrightarrow b \\ \text{If } a, A \hookrightarrow b \text{ and } a \hookrightarrow b, \sim A, \text{ then } a \hookrightarrow b \end{gathered} \tag{AN}$$

Attack relations satisfying the above postulates will be called *N-attack relations*. It turns out that the latter are inter-definable with a particular kind of consequence relations.

Recall that a *Scott consequence relation*, known also as a multiple-conclusion consequence relation [Shoesmith and Smiley, 1978; Gabbay, 1981; Segerberg, 1982; Wojcicki, 1988], is a binary relation between *sets* of propositions that is required to satisfy the following will-known postulates:

(Reflexivity) $A \Vdash A$;

(Monotonicity) If $a \Vdash b$ and $a \subseteq a'$, $b \subseteq b'$, then $a' \Vdash b'$;

(Cut) If $a \Vdash b, A$ and $a, A \Vdash b$, then $a \Vdash b$,

In this logical framework, our target consequence relations can be described as follows:

Definition 41. *A* Belnap consequence relation *in a propositional language with a global negation* $\sim$ *is a Scott consequence relation satisfying the following two Double Negation rules for the global negation:*

$$A \Vdash \sim\sim A \qquad \sim\sim A \Vdash A.$$

[9]This negation connective played a prominent role in Belnap's information lattices [Belnap, 1977].

For any set u of propositions, we will denote by $\sim u$ the set $\{\sim A \mid A \in u\}$. Now, for a given N-attack relation, we can define the following consequence relation:

$$a \Vdash b \equiv\ a \hookrightarrow \sim b \tag{CA}$$

Similarly, for any Belnap consequence relation we can define the corresponding attack relation as follows:

$$a \hookrightarrow b \equiv\ a \Vdash \sim b \tag{AC}$$

As has been shown in [Bochman, 2003a], the above definitions establish an exact equivalence between N-attack relations and Belnap consequence relations. This correspondence allows us to represent an assumption-based argumentation framework from [Bondarenko *et al.*, 1997] entirely in the framework of attack relations (see below).

N-attack relations allow to provide simpler alternative descriptions of negative and positive argumentation. Thus, the rule Import of negative argumentation for such attack relations is equivalent to the condition

$$A, \sim A \hookrightarrow \emptyset.$$

whereas the rule Export of positive argumentation is equivalent to

$$\emptyset \hookrightarrow A, \sim A.$$

The above condition explicitly says that any argument should be either rejected or accepted. As could be expected, it is equivalent also to the principle of *reasoning by cases*:

(**Factoring**) If $a, A \hookrightarrow b$ and $a, \sim A \hookrightarrow b$, then $a \hookrightarrow b$.

Assumptions versus factual propositions. Though the global negation $\sim$ is a logically well-defined connective, it implicitly interferes with the main principle of argumentation that presupposes an asymmetric treatment of acceptance and rejection for arguments. Indeed, if A is an argument, then $\sim A$ cannot already be viewed as an argument, since otherwise presumptive acceptance of $\sim A$ would directly imply presumptive *rejection* of A itself!

The emerging problem immediately reminds us, however, that our commonsense epistemic states are not homogeneous: in addition to normality assumptions (that can be viewed as primitive arguments), they contain also ordinary factual claims. Furthermore, the latter have in a sense an opposite nature as compared to arguments. Namely, they are presumably rejected unless we have reasons for their acceptance.

A simple and perhaps the most natural way of resolving the above issues consists in a clear separation between assumptions and factual propositions; it has been actually implemented in the assumption-based argumentation of [Bondarenko *et al.*, 1997].

Assumption-based argumentation (ABA). Slightly changing the formulation of [Bondarenko *et al.*, 1997], an assumption-based argumentation framework can be defined as a triple consisting of an underlying deductive system (including the current set of beliefs), a distinguished subset of propositions Ab called *assumptions*, and a mapping from Ab to the set of all propositions of the language that determines the *contrary* $\overline{A}$ of any assumption A.

Now, the underlying deductive system can be expressed directly in the framework of N-attack relations by identifying deductive rules $a \vdash A$ with attacks of the form $a \hookrightarrow \sim A$. Furthermore, the global negation $\sim$ can also serve as a faithful logical formalization of the operation of taking the contrary. More precisely, given an arbitrary underlying language $\mathcal{L}$ that does not contain $\sim$, we can *define* assumptions as propositions of the form $\sim A$, where $A \in \mathcal{L}$. Then, since $\sim$ satisfies double negation, a negation of an assumption will be a proposition from $\mathcal{L}$. Accordingly, N-attack relations can be seen as a proper generalization of the assumption-based framework.

Remark 42. *It should be mentioned that our representation assign a bit more structure and properties to assumptions and the contrary mapping than it was originally assumed in ABA. Still, it can be verified that this extended representation is fully conservative with respect to the applications of this argumentation theory to other nonmonotonic formalisms, described in [Bondarenko* et al.*, 1997].*

In [Bondarenko *et al.*, 1997], the connection between (assumption-based) argumentation and main nonmonotonic formalisms has been established by showing that these nonmonotonic systems can be viewed as assumption-based frameworks just by defining assumptions and their contraries. As a partial converse of these results, we are going to show below that many of these formalisms constitute actually primary instantiations of propositional argumentation in appropriately chosen logical languages.

6 Default argumentation

Taking seriously the idea of propositional argumentation, it is only natural to make further steps toward extending the underlying language of arguments to the usual classical propositional language. These steps should be coordinated, however, with the inherently four-valued nature of the attack relation. And the way to do this amounts to requiring that the relevant classical connectives should behave in a usual classical way with respect to both acceptance and rejection of arguments.

As a first such connective, we introduce the *conjunction* $\wedge$ of arguments that is determined by the following familiar semantic conditions:

$$A \wedge B \text{ is accepted iff } A \text{ is accepted and } B \text{ is accepted}$$
$$A \wedge B \text{ is rejected iff } A \text{ is rejected or } B \text{ is rejected}$$

As can be seen, $\wedge$ behaves as an ordinary classical conjunction with respect to acceptance and rejection of arguments. On the other hand, it is a four-valued connective, since the above conditions determine a four-valued truth-table for conjunction in the Belnap's interpretation of four-valued logic (see [Belnap, 1977]). The following postulates provide a simple syntactic characterization of this connective for attack relations:

$$\begin{aligned} a, A \wedge B \hookrightarrow b \ &\text{ iff } \ a, A, B \hookrightarrow b \\ a \hookrightarrow A \wedge B, b \ &\text{ iff } \ a \hookrightarrow A, B, b \end{aligned} \tag{$A_\wedge$}$$

Collective attack relations satisfying these postulates will be called *conjunctive*. The next result shows that they give a complete description of the four-valued conjunction.

Corollary 43. *An attack relation is conjunctive if and only if it coincides with $\hookrightarrow_I$, for some set of four-valued interpretations I in a language with the four-valued conjunction $\wedge$.*

An immediate benefit of introducing conjunction into the language of argumentation is that any finite set of arguments a becomes reducible to a single argument $\bigwedge a$:

$$a \hookrightarrow b \quad \text{iff} \quad \bigwedge a \hookrightarrow \bigwedge b.$$

As a result, the collective attack relation in this language is reducible to an attack relation between individual arguments, just as it has been assumed in [Dung, 1995b].

Having a conjunction at our disposal, we only have to add a classical negation $\neg$ in order to obtain a full classical language. Moreover, since sets of arguments are reducible to their conjunctions, we can represent the resulting argumentation theory using just a binary attack relation on classical formulas.

As a basic condition on argumentation in the classical propositional language, we will require only that the attack relation should respect the classical entailment $\vDash$ in the precise sense of being monotonic with respect to $\vDash$ on both sides.

Definition 44. *A* propositional attack relation *is a relation* $\hookrightarrow$ *on the set of classical propositions satisfying the following postulates:*

(Left Strengthening) *If* $A \vDash B$ *and* $B \hookrightarrow C$*, then* $A \hookrightarrow C$*;*

(Right Strengthening) *If* $A \hookrightarrow B$ *and* $C \vDash B$*, then* $A \hookrightarrow C$*;*

(Truth) $\mathbf{t} \hookrightarrow \mathbf{f}$*;*

(Falsity) $\mathbf{f} \hookrightarrow \mathbf{t}$*.*

Left Strengthening says that logically stronger arguments should attack any argument that is attacked already by a logically weaker argument, and similarly for Right Strengthening. Truth and Falsity postulates characterize the limit cases of argumentation by stipulating that any tautological argument attacks any contradictory one, and vice versa.

There exists a simple definitional way of extending the above attack relation to a collective attack relation between arbitrary sets of propositions. Namely, for any sets u, v of propositions, we can define $u \hookrightarrow v$ as follows:

$$u \hookrightarrow v \;\equiv\; \text{there exist finite } a \subseteq u, b \subseteq v \text{ such that } \bigwedge a \hookrightarrow \bigwedge b$$

The resulting attack relation will satisfy the properties of collective argumentation, as well as the postulates $(A_\wedge)$ for conjunction.

Finally, in order to acquire full expressive capabilities of the argumentation theory, we can add the global negation $\sim$ to the language. Actually, a rather simple characterization of the resulting collective argumentation theory can be obtained by accepting the basic postulates AN for $\sim$, plus the following rule that permits the use of classical entailment in attacks:

Classicality If $a \vDash A$, then $a \hookrightarrow \sim A$ and $\sim A \hookrightarrow a$.

It can be verified that the resulting system satisfies all the postulates for propositional argumentation. The system will be used later for a direct representation of default logic.

6.1 Logical semantics

A semantic interpretation of propositional attack relations can be obtained by generalizing four-valued interpretations to pairs (u, v) of deductively closed theories, where u is the set of accepted propositions, while v the set of propositions that are not rejected. Such pairs will be called *bimodels*, while a set of bimodels will be called a *binary semantics*.

Definition 45. *An attack* $A \hookrightarrow B$ *will be said to be* valid *in a binary semantics* $\mathcal{B}$ *if there is no bimodel* (u, v) *from* $\mathcal{B}$ *such that* $A \in u$ *and* $B \in v$.

We will denote by $\hookrightarrow_{\mathcal{B}}$ the set of attacks that are valid in a semantics $\mathcal{B}$. This set forms a propositional attack relation. Moreover, the following result shows that propositional attack relations are actually complete for the binary semantics.

Theorem 46. $\hookrightarrow$ *is a propositional attack relation if and only if it coincides with* $\hookrightarrow_{\mathcal{B}}$, *for some binary semantics* $\mathcal{B}$.

6.2 Default logic

Now we will show that propositional argumentation provides a direct representation of Reiter's default logic [Reiter, 1980].

Given a system of propositional argumentation in the classical language augmented with the global negation $\sim$, we will interpret Reiter's default rule $a{:}b/A$ as an attack[10]

$$a, \sim\neg b \hookrightarrow \sim A,$$

or, equivalently, as a rule $a, \sim\neg b \Vdash A$ of the associated Belnap consequence relation. Similarly, an axiom A of a default theory will be interpreted as an attack $\mathbf{t} \hookrightarrow \sim A$. For a default theory Δ, we will denote by $tr(\Delta)$ the corresponding argument theory obtained by this translation.

By our general agreement, by *assumptions* we will mean propositions of the form $\sim A$, where A is a classical proposition. For a set u of classical propositions, we will denote by $\tilde{u}$ the set of assumptions $\{\sim A \mid A \notin u\}$. Finally, a set w of assumptions will be called *stable* in an argument theory Δ if, for any assumption A, $A \in w$ iff $w \not\hookrightarrow_{\Delta} A$, where $\hookrightarrow_{\Delta}$ is the least propositional attack relation containing Δ. Then we have

Theorem 47. *A set* u *of classical propositions is an extension of a default theory* Δ *if and only if* $\tilde{u}$ *is a stable set of assumptions in* $tr(\Delta)$.

The above result is similar to the corresponding representation result in [Bondarenko *et al.*, 1997, Theorem 3.10], but it is much simpler, and is formulated entirely in the framework of propositional attack relations. The simpler representation was made possible due to the fact that propositional attack relations already embody the deductive capabilities treated as an additional ingredient in assumption-based frameworks.

[10] As before, we use set notation according to which $\neg b$ denotes the set $\{\neg B \mid B \in b\}$.

7 Probative and causal argumentation

We will introduce now some stronger propositional attack relations that satisfy further reasonable postulates:

(Left Or) If $A \hookrightarrow C$ and $B \hookrightarrow C$, then $A \vee B \hookrightarrow C$;

(Right Or) If $A \hookrightarrow B$ and $A \hookrightarrow C$, then $A \hookrightarrow B \vee C$;

(Self-Defeat) If $A \hookrightarrow A$, then $\mathbf{t} \hookrightarrow A$.

Definition 48. *A propositional attack relation will be called* probative *if it satisfies Left Or,* basic*, if it also satisfies Right Or, and* causal*, if it is basic and satisfies Self-Defeat.*

Probative argumentation allows for reasoning by cases. Its semantic interpretation can be obtained by restricting bimodels to pairs (α, v), where α is a world (maximal classically consistent set). The corresponding binary semantics will also be called *probative*. Similarly, the semantics for basic argumentation is obtained by restricting bimodels to world pairs (α, β); such a binary semantics will be called *basic*. Finally, the *causal* binary semantics is obtained from the basic semantics by requiring further that (α, β) is a bimodel only if (β, β) is also a bimodel.

Corollary 49. *A propositional attack relation is probative [basic, causal] iff it is determined by a probative [resp. basic, causal] binary semantics.*

Basic propositional argumentation can already be given a purely four-valued semantic interpretation, in which the classical negation $\neg$ has the following semantic description:

$$\neg A \text{ is accepted iff } A \text{ is not accepted}$$
$$\neg A \text{ is rejected iff } A \text{ is not rejected}$$

A syntactic characterization of this connective in collective argumentation can be obtained by imposing the rules

$$\begin{gathered} A, \neg A \hookrightarrow \qquad \hookrightarrow A, \neg A \\ \text{If } a, A \hookrightarrow b \text{ and } a, \neg A \hookrightarrow b \text{ then } a \hookrightarrow b \\ \text{If } a \hookrightarrow b, A \text{ and } a \hookrightarrow b, \neg A \text{ then } a \hookrightarrow b \end{gathered} \qquad (A_\neg)$$

Then a basic propositional attack relation can be alternatively described as a collective attack relation satisfying the rules $(A_\wedge)$ and $(A_\neg)$. Moreover, the global negation $\sim$ can be added to this system just by adding the corresponding postulates (AN). It turns out, however, that the global negation is *eliminable* in this setting via to the following reductions:

$$a, \sim A \hookrightarrow b \equiv a \hookrightarrow b, \neg A \qquad a \hookrightarrow \sim A, b \equiv a, \neg A \hookrightarrow b$$
$$a, \neg\sim A \hookrightarrow b \equiv a \hookrightarrow b, A \qquad a \hookrightarrow \neg\sim A, b \equiv a, A \hookrightarrow b \qquad (R_\sim)$$

As a result, the basic attack relation can be safely restricted to an attack relation in a classical language.

Finally, the rule Self-Defeat of causal argumentation gives a formal representation for an often expressed desideratum that self-conflicting arguments should not participate in defeating other arguments (see, e.g., [Bondarenko *et al.*, 1997]). This aim is achieved in our setting by requiring that such arguments are attacked even by tautologies, and hence by any argument whatsoever.

7.1 Argumentation vs. causal reasoning

Probative attack relations turn out to be equivalent to general production inference relations from [Bochman, 2003b; Bochman, 2004a], a variant of input-output logics from [Makinson and van der Torre, 2000].

A *production inference relation* is a relation $\Rightarrow$ on the set of classical propositions satisfying the following rules:

(Strengthening) If $A \vDash B$ and $B \Rightarrow C$, then $A \Rightarrow C$;

(Weakening) If $A \Rightarrow B$ and $B \vDash C$, then $A \Rightarrow C$;

(And) If $A \Rightarrow B$ and $A \Rightarrow C$, then $A \Rightarrow B \wedge C$;

(Truth) $\mathbf{t} \Rightarrow \mathbf{t}$;

(Falsity) $\mathbf{f} \Rightarrow \mathbf{f}$.

A production rule $A \Rightarrow B$ can be informally interpreted as saying that A *causes*, or *explains* B. A characteristic property of production inference is that reflexivity $A \Rightarrow A$ does not hold for it. Production rules are extended to rules with sets of propositions in premises by requiring that $u \Rightarrow A$ holds for a set u of propositions iff $\bigwedge a \Rightarrow A$, for some finite $a \subseteq u$. $\mathcal{C}(u)$ will denote the set of propositions produced by u:

$$\mathcal{C}(u) = \{A \mid u \Rightarrow A\}$$

The production operator $\mathcal{C}$ plays much the same role as the usual derivability operator for consequence relations.

A production inference relation is called *basic*, if it satisfies

(Or) If $A \Rightarrow C$ and $B \Rightarrow C$, then $A \vee B \Rightarrow C$.

and *causal*, if it is basic and satisfies, in addition

(Coherence) If $A \Rightarrow \neg A$, then $A \Rightarrow \mathbf{f}$.

It has been shown in [Bochman, 2003b] that causal inference relations provide a complete description of the underlying logic of causal theories from [McCain and Turner, 1997] (see also [Giunchiglia *et al.*, 2004]).

It turns out that the binary semantics, introduced earlier, is appropriate also for interpreting production inference:

Definition 50. *A rule $A \Rightarrow B$ is* valid *in a binary semantics $\mathcal{B}$ if, for any bimodel $(u, v) \in \mathcal{B}$, $A \in u$ only if $B \in v$.*

As has been shown in [Bochman, 2004a], the above semantics is adequate for production inference relations. Moreover, the semantics for basic production inference can be obtained by restricting bimodels to world pairs (α, β), while the semantics for causal inference is obtained by requiring, in addition, that (α, β) is a bimodel only if (α, α) is also a bimodel.

Now, the correspondence between probative argumentation and production inference can be established directly on the syntactic level using the following definitions:

$$A \Rightarrow B \;\equiv\; \neg B \hookrightarrow A; \tag{PA}$$

$$A \hookrightarrow B \;\equiv\; B \Rightarrow \neg A. \tag{AP}$$

Under these correspondences, the rules of a probative attack relation correspond precisely to the postulates for production relations. Moreover, the correspondence extends also to a correspondence between basic and causal argumentation, on the one hand, and basic and causal production inference, on the other. Hence the following result is straightforward.

Lemma 51. *If $\hookrightarrow$ is a probative [basic, causal] attack relation, then (PA) determines a [basic, causal] production inference relation, and vice versa, if $\Rightarrow$ is a [basic, causal] production inference relation, then (AP) determines a probative [basic, causal] attack relation.*

Remark 52. *A seemingly more natural correspondence between propositional argumentation and production inference can be obtained using the following definitions:*

$$A \Rightarrow B \;\equiv\; A \hookrightarrow \neg B \qquad A \hookrightarrow B \;\equiv\; A \Rightarrow \neg B.$$

By these definitions, A explains B if it attacks $\neg B$*, and vice versa, A attacks B if it explains* $\neg B$*. Unfortunately, this correspondence, though plausible by itself, does not take into account* the intended understanding of arguments as (negative) assumptions. *As a result, it cannot be extended directly to the correspondence between the associated nonmonotonic semantics, described below.*

As our next result, we will establish a correspondence between the nonmonotonic semantics of causal inference relations and that of causal argumentation.

The nonmonotonic semantics of a causal inference relation is a set of its *exact worlds*, namely worlds α such that $\alpha = \mathcal{C}(\alpha)$ (see [Bochman, 2004a]). Such a world satisfies the rules of the causal relation, and any proposition that holds in it is explained by the causal rules.

A *causal theory* is an arbitrary set of production rules. By a nonmonotonic semantics of a causal theory Δ we will mean the exact worlds of the least causal relation containing Δ.

The correspondence between exact worlds and stable sets of assumptions is established in the next theorem.

Theorem 53. *If* Δ *is a causal theory, and* Δ_a *its corresponding argument theory given by (AP), then a world* α *is an exact world of* Δ *iff* $\tilde{\alpha}$ *is a stable set of assumptions in* Δ_a*.*

The above result shows, in effect, that propositional argumentation subsumes causal reasoning as a special case. Moreover, it can be shown that causal attack relations constitute a strongest argumentation system suitable for this kind of nonmonotonic semantics.

7.2 Abstract dialectical frameworks (ADFs)

As we are going to show in this section, Abstract Dialectical Frameworks [Brewka and Woltran, 2010; Brewka *et al.*, 2013 can be viewed, in effect, as yet another bridge between argumentation and causal reasoning.

We will restrict our descriptions below only to the features of ADFs that are relevant for our exposition.

Abstract Dialectical Frameworks have been introduced as an abstract argumentation formalism purported to capture more general forms of argument interaction than just attacks among arguments. To achieve this, each argument (or statement) in an ADF is associated with an *acceptance condition*, which is some propositional function determined by arguments that are linked to it. Using such acceptance conditions, ADFs allow to express that arguments may jointly support another argument, or that two arguments may jointly attack a third one, and so on. Dung's argumentation frameworks are recovered in this setting by acceptance condition saying that an argument is accepted if none of its parents is.

Formally, an abstract dialectical framework is a directed graph whose nodes represent statements or positions which can be accepted or not. The links represent dependencies: the status of a node s only depends on the status of its parents (denoted $par(s)$), that is, the nodes with a direct link to s. In addition, each node s has an associated acceptance condition C_s specifying the exact conditions under which s is accepted. C_s is a function assigning to each subset of $par(s)$ one of the truth values $\mathbf{t}, \mathbf{f}$. Intuitively, if for some $R \subseteq par(s)$ we have $C_s(R) = \mathbf{t}$, then s will be accepted provided the nodes in R are accepted and those in $par(s) \setminus R$ are not accepted.

Definition 54. *An abstract dialectical framework is a tuple $D = (S, L, C)$ where*

- *S is a set of statements (positions, nodes),*
- *$L \subseteq S \times S$ is a set of links,*
- *$C = \{C_s\}_{s \in S}$ is a set of total functions $C_s : 2^{par(s)} \to \{\mathbf{t}, \mathbf{f}\}$, one for each statement s. C_s is called acceptance condition of s.*

A more 'logical' representation of ADFs can be obtained simply by assigning each node s a *classical* propositional formula corresponding to its acceptance condition C_s (see [Ellmauthaler, 2012]). In this case we can tacitly assume that the acceptance formulas implicitly specify the parents a node depends on. It is then not necessary to give the links L, so an ADF D amounts to a tuple (S, C) where S is a set of statements, and C is a set of propositional formulas, one for each statement from S. The notation $s[C_s]$ has been used by the authors to denote the fact that C_s is the acceptance condition of s.

A two-valued interpretation v is a (two-valued) *model* of an ADF (S, C) whenever for all statements $s \in S$ we have $v(s) = v(\varphi_s)$, that is, v maps exactly those statements to true whose acceptance conditions are satisfied under v. This notion of a model provides a natural semantics for ADFs. In addition to this semantics, however, the authors define appropriate generalizations for all the major semantics of Dung's argumentation frameworks. Following the 'revised' description in [Brewka *et al.*, 2013], all these semantics are defined by generalizing the two-valued interpretations to three-valued ones. All of them are formulated using the basic operator Γ_D over three-valued interpretations that was introduced, in effect, already in [Brewka and Woltran, 2010]. For an ADF D and a three-valued interpretation v, the interpretation $\Gamma_D(v)$ is given by the mapping

$$s \mapsto \prod \{w(\varphi_s) \mid w \in [v]_2\},$$

where $[v]_2$ is the set of all two-valued interpretations that extend v.

For each statement s, the operator Γ_D returns the consensus truth value for its acceptance formula φ_s, where the consensus takes into account all possible two-valued interpretations w that extend the input valuation v. Then two-valued models of D are precisely those classical interpretations that are fixed points of Γ_D.

Taken in its full generality, however, the operator Γ_D allows to define generalizations of all the major Dung's argumentation semantics as follows.

A three-valued interpretation v for an ADF D is

- admissible iff $v \leq_i \Gamma_D(v)$;
- complete iff $\Gamma_D(v) = v$;
- preferred iff it is $\leq_i$-maximal admissible.
- grounded iff it is the least fixpoint of Γ_D.

As has been shown, the above definitions provide proper generalizations of the corresponding semantics for Dung's argumentation frameworks and, moreover, preserve much of the properties and relations of the latter.

7.2.1 The causal representation

We will describe now a uniform and modular translation of ADFs into the causal calculus. Actually, the key to this translation can be found in the striking similarity between the official definition of an ADF and the notion of a *causal model*, used by Judea Pearl in [Pearl, 2000]. Causal models are defined as triples $M = \langle U, V, F \rangle$, where U is a set of *exogenous* variables, V is a finite set of *endogenous* variables, while F is a set of functions that determine the values of each endogenous variable in terms of other variables.

Symbolically, F is represented as a set of *structural* equations

$$v_i = f_i(pa_i, u_i) \quad i = 1, \ldots, n$$

where pa_i is any realization of the unique minimal set of variables PA_i in $V \backslash \{V_i\}$ (parents) sufficient for representing f_i, and similarly for $U_i \subseteq U$.

In Pearl's account, every instantiation $U = u$ of the exogenous variables determines a particular "causal world" of the causal model. Such worlds stand in one-to-one correspondence with the solutions to the above equations in the ordinary mathematical sense. However, structural equations also encode causal information in their very syntax by treating the variable on the left-hand side of = as the effect and treating those on the right as causes. Accordingly, the equality signs in structural equations convey the asymmetrical relation of "is determined by".

Being restricted to the classical propositional language, Pearl's notion of a causal model can be reduced to the following notion of a Boolean causal model that has been used in [Bochman and Lifschitz, 2015]:

Definition 55. *Assume that the set of propositional atoms is partitioned into a set of* exogenous *atoms and a finite set of* endogenous *atoms.*

- *A* Boolean structural equation *is an expression of the form $p = F$, where p is an endogenous atom and F is a propositional formula in which p does not appear.*
- *A* Boolean causal model *is a set of Boolean structural equations $p = F$, one for each endogenous atom p.*

As can be seen, the above definition is much similar to the logical reformulation of ADFs, with structural equations $p = F$ playing essentially the same role as the acceptance conditions $p[F]$. The differences are that only endogenous atoms are determined by their associated conditions in causal models, but on the other hand, there are no restrictions on appearances of atoms on both sides in ADF's acceptance conditions. Furthermore, plain (two-valued) models of ADFs correspond precisely to causal worlds of the causal model, as defined in [Bochman and Lifschitz, 2015]:

Definition 56. *A* solution *(or a* causal world*) of a Boolean causal model M is any propositional interpretation satisfying the equivalences $p \leftrightarrow F$ for all equations $p = F$ in M.*

Now, a modular representation of Boolean causal models as causal theories of the causal calculus has been given in [Bochman and Lifschitz, 2015], and it can now be seamlessly transformed into the following causal representation of ADFs:

Definition 57 (*Causal representation of an ADF*)**.** *For any ADF D, Δ_D is the causal theory consisting of the rules*

$$F \Rightarrow p \quad \textit{and} \quad \neg F \Rightarrow \neg p$$

for all acceptance conditions $p[F]$ in D.

The above representation is fully modular, and it will be taken as a uniform basis for the correspondences described in this section.

To begin with, the correspondence results from [Bochman and Lifschitz, 2015] immediately imply

Theorem 58. *The two-valued semantics of an ADF D corresponds precisely to the causal nonmonotonic semantics of Δ_D.*

As a consequence, the full system of causal inference provides a precise logical basis for this nonmonotonic semantics.

Furthermore, it can be shown that the above causal representation also survives the transition to three-valued models of ADFs. An essential precondition of this causal representation, however, amounts to transforming the underlying semantic interpretations of ADFs in terms of three-valued models into ordinary classical logical descriptions. In fact, the very possibility of such a classical reformulation stems from the crucial fact that the basic operator Γ of an ADF, described earlier, is defined, ultimately, in terms of ordinary classical interpretations extending a given three-valued one.

Any three-valued interpretation v on the set of statements S can be faithfully encoded using an associated set of literals $[v] = S_0 \cup \neg S_1$ such that $S_0 = \{p \in S \mid v(p) = \mathbf{t}\}$ and $S_1 = \{p \in S \mid v(p) = \mathbf{f}\}$. Moreover, this set of literals generates a unique deductively closed theory $\mathrm{Th}([v])$ that corresponds in this sense to the source three-valued interpretation v. Conversely, let us say that a deductively closed set u is a *literal theory*, if it is a deductive (classical) closure of some set of literals. Then the latter set of literals will correspond to a unique three-valued interpretation v such that $u = \mathrm{Th}([v])$. These simple facts establish a precise bi-directional correspondence between three-valued interpretations and classical literal theories.

Now, a broader correspondence between various semantics of ADFs and general nonmonotonic semantics of the causal calculus arises from the fact that the operator Γ of an ADF naturally corresponds to a particular causal operator of the associated causal theory.

Let L denote the set of classical literals of the underlying language. We will denote by $\mathcal{C}^L$ the restriction of a causal operator $\mathcal{C}$ to literals, that is, $\mathcal{C}^L(u) = \mathcal{C}(u) \cap L$. Now, it can be shown that the operator Γ of ADFs corresponds precisely to this 'literal restriction' of the causal operator associated with a basic production inference:

Lemma 59. *For any three-valued interpretation v,*

$$[\Gamma_D(v)] = \mathcal{C}_D^L([v]),$$

where $\mathcal{C}_D$ is a basic production operator corresponding to Δ_D.

The above equation has immediate consequences for the broad correspondence between the semantics of ADFs that are defined in terms of the operator Γ_D and natural sets of propositions definable wrt associated causal theory. Thus, we have

Theorem 60. *Complete models of an ADF D correspond precisely to the fixed points of $\mathcal{C}^L_D$:*

$$v = \Gamma_D(v) \quad \textit{iff} \quad [v] = \mathcal{C}^L_D([v])$$

As a result, we immediately conclude that preferred models of an ADF correspond to maximal fixpoints of $\mathcal{C}^L_D$ (with respect to set inclusion), while the grounded model corresponds to the least fixpoint of $\mathcal{C}^L_D$.

Further details about these correspondences are discussed in [Bochman, 2016].

7.3 Logic programming

To complete the circle of representations, described in this study, we will show in this section that the formalism of logic programming itself, which could be seen as one of the main sources of Dung's argumentation theory, can also be viewed as a very specific kind of propositional argumentation.

A *general logic program* Π is a set of rules of the form[11]

$$\mathbf{not}\, d, c \leftarrow a, \mathbf{not}\, b \tag{*}$$

where a, b, c, d are finite sets of propositional atoms. These are program rules of a most general kind that contain disjunctions and negations as failure **not** in their heads. As has been shown in [Bochman, 2004b], general logic programs are representable as causal theories obtained by translating program rules (*) as causal rules

$$d, \neg b \Rightarrow \bigwedge a \to \bigvee c,$$

and adding a formalization of the Closed World Assumption:

(Default Negation) $\neg p \Rightarrow \neg p$, for any propositional atom p.

Now, due to the correspondence between causal reasoning and argumentation, this causal theory can be transformed (using (PA)) into an argument theory that consists of attacks

$$a, \neg c \hookrightarrow \neg b, d \tag{AL}$$

plus the 'argumentative' Closed World Assumption:

(Default Assumption) $p \hookrightarrow \neg p$, for any atom p.

Let $tr(\Pi)$ denote the argument theory obtained by this translation from a logic program Π. Then we obtain

[11] As before, $\mathbf{not}\, a$ denotes the set $\{\mathbf{not}\, A \mid A \in a\}$.

Theorem 61. *A set u of propositional atoms is a stable model of a logic program Π iff $\tilde{u}$ is a stable set of assumptions in $tr(\Pi)$.*

It is interesting to note that, due to the reduction rules ($R_\sim$) for the global negation $\sim$, described earlier, the above representation (AL) of the program rules is equivalent to $a, \sim b \hookrightarrow \sim c, d$, and therefore to the inference rules

$$a, \sim b \Vdash c, \sim d$$

of the associated Belnap consequence relation. For normal logic programs (single atoms in heads), this latter representation coincides with that given in [Bondarenko *et al.*, 1997].

8 Conclusions

The main objective of this study consisted in showing that both logic and nonmonotonic reasoning constitute two distinct, but essential, components of argumentation. On the way, we have shown that propositional argumentation suggests a viable and useful extension of the abstract argumentation theory that allows us to endow argumentation with full-fledged logical capabilities. The resulted theory has allowed us, in particular, to provide a systematic description of a large number of argumentation and nonmonotonic formalisms, such as assumption-based argumentation, abstract dialectical frameworks, default logic, logic programming and causal reasoning. It is natural to expect that further development of this approach to argumentation may bring additional theoretical and practical benefits.

One of the basic tasks that still need to be resolved with respect to the suggested formalism of propositional argumentation is a *systematic* connection of the latter with the more 'standard' approach of *structural argumentation* in which arguments are represented directly as derivations (proofs) constructed from strict and defeasible inference rules. Recall that both kinds of formalisms, propositional and proof-theoretic ones, are peacefully coexisting in the majority of traditional logical systems, so its only natural to expect that the same kind of correspondence could be established also for the formal argumentation theory at its present, essentially nonmonotonic stage of development.

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