A grainless semantics for concurrent separation logic

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Géométrie du Calcul
GEOCAL '06

Shared-memory programs

- Parallel composition

  $c_1 \mid\mid c_2$

- Conditional critical region

  with r **when** b **do** c

- Local resource declaration

  resource r **in** c

Traditional semantics

- Program denotes a set of *traces*
- Trace = sequence of *atomic actions*
- Parallel composition = *fair interleaving*
- Resource acquisition is *mutually exclusive*

Granularity

Traditional models assume a *default level of granularity* for *atomic actions*

- integer assignments — *coarse*
- reads and writes to integer variables — *fine*
- reads and writes to machine words — *word-level*

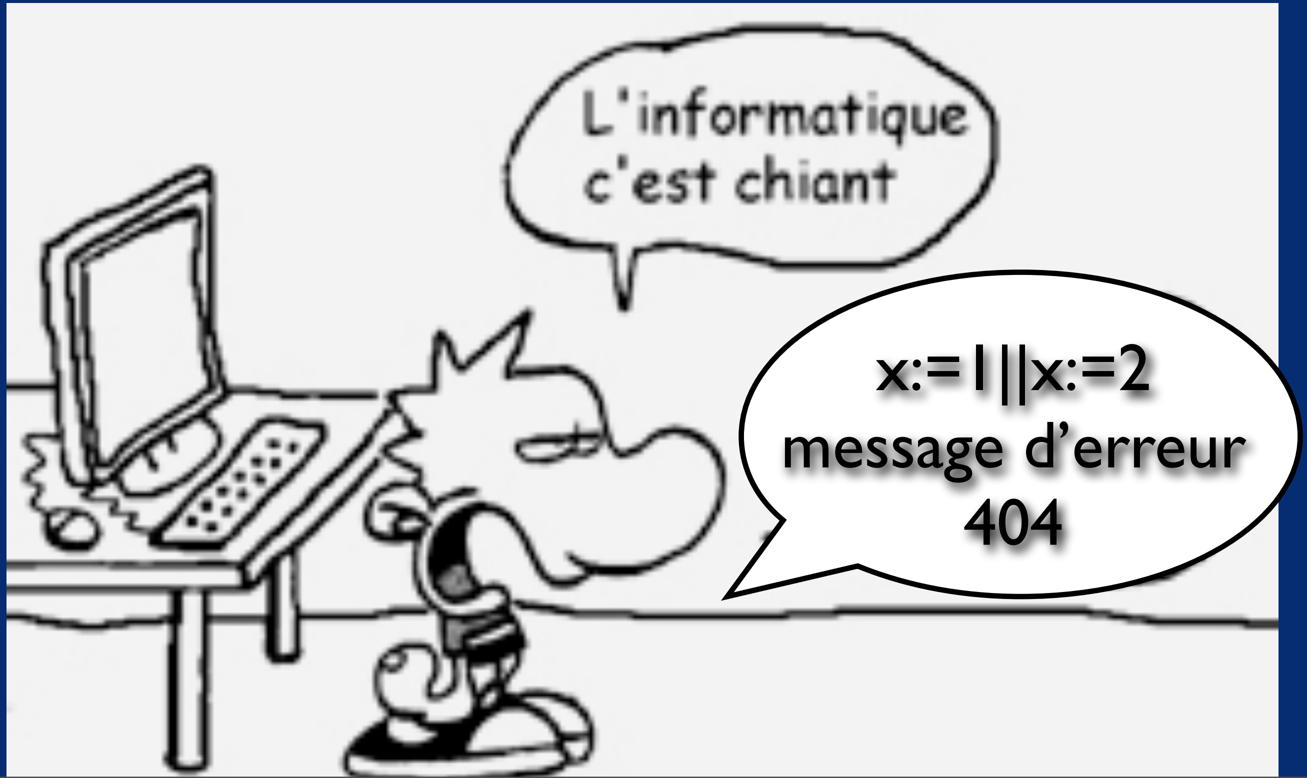
Example

x := x+1 || x := x+2

- coarse *x increases by 3*
- fine *x increases by 1, 2, or 3*
- word *depends on word size*

Danger

A *concurrent write*
to a variable being used by another process ...



Danger

A *concurrent write*
to a variable being used by another process ...

- race condition
- unpredictable results

Status quo

- Traditional models *ignore* race conditions
  by assuming that atomic actions never overlap
- Each supports compositional reasoning
  for *race-free* programs
- But they each interpret *racy* programs differently

Race-freedom

- Partial correctness behavior of a *race-free* program is *independent* of granularity

Should be able to abstract away from granularity

... but traditional models don't do this!

The Dijkstra Principle

Similarly...

... processes should be *loosely connected*;
... apart from the (rare) moments of explicit communication, processes are to be regarded as *completely independent* of each other

The Dijkstra Principle

That is...

... should be able to abstract away from what happens between synchronizations

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... should be able to abstract away from what happens between synchronizations

WARNING
Not reflected in design of traditional models

Traditional logic

... based on Dijkstra's Principle

Owicki/Gries '76

$$\Gamma \vdash \{p\}\ c\ \{q\}$$

- Resource-sensitive partial correctness
  - Γ specifies *resources* r_i, *protection lists* X_i, and *invariants* R_i
  - p, q describe *unprotected* variables
- Static constraints guarantee race-freedom
  - *critical variables* must be protected
  - protected variables only allowed inside region

Parallel rule

Owicki/Gries

$$\frac{\Gamma \vdash \{p_1\}\ c_1\ \{q_1\} \qquad \Gamma \vdash \{p_2\}\ c_2\ \{q_2\}}{\Gamma \vdash \{p_1 \wedge p_2\}\ c_1 \| c_2\ \{q_1 \wedge q_2\}}$$

provided

$$\mathrm{free}(p_1, q_1) \cap \mathrm{writes}(c_2) = \varnothing$$
$$\mathrm{free}(p_2, q_2) \cap \mathrm{writes}(c_1) = \varnothing$$
$$\mathrm{free}(c_1) \cap \mathrm{writes}(c_2) \subseteq \mathrm{owned}(\Gamma)$$
$$\mathrm{free}(c_2) \cap \mathrm{writes}(c_1) \subseteq \mathrm{owned}(\Gamma)$$

critical variables are protected

Resource rules

Owicki/Gries

$$\frac{\Gamma \vdash \{(p \wedge R) \wedge b\}\ c\ \{q \wedge R\}}{\Gamma, r(X){:}R \vdash \{p\}\ \textbf{with}\ r\ \textbf{when}\ b\ \textbf{do}\ c\ \{q\}}$$

$$\frac{\Gamma, r(X){:}R \vdash \{p\} c \{q\}}{\Gamma \vdash \{p \wedge R\}\ \textbf{resource}\ r\ \textbf{in}\ c\ \{q \wedge R\}}$$

(subject to well-formedness conditions)

Validity

Definition

$\Gamma \vdash \{p\}c\{q\}$ is *valid* if:

Every finite computation of c
in an environment that respects Γ,
from $p \wedge R_1 \wedge \ldots \wedge R_n$,
respects Γ, is race-free,
and ends in $q \wedge R_1 \wedge \ldots \wedge R_n$

(made formal using a traditional model)

Soundness
Theorem

- Owicki-Gries logic is *sound* for pointer-less programs
  - Every provable formula is *valid*

Can use *any* of the
traditional semantic models...

Compositionality

Theorem

Traditional traces support compositional reasoning

- Let c_1 and c_2 be code fragments that denote the *same trace set*
- Let $C[-]$ be a program context
- If $\Gamma \vdash \{p\}\ C[c_1]\ \{q\}$ is valid, so is $\Gamma \vdash \{p\}\ C[c_2]\ \{q\}$

same traces

$\Rightarrow$

same behavior, in all contexts

Semantic problems

Traditional models don't help much!

These models...

- make too many distinctions

$$[\![x{:=}x{+}1; x{:=}x{+}1]\!] \neq [\![x{:=}x{+}2]\!]$$

$$[\![x{:=}1; y{:=}2]\!] \neq [\![y{:=}2; x{:=}1]\!]$$

- do not reflect Dijkstra's principle

  ... contain unnecessary traces

- suffer from combinatorial explosion

  ... involve unnecessary interleavings

Logic problems

Traditional logic doesn't generalize...

- Owicki-Gries logic is *not sound for pointer programs*
- Static constraints cannot prevent pointer races, because of *aliasing*

[x]:=1 || [y]:=2 *races* if x and y are aliases	dispose x || dispose y *races* if x and y are aliases
concurrent update	*dangling pointer*

Pointer programs

- Lookup

  $i := [e]$

- Update

  $[e] := e'$

- Allocation

  $i :=$ **cons** $(e_1, \ldots, e_n)$

- Disposal

  dispose e

Reasoning about pointers

Reynolds '02

- Hoare-style rules for *sequential* pointer-programs
- State = store + heap
- Pre- and post-conditions drawn from *separation logic*
  - emp : heap is empty
  - $e \mapsto e'$: singleton heap
  - $p_1 \star p_2$: heap can be split so that p_1 and p_2 hold *separately*

A proposal

O'Hearn '02

- Combine Owicki-Gries with separation logic
  - Let resource invariants be *separation logic* formulas
  - Use ✶ strategically to prevent aliasing

an apparently simple idea
with deep consequences

Parallel rule

O'Hearn '02

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$\star$ for $\wedge$

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same as before

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Resource rules

O'Hearn '02

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An intuitive definition,
in need of formalization...

Ownership transfer

- The logic allows proofs in which *ownership* of a pointer *transfers* implicitly between processes and resources, based on resource invariants

  - for each *available* resource, invariant holds *in a sub-heap*
  - when *acquiring* a resource, process assumes invariant, *claims* ownership of the protected variables + sub-heap
  - when *releasing* a resource, process guarantees that invariant holds in some sub-heap, *cedes* ownership

Example

1-place shared buffer

PUT :: **with** buf **when** full=0 **do** (z := x; full := 1)

GET :: **with** buf **when** full=1 **do** (y := z; full := 0)

Let Γ = buf(z,full): $(\mathrm{full}=1 \wedge z \mapsto \_\ ) \vee (\mathrm{full}=0 \wedge \mathrm{emp})$

$\Gamma \vdash \{x \mapsto \_\}\ \mathrm{PUT}\ \{\mathrm{emp}\}$

$\Gamma \vdash \{\mathrm{emp}\}\ \mathrm{GET}\ \{y \mapsto \_\}$

ownership
passes
from left to right

$\Gamma \vdash \{x \mapsto \_\}\ \mathrm{PUT} \parallel (\mathrm{GET};\ \mathrm{dispose}\ y)\ \{\mathrm{emp}\}$

Example, take 2

1-place shared buffer

Using a different invariant...

$\Gamma' = \text{buf(z,full)}: (\text{full}=1 \wedge \text{emp}) \vee (\text{full}=0 \wedge \text{emp})$

$\Gamma' \vdash \{x \mapsto \_\}\ \text{PUT}\ \{x \mapsto \_\}$

$\Gamma' \vdash \{\text{emp}\}\ \text{GET}\ \{\text{emp}\}$

no transfer
of ownership

$\Gamma' \vdash \{x \mapsto \_\}\ (\text{PUT; dispose } x) \parallel \text{GET}\ \{\text{emp}\}$

Example, take 3

1-place shared buffer

And we cannot prove a racy program...

dangling pointer

$\Gamma \vdash \{x \mapsto \_\}$ (PUT; dispose x) || (GET; dispose y) {...}

not provable, for any Γ

ownership cannot go both ways!

Soundness?

- Far from obvious! Ownership is a tricky concept...
  - cannot rely on Owicki/Gries
  - resource invariants must be restricted (Reynolds '02)
- Need a semantics
  - must account for ownership transfer
  - ideally, should be *grainless*
- Traditional models won't work...

A new semantics

based on Dijkstra's principle

Footstep traces

- No interference except on synchronization
  - *built into structure of traces*
- Treats race condition as *disaster*
  - *built into definition of interleaving*
- Independent of granularity
  - *abstracts away from state changes between synchronizations*

Advantages

- Succinctness

  big steps, fewer traces, fewer interleavings

- Simplicity

  supports "sequential" reasoning
  for synchronization-free code

- Soundness

  can be used to prove soundness of
  concurrent separation logic

States

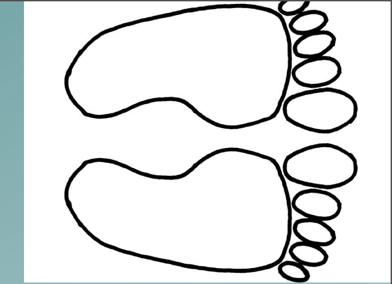
Definition

A state specifies values for identifiers and heap cells

- $St = Var \rightharpoonup V_{int}$ *states*
- $Var = Ide \uplus Loc$ *variables*
- $Loc \subseteq V_{int}$ *heap cells*

Let σ range over the set of states...

Footsteps



A *footstep* $(\sigma, \sigma')_X$ describes the *footprint* of a sequence of state changes:

change σ to σ',
while only *reading* the variables in X

$(\sigma, \sigma')_X$

Resource actions

- *try*(r) unsuccessful attempt
- *acq*(r) acquisition
- *rel*(r) release

Used to model synchronization

Traces

loosely connected
sequences of actions

- Built from *catenable* actions

$$([x{:}0], [x{:}1])_{\emptyset} \asymp ([x{:}1, y{:}0], [x{:}1, y{:}1])_{\{x\}}$$

- Consecutive footsteps get *stumbled* together

$$([x{:}0], [x{:}1])_{\emptyset} \; ; ([x{:}1, y{:}0], [x{:}1, y{:}1])_{\{x\}} = ([x{:}0, y{:}0], [x{:}1, y{:}1])_{\emptyset}$$

$$([v{:}0], [\ ])_{\emptyset} \; ; ([v{:}0], [\ ])_{\emptyset} = ([v{:}0], \textit{abort})$$

- Interference only on synchronization

$$(\sigma_0, \sigma_0')_{X_0} \; \mathrm{acq}(r) \; (\sigma_1, \sigma_1')_{X_1} \; \mathrm{rel}(r) \; (\sigma_2, \sigma_2')_{X_2} \; \ldots$$

Semantics

- A command denotes a set of *traces*

$$[\![c]\!] \subseteq Tr$$

- Defined by structural induction on c

$$[\![c_1;c_2]\!] = \{\ \alpha_1;\alpha_2 \mid \alpha_1 \in [\![c_1]\!],\ \alpha_2 \in [\![c_2]\!],\ \alpha_1 \asymp \alpha_2\ \}$$

$$[\![c_1 \| c_2]\!] = \bigcup \{\ \alpha_1 \| \alpha_2 \mid \alpha_1 \in [\![c_1]\!],\ \alpha_2 \in [\![c_2]\!]\ \}$$

resource-sensitive, race-detecting,
fair interleaving

Examples

$[\![x{:=}1;\ y{:=}2]\!] = [\![y{:=}2;\ x{:=}1]\!]$

$= \{\ ([x{:}v, y{:}v'], [x{:}1, y{:}2])_{\emptyset} \mid v, v' \in V_{int}\ \}$

$[\![x{:=}x{+}1 \parallel x{:=}x{+}2]\!]$

$= \{\ ([x{:}v], \mathit{abort}) \mid v \in V_{int}\ \}$

$[\![$ resource r in

with r do x:=x+1 || with r do x:=x+2 $]\!]$

$= \{\ ([x{:}v], [x{:}v{+}3])_{\emptyset} \mid v \in V_{int}\ \}$

Theorem

Footstep traces
allow sequential reasoning
for resource-free programs

Every trace of a *resource-free program*
is a single footstep

c is resource-free if $res(c)=\varnothing$ (no free resource names)

Every sequential program is resource-free

Race-free

Definition

c is *race-free from* σ

iff

$\forall \alpha \in [\![c]\!].\ \neg\,(\sigma \overset{\alpha}{\Rightarrow} \mathit{abort})$

no trace leads to an error

Example

full := 0; **resource** buf **in**
(x := **cons**(-); PUT; **dispose** x) || GET

race-free

has footstep traces

([full:_, x:_, y:_, z:_], [full:0, x:v, y:v, z:v])$_{\varnothing}$

Example

full := 0;
resource buf **in**
(x := **cons**(-); PUT) || (GET; **dispose** y)

ownership transfer

race-free

... has the same trace set!

(not true in traditional models)

Explosion?

N^2 2^N $N!$

N puts,
N gets

full := 0;
resource buf **in**
    (x := **cons**(-); PUT)^N || (GET; **dispose** y)^N

independent of N

... has the same trace set!

(not true in traditional models)

Racy example

full := 0; x := **cons**(-);
resource buf **in**
(PUT; **dispose** x) || (GET; **dispose** y)

has traces

([full:_, x:_, y:_, z:_], *abort*)

race condition modelled correctly

Soundness
Theorem

Brookes '04

- Every provable formula of concurrent separation logic is *valid*, provided resource invariants are *precise*
  - Footstep traces permit rigorous treatment of *ownership transfer*
  - Show that each inference rule preserves validity
    - proof reveals crucial role of *precision*

⇒ *Every provable program is race-free*

Compositionality

Theorem

Footstep traces support compositional reasoning for parallel pointer-programs

- Let c_1 and c_2 be code fragments with the same *footstep traces*
- Let $C[-]$ be a program context
- If $\Gamma \vdash \{p\}C[c_1]\{q\}$ is valid, so is $\Gamma \vdash \{p\}C[c_2]\{q\}$

same traces

$\Rightarrow$

same behavior, in all contexts

Advantages

significant help for compositional reasoning!

Footstep trace semantics...

- makes fewer distinctions

$$[\![x{:=}x{+}1; x{:=}x{+}1]\!] = [\![x{:=}x{+}2]\!]$$

$$[\![x{:=}1; y{:=}2]\!] = [\![y{:=}2; x{:=}1]\!]$$

- embodies Dijkstra's principle

  ... *only loosely connected traces*

- ameliorates the combinatorial explosion

  ... *fewer interleavings*

Conclusions

- Semantics abstracts away from inessential details between synchronizations
  - *facilitates reasoning about loosely connected processes*
- Logic provides safe reasoning about concurrency + pointers
  - *every provable program is race-free*

Ideas should be more widely applicable