

A BRIEF HISTORY OF SHARED MEMORY

STEPHEN BROOKES
CMU

OUTLINE

- ✱ **Revisionist history**
 - ✱ Rational reconstruction of early models
 - ✱ Evolution of recent models
- ✱ **A unifying framework**
 - ✱ Fault-detecting trace semantics
- ✱ **Some general results**
 - ✱ Soundness of fault-avoiding logics

FRAMEWORK

- ✻ An abstract notion of *state* and *action*
- ✻ A recipe for constructing *denotational* models
 - ✻ sequential programs
 - ✻ shared memory parallel programs
- ✻ Designed to support *compositional* reasoning
 - ✻ *fault-avoiding correctness*
 - ✻ *rely/guarantee properties*

STATE AND ACTION

DEFINITION

A *state model* is a tuple $(S, A, \rightarrow, \#)$
with

$$\begin{array}{ll} S = (S, \otimes) & \text{states} \\ A & \text{actions} \\ \rightarrow \subseteq S \times A \times S^\dagger & \text{footprint} \\ S^\dagger = S \uplus \{\text{error}\} & \end{array}$$

and

$$\begin{array}{ll} \otimes : S \times S \rightarrow S & \text{compatibility} \\ \# \subseteq A \times A & \text{independence} \end{array}$$

satisfying natural axioms ...

STATE AXIOMS

- ✱ (S, \otimes) is a partial commutative monoid...

$$\begin{aligned}\sigma \otimes \tau &\simeq \tau \otimes \sigma \\ \rho \otimes (\sigma \otimes \tau) &\simeq (\rho \otimes \sigma) \otimes \tau\end{aligned}$$

- ✱ ... with unique decomposition

$$\sigma \otimes \sigma_1 = \sigma \otimes \sigma_2 \Rightarrow \sigma_1 = \sigma_2$$

FOOTPRINT AXIOMS

- ✻ *Successful action has unique cause*

For all σ, λ

at most one σ_1 such that

$$\sigma_1 \xrightarrow{\lambda} \sigma'_1, \sigma = \sigma_1 \otimes \sigma_2$$

- ✻ *Failure is irrevocable*

If $\sigma_1 \otimes \sigma_2 \xrightarrow{\lambda} error,$

then $\sigma_1 \xrightarrow{\lambda} error$

INDEPENDENCE AXIOMS

- ☼ *Independence implies non-interfering footprints*

$$\lambda_1 \# \lambda_2 \quad \& \quad \sigma_1 \xrightarrow{\lambda_1} \sigma_1' \quad \& \quad \sigma_2 \xrightarrow{\lambda_2} \sigma_2' \\ \& \quad \sigma = \sigma_1 \otimes \tau_1 = \sigma_2 \otimes \tau_2$$

implies

$$\exists \tau_1', \tau_2'. \quad \sigma_1' \otimes \tau_1 = \sigma_2 \otimes \tau_2' \\ \& \quad \sigma_2' \otimes \tau_2 = \sigma_1 \otimes \tau_1' \\ \& \quad \sigma_1' \otimes \tau_1' = \sigma_2' \otimes \tau_2'$$

- ☼ *Symmetry*

$$\lambda_1 \# \lambda_2 \text{ implies } \lambda_2 \# \lambda_1$$

ENABLING

For any state model we can derive an *enabling* relation

$$\Rightarrow \subseteq S^\dagger \times A \times S^\dagger$$

Let $\sigma \stackrel{\lambda}{\Rightarrow} \sigma'$ iff $\exists \sigma_1, \sigma_1', \sigma_2 \in S$.

$$\sigma = \sigma_1 \otimes \sigma_2$$

$$\& \quad \sigma' = \sigma_1' \otimes \sigma_2$$

$$\& \quad \sigma_1 \stackrel{\lambda}{\rightarrow} \sigma_1'$$

Let $\sigma \stackrel{\lambda}{\Rightarrow} error$ iff $\sigma \stackrel{\lambda}{\rightarrow} error$ or $\sigma = error$

CONSEQUENCES

✱ FRAME

$$\sigma_1 \stackrel{\lambda}{\Rightarrow} \tau_1 \neq \text{error} \quad \& \quad \sigma_1 \otimes \sigma_2 \stackrel{\lambda}{\Rightarrow} \tau$$

implies

$$\tau = \tau_1 \otimes \sigma_2$$

✱ SAFETY MONOTONICITY

$$\sigma_1 \otimes \sigma_2 \stackrel{\lambda}{\Rightarrow} \text{error}$$

implies

$$\sigma_1 \stackrel{\lambda}{\Rightarrow} \text{error}$$

CONSEQUENCES

✱ *Independent actions don't interfere*

If $\lambda_1 \# \lambda_2$ then

$$\sigma \xRightarrow{\lambda_1} \tau_1, \sigma \xRightarrow{\lambda_2} \tau_2$$

implies

$$\exists \tau. \tau_1 \xRightarrow{\lambda_2} \tau, \tau_2 \xRightarrow{\lambda_1} \tau$$

EXAMPLE

global transition traces

- ☼ $S = (\text{Ide} \rightarrow V) \cup \{\mathbb{1}\}$

- ☼ $\sigma \otimes \mathbb{1} = \sigma = \mathbb{1} \otimes \sigma$

- ☼ $A = S \times S$

- ☼ $\begin{matrix} (\sigma, \tau) \\ \sigma \rightarrow \tau \end{matrix}$

- ☼ $(\sigma_1, \tau_1) \# (\sigma_2, \tau_2) \text{ iff } \sigma_1 = \tau_1 = \sigma_2 = \tau_2$

cf. Park 1979

EXAMPLE

local transition traces

✱ $S = \text{Ide} \xrightarrow{\text{fin}} V$

✱ $-\otimes-$ disjoint union

✱ $A = \{(\sigma, \tau) \mid \text{dom } \sigma = \text{dom } \tau\}$

✱ $\overset{(\sigma, \tau)}{\sigma} \rightarrow \tau$

✱ $\overset{(\sigma, \tau)}{\sigma_1} \rightarrow_{\text{error}}$ iff $\sigma_1 \upharpoonright \text{dom}(\sigma) = \sigma \upharpoonright \text{dom}(\sigma_1) \subset \sigma$

✱ $(\sigma_1, \tau_1) \# (\sigma_2, \tau_2)$ iff $\text{dom}(\sigma_1) \cap \text{dom}(\sigma_2) = \emptyset$

cf. LICS 1996

EXAMPLE

action traces, shared store

- ✱ $S = \text{Ide} \rightarrow_{fin} V$
- ✱ $-\otimes-$ disjoint union
- ✱ $A = \{i=v, i:=v \mid i \in \text{Ide}, v \in V\}$
- ✱ $[i:v] \xrightarrow{i=v} [i:v]$
- ✱ $[i:v] \xrightarrow{i:=v'} [i:v']$
- ✱ $\sigma \xrightarrow{i=v, i:=v'} \text{error}$ iff $i \notin \text{dom}(\sigma)$
- ✱ $\neg(i:=v \# i=v'), \neg(i:=v \# i:=v')$

cf. CONCUR 2002

EXAMPLE

action traces, shared mutable state

- ☀ $S = \text{Store} \times \text{Heap}$
- ☀ $-\otimes-$ disjoint union, componentwise
- ☀ $A = A_{\text{store}} \cup \{[l]=v, [l]:=v, \text{alloc}(l,v), \text{disp } l\}$
- ☀ $([], []) \xrightarrow{\text{alloc}(l,v)} ([], [l:v])$
- ☀ $([], [l:v]) \xrightarrow{\text{disp } l} ([], [])$
- ☀ $(s, h) \xrightarrow{\text{disp } l} \text{error}$ iff $l \notin \text{dom}(h)$
- ☀ $\neg(\text{disp } l \# \text{disp } l)$

cf. CONCUR 2004

EXAMPLE

permissions

- ✱ $S = \text{Ide} \xrightarrow{\text{fin}} V \times \mathbb{P}, \quad (\mathbb{P}, \oplus, \top)$ a *permission algebra*
- ✱ $\text{-}\otimes\text{-}$ *combines* permissions, when *compatible*
- ✱ $A = \{(i=v, \pi), (i:=v, \top) \mid \pi \in \mathbb{P}, v \in V\}$
- ✱ $[i:(v, \pi)] \xrightarrow{i=v, \pi} [i:(v, \pi)]$
- ✱ $[i:(v, \top)] \xrightarrow{i:=v', \top} [i:(v', \top)]$
- ✱ $\sigma \xrightarrow{i=v, \pi} \text{error}$ iff $i \notin \text{dom}(\sigma)$
- ✱ $\sigma \xrightarrow{i:=v', \top} \text{error}$ iff $\neg \exists v. (i, (v, \top)) \in \sigma$
- ✱ $(i=v, \pi_1) \# (i=v, \pi_2)$ when $\pi_1 \oplus \pi_2$ defined

TRACES

- ✱ A trace is a finite or infinite sequence of actions
- ✱ α is (sequentially) *executable* iff $\exists \sigma. \sigma \stackrel{\alpha}{\Rightarrow} \cdot$
- ✱ Let $\alpha \frown \beta$ iff $\alpha\beta$ executable
- ✱ Let $\text{Tr}(A) \subseteq \wp(A^\infty)$ be sets of executable traces

SEMANTIC RECIPE

for sequential programs

- Given a state model $\Sigma = (S, A, \rightarrow, \#)$ we can define a *trace semantics*

$$\llbracket - \rrbracket_{\Sigma} : \text{Com} \rightarrow \text{Tr}(A)$$

$$\llbracket - \rrbracket_{\Sigma} : \text{Exp}_{int} \rightarrow \text{Tr}(A) \times V_{int}$$

$$\llbracket - \rrbracket_{\Sigma} : \text{Exp}_{bool} \rightarrow \text{Tr}(A) \times V_{bool}$$

by structural induction

- $\llbracket c \rrbracket_{\Sigma}$ is set of executable traces

SEMANTIC CLAUSES

- ✻ $\llbracket c_1; c_2 \rrbracket = \llbracket c_1 \rrbracket \llbracket c_2 \rrbracket$
 $= \{ \alpha \beta \mid \alpha \in \llbracket c_1 \rrbracket, \beta \in \llbracket c_2 \rrbracket, \alpha \frown \beta \}$
- ✻ $\llbracket \text{while } b \text{ do } c \rrbracket =$
 $(\llbracket b \rrbracket_{\text{true}} \llbracket c \rrbracket)^* \llbracket b \rrbracket_{\text{false}} \cup (\llbracket b \rrbracket_{\text{true}} \llbracket c \rrbracket)^\omega$

FAULT-AVOIDING CORRECTNESS

DEFINITION

$\{p\}c\{q\}$ is **VALID** iff

$$\forall \sigma \in S. \forall \alpha \in [c]. \forall \sigma'.$$

$$\sigma \models p \ \& \ \sigma \xrightarrow{\alpha} \sigma' \text{ implies } \sigma' \neq \text{error} \ \& \ \sigma' \models q$$

*every finite execution of c ,
from a state satisfying p ,
is error-free,
and ends in a state satisfying q*

VALIDATION THEOREM

For all sequential programs,

$$\llbracket c_1 \rrbracket = \llbracket c_2 \rrbracket$$

implies

$$\forall C. \forall p, q.$$

$\{p\}C[c_1]\{q\}$ **valid** iff $\{p\}C[c_2]\{q\}$ **valid**

*sequential commands with
the same executable traces
satisfy the same formulas,
in all sequential contexts*

PARALLEL PROGRAMS

- ✻ $c_1 \parallel c_2$ *shared memory*
- ✻ **with** r **when** b **do** c *conditional critical region*
- ✻ **resource** r **in** c *local resource*

$r \in \text{Res} = \text{set of } \textit{resource names}$

RESOURCE ACTIONS

- ✻ $\Delta = \{\text{try } r, \text{acq } r, \text{rel } r \mid r \in \text{Res}\}$
- ✻ Each resource is *exclusive*
 - ✻ *acquired* by at most one process at a time
 - ✻ *available* when not currently acquired
 - ✻ process must *acquire* before *release*,
keeps *trying* when unavailable

WELL-RESOURCED

✻ A sequence $\alpha \in (A \cup \Delta)^\omega$ is *well-resourced* iff

$$\forall r. \alpha \upharpoonright \{\text{acq } r, \text{rel } r\} \leq (\text{acq } r \ \text{rel } r)^\omega$$

acquires before releases

RESOURCE CONSTRAINTS

- ✱ Ability to do resource actions depends on resource sets R_1 held by *process*, R_2 held by *environment*
- ✱ These sets start empty and stay disjoint...

$$R_1 \xrightarrow[\text{R}_2]{\text{acq } r} R_1 \cup \{r\} \quad \text{iff} \quad r \notin R_1 \cup R_2$$

$$R_1 \xrightarrow[\text{R}_2]{\text{rel } r} R_1 - \{r\} \quad \text{iff} \quad r \in R_1$$

$$R_1 \xrightarrow[\text{R}_2]{\text{try } r} R_1$$

RACE CONDITIONS

- ✱ Concurrent execution of non-independent actions may yield unpredictable results
- ✱ Introduce an action *abort* to model such races
- ✱ Let $A^\dagger =_{\text{def}} A \cup \{abort\}$
- ✱ Define $\sigma \xrightarrow{abort} \sigma'$ iff $\sigma' = error$

SEMANTIC RECIPE

for parallel programs

- Let $\text{Tr}(A, \Delta)$ be sets of well-resourced traces over $A^\dagger \cup \Delta$

$$\text{Tr}(A, \Delta) \subseteq \wp(A^\dagger \cup \Delta)^\infty$$

- A parallel program will denote a set of well-resourced traces

$$\llbracket - \rrbracket : \text{Com} \rightarrow \text{Tr}(A, \Delta)$$

PARALLEL COMPOSITION

$$\llbracket c_1 \parallel c_2 \rrbracket = \llbracket c_1 \rrbracket_{\emptyset} \parallel_{\emptyset} \llbracket c_2 \rrbracket$$

- ✱ $\alpha \parallel_{R_1 R_2} \beta$ *resource-sensitive, race-detecting* fair merges
- ✱ Can be characterized as a *greatest fixed point*

$$(\lambda_1 \alpha) \parallel_{R_1 R_2} (\lambda_2 \beta) =$$

$$\{ \lambda_1 \gamma \mid R_1 \xrightarrow[R_2]{\lambda_1} R'_1, \gamma \in \alpha \parallel_{R'_1 R_2} (\lambda_2 \beta) \}$$

$$\cup \{ \lambda_2 \gamma \mid R_2 \xrightarrow[R_1]{\lambda_2} R'_2, \gamma \in (\lambda_1 \alpha) \parallel_{R_1 R'_2} \beta \}$$

$$\cup \{ abort \mid \neg(\lambda_1 \# \lambda_2) \}$$

REGION

$$\llbracket \text{with } r \text{ when } b \text{ do } c \rrbracket = \text{wait}^* \text{ enter} \cup \text{wait}^\omega$$

$$\ast \text{ wait} = \{\text{try } r\} \cup (\text{acq } r) \llbracket b \rrbracket_{\text{false}} (\text{rel } r)$$

$$\ast \text{ enter} = (\text{acq } r) \llbracket b \rrbracket_{\text{true}} \llbracket c \rrbracket (\text{rel } r)$$

LOCAL RESOURCE

$$\llbracket \text{resource } r \text{ in } c \rrbracket = \{ \alpha \setminus r \mid \alpha \in \llbracket c \rrbracket_r \}$$

✱ $\alpha \setminus r$ obtained by erasing $\{\text{acq } r, \text{rel } r, \text{try } r\}$

✱ $\alpha \in \llbracket c \rrbracket_r$ iff $\alpha \in \llbracket c \rrbracket$ and

$$\alpha \uparrow r \leq (\text{acq } r (\text{try } r)^\infty \text{rel } r)^\infty$$

resource not accessible by environment

FAULT-AVOIDING CORRECTNESS

DEFINITION

$\{p\}c\{q\}$ is **VALID** iff

$$\forall \sigma \in S. \forall \alpha \in [c]. \forall \sigma'.$$

$$\sigma \models p \ \& \ \sigma \xrightarrow{\alpha} \sigma' \ \text{implies} \ \sigma' \neq \text{error} \ \& \ \sigma' \models q$$

(as before)

VALIDATION THEOREM

For all parallel programs,

$$\llbracket c_1 \rrbracket = \llbracket c_2 \rrbracket$$

implies

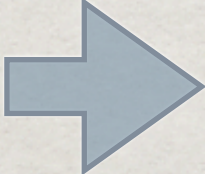
$$\forall C. \forall p, q.$$

$\{p\}C[c_1]\{q\}$ **valid** iff $\{p\}C[c_2]\{q\}$ **valid**

*parallel commands with
the same **traces**
satisfy the same formulas,
in all **parallel** contexts*

EXAMPLES

... rational reconstruction

STATE MODEL  SEMANTICS

global $S \times S$	global transition traces
local $S \times S$	local transition traces
reads/writes	store action traces
store+heap	store/heap action traces
permissive state	permissive action traces

Park '79

LICS '96

CONCUR' 02

CONCUR' 04

MFPS '05

EXECUTABLE TRACES

- ✱ Validity of $\{p\}c\{q\}$ depends only on the *executable* traces of c
- ✱ But the *executable* traces of $c_1 \parallel c_2$ cannot be derived from the *executable* traces of c_1 and c_2
- ✱ So our semantic recipe for $c_1 \parallel c_2$ includes *non-sequential* traces
- ✱ But how non-sequential do we need to be?

DIJKSTRA'S PRINCIPLE

- ✻ A rule for designing correct concurrent programs

*“... regard processes as independent,
except when they synchronize”*

- ✻ Suggests working with “almost sequential” traces...

ALMOST SEQUENTIAL

... sequential except at synchronizations

- ✱ A trace α is *almost sequential* iff

$$\alpha \setminus \{\text{try}, \text{rel}\} = \alpha_1 (\text{acq } r_1) \alpha_2 (\text{acq } r_2) \dots$$

where each $\alpha_n \in A^\infty$ is *sequential*

- ✱ The *almost sequential* traces of $c_1 \parallel c_2$ are fair merges of *almost sequential* traces of c_1 and c_2
- ✱ Easy to adjust semantic clauses to obtain just the *almost sequential* traces

$$[[c]]_{as} \subseteq [[c]]$$

VALIDATION THEOREM

(improved)

For all parallel programs,

$$\llbracket c_1 \rrbracket_{as} = \llbracket c_2 \rrbracket_{as}$$

implies

$$\forall C. \forall p, q.$$

$\{p\}C[c_1]\{q\}$ **valid** iff $\{p\}C[c_2]\{q\}$ **valid**

*parallel commands with
the same almost sequential traces
satisfy the same formulas,
in all parallel contexts*

EQUIVALENT TRACES

*... same effect, same resource protocol,
in all contexts*

✻ For $\alpha, \beta \in A^\infty$ let $\alpha \approx \beta$ iff

$$|\alpha| = |\beta| \text{ and } \forall \lambda. (\alpha \# \lambda \Leftrightarrow \beta \# \lambda)$$

where $|\alpha| = \{(\sigma, \sigma') \mid \sigma \xrightarrow{\alpha} \sigma'\}$

✻ Extend to $\text{Tr}(A, \Delta)$ so that $\alpha \approx \beta$ iff

$$\alpha = \alpha_1 \delta_1 \dots \alpha_n \delta_n \dots$$

$$\beta = \beta_1 \delta_1 \dots \beta_n \delta_n \dots$$

where each $\alpha_i \in (A^\dagger)^\infty$, $\delta_i \in \Delta^+$

and $\forall n. \alpha_n \approx \beta_n$

EQUIVALENT TRACE SETS

☼ Let $T_1 \approx T_2$ iff

$$\forall \alpha \in T_1. \exists \beta \in T_2. \alpha \approx \beta$$

and

$$\forall \beta \in T_2. \exists \alpha \in T_1. \alpha \approx \beta$$

VALIDATION THEOREM

(improved again)

For all parallel programs,

$$\llbracket c_1 \rrbracket \approx \llbracket c_2 \rrbracket$$

implies

$$\forall C. \forall p, q.$$

$\{p\}C[\llbracket c_1 \rrbracket]\{q\}$ **valid** iff $\{p\}C[\llbracket c_2 \rrbracket]\{q\}$ **valid**

*parallel commands with
equivalent trace sets
satisfy the same formulas,
in all parallel contexts*

FOOTSTEP TRACES

- ✱ Obtained from *action trace* model by quotient

- ✱ Traces have form

$$(\sigma_1, \sigma_1')_{X_1} \delta_1 (\sigma_1, \sigma_1')_{X_2} \delta_2 \dots$$

where each X_i is a *read-only* set

- ✱ For all parallel programs

$$\llbracket c_1 \rrbracket_{f\delta} = \llbracket c_2 \rrbracket_{f\delta} \quad \text{iff} \quad \llbracket c_1 \rrbracket_{a\delta} \approx \llbracket c_2 \rrbracket_{a\delta}$$

cf. MFPS '06

ADVANTAGES

- ✻ For a *synchronization-free* parallel program the footstep traces form a non-deterministic relation on states
- ✻ Taming the combinatorial explosion

VALIDATION THEOREM

(final version)

For all parallel programs

$$\llbracket c_1 \rrbracket_{f'} = \llbracket c_2 \rrbracket_{f'}$$

implies

$$\forall C. \forall p, q.$$

$\{p\}C[c_1]\{q\}$ **valid** iff $\{p\}C[c_2]\{q\}$ **valid**

*parallel commands with
the same footprint traces
satisfy the same formulas,
in all parallel contexts*

COMPOSITIONALITY

- ✱ Semantic model is compositional and supports reasoning about fault-avoiding partial correctness
- ✱ But partial correctness properties of $c_1 \parallel c_2$ cannot be deduced from partial correctness properties of c_1 and c_2
- ✱ For a compositional *logic*, we need to work with more general formulas
 - ✱ fault-avoiding *rely/guarantee* properties

FAULT-AVOIDING LOGICS

$$\Gamma \vdash \{p\}c\{q\}$$

- ✱ Γ specifies *protection rules* and *resource invariants*
- ✱ Rely/guarantee interpretation...

every finite *interactive execution* of c ,
in an *environment that respects* Γ ,
from a state satisfying p ,
respects Γ , is *error-free*,
and ends in a state satisfying q

- ✱ Implies fault-avoiding correctness

EXAMPLES

- ✻ Separation logic
sequential pointer-programs
Reynolds
- ✻ Simple shared memory
shared memory parallel, no pointers
Owicki/Gries
- ✻ Concurrent separation logic
shared memory parallel, pointers
O'Hearn
- ✻ Permissions logic
shared memory parallel, pointers
Bornat et al

VALIDITY

Definition

$\Gamma \vdash \{p\}c\{q\}$ is **VALID** iff

$$\forall \sigma \in S_{\Gamma}. \forall \alpha \in \llbracket c \rrbracket . \forall \sigma'.$$

$\sigma \models p$ & $\sigma \xrightarrow{\alpha}_{\Gamma} \sigma'$ implies $\sigma' \neq \text{error}$ & $\sigma' \models q$

*every finite interactive execution of c ,
in an environment that respects Γ ,
from a state satisfying p ,
respects Γ , is error-free,
and ends in a state satisfying q*

INTERACTIVE VALIDATION THEOREM

For all parallel programs

$$\llbracket c_1 \rrbracket_{f\delta} = \llbracket c_2 \rrbracket_{f\delta}$$

implies

$$\forall C. \forall \Gamma, p, q.$$

$\Gamma \vdash \{p\} C[c_1] \{q\}$ **valid** iff $\Gamma \vdash \{p\} C[c_2] \{q\}$ **valid**

*parallel commands with
the same footprint traces
satisfy the same rely/guarantee formulas,
in all parallel contexts*

SEPARATION LOGIC

✱ $S = \text{Store} \times \text{Heap}$

✱ $(s, h) \models p_1 \star p_2$ iff $\exists s_1, s_2. \exists h_1 \perp h_2.$

$$s = s_1 \cup s_2, \quad h = h_1 \uplus h_2,$$

$$(s_1, h_1) \models p_1 \quad \& \quad (s_2, h_2) \models p_2$$

✱ p is *precise* iff

$$\forall (s, h). \exists \text{ at most one } h' \subseteq h \\ \text{such that } (s, h') \models p$$

✱ $\Gamma = r_1(X_1):I_1, \dots, r_n(X_n):I_n$

X_j disjoint, I_j precise, ...

PARALLEL RULE

$$\frac{\Gamma \vdash \{p_1\}c_1\{q_1\} \quad \Gamma \vdash \{p_2\}c_2\{q_2\}}{\Gamma \vdash \{p_1 \star p_2\}c_1 \parallel c_2\{q_1 \star q_2\}}$$

provided

$$\text{free}(c_1) \cap \text{writes}(c_2) \subseteq \text{owned}(\Gamma)$$

$$\text{free}(c_2) \cap \text{writes}(c_1) \subseteq \text{owned}(\Gamma)$$

$$\text{free}(p_2, q_2) \cap \text{writes}(c_1) = \emptyset$$

$$\text{free}(p_1, q_1) \cap \text{writes}(c_2) = \emptyset$$

REGION RULE

$$\frac{\Gamma \vdash \{(p \star I) \wedge b\} c \{q \star I\}}{\Gamma, r(X):I \vdash \{p\} \text{with } r \text{ when } b \text{ do } c \{q\}}$$

RESOURCE RULE

$$\frac{\Gamma, r(X):I \vdash \{p\} c \{q\}}{\Gamma \vdash \{p \star I\} \text{with } r \text{ when } b \text{ do } c \{q \star I\}}$$

SOUNDNESS THEOREM

- ✻ Each rule of concurrent separation logic is valid

Use semantic model to formalize

- local state
- ownership transfer

Proof reveals key role of *precision*

SIMILARLY...

- ✻ Soundness proofs for

- ✻ Owicki-Gries

- ✻ permissions logic

based on appropriate choice of state model

CONCLUSIONS

- ✱ A general, abstract notion of *state model*
- ✱ A recipe for constructing semantic models
- ✱ Suitable for compositional reasoning
 - ✱ fault-avoiding partial correctness
 - ✱ rely/guarantee partial correctness properties
- ✱ Soundness proofs for fault-avoiding logics

FUTURE RESEARCH

- ✱ **Fault-avoiding logics**

- ✱ total correctness
- ✱ safety and liveness

- ✱ **Semantic models**

- ✱ full abstraction?

- ✱ **Synchronization**

- ✱ other primitives
- ✱ abstract model?