Teaching Statement

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My motivation in teaching mathematical subjects is to stimulate intellectual growth among the students, bringing them closer to being productive contributers to the research community. At the graduate level, teaching is a process of planting seeds, which will eventually grow into fruitful collaborations and theorems in the future. At the undergraduate level, teaching a mathematical subject is about inducing the students to develop certain skills, while at the same time conveying the sense of beauty that underlies the discipline.

Lecturing

One thing I have learned is that interaction is essential for education, particularly in theory-oriented courses. Students learn the material best when they are themselves rediscovering it, and it is the teacher's role in this process to guide the student toward these discoveries. In college, as I became more senior, I was frequently asked to tutor less-experienced students, helping them to understand basic concepts and techniques. Through this process, I learned the skill of guiding the students to answer their own questions, turning their question into a sequence of simpler ones that they can answer. Whenever possible, I believe this technique should be used in the classroom, and in less formal interactions with students, as its effectiveness has been proven time and again, both anecdotally and in scientific studies.

Of course, the Socratic style of teaching is not always appropriate, and sometimes one must resort to the traditional lecture format. In these cases, it is especially important to plan carefully, and make every effort to anticipate the mental process of the students. During the lecture, it is important to be mindful of the students' reactions and expressions in the classroom, always trying to guess at how much they understand and what the gaps are. For instance, when the room is quiet, and nobody is asking questions or making comments, this is often a sign that they are not following. To avoid this, it is important to maintain a constant friendly dialogue with the students, to develop an inviting environment where the students are encouraged to express their lack of comprehension, not hide it. For instance, in the undergraduate class Modern Computer Algebra at CMU, which I was a teaching assistant for, the instructor would intentionally plant a mistake in the lecture, offering a bounty for anyone who could find it, so that the students were induced to question anything they did not understand.

Something I learned from interaction with Steven Rudich (in his "Technically Speaking" class, taught at CMU) is that part of giving a good lecture is telling a good mystery story. It begins with an intriguing puzzle, posed to the students rhetorically; they cannot help but wonder how such a puzzle might be solved. Then over the course of the lecture, at moments when the solution seems impossible, gradually hints are revealed. By the end of the lecture, the students and instructor together have been on an investigative journey, feeling they have witnessed the discovery of this solution, that they were a part of it. This type of lecture lingers in the students' memories for years, certainly conveying the technical content, but simultaneously serving to engage and inspire. Over the course of taking his class, I was able to master this skill by practicing in my in-class presentations, and have since found it a most-effective strategy in guest lectures I gave in the CMU Graduate Algorithms class (taught by Manuel Blum, Spring 2011) and the Modern Computer Algebra class (taught by Victor Adamchik, Fall 2011) at CMU.

As for planning the lectures themselves, there are at least two distinct approaches to teaching courses on theoretical subjects, and I personally strive for a good mix of the two. One approach is result-driven. For each topic covered in the class, following the result-driven approach, one selects the most significant results in that area, and makes it the goal of the course to present these results, their proofs, and examples illustrating what the implications of these results are. The intention behind this approach is that students should become familiar with everything that is known about a topic, understanding the "road map" of the area, so that (for instance) in the future they can place any new results in context, and appreciate their significance. In the process, the students become aware of some of the history of an area, its open problems and directions, and the narrative of what leads to what, and why we are interested in that.

The other approach to designing a theory course is technique-driven. In some cases, there are important techniques that, while present in the proof of some major results, would be better taught in the context of proving a lesser result. In the technique-driven approach, rather than starting with a list of major results, the seed of the course is a list of major techniques, which can be useful to the students in their future careers. The motivation behind this approach is that the job of the course is to prepare the students for careers requiring mathematical skills, and in that context they will inevitably find a few core tools quite useful, so that the course should be focused around these tools. For instance, the practical skill of recognizing how to transform a problem into one where the Chernoff bound for Bernoulli samples can be applied might not be considered a core result in a probability theory class, but it is so widely useful that it is certainly worth covering.

Of course, the best scenario is when these two approaches coincide, so that the proofs of the major results are also perfect case studies in the use of a particular technique. However, when the two approaches split, it is best to provide a good mixture, covering the important results, but also focusing a good fraction of lectures on techniques. One approach that seems particularly effective is to focus the homework assignments on techniques, while the lectures focus largely on the major results. However,

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by mixing in a few lectures on techniques, the students seem to get far more out of the class. So it is important never to focus on one of these two approaches to the exclusion of the other.

Related to this issue, one way many theory lectures run astray is to focus too much on presenting a correct formal argument, and not enough on the concepts and ideas that underly those formal arguments. That is, in presenting a step-by-step proof of a major theorem, it may well be the case that most of the students can follow every step, yet still not understand at all what the general idea of the proof is, or even what the theorem is really about. One thing I learned from Avrim Blum's lectures is that the key to cutting through this problem is to, in addition to the formal details, take a moment to explain at a purely intuitive and informal level what the proof is about, what the key insight that cracks open the solution is, and why we care about this theorem at all. Often the students get far more out of these little asides than from the formal proof itself. This is partly because this key insight often also reflects some aspect of an idealized creative process that could potentially explain how the original proof was discovered.

For me, FOCS, STOC, and the CMU Theory Lunch are also good settings to learn teaching skills. I benefited substantially from attending Shafi Goldwasser's Piore Award Lecture at FOCS 2011; she held a large audience's constant attention over more than 40 minutes, exciting the audience with spontaneous jokes, with a perfect balance between intensive information flow and motivating examples. I also enjoyed the style of Silvio Micali's seminar talk at CMU in fall 2011. At times in the talk, it almost felt like an open discussion, with everyone discussing freely on the topic; but when a question would naturally arise from the discussion, Silvio's next slide would somehow always magically capture the exact answer to the problem raised. It seems what was most likely happening was that Silvio had anticipated the precise natural progression for this discussion, including the gaps that required him to provide the next step, and prepared slides just to help the discussion along at these times.

Evaluation

Evaluation can be an important part of the education process, as a motivating factor, a way to catch students who are slipping, a way to identify stars, and perhaps most importantly, as a way for the students to practice the mathematical skills being taught in the class. On this latter note, it has been my observation, particularly in the CMU Graduate Algorithms class I was a teaching assistant for (taught by Manuel Blum, Spring 2011), that the emotional struggle of working through homework problems is an essential part of learning Mathematics. This forces the student to develop an understanding of the ideas behind the formality. In the process, the student develops critical problem solving skills, which are virtually impossible to teach in any other way than by doing: by solving problem after problem, learning to recognize patterns by a succession of "aha" moments. This problem solving skill is often the most valuable thing the students will take away from a theory class, in terms of usefulness in their future careers.

Another creative use of evaluation, which I learned from Manuel Blum, is to get students involved in the lecture. At various times in the lecture, Manuel would stop, and ask the students to discuss amongst themselves and try to resolve some point, or work out some example, essentially letting the students take the lead. When someone finally came up with a correct answer, Manuel would write down which student it was, and then use this information when determining grades.

Beyond the Classroom

So far, I have been describing my attitude toward classroom teaching. But teaching extends beyond the classroom, into most aspects of academic life. Generally, teaching is an essential part of an academic environment, since research is mostly about learning new things. Furthermore, research is a community effort, so that having a smarter, better-educated community of colleagues is in all of our best interests. Especially in mathematical disciplines, the graduate students sitting in lecture today are likely to be proving the next major results only a few short years from now. So an instructor should approach the task of graduate-level teaching as being part of the process of cultivating her own future colleagues. By properly preparing them for the challenges they will face in their future research endeavors, the teacher is contributing to the right combination of circumstances for the next breakthrough, by one of those very students.

The other aspect of teaching is its effect on the teacher herself. I learned long ago that the best way for me to learn something is by preparing to teach it to someone else. Anticipating the questions of students forces one to consider a topic from many different angles. And the need to clearly explain a topic forces one to distill it down to its essence. By tutoring students, preparing guest lectures and conference presentations, interacting with students in office hours, writing and grading homework problems, and addressing student questions, I have been induced to see new perspectives and insights into my discipline, which I otherwise might have missed. This is just one of the many reasons I find teaching a fulfilling and essential part of my academic life.

My teaching skills are continually improving, and I am constantly learning new approaches, attending workshops and seminars on the subject, and practicing these skills via guest lectures and other presentations. I plan for this growth to continue in the future, as I move into the next phase of my career as a postdoctoral fellow, where teaching will take an even more prominent role in my daily life.