CFLs, Finite Unary Sets, DFAs, NFAs

Where this Talk Came From

This talk is based on the paper
Simulating Finite Automata with Context-Free Grammars by
Domaratzki, Pighizzini, Shallit. Information Processing Letters,
Volume 84, 2002,339-344.

CFLs for Finite Unary Sets

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

$G_0 \rightarrow e$ $\qquad L(G_0) = \{e\}$

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

$G_0 \to e$	$L(G_0) = \{e\}$
$G_1 \to a$	$L(G_1) = \{a\}$

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

$G_0 \to e \qquad L(G_0) = \{e\}$

$G_1 \to a \qquad L(G_1) = \{a\}$

$G_2 \to G_1 G_1 \qquad L(G_2) = \{aa\}$

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

$G_0 \to e \qquad L(G_0) = \{e\}$

$G_1 \to a \qquad L(G_1) = \{a\}$

$G_2 \to G_1 G_1 \qquad L(G_2) = \{aa\}$

$G_3 \to G_2 G_1 \qquad L(G_3) = \{aaa\}$

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

$G_0 \rightarrow e \qquad L(G_0) = \{e\}$

$G_1 \rightarrow a \qquad L(G_1) = \{a\}$

$G_2 \rightarrow G_1 G_1 \qquad L(G_2) = \{aa\}$

$G_3 \rightarrow G_2 G_1 \qquad L(G_3) = \{aaa\}$

$G_4 \rightarrow G_3 G_1 \qquad L(G_4) = \{aaaa\}$

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

$G_0 \rightarrow e \qquad L(G_0) = \{e\}$

$G_1 \rightarrow a \qquad L(G_1) = \{a\}$

$G_2 \rightarrow G_1 G_1 \qquad L(G_2) = \{aa\}$

$G_3 \rightarrow G_2 G_1 \qquad L(G_3) = \{aaa\}$

$G_4 \rightarrow G_3 G_1 \qquad L(G_4) = \{aaaa\}$

$G_5 \rightarrow G_4 G_1 \qquad L(G_5) = \{aaaaa\}$

CFG's for $\{e, a, a^2, a^3, a^4, a^5\}$

We do $n = 5$.

$G_0 \to e \qquad L(G_0) = \{e\}$

$G_1 \to a \qquad L(G_1) = \{a\}$

$G_2 \to G_1 G_1 \qquad L(G_2) = \{aa\}$

$G_3 \to G_2 G_1 \qquad L(G_3) = \{aaa\}$

$G_4 \to G_3 G_1 \qquad L(G_4) = \{aaaa\}$

$G_5 \to G_4 G_1 \qquad L(G_5) = \{aaaaa\}$

Can generalize to get $G_0, \ldots, G_r$ with $L(G_i) = \{a^i\}$.

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$F_0 \to e \qquad L(F_0) = \{e\}.$

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$$F_0 \to e \qquad L(F_0) = \{e\}.$$
$$F_1 \to G_5 \qquad L(F_1) = \{a^5\}.$$

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$F_0 \rightarrow e$ $\qquad L(F_0) = \{e\}.$

$F_1 \rightarrow G_5$ $\qquad L(F_1) = \{a^5\}.$

$F_2 \rightarrow F_1 F_1$ $\qquad L(F_2) = \{a^{10}\}.$

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$$
\begin{array}{ll}
F_0 \to e & L(F_0) = \{e\}. \\
F_1 \to G_5 & L(F_1) = \{a^5\}. \\
F_2 \to F_1 F_1 & L(F_2) = \{a^{10}\}. \\
F_3 \to F_2 F_1 & L(F_3) = \{a^{15}\}.
\end{array}
$$

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$$
\begin{array}{ll}
F_0 \to e & L(F_0) = \{e\}. \\
F_1 \to G_5 & L(F_1) = \{a^5\}. \\
F_2 \to F_1 F_1 & L(F_2) = \{a^{10}\}. \\
F_3 \to F_2 F_1 & L(F_3) = \{a^{15}\}. \\
F_4 \to F_3 F_1 & L(F_4) = \{a^{20}\}.
\end{array}
$$

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$$
\begin{array}{ll}
F_0 \rightarrow e & L(F_0) = \{e\}. \\
F_1 \rightarrow G_5 & L(F_1) = \{a^5\}. \\
F_2 \rightarrow F_1 F_1 & L(F_2) = \{a^{10}\}. \\
F_3 \rightarrow F_2 F_1 & L(F_3) = \{a^{15}\}. \\
F_4 \rightarrow F_3 F_1 & L(F_4) = \{a^{20}\}. \\
F_5 \rightarrow F_4 F_1 & L(F_5) = \{a^{25}\}.
\end{array}
$$

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$F_0 \rightarrow e \qquad L(F_0) = \{e\}.$
$F_1 \rightarrow G_5 \qquad L(F_1) = \{a^5\}.$
$F_2 \rightarrow F_1 F_1 \qquad L(F_2) = \{a^{10}\}.$
$F_3 \rightarrow F_2 F_1 \qquad L(F_3) = \{a^{15}\}.$
$F_4 \rightarrow F_3 F_1 \qquad L(F_4) = \{a^{20}\}.$
$F_5 \rightarrow F_4 F_1 \qquad L(F_5) = \{a^{25}\}.$
Can generalize to get $F_0, \ldots, F_k$ with $L(F_i) = \{a^{ik}\}$.

CFG's for $\{e, a^5, a^{10}, a^{15}, a^{20}, a^{25}\}$

$F_0 \rightarrow e \qquad L(F_0) = \{e\}.$
$F_1 \rightarrow G_5 \qquad L(F_1) = \{a^5\}.$
$F_2 \rightarrow F_1 F_1 \qquad L(F_2) = \{a^{10}\}.$
$F_3 \rightarrow F_2 F_1 \qquad L(F_3) = \{a^{15}\}.$
$F_4 \rightarrow F_3 F_1 \qquad L(F_4) = \{a^{20}\}.$
$F_5 \rightarrow F_4 F_1 \qquad L(F_5) = \{a^{25}\}.$
Can generalize to get $F_0, \ldots, F_k$ with $L(F_i) = \{a^{ik}\}$.
We will use $k = r$ so $F_0, \ldots, F_r$ with $L(F_i) = \{a^{ir}\}$.

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$E_0 \rightarrow e \qquad L(E_0) = \{e\}.$

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$E_0 \rightarrow e \qquad L(E_0) = \{e\}.$

$E_1 \rightarrow F_5 \qquad L(E_1) = \{a^{25}\}.$

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$E_0 \to e \qquad L(E_0) = \{e\}.$

$E_1 \to F_5 \qquad L(E_1) = \{a^{25}\}.$

$E_2 \to E_1 E_1 \qquad L(E_2) = \{a^{50}\}.$

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$$
\begin{array}{ll}
E_0 \to e & L(E_0) = \{e\}. \\
E_1 \to F_5 & L(E_1) = \{a^{25}\}. \\
E_2 \to E_1 E_1 & L(E_2) = \{a^{50}\}. \\
E_3 \to E_2 E_1 & L(E_3) = \{a^{75}\}.
\end{array}
$$

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$$
\begin{array}{ll}
E_0 \rightarrow e & L(E_0) = \{e\}. \\
E_1 \rightarrow F_5 & L(E_1) = \{a^{25}\}. \\
E_2 \rightarrow E_1 E_1 & L(E_2) = \{a^{50}\}. \\
E_3 \rightarrow E_2 E_1 & L(E_3) = \{a^{75}\}. \\
E_4 \rightarrow E_3 T_1 & L(E_4) = \{a^{100}\}.
\end{array}
$$

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$$
\begin{array}{ll}
E_0 \to e & L(E_0) = \{e\}. \\
E_1 \to F_5 & L(E_1) = \{a^{25}\}. \\
E_2 \to E_1 E_1 & L(E_2) = \{a^{50}\}. \\
E_3 \to E_2 E_1 & L(E_3) = \{a^{75}\}. \\
E_4 \to E_3 T_1 & L(E_4) = \{a^{100}\}. \\
E_5 \to E_4 T_1 & L(E_5) = \{a^{125}\}.
\end{array}
$$

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$E_0 \to e \qquad L(E_0) = \{e\}$.
$E_1 \to F_5 \qquad L(E_1) = \{a^{25}\}$.
$E_2 \to E_1 E_1 \qquad L(E_2) = \{a^{50}\}$.
$E_3 \to E_2 E_1 \qquad L(E_3) = \{a^{75}\}$.
$E_4 \to E_3 T_1 \qquad L(E_4) = \{a^{100}\}$.
$E_5 \to E_4 T_1 \qquad L(E_5) = \{a^{125}\}$.
Can generalize to get $E_0, \ldots, E_k$ with $L(E_i) = \{a^{ik_1k_2}\}$.

CFG's for $\{e, a^{25}, a^{50}, a^{75}, a^{100}, a^{125}\}$

$E_0 \to e \qquad L(E_0) = \{e\}.$
$E_1 \to F_5 \qquad L(E_1) = \{a^{25}\}.$
$E_2 \to E_1 E_1 \qquad L(E_2) = \{a^{50}\}.$
$E_3 \to E_2 E_1 \qquad L(E_3) = \{a^{75}\}.$
$E_4 \to E_3 T_1 \qquad L(E_4) = \{a^{100}\}.$
$E_5 \to E_4 T_1 \qquad L(E_5) = \{a^{125}\}.$
Can generalize to get $E_0, \ldots, E_k$ with $L(E_i) = \{a^{ik_1 k_2}\}$.
We will use $k_1 = k_2 = r$ so $E_0, \ldots, E_r$ with $L(T_i) = \{a^{ir^2}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_0 S_0$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \to E_0 S_0$

$S_0 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

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$S_0 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_0 S_0$

$S_0 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \to E_0 S_0$

$S_0 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \to F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^5, a^6, a^7, a^8, a^9\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \to E_0 S_0$

$S_0 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \to F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^5, a^6, a^7, a^8, a^9\}$

$S_0 \to F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_0 S_0$

$S_0 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^5, a^6, a^7, a^8, a^9\}$

$S_0 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{10}, a^{11}, a^{12}, a^{13}, a^{14}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_0 S_0$

$S_0 \rightarrow F_0 G_0 \mid F_1 G_0 \mid F_2 G_0 \mid F_3 G_0 \mid F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \rightarrow F_0 G_1 \mid F_1 G_1 \mid F_2 G_1 \mid F_3 G_1 \mid F_4 G_1$

From this S generates $\{a^5, a^6, a^7, a^8, a^9\}$

$S_0 \rightarrow F_0 G_2 \mid F_1 G_2 \mid F_2 G_2 \mid F_3 G_2 \mid F_4 G_2$

From this S generates $\{a^{10}, a^{11}, a^{12}, a^{13}, a^{14}\}$

$S_0 \rightarrow F_0 G_3 \mid F_1 G_3 \mid F_2 G_3 \mid F_3 G_3 \mid F_4 G_3$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_0 S_0$

$S_0 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^5, a^6, a^7, a^8, a^9\}$

$S_0 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{10}, a^{11}, a^{12}, a^{13}, a^{14}\}$

$S_0 \rightarrow F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$

From this S generates $\{a^{15}, a^{16}, a^{17}, a^{18}, a^{19}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_0 S_0$

$S_0 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^5, a^6, a^7, a^8, a^9\}$

$S_0 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{10}, a^{11}, a^{12}, a^{13}, a^{14}\}$

$S_0 \rightarrow F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$

From this S generates $\{a^{15}, a^{16}, a^{17}, a^{18}, a^{19}\}$

$S_0 \rightarrow F_0 G_4 \quad | \quad F_1 G_4 \quad | \quad F_2 G_4 \quad | \quad F_3 G_4 \quad | \quad F_4 G_4$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{e, \ldots, a^{24}\}$.

Recall: $L(E_0) = \{e\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \to E_0 S_0$
$S_0 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$
From this S_0 generates $\{e, a, a^2, a^3, a^4\}$

$S_0 \to F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$
From this S generates $\{a^5, a^6, a^7, a^8, a^9\}$

$S_0 \to F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$
From this S generates $\{a^{10}, a^{11}, a^{12}, a^{13}, a^{14}\}$

$S_0 \to F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$
From this S generates $\{a^{15}, a^{16}, a^{17}, a^{18}, a^{19}\}$

$S_0 \to F_0 G_4 \quad | \quad F_1 G_4 \quad | \quad F_2 G_4 \quad | \quad F_3 G_4 \quad | \quad F_4 G_4$
From this S generates $\{a^{20}, a^{21}, a^{22}, a^{23}, a^{24}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\qquad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\qquad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_1 S_1$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\qquad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \to E_1 S_1$

$S_1 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_1 S_1$

$S_1 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \to E_1 S_1$

$S_1 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \to F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_1 S_1$

$S_1 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{30}, a^{31}, a^{32}, a^{33}, a^{34}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_1 S_1$

$S_1 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{30}, a^{31}, a^{32}, a^{33}, a^{34}\}$

$S_1 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

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$S \to E_1 S_1$

$S_1 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \to F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{30}, a^{31}, a^{32}, a^{33}, a^{34}\}$

$S_1 \to F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{35}, a^{36}, a^{37}, a^{38}, a^{39}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

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$S_1 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{30}, a^{31}, a^{32}, a^{33}, a^{34}\}$

$S_1 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{35}, a^{36}, a^{37}, a^{38}, a^{39}\}$

$S_1 \rightarrow F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_1 S_1$

$S_1 \rightarrow F_0 G_0 \mid F_1 G_0 \mid F_2 G_0 \mid F_3 G_0 \mid F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \rightarrow F_0 G_1 \mid F_1 G_1 \mid F_2 G_1 \mid F_3 G_1 \mid F_4 G_1$

From this S generates $\{a^{30}, a^{31}, a^{32}, a^{33}, a^{34}\}$

$S_1 \rightarrow F_0 G_2 \mid F_1 G_2 \mid F_2 G_2 \mid F_3 G_2 \mid F_4 G_2$

From this S generates $\{a^{35}, a^{36}, a^{37}, a^{38}, a^{39}\}$

$S_1 \rightarrow F_0 G_3 \mid F_1 G_3 \mid F_2 G_3 \mid F_3 G_3 \mid F_4 G_3$

From this S generates $\{a^{40}, a^{41}, a^{42}, a^{43}, a^{44}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_1 S_1$

$S_1 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{30}, a^{31}, a^{32}, a^{33}, a^{34}\}$

$S_1 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{35}, a^{36}, a^{37}, a^{38}, a^{39}\}$

$S_1 \rightarrow F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$

From this S generates $\{a^{40}, a^{41}, a^{42}, a^{43}, a^{44}\}$

$S_1 \rightarrow F_0 G_4 \quad | \quad F_1 G_4 \quad | \quad F_2 G_4 \quad | \quad F_3 G_4 \quad | \quad F_4 G_4$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{25}, \ldots, a^{44}\}$.

Recall: $L(E_1) = \{a^{25}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_1 S_1$

$S_1 \rightarrow F_0 G_0 \mid F_1 G_0 \mid F_2 G_0 \mid F_3 G_0 \mid F_4 G_0$

From this S generates $\{a^{25}, a^{26}, a^{27}, a^{28}, a^{29}\}$

$S_1 \rightarrow F_0 G_1 \mid F_1 G_1 \mid F_2 G_1 \mid F_3 G_1 \mid F_4 G_1$

From this S generates $\{a^{30}, a^{31}, a^{32}, a^{33}, a^{34}\}$

$S_1 \rightarrow F_0 G_2 \mid F_1 G_2 \mid F_2 G_2 \mid F_3 G_2 \mid F_4 G_2$

From this S generates $\{a^{35}, a^{36}, a^{37}, a^{38}, a^{39}\}$

$S_1 \rightarrow F_0 G_3 \mid F_1 G_3 \mid F_2 G_3 \mid F_3 G_3 \mid F_4 G_3$

From this S generates $\{a^{40}, a^{41}, a^{42}, a^{43}, a^{44}\}$

$S_1 \rightarrow F_0 G_4 \mid F_1 G_4 \mid F_2 G_4 \mid F_3 G_4 \mid F_4 G_4$

From this S generates $\{a^{45}, a^{46}, a^{47}, a^{48}, a^{49}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

Recall: $L(E_2) = \{a^{50}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

Recall: $L(E_2) = \{a^{50}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_2 S_2$

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$S \rightarrow E_2 S_2$

$S_2 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

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From this S generates $\{a^{50}, a^{51}, a^{52}, a^{53}, a^{54}\}$

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$S_2 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

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$S_2 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{55}, a^{56}, a^{57}, a^{58}, a^{59}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

Recall: $L(E_2) = \{a^{50}\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_2 S_2$

$S_2 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

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$S_2 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{55}, a^{56}, a^{57}, a^{58}, a^{59}\}$

$S_2 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{60}, a^{61}, a^{62}, a^{63}, a^{64}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

Recall: $L(E_2) = \{a^{50}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_2 S_2$

$S_2 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{50}, a^{51}, a^{52}, a^{53}, a^{54}\}$

$S_2 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{55}, a^{56}, a^{57}, a^{58}, a^{59}\}$

$S_2 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{60}, a^{61}, a^{62}, a^{63}, a^{64}\}$

$S_2 \rightarrow F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

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$S \rightarrow E_2 S_2$

$S_2 \rightarrow F_0 G_0 \mid F_1 G_0 \mid F_2 G_0 \mid F_3 G_0 \mid F_4 G_0$

From this S generates $\{a^{50}, a^{51}, a^{52}, a^{53}, a^{54}\}$

$S_2 \rightarrow F_0 G_1 \mid F_1 G_1 \mid F_2 G_1 \mid F_3 G_1 \mid F_4 G_1$

From this S generates $\{a^{55}, a^{56}, a^{57}, a^{58}, a^{59}\}$

$S_2 \rightarrow F_0 G_2 \mid F_1 G_2 \mid F_2 G_2 \mid F_3 G_2 \mid F_4 G_2$

From this S generates $\{a^{60}, a^{61}, a^{62}, a^{63}, a^{64}\}$

$S_2 \rightarrow F_0 G_3 \mid F_1 G_3 \mid F_2 G_3 \mid F_3 G_3 \mid F_4 G_3$

From this S generates $\{a^{65}, a^{66}, a^{67}, a^{68}, a^{69}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

Recall: $L(E_2) = \{a^{50}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_2 S_2$

$S_2 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{50}, a^{51}, a^{52}, a^{53}, a^{54}\}$

$S_2 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{55}, a^{56}, a^{57}, a^{58}, a^{59}\}$

$S_2 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{60}, a^{61}, a^{62}, a^{63}, a^{64}\}$

$S_2 \rightarrow F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$

From this S generates $\{a^{65}, a^{66}, a^{67}, a^{68}, a^{69}\}$

$S_2 \rightarrow F_0 G_4 \quad | \quad F_1 G_4 \quad | \quad F_2 G_4 \quad | \quad F_3 G_4 \quad | \quad F_4 G_4$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{50}, \ldots, a^{74}\}$.

Recall: $L(E_2) = \{a^{50}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_2 S_2$

$S_2 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{50}, a^{51}, a^{52}, a^{53}, a^{54}\}$

$S_2 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{55}, a^{56}, a^{57}, a^{58}, a^{59}\}$

$S_2 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{60}, a^{61}, a^{62}, a^{63}, a^{64}\}$

$S_2 \rightarrow F_0 G_3 \quad | \quad F_1 G_3 \quad | \quad F_2 G_3 \quad | \quad F_3 G_3 \quad | \quad F_4 G_3$

From this S generates $\{a^{65}, a^{66}, a^{67}, a^{68}, a^{69}\}$

$S_2 \rightarrow F_0 G_4 \quad | \quad F_1 G_4 \quad | \quad F_2 G_4 \quad | \quad F_3 G_4 \quad | \quad F_4 G_4$

From this S generates $\{a^{70}, a^{71}, a^{72}, a^{73}, a^{74}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

Recall: $L(E_3) = \{a^{75}\}$ $\qquad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

Recall: $L(E_3) = \{a^{75}\}$ $\qquad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_3 S_3$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

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$S \rightarrow E_3 S_3$

$S_3 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

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$S \rightarrow E_3 S_3$

$S_3 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{75}, a^{76}, a^{77}, a^{78}, a^{79}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

Recall: $L(E_3) = \{a^{75}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

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$S_3 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{75}, a^{76}, a^{77}, a^{78}, a^{79}\}$

$S_3 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

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$S_3 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{75}, a^{76}, a^{77}, a^{78}, a^{79}\}$

$S_3 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{80}, a^{81}, a^{82}, a^{83}, a^{84}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

Recall: $L(E_3) = \{a^{75}\}$ $\quad$ $L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_3 S_3$

$S_3 \rightarrow F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{75}, a^{76}, a^{77}, a^{78}, a^{79}\}$

$S_3 \rightarrow F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{80}, a^{81}, a^{82}, a^{83}, a^{84}\}$

$S_3 \rightarrow F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

Recall: $L(E_3) = \{a^{75}\}$ $\qquad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \to E_3 S_3$

$S_3 \to F_0 G_0 \quad | \quad F_1 G_0 \quad | \quad F_2 G_0 \quad | \quad F_3 G_0 \quad | \quad F_4 G_0$

From this S generates $\{a^{75}, a^{76}, a^{77}, a^{78}, a^{79}\}$

$S_3 \to F_0 G_1 \quad | \quad F_1 G_1 \quad | \quad F_2 G_1 \quad | \quad F_3 G_1 \quad | \quad F_4 G_1$

From this S generates $\{a^{80}, a^{81}, a^{82}, a^{83}, a^{84}\}$

$S_3 \to F_0 G_2 \quad | \quad F_1 G_2 \quad | \quad F_2 G_2 \quad | \quad F_3 G_2 \quad | \quad F_4 G_2$

From this S generates $\{a^{85}, a^{86}, a^{87}, a^{88}, a^{89}\}$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

Recall: $L(E_3) = \{a^{75}\}$ $\quad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_3 S_3$

$S_3 \rightarrow F_0 G_0 \mid F_1 G_0 \mid F_2 G_0 \mid F_3 G_0 \mid F_4 G_0$

From this S generates $\{a^{75}, a^{76}, a^{77}, a^{78}, a^{79}\}$

$S_3 \rightarrow F_0 G_1 \mid F_1 G_1 \mid F_2 G_1 \mid F_3 G_1 \mid F_4 G_1$

From this S generates $\{a^{80}, a^{81}, a^{82}, a^{83}, a^{84}\}$

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From this S generates $\{a^{85}, a^{86}, a^{87}, a^{88}, a^{89}\}$

$S_3 \rightarrow F_0 G_3 \mid F_1 G_3 \mid F_2 G_3 \mid F_3 G_3 \mid F_4 G_3$

CFG for $\{e, a, a^2, \ldots, a^{125}\}$: Just $\{a^{75}, \ldots, a^{100}\}$.

Recall: $L(E_3) = \{a^{75}\}$ $\qquad L(F_i) = \{a^i\}$ qquad $L(G_i) = \{a^{5i}\}$.

$S \rightarrow E_3 S_3$

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How Many NonTerminals Were Used?

We generates $\{e, a, a^2, \ldots, a^{124}\}$ with a CFG with the following nonterminals:
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Thm Let $A = \{e, a, \ldots, a^{124}\}$. There is a CFG G such that $L(G) = A$ and $|G| = 23$. (We will generalize this later.)

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Thm Let $n \in \mathrm{N}$. Let $A \subseteq \{e, a, \ldots, a^{n-1}\}$. There is a CFG G such that $L(G) = A$ and $|G| = O(n^{1/3})$.

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Thm For all n there exists $A \subseteq \{e, a, \ldots, a^{n-1}\}$ such that if CFG G has $L(G) = A$ then $|G| = \Omega(n^3)$.
The proof is not constructive.

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For there to be some A that no CFG on t nonterminals generates we need $2^{t^3+t} < 2^n$. Take $t = 0.5n^{1/3}$.

DFAs for Finite Unary Sets