Logic Seminar Computability and Ramsey Cheat Sheet

RAMSEY THEORY DEFINITIONS AND STATEMENTS

Def 0.1 If $\mathrm{COL} : \binom{\mathsf{N}}{2} \to [c]$ then a *homog set* is $H \subseteq \mathsf{N}$ such that COL restricted to H is constant.

Theorem 0.2 *For all* $\mathrm{COL} : \binom{\mathsf{N}}{2} \to [c]$ *there exists an infinite homog set.*

THE ARITHMETIC HIERARCHY

Notation 0.3

1. $A \in \Sigma_0$ if A is computable. $A \in \Pi_0$ if A is computable.
2. For $i \geq 1$ $A \in \Sigma_i$ is there exists $B \in \Pi_{i-1}$ such that $A = \{x \mid (\exists y)[(x,y) \in B]\}$
3. For $i \geq 1$ $A \in \Pi_i$ is there exists $B \in \Sigma_{i-1}$ such that $A = \{x \mid (\forall y)[(x,y) \in B]\}$

1. $\Sigma_0 \subset \Sigma_1 \subset \Sigma_2 \subset \cdots$. AND $\Pi_0 \subset \Pi_1 \subset \Pi_2 \subset \cdots$.
2. If $A \leq_{\mathrm{T}} \Sigma_2$ or $A \leq_{\mathrm{T}} \Pi_2$ then $A \in \Sigma_3$.

TURING MACHINES

Notation 0.4

1. $M_0, M_1, \ldots$ is a standard list of Turing Machines (TMs).
2. $M_{e,s}(x)$ means that we run M_e for s steps. $M_e(x) \downarrow$ means that M_e halt on input x.
3. $W_e = \{x \colon M_e(x) \downarrow\}$. $W_{e,s} = \{x \colon M_{e,s}(x) \downarrow\}$.
4. NOTE: $W_0, W_1, \ldots$ is a list of all Σ_1 sets.