Comments on
Oblivious Classes Revisited: Lower Bounds and Hierarchies
Paper by Gajulapali, Li, Volkovich

Comments by by Gasarch

I read enough of the paper to understand the statement of the results and also see how my paper came into it. The rest looks to hard for me (that is NOT a complaint about your paper, more a statement of my time, energy, and knowledge) so thats where I stopped.

My comments might help you make the better better.

I also asked some questions that you can email me answers to.

1. The abstract should begin by saying something about what *oblivious* means.

2. Page 1. S_2P. I think the 2 is because there are 2 proves. But since one is a YES-prove and the other is a NO-prover, there is no obvious definition for S_3P. SO- is there an S_3P? And if not, then is S_2P one of those awful names that we have to use since its traditional? I suspect this comment will not change the paper; however, I want to know whats going on here.

3. Page 2. Inconsistent notation: The obvious version of NP is ONP, but the oblivious version of S_2P is O_2P.

4. Page 2. O_2P needs some clarification. You said that oblivious means that for all n there is ONE string for the witness. But for S_2P you need a witness for both the case where $x \in A$ and for $x \notin A$. So do you still have just one string that is a witness for both membership and non-membership? Or do you have two strings- one that is a witness for all $x \in A$ and one that is a witness for all $x \notin A$.

5. Page 2 (this is probably hopeless) Can you give any examples of NON-SPARSE sets that are naturally in ONP or O_2P? Were there any particular problems that motivated the definition?

6. Page 2 *In particular it was shown that* $\mathrm{NP} \cap \mathrm{SPARSE} \subseteq \mathrm{ONP}$.

   That sentence makes it seem that this is not a trivial statement. I think it is (though I could be wrong):

   If A is sparse then the one string that is a witness for all of them is a coding of the set of strings

   (x, w_x) where $x \in A$ and w_x is the witness.

   So is it that easy OR am I missing something?

   If it is that easy then perhaps say

   In particular it is easy to see that $\mathrm{NP} \cap \mathrm{SPARSE} \subseteq \mathrm{ONP}$.

or

In particular they observed that NP $\cap$ SPARSE $\subseteq$ ONP.

7. Page 2. You refer to BPP $\subseteq$ O_2P as a *non-oblivious containment*. I do not know what that phrase means.

8. Page 2. The result BPP $\subseteq$ O_2P needs a reference (that just means I had never seen it before, though perhaps it is better known than I realize).

9. Page 2. While I am flattered and surprised that you mention my Grid Coloring problem, virtually all complexity problems that come out of Ramsey Theory are in ONP. Here are a few more:

   Recall that $\binom{[n]}{a}$ is the edges of the complete a-hypergraph.

   (a) The standard Ramsey Theory:

   $$\{(1^n, 1^k) \mid \text{ there is a 2-coloring of } \binom{[n]}{2} \text{ without a monochromatic } K_k\}.$$

   Can extend to more colors and to hypergraphs.

   Can also replace K_k with some other graph like C_k or W_k or P_k.

   For some of those the problem is in P.

   (b) (From the Gallai-Witt Theorem) Rather than have the coloring avoid mono rectangles (my problem) have it avoid Theory is to avoid mono SQUARES. You can also try to avoid mono k-grids. You can also do all of this in k-dimensions.

   (c) (From Van der Warden's Theorem)

   $$\{(1^n, 1^k) \mid \text{ there is a 2-coloring of } [n] \text{ without a mono arithmetic sequence of length } k\}.$$

   (d) This is a problem I made up, thought its a generalization of a problem in one of Dennis Shasha's puzzle books. It was at one time on the Wikipedia page of NP-intermediary problem. They got it from my blog. Its not there anymore, perhaps because I blogged that about it being there as being stupid since ANY sparse problem that in NP that is not known to be in P is a candidate for being NP-intermediary.

   After that to-long preamble, here is the problem:

   $$\{(1^n, 1^k) \mid \text{ you can partition } \{1, \ldots, n\} \text{ into } k \text{ boxes so that no box has } x, y, z \text{ with } x + y = z\}$$

One can replace $x + y = z$ with any equation on any number of variables, so long as its easy to verify that its not solved.

Later in the paper (Page 5) you say that my GC problem is the only natural problem you know of. I first though *that can't be right* but then I thought that YES, the problems above are all problems I've thought about but nobody else seems to have. I'm not so much bragging as wondering *why has nobody else looked at these problems.* One reason- they are HARD! Likely not NP-complete since they are sparse, and likely not in P just cause.

ANYWAY, back to YOUR paper:

You might want to say:

Grid coloring problem (defined in [AGL23]) and other similar problems in Ramsey Theory.

10. Page 3.

    In particular, Korten [Kor22] showed that solving Avoid *would result in ... Ramsey Graphs [Rad21] ...*

    (a) By *Solving* Avoid do you mean obtaining a polynomial time algorithm for Avoid? Or perhaps a randomized algorithm (over time people are accepting randomized algorithms as a notion of efficient). Also see next item.

    (b) Consider the result

        Solving Avoid $\rightarrow$ better 2-source extractors.

        How to interpret it? If we don't think there are better 2-source extractors then we think Avoid cannot be solved. If we think there are better 2-source extractors it means that we should work really hard to solve Avoid .

        If we think Avoid cannot be solved by that there are better 2-source extractors then its not clear how to interpret the result.

    (c) This is just my curiosity and I don't think it leads to a change in the paper: the reference to Ramsey Graphs, do you mean constructive lower bounds on the Ramsey Numbers? It is known that 2-source extractors yield better constructive Ramsey Numbers so is there proof really

        Solving Avoid $\rightarrow$ better 2-source extractors $\rightarrow$ Ramsey.

11. Page 3. The Time Hierarchy Theorem. This seems to be independent of the rest of the paper. You should add to the end of this section that you will prove a time hierarchy theorem for $\mathrm{O_T IME}$ later in the paper.

12. Page 4. I think by $\Sigma_4\mathrm{P}$ you mean Σ_4^p. Same for $\Sigma_2\mathrm{P}$.

13. Page 4.

    Nonetheless, the last word has been said yet about $\mathrm{S_2P}$.

    This is a jumble of words that might be missing a *not*.

14. Page 4.

    However, prior to our to result to the best of our knowledge, we could not prove ...

    You are missing a comma after *result*.

    More important- *we could not prove*. By *we* do you really mean Gajulapalli-Li-Volkovich or do you mean that nobody could prove it?

15.