Spectral graph theory and its applications

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Spectral graph theory—the study of the eigenvectors and eigenvalues of matrices associated with graphs—is a large field with many beautiful results. Most of the work in this area has been descriptive, determining how combinatorial features of a graph are revealed by its spectra. On the other hand, many applications require us to control the spectra of a graph. One of the main goals of the course will be to use descriptive results to improve efforts at control of spectra. We will not restrict ourselves to graphs, and will generally consider how combinatorial features of matrices affect their spectra. We will consider application areas including quantum computation, error-correcting codes, graph partitioning and preconditioning. I hope that some in the class will help me develop spectral hypergraph theory, an area in which I have many conjectures and few theorems.

The exact list of topics to be covered will be dictated by student interest. Possible topics include:

1. Spectral graph drawing
   Relations to the work of Tutte, and Colin de Verdiere embeddings of graphs.
2. Using spectra to test graph isomorphism.
3. Spectra of special graphs
   - (a) Strongly regular graphs.
   - (b) Cayley graphs, and connections to group representation theory.
   - (c) Path graphs, and discretizations of the continuous Laplacian.
4. The second eigenvalue of a graph
   - (a) Cheeger's inequality, with three proofs.
   - (b) Spectral graph partitioning.
   - (c) The second eigenvalue of planar graphs.
   - (d) The diameter of graphs, with applications to concentration of measure.
   - (e) Expansion in graphs.
   - (f) Pseudo-random phenomena in graphs.
5. Eigenvalues and eigenvectors of random graphs
   - (a) Continuous distributions, extreme eigenvalues.
   - (b) Concentration of eigenvalues.
   - (c) Discrete distributions, many conjectures.
   - (d) Spectral Partitioning of semi-random graphs.
   - (e) Finding Cliques hidden in random graphs.
   - (f) Coloring random 3-colorable graphs.
   - (g) Free probability, simplified.

6. Preconditioning matrices.
    (a) Preconditioning with spanning trees.
    (b) Sparsifying Laplacian matrices.
    (c) Preconditioning Laplacian matrices.
    (d) Preconditioning matrices from hypergraphs weighted by elasticity.
7. Quantum computation.
    (a) Quantum computation in one lecture.
    (b) Discrete proof of the adiabatic theorem.
    (c) Analysis of the adiabatic algorithm in special cases.
8. Constructing error-correcting codes from expanding graphs.