Distributed Beamforming for Relay Networks Based on Second-Order Statistics of the Channel State Information

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***Abstract*—In this paper, the problem of distributed beamforming is considered for a wireless network which consists of a transmitter, a receiver, and r relay nodes. For such a network, assuming that the second-order statistics of the channel coefficients are available, we study two different beamforming design approaches. As the first approach, we design the beamformer through minimization of the total transmit power subject to the receiver quality of service constraint. We show that this approach yields a closed-form solution. In the second approach, the beamforming weights are obtained through maximizing the receiver signal-to-noise ratio (SNR) subject to two different types of power constraints, namely the total transmit power constraint and individual relay power constraints. We show that the total power constraint leads to a closed-form solution while the individual relay power constraints result in a quadratic programming optimization problem. The later optimization problem does not have a closed-form solution. However, it is shown that using semidefinite relaxation, this problem can be turned into a convex feasibility semidefinite programming (SDP), and therefore, can be efficiently solved using interior point methods. Furthermore, we develop a simplified, thus suboptimal, technique which is computationally more efficient than the SDP approach. In fact, the simplified algorithm provides the beamforming weight vector in a closed form. Our numerical examples show that as the uncertainty in the channel state information is increased, satisfying the quality of service constraint becomes harder, i.e., it takes more power to satisfy these constraints. Also our simulation results show that when compared to the SDP-based method, our simplified technique suffers a 2-dB loss in SNR for low to moderate values of transmit power.**

***Index Terms*—Convex feasibility problem, distributed beamforming, distributed signal processing, relay networks, semidefinite programming.**

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I. Introduction

The explosive growth of research in wireless communications has been inspired by the demand for developing affordable bandwidth-efficient technologies to provide users with wireless access anywhere anytime. To develop such technologies, various means of diversity, including time, frequency, code, and space have to be exploited. These types of diversity have been well studied in the literature. Recently another type of diversity, namely multiuser cooperation diversity, has attracted attentions in the research community [1]–[3]. Communication based on user cooperation, often called cooperative communications, exploits the spatial diversity of multiuser systems without the need for using multiple antennas at each user [4]. In cooperative communications, users relay each other's messages thereby providing multiple paths from the source to the destination.

Emerging wireless technologies, such as sensor and relay networks, have found applications in cooperative communications. In fact, users of a wireless network can cooperate by relaying each other's messages thus improving the communications reliability. However, the limited communication resources, such as battery lifetime of the devices and the scarce bandwidth, challenge the design of such cooperative communication schemes. Therefore, while ensuring that each user receives a certain quality of service (QoS), one is often confronted with the challenge that communication resources are subject to stringent constraints.

Various cooperative communication schemes have been presented in the literature. A three-node network is considered in [5], where one of the nodes relays the messages of another node towards the third one. For such a network, different cooperative protocols are then developed and the outage and ergodic capacities are analyzed. This analysis was later extended in [6] to the case of a relay network where the relay nodes as well as the receiving and transmitting nodes were equipped with multiple antennas. The common assumption used in [5] and [6] is that the perfect *instantaneous* channel state information (CSI) is available at the receiver as well as at the relaying nodes.

Other examples of cooperative communication schemes are amplify-and-forward [3], coded-cooperation [7], and compress-and-forward [8]. Of all these schemes, the amplify-and-forward approach, due to its simplicity, is of particular interest. Recently, the amplify-and-forward approach has been extended to develop space-time coding strategies for relay networks, thereby opening a new research avenue called distributed space-time coding [9]–[15]. While the aforementioned

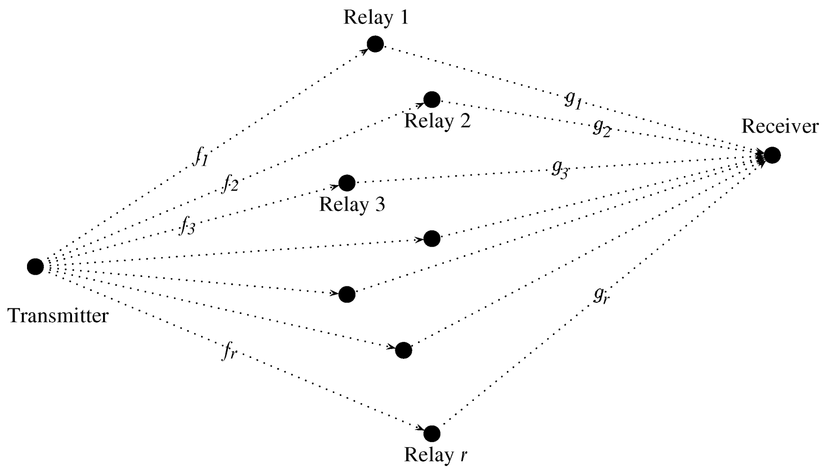

Fig. 1. Relay network.

cooperative approaches assume different levels of CSI availability in the network, they all share the common assumption that the relay nodes operate at their maximum allowable power.

For different relaying strategies, the problem of power allocation between the source and the relay node(s) has been well studied in the literature [4]. In [16], the problem of optimal power allocation is considered in the context of coherent combining the relay signals under the aggregate relay power constraint. This approach assumes that the relays have the perfect knowledge of both their receive and transmit *instantaneous* CSI.

In [17] and [18], a distributed beamforming strategy has been developed for the case where the relaying nodes cooperate to build a beam towards the receiver under individual relay power constraints. To do so, each relay multiplies its received signal by a complex weight and retransmits it. In this scheme, the amplitude and the phase of the transmitted signals are properly adjusted such that they are constructively added up at the receiver. While assuming that the power of each individual relay is limited, it is assumed in [17] that each relay knows the instantaneous CSI for both backward (transmitter to the relay) and forward (relay to the receiver) links. Using such an assumption, the network beamforming approach is simplified to a distributed power control method. In fact, each relay matches the phase of its weight vector to the total phase of the backward and forward links. Therefore, only the amplitudes of the complex weights remain to be determined. These amplitudes are then obtained through maximizing the signal-to-noise ratio (SNR) at the receiver while guaranteeing that the individual relay powers meet the corresponding constraints. Interestingly enough, such a maximization results in relay powers that are not necessarily at their maximum allowable values. The relaying schemes developed in [16] and [17] are based on the availability of instantaneous CSI, and therefore, they do not allow any uncertainty in the channel modeling.

In this paper, we consider the problem of distributed beamforming under the assumption that the *second-order statistics* of the channel coefficients are available. Such an assumption allows us to consider uncertainty in the channel modeling through introducing the covariance matrices of the channel coefficients. Based on this assumption, we develop two distributed beamforming algorithms. As the first approach, we aim to minimize the total transmit power required in the relay network subject to a constraint which guarantees that the receiver QoS (measured by the receiver SNR) remains above a predefined threshold. We show that this approach results in a closed-form solution for the beamforming weights. In the second approach, our goal is to maximize the receiver SNR subject to two different types of power constraints: aggregate power constraint as well as individual relay power constraints. We show that in the case of constrained aggregate power, the beamforming problem has a closed-form solution. We also show that in the case of individual relay power constraints, the beamforming problem can be approximately written as a semidefinite programming (SDP) problem which can be efficiently solved using interior point methods. Furthermore, to avoid the computational complexity of SDP, we present a simplified (but suboptimal) technique which provides the beamforming weight vector in a closed form.

The remainder of the paper is organized as follows. In Section II, we present the data model. The power-minimization-based beamforming technique is developed in Section III. Section IV presents the SNR-maximization-based beamforming algorithms. Simulation results are provided in Section V, and concluding remarks are given in Section VI.

II. System Model

Consider a wireless network which consists of a transmitter, a receiver, and r relay nodes, as shown in Fig. 1. We assume that due to the poor quality of the channel between the transmitter and receiver, there is no direct link between them. As a result, the transmitter deploys the relay nodes to communicate with the receiver. Each relay has a single antenna for both transmission and reception. Assuming a flat fading scenario, let f_i denote the channel coefficient from the transmitter to the ith relay and g_i represent the channel coefficient from the ith relay to the receiver. We also assume that the second-order statistics of the channel coefficients $\{f_i\}_{i=1}^r$ and $\{g_i\}_{i=1}^r$ are known. In fact, we model f_i and g_i as random variables with known second-order statistics.

We herein study a two-step amplify-and-forward (AF) protocol. During the first step, the transmitter broadcasts to the relays the signal $\sqrt{P_0}s$, where s is the information symbol and

P_0 is the transmit power. We assume that $E\{|s|^2\} = 1$, where $E\{\cdot\}$ represents the statistical expectation, and $|\cdot|$ denotes the amplitude of a complex number. The signal x_i received at the ith relay is given by

$$x_i = \sqrt{P_0} f_i s + \nu_i \tag{1}$$

where ν_i is the noise at the ith relay whose variance is known to be σ_ν^2.

During the second step, the ith relay transmits the signal y_i which can be expressed as

$$y_i = w_i x_i \tag{2}$$

where w_i is the complex beamforming weight used by the ith relay. At the destination, the received signal can be written as

$$z = \sum_{i=1}^{r} g_i y_i + n \tag{3}$$

where z is the received signal and n is the receiver noise whose variance is known to be σ_n^2. Using (1) and (2), we can rewrite (3) as

$$\begin{aligned} z &= \sum_{i=1}^{r} g_i w_i x_i + n \\ &= \underbrace{\sqrt{P_0}\sum_{i=1}^{r} w_i f_i g_i s}_{\text{signal component}} + \underbrace{\sum_{i=1}^{r} w_i g_i \nu_i + n}_{\text{total noise, } n_T}. \end{aligned} \tag{4}$$

Our goal is to obtain the weight coefficients $\{w_i\}_{i=1}^r$ such that the SNR at the receiver is either maximized subject to some power constraint(s) or kept above a certain threshold while minimizing the total transmit power.

III. Power Minimization

In this section, we aim to find the beamforming weights $\{w_i\}_{i=1}^r$ such that the total relay transmit power P_T is minimized while maintaining the receiver QoS at a certain level, i.e., the receiver SNR is required to be larger than a certain predefined threshold $\gamma > 0$. Mathematically, we solve the following optimization problem:

$$\begin{aligned} &\min \quad P_T \\ &\text{subject to} \quad \text{SNR} \geq \gamma \end{aligned} \tag{5}$$

where SNR is defined as the ratio of the signal power P_s to the noise power P_n. The total relay transmit power P_T can be obtained as

$$\begin{aligned} P_T &= \sum_{i=1}^{r} E\left\{|y_i|^2\right\} \\ &= \sum_{i=1}^{r} |w_i|^2 E\left\{|x_i|^2\right\} \\ &= \mathbf{w}^H \mathbf{D} \mathbf{w} \end{aligned} \tag{6}$$

where $(\cdot)^H$ represents Hermitian transpose and the following definitions are used:

$$\begin{aligned} \mathbf{w} &\triangleq [w_1 \; w_2 \; \ldots \; w_r]^T \\ \mathbf{D} &\triangleq P_0 \mathrm{diag}\left(\left[E\left\{|f_1|^2\right\} \; E\left\{|f_2|^2\right\} \; \cdots \; E\left\{|f_r|^2\right\}\right]\right) + \sigma_\nu^2 \mathbf{I}. \end{aligned}$$

Here, $(\cdot)^T$ denotes the transpose operator, $\mathrm{diag}(\mathbf{a})$ represents a diagonal matrix whose diagonal entries are the entries of the vector $\mathbf{a}$, and $\mathbf{I}$ is the identity matrix.

Using (4) and assuming that the relay noises $\{\nu_i\}_{i=1}^r$, the receiver noise n, and the channel coefficients $\{g_i\}_{i=1}^r$ are all independent from each other, the total noise power P_n can then be obtained as

$$\begin{aligned} P_\mathrm{n} &= E\left\{|n_T|^2\right\} \\ &= E\left\{\sum_{i,j=1}^{r} w_i w_j^* g_i g_j^*\right\} \underbrace{E\left\{|\nu_i|^2\right\}}_{\sigma_\nu^2} + E\left\{|n|^2\right\} \\ &= \mathbf{w}^H \mathbf{Q} \mathbf{w} + \sigma_n^2 \end{aligned} \tag{7}$$

where $(\cdot)^*$ represents complex conjugate and the following definitions are used:

$$\begin{aligned} \mathbf{Q} &\triangleq \sigma_\nu^2 E\{\mathbf{g}\mathbf{g}^H\} \\ \mathbf{g} &\triangleq [g_1 \; g_2 \; \cdots \; g_r]^T. \end{aligned}$$

Also, using (4), the signal component power P_s can be obtained as

$$\begin{aligned} P_\mathrm{s} &= E\left\{P_0 \left|\sum_{i=1}^{r} w_i f_i g_i\right|^2 |s|^2\right\} \\ &= P_0 E\left\{\sum_{i,j=1}^{r} w_i w_j^* f_i g_i f_j^* g_j^*\right\} \underbrace{E\left\{|s|^2\right\}}_{1} \\ &= \mathbf{w}^H \mathbf{R} \mathbf{w} \end{aligned} \tag{8}$$

where $\mathbf{R}$ is the correlation matrix of the vector $\mathbf{h} = [f_1 g_1 \; f_2 g_2 \; \ldots \; f_r g_r]^T = \mathbf{f} \odot \mathbf{g}$ and $\odot$ represents the element-wise Schur–Hadamard product, that is,

$$\mathbf{R} \triangleq P_0 E\{\mathbf{h}\mathbf{h}^H\} = P_0 E\left\{(\mathbf{f} \odot \mathbf{g})(\mathbf{f} \odot \mathbf{g})^H\right\}. \tag{9}$$

Using (6), (7), and (8), the optimization problem in (5) can be written as

$$\begin{aligned} &\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{D} \mathbf{w} \\ &\text{subject to} \quad \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\sigma_n^2 + \mathbf{w}^H \mathbf{Q} \mathbf{w}} \geq \gamma \end{aligned} \tag{10}$$

or, equivalently, as

$$\begin{aligned} &\min_{\tilde{\mathbf{w}}} \quad \|\tilde{\mathbf{w}}\|^2 \\ &\text{subject to} \quad \tilde{\mathbf{w}}^H \mathbf{D}^{-1/2}(\mathbf{R} - \gamma \mathbf{Q})\mathbf{D}^{-1/2}\tilde{\mathbf{w}} \geq \gamma \sigma_n^2 \end{aligned} \tag{11}$$

where we have changed the optimization variable to $\tilde{\mathbf{w}} = \mathbf{D}^{1/2}\mathbf{w}$.

It is worth mentioning that if γ is chosen such that $(\mathbf{R}-\gamma\mathbf{Q})$ is negative definite, then the optimization problem in (11) becomes infeasible.

One can easily show that the inequality constraint in (11) is satisfied with equality at the optimum, for otherwise, the optimal $\tilde{\mathbf{w}}$ could be scaled down to satisfy the constraint with equality, thereby decreasing the objective function and contradicting optimality. Therefore, we can rewrite (11) as

$$\min_{\tilde{\mathbf{w}}} \quad \|\tilde{\mathbf{w}}\|^2 \quad \text{subject to} \quad \tilde{\mathbf{w}}^H\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\tilde{\mathbf{w}} = \gamma\sigma_n^2. \tag{12}$$

The Lagrange multiplier function can now be defined as

$$L(\tilde{\mathbf{w}},\lambda) \triangleq \|\tilde{\mathbf{w}}\|^2 - \lambda\left(\tilde{\mathbf{w}}^H\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\tilde{\mathbf{w}} - \gamma\sigma_n^2\right). \tag{13}$$

Using the following definition for differentiation of $L(\tilde{\mathbf{w}},\lambda)$ with respect to $\mathbf{w}^H$:

$$\frac{\partial L(\tilde{\mathbf{w}},\lambda)}{\partial \tilde{\mathbf{w}}^H} \triangleq \frac{1}{2}\left(\frac{\partial L(\tilde{\mathbf{w}},\lambda)}{\partial \Re\tilde{\mathbf{w}}} + j\frac{\partial L(\tilde{\mathbf{w}},\lambda)}{\partial \Im\tilde{\mathbf{w}}}\right)$$

where $\Re$ and $\Im$ denote the real and imaginary parts, we obtain that

$$\frac{\partial L(\tilde{\mathbf{w}},\lambda)}{\partial \tilde{\mathbf{w}}^H} = \tilde{\mathbf{w}} - \lambda\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\tilde{\mathbf{w}}. \tag{14}$$

Equating $\partial L(\tilde{\mathbf{w}},\lambda)/\partial\tilde{\mathbf{w}}^H$ to zero, we obtain that

$$\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\tilde{\mathbf{w}} = \frac{1}{\lambda}\tilde{\mathbf{w}}. \tag{15}$$

It follows from (15) that $\tilde{\mathbf{w}}$ should be chosen as one of the eigenvectors of the matrix $\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}$ and $1/\lambda$ is the corresponding eigenvalue. Multiplying both sides of (15) with $\lambda\tilde{\mathbf{w}}^H$ yields

$$\|\tilde{\mathbf{w}}\|^2 = \tilde{\mathbf{w}}^H\tilde{\mathbf{w}} = \lambda\tilde{\mathbf{w}}^H\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\tilde{\mathbf{w}} = \lambda\gamma\sigma_n^2 \tag{16}$$

where in the last equality we have used the constraint in (12). It follows from (16) that minimizing $\|\tilde{\mathbf{w}}\|^2$ amounts to minimizing λ (or, equivalently, maximizing $1/\lambda$). This means that $1/\lambda$ has to be selected as the largest eigenvalue of $\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}$. As a result, the solution to (5) is given by

$$\tilde{\mathbf{w}}_1 = \beta\mathbf{u} \tag{17}$$

where

$$\mathbf{u} = \mathcal{P}\left\{\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\right\}. \tag{18}$$

Here $\mathcal{P}\{\cdot\}$ represents the normalized principal eigenvector of a matrix, and β is a scalar which is chosen to satisfy the equality constrain in (12), i.e., $\beta = \left(\gamma\sigma_n^2/(\mathbf{u}^H\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\mathbf{u})\right)^{1/2}$.

Eventually, the optimum beamforming weight vector can be written as

$$\mathbf{w}_1 = \left(\frac{\gamma\sigma_n^2}{\mathbf{u}^H\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\mathbf{u}}\right)^{1/2} \times\mathbf{D}^{-1/2}\mathcal{P}\left\{\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\right\}. \tag{19}$$

The minimum total relay transmit power for any feasible γ is given by

$$P_T^{\min}(\gamma) = \frac{\gamma\sigma_n^2}{\lambda_{\max}\left(\mathbf{D}^{-1/2}(\mathbf{R}-\gamma\mathbf{Q})\mathbf{D}^{-1/2}\right)} \tag{20}$$

where $\lambda_{\max}(\cdot)$ denotes the principal eigenvalue of a matrix.

IV. SNR Maximization

In this section, we consider a different approach to obtain the beamforming weight vector. Our goal is to maximize the receiver SNR subject to two different types of relay power constraints. We first study the case where the total relay transmit power is constrained, and then investigate the scenario where the individual relay transmit powers are limited.

A. Total Power Constraint

In this subsection, we aim to maximize the SNR subject to a constraint on the total transmit power. That is, we solve the following optimization problem:

$$\max_{\mathbf{w}} \quad \text{SNR} \quad \text{subject to} \quad P_T \le P_T^{\max}. \tag{21}$$

where $P_T^{\max}$ is the maximum allowable total transmit power. Using (6), (7), and (8), the optimization problem (21) can be rewritten as

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\sigma_n^2 + \mathbf{w}^H\mathbf{Q}\mathbf{w}} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{D}\mathbf{w} \le P_T^{\max}. \tag{22}$$

To solve (22), let us write the weight vector $\mathbf{w}$ as

$$\mathbf{w} = \sqrt{p}\mathbf{D}^{-1/2}\tilde{\mathbf{w}} \tag{23}$$

where $\tilde{\mathbf{w}}$ satisfies $\tilde{\mathbf{w}}^H\tilde{\mathbf{w}} = 1$. The optimization problem (22) can be rewritten as

$$\max_{p,\tilde{\mathbf{w}}} \quad \frac{p\tilde{\mathbf{w}}^H\tilde{\mathbf{R}}\tilde{\mathbf{w}}}{\sigma_n^2 + p\tilde{\mathbf{w}}^H\tilde{\mathbf{Q}}\tilde{\mathbf{w}}} \quad \text{subject to} \quad \|\tilde{\mathbf{w}}\|^2 = 1 \text{ and } p \le P_T^{\max} \tag{24}$$

where the following definitions are used:

$$\tilde{\mathbf{R}} = \mathbf{D}^{-1/2}\mathbf{R}\mathbf{D}^{-1/2}$$
$$\tilde{\mathbf{Q}} = \mathbf{D}^{-1/2}\mathbf{Q}\mathbf{D}^{-1/2}.$$

As the objective function in (24) is monotonically increasing in p, for any value of $\tilde{\mathbf{w}}$, this objective function is maximized for $p = P_T^{\max}$. Hence, the optimization problem in (24) can be simplified as

$$\max_{\tilde{\mathbf{w}}} \quad \frac{P_T^{\max}\tilde{\mathbf{w}}^H\tilde{\mathbf{R}}\tilde{\mathbf{w}}}{\sigma_n^2 + P_T^{\max}\tilde{\mathbf{w}}^H\tilde{\mathbf{Q}}\tilde{\mathbf{w}}} \quad \text{subject to} \quad \|\tilde{\mathbf{w}}\|^2 = 1 \tag{25}$$

or, equivalently, as

$$\max_{\tilde{\mathbf{w}}} \quad \frac{P_T^{\max}\tilde{\mathbf{w}}^H\tilde{\mathbf{R}}\tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H\left(\sigma_n^2\mathbf{I} + P_T^{\max}\tilde{\mathbf{Q}}\right)\tilde{\mathbf{w}}} \quad \text{subject to} \quad \|\tilde{\mathbf{w}}\|^2 = 1. \tag{26}$$

It is well known [23] that the objective function in (26) is *globally* maximized when $\tilde{\mathbf{w}}$ is chosen as the principal generalized eigenvector of $(\tilde{\mathbf{R}}, \sigma_n^2\mathbf{I}+P_T^{\max}\tilde{\mathbf{Q}})$, or, equivalently, as the principal eigenvector of the matrix $(\sigma_n^2\mathbf{I} + P_T^{\max}\tilde{\mathbf{Q}})^{-1}\tilde{\mathbf{R}}$. It is easy to show that such a global maximizer of the objective function in (26) can be normalized to satisfy the unit-norm constraint in (26). Therefore, the solution to (26) is given by

$$\tilde{\mathbf{w}}_2 = \mathcal{P}\left\{\left(\sigma_n^2\mathbf{I} + P_T^{\max}\tilde{\mathbf{Q}}\right)^{-1}\tilde{\mathbf{R}}\right\}. \tag{27}$$

As a result, the beamforming weight vector can be written as

$$\mathbf{w}_2 = \sqrt{P_T^{\max}}\mathbf{D}^{-1/2}\mathcal{P}\left\{\left(\sigma_n^2\mathbf{I} + P_T^{\max}\mathbf{D}^{-1/2}\mathbf{Q}\mathbf{D}^{-1/2}\right)^{-1} \times\mathbf{D}^{-1/2}\mathbf{R}\mathbf{D}^{-1/2}\right\} \tag{28}$$

and the maximum achievable SNR can be expressed as

$$\mathrm{SNR}_{\max}\left(P_T^{\max}\right) = P_T^{\max} \lambda_{\max}\left(\left(\sigma_n^2\mathbf{I}+P_T^{\max}\mathbf{D}^{-1/2}\mathbf{Q}\mathbf{D}^{-1/2}\right)^{-1}\mathbf{D}^{-1/2}\mathbf{R}\mathbf{D}^{-1/2}\right). \tag{29}$$

B. Individual Power Constraint

In this subsection, we consider a different type of power constraint. More specifically, we consider the case where each relay node is restricted in its transmit power. Such a case is of particular interest when the relay nodes are restricted in their battery lifetimes. In this case, we aim to solve the following optimization problem:

$$\max_{\mathbf{w}} \quad \mathrm{SNR} \quad \text{subject to} \quad \mathbf{D}_{ii}|w_i|^2 \le P_i \text{ for } i = 1,2,\ldots,r \tag{30}$$

or, equivalently

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\sigma_n^2 + \mathbf{w}^H\mathbf{Q}\mathbf{w}} \quad \text{subject to} \quad \mathbf{D}_{ii}|w_i|^2 \le P_i \quad \text{for } i = 1,2,\ldots,r \tag{31}$$

where P_i is the maximum allowable transmit power of the ith relay, and $\mathbf{D}_{ii}$ is the ith diagonal entry of the matrix $\mathbf{D}$. Using the definition $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$, the optimization problem in (31) can be written as

$$\begin{aligned} \max_{\mathbf{X}} \quad & \frac{\mathrm{tr}(\mathbf{R}\mathbf{X})}{\sigma_n^2 + \mathrm{tr}(\mathbf{Q}\mathbf{X})} \\ \text{subject to} \quad & \mathbf{D}_{ii}\mathbf{X}_{ii} \le P_i \quad \text{for } i = 1,2,\ldots,r \\ \text{and} \quad & \mathrm{rank}\,\mathbf{X} = 1,\ \mathbf{X} \succeq 0 \end{aligned} \tag{32}$$

or, equivalently, as

$$\begin{aligned} \max_{\mathbf{X},t} \quad & t \\ \text{subject to} \quad & \mathrm{tr}\left(\mathbf{X}(\mathbf{R} - t\mathbf{Q})\right) \ge \sigma_n^2 t \\ \text{and} \quad & \mathbf{X}_{ii} \le P_i/\mathbf{D}_{ii} \quad \text{for } i = 1,2,\ldots,r \\ \text{and} \quad & \mathrm{rank}\,\mathbf{X} = 1,\ \mathbf{X} \succeq 0 \end{aligned} \tag{33}$$

where $\mathrm{tr}(\cdot)$ represents the trace of a matrix and $\mathbf{X} \succeq 0$ means that $\mathbf{X}$ is constrained to be a symmetric positive semidefinite matrix. The optimization problem in (33) is not convex and may thus not be amenable to a computationally efficient solution. Let us ignore the rank constraint in (33). That is, using a semidefinite relaxation, we aim to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{X},t} \quad & t \\ \text{subject to} \quad & \mathrm{tr}\left(\mathbf{X}(\mathbf{R} - t\mathbf{Q})\right) \ge \sigma_n^2 t \\ \text{and} \quad & \mathbf{X}_{ii} \le P_i/\mathbf{D}_{ii} \quad \text{for } i = 1,2,\ldots,r \\ \text{and} \quad & \mathbf{X} \succeq 0. \end{aligned} \tag{34}$$

Due to the relaxation, the matrix $\mathbf{X}^*$ obtained by solving the optimization problem in (34) will not be of rank one in general. If $\mathbf{X}^*$ happens to be rank one, then its principal eigenvector yields the optimal solution to the original problem.

Note that the optimization problem in (34) is quasi-convex. In fact, for any value of t, the feasible set in (34) is convex. Let $t_{\max}$ be the maximum value of t obtained by solving the optimization problem (34). If, for any given t, the convex feasibility problem [19]

$$\begin{aligned} \text{find} \quad & \mathbf{X} \\ \text{such that} \quad & \mathrm{tr}\left(\mathbf{X}(\mathbf{R} - t\mathbf{Q})\right) \ge \sigma_n^2 t \\ \text{and} \quad & \mathbf{X}_{ii} \le P_i/\mathbf{D}_{ii} \quad \text{for } i = 1,2,\ldots,r \\ \text{and} \quad & \mathbf{X} \succeq 0 \end{aligned} \tag{35}$$

is feasible, then we have $t_{\max} \ge t$. Conversely, if the convex feasibility optimization problem (35) is not feasible, then we conclude $t_{\max} < t$. Therefore, we can check whether the optimal value $t_{\max}$ of the quasi-convex optimization problem in (34) is smaller than or greater than a given value t by solving the convex feasibility problem (35).

Based on this observation, we can use a simple algorithm to solve the quasi-convex optimization problem (34) using bisection technique, solving a convex feasibility problem at each step. We assume that the problem is feasible, and start with an interval $[l,\ u]$ known to contain the optimal value $t_{\max}$. We then solve the convex feasibility problem at its midpoint $t = (l + u)/2$, to determine whether the optimal value is larger or smaller than t. We update the interval accordingly to obtain a new interval. That is, if t is feasible, then we set $l = t$, otherwise, we choose $u = t$ and solve the convex feasibility problem in (35) again.

Remark 1: In order for any bisection method to achieve a global optimum, it is required that the feasible values of the search parameter constitute a connected set, otherwise the algorithm can lead to a local optimal. In our problem, feasible values of t are the same as the set (denoted by $\mathcal{R}$) of the achievable objective values of the optimization problem in (32) when the rank constraint is relaxed. The set $\mathcal{R}$ is certainly connected because the objective function in (32) is a continuous function which maps any connected set (in this case the convex feasible region of (35)) to another connected set.

Remark 2: To choose the initial value for l and u, one can select $l = 0$ and $u = \mathrm{SNR}_{\max}(P_T^{\max})$ where $P_T^{\max} = \sum_{i=1}^{r} P_i$ is chosen. In fact, one can easily show that the maximum achievable SNR with the total power constraint $P_T^{\max} = \sum_{i=1}^{r} P_i$ is larger than or equal to the maximum SNR achieved by solving (31).

Remark 3: To solve the convex feasibility problem (35), one can use the well-studied interior-point-based methods. For example, the SeDuMi [20] is an interior point-method-based package which produces a feasibility certificate if the problem is feasible.

Remark 4: Once the maximum feasible value for t is obtained, one can replace it into (34). This turns (34) into a convex problem which can be solved efficiently using interior-point-based methods.

Remark 5: It is worth mentioning that our problem formulation is applicable to both random and deterministic channel cases. In the case of random channels, our beamforming methods may not be optimal for individual channel realizations, rather our techniques are designed to be optimal in a statistical sense. Naturally our algorithms may perform poorly when they are applied for a specific channel realization. In order to design beamforming techniques for a specific channel realization, one needs to know the channel coefficients precisely. In this case, our formulation is still applicable, however, if the channel coefficients deviate slightly from their nominal values by unknown random fluctuations, the performance of our beamforming algorithms can become degraded drastically. In such a channel modeling, the maximum SNR can be very low due to lack of coherent combining of relay signals at the receiver. This is a well-known phenomenon and has been studied in the recent literature where robust beamforming has been of primary concern (see, for example, [23] and references therein). To compensate the lack of coherence in relay signals, one of the two following approaches can be taken: one can model the channel deviations from their nominal values into the correlation matrices and design a beamforming technique which is statistically optimal. Obviously, the receiver SNR will be smaller than that for the known channel case. Alternatively, one can use a robust technique which guarantees the worst-case performance for all channel coefficients that belong to an uncertainty set. Naturally the "size" of the uncertainty set determines the SNR loss compared to the known channel (coherent combining) case. Recently, worst-case optimization-based beamforming has been the focus of several studies, see [23] and [25]. The results developed in [23] can be straightforwardly applied to the beamforming techniques developed in this paper to provide robustness against unknown mismatch between the presumed and the actual channel coefficients, thereby protecting the performance against the lack of perfect coherent combining.

Remark 6: In semidefinite relaxation, the solution may not be rank-1 in general simply because the feasible set of the optimization problem (34) is a subset of that of the optimization problem (33). Interestingly, in our extensive simulation results, we never encountered a case where the solution to the SDP problem had a rank higher than one. For the cases where the SDP problem has a solution with rank higher than one, several *randomization techniques* have been proposed in the literature which use the solution to the SDP problem to provide a good approximation to the rank-1 problem [21]. The basic idea in randomization is to use $\mathbf{X}^*$ to generate a set of candidate weight vectors $\{\mathbf{w}_k\}$ and then select the best solution among these candidates. One such randomization technique, eigendecomposes $\mathbf{X}^*$ as $\mathbf{X}^* = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H$ and chooses $\mathbf{w}_k = \mathbf{U}\boldsymbol{\Lambda}\mathbf{v}_k$, where $\mathbf{v}_k$ is a vector of zero-mean, unit-variance complex circularly symmetric uncorrelated Gaussian random variables. That is $\mathbf{w}_k$'s are samples from the complex Gaussian distribution $\mathcal{N}(0, \mathbf{X}^*)$.

For cases where the SDP problem has a solution with rank higher than one, it is possible to establish a bound for performance of the randomization technique. To show this, consider the problem

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\sigma_n^2 + \mathbf{w}^H\mathbf{Q}\mathbf{w}} \qquad \text{subject to} \quad \mathbf{w}^H\mathbf{G}_i\mathbf{w} \le 1, \quad i = 1, 2, \ldots, r \tag{36}$$

where $\mathbf{G}_i$ is a matrix with all zero entries except for the ith diagonal element which is equal to $\mathbf{D}_{ii}/P_i$. The SDP relaxation can be written as

$$\max_{\mathbf{w}} \quad \frac{\mathrm{tr}(\mathbf{R}\mathbf{X})}{\sigma_n^2 + \mathrm{tr}(\mathbf{Q}\mathbf{X})} \qquad \text{subject to} \quad \mathrm{tr}(\mathbf{G}_i\mathbf{X}) \le 1,\ \mathbf{X} \succeq 0,\ i = 1, 2, \ldots, r. \tag{37}$$

Using bisection, we can solve the SDP relaxation in polynomial time yielding an optimal $\mathbf{X}^* \succeq 0$ and a μ^* satisfying

$$\mathrm{tr}(\mathbf{R}\mathbf{X}^*) = \mu^*\left(\mathrm{tr}(\mathbf{Q}\mathbf{X}^*) + \sigma_n^2\right). \tag{38}$$

Clearly, μ^* is an upper bound for the optimal value of (36).

Now consider the nonconvex quadratic optimization problem

$$\max_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}\mathbf{w} - \mu^*\left(\mathbf{w}^H\mathbf{Q}\mathbf{w} + \sigma_n^2\right) \qquad \text{subject to} \quad \mathbf{w}^H\mathbf{G}_i\mathbf{w} \le 1, \quad i = 1, 2, \ldots, r. \tag{39}$$

Its SDP relaxation can be written as

$$\max_{\mathbf{w}} \quad \mathrm{tr}(\mathbf{R}\mathbf{X}) - \mu^*\left(\mathrm{tr}(\mathbf{Q}\mathbf{X}) + \sigma_n^2\right) \qquad \text{subject to} \quad \mathrm{tr}(\mathbf{G}_i\mathbf{X}) \le 1, \mathbf{X} \succeq 0,\ i = 1, 2, \ldots, r. \tag{40}$$

By the definition of μ^*, it follows that $\mathbf{X}^* \succeq 0$ is a global optimal solution for (40). Let us sample from the complex Gaussian distribution $\mathcal{N}(0, \mathbf{X}^*)$. By the result of [22], we can generate in randomized polynomial time an approximate solution $\hat{\mathbf{w}}$ satisfying

$$\hat{\mathbf{w}}^H\mathbf{R}\hat{\mathbf{w}} - \mu^*\hat{\mathbf{w}}^H\mathbf{Q}\hat{\mathbf{w}} \ge c\left(\mathrm{tr}(\mathbf{R}\mathbf{X}^*) - \mu^*\mathrm{tr}(\mathbf{Q}\mathbf{X}^*)\right)$$

where $c = O((\log r)^{-1})$ is a constant. In light of (38), we further obtain

$$\hat{\mathbf{w}}^H \mathbf{R} \hat{\mathbf{w}} - \mu^* \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}} \geq c\mu^* \sigma_n^2$$

implying

$$\hat{\mathbf{w}}^H \mathbf{R} \hat{\mathbf{w}} - c\mu^* \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}} \geq c\mu^* \sigma_n^2 + (1-c)\mu^* \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}} \geq c\mu^* \sigma_n^2$$

where the last step follows from the positive semidefiniteness of $\mathbf{Q}$. Rearranging the terms, we obtain

$$\frac{\hat{\mathbf{w}}^H \mathbf{R} \hat{\mathbf{w}}}{\sigma_n^2 + \hat{\mathbf{w}}^H \mathbf{Q} \hat{\mathbf{w}}} \geq c\mu^*$$

implying that $\hat{\mathbf{w}}$ is a c-optimal solution of (36). In other words, the SDP relaxation approach provides a $c = O((\log r)^{-1})$ approximation to the nonconvex fractional quadratic optimization problem (36).

As was shown above, designing the beamformer based on the SNR maximization with individual relay power constraints requires an iterative procedure where, at each step, a convex feasibility problem is solved. We now turn (31) into an optimization problem which can be solved easily without significant computational complexity. To do so, we ignore σ_n^2 in the numerator of the objective function in (31) and aim to solve the following optimization problem:

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\mathbf{w}^H \mathbf{Q} \mathbf{w}} \quad \text{subject to} \quad |w_i|^2 \leq P_i / \mathbf{D}_{ii}. \tag{41}$$

In fact, the objective function in (41) is an upper bound to the objective function in (31).

The objective function in (41) is *globally* maximized for

$$\mathbf{w} = \eta \mathbf{v} \tag{42}$$

where $\mathbf{v}$ is the normalized principal eigenvector of the matrix $\mathbf{Q}^{-1}\mathbf{R}$ and η can be any scalar parameter. If we choose

$$\eta = \frac{1}{\left(\sqrt{\mathbf{D}_{kk}} |v_k|\right) / \sqrt{P_k}} \tag{43}$$

where v_k denotes the kth entry of $\mathbf{v}$ and

$$k = \arg \max_{1 \leq i \leq r} \frac{\mathbf{D}_{ii} |\mathbf{v}_i|^2}{P_i} \tag{44}$$

then, the global maximizer in (42) becomes the solution to (41) as well.

V. Simulation Results

In our numerical examples, we consider a network with 20 relay nodes ($r = 20$). The channel coefficients f_i and g_j are assumed to be independent from each other for any i and j. We also assume that the channel coefficient f_i can be written as

$$f_i = \bar{f}_i + \tilde{f}_i \tag{45}$$

where $\bar{f}_i$ is the mean of f_i and $\tilde{f}_i$ is a zero-mean random variable. We assume that $\tilde{f}_i$ and $\tilde{f}_j$ are independent for $i \neq j$. For any f_i, we choose $\bar{f}_i = e^{j\theta_i} / \sqrt{1+\alpha_f}$ and $\text{var}(\tilde{f}) = \alpha_f / (1+\alpha_f)$, where θ_i is a uniform random variable randomly chosen from the interval [0, 2π] and α_f is a parameter which determines the level of uncertainty in the channel coefficient f_i. Note that as $E\{|f_i|^2\} = 1$, if α_f is increased, the variance of the random component $\tilde{f}_i$ is increased while the mean $\bar{f}_i$ is decreased. This, in turn, means that the level of the uncertainty in the channel coefficient f_i is increased.

Similarly, we model the channel coefficient g_i as

$$g_i = \bar{g}_i + \tilde{g}_i \tag{46}$$

where $\bar{g}_i$ is the mean of g_i and $\tilde{g}_i$ is a zero-mean random variable. We assume that $\tilde{g}_i$ and $\tilde{g}_j$ are independent for $i \neq j$. For any g_i, we choose $\bar{g}_i = e^{j\phi_i} / \sqrt{1+\alpha_g}$ and $\text{var}(\tilde{g}) = \alpha_g / 1 + \alpha_g$, where ϕ_i is a uniform random variable chosen from the interval [0, 2π] and α_g is a parameter which determines the level of uncertainty in the channel coefficient g_i.

Based on this channel modeling, we can write the (i, j) entry of the matrices $\mathbf{R}$ and $\mathbf{Q}$, respectively, as

$$[\mathbf{R}]_{i,j} = P_0 \left(\bar{f}_i \bar{f}_j^* + \frac{\alpha_f}{1+\alpha_f} \delta_{ij} \right) \left(\bar{g}_i \bar{g}_j^* + \frac{\alpha_g}{1+\alpha_g} \delta_{ij} \right)$$

$$[\mathbf{Q}]_{i,j} = \sigma_\nu^2 \left(\bar{g}_i \bar{g}_j^* + \frac{\alpha_g}{1+\alpha_g} \delta_{ij} \right)$$

where δ_{ij} is the Kronecker function. It is worth mentioning that the distributions of $\tilde{f}_i$ and $\tilde{g}_i$ do not play a role, for our algorithms use only the second-order statistics of $\tilde{f}_i$ and $\tilde{g}_i$ and not their distributions. Also, we consider asymptotic regimes where the correlation matrices $\mathbf{R}$ and $\mathbf{Q}$ are exactly known, and therefore, we do not need to generate the channel coefficients. Alternatively, one can obtain the sample estimates of these correlation matrices from a finite number of samples of the channel coefficients which are generated randomly. In this case, the mismatch between the true and the sample correlation matrices may degrade the performance of our beamforming techniques. To cope with such performance degradation, one has to resort to robust techniques proposed in [23], where positive and negative diagonal loading techniques are used to compensate for the lack of the precise knowledge of the correlation matrices.

Throughout our numerical examples, the transmit power P_0 is assumed to be the same as receiver noise power which is 0 dBW.

A. Power Minimization

Fig. 2 shows the minimum total relay transmit power, $P_T^{\min}(\gamma)$ versus the SNR threshold γ for $\alpha_f = -5$ dB and for different values of α_g. Fig. 3 illustrates $P_T^{\min}(\gamma)$ versus γ for $\alpha_g = -5$ dB and for different values of α_f. In these figures, the transmit powers have been plotted only for those values of γ that are feasible.

As can be seen from these figures, when the uncertainty in f_i and g_i coefficients (measured, respectively, by α_f and α_g) is increased, it becomes exceedingly difficult to guarantee that the SNR is above a certain threshold γ. That is, as α_f (or α_g) is increased, it takes more power to ensure that the SNR is above a certain (feasible) γ. Also, as α_f (or α_g) is increased, the maximum feasible value of γ is decreased.

As can be seen from Figs. 2 and 3, the transmit power increases drastically near some limiting γ. This limiting value of γ is the one which makes the optimization problem infeasible.

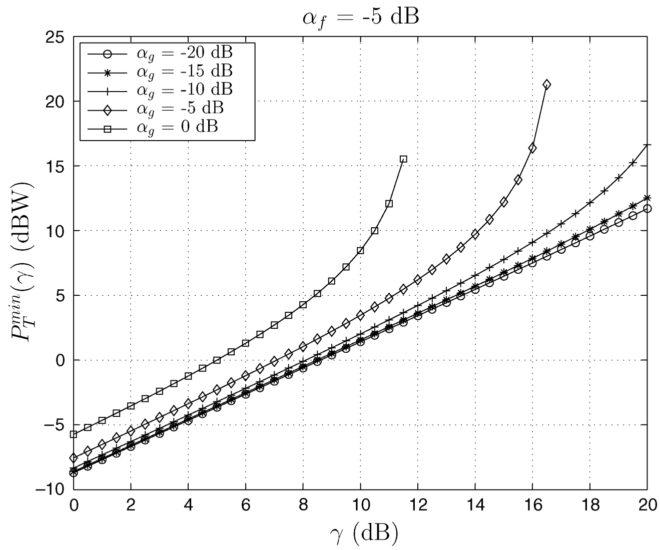

Fig. 2. Minimum total relay transmit power versus SNR threshold γ, for different values of α_g and for $\alpha_f = -5$ dB.

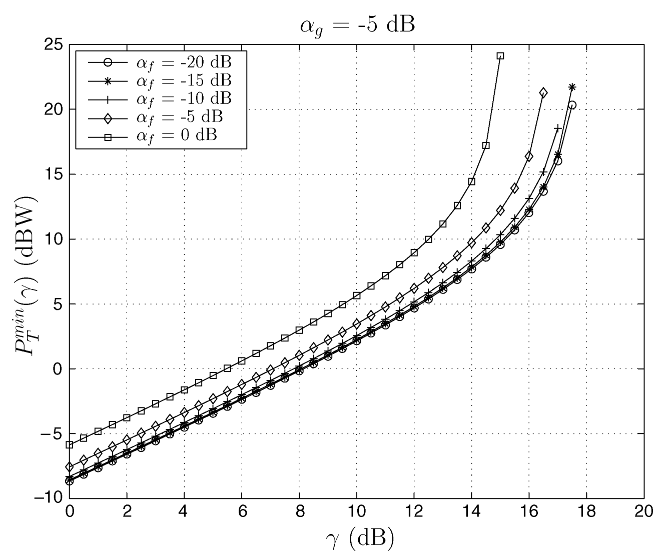

Fig. 3. Minimum total relay transmit power versus SNR threshold γ, for different values of α_f and for $\alpha_g = -5$ dB.

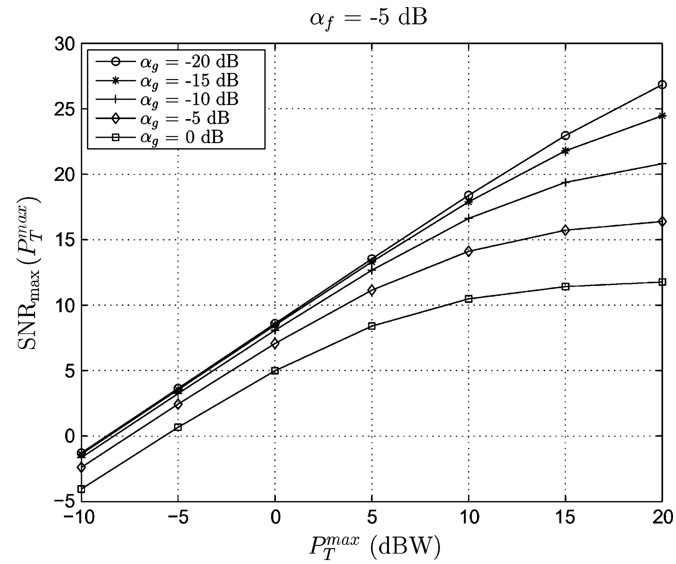

Fig. 4. Maximum achievable SNR versus the maximum allowable total transmit power $P_T^{\max}$ for different values of α_g and for $\alpha_f = 0$ dB.

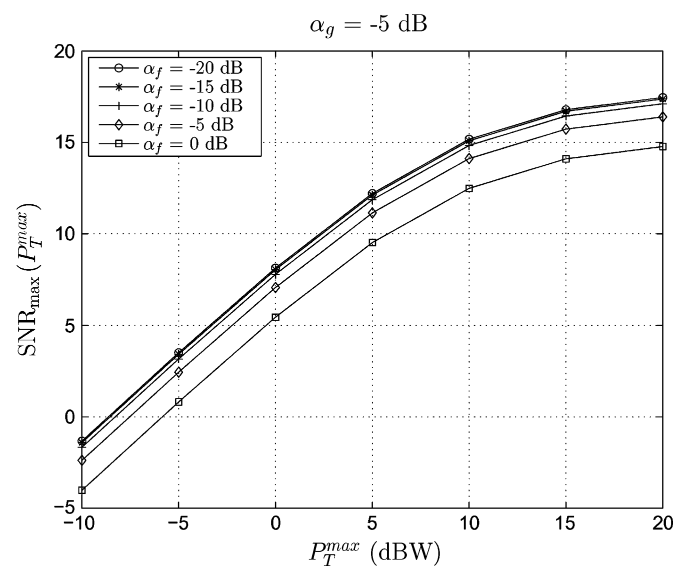

Fig. 5. Maximum achievable SNR versus the maximum allowable total transmit power $P_T^{\max}$ for different values of α_f and for $\alpha_g = 0$ dB.

B. SNR Maximization

In Fig. 4, we have plotted the maximum achievable SNRs, given as in (29), versus the maximum allowable total transmit power $P_T^{\max}$ for $\alpha_f = -5$ dB and for different values of α_g. In Fig. 5, we have shown the maximum achievable SNRs versus $P_T^{\max}$ for $\alpha_g = -5$ dB and for different values of α_f. As can be seen from these figures, for any given $P_T^{\max}$, the maximum achievable SNR is decreased as the uncertainty in the f_i (or in the g_i) coefficients is increased.

In the next numerical example, we consider the case where the individual relay nodes are limited in their transmit powers. We assume that the relay nodes are divided into two groups. The relay nodes in each group have the same maximum allowable transmit power, while the maximum allowable transmit power for one group is twice that for the other group, that is, $P_1 = P_2 = \cdots = P_{10} = 2P_{11} = 2P_{12} = \cdots = 2P_{20}$. We use the SDP-based technique proposed in Section IV-B to obtain the optimum value for matrix $\mathbf{X}$, say $\mathbf{X}^*$. We have investigated the solution to SDP problem for different values of θ_i and ϕ_i, for different maximum allowable transmit powers, and for different values of α_f and α_g. In our intensive simulation examples, we have observed that the matrix $\mathbf{X}^*$ is always rank one, and therefore, no randomization technique is required. As a result, the optimum value for the vector $\mathbf{w}$ is the same as the principal eigenvector of $\mathbf{X}^*$. Fig. 6 shows the maximum achievable SNRs, when the individual relay nodes have the aforementioned power constraints, versus the total relay transmit power $P_T = \sum_{i=1}^{r} P_i$, for $\alpha_f = -5$ dB and for different values of α_g. Fig. 7 illustrates the maximum achievable SNRs versus P_T for $\alpha_g = -5$ dB and for different values of α_f. For this example, we have also plotted the performance of the simplified technique in Figs. 8 and 9. As can be seen from Figs. 6–9, for any given P_T, the maximum achievable SNR of both the SDP-based

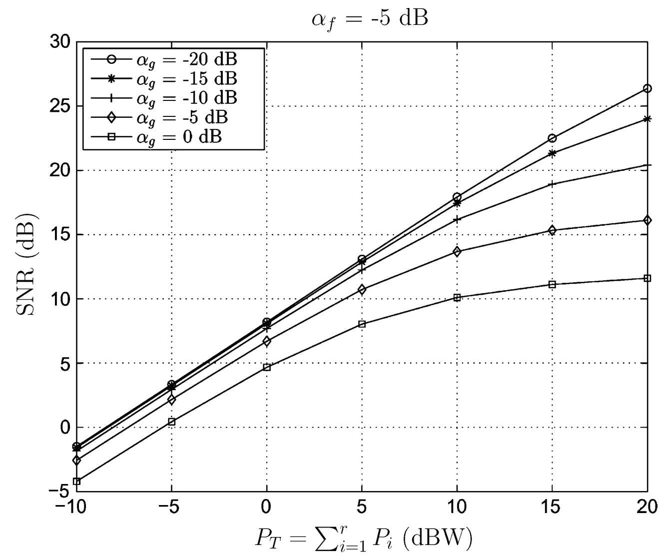

Fig. 6. Maximum achievable SNR, with individual relay power limits $\{P_i\}_{i=1}^{r}$, versus the transmit power $P_T = \sum_{i=1}^{r} P_i$ for different values of α_g and for $\alpha_f = 0$ dB.

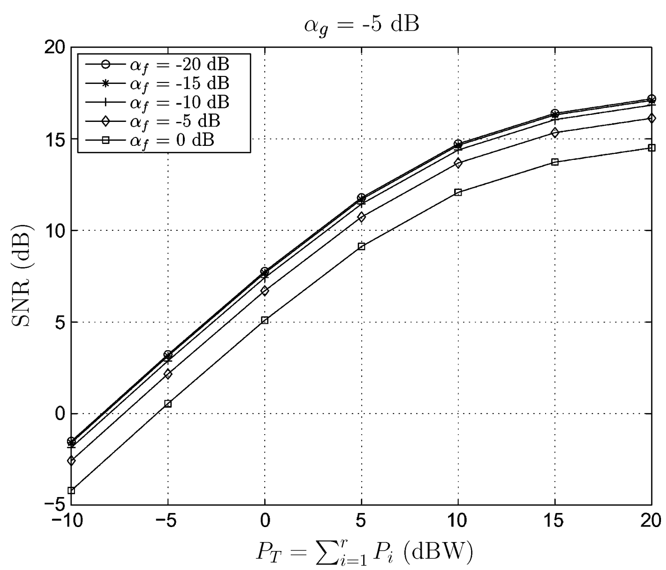

Fig. 7. Maximum achievable SNR, with individual relay power limits $\{P_i\}_{i=1}^{r}$, versus the transmit power $P_T = \sum_{i=1}^{r} P_i$ for different values of α_f and for $\alpha_g = 0$ dB.

technique and the simplified method is decreased when the uncertainty in f_i (or in g_i) coefficients is increased. In Fig. 10, we compare the performance of the techniques developed in this paper for SNR maximization for $\alpha_f = \alpha_g = -5$ dB. As can be seen from this figure, in this example, the maximum achievable SNR under constrained total transmit power and that under constrained individual replay powers are very close to each other. It can also be seen that when the individual relay powers are constrained, the simplified method suffers a 2-dB loss in SNR as compared to the SDP-based technique for low to moderate values of P_T. For large values of P_T, the simplified method has a maximum SNR close to that of the SDP-based approach.

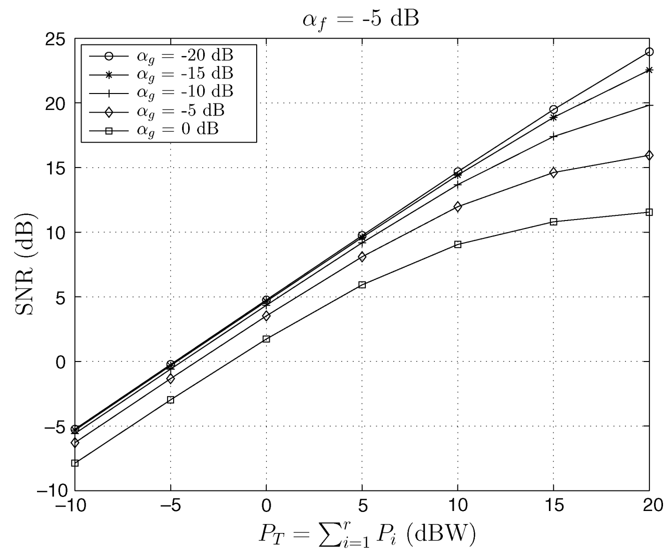

Fig. 8. Maximum achievable SNR of the simplified technique, with individual relay power limits $\{P_i\}_{i=1}^{r}$, versus the transmit power $P_T = \sum_{i=1}^{r} P_i$ for different values of α_g and for $\alpha_f = 0$ dB.

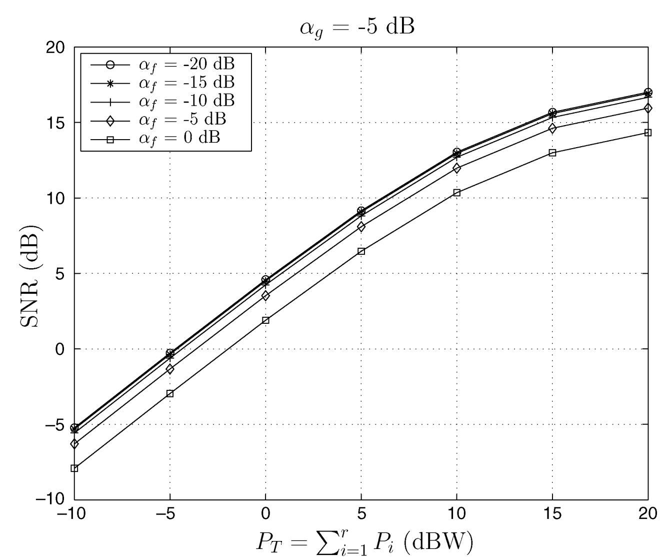

Fig. 9. Maximum achievable SNR of the simplified technique, with individual relay power limits $\{P_i\}_{i=1}^{r}$, versus the transmit power $P_T = \sum_{i=1}^{r} P_i$ for different values of α_f and for $\alpha_g = 0$ dB.

VI. Conclusions

In this paper, we studied the problem of distributed beamforming in a network which consists of a transmitter, a receiver and r relay nodes. Assuming that the second-order statistics of the channel coefficients are available, we considered two different approaches to beamforming design. As the first approach, we designed the beamformer through minimization of the total transmit power subject to a constraint which guarantees the receiver quality of service. We showed that this approach yields a closed-form solution. In the second approach, we obtained the beamforming weights through maximizing the receiver SNR subject to two different types of power constraints, namely total transmit power constraint and individual relay power constraints. We herein have shown that the total power constraint leads to a closed-form solution while the individual

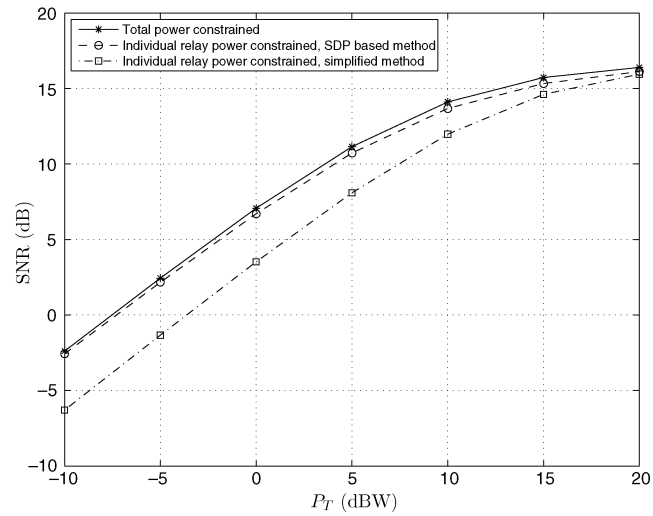

Fig. 10. Maximum achievable SNR versus the total transmit power P_T for different methods.

relay power constraints result in a quadratic programming optimization problem. The later optimization problem does not have a closed-form solution. However, it is shown that using semidefinite relaxation, it can be turned into a convex feasibility semidefinite programming, and therefore, can be efficiently solved using interior point methods. Furthermore, we presented a simplified (but suboptimal) technique which can be used to avoid the computational complexity of semidefinite programming. Our simplified algorithm provides the beamforming weight vector in a closed form. Simulation results show that when compared to the semidefinite programming-based method, our simplified technique suffers a 2-dB loss in SNR for low to moderate values of transmit power.

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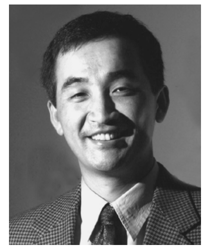

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