EE 570: Location and Navigation

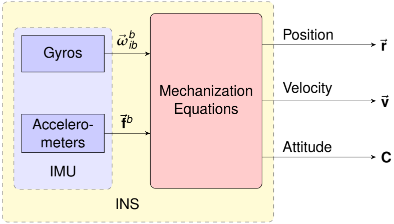
INS/GPS Integration

Aly El-Osery

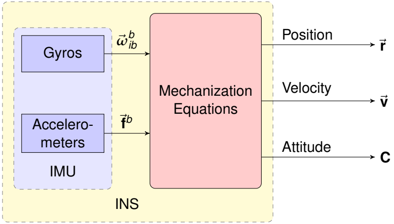
Electrical Engineering Department, New Mexico Tech
Socorro, New Mexico, USA

April 29, 2011

Need for Integration

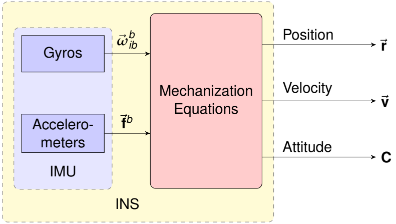


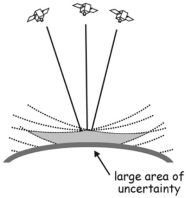
Need for Integration



Advantages	Disadvantages
Immune to RF Jaming	Drifts
High data rate	Errors are time dependent
High accuracy in short term	Need Initialization

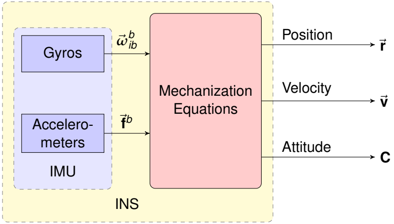
Need for Integration



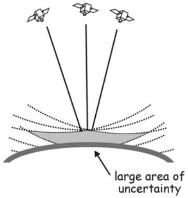


Advantages	Disadvantages
Immune to RF Jaming	Drifts
High data rate	Errors are time dependent
High accuracy in short term	Need Initialization

Need for Integration

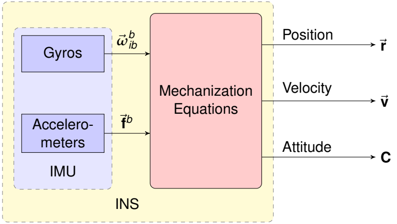


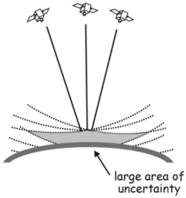
Advantages	Disadvantages
Immune to RF Jaming	Drifts
High data rate	Errors are time dependent
High accuracy in short term	Need Initialization



Advantages	Disadvantages
Erros time-indep.	Sensitive to RF Interference
No initialization	No attitude information

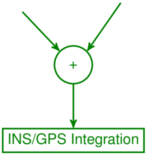
Need for Integration



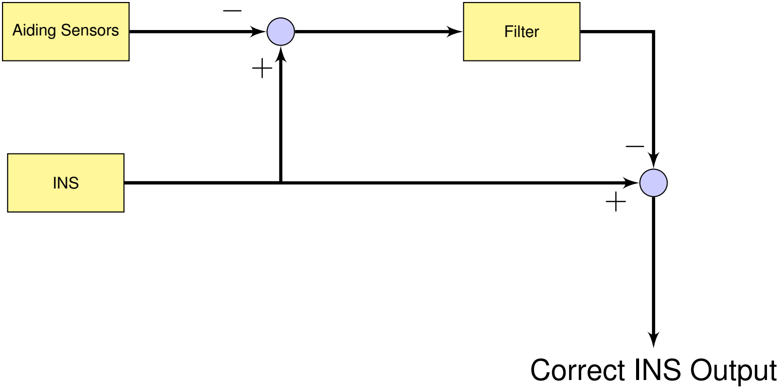


Advantages	Disadvantages
Immune to RF Jaming	Drifts
High data rate	Errors are time dependent
High accuracy in short term	Need Initialization

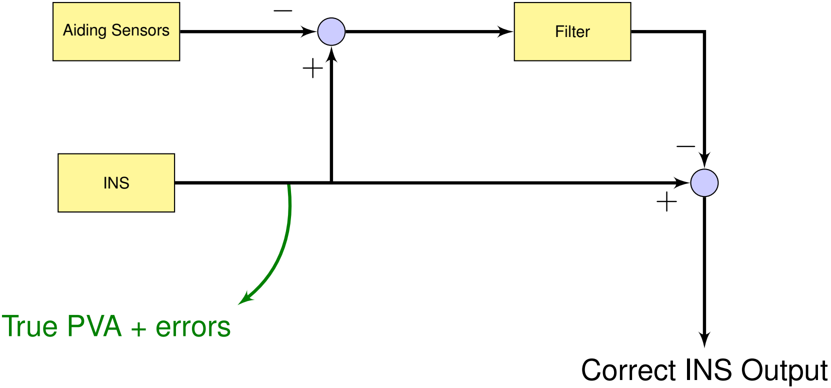
Advantages	Disadvantages
Erros time-indep.	Sensitive to RF Interference
No initialization	No attitude information



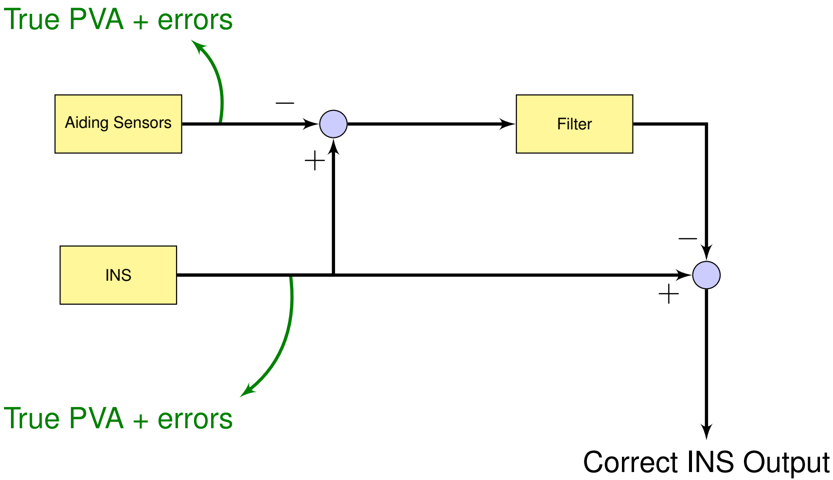
Open-Loop Integration



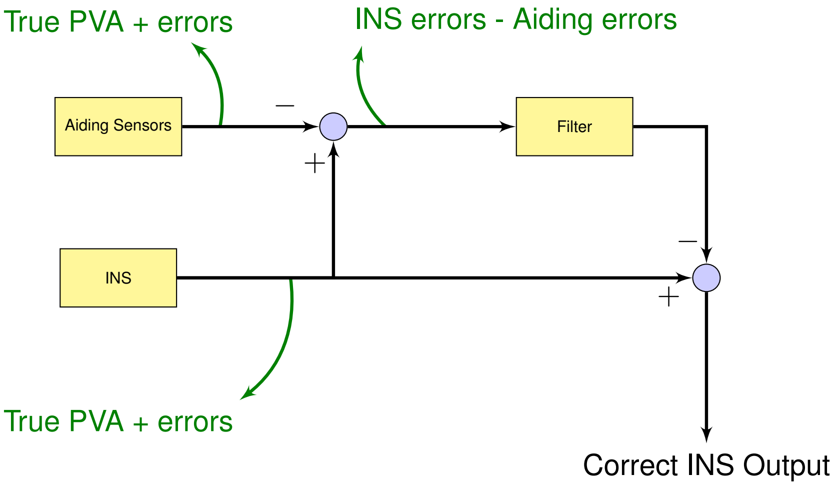
Open-Loop Integration



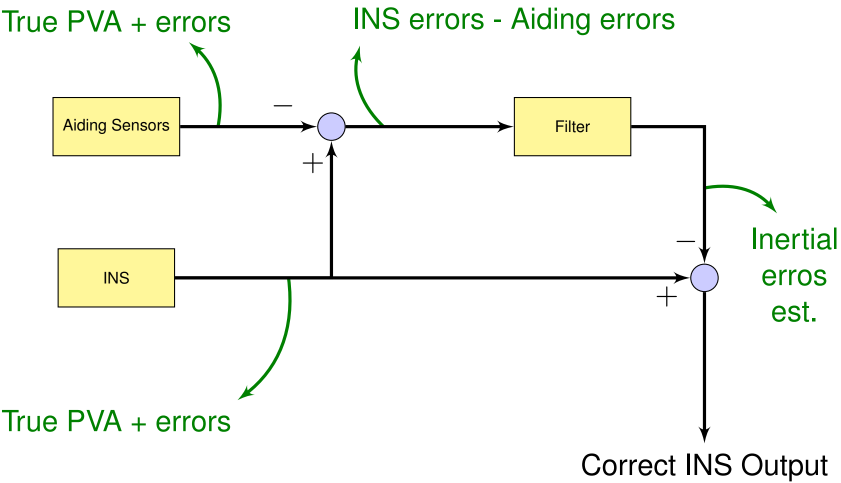
Open-Loop Integration



Open-Loop Integration

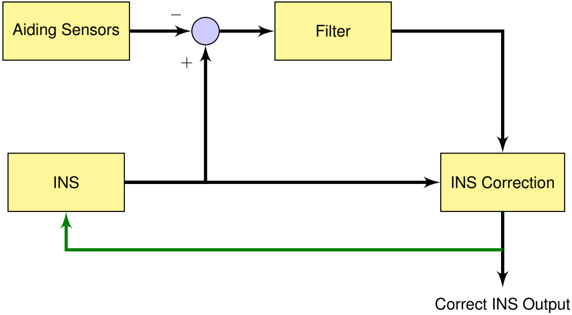


Open-Loop Integration

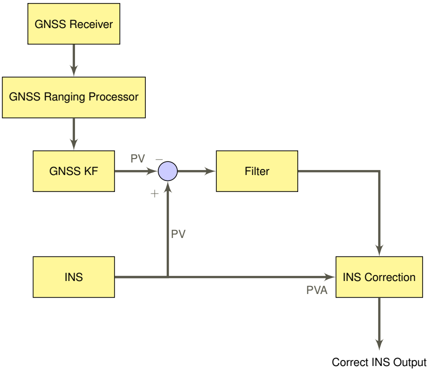


Closed-Loop Integration

If error estimates are fedback to correct the INS mechanization, a reset of the state estimates becomes necessary.

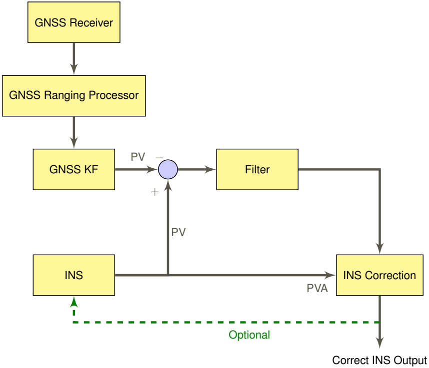


Loosely Coupled Integration



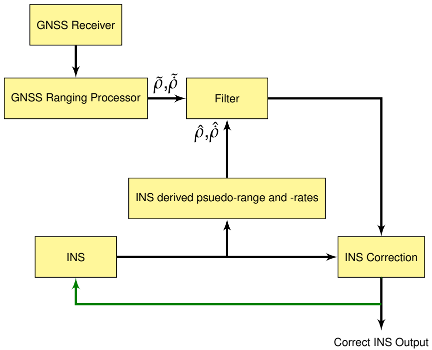
- Simple
- Cascade KF therefore integration KF BW must be less than that of GNSS KF (e.g. update interval of 10s)
- Minimum of 4 satellites required

Loosely Coupled Integration



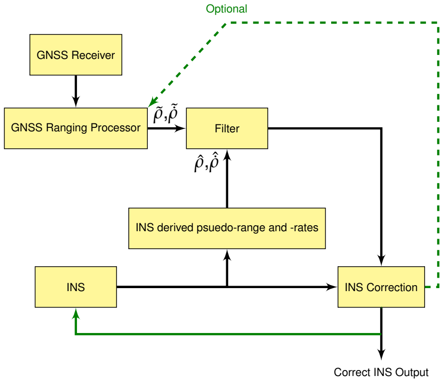
- Simple
- Cascade KF therefore integration KF BW must be less than that of GNSS KF (e.g. update interval of 10s)
- Minimum of 4 satellites required

Tightly Coupled Integration



- No cascade KF
- KF BW must be kept less than the GNSS tracking loop
- Does not require 4 satellites

Tightly Coupled Integration



- No cascade KF
- KF BW must be kept less than the GNSS tracking loop
- Does not require 4 satellites

INS Derived Psuedo-Range and -Rates

$$\hat{\rho}_{Cj,k} = \sqrt{[\hat{\vec{\mathbf{r}}}^e_{esj}(\tilde{t}_{st,j,k}) - \hat{\vec{\mathbf{r}}}^e_{ea,kj}]^T[\hat{\vec{\mathbf{r}}}^e_{esj}(\tilde{t}_{st,j,k}) - \hat{\vec{\mathbf{r}}}^e_{ea,kj}]} + \delta\hat{\rho}_{rc,k} + \delta\rho_{ie,j} \tag{1}$$

$$\hat{\dot{\rho}}_{Cj,k} = \hat{\vec{\mathbf{u}}}^{e\,T}_{as,j,k}[\hat{\vec{\mathbf{v}}}^e_{esj}(\tilde{t}_{st,j,k}) - \hat{\vec{\mathbf{v}}}^e_{ea,kj}]^T + \delta\hat{\dot{\rho}}_{rc,k} + \delta\dot{\rho}_{ie,j} \tag{2}$$

where

$$\vec{\mathbf{u}}^e_{as,j} = \frac{\vec{\mathbf{r}}^e_{es,j}(t_{st,j}) - \vec{\mathbf{r}}^a_{as,j}(t_{sa})}{\|\vec{\mathbf{r}}^e_{es,j}(t_{st,j}) - \vec{\mathbf{r}}^a_{as,j}(t_{sa})\|} \tag{3}$$

Observability

- Attitude and acceleration errors are observable through growth in velocity and position errors.
- In level accelertion, heading error only produces velocity error, therefore requires significant maneuvering.
- If level and not accelerating, vertical accel bias is the only cause of vertical velocity error growth.

ECEF Error Mechanization (loosely coupled)

Assuming errors are due to biases that are modeled as WGN.

$$\begin{pmatrix} \delta\dot{\vec{\psi}}^e_{eb} \\ \delta\dot{\vec{\mathbf{v}}}^e_{eb} \\ \delta\dot{\vec{\mathbf{r}}}^e_{eb} \\ \dot{\vec{\mathbf{b}}}_a \\ \dot{\vec{\mathbf{b}}}_g \end{pmatrix} = \mathbf{F}(t) \begin{pmatrix} \delta\vec{\psi}^e_{eb} \\ \delta\vec{\mathbf{v}}^e_{eb} \\ \delta\vec{\mathbf{r}}^e_{eb} \\ \vec{\mathbf{b}}_a \\ \vec{\mathbf{b}}_g \end{pmatrix} = \mathbf{F}(t)\vec{\mathbf{x}}(t) \tag{4}$$

where

$$\mathbf{F}(t) = \begin{pmatrix} -[\vec{\omega}^e_{ie}\times] & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \mathbf{C}^e_b \\ -[\mathbf{C}^e_b\vec{\mathbf{f}}^b_{ib}\times] & -2\Omega^i_{ie} & \frac{2g_0}{\|\vec{\mathbf{r}}^e_{eb}\| r^e_{eS}}[\vec{\mathbf{r}}^e_{eb}(\vec{\mathbf{r}}^e_{eb})^T]\delta\vec{\mathbf{r}}^e_{eb} & \mathbf{C}^e_b & 0_{3\times3} \\ 0_{3\times3} & \mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{pmatrix}$$

Kalman Filter

$$\hat{\vec{\mathbf{x}}}_{k|k-1} = \mathbf{\Phi}_{k-1}\hat{\vec{\mathbf{x}}}_{k-1|k-1} \quad (5)$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{\Phi}_{k-1}^T \quad (6)$$

$$\hat{\vec{\mathbf{x}}}_{k|k} = \hat{\vec{\mathbf{x}}}_{k|k-1} + \mathbf{K}_k(\vec{\mathbf{z}}_k - \mathbf{H}_k\hat{\vec{\mathbf{x}}}_{k|k-1}) \quad (7)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{P}_{k|k-1}\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T \quad (8)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (9)$$

Kalman Filter

Project Ahead

$$\hat{\vec{\mathbf{x}}}_{k|k-1} = \mathbf{\Phi}_{k-1}\hat{\vec{\mathbf{x}}}_{k-1|k-1} \tag{5}$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{\Phi}_{k-1}^T \tag{6}$$

$$\hat{\vec{\mathbf{x}}}_{k|k} = \hat{\vec{\mathbf{x}}}_{k|k-1} + \mathbf{K}_k(\vec{\mathbf{z}}_k - \mathbf{H}_k\hat{\vec{\mathbf{x}}}_{k|k-1}) \tag{7}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{P}_{k|k-1}\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T \tag{8}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1} \tag{9}$$

Kalman Filter

$$\hat{\vec{\mathbf{x}}}_{k|k-1} = \mathbf{\Phi}_{k-1}\hat{\vec{\mathbf{x}}}_{k-1|k-1} \quad (5)$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{\Phi}_{k-1}^{T} \quad (6)$$

Update

$$\hat{\vec{\mathbf{x}}}_{k|k} = \hat{\vec{\mathbf{x}}}_{k|k-1} + \mathbf{K}_k\left(\vec{\mathbf{z}}_k - \mathbf{H}_k\hat{\vec{\mathbf{x}}}_{k|k-1}\right) \quad (7)$$

$$\mathbf{P}_{k|k} = \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k\right)\mathbf{P}_{k|k-1}\left(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k\right)^{T} + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^{T} \quad (8)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^{T}\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{T} + \mathbf{R}_k\right)^{-1} \quad (9)$$

Closed-Loop Kalman Filter

Since the errors are being fedback to correct the INS, the state estimate must be reset after each INS correction.

$$\hat{\vec{\mathbf{x}}}_{k|k-1} = 0 \tag{10}$$

$$\mathbf{P}_{k|k-1} = \mathbf{Q}_{k-1} + \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{\Phi}_{k-1}^T \tag{11}$$

$$\hat{\vec{\mathbf{x}}}_{k|k} = \mathbf{K}_k\vec{\mathbf{z}}_k \tag{12}$$

$$\mathbf{P}_{k|k} = \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k\right)\mathbf{P}_{k|k-1}\left(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k\right)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T \tag{13}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \tag{14}$$

Discretization

$$\Phi_{k-1} \approx \mathbf{I} + \mathbf{F}\tau_s \tag{15}$$

$$\mathbf{Q} = \begin{pmatrix} n_{rg}^2\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & n_{ag}^2\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & n_{bad}^2\mathbf{I}_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & n_{bgd}^2\mathbf{I}_{3\times3} \end{pmatrix} \tau_s \tag{16}$$

where τ_s is the sample time, n_{rg}^2, n_{ag}^2, n_{bad}^2, n_{bgd}^2 are the PSD of the gyro and accel random noise, and accel and gyro bias variation, respectively.

ECEF INS/GNSS Loosely Coupled

$$\vec{\mathbf{z}}_k^e = \begin{pmatrix} \tilde{\vec{\mathbf{r}}}_{GPS} - \hat{\vec{\mathbf{r}}}_{eb}^e \\ \tilde{\vec{\mathbf{v}}}_{GPS} - \hat{\vec{\mathbf{v}}}_{eb}^e \end{pmatrix} \tag{17}$$

$$\mathbf{H} = \begin{pmatrix} 0_{3\times3} & 0_{3\times3} & -\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -\mathbf{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{pmatrix} \tag{18}$$

Theoretically, the lever arm from the INS to the GNSS antenna needs to be included, but in practice, the coupling of the attitude errors and gyro biases into the measurement through the lever arm is week.

ECEF INS/GNSS Tightly Coupled

Pseudo-ranges are used instead of XYZ.

$$\vec{\mathbf{z}} = \begin{pmatrix} \vec{\mathbf{z}}_\rho \\ \vec{\mathbf{z}}_{\dot\rho} \end{pmatrix} \tag{19}$$

where

$$\vec{\mathbf{z}}_\rho = (\tilde\rho_{C1} - \hat\rho_{C1}, \tilde\rho_{C2} - \hat\rho_{C2}, \ldots, \tilde\rho_{Cn} - \hat\rho_{Cn}) \tag{20}$$

$$\vec{\mathbf{z}}_{\dot\rho} = (\tilde{\dot\rho}_{C1} - \hat{\dot\rho}_{C1}, \tilde{\dot\rho}_{C2} - \hat{\dot\rho}_{C2}, \ldots, \tilde{\dot\rho}_{Cn} - \hat{\dot\rho}_{Cn}) \tag{21}$$

$$\vec{\mathbf{x}}(t) = \begin{pmatrix} \delta\vec{\boldsymbol{\psi}}^e_{eb} & \delta\vec{\mathbf{v}}^e_{eb} & \delta\vec{\mathbf{r}}^e_{eb} & \vec{\mathbf{b}}_a & \vec{\mathbf{b}}_g & \delta\rho_{rc} & \delta\dot\rho_{rc} \end{pmatrix}^T \tag{22}$$

$\tilde\rho_{Cj}$, and $\tilde{\dot\rho}_{Cj}$ and $\hat\rho_{Cj}$, and $\hat{\dot\rho}_{Cj}$ are the psuedo-ranges and rates obtained from the GNSS and INS, respectively, for the *j*th satallite. These equations are none linear and an EKF needs to be used. $\delta\rho_{rc}$ and $\delta\dot\rho_{rc}$ are the clock bias and drift.

Tightly Coupled Linearized Measurement Matrix

$$\mathbf{H}^e = \left(\begin{array}{ccccccc} 0_{1\times3} & 0_{1\times3} & \vec{\mathbf{u}}^{e\,T}_{as,1} & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\ 0_{1\times3} & 0_{1\times3} & \vec{\mathbf{u}}^{e\,T}_{as,2} & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1\times3} & 0_{1\times3} & \vec{\mathbf{u}}^{e\,T}_{as,n} & 0_{1\times3} & 0_{1\times3} & 1 & 0_{1\times3} \\ \hline 0_{1\times3} & \vec{\mathbf{u}}^{e\,T}_{as,1} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 \\ 0_{1\times3} & \vec{\mathbf{u}}^{e\,T}_{as,2} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1\times3} & \vec{\mathbf{u}}^{e\,T}_{as,n} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 0_{1\times3} & 1 \end{array}\right) \quad (23)$$