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GUIDANCE & COMPUTER SCHEME FOR MANNED LUNAR LANDING

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Abstract

A

(1) Statement of the Problem.

A continuous thrust is employed for braking a vehicle from orbit to landing. The nonlinear guidance and computer scheme can be carried by the space vehicle to eliminate the inherent delay of an earth-bound radio command system from the moon. Altitude and angular momentum for the computer scheme can be measured on board by radar and gyroscopic equipment operated by an astronaut. A bull's-eye landing is guaranteed by using the proposed system without tracking stations on the moon.

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(2) New Method of Approach.

(a) Variational Equations.

The following equations describing the dynamical behavior of the rocket-powered vehicle are given by the author in the paper "Nonlinear Guidance System for Descent Trajectories," AIAA Journal, Vol. 1, No. 8, August 1963, pp. 1958-1960.

$$\frac{d\theta}{dk} = \frac{u^3}{2a_\theta} \quad \text{and} \qquad (1)$$

$$\frac{d^2u}{dk^2} + \left[\frac{1}{2k} + \frac{\frac{d}{dk}(2a_\theta u^{-3})}{2a_\theta u^{-3}}\right]\frac{du}{dk} + \left(\frac{u^3}{2a_\theta}\right)^2\left[u - \frac{g_o}{ku_o^2} + \frac{a_r}{ku^2}\right] = 0 \qquad (2)$$

The variational equations are, (new results)

$$\frac{d\Delta\theta}{dk} = 3\frac{u^3}{2a_\theta}\frac{\Delta u}{u} - \frac{u^3}{2a_\theta}\frac{\Delta a_\theta}{a_\theta} \quad \text{and} \qquad (3)$$

$$\frac{d^2\Delta u}{dk^2} + A(k)\frac{d\Delta u}{dk} + B(k)\Delta u = C(k)\Delta a_\theta + D(k)\Delta a_r + E(k)\frac{d\Delta a_\theta}{dk} \qquad (4)$$

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where A(k), B(k), C(k), D(k) and E(k) are functions of k.

(b) Computer Scheme.

The particular computer scheme follows Figure 1 where u_m and k_m are the measured inverse radius and specific momentum of the vehicle, respectively. The computer inverse radius u_k is determined by a particular program in terms of k_m.

(i) The angular error $\Delta\theta$.

Assuming no guidance errors ($\delta a_\theta \to 0$, $\delta a_r \to 0$) and measurement errors ($\delta u \to 0$, $\delta k \to 0$), then

$$a_\theta = a_{\theta c} \quad \text{and} \quad u_m = u \tag{5}$$

The variational equation of a_θ and $\frac{d\theta}{dk}$ can be written as

$$\Delta a_\theta = \Delta a_{\theta c} = \frac{\partial a_{\theta c}}{\partial u_m} \Delta u_m = \frac{\partial a_{\theta c}}{\partial u_m} = \Delta u \tag{6}$$

$$\frac{d}{dk}(\Delta\theta) = \frac{u^2 \Delta u}{2a_\theta} \left(3 - \frac{u}{a_\theta} \frac{\partial a_{\theta c}}{\partial u_m}\right) \tag{7}$$

If nonlinear guidance is employed in determining the trajectories, the computer is supposed to calculate the reference specific force according to the program, i.e.,

$$a_{\theta c} = \frac{u_m^3}{2\beta}, \qquad \text{or} \qquad a_\theta = \frac{u^3}{2\beta} \tag{8}$$

By evaluating the partial derivative and substituting in Equation (7) we obtain, (new results)

$$\frac{d}{dk}(\Delta\theta) = 0 \tag{9}$$

If the initially perturbed quantity is $\Delta\theta_b$, then for the entire landing operation

$$\Delta\theta = \Delta\theta_b = \text{constant} \tag{10}$$

(ii) The radial error Δu.

With the previous assumptions of zero measurement and guidance errors, the variational equation in u is

$$\frac{d^2\Delta u}{dk^2} + \frac{1}{2k}\frac{d\Delta u}{dk} + \beta^2[1 - \frac{2a_r}{ku^3}]\Delta u = - \frac{\beta^2}{ku^2}\Delta a_r \tag{11}$$

The control variable a_r is generated from the feedback signals u_m, k_m, and u_k. Because $k_m(=k)$ is considered to be an independent variable, it follows that

$$\Delta k_m = \Delta k = 0 \tag{12}$$

provided $\delta k = 0$, i.e., no measurement error in k.

The variational equation for a_r is

$$\Delta a_r = \Delta a_{rc} = \frac{\partial a_{rc}}{\partial u_m}\Delta u_m = \frac{\partial a_{rc}}{\partial u_m}\Delta u \tag{13}$$

The radial specific force a_r is chosen to be

$$a_r = a_{rc} = \frac{g_o u_m^2}{u_o^2} - k_m u_m^3 + \lambda^2 k_b u_m^2(u_k - u_o) \tag{14}$$

Differentiating Equation (14) with respect to u_m, one obtains the value of Δa_r in Equation (13).

Substituting this value and Equation (14) into Equation (11) yields, (new results)

$$\frac{d^2\Delta u}{dk^2} + \frac{1}{2k}\frac{d\Delta u}{dk} = 0 \tag{15}$$

If the initial conditions are

$$\Delta u\Big|_{k = k_b} = \Delta u_b \quad \text{and} \quad \frac{d\Delta u}{dk}\Big|_{k = k_b} = 0 \tag{16}$$

then the solution for the entire landing operation is

$$\Delta u = \Delta u_b = \text{constant} \tag{17}$$

(iii) The velocity errors ΔV_θ and ΔV_r.

The perturbed value of V_θ can be found as

$$\Delta V_\theta = -k^{\frac{1}{2}} \Delta u \quad \text{and} \quad \Delta V_\theta \Big|_{k = 0} = 0 \tag{18}$$

The radial velocity may be determined as

$$V_r = k^{\frac{1}{2}} \frac{du/dk}{d\theta/dk} \tag{19}$$

Noting that $\frac{d\Delta\theta}{dk} = 0$, the error of radial velocity becomes

$$\Delta V_r = k^{\frac{1}{2}} \frac{d\Delta u}{dk} \frac{dk}{d\theta} \tag{20}$$

If the initial condition $\frac{d\Delta u}{dk}\Big|_{k = k_b} = 0$ as indicated in Equation (16), then

$$\Delta V_r = 0 \text{ for all } k. \tag{21}$$

(3) Important Conclusions.

(a) The variation equations are completely general in scope for any two-dimensional problem for any system with variable angular momentum.

(b) By choosing the computer scheme and the nonlinear guidance for specific forces a_θ and a_r, the angular and radial errors are small and the velocity errors are zero, if the system starts with specific perturbed initial conditions.

(c) The computer scheme can be carried out entirely on board the vehicle without requiring any tracking station on the moon.

All above are new materials not published or presented elsewhere.

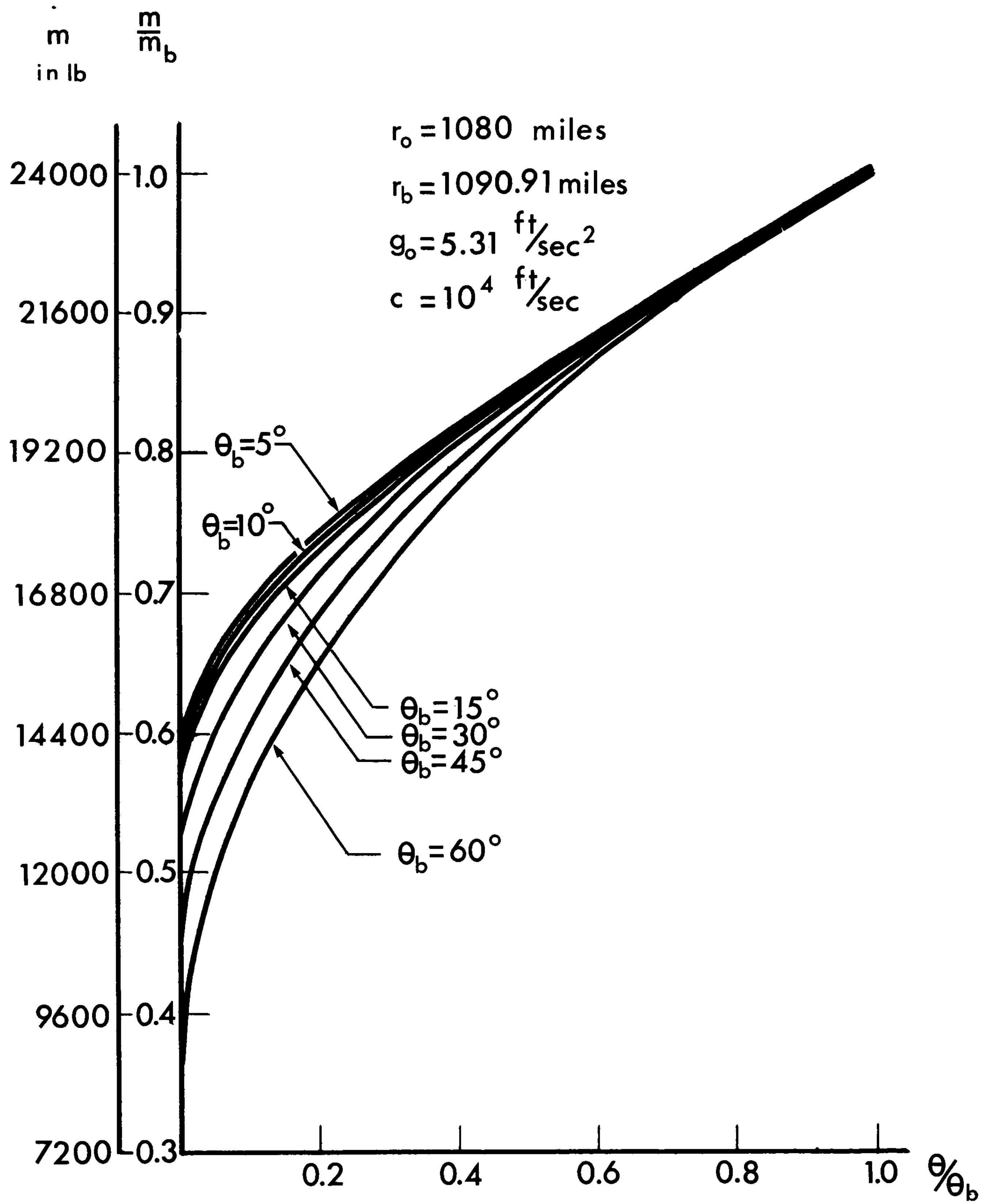


Fig 1 Mass Ratio vs θ/θ_b

(m_b = initial mass)

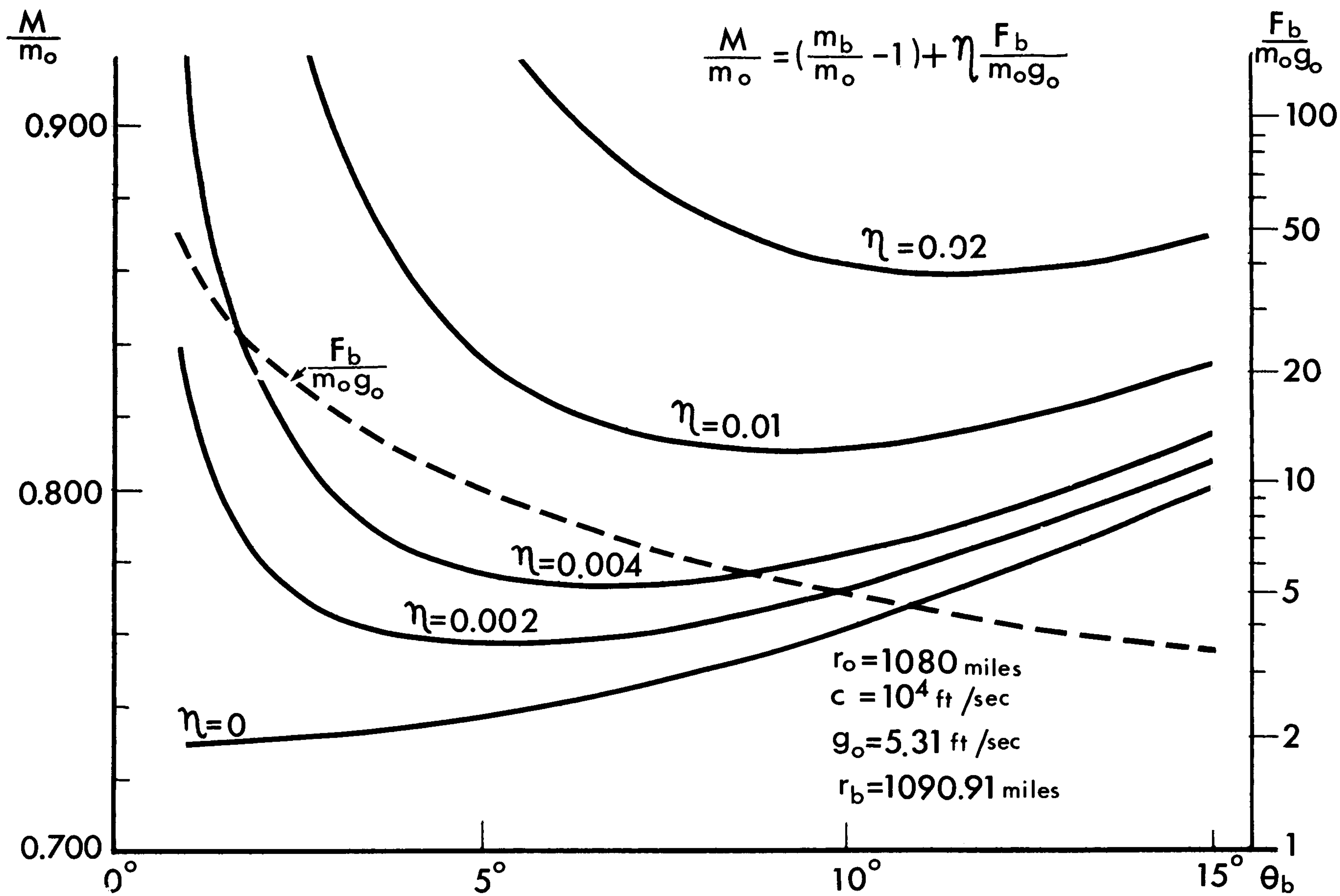
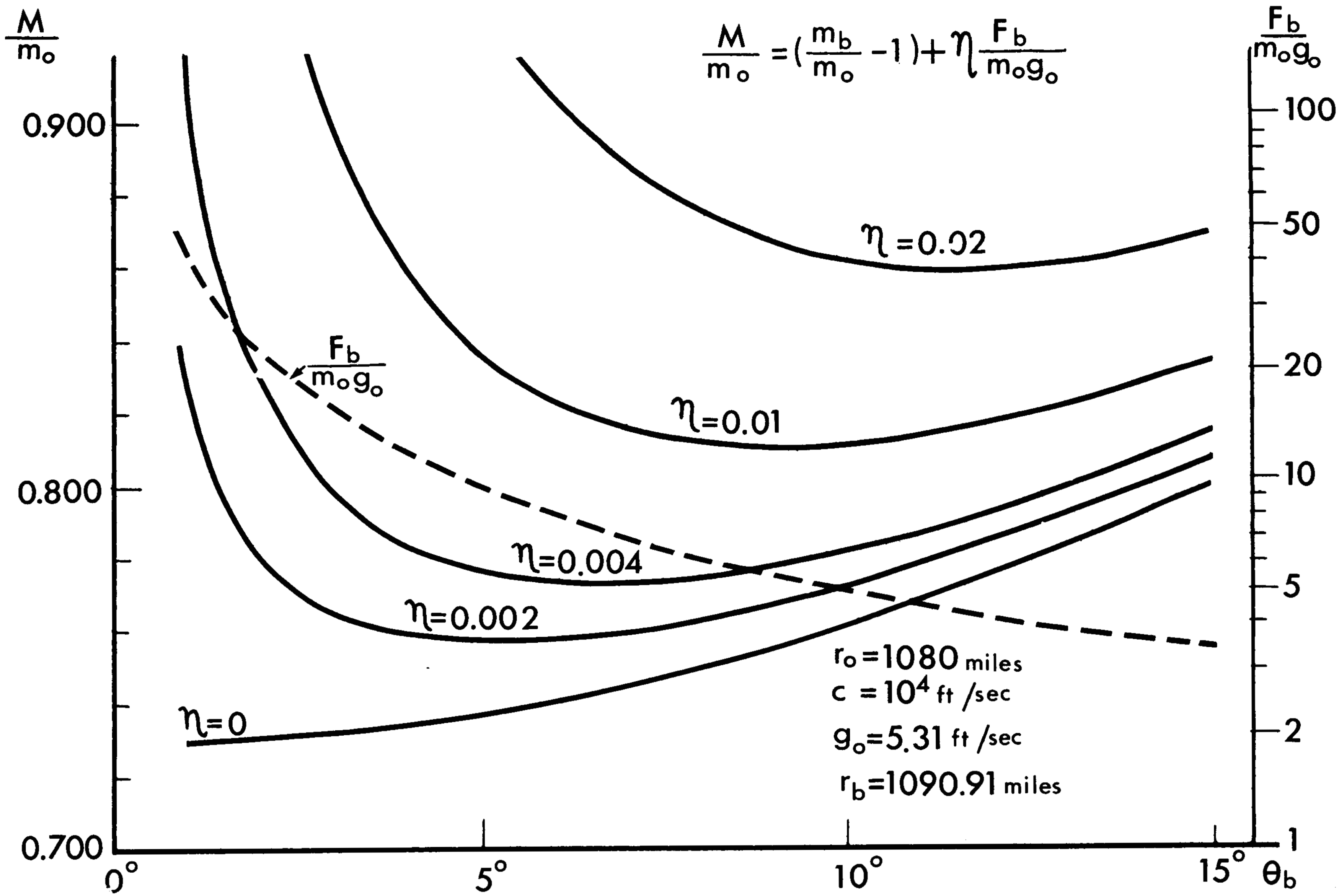


Fig 2 Combined Mass of Fuel and Engine vs θ_b



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