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PROJECT APOLLO

TASK MSC/TRW A-89

SITE ACCESSIBILITY ANALYSIS FOR ADVANCED LUNAR MISSIONS

FINAL REPORT
VOLUME I
SUMMARY

Prepared for
Advanced Spacecraft Technology Division
National Aeronautics and Space Administration
Manned Spacecraft Center
Houston, Texas
Contract NAS 9-4810

TRW SYSTEMS

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HOUSTON, TEXAS

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FOREWORD

This final technical report is submitted to NASA/MSC by TRW Systems in accordance with Task A-89 of the Apollo Mission Trajectory Control Program, Contract NAS 9-4810.

This report consists of two volumes, each of which is self-contained. Volume I summarizes the results of the two-impulse study and presents a simplified version of the graphical method for determining approximate lunar areas of accessibility for mission planning purposes. Volume II presents a complete description of the two-impulse scheme, including the detailed graphical method of determining lunar site accessibility.

In order to minimize the inclusion of non-essential data in these two volumes, several internal reports were documented under this task and are available on request.

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CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (Continued)

ILLUSTRATIONS (Continued)

ILLUSTRATIONS (Continued)

1. INTRODUCTION

This summary volume presents a simplified and relatively rapid mission analysis procedure related to lunar site accessibility. It is a graphical procedure oriented towards use by the mission planner at the management level.

The simplified mission analysis procedure, in conjunction with the graphical data included in this summary volume, will provide sufficient accuracy to allow the mission planner to develop insight into the relationships between lunar site accessibility and mission requirements and constraints. This allows the mission planner to coordinate these relationships for effective mission design. Mission considerations may include

- The geometrical relationships and constraints between site accessibility and

  LM surface stay time

  LM plane change capability

  LM abort requirements

  CSM orbit requirements

  CSM plane changes

- The ΔV relationships between site accessibility and

  CSM orbit stay time

  Abort requirements

  CSM plane changes

  Translunar and transearth flight times

  Spacecraft performance capability

- The optimization of various parameters

  Service module propellant (ΔV)

  Mission duration

  Translunar or transearth flight time

  Surface stay time

The mission analysis procedure presented here consists of three basic steps: (1) the determination of the various geometrical constraints upon site accessibility, (2) the determination of the ΔV constraints or requirements upon accessibility, and (3) the graphical procedure which consists of the manipulation or interpretation of the results of the first two steps to provide the answer or data for the specific mission consideration. These basic steps are discussed in Section 4. The detailed procedure for two example cases is presented in Section 5.

The translunar and transearth velocity data presented here and used in the graphical procedure, represent optimized two-impulse transfers to and from the moon. These transfers provide considerable savings in SM fuel when compared with single-impulse transfers which are presently planned for the Apollo mission. A complete description of the optimization technique, the computer program, and the two-impulse data generated with this program may be found in Volume II. The mission analysis procedure, however, is independent of the mode of orbit transfer. The only requirement is velocity data in the proper format, whether it be single-impulse or multi-impulse.

It is recommended that the mission analyst who will be concerned with the more refined aspects treated in Volume II (Reference 1) read the introductory discussion of site accessibility and description of the graphical mission analysis procedure in this summary volume prior to reading Volume II.

2. TRAJECTORY GEOMETRY

The trajectory profile assumed for the lunar missions discussed here is closely related to that of the Apollo mission. The significant difference, which allows considerably more accessibility at the moon and longer LM surface stay times, is the greater flexibility allowed for the translunar and transearth transfer trajectories. For example, the free-return circumlunar constraint on the translunar phase has been removed.* The effect of this constraint is to force the approach hyperbola at the moon to lie near the moon's equator (within 15 degrees) thus requiring large yaw penalties at lunar orbit insertion (LOI) to achieve higher latitudes. In addition to this yaw penalty, the translunar flight times for circumlunar trajectories are relatively short (65 to 85 hours) resulting in higher approach velocities at the moon when compared with the longer flight times (up to 132 hours) for the non-free return trajectories.

Additional reduction in the SM fuel requirements is obtained by allowing the CSM to perform a maneuver between translunar injection (TLI) and LOI. This maneuver has been defined here as the two-impulse transfer; the TLI is the first and LOI the second. A similar additional impulse may be used after transearth injection (TEI) to also reduce the fuel requirements to return to earth. The optimization of these two-impulse transfers have been performed and the data are presented in Section 4. They will be briefly described in this section; however, a complete discussion may be found in Volume II.

All other Apollo trajectory constraints remain essentially unchanged, including the launch and earth-orbit phase and the reentry phase. The specific ground rules and assumptions have been listed in Section 3.

*It is possible to maintain the free-return constraint for a considerable time after translunar injection along a high pericynthion circumlunar trajectory, and then utilize an SM impulse in the earth phase to get on one of the optimum two-impulse trajectories described here. The degradation in site accessibility utilizing this "three-impulse" mode would be negligible. (Reference 2)

2.1 PHYSICAL MODEL

The physical model assumes that the trajectory consists of patched conics as depicted in Figure 2-1. Thus, the moon's gravitational field extends out to a distance of approximately 30,000 nautical miles. This limit is represented by a sphere whose center is at the moon and which will move with the moon. Since the earth and sun's gravity is neglected within this "sphere of action" (MSA), (Reference 3), all spacecraft free-flight motion can be represented by conic sections with the moon at one focus. These are called moon phase conics. A similar situation exists outside the MSA where it is assumed that only the earth's gravitation is important. Here, the conics will be earth centered and hence, called earth phase conics, as indicated in Figure 2-1.

A complete translunar trajectory is generated by patching an earth centered and a moon centered conic at the MSA so that they have the same position and velocity at this point (Point B in Figure 2-1). The seeming discontinuity at this point is caused by the relative motion of the MSA (and hence, the conic within it) with respect to the earth centered conic. Thus, in order to ensure continuity in the velocity vector at Point B, the moon's velocity relative to the earth must be subtracted from the vehicle's velocity relative to the earth to obtain the vehicle's velocity relative to the moon. This is depicted in the velocity vector diagram shown in the lower right-hand corner.

2.2 TWO-IMPULSE TRANSFER

The two-impulse transfers to and from the moon are shown in Figure 2-2. It has been shown (References 1, 4) that minimum total ΔV requirements will generally occur when both impulses are within the MSA. With this restriction, the problem for the translunar transfer may then be stated as follows:

> Given a fixed day of launch (or lunar distance), translunar flight time from TLI to LOI, and a fixed inclination and node of the 80-nautical mile CSM parking orbit, find the minimum total two-impulse ΔV required in the MSA to enter this parking orbit.

In generating the two-impulse data, it was assumed that launch conditions at the earth are a 90-degree azimuth from Cape Kennedy and translunar injection from a 100-nautical mile parking orbit. The launch opportunity chosen is the one resulting in the earth phase trajectory lying nearly in the moon's orbit plane. Reentry conditions are identical to those presently planned for Apollo. That is, the transearth trajectory is targeted to reentry at 400,000 feet with a velocity path angle of -6.4 degrees. Touchdown is assumed to be at the center of the Apollo footprint. Also, the earth phase conic is assumed to lie nearly in the moon's orbit plane. *

A planar view of the two-impulse transfers to and from the moon are shown in Figure 2-2. The patching point B, discussed in Figure 2-1, is also shown here. Considering drawing (A) first, the translunar injection is targeted to a pericynthion altitude which may vary from 40 nautical miles (a lower limit constraint) to 26,000 nautical miles. This altitude is called the virtual pericynthion (Point C). The targeted moon phase inclination may also vary from 0 to 180 degrees. The first impulse within the MSA (shown as B′) may be anywhere on this moon centered hyperbola, including beyond virtual pericynthion. Also, this maneuver may be out-of-plane as required to intersect the desired parking orbit at Point C′. The pericynthion altitude is restricted to lie between 40 and 80 nautical miles. The second impulse (LOI) which may also be out-of-plane occurs at C′.

For a fixed flight time from TLI to LOI and a fixed CSM orbit, the optimization to minimize the sum of the two impulses at Point B′ and C′ is performed by varying the virtual pericynthion altitude at C, the inclination of this hyperbola, the position of the first impulse (Point B′) and the position of the LOI on the orbit. Also, the flight time to virtual pericynthion is varied to ensure that a true minimum velocity is found Thus, this optimization represents a five parameter search.

A precisely mirror image situation occurs for the optimization of the two-impulse transearth transfer shown in drawing (B). The first impulse

*This assumption, which has been made throughout this study, does not considerably degrade the two-impulse results which are presented. It can be shown (Reference 1) that an out-of-plane launch or reentry perturbs the approach (or return) hyperbolic moon centered asymptote by less than 5 degrees. The actual effect of this perturbation on ΔV may be found by a technique presented in Volume II.

(TEI) will occur at B′ and the second at C′. The optimization parameters are identical with those for the translunar case. One variation is that if TEI occurs past pericynthion, the 40-nautical mile altitude constraint on this hyperbola need not be imposed. A similar argument applies to the second or outer hyperbola from C′ and on.

The complexity of this optimization problem requires that short-cuts be taken whenever possible and justified. One, mentioned above, is that it is sufficient to consider that the earth centered conics lie essentially in the moon's orbit plane. Two others are based on symmetry. The first is that symmetry exists relative to the moon's orbit plane, so that the two-impulse results for a given CSM orbit will be the same as the results of a similar orbit where the ascending and descending nodes are interchanged. Thus, the two-impulse ΔV requirements for a given orbit will be the same for an orbit of the same inclination with the node displaced 180 degrees.

Second, if the moon is at apogee (the results presented here are for this situation), symmetry exists between translunar and transearth transfers for a given flight time. The only difference in the earth phase conics will be perigee distance; i.e., 100 nautical miles for the translunar and approximately 20 nautical miles (vacuum) for the transearth. The effect of this variation on the two-impulse velocity requirements, however, does not warrant completely reoptimizing the transearth transfers. Using symmetry, then, the translunar two-impulse results for a CSM orbit whose node and inclination are Ω_c^* and i_c^* (see footnote below), respectively, will be equal to the transearth velocity requirements for a CSM orbit with a node location of $-\Omega_c^*$ and inclination of i_c^* (same) for the same flight time.

The two-impulse translunar and transearth optimized ΔV are presented in a graphical form suitable for use in the mission analysis procedure.

Ω_c^* and i_c^* are defined in Volume II to be the node and inclination in moon orbit plane coordinates. However, for the simplified procedure, lunar librations and the inclinations of the moon's equator to the moon's orbit plane are neglected, so that Ω_c^* and i_c^* are equivalent to node longitude and inclination with respect to the lunar equator.

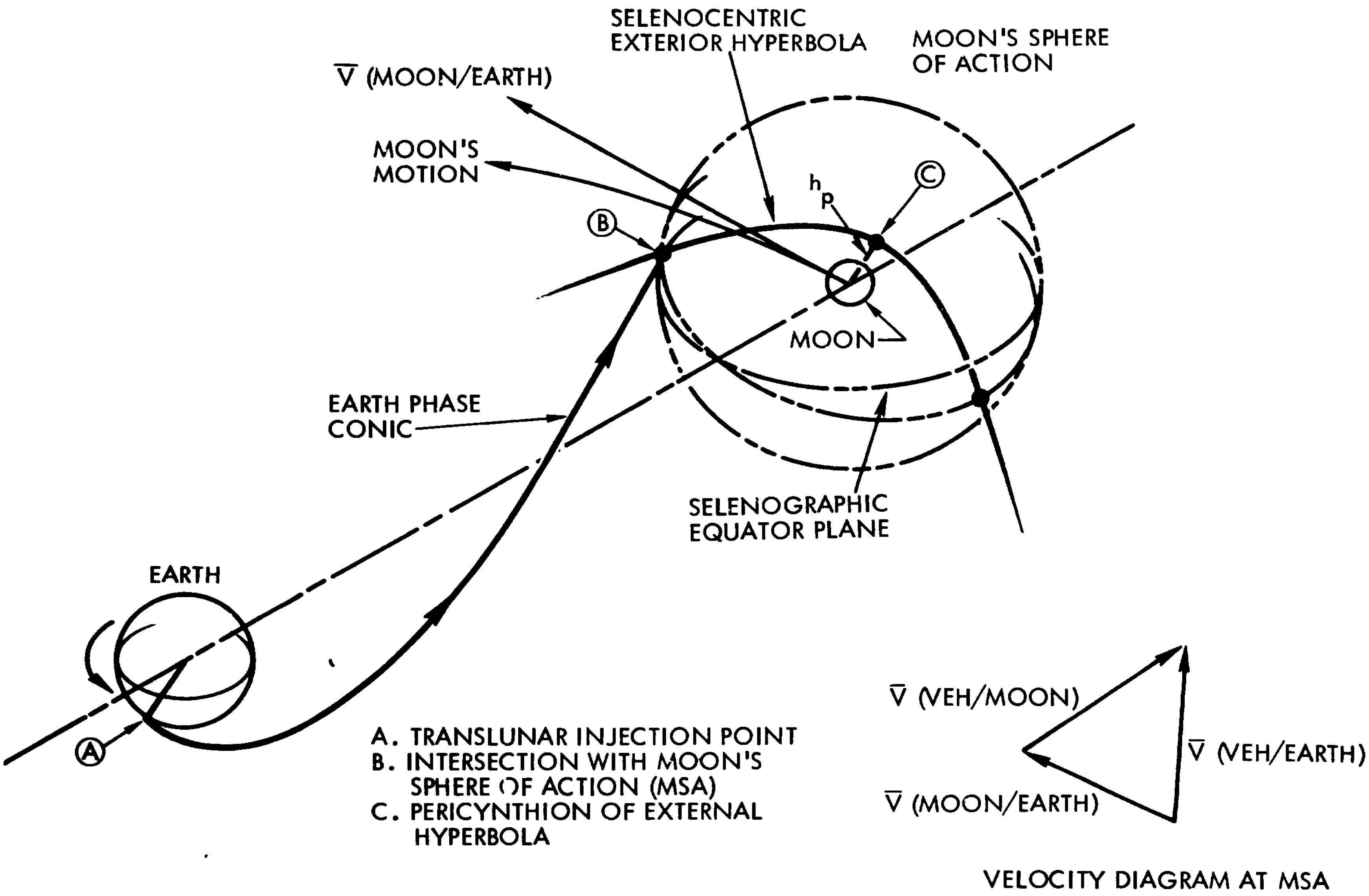

Figure 2-1. Earth-Moon Patched Conic Geometry

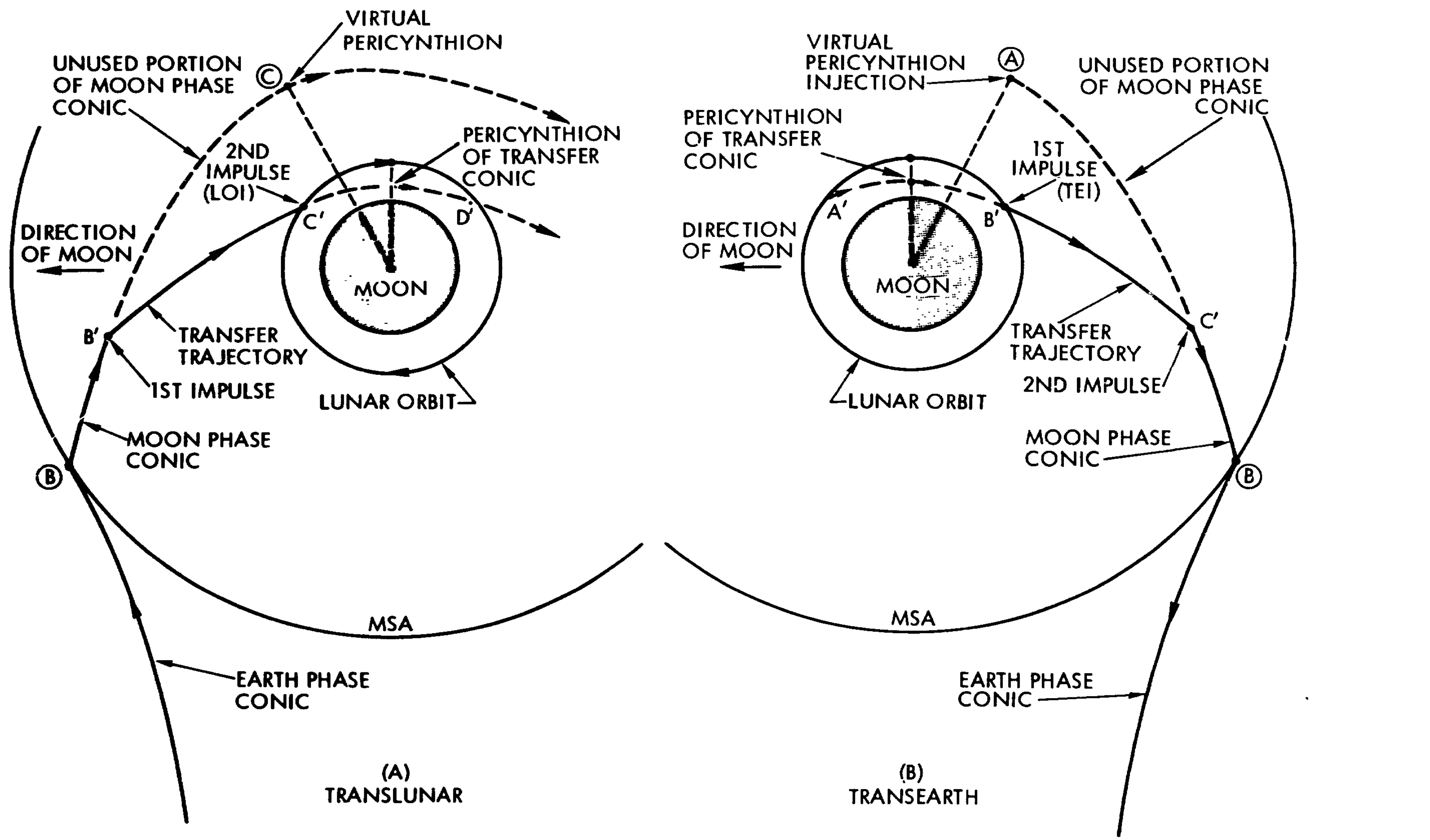

Figure 2-2. Two-impulse Trajectory Profile

3. GROUNDRULES AND ASSUMPTIONS

The data that are used or derived for use in the graphical procedure are affected by the groundrules and assumptions listed below. However, the accuracy achieved by simplifying the procedure is sufficient for mission planning purposes. The groundrules and assumptions are listed as follows:

- A Cape Kennedy launch at a 90-degree azimuth, with an injection into the translunar trajectory during the third earth parking orbit
- Earth phase of the translunar and transearth trajectories lies nearly in the moon's orbit plane
- Virtual pericynthion altitude between 40 to 26,000 nautical miles
- The moon is at maximum (and assumed constant) distance from the earth at the time of LOI and TEI.
- Time from translunar injection to lunar parking orbit and time from transearth injection to earth reentry are varied in 12-hour increments from 60 to 132 hours in generating the ΔV data.
- Only retrograde orbits are considered.*
- Lunar parking orbit altitude 80 nautical miles
- A patched conic trajectory model has been used.
- No midcourse corrections are provided for the ΔV data.
- Inclination of moon's equator to the moon's orbit plane is assumed zero.
- Lunar librations are neglected.

*Although a slight ΔV advantage may be attained by a choice of a retrograde or posigrade orbit for a given mission, posigrade orbits are not considered due to the fact that surface stay time is reduced because of nodal regression. Also, this ΔV advantage is not as significant as the ΔV differences associated with the assumptions made in the simplified mission analysis procedure (i.e., constant earth moon distance, no lunar librations, and zero inclination of moon's equator to moon's orbit plane).

- The translunar ΔV for a given orbit is the same as that for the transearth ΔV required of an orbit whose inclination is the same and whose node is negative that of the translunar orbit (i. e., mirror image).
- CSM lunar orbit inclination and node variations due to the moon's oblateness are neglected.
- Only northern* site latitudes are considered.

Modifications (for use with the analysis procedure) to the data to account for variations in earth-moon distance and lunar librations are discussed in Volume II.

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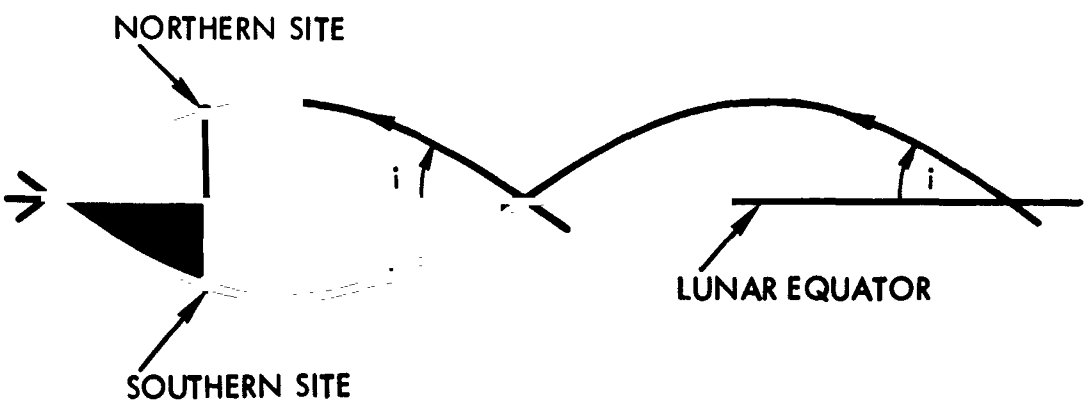
*A mirror image symmetry exists so that an analysis of a southern latitude site is made by assuming that it is a northern latitude. All geometrical constraints, plane change magnitudes (the direction of plane changes, however, are reversed), and ΔV requirements are the same. However, for southern latitudes, the CSM orbit ascending node is displaced 180 degrees (the ΔV requirement is the same), from that for a site of the same longitude and a northern latitude of the same magnitude. This is apparent from the diagram below:



4. SITE ACCESSIBILITY ANALYSIS

There are three logical steps in site accessibility analysis, which is the basis for the graphical mission analysis procedure described in Section 5. They are as follows:

Step I — The determination of geometrical constraints (or requirements) upon site accessibility

Step II — The determination of CSM ΔV constraints upon site accessibility

Step III — The interpretation of the results of Steps I and II in determining site accessibility for a given site and mission, or the generation of contours of lunar surface accessibility

Step I consists basically of determining the CSM orbits that will assure LM-CSM rendezvous capability for a given site latitude, surface stay time and ΔV capability.

Step II consists of determining what CSM orbits are achievable for a given mission profile and spacecraft ΔV capability.

Step III is the graphical procedure in which the data from Steps I and II are interpreted to determine accessibility.

Before discussing the graphical procedure in detail, it will be necessary (to fully understand the mission analysis procedure) to discuss the relationships between mission requirements or constraints and Steps I and II.

4.1 ACCESSIBILITY GEOMETRICAL CONSTRAINTS

There are specific geometrical relationships between landing site latitude, LM stay time, CSM orbit inclination, LM abort requirements and plane change capability*, so that certain constraints exist upon site accessibility. It is essential, in understanding the mission analysis procedure,

*Although LM plane change capability is actually a performance limitation, it is equivalent to a geometric accessibility constraint, since the LM is a separate stage and, therefore, is not included in the CSM ΔV optimization.

that the nature and reasons for these geometrical constraints be well understood.

One basic geometrical relationship is that at the time of LM landing (the LM may descend to the lunar surface after a few revolutions of the CSM orbit following LOI, or even several days afterwards), the landing site lies in the plane of the CSM orbit. From this point on (unless the site is at one of the poles, or the CSM orbit inclination is zero-- the site thereby being on the equator), the site will drift eastward out of the CSM orbit plane. This relative drift is caused by the rotation of the moon about its axis (13.2 degrees per day).*

The various geometrical relationships can be understood with the aid of Figures 4-1 and 4-2. Figure 4-1 shows a typical orbit-site geometry. Points A and B correspond to the position of the landing site at LM arrival and departure, respectively. Shown are four retrograde orbits passing through the site at arrival.

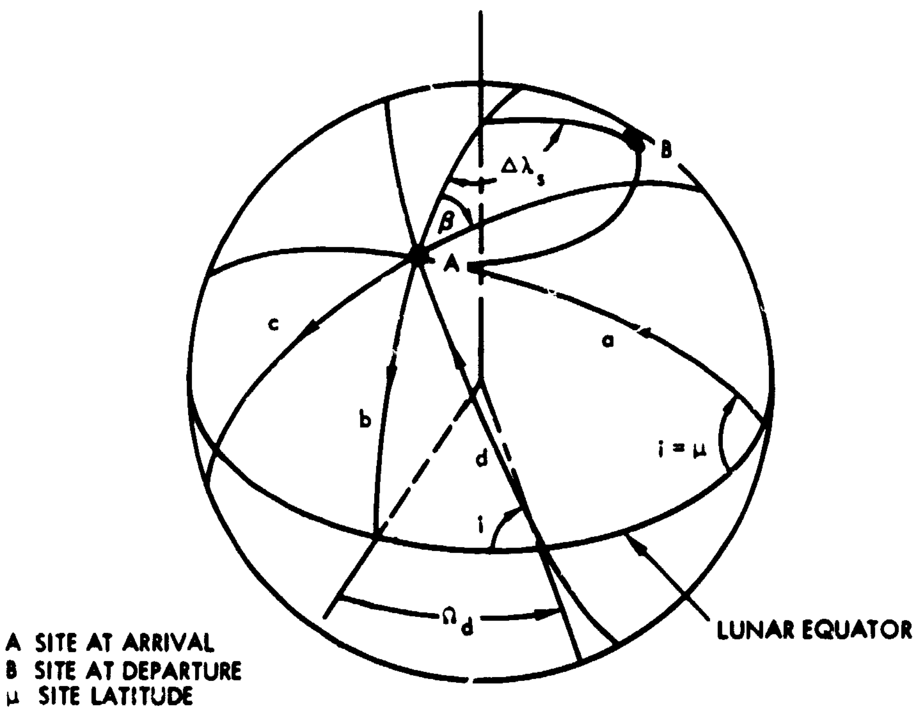


Figure 4-1. Lunar Orbit-Site Geometry

*Nodal regression caused by the moon's oblateness is neglected in the simplified mission analysis procedure.

Orbit a is the minimum inclination CSM orbit in which the inclination* i is equal in magnitude to the landing site latitude, μ. Orbit b is the maximum inclination (polar) orbit; whereas, c and d are orbits of intermediate inclinations. Ω_d is the longitudinal displacement of the ascending node of the CSM orbit relative to the landing site longitude measured eastward from the site longitude. The angle $\Delta\lambda_s$ is the eastward site longitude displacement corresponding to the surface stay time. Since all possible CSM orbits correspond to a rotation about the about the initial site vector (the radius vector through the site at arrival), β is defined for physical clarity to be the angle between the CSM orbit and the site meridian at arrival (see Figure 4-1).

During the time that the LM remains on the moon, the dihedral angle θ_e between the vector through the site and the CSM orbit plane will vary. Beginning with the time of descent, as stay time increases the site will move eastward, and the plane change will initially increase and then vary depending upon the geometry. θ_e will be a function of latitude, inclination and stay time, but not of site longitude.

The equation for θ_e can be shown to be:**

$$\sin \theta_e = \cos \mu \left[\sin \mu \sin \beta \, (1 - \cos \Delta\lambda_s) - \sin \Delta\lambda_s \cos \beta \right] \quad (1)$$

where

$$\sin \beta \cos \mu = \cos i \quad (2)$$

Figure 4-2, which is a view of Figure 4-1 looking down upon the north pole region, depicts the geometrical relationships between the CSM orbit, surface stay time and plane change capability for a given site

*The classical definition of inclination will not be used here, but will always be taken between 0 and 90 degrees and the orbit indicated as retrograde.

**It should be noted that θ_e can have negative values as well as positive values. This merely means that θ_e is positive if the plane change as measured from the orbit to the site has a northward component and is negative if it is heading southward.

latitude. For a given stay time (corresponding to a site longitudinal displacement of $\Delta\lambda_s$) and a maximum plane change capability θ_m, it is seen from Figure 4-2 that orbit 1 is the highest CSM orbit inclination possible that will satisfy the continuous abort capability requirement. This corresponds to point a in which the maximum plane change θ_m occurs. On the other hand, orbit 2 is the lowest inclination orbit that will satisfy the abort requirement. This corresponds to the maximum plane change θ_m occurring at the point of LM departure B. It becomes apparent, then, that all CSM orbits lying between 1 and 2 of Figure 4-2 will satisfy the continuous abort requirement so that there will be, in general, a range of CSM orbits that will satisfy the stay time and plane change requirements for a given site latitude.

Figure 4-3* is a typical plot depicting the geometrical relationships described above. These curves are generated from Equations (1) and (2). Figure 4-3 can be related to Figure 4-2 as follows: First, it is noted that i, μ, and Ω_d are related as shown in Figure 4-4, so that following a line of constant μ (dotted curves of Figure 4-3), i will vary as shown in Figure 4-4 as Ω_d increases from zero to 180 degrees. Consider point a of Figure 4-3.

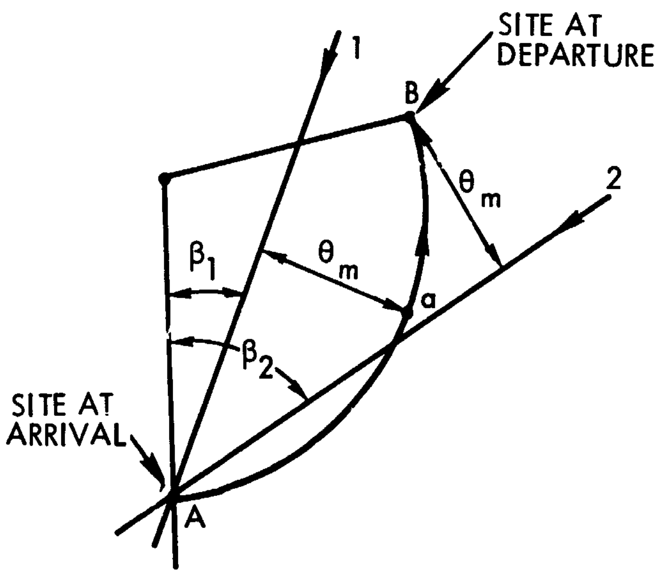


Figure 4-2. Lunar Orbit-Plane Change Geometry

*The geometrical constraints curves are presented in i-Ω_d coordinates for use in the mission analysis graphical procedure described in Section 5.

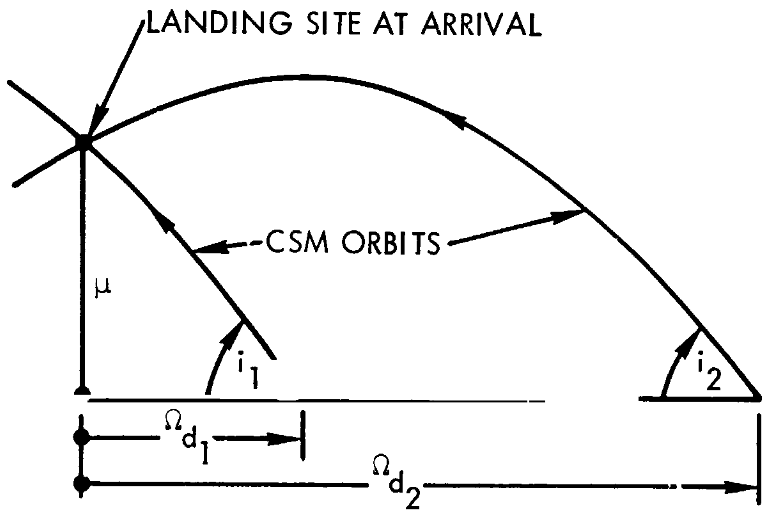


Figure 4-4. μ, i and Ω_d Geometry

This corresponds to the maximum CSM orbit inclination that will provide continuous abort capability for a site latitude of 20 degrees, plane change capability of 4 degrees, and a surface stay time of 5 days. Point "a" corresponds to orbit 1 of Figure 4-2. Following the $\mu = 20$-degree curve from point a to point b of Figure 4-3, all CSM orbits that satisfy the 5-day stay time requirement are traversed. This traversal corresponds to increasing β from β_i to β_2 as shown in Figure 4-2. Referring to Figure 4-1, it is seen that this change in β lowers the orbit inclination and shifts the nodes westward. This is also apparent from Figure 4-3.

It is noted from Figure 4-3 that there are site latitudes which are not obtainable for larger surface stay times. This is simply a result of the fact that the total plane change variation exceeds the plane change capability for that given stay time. This is depicted in Figure 4-5. For a maximum plane change capability of θ_m, the continuous abort capability is satisfied for the stay time corresponding to the site traversal from point A to point b. However, the plane change capability is exceeded from point b to B. It is obvious that no CSM orbit will satisfy this geometry, unless the plane change capability is increased.

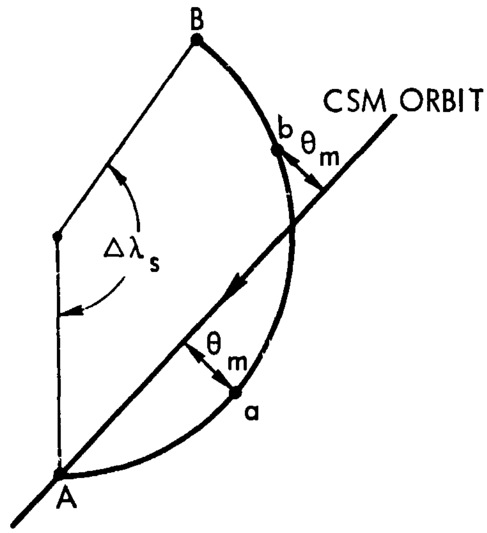


Figure 4-5. Plane Change Geometry - θ_m Exceeded

The right boundary curve of Figure 4-3 corresponds to orbit 1 of Figure 4-2. This boundary curve will also remain the same for stay times of 5 to 8 days. This becomes apparent by considering Figure 4-6. Orbit 1 and point B correspond to the stay time in which the plane change equals the maximum capability θ_m. It is seen that this geometry remains fixed for that range of stay times from point B to B''. The left boundaries of Figure 4-3 correspond to orbit 2 of Figure 4-2. Consider Figure 4-7: for the stay time corresponding to point a, the range of allowable CSM orbits lies between orbits 1 and 2. For a longer stay line corresponding to point b, it is seen that the range becomes smaller and boundary 2 approaches boundary 1 to position 2' (the left curve of Figure 4-3 shifts towards the right boundary). As stay time increases to that corresponding to point B, then there is only one CSM orbit that will satisfy the geometry. This corresponds to the intersection of the boundaries. Longer stay times then become impossible for that geometry. Figure 4-18 is a plot relating maximum allowable stay times as a function of site latitude for various plane change capability values.

To eliminate the necessity of generating the geometrical constraint curves for the graphical mission analysis procedure, graphs* corresponding to Figure 4-3 have been constructed for various stay times and plane change capabilities and appear in Figures 4-19 through 4-23. Figures 4-18 through 4-51 are located in the appendix of working graphs.

Consider the case in which the LM plane change capability is small or zero. Essentially all necessary plane changes must then be made by the CSM (CSM plane changes are discussed in Section 4.2.3). Referring to Figure 4-8, it is seen that the geometry** for this simple case becomes obvious. For a given site latitude μ and a given stay time $\Delta\lambda_s$, the symmetry depicted in Figure 4-8 must exist so that the site drifts into the CSM orbit plane of the end of the desired stay time. All the CSM orbit nodes*** must be coincident for a given stay time.

Figure 4-8 is depicted in Figure 4-9 in a form consistent with the graphical mission analysis procedure in which the CSM orbit inclination i is plotted as a function of the ascending node longitude displacement Ω_d from the landing site longitude. Figure 4-9 shows that the locus of points satisfying the geometry of Figure 4-8 is a vertical straight line for a given stay time. For example, if a 6-day stay time is desired, then the CSM orbit node must lie 130 degrees east of the site longitude. If a site latitude 20 degrees is also desired, then the orbit inclination must be 25 degrees (point A of Figure 4-9).

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*These graphs, which have been extracted from Volume II, also include nodal regression.

**In fact this geometry would be desirable. It is not unreasonable to expect that for any lunar mission, that CSM orbit will be selected which intersects the landing site at the nominal time of LM ascent. This minimizes LM propellant requirements for ascent. Any plane changes, then, will be made by the CSM or LM only in the event of an abort or a non-nominal LM lift-off time.

***The longitudinal node displacement from the site longitude, Ω_d is given by the expression $\Omega_d = 90 + \Delta\lambda_s/2$ (deg)

- CSM plane changes
- CSM total ΔV capability
- CSM abort requirements

4.2.1 Translunar and Transearth ΔV Requirements

The translunar and transearth CSM ΔV requirements will depend upon the inclination and node of the CSM parking orbit and the time elapsed between LOI and TEI, neglecting at the moment, any other constraints or requirements.

Figures 4-24 through 4-37 show the translunar and transearth ΔV requirements displayed as inclination versus node for various values of constant ΔV. The curves are generated for flight times from 60 to 132 hours at 12-hour intervals for retrograde orbits. The velocity curves are relative to the earth-moon plane coordinates, which for the basic procedure are assumed coincident with selenographic coordinates.

To understand the relationship between the translunar ΔV requirements of a given CSM parking orbit to the transearth ΔV requirements of the same orbit, consider the velocity curves of Figure 4-10, in which the translunar and transearth ΔV curves are shown for flight times of 96 and 72 hours, respectively. If, for example, it is desired to achieve a CSM orbit with an inclination of 30 degrees and an ascending node longitude of 62.5° East (or 117.5° West), corresponding to point A, then the required translunar ΔV will be 3800 feet per second. If the CSM orbit stay time is zero (TEI occurs immediately after LOI), the geometry at LOI and TEI is the same, so that the transearth ΔV can be found at point B (which corresponds to the same orbit as point A) to be 3400 feet per second. The translunar and transearth ΔV for points A' and B', which correspond to an orbit inclination of 15 degrees and ascending node longitude of 15° East (or 165° West) will be 3300 and 3240 feet per second, respectively.

For this simple case, the two curves can be overlayed with coincident scales, and the translunar and transearth ΔV can be read simultaneously for any orbit. Points A and B (and A' with B') will be coincident. However, for a given CSM orbit stay time, the earth-moon geometry changes so that the interpretation of these curves must be modified. This is discussed in the following section.

4.2.2 CSM Orbit Stay Time

During the CSM orbit stay time, the inclination will remain the same*; however, the earth moon line (the orbit plane will remain inertially fixed) will rotate eastward through some angle (or true anomaly, η_m). Figure 4-11 depicts this motion for some given orbit stay time. The true anomaly is the inertial angle that the moon rotates during the orbit stay time (13.2 degrees per day).

Once η_m has been determined, it is then possible to associate the translunar and transearth velocity requirements. For example, assume a true anomaly η_m of 30 degrees corresponding to a CSM orbit stay time of approximately 2.3 days. It is seen from Figure 4-11 that the earth-moon line has moved 30 degrees eastward (the orbit node has moved westward 30 degrees), so that at TEI the longitude of the ascending node corresponding to point A is now 32.5 degrees East (See Figure 4-10) corresponding to point C. The transearth velocity requirement is now 3150 feet per second. For point A', the ascending node has moved from 15° East to 15° West longitude (or from 165° West to 165° East Longitude), where the transearth ΔV required is now 3600 feet per second.

It becomes apparent, then, that if the transearth velocity curves are overlayed on the translunar curves with coincident scales, the translunar and transearth ΔV requirements can be read off simultaneously for an CSM orbit (any node-inclination combination) for a zero orbit stay time. As CSM orbit stay time begins to increase, the origin (which coincides with the earth-moon line) of the transearth ΔV overlay shifts eastward (to the right) relative to the translunar plot. For the case above (η_m = 30 degrees) the origin of the transearth ΔV overlay is coincident with the 30-degree longitude of the translunar plot. In Figure 4-10, points A and A' will become coincident with points C and C', respectively. It is noted that to read off values to the right of η_m equal to 30 degrees on the translunar

*The inclination will change slightly relative to the earth moon plane as a result of the moon's oblateness; however, the change will be at most two degrees for a 14-day orbit stay time, and is therefore neglected here.

plot, the left origin of the transearth scale is coincident with the 30-degree longitude of the translunar plot; whereas, to read off values to the left, the right origin of the transearth scale is placed coincident with the 30-degree translunar longitude.

If continuous CSM abort is required for a mission, then the transearth ΔV requirements must be investigated throughout the CSM orbit stay time to find the maximum ΔV condition. This is discussed in Section 4.2.5.

4.2.3 CSM Orbit Plane Changes

The consideration of a CSM plane change during lunar orbit can significantly enhance site accessibility. This, in effect, increases the LM plane change capability by the amount performed by the CSM. However, any CSM plane change made while in the lunar parking orbit will not only reduce the CSM fuel available for TEI but also will change the orbit inclination and node position thereby changing the transearth velocity requirements. In addition, if continuous CSM and LM abort capability is required for the mission, then the transearth ΔV will also be a function of the time at which the CSM plane change is made. The effects of abort requirements are discussed in Section 4.2.5.

The basic mission analysis procedure described in Section 5 includes CSM plane changes for the specific geometry depicted in Figures 4-8 and 4-9 in which the initial CSM orbit plane includes the landing site at arrival and at the nominal time of departure.* The typical plane change geometry is depicted in Figure 4-12. If a plane change is made at some time after arrival (point α of Figure 4-12), the required** plane change is θ_e resulting in a CSM orbit with a different inclination and node position. It

*See second footnote on Page 4-7.

**For simplicity, it is assumed that all the plane change is performed by the CSM. If a combined plane change is performed by the CSM and the LM, then the CSM plane change ΔV and resulting changes in orbit inclination and node position will be less than that presented in the graphical data of this volume (Satisfactory approximations can be made, however, with this data). Combined plane changes are treated in Volume II.

is assumed that the plane change occurs 90 degrees before (or after) the point of closest approach of the CSM to the landing site corresponding to point P' of Figure 4-12. It is apparent from this figure that as stay time increases from the time of arrival (at which time θ_e is zero and the optimum point for a plane change is at point P) to one-half the total stay time, the plane change θ_e is a maximum (and equal to i - μ), where

$$\tan i = \frac{\tan \mu}{\cos \lambda_s/2} \qquad (3)$$

and the optimum point for a plane change is on the equator (at which time a plane change results in a change in inclination with no node shift). Symmetry exists for the remainder of the stay time.

Figures 4-38 through 4-43 show the plane change angle θ_e versus time and site latitude for various stay times. Figures 4-44 through 4-49 show the changes in the CSM orbit inclination and node longitude versus time and site latitude for various stay times. Figure 4-50 shows the plane change ΔV versus plane change angle θ_e for an 80-nautical mile circular orbit. The plane change ΔV is also given by the expression:

$$\text{Plane change } \Delta V = 10,580 \sin \theta_e/2 \qquad (4)$$

It should be noted from these figures that as the stay time approaches 13.7 days corresponding to a site longitude displacement of 180 degrees, the geometry becomes somewhat unrealistic. A site displacement of 180 degrees requires a polar orbit to satisfy the requirement that the site at arrival and nominal departure be in the CSM orbit plane. This means that a plane change up to 90 degrees may be required for an abort. On the other hand, if a site latitude of 20 degrees, for example, were considered for a stay time of 13.7 days, a minimum inclination orbit of 20 degrees would require a maximum plane change of 40 degrees.

The use of the graphical data of Figures 4-38 through 4-50 is discussed in Sections 4.2.5.2 and 5.1.1.

4.2.4 Spacecraft Performance Capability

In order to determine whether the spacecraft is capable of satisfying the velocity requirements for a mission under consideration, it is convenient to relate the ΔV available for plane changes and transearth injection to the ΔV required to achieve the desired CSM orbit. This relationship between translunar and transearth ΔV for a given spacecraft configuration can be determined from the following expression:

$$V_{TE} = g_o I_{sp} \ln [(kW_o - W_{LM})/W_{CSM}] \tag{5}$$

where

k	$= e^{-\Delta V_{TL}/g_o I_{sp}}$
V_{TE}	= transearth ΔV available (ft/sec)
V_{TL}	= translunar ΔV used (ft/sec)
g_o	= gravitational constant (32.174 ft^2/sec)
I_{sp}	= specific impulse (sec)
W_o	= weight after TLI without spacecraft-launch vehicle adapter (SLA) (lb)
W_{LM}	= Lunar module weight discarded (lb)
W_{CSM}	= Command and service module weight (lb)

The spacecraft configuration used through this report has the following characteristics:

W_o	= 94,548 lb (without SLA)
W_{LM}	= 32,000 lb
W_{CSM}	= 23,562 lb
I_{SP}	= 313 sec

Equation (5) then becomes

$$\Delta V_{TE} = 10,070 \ln (4.013\ k - 1.358) \tag{6}$$

Using the V_{TL} versus k and the lnx versus x curves of Figures 4-13 and 4-14, respectively, a table can be constructed as follows:

k	ΔV_{TL} (Fig 4-14)	4.013k - 1.358	ln(3.97k-1.35) (Fig 4-15)	ΔV_{TE}
0.60	5140	1.050	0.049	490
0.65	4340	1.250	0.223	2240
0.70	3590	1.451	0.372	3740
0.75	2900	1.652	0.502	5050
0.80	2250	1.852	0.616	6200

This table can now be used to plot the performance curve shown in Figure 4-15. If the translunar and transearth midcourse correction ΔV's are to be included in the generation of the capability curve of Figure 4-15, it is apparent that this is achieved by simply shifting the curve to the left an amount equal to the transearth midcourse ΔV (resulting in Curve A') and then lowering the resulting curve an amount equal to the translunar midcourse ΔV (resulting in Curve B.) Curve B will be used in the generation of the ΔV constraint data in Section 5, which corresponds to the following typical Apollo values:

Translunar midcourse ΔV = 162 ft/sec

Transearth midcourse ΔV = 94 ft/sec

If a spacecraft of different weight distributions than that shown for the example spacecraft of Figure 4-15 is considered for a mission, then a new capability curve can be generated as described above with the aid of Figures 4-13 and 4-14.

The procedure in using the performance capability curve of Figure 4-15 for a specific site analysis or an accessibility contour generation is discussed in Section 5.

4.2.5 Continuous Abort Requirements

A continuous abort capability exists for a mission if there is sufficient ΔV capability remaining after LOI for an earth return at any time (i.e., once per CSM revolution) during the lunar orbit stay time. Therefore, if continuous abort is required for a mission, it will be necessary to consider the transearth ΔV requirements not only at the end of the parking orbit stay time (at TEI) but also throughout the stay time from LOI on. This results from the fact that the earth-moon geometry is continually changing with time so that transearth velocity requirements are also changing. In addition, the effects of any required CSM plane change upon the transearth ΔV requirements must be considered.

Depending upon the desired CSM orbit inclination, nodal position, and orbit stay time, the transearth velocity requirements during CSM orbit stay time can behave as described in the following four cases:

Case 1. Increase to a maximum and then decrease

Case 2. Continually increase after LOI

Case 3. Continually decrease after LOI

Case 4. Decrease to a minimum and then increase

The continuous abort requirement with and without CSM plane changes will be considered.

4.2.5.1 Continuous Abort Without CSM Plane Change

The behavior of the transearth ΔV requirements as a function of orbit stay time is readily determined from the transearth ΔV curves. Figure 4-16 depicts the four cases listed above for a transearth flight time of 72 hours. For reference, the translunar ΔV curves for a flight are also shown. An orbit stay time of five days is assumed, corresponding to a westward node traversal of approximately 66 degrees. Considering Case 1 of Figure 4-15, point A_1 corresponds to a retrograde CSM orbit of 14 degrees inclination and ascending node longitude of 172 degrees East or 8 degrees West. The translunar ΔV required is 3200 ft/sec. If an abort were required immediately after LOI, the required transearth ΔV would be 3500 feet per second. As orbit stay time increases, the ascending node of the orbit moves westward, relative to the earth moon line, and the

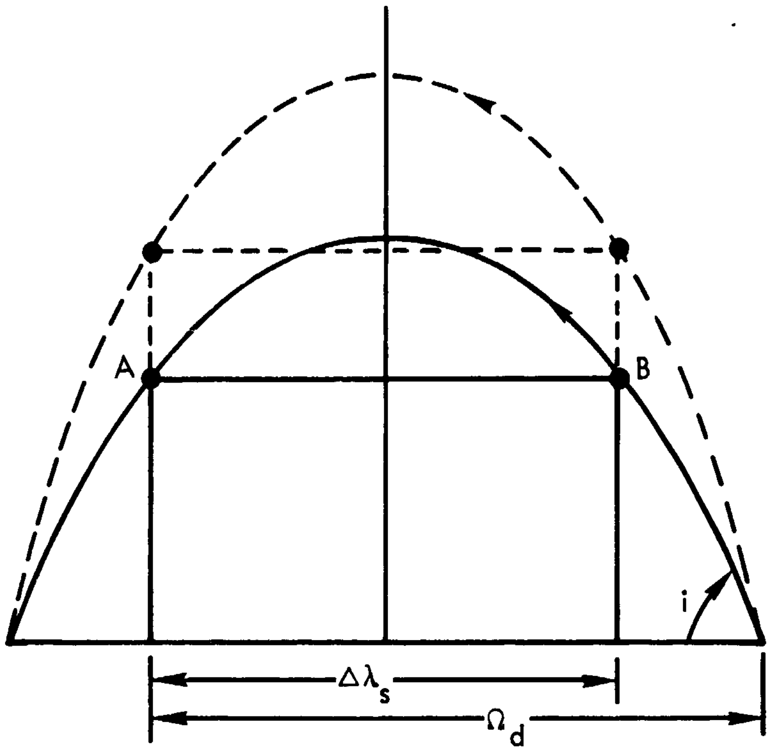

Figure 4-8. Specific CSM Orbit-Site Geometry

The relationship between the geometrical accessibility constraints and ΔV constraints now becomes apparent. For a given mission profile the achievement of a specific CSM orbit inclination and nodal longitude will require a specific total CSM ΔV for translunar and transearth ΔV and any CSM plane changes. If this lies within the performance capability of the CSM and satisfies all mission constraints, then the site under consideration for this mission profile is deemed accessible. If not, it is inaccessible. However, accessibility may possibly be achieved if the mission profile or the capability of the CSM is appropriately modified.

4.2 ΔV ACCESSIBILITY CONSTRAINTS

For a given mission under consideration, the ΔV constraints upon site accessibility will be dictated by

- Translunar ΔV requirements
- Transearth injection ΔV requirements
- CSM orbit stay time

transearth ΔV continually increases to a maximum of 3700 feet per second at point B_1 and then decreases to 3550 feet per second at the end of five days (point C_1). For this case 3700 feet per second would be budgeted for the transearth ΔV for continuous abort. Case 2 depicts an orbit (with an inclination of 45 degrees and ascending node longitude of 29° East or 151° West). The transearth ΔV continually increases throughout the orbit stay time so that the transearth ΔV required (5300 feet per second) at TEI (point C_2) would be budgeted. Case 3 depicts an orbit in which the transearth ΔV continually decreases so that the maximum ΔV occurs immediately after LOI. The transearth ΔV that would be budgeted would then be 4350 feet per second, corresponding to point A_3. Case 4 depicts an orbit in which the ΔV decreases to a minimum at point B_4 and then continually increases. The highest ΔV value of 3800 feet per second, corresponding to point C_4, would be budgeted for this case.

When the ΔV requirements are determined for a specific orbit, the spacecraft capability curve of Figure 4-15 is used to determine whether the velocity requirements are achievable.

The generation of ΔV constraint contours, which is the locus of CSM orbits that consume all CSM propellant for a given mission with continuous abort capability, is described in Section 5. 1. 2.

4. 2. 5. 2 Continuous Abort with CSM Plane Change

To determine the maximum ΔV requirements for an earth return for the case in which the CSM is to execute the plane change in an abort situation, the time at which the plane change occurs must be considered, since this affects the subsequent transearth ΔV variation with time to TEI.

The effect of a CSM plane change upon the subsequent transearth ΔV requirements can be understood by considering the cases depicted in Figure 4-17. Case (a) depicts Case 2 in the previous section, in which the transearth ΔV continually increases after LOI. Point 2 corresponds to half the surface stay time at which time the plane change angle is a maximum. If a plane change is made at this time there is no node shift of the CSM orbit, since the plane change is made when the CSM is over the equator (see Figure 4-12). A specific example may be considered in discussing plane changes at points 1 and 3. A surface stay time of 6 days and a site latitude of 30 degrees are assumed. This is found from Figure 4-9, Equation 3, or from Figure 4-40 which shows the plane change angle versus stay time (the maximum plane change angle occurs after half the stay time has elapsed, and is equal to i - μ). Considering point 1, it may be assumed that this corresponds to a plane change made after 1.5 days have elapsed. From Figure 4-40 it is seen that the required CSM plane change is 5.0 degrees. The resulting change in CSM orbit inclination and node longitude is found from Figure 4-46 in which the inclination is lowered 4.8 degrees, and the ascending node has shifted eastward 2.8 degrees. If the plane change is made at point 3, which is after half the stay time has elapsed, the node shift is then westward. The direction of node shift is also apparent from inspection of Figure 4-12. The symmetry in Figures 4-38 through 4-49 may be noted with respect to the surface stay time midpoint.

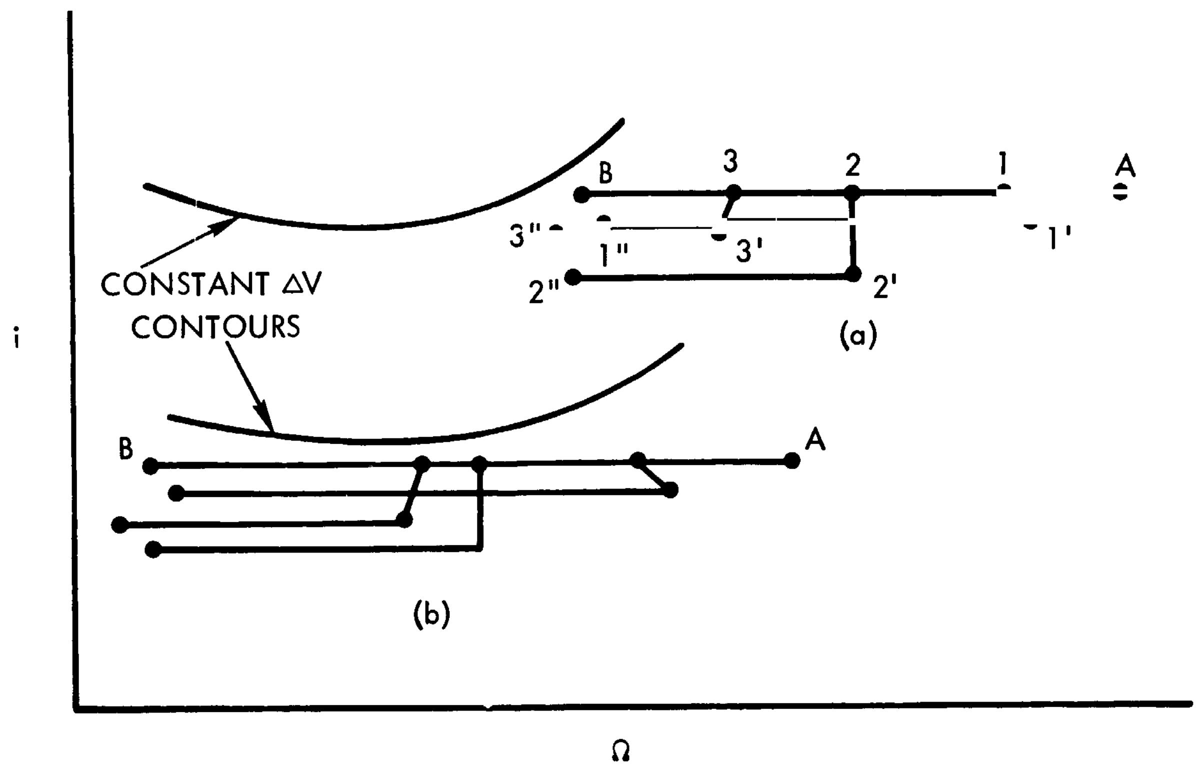

Figure 4-17. Sample Cases - Continuous Abort with CSM Plane Change

Referring again to Figure 4-17 (a), the resulting ΔV requirements at TEI, then, correspond to the end points 1'', 2'',or 3''. However, the worst ΔV condition for this case is that in which a LM abort is required at the stay time midpoint, requiring the maximum CSM plane change, and in which TEI occurs at the nominal time. This is due to the fact that any reduction in transearth ΔV resulting from a CSM plane change will be less than the ΔV required to perform the plane change. This is apparent by inspection of the translunar and transearth ΔV curves of Figures 4-31 to 4-37, where the maximum ΔV gradient is approximately 67 feet per second per degree plane change, corresponding to a flight time of 60 hours. The ΔV gradients become smaller for longer flight times. The ΔV required per degree plane change for a circular 80 nautical miles orbit is approximately 92 feet per second. The larger the plane change, the larger the difference, or net ΔV penalty will be. For the case depicted in Figure 4-17 (b), the worst ΔV abort case is the same as that stated above.

The following statements are apparent in determining the worst ΔV case for continuous abort with CSM plane change.

- If the transearth ΔV continually increases after LOI, then the worst ΔV case is one in which the maximum plane change is made (after half of the stay time has elapsed).
- If the transearth ΔV reaches a maximum well after half the stay time, then the worst case is one in which the maximum plane change is made.

For other cases, the worst ΔV abort case is determined by examining the ΔV requirements throughout the stay time. Some cases may be apparent. For example, if the transearth ΔV requirements continually decrease throughout the stay time, then it should be determined whether the plane change ΔV increase versus stay time or the transearth ΔV decrease versus stay time is greater. If the transearth ΔV reduction versus stay time is greater, then the worst abort case is at LOI. If not, then the worst case if found by determining the ΔV requirements for several time points, using the CSM plane change data in Section 8.

The CSM ΔV required versus plane change angle is shown in Figure 4-50 (which is obtained from Equation 4).

4.2.6 Example ΔV Constraint Curves

Figure 4-51 shows the envelope of CSM inclinations and nodal locations that are achievable within the framework of the spacecraft capability defined in Figure 4-15, a 14-day maximum total mission time, and a continuous abort requirement without CSM plane changes. The curves are drawn for three lunar stay times (time elapsed between lunar orbit insertion and transearth injection). For a given stay time, the range of possible orbits lies between the corresponding stay time boundaries.

The right-hand boundary is the result of slow ($\approx 132^h$) transearth times. It represents the locus of cases where CSM abort follows immediately after LOI. This constraint is a function of the spacecraft total ΔV capability and is not a function of stay time (for the values of stay time considered). The boundaries on the left hand are the result of slow ($\approx 132^h$) translunar flight times with return flight time not being particularly critical. The continuous abort requirement (corresponding to zero stay time) lies to the left of the 2, 3, and 5-day stay time lines and is not shown.

A drawback to the form in which the data are shown in Figure 4-51 is that no information is shown for the combination of outbound and return trajectory flight times that will yield the desired inclination and node. For this information the supporting data has been made available.

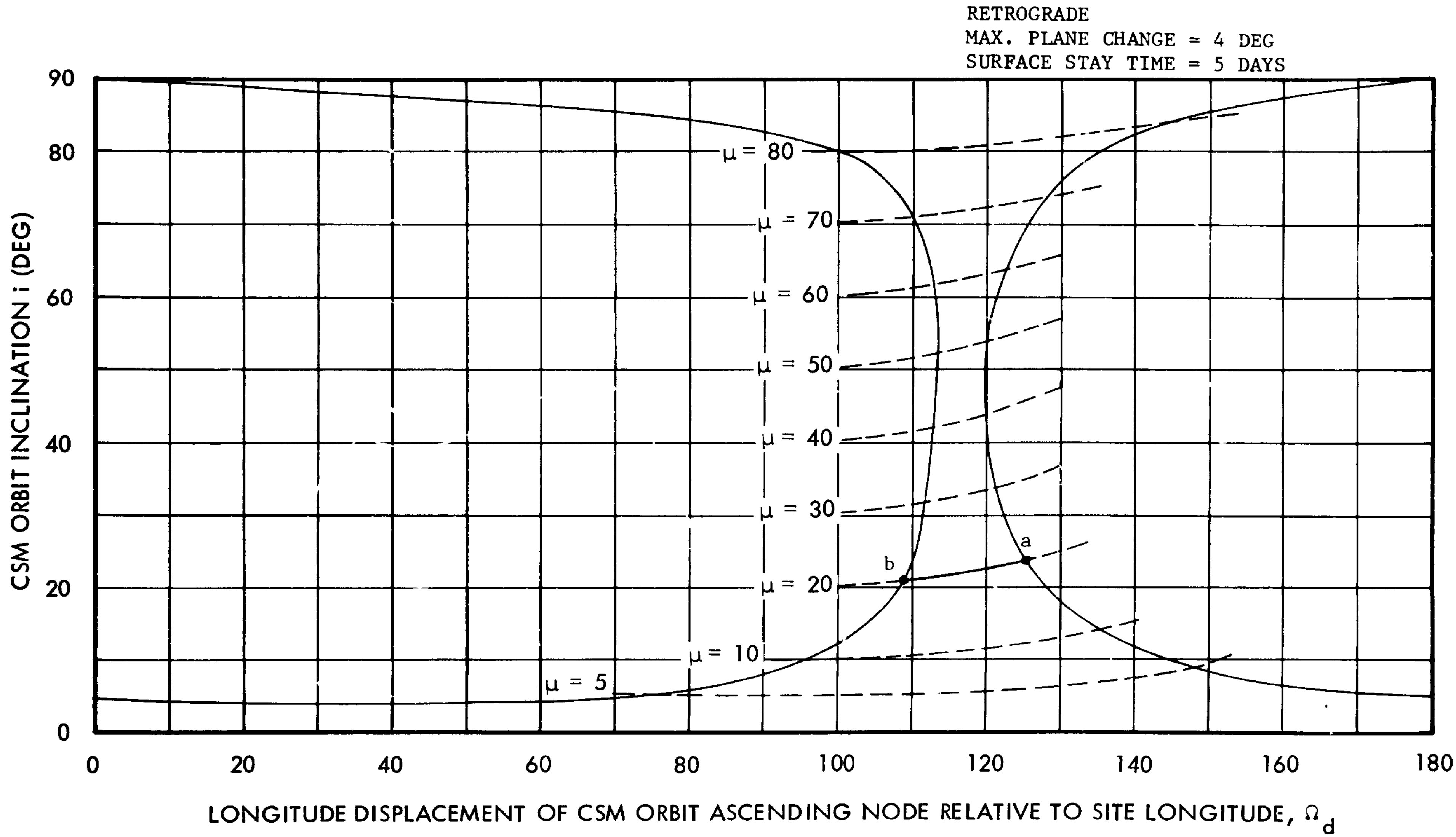

Figure 4-3. Example Geometric Constraint Curves

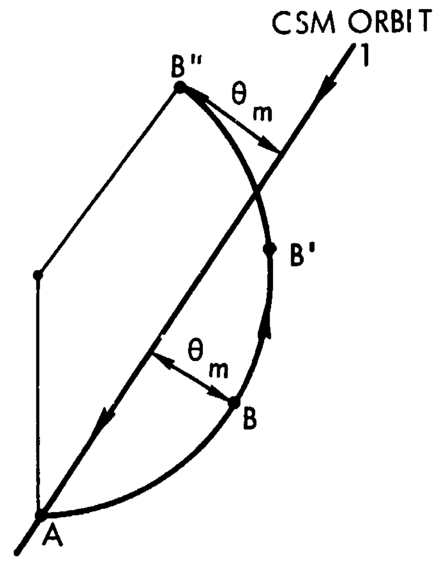

Figure 4-6. Right Boundary Plane Change Geometry

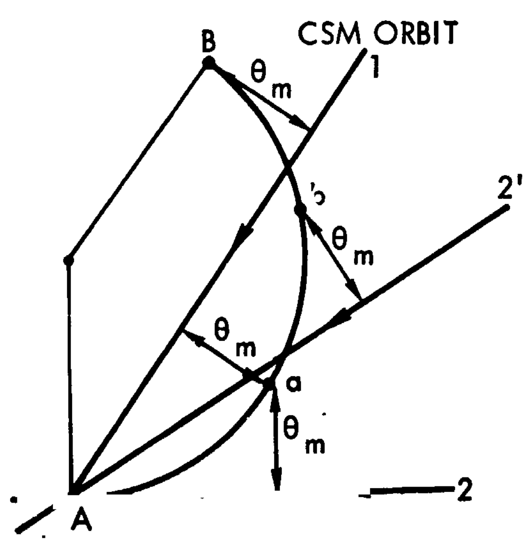

Figure 4-7. Left Boundary Plane Change Geometry

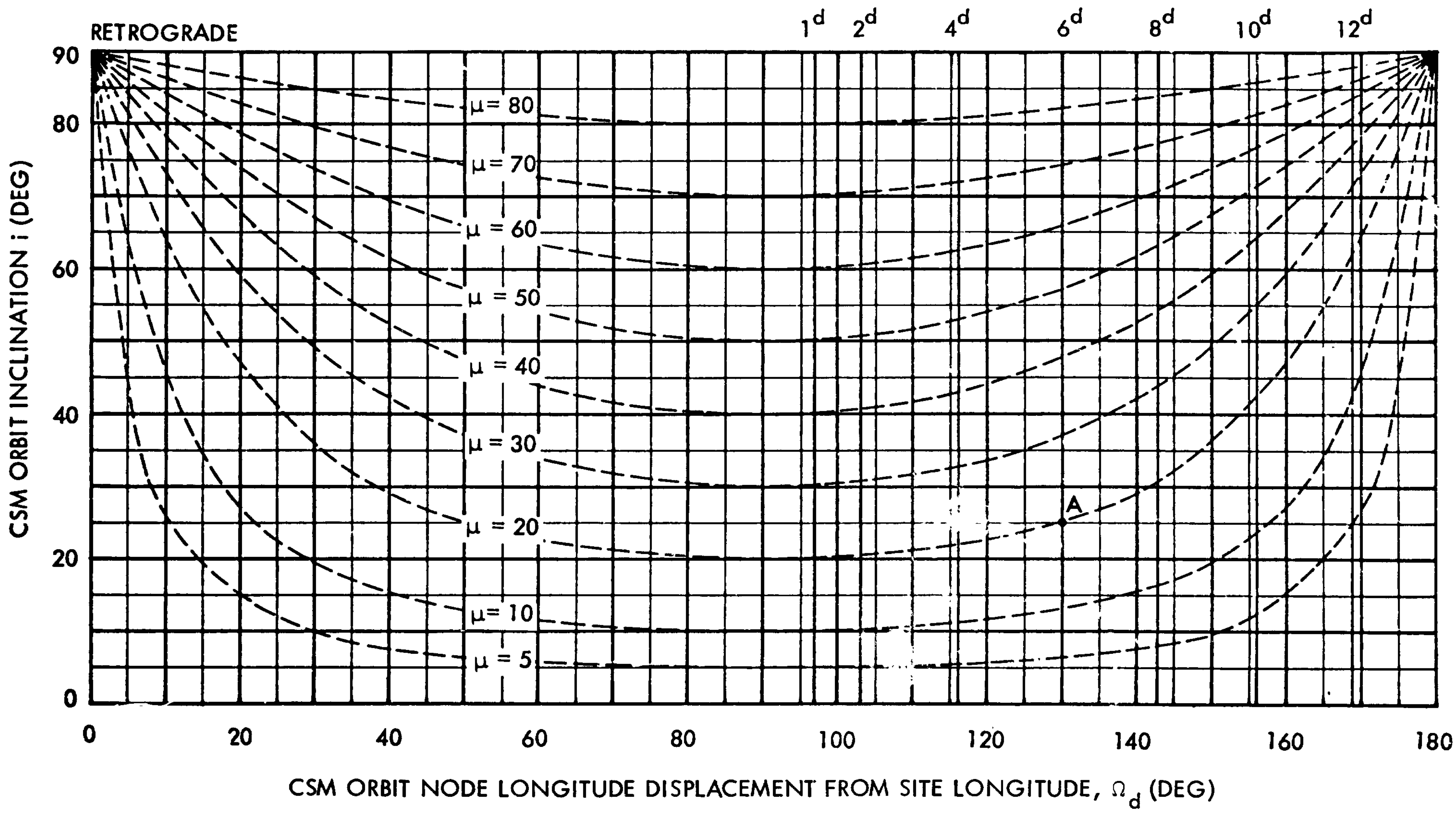

Figure 4-9. Zero LM Plane Change; i versus Ω_d for Various Surface Stay Times

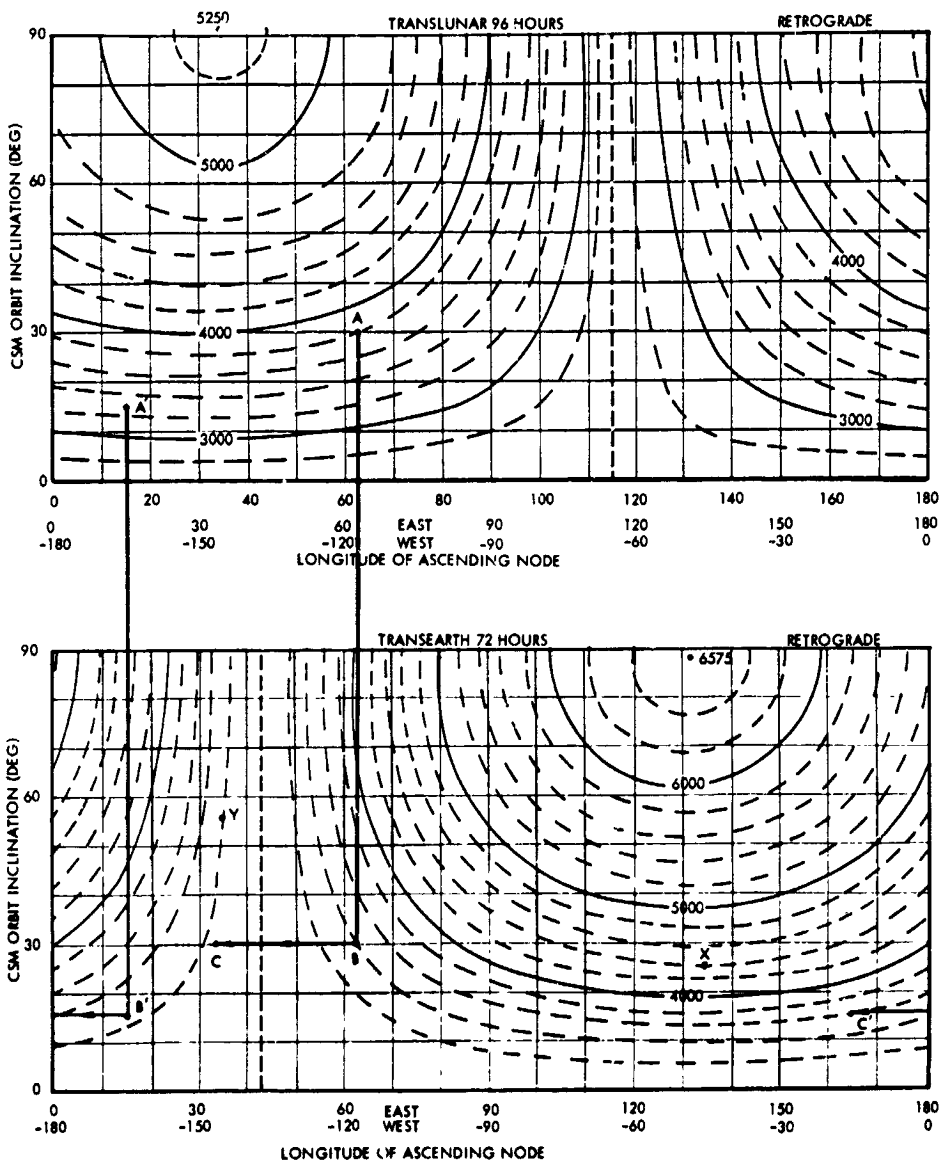

Figure 4-10. Translunar and Transearth ΔV for 96- and 72-Hour Flight Times, Respectively

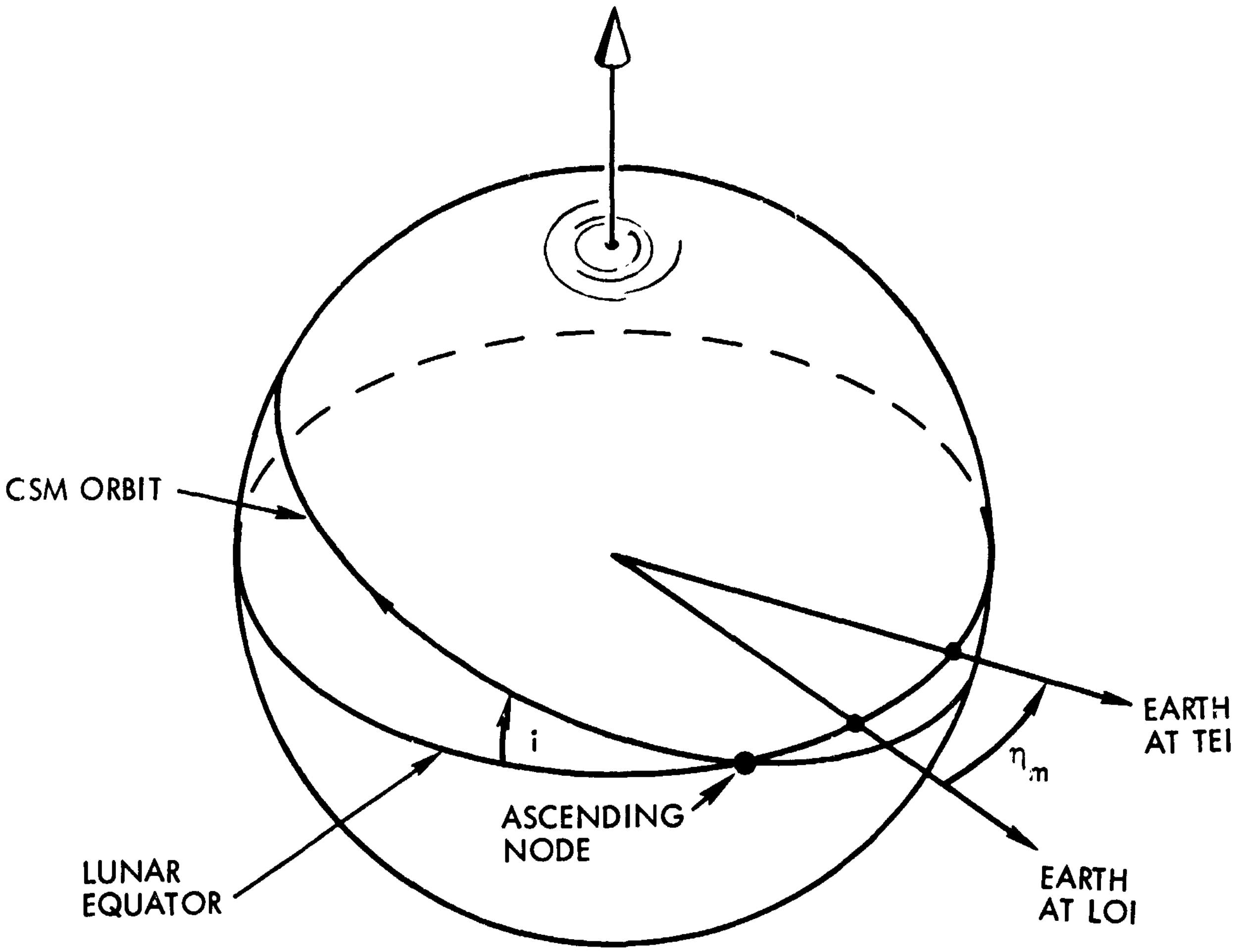

Figure 4-11. Lunar Orbit and Earth Moon Geometry at LOI and TEI

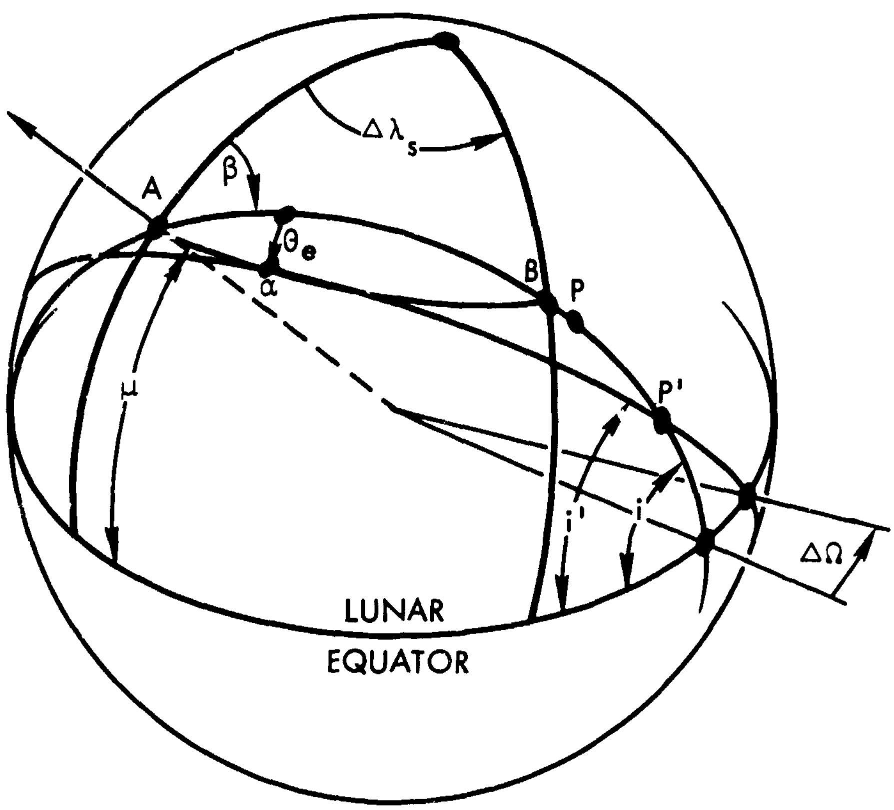

Figure 4-12. CSM Plane Change Geometry

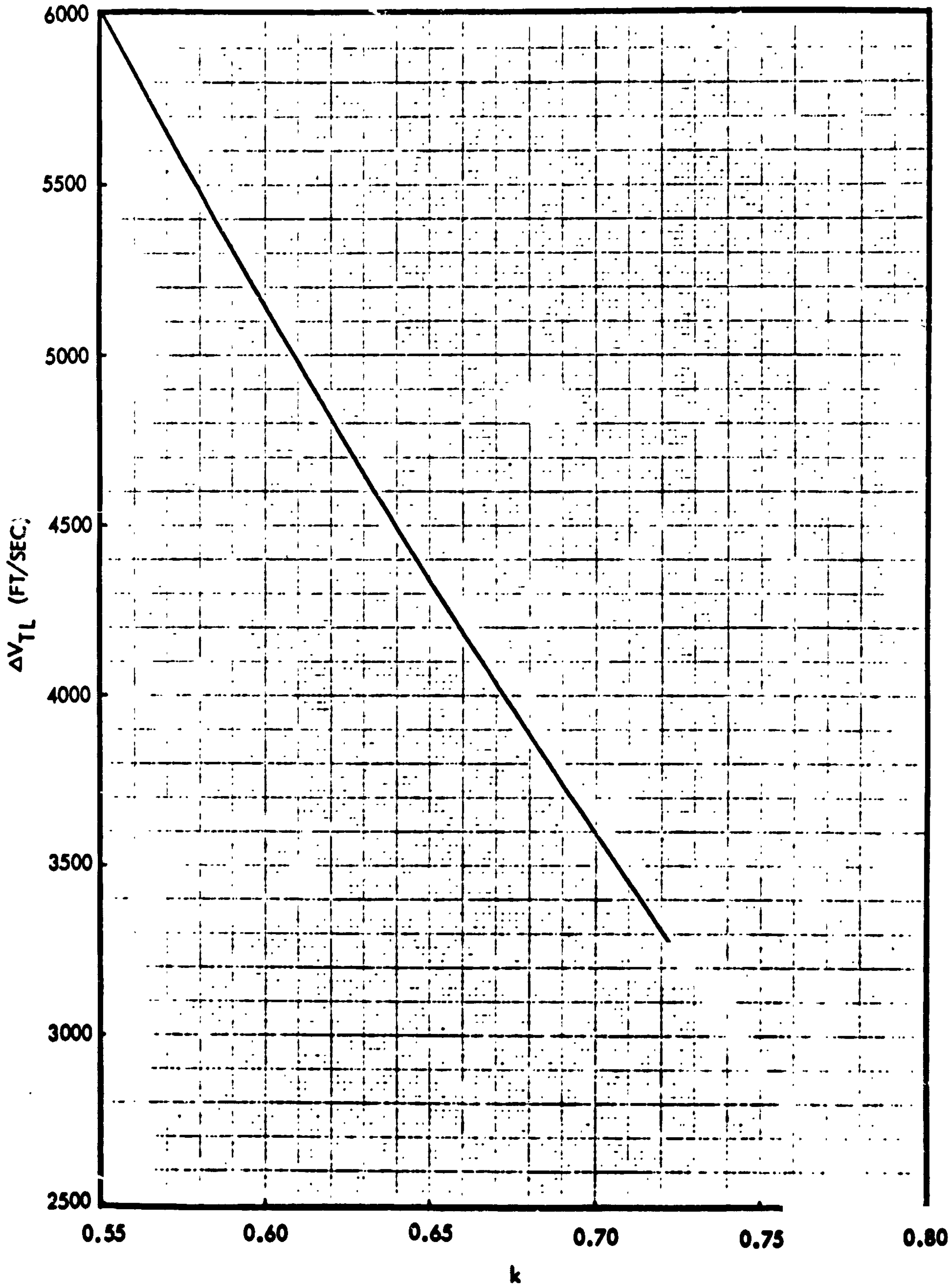

Figure 4-13. Translunar ΔV versus k

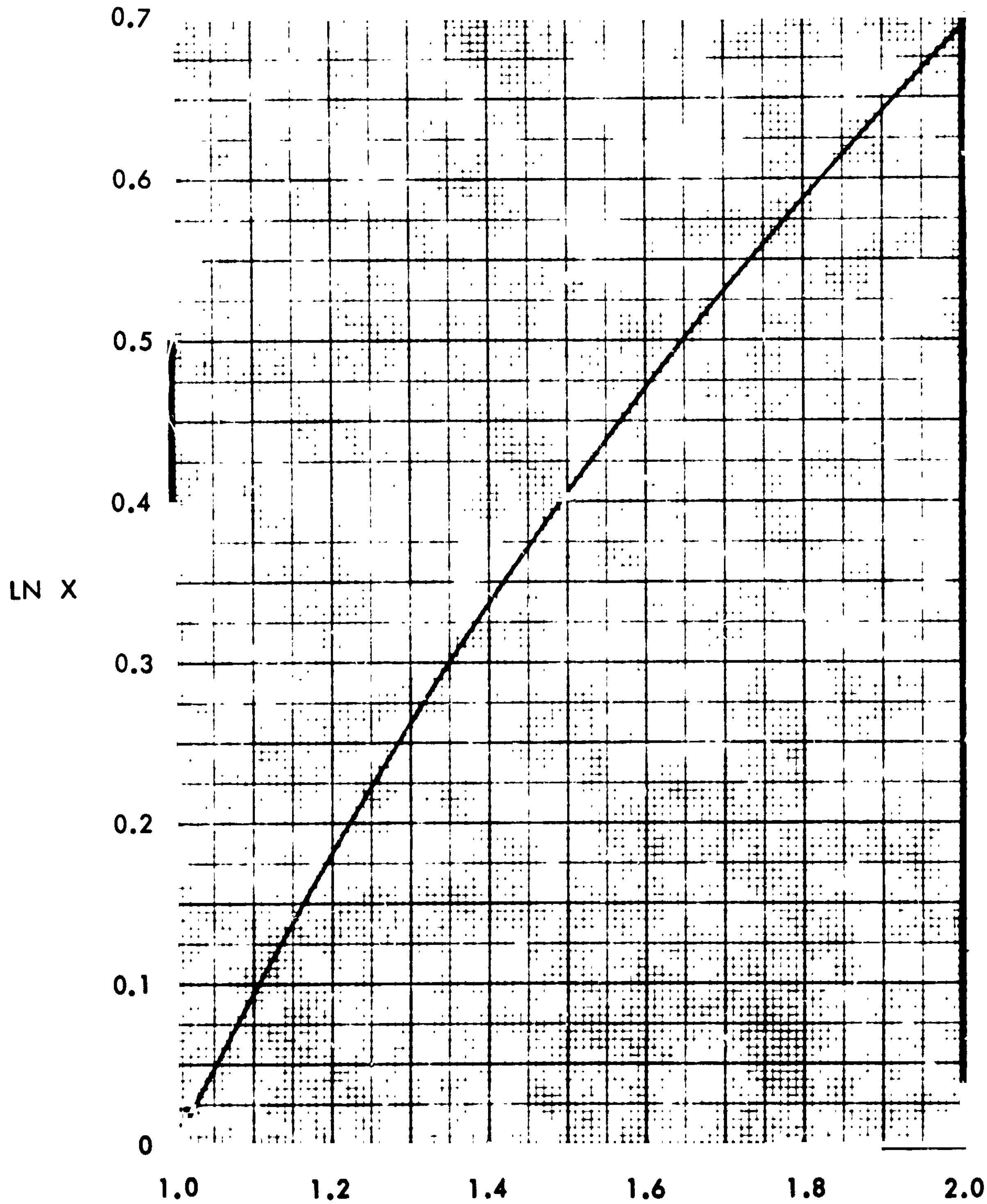

Figure 4-14. Ln X versus X

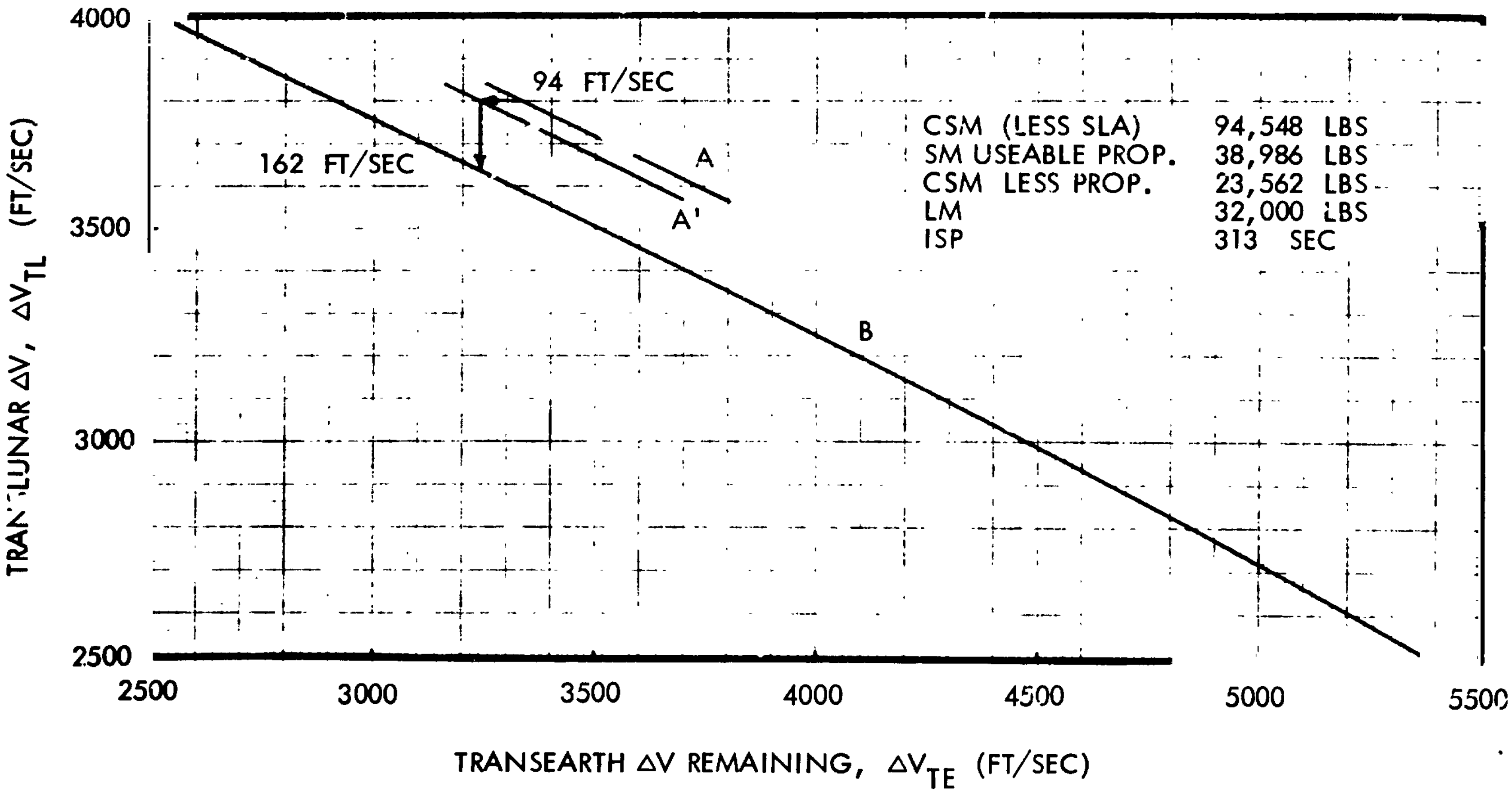

Figure 4-15. Spacecraft Performance Capability

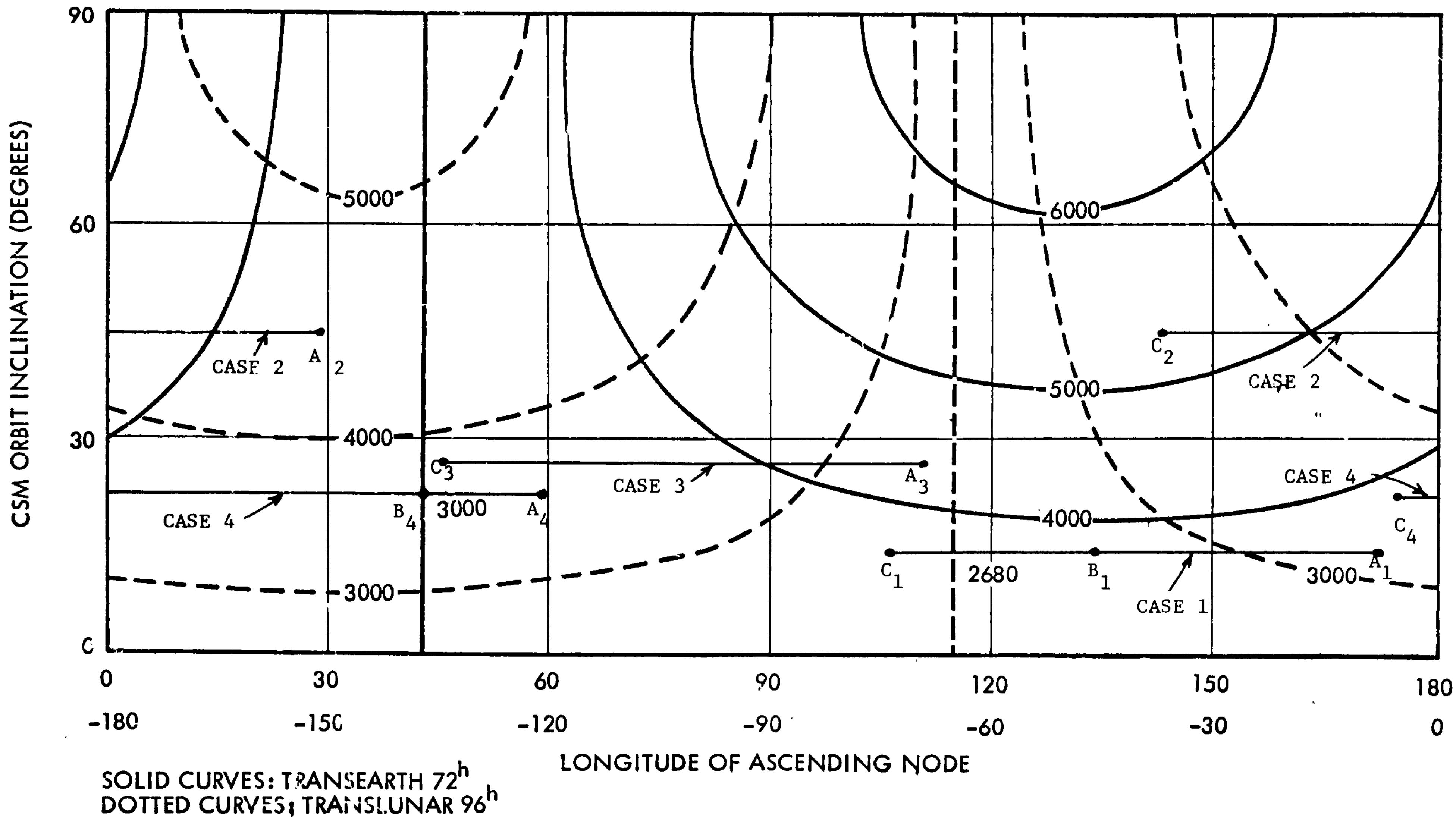

Figure 4-16. Sample Cases; or Continuous Abort Without CSM Plane Change

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5. BASIC MISSION ANALYSIS PROCEDURE

With the graphical data contained in this report and a full understanding of the geometrical and ΔV accessibility constraints, an inexhaustible variety of missions can be analyzed with relative ease. The procedure may be apparent from the discussion of Section 4; however, specific cases will be cited in the following sections to review the procedure details.

5.1 GRAPHICAL PROCEDURE

Two types of missions will be considered in the example cases. The first will be an accessibility analysis of a specific site, and the second will be the generation of an accessibility contour for an example mission.

5.1.1 Specific Site Analysis

The specific site selected for the example is Aristarchus, which is located at 47.5° W. longitude and 23.8° N. latitude. Two example cases will be considered for this site; Example 1 is the determination of a maximum allowable stay time, and Example 2 is the determination of a minimum achievable total mission duration.

Example 1: It is desired to determine the maximum surface stay time ach' ·able for Aristarchus with the following mission requirements:

- Maximum mission duration of 14 days
- Continuous abort capability with plane change to be performed by the CSM
- No LM plane change
- LM descent to occur 12 hours after LOI
- Site to be in CSM orbit plane at nominal (i. e., no abort occurrence) time of LM ascent
- TEI to nominally occur 12 hours after LM ascent
- Translunar and transearth flight times to be within the range of 60 to 132 hours
- Spacecraft configuration and midcourse ΔV requirements as shown in Curve B of Figure 4-15

The approach taken here will be first to assume a surface stay time and determine the optimum (minimum fuel) flight profile satisfying the mission requirements. If this is within the capability of the spacecraft, then a longer stay time is assumed, and the optimum profile determined; if not, then a shorter stay time is assumed. The maximum stay time is then determined by interpolation (or extrapolation) of the above data by finding that surface stay time which just depletes the spacecraft propellant for the minimum fuel flight profile. In accordance with the three basic steps of the graphical procedure, this is accomplished as follows:

Step I: Determination of Geometrical Constraints

The CSM orbit which satisfies the required geometry can be obtained from Figure 4-4; or more accurately*, from Equation (3) on page 4-12 and the equation in the footnote on page 4-7. The results of interest are listed in Table 5-1 for various surface stay times. The longitude of the CSM orbit ascending node is adjusted to account for the required 12-hour wait between LOI and LM descent (since the site must lie in the CSM orbit plane at LM descent). A 12-hour wait requires placing the ascending node at LOI 6.6 degrees further eastward.

Step II: Determination of ΔV Requirements

The graphical data necessary for the determination of the ΔV requirements are

- Translunar ΔV data (Figures 4-24 through 4-30)
- Transearth ΔV data (Figures 4-31 through 4-34)
- CSM plane change data (Figures 4-38 through 4-50)
- Spacecraft capability curve (Figure 4-15)

*The required CSM orbit inclination can also be obtained from the CSM plane change data.

Table 5-1. CSM Orbit Parameters, Aristarchus Example 1

	Surface Stay Time (day)			
	2	4	5	6
$\Delta\lambda s/2$ (deg)	13.2	26.4	33.0	39.5
i (deg)	24.4	26.2	27.7	29.8
Ω_d (deg)	103.2	116.4	123.0	129.5
Longitude of Ascending Node at LM Descent	55.7°E	68.9°E	75.5°E	82.0°E
Longitude of Ascending Node at LOI	62.3°E	75.5°E	82.1°E	88.6°E
Longitude of Ascending Node at Nominal TEI	22.8°E	9.6°E	3.0°E	3.7°W
Maximum Plane Change (deg) - From Figures 4-38 through 4-40, or Figures 4-44 through 4-46, or Equation (3)	0.6	2.4	3.9	6.0
ΔV for Maximum Plane Change (ft/sec) From Figure 4-50 or Equation (4)	55	220	360	550

The optimum (minimum ΔV) flight profile is to be found for each value of surface stay time considered. This is accomplished by assuming a translunar flight time and determining the transearth ΔV requirements for various transearth flight times to obtain the minimum. This is repeated for other translunar flight times to determine the over-all minimum ΔV flight profile.

For example, consider a surface stay time of 2 days. From Table 5-1, it is seen that the required CSM orbit is inclined 24.4 degrees with an ascending node longitude at LOI of 62.3° East. The ascending node longitude at TEI is 22.8° East corresponding to a CSM orbit stay time of 3 days. A translunar flight time of 132 hours is assumed. From Figure 4-30 it is found that the translunar ΔV required to achieve this CSM orbit is 3080 feet per second. From

Figure 4-15 it is found that the transearth ΔV available is 4320 feet per second. The transearth ΔV requirements for various flight times are now considered to determine the minimum transearth ΔV for the 132-hour translunar flight time. A 60-hour transearth flight time is considered. It is seen from Figure 4-31 (by overlaying upon the translunar curve) that the transearth ΔV immediately after LOI is 3880 feet per second, then decreases to a minimum of 3500 feet per second, and then increases to 3580 feet per second at the end of the 3-day CSM orbit stay time. It is noted that after LOI, the transearth ΔV decreases at a faster rate than the plane change ΔV is increasing. This is ascertained by comparing Figure 4-31 with Figures 4-38 and 4-50 (or Equation (4)). The worst abort case*, then, is immediately after LOI requiring 3880 feet per second. This gives a ΔV margin (ΔV remaining) or 440 feet per second. A 72-hour transearth flight time (Figure 4-32) is considered. It is seen that the transearth ΔV immediately after LOI is 3300 feet per second, decreases to a minimum of 3000 feet per second, and increases to 3300 feet per second at the end of the 3-day CSM orbit stay time. The worst abort case*, then, is that in which a LM abort is required at the maximum plane change angle and in which TEI occurs at the nominal time of 3 days after LOI. The maximum plane change angle is 0.6 degree, requiring a CSM ΔV of 55 feet per second. This also reduces the CSM orbit inclination by 0.6 degree, so that the transearth ΔV is now reduced from 3300 to 3290 feet per second. The maximum ΔV for this case is then 3290 + 55 = 3345 feet per second. The ΔV margin is now 995 feet per second. The above steps are repeated until the minimum transearth ΔV is found for the assumed translunar flight time of 132 hours.

Other translunar flight times are assumed for the given stay time of 2 days, and the over-all minimum ΔV is then determined for that flight time. The optimum flight profile for the 2-day surface stay time corresponds to translunar and transearth flight times of 132 and

*See Section 4.2.5

84 hours, respectively, resulting in a ΔV margin of 1000 feet per second.

The above is repeated for surface stay times of 4, 5, and 6 days (in which the ΔV margin for the minimum ΔV flight profile becomes negative, so that 6 days cannot be achieved).

Step III: Interpretation of Results

The results of the above computations are presented in Figure 5-1 in which the ΔV margin for the minimum ΔV flight profile is plotted versus surface stay time. It is seen that the maximum surface stay time is 5.7 days which is that corresponding to a zero ΔV margin (total propellant depletion). The translunar and transearth flight times for the 5.7-day surface stay time are approximately 111 and 64 hours, respectively, resulting in a total mission duration of approximately 14 days. The rapid decrease in ΔV margin after 4 days is caused by the 14-day total mission duration constraint.

<u>Example 2</u>: It is desired to determine the minimum total mission duration flight profile for a 2-day surface stay time at Aristarchus and for the same mission requirements of Example 1.

The minimum total mission duration flight profile is defined here to mean the minimum combined translunar and transearth flight times for the nominal mission in which no abort occurs. However, continuous abort capability is still required.

The approach taken is to determine the transearth ΔV available after LOI versus translunar flight time. This is done using the translunar ΔV curves (Figures 4-24 through 4-30) to determine the ΔV required (versus flight time) to attain the required CSM orbit (from Table 5-1: inclination of 24.4 degrees and ascending node longitude of 62.3° E.) and the spacecraft capability curve of Figure 4-15. The results are plotted in Figure 5-2. In addition, the transearth ΔV at nominal TEI (node longitude = 22.8° E.) versus transearth flight time with and without abort is plotted in Figure 5-2. For this mission, the worst abort case for transearth flight times of 72 to 120 hours is one in which a LM abort is required for the maximum CSM plane change and in which TEI occurs at the nominal time. The two curves for abort and no abort in Figure 5-2 are vertically displaced from 35 to 45 feet per second, which corresponds to the difference between the maximum plane change ΔV of 55 feet per second and the reduction in transearth ΔV at nominal TEI of 10 to 20 feet per second.

The curves of Figure 5-2 are interpreted as follows. For a translunar flight time of 97.5 hours, for example, the ΔV available after LOI is 3400 feet per second. This means that a continuous abort is always possible and that abort (worst case) transearth flight times between 67.5 and 97.5 hours can be achieved. For the nominal case of no abort, the minimum transearth flight time that can be achieved with 3400 feet per second ΔV is 65.5 hours giving a combined translunar plus minimum transearth flight time of 165 hours.

It is noted from Figure 5-2 that the minimum allowable translunar flight time is 95.5 hours, which results in a transearth ΔV availability of 3320 feet per second. This corresponds to the minimum allowable ΔV to provide continuous abort capability. This case corresponds to a transearth abort flight time of 80 hours. For the nominal TEI time, 3420 feet per second gives a transearth flight time of 69.5 hours giving a combined total of 165 hours.

The minimum total flight time can be found by plotting the sum of the translunar and transearth flight times versus the translunar flight time (for translunar flight times greater than 95.5 hours to assure continuous abort capability) as done above. A minimum combined flight time of 163 hours is achieved corresponding to a translunar flight time between 98 and 99 hours and a transearth flight time between 65 and 64 hours, respectively. However, the accuracy of the curves within the region of 60 to 72 hours is questionable, so that the determination of the minimum flight time by inspection of Figure 5-2 is adequate.

5.1.2 Accessibility Contour Generation

An accessibility contour is the locus of points that separate the accessible and inaccessible areas of the lunar surface. The generation of this contour requires the generation of the geometrical (Step I) and the ΔV (Step II) constraint curves.

Two important features concerning these constraint curves will now be re-stated. A geometrical constraint curve (Figures 4-19 through 4-23) shows all CSM orbits in the form of inclination versus node longitude displacement (relative to the site longitude) that satisfy the stay time, LM plane change capability, and continuous abort requirement. These geometric constraint curves are independent of site longitude, but can be interpreted as those corresponding to site longitudes of zero. For example, if a CSM orbit node displacement of 120 degrees is considered for a site longitude of zero, the ascending node of the orbit is then 120° East*. On the other hand, if a site longitude of 20° West is considered, then the orbit node will be 120° east of the site corresponding to a longitude of 100° East.

Now the question arises: What CSM orbits that satisfy the geometric constraints can be achieved by the spacecraft? The ΔV constraint curve answers this question, since it shows the locus of all CSM orbits (i versus Ω) that deplete all available CSM propellant.

The generation (Step II) of the ΔV constraint curve and the manipulation (Step III) of the geometric and ΔV constraint curves to obtain the site accessibility contour will now be described for the following example. Consider the following mission:

- Total mission duration = 14 days
- Translunar flight time = 96 hours
- Transearth flight time = 72 hours
- Time in lunar orbit (retrograde) = 7 days
- Surface stay time = 5 days
- LM descent 1 day after LOI

*It is recalled that only retrograde orbits are considered, so that the CSM orbit ascending node will always be east of the site.

- LM plane change capability = 4 degrees
- No CSM plane changes
- Continuous abort capability
- Translunar and transearth midcourse ΔV of 162 and 94 feet per second, respectively
- Spacecraft configuration of Figure 4-15

Step I: Geometric Constraint Curve

Figure 4-3 is the geometric constraint curve to be used for this mission.

Step II: Generation of the ΔV Constraint Curve

The graphical data needed to generate the ΔV constraint curve are

- Translunar ΔV curve for 96-hour flight time (Figure 4-27)
- Transearth ΔV overlay curve for 72-hour flight time (Figure 4-32)
- Spacecraft capability curve (Figure 4-15)

The 72-hour transearth ΔV curve is now overlayed on the 96-hour translunar ΔV curve with scales coincident as shown in Figure 5-3. To determine the locus of CSM orbits that consume all available propellant, a translunar ΔV of 3000 feet per second is assumed. From the spacecraft capability curve of Figure 4-15, the transearth ΔV available is 4470 feet per second. A 7-day orbit stay time corresponds to a westward node shift of 92.3 degrees, so that CSM orbit is to be found in which the maximum transearth ΔV during this time interval if 4470 feet per second. This is conveniently done by cutting or marking the edge of a piece of cardboard or paper equal to 92.3 degrees on the longitude scale. The right edge of this paper is shifted along the translunar ΔV contour equal to 3000 feet per second until the orbit is found in which the maximum transearth ΔV for that CSM corresponding to point A will vary as the node shifts from A to C in the 7 days. The maximum ΔV corresponding to point B is seen to be less than the required 4470 feet per second. This line is now shifted upward, keeping the right edge on the 3000-foot per second translunar ΔV contour until the maximum transearth ΔV is 4470 feet per second, corresponding to point B'. That orbit with inclination and node corresponding to point A', then, is a point on the ΔV accessibility constraint curve. It is convenient to place a vellum on the overlays and mark these points. This process is repeated by assuming other values of ΔV_{TL} until the curve of Figure 5-4 is obtained. The interpretation of this curve is that all CSM orbits in the region above the curve cannot be achieved for the spacecraft of Figure 4-15.

Step III: Generation of Site Accessibility Contour

Site accessibility is determined by overlaying the ΔV constraint curve (vellum) Figure 5-4, on the geometrical constraint curve, Figure 4-3, as shown in Figure 5-5. Figure (a) corresponds to a site longitude of 3° East (the position of the zero of the geometrical constraint curve on the scale of the ΔV constraint curve indicates the site longitude). It is noted in Figure (a) that site latitudes from zero to 36 degrees (point A) can be achieved for a site longitude of 3° East. Figure (b) shows a site longitude of 20° East (or 160 degrees West), in which all latitudes are accessible up to a maximum of 26 degrees. For a longitude of 5° West (or 175° East) the maximum latitude is 34 degrees (Figure c). For 20° West, or 160° East, the maximum latitude is 26 degrees.

As the overlays are displaced relative to each other, and the latitudes recorded, the resulting accessibility contour of Figure 5-6 is obtained. It should be pointed out that the resulting curve of Figure 5-6 was constructed with the assumption that LM descent occurred at LOI. For the example case, then, in which the LM descends one day after LOI, the curve of Figure 5-6 must be shifted to the left 13.2 degrees. For example, if there is no waiting period between LOI and LM descent, the maximum achievable site latitude for a 50-degree East longitude is 20 degrees (from Figure 5-6). For a one day wait, the 20 maximum latitude corresponds to a longitude of 63.2 degrees East.

5.2 MISSION ANALYSIS CONSIDERATIONS

A lunar accessibility analysis will, in general, fall under one of the following categories:

- Accessibility Contour Generation
- Specific Site Analysis
- Parameter Optimization
- Mission Trade-offs

5.2.1 Accessibility Contour Generation

In general, an accessibility contour is generated for a given spacecraft capability to determine just what portion of the lunar surface is accessible to satisfy given mission requirements. Such a profile may appear like that shown in Figure 5-6. The generation of this contour is discussed in Section 5.1.2. The following questions concerning the characteristics of this accessibility contour may arise.

1) How does a given change in LM plane change capability, orbit stay time, surface stay time, spacecraft capability, ΔV budget, mission duration, translunar flight time, transearth flight time, etc., change the accessibility contour?

2) How does a change in abort requirements affect the contour?

3) How does a CSM plane change maneuver affect the contour?

4) How does positioning the surface stay time interval within the CSM orbit stay time interval affect the contour?

These questions can be answered by use of the graphical procedure with appropriate changes in values or requirements.

The generation of various contours are extremely useful to the mission analyst in that he can develop an understanding of the relationships between accessibility and mission profile modifications.

5.2.2 Specific Site Analysis

If a mission planner is concerned with designing a mission with respect to a given site, then three basic questions become apparent.

1) Is this site attainable for a given spacecraft capability?

2) What is the minimum total ΔV required to attain accessibility for a given mission profile?

3) How does the total ΔV vary with mission changes or parameter variations?

Some of the above questions may possibly be answered by any contours that may have been previously generated. For example, if an accessibility contour were generated based upon spacecraft performance alone, then question (1) can readily be answered. If accessibility contours were generated for various values of propellant margin for the mission profile and spacecraft capability, then question (2) can be answered directly. Many specific questions concerning a given site are readily answered by the graphical procedure.

5.2.3 Parameter Optimization

Several optimization considerations which may cause concern for a given mission are

1) Maximization of stay time for a specific site

2) Minimization of total ΔV

3) Minimization of translunar, transearth or total mission duration

These optimizations can be performed with the basic procedure, although several iterations may be necessary to achieve optimization. The disadvantage of any additional time that may be necessary to perform a specific optimization would be offset by the advantage of gaining insight into the relationships between the parameters varied and site accessibility.

5.2.4 Mission Trade-offs

Although minimum total ΔV is one of the goals of a mission design, there are several trade-offs to be considered for a mission under consideration, some of which may be

- Surface stay time versus ΔV penalty
- ΔV gained by performing a CSM plane change versus desirability of the additional SFS burn
- Accessibility enhancement versus relaxation of continuous abort requirements (such as intermittent abort)

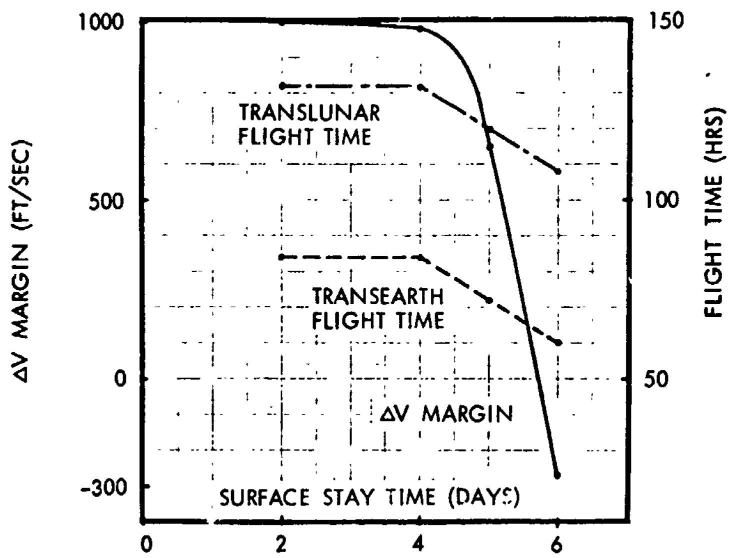

Figure 5-1. ΔV Margin versus Surface Stay Time Aristarchus, Example 1

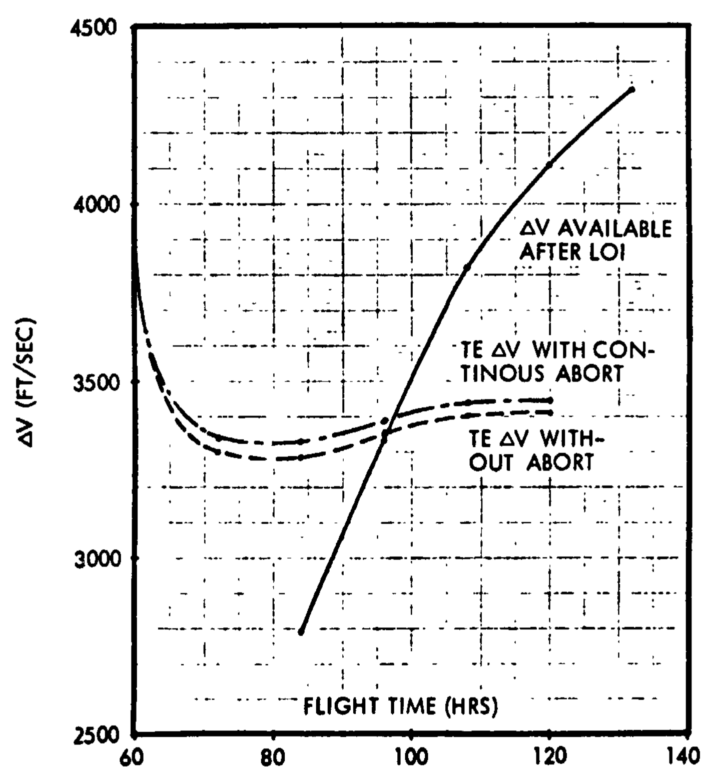

Figure 5-2. ΔV Requirements versus Flight Time - Aristarchus, Example 2

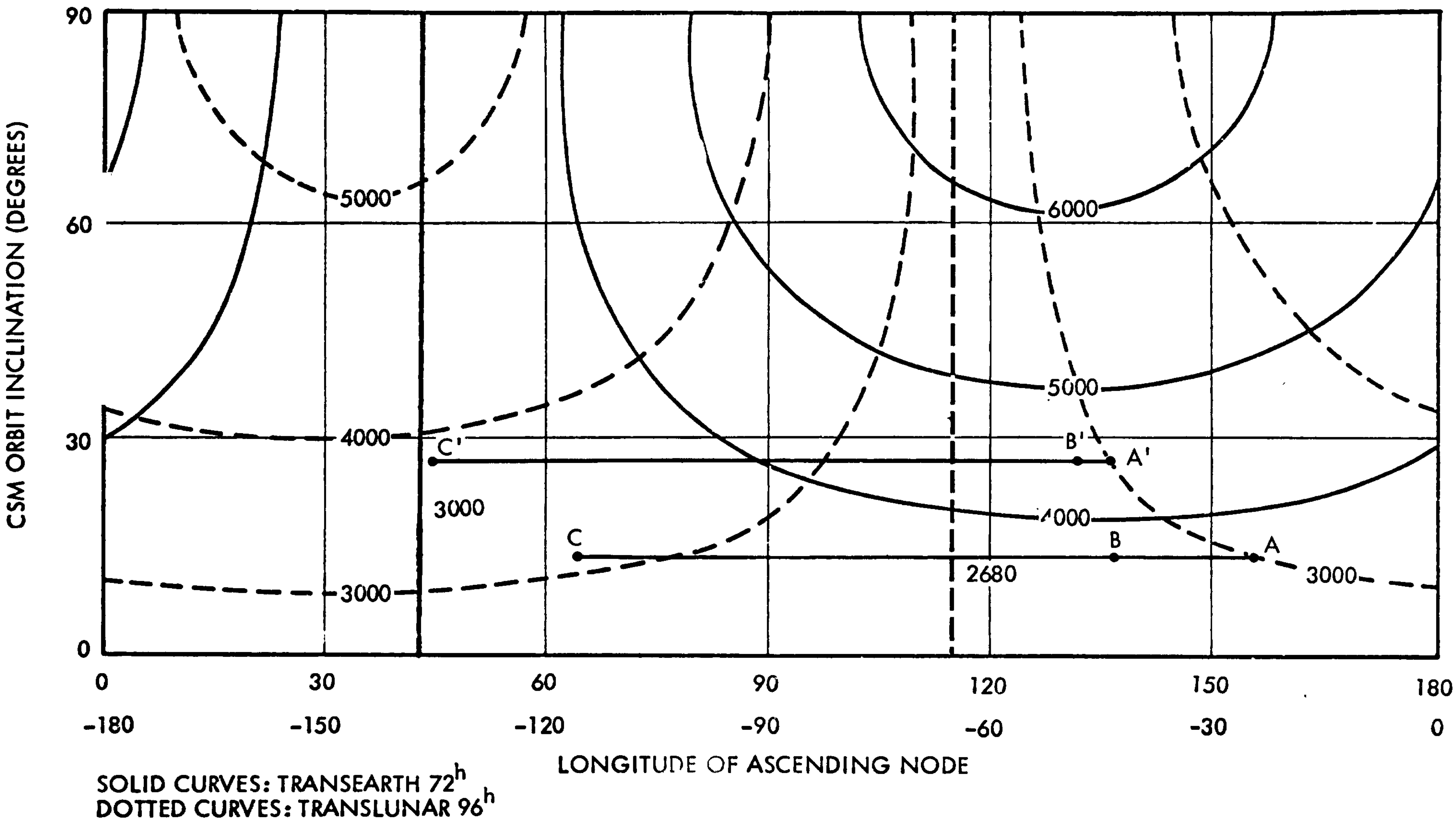

Figure 5-3. Procedure Example for Generating ΔV Constraint Curve

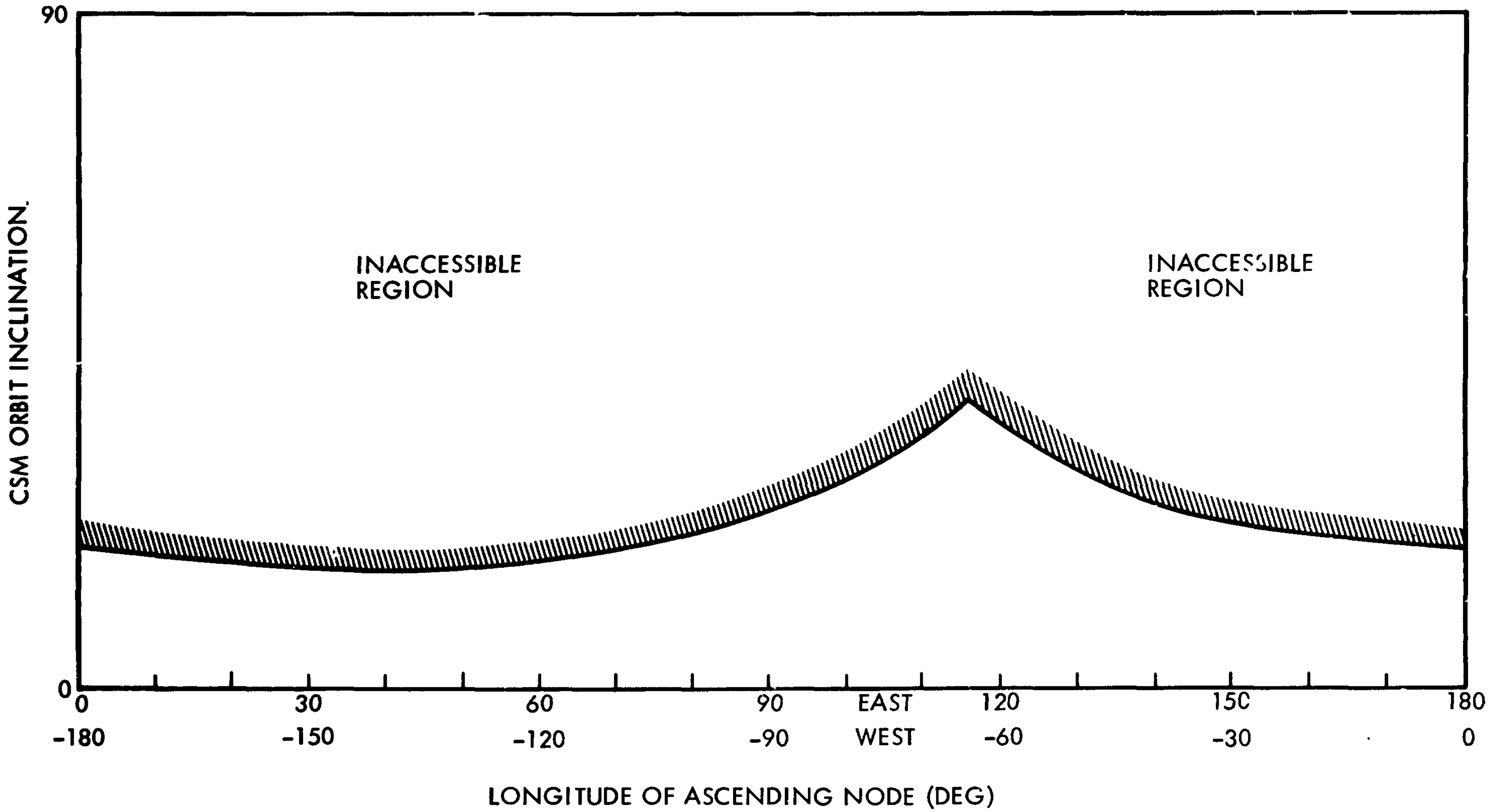

Figure 5-4. ΔV Constraint Curve for Example Mission

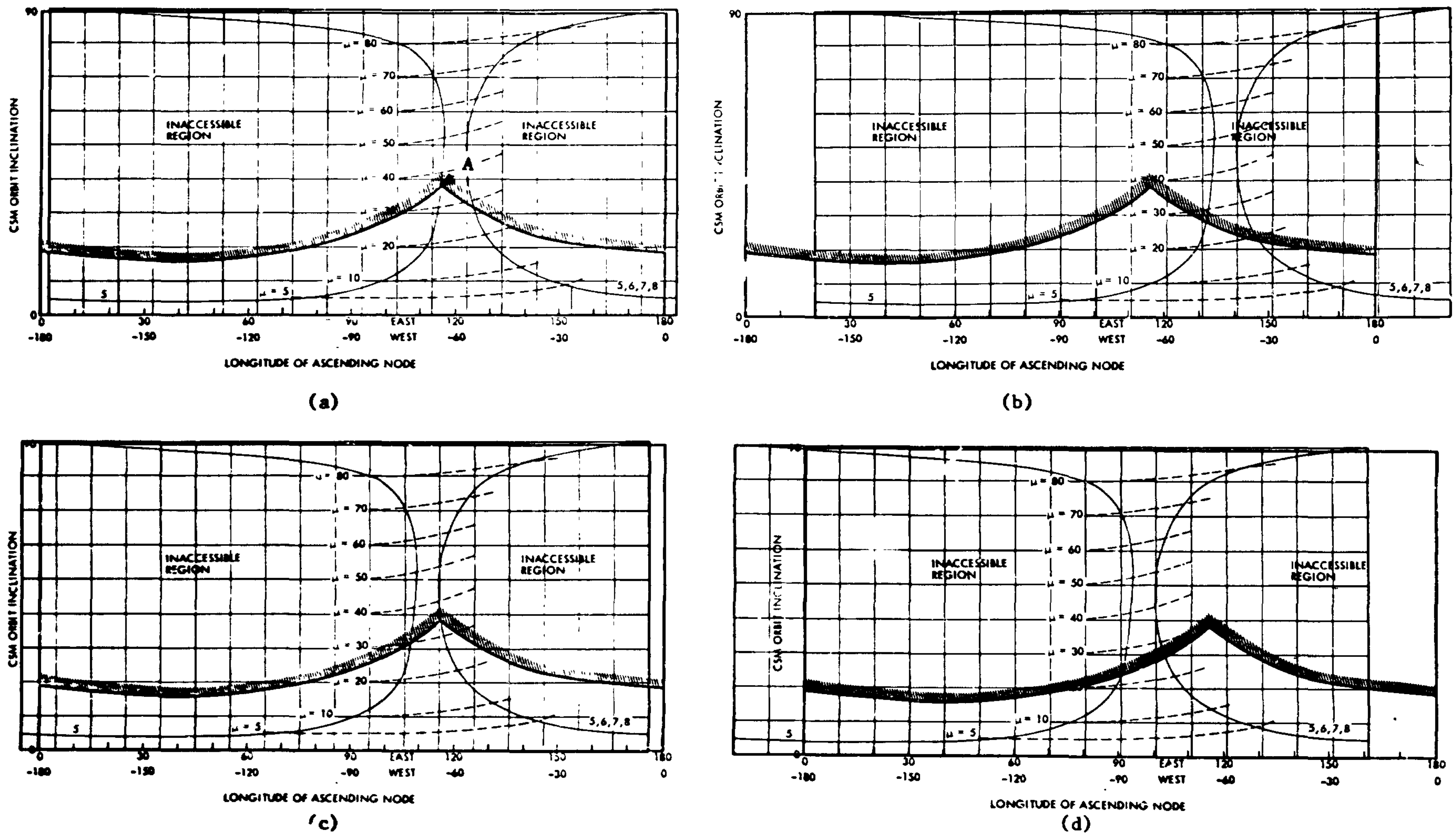

Figure 5-5. Procedure Examples for Generating Site Accessibility Contour

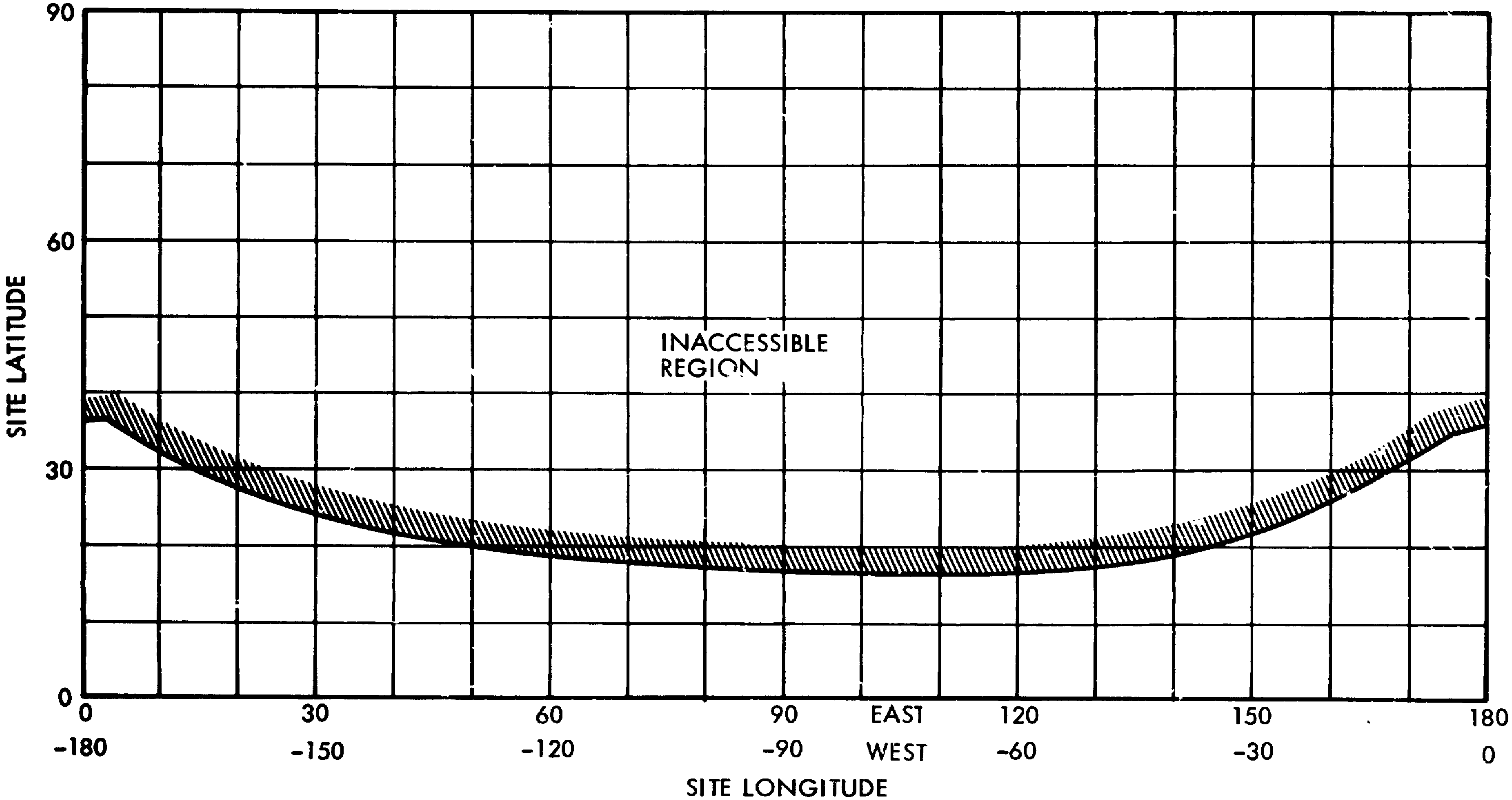

Figure 5-6. Site Accessibility Contour for Example Mission

6. CONCLUSIONS AND REMARKS

The usefulness of the basic graphical site accessibility analysis procedures discussed above for mission analysis and planning purposes is apparent. The accuracy, considering the assumptions upon which this simplified procedure is based, is sufficiently good to allow the mission planner to develop insight into the nature and extent of the effects of the many mission requirements and constraints upon lunar site accessibility. If, however, more accuracy is desired, then the data and procedures presented in Volume II can be used.

In addition, it is expected that computer program development activity will be necessary for accurate mission analysis and planning for future Apollo and AAP missions. Insight gained from a thorough knowledge of the lunar accessibility analysis technique described in these two volumes including the associated limitations will be valuable in determining what these program development requirements should be.

APPENDIX

Working Graphs

(Figures 4-18 through 4-51)

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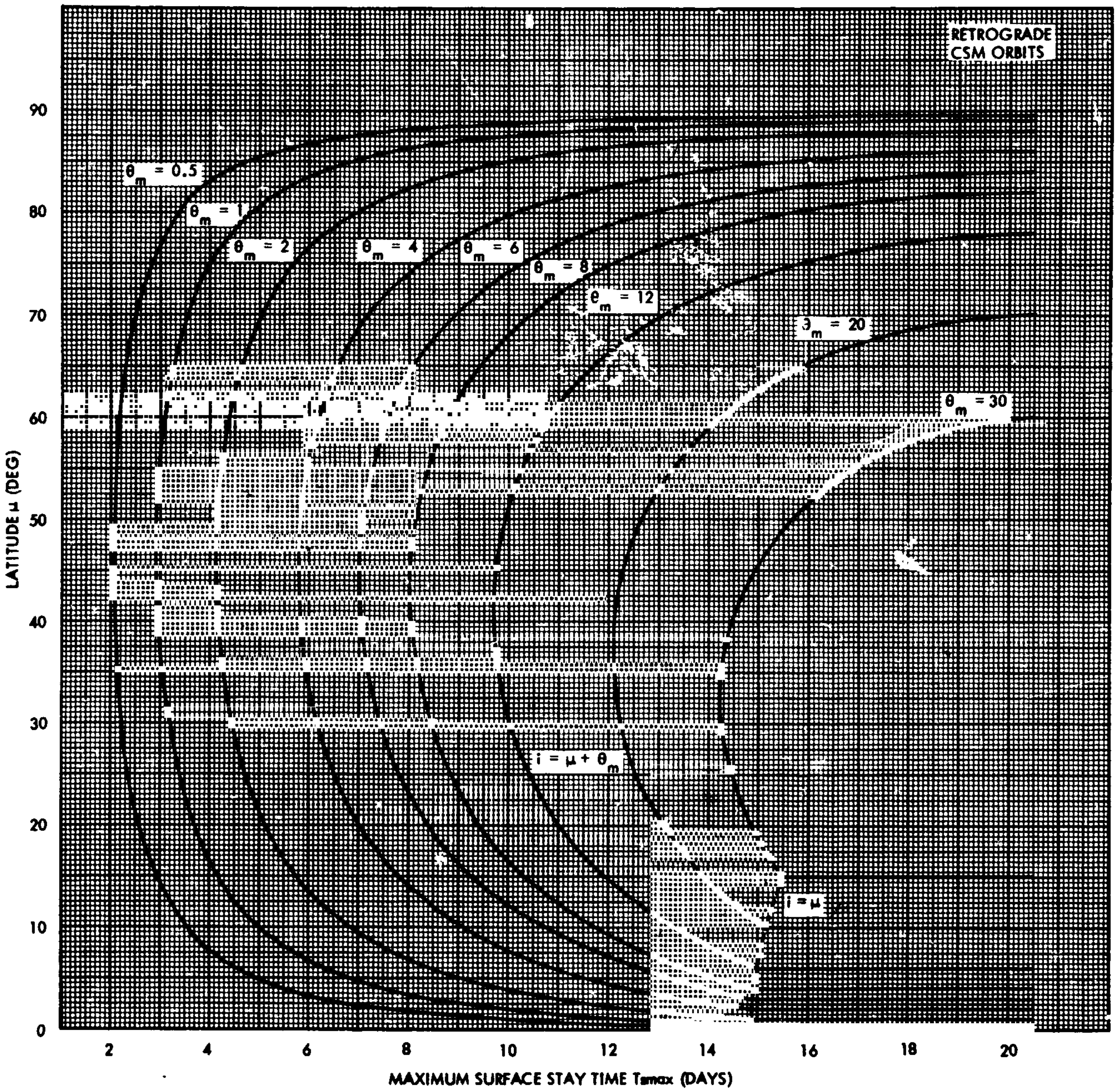


Figure 4-18. Maximum Lunar Surface Stay Time versus Site Latitude - Retrograde Orbits

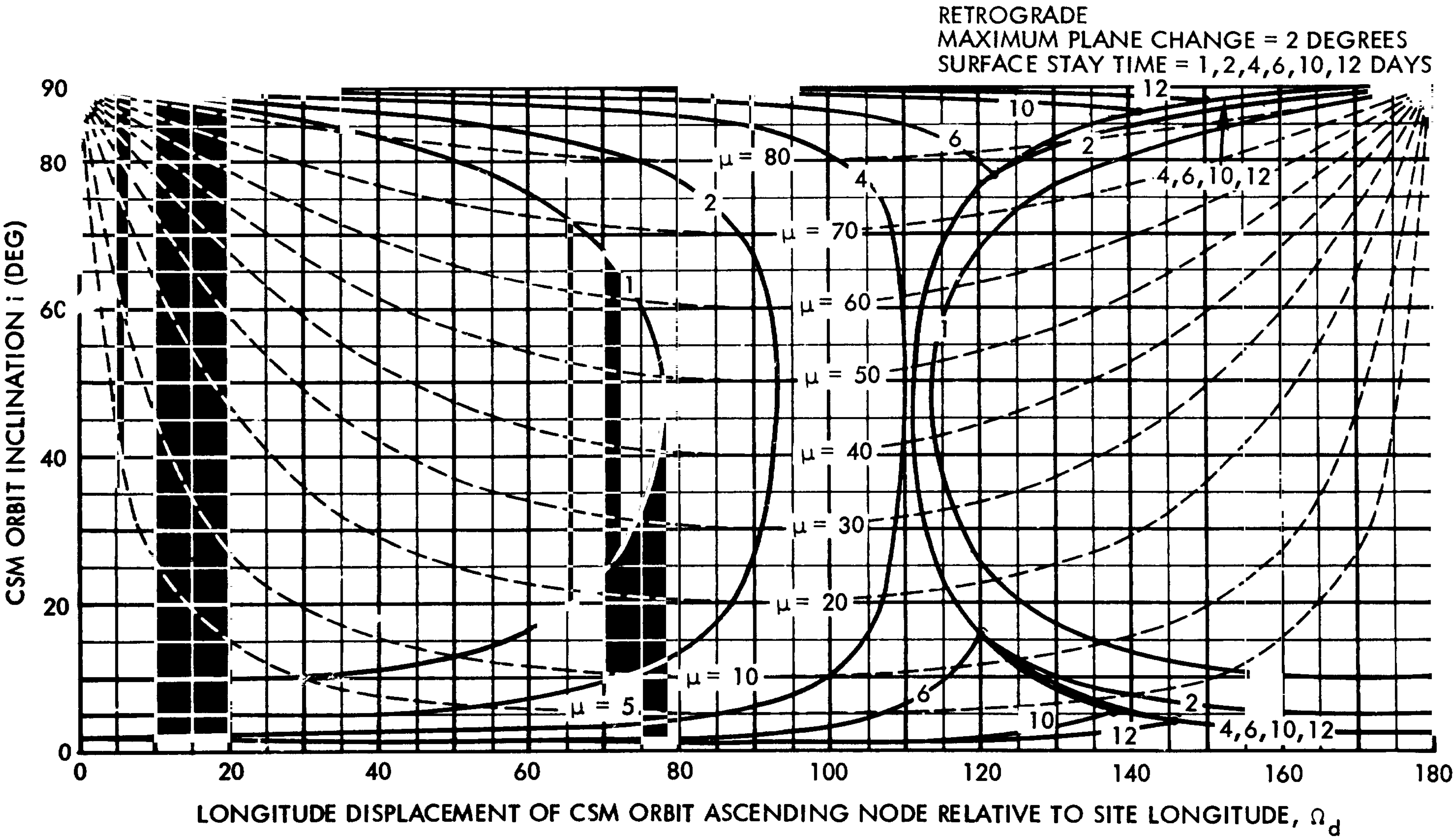


Figure 4-19. Geometric Constraints for LM Plane Change Capability of 2 Degrees and Surface Stay Times of 1, 2, 4, 6, 10, and 12 Days

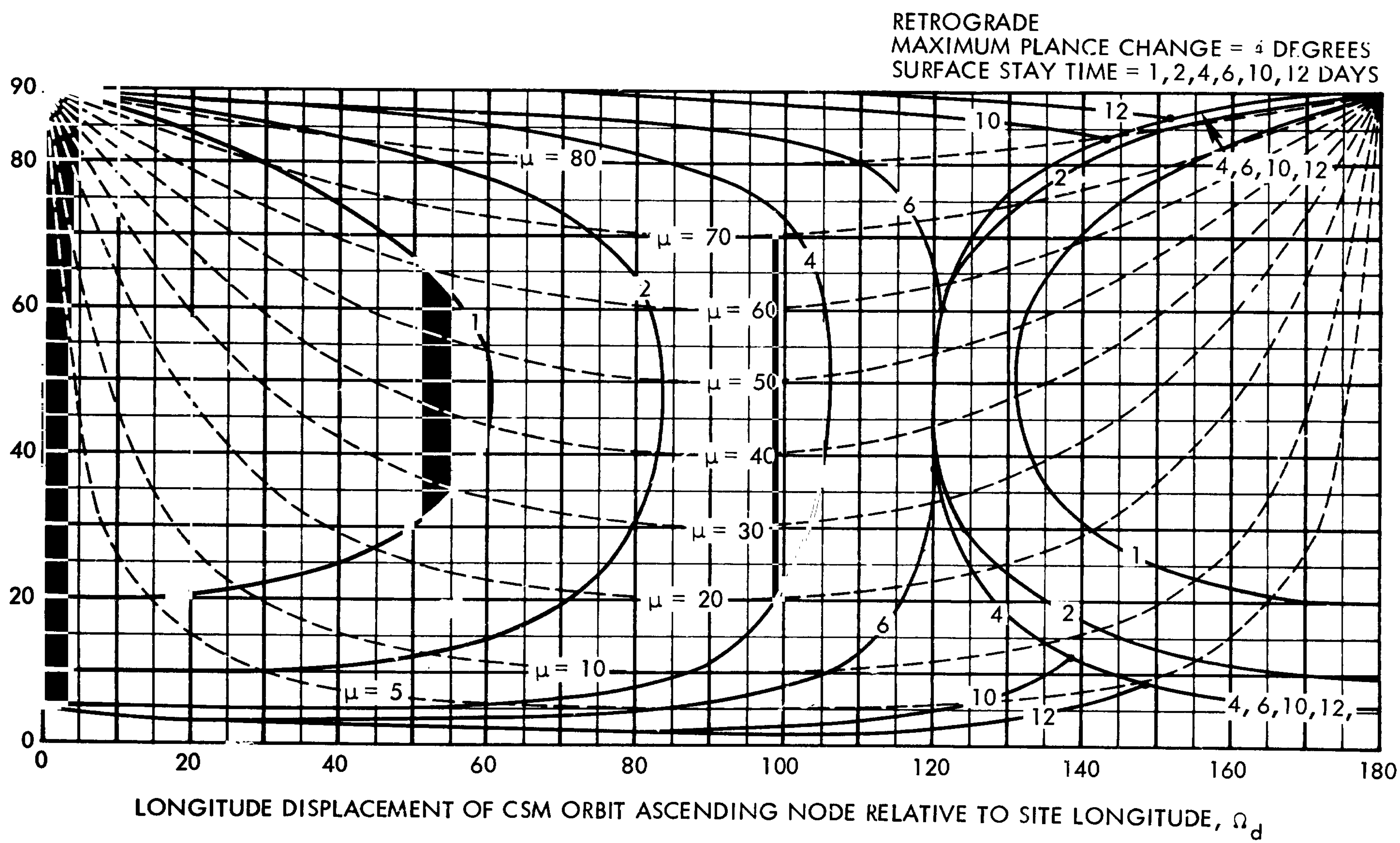


Figure 4-20. Geometric Constraints for LM Plane Change Capability of 4 Degrees and Surface Stay Times of 1, 2, 4, 6, 10, and 12 Days

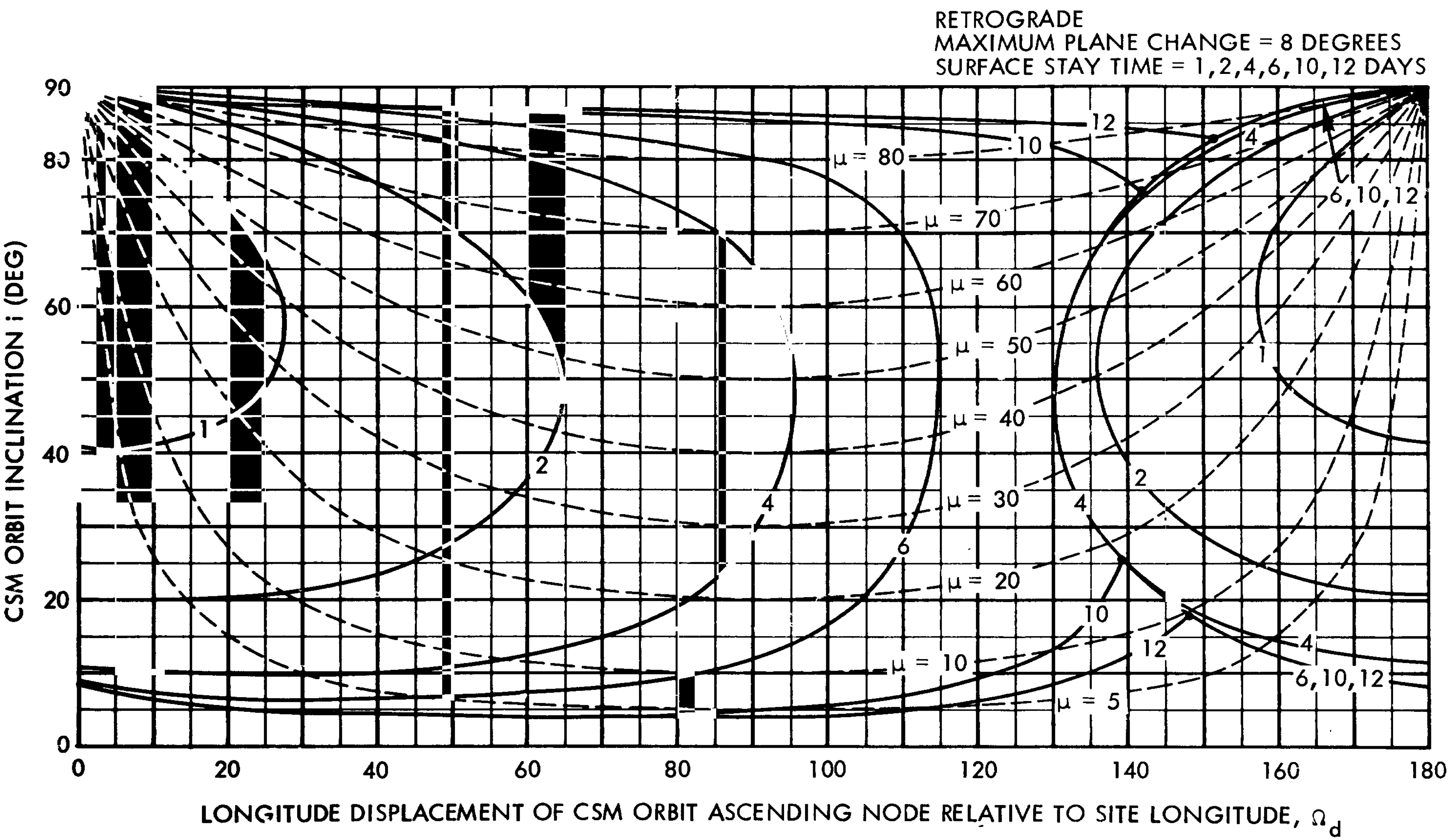


Figure 4-21. Geometric Constraints for LM Plane Change Capability of 8 Degrees and Surface Stay Times of 1, 2, 4, 6, 10, and 12 Days

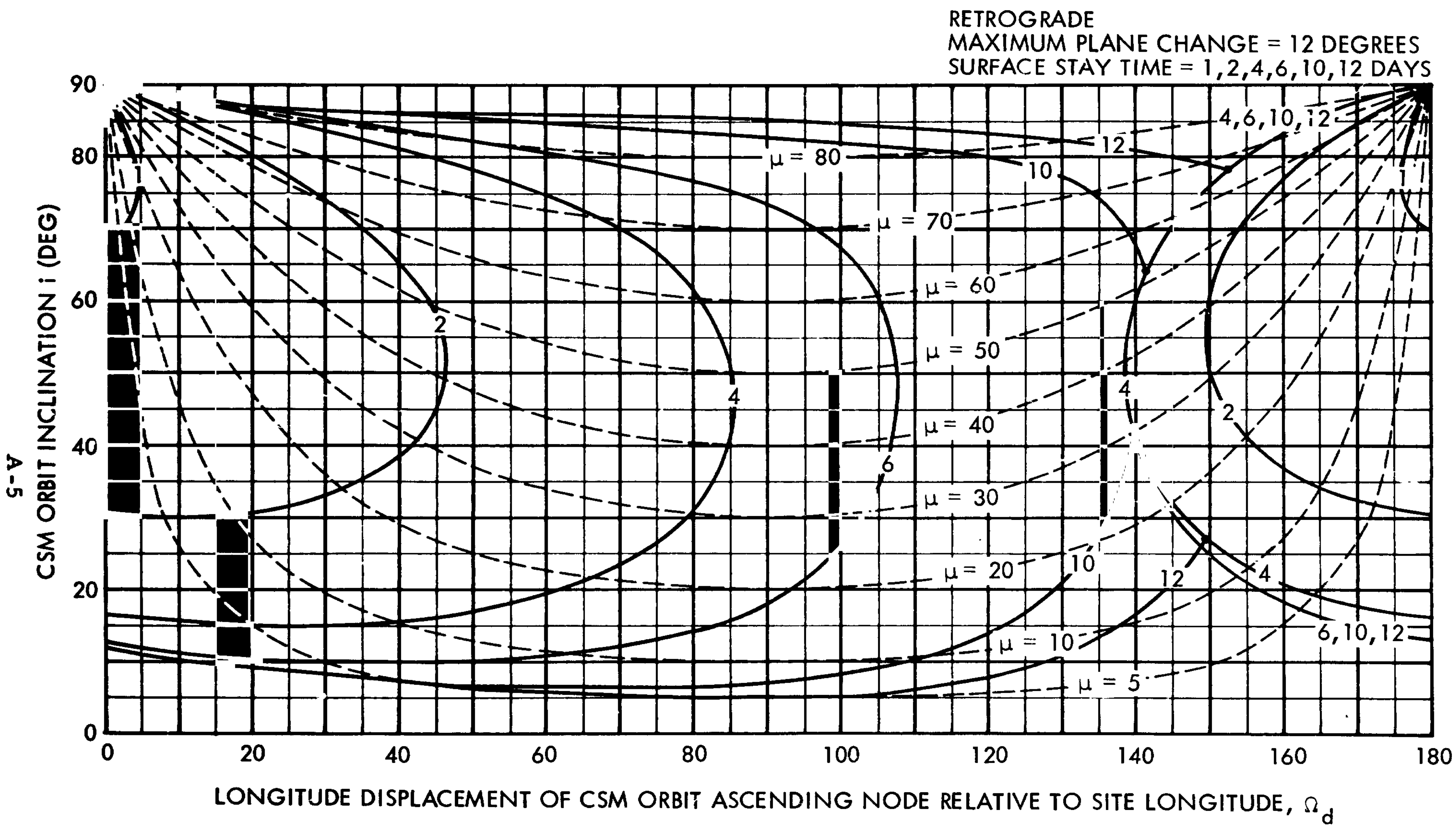


Figure 4-22. Geometric Constraints for LM Plane Change Capability of 12 Degrees and Surface Stay Times of 1, 2, 4, 6, 10, and 12 Days

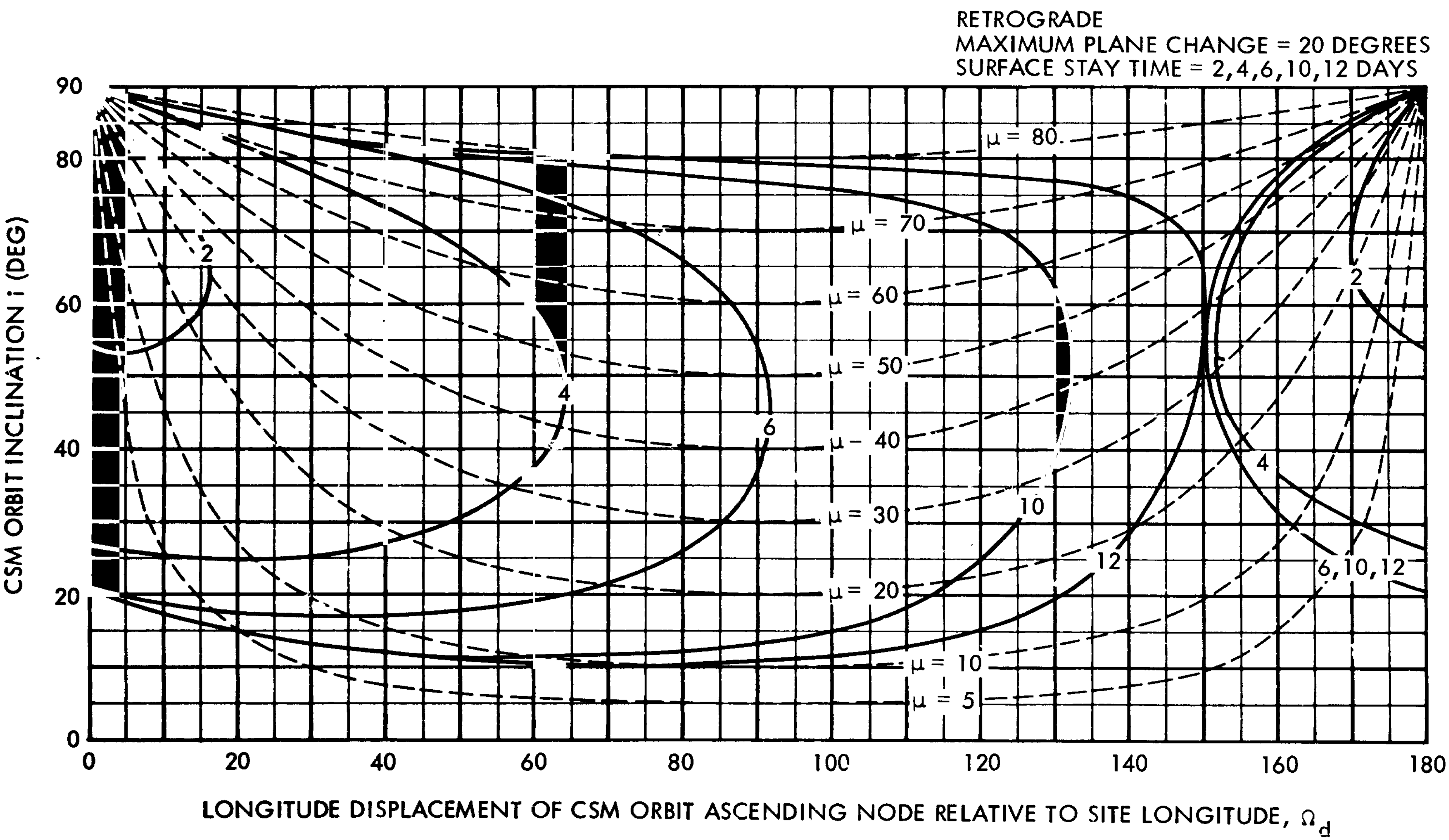


Figure 4-23. Geometric Constraints for LM Plane Change Capability of 20 Degrees and Surface Stay Times of 2, 4, 6, 10, and 12 Days

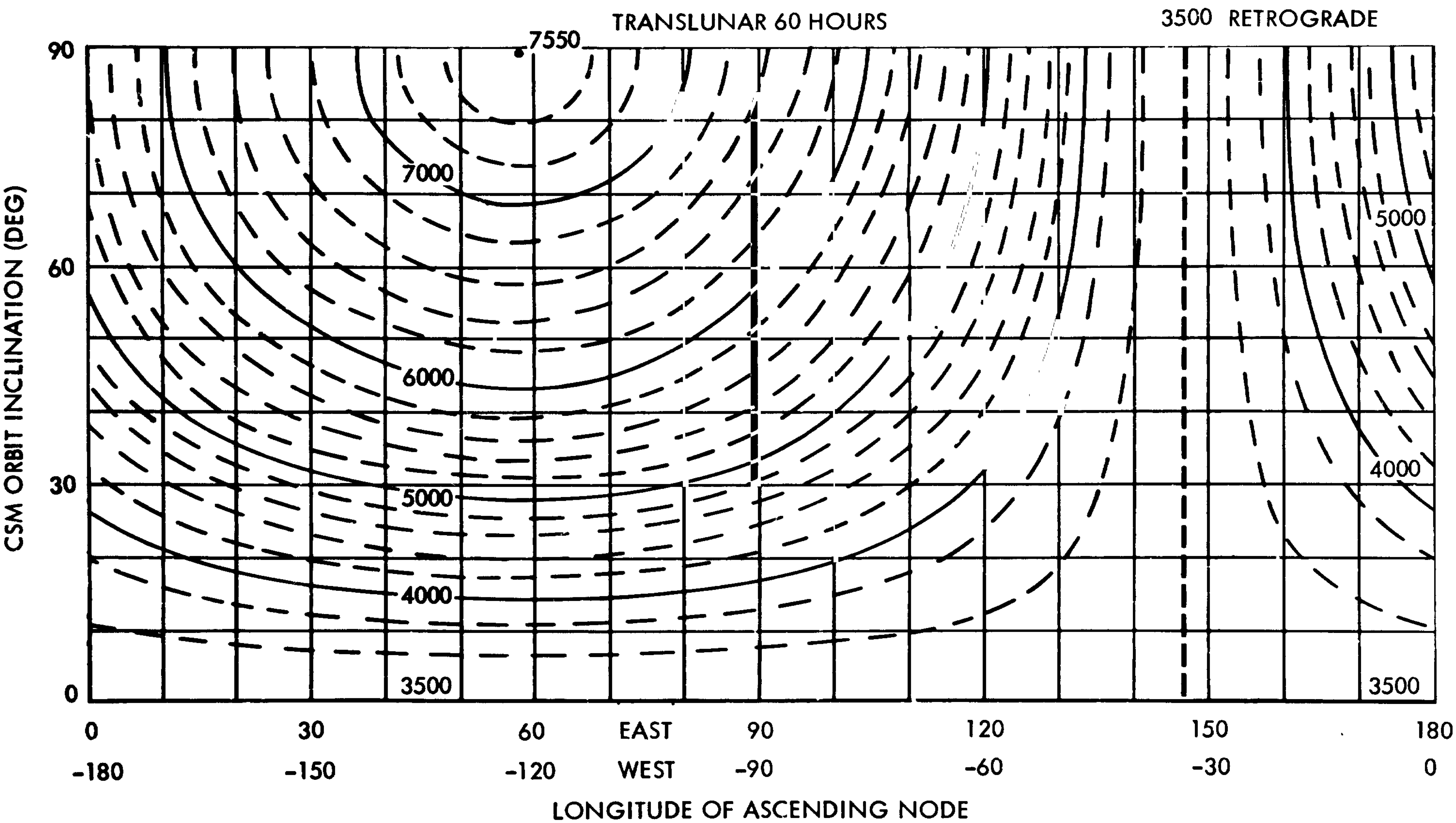


Figure 4-24. Translunar ΔV Requirements; 60-Hour Flight Time

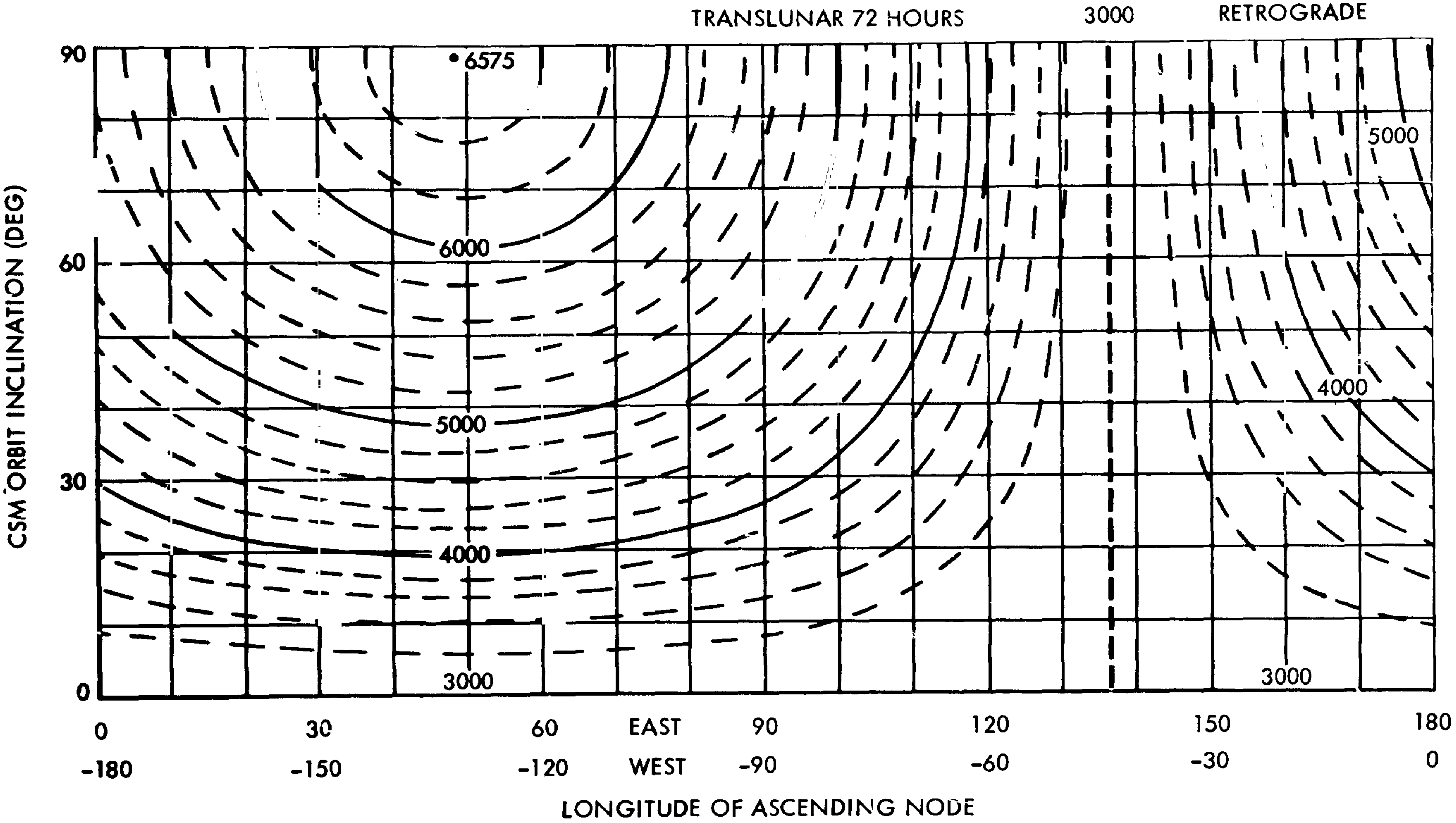


Figure 4-25. Translunar ΔV Requirements; 72-Hour Flight Time

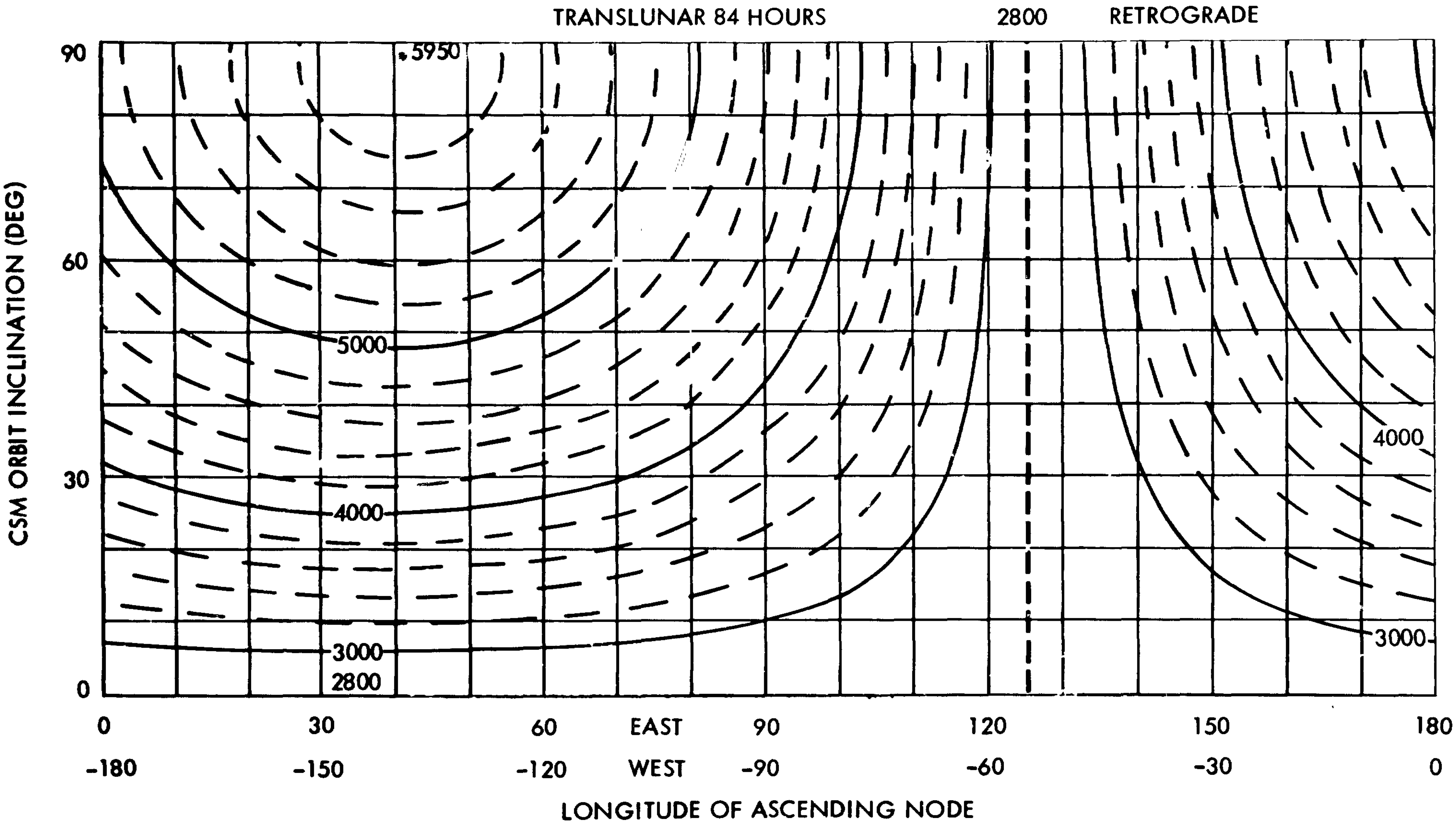


Figure 4-26. Translunar ΔV Requirements; 84-Hour Flight Time

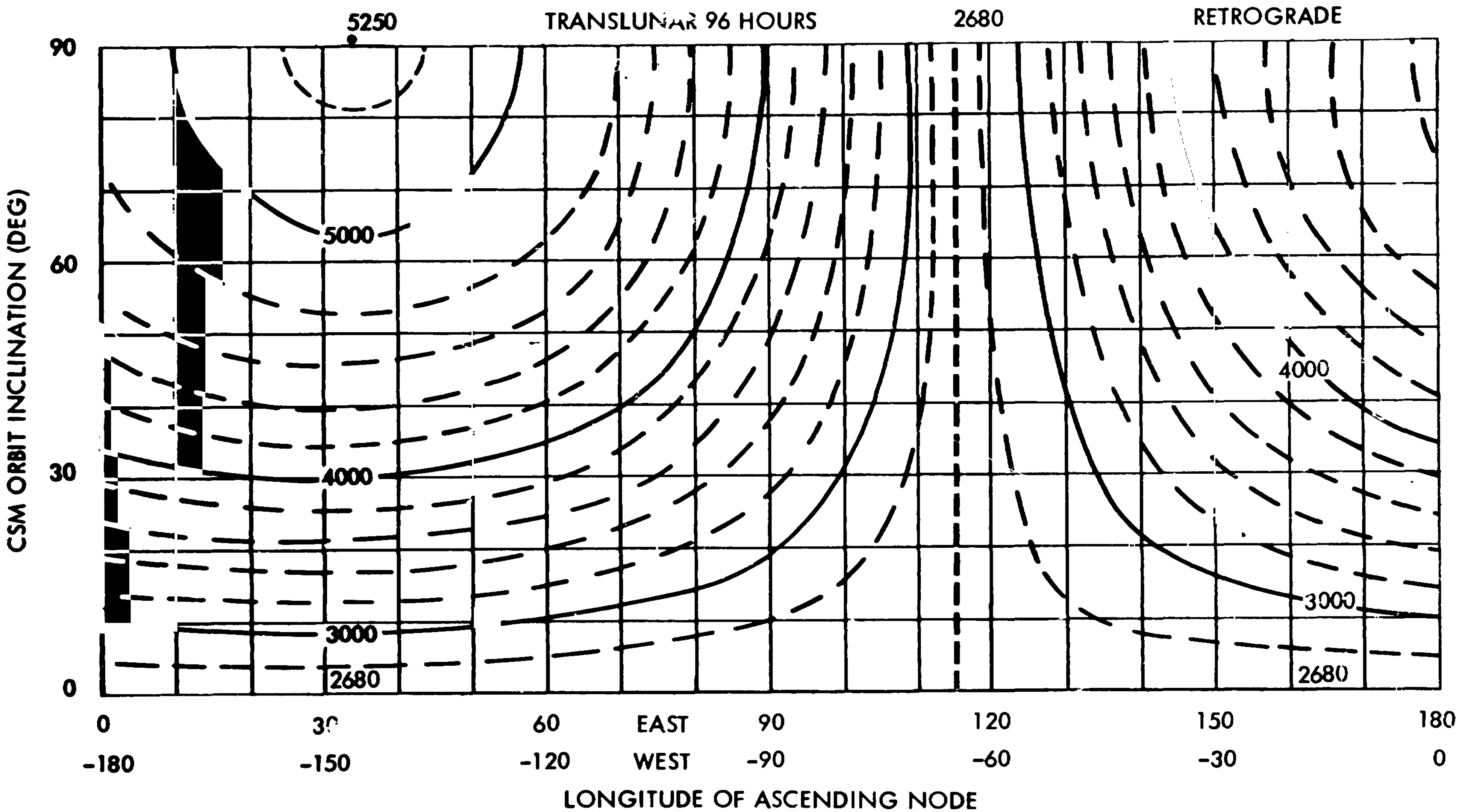


Figure 4-27. Translunar ΔV Requirements; 96-Hour Flight Time

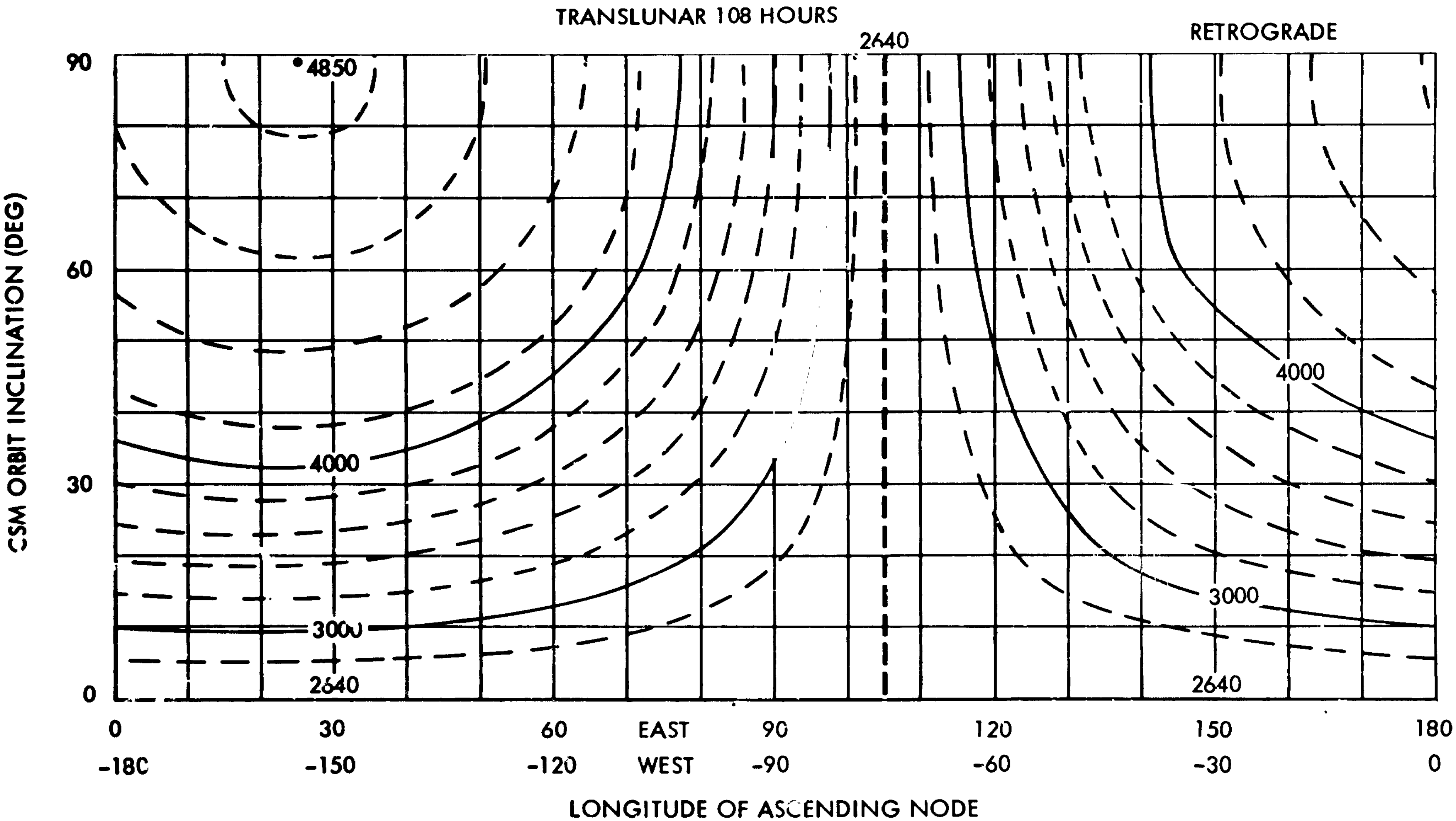


Figure 4-28. Translunar ΔV Requirements; 108-Hour Flight Time

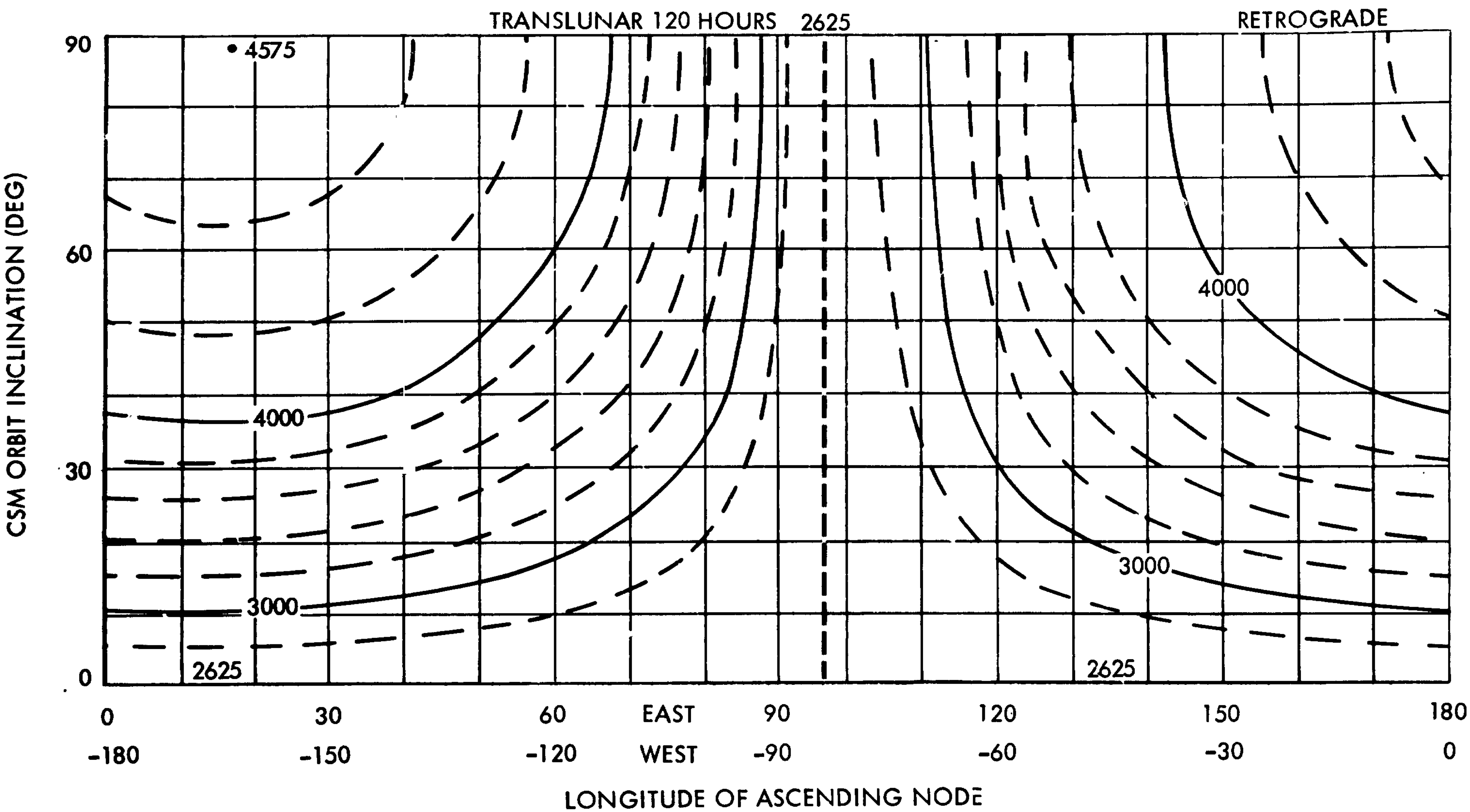


Figure 4-29. Translunar ΔV Requirements; 120-Hour Flight Time

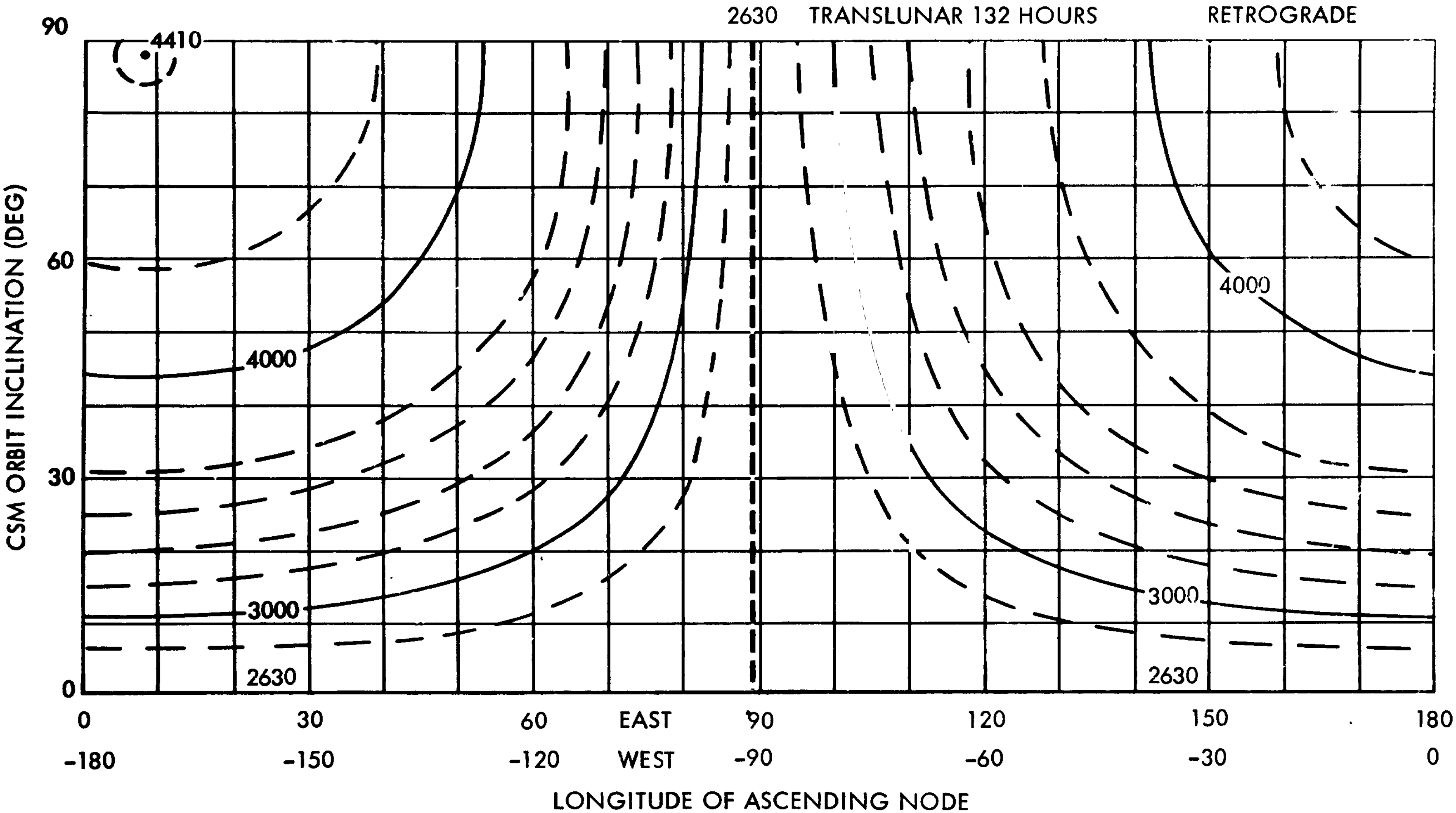


Figure 4-30. Translunar ΔV Requirements; 132-Hour Flight Time

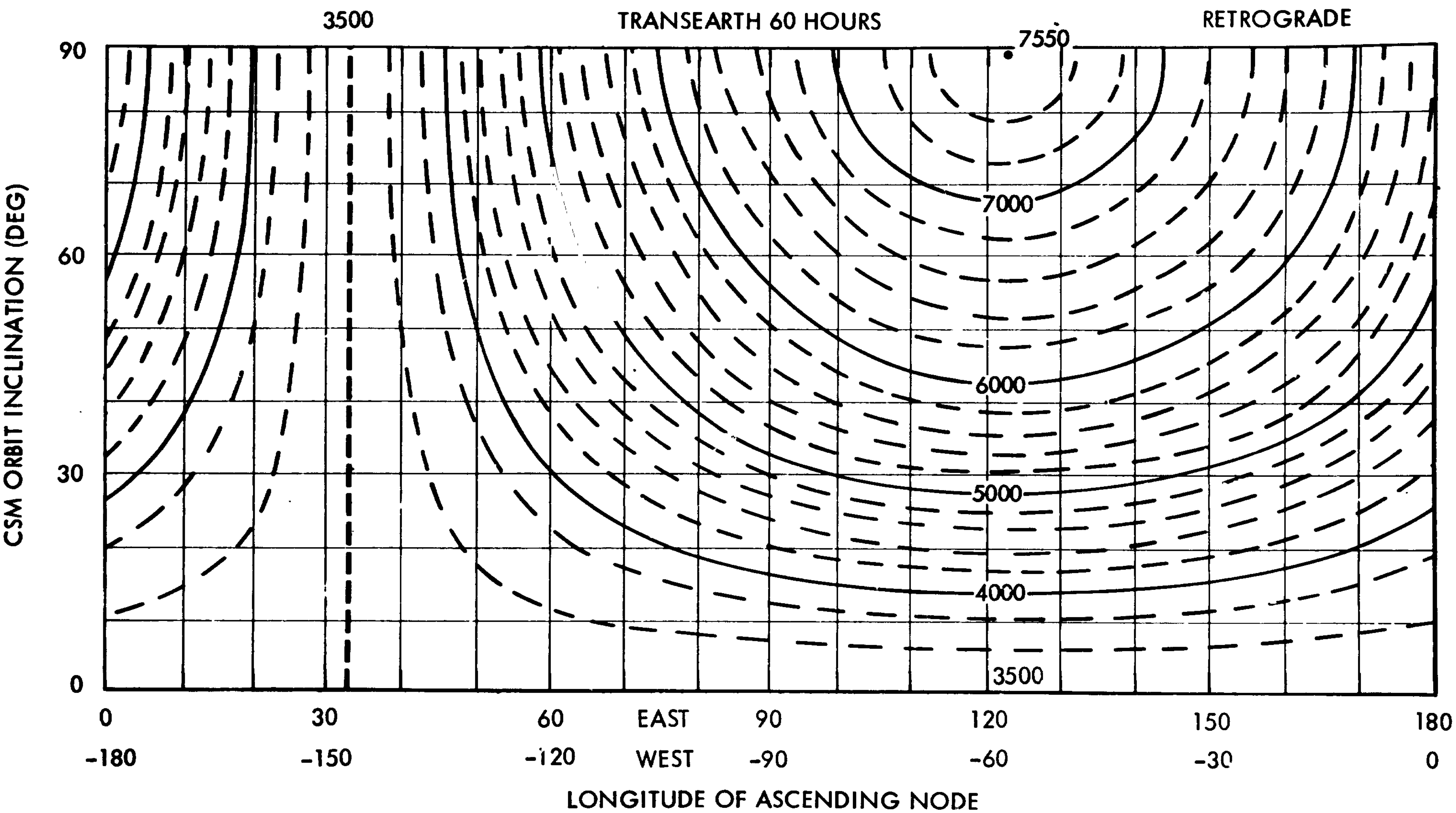


Figure 4-31. Transearth ΔV Requirements; 60-Hour Flight Time

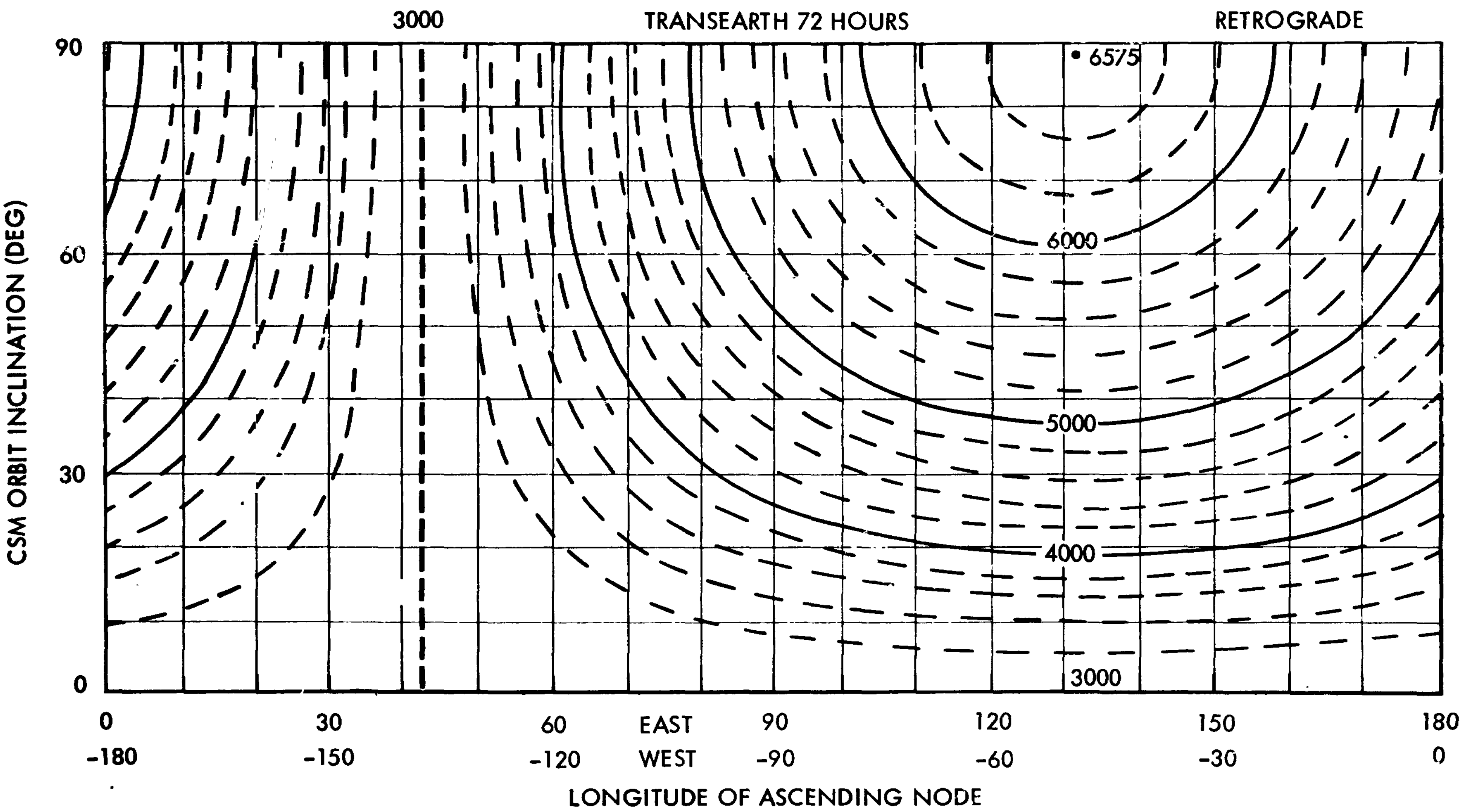


Figure 4-32. Transearth ΔV Requirements; 72-Hour Flight Time

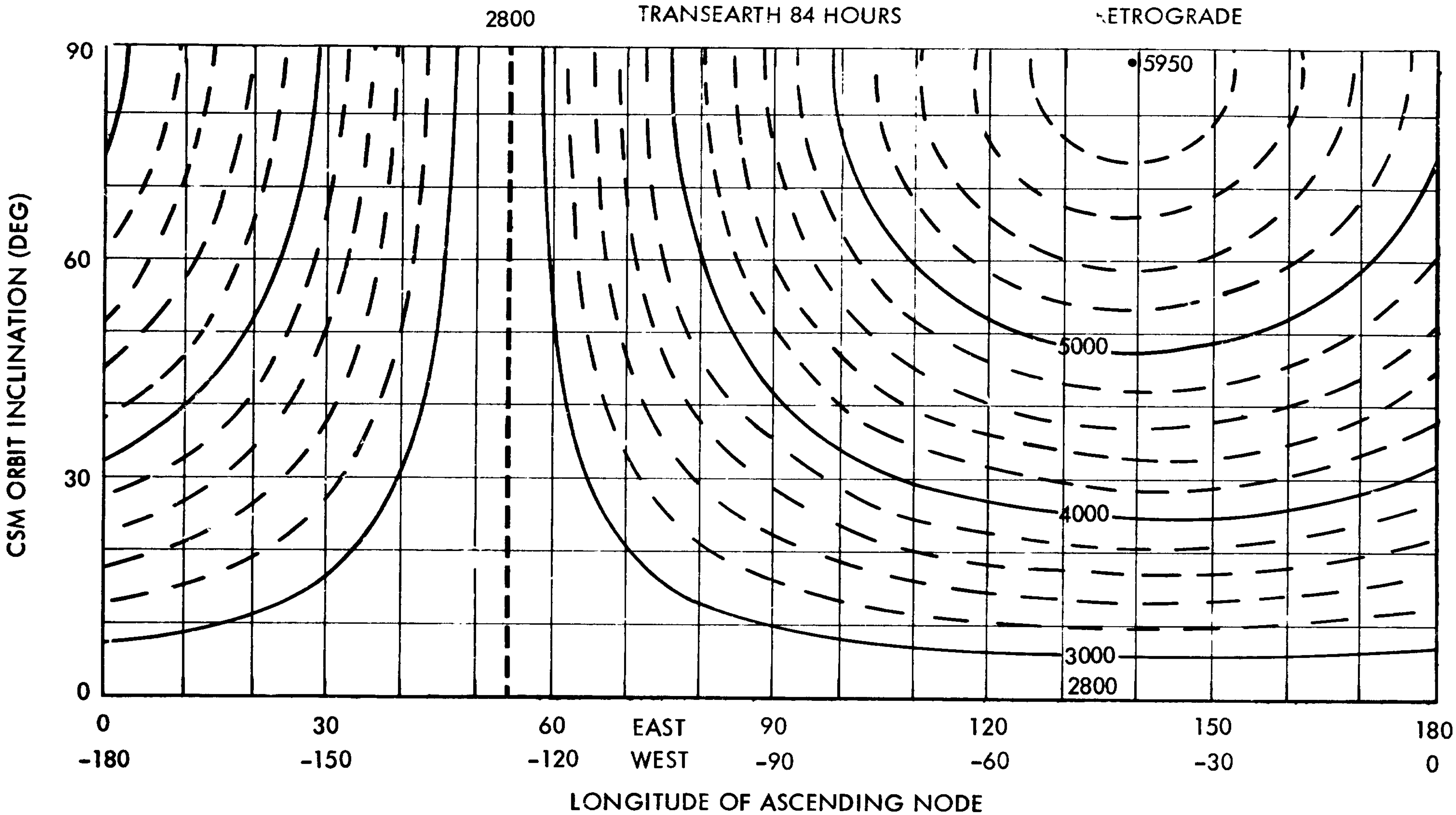


Figure 33. Transearth ΔV Requirements; 84-Hour Flight Time

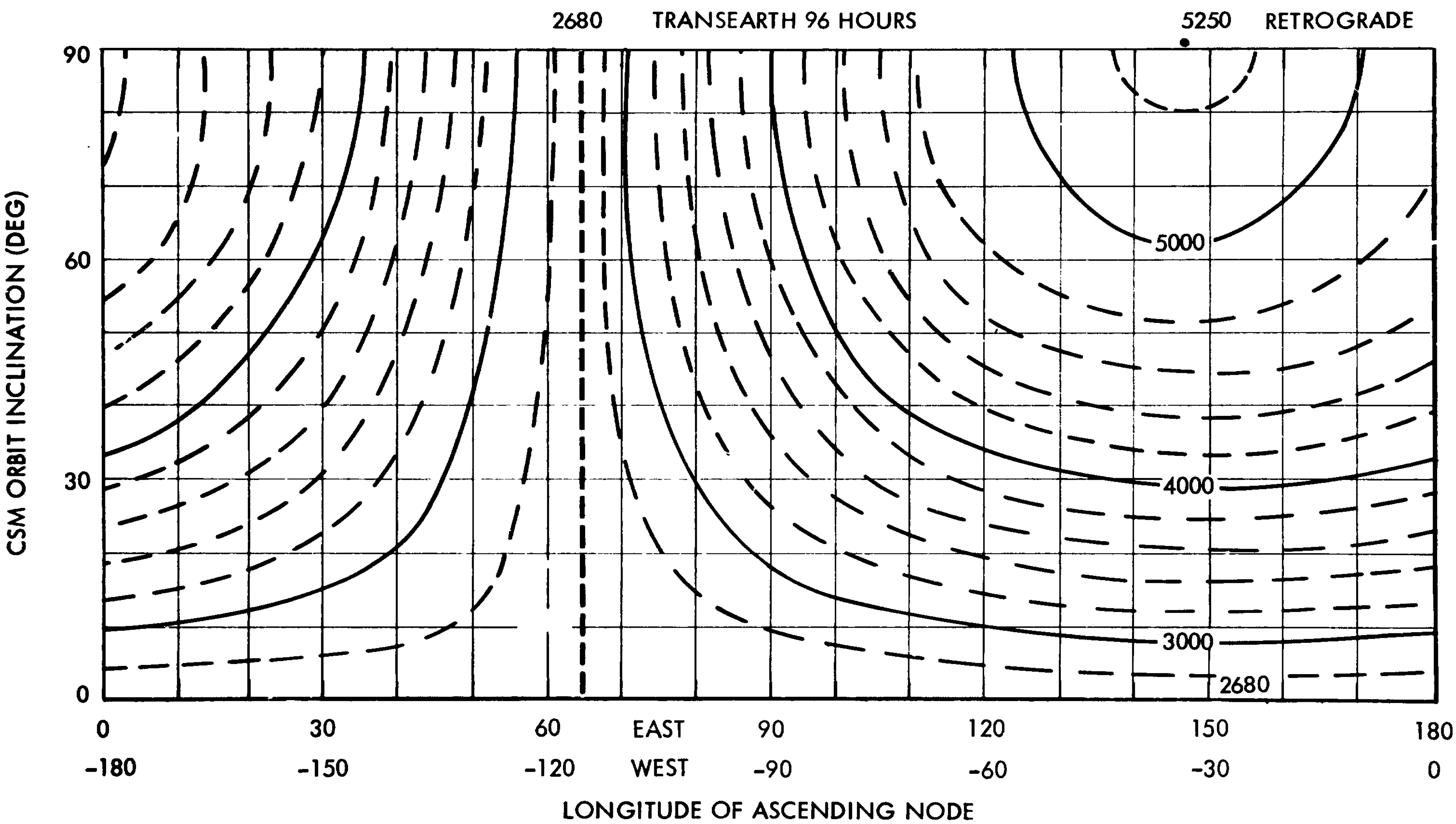


Figure 4-34. Transearth ΔV Requirements; 96-Hour Flight Time

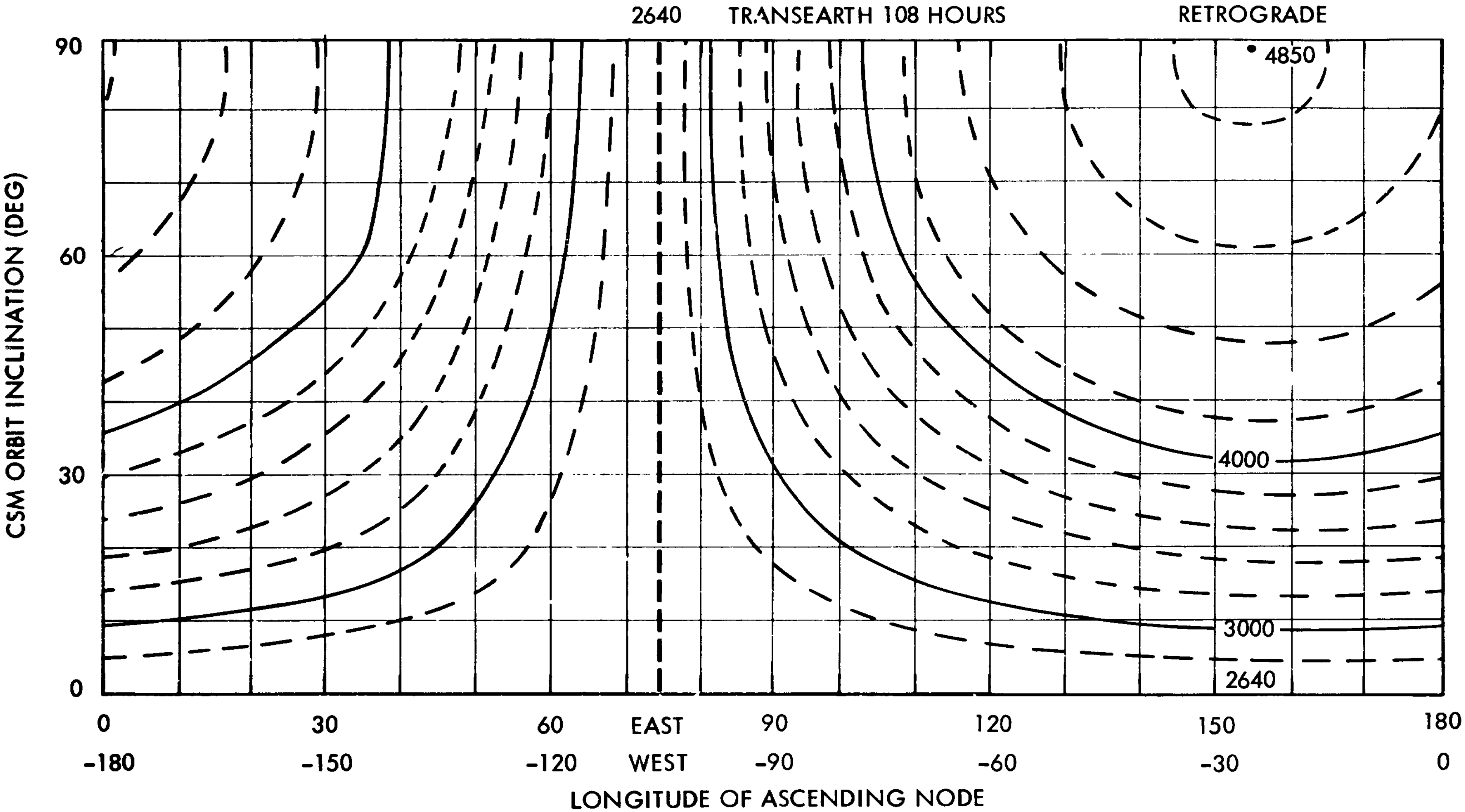


Figure 4-35. Transearth ΔV Requirements; 108-Hour Flight Time

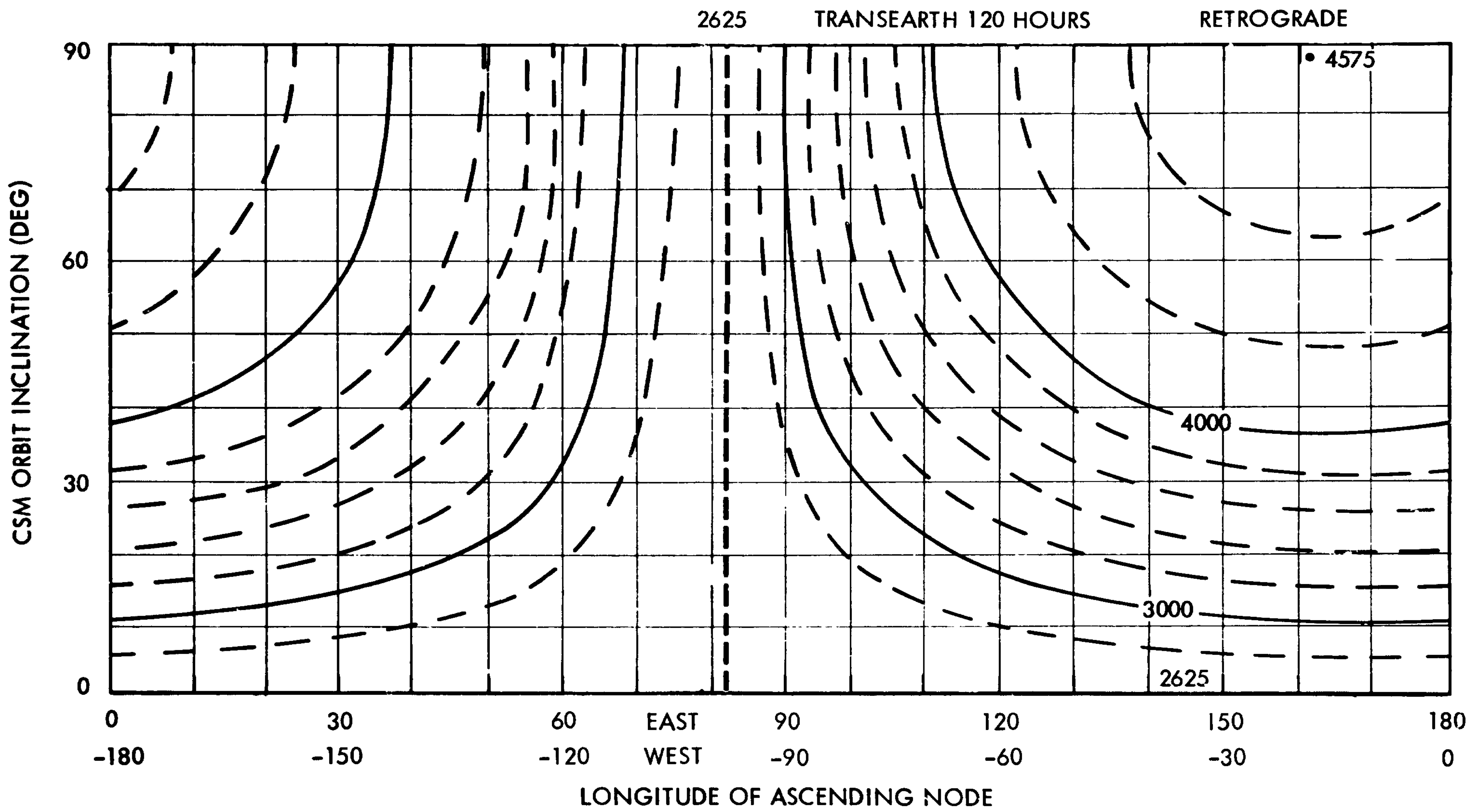


Figure 4-36. Transearth ΔV Requirements; 120-Hour Flight Time

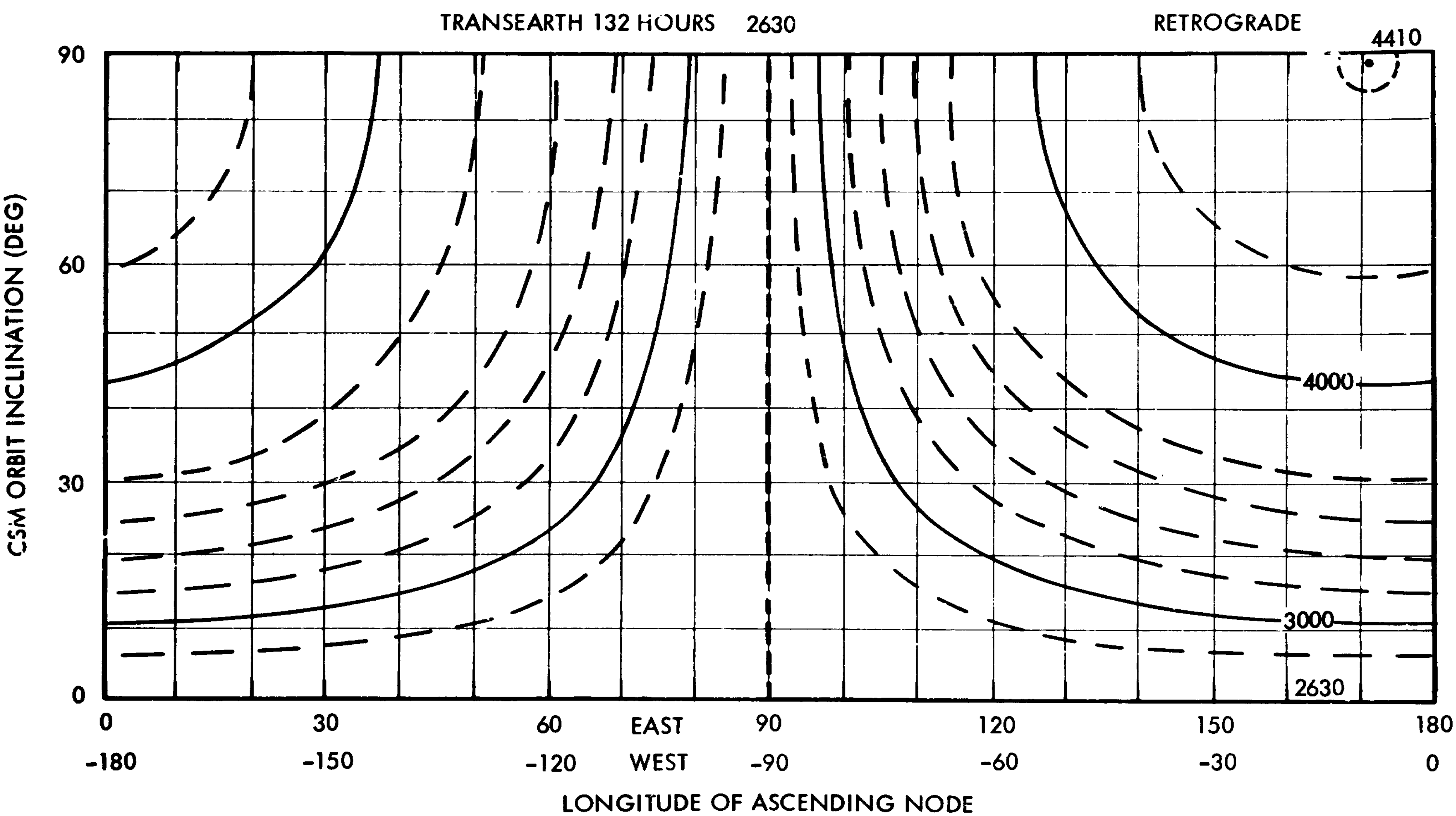
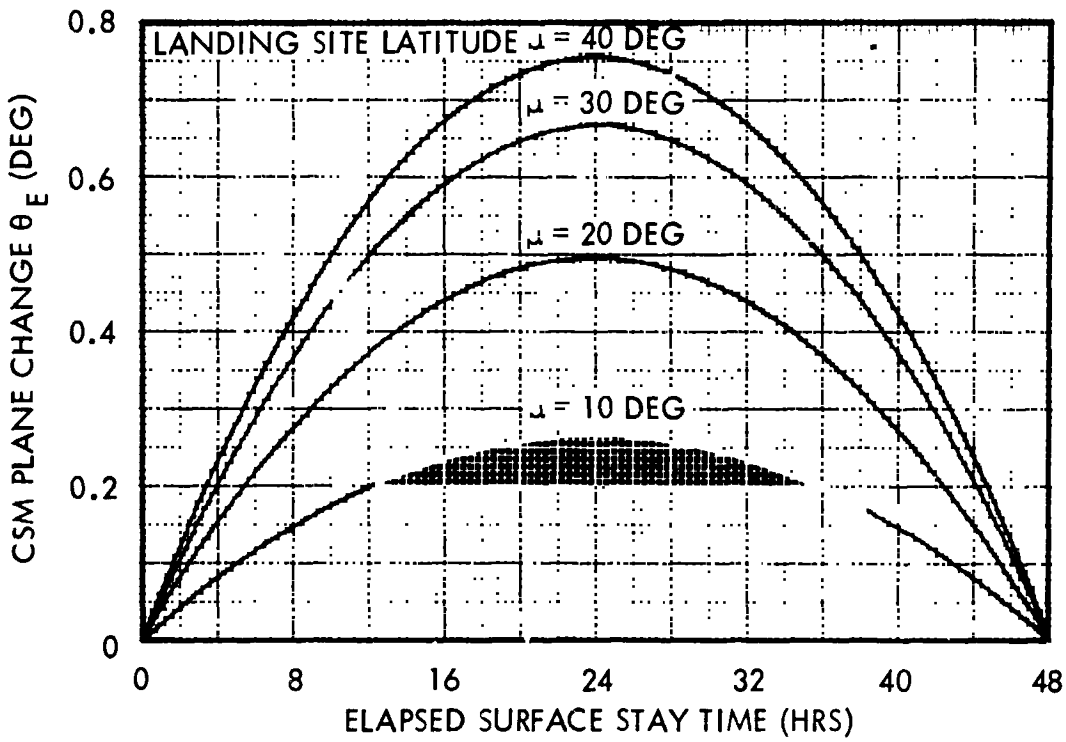


Figure 4-37. Transearth ΔV Requirements; 132-Hour Flight Time



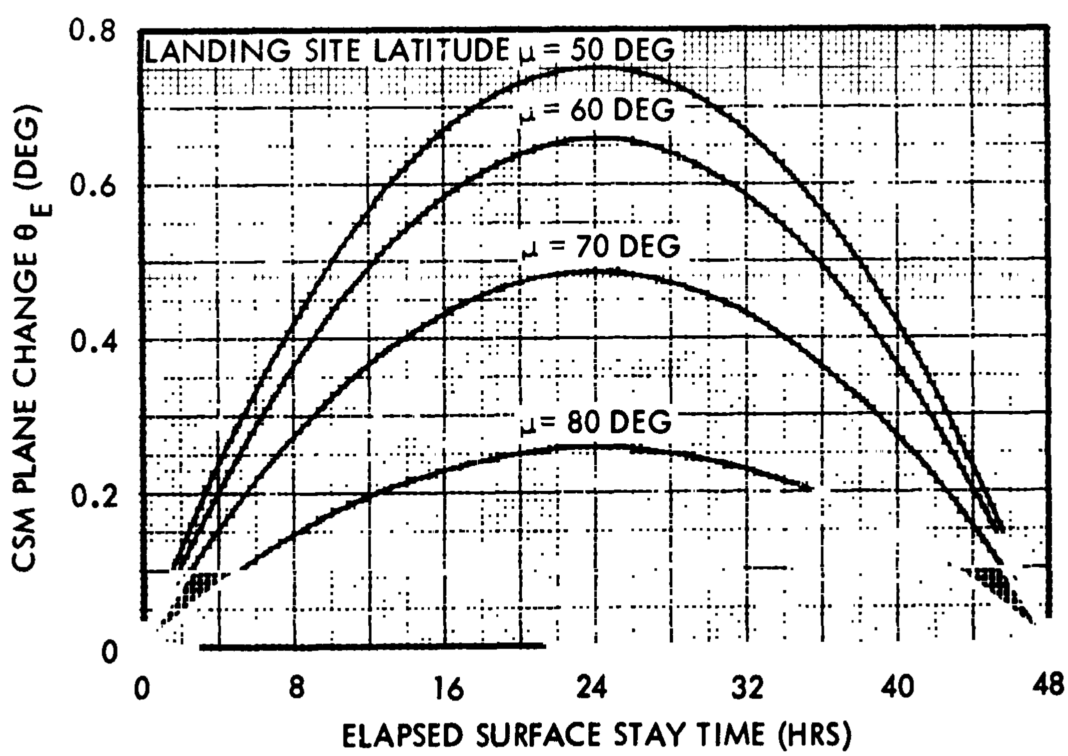
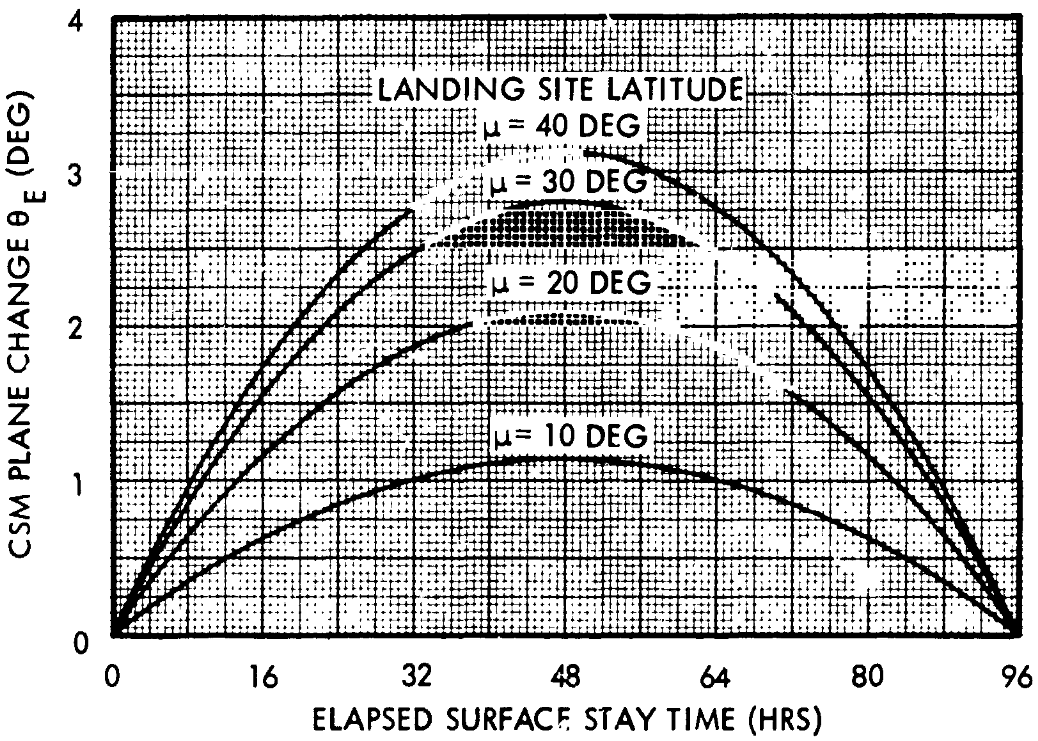


Figure 4-38. CSM Plane Change Angle versus Surface Stay Time for Various Site Latitudes; 2-Day Total Stay Time for Zero LM Plane Change Geometry



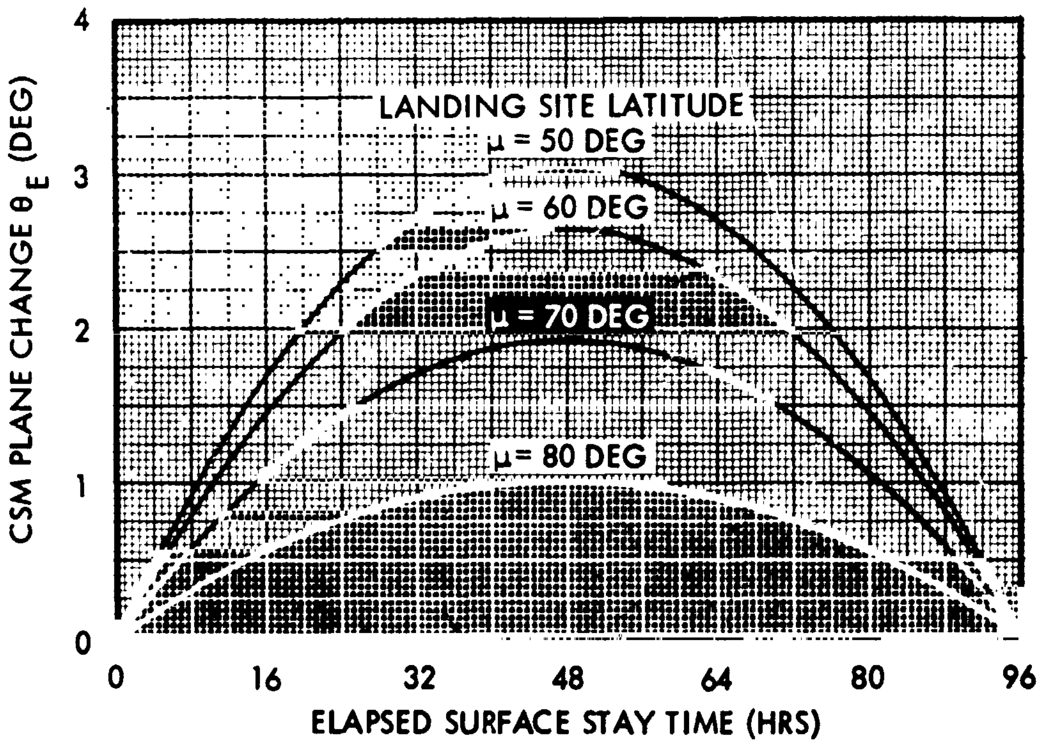


Figure 4-39. CSM Plane Change Angle versus Surface Stay Time for Various Site Latitudes; 4-Day Total Stay Time for Zero LM Plane Change Geometry

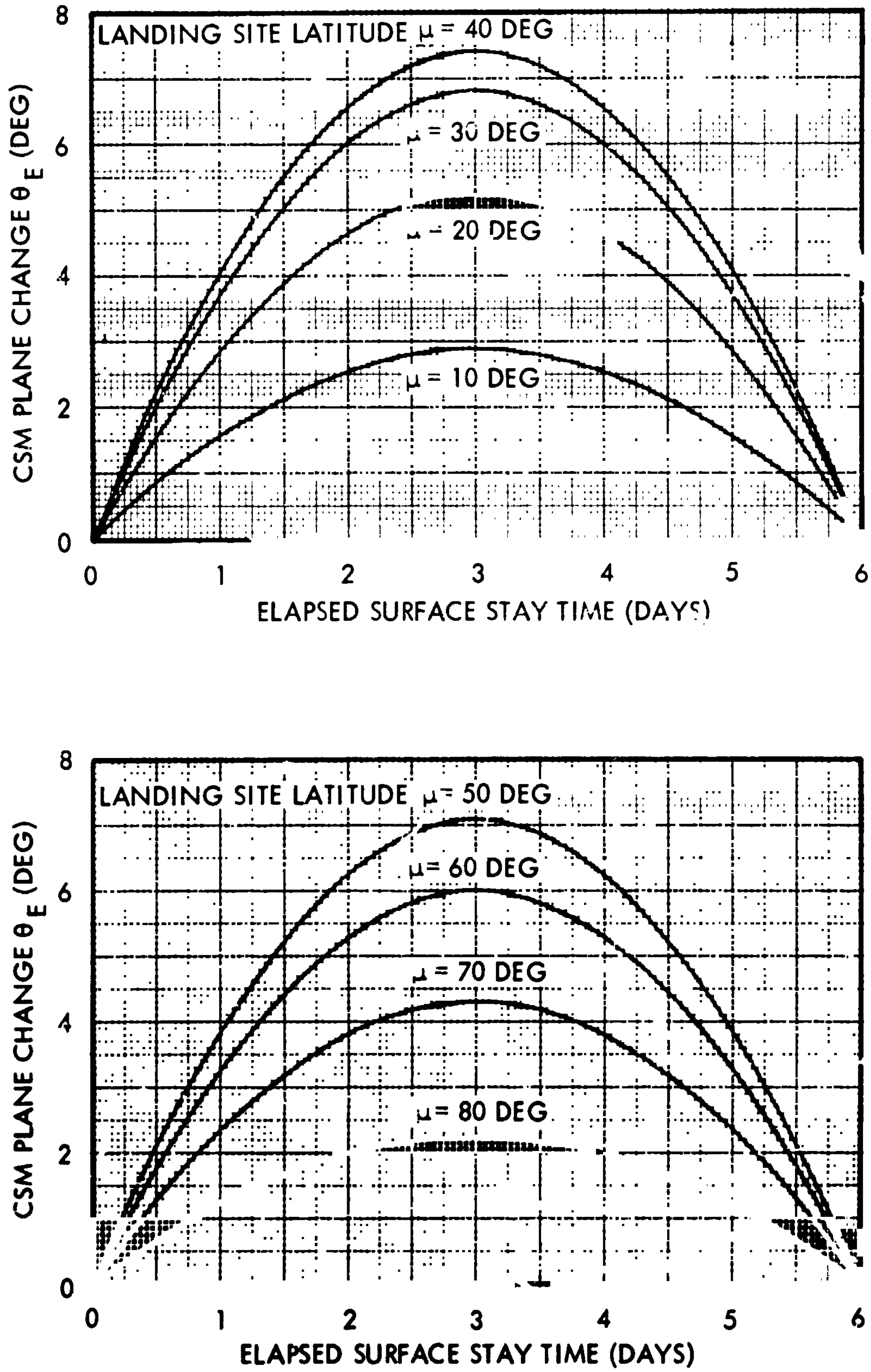
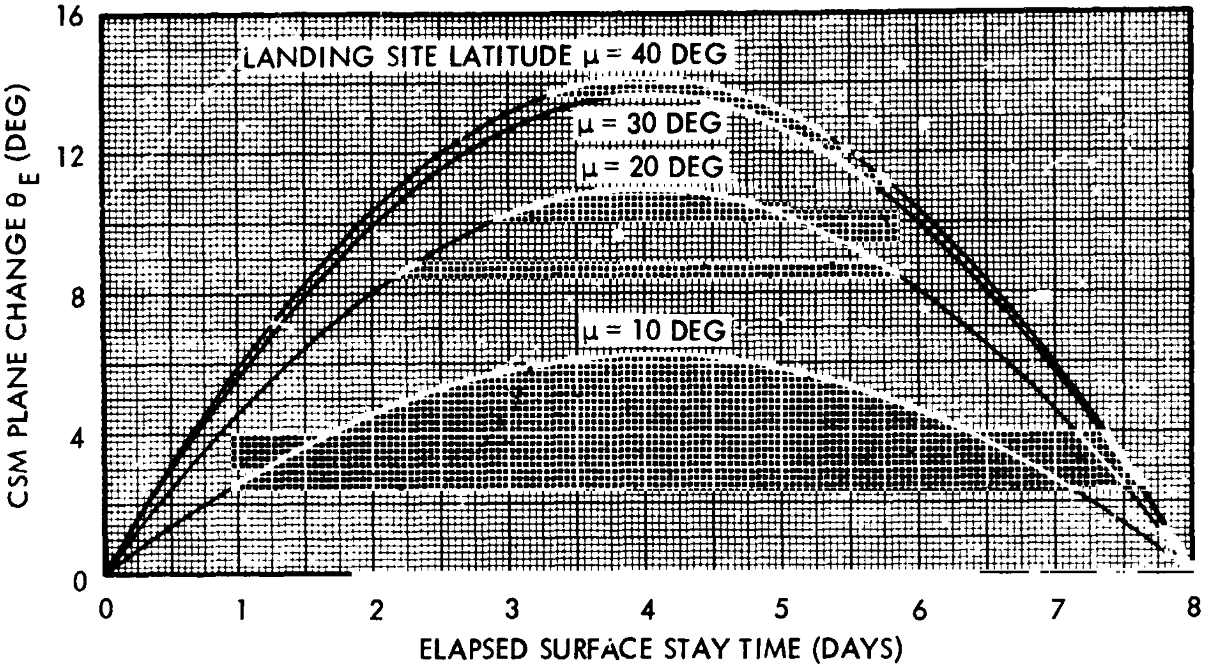


Figure 4-40. CSM Plane Change Angle versus Surface Stay Time for Various Site Latitudes; 6-Day Total Stay Time for Zero LM Plane Change Geometry



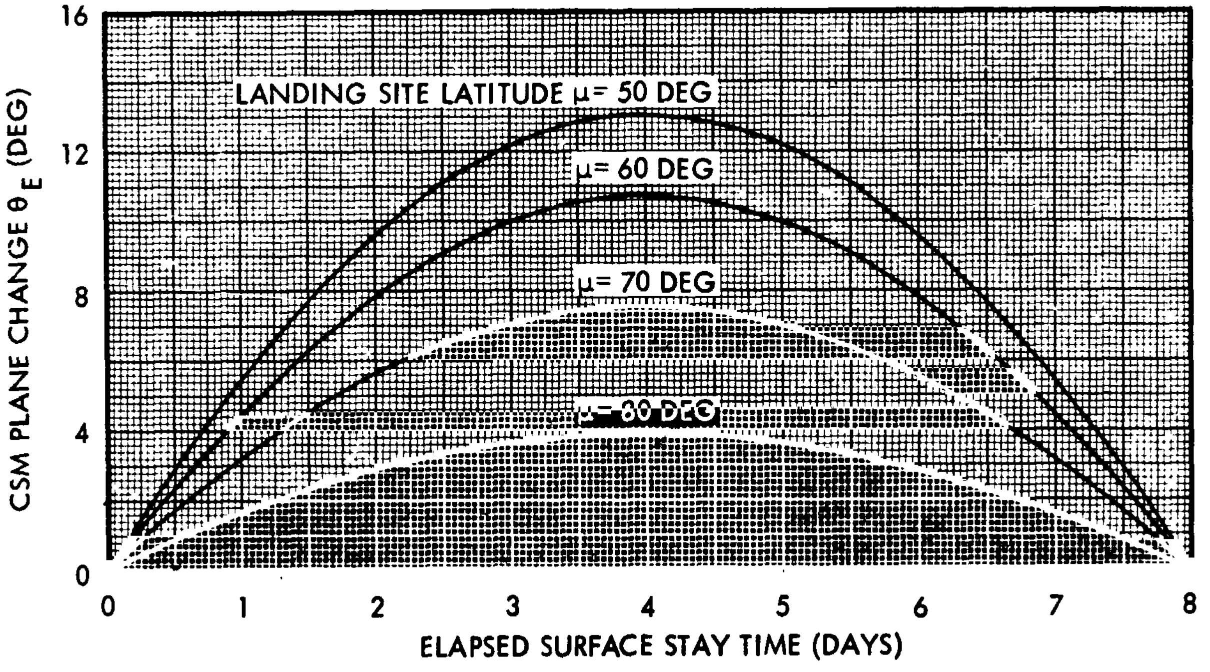


Figure 4-41. CSM Plane Change Angle versus Surface Stay Time for Various Site Latitudes; 8-Day Total Stay Time for Zero LM Plane Change Geometry

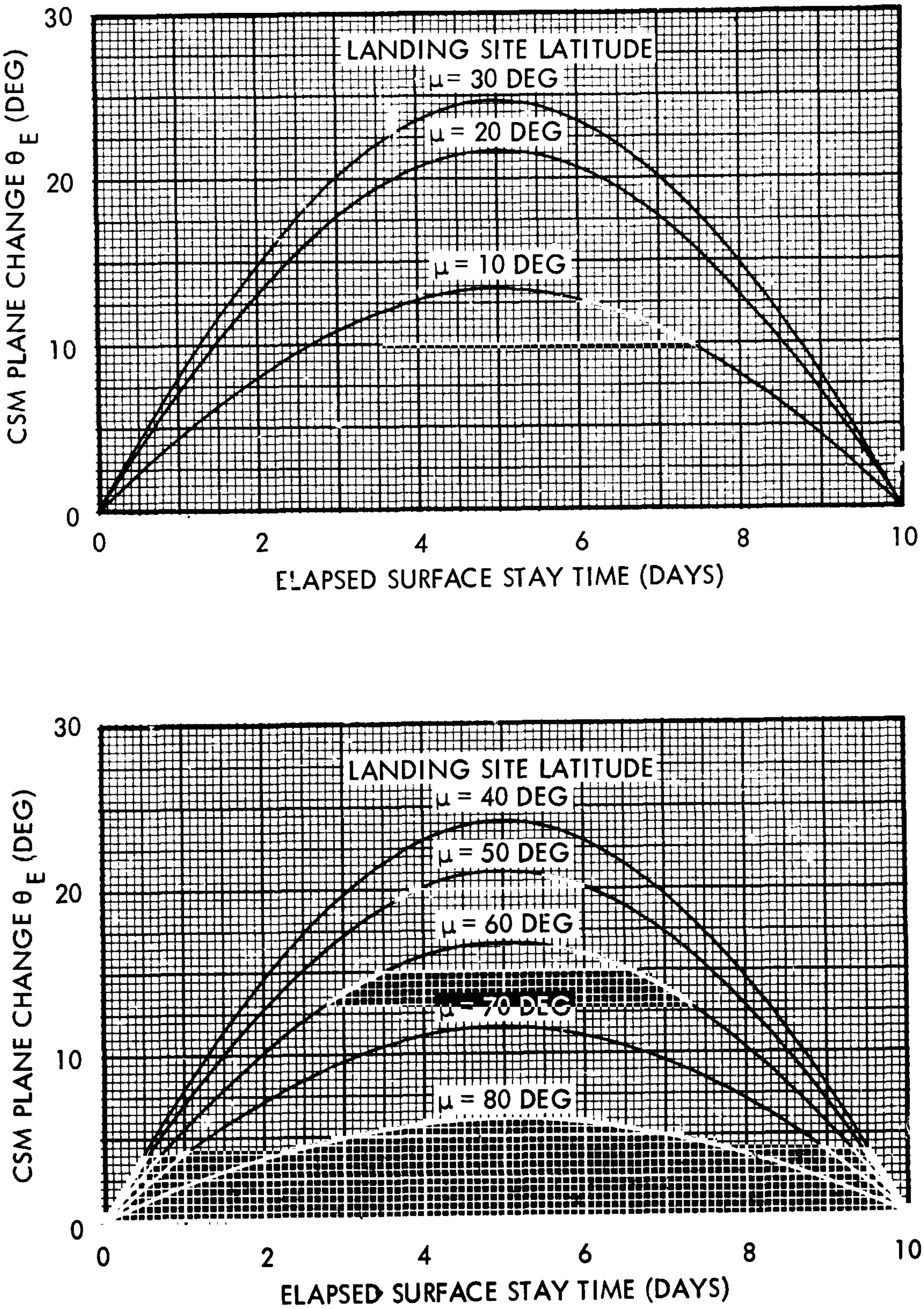
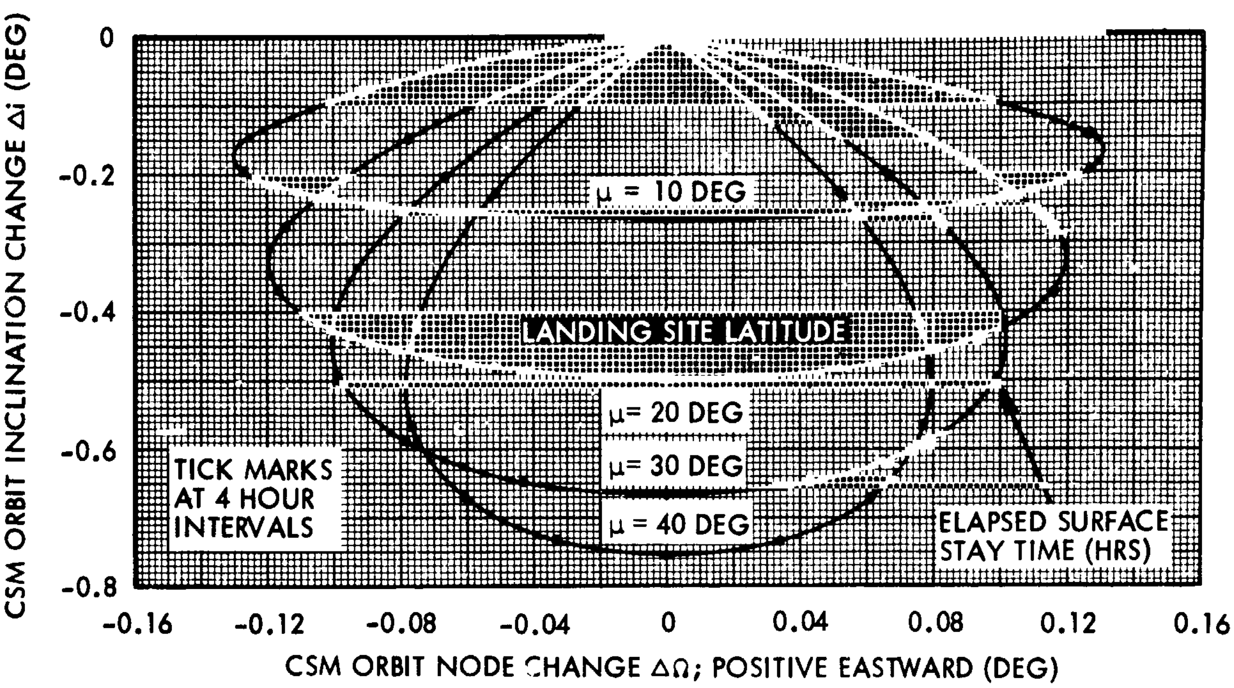


Figure 4-42. CSM Plane Change Angle versus Surface Stay Time for Various Site Latitudes; 10-Day Total Stay Time for Zero LM Plane Change Geometry



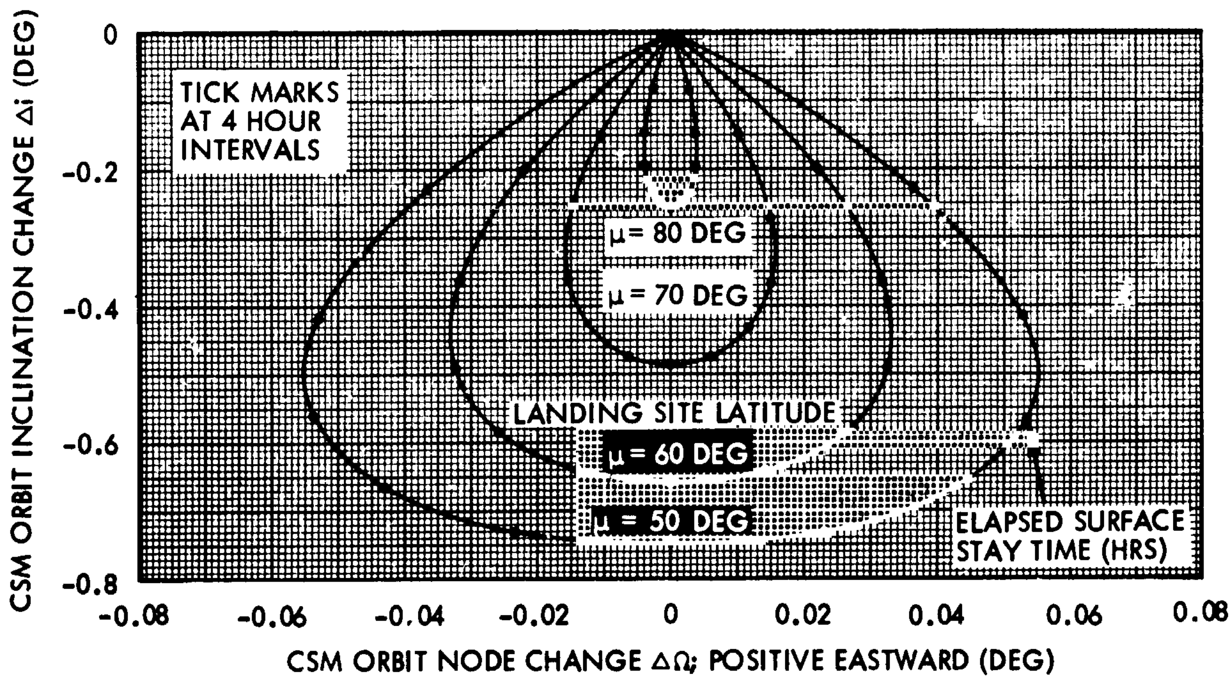
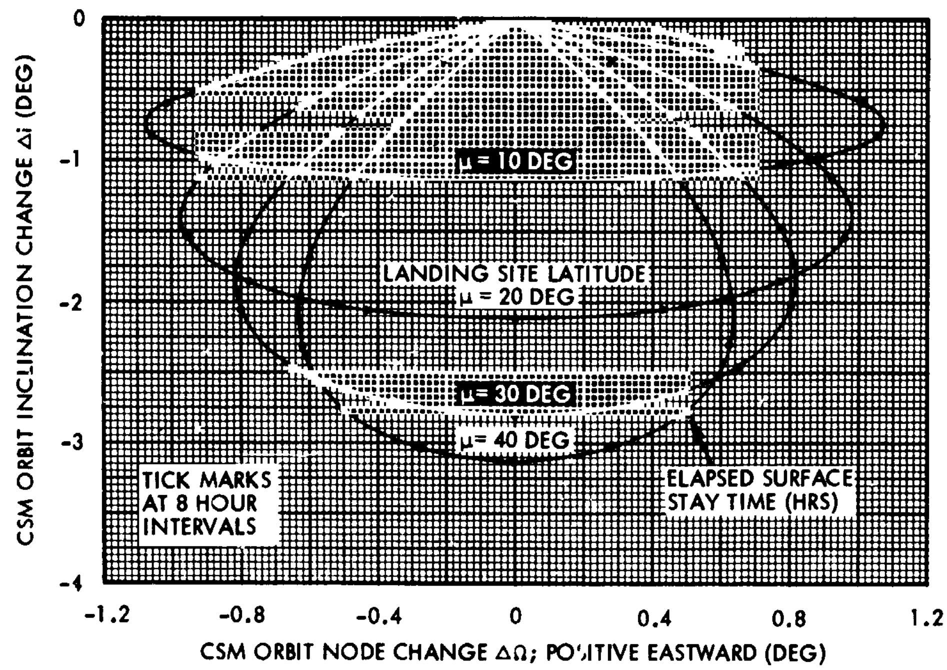


Figure 4-44. CSM Plane Change Effect Upon Inclination and Node of CSM Orbit versus Surface Stay Time for Various Site Latitudes; 2-Day Total Stay Time for Zero LM Plane Change Geometry



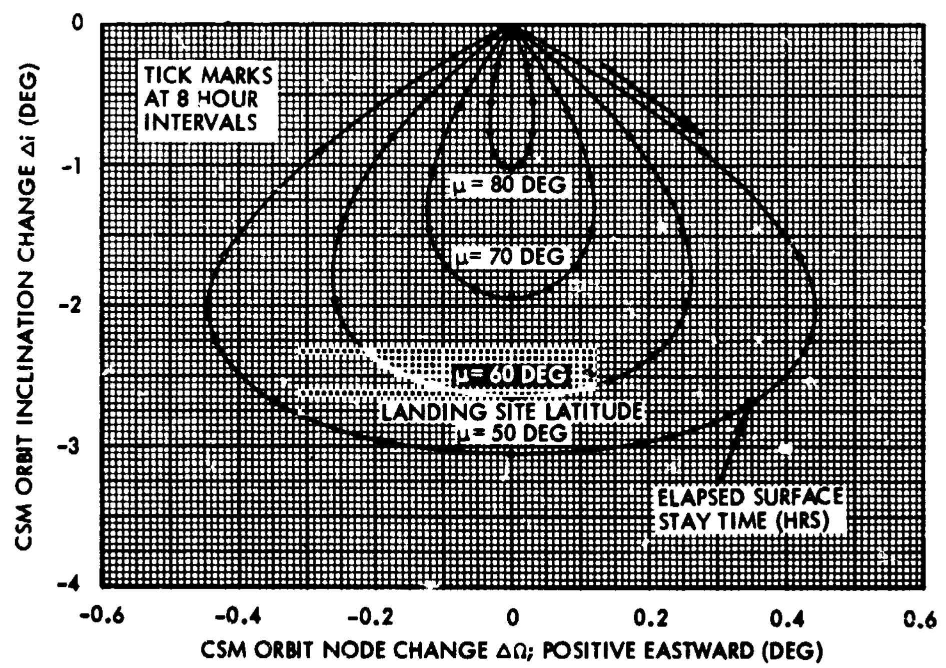
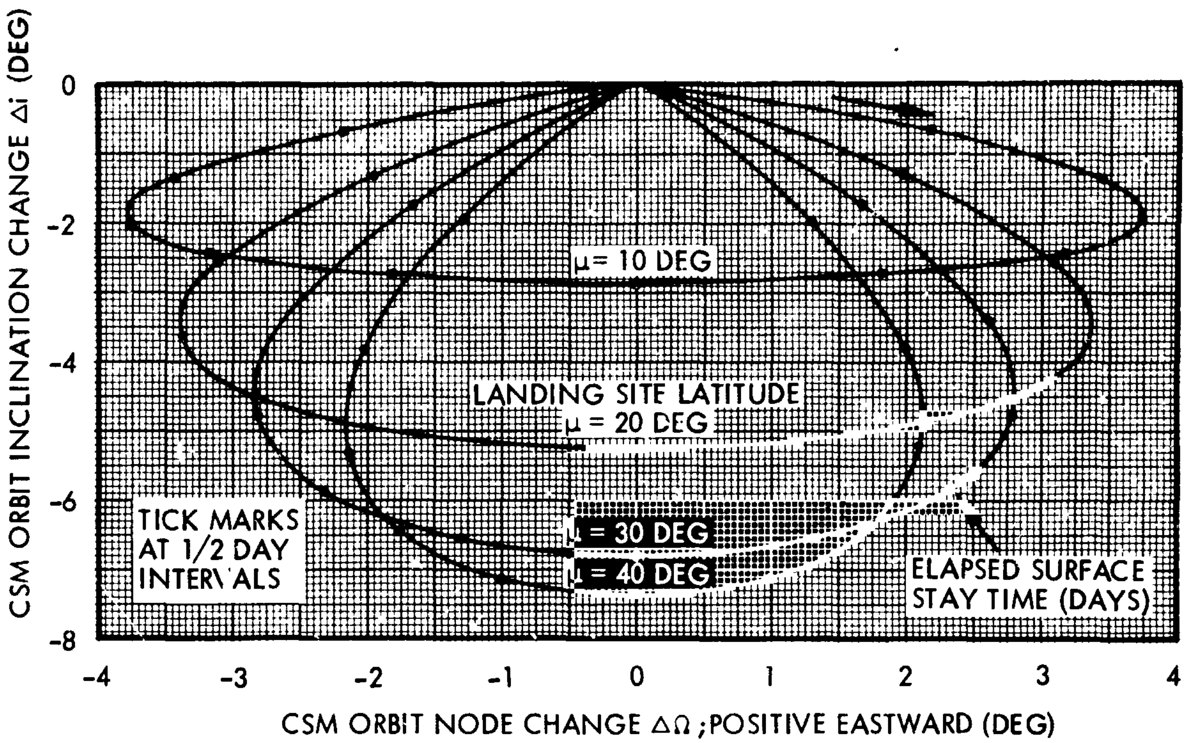


Figure 4-45. CSM Plane Change Effect Upon Inclination and Node of CSM Orbit versus Surface Stay Time for Various Site Latitudes; 4-Day Total Stay Time for Zero LM Plane Change Geometry



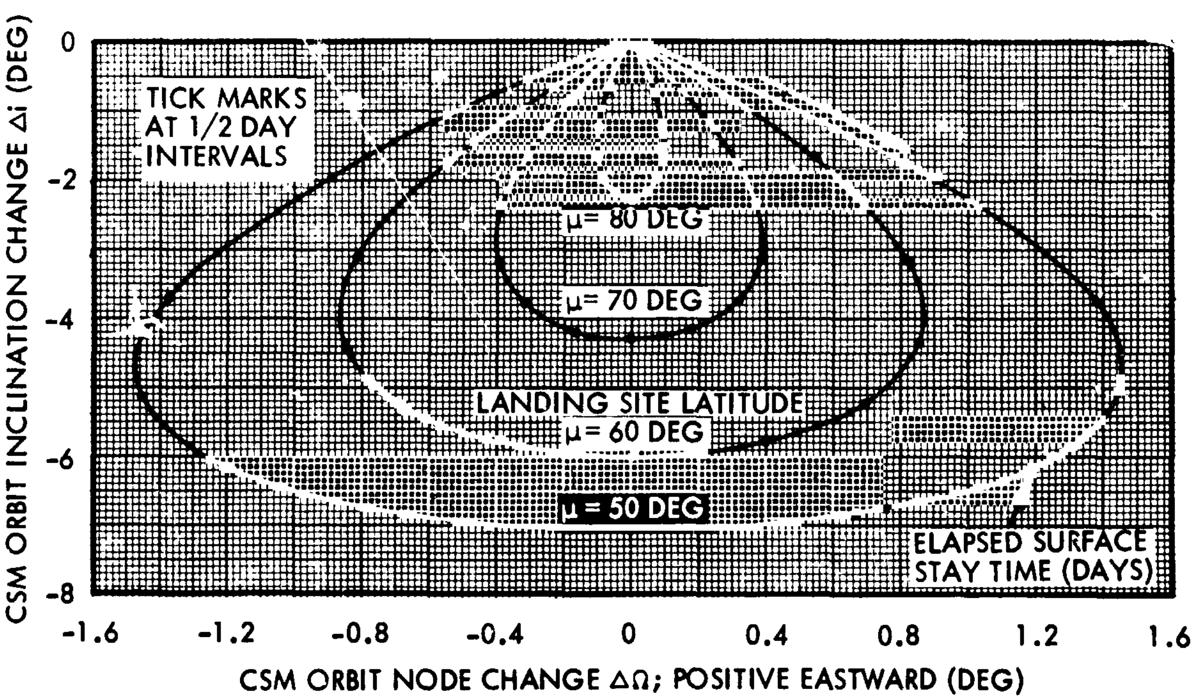
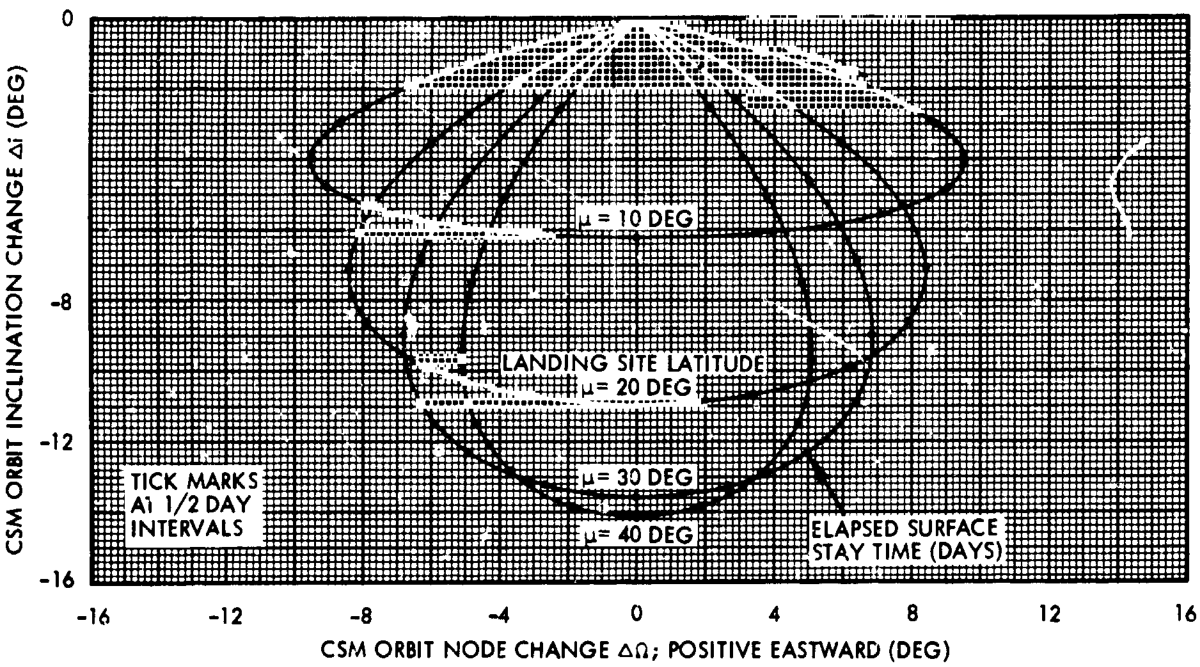


Figure 4-46. CSM Plane Change Effect Upon Inclination and Node of CSM Orbit versus Surface Stay Time for Various Site Latitudes; 6-Day Total Stay Time for Zero LM Plane Change Geometry



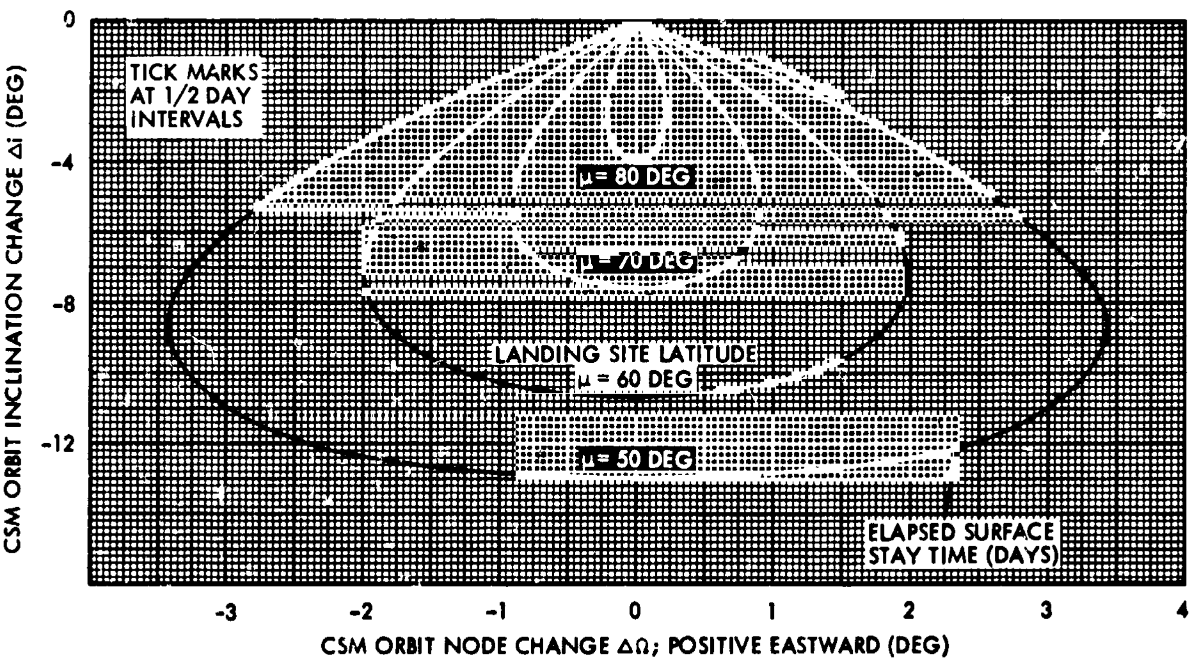
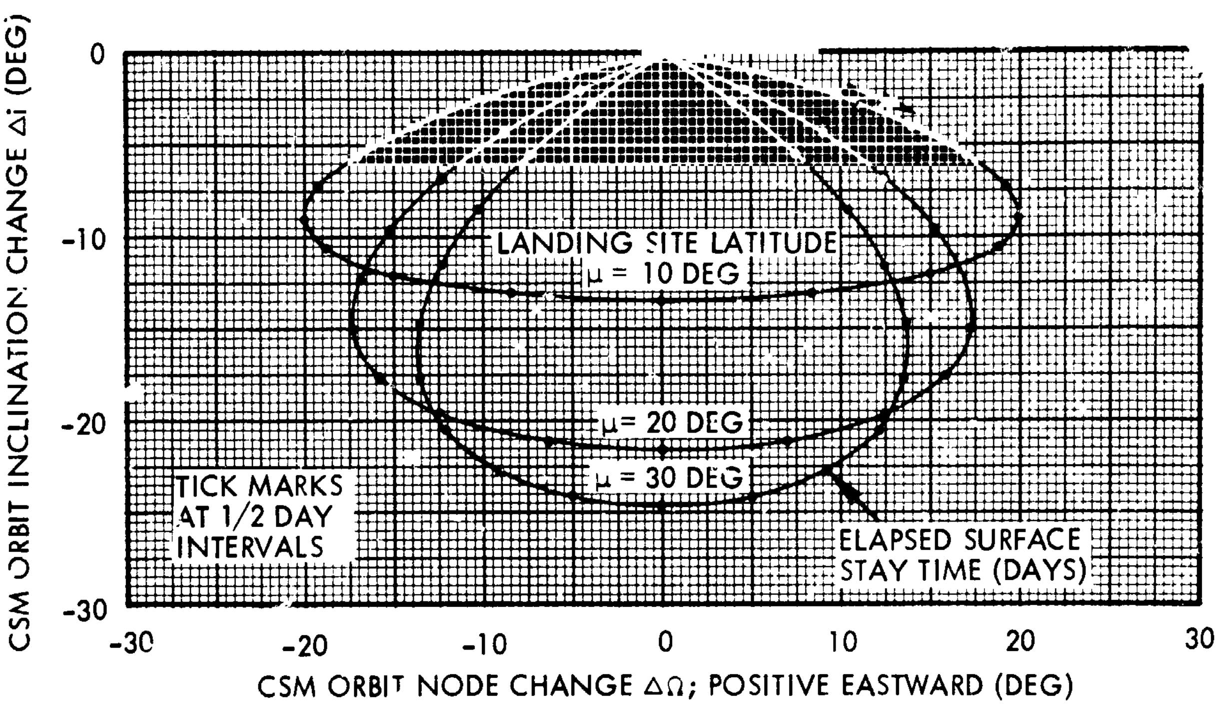


Figure 4-47. CSM Plane Change Effect Upon Inclination and Node of CSM Orbit versus Surface Stay Time for Various Site Latitudes; 8-Day Total Stay Time for Zero LM Plane Change Geometry



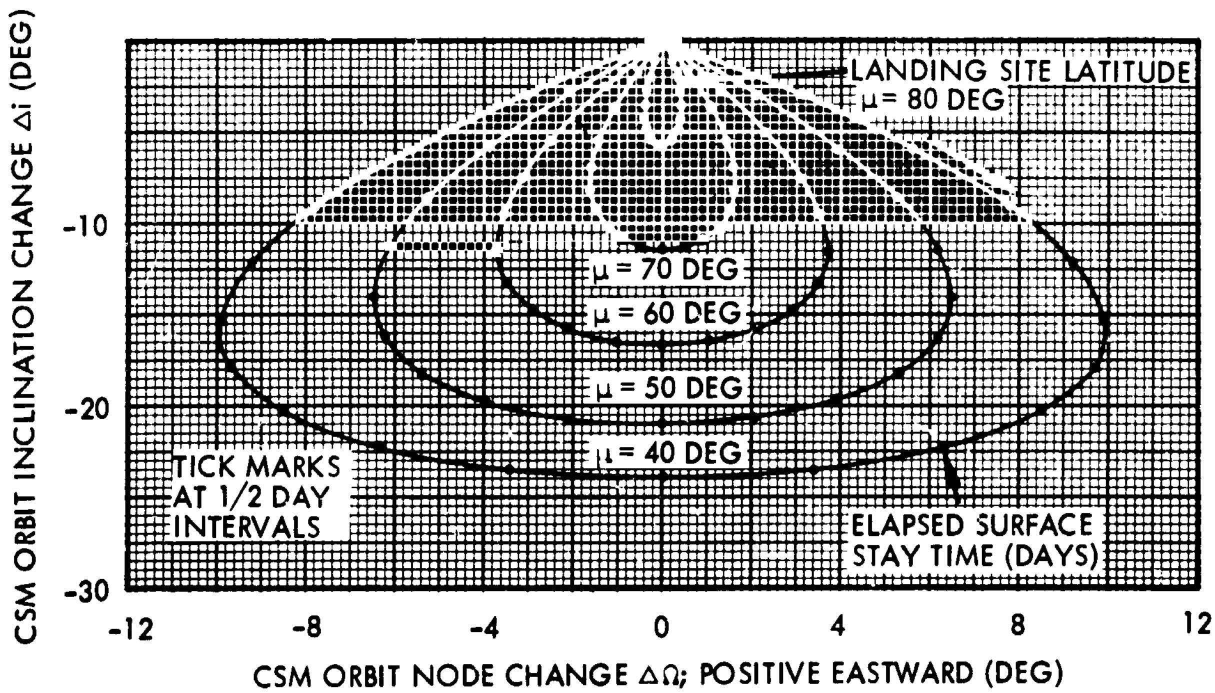
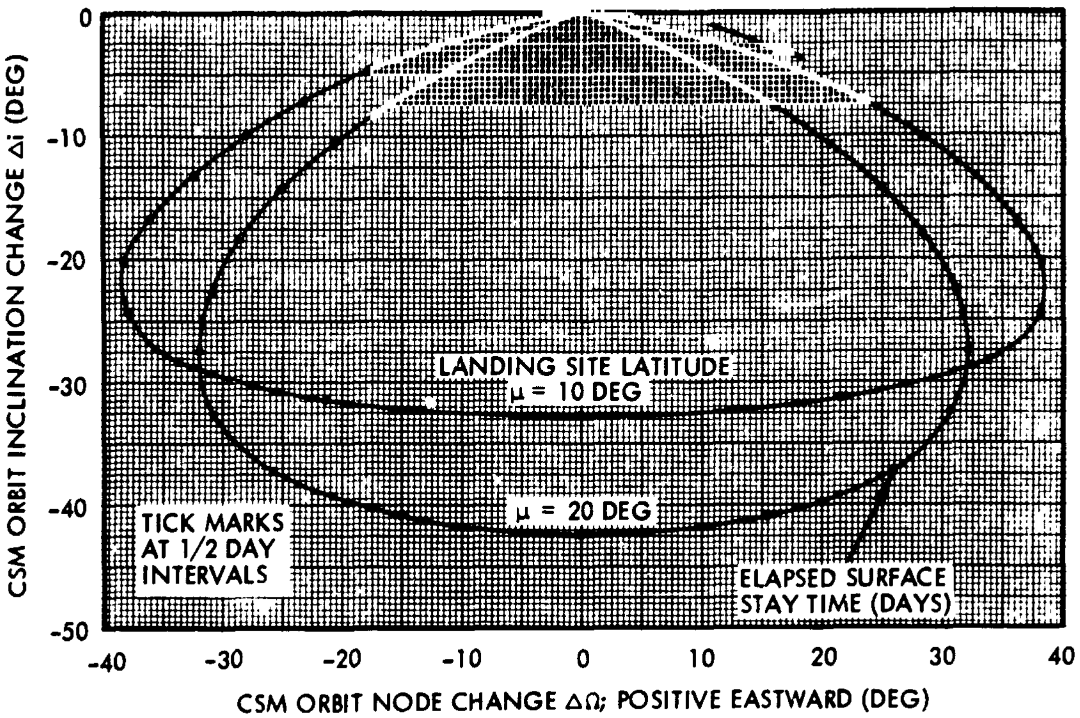


Figure 4-48. CSM Plane Change Effect Upon Inclination and Node of CSM Orbit versus Surface Stay Time for Various Site Latitudes; 10-Day Total Stay Time for Zero LM Plane Change Geometry



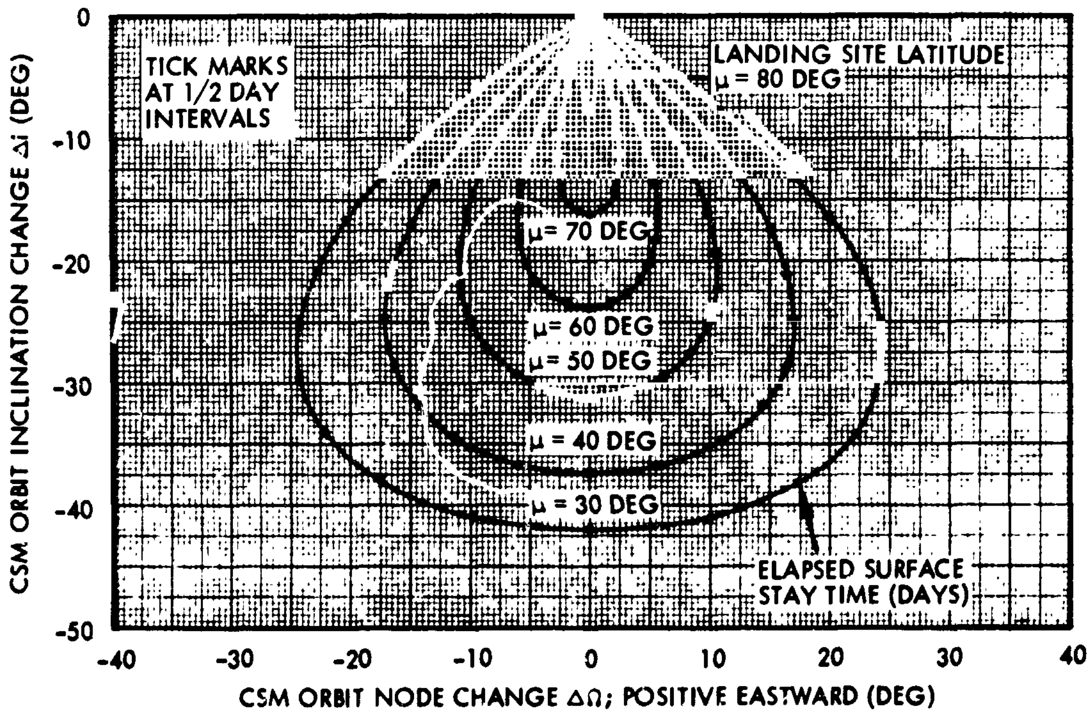


Figure 4-49. CSM Plane Change Effect Upon Inclination and Node of CSM Orbit versus Surface Stay Time for Various Site Latitudes; 12-Day Total Stay Time for Zero LM Plane Change Geometry

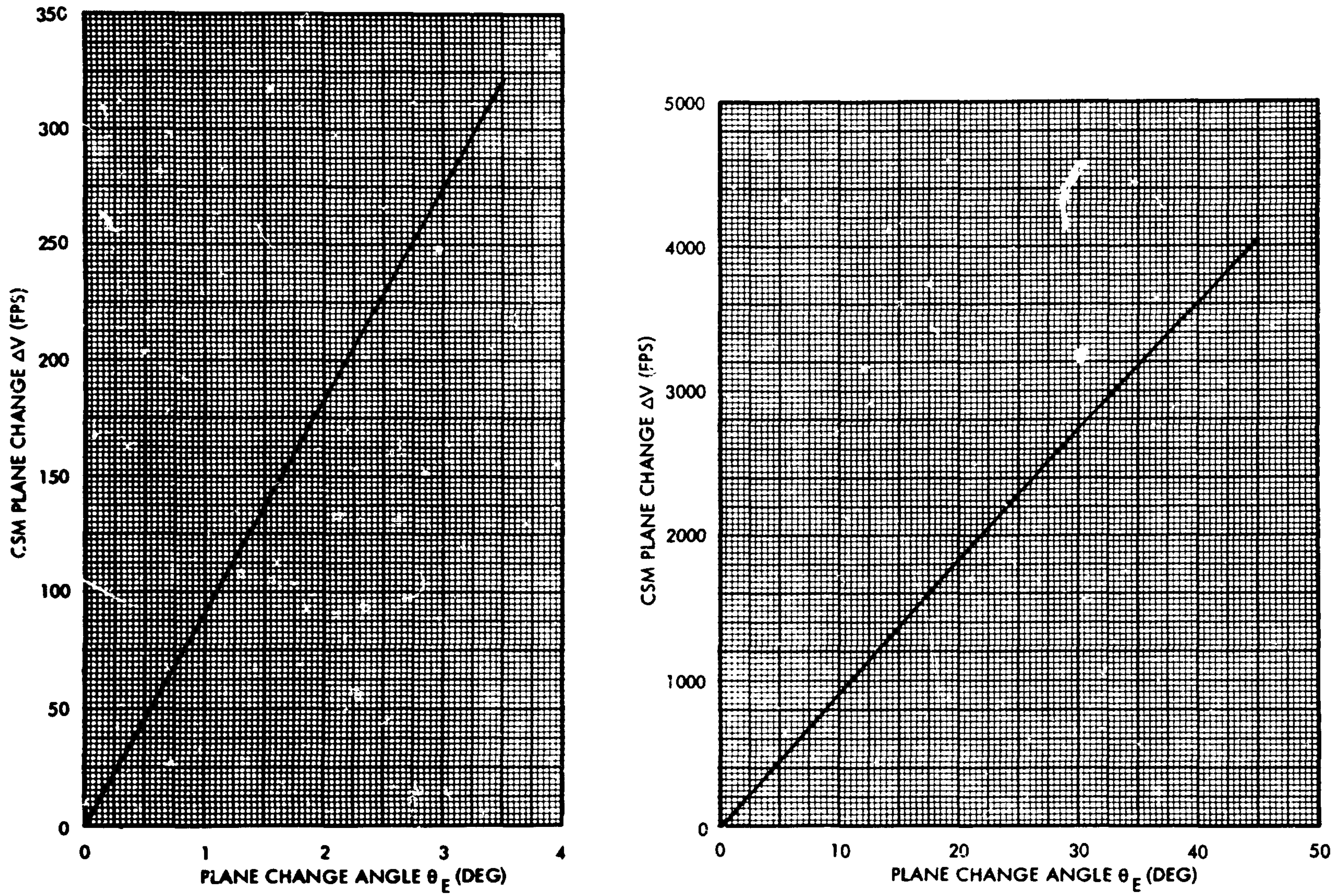


Figure 4-50. CSM Plane Change ΔV versus Plane Change Angle for 80-nautical mile Circular Orbit

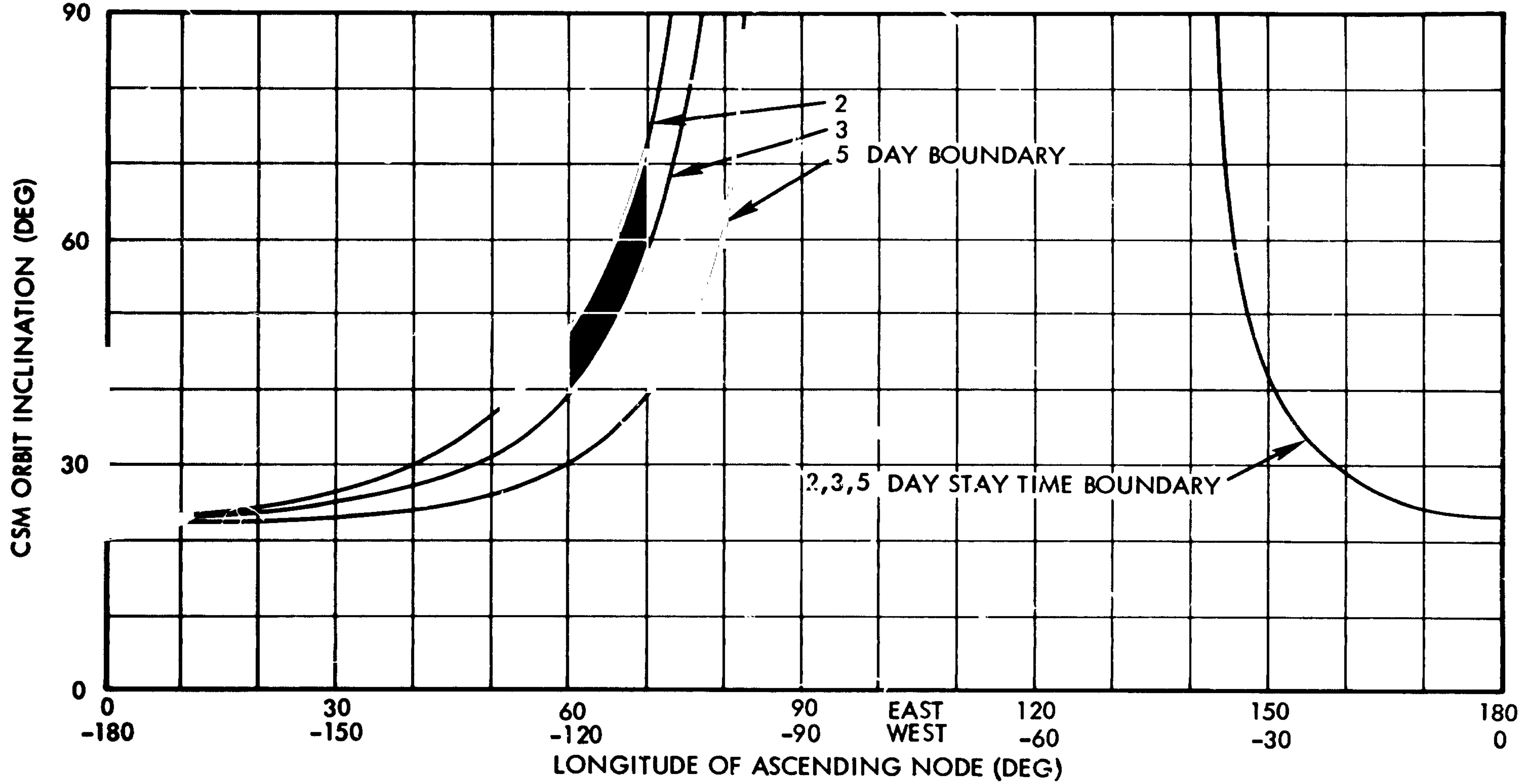


Figure 4-51. Possible Lunar Parking Orbits, Continuous Abort, 14-day Minimum

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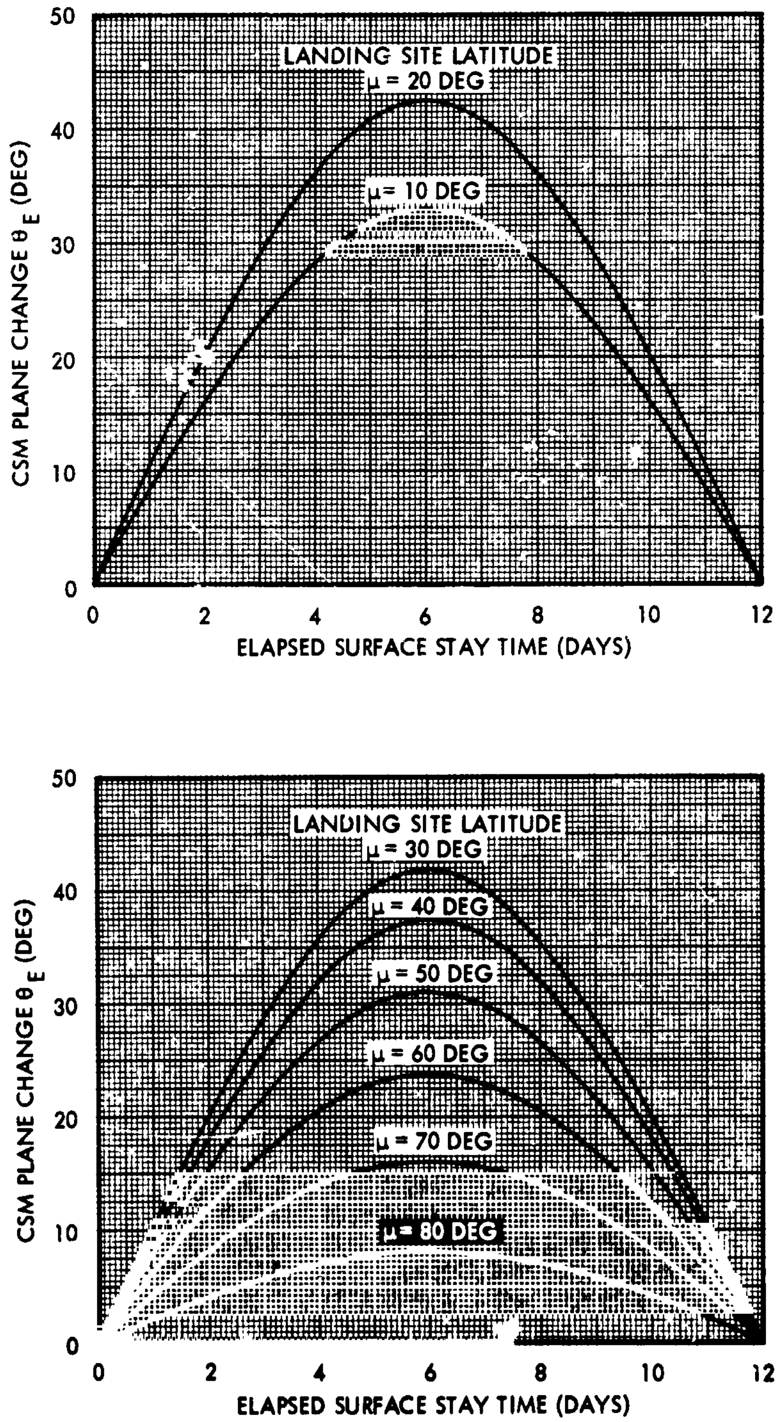

Figure 4-43. CSM Plane Change Angle versus Surface Stay Time for Various Site Latitudes; 12-Day Total Stay Time for Zero LM Plane Change Geometry