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MANNED SPACECRAFT CENTER
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GUIDANCE LAWS FOR CONTROLLING OFF-NOMINAL LM POWERED DESCENT TRAJECTORIES BACK TO THE NOMINAL

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SUMMARY

Additional guidance laws, compatible with the present LM descent quadratic guidance laws, are presented which will greatly reduce the trajectory dispersions that result from off-nominal conditions of the LM descent, such as navigation error. Also, the fuel penalty asscciated with descent engine thrust variation in the FTP (fixed throttle position) mode is practically eliminated. The laws presented are designed to guide the vehicle back to the desired nominal conditions once the off-nominal conditions are detected, such as navigation-altitude error as detected by the landing radar.

The quadratic guidance laws compute the TGO (time-to-go) to an aim point and guide the vehicle to achieve that aim point at TGO equal zero. The guidance laws presented herein perform the additional function of controlling the altitude-time profile and the range-range rate profile to a specified nominal profile for TGO greater than zero. The range-range rate control requires a different guidance law in the fixed throttle regime than in the throttleable regime. The altitude control and the range rate control in the throttleable regime are termed "delta-guidance" because an additional explicit term is added to the quadratic acceleration command. The range rate control in the fixed throttle regime is referred to as "velocity control throttle logic" and involves an empirical control scheme. Each of these functions is independent of the others in that any one or all of them could be used in the LM descent guidance to produce the respective desired results.

These functions were evaluated using error cases chosen to give large dispersions. Relative to the same nominal quadratic guidance trajectory and error conditions, the additional guidance functions resulted in the following improvements:

1. Depending on how it was implemented, delta guidance in the vertical axis reduced altitude dispersions as much as 25 percent at a RGO (range-to-go) of 20,000 ft and as much as 80 percent at an RGO of 2000 ft.

2. With delta guidance in the horizontal axis, the desired approach velocity near the manual takeover point (RGO = 2000 ft) can be maintained for a throttle down time well past high gate. This would allow a later nominal throttle down time with a resulting fuel saving.

3. Velocity control throttle logic can result in a fuel saving corresponding to 70 ft/sec of characteristic velocity.

INTRODUCTION

The LM descent guidance utilizes a quadratic acceleration guidance law which guides the vehicle to achieve specific aim point conditions at a TGO of zero. The descent guidance must accommodate a non-linear descent engine thrust control characteristic which requires that the engine be set at a FTP near maximum thrust during the initial part of the powered descent. Later, when the commanded thrust falls below the throttle recovery point (nominally 57 percent of maximum thrust), the thrust changes abruptly (throttles down) to the commanded value and follows the command as long as it remains below the 63 percent level. Trajectories are designed for nominal conditions of level terrain, no navigation or IMU errors, and a nominal thrust profile in the FTP region. Off-nominal trajectories have the following types of deviations:

a. Altitude deviations caused by navigation-IMU errors and deviations in terrain altitude and slope.

b. Deviations in forward velocity associated with low engine thrust in FTP and late throttle recovery.

c. Deviations in fuel used associated with variations in the throttle recovery time and FTP thrust.

The technique of projecting the guidance aim point beyond the desired aim conditions as presented in reference 1 is being incorporated in the descent guidance. This technique reduces the vehicle pitch sensitivity and trajectory altitude dispersions that result from terrain altitude deviation, but the trajectory altitude dispersions resulting from terrain slope and navigation-IMU errors are aggravated (also shown in reference 1). The delta guidance equations described herein control the trajectory closer to nominal by generating additional acceleration command terms and a velocity control throttle logic which would standardize the throttle recovery time. These additional guidance laws guide the vehicle back to the nominal trajectory from the off-nominal conditions that can be detected; i.e., they do not correct for down and crossrange navigation errors, because the LM guidance computer is unaware of such errors.

The results presented are considered preliminary in that they are just a few examples of some of the tests conducted. A test plan is presently being prepared to thoroughly evaluate the techniques presented herein and to provide answers to questions left unanswered in this report, such as fuel saving, sensitivity, etc.

DISCUSSION

Cubic Guidance

The quadratic guidance constrains the descent trajectory at the aim points in position, velocity, and acceleration in three axes. The next derivative, jerk, is also constrained in two axes (forward and lateral) by computing TGO so that the desired final forward jerk is achieved, and by rotating the guidance coordinate frame to achieve zero final lateral jerk. Now with TGO and the coordinate frame defined, there is no degree of freedom left to constrain the vertical axis any further, unless the order of the acceleration command is increased to cubic or other changes made; i.e., with delta guidance. However, cubic guidance increases the radar-terrain sensitivity. This report will show that delta guidance can do a better job than cubic and can do so with a selectable radar-terrain sensitivity.

Delta Guidance

Additional constraints on the trajectory can be obtained not only by increasing the order of the command acceleration, but also by changing its form to

$$\text{ACCEL CMD} = \text{QUAD ACCEL CMD} + \text{DELTA}$$

where delta can now be considered as either a velocity or position command function. For example, Mr. McSwain's original concept of delta (Δ) was

$$\Delta = K_1 (h_c - h) + K_2 (\dot{h}_c - \dot{h})$$

where $h_c = -K_3$ RZG = position command

$\dot{h}_c = -K_3$ VZG = velocity command

and

VZG = forward (Z) component of velocity in guidance coordinates.

RZG = forward (Z) component of position in guidance coordinates.

This rate damped altitude control system would be useful for the final approach phase, if the nominal h vs RZG trajectory were linear. The derivation of delta guidance that follows will maintain the above form, but commanded range, R_c, and commanded velocity, V_c, will contain explicit equations based on a quadratic nominal trajectory.

Relating again to cubic guidance which constrains the final vertical jerk, delta guidance can achieve the same constraint by a $K_1 = 0$ above, and an equation for V_c that would specify what the vertical velocity should be for trajectory conditions given at any time-to-go. This velocity control system would then force the actual velocity (V) to V_c, at which point the quadratic guidance would proceed on with a zero delta and meet the final vertical jerk. The advantage of this over cubic is that the gain K_2 can be selected to either tighten the control of the trajectory or to relax it to reduce radar-terrain sensitivity.

Delta (Δ) Guidance Logic

Definitions:

i = 1 or 3 for axes (1) or (3); i.e., x or z

n = aim point number, 1 for high gate, 2 for low gate

D = denotes desired aim point conditions

G - denotes guidance axis coordinates

JDG (i,n) and SDG (i,n) are six new desired quantities of final jerk and snap.

JDG (3,1) and (3,2) already exist in the LM guidance. Jerk, of course, is the rate of change of acceleration at TGO = ZERO and snap is the next higher derivative.

R, V, and A are position, velocity, and acceleration; i.e., RDG (1,2) is the desired component of R along the X-axis at low gate, and RG (1) is the LGC knowledge of X at the time-to-go to the aim point.

Given that the total command acceleration $\underline{AG}$ vector is

$$\underline{AG} = \underline{AG}\ (\text{Quadratic}) + \underline{\Delta} \qquad (1)$$

The objective of delta is to control to the nominal trajectory; and therefore, the control function delta is given the form of a rate damped position control system, or a rate command system (i.e., $K_1 = 0$).

$$\underline{\Delta} = K_1 \, (\underline{RDES} - \underline{RG}) + K_2 \, (\underline{VDES} - \underline{VG}) \qquad (2)$$

where $K_1 = Wn^2$

Wn - Natural frequency

$K_2 = 2\zeta W_n$

ζ = Damping coefficient .

The relationship shown of the gains K_1 and K_2 is an approximation. Appendix I contains an exact derivation of delta but the added complexity is probably not justified.

The task now is to define the desired trajectory conditions in terms of (RDES) and (VDES).

From the equation for final jerk for quadratic guidance of reference 2 ($-C_2$ on page 61), the velocity that should exist at any TGO so that the specified desired final jerk would exist, can be shown to be

$$\text{VDES}(i) = \frac{4}{\text{TGO}} \, (\text{RDG}(i,n) - \text{RG}(i)) - 3 \, \text{VDG}(i,n) + \text{ADG}(i,n) \cdot \text{TGO} - \frac{\text{JDG}(i,n) \cdot \text{TGO}^2}{6} \qquad (3)$$

A velocity control system consisting of just the K_2 part of delta could be used which then constrains the final jerk in the same manner that cubic guidance would, except that the added advantage here is the sensitivity control with the coefficient K_2.

Constraining final jerk will tend to reduce trajectory dispersions close to the terminus of the trajectory, but will not help much at large time-to-go. The K_1 part of delta is used to control to the nominal position at all TGO.

If all of the coefficients (AD, JD, and SD) of the quadratic acceleration for a nominal trajectory are specified, then the nominal position at any given TGO will be

$$\text{RDES}(i) = \text{RDG}(i,n) - \text{VDG}(i,n) \cdot \text{TGO} + \text{ADG}(i,n) \cdot \text{TGO}^2/2 - \text{JDG}(i,n) \cdot \text{TGO}^3/6 + \text{SDG}(i,n) \cdot \text{TGO}^4/24 \qquad (4)$$

To limit attitude excursions introduced by delta in driving to a nominal trajectory, a limit on the delta control acceleration would be desirable.

$$\Delta(i) = \left[K_1 \left(RDES(i) - RG(i)\right) + K_2 \left(VDES(i) - VG(i)\right)\right]_{limit} \qquad (5)$$

Delta guidance was mainly devised for control of altitude or X-axis trajectory dispersions, but it also has application to the forward Z-axis (delta for lateral axis would not be desirable, for quadratic guidance operates satisfactorily there). There is an important factor for horizontal delta; i.e., K_1 must be a negative number. (K_1 and K_2 are positive for vertical delta.) K_2 is immaterial for horizontal delta for VDES will, by definition, always equal VG; because TGO is computed so that final jerk is constrained. The TGO calculation, constrained in the forward axis, is also the cause of the requirement for K_1 to be negative for horizontal delta. The mathematical proof of this statement is rather involved and will not be made here. The proof lies in the fact that for a given range deviation from the nominal range-range rate profile, TGO will readjust itself and RDES, so that the range error will take on a different sign from that of the initial deviation.

The following conditions were applied to delta guidance in the simulation from which the results of this report were obtained.

a. Vertical delta guidance started at the nominal time of radar acquisition and used $K_1 = .0025$, $K_2 = 0.1$.

b. Horizontal delta started after throttle recovery and used $K_1 = -.0025$, $K_2 = -0.1$.

c. Delta guidance stopped when $-RZG < 2000$ ft.

d. Acceleration limit was set at ± 1 ft/sec^2.

Velocity Control Throttle Logic

The reduction of variation of the throttle recovery point and associated fuel penalties is the objective of this logic. This can be achieved by commanding a lower (than FTP) throttleable thrust level, when a higher than desired acceleration is detected. A return to the FTP level is made when the desired empirical conditions are achieved.

The original control concept as related to the authors by H. E. Smith of MSC was that of precomputing the quadratic thrust command vs TGO profile that would exist for a three-sigma low FTP engine and to store this

in the guidance computer. An engine with a higher thrust level in FTP would then produce a thrust command which at FTP ignition would be the same, but would get lower in time. With associated deadband and the stored profile for the switching logic, the engine would be pulsed down until the thrust command built up to or beyond the stored profile. This tends to produce a standard final throttle recovery point.

There are two other alternatives for the stored profile used by this throttle logic--VZG versus RZG or VZG versus TGO. The main action taking place during the FTP phase is that of reducing the forward Z component of velocity in guidance coordinates (VZG) to the point where the remainder of the descent can be performed with a throttleable thrust level. The throttle recovery conditions can then be standardized by controlling the Z components of range and range rate to the stored profile. Also, it can be shown because of the way in which TGO is computed, that controlling to VZG vs TGO would produce identical results; i.e., whenever VZG vs RZG is satisfied, VZG vs TGO will also be satisfied. The TGO profile should be easier to empirically curve fit, as shown in figure 1. Note that thrust command vs TGO might be the easiest to fit in that it is essentially linear from TGO = 250 to 600. Control to a thrust commanded profile was not used for the results presented due to this parameter's sensitivity to errors in all three coordinate axes.

The reason for considering this logic for the Z-axis only is that there may be some undesirable effects on the thrust command profile from the X-axis when radar updates start. Both of these profiles will be studied further. The results presented herein will be for the VZG - TGO profile only.

TEST PLAN

Tests conducted during the developmental stage of delta guidance and velocity control throttle logic will be presented. A test plan to thoroughly evaluate these concepts will later be conducted using a more standard set of conditions. The trajectories used in this report vary widely from the old set of high gate targets (using projection technique), one-phase targeting, and the latest modified two-phase. The intent of the tests presented is to show a few examples of the ability of these "control to nominal trajectory" concepts of (1) reducing altitude dispersions with vertical delta, (2) maintaining desired forward velocity in the area of manual takeover with horizontal delta, and (3) standardizing the throttle recovery point with velocity control throttle logic. Tests were conducted on the all-digital LM descent program of reference 3.

Off-Nominal Conditions

Two off-nominal conditions were used: (1) FTP thrust low, navigation and IMU errors that produce vehicle low, and terrain slope of 1° that produces apparent vehicle high (TLVL-1°), and (2) vice versa (THVH+1°). The actual FTP thrust profiles were:

$$TL = 9450 + 0.42568\ t$$

$$TN = 9712.5 + 0.56757\ t$$

$$TH = 9807 + 0.59595\ t$$

where t = time from FTP ignition.

The navigation-IMU errors were those described in reference 1; i.e., the errors were combined in such a manner as to produce an altitude error of $\pm$3000 ft at 300 sec after FTP ignition, and a velocity error ($\dot{h}$) of 20 ft/sec at high gate (480 sec after ignition). These error conditions (3-Sigma) are based on an error probability analysis conducted some years ago--there may be a lower sigma number (i.e., higher probability) associated with these conditions today.

Tests Conducted

1. One-phase trajectory shaped to old high gate-low gate final approach with:

   a. Vertical delta guidance

   b. Complex delta of Appendix I.

2. Old high gate trajectory (reference 4) except high gate projected 50 sec and low gate projected 10 sec. Three runs each of nominal, THVH-1°, and TLVL+1°.

   a. Just quadratic guidance

   b. Cubic guidance in vertical axis only.

3. Same as 2, only landing site shifted 40,000 ft closer to vehicle at ignition.

   a. Just quadratic guidance

b. Vertical delta guidance

(1) $K_1 = 0$, $K_2 = 0.1$

(2) $K_1 = 0.0025$, $K_2 = 0.1$ - After high gate only

(3) " " - After throttle recovery

(4) " " - After nominal radar acquisition point.

4. Modified two-phase type trajectory with low final approach speeds ($\dot{Z}G = 65$ @ $ZG = 2000$) and nominal throttle recovery at TGO = 80 sec from high gate (not quite latest reference trajectory because of negative jerk in final approach). TLVL+1°

a. Vertical delta guidance

b. Vertical and horizontal delta guidance

5. A one-phase trajectory using velocity control throttle logic and vertical and horizontal delta guidance.

a. TL
b. TN
c. TH
d. THVH-1°
e. TLVL+1°
f. TL with landing site shifted 20,000 ft closer to vehicle at ignition.

The VZG_c versus TGO profile for the throttle logic was obtained by forming the following function:

$$VZG_c = AT^2 + BT + C + \frac{D}{T} + \frac{E}{T^2} + \frac{F}{T^3}$$

$$T = TGO$$

A smaller number of terms for generating the profile would probably be satisfactory. The velocity command profile used represented the maximum velocity at any time that was obtained from a series of runs containing 162 combinations of error sources; i.e., thrust deviation, navigation errors, IMU errors, terrain slope, etc. The throttle logic consisted of an engine pulse down to a constant thrust command of 63 percent when the actual velocity (VZG) was less than VZG_c by 20 ft/sec. For the vertical (or radial) acceleration control during this pulse down mode, the engine was assumed to still be at the FTP thrust level (this prevents pitch transients during engine pulses). The FTP mode was reentered when VZG was equal to or greater than VZG_c.

DISCUSSION OF RESULTS

Vertical Delta Guidance

Trajectory Shaping. - The one-phase concept of LM descent targeting consists actually of two phases with different aim conditions for each phase, with the exception that the position conditions are identical and located near the landing site. One problem of this concept is that of obtaining a desired shaping of the final approach trajectory. The data of figure 2 demonstrates two items: (1) the ability with vertical delta guidance of providing a final approach for one-phase identical to that of the old high gate-low gate targeting, and (2) a comparison between the main delta guidance of this report (euqation 5) and the complex delta guidance of Appendix I (equation 13). Both delta guidance systems converged well to the desired final approach, which leads to the conclusion that the delta equation (5)(with its limited acceleration increment; i.e., attitude transient) is satisfactory.

Altitude Dispersions. - The altitude dispersions during the final approach at two locations and characteristic velocities are tabulated in Tables I and II. The net dispersion, which is the difference in altitude between the high and low off-nominal conditions, is shown for quadratic guidance by itself as Case F. The more constraining cubic guidance (Case G) does lower these dispersions slightly.

The nominal trajectories of Tables I and II are basically the old high gate trajectory of reference 4, except the aim points are projected (50 sec for high gate and 20 sec for low gate). The throttle recovery point on the trajectories of Table I was too soon; and therefore, the ΔV was over the budget. To correct this, the landing site was shifted 40,000 ft closer to the vehicle at ignition for the trajectories of Table II. The later throttle recovery point increases the dispersions as can be seen by comparing Cases F and A.

When delta guidance is flown with $K_1 = 0$ (Case B), the effect is the same as with cubic guidance--terminal jerk constrained. The dispersions for Case B were lower than the cubic guidance, Case G, even though the dispersions for Case A were larger than Case F. This partial delta guidance can therefore do a better job than cubic guidance.

The progressively better conditions going from Cases A to E on Table II show that it is best to have the full vertical delta guidance (K_1 and K_2), and starting as soon as possible; i.e., at nominal radar acquisition. The dispersions of Case E are 25 percent and 80 percent lower at 20,000 and 2,000 ft range than Case A. Comparing Cases A and E shows a slight ΔV saving on the nominal for delta guidance. The extra 20 ft/sec for delta guidance and THVH-1° is insignificant because this would be RSS'd with other ΔV dispersions for the fuel budget.

Horizontal Delta Guidance

One of the best ways of saving fuel for a LM descent is to shorten the time between throttle recovery and landing. The problem, though, is that when low FTP thrust levels are encountered, and if throttle recovery hasn't occurred well before high gate, the forward velocity during the final approach can be far off nominal (i.e., high). The object now is not to show how much later throttle recovery can occur if horizontal delta is used, but merely to demonstrate for one particular case where throttle recovery was well past high gate that horizontal delta guidance maintained the desired forward velocity of 65 ft/sec at a range-to-go of 2000 ft.

The nominal forward velocity-range profile is shown on figure 3. The nominal throttle recovery is about 80 sec prior to high gate, and the forward velocity is about 65 ft/sec at a range of 2000 ft. With the TLVL+1° condition, throttle recovery occurs at 10,000 ft range--well past the nominal high gate point at 26,000 ft range. Using just vertical delta guidance (same results expected if no delta used), figure 3 shows the higher forward velocity profile with approximately 85 ft/sec at a range of 2000 ft. And with horizontal delta guidance, which starts acting at throttle recovery, the forward velocity is back to the nominal 65 ft/sec at a range of 2000 ft.

Velocity Control Throttle Logic

The data for the one-phase trajectory that used the throttle logic and delta guidance is shown on Table III. The throttle logic produced no effect on the 3-sigma low thrust engine because the engine was not pulsed down until the final throttle recovery which occurred exactly at high gate. The ability of the throttle logic to standardize the throttle recovery point is demonstrated with the thrust nominal and 3-sigma high runs where the time variation of throttle recovery was only 8 sec, and the ΔV only 2 ft/sec deviation between the three runs. The worst ΔV deviation from nominal (TN) was 9 ft/sec for the TLVL+1° condition.

The number of engine pulses prior to the final throttle recovery point is tabulated and the thrust-time profiles are shown on figure 4. The maximum number of ten pre-throttle recovery engine pulses might be undesirable. No attempt at this point has been made to reduce the number of pulses, but changing the deadband switching criteria from a constant 20 ft/sec should decrease the number of pulses. Also, the number of pulses might be decreased and the accuracy of throttle recovery maintained if the switching level (deadband) were a function of TGO.

The exact value of this throttle logic in terms of ΔV cannot be stated, but an estimate can be made by relating the results of Table III to the test case E of Table II. If the ΔV budget were based on the worst

condition (according to Table III) of TLVL+1° (6466 for Case E), the saving would be the ΔV difference of that case and the nominal (6539) for Case E; i.e., 73 ft/sec. Other considerations though are the amount of ΔV RSS'd in the fuel budget for high deviations from nominal, and also, the fuel saving of later throttle recovery times that might be possible with horizontal delta guidance.

An indication of how the throttle logic might behave in the presence of LGC known off-nominal conditions at PDI is intended with Case 6 of Table III. The landing site was shifted 20,000 ft away from the vehicle, which is approximately equivalent to a PDI ignition four seconds too early. At FTP ignition for this run, the throttle logic immediately prevented the engine from going to FTP, until the correct range-range rate was obtained. The final throttle recovery point was within 10 sec of the desired time.

Two last items can be seen on Table III. The forward velocity was held to within 0.1 ft/sec of the desired at a range of 2000 ft due to the horizontal delta guidance. The altitude dispersions were low due to vertical delta guidance--these off-nominal test conditions on the one-phase trajectory without delta guidance resulted in unsuccessful runs.

CONCLUSIONS

Delta guidance, which computes an additional term that is added to the quadratic acceleration command, is considered (for the following conclusions) to operate from the nominal radar acquisition to a range of 2000 ft with an acceleration command authority limit of ± 1 ft/sec^2.

1. Delta guidance when used in the vertical (X) axis for LM powered descent can reduce the altitude dispersions by 25 percent at a range to the landing site of 20,000 ft, and by 80 percent at a range of 2000 ft.

2. Delta guidance when used in the horizontal (Z) axis has the ability to save budgeted fuel by maintaining the desired forward final approach speeds for late throttle recovery times.

3. The velocity control throttle logic has the ability to save budgeted fuel by standardizing the throttle recovery point. A ΔV saving of 70 ft/sec or greater is possible. The throttle logic maintains a standard range-range rate profile by comparing actual conditions with stored conditions of either range rate (VZG) - TGO, range-range rate, or range-TGO. The throttle logic commands a low throttleable level of thrust when the actual control parameter exceeds a preset level below the desired condition--FTP is reentered when the desired condition is exceeded.

APPENDIX

DERIVATION OF AN EXACT VERTICAL DELTA GUIDANCE TO PRODUCE SPECIFIED CONTROL RESPONSE OF FREQUENCY AND DAMPING

(This form not recommended due to added complexity and lack of limiting of system response; i.e., acceleration increment)

Given the total acceleration command of

$$AG = AG(QUAD) + \Delta = ADG - JFG \cdot TGO + SFG \cdot TGO^2/2 + \Delta \quad (1)$$

where the final jerk and snap (JFG and SFG) are no longer constants if Δ is applied. But still, the following velocity and position equations are valid at each computation interval:

$$VG = VDG - ADG \cdot TGO + JFG \cdot TGO^2/2 - SFG \cdot TGO^3/6 \quad (2)$$

$$RG = RDG - VDG \cdot TGO + ADG \cdot TGO^2/2 - JFG \cdot TGO^3/6 + SFG \cdot TGO^4/24 \quad (3)$$

because for the computation of AG(QUAD) the JFG and SFG terms are computed to satisfy the above equations (see reference 2).

The object of delta guidance is to control to the nominal position on the trajectory for the specified TGO. This nominal position is:

$$RDES = RDG - VDG \cdot TGO + ADG \cdot TGO^2 - JDG \cdot TGO^3/6 + SDG \cdot TGO^4/24 \quad (4)$$

where JDG and SDG are new constants of desired final jerk and snap.

The quantity which should be controlled to zero is:

$$Q \equiv RDES - RG = -(JDG - JFG)\ TGO^3/6 + (SDG - SFG)\ TGO^4/24 \quad (5)$$

Reference 2 derives the following:

$$JFG = -18VDG/TGO^2 - 6VG/TGO^2 + 24(RDG - RG)/TGO^3 + 6ADG/TGO \quad (6)$$

$$SFG = -48VDG/TGO^3 - 24VG/TGO^3 + 72(RDG - RG)/TGO^4 + 6ADG/TGO^2 \quad (7)$$

by substitution of (6) and (7) into (5)

$$Q = -JDG \cdot TGO^3/6 + SDG \cdot TGO^4/24 - VDG \cdot TGO + RDG - RG + ADG \cdot TGO^2/2 \quad (8)$$

the next two derivations of Q are needed to obtain a control law

$$\dot{Q} = JDG \cdot TGO^2/2 - SDG \cdot TGO^3/6 + VDG - VG - ADG \cdot TGO \quad (9)$$

$$\ddot{Q} = -JDG \cdot TGO + SDG \cdot TGO^2/2 - AG + ADG \quad (10)$$

by substitution of (1), (6), and (7) into (10)

$$\ddot{Q} = -JDG \cdot TGO + SDG \cdot TGO^2/2 + 6(VDG + VG)/TGO - 12(RDG - RG)/TGO^2 - \Delta \quad (11)$$

A regular position command rate damped control law can now be established as:

$$\ddot{Q} = -K_1 Q - K_2 \dot{Q} \quad (12)$$

where $K_1 = W_n^2$

W_n = natural frequency

$K_2 = 2\zeta W_n$

ζ = damping coefficient

Delta can now be solved from equations (8), (9), (11), and (12)

$$\Delta = -\left(TGO - \frac{K_2 TGO^2}{2} + \frac{K_1 TGO^3}{6}\right) JDG + \left(TGO^2 - \frac{K_2 TGO^3}{3} + \frac{K_1 TGO^4}{12}\right) \frac{SDG}{2} + \left(\frac{6}{TGO} + K_2 - K_1 TGO\right) VDG + \left(\frac{6}{TGO} - K_2\right) VG + \left(K_1 - \frac{12}{TGO^2}\right) (RDG - RG) + \left(\frac{K_1 TGO^2}{2} - K_2 TGO\right) ADG \quad (13)$$

The further substitution of (6), (7), and (13) into (1) gives

$$AG = (\text{first two terms of } \Delta \text{ above}) + K_2(VDG - VG) - K_1 \cdot TGO \cdot VDG$$

$$+ K_1(RDG - RG) + (1 - K_2 TGO + K_1 TGO^2/2)\ ADG.$$

TABLE I(a). - REDUCTION IN DISPERSIONS WITH CUBIC GUIDANCE AND MIT 50 SEC PROJ. TARGET WITH NO L.S. SHIFT

Case	Altitude at 20000' Range, ft.			Net Dispersion (ft)	Altitude at 2000' Range, ft.			Net Dispersion (ft)
	Nominal	THVH-1	TLVL+1		Nominal	THVH-1	TLVL+1	
F	5641	7174	4839	2335	578	953	490	463
G	5656	7009	4911	2098	554	815	427	388

TABLE I(b). - TOTAL DELTA V

Case	Delta V at touchdown (ft/sec)		
	Nominal	THVH-1	TLVL+1
F	6598	6629	6523
G	6591	6626	6515

Case

F Quadratic Guidance

G Cubic Guidance in the vertical axis only

TABLE II(a). - REDUCTION IN DISPERSIONS WITH DELTA GUIDANCE AND MIT 50 SEC PROJ. TARGET WITH 40K L.S. SHIFT

Case	Altitude at 20000' Range, ft.			Net Dispersion (ft)	Altitude at 2000' Range, ft.			Net Dispersion (ft)
	Nominal	THVH-1	TLVL+1		Nominal	THVH-1	TLVL+1	
A	5601	7444	4153	3291	611	1011	225	786
B	5593	7159	4156	3003	617	770	426	344
C	5534	7183	4359	2824	614	678	498	180
D	5534	6992	4450	2542	614	676	498	178
E	5465	6498	4489	2459	590	674	512	162

TABLE II(b). - TOTAL DELTA V

Case	*Delta V at touchdown (ft/sec)		
	Nominal	THVH-1	TLVL+1
A	6542	6571	6432
B	6542	6586	6452
C	6541	6589	6454
D	6544	6586	6453
E	6539	6590	6466

* Δ V starts at FTP ignition

Case

A Quadratic Guidance
B " "
C " "
D " "
E " "

with Delta Guidance ($\Delta = K_2 \{V_{DES} - VG\}$)

($\Delta = K_1 \{R_{DES} - RG\} + K_2 \{V_{DES} - VG\}$) after high gate
" " " after throttle recovery
" " " after radar acquisition

TABLE III. - A ONE-PHASE TRAJECTORY USING VELOCITY CONTROL THROTTLE LOGIC WITH DELTA GUIDANCE IN THE HORIZONTAL AND VERTICAL AXES

	Conditions	Altitude at RGO = 20K (ft)	Altitude at RGO = 2K (ft)	Delta V at Touchdown (ft/sec)	Throttle Recovery time (+ before h.g.) (sec)	Number of Engine Pulses	Forward Velocity at RGO = 2K (ft/sec)
1	TL	5761	593	6609	0	0	84.4
2	TN	5820	556	6611	+4	8	84.4
3	TH	5676	569	6610	+8	9	84.4
4	THVH-1	7801	638	6614	+6	10	84.5
5	TLVL+1	3532	512	6620	-18	2	84.3
6	* TL	5675	573	6601	-10	1	84.4

*Landing site shifted 20,000 ft further from vehicle @ ignition

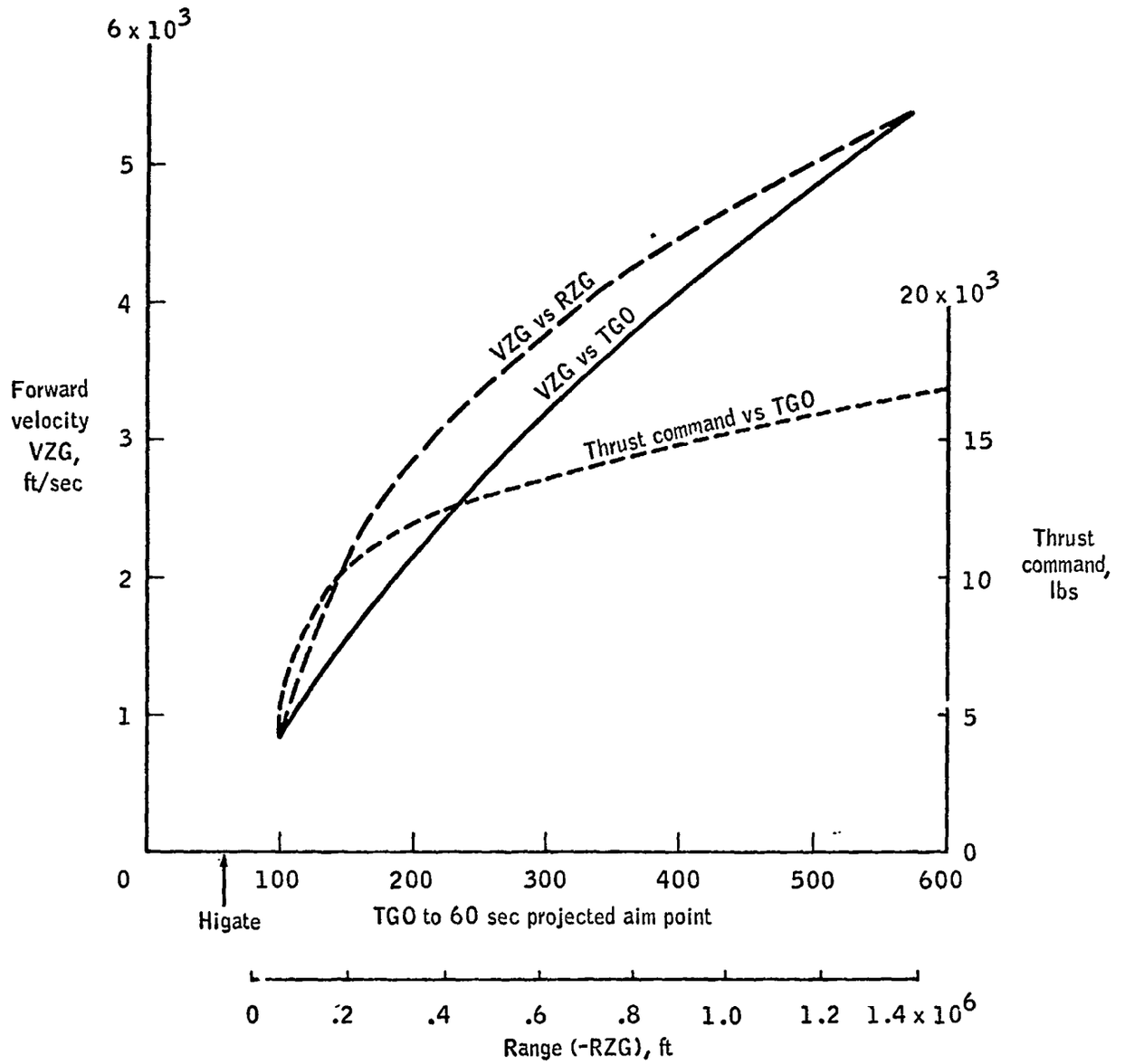

Figure 1. - Profiles considered for velocity control throttle logic. Data from latest unpublish MPAD ref traj as of 2/17/69, with 3-sigma low thrust engine.

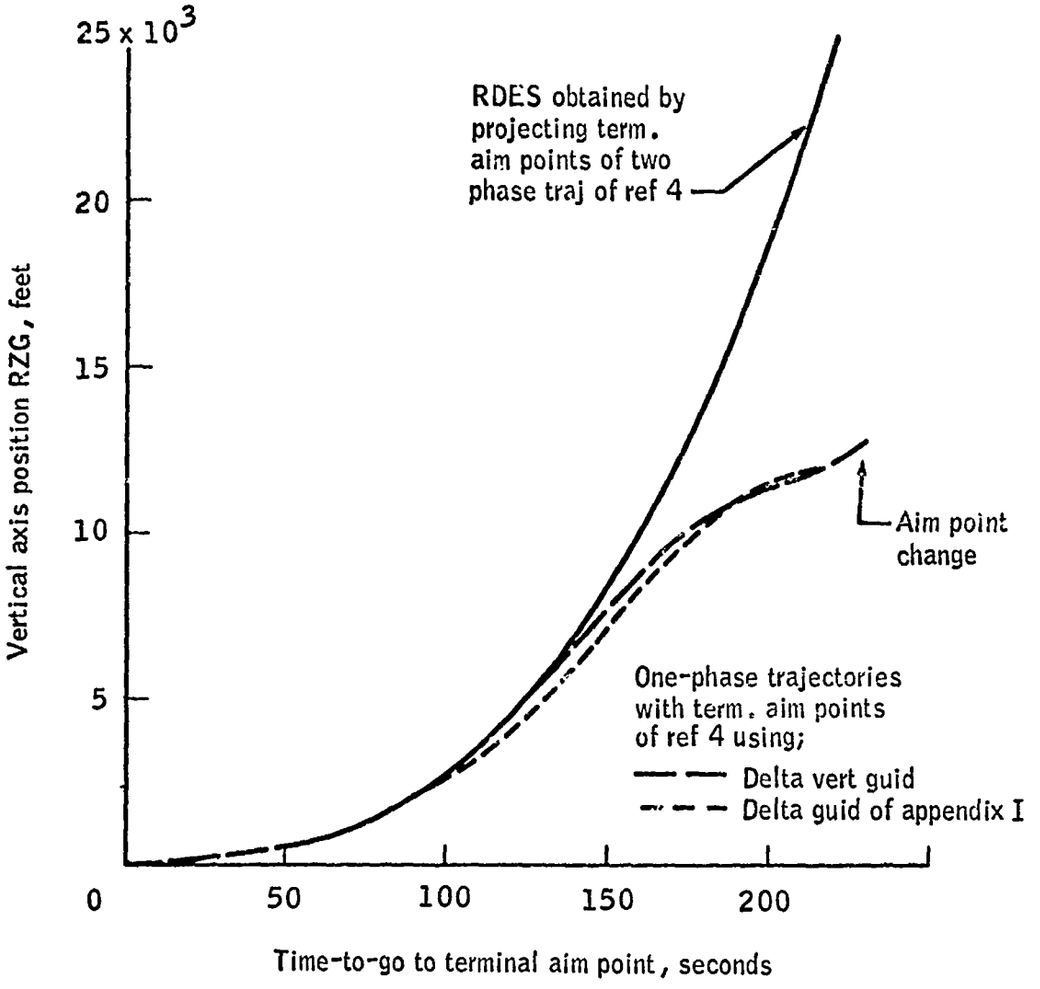

Figure 2. - A one-phase trajectory shaped to old high gate-low gate final approach with delta guidance.

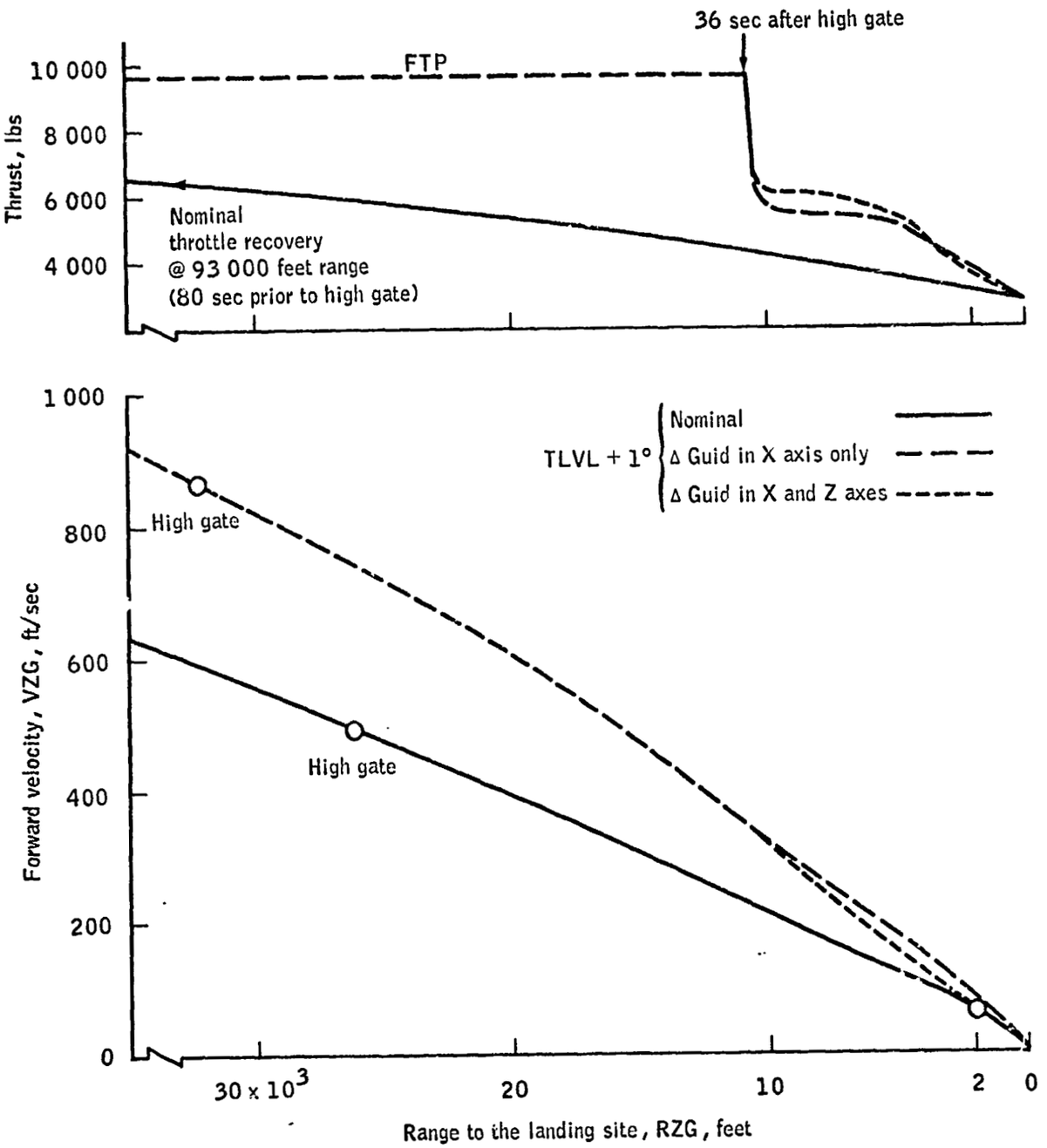

Figure 3. - Maintain desired forward approach speed with Delta horizontal guidance when throttle recovery occurs after high gate for TLVL + 1° condition.

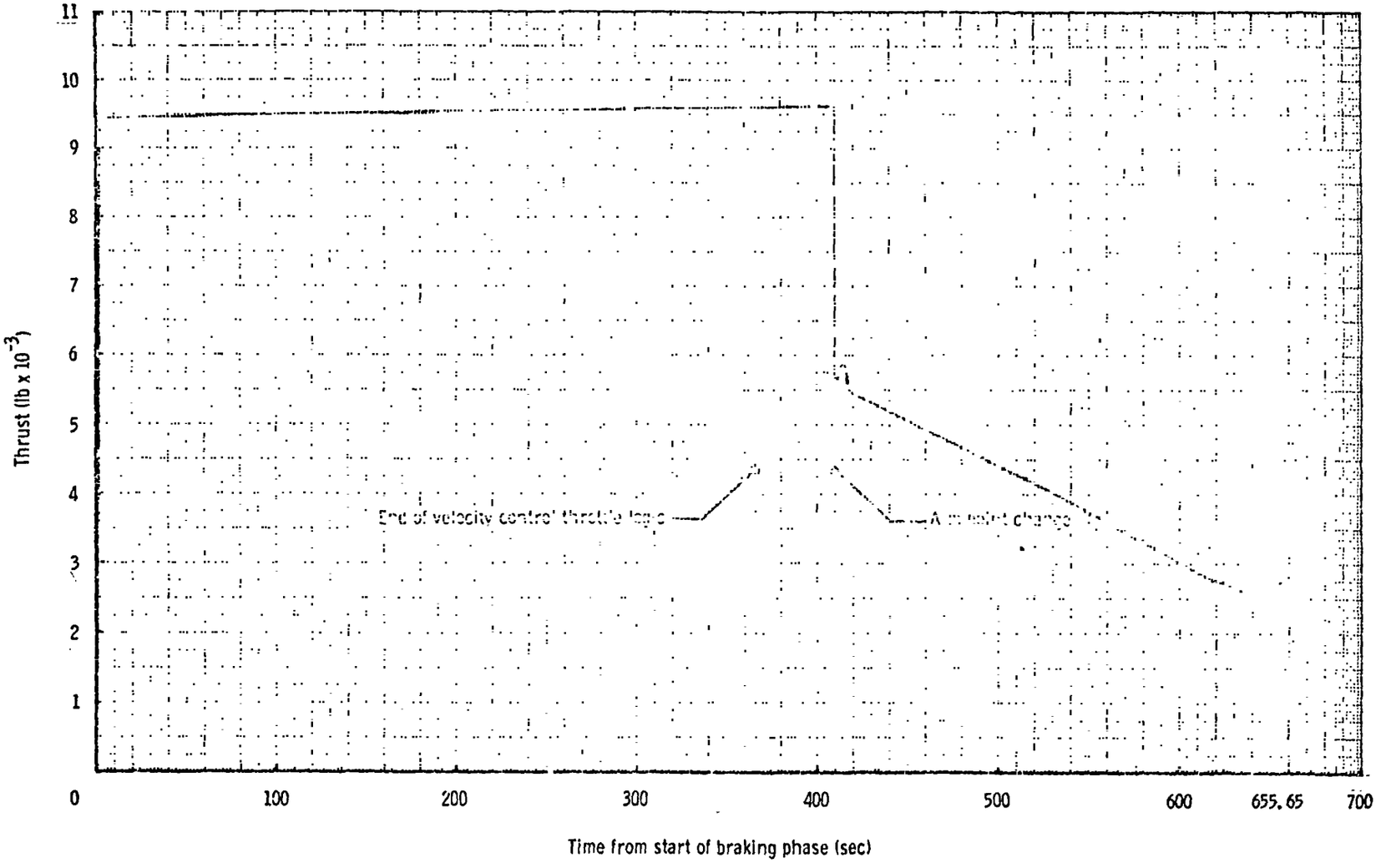

Figure 4 (a). - Thrust profile for low FTP thrust engine using velocity control throttle logic.

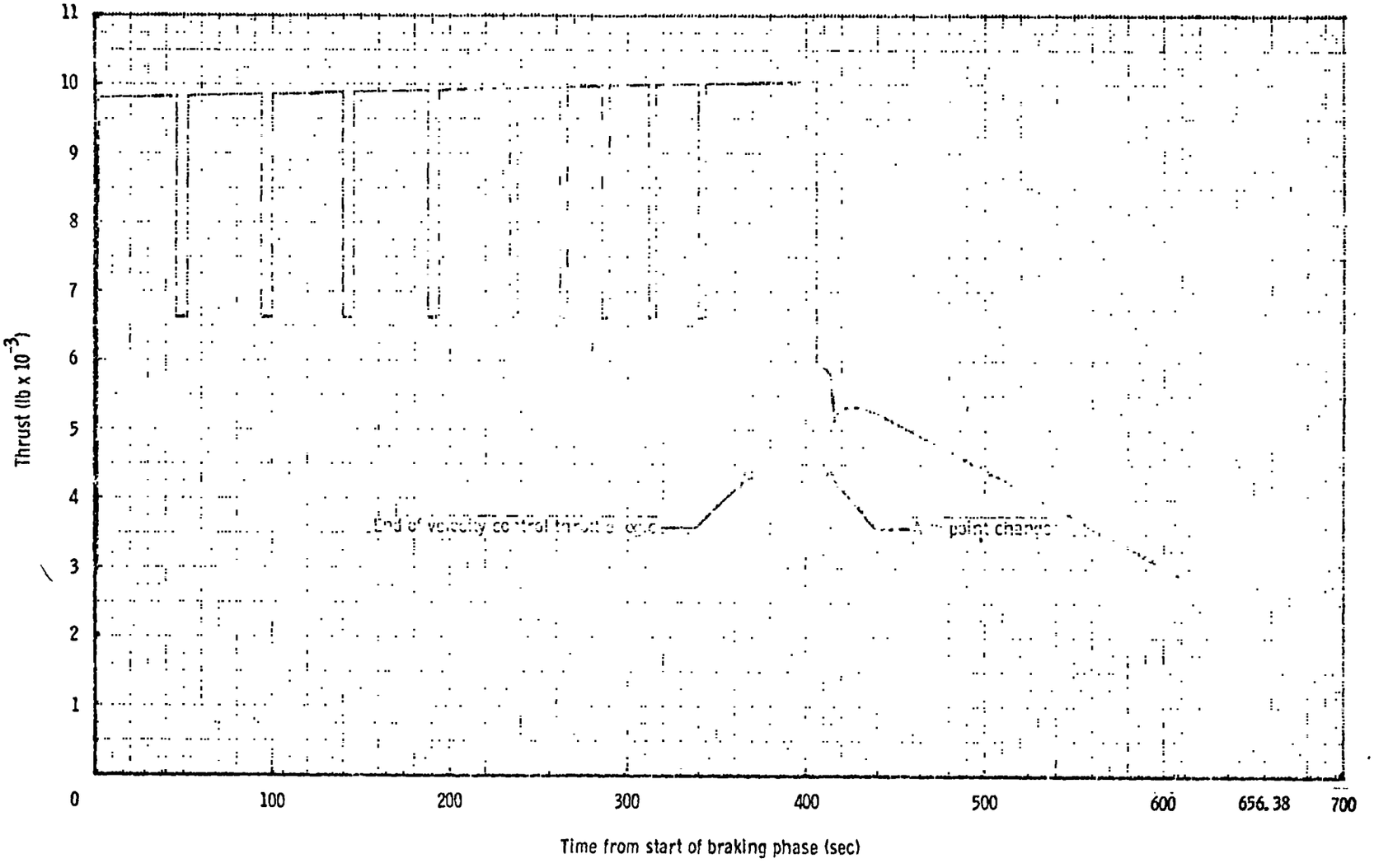

Figure 4 (c). - Thrust profile for high FTP thrust engine using velocity control throttle logic.

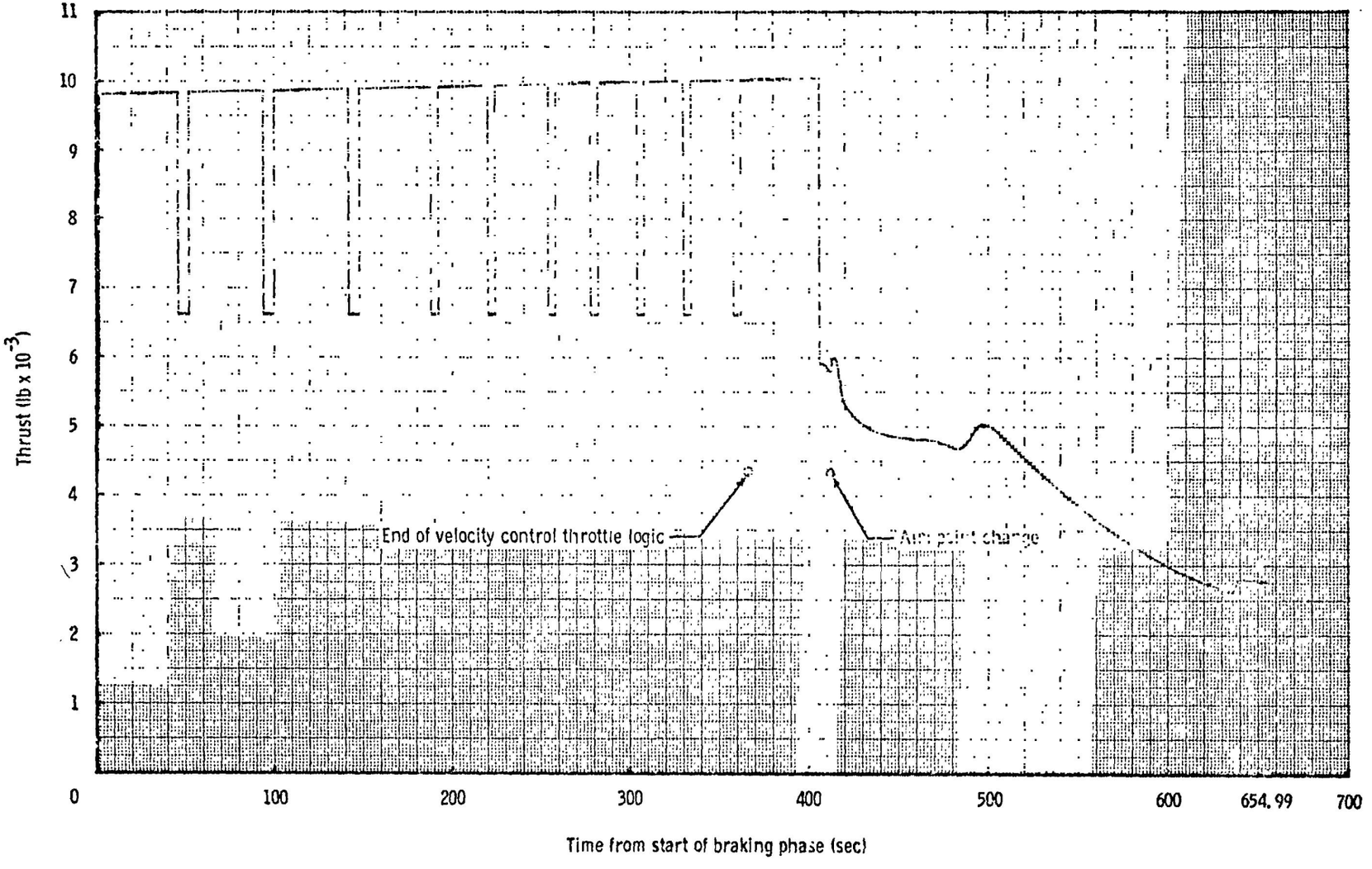

Figure 4 (d). - Thrust profile for THVH -1° using velocity control throttle logic.

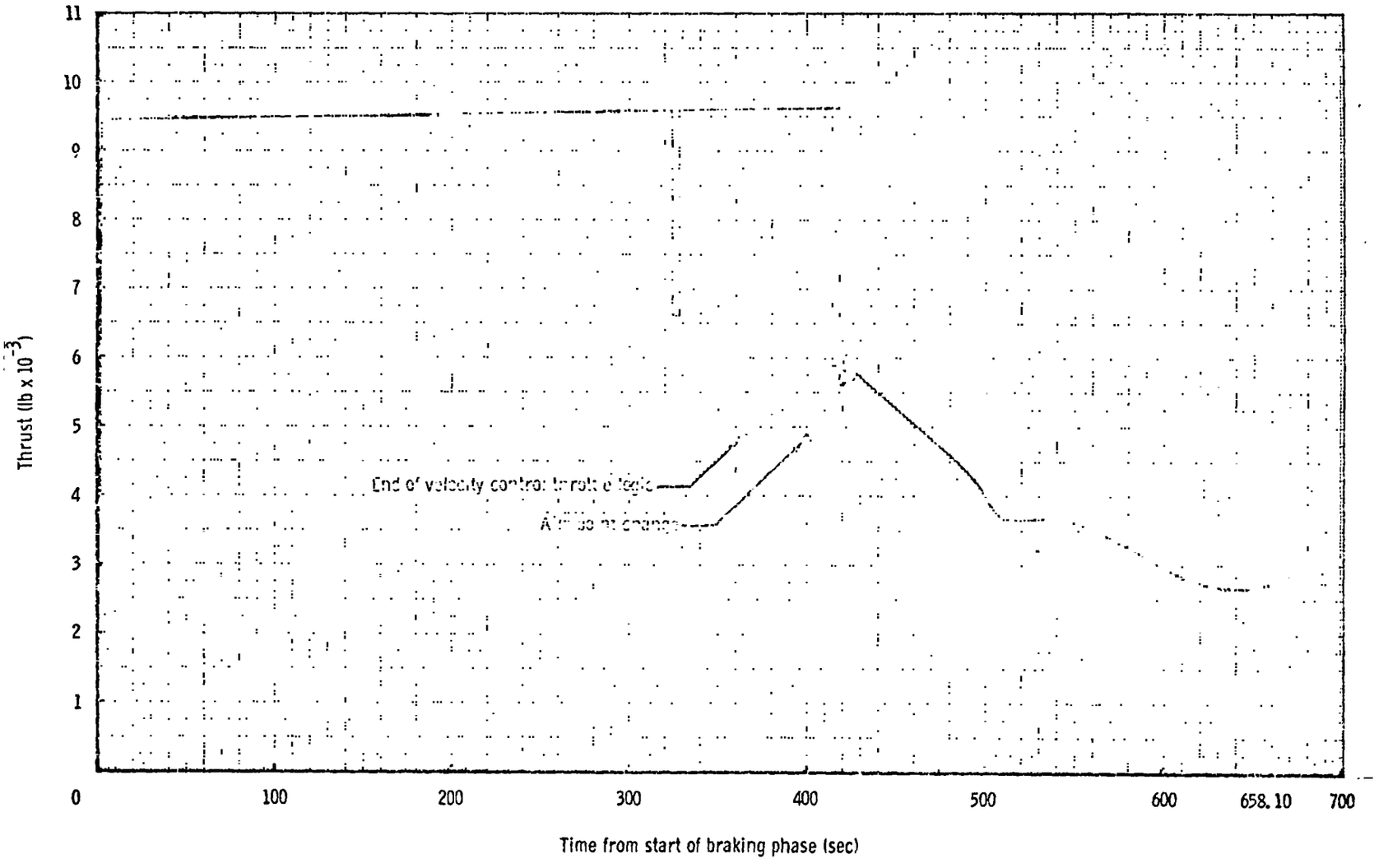

Figure 4 (e). - Thrust profile for TLVL +1° using velocity control throttle logic.

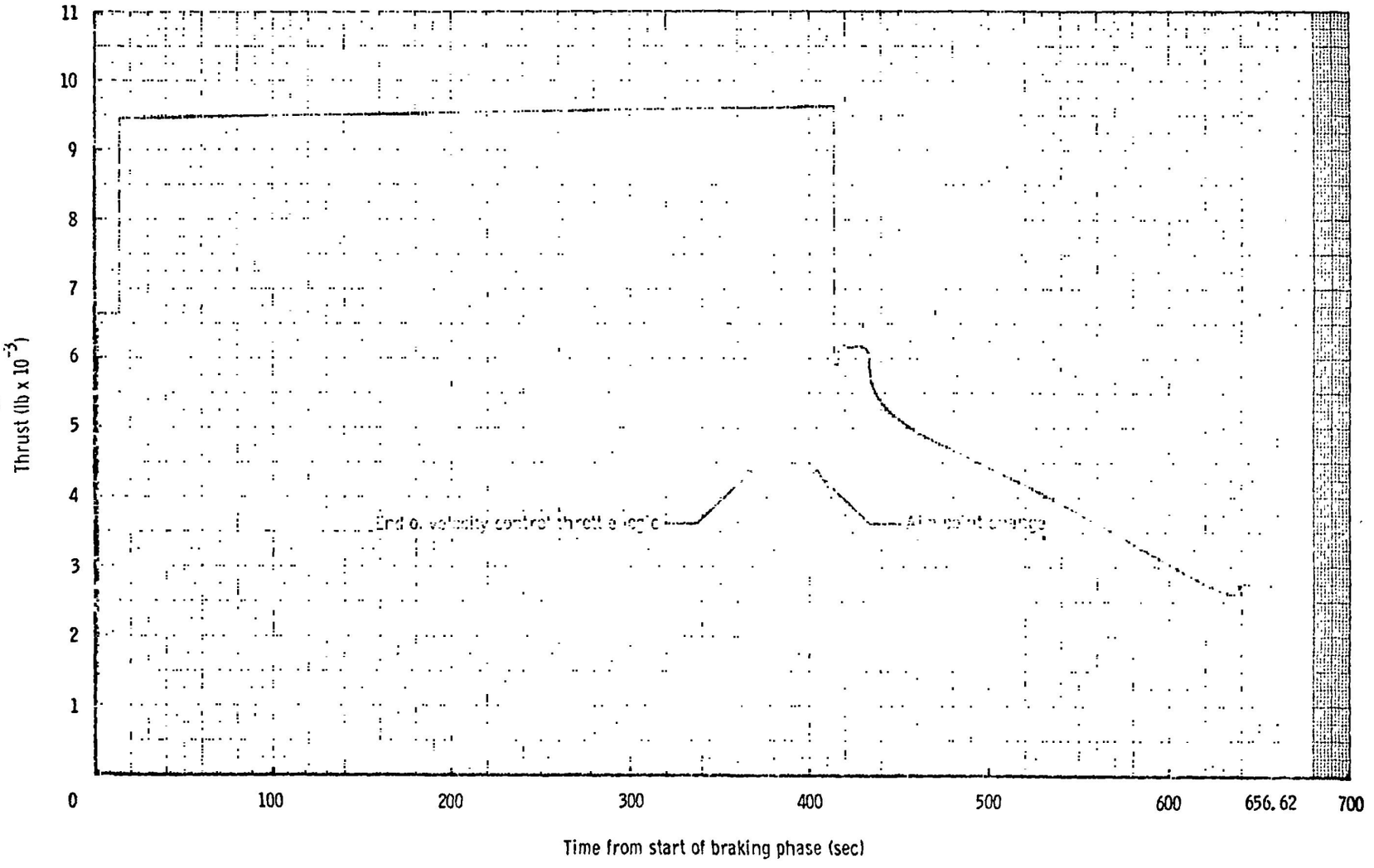

Figure 4 (f). - Thrust profile for low FTP thrust engine with the landing point shifted 20 000 ft away from the vehicle using velocity control throttle logic.

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