(202)

05952-H364-R0-00

(yes)

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MSC INTERNAL NOTE NO. 68-FM-46

February 18, 1968

FORMULATION OF RENDEZVOUS NAVIGATION PLANS FOR MISSIONS D, E, AND G

By Navigation Analysis Section
TRW Systems Group

MSC Task Monitor
P. T. Pixley

MISSION PLANNING AND ANALYSIS DIVISION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

NASA - Manned Spacecraft Center

RELEASE APPROVAL

1. Type of Document: Internal note

2. Identification: 68-FM-46

Page 1 of 2 Pages

TO:

3. FROM:
Division: Mission Planning and Analysis
Branch: Mathematical Physics
Section

4. Title or Subject: FORMULATION OF RENDEZVOUS NAVIGATION PLANS FOR MISSIONS D, E, AND G

Date of Paper: February 18, 1968

5. Author(s): Navigation Analysis Section, TRW Systems
MSC Task Monitor: P. T. Pixley

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FORMULATION OF RENDEZVOUS NAVIGATION PLANS FOR MISSIONS D, E, AND G	February 18, 1968

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05952-H364-R0-00

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By

NAVIGATION ANALYSIS SECTION, TRW SYSTEMS GROUP

FEBRUARY 18, 1968

MISSION PLANNING AND ANALYSIS DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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CONTENTS

NOMENCLATURE

A	Matrix of regression coefficients for the dynamical state
AGS	Abort Guidance System
B	Matrix of regression coefficients for the systematic errors
GSI	Ground state initialization
GSU	Ground state update
J	Data normal matrix
JCP	Joint cumulative probability (defined in Appendix A)
k_{Di}	Dimensionless coefficient of drag for the i^{th} vehicle
L	First difference of JCP
MSFN	Manned Spaceflight Network
OSR	Onboard state reinitialization
PNGS	Primary Navigation and Guidance System
PRS	Probability of rendezvous success
Q	Second difference of JCP
RSS	Root sum of squares
RTCC	Real-time Computing Center
S_i	i^{th} safety factor
SOE	Solve on everything
t	Time
(U, V, W)	Orthonormal triad of vectors defining the orbit plane coordinates
v_{DCA}	Relative velocity at the distance of closest approach
W	Diagonal weighting matrix
WLS	Weighted least squares
X	Dynamical state vector or bounds vector for JCP
Z	Systematic error vector

ΔH	Concentric altitude difference for coelliptic orbits
ΔV	Increment in speed of vehicle due to a powered maneuver
δk_{Di}	Error in coefficient of drag for the i^{th} vehicle
$\delta\mu_{\oplus, ☽}$	Error in gravitational constant for the earth or the moon
Λ	Covariance matrix
μ	Gravitational constant
ρ_{DCA}	Distance of closest approach
σ^2	Variance
ω_k	k^{th} preset data weight
ω_{rr}	Preset weight for spacial errors during rendezvous
ω_{rv}	Preset weight for velocity errors during rendezvous

1. INTRODUCTION AND SUMMARY

TRW will generate TAPP program simulations of the rendezvous maneuver sequences in Missions D, E, and G with various navigation plans for each mission. The navigation plan specifies the data incorporation scheme which provides estimates for the states of both vehicles and consequently determines the maneuvers. A navigation plan is characterized by the following:

1) The choice of onboard versus ground computer estimates,

2) The source and type of a priori knowledge in each fit,

3) The type and amount of tracking data used in each fit, and

4) The set of parameters to be estimated by the fit.

The onboard data incorporation system is a Kalman filter, which is simulated by a weighted least squares (WLS) estimator. If relative tracking alone provides inadequate navigation, a ground state update can be provided by the Real-time Computing Center (RTCC) using Manned Spaceflight Network (MSFN) tracking data.

When relative tracking data is combined with a ground state update, the onboard estimate becomes dependent on the choice of the preset a priori weights, ω_{rr} and ω_{rv} (for the rendezvous mode), as well as the preset data weights, ω_k. The data weights enter each data incorporation directly, but the effect of the a priori weight is felt only indirectly after the first incorporation, which also updates the covariance of the error in the onboard estimate of the state of the active vehicle. The simulations will be used to determine the values of the preset weights, to select the most acceptable navigation plan, and to provide an adequate understanding of the navigational accuracy which will be available for each mission.

The real world covariance matrices for the errors in the estimates of both vehicles and the estimate of their relative state will be computed for various trial weights in order to determine a preliminary choice of values for the preset weights. Given a set of minimum requirements for successful rendezvous, the probability of satisfying these requirements will be computed by the TAPP VI Monte Carlo Processor in order to provide an acceptability criterion for the navigational accuracy analysis. A final determination of the preset weights can be obtained by maximizing the probability of satisfying the rendezvous requirements over a sufficiently large set of trial weights. The Monte Carlo statistics also provide a means of comparing the various navigation plans after the weights are optimized for each plan. The actual preset values of the weights to be used for a particular mission should be the best available optimization of the weights for the most acceptable navigation plan.

Section 2 gives the proposed technical approach for the simulation of the orbital navigation problem of the rendezvous missions. More detailed discussions of the event sequences for Missions D, E, and G are given in Appendixes C, D, and E. The various data incorporation schemes are given in Appendix A. Appendix B gives the formulation of a criterion for the acceptability of navigational accuracy in terms of a joint cumulative probability (JCP) assuming that the requirements for successful rendezvous can be expressed in terms of bounded physical quantities such as fuel mass. The JCP criterion provides a means of determining the preset weights by Monte Carlo procedures; however, it is expected that an adequate set of preset weights will be obtained from the covariance matrix analysis outlined in Section 2.3.

2. THE TECHNICAL APPROACH

The most acceptable navigation plan for each rendezvous profile will be selected from several proposed plans. The relative merit of different plans can only be determined after the optimal values for the onboard preset weighting parameters associated with each plan (a priori estimate weights and relative tracking data weights) are determined. Specifications of preset parameter values are not necessary for navigation plans which do not require that onboard tracking data be processed by the onboard computer.

The selection of the most acceptable values for the preset weights and of the most acceptable navigation plan itself is carried out in two phases. The initial phase is an evaluation of the covariance matrices for the estimates of the relative states at nominal maneuver times and at the nominal time of rendezvous. This evaluation will be carried out for trial sets of weighting parameters and for a particular navigation plan in order to select preliminary optimal weights for that navigation plan. A preliminary set of weights will be determined for each navigation plan of importance to a particular mission profile.

The covariance matrices for the error in the estimate of the relative state, which are obtained when the preliminary weights are used, will then be compared in order to determine the most acceptable navigation plan. An optimal or ideal set of covariance matrices for the error in the estimate of the relative state may be computed by assuming that the onboard computer; (1) solves for all systematic error parameters, (2) weights the relative tracking observations with the real-world variances (real-world weights), and (3) weights the a priori MSFN estimates with the correct real-world covariance matrices.

The covariance of the onboard estimate should be close to the covariance of the ideal solve-on-everything (SOE) covariance for each maneuver if the preset weights are chosen optimally. The comparison of the covariances and the preliminary choice of preset weights will be made by an engineering evaluation using program output.

Phase two of the study will attempt to refine the preliminary estimate of the preset weights determined in phase one and to evaluate the selected navigation plan by examining Monte Carlo statistics of the ΔV cost for each maneuver and the distance of closest approach at rendezvous when the preliminary preset weights are used.

2.1 REAL WORLD COVARIANCES FOR THE RTCC ESTIMATES

The MSFN tracking data goes to the RTCC computer, which generates RTCC estimates and associated fit-world covariances for the states of both vehicles in real time. The RTCC estimates are used for the purposes of ground-based flight control and can be transmitted to the computer on board either vehicle to provide a RTCC update prior to a computed maneuver. Onboard navigation is based on the ability of the onboard computer to propagate estimates of both states, to receive their RTCC updates, and to incorporate the onboard observations.

For purposes of navigational accuracy analysis, it is necessary to compute the real-world covariances of the updates in order to determine the quality of the fit-world estimate which can have systematic errors. If all systematic error variables are solved for, the fit-world covariance is the real-world covariance, but in practice the fit-world covariance underestimates the errors (i.e., neglects systematic errors). The present study is concerned with the navigational accuracy which can be obtained from various alternative combinations of RTCC updates and relative tracking for rendezvous purposes. Consequently, an estimate of the relative state and its real-world covariance will be computed for the RTCC updates, the onboard updates, and an idealized onboard estimate, which includes all systematic errors.*

Rendezvous navigation usually requires a RTCC update, but the subsequent data incorporations are degraded by the use of a preset a priori covariance, Λ_p, for the RTCC update. Let

$$\Lambda_p^{-1} = \begin{pmatrix} \omega_1 \bar{\bar{1}} & 0 \\ 0 & \omega_2 \bar{\bar{1}} \end{pmatrix}$$

where $\bar{\bar{1}}$ is a 3 X 3 identity matrix and $\omega_1 = \omega_{rr}$ and $\omega_2 = \omega_{rv}$. (Notice that ω_{rr} and ω_{rv} are weights rather than RMS errors).

*This will be denoted an SOE-fit or SOE estimate.

Ideally, the RTCC estimate $\delta X_{1E}(G, t_U)$* for vehicle (V_1) at an update time t_U should have an a priori covariance, $\Lambda_G^{x_1 x_1}$, which is the real-world covariance for the MSFN tracking of vehicle (V_1) up to t_U. This would overcome the loss of tracking information which occurs when the preset a priori is used, but it neglects the correlation of the errors in the active and the passive vehicles due to station location errors and other errors which influence the estimates of both states. Consequently, for rendezvous navigation, the ideal onboard data incorporation procedure should provide SOE estimates, $\delta x_{SOE}(V_1, t_M)$, onboard the active vehicle (V_1) at maneuver times t_M.

The onboard SOE state,

$$x_{SOE} = (x_1, x_2, z_V)$$

includes both vehicles and all systematic errors (z_V) which can occur in the onboard data incorporation (Table 2, see Section 2.2).

$$z_V = (z_D, z_V')$$

where z_D gives the dynamical systematic errors, and z_V' gives the non-dynamical systematic error.

The corresponding SOE state for the MSFN tracking is (x_1, x_2, z_G), however, this state has too many variables to be estimated (Table 1, see next page). Notice that the same dynamical systematic errors (z_D) occur in both z_G and z_V, so that

$$z_G = (z_D, z_G')$$

and

$$z_D = z_G \cap z_V = (\delta\mu, \delta k_{D1}, \delta k_{D2}).$$

Thus, z_V' contains the non-dynamical systematic errors of z_V and is uncorrelated with the RTCC estimates. Similarly, z_G' contains the

*G denotes ground systems or MSFN tracking and E denotes an estimated state as opposed to an actual or reference state.

Table 2-1. MSFN Systematic Error Vector, z_G

Non-Dynamical, z_G'	S-Band Data Biases/Station
	Range Rate Bias
	X_{30} Angle Bias
	Y_{30} Angle Bias
	X_{85} Angle Bias
	Y_{85} Angle Bias
	C-Band Data Biases/Station
	Range Bias
	Elevation Bias
	Azimuth Bias
	Station Location Errors/Station
	Longitude Bias
	Latitude Bias
	Height Bias
Dynamical, z_D	$\delta\mu_{\oplus}$, Error in Gravitational Constant for the Earth
	or $\delta\mu_{☽}$, Error in Gravitational Constant for the Moon
	δk_{D1}, Error in the Dimensionless Coefficient of Drag for Vehicle V_1
	δk_{D2}, Error in the Dimensionless Coefficient of Drag for Vehicle V_2

non-dynamical systematic errors of z_G. Consequently, the RTCC part of the a priori covariance for an onboard SOE fit is the real-world covariance of the estimate $\delta X_{1E}(G, t_U)$ and has the form

$$\Lambda(G, t_U) = \begin{pmatrix} \Lambda_G^{x_1x_1} & \Lambda_G^{x_1x_2} & \Lambda_G^{x_1z_D} \\ \Lambda_G^{x_2x_1} & \Lambda_G^{x_2x_2} & \Lambda_G^{x_2z_D} \\ \Lambda_G^{z_Dx_1} & \Lambda_G^{z_Dx_2} & \Lambda_G^{z_Dz_D} \end{pmatrix}$$

where

$$\Lambda_G^{x_ix_j} = J_{x_ix_i}^{-1} \left[J_{x_iz_G} \Lambda^{z_Gz_G} J_{z_Gx_j} + \tilde{J}_{x_ix_j} \right] J_{x_jx_j}^{-1}$$

and

$$\Lambda_G^{x_iz_D} = J_{x_ix_i}^{-1} \, J_{x_iz_D} \, \Lambda^{z_Dz_D}$$

are real-world covariances for the RTCC estimates, if

$$J_{x_ix_i} = \left(A_i^T W A_i\right)_G \text{ for } i = 1, 2 .$$

$$\tilde{J}_{x_ix_j} = \left(A_i^T W M_{ij} W A_j\right)_G$$

where

$$M_{ij} = E\left(n_i n_j^T\right) = \begin{cases} M_{ii} \text{ if } i = j \\ 0 \text{ if } i \neq j . \end{cases}$$

M_{ii} is the noise covariance matrix of the zero mean noise vector n_i for the i^{th} vehicle. A diagonal form of M_{ii} can be input to TAPP IV in order to compute $\tilde{J}_{x_ix_i}$.

$$J_{x_iz_G} = \left(A_i^T W B_i\right)_G = \left(J_{x_iz_D}, J_{x_iz_G'}\right)$$

where $J_{x_iz_D}$ is the partition of $J_{x_iz_G}$ for the dynamical systematic errors. $\Lambda^{z_Gz_G}$ is the diagonal matrix of variances for the systematic errors in

RTCC estimates,* and $\Lambda^{z_D z_D}$ is the submatrix of variances for the dynamical systematic errors z_D. There are six or seven non-dynamical errors for each MSFN station (three or four data biases and three station location biases) in addition to the three dynamical errors, making a total of $7n_s + 6n_c + 3$ systematic errors for an RTCC update utilizing data from n_s different S-band stations and n_c different C-band stations.

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*Correlations cause non-diagonal terms. For example, co-located stations have both independant and correlated errors in longitude, latitude, and height above the geoid. Each pair of correlated variables has zero mean random errors $\delta_{1,2}$ of the form

$$\delta_1 = \epsilon_1 + \epsilon_{CO}$$

$$\delta_2 = \epsilon_2 + \epsilon_{CO}$$

where $\epsilon_{1,2}$ represents the uncorrelated errors of measurement from the common fiducial point to the station and ϵ_{CO} is the correlated part of the errors $\delta_{1,2}$ due to the error in location of the fiducial point. The 6 X 6 covariance matrix for these variables is given by

$$\Sigma_2 = \begin{pmatrix} \Sigma_\lambda & 0 & 0 \\ 0 & \Sigma_\theta & 0 \\ 0 & 0 & \Sigma_h \end{pmatrix}$$

where $\Sigma_{\lambda,\theta,h}$ are 2 X 2 covariance matrices of the form

$$\begin{pmatrix} \overline{\epsilon_{CO}^2} + \overline{\epsilon_1^2} & \overline{\epsilon_{CO}^2} \\ \overline{\epsilon_{CO}^2} & \overline{\epsilon_{CO}^2} + \overline{\epsilon_2^2} \end{pmatrix} .$$

The correlated errors are described by a third variance $\overline{\epsilon_{CO}^2}$. Drag provides a similar example, if the atmospheric variations are correlated, because the vehicles are close to each other.

It is also desirable to compute the real-world covariance, $\Lambda_{REL}(G, t_U, t_M)$, for the errors in the estimate of the relative state at maneuver time t_M as determined from the MSFN data at update time t_U.

Let the actual relative state and the estimated relative state be given by

$$\delta X_{rA} = \delta X_{2A} - \delta X_{1A} \text{ and}$$

$$\delta X_{rE} = \delta X_{2E}(G, t_U, t_M) - \delta X_{1E}(G, t_U, t_M),$$

so that the real-world covariance of the relative state is given by

$$\Lambda_{REL}(G, t_U, t_M) = E\left[(\delta X_{rE} - \delta X_{rA})(\delta X_{rE} - \delta X_{rA})^T\right]$$

$$= \Lambda_{(Gt_U t_M)}^{x_1 x_1} + \Lambda_{(Gt_U t_M)}^{x_2 x_2} - \Lambda_{(Gt_U t_M)}^{x_1 x_2} - \Lambda_{(Gt_U t_M)}^{x_2 x_1}$$

where

$\Lambda_{(Gt_U t_U)}^{x_i x_j} = \Lambda_G^{x_i x_j}$ for the specific case $t_M = t_U$, but in general

$$\Lambda_{(Gt_U t_M)}^{x_i x_j} = T_i \Lambda_G^{x_i x_j} T_j^T - T_i \Lambda_G^{x_i z_D} U_j^T$$

$$- U_i \Lambda_G^{z_D x_j} T_j^T + U_i \Lambda_G^{z_D z_D} U_j^T$$

if t_M and t_U belong to the same free flight segment so that

$$T_i = \partial X_i(t_M)/\partial X_i(t_U) \text{ and}$$

$$U_i = \partial X_i(t_M)/\partial z_D \text{ for } i = 1, 2.$$

Substituting the explicit expressions for $\Lambda_G^{x_i x_j}$ and $\Lambda_G^{x_i z_D}$ gives

$$\Lambda_{REL}(G, t_U, t_M) = T_1 J^{-1}_{x_1x_1} \tilde{J}_{x_1x_1} J^{-1}_{x_1x_1} T_1^T + T_2 J^{-1}_{x_2x_2} \tilde{J}_{x_2x_2} J^{-1}_{x_2x_2} T_2^T$$

$$+ \Psi \Lambda^{z_G' z_G'} \Psi^T + \Phi \Lambda^{z_D z_D} \Phi^T + (U_2 - U_1) \Lambda^{z_D z_D} (U_2 - U_1)^T$$

where

$$\Psi = T_2 J^{-1}_{x_2x_2} J_{x_2 z_G'} - T_1 J^{-1}_{x_1x_1} J_{x_1 z_G'}$$

and

$$\Phi = T_2 J^{-1}_{x_2x_2} J_{x_2 z_D} - T_1 J^{-1}_{x_1x_1} J_{x_1 z_D} - U_2 + U_1 .$$

The first two terms of Λ_{REL} are due to vehicles V_1 and V_2 independently, while the last three terms represent correlation effects which depend on the systematic errors.

2.2 REAL-WORLD COVARIANCES FOR THE ONBOARD ESTIMATES

The covariances of the errors in the estimate of the relative state provided by the estimates made onboard the active vehicle can be computed for various trial weights and compared with each other and with the covariances of the estimate of the state obtained from the SOE estimate. This comparison leads to a choice of optimal values for the preset weights. The SOE estimate

$$x_{SOE} = (x_1, \ x_2, \ z_V)$$

gives an ideal onboard estimate of the relative state $x_{REL} = x_2 - x_1$.

The SOE covariance for the errors in the estimate made on board vehicle V_1 at maneuver time t_M is given by

$$\Lambda_{SOE} = \left[J(V_1, t_M) + \Lambda_p^{-1} (V_1, t_M) \right]^{-1}$$

where

$$J(V_1, t_M) = \begin{pmatrix} A_1^T W A_1 & A_1^T W A_2 & A_1^T W B_V \\ A_2^T W A_1 & A_2^T W A_2 & A_2^T W B_V \\ B_V^T W A_1 & B_V^T W A_2 & B_V W B_V \end{pmatrix} = \begin{pmatrix} J^V_{x_1 x_1} & J^V_{x_1 x_2} & J^V_{x_1 z_V} \\ J^V_{x_2 x_1} & J^V_{x_2 x_2} & J^V_{x_2 z_V} \\ J^V_{z_V x_1} & J^V_{z_V x_2} & J^V_{z_V z_V} \end{pmatrix}$$

is the normal matrix for the relative tracking in the free-flight segment, (t_{M-1}, t_M), from the previous maneuver time t_{M-1}, to the current maneuver time, t_M; and

$$\Lambda_p(V_1, t_M) = \begin{cases} \Lambda'(G, t_U) \text{ if } t_{M-1} = t_U \text{ or} \\ \Lambda_{SOE} (V_1, t_{M-1}) \text{ otherwise,} \end{cases}$$

is the a priori covariance for the estimate at t_M. The a priori covariance matrix, $\Lambda'(G, t_U)$, for the segment starting with a ground state update is formed from the previously calculated MSFN covariances $\Lambda(G, t_U)$

and the uncorrelated non-dynamical error sources due to the components of z'_V (See Table 2).

$$\Lambda'(G, t_U) = \begin{pmatrix} \Lambda(G, t_U) & 0 \\ 0 & \Lambda_{z'_V z'_V} \end{pmatrix}$$

Table 2-2. Onboard Systematic Error Vector, z_V

Type	Components
Non-Dynamical z'_V	<u>RR Data Biases</u>
	Range
	Range Rate
	Shaft Angle
	Trunnion Angle
	<u>IMU Errors</u>
	$\delta\alpha_{1,2,3}$ initial misalignment
	$\beta_{1,2,3}$ three-axis drift
	<u>Or SXT Data Biases</u>
	Shaft Angle
	Trunnion Angle
Dynamical, z_D	$\delta\mu_\oplus$
	or $\delta\mu_☽$
	δk_{D1}
	δk_{D2}

If Λ_{SOE} is partitioned into submatrices which are similar to those of $J(V_1, t_M)$, then

$$\Lambda_{SOE} = \begin{pmatrix} \Lambda_v^{x_1x_1} & \Lambda_v^{x_1x_2} & \Lambda_v^{x_1z_V} \\ \Lambda_v^{x_2x_1} & \Lambda_v^{x_2x_2} & \Lambda_v^{x_2z_V} \\ \Lambda_v^{z_Vx_1} & \Lambda_v^{z_Vx_2} & \Lambda_v^{z_Vz_V} \end{pmatrix}$$

and

$$\Lambda_{REL}(SOE, t_U, t_M) = \Lambda_v^{x_1x_1} + \Lambda_v^{x_2x_2} - \Lambda_v^{x_1x_2} - \Lambda_v^{x_2x_1}$$

gives the covariance of the relative state as determined from X_{SOE} at maneuver time t_M after an update at time t_U. $\Lambda_{REL}(SOE, t_U, t_M)$ can be compared to $\Lambda_{REL}(G, t_U, t_M)$ to observe the effect of incorporating the relative tracking in an SOE fit.

There is also a known bias in the onboard estimates due to a neglect of the drag forces. Consequently the error in the onboard estimate, $\delta\vec{X}_{iE} - \delta\vec{X}_{iA}$, has a time dependent non-zero mean given by

$$\hat{\vec{\mu}}_i(V_1, t) = \left(\Lambda_p^{-1} + A_i^T W A_i\right)_{V_1}^{-1} \left(A^T W B_{k_{Di}}\right)_{V_1}$$

for the i^{th} vehicle if the observations made on board vehicle V_1 are reduced to the data epoch, t. $B_{k_{Di}}$ is the column of the regression matrix for the systematic errors, corresponding to the error in the coefficient of drag, k_{Di}. This expression is multiplied by the unit factor $\Delta k_{Di} = 1$, which represents the fact that the entire drag effect is neglected by the onboard computer.

2.3 PRELIMINARY ESTIMATE OF OPTIMAL WEIGHTS

The choice of weights, ω_k, determines the performance of the onboard filter. The real-world covariance matrix of the errors in the estimate of the relative state defined by the estimates onboard vehicle (V_1) at time t_M is given by

$$\Lambda_{Rr} = \Lambda_{R11} + \Lambda_{R22} - \Lambda_{R21} - \Lambda_{R12}$$

where

$$\Lambda_{R\gamma\gamma'} = E\left[\left(\delta X^1_{E\gamma} - \delta X^1_{A\gamma}\right)\left(\delta X^1_{E\gamma'} - \delta X^1_{A\gamma'}\right)^T\right]_{t_M}$$

can be evaluated in terms of the weights ω_k, the normal matrices, and the real-world variances of the systematic errors. $\omega_k = 1 \cdots p$ includes the two a priori weights and the appropriate data weights for the onboard observations.

The Λ_{Rr} can be evaluated in $(U_1, V_1, W_1)_{t_M}$ coordinates for any n sets of trial weights, $\omega_k^{(j)}$, with $j = +1, +2 \cdots, +n$. After assuming some reasonable starting point $\omega_k^{(o)}$, it is desirable to explore a cluster of weightings which can be expressed in terms of its center $\omega_k^{(o)}$ and fractional variation f. Given a particular j belonging to the set $\left|\pm 1, \cdots, \pm p\right|$, then the j^{th} trial set of p weights $\left|\omega_k^{(j \neq o)}\right|$ can be generated by the formula

$$\omega_k^{(j\neq o)} = \omega_k^{(o)} \left[1+(j/|j|)\, f\, \delta_{|j|,k}\right] \text{ with } k = 1, \cdots, p.$$

If one or more of the weights seems to be far from optimal, a new choice of $\omega_k^{(o)}$ is required so that the process can be repeated with a new cluster. If the cluster seems to include the optimal, a final run can be made with a best guess for $\omega_k^{(o)}$ and a smaller variation.

The preliminary choice of weights is made by an external comparison of the various Λ_{Rr} and the Λ_{REL} at each maneuver time t_M. It is assumed that a choice of weights which minimizes the RSS uncertainty of velocity,

$$\sigma^2_{v_M} = \Lambda_{Rr,44} + \Lambda_{Rr,55} + \Lambda_{Rr,66},$$

for each maneuver and keeps the distance of closest approach within reasonable bounds, will minimize ΔV_{Tot}, but the Monte Carlo procedure is required to prove the point. Currently, there are no plans for mechanizing the search for the preliminary optimum, because no logically satisfactory criterion exists within the covariance analysis, and it seems desirable to examine the numerical situation as fully as possible.

The search procedure can be simplified by assuming a relationship between the a priori weights ω_1 and ω_2 and Pace's optimal a priori, (Reference 1) $\Lambda_p^{(Opt)}$, which is a function of the fit-world data weights $\omega_{k>2}$. $\Lambda_p^{(Opt)}$ is such that the real-world covariance of the associated estimate is a minimum with respect to arbitrary symmetric variations of Λ_p. $\Lambda_p^{(Opt)}$ is given by

$$\Lambda_p^{(Opt)} = \left[J^v_{x_1x_1} \Lambda_G^{x_1 z_D} J^v_{z_D x_1} + J^v_{x_1 z_v} \Lambda^{z_v z_v} J^v_{z_v x_1} + \tilde{J}^v_{x_1 x_1} \right]^{-1} \left[J^v_{x_1 x_1} \Lambda_G^{x_1 x_1} + J^v_{x_1 z_D} \Lambda_G^{z_D x_1} \right].$$

The weighting matrix W for the observations made on board vehicle V_1 can be written

$$W_{ij} = \omega_k \delta_{ij}$$

where k is such that the k^{th} data type includes the i^{th} observation.

After substitutions for the W matrix,

$$J^v_{x_1x_1} = \left(A_1^T W A_1\right)_{V_1} = \sum_{k>2} \omega_k \left(\sum_{i\epsilon k} A_1^T A_1\right)_{V_1}$$

$$J^v_{x_1 z_v} = \left(A_1^T W B_1\right)_{V_1} = \sum_{k>2} \omega_k \left(\sum_{i\epsilon k} A_1^T B_1\right)_{V_1} = \left(J^v_{x_1 z_D}, \; J^v_{x_1 z_v'}\right)$$

and

$$\tilde{J}^v_{x_1x_1} = A_1^T W M_{11} W A_1 = \sum_{k,k'>2} \omega_k \omega_{k'} \tilde{J}_{kk'}.$$

This gives the normal matrices for relative tracking made onboard vehicle (V_1) explicitly in terms of the data weights $\omega_{k>2}$.

$\Lambda_G^{z_D x_1}$ and $\Lambda_G^{x_1 x_1}$ are previously computed blocks of $\Lambda(G, t_V)$. Let

$$\omega_1 = \omega_{rr} \text{ and } \omega_2 = \omega_{rv} \text{ so that}$$

$$\Lambda_p^{-1} = \begin{pmatrix} \omega_1 \bar{\bar{1}} & 0 \\ 0 & \omega_2 \bar{\bar{1}} \end{pmatrix}$$

gives the preset a priori covariance. We can construct an estimate of $\omega_{1,2}$ from the $\Lambda_p^{(Opt)}$ matrix by assuming that

$$\omega_1 = \left(\Lambda_{p,11}^{(Opt)} + \Lambda_{p,22}^{(Opt)} + \Lambda_{p,33}^{(Opt)}\right)^{-1}$$

and

$$\omega_2 = \left(\Lambda_{p,44}^{(Opt)} + \Lambda_{p,55}^{(Opt)} + \Lambda_{p,66}^{(Opt)}\right)^{-1}$$

However, it cannot be proved that this relationship will necessarily lead to the optimum values for $\omega_{1,2}$. The $\omega_{1,2}$, obtained from $\Lambda_p^{(Opt)}$ using the assumed trial set of data weights, $\omega_{k>2}$ can be used to calculate the covariance matrices for the errors in the estimate of the relative state at the maneuver times.

2.4 SUMMARY OF COVARIANCE OPTIMIZATION PROCEDURE

1) Compute MSFN normal matrices and transition matrices using TAPP IV

2) Compute real-world covariances for the MSFN updates (i. e., $\Lambda(G, t_U)$ and $\Lambda_{REL}(G, t_U, t_M)$)

3) Compute onboard normal matrices for relative tracking using TAPP IV

4) Compute SOE covariances for the onboard data incorporation (i. e., Λ_{SOE}, $\Lambda_{REL}(SOE, t_U, t_M)$, and $\hat{\vec{\mu}}_i(V_1, t_M)$ for earth orbits)

5) Compare $\Lambda_{REL}(G, t_U, t_M)$ and $\Lambda_{REL}(SOE, t_U, t_M)$ to determine maximum value of onboard data

6) Compute $\Lambda_p^{(Opt)}$ using real-world data weights to obtain a dimensionless parameterization of $\Lambda_p^{(Opt)}$ for arbitrary weights. (Examine the semi-traces to determine if the ratio is roughly 1000/1 as expected)

7) Compute real-world covariances Λ_{Rr} for the relative state of the onboard estimates for an adequate set of alternate weights. For 5 variable weights and 5 maneuvers, this means outputting $(2n+1)x5 = 11x5 = 55$ covariance matrices of 21 components each. After selecting the optimum weights, Λ_{Rr} can be transformed to a normalized covariance for the errors in the estimate of the Keplerian elements.

8) Output and compare $\Lambda_{REL}(SOE, t_U, t_M\ j)$ where j indexes the alternate sets of weights). The comparison may be restricted to one or two components of the covariance matrices, primarily the RSS velocity components. Even so, a multivariate trade-off is required in order to pick the optimum and therefore, the success of the procedure will depend on the numerical situations. (This gives the preliminary estimates for the preset weights)

9) If further information is desired, go back to step (7) and compute Λ_{Rr} with a new set of weights; otherwise, proceed to the Monte Carlo procedure.

2.5 MONTE CARLO STATISTICS AND FINAL ESTIMATE OF OPTIMUM WEIGHTS

At least one TAPP VI Monte Carlo run is needed to obtain statistics on the ΔV cost for each maneuver, and if the results of this run suggest a need for an improved optimization of the preset weights, then a series of Monte Carlo runs can be made to provide a Monte Carlo optimization by the procedure described in Appendix B.

At present no precise statement of statistical confidence level requirements is available, but the first run can be made with a sample size $n_s = 100$ and with the previously determined preliminary weights, ω_p. The Monte Carlo processor can output the median values

$$X_M = [\Delta V_{Tot}(\vec{\omega}_p)]_{Median}$$

and

$$Y_M = [\rho_{DCA}(\vec{\omega}_p)]_{Median}$$

for the variables ΔV_{Tot} and ρ_{DCA}.

$$\Delta V_{Tot} = \sum_M \Delta V_M$$

where M denotes a maneuver and

$$\Delta V_{TPF} = 2\, v_{DCA} \text{ for } M = TPF.$$

ρ_{DCA} is the distance of closest approach as determined by the MCC maneuver and neglecting the man-in-the-loop part of the pre-TPF maneuvering. v_{DCA} is the relative velocity at closest approach. The term $2\, v_{DCA}$ is an estimate of ΔV_{TPF}. The median values X_M and Y_M define a joint cumulative probability, JCP, which is also output by the Monte Carlo processor.

$$JCP = n_J / n_s,$$

where n_J is the number of trajectories which satisfy the bounds

$$\Delta V_{Tot} \leq X_M$$

and

$$\rho_{DCA} \leq Y_M.$$

It is also desirable to compute JCP for more realistic bounds X_A and Y_A in order to obtain an estimate of the probability for rendezvous with preliminary weights.

Means and variances will be output for the actual deviation and the error in the estimate for the states of both vehicles at each maneuver time in UVW and Keplerian coordinates. The means and variances of the relative state will be output for each maneuver time in the UVW system of the active vehicle. The joint cumulative probability defined in Appendix B will be computed for the preliminary weights. Hopefully, the preliminary estimate will be sufficiently accurate for practical purposes. The means and variances obtained from the covariances can be compared to those obtained from the Monte Carlo processor to determine the effect of nonlinearities.

The Monte Carlo Methods have several advantages over the covariance methods based on TAPP IV:

1) The Monte Carlo simulation computes statistics of maneuver fuel cost and target parameter accuracy. In the Covariance matrix approach it is tacitly assumed that optimizing the tracking accuracy at the maneuver times minimized fuel cost. This assumption will be evaluated using Monte Carlo results.

2) The Monte Carlo simulation includes errors of execution for each maneuver, whereas the covariance method as presently conceived will assume perfect burns.

3) The Monte Carlo simulation correctly computes the maneuver times, but the TAPP IV simulation assumes nominal maneuver times.

4) The TAPP IV program does not provide statistics for the Kepler elements directly, but must resort to a Monte Carlo subroutine.

2.6 THE MOST ACCEPTABLE NAVIGATION PLAN

The choice of navigation plan is chiefly based on the fuel requirement statistics or the nearly equivalent ΔV cost statistics, assuming that the rendezvous requirements can be satisfied. The rendezvous requirements are principally concerned with the TPF maneuver, which must fall within a height-velocity box or, in most cases, within a maximum distance of closest approach. A comparison of the fuel costs for various navigation plans requires at least one Monte Carlo run; however, the TAPP-IV-MOFIT series of programs can provide a fairly accurate statistical description of the mission in every respect except fuel cost.

2.7 PROGRAM REQUIREMENTS

This analysis will utilize the existing TRW TAPP IV, FASTAP I, FASTAP II, TAPP VI, and TAPP processor programs as well as a special program written in the MOFIT language. FASTAP I and II combine the information from several TAPP IV tapes and write it on one FASTAP II output tape, which is input for the "MOFIT program" and the TAPP VI program. The "MOFIT program" (actually a special program coded in MOFIT) calculates the covariance matrices of the errors in the estimate of the relative state. The TAPP VI program generates a Monte Carlo simulation of the mission outputs to the processor program, which generates sample statistics for the TAPP VI output variables.

2.7.1 TAPP IV

Several TAPP IV tapes will be required for each mission to be analyzed. These tapes will contain tracking normal matrices and covariance matrix propagation matrices required to simulate various onboard updates based on MSFN and relative tracking data prior to and during the rendezvous sequence. TAPP IV generates normal matrices (denoted A^TWA) and systematic error matrices (denoted A^TWB) for the relative tracking and the MSFN tracking for each rendezvous segment for both vehicles. Each relative tracking data type (i) will constitute a separable data set, and the normal matrix due to that data type alone will be available, with the real-world noise variance σ_i^2, so that $(A^TWA)_i = (A^TA)_i/\sigma_i^2$. The value of $(A^TWA)_i$ for an arbitrary data weight, $\tilde{W}_i$, can be obtained from $(A^TWA)_i$ by multiplying it by the dimensionless quantity $\tilde{W}_i/W_i$, which can be input to the "MOFIT program" as needed. Consequently, only one set of TAPP IV tapes is needed to consider many different trial values for the weighting parameters.

2.7.2 FASTAP I, II

FASTAP I will merge the various TAPP IV tapes at their corresponding print times (Reference 1). Two sets of propagation matrices, one for each vehicle, will be required on the FASTAP I output tape. These matrices are needed to propagate the covariance of the relative state from maneuver to maneuver. The FASTAP I output tape, which contains all the desired information from the TAPP IV tapes, is then input to the

FASTAP II program. FASTAP II (Reference 1) adds normal matrices and augments systematic error matrices (if necessary) for each of the FASTAP I prints. It is the FASTAP II output tape which will be input to the MOFIT program.

2.7.3 MOFIT Based Programs

Several special purpose programs will be written in the MOFIT language, for the purpose of calculating various covariance matrices. A separate MOFIT run will be made for each preliminary optimization.

The MOFIT programs will be developed in stages or phases, each of which will perform a different function. Until better names are selected, we can designate the programs as:

I. Calculate fit-world and real-world covariance matrices for the possible updates based on MSFN data.

II. Calculate the covariance matrices for the error in the estimate of the relative state at the maneuver times when the active vehicle solves on all the systematic error parameters normally associated with the onboard filter. (This simulates an optimal onboard filter.)

III. Generate the optimum a priori covariance matrices which should be used onboard when the statistics of the systematic error sources is known. (These matrices are a function of the trial weights.) Extract two diagonal weighting parameters for the fit-world estimate and calculate the resulting covariance matrices of the relative states at the maneuver times.

2.7.4 TAPP VI

The weighting parameter values determined by the covariance matrix study will be used to generate navigation input for the TAPP VI Monte Carlo program (Reference 2). This program computes statistics on maneuver fuel cost and distance of closest approach, assuming a sample of several hundred trajectories for the mission, which results from randomly selected values for the guidance and navigation error sources.

A separate TAPP VI program will be assembled for each mission profile to be analyzed. These simulations are made up of prop boxes (event simulation routines) obtained from an existing prop-box library.

2.7.5 Processor Program

The TAPP VI program writes an output tape containing the values for parameters of interest generated in the Monte Carlo runs. This tape is input to the processor program (Reference 3) which computes statistics on the output parameters. Among the statistics available are

1) Sample means
2) Sample variances
3) Cumulative distribution plots

APPENDIX A

OPTIONAL NAVIGATIONAL SCHEMES, ASSUMING EXISTING APOLLO SYSTEMS AND MISSION PLANS

A-1. The Optional Navigational Schemes

a) All maneuvers are computed by the onboard computer* using only the MSFN tracking of the vehicles. The MSFN observations are processed at RTCC and the ground-state estimate of both vehicles is transmitted to the vehicles by the uplink channel. Alternatively, the maneuvers can be computed at RTCC and transmitted to the vehicle.

b) No ground-state estimate is transmitted to the vehicles during the rendezvous sequence. The onboard computer incorporates the onboard observations to estimate the relative state vector and to provide the maneuver computations. The preset observational weights influence each data incorporation and the preset a priori weights influence the estimated state after a reinitialization of the error transition matrix.

c) Kalman Filter incorporation of onboard observations plus an optional ground-state update. This is the normal means of computing the rendezvous maneuvers.

d) Ground computer incorporation of MSFN observations plus an optional onboard state update (or initialization). The onboard computer operates as in scheme (c). This alternative only applies to Mission G.

A-2. The Communications Channels

a) The command uplink can transmit a ground-state estimate (GSU) of both vehicles to either vehicle, and although an operational channel exists for the covariance matrices, it is assumed to be of no practical value.

b) The downlink telemetry can transmit the onboard observations and the onboard states to RTCC. The ground computer is currently able to use the onboard states for initialization(GSI) and a program modification would enable it to process the associated covariance matrix. Lunar Landing Site tracking will be processed on the ground, but no other ground incorporation of the relative tracking is expected.

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*Whenever possible; i.e., the LGC can compute CSI, CDH, TPI, and MC maneuvers and the AGC only TPI and MC.

A-3. The Onboard Navigational Options

a) The observing vehicle has a choice of observations and if the other vehicle is being observed, the observation can be incorporated into the estimate of either vehicle as desired by the operator onboard the observing vehicle.

b) The ground state updates (GSU) can be accepted or rejected.

c) The onboard state can be reinitialized (OSR) by giving the error transition matrix its preset values. Reinitialization occurs after a GSU, a change of tracking program, or an astronaut command. Reinitialization after a maneuver is resorted to if the errors of execution are large.

d) Various biases* can be solved for optionally by varying the dimension of the state from 6 to 9. This variable must be specified when using a WLS simulation of the Kalman Filter.

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*In the LGC, the shaft and trunnion biases; in the CMC, the landing site position biases.

APPENDIX B

NAVIGATIONAL ACCURACY ACCEPTABILITY CRITERION

B-1. A Discussion of Navigation Accuracy Acceptability

The principal criterion for the acceptability of the navigational system is the probability of successful rendezvous. In general, rendezvous is achieved by a sequence of powered maneuvers determined by the guidance and navigation system. Mission analysis generates the maneuver requirements, the tracking accuracy requirements, and the reference trajectory which should be able to satisfy the mission objectives using given flight hardware and without violating any constraints. A rendezvous failure occurs if a combination of tracking errors and errors of execution (due to imperfect burns) causes the violation of a mission constraint or the failure to achieve docking with the available fuel. Rendezvous is successful if an acceptable distance of closest approach is achieved with the available fuel and the available tracking statistics. The probability of successful rendezvous is related to the joint cumulative probability that the constraints are satisfied with the available fuel when the sample of actual and estimated trajectories is determined by the distribution of tracking errors and errors of execution. The joint cumulative probability (JCP) can be obtained from a TAPP simulation of the mission after currently planned modifications are installed in the TAPP processor. In principle, a large number of mission constraints may exist which should be included in the definition of the JCP; however, in practice one, two, or three of them will be in control, and a working approximation for the probability of success can be computed from the minimum requirements.

Alternative measures of navigational accuracy acceptability can be obtained from the TAPP statistics on the individual fuel or mission objective variables. (An optimum a priori matrix can be determined analytically by minimizing the real-world covariance for an assumed set of data weights; however, the optimum a priori cannot be expressed in terms of the preset a priori weights.) An optimum choice of preset parameters will exist for a restricted class of errors in state, but the analytical solution is not available. However, it is possible to compute the sensitivity of the estimate to variations of the preset parameters.

B-2. Optimizing the Preset Parameters

The preset weights which influence rendezvous navigation are ω_{rr}, ω_{rv}, and the appropriate data weights for the onboard observations. These parameters can be optimized by maximizing the estimated probability of successful rendezvous, which is the joint cumulative probability JCP. Let

$$JCP = P(x_1 \le X_1, \ldots, x_n \le X_n) = f(\vec{X}, \vec{\omega})$$

where $\vec{X}$ is the bounds vector, and $\vec{\omega}$ is the vector of preset weights with the understanding that

$$\omega_1 = \omega_{rr} \sim 1000 \text{ ft}$$

$$\omega_2 = \omega_{rv} \sim 1 \text{ ft/sec and}$$

$$\omega_{k>2} \sim \left(\overline{\alpha^2_{k>2}}\right)^{-1}$$

where $\overline{\alpha_k^2}$ is a real world data variance. The choice of variables $x_1 \ldots x_n$ determines the acceptability criterion. The probability of rendezvous success is approximated by PRS, which is given by

$$PRS = f(\vec{X}_{Op}, \vec{\omega}_{Op})$$

where $\vec{X}_{Op}$ is the vector of operational bounds, and $\vec{\omega}_{Op}$ is the vector of operational flight values for the preset weights. In practice, we expect to have safety factors, S_i, such that

$$S_i = X_{Op_i}/X_M >> 1 \text{ for } i = 1 \ldots n$$

if the X_{M_i} are the median values of x_i. Consequently,

$$f(\vec{X}_M, \vec{\omega}_{Op}) << PRS \le 1.$$

The value of $\vec{\omega}$ which maximizes $f(\vec{X}_M \vec{\omega})$ is defined to be the optimum $\vec{\omega}$ for median bounds. The median bounds optimum will not differ from the true optimum by a significant amount and, in the limiting case of no safety factor,

$$X_{Op_i} \rightarrow X_{M_i} \text{ and}$$

the median bounds optimum becomes the true optimum. It is clear that an optimum exists, but uniqueness can be questioned. Hopefully, the usual dispersion of trajectories will lead to a single optimum, but several local maxima corresponding to relatively optimum $\vec{\omega}$ may occur.

If $\vec{\omega}_{OPT}$ is such that

$$f(\vec{X}_M, \vec{\omega}_{OPT}) = \underset{\omega_j}{\text{Max}}\ f(\vec{X}_M, \vec{\omega}), \text{ then}$$

$$\partial f(\vec{X}_M, \vec{\omega}_{OPT})/\partial\omega_j = 0 \text{ and}$$

$$\partial^2 f(\vec{X}_M, \vec{\omega}_{OPT})/\partial\omega_j^2 = Q_{jj} < 0.$$

where

$$j = 1, 2 \ldots D = \dim(\vec{\omega}).$$

Let $(P_1, P_2, \ldots P_D)$ be a permutation of the integers $(1, 2 \ldots D)$ such that

$$Q_{P_j P_j} \leq Q_{P_{j+1} P_{j+1}}.$$

If P_j is known, $\vec{\omega}_{OPT}$ can be determined efficiently by a one dimensional quadratic fitting procedure applied to the components of $\vec{\omega}$ in the order of P_j. Given the preliminary estimate of the preset weights $\vec{\omega}_p$ and step size Δ_1, compute

$$JCP_o = f(\vec{X}_M, \vec{\omega}_p)$$

and

$$JCP_{\pm 1} = f(\vec{X}_M, \vec{\omega}_{\pm 1}) \text{ with}$$

$$(\vec{\omega}_{\pm 1})_j = (\vec{\omega}_p)_j \pm \Delta_1 \delta_{jP_1}.$$

Let

$$Q^1_{P_1 P_1} = (JCP_1 + JCP_{-1} - 2JCP_o)/\Delta_1^2 \text{ and}$$

$$L^1_{P_1} = (JCP_1 - JCP_{-1})/(2\Delta_1)$$

so that the first final approximation to $\vec{\omega}_{OPT}$ is given by

$$(\vec{\omega}_{f\ 1})_j = (\vec{\omega}_f)_j + \Delta_{f\ 1}\ \delta_{jP_1} \text{ with}$$

$$\Delta_{f\ 1} = -L^1_{P_1}/Q^1_{P_1 P_1} \text{ if } Q_{P_1 P_1} < 0.$$

The associated correction to JCP is given by

$$JCP_{OPT\,1} = JCP_o - \frac{1}{2}\left(L^1_{P_1}\right)^2 \left(Q^1_{P_1 P_1}\right)^{-1}.$$

Compute the corresponding quantities for $P_2 \ldots P_D$ and then recompute for P_1. If

$$\omega_{f\,2,\,P_1} \neq \omega_{f\,1,\,P_1} \quad \text{and}$$

$$JCP_{OPT,\,D+1} \geq JCP_{OPT\,1} + \epsilon,$$

the procedure should be repeated. This process will converge, but it may converge to a relative maximum instead of the absolute maximum. The previous optimization procedure will rule out most of these problems.

APPENDIX C

LM ACTIVE RENDEZVOUS IN EARTH ORBIT MISSION D

Mission Reference:

1. "H. L. Conway et al, "AS-504/CSM-103/LM-3 Mission Profile," MSC-IN67-FM-184, 28 November 1967.
2. Abstracts of Meeting on Rendezvous for Mission D, Data Priority Panel, MSC, 3 January 1968.

C-1. Mission Simulation

Mission D simulates the lunar landing mission in earth orbit. The reference trajectory is not yet available; however, Mission Reference 1 for the D mission shows a maneuver sequence and associated rendezvous geometry which is very similar to the E mission. Mission D has an additional insertion burn between the separation and the CSI burns; however, its nominal maneuver times are nearly the same as the corresponding nominal maneuver times for the E mission in ground elapsed time (g. e. t.). The simulated segment of Mission D extends from three revolutions before the LM/CSM separation to final rendezvous. The non-nominal CSM maneuvers require sextant tracking and a communications channel for LM maneuver data, which probably will not be used and do not require simulation.

The LM relative tracking is provided by the LM rendezvous radar, which can operate throughout the free-flight phase of the LM except during the dead times due to the maneuvers. Ground state updates of both states to both vehicles will be provided prior to the separation and CSI_2 maneuvers. IMU alignment will be performed prior to the CSI_1 and during the TPI_1 maneuvers. The TPI_1 maneuver is not executed in the D and E missions.

C-2. Prop Box Listing

The routines developed under A-54. 2 and used in the TAPP VI simulation include:

RETARGET	retargets LM insertion based on the estimated CSM state (Reference 4)

FFP	propagates state vectors through free-flight segments of the trajectory and updates a priori covariance matrices. (Reference 5)
TRAK	simulates ground and onboard tracking updates. (Reference 6)
PRETPI	calculates time of TPI based on LM and CSM estimate states.
DVNCC, NSR	calculates the ΔV required at NCC, NSR (may be zero) (Reference 7).
CSI/CDH	PROP 28 calculates the ΔV at CSI and CDH as well as the time of CDH.
DVTPI	calculates the ΔV required at TPI.
DVMCC	calculates the ΔV required at MCC.
DOBURN	applies impulsive burn to vehicle, simulates execution errors, etc.
BRAKE	differences actual velocity vectors at TPF to approximate braking ΔV_{TPT}.
TCLOSE	calculates the time of closest approach, the distance of closest approach, and the relative velocity, v_{DCA}, at closest approach. (Reference 8)

C-3 Ground Rules for Mission D

(See Section D-2 for LM rendezvous radar dead time.)

Dead times for CSM-sextant sightings are as follows:

1) 10 minutes prior to 6 minutes after an SPS burn

2) 14 minutes prior to the TPI burn

3) 7 minutes prior to the MCC burn

4) 10 minutes prior to the TPF burn

5) From an RCS burn to 5 minutes after the burn.

C-4. The Sequence of Events

Same as Mission E (Section D-3 and Figure D-1).

APPENDIX D

LM ACTIVE RENDEZVOUS IN EARTH ORBIT, MISSION E

Mission References:

1. FM6/Rendezvous Analysis Branch, "Present Third Manned Saturn V LM Mission Profile," MSC-67-FM64-162, 22 September 1967.

   "Presentation of LM-Active Rendezvous Profiles for the Third Saturn V Manned LM Mission," MSC-67-FM64-164, 22 September 1967.

   Kenneth A. Young, "Presentation of Rendezvous Profiles for the Second Manned-LM Mission (E)," 67-FM64-187, 20 October 1967.

2. Guerro, J. J. "Reference Trajectory for AS-503A," 006882-FM6-E051-1633K, 17 October 1967.

D-1. Mission Simulation

The simulated segment extends from the beginning of the 4th period at 92 hours g. e. t. to rendezvous and docking and includes AGS controlled approach to rendezvous prior to the actual rendezvous using PNGS controls. Currently, no AGS Prop-Boxes are available, and the AGS maneuvers will be simulated with PNGS Prop-Boxes, unless a further request for AGS simulation occurs. The B-2 profile, which is an earth orbit simulation of the third apsis lunar abort from powered descent, will be simulated. There are DPS, APS, and RCS powered maneuvers (see Figure D-1), which have separate fuel storage limits. Consequently, the joint cumulative probability JCP should be a function of the four variables ΔV_{DPS}, ΔV_{APS}, ΔV_{RCS}, and ρ_{DCA}; however, the ΔV_{DPS} variable will be omitted, because the DPS rendezvous is not completed and the DPS burns have little influence on the final rendezvous operation.

The rendezvous radar will provide relative tracking continuously after insertion, subject to the constraints given in Reference 9.

D-2. Ground Rules for Mission E

a) Dead times for rendezvous radar measurements.

   1) 12 minutes prior to 4 minutes after an APS burn

   2) 12 minutes prior to the CSI-RCS burn

   3) 7 minutes prior to RCS burns other than CSI

   4) From an RCS burn to 2 minutes after the burn

b) Rendezvous Radar acquisition occurs at 400 nautical miles unless obstructed

c) The LGC accepts a new set of observations every 1.25 minutes.

D-3. The Sequence of Events

1) Initialize the LM/CSM at 92 hours g.e.t. with inputs for its nominal state and both covariances

2) Track and propagate to nominal time of separation

3) Make given separation and insertion burns with execution errors. (Omit insertion burn for Mission E but include it for Mission D.)

4) Track and propagate to pre-CSI_1 epoch

5) Compute CSI_1 and $^tCDH_1$

6) Track and propagate to $^tCSI_1$

7) Execute CSI_1

8) Track and propagate to $^tCDH_1$

9) Compute and execute CDH_1

10) Track and propagate pre-TPI epoch

11) Compute TPI_1

12) Track and propagate to pre-CSI_2 epoch

13) Make a ground-state update and incorporate pre-CSI_2 relative tracking

14) Compute CSI_2 and $^tCDH_2$

15) Track and propagate to $^{t}CSI_2$

16) Execute CSI_2

17) Track and propagate to $^{t}CDH_2$

18) Compute and execute CDH_2

19) Compute TPI_2

20) Track and propagate to $^{t}TPI_2$

21) Execute TPI_2

22) Track and propagate to pre-MCC epoch

23) Compute MCC

24) Propagate to MCC

25) Execute MCC

26) Compute DCA

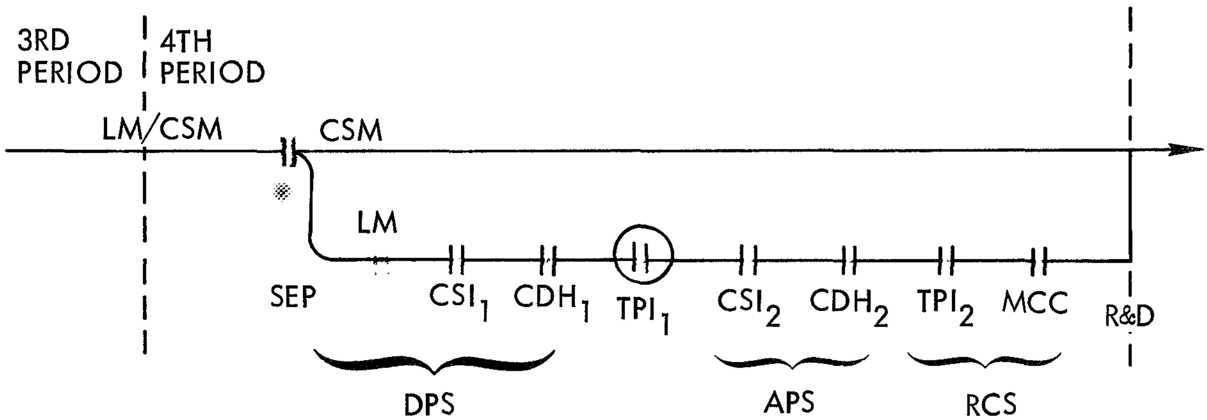

Figure D-1. Mission E (B-2 Profile)

APPENDIX E

LM ACTIVE CONCENTRIC ASCENT TO LUNAR RENDEZVOUS, MISSION G (LUNAR LANDING)

Mission References:

1) "Apollo Reference Mission Program, Version ARM 05, User's Manual and Test Cases," TRW 05952-H235-R0-00, 26 June 1967.

2) Alphin, J. H., "Reference Trajectory for AS-504," 000048-FM6-E023-1913N, 4 October 1967.

E-1. Mission Simulation

The simulated segment extends from LM insertion to rendezvous and docking and includes five RCS maneuvers (i.e., CSI, CDH, TPI, MCC, TPF. See Figure E-1). This neglects the CSM planar correction just prior to LM ascent.

The relative tracking is provided by the LM rendezvous radar, which will be within the 400-nautical mile limiting range during the simulated segment. The CSM state can be determined by MSFN tracking or by lunar tracking with the rendezvous radar during the LM's lunar stay. An option to generate the lunar tracking of the CSM may be considered, if a comparison is desired with the previously generated MSFN tracking. A second option for simulation of the powered LM ascent phase using the GAHSP (Reference 9) prop box may be considered, if the required covariances of the actual and estimated errors at LM insertion time are not available.

E-2. Ground Rules for Mission G:

a) The difference of the orbital radii for the segment from CDH to TPI is called ΔH and must be 15-50 nautical miles.

b) Time of TPI must be 120 minutes after the time of insertion ± 10 minutes.

c) Allow at least 25 minutes between Insertion, CSI, CDH, and TPI maneuvers.

d) Dead times for rendezvous radar measurement (see Reference 9):

1) 12 minutes prior to 4 minutes after an APS burn

2) 12 minutes prior to the CSI-RCS burn

3) 7 minutes prior to RCS burns with the exception or the CSI burn.

4) From an RCS burn to 2 minutes after the burn

e) Rendezvous radar acquisition occurs at 400 nautical miles and provides a new set of observations every 1.25 minutes

f) The new nominal CSM orbit is 60 X 60 nautical miles.

E-3. The Sequence of Events

1) Initialize

The LM and CSM will be initialized at nominal LM injection time. The CSM estimate state will be set equal to its nominal at LM injection. This obviates the need of retargeting the LM. The actual CSM state at LM injection is obtained by sampling the distribution represented by its estimate error covariance matrix. The LM estimate and actual states will be found by sampling the distribution represented by the LM insertion covariance matrix. The LM launch is targeted to its nominal. A total of three matrices must be input for these computations.

The propellant weights, static performance errors, and static systematic errors will be selected for both vehicles.

2) Propagate and Track to t_{CSI}

The LM state is updated by incorporating pre-CSI tracking. The CSM states are propagated without tracking. The CSI maneuver is based on a fixed time.

3) Compute CSI ΔV and t_{CDH}

The ΔV at CSI is computed as well as the time of CDH. This is done in the PROP 28 CSI/CDH prop box.

4) Execute CSI

This is done with DOBURN.

5) <u>Propagate and Track to t_{CDH}</u>

The LM state is updated by pre-CDH relative tracking. The CSM states are propagated without tracking. The value of t_{CDH} was computed in Step 3.

6) <u>Compute CDH ΔV</u>

The ΔV at CDH is computed by the previously employed PROP 28 CSI/CDH. However, the time of CDH is not recomputed. The value calculated in Step 3 is used. The pre-CDH relative tracking is only used for the ΔV_{CDH} computation.

7) <u>Execute CDH at t_{CDH}</u> .

This is to be done with DOBURN.

8) <u>Propagate and Track to pre-TPI Epoch</u>

The LM state is updated by relative tracking, and the CSM states are propagated without tracking.

9) <u>Compute t_{TPI}</u>

This Prop-box computes t_{TPI} base on the estimate states of both vehicles.

10) <u>Compute ΔV_{TPI}</u>

This is computed in the Prop-box DVTPI.

11) <u>Execute TPI</u>

This is done with DOBURN.

12) <u>Propagate and Track to Pre-MCC Epoch</u>

The LM is updated by pre-MCC relative tracking and the CSM states are propagated without tracking to t_{MCC}. The midcourse correction is a fixed time after TPI.

13) <u>Compute MCC</u>

The burn is computed in DVMCC and executed by DOBURN.

14) <u>Propagate to MCC</u>

15) <u>Execute MCC</u>

16) <u>Compute DCA</u>

The distance of closest approach is computed (Reference 6).

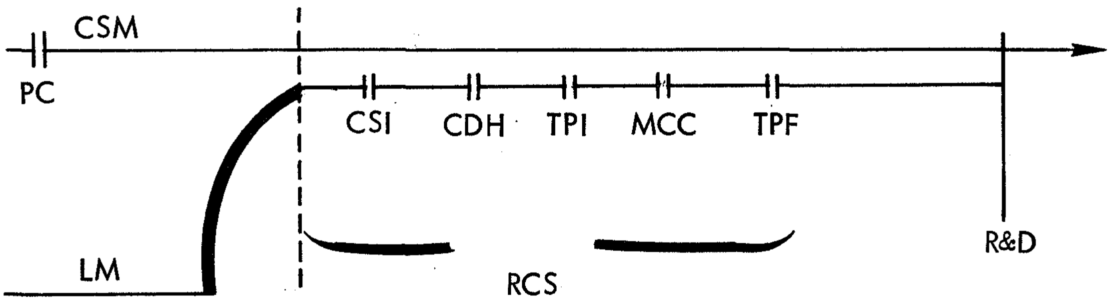

Figure E-1. Mission G (Concentric Ascent to Rendezvous)

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