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STUDY OF POWERED DESCENT TRAJECTORIES FOR MANNED LUNAR LANDINGS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
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STUDY OF POWERED DESCENT TRAJECTORIES FOR MANNED LUNAR LANDINGS

Prepared by: Floyd V. Bennett
Floyd V. Bennett
STD, Guidance Analysis Branch

Thomas G. Price
Thomas G. Price
STD, Guidance Analysis Branch

Authorized for Distribution:

Maxime A. Faget
Assistant Director
for Engineering and Development

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SUMMARY

The design powered descent trajectory for the LEM landing mission is divided into three operational phases: an initial fuel optimum phase; a landing approach transition phase; and a final translation and touchdown phase. An analysis of this descent indicates that several operational tradeoffs are available.

INTRODUCTION

The powered descent and landing from lunar orbit is perhaps the most critical part of the lunar landing mission. Because of the great effect upon the earth launch booster requirements and the payload delivered to the lunar surface, the weight of the fuel expended during the maneuver must be minimized. However, it is expected that a major portion of the maneuver, in particular the final landing approach, will be under the control of the crew. Control by the crew will logically add to the mission reliability because their judgment faculties can assess the suitability of the surface for landing and their control capabilities can be applied to provide a degree of flexibility impossible with an automatic system. In order for the crew to be able to properly perform these functions, the trajectory characteristics of the landing approach, the attitude of the spacecraft, abort considerations, the visibility limits of the spacecraft windows and perhaps other factors must be accomodating. Because some or all of these factors may be in conflict with the need for minimum fuel expenditure, it is important that the tradeoffs be well understood. The final selection of the mission design will require an understanding of the operational nature of the maneuver.

Future fixed-base and free-flight simulations are expected to provide a better understanding of the operational nature of the maneuver. In the meantime, however, an analytical examination of the tradeoffs which are initially evident, coupled with a logical development of criteria to be applied to the landing maneuver, is felt to be an important and necessary step. The purpose of this report is to present an analytical study of the complete lunar landing maneuver and to examine the maneuver characteristics in light of assumed operational criteria.

SYMBOLS

CM	Command Module
g	Acceleration due to gravity
h	Altitude
h_t	Initial altitude of landing approach phase
h_p	Pericynthion altitude of descent transfer
$\dot{h}$	Vertical velocity
I_{sp}	Specific impulse
LEM	Lunar Excursion Module
R	Radius of moon
t	Time
T	Thrust
TR	Throttle ratio
T/W	Thrust-to-weight ratio of engine (earth weight)
V	Velocity
V_c	Characteristic velocity
W	Weight
x	Horizontal range along lunar surface
$\dot{x}$	Horizontal velocity at lunar surface ($= \frac{R}{Rth} V\cos\gamma$)
β	Look angle to landing site with respect to vehicle attitude axis
γ	Flight path angle with respect to local horizon
θ	Pitch attitude with respect to local horizon

Subscripts

max	Maximum
0	Initial condition
f	Fuel
t	Condition for landing approach transition
min	Minimum

PHASES OF POWERED DESCENT

The powered descent portion of the lunar landing mission is a continuous-thrust maneuver of several minutes duration and is initiated at or near the pericynthion of the descent transfer orbit (see fig. 1). This maneuver may logically be described in three phases as follows:

1. Fuel Optimum Descent - In this initial phase, far from the landing site, the important consideration is optimum fuel performance. This phase is continued to a point where a modification to the trajectory is necessary to allow the crew to assess the approach to the landing site. This latter point is as yet undefined and is subject to tradeoffs to be examined in this report.

2. Landing Approach Transition - This second phase succeeds the Fuel Optimum Descent phase down to the initiation of the final landing approach. It is during this phase that the assessment of the landing area is made by the crew.

3. Final Translation and Touchdown - This is the terminal phase of the descent trajectory from the end of the Landing Approach Transition to touchdown. This phase takes place close to the lunar surface, and because of flight safety considerations, uses relatively low velocities and conservative attitude deviations to translate and descend to the final touchdown point.

A sketch of the powered descent is given in Figure 2.

Although the lunar landing descent maneuver is still in the planning stage, considerable thought has been given to determining the important operational factors of each of the three phases. These factors are discussed in the next section.

OPERATIONAL FACTORS

The major criteria for the entire powered descent are flight safety and economy of fuel utilization. The important operational factors for establishing these criteria are presented for each phase of flight.

Fuel Optimum Descent.- During this phase primary concern is focused on fuel consumption. Two factors which most influence the fuel performance are the altitude at initiation of this phase and the thrust level used during this phase. The thrust is assumed to be constant for this phase.

The initial altitude should be as low as possible from the standpoint of fuel consumption (see fig. 3). (Figure 3 was obtained from an unpublished MSC internal memorandum by Thomas Price and Donald J. Jezewski, and is based on a calculus of variation technique reported in reference 1.) Also, this altitude should be as low as possible in order to keep the time of flight during which the LEM is on a surface collision course as short as possible. However, consideration of lunar mountains, which extend up to 20,000 feet, and consideration of a margin of safety for guidance errors put a lower constraint on the initial altitude. Thus, a good compromise between fuel and safety requirements for the initial altitude appears to be about 50,000 feet. (One further consideration on the initial altitude is the ability of the crew to survey the landing site in the event that orbital reconnaissance prior to the landing maneuver is desired. Here again then, the requirement would be for a reasonably low altitude.)

Having established the initial altitude, the engine T/W_o or thrust level which yields minimum fuel consumption can also be established from figure 3. The fuel optimum T/W_o can be seen to be about 0.7. However, consideration of the thrust level used in this phase must also take into account the throttling capability of the engine in order to produce the minimum thrust level desired for hover and translation. (For the present analytical study it is assumed that the engine is operated at maximum thrust level during this phase; however, operationally, it would be desirable to operate slightly under this value in order to have some reserve capability for abort situations.) A satisfactory value for the minimum thrust level at which control can be maintained during the translation phase is that necessary to support $\frac{3}{4}$ of the lunar weight of the LEM at that point. Preliminary calculations indicate that the LEM weight will be reduced by about one-half during the powered descent; hence, it can be seen from figure 4 for $T/W_o = 0.7$ (for fuel optimum) that the

throttle ratio is about 11 to 1. However, present state-of-the-art for engine design indicates this ratio is too high. A ratio of about 9 to 1 is the maximum that should be planned. Thus, in order to reduce the throttle ratio it is evident from figure 4 that the T/W_o must be reduced. It was shown in figure 2, however, that the fuel requirements increase with decreasing T/W_o; (below 0.7), therefore, a tradeoff between T/W_o and throttle ratio must be made. From figure 3 it can be seen that only a slight fuel penalty is paid by dropping the T/W_o to 0.4, whereas for reductions below this level, drastic penalties are incurred. For a T/W_o of 0.4 the throttling ratio is only 6.5 to 1 which is well within the state-of-the-art capability. Hence, a T/W_o of 0.4 appears to be a satisfactory compromise for this phase of the descent.

Landing Approach Transition.- The factors important to this phase of the descent are those which relate to the pilot's ability to assess the general suitability of the landing area and those that may cause his controlling task to be more or less critical. The full extent of these factors will be known only after operational experience is obtained from fixed-base and free-flight simulation tests. The factors known to have some importance during this part are the ability of the pilot to adequately view the landing site, the time that he has to do this, and the complexity of his control task. The latter factor would indicate the desire to have a minimum of required attitude changes and to have the approach velocities, particularly rate-of-descent, such that the control problem of the pilot does not require an undue amount of attention. On this basis, then, it is felt reasonable to constrain the trajectory during this phase by holding the attitude and throttle settings constant. Thus, the three parameters that may be varied during this phase that will directly influence the operational factors discussed above are (1) spacecraft attitude, (2) throttle settings, and (3) the altitude at which this phase of the descent is initiated.

Final Translation and Touchdown.- It is during this portion that the final selection of the landing point is made and the landing completed. Generally speaking, this phase should avoid radical maneuvering and should be compatible with the requirements associated with an abort. Thus, the important operational factors for this phase are the initial altitude, the spacecraft attitude, horizontal velocity, vertical descent rate, and flight time. Limitations must be imposed on the spacecraft attitude, horizontal velocity, and vertical descent rate for reasons of flight safety and ease of control. Likewise, a limitation must be imposed on flight time for reasons of fuel consumption. However, within these limitations it is desired that the obtainable areas for landing be made as large as is feasible.

This phase of the descent is included primarily for completeness; hence, no parametric study of the important factors is intended. See the Scope of Calculations for limitations used in this phase.

DESIGN LANDING SEQUENCE

In order to provide a better understanding of elements of the descent maneuver as they are discussed in the following sections, the following description of the landing sequences is offered:

The maneuver is initiated approximately at pericynthion altitude at a preselected position about 200 miles from the intended landing point. A constant thrust near the maximum capability of the descent engine will be utilized throughout this part of the trajectory. The trajectory will be shaped by the guidance logic to follow a near-fuel-optimum path to certain altitude, position and velocity condition predetermined as the desired initial conditions for the Landing Approach Transition. Upon reaching the desired conditions for the start of the Landing Approach Transition, the spacecraft attitude and throttle setting will be changed in accordance with the preselected values of these parameters. Closed loop guidance could call for some modification to the preselected values but these are not expected to be radical changes. During the Landing Approach Transition the pilot assesses the landing area and continues to update this assessment as the range is decreased. In addition, the pilot will judge the suitability of the landing approach trajectory to assure himself that the approach is safe and the guidance system is working (a qualitative evaluation). If the pilot is dissatisfied with the approach, he may either abort or take over control of the spacecraft and suitably modify the trajectory. If the predicted landing area appears generally suitable the pilot will continue the approach to attain the desired initial conditions of the Final Translation and Touchdown phase. Upon reaching the initial condition of this final part the pilot must then decide upon the final landing position and control the trajectory to obtain this position within the time allotted for the maneuver. Some flexibility for minor changes in the landing position remain up until a hover position is reached at a low altitude just prior to touchdown.

SCOPE OF CALCULATIONS

General.- In an effort to keep the results of this study independent of CM orbit altitude and of the type of orbital descent transfer, the initial conditions of the powered descent were circular orbit

conditions at 50,000 feet (V = 5,484 fps, γ = 0°). (The choice of 50,000 feet for the initial altitude was established in the section on Operational Factors.) The primary effect of different transfer trajectories on the powered descent would be only to change the magnitude of the initial velocity at 50,000 feet and consequently, change the characteristic velocity required by an equal amount. For example, for a Hohmann transfer from the CM in an 80-n.m. circular orbit, the initial velocity would be increased by 98 fps over circular orbit speed at 50,000 feet, and an equiperiod transfer from the same altitude would require an initial increase of 190 fps.

Fuel Optimum Phase.- The fuel optimum portion of this descent is based on the calculus of variation technique reported in reference 1 for two-dimensional motion, a circular gravitational body, and a constant level of thrust. The T/W_o is 0.40, as established in the Operational Factors Section. The specific impulse, Isp equals 315 seconds throughout. The final conditions for the fuel optimum phase are specified by the desired conditions at the transition altitude for the landing approach phase.

Landing Approach Transition.- For the landing approach phase, the three major parameters were varied in the following manner. Three values for the transition altitude, h_t, were considered: h_t = 5,000; 10,000; and 15,000 feet. The attitude, θ_t, was varied from 90° (vertical attitude) for best visibility, to 140° (which is approaching the condition for the fuel optimum descent to the surface). The throttle setting was varied from 0.75 to as low as 0.30 (throttle setting was 1.00 for the fuel optimum phase). The equations of motion for this phase are based on the same assumptions that were used in the fuel optimum phase.

Final Translation and Touchdown.- This phase of the descent is included primarily for completeness; hence no parametric study of the important factors is intended. Instead, a set of numbers and limitations were chosen to be generally compatible with the task. The initial conditions for this phase are assumed to be: altitude = 1,000 feet; velocity = 75 fps; and flight path angle = 0°. Flight time is limited to a maximum of 2 minutes; attitude excursions in pitch and roll are limited to ±30°; and horizontal velocity is limited to a maximum of 75 fps. The vertical descent rate is limited to a maximum of 20 fps and for purposes of standardizing the calculations, an altitude profile of the required descent rates given in figure 5 is used for all descents. A minimum of 15 seconds of the allotted 2-minute flight time is allowed to descend from the 50-ft hover point shown in figure 5. Finally, in order to stay reasonably compatible with the abort situation the LEM is not allowed to have motion in the direction opposite to that of the CM; however, it is allowed to establish a heading of ±90° out of the plane

of CM motion. Alleviation of any or all of the limitations cited herein may well be possible after the experience is gained through simulation, but at the present time they are considered reasonable.

The equations of motion for this phase are based on three-dimensional motion, a flat gravitational body, a variable level of thrust, and a constant spacecraft mass.

RESULTS AND DISCUSSION

The discussion of the powered descent is divided into two parts. The first part concerns the descent from the initial 50,000 feet altitude down to 1,000 feet altitude and includes the fuel optimum phase and the landing approach transition phase. The second part is concerned with the final translation and touchdown from 1,000 feet to the surface.

Descent to 1,000 Feet Altitude

The initial conditions for this portion of the descent are circular orbit conditions at 50,000 feet and the terminal conditions are 1,000 feet altitude with a velocity of 75 fps directed along the horizontal.

Fuel Requirements.- In discussing the results of the calculations for the design powered descent, the first consideration is to determine the characteristic velocity which is a measurement of fuel consumption. For reference purposes a constant-thrust fuel optimum descent (to 1,000 feet) was calculated and found to require a characteristic velocity of 5,627 fps. The time history of this fuel optimum descent is given in figure 6. The characteristic velocity requirements for the design landing technique is presented in figure 7 as a function of the major parameters for the landing approach transition maneuver, namely, h_t, θ_t, and TR_t. It is evident from this figure that in order to attain the value for each of the three parameters which yields the best operational feature, that the fuel requirement would be prohibitive. For example, it was desired to make the transition maneuver at as high an altitude as possible in order to provide adequate time to assess the landing area; however, for corresponding values of TR_t and θ_t, the fuel requirements increase with h_t. In a similar manner the ideal attitude for maximum visibility (90° or vertical) and the ideal low value of TR_t for establishing low descent rates and improving the abort situation are both very costly in fuel. These results are not too surprising, however, since the pitch angle for the fuel optimum case in which the

throttle is full open (TR = 1.00) was found to vary from 164° to 154° in the final approach (see fig. 6). Thus, since the ideal conditions are too costly (in fuel) to obtain, it remains to investigate the trajectories which can be obtained with only a moderate increase in characteristic velocity over the fuel optimum case, say 200 to 300 fps. The conditions for six such trajectories chosen for further investigation are listed in table I.

Visibility.- For consideration of the operational problem of landing site visibility, time histories of the look angle, β, defined as the angle between the attitude axis and the line of sight to the landing site, are shown in figure 8. The landing site is assumed to be at a point 3,000 feet downrange of the 1,000 feet altitude point (see next section on Final Translation and Touchdown). It is apparent from this figure that all of these trajectories except trajectory (a), which has a high TR_t (0.75), yield considerably more visibility in both magnitude and time than does the fuel optimum descent, shown for reference purposes. Trajectory (a) has very poor visibility up to only 38 seconds prior to reaching the 1,000-ft altitude point. Trajectory (f) yields the best visibility of the six; however, since it has the lowest throttle ratio (0.30) and an attitude nearest the vertical (120°) it also requires more fuel than the others.

Vertical Descent Rate.- As stated in the section on Operational Factors, it is desirable to have a low rate of descent in order to ease the pilot's control problem. Time histories of the vertical velocity for the six trajectories are presented in figure 9. Once again, improvement over the fuel optimum case is found in all of these trajectories except trajectory (a). For trajectory (a) the pitch attitude of 140° and the TR_t of 0.75 combined to produce vertical descent rates higher than the fuel optimum. For trajectories (b) through (f), considerable reductions in the vertical velocity are realized with the greatest reductions found in trajectory (f). In order to get a feel for the magnitudes of these descent rates it should be mentioned that jet aircraft instrument landings have a descent rate of about 4,000 fpm. Trajectories (b), (c), (d), and (e) have a rate of about 100 fps (or 6,000 fpm) at 5,000 feet altitude and for trajectory (f) the rate is about 4,000 fpm at 5,000 feet. However, it must be remembered that trajectory (f) requires more fuel than the others.

Time to Assess Landing Area.- Another operational factor to be considered is the time available to assess the landing area. It can be seen from the time histories of figure 9 that trajectories (b) through (e) yield a slightly longer time at low altitudes than does the fuel optimum (60 seconds compared to 45 seconds to descend from 5,000 feet). Again, the more expensive trajectory (f) is greatly improved over the

fuel optimum requiring 2-minutes to descend from 5,000 feet.

Horizontal Velocity.- In figure 10 the time histories of horizontal velocity are presented. Notice that this velocity is quite high for the fuel optimum trajectory, (1,000 fps at only 5,000 feet altitude). The operationally designed trajectories, however, yield a substantial reduction (for example, trajectories (d) and (e) have a horizontal velocity of only 450 fps at 5,000 feet). Thus, by reducing the horizontal velocity (as well as the vertical velocity) these descent trajectories not only make the landing safer, but also improve initial conditions in the event that it becomes necessary for the crew to control the LEM on the back-up or abort guidance mode.

Design Powered Descent.- From the preceding discussion it is evident that several trajectories meet the operational requirements set up for the design of the powered descent. Profiles of the landing approach transition phase for a few of these descents are shown in figure 11. The descents from 50,000 feet are tabulated in table II. Trajectory (a) was not included because of poor visibility and trajectory (f) was not included because of the additional fuel requirement. It is not to be implied that the trajectories of figure 11 are the only desirable descents, but rather, any other descents should have at least as good operational features as these while maintaining as low fuel requirements as possible.

Final Translation and Touchdown

The initial conditions for this final phase of the descent are 1,000 feet altitude with a velocity of 75 fps directed along the horizontal. Terminal conditions are touchdown on the lunar surface with an impact velocity of 6.7 fps or less.

Descent Trajectories.- Based on the limitations given in the scope of calculations, the minimum range, nominal, and maximum range descent trajectories were calculated. The minimum range descent is based on the minimum range required for reducing the forward velocity to zero. The nominal descent is based on achieving a range of about 3,000 feet for normal maneuvering. And finally, the maximum range descent is based on holding the maximum forward velocity for as long as possible. These descents are illustrated in figure 12. The minimum and maximum ranges were found to be about 1,100 and 6,800 feet, respectively. The characteristic velocity for these descents is given in table III.

Maximum Footprint.- One of the desired operational features was to have an obtainable landing area or footprint as large as possible. The maximum landing footprint was established by rotating the 75 fps velocity vector to the desired direction and holding it as long as possible. In order to build the velocity in the desired direction as rapidly as possible, the thrust is applied in the direction $90° + \varphi/2$, where φ is the

desired out-of-plane direction angle. For abort considerations, this out-of-plane angle is limited to 90°. Subject to the limitations given in the Scope of Calculations, the maximum landing footprint is shown in figure 13. (This footprint represents an analytical maximum assuming no piloting errors, and hence, is expected to be slightly larger than an operational obtainable limit.) Notice that the imposed limitations did not restrict the out-of-plane range appreciably, as 5,370 feet was obtained at the maximum out-of-plane angle of 90°. The maximum range, as shown previously, was 6,800 feet for the in-plane case. The 90° out-of-plane maneuver required the maximum characteristic velocity, 693 fps, (see table III). A three-dimensional view of the maximum footprint with associated trajectories is shown in figure 14 (only half of footprint shown, symetrical about forward range axis).

CONCLUDING REMARKS

An analysis of the powered descent portion of the LEM landing mission has been presented with special emphasis on the tradeoffs imposed by various operational considerations. The design landing trajectory was divided into three operational phases. The initial phase is primarily concerned with fuel economy, the second phase is the landing approach transition which emphasizes pilot control, and the final phase is the translation and touchdown which is concerned with obtaining as large a landing area as possible. Flight safety and fuel economy are overriding criteria throughout all phases of the descent. This design landing technique was found to yield several trajectories with satisfactory operational features which allow the pilot adequate control of the final approach and at the same time satisfy the flight safety and fuel economy criteria. It was also shown that the out-of-plane range capability during translation is nearly as great as the in-plane capability.

REFERENCES

1. Miele, A.: General Variational Theory of the Flight Paths of Rocket Aircraft, Missiles, and Satellite Carriers, AFOSR-TN-58-246 (AD-154 148).

TABLE I.- CONDITIONS FOR SAMPLE OPERATIONAL TRAJECTORIES

Trajectory	TR_t	h_t ft	θ_t deg	V_c fps
a	0.75	5,000	140	5,664
b	0.50	15,000	140	5,800
c	0.50	10,000	140	5,761
d	0.40	15,000	130	5,910
e	0.40	10,000	130	5,854
f	0.30	5,000	120	5,944

TABLE II.- TABULATION OF LEM DESCENTS* - Continued

Initial conditions: Circular orbit at 50,000 ft; Final Conditions: h = 1,000 ft., V = 75 fps, $\gamma = 0°$

Trajectory b ($TR_t = 0.50$, $h_t = 15{,}000$ ft., $\theta_t = 140°$)

t sec	h ft	x ft	θ deg	V fps	$-\gamma$ deg	$-\dot{h}$ fps	$\dot{x}$ fps
0	1,000	0	140.00	75.00	0	0	75.00
10.00	1,106	1,193	140.00	164.85	7.356	21.11	163.46
20.00	1,419	3,266	140.00	254.38	9.373	41.43	250.92
30.00	1,932	6,208	140.00	342.95	10.248	61.02	337.37
42.50	2,843	11,093	140.00	452.21	10.775	84.54	444.02
57.50	4,315	18,701	140.00	581.17	11.059	111.48	569.95
77.50	6,888	31,751	140.00	749.56	11.188	145.43	734.43
97.50	10,119	48,052	140.00	914.03	11.194	177.45	895.05
117.50	13,975	67,528	140.00	1,074.76	11.149	207.82	1,051.89
122.35	15,000	72,722	163.23**	1,113.19	11.135	214.98	1,089.37
127.35	16,084	78,410	163.64	1,209.12	10.414	218.57	1,185.86
152.35	21,673	113,980	165.66	1,677.75	7.732	225.73	1,656.20
202.35	32,490	219,190	169.45	2,562.73	4.470	199.72	2,540.46
262.35	42,523	401,280	173.57	3,543.06	2.119	131.03	3,514.43
302.35	46,747	554,050	176.07	4,153.04	1.112	80.57	4,118.49
342.35	49,064	730,340	178.36	4,731.49	0.450	37.12	4,690.98
382.35	49,911	928,940	180.45	5,280.60	0.089	8.23	5,234.77
397.00	50,000	1,014,000	181.10	5,483.47	0	0	5,435.00

*Tabulated in reverse.

**Instantaneous change in θ for landing approach transition.

TABLE II.- TABULATION OF LEM DESCENTS* - Continued

Trajectory c (TR_t = 0.50, h_t = 10,000 ft., θ_t = 140°)

t sec	h ft	x ft	θ deg	V fps	-γ deg	$-\dot{h}$ fps	$\dot{x}$ fps
0	1,000	0	140.00	75.00	0	0	75.00
10.00	1,105	1,192	140.00	164.50	7.279	20.84	163.15
21.25	1,467	3,579	140.00	264.76	9.426	43.36	261.12
31.25	1,997	6,621	140.00	352.86	10.220	62.61	347.14
43.75	2,925	11,626	140.00	461.35	10.703	85.72	453.27
53.75	3,872	16,578	140.00	547.30	10.901	103.50	537.06
78.75	6,992	32,582	140.00	757.32	11.079	145.53	742.29
88.75	8,527	40,408	140.00	839.61	11.088	161.47	822.70
97.51	10,000	47,920	160.56**	910.91	11.081	175.07	892.36
107.51	11,834	57,816	161.48	1,105.23	9.945	190.87	1,086.37
132.51	16,952	90,909	163.73	1,577.30	7.840	215.17	1,557.93
152.51	21,335	125,730	165.49	1,941.65	6.548	221.41	1,921.79
192.51	30,015	216,630	168.87	2,637.95	4.513	207.56	2,616.00
252.51	40,808	402,980	173.61	3,610.39	2.333	146.94	3,581.76
292.51	45,685	558,360	176.55	4,216.00	1.315	96.73	4,181.39
352.51	49,380	834,820	180.59	5,066.92	0.338	29.90	5,023.33
383.00	50,000	1,000,000	182.50	5,483.47	0	0	5,435.00

*Tabulated in reverse.

**Instantaneous change in θ for landing approach transition.

TABLE II.- TABULATION OF LEM DESCENTS* - Continued

Trajectory d (TR_t = 0.40, h_t = 15,000 ft., θ_t = 130°)

t sec	h ft	x ft	θ deg	V fps	-γ deg	$-\dot{h}$ fps	$\dot{x}$ fps
0	1,000	0	130.00	75.00	0	0	75.00
10.00	1,093	1,051	130.00	136.38	7.796	18.50	135.09
20.00	1,368	2,700	130.00	198.05	10.583	36.37	194.64
30.00	1,818	4,942	130.00	259.31	11.940	53.65	253.62
42.50	2,620	8,569	130.00	335.08	12.835	74.44	326.56
57.50	3,917	14,116	130.00	424.74	13.380	98.29	412.93
77.50	6,186	23,511	130.00	542.13	13.694	128.34	526.14
97.50	9,038	35,148	130.00	657.10	13.787	156.59	637.16
117.50	12,439	48,983	130.00	769.73	13.769	183.20	745.98
127.50	14,335	56,711	130.00	825.20	13.736	195.94	799.59
130.86	15,000	59,428	162.62**	843.71	13.723	200.15	817.48
150.86	19,175	79,708	164.23	1,231.34	10.063	215.16	1,208.34
195.86	28,913	153,040	167.63	2,061.41	5.849	210.06	2,040.33
245.86	38,405	276,720	171.08	2,919.75	3.237	164.86	2,895.59
305.86	46,097	479,200	174.79	3,872.13	1.343	90.73	3,840.02
345.86	48,788	644,650	177.01	4,465.30	0.581	45.27	4,427.19
385.86	49,891	832,980	179.03	5,028.06	0.143	12.56	4,984.44
419.00	50,000	1,005,000	180.40	5,483.47	0	0	5,435.00

*Tabulated in reverse.

**Instantaneous change in θ for landing approach transition.

TABLE II.- TABULATION OF LEM DESCENTS* - Concluded

Trajectory e (TR_t = 0.40, h_t = 10,000 ft., θ_t = 130°)

t sec	h ft	x ft	θ deg	V fps	$-\gamma$ deg	$-\dot{h}$ fps	$\dot{x}$ fps
0	1,000	0	130.00	75.00	0	0	75.00
10.00	1,091	1,049	130.00	136.00	7.652	18.11	134.77
20.00	1,360	2,694	130.00	197.28	10.397	35.60	193.99
30.00	1,801	4,927	130.00	258.13	11.734	52.50	252.66
42.50	2,585	8,540	130.00	333.41	12.617	72.83	325.21
57.50	3,854	14,064	130.00	422.50	13.152	96.13	411.14
77.50	6,073	23,417	130.00	539.15	13.459	125.49	523.78
97.50	8,861	35,000	130.00	653.41	13.546	153.04	634.25
104.72	10,000	39,719	160.24**	694.07	13.547	162.58	673.58
124.72	12,639	52,052	161.59	989.72	10.937	194.27	1,066.66
149.72	18,744	89,769	164.20	1,560.02	7.900	214.43	1,540.15
199.72	29,500	189,360	168.31	2,452.76	4.842	207.02	2,431.43
239.72	37,139	300,040	171.40	3,120.89	3.158	171.94	3,095.98
299.72	45,340	513,990	175.67	4,055.99	1.410	99.83	4,022.78
339.72	48,379	686,550	178.28	4,639.12	0.658	53.27	4,599.78
379.72	49,745	881,590	180.68	5,192.66	0.193	17.49	5,147.72
402.50	50,000	1,004,000	182.00	5,483.47	0	0	5,435.00

*Tabulated in reverse.

**Instantaneous change in θ for landing approach transition.

TABLE III.- CHARACTERISTIC VELOCITY REQUIREMENTS
FOR FINAL TRANSLATION AND TOUCHDOWN

Initial Conditions: $h = 1{,}000$ ft
$V = 75$ fps
$\gamma = 0°$

Out-of-plane angle, deg	Type of Descent	V_c, fps
0	min. range	636
0	nominal	621
0	max. range	663
30	max. range	674
45	max. range	679
60	max. range	684
90	max. range	693

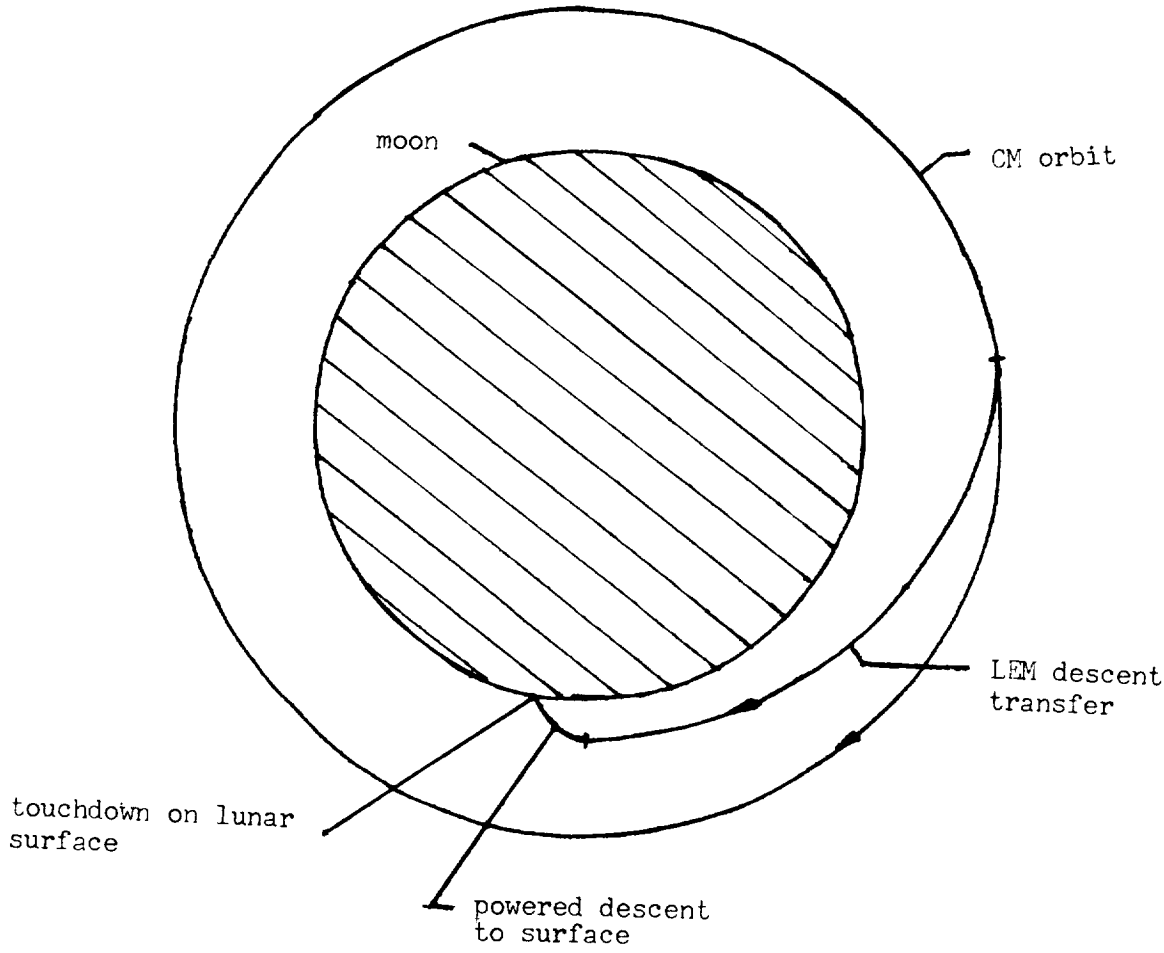


FIGURE 1.- SKETCH OF LEM DESCENT

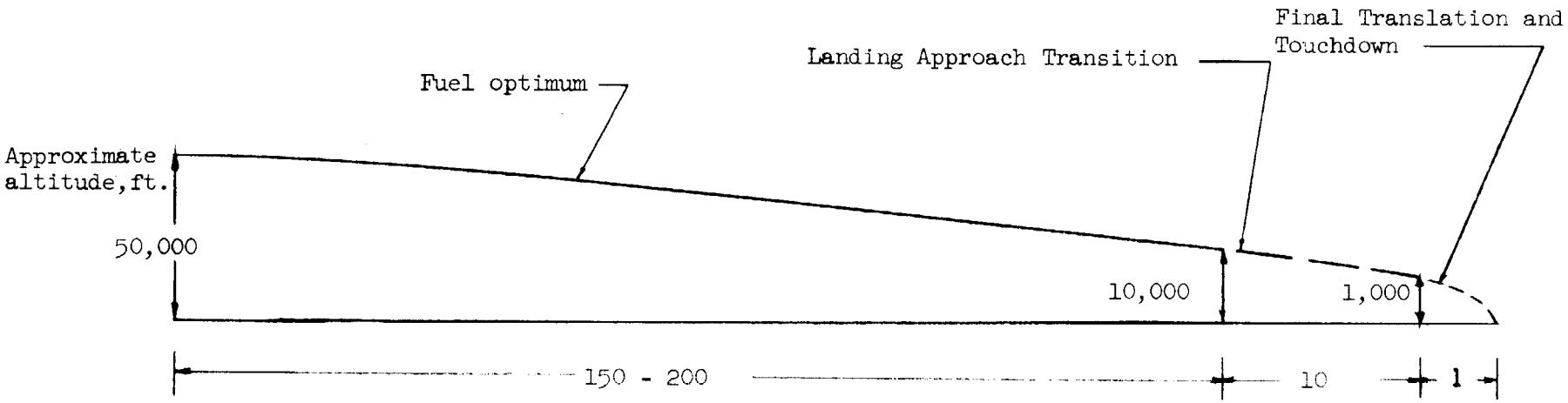


FIGURE 2.- THREE PHASES OF POWERED DESCENT

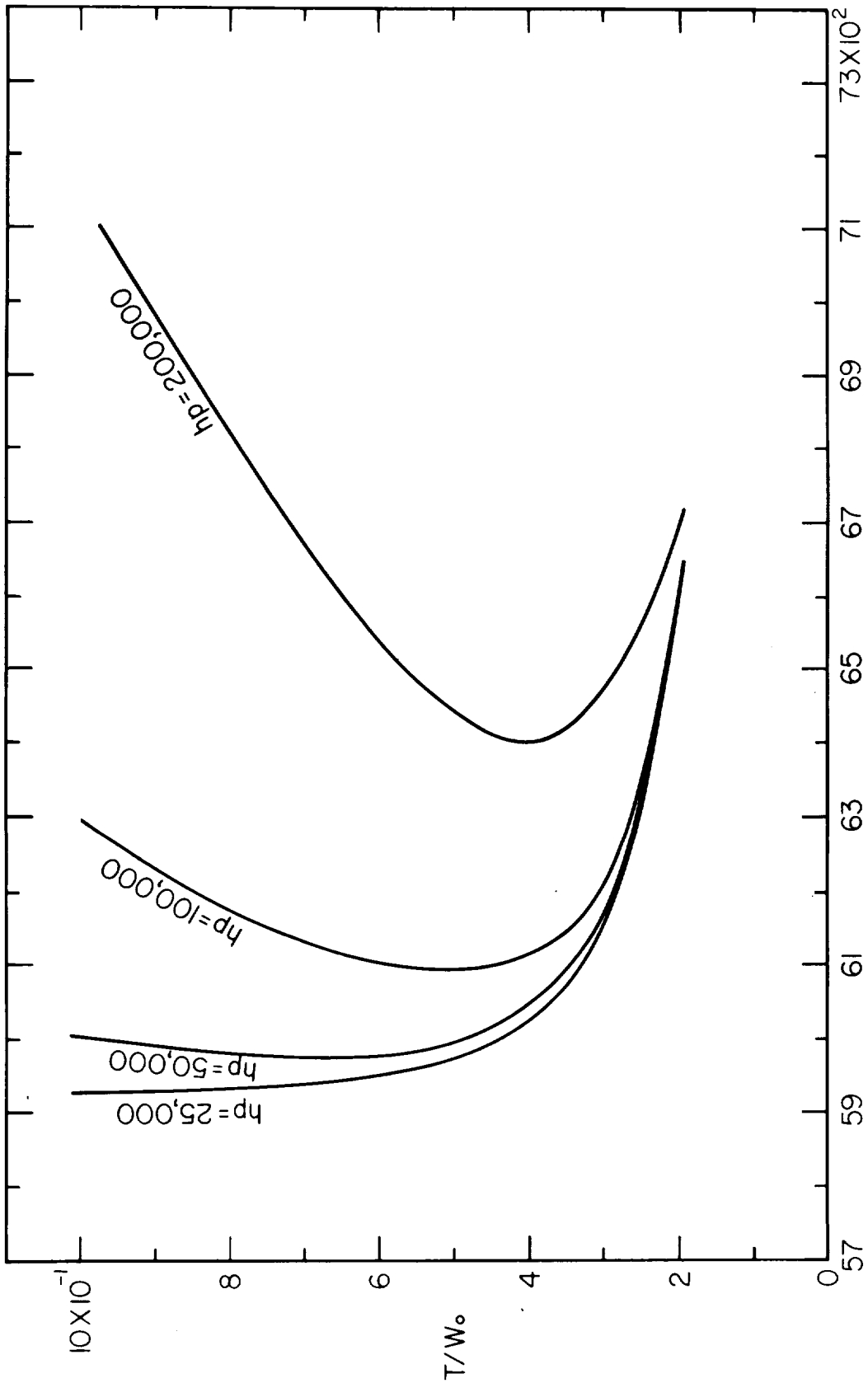


FIGURE 3.- CHARACTERISTIC VELOCITY FOR FUEL OPTIMUM POWERED DESCENTS FROM PERICYNTHION ALTITUDE OF DESCENT TRANSFER WITH APOCYNTHION ALTITUDE OF 100 N. MI.

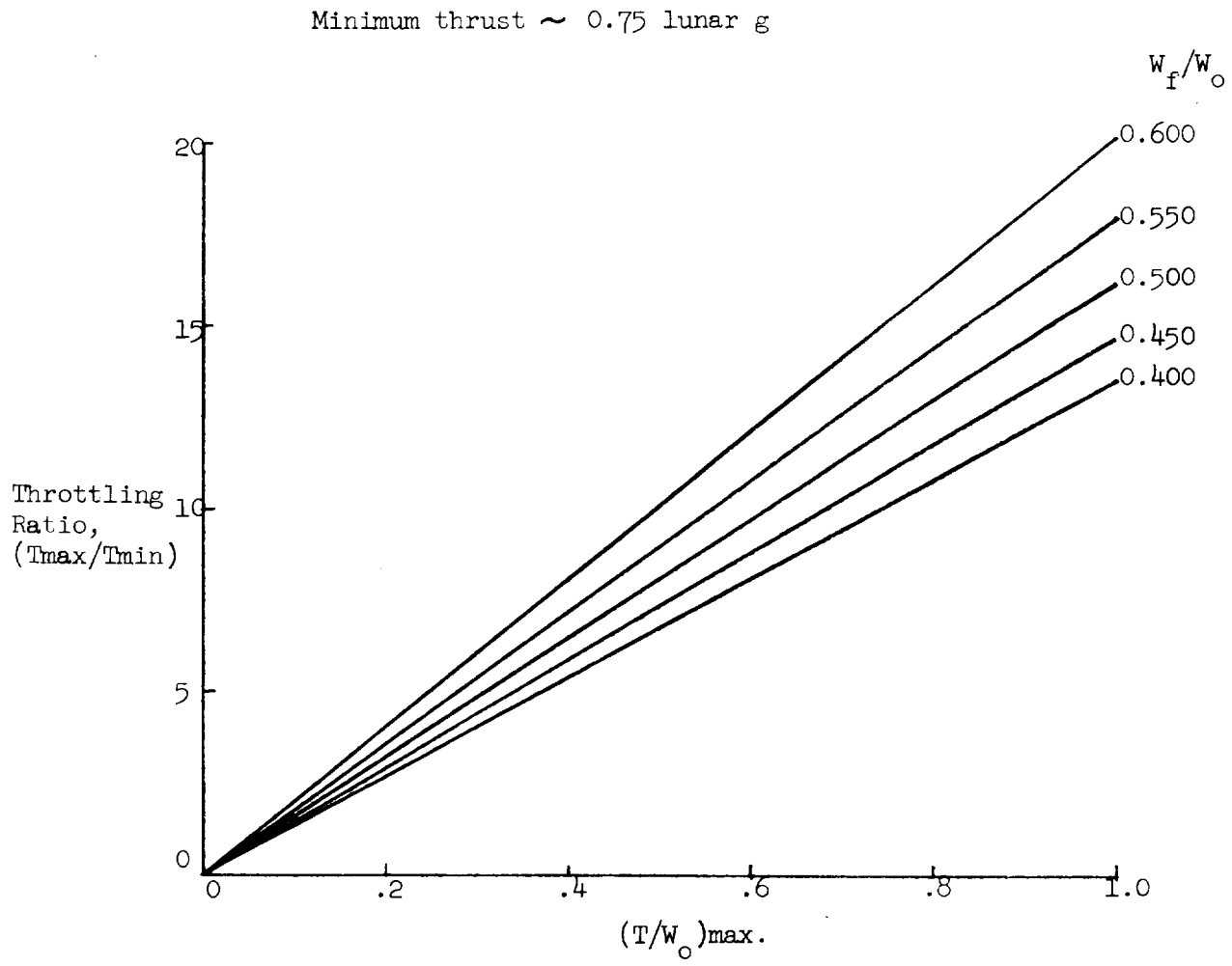


FIGURE 4.- VARIATION OF THROTTLING RATIO WITH MAXIMUM T/W_o FOR SEVERAL FUEL CONSUMPTION RATIOS.

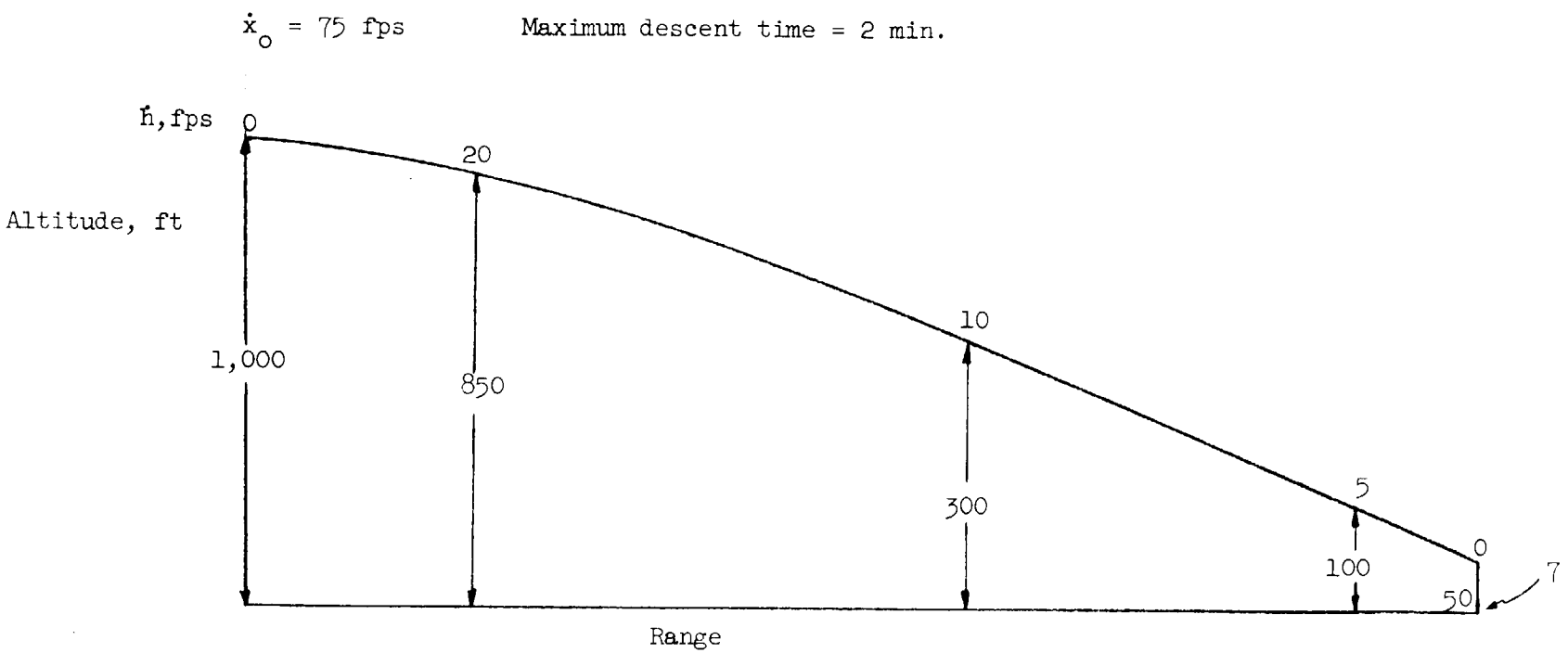


FIGURE 5.- PROFILE OF VERTICAL DESCENT RATES FOR FINAL TRANSLATION AND TOUCHDOWN PHASE.

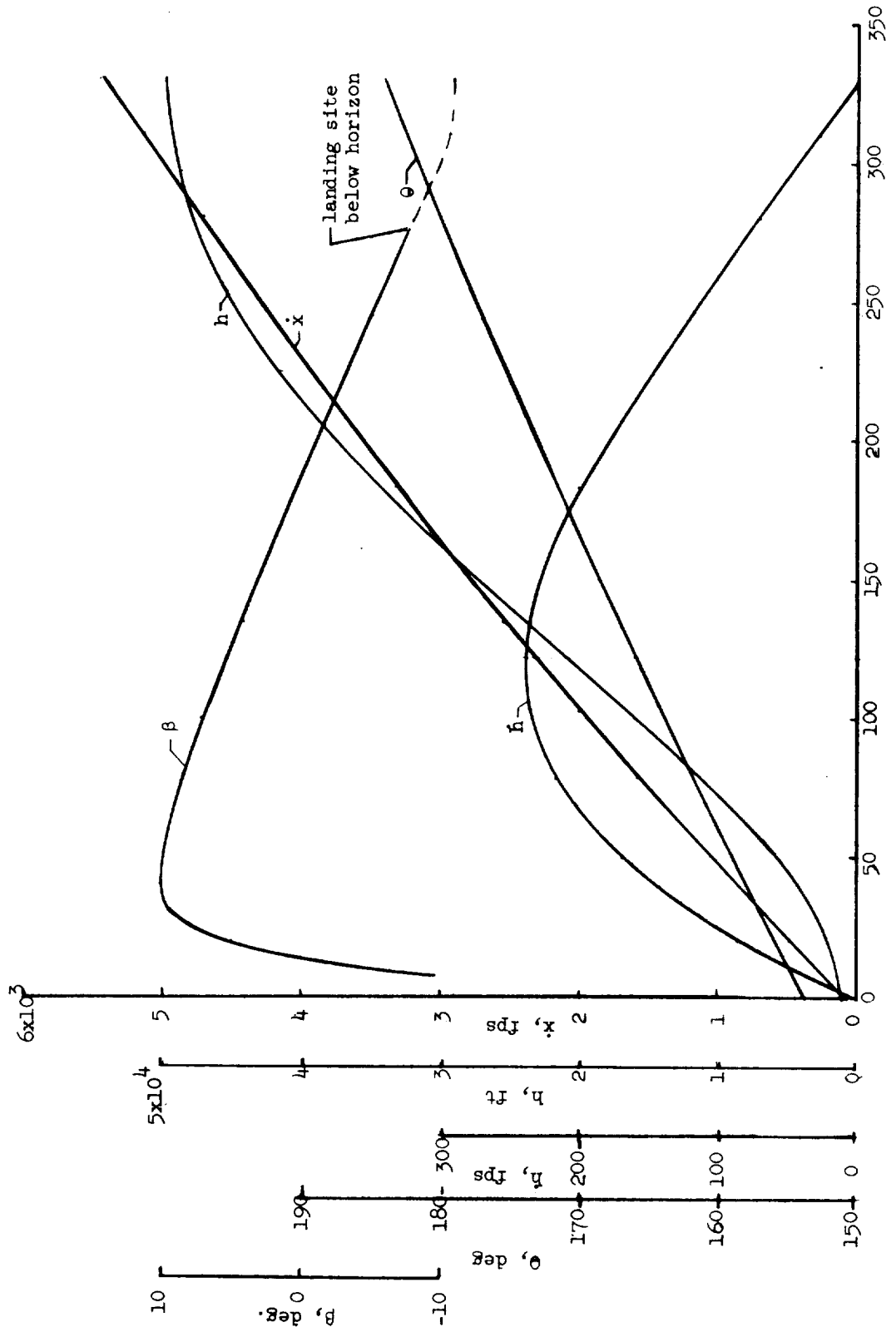


FIGURE 6.- TIME HISTORY OF FUEL OPTIMUM LANDING FROM 50,000 FT. CIRCULAR ORBIT.
($T/W_o = 0.4$, $I_{sp} = 315$ sec.)

FINAL CONDITIONS: $h = 1,000$ FT.; $\dot{x} = 75$ fps; $\dot{h} = 0$ fps.

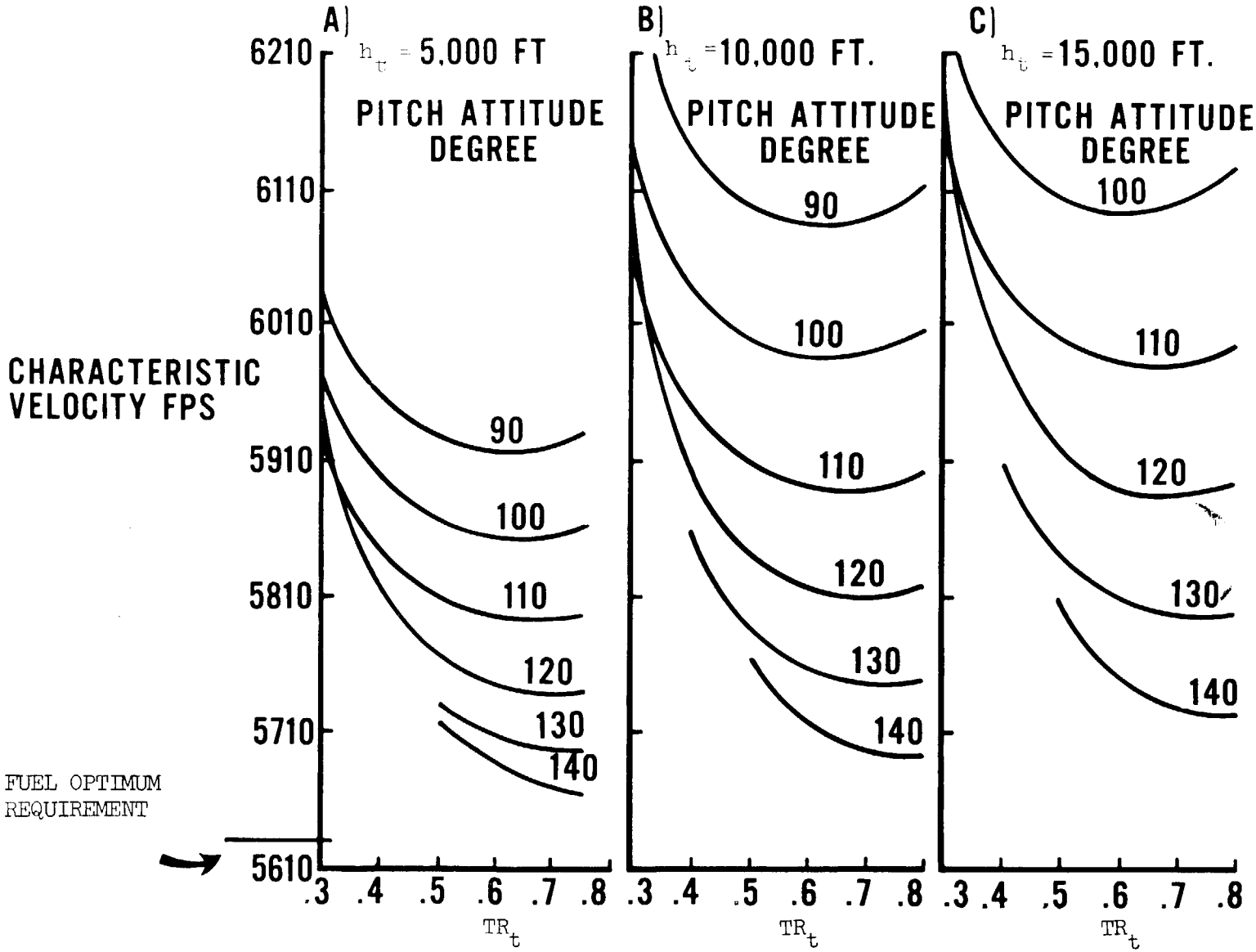


FIGURE 7.- CHARACTERISTIC VELOCITY FOR DESIGN POWERED DESCENT
(INITIAL CONDITIONS: 50,000 FT. CIRCULAR ORBIT. FINAL CONDITIONS: h = 1,000 FT. V = 75 fps, γ = 0.)

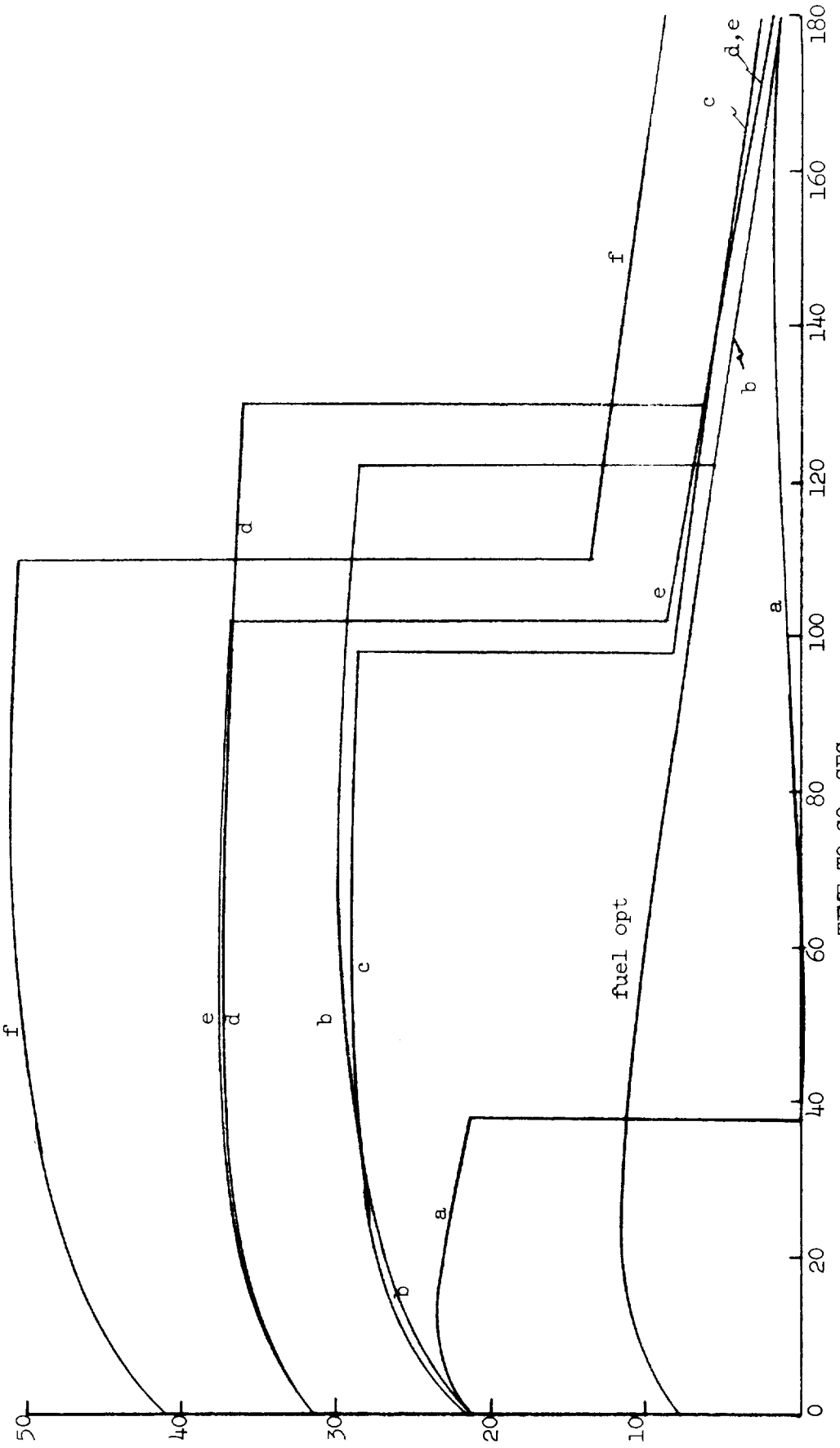


FIGURE 8.- TIME HISTORY OF LOOK ANGLE TO NOMINAL LANDING SITE.
(3,000 FT. DOWNRANGE OF 1,000 FT. ALTITUDE POINT)

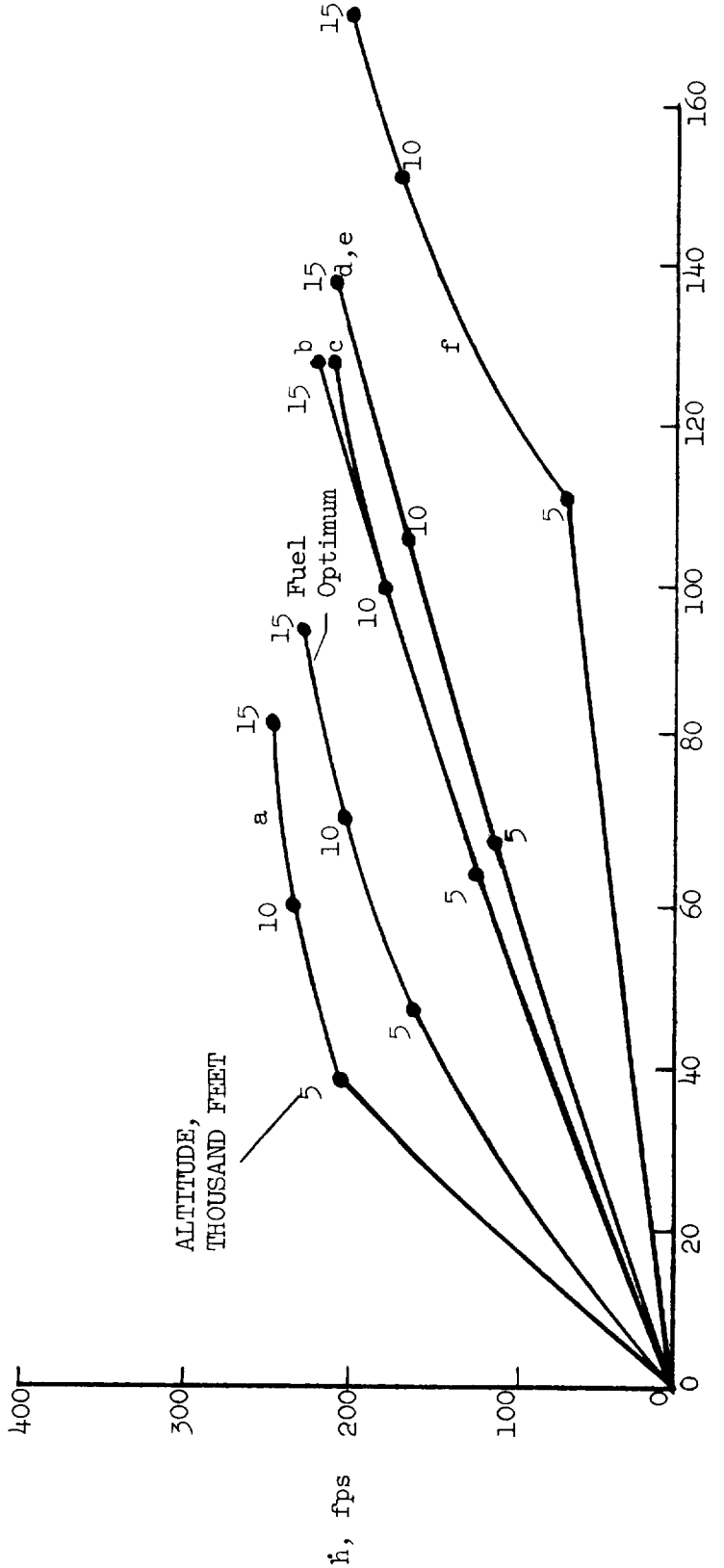


FIGURE 9.- TIME HISTORY OF VERTICAL VELOCITY.

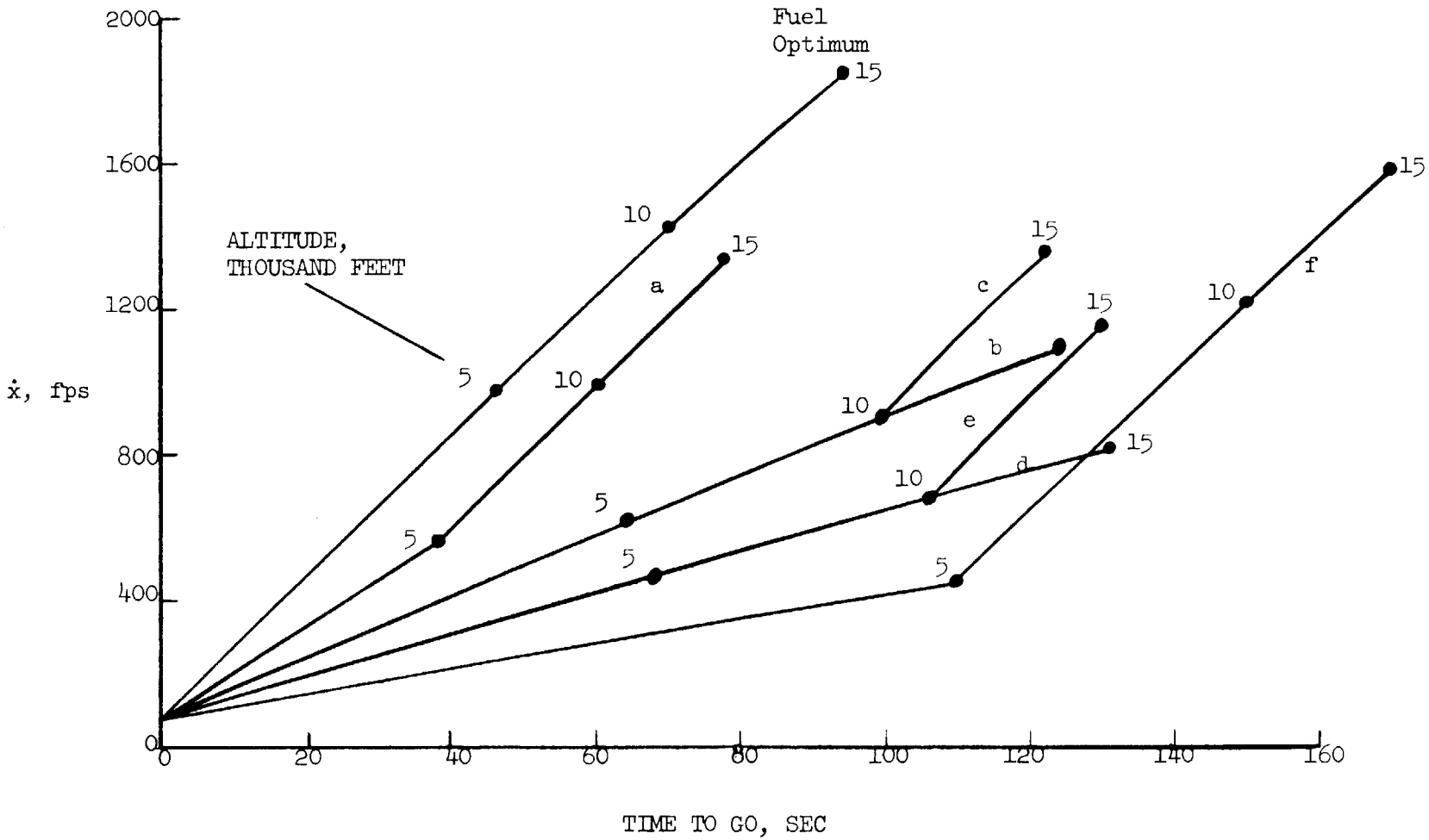
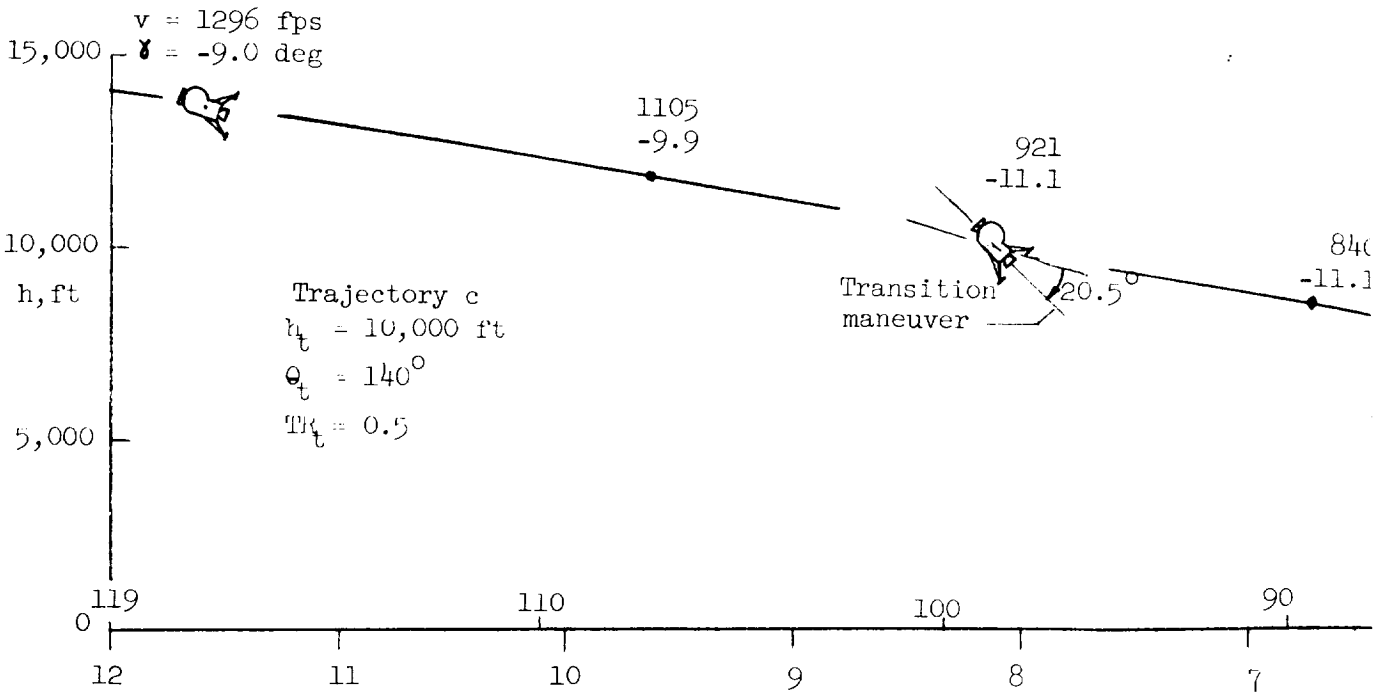


FIGURE 10.- TIME HISTORY OF HORIZONTAL VELOCITY.



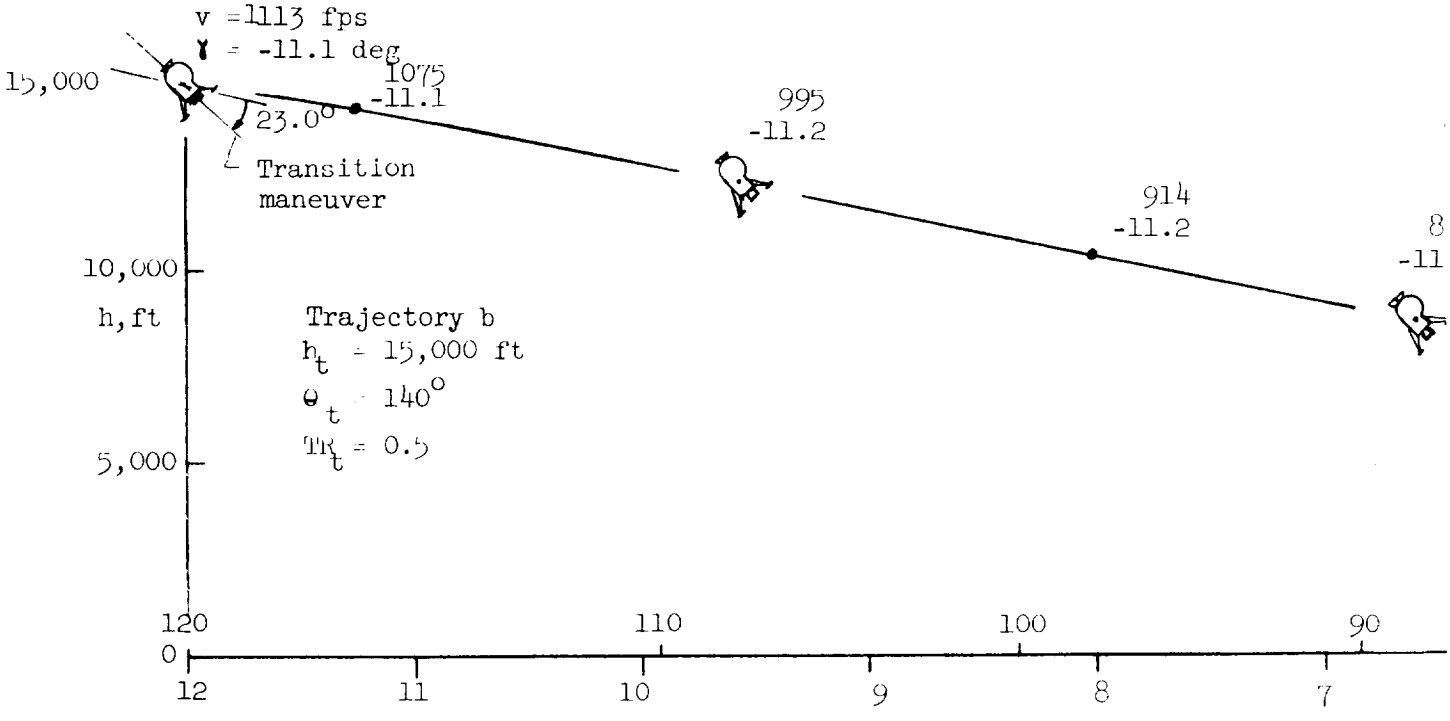
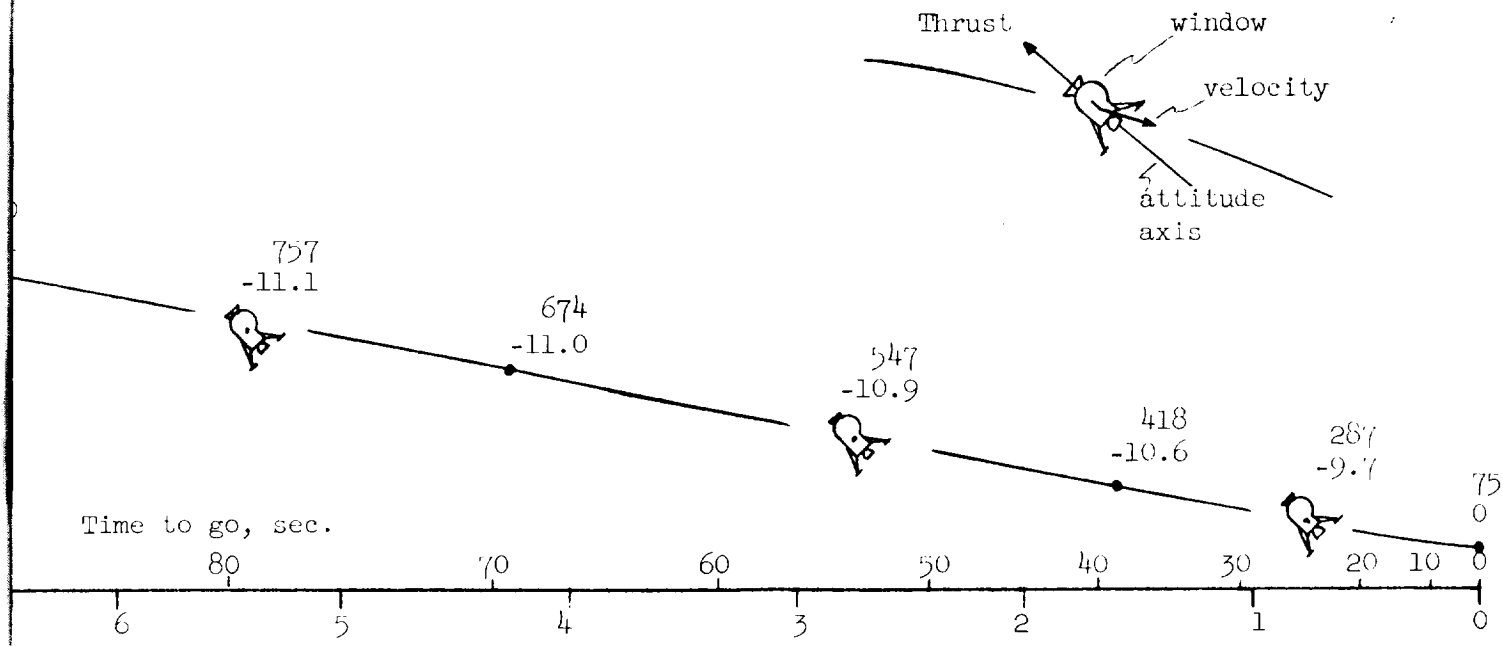


FIGURE 11.- PROFILES OF SEVERAL LANDI

FOLDOUT FRAME /



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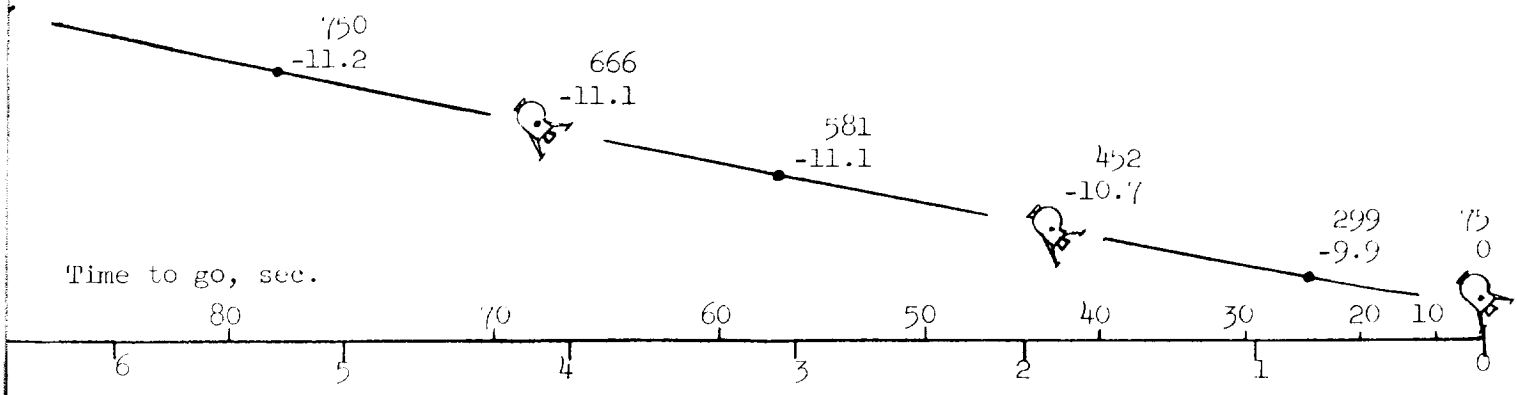
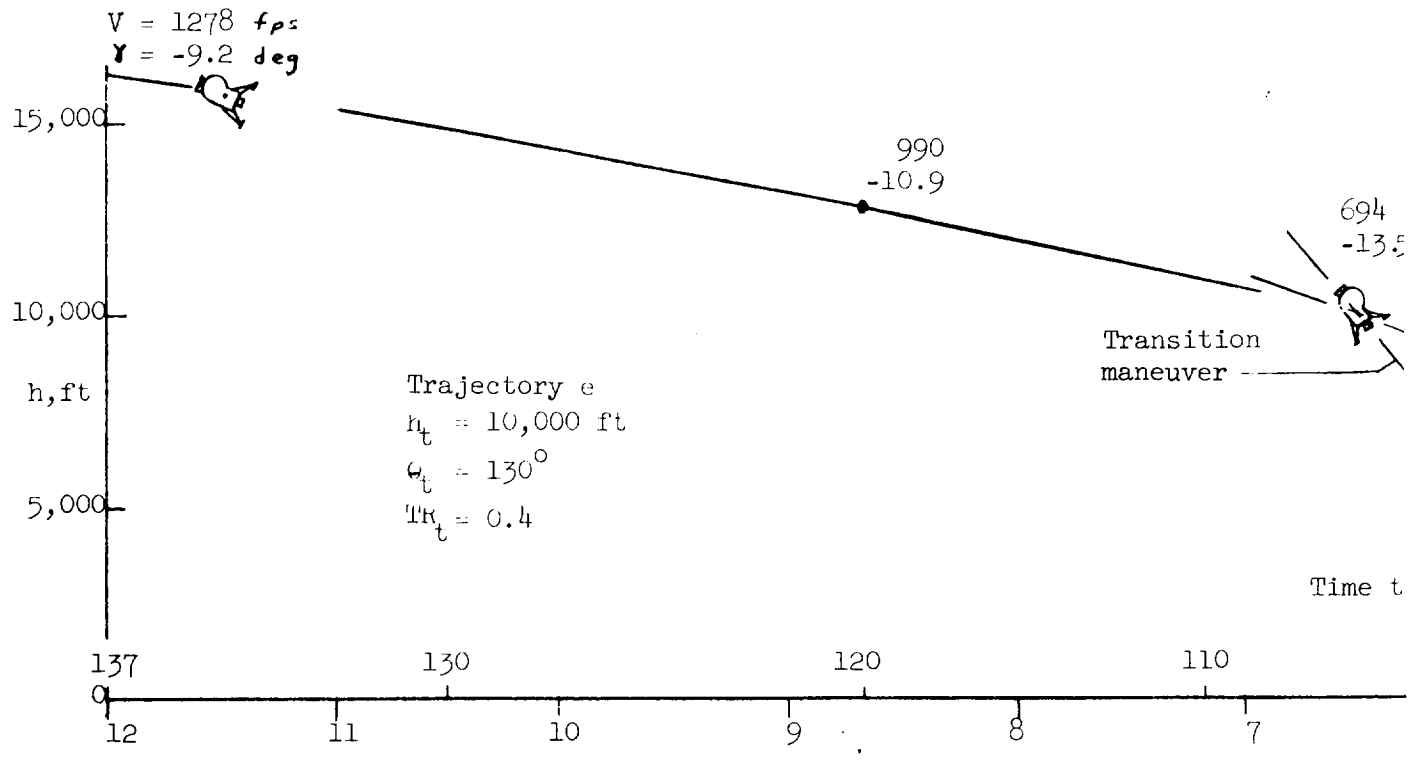


o go, n.mi.

NG APPROACH TRANSITION TRAJECTORIES.

FOLDOUT FRAME 2



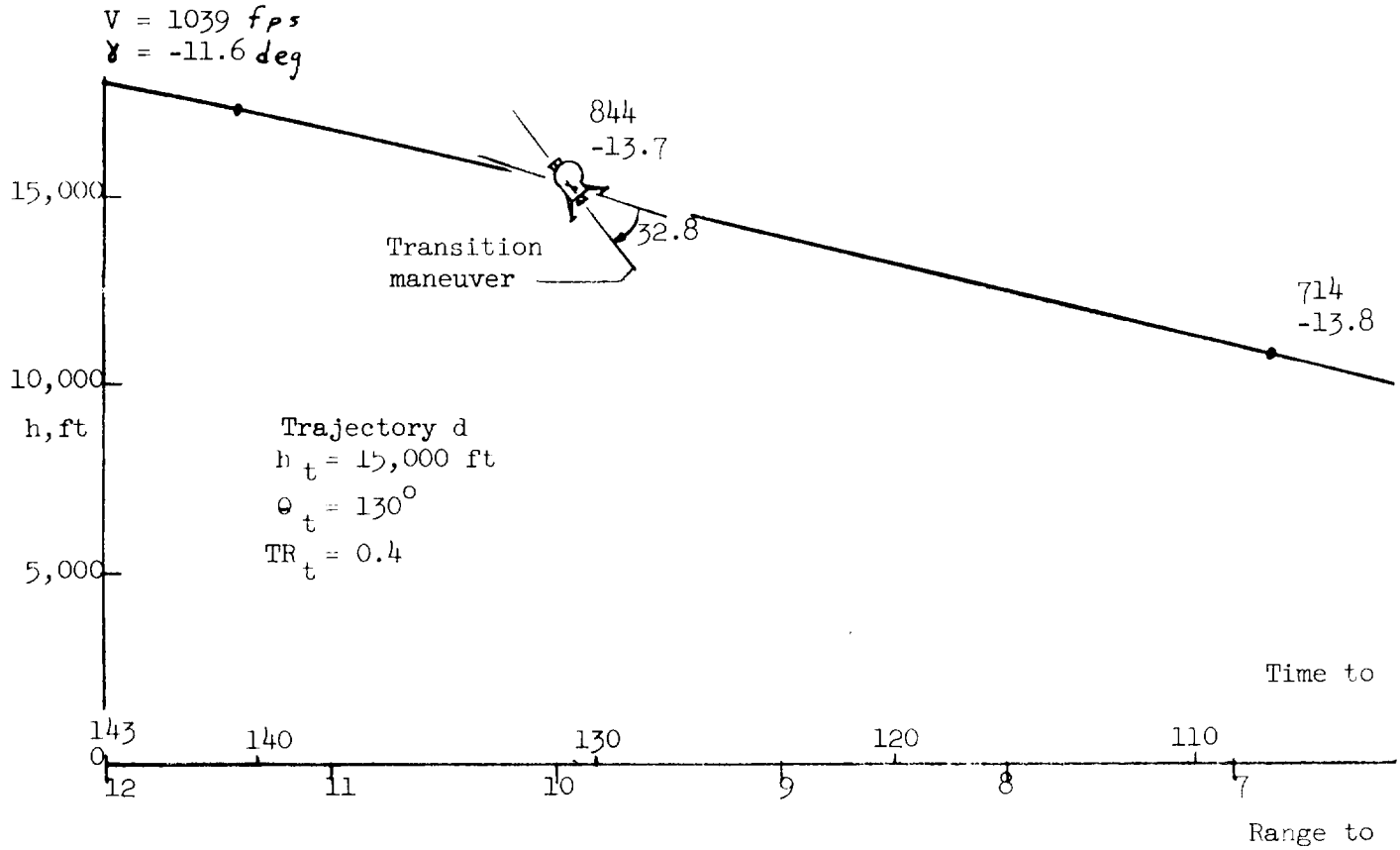
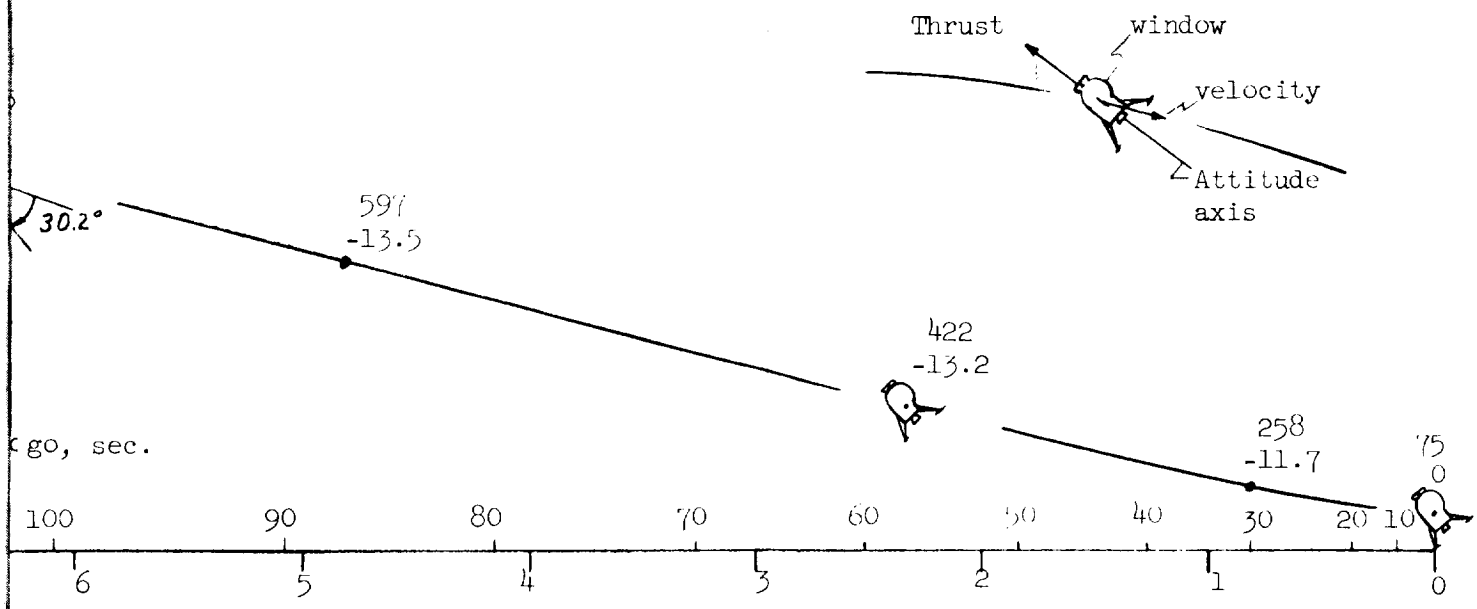
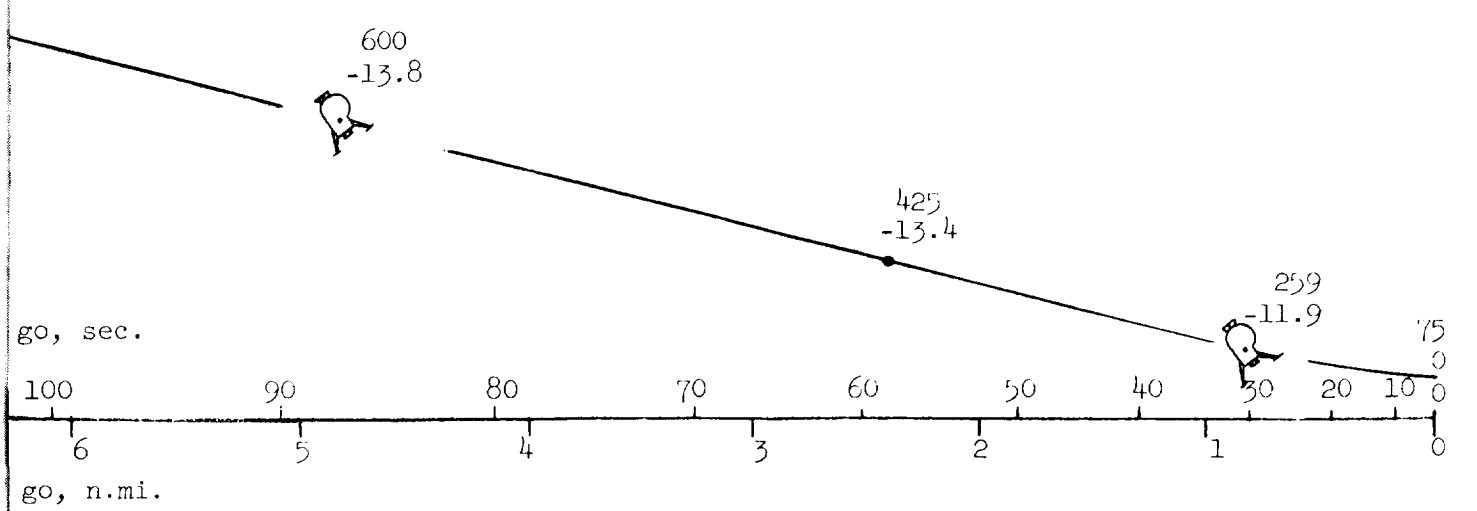


FIGURE 11.- PROFILES OF SEVERAL LANDING APPROA

FOLDOUT FRAME /





CH TRANSITION TRAJECTORIES - Concluded.

FOLDOUT FRAME 2

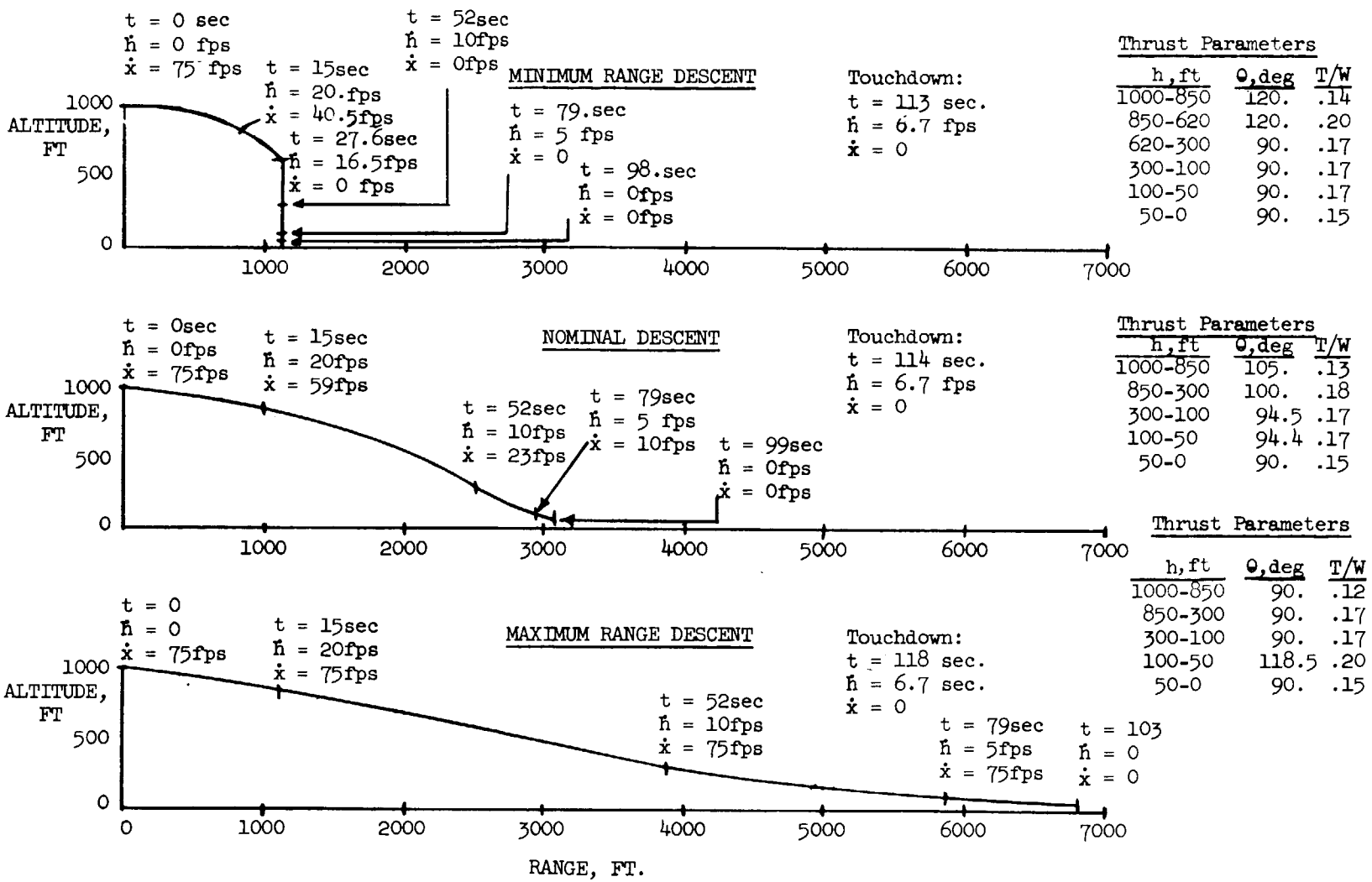


FIGURE 12.- LEM DESCENTS FROM 1,000 FT.
(Limitations: $\Delta\theta_{max} = \pm 30°$, $\dot{h}_{max} = 20$ fps)

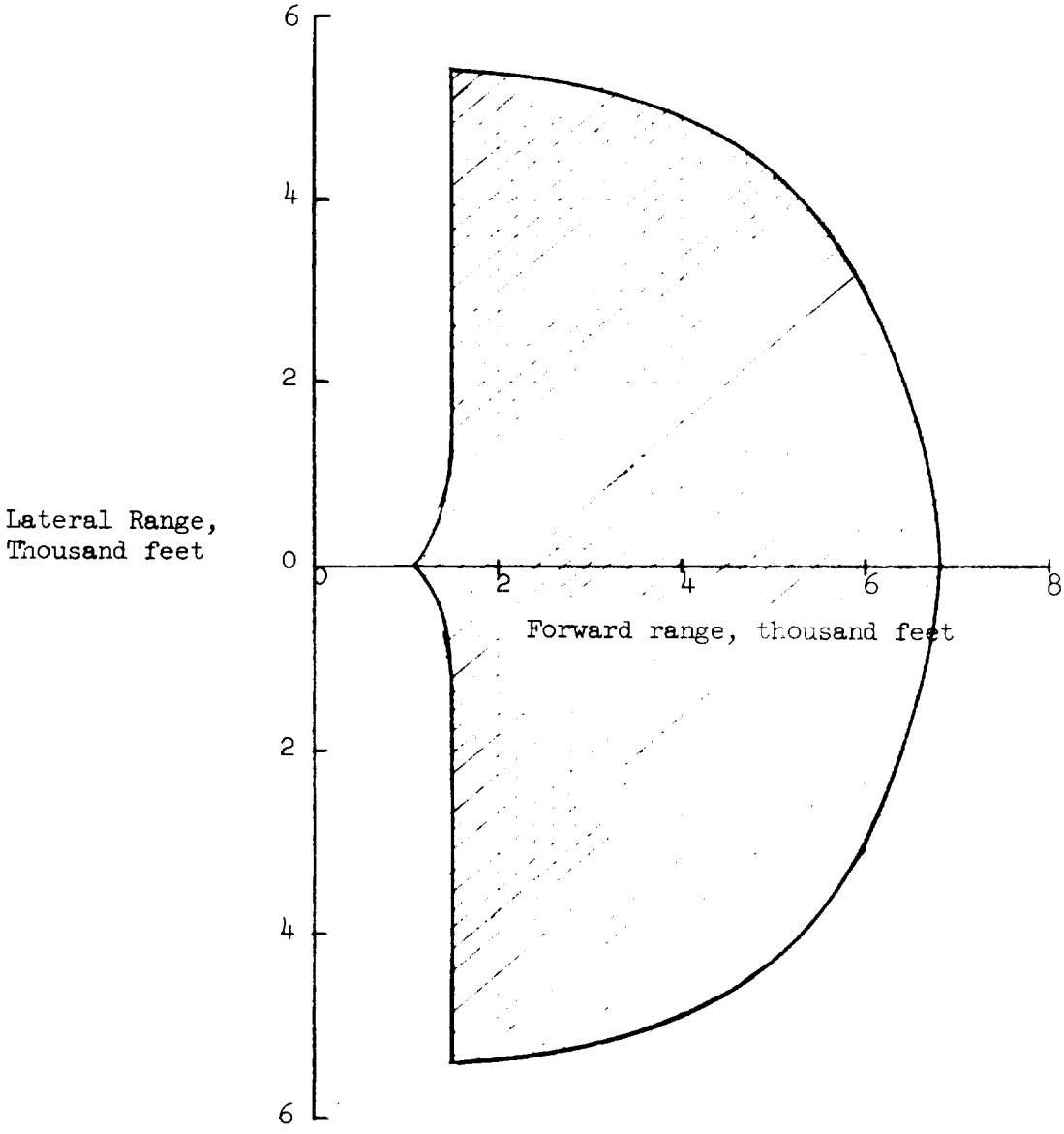


FIGURE 13.- MAXIMUM FOOTPRINT FOR LEM DESCENT FROM 1,000 FT.
(V_o = 75 fps; γ_o = 0°)

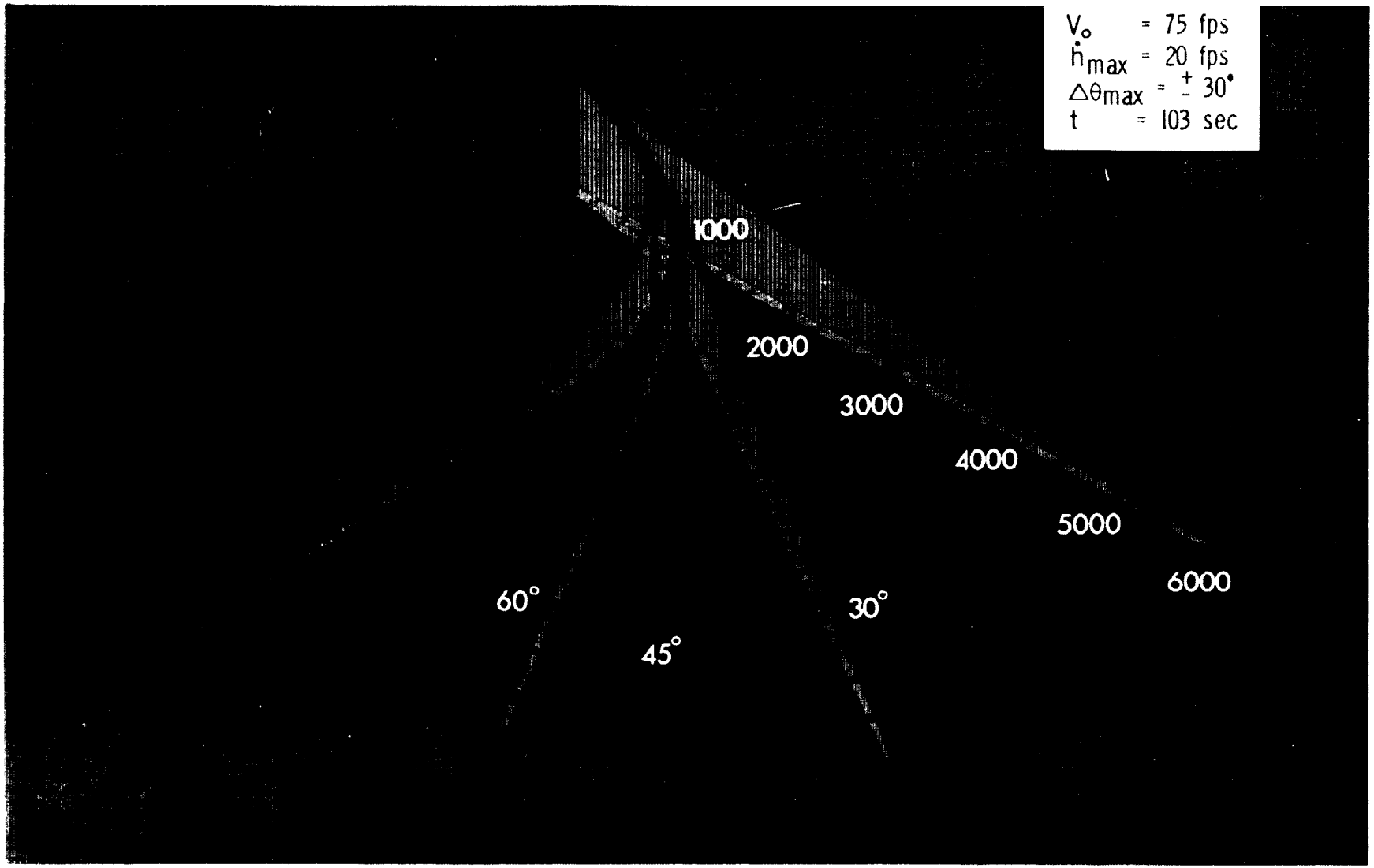


FIGURE 14.- MAXIMUM FOOTPRINT FOR LEM DESCENT FROM 1,000 FT.
(Only half of footprint shown, symmetrical about forward axis.)