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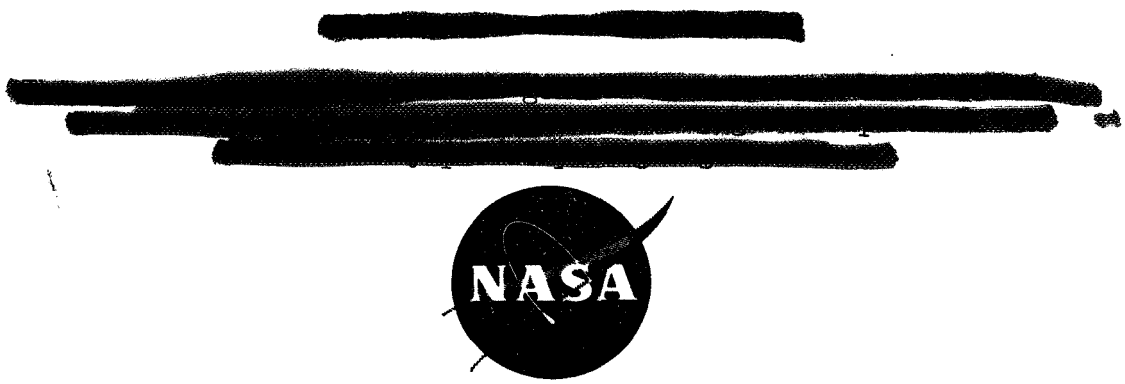
NASA Program Apollo Working Paper No. 1104

SIMPLIFIED ANALYSIS OF LEM MIDCOURSE CORRECTIONS



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
Houston, Texas

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SIMPLIFIED ANALYSIS OF LEM MIDCOURSE CORRECTIONS

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ABSTRACT

Midcourse velocity correction schedules for the LEM rendezvous with the CM in lunar orbit are presented on a constant error basis in order to identify and establish the trends of the important parameters. These parameters are found to be number of midcourse corrections and time of the final correction. The direction of the velocity error does not affect the correction schedule but does affect the magnitude of the required guidance velocity.

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SUMMARY

Results are presented from an investigation of LEM midcourse guidance procedures for rendezvous with the CM in lunar orbit. The analysis is carried out in three dimensions using linearized equations of motion, and a comparison is made between the results given by the linearized equations and the exact two-body equations. The results presented are for constant velocity errors in order to simplify the analysis. Two types of LEM transfers are investigated; a 180° in-plane transfer and a 220° out-of-plane transfer. It is shown that for constant velocity errors an optimum midcourse correction schedule can be found and that this schedule depends on the number of corrections and the time of the last correction. The direction of the velocity error does not affect the correction schedule but does affect the magnitude of the required guidance velocity.

INTRODUCTION

Midcourse guidance corrections are corrections applied to a spacecraft trajectory in order to guide to some desired terminal conditions. These corrections are required because of inherent inaccuracies of the guidance system (for example, propulsion and guidance hardware limitations, determination of astrodynamic constants, and crew limitations). In order to determine the midcourse guidance requirements for a particular problem statistical techniques must be utilized since the guidance errors must be assumed to be of a random character. However, statistical studies are quite complex in utilization as well as in interpretation; thus it would be desirable to determine on a fixed error basis what the important parameters are, and to determine if an optimum schedule can be determined for these known errors. The purpose of the present report is to present the results of a constant error study for LEM midcourse guidance during the ascent transfer to the CM in lunar orbit.

The equations of motion used in this investigation assume a linear gravity field and are referenced to a rotating spherical coordinate system with origin in the CM, see reference 1. These approximate equations of motion are used to simplify the analysis. For the present analysis they were assumed to be adequate for establishing trends and the relative importance of the various parameters. Furthermore, comparison is shown in the discussion of the present report between solutions utilizing this approximate technique and an exact two-body method.

In this study two types of ascent transfers are investigated; namely, the 180° Hohmann transfer for the normal coplanar operation and a 220° transfer for operating at the maximum out-of-plane LEM design limit of 2°. These transfers are initiated with an error of specified magnitude and direction and each correction is subsequently initiated with this same error.

SYMBOLS

A	Acceleration of LEM with respect to CM
$\bar{i}, \bar{j}, \bar{k}$	Unit vectors along x, y, z axis, respectively
n	Number of midcourse corrections
t	Time
t_f	Time of final correction
V	Velocity of LEM prior to correction impulse
V_I	Transfer velocity required to intercept CM in specified time
V_{to}	Relative terminal velocity of the LEM with respect to the CM for the case of no errors
V_{te}	Relative terminal velocity of the LEM with respect to the CM for the case of errors
ΔV_c	Applied correction velocity
ΔV_g	Total guidance velocity
x, y, z	Coordinates, defined in figure 2
δ	Fraction of remaining time to intercept
σ	Velocity error
τ	Time to intercept (transfer time)
ω	Angular rate of rotation of target-centered coordinate system

$(\dot{})$	Derivative of ( ) with respect to t
$(\ddot{})$	Derivative of $(\dot{})$ with respect to t
$(\bar{})$	Denotes vector

Subscripts

o	Conditions when t = 0
I	Pertaining to intercept

METHOD OF ANALYSIS

In this section brief descriptions of the midcourse correction technique and of the linearized equations of motion are given.

Midcourse Correction Technique.- As indicated earlier, because of errors inherent in any space guidance system it is necessary to perform corrections periodically to a spacecraft trajectory in order to attain the desired terminal conditions (rendezvous), see figure 1. The error and correction technique utilized in the present investigation is as follows. At initiation of the intercept transfer and at application of each correction velocity, a fixed error velocity of specified magnitude and direction is also applied. The total time of the transfer is held constant (equal to the transfer time for no velocity errors) in order to keep the study from becoming too complex. Application of the corrections is scheduled on a fixed percentage of the remaining transfer time (that is, correction time = δ (τ - time of previous correction)) except the final correction time which is specified independently of prior correction times. The applied correction velocity vector ΔV_c is defined on an impulsive basis as:

$$\overline{\Delta V}_c = \bar{V}_I - \bar{V} \tag{1}$$

where $\bar{V}$ is the velocity vector of the LEM prior to the correction impulse and $\bar{V}_I$ is the transfer velocity vector required to intercept the CM in the specified time.

The total guidance velocity requirement ΔV_g is defined as the sum of the correction velocities plus the difference between the terminal velocity of the transfer with errors V_{te} and the terminal velocity of

the transfer with no errors V_{to}; that is:

$$\Delta V_g = \sum_{i=1}^{n} \left(\overline{\Delta V}_c\right)_i + \left|\overline{V}_{te}\right| - \left|\overline{V}_{to}\right| \tag{2}$$

The equations of motion used to determine the velocities required to solve equations (1) and (2) are discussed briefly in the next section.

Linearized Equations of Motion.- The equations of motion used in the present study are based on a linear gravity approximation and are derived in reference 1. For a rotating axis system centered in the CM, (figure 2) the linearized equations are:

$$\begin{aligned} \ddot{x} + 2\omega\dot{z} &= A_x \\ \ddot{y} + \omega^2 y &= A_y \\ \ddot{z} - 2\omega\dot{x} - 3\omega^2 z &= A_z \end{aligned} \tag{3}$$

The solutions to these equations for coasting flight ($A_x = A_y = A_z = 0$) are, from reference 1,

$$\begin{aligned} x &= x_o + \frac{\dot{x}_o}{\omega}(4 \sin \omega t - 3\omega t) - 6z_o(\omega t - \sin \omega t) - \frac{2\dot{z}_o}{\omega}(1 - \cos \omega t) \\ y &= y_o \cos \omega t - \frac{\dot{y}_o}{\omega} \sin \omega t \\ z &= \frac{2\dot{x}_o}{\omega}(1 - \cos \omega t) + z_o(4 - 3 \cos \omega t) + \frac{\dot{z}_o}{\omega} \sin \omega t \end{aligned} \tag{4}$$

The components of the velocity vector $\overline{V}$ are:

$$\begin{aligned} \dot{x} &= \omega\left[2\left(2\frac{\dot{x}_o}{\omega} + 3z_o\right) \cos \omega t - \frac{2\dot{z}_o}{\omega} \sin \omega t - 6z_o - \frac{3\dot{x}_o}{\omega}\right] \\ \dot{y} &= y_o \cos \omega t + \frac{\dot{y}_o}{\omega} \sin \omega t \\ \dot{z} &= \omega\left[\left(2\frac{\dot{x}_o}{\omega} + 3z_o\right) \sin \omega t + \frac{\dot{z}_o}{\omega} \cos \omega t\right] \end{aligned} \tag{5}$$

Also from reference 1, the components of the LEM transfer velocity vector $\underline{V}_I$ required to intercept the CM in time τ are:

$$\dot{x}_I = \frac{\omega}{\Delta}\left\{-x_o \sin \omega\tau + z_o\left[6\,\omega\tau \sin \omega\tau - 14\,(1 - \cos \omega\tau)\right]\right\} - \dot{x}_o$$

$$\dot{y}_I = -\omega y_o \cot \omega\tau \qquad (6)$$

$$\dot{z}_I = \frac{\omega}{\Delta}\left[2x_o\,(1 - \cos \omega\tau) + z_o\,(3\,\omega\tau \cos \omega\tau - 4 \sin \omega\tau)\right] - \dot{z}_o$$

where

$$\Delta = 8\,(1 - \cos \omega\tau) - 3\,\omega\tau \sin \omega\tau.$$

Due to the simplicity of equations (6), it has often been proposed (reference 1) that these expressions be used for intercept guidance equations. For the present investigation these equations are considered to be adequate since the main purpose is only to establish the important parameters and the trends of the results with variations of these parameters. Also, the correction velocity has been defined by equation (1) as the difference between the velocity of the LEM at some time t (equations (5)) and the transfer velocity required at that time to intercept the CM in some time τ (equations (6)); thus, since the same assumptions are made in each calculation and the calculations are differenced, the errors in these velocity calculations will tend to cancel, thereby giving good results for the correction velocity. A further investigation of these errors which is beyond the scope of the present study is, however, deemed desirable.

SCOPE OF CALCULATIONS

The method of analyzing midcourse corrections presented in the preceding section is applied to that phase of the Apollo Mission concerned with LEM rendezvous with the CM in lunar orbit. Two types of LEM transfer trajectories are investigated; namely, the 180° Hohmann transfer for the normal in-plane launch and a 220° transfer for operating at the maximum out-of-plane LEM design limit of 2°, see figure 3. The CM is assumed to be in a circular orbit at 80 nautical miles altitude. The pericynthion altitude of the LEM transfer orbits is assumed to be 50,000 feet. The transfer time for the in-plane transfer is 3484 sec and for the out-of-plane transfer is 4340 sec. The terminal velocity with no errors, V_{to}, is 97 fps and 372 fps for the in-plane and out-of-plane transfers,

respectively.

As stated in the section on "Midcourse Correction Technique" a constant error velocity is added at initiation of the intercept transfer and application of each correction. The magnitude of the error velocity used in the present investigation is 8.6 fps. This value represents an average between higher injection errors and smaller midcourse errors. The direction cosines defining the directions studied are shown in the table following.

VELOCITY ERROR DIRECTION COSINES

Case	x	y	z
1	1	0	0
2	0	1	0
3	0	0	1
4	.577	.577	.577
5	.707	0	.707

For each of these error directions, guidance to intercept is attempted with 2, 3, and 4 corrections. Three values for the time of the final correction, 5, 10, and 15 minutes from intercept, are investigated. All other corrections are made after the LEM has traversed a specified constant fraction of remaining transfer time. The range studied for this fraction of remaining transfer time is 0.1 to 0.9.

RESULTS AND DISCUSSION

The results and discussion are given in two parts. The first part is concerned with the 180° in-plane transfer while the second is concerned with the 220° out-of-plane transfer.

180° Transfer

The variation of the guidance velocity requirements for the in-plane transfer with the correction time is presented in figure 4 for all five

directions of the error velocity. The effect of each parameter is shown in this figure and is discussed briefly in the following subsections:

Effect of time of final correction.- As stated earlier, the final correction time is specified independently of prior correction times. As shown in figure 4, the guidance velocity is decreased by applying the final correction as early as possible. This is because the velocity required to correct a given error in a given transfer time increases as an inverse function of the range between the two spacecraft. Also, from figure 4 it can be seen that correction schedules for minimum guidance velocity exist and that this schedule is a function of the final correction time. Hereafter the correction schedule associated with minimum guidance velocity will be referred to as optimum. In general, executing the final correction earlier decreases the fraction of remaining transfer time which means each of the other corrections are executed earlier. It should also be pointed out that the time of the final correction has a direct relationship to the miss distance at desired intercept time; that is, the earlier the time of the last correction, the greater is the miss distance, (see figure 5). Thus, a compromise on the time of the final correction must be made in order to keep guidance velocity and the miss distance within reasonable bounds.

Effect of number of corrections.- The number of corrections greatly affects the guidance velocity requirements as well as the optimum correction schedule, see figure 4. For all cases except that for which the error velocity is entirely out-of-plane ($\bar{\sigma} = 8.6\ \bar{j}$) the guidance velocity decreases as the number of corrections is increased to 3 or 4. However, the number of corrections was varied sufficiently to establish that increasing the number of corrections beyond 4 yields little or no further decrease in ΔV_g. Also, increasing the number of corrections decreases the time of the optimum correction schedule. For the case of out-of-plane velocity error only, it can be seen that the number of corrections has little effect on ΔV_g or correction time. However, for this case, a slight increase in ΔV_g is noted as the number of corrections is increased.

Effect of error velocity direction.- It is evident from figure 4 that the direction of the velocity error has a significant effect on the guidance velocity requirements. For clarity, sample results are cross-plotted in figure 6 which illustrate the effect of the error velocity direction. The most severe velocity requirements result from velocity errors in the xz-plane, ($\bar{\sigma} = 6.08\ \bar{i} + 6.08\ \bar{k}$) whereas the out-of-plane errors ($\bar{\sigma} = 8.6\ \bar{j}$) produced the lowest requirements. In general, the error velocity direction has little effect on the optimum correction schedule.

220° Transfer

The variation of the guidance velocity requirements for the out-of-plane transfer with the correction time is presented in figure 7 for all five directions of the error velocity. These results are in general agreement with those for the in-plane case. One exception is the results for the out-of-plane velocity error ($\overline{\sigma} = 8.6\ \overline{j}$). For the 220° transfer the results of figure 7 indicate the out-of-plane velocity errors should be corrected as soon as possible, while the 180° transfer results (figure 4) showed little variation with correction time for the out-of-plane error (slight decrease in ΔV_g as correction time increased). However, the out-of-plane velocity error, as in the 180° transfer, requires the least amount of ΔV_g of all the directions considered, see figure 8. Also, the errors in the xz-plane still require the largest ΔV_g. Finally, the miss distances for the 220° transfer are shown in figure 9 to be nearly equal to those occurring in the 180° transfer. In general, the optimum correction schedule for the 220°, out-of-plane transfer is the same as that for the 180°, in-plane transfer.

COMPARISON WITH EXACT TWO-BODY SOLUTION

An example of the accuracy of the linearized equations of motion is shown in figure 10. The solid lines in this figure were obtained using the more complicated but exact two-body equations. Results are shown for 2, 3, and 4 corrections in each case with the final correction being made 5 minutes from intercept. It is evident from this figure that the linearized equations give good agreement for predicting an optimum correction schedule and that the guidance velocity prediction agrees to an accuracy of about 10 percent or less.

CONCLUDING REMARKS

An investigation of LEM midcourse guidance procedures for rendezvous with the CM in lunar orbit is reported. This investigation is based on constant velocity errors and linearized equations of motion in order to simplify the analysis and at the same time establish trends of the important parameters. A comparison of the results of this investigation with results obtained from exact two-body equations indicates this method is valid for establishing these trends.

The optimum correction schedule was found to depend on the number of corrections and the time of the final correction. The two types of LEM transfer trajectories investigated, 180° in-plane and 220° out-of-plane transfers, were found to yield nearly the same optimum midcourse correction schedule; namely, 3 or 4 corrections with the final correction scheduled 10 to 15 minutes before the end of the transfer and all prior corrections scheduled at 0.20 to 0.40 of the remaining transfer time.

The direction of the error velocity was found to have a large effect on the magnitude of the guidance velocity and miss distance. The in-plane velocity errors produce the largest guidance velocity and miss distance, while the out-of-plane errors yield the smallest correction and miss distance. Miss distance is, as expected, primarily a function of the time of the final correction -- the earlier the final correction, the larger the miss distance.

REFERENCE

1. Robert C. Stapleford, "A Study of the Two Basic Approximations in the Impulsive Guidance Techniques for Orbital Rendezvous", Systems Technology Incorporated Tech. Rept. No. ASD-TDR-62-83.

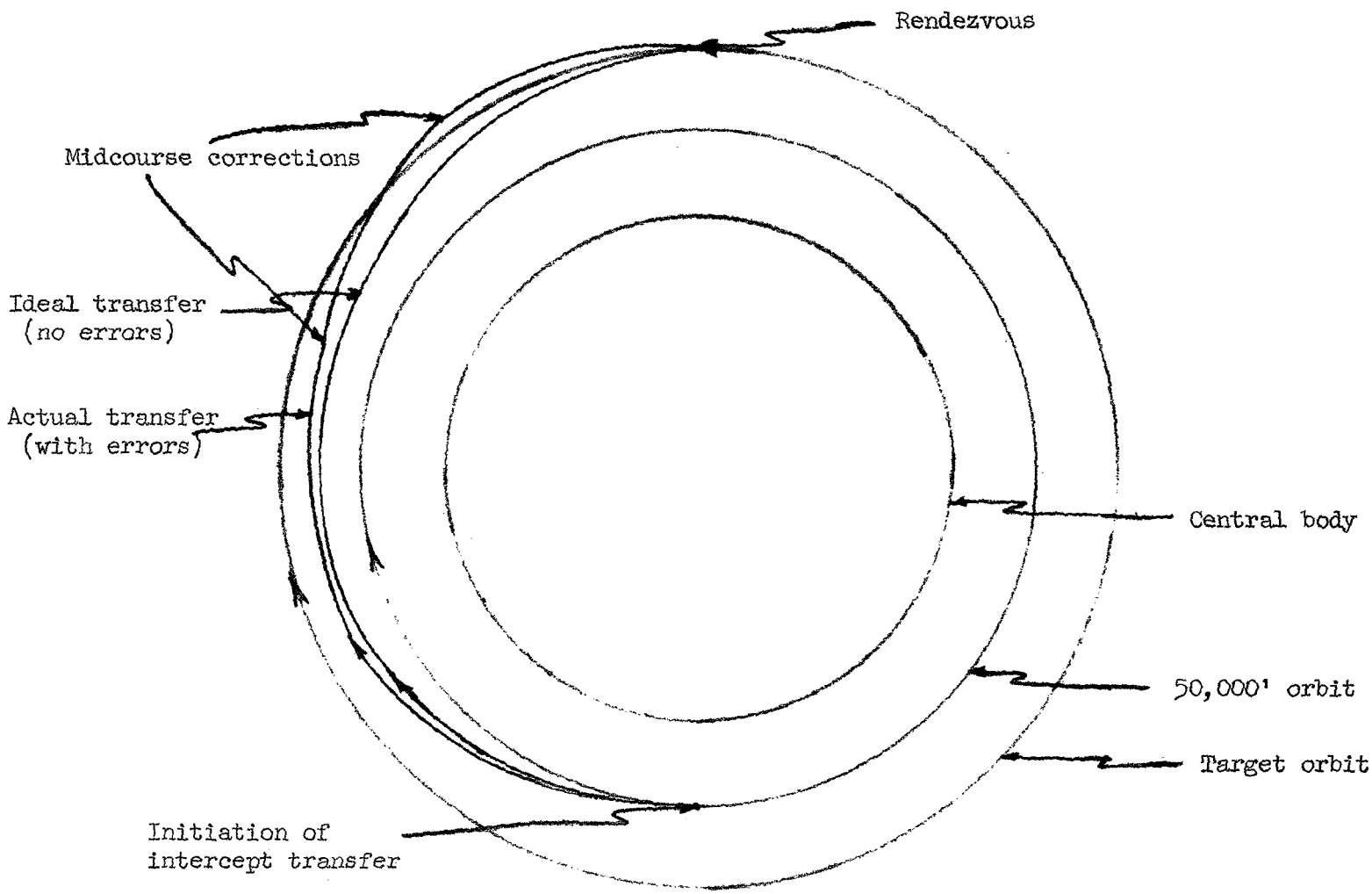


Figure 1.- Sketch of midcourse corrections procedure

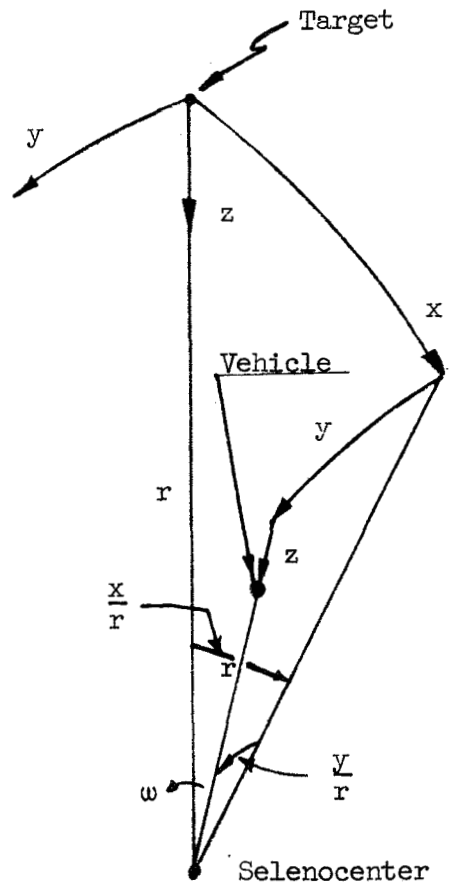
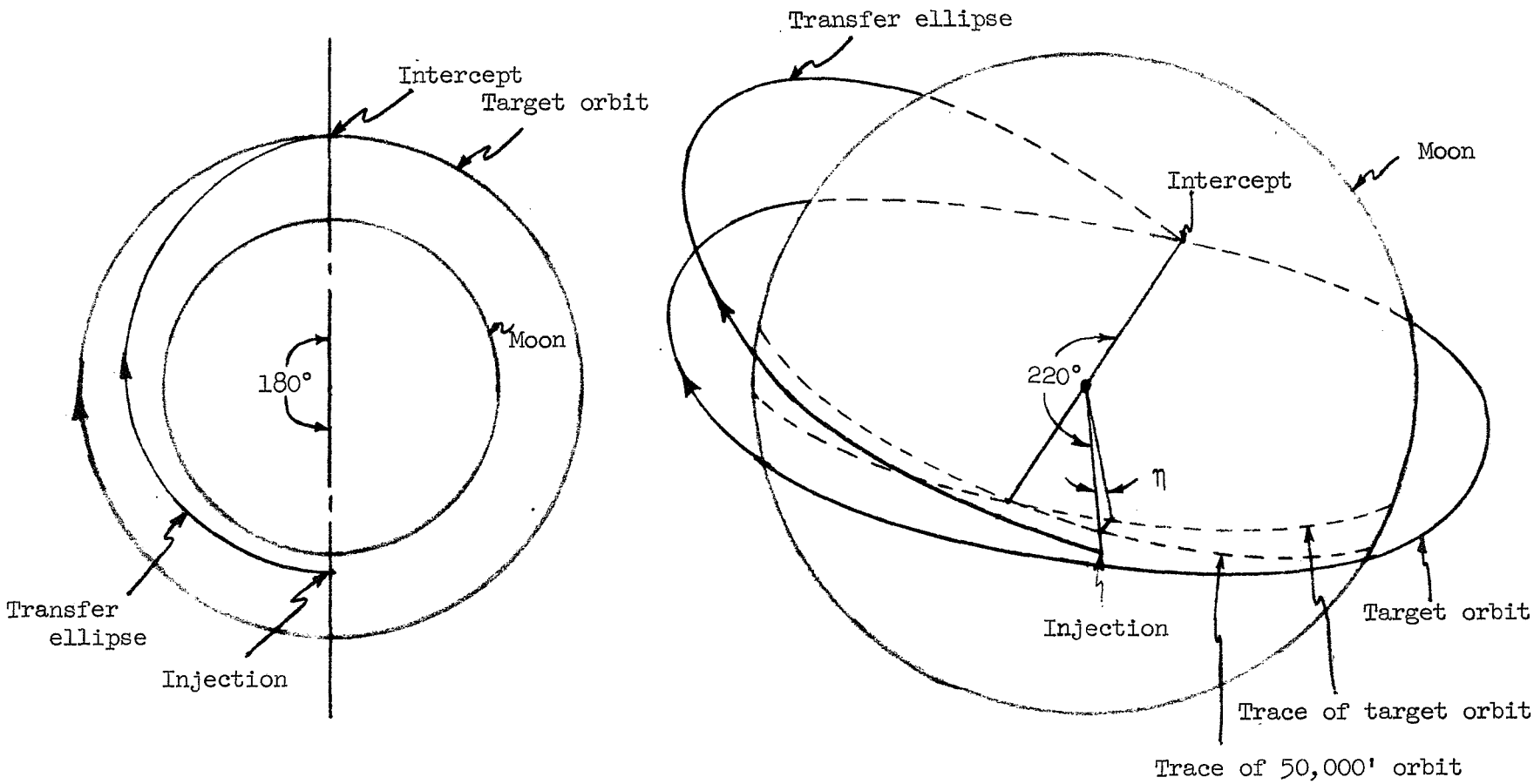


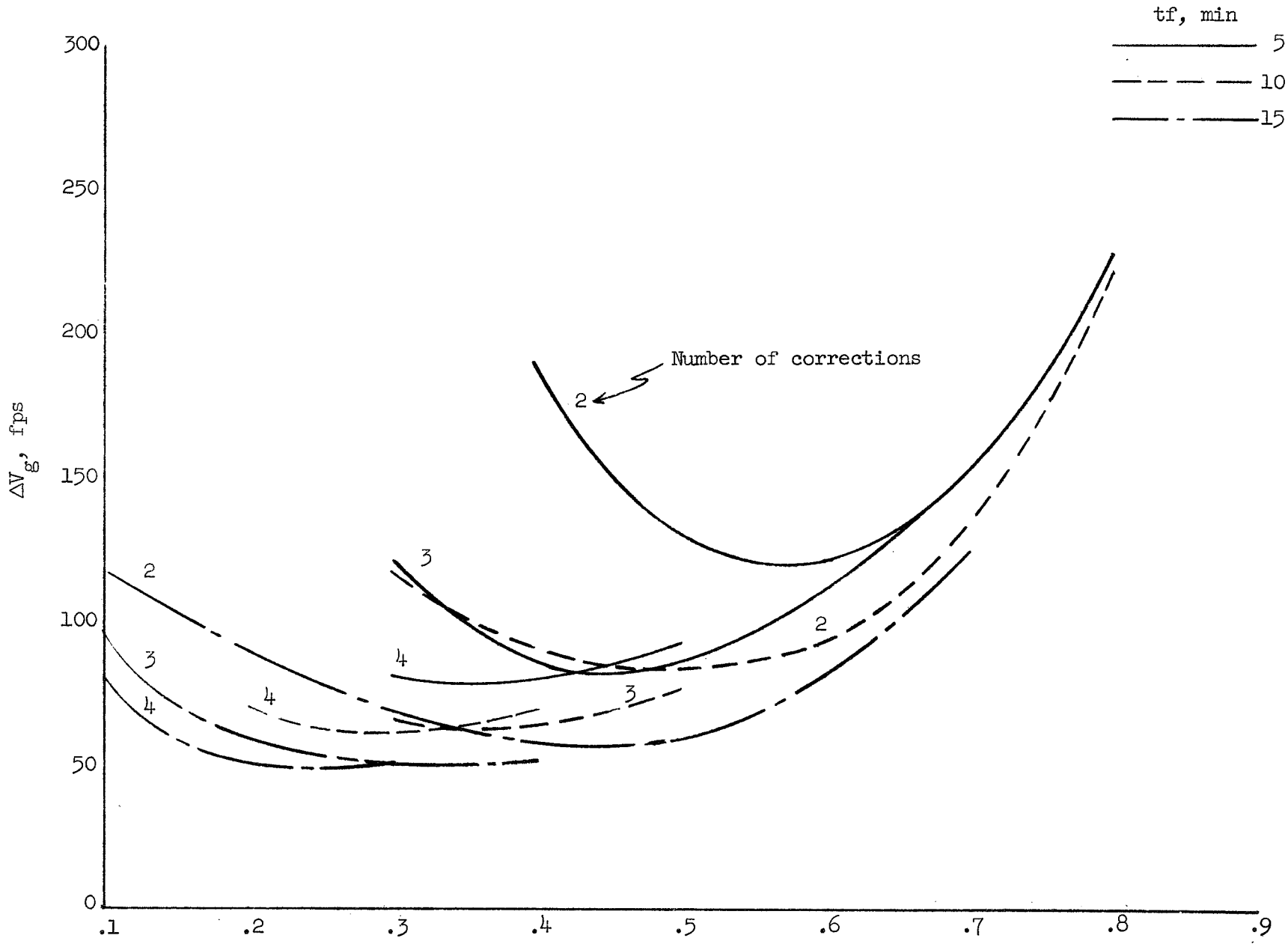
Figure 2.- Target centered rotating spherical coordinate system



(a) Hohmann (in-plane)

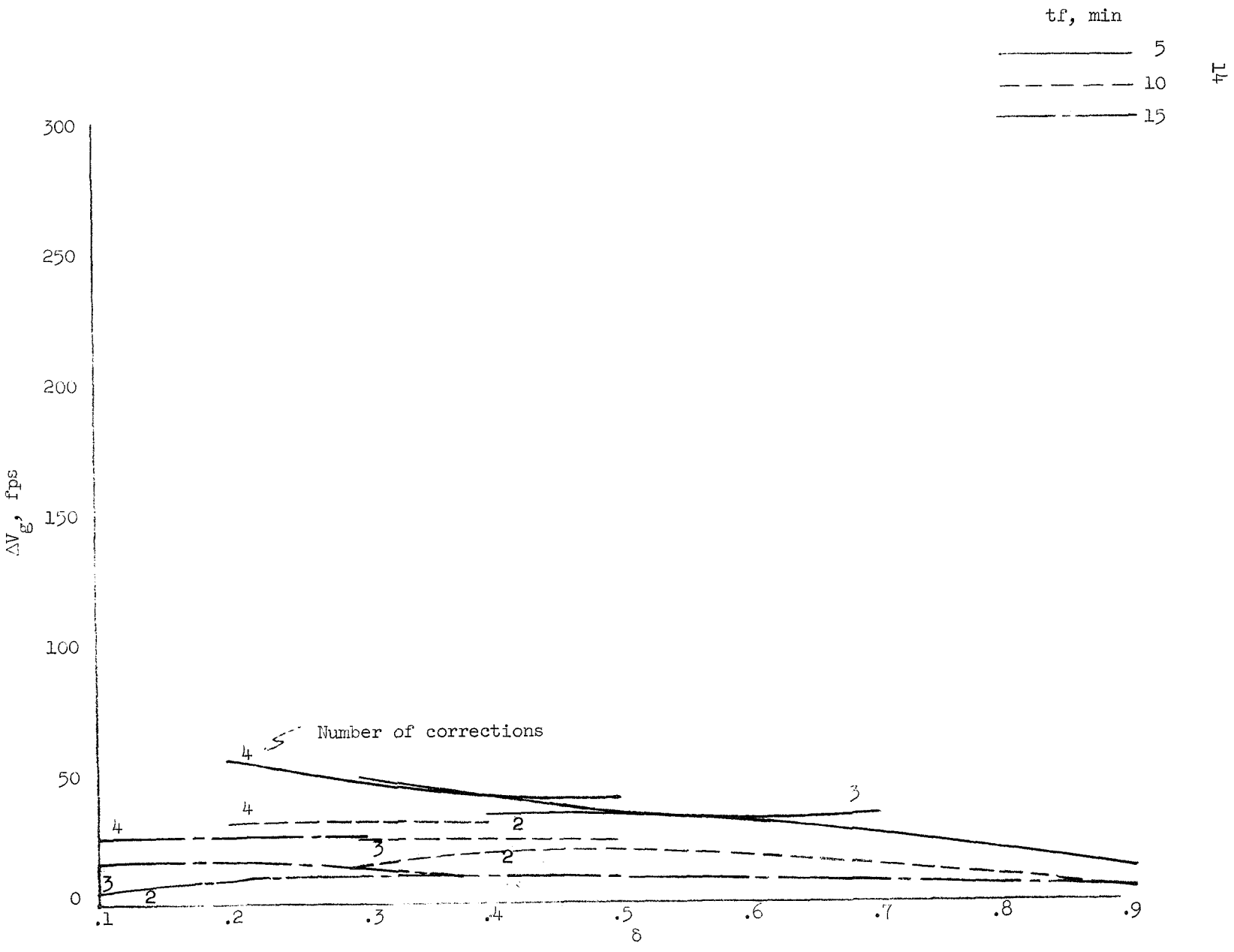
(b) 220° transfer (out-of-plane)

Figure 3.- Types of LEM transfer orbits investigated



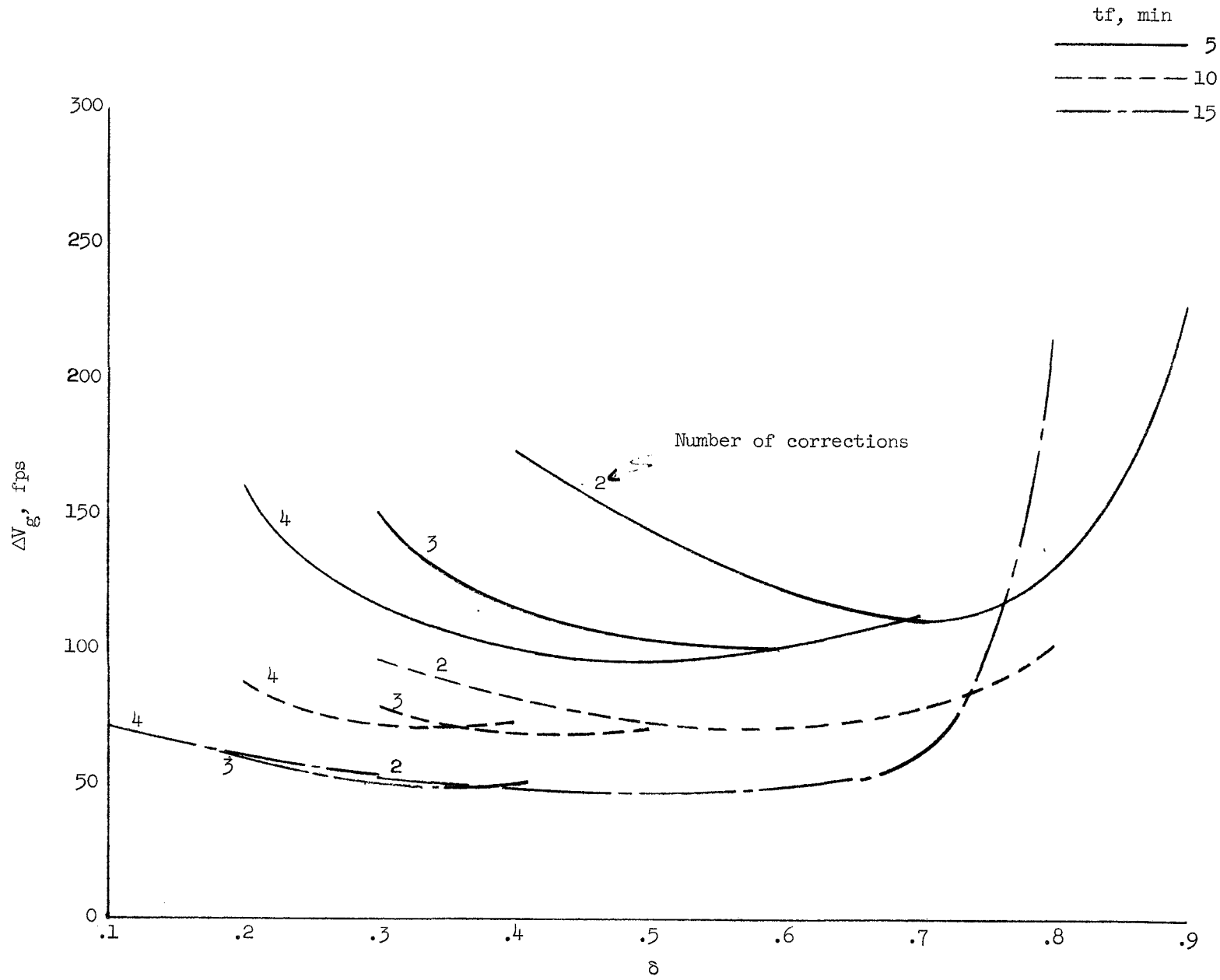
(a) Case 1 $\bar{\sigma} = 8.6\ \bar{i}$

Figure 4.- Variation of guidance velocity with correction schedule time for 180° transfer



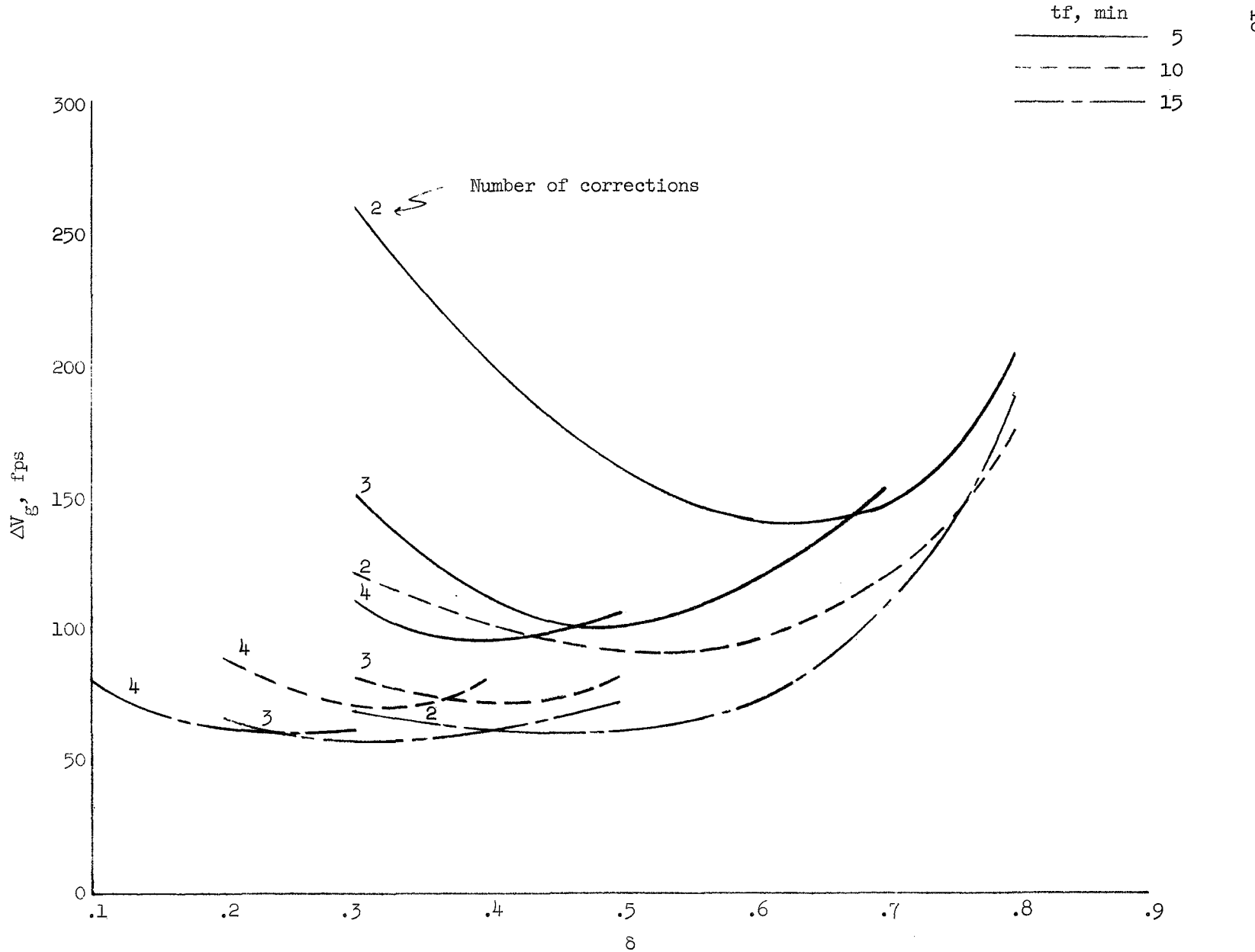
(b) Case 2 $\bar{\sigma} = 8.6\ \bar{j}$

Figure 4.- Continued



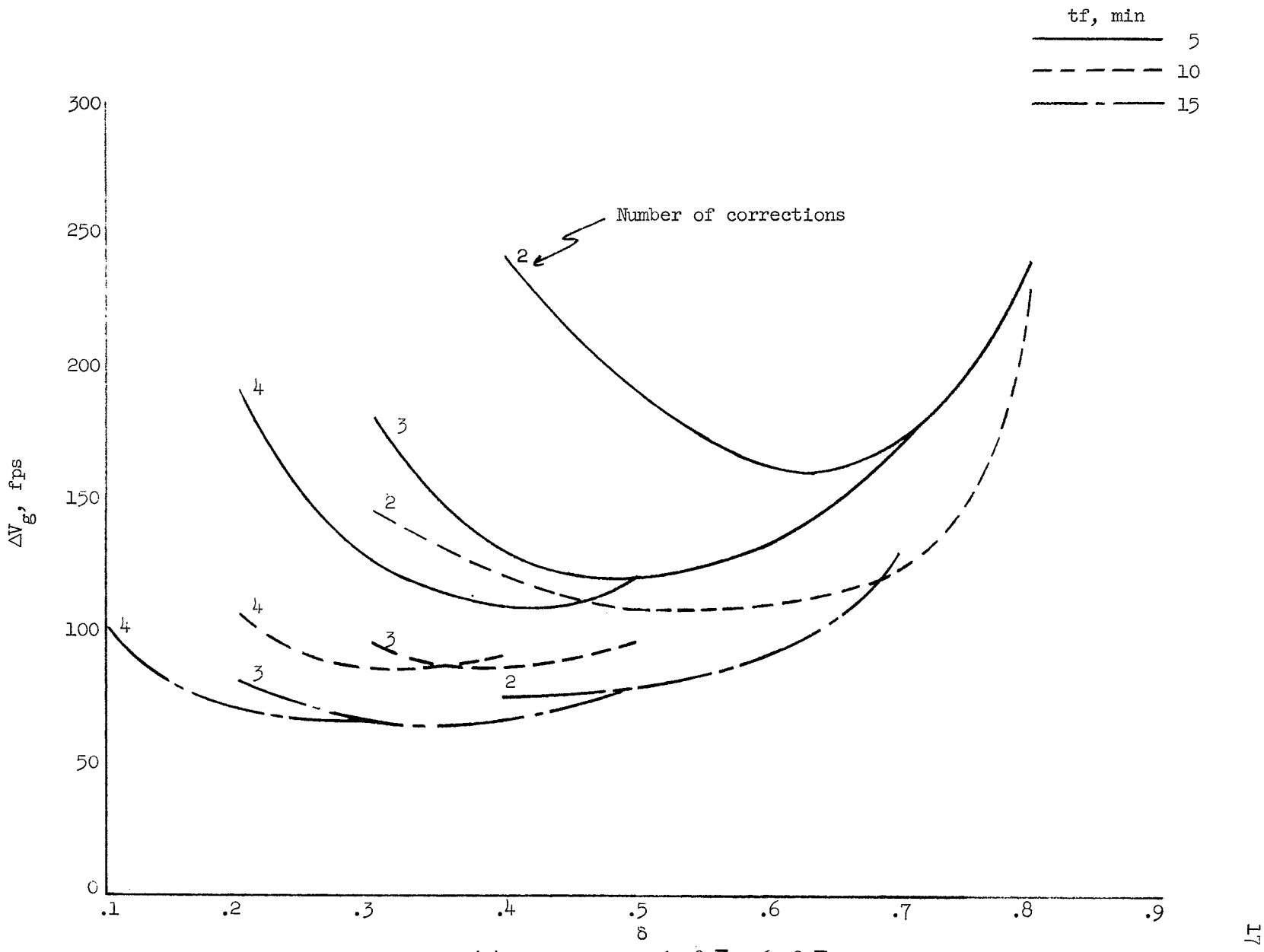
(c) Case 3 $\bar{\sigma} = 8.6\,\bar{k}$

Figure 4.- Continued



(d) Case 4 $\bar{\sigma} = 5.0\,\bar{i} + 5.0\,\bar{j} + 5.0\,\bar{k}$

Figure 4.- Continued



(e) Case 5 $\bar{\sigma} = 6.08\,\bar{i} + 6.08\,\bar{k}$

Figure 4.- Concluded

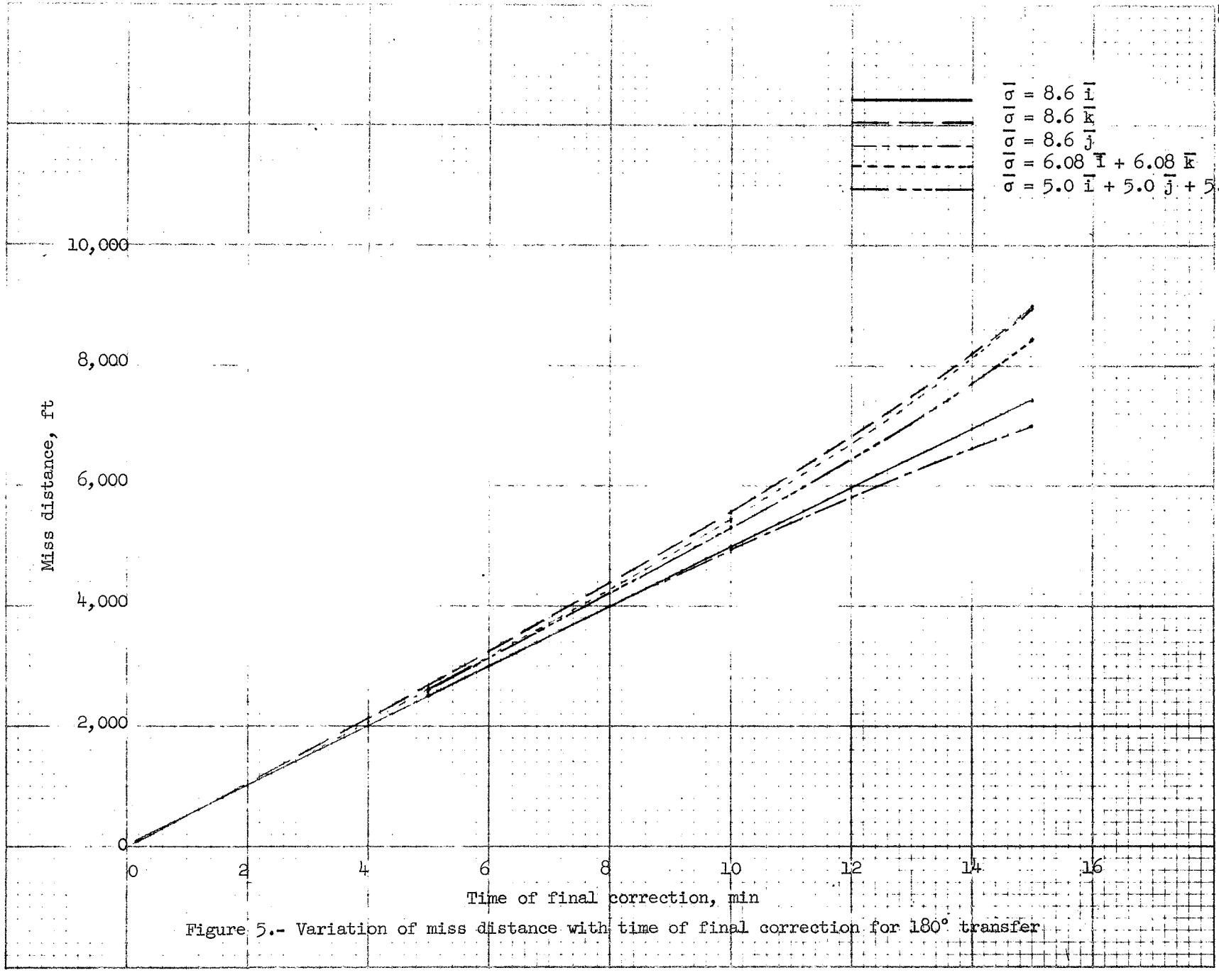


Figure 5.- Variation of miss distance with time of final correction for 180° transfer

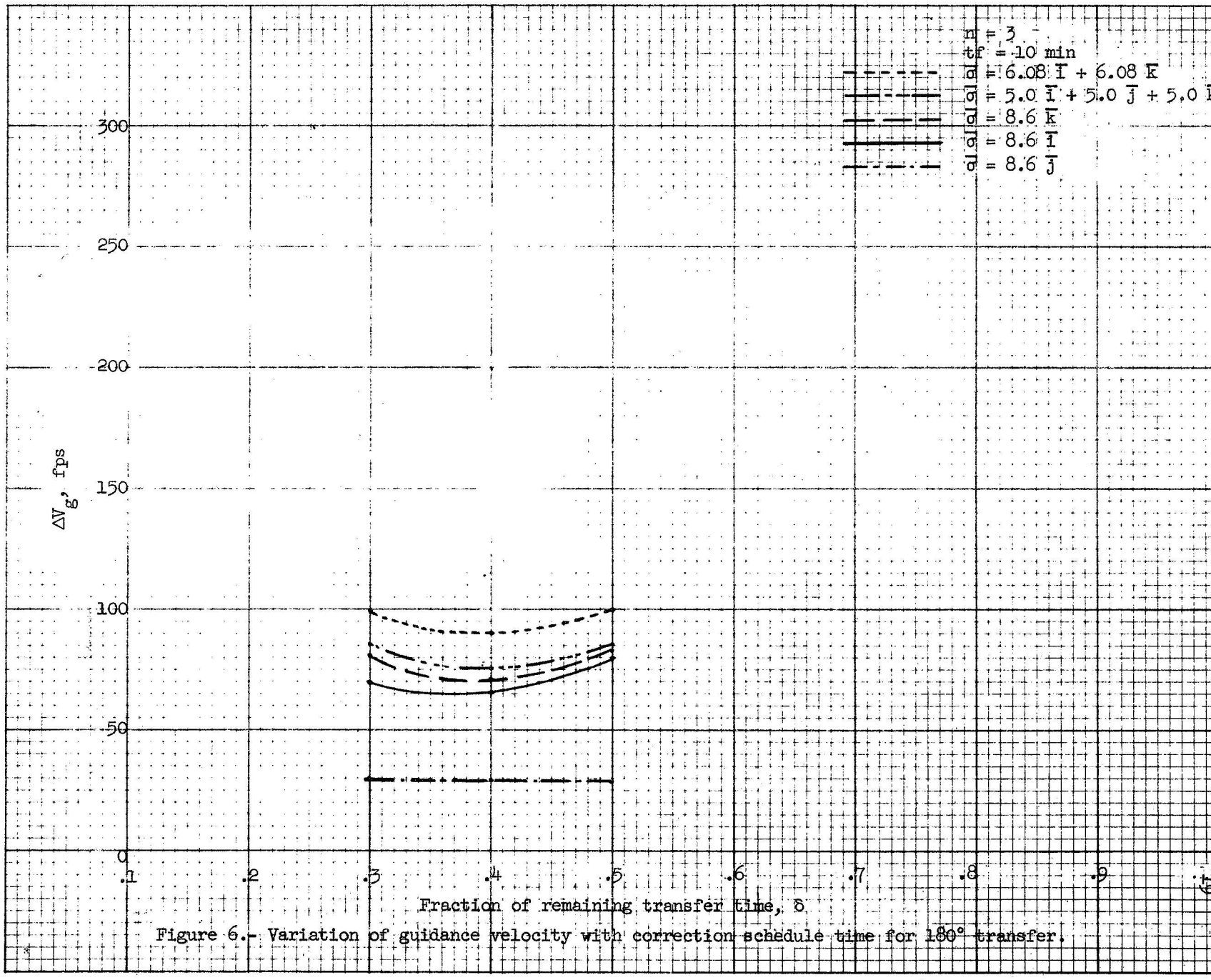
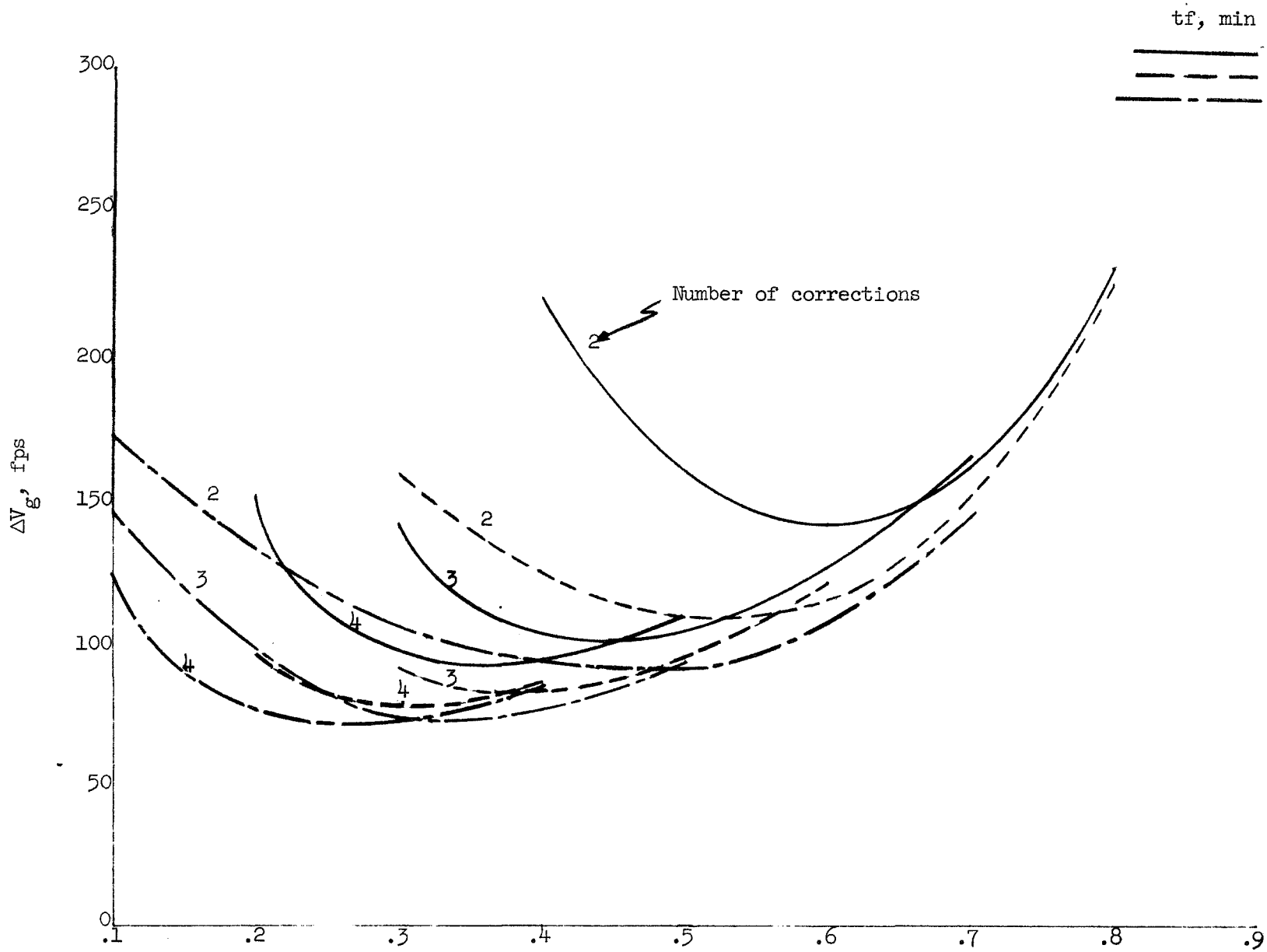


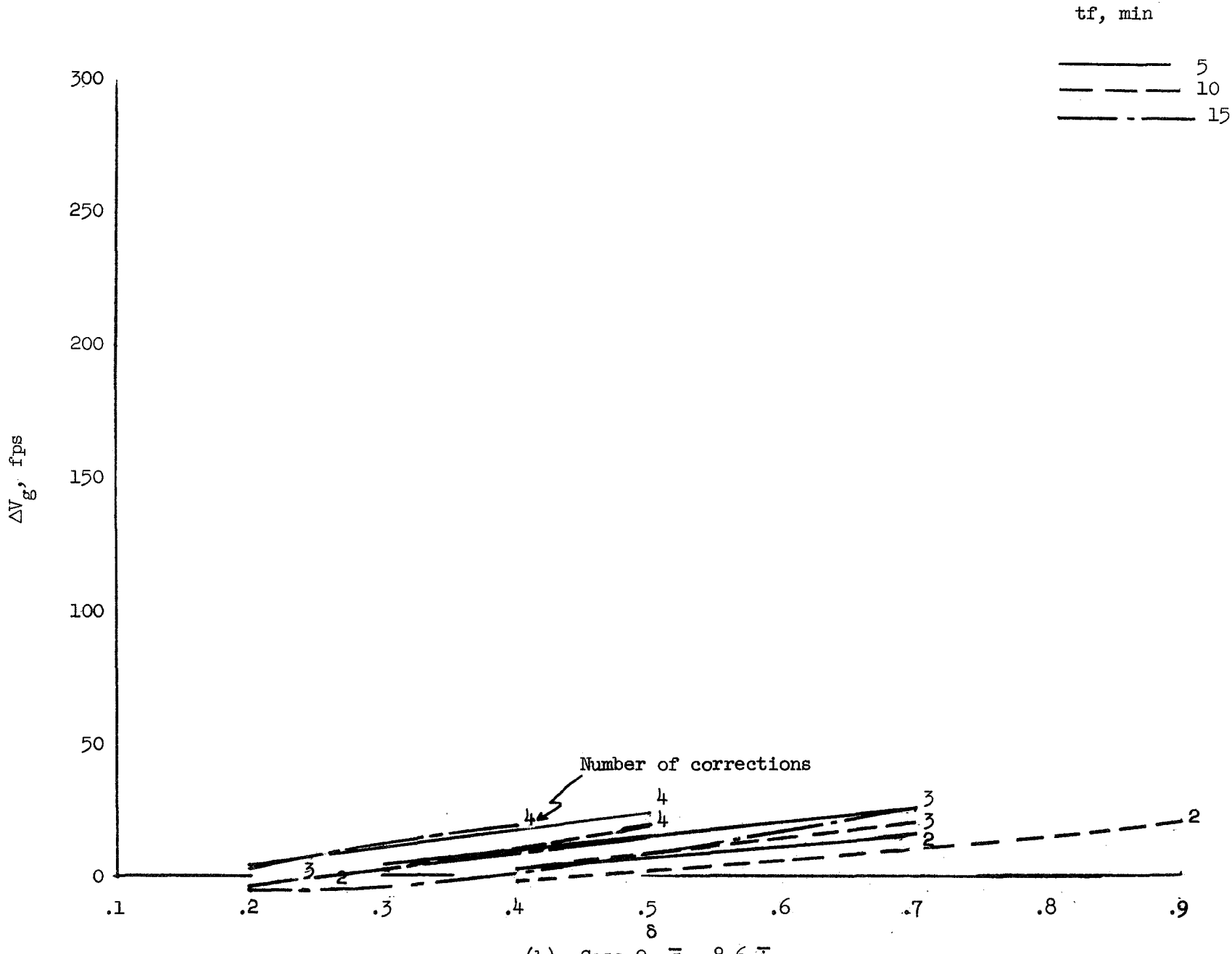
Figure 6.- Variation of guidance velocity with correction schedule time for 180° transfer.

BEE 10x10



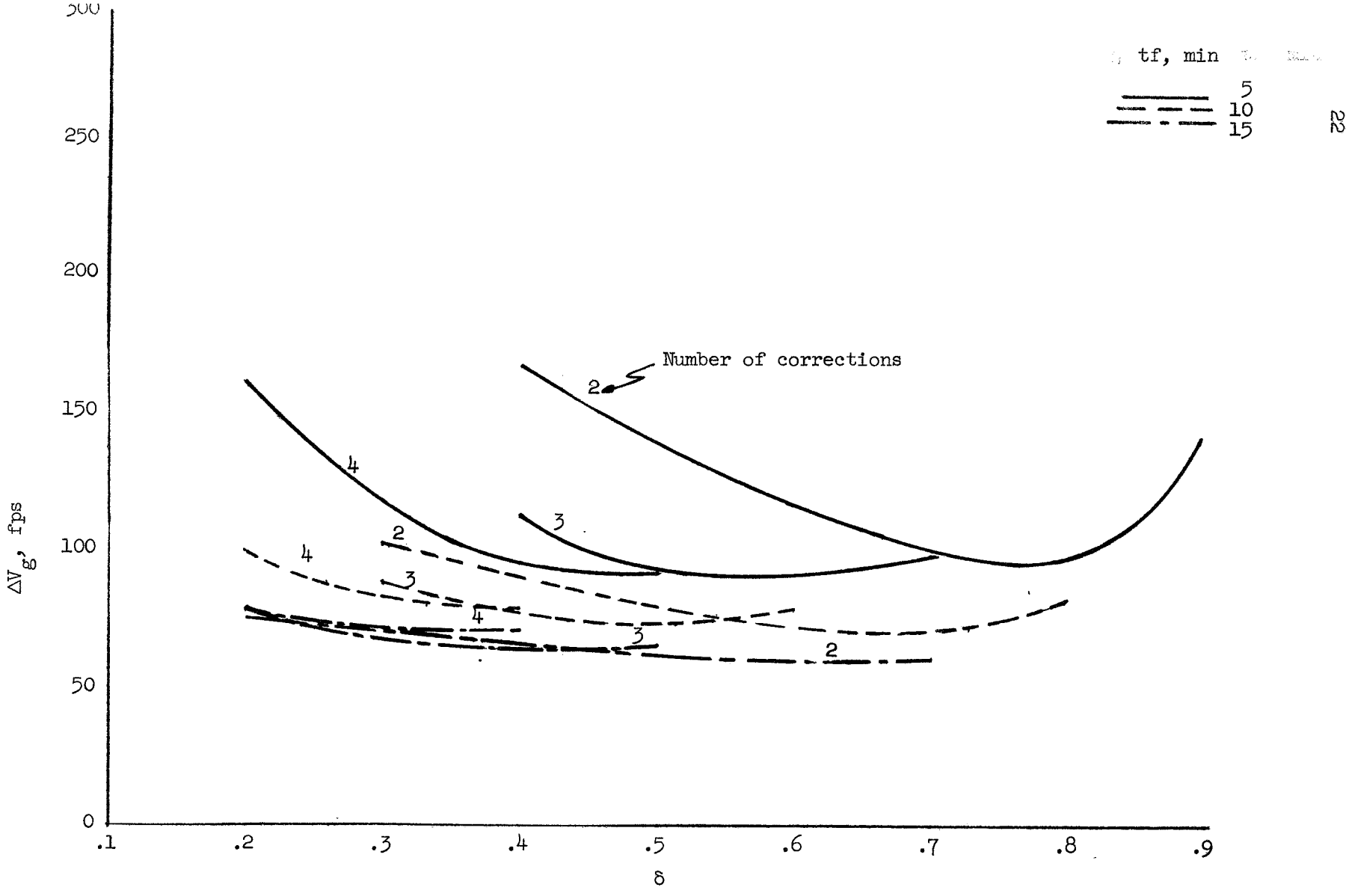
(a) Case 1 $\bar{\sigma} = 8.6\ \bar{I}$

Figure 7.- Variation of guidance velocity with correction schedule time for 220° transfer



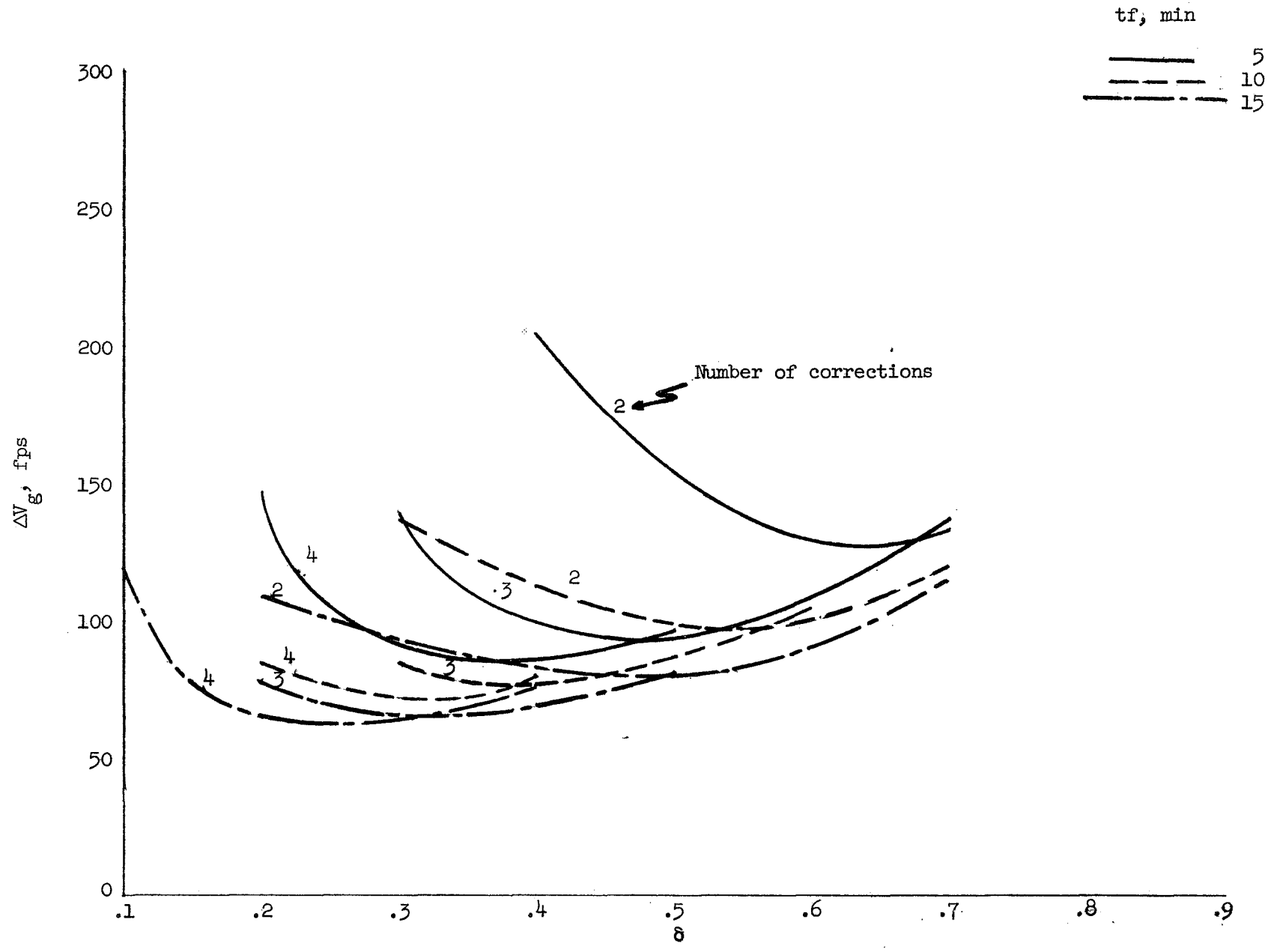
(b) Case 2 $\bar{\sigma} = 8.6\ \bar{j}$

Figure 7.- Continued



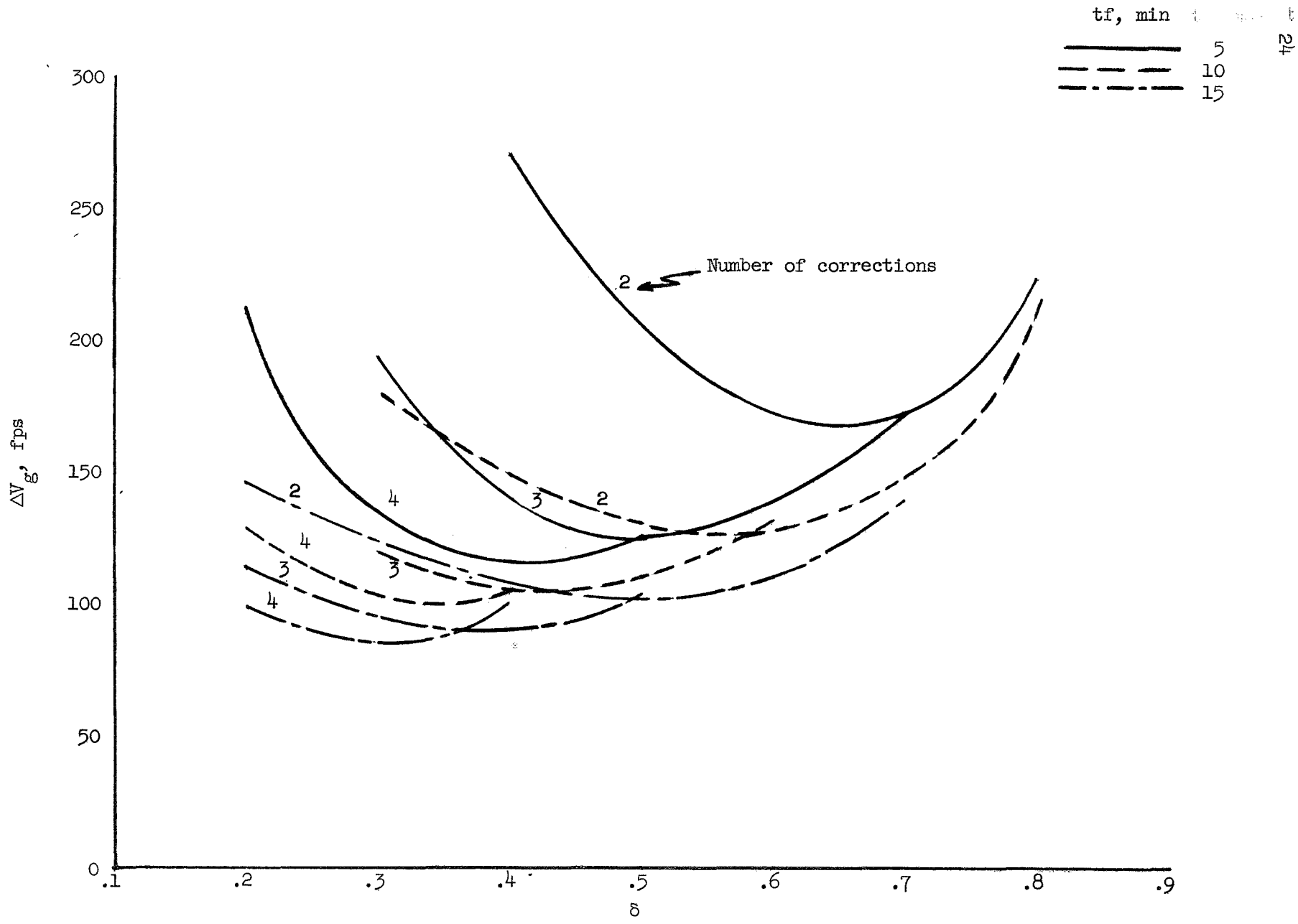
(c) Case 3 $\bar{\sigma} = 8.6\ \bar{k}$

Figure 7.- Continued



(d) Case 4 $\bar{\sigma} = 5.0\,\bar{I} + 5.0\,\bar{J} + 5.0\,\bar{K}$

Figure 7.- Continued



(e) Case 5 $\bar{\sigma} = 6.08\,\bar{I} + 6.08\,\bar{k}$

Figure 7.- Concluded

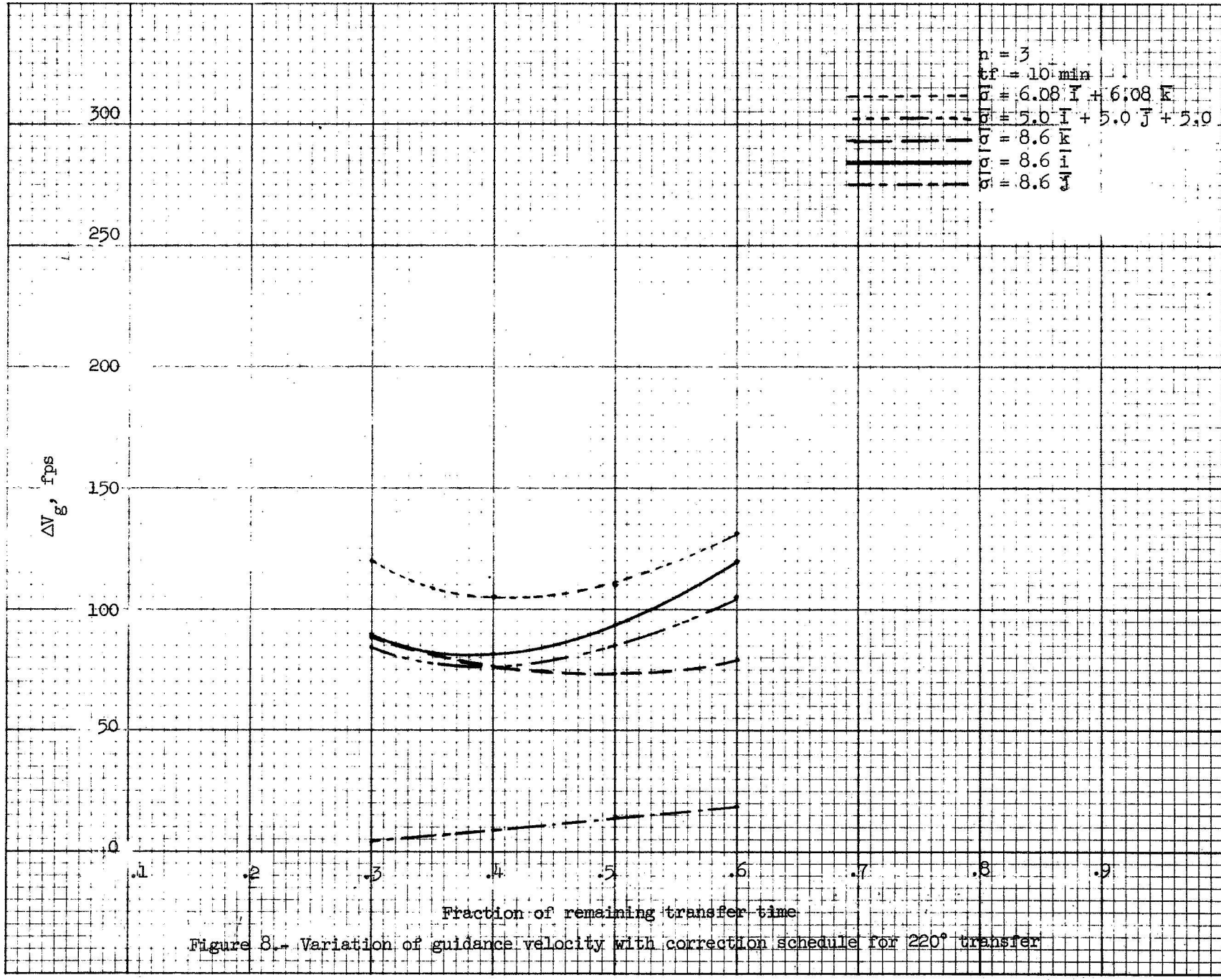


Figure 8.- Variation of guidance velocity with correction schedule for 220° transfer

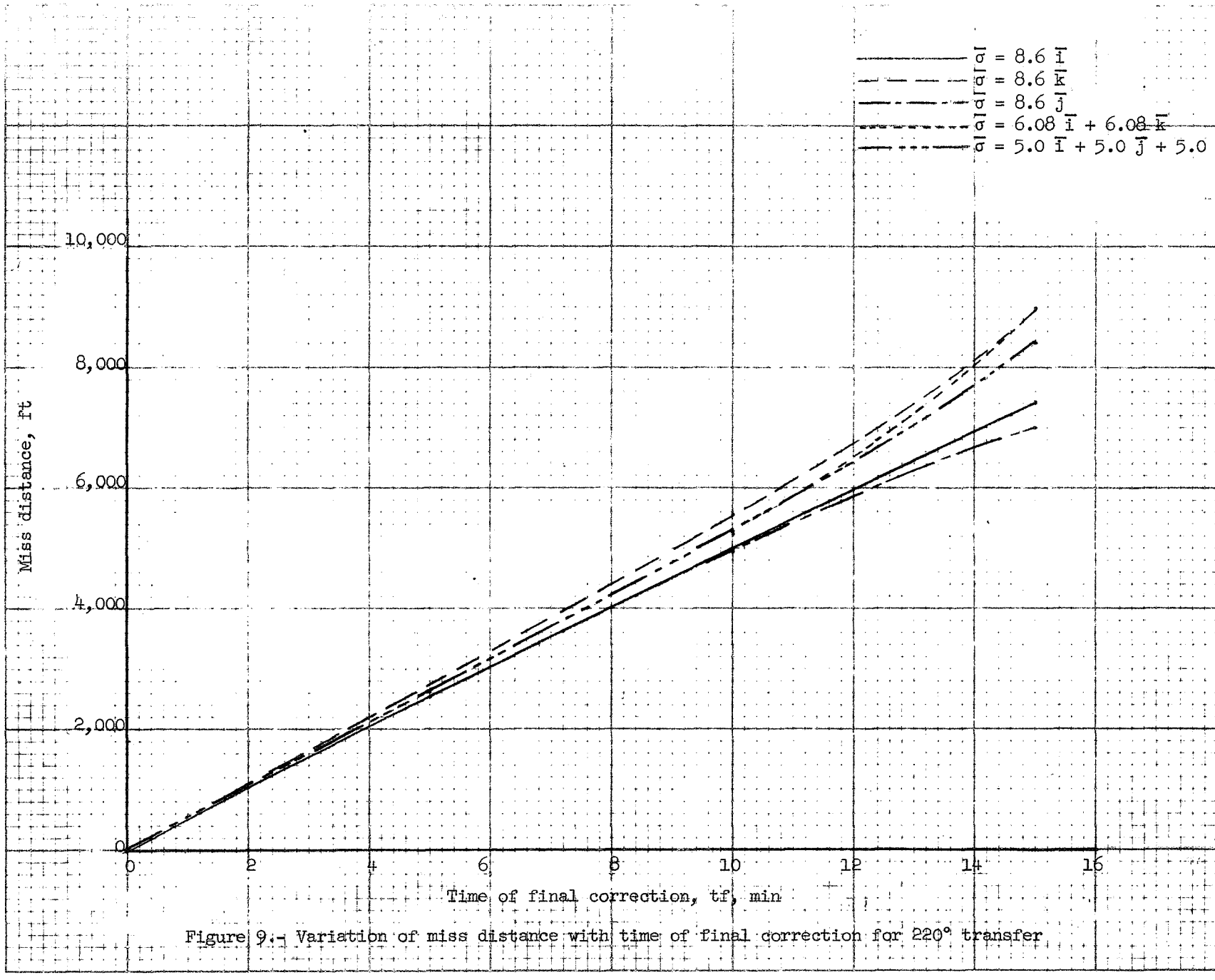


Figure 9.- Variation of miss distance with time of final correction for 220° transfer

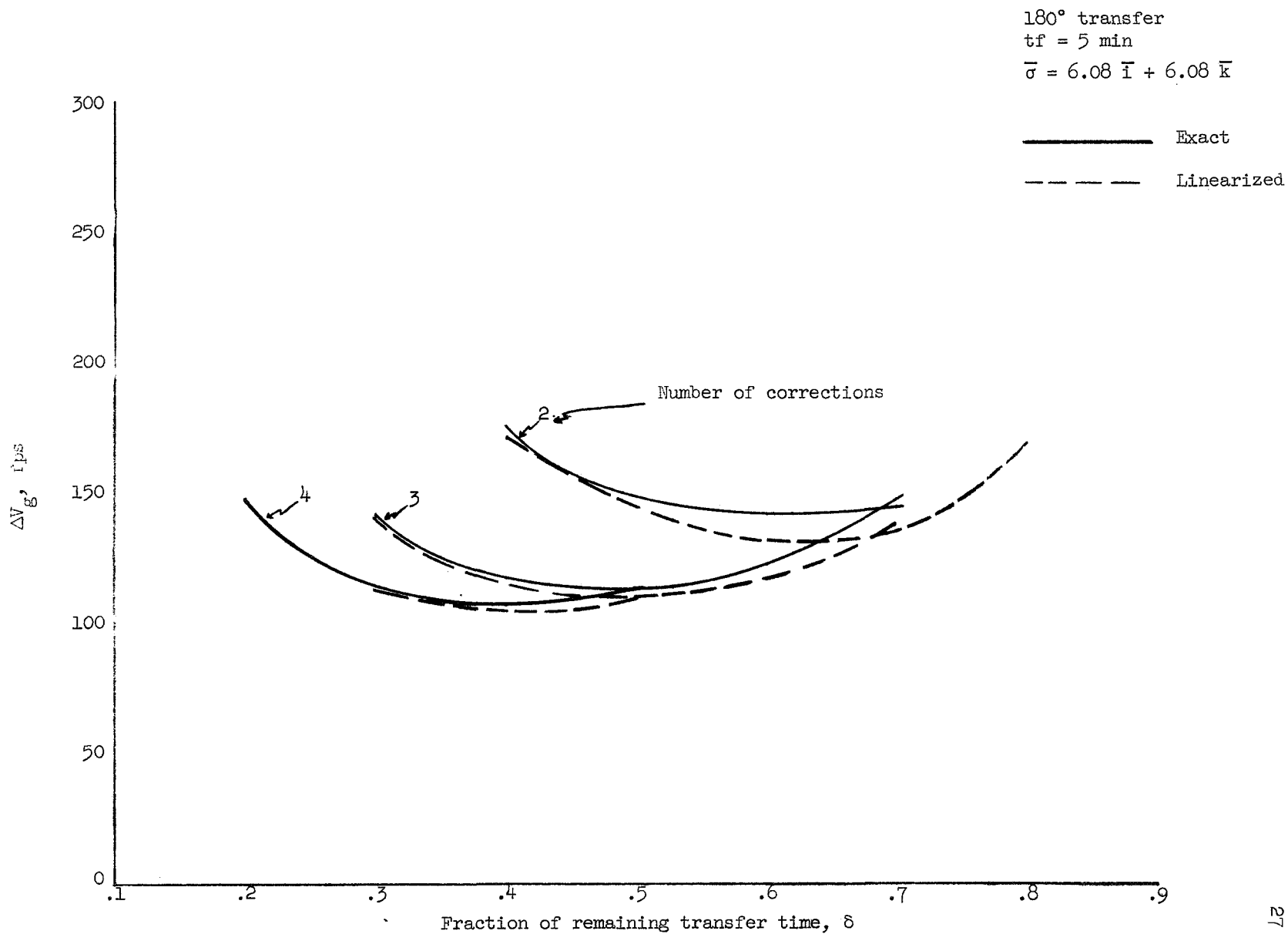


Figure 10.- Comparison of the results obtained from the linearized equations of motion with those of the exact equations