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RELATIVISTIC TIME DILATION ON LUNAR FLIGHTS

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16. Abstract

This report describes briefly the theory of relativistic time dilation. The theory is applied to the Apollo 8 flight to the moon and return. It is found that the astronauts aged some 335 μ-sec relative to an earth-being at Cape Kennedy.

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RELATIVISTIC TIME DILATION ON LUNAR FLIGHTS

A gravitational field in space is described in a system of coordinates, x^1, x^2, x^3, x^4, where $x^4 = ct$, and t represents coordinate time (c is the speed of light), by the relationship

$$ds^2 = g_{ij}\, dx^i\, dx^j \quad , \tag{1}$$

where i and j are summed from 1 to 4 by use of Einstein's summation convention. The relationship holds between each locally measured interval, ds, and the corresponding coordinate difference, dx^i. The quantities, g_{ij}, define the metric or physical properties of space. For events occurring at a single point, the locally measured interval is given by

$$ds = c\, d\tau \quad , \tag{2}$$

where $d\tau$ denotes the proper time at that point.

For a spherically symmetric stationary distribution of mass about the origin, the g_{ij} are given by the Schwarzschild solution to the field equations of general relativity:[1]

$$g_{44} = 1 + 2K \quad , \tag{3}$$

$$g_{4\mu} = g_{\mu 4} = 0 \quad , \tag{4}$$

and

$$g_{\mu\nu} = g_{\nu\mu} = -\delta_{\mu\nu} + \frac{2K}{1 + 2K}\left(\frac{x^\mu x^\nu}{r^2}\right) \quad , \tag{5}$$

where μ, $\nu = 1, 2, 3$ and $\delta_{\mu\nu} = 1$ $(\mu = \nu)$; $\delta_{\mu\nu} = 0$ $(\mu \neq \nu)$.

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1. These values for the g_{ij} are for a radius greater than or equal to the radius of the spherical distribution of mass.

The coordinates $x^{1,2,3}$ are Cartesian and

$$\bar{r} = x^1 \bar{i} + x^2 \bar{j} + x^3 \bar{k} \; , \; r = \left[(x^1)^2 + (x^2)^2 + (x^3)^2 \right]^{\frac{1}{2}} \tag{6}$$

$$K = -\frac{GM}{c^2 r} \quad . \tag{7}$$

The factor G is the gravitational constant and M is the mass of the spherically symmetric mass distribution. The normalized velocity of a point moving with respect to the origin is defined as

$$\beta = \frac{v}{c} = \frac{1}{c} \left[\left(\frac{dx^1}{dt} \right)^2 + \left(\frac{dx^2}{dt} \right)^2 + \left(\frac{dx^3}{dt} \right)^2 \right]^{\frac{1}{2}} \quad . \tag{8}$$

By substitution of equation (2), the g_{ij} , and equation (8) into equation (1), and neglecting terms higher in order than K and β^2 yields

$$\frac{d\tau}{dt} = (1 + 2K - \beta^2)^{\frac{1}{2}} \tag{9}$$

for the relationship between proper time and coordinate time for a point moving with normalized velocity β in a Schwarzschild field.

Within the realm of our forthcoming considerations, the earth is sufficiently spherical to be considered to yield a Schwarzschild field. A spaceship going to the moon would be governed in its flight trajectory predominantly by the earth's Schwarzschild field. However, as the spaceship gets closer to the moon, the Schwarzschild field is pertrubed by the gravitational field of an assumed spherically symmetric moon. Hence, if $\bar{r}_m$ is the position of the moon and m is the moon's mass, then define

$$\bar{r}' = \bar{r}_m - \bar{r} \quad , \quad r' = |\bar{r}'| \quad , \tag{10}$$

and the moon's normalized gravitational potential is given by

$$\Psi = -\frac{GM}{c^2 r'} \quad . \tag{11}$$

The effects of earth oblateness, the sun's potential, solar pressure, etc., will be taken to be negligible compared to the effects of a zero-order earth and a zero-order moon. In view of the effect in equation (11), we will replace the factor K in equation (9) by $K + \Psi$, i.e.,

$$K \rightarrow K + \Psi \quad . \tag{12}$$

The subscript e will be used to designate the relationship between proper time and the coordinate time of a point on the earth's surface corresponding to the latitude of Cape Kennedy. The subscript s will be used to designate the relationship between proper time and coordinate time of a space traveler moving in the earth-moon field. We have

$$\left(\frac{d\tau}{dt}\right)_e = \left(1 + 2\,K_e - \beta^2{}_e\right)^{\frac{1}{2}} \quad , \tag{13}$$

$$\left(\frac{d\tau}{dt}\right)_s = \left(1 + 2K_s - \beta^2{}_s\right)^{\frac{1}{2}} \quad . \tag{14}$$

The proper time difference between the separation and return of a space traveler journeying to the moon and back can be defined by

$$\Delta\tau = \int_0^T \left[\left(\frac{d\tau}{dt}\right)_e - \left(\frac{d\tau}{dt}\right)_s\right] dt \tag{15}$$

where T is the total time of separation. An expansion through the first order of equations (13) and (14) and substitution into equation (15) yields

$$\Delta\tau = \int_0^T \left[\frac{1}{2}\left(2K_e - \beta^2{}_e\right) - \frac{1}{2}\left(2K_s - \beta^2{}_s\right)\right] dt \tag{16}$$

or

$$\frac{d(\Delta\tau)}{dt} = \frac{1}{2}\left(2K_e - \beta^2{}_e\right) - \frac{1}{2}\left(2K_s - \beta^2{}_s\right) \quad . \tag{17}$$

The following terms are defined:

$G = 6.67 \times 10^{-11}\ m^3/(kg\text{-}sec^2)$, universal gravitational constant

$r_o = 6.37 \times 10^6$ m, average earth radius

$M = 5.983 \times 10^{24}$ kg, earth's mass

$c = 3.00 \times 10^8$ m/sec, velocity of light

$\omega = 7.27 \times 10^{-5}$ rad/sec, earth's rate of rotation

$m = 7.347 \times 10^{22}$ kg, moon's mass.

The latitude of Cape Kennedy is about 27° above the equator and the speed of Cape Kennedy in a space-fixed frame of reference is approximately $(\omega r_o) \cos(27°)$. So we calculate

$$\frac{1}{2}\left(2K_e - \beta^2{}_e\right) \approx -6.97 \times 10^{-10} \tag{18}$$

for all times throughout separation.

The speed of the spaceship in a 100 nautical-mile orbit is about 7.8×10^3 m/sec. Hence, a calculation yields

$$\frac{1}{2}\left(2K_s - \beta^2{}_s\right) \approx -1.02 \times 10^{-9} \quad . \tag{19}$$

The spaceship remains in the 100 nautical-mile orbit for about 2 1/2 hours before insertion into an earth-moon trajectory. Therefore, for the first 2 1/2 hours we have

$$\frac{d(\Delta\tau)}{dt} = -6.97 \times 10^{-10} - (-1.02 \times 10^{-9}) \approx 3.2 \times 10^{-10} \quad . \tag{20}$$

After insertion into an earth-moon trajectory and lunar orbit, the curves of Figure 1 show the actual distance-velocity profile for the Apollo 8 spaceship. These curves, along with the time, distance, and velocity data of Table 1, were furnished by Mr. William D. McFadden of Mr. Robert Benson's group of Aero-Astrodynamics Laboratory at MSFC, Huntsville, Alabama. From these curves and data, $d(\Delta\tau)/dt$ was ultimately computed for each value of universal time[2] in Table 1. Figure 2 is a plot of $d(\Delta\tau)/dt$ as a function of universal time from launch to insertion into lunar orbit. Also, Figure 2 shows $d(\Delta\tau)/dt$ as a function of universal time for a typical lunar orbit. We compute the shaded area under the curves to obtain:

$\Delta\tau_1 \approx 3.30$ μ-sec, from launch to about 2 3/4 hours after launch

$\Delta\tau_2 \approx -158.00$ μ-sec, from 2 3/4 hours after launch to about 71 hours after launch

$\Delta\tau_3 \approx -2.30$ μ-sec, for a typical lunar orbit of about 2.10 hours.

We note that insertion into lunar orbit occurred about 71 hours after launch from Cape Kennedy. We assume that the return trip from the moon to the earth is nearly symmetrical and that there were approximately 10 lunar orbits. Hence, we compute the integral, equation (16), to be

$$\Delta\tau \approx \int_0^T \left[\frac{1}{2}\left(2K_e - \beta^2{}_e\right) - \frac{1}{2}\left(2K_s - \beta^2{}_s\right)\right] dt$$

$$= \Delta\tau_1 + 2(\Delta\tau_2) + 10(\Delta\tau_3) \tag{21}$$

or

$$\Delta\tau \approx 3.30 + 2(-158.00) + 10(-2.30) \approx -335.00 \ \mu\text{-sec} \quad . \tag{22}$$

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2. Astronomical observations are usually referred to universal time, or U. T., which is the mean solar time at the meridian of Greenwich.

This means that the astronauts aged 335 μ-sec more than an earth man at Cape Kennedy during their trip to the moon and back. It is interesting to note that if an astronaut made a similar trip to the moon and back every week for a period of time between 50 and 60 years, he would only age about one second more than if he had remained on earth. I do not think he would have to worry about old age creeping up on him for making so many trips to the moon!

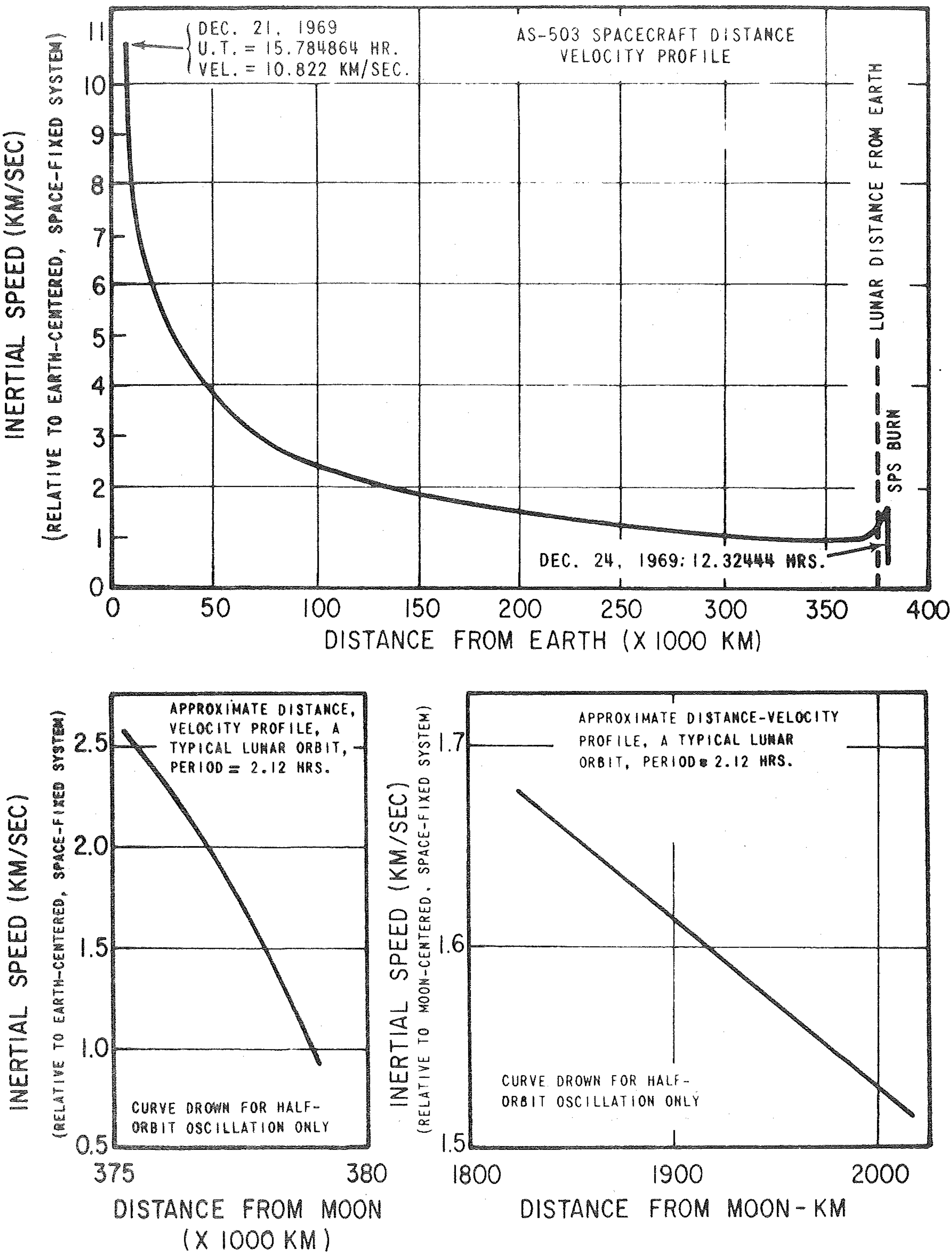

Figure 1. Distance, time, velocity profiles for Apollo 8.

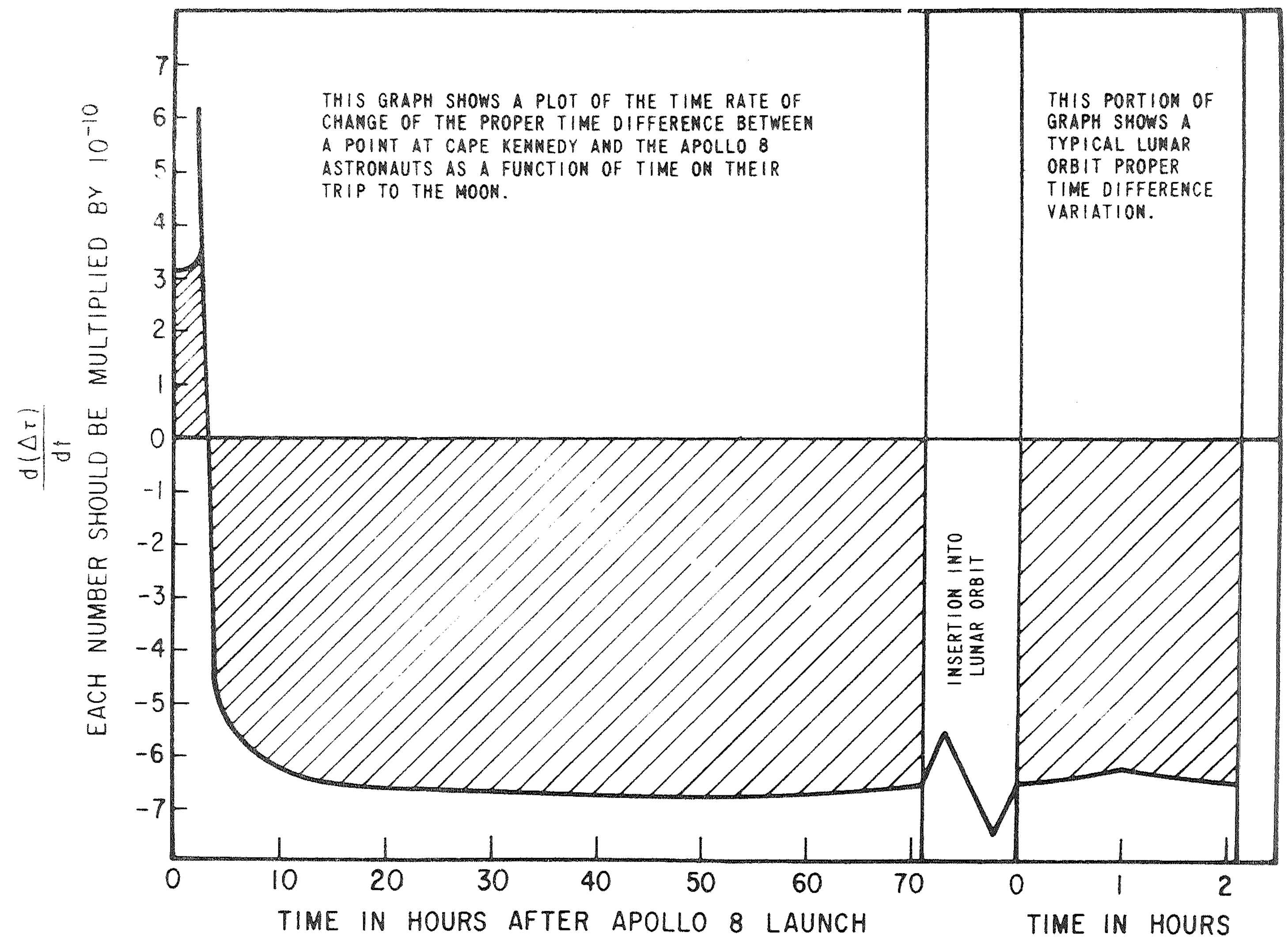

Figure 2. Proper time difference as a function of universal time.

TABLE 1. RELATIVISTIC TIME DILATION DATA FOR APOLLO 8

UNIVERSAL TIME AFTER LAUNCH			r	r'	SPACECRAFT VELOCITY	$\frac{1}{2}(2K_e-\beta_e^2)$	$2K_s$	$-\beta_s^2$	$\frac{1}{2}(2K_s-\beta_s^2)$	$\frac{d(\Delta\tau)}{dt}$
Hours	Minutes	Seconds	Meters	Meters	m/sec					
0.00	0	0	6.37×10^{6}	3.720×10^{8}	4.13×10^{2}	-6.97×10^{-10}	-1.392×10^{-9}	-1.89×10^{-12}	-6.97×10^{-10}	0.00
0.15	9	5.4×10^{2}	6.55×10^{6}	3.719×10^{8}	7.80×10^{3}	-6.97×10^{-10}	-1.352×10^{-9}	-6.76×10^{-10}	-1.02×10^{-9}	3.20×10^{-10}
2.45	147	8.82×10^{3}	6.55×10^{6}	3.719×10^{8}	7.80×10^{3}	-6.97×10^{-10}	-1.352×10^{-9}	-6.76×10^{-10}	-1.02×10^{-9}	3.20×10^{-10}
* 2.50	150	9.00×10^{3}	6.60×10^{6}	3.719×10^{8}	10.82×10^{3}	-6.97×10^{-10}	-1.344×10^{-9}	-1.30×10^{-9}	-1.32×10^{-9}	6.20×10^{-10}
2.61	157	9.42×10^{3}	7.80×10^{6}	3.707×10^{8}	10.00×10^{3}	-6.97×10^{-10}	-1.137×10^{-9}	-1.11×10^{-9}	-1.13×10^{-9}	4.30×10^{-10}
2.92	175	1.05×10^{4}	1.33×10^{7}	3.652×10^{8}	7.62×10^{3}	-6.97×10^{-10}	-6.67×10^{-10}	-6.45×10^{-10}	-6.56×10^{-10}	-0.41×10^{-10}
3.72	223	1.338×10^{4}	2.77×10^{7}	3.508×10^{8}	5.22×10^{3}	-6.97×10^{-10}	-3.20×10^{-10}	-3.03×10^{-10}	-3.12×10^{-10}	-3.85×10^{-10}
4.92	295	1.77×10^{4}	4.52×10^{7}	3.333×10^{8}	4.01×10^{3}	-6.97×10^{-10}	-1.96×10^{-10}	-1.79×10^{-10}	-1.88×10^{-10}	-5.09×10^{-10}
6.92	415	2.49×10^{4}	6.90×10^{7}	3.095×10^{8}	3.18×10^{3}	-6.97×10^{-10}	-1.29×10^{-10}	-1.12×10^{-10}	-1.21×10^{-10}	-5.76×10^{-10}
9.92	595	3.57×10^{4}	9.82×10^{7}	2.803×10^{8}	2.57×10^{3}	-6.97×10^{-10}	-9.03×10^{-11}	-7.34×10^{-11}	-8.19×10^{-11}	-6.15×10^{-10}
15.92	955	5.73×10^{4}	1.449×10^{8}	2.336×10^{8}	1.99×10^{3}	-6.97×10^{-10}	-6.12×10^{-11}	-4.40×10^{-11}	-5.26×10^{-11}	-6.44×10^{-10}
24.92	1495	8.97×10^{4}	2.004×10^{8}	$1.7[illegible]1\times10^{8}$	1.58×10^{3}	-6.97×10^{-10}	-4.48×10^{-11}	-2.77×10^{-11}	-3.63×10^{-11}	-6.61×10^{-10}
34.92	2095	1.257×10^{5}	2.593×10^{8}	1.192×10^{8}	1.26×10^{3}	-6.97×10^{-10}	-3.51×10^{-11}	-1.76×10^{-11}	-2.64×10^{-11}	-6.71×10^{-10}
54.92	3295	1.977×10^{5}	3.288×10^{8}	4.97×10^{7}	0.97×10^{3}	-6.97×10^{-10}	-2.92×10^{-11}	-1.05×10^{-11}	-1.99×10^{-11}	-6.77×10^{-10}
60.18	3611	2.167×10^{5}	3.514×10^{8}	2.71×10^{7}	0.94×10^{3}	-6.97×10^{-10}	-2.93×10^{-11}	-9.82×10^{-12}	-1.96×10^{-11}	-6.77×10^{-10}
** 71.34	4280	2.568×10^{5}	3.784×10^{8}	1.825×10^{6}	1.05×10^{3}	-6.97×10^{-10}	-8.31×10^{-11}	-1.23×10^{-11}	-4.77×10^{-11}	-6.49×10^{-10}
			DATA FOR A TYPICAL LUNAR ORBIT							
0.00	0	0	3.788×10^{8}	1.825×10^{6}	1.04×10^{3}	-6.97×10^{-10}	-8.31×10^{-11}	-1.20×10^{-11}	-4.75×10^{-11}	-6.49×10^{-10}
0.35	21	1260	3.776×10^{8}	1.890×10^{6}	1.55×10^{3}	-6.97×10^{-10}	-8.11×10^{-11}	-2.67×10^{-11}	-5.39×10^{-11}	-6.43×10^{-10}
0.70	42	2520	3.764×10^{8}	1.955×10^{6}	2.06×10^{3}	-6.97×10^{-10}	-7.93×10^{-11}	-4.72×10^{-11}	-6.32×10^{-11}	-6.34×10^{-10}
1.05	63	3780	3.752×10^{8}	2.020×10^{6}	2.57×10^{3}	-6.97×10^{-10}	-7.75×10^{-11}	-7.34×10^{-11}	-7.55×10^{-11}	-6.21×10^{-10}
1.40	84	5040	3.764×10^{8}	1.955×10^{6}	2.06×10^{3}	-6.97×10^{-10}	-7.93×10^{-11}	-4.72×10^{-11}	-6.32×10^{-11}	-6.34×10^{-10}
1.75	105	6300	3.776×10^{8}	1.890×10^{6}	1.55×10^{3}	-6.97×10^{-10}	-8.11×10^{-11}	-2.67×10^{-11}	-5.39×10^{-11}	-6.43×10^{-10}
2.10	126	7560	3.788×10^{8}	1.225×10^{6}	1.04×10^{3}	-6.97×10^{-10}	-8.31×10^{-11}	-1.20×10^{-11}	-4.75×10^{-11}	-6.49×10^{-10}

* U.T. = DEC. 21, 1969: 15.785 hrs. (EARTH-LUNAR TRAJECTORY INSERTION TIME)
** U.T. = DEC. 24, 1969: 12.320 hrs. (LUNAR ORBIT INSERTION TIME)

APPROVAL

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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