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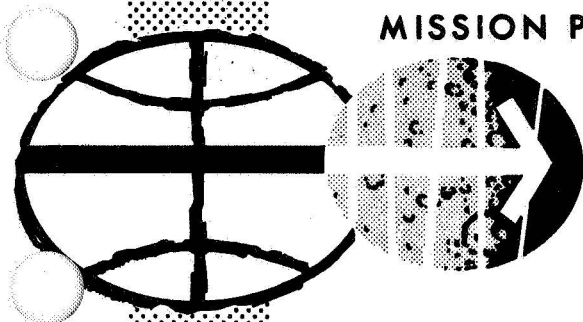
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**A METHOD FOR STAR VECTOR  
DETERMINATION FROM ALINEMENT  
OPTICAL TELESCOPE SIGHTING**

Mathematical Physics Branch

MISSION PLANNING AND ANALYSIS DIVISION

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PROJECT APOLLO

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FROM ALINEMENT OPTICAL TELESCOPE SIGHTING

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SUMMARY

A method is defined for star vector determination from alinement optical telescope (AOT) sightings from the lunar module (LM) on the lunar surface. This formulation will be coded into the Real-Time Computer Complex (RTCC) off-line AOT and gravity (AOT+G) bench program, the Gravity-Optics LM Attitude and Position (GOLAP) program.

INTRODUCTION

The star vector determination in the GOLAP program is currently computed by either of the two techniques defined in reference 1. This program is the bench program used for verification of the RTCC off-line AOT+G program which determines the LM lunar surface position using AOT star sightings and the inertial measurement unit (IMU) measured gravity direction. This note presents a third method of determining the measured star vector in LM body coordinates when the measurement data consist of a spiral measurement, a cursor measurement, and a time associated with each measurement. This technique will be added to the GOLAP program and will serve as the basis for the formulation of the changes to reference 2 if such a change should be desired at a later date.

The appendix contains a more detailed discussion of coordinate systems and coordinate transformations of option 2 of reference 1.

DISCUSSION

A possibility exists that the data available for determining star vector measurements on the lunar surface for Apollo 15 and subsequent missions will be spiral measurements, cursor measurements, and the time associated with each measurement. The following paragraphs define an

additional capability which will be added to the bench program, GOLAP, to account for this possibility. Much of the logic and many of the computations contained in option 2 of reference 1 are the same as for those defined herein.

The rotation of the celestial sphere with respect to the LM navigation base is the negative of the lunar rotation vector and is given at any time by the following equation.

$$\underline{V} = -[\text{NBSM}]^T [\text{REFSMMAT}] [\text{LIB}]^T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

where

[NBSM] is the transformation through the gimbal angles, defined in reference 1 (one set after LM landing)

[REFSMMAT] is the reference to the stable member transformation matrix

[LIB] is the reference to the selenographic transformation matrix

In principle, the preceding three matrices should all be valid at the same time; however, in practice, some relaxation of this restriction is acceptable. Since only the third row of the LIB matrix is actually used, and since the third row is essentially constant during the lunar stay, any libration matrix valid during the lunar stay is acceptable. In addition, platform misalignment is reflected as an error in REFSMMAT which results in a small error in the lunar spin axis in body coordinates. Since spiral and cursor measurements usually occur near each other in time, the platform misalignment will result in a small rotation about an axis whose error in direction is small. The effect of this error will be neglected. It is noted that even though the inertial orientation of the platform (REFSMMAT) and NBSM may change, it is not necessary to update the vector  $\underline{V}$  since the lunar rotation vector with respect to the LM will be constant. If shifting of the LM is suspected, then REFSMMAT and NBSM should be updated if new values are available.

Given a cursor measurement (YROT) at time  $T_Y$ , and a spiral measurement (SROT) on the same star at time  $T_S$ , the cursor measurement can be transformed to the "rotated optics" coordinate frame by

$$\underline{V}_C = [\text{ROTOP}]^T [\text{OPNB}]^T [\text{ROTVEC}] [\text{OPNB}] \underline{V}_{CO}$$

where

[OPNB] is the optics to navigation base coordinate transformation discussed in the appendix

[ROTOP] is the rotated optics to optics coordinate transformation discussed in the appendix

[ROTVEC] is the transformation which rotates the vector normal to the plane defined by the cursor measurement about the lunar spin axis in navigation base coordinates by an amount equal to the lunar rotation between cursor and spiral measurements. This matrix is given by

$$[\text{ROTVEC}] = I + \sin \phi [\underline{V}] + (1 - \cos \phi) [\underline{V}]^2$$

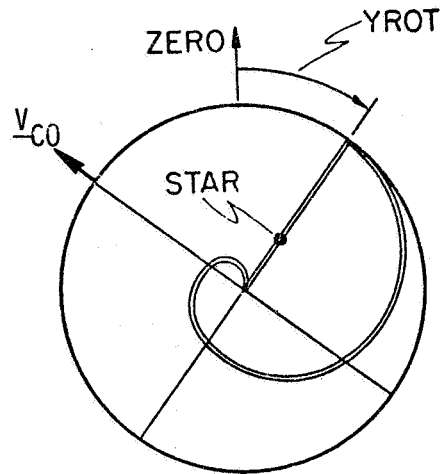
where

$$[\underline{V}] = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

$\phi = \omega(T_S - T_Y)$ ,  $\omega$  is the lunar rotation rate

$$\underline{v}_{CO} = \begin{pmatrix} \cos \text{ YROT} \\ \sin \text{ YROT} \\ 0 \end{pmatrix}$$

is the unit vector normal to the plane defined by the cursor measurement (i.e., the plane containing the star and the center of the AOT field of view) in optics coordinates as illustrated in the following figure.



Cursor measurement.

A spiral measurement defines the locus vectors given by

$$\underline{V}_S = \begin{pmatrix} \sin \theta & \sin \frac{\theta}{12} \\ \cos \theta & \sin \frac{\theta}{12} \\ & \cos \frac{\theta}{12} \end{pmatrix} \quad 0 \leq \theta \leq 360^\circ$$

in the rotated optics coordinate frame. The measured vector is the one normal to  $\underline{V}_C$ ; i.e.,

$$\underline{V}_C \cdot \underline{V}_S = 0$$

The value of  $\theta$  for which the above equation holds can be found iteratively by letting

$$f_i = \underline{V}_C \cdot \underline{V}_{Si}$$

where

$\underline{V}_{Si}$  is the value of  $\underline{V}_S$  for the  $i$ th iteration

and

$$\theta_1 = \text{SROT} - \text{YROT}$$

if  $\theta_1 < 0$ ,

$$\theta_1 = \theta_1 + 360^\circ$$

$$\theta_0 = \theta_1 + 1^\circ$$

then

$$\Delta\theta_i = \frac{\theta_i - \theta_{i-1}}{f_i - f_{i-1}} f_i$$

if

$$\Delta\theta_i > 3^\circ$$

$$\Delta\theta_i = 3^\circ$$

$$\Delta\theta_i < -3^\circ$$

$$\Delta\theta_i = -3^\circ$$

$$\theta_{i+1} = \theta_i - \Delta\theta_i$$

If  $|\Delta\theta_i| > 0.001^\circ$  continue to iterate, otherwise the measured vector in body coordinates is

$$\underline{\text{LOS}}_b = [\text{OPNB}] [\text{ROTOP}] \underline{V}_S(\theta_f)$$

at the time of the spiral measurement. The angle  $\theta_f$  is the final value of the preceding iteration.

This technique for determining  $\underline{\text{LOS}}_b$  will be added to GOLAP as option 3.

## CONCLUSIONS

A method for star vector determination from AOT sightings has been defined. This formulation will be coded into the RTCC off-line AOT+G bench program GOLAP.



APPENDIX  
COORDINATE SYSTEMS AND COORDINATE TRANSFORMATIONS FOR  
STAR VECTOR DETERMINATION ON THE LUNAR SURFACE



APPENDIX  
COORDINATE SYSTEMS AND COORDINATE TRANSFORMATIONS FOR  
STAR VECTOR DETERMINATION ON THE LUNAR SURFACE

INTRODUCTION

The RTCC off-line AOT+G program star vector determination routine transforms cursor measurement data to a rotated optics coordinate system defined by a cursor mark. In order to do this, it is convenient to use certain coordinate systems and rotation matrices which have not been previously documented. This appendix describes these coordinate systems and rotation matrices. It is emphasized that these coordinate systems have been arbitrarily selected and do not constitute a unique formulation. It is only necessary that the rotation matrices properly describe relationships between the selected coordinate systems. Coordinate systems not defined in this appendix are defined in reference 3.

OPTICS COORDINATE SYSTEM

The optics coordinate system is defined with the AOT in any detent and the reticle rotation equal to zero. The axes of this coordinate system are as follows.

- a. The Z-axis is along the center of the field of view.
- b. The Y-axis is along the double-lined cursor used for sighting on the lunar surface.
- c. The X-axis completes the triad.

This coordinate system is represented in figure A-1.

## ROTATED OPTICS COORDINATE SYSTEM

The rotated optics coordinate system is similar to the optics coordinate system. The difference is that the reticle need not be in the zero position. This rotation through an angle SROT is required for surface alinements. This coordinate system is represented in figure A-2.

## OPTICS TO NAV BASE TRANSFORMATION MATRIX

The optics to nav base transformation can be thought of as two separate rotations. The first of these rotations relates the center of the optics field of view to the nav base. The optics azimuth and elevation are defined in reference 3. The center-of-optics to nav base transformation is given by

$$[\text{CFOV}] = \begin{bmatrix} 0 & \cos EL & \sin EL \\ -\cos AZ & -(\sin EL)\sin AZ & (\cos EL)\sin AZ \\ \sin AZ & -(\sin EL)\cos AZ & (\cos EL)\cos AZ \end{bmatrix}$$

where AZ and EL are the azimuth and elevation of the appropriate detent.

A rotation of the star field in detents other than detent number two results from the construction of the hardware. This rotation about the center of the field of view is given in reference 3 by equation 6.3.4. In matrix rotation, this rotation is equivalent to

$$[\text{STRT}] = \begin{bmatrix} \cos R & -\sin R & 0 \\ \sin R & \cos R & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where

$R = AZ_2 - AZ_N$ , where  $AZ_2$  is the azimuth of detent two and  $AZ_N$  is the azimuth of the appropriate detent.

The complete rotation from the optics to nav base is given by

$$[\text{OPNB}] = [\text{CFOV}] [\text{STRT}]$$

## ROTATED OPTICS TO OPTICS TRANSFORMATION MATRIX

It can readily be seen from figure A-2 that the rotated optics to optics transformation matrix is given by

$$[\text{ROTOP}] = \begin{bmatrix} \cos \text{SROT} & -\sin \text{SROT} & 0 \\ \sin \text{SROT} & \cos \text{SROT} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

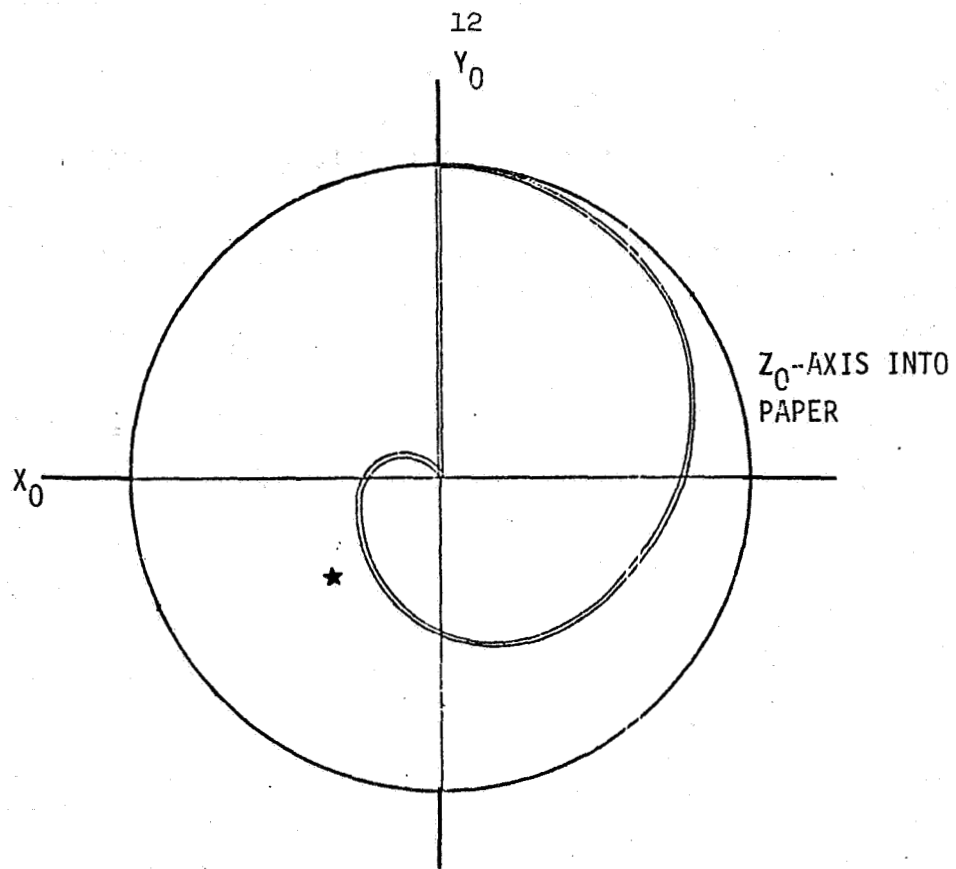


Figure A-1.- Optics coordinate system.

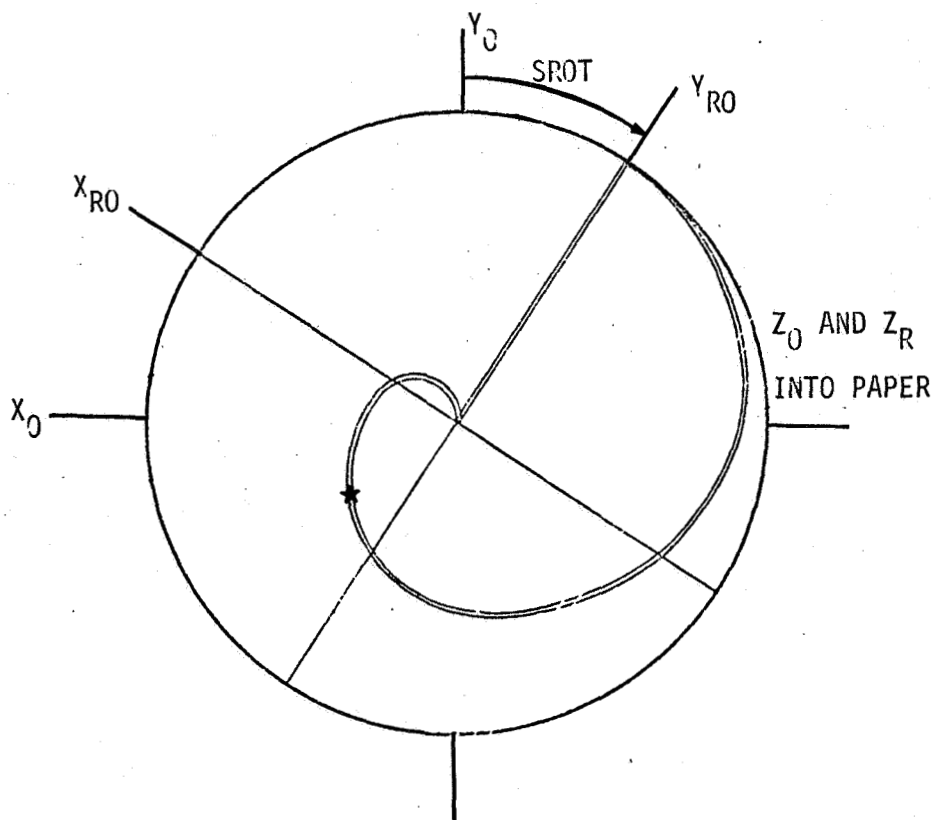


Figure A-2.- Rotated optics coordinate system.

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