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A Comparison of the Readout Resolutions of the Proportional Elastance vs Constant Elastance Torque-to-Balance Systems

by
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September 19

MIT

CAMBRIDGE 39, MASSACHUSETTS

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A COMPARISON OF THE READOUT RESOLUTIONS OF THE PROPORTIONAL ELASTANCE VS CONSTANT ELASTANCE TORQUE-TO-BALANCE SYSTEMS

ABSTRACT

This report establishes voltage signal-to-noise ratios of the read-out voltage for both the constant elastance torque-to-balance system and proportional elastance torque-to-balance system. Results show that resolution of low level torques is limited by servo electronics noise for the constant elastance system whereas resolution of low level torques is limited by gyro "noise torques" for the proportional elastance system. These "noise torques" are precisely those error torques which require investigation to further improve gyro performance.

Thus advantages of the proportional elastance torque-to-balance system are established.

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A COMPARISON OF THE READOUT RESOLUTIONS OF THE PROPORTIONAL ELASTANCE VS CONSTANT ELASTANCE TORQUE-TO-BALANCE SYSTEMS

Introduction

The purpose of this paper is to compare the readout signal-to-noise ratio of the proportional elastance torque-to-balance system to the readout signal-to-noise ratio of the constant elastance torque-to-balance system.* The signal and noise terms are shown in each equation and a voltage signal-to-noise ratio is developed. From a study of the frequencies contained in the torque terms and those contained in the noise terms, the proportional elastance system is shown to have a considerably higher resolution than the constant elastance system.

1. The Basic Loop for Both Systems

The basic servo loop for both systems is shown in Fig. 1.

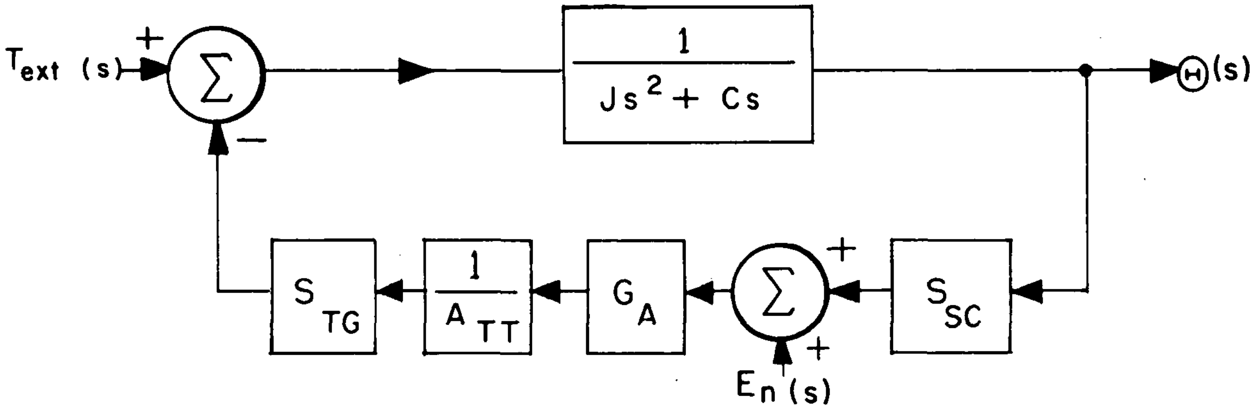

Fig. 1 Basic servo loop

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* See Instrumentation Laboratory Report R-367 "A Multirange Precision Torque Measuring Device" - by P. J. Gilinson, Jr., C. R. Dauwalter, and J. A. Scoppettuolo for further description of systems.

The difference between the two systems is that for the constant elastance system the term $\frac{1}{A_{TT}}$ is constant, whereas for the proportional elastance system, the term $\frac{1}{A_{TT}}$ is variable.

The external torque, $T_{ext}(s)$, contains two terms; the signal term designated $T_{sig}(s)$; and the noise term designated $T_n(s)$. Thus:

$$T_{ext}(s) = T_{sig}(s) + T_n(s) \tag{1}$$

$\Theta(s)$ is the angle of the gyro float shaft.

$E_n(s)$ is the equivalent noise voltage of the amplifier referred to the input. This voltage includes the shot and thermal noises of the amplifier.

$E_{RO}(s)$ is the read-out voltage seen at the output of the feedback loop amplifier. The signal is generally processed through a phase-sensitive device which has as a reference a signal proportional to the excitation of the torque generator primary coil. The two signals (vis-a-vis the phase sensitive reference voltage and $E_{RO}(s)$) are effectively multiplied and a torque read-out obtained.

$E_{RO}(s)$ contains two terms; the signal term $E_{ROS}(s)$ and the noise term $E_{RON}(s)$, so that:

$$E_{RO}(s) = E_{ROS}(s) + E_{RON}(s) \tag{2}$$

The gyro under test is shown in the top block of Fig. 1 and represents the gyro mass and fluid damping. Elastic restraint torques in the gyro are assumed to be either compensated or leveled out and are not considered for the present. The gyro transfer function is here shortened to H(s) - i. e. :

$$H(s) = \frac{1}{Js^2 + Cs} \tag{3}$$

S_{SG} and S_{TG} are the respective sensitivities of the signal and torque generators. G_A is the gain of the feedback amplifier. The attenuation term is written $\frac{1}{A_{TT}}$ so that when a considerable attenuation is introduced $\frac{1}{A_{TT}}$ becomes small, as is normally the case.

The basic loop is now rotated slightly so that $E_{RO}(s)$, the read-out voltage, appears on the right hand side of the page (see Fig. 2).

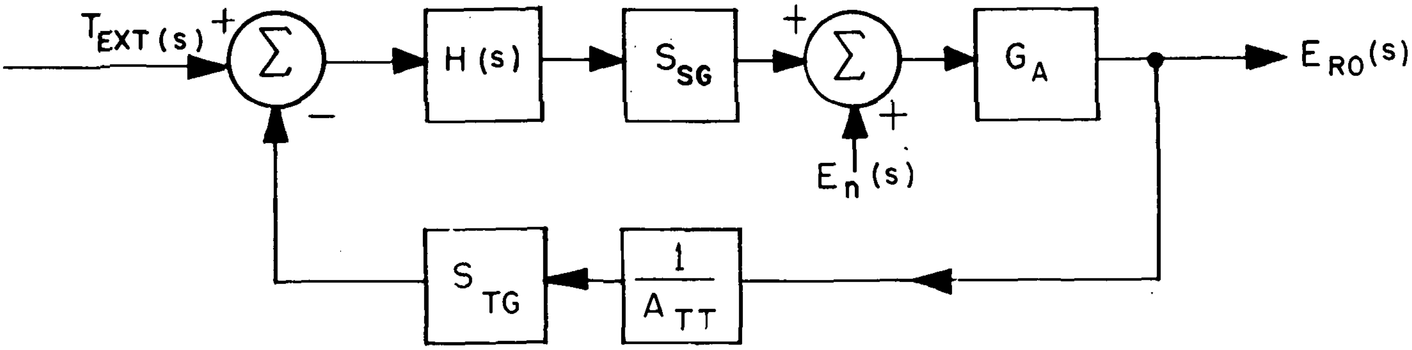

Fig. 2 Basic servo loop for both constant and proportional elastance systems

The read-out voltage is

$$E_{RO}(s) = \frac{S_{SG}\, G_A\, H(s)\, T_{ext}(s)}{1 + \frac{S_{SG}\, S_{TG}\, G_A\, H(s)}{A_{TT}}} + \frac{G_A\, E_n(s)}{1 + \frac{S_{SG}\, S_{TG}\, G_A\, H(s)}{A_{TT}}} \tag{4}$$

Substituting the gyro dynamics for H(s) this voltage becomes:

$$E_{RO}(s) = \frac{\frac{S_{SG}\,G_A}{\left(Js^2 + Cs\right)} \cdot T_{ext}(s)}{1 + \frac{S_{SG}\,S_{TG}\,G_A}{A_{TT}\left(Js^2 + Cs\right)}} + \frac{G_A\,E_n(s)}{1 + \frac{S_{SG}\,S_{TG}\,G_A}{A_{TT}\left(Js^2 + Cs\right)}} \qquad (5)$$

Revising eq (5) the final form of the read-out voltage for either system is:

$$E_{RO}(s) = \frac{S_{SG}\,G_A\,T_{ext}(s)}{\left(Js^2 + Cs\right) + \frac{S_{SG}\,S_{TG}\,G_A}{A_{TT}}}$$

$$+ \frac{G_A\,E_n(s)\left(Js^2 + Cs\right)}{\left(Js^2 + Cs\right) + \frac{S_{SG}\,S_{TG}\,G_A}{A_{TT}}} \qquad (6)$$

Equation (6) is now analyzed on the basis of a constant elastance system.

2: The Signal-to-Noise Ratio of the Read-out Voltage, $E_{RO}(s)$, for the Constant Elastance System

The servo system in Fig. 2 becomes a constant elastance system when A_{TT} is constant. Thus A_{TT} is now fixed and analysis proceeds with recognition of this constraint. For the present, $T_{ext}(s)$ is assumed to contain only signal torques so that:

$$T_{ext}(s) = T_{sig}(s) \qquad (7)$$

$$E_{RO}(s) = \frac{S_{SG} G_A T_{sig}(s)}{\left(Js^2 + Cs\right) + \frac{S_{SG} S_{TG} G_A}{A_{TT}}} + \frac{G_A \left(Js^2 + Cs\right) E_n(s)}{\left(Js^2 + Cs\right) + \frac{S_{SG} S_{TG} G_A}{A_{TT}}} \tag{8}$$

The first term on the right hand side of eq (8) represents pure signal in the read-out voltage, and the second term represents pure noise.

For a constant signal torque applied to the gyro float the signal torque expression is

$$T_{sig}(s) = \frac{T_{ext}}{s} \tag{9}$$

and the signal term of the read-out voltage is:

$$E_{ROS}(s) = \frac{S_{SG} G_A T_{ext}}{s\left[\left(Js^2 + Cs\right) + \frac{S_{SG} S_{TG} G_A}{A_{TT}}\right]} \tag{10}$$

Since the signal is read-out after all step transients have decayed, $E_{ROS}(s)$ is examined for the condition $s \to 0$, since $t \to \infty$, $s \to 0$, or

$$E_{ROS}(s)\Bigg|_{\substack{s \to 0 \\ t \to \infty}} \cong \frac{S_{SG} G_A T_{ext}}{\frac{S_{SG} S_{TG} G_A}{A_{TT}} \cdot s} = \frac{A_{TT} T_{ext}}{S_{TG}} \cdot \frac{1}{s} \tag{11}$$

Equation (11) transforms in the time domain to a constant.

$$E_{ROS}(s)\Big|_{s\to 0} \longleftrightarrow e_{ROS}(t)\Big|_{t\to\infty} = \frac{A_{TT}\,T_{ext}}{S_{TG}} \qquad (12)$$

Since the attenuation, A_{TT}, in the constant elastance system is fixed, eq (12) shows that as the torque level becomes small, so also does the signal level of the read-out voltage.

The second term on the right hand side of eq (8) is a pure noise term. $E_n(s)$ is assumed to have some value (in the frequency domain) from some low frequency, not including dc, to some high frequency above the break frequency of the gyro response, centered at such a frequency that the approximation

$$E_{RON}(s) \cong \frac{G_A\left(Js^2 + Cs\right)E_n(s)}{\left(Js^2 + Cs\right)} = G_A\,E_n(s) \qquad (13)$$

can be made.

Emphasis is placed on the fact that this noise contribution to the read-out voltage need not be carefully analyzed so long as we know its origin and accept its existence as part of the read-out voltage.

The total read-out voltage, with the above considerations, is

$$E_{RO}(s) \cong \frac{A_{TT}\,T_{ext}(s)}{S_{TG}} + G_A\,E_n(s) \qquad (14)$$

Equation (14) represents the read-out voltage under actual measurement conditions, the first term on the right hand side being signal and the $G_A\,E_n(s)$ term noise.

The frequency domain voltage signal-to-noise ratio is

$$\frac{S}{N}(s) = \frac{A_{TT} T_{ext}(s)}{G_A S_{TG} E_n(s)} \tag{15}$$

Equation (15) shows that as the external torque goes down in magnitude, with A_{TT} fixed, the signal-to-noise ratio is also proportionately reduced. Equation (15) shows that small torque levels are difficult to measure with the constant elastance system since the signal-to-noise ratio degenerates in direct proportion to the torque level.

3. The Signal-to-Noise Ratio of the Read-Out Voltage, $E_{RO}(s)$, for the Proportional Elastance System

The servo system in Fig. 2 becomes a proportional elastance system when A_{TT} is made variable. Thus A_{TT} is now variable and analysis proceeds with recognition of this condition.

Setting $T_{ext}(s) = T_{sig}(s)$ yields the same expression for the read-out voltage for the proportional elastance system as that for the constant elastance system, viz:

$$E_{RO}(s) = \frac{S_{SG} G_A T_{sig}(s)}{\left(J^2 + Cs\right) + \frac{S_{SG} S_{TG} G_A}{A_{TT}}} + \frac{G_A \left(Js^2 + Cs\right) E_n(s)}{\left(Js^2 + Cs\right) + \frac{S_{SG} S_{TG} G_A}{A_{TT}}} \tag{16}$$

Signal and noise terms in eq (16) are left and right respectively, as before.

For constant signal torque input the signal term of the read-out voltage is identical with the signal term of the constant elastance system.

$$E_{ROS}(s) \cong \frac{A_{TT} T_{ext}(s)}{S_{TG}} \tag{17}$$

Since the same noise is present in both systems, the argument concerning the noise is not repeated except to state that for large A_{TT} the noise term in eq (16) can most certainly be approximated by

$$E_{RON}(s) \cong G_A E_n(s) \tag{18}$$

Thus the signal-to-noise ratio expression for the proportional elastance system is also identical to that for the constant elastance system. This ratio is

$$\frac{S}{N}(s) = \frac{A_{TT} T_{ext}(s)}{G_A S_{TG} E_n(s)} \tag{19}$$

Note that in eq (19) A_{TT} is variable, and that the proportional elastance system is pertinent here.

Clearly, as $T_{ext}(s)$ goes down in magnitude, if A_{TT} is increased "proportionally", the signal-to-noise ratio remains constant. This is the significant feature of the proportional elastance system. Thus low torque levels can be measured easily since the signal-to-noise ratio can be maintained high and nearly constant at very low torque levels. This result shows clearly the advantage of the proportional elastance torque measuring system.

4. Consideration of the Signal-to-Noise Ratio for the Proportional Elastance Torque-to-Balance System when $T_{ext}(s)$ Contains both Signal and Noise Terms

When the external torque input to the proportional elastance system contains both signal and noise terms, the read-out voltage, $E_{RO}(s)$, becomes

$$E_{RO}(s) = \frac{S_{SG}\,G_A\left[T_{sig}(s) + T_n(s)\right]}{Js^2 + Cs + \dfrac{S_{SG}\,S_{TG}\,G_A}{A_{TT}}} + \frac{G_A\left(Js^2 + Cs\right)E_n(s)}{Js^2 + Cs + \dfrac{S_{SG}\,S_{TG}\,G_A}{A_{TT}}} \tag{20}$$

which is rewritten

$$E_{RO}(s) = \frac{S_{SG}\,G_A\,T_{sig}(s)}{Js^2 + Cs + \dfrac{S_{SG}\,S_{TG}\,G_A}{A_{TT}}} + \frac{S_{SG}\,G_A\,T_n(s)}{Js^2 + C_s + \dfrac{S_{SG}\,S_{TG}\,G_A}{A_{TT}}} + \frac{G_A\left(Js^2 + Cs\right)E_n(s)}{Js^2 + Cs + \dfrac{S_{SG}\,S_{TG}\,G_A}{A_{TT}}} \tag{21}$$

The middle term on the right hand side of eq (21) is the only new term in the read-out voltage expression and is a pure noise term. The first and last terms have already been discussed.

Considering $T_{sig}(s)$ to be a low level constant torque, so that A_{TT} is large as before, the signal term in eq (21) is

$$E_{ROS}(s) = \frac{A_{TT}\,T_{ext}(s)}{S_{TG}} \tag{22}$$

and the noise term is

$$E_{RON}(s) = \frac{S_{SG} G_A T_n(s)}{Js^2 + Cs + \frac{S_{SG} S_{TG} G_A}{A_{TT}}} + \frac{G_A \left(Js^2 + Cs\right) E_n(s)}{Js^2 + Cs + \frac{S_{SG} S_{TG} G_A}{A_{TT}}} \tag{23}$$

The second term on the right hand side of eq (23) is again approximated by $G_A E_n(s)$ for large A_{TT} so that the noise term is revised to

$$E_{RON}(s) = \frac{S_{SG} G_A T_n(s)}{Js^2 + Cs + \frac{S_{SG} S_{TG} G_A}{A_{TT}}} + G_A E_n(s) \tag{24}$$

$T_n(s)$ in eq (24) is the external "noise torque" and has a spectrum in the frequency domain.

When $T_{sig}(s)$ and $T_n(s)$ are of the same order of magnitude, $T_n(s)$ and the appropriate loop response cause a considerable contribution to $E_{RON}(s)$ (and therefore $E_{RO}(s)$ so as to disguise $T_{sig}(s)$ in $E_{RO}(s)$.

The point being established is that the ultimate resolution of the proportional elastance torque-to-balance system is determined by gyro internal noise torques (drift, uncertainty, restraint, flex lead, temperature, mass unbalance, etc.) and not noise due to the electronics of the servo loop. The constant elastance system, on the other hand, has its ultimate resolution determined by noise in the electronics plus gyro noise. Thus the proportional elastance torque-to-balance system reads all torques due to gyro parameters only. Gyro performance can thus be reliably improved since some of these "noise torques" are predictable and can therefore be removed through improved design. Otherwise further compensation for these torques can be added.

5. Summary of Results

The results of the above analysis show clearly that the proportional elastance torque-to-balance system can read small torques with high signal-to-noise ratio without having any appreciable electronic servo loop noise in the read-out signal, $E_{RO}(s)$. All torques other than signal torques can thus be read and discerned easily. Further gyro engineering progress is therefore possible. However, the constant elastance torque-to-balance system only provides accurate torque read-outs down to levels that are limited by the noise level of the servo electronics, causing a lower limiting level of small torque discernability. Clearly this puts a lower level on gyro engineering progress since gyro torques below this level cannot be read-out and can not, therefore, be analyzed.

This report shows theoretically the advantages of the proportional elastance torque-to-balance system.

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