~~Bellcomm~~ / V. Mummert

FM file

68-FM-256

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MSC INTERNAL NOTE NO. 68-FM-256

October 11, 1968

VERIFICATION OF SUNDISK ORBITAL NAVIGATION PROGRAM

Mathematical Physics Branch

MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

(NASA-TM-X-69716) VERIFICATION OF SUNDISK ORBITAL NAVIGATION PROGRAM (NASA) 112 p

N74-70626

Unclas
00/99 16149

MSC INTERNAL NOTE NO. 68-FM-256

PROJECT APOLLO

VERIFICATION OF SUNDISK ORBITAL NAVIGATION PROGRAM

By Richard E. Eckelkamp
Mathematical Physics Branch

October 11, 1968

MISSION PLANNING AND ANALYSIS DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

Approved: ______________________
James C. McPherson, Chief
Mathematical Physics Branch

Approved: ______________________
John P. Mayer, Chief
Mission Planning and Analysis Division

CONTENTS

TABLES

VERIFICATION OF SUNDISK ORBITAL NAVIGATION PROGRAM

By Richard E. Eckelkamp

SUMMARY

This paper presents a detailed explanation and test verification of SUNDISK 282 orbital navigation program 22 (P22). Included are a step-by-step analysis of the coding and flow charts, a summary of the equations utilized, an outline of astronaut procedures, and charted results of bit-by-bit test cases.

INTRODUCTION

Orbital navigation, as contained in the roped version 282 of SUNDISK for the command module computer (CMC), will be exercised for the first time on Apollo 7. This report fulfills a need of gathering and interpreting the official documentation concerning the program under one cover.

In orbital navigation, optical sightings on landmarks are statistically weighed to correct the vehicle position and velocity in the CMC as well as the position of the landmarks. This process can thus determine both the orbit and the landing site.

The explanation of P22 will begin with a summation of the formulation found in parts 5.1 to 5.2.8 of Section V of the GSOP (ref. 1). A line-by-line interpretation of the coding, as presented in reference 2, will then be given. The MIT flow charts are available to aid in the understanding of the coding. A resume of astronaut procedures and the test results follows.

The Kalman-Schmidt Filter

Accurate estimates of the state vectors of bodies relative to particular reference systems may be computed by integration of the equations of state complemented with processing of navigational observations. One method of processing observations, the Kalman-Schmidt filter, adapted from filter theory in electrical engineering, statistically weighs single observations and uses them to correct state vectors while smoothing observational noise.

In orbital navigation the state vector consists of the command module's position and velocity and the position of a given landmark in an inertial coordinate system. Telescope or sextant observations of the landmark relative to an inertial platform constitute the measurement.

Before presenting detailed information of the navigation processes, a brief outline of the Kalman-Schmidt method is useful.

To correct the state at time t_i

a) Make an observation Q_i at t_i

b) Integrate the state vectors as stored in the vehicle computer to t_i

$$\int \begin{bmatrix} \underline{r}_c \\ \\ \underline{v}_c \\ \\ \underline{r}_\ell \end{bmatrix}_{i-1} = \begin{bmatrix} \underline{r}_c \\ \\ \underline{v}_c \\ \\ \underline{r}_\ell \end{bmatrix}_{i}$$

where $\underline{r}_c$ = position vector of command module

$\underline{v}_c$ = velocity vector of command module

$\underline{r}_\ell$ = position vector of landmark

c) Compute from this state vector the estimated observation $\hat{Q}_i$

d) Find the observation residual $Q_i - \hat{Q}_i = \delta Q_i$

e) Compute the weighting factor $\underline{\omega}_i$ for the residual at t_i

This factor is obtained by propagating the square root of the state convariance matrix (or matrix of state uncertainty) to t_i.

f) Correct the state

$$\begin{bmatrix} \underline{r}_c \\ \underline{v}_c \\ \underline{r}_\ell \end{bmatrix}_i^+ = \begin{bmatrix} \underline{r}_c \\ \underline{v}_c \\ \underline{r}_\ell \end{bmatrix} + \underline{\omega}_i \delta Q_i$$

g) Also correct, or update, $\underline{\omega}_i$ to $\underline{\omega}_i^+$, where the plus denotes the updated quantity.

This statistically weighting of single observations for vector correction works well and is equivalent to the modified weighted least square (WLS) method used for ground-based navigation. Generally, the number of observations required to reduce initial errors to within the system noise level and maintain this level of accuracy varies with mission phase and navigation requirements.

SUMMATION OF ORBITAL NAVIGATION EQUATIONS

Coordinate Systems

To locate any body at a given time a coordinate system is necessary. For the Apollo program, the basic reference system is the nearest Besselian year (NBY) coordinate system.

The NBY system is defined in an earth-centered inertial Cartesian system. The X-axis is along the line of intersection of the mean equatorial plane and the mean orbit plane of the earth (equinox); Z is along the mean north pole; and Y completes the right-handed triad - all defined at the time of the beginning of the NBY. Vectors dated after the Julian date of June 30 are referenced to the following year. For example, the Besselian year 1968 begins on Julian day January 1.283, 1968; the Besselian year 1969, January 0.525, 1969. The center of this system is translated to the moon's center when the vehicle is in lunar reference.

As stated in reference 3, the CMC programs, including orbital navigation, utilize an approximate system to locate earth fixed targets. The transformation from this system to the NBY system for a state vector $\tilde{\underline{r}}_\ell$ in this CMC system is

$$\underline{r}_{\ell NBY} = \begin{bmatrix} 1 & 0 & -A_y \\ 0 & 1 & A_x \\ A_y & -A_x & 1 \end{bmatrix} \begin{bmatrix} \cos A_z & -\sin A_z & 0 \\ \sin A_z & \cos A_z & 0 \\ 0 & 0 & 1 \end{bmatrix} . \; \tilde{\underline{r}}_\ell$$

where $\tilde{\underline{r}}_\ell$ and $\underline{r}_{\ell NBY}$ are column vectors. The onboard state vector $\tilde{\underline{r}}_\ell$ is first rotated about the true Z-axis by an angle $-A_z$ where

$$A_z = A_{zo} + \omega(t + t_{ephem})$$

A_{zo} = angle between the X-axis and the Greenwich meridian at midnight just prior to the year preceding the reference Besselian year

t_{ephem} = elapsed time between July 1 and the time that the CMC clock was zeroed

t = time indicated in the CMC clock

ω = sidereal rotation rate of the earth

The transformation is completed by making small angular rotations about the X- and Y- axes. The values of A_x and A_y are constant for a given mission. These introduce an approximation. The resulting errors are proportional to the elapsed time between t and the time associated with the evaluation of A_x and A_y.

Orbital Integration

The state vector for the CSM is periodically integrated using either the conic or the Encke method. In the conic mode, the perturbing acceleration, $\underline{a}_d(t)$, in the basic equation

$$\frac{d^2}{dt^2}\underline{r}(t) + \frac{\mu_e}{r^3}\underline{r}(t) = \underline{a}_d(t)$$

is omitted. In the precision mode, an oscillating conic is defined at t_o, again ignoring $\underline{a}_d(t_o)$. The perturbing acceleration is then integrated separately for any $t_o + \Delta t$ using Nystrom's method. When the perturbed portion of the vector reaches a certain magnitude, currently defined as 8 kilometers, a new conic is defined, a procedure called rectification. The time of the new conic, t_o', is

$t_o' = t_o + n\Delta t$, where $n\Delta t$ is the elapsed time since the last rectifications at t_o.

The square root of the state covariance matrix, W, is also propagated forward. The covariance, or correlation matrix, is defined for orbital navigation as

$$E(t) = \begin{bmatrix} \underline{\varepsilon}\,\underline{\varepsilon}^T & \underline{\varepsilon}\,\underline{\eta}^T & \underline{\varepsilon}\,\underline{\beta}^T \\ \underline{\eta}\,\underline{\varepsilon}^T & \underline{\eta}\,\underline{\eta}^T & \underline{\eta}\,\underline{\beta}^T \\ \underline{\beta}\,\underline{\varepsilon}^T & \underline{\beta}\,\underline{\eta}^T & \underline{\beta}\,\underline{\beta}^T \end{bmatrix}_{9 \times 9}$$

where $\underline{\varepsilon}$, $\underline{\eta}$, and $\underline{\beta}$ are partial estimates of the errors in CSM position, CSM velocity, and landmark positive, respectively. The W matrix is defined by

$$E(t) = W(t)W(t)^T$$

(T denotes transpose) and extrapolated by numerical integration of

$$\frac{d}{dt}W(t) = \begin{bmatrix} 0 & I & 0 \\ G(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{9 \times 9} W(t)$$

where

$$G(t) = \frac{\mu_e}{r^5(t)} [3\underline{r}(t)\underline{r}(t)^T - \underline{r}^2(t)I]_{3 \times 3}$$

I = 3 × 3 identity matrix

0 = 3 × 3 zero matrix

Despite the use of rectification, initial errors in the state and dynamic biases neglected in the equations, for example, drag, eventually cause the CMC vector to diverge from the actual vector. The W matrix is subsequently affected the same. Navigation measurements are utilized to correct both the state and the W matrix. In orbital navigation landmark sightings are processed through the Kalman-Schmidt filter for this purpose.

Use of Landmark Sightings in the Kalman-Schmidt Filter

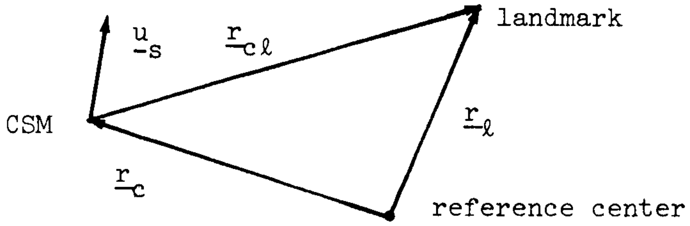


Figure 1.

Given that a mark relative to the navigational base coordinate system has been made on a landmark and transmitted to the CMC,program 22 will utilize this data to correct the state vector.

As outlined in the introduction, an observation is computed for the same time as the observed data. The difference between the actual and computed observation, δQ_i, is weighted with $\underline{\omega}_i$ and added to the state vector.

<u>The b-vector</u>.- In the computation of $\underline{\omega}_i$, where

$$\underline{\omega}^T = \frac{1}{z^2 + \bar{\alpha}^2} \, \underline{z}^T W^T \tag{1}$$

$$\underline{z} = W^T \underline{b}$$

the b-vector, defined as

$$\underline{b} = \frac{\partial(\text{COMPUTED OBSERVATION})}{\partial(\text{COMPUTED STATE})} = \frac{\partial Q_i}{\partial X_i} \tag{2}$$

indicates how much and in which direction the state will change for a given change in the observation. Referring to figure 1 and following the derivation from reference 3, let $\underline{u}_s$ be a unit vector pointing toward an imaginary star.

Assume $\underline{u}_s \times \underline{r}_{c\ell} \neq 0$.

To get the $\underline{b}$ vector, take the dot product

$$r_{c\ell} \cos\theta = \underline{r}_{c\ell} \cdot \underline{u}_s$$

Take the differential:

$$\delta r_{c\ell} \cos\theta - r_{c\ell} \sin\theta \, d\theta = \delta \underline{r}_{c\ell} \cdot \underline{u}_s \tag{3}$$

Now,

$$r_{c\ell} = [\underline{r}_{c\ell} \cdot \underline{r}_{c\ell}]^{1/2}$$

$$\delta r_{c\ell} = 1/2 [\underline{r}_{c\ell} \cdot \underline{r}_{c\ell}]^{-1/2} [\delta \underline{r}_{c\ell} \cdot \underline{r}_{c\ell} + \underline{r}_{c\ell} \cdot \underline{r}_{c\ell}]$$

$$\delta r_{c\ell} = \frac{\underline{r}_{c\ell} \cdot \sigma\underline{r}_{c\ell}}{r_{c\ell}} \qquad (4)$$

Substituting (4) into (3) and rearranging,

$$\frac{\underline{r}_{c\ell} \cdot \delta\underline{r}_{c\ell}}{r_{c\ell}} \cos\theta - r_{c\ell} \sin\theta \, d\theta = \delta\underline{r}_{c\ell} \cdot \underline{u}_s$$

$$\delta\theta = \left[\frac{\underline{r}_{c\ell}\cos\theta}{r_{c\ell}} - \underline{u}_s\right] \frac{-\ 1}{r_{c\ell}\sin\theta} \cdot \delta\underline{r}_{c\ell}$$

Now if we choose $\underline{u}_s$ perpendicular to $\underline{r}_{c\ell}$

$$\delta\theta = - \frac{\underline{u}_s}{r_{c\ell}} \cdot \delta\underline{r}_{c\ell}$$

This is the relation which defines $\underline{b}$ for updating the relative position $\underline{r}_{c\ell}$. Since the CMC updates $\underline{r}_c$ and $\underline{r}_\ell$, note

$$\underline{r}_c + \underline{r}_{c\ell} = \underline{r}_\ell$$

$$\delta\underline{r}_c + \delta\underline{r}_{c\ell} = \delta\underline{r}_\ell$$

Then (3) becomes

$$\delta\theta = \frac{-u_s}{r_{c\ell}} \cdot [\delta\underline{r}_\ell - \delta\underline{r}_c]$$

$$= \frac{u_s}{r_{c\ell}} \cdot [\delta\underline{r}_c - \delta\underline{r}_\ell]$$

The $\underline{b}$ - vector can be written compactly as

$$\underline{b} = \frac{1}{r_{c\ell}} \begin{bmatrix} \underline{u}_s \\ 0 \\ -\underline{u}_s \end{bmatrix}$$

Each observation point is incorporated twice, since two degrees of freedom are available perpendicular to $\underline{r}_{c\ell}$. For the first direction of correction,

$$\underline{u}_s^{(1)} = \text{unit} \left\{ \underline{r}_{c\ell} \times \left[\underline{r}_{c\ell} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] \right\}$$

In case the computer computation is difficult resulting from $\underline{r}_{c\ell}$ being parallel or near-parallel to $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, use

$$\underline{u}_s^{(1)} = \text{unit} \left\{ \underline{r}_{c\ell} \times \left[\underline{r}_{c\ell} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \right\}$$

For the second correction use $\underline{u}_s^{(2)} = \text{unit} \left[\underline{r}_{c\ell} \times \underline{u}_s^{(1)} \right]$,

where the plus indicates that $\underline{r}_{c\ell}$ has been updated by the first correction.

<u>The W matrix and α^2</u>.- The 9-by-9 W matrix in equation (1) represents a numerical estimate of the uncertainties associated with the mathematical description space. These uncertainties include noises and bias on the state vector and prediction models (lack of drag, for example) and limitations of their representation for programming. For convenience, the W matrix may be divided into nine 3-by-3 matrices:

$$W = \begin{bmatrix} W_0 & W_1 & W_2 \\ W_3 & W_4 & W_5 \\ W_6 & W_7 & W_8 \end{bmatrix} \tag{6}$$

At the beginning of a navigational sequence, this matrix must be set to an initial value representing, among other things, an estimate of state uncertainty. Each 3-by-3 matrix is taken initially to be a diagonal, since the initial self-correlations of vehicle position, vehicle velocity, and landmark position, W_0, W_4, W_8 respectively, cannot be accurately determined a priori.

Further, the initial correlation between W_0, W_4, and W_8 are simplified. The correlation between W_0 and W_4 represented by W_1 and W_3, are taken to be zero. For a known landmark, correlations between W_8, W_0, and W_4 are also taken to be zero.

For an unknown landmark (unℓ), however,

$$W_6 = K_0 W_0$$

$$W_7 = K_1 W_1$$

$W_8 = w_{un\ell} I + K_2 W_0$, where K_0, K_1, K_2, and $w_{un\ell}$ are constants.

These artificial correlations attempt to reflect the fact that the initial value of the unknown landmarks position will be heavily affected by errors in the vehicle state. In practice, K_0 and K_1 are set to zero in SUNDISK. For the program COLOSSUS a more realistic approach exists for the definition of errors associated with unknown landmarks.

As a marking sequencing continues, the numerical size and characteristics of the W matrix change. Correlations between the various elements grow. After each state vector update from mark data, the W matrix is also updated. The W matrix is extrapolated in time by integration, as outlined in the introduction.

The term $\overline{\alpha}^2$ in equation (1) is a constant representing the uncertainties of the observational space. Noises and biases of the observing instruments and the observer are included.

It should be emphasized that since the filter employed in orbital navigation is nonoptimal, i.e., all known dynamic forces and data errors are not modeled, and the numerical values of $\overline{\alpha}^2$ and the W matrix do not reflect the actual estimates of instrument and state vector error. Further, since the filter is linear, the relative value of W and $\overline{\alpha}^2$ is the principle factor affecting the operation of the filter. For example, the filter will operate identically if values of W_0 and $\overline{\alpha}^2{}_0$ or $k W_0$ and $k^2 \alpha^2$ are utilized, where k is any constant within the numerical limits of the computer.

The observation residual.- Recall from the introduction the equation for correcting the state vector:

$$\begin{bmatrix} \underline{r}_c \\ \underline{v}_c \\ \underline{r}_\ell \end{bmatrix}_i^+ = \begin{bmatrix} \underline{r}_c \\ \underline{v}_c \\ \underline{r}_\ell \end{bmatrix} + \underline{\omega}_i \delta Q_i \qquad (7)$$

One must compute δQ_i:

$$\delta Q = Q_i - \hat{Q}_i$$

Let $\underline{u}_m$ be a unit vector in the NBY coordinate system pointing along the measured line of sight. Since the optical measurement occurs in the navigation base coordinate system, the transformation to NBY coordinates is

$$\underline{u}_{m_{NBY}} = [\text{REFSMMAT}]^T \; [\text{GIMBLE}] \; [\text{NAVGMBL}] \; \underline{u}_{m_{NBS}}$$

where the matrix [REFSMMAT] transforms from NBY to IMU coordinates, [GIMBLE], from body to IMU coordinates, and [NAVGMBLE], from navigation base to body coordinates. Superscript T denotes a transpose.

For the actual observation,

$$\underline{u}_s \cdot \underline{u}_m = \cos\theta$$

$$\cos^{-1}(\underline{u}_s \cdot \underline{u}_m) = \theta_{ACTUAL}.$$

For the computed observation

$\cos^{-1}(\underline{u}_s \cdot \underline{u}'_m) = \theta_{COMPUTED}$, where the prime denotes a computed unit vector.

Now

$$\delta Q = \theta_{ACTUAL} - \theta_{COMPUTED} = \cos^{-1}(\underline{u}_s \cdot \underline{u}_m) - \cos^{-1}(\underline{u}_s \cdot \underline{u}'_m)$$

$$\delta Q = \cos^{-1}(\underline{u}_s \cdot \underline{u}_m) - \frac{\Pi}{2}$$, since $\underline{u}_s$ is perpendicular to the estimated line of sight, $\underline{u}_m$.

The CMC's Implementation of the Kalman Filter

To utilize equation (7), up to five marks are made with the telescope or sextant on a particular landmark. A CMC routine, auto-optics, can aid the astronaut by aiming the observing instrument at the estimated landmark position. (The accuracy of the aim is proportional to the accuracy of the CMC state vector.) A residual, δQ, is computed with equation (8) and the b vectors are computed.

For computational purposes the W matrix is divided into 27 column vectors within the CMC. For example, W_0 in equation (6) may be written

$$W_0 = \begin{bmatrix} \underline{w}_0^T \\ \underline{w}_1^T \\ \underline{w}_2^T \end{bmatrix}$$

The nine dimensional quantities $\underline{b}$, $\underline{\omega}$, and $\underline{z}$ are also divided as

$$\underline{b} = \begin{bmatrix} \underline{b}_0 \\ \underline{b}_1 \\ \underline{b}_2 \end{bmatrix} \qquad \underline{\omega} = \begin{bmatrix} \underline{\omega}_0 \\ \underline{\omega}_1 \\ \underline{\omega}_2 \end{bmatrix} \qquad \underline{z} = \begin{bmatrix} \underline{z}_0 \\ \underline{z}_1 \\ \underline{z}_2 \end{bmatrix}$$

The Kalman computations are then performed as

$$\underline{z}_i = \sum_{j=0}^{2} W_{i+3j}\underline{b}_j \text{ , } i = 0, 1, 2 \qquad (8)$$

$$\underline{\omega}_i^T = \beta \sum_{j=0}^{2} \underline{z}_j^T W_{3i+j} \tag{9}$$

where $$\beta = \frac{1}{\left(\sum_{j=0}^{2} \underline{z}_j \cdot \underline{z}_j + \bar{\alpha}^2\right)}$$

$$\delta\underline{x}_i = \delta Q \underline{\omega}_i, \text{ where } \begin{bmatrix} \delta\underline{x}_0 \\ \delta\underline{x}_1 \\ \delta\underline{x}_2 \end{bmatrix} = \begin{bmatrix} \delta\underline{r}_c \\ \delta\underline{v}_c \\ \delta\underline{r}_\ell \end{bmatrix} \tag{10}$$

Verification of the Kalman factor.- To verify that $\underline{\omega}_i$, defined in equation (9), is the Kalman weighting factor, which is normally defined as

$$\underline{\omega} = E\underline{M}^T\left(\underline{M}E\underline{M}^T + \bar{\alpha}^2\right)^{-1} \tag{11}$$

begin with the compact definition given in reference 1, as

$$\underline{\omega} = \frac{1}{z^2 + \bar{\alpha}^2}\underline{z}^T W^T$$

which implies

$$\underline{\omega} = \frac{1}{z^2 + \bar{\alpha}^2} W\underline{z} \tag{12}$$

To get (12) expand the Kalman equation (11) using MIT terminology

$$\begin{aligned} \underline{\omega} &= WW^T\underline{M}^T\left(\underline{M}WW^T\underline{M}^T + \bar{\alpha}^2\right)^{-1} \\ \underline{\omega} &= WW^T\underline{b}^T\left(\underline{b}WW^T\underline{b}^T + \bar{\alpha}^2\right)^{-1} \end{aligned} \tag{13}$$

From reference (1),

$$\underline{z} = W^T\underline{b}$$

Since the basic Kalman filter in (11) considers $\underline{b}$ as a row vector,

$$\underline{z} = W^T\underline{b}^T \tag{14}$$

Substituting (14) in (13)

$$\underline{\omega} = W\underline{z}(\underline{zz}^T + \bar{\alpha}^2)^{-1}$$

which is MIT's formulation (12)

$$\underline{\omega} = \frac{1}{z^2 + \bar{\alpha}^2} W\underline{z}$$

Verification of the W updating equation.- As noted in the introduction, the W matrix is updated after each incorporation of mark data. In CMC notation the update is

$$\underline{w}_{i+9j} = \underline{w}_{i+9j} - \gamma z i \underline{\omega}_j \qquad \begin{aligned} i &= 0, 1, \ldots, 8 \\ j &= 0, 1, 2 \\ \gamma &= \frac{1}{1 + \sqrt{\beta\alpha^2}} \end{aligned} \tag{15}$$

To verify that equation (15) follows from Kalman theory, proceed as follows:

First equation (15) may be written (ref. 1) as

$$W^+ = W - \frac{\underline{\omega}\underline{z}^T}{1 + \sqrt{\dfrac{\bar{\alpha}^2}{z^2 + \bar{\alpha}^2}}} \tag{16}$$

The Kalman update is defined as

$$E^+ = (I - \underline{\underline{\omega M}})E$$

Expanding in MIT notation,

$$\begin{aligned} (WW^T)^+ &= WW^T - \underline{\underline{\omega b}}WW^T \\ &= WW^T - WW^T\underline{b}^T(\underline{b}WW^T\underline{b}^T + \alpha^2)^{-1}\underline{b}WW^T \end{aligned}$$

Now following the derivation in reference 4,

$$(WW^T)^+ = W[I - \underline{z}(z^2 + \bar{\alpha}^2)^{-1}\underline{z}^T]W^T \qquad (17)$$

In order for equation (17) to be valid, the right side of the equation must consist of some quantity times its transpose to update W. Otherwise, only E could be updated and the CMC W scheme would be invalid. Assume this quantity to be of the form

$$W[I - \beta\underline{\underline{z}}\underline{\underline{z}}^T] \qquad (18)$$

where β must be found.

Equating equation (17) to equation (18) times the transpose of equation (18):

$$W[I - \underline{z}(z^2 + \bar{\alpha}^2)^{-1}\underline{z}^T]W^T = W[I - \beta\underline{\underline{z}}\underline{\underline{z}}^T][I - \beta\underline{\underline{z}}\underline{\underline{z}}^T]W^T$$

$$I - \underline{\underline{z}}\underline{\underline{z}}^T(z^2 + \bar{\alpha}^2)^{-1} = I - 2\beta\underline{\underline{z}}\underline{\underline{z}}^T + \beta^2\underline{\underline{z}}\underline{\underline{z}}^T\underline{\underline{z}}\underline{\underline{z}}^T$$

$$\beta^2z^2 - 2\beta + (z^2 + \bar{\alpha}^2)^{-1} = 0$$

Using the quadratic formula

$$\beta = \frac{2 \pm \sqrt{4 - 4z^2(z^2 + \bar{\alpha}^2)^{-1}}}{2z^2}$$

$$= \frac{1 \pm \sqrt{1 - z^2(z^2 + \bar{\alpha}^2)^{-1}}}{z^2}$$

In order to limit the amount by which W is decreased during each update, take the smaller β.

Multiply by $\dfrac{\sqrt{z^2 + \bar{\alpha}^2}}{\sqrt{z^2 + \bar{\alpha}^2}}$

$$\beta = \frac{\sqrt{z^2 + \bar{\alpha}^2} - \sqrt{z^2 - \bar{\alpha}^2 - z^2}}{z^2\sqrt{z^2 + \bar{\alpha}^2}}$$

$$= \frac{\sqrt{z^2 + \bar{\alpha}^2} - \bar{\alpha}^2}{z^2\sqrt{z^2 + \bar{\alpha}^2}}$$

Since $W^+ = \Delta \qquad W^+ = W + \Delta W = W[I - \beta \underline{z}\underline{z}^T]$

$$\Delta W = -W\beta \underline{z}\underline{z}^T$$

Using (19)

$$\Delta W = -W\left[\frac{\sqrt{z^2 + \bar{\alpha}^2} - \alpha}{z^2\sqrt{z^2 + \bar{\alpha}^2}}\right]\underline{z}\underline{z}^T$$

$$= -W\underline{z}\underline{z}^T\left[\frac{\sqrt{(z^2 + \bar{\alpha}^2} - \alpha)\left(\sqrt{z^2 + \bar{\alpha}^2} + \alpha\right)}{z^2\sqrt{z^2 + \bar{\alpha}^2}\left(\sqrt{z^2 + \bar{\alpha}^2} + \alpha\right)}\right]$$

$$= -W\underline{z}\underline{z}^T \frac{1}{z^2 + \bar{\alpha}^2 + \alpha\sqrt{z^2 + \bar{\alpha}^2}}$$

$$= -W\underline{z}\underline{z}^T \frac{1}{(z^2 + \bar{\alpha}^2)\left[1 + \sqrt{\frac{\bar{\alpha}^2}{z^2 + \bar{\alpha}^2}}\right]} \qquad (21)$$

Regrouping,

$$\Delta W = \frac{-W\underline{z}}{z^2 + \bar{\alpha}^2} \; \frac{\underline{z}^T}{1 + \sqrt{\dfrac{\bar{\alpha}^2}{z^2 + \bar{\alpha}^2}}}$$

$$= \frac{-W\underline{z}^T}{1 + \sqrt{\dfrac{\bar{\alpha}^2}{z^2 + \bar{\alpha}^2}}}$$

Therefore, from (20) and (21)

$$W^+ = W - \frac{W\underline{z}^T}{1 + \sqrt{\dfrac{\bar{\alpha}^2}{z^2 + \bar{\alpha}^2}}}$$

which is equation (16).

Unknown landmarks.- The orbital navigation program can also use optical marks made on unknown landmarks. In this mode of operation the program uses the first mark to define the location of the landmark.

Following the derivation in reference 3, consider the plane determined by the planet center, landmark, and vehicle shown in figure 2 below.

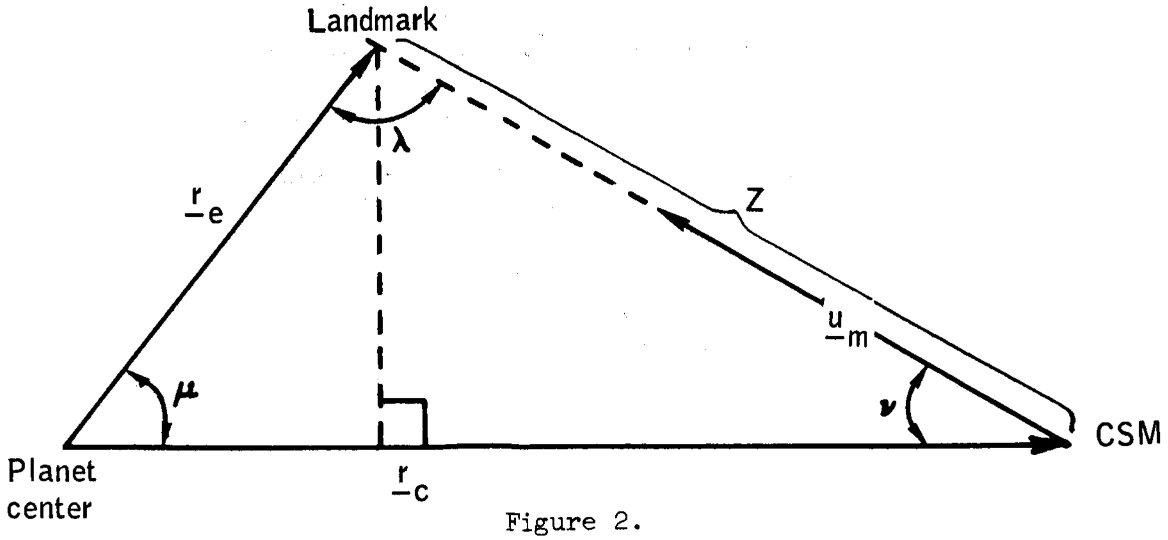


Figure 2.

By vector addition

$$\underline{r}_\ell = \underline{r}_c + \underline{z} \tag{22}$$

To find $\underline{z}$ and thus, $\underline{r}_\ell$, notice

$$z \cos \nu + r_\ell \cos \mu = r_c$$

$$z = \frac{r_c - r_\ell \cos \mu}{\cos \nu} = \frac{r_c}{\cos \nu}\left[1 - \frac{r_\ell}{r_c} \cos \mu\right] \tag{23}$$

Using the law of sines,

$$\frac{\sin \lambda}{r_c} = \frac{\sin \nu}{r_\ell} \quad \text{and} \quad \sin \lambda = \frac{r_c}{r_\ell} \sin \nu \tag{24}$$

and using the identity $\cos^2 \lambda + \sin^2 \lambda = 1$,

$$\cos \lambda = \sqrt{1 - \sin^2 \lambda} = \sqrt{1 - \left(\frac{r_c}{r_\ell}\right)^2 \sin^2 \nu} \tag{25}$$

Now, from the figure

$$\cos \mu = \cos[\pi - (\lambda + \nu)] = -\cos(\lambda + \nu)$$

$$\cos \mu = \sin \lambda \sin \nu - \cos \lambda \cos \nu \tag{26}$$

Combining (24), (25), and (26)

$$\cos \mu = \frac{r_c}{r_\ell} \sin^2 \nu + \cos \nu \sqrt{1 - \left(\frac{r_c}{r_\ell}\right)^2 \sin^2 \nu}$$

$$\frac{r_\ell}{r_c} \cos \mu = \sin^2 \nu + \cos \nu \frac{r_\ell}{r_c} \sqrt{1 - \left(\frac{r_c}{r_\ell}\right)^2 \sin^2 \nu}$$

Recall equation (23)

$$z = \frac{r_c}{\cos \nu}\left[1 - \frac{r_\ell}{r_c} \cos \mu\right]$$

$$z = \frac{r_c}{\cos \nu} \left[1 - \sin^2 \nu - \cos \nu \, \frac{r_\ell}{r_c} \sqrt{1 - \left(\frac{r_c}{r_\ell}\right)^2 \sin^2 \nu} \right]$$

$$= \frac{r_c}{\cos \nu} \left[\cos^2 \nu - \cos \nu \, \frac{r_\ell}{r_c} \sqrt{1 - \left(\frac{r_c}{r_\ell}\right)^2 \sin^2 \nu} \right]$$

$$= r_c \left[\cos \nu - \sqrt{\left(\frac{r_\ell}{r_c}\right)^2 - \sin^2 \nu} \right]$$

So equation (22) becomes

$$\underline{r}_\ell = \underline{r}_c + r_c \left[\cos \nu - \sqrt{\left(\frac{r_\ell}{r_c}\right) - \sin^2 \nu} \right] \underline{u}_m \qquad (27)$$

where $\underline{u}_m$ is the measured line of sight.

Looking at the result for the landmark position, equation (27), one notices the r_ℓ, the magnitude of the vector, is required. Since this is unavailable for an unknown landmark, r_ℓ, is taken to be the mean radius of the planet.

This approximation introduces considerable inaccuracies into the landmark position estimate. For the remainder of the marks, the landmark is considered known. The resulting updates decrease the landmark error.

Having reviewed and approved the equations used in P22, this paper will next provide an explanation of the coding of these equations.

THE CMC CODING

Review of the coding for the orbital navigation program has been accomplished chiefly through analysis of the MIT flow charts and study of the decoding in reference 2. The Program 22 flow charts are presented in Appendix A.

The coding analysis will follow through reference 2 line by line.

Explanation of each line will be given to the right of that line.

1. Start program

<u>PROG 22</u>

CODE	EXPLANATION
2. Perform R02	2. IMU STATUS CHECK; if REFSM = 1, IMU orientation is known by computer and proceed. if REFSM = 0 and IMU is on, perform P51 to determine alignment. if REFSM = 0 and IMU is off, go to POS to start up GNCS.
3. Set TARG2FLG = 1 (bit 9 of FLAGWRD 1)	3. Target is a landmark.
4. Set TARG1FLG = 0 (bit 10 of FLAGWRD 1)	4. Target is not LEM.
5. Set RNDVZFLG = 0 (bit 7 of FLAGWRD 0)	5. P20 is not running.
6. $T_{decl} = T_{now}$	6. Time to which integration is to be performed is present time.
7. Perform CSM Conic	7. CSM R & V are integrated conically.

8. PMGA = $\cos^{-1}[\lvert \text{unit}\,(\underline{V}_{att} \times \underline{R}_{att}) \cdot \text{REFSMMAT}_3 \rvert]$	8. Assuming X of spacecraft is in orbit plane, this checks to see if IMU orientation is satisfactory for P22. PMGA (displayed) is maximum middle gimbal angle possible. If > 60°, and time permits, IMU should be realigned (P52), then key in V37E22E and proceed. If not, proceed. Maneuver to acquire landmark.
9. Set RENDWFLG (bit 1 of FLAGWRD 5)	9. W matrix is invalid.
10. TS = 0645_{vn}	10. Machine channeling for 8. (display) (specifies VN PATTERN for display routine).
11. Perform GOFLASHR	11. Display of 8; exit to realign or continue as decided.
12. TS = 011_2 Perform BLANKET	12. Display interface routine.

PROG. 22A

13. MARKINDX = 5	13. Communication cell with R53.
14. TS = 00011_8	14. Channeling to display.
15. Proceed to GOPERF1	15. Display interfacing for choice of auto-optics (R52).
16. If No R52, proceed. If Yes R52, perform R52 and R53 within and proceed to 18.	16. Monitor and respond to landmark parameters. R53 is called within R52 automatically or by selection of manual optics. SCT trunnion or shaft and trunnion may be driven toward CMC's estimate of landmark position.

17.	Perform R53	17.	Marks are made; then accepted or rejected; up to 5 unrejected marks may be stored.
18.	NUM8NN = QPRET_{mk}	18.	Total number of marks taken = total number of marks taken.
19.	NUMBKK = 1	19.	Serial no. of mark being processed.
20.	S22LOC = SVMRKDAT	20.	Buffer cell location of 1st mark data to be processed.
21.	SVMRKDAT + i = $E_{\text{MARKSTAT} + i}$ $i = 0, 1, \ldots 35$	21.	35 cells divided into 5 sets, 1 for each mark, containing (TIME 2:1) (TIME 1:2) (CDU_y), (CDUS), (CDU_z) (CDUT), (CDU_x) composing a table to avoid loss if restart occurs.
22.	Perform 22 LMKID.	22.	--
23.	22LMKID a. LANDMARK = bits 9-4 of 22LMBDO	23.	a. ID for first mark is placed in 22 LMBDO and coded as A, B, C, D, and E where A = 1 known = 2 unknown B = 1 coordinates stored = 2 coordinates not stored CD = landmark serial number E = 1 (earth landmark) and initialized as A = 1, B = 1, CD = ID, E = 1

b. if LNDMKSTR = 0, (bit 7 of FLAGWRD 2) LANDMARK = LANDMARK + 2^{10}	b. If landmark coordinates are not stored, B = 2.
c. if LNDMKSTR = 1, LANDMARK = LANDMARK + 2^{9}	c. If landmark is stored, B = 1.
d. if LNDKNOWN = 0, (bit 8 of FLAGWRD 6) LANDMARK = LANDMARK + 2^{13}	d. If landmark is unknown, A = 2.
e. if LNDKNOWN = 1 LANDMARK = LANDMARK + 2^{12}	e. If landmark is known, A = 1.
f. LANDMARK = LANDMARK + 1	f. Landmark is an earth landmark.
g. TS = 0570_{VN}	g. Machine channeling for display.
h. Perform GOFLASHR	h. Displays LANDMARK, proceed or terminate and load new data.
i. 22LMBDO = bit 9-4 of LANDMARK	i. Corrected ID is placed back in 22LMBDO.
j. Set bit 7 (LNDMKSTR) of FLAGWRD 2 = bit 10 of LANDMARK	j. Landmark stored or not stored.
k. Set bit 8 (LNDKNOWN) of FLAGWRD 6 = bit 13 of LANDMARK	k. Landmark known or unknown.
24. Perform MKRELEAS	24. --
25. MKRELEAS	25. --
a. Release VAC areas assigned to marks.	a. VAC area is now available for other use.
b. MARKSTAT = 0	b. Mark storage cells are now available for a future set of marks.
c. Inhibit interrupts	c. Following sequence will not be interrupted.

d. Set bit 9 of OPTMODES = 0	d. Optics switched from computer control mode.
e. OPTIND = -1	e. Driving of optics is bypassed.
f. Set bit 2 of channel 12 = 0	f. Disable optics CDU error counter.
g. Return	g. --
26. If ORBWOK (bit 6 of FLAGWORD 3) = 1, set WDIM9INC (bit 9 of FLAGWORD 5) = 1	26. If W matrix is valid for orbital navigation, it is set as 9 × 9 for incorporation purposes.
27. If ORBWOK = 0 $W_i = 0$ (i = 0, 1, ..., 53)	27. If W matrix is invalid for orbital navigation, ...
a. $W_0 = C_{WORBPS}$ $W_4 =$ " $W_8 =$ " $W_{36} = C_{WORBVL}$ $W_{40} =$ " $W_{44} =$ "	a. Reinitialize upper 6 × 9 to a diagonal in the upper left 6 × 6.
b. Set ORBWOK = 1	b. W matrix is valid for orbital navigation.
c. Set WDIM9INC = 0	c. W matrix is not a 9 × 9 for incorporation purposes.
28. Perform S22FLAGS	28. Preparation for integration.
a. Perform INSTALL	a. Grabs integration package and secures it from other users.
b. $T_{decl} = E_{S22LOC\ d_p}$	b. Integration will be up to time of 1 mark, stored in E.

	c. Set bits 5(STATEUP), 3(CSMINT), 2(WDIMEN 9), and 1(WMATINT) of FLAGWRD 3 = 1	c. CM permanent state vector and 9 × 9 W matrix are to be integrated.
	d. Let CONINT (bit 4 of FLAGWRD 3) = 0	d. Encke integration.
	e. Return	e. --
29.	Set WDIMEN9 (bit 2 of FLAGWRD 3) = 0	29. W matrix is 6 × 6.
30.	If WDIM9INC = 0, set WMATINT (bit 1 of FLAGWRD 3) = 0	30. W matrix is to be integrated.
31.	Perform INTEGRV	31. Integration.
32.	$\text{CSMPO}\underline{\text{S}} = \underline{R}_{att}$	32. Answer.
33.	If LNDKNOWN = 1, proceed to 22LMK1. if LNDKNOWN = 0, Proceed	33. If landmark is known, proceed to 22LMK1; otherwise, proceed [We are now treating an unknown landmark]
34.	MARKDATA = S22LOC	34. Address of next mark data.
35.	$W_i = 0$ (i = 54 - 80)	35. - 39. Lower right 3 × 3 of W matrix is initialized for unknown landmark tracking. $K_2 = 1$
36.	$W_{72} = K_{wun}$	
37.	$W_{76} = K_{wun}$	
38.	$W_{80} = K_{wun}$	
39.	$[W_8] = [W_8] + K_2\ [W_0]$	
40.	X 1 = - MARKDATA	40. - 42. Mark data is shoved into buffers for computations.
41.	MARKDOWN = i = $E_{MARKDATA + i}$, (i = 0-6)	
42.	S1 = MARKDATA + 2	
43.	Perform SXTNB	43. TS converted to double precision for pointing vector computation.

44.	Perform NBSM	44.	Computes pointing vector in IMU coordinates.
45.	$\underline{UM} = \underline{TS}$ [REFSMMAT]	45.	Pointing vector in NBY coordinates.
46.	TS_1 = unit $\underline{CSMPOS}$	46.	For 50.
47.	$ALPHAV_z = TS_{1,z}$	47.	For 48.
48.	Perform GETERAD	48.	Compute Fischer radius at latitude of landmark.
49.	$ORBCOSA = -\underline{U}_M \cdot \underline{TS}.$	49.	For 50.
50.	$ALPHAV = CSMPO\underline{S} = \lvert COMPO\underline{S}\rvert \left[ORBCOSA - \sqrt{\left(\frac{ERADM}{\lvert COMPO\underline{S}\rvert}\right)^2 -(1 - ORBCOSA)^2}\,\right]\underline{UM}$	50.	First estimate of $\underline{r}_1$.
51.	Delay 2 secs (via DELAYJOB)	51.	--
52.	Proceed to S22B1G2	52.	--

<u>S22B1G2</u>

a.	Set ERADFISC (bit 13 of FLGWRD1) = 1	a.	Use Fischer ellipsoid radius.
b.	$TS = E_{S22LOC_{dp}}$	b.	Time of mark.
c.	Perform LAT - LONG	c.	$\underline{r}_1$ converted to latitude, longitude, and attitude.
d.	If NUM8NN - NUM8KK $\leq$ 0, go to 9DWT06DW	d.	If this is last mark, go to ......
e.	NUMBKK = NUMBKK + 1	e.	Serial number of mark date is increased by one.
f.	S22LOC = S22LOC + 7	f.	Starting address of next mark data.
g.	Proceed to S22B1G1	g.	--

[Now if we had marked on a known landmark, we would be back at 33 and starting 22LMK1]

22 LMK1

a.	If LNDMKSR (bit 7 of FLAGWRD2) = 0, proceed to 22LMKDAT; otherwise, proceed.	a.	If landmark coordinates are not stored, ...
b.	TS = bits 14 - 4 of 22LMB00, shifted right 3 places	b.	--
c.	X1 = LLATAB + 6TS	c.	Address of landmark table entry.
d.	$\underline{LAT} = \underline{E}_{X1}$	d.	Latitude, longitude, and altitude stored.
e.	Proceed to S22.16A	e.	--

[If coordinates had not been stored in a above]

22LMKDAT

a.	LANDALT = ALT/K_{2dcptl}	a.	Scale altitude for display; store in LANDALT.
b.	LANDLONG = 1/2 LONG	b.	Longitude/2 displayed for better accuracy.
c.	TS = 0689_{vn}	c.	Channeling for display.
d.	Proceed to GOFLASH;	d.	Display of landmark coordinates.
	(1) If terminate proceed to GOTOPOOH		
	(2) If proceed, proceed to S22.16		(2) if accept data, ...
	(3) Otherwise, proceed.		(3) if reject data, ...
e.	ALT = K_{2dcptl} LANDALT	e., f.	Data has been modified. This is rescaling as above.
f.	LONG = 2LONDLONG		

g. Proceed to c	g. Recycle from c till satisfied.

S22.16A

a. $W_i = 0$ (i = 54-80)	
b. $W_{72} = C_{Wmk}$	a-d. Initialize lower right 3 × 3 for known landmark incorporation.
c. $W_{76} = C_{Wmk}$	
d. $W_{80} = C_{Wmk}$	
e. If LNDMKSTR = 1, $[W_8] = K_{3d4} [W_8]$	e. If landmark is stored, reduce W_8.
f. Proceed to S22BI61	f. --

S22B1G1

53. Set ERADFISC (Bit 13 of FLAGWRD 1) = 1	53. Use Fischer ellipsoid radius for coordinate computations.
54. $TS = E_{S22LOC_{dp}}$	54. Time of mark.
55. Perform LALOTORV	55. Gets A_x, A_y, A_z, and $\underline{r}_\ell$.
56. $X78\underline{9} = K_{rfactl}$ ALPHAV	56. Rescales $\underline{r}_\ell$ to kilometers.
57. Perform S22FLAGS	57. Sets flags for and time of next mark for integration.
58. Perform lNTEGRV	58. W matrix and CSM vector are integrated to specific time.
59. $CSMPO\underline{S} = \underline{R}_{att}$	59. Answer.
60. MARKDATA = S22LOC	60. Address of next mark data.
61. X1 = - MARKDATA	
62. MARKDOWN + i = $E_{MARKDATA + i}$ i = (0 - 6)	61-63. Mark data is shoved into buffers for computations.
63. S1 = MARKDATA + 2	

64.	Perform SXTNB	64.	TS converted to double precision for pointing vector computation.
65.	Perform NBSM	65.	Computes pointing vector in IMU coordinates.
66.	$\underline{UM} = \underline{TS}$ [REFSMMAT]	66.	Pointing vector in NBY coordinates.
67.	$\underline{RCLP} = \underline{X789} - K_{rfact1}\ \underline{CSMPOS}$	67.	Relative landmark vector.
68.	Set FSTINCRP = 1 (bit 11 of FLAGWRD5)	68.	First incorporation of measurement data is being made.
69.	$\underline{TS} = \underline{RCLP} * \text{UNIT } \underline{Z}$	69.	TS now contains star unit vector $\underline{U}_s^- (o)$.
70.	If $\lvert TS \rvert < 2^{-6}$ km, $\underline{U}$ STAR = UNIT $\underline{Y}$	70.	If 69 overflows, $\underline{U}_s^{(o)} = \begin{bmatrix}0\\1\\0\end{bmatrix}_{NBY}$
71.	If $\lvert TS \rvert \geq 2^{-6}$ km, $\underline{USTAR}$ = UNITT$\underline{S}$	71.	If 69 doesn't overflow, ...
72.	Proceed to S22B164	72.	--

<u>S22B164</u>

73.	$VARIANCE = (\underline{RCLP})^2 (K_{SCTVAR} + K_{IMUVAR})$	73.	$\bar{\alpha}^2$
74.	Perform BVECTORS	74.	--
75.	<u>BVECTORS</u> a. $\underline{BVECTOR}_0 = \text{unit}(\text{UNIT } \underline{RCLP} * \underline{USTAR})$ b. $\underline{USTAR} = \underline{BVECTOR}_0$ c. $DELTAQ = K_{2pi}\ \lvert \underline{RCLP} \rvert$ $[\cos^{-1} (\underline{BVECTOR}_0 \cdot \underline{UM}) - 1/4]$, $(1/4 = 90^\circ)$	75.	BVECTOR and other intermediate Kalman computations.

	d. $\underline{\text{BVECTOR}}$, = 0		
	e. $\underline{\text{BVECTOR}}_2 = 0$		
	f. RETURN		
76.	$\underline{\text{BVECTOR}}_2 = -\underline{\text{USTAR}}$	76.	BVECTOR for update of landmark.
77.	Set WD1M91NC (bit 9 of FLAGWRD5) = 1	77.	W matrix is 9 × 9 for incorporation purposes.
78.	Set CSMUPDT (bit 8 of FLAGWRD1) = 1	78.	CSM state vector is to be updated.
79.	Perform 1NCORP1	79.	--
80.	<u>1NCORP1</u>	80.	--
	a. $\underline{z}_0 = [W_0]\ \underline{\text{BVECTOR}}_0 + [W_3]\ \underline{\text{BVECTOR}}_1 + [W_6]\ \underline{\text{BVECTOR}}_2$	a-c.	Computation of $\underline{z}$, where $\underline{z} = W^T\underline{b}$
	b. $\underline{z}_1 = [W_1]\ \underline{\text{BVECTOR}}_0 + [W_4]\ \underline{\text{BVECTOR}}_1 + [W_7]\ \underline{\text{BVECTOR}}_2$		
	c. $\underline{z}_2 = [W_2]\ \text{BVECTOR} + [W_5]\ \underline{\text{BVECTOR}}_1 + [W_8]\ \underline{\text{BVECTOR}}_2$		
	d. If WD1M91NC = 0, $\underline{z}_2 = 0$	d.	If W matrix is not a 9 × 9 for incorporation purposes.
	e. $\text{LITLA} = \underline{z}_0^{\,2} + \underline{z}_1^{\,2} + \underline{z}_2^{\,2} + \text{VARIANCE}$	e.	$z^2 + \bar{\alpha}^2$
	f. $\text{TS} = \sqrt{\text{LITLA VARIANCE}}$	f-h.	Intermediate steps in computing the Kalman.
	g. GAMMA = 1/(TS + LITLA)		
	h. DELQDA = DELTAQ/LITLA		

i.	$\underline{\text{OMEGA}}_0 = \underline{z}_0\,[W_0] + \underline{z}_1\,[W_1] + \underline{z}_2\,[W_2]$	i-n.	More intermediate steps and state correction computations.
j.	$\underline{\text{OMEGA}}_1 = \underline{z}_0\,[W_3] + \underline{z}_1\,[W_4] + \underline{z}_2\,[W_5]$		
k.	$\underline{\text{OMEGA}}_2 = \underline{z}_0\,[W_6] + \underline{z}_1\,[W_7] + \underline{z}_2\,[W_8]$		
l.	$\text{DELTA } \underline{X}_0 = \text{DELQDA } \underline{\text{OMEGA}}_0$		
m.	$\text{DELTA } \underline{X}_1 = \text{DELQDA } \underline{\text{OMEGA}}_1$		
n.	$\text{DELTA } \underline{X}_2 = \text{DELQDA } \underline{\text{OMEGA}}_2$		
o.	RETURN		
81.	$\text{DSTEM1}_{dp} = K_{kmmtr}\ 2\ \lvert\text{DELTA } \underline{X}_0\rvert$	81,82.	Scaling for display.
82.	$\text{DSTEM1} + 2_{dp} = K_{kmmcs}\ 2\ \lvert\text{DELTA } \underline{X}_1\rvert$		
83.	TS = 0649_{vn}	83-85.	Display routines and incorporation decision.
84.	Perform GOFLASHR If terminate, proceed to fourth line of S22B1G2; if proceed, go to 87; otherwise, proceed		
85.	TS = 00_2 and perform BLANKET		
86.	END of JOB	86.	--
87.	Perform 1NCORP2	87.	--

88. <u>1NCORP2</u>	88. --
a. EGRESS = return address	a. --
b. Perform INTSTALL	b. Grabs and secures orbital package for matrix updating procedures.
c. $\underline{\mathrm{OMEGAM}}_0 = \mathrm{GAMMA}\ \underline{\mathrm{OMEGA}}_0$	c-e. --
d. $\underline{\mathrm{OMEGAM}}_1 = \mathrm{GAMMA}\ \underline{\mathrm{OMEGA}}_1$	
e. $\underline{\mathrm{OMEGAM}}_2 = \mathrm{GAMMA}\ \underline{\mathrm{OMEGA}}_2$	
f. Set INTPHS (bit 13 of FLAGWRD5) = 1	f. To preclude loss of integrating package to priority if a restart occurs.
g. $[W_0] = [W_0] - \underline{Z}_0\ \underline{\mathrm{OMEGAM}}_0$	g-j. W matrix is updated.
h. $[W_1] = [W_1] - \underline{Z}_1 \underline{\mathrm{OMEGAM}}_0$	
i. $[W_3] = [W_3] - \underline{Z}_0\ \underline{\mathrm{OMEGAM}}_1$	
j. $[W_4] = [W_4] - \underline{Z}_1\ \underline{\mathrm{OMEGAM}}_1$	
k. If WD1M91NC = 1, $[W_2] = [W_2] - \underline{Z}_2\ \underline{\mathrm{OMEGAM}}_0$ $[W_5] = [W_5] - \underline{Z}_2\ \underline{\mathrm{OMEGAM}}_1$ $[W_6] = [W_6] - \underline{Z}_0\ \underline{\mathrm{OMEGAM}}_2$ $[W_7] = [W_7] - \underline{Z}_1\ \underline{\mathrm{OMEGAM}}_2$ $[W_8] = [W_8] - \underline{Z}_2\ \underline{\mathrm{OMEGAM}}_2$	k. Additional W matrix updating for a 9 x 9
l. If CSMUPDT = 1 (bit 8 of FLGWRD1)	l. If the CSM state vector is to be updated, do so.

(1) $\underline{TS}_1 = \underline{DELTAV}_{cm} + \underline{DELTAX}_o$ (2) $\underline{TS}_2 = \underline{NUV}_{cm} + \underline{DELTAX}_1$ (3) If there is no overflowing in computing $\underline{TS}_1$ and/or $\underline{TS}_2$: $\underline{DELTAV}_{cm} = \underline{TS}_1$ $\underline{NUV}_{cm} = \underline{TS}_2$	(1)-(3). For no overflow condition add weighted residual to perturbation portion of vector.
(4) If overflow has occurred in computing TS_1 and/or TS_2: $\underline{R}_{rectcm} = RVC_{cm} + DELTAV_{cm} + \underline{DELTAX}_o$ $\underline{RCV}_{cm} = \underline{R}_{rectcm}$ $\underline{V}_{rectcm} = \underline{VCV}_{cm} + NUV_{cm} + \underline{DELTAX}_1$ $\underline{VCV}_{cm} = \underline{V}_{rectcm}$ $\underline{DELTAV}_{cm} = 0$ $NUV_{cm} = 0$ $T_{ccm} = 0$ $XKEP_{cm} = 0$	(4) For overflow condition add weighted residual to the whole state vector, i.e., the conic portion plus the perturbed portion, and consider this update as a new rectification.
(5) If WD1M91NC = 1 $X\ 78\underline{9} = X\ 78\underline{9} + \underline{DELTAX}_2$	(5) If matrix is 9 × 9, update the landmark coordinates.

(6) Perform PTOACSM	(6) New vector is labeled as the permanent state vector.
(7) $\underline{R} = K_{rfctr}\,(RC\underline{V} + TDELTA\underline{V})$	(7)-(9) --
(8) $\underline{V} = K_{vfctr}\,(VC\underline{V} + TNU\underline{V})$	
(9) $T_{pptm} = T_{et}$	
m. If CSMUPOT = 0	m. If LM state vector is to be updated, perform 1, but with LM vectors.
n. QPRET = EGRESS	n. --
o. PROCEED TO INTWAKE	o. Which returns
89. $CSMPO\underline{S} = K_{1kb}\,15\,(RCV_{\underline{cm}} + DELTAV_{\underline{cm}})$	89. CSM position vector rescaled
90. If FSTINCRP (bit 11 of FLAGWRD 5) = 1 $RCL\underline{P} = X\,78\underline{9}\ -$ $- K_{rfact1}\,CSMPO\underline{S}$ Proceed to S22B164	90. If first incorporation of a measurement, compute relative landmark position and recycle to S22B164
91. Proceed to S22B162.	91. Recycle to S22B162 to process next mark
[Within S22B162. 52.d. If NUM8NN - NUM8KK ≤ 0, go to 9DWT06DW]	[If all marks have been processed go to 9DWT06DW]
92. 9DWT06DW	92. Converts upper 6 × 9 of 9 × 9 matrix to an equivalent 6 × 6

93. S22.1P5	93.
a. If LNDMKSTR = 0 (bit 7 of FLAGWRD 2)	a. If landmark is not stored, serial number of cell where landmark data is stored is kept as one, since only one landmark can be stored in Sundisk
(1) If 22LMBO = K_{22mdmx}, 22LMBDO = 0	
(2) If 22LMBD ≠ K_{22mbmx}, 22LMBDO = 22LMBD	
b. Proceed to S22EX3	b. --
94. S22EX3	94.
a. LANDMARK = 22LMBDO	a. Landmark code = number in 22LMBDO
b. Proceed to S22EX33	b. --
95. S22EX33	95.
a. TS = 6570_{vn}	a. For display of landmarks ID.
b. Perform GOFLASHR	b. --
(1) If terminate, proceed to 22TERM,	(1) If one does not wish to see display of updated landmark, ...
(2) If proceed, go to 1;	(2) If one wishes to see them, ...
(3) otherwise, proceed to S22EX3	(3) Otherwise, recycle....
c. TS = 101_2 and perform BLANKET	c. Kills display
d. End of job	d. --
e. LANDALT = ALT/$K_{2dcptle}$	e. Scaling for display
f. LANDLONG = 1/2 LONG	f. For display

g. TS = 0689_{vn}	g. Verb-noun flash for display of updated landmark coordinates
h. Proceed to GOFLASH;	h.
(1) if terminate, proceed to 22TERM;	(1) If one doesn't want updated landmark coordinates to be stored as the landing site, ...
(2) if proceed, proceed;	(2), (3) --
(3) otherwise, go to i.	
i. TS = bits 14-4 of 22LMBDO, right shifted 3 places	i-k. Landmark data is stored.
j. X1 = LLATAB + 6 TS	
k. $\underline{E}_{X1} = \underline{LAT}$	
l. If LNDMKSTR (bit 7 of flagword 2) = 0 22LMBDO = 22LMBDO + K_{1b11}	l. If landmark is not tagged as stored, indicate that one has now been stored.
m. Proceed to 22TERM	m. --
96. <u>22TERM</u>	96.
a. TS = 00017_8	a. --
b. Proceed to 60PERF1;	b.
(1) If terminate, proceed to GOTOPOOH;	(1) and (2) P22 is finished.
(2) If proceed, proceed to GOTOPOOH;	
(3) Otherwise, proceed to PROG22A	(3) Recycle to perform another sighting

Results of Coding Review

Flow chart comparison.- A comparison of the above coding and the MIT flow charts in appendix A reveals no differences except occasional insignificant order changing of steps and the omission of the W_8 downgrade (page 9 of appendix A) for a stored landmark in the coding. In this latter instance, as at other times during the review, reference to the actual SUNDISK revision 282 listing was used to resolve difficulties. The W_8 downgrade is not contained in the 282 listing.

Comparison of equation and astronaut procedure to coding.- Besides the comparison between coding and flow charts, checks have been made between coding and equations, and coding and astronaut procedural interfaces, as outlined in reference 5 and reproduced in appendix B. In all instances, the comparisons were satisfactory. P22's interfaces with R52, R53, R02, CSMCONIC, P52, and P00 also appear satisfactory.

Illegal interfaces.- Not all interfaces with P22 are valid, however. Generally, none of the P30's or P40's (P3X or P4X) should be called while P22 is running, or vice versa, due to shared erasable cells (ref. 6). Verb 82 cannot be used during P22 for the same reason (ref. 7). Programs 22 and 20 cannot be operated simultaneously since P22 initializes the W matrix to a 9-by-9 (ref. 8). Program 22 or any program cannot be called by verb 37 if integration is in process, or in P05 until the time specified in reference 8. Before running any program, reference 8 and its updates should be read.

ASTRONAUT PROCEDURES

Crew procedures for exercising the orbital navigation program are presented in appendices B and C. Appendix B contains the procedures and interfaces among the crew, CMC, and the ground, as written by MIT in reference 5. Appendix C gives the concentrated crew procedures for P22 which are to be carried onboard the CM during Apollo 7 (ref. 9).

Comparison of appendices B and C reveals no differences save the omission of the IMU status check (R02) in appendix C.

BIT-BY-BIT TESTING

The final "roped" version of SUNDISK 282 has been tested on the MSC bit-by-bit simulator. This simulator contains an exact representation of the CMC, i.e., the machine language coding is the same.

Results of the bit-by-bit tests for orbital navigation are contained in appendices D and E. The tables (ref. 10) contain comparisons between the W-matrix, b-vector, and state vectors as computed by SUNDISK 282 P22 and by a reliable onboard bench program outlined in reference 3.

Two bit-by-bit cases were run. The first case (appendix D) involved sightings on two known landmarks. The second case (appendix E), is a rerun of the first case with the first landmark considered unknown. In both cases, only three marks could be taken on the second landmark due to constraints within the environmental program associated with the bit-by-bit simulator.

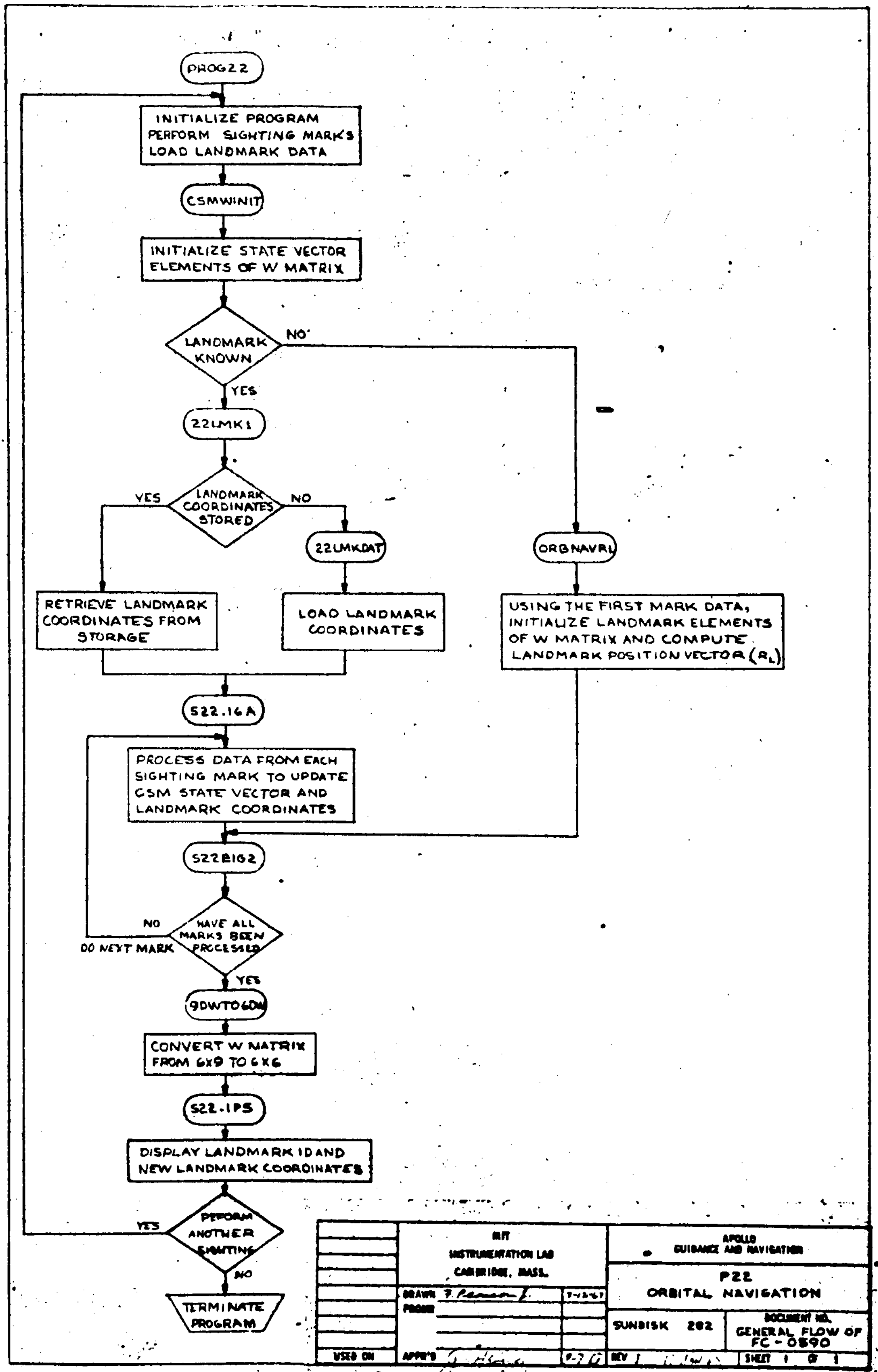
For both cases the results are excellent. Differences between the SUNDISK program and the engineering simulations are small and can be attributed to differences between fixed and floating point machines. These results, added to other studies of P22 with the engineering simulator, indicate the program operates correctly and performs well in flight configuration.

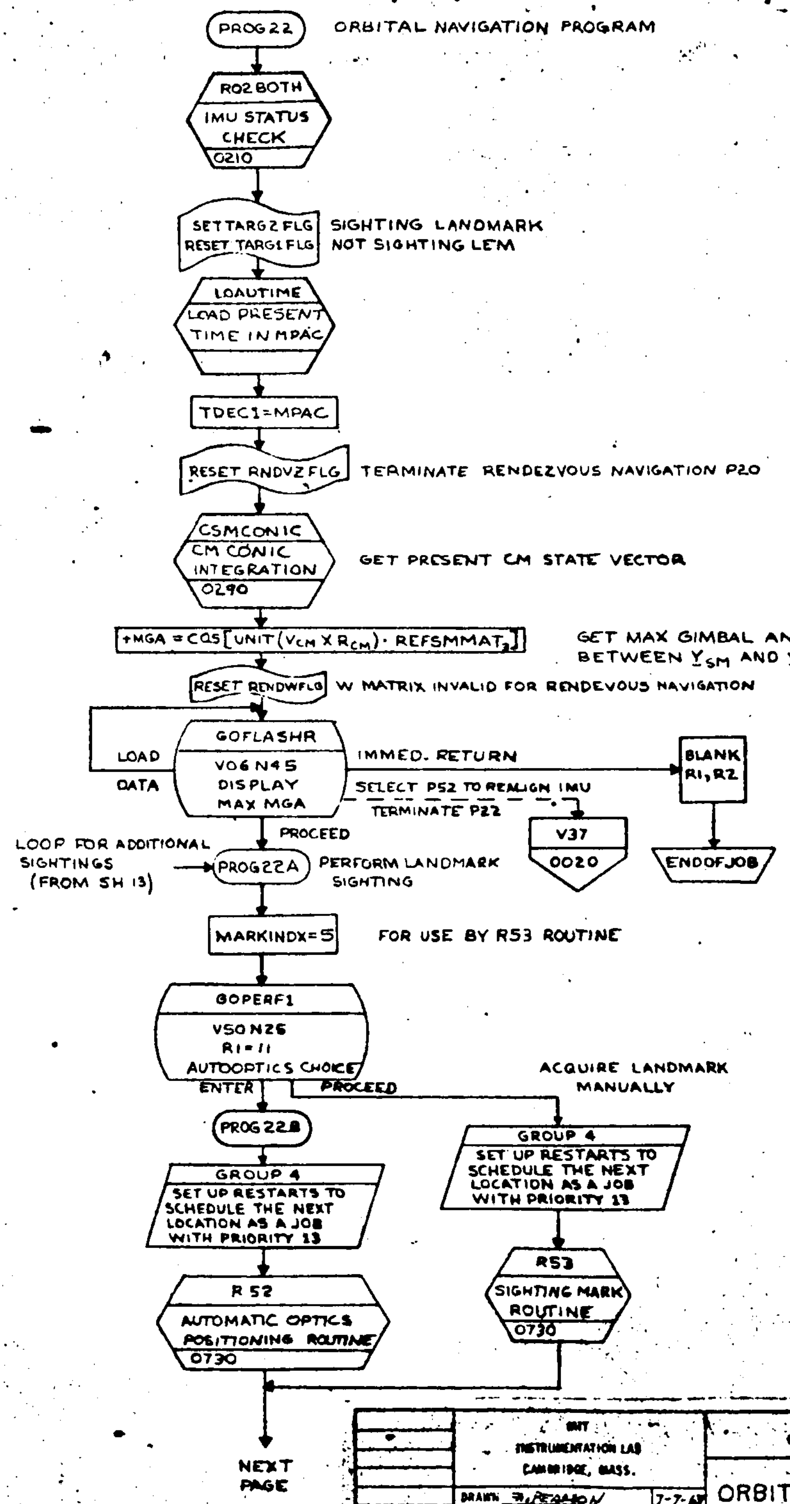
CONCLUSION

From the study of equations, coding, astronaut procedures, and testing which has been presented, SUNDISK 282 orbital navigation program. P22 has been verified as valid for earth orbital flight.

APPENDIX A

MIT FLOWCHARTS FOR SUNDISK REV 212 P22


PROG22
INITIALIZE PROGRAM
PERFORM SIGHTING MARKS
LOAD LANDMARK DATA
CSMWINIT
INITIALIZE STATE VECTOR
ELEMENTS OF W MATRIX
LANDMARK KNOWN
NO
YES
22LMK1
LANDMARK COORDINATES STORED
YES
NO
22LMKDAT
ORBNAVRL
RETRIEVE LANDMARK COORDINATES FROM STORAGE
LOAD LANDMARK COORDINATES
USING THE FIRST MARK DATA, INITIALIZE LANDMARK ELEMENTS OF W MATRIX AND COMPUTE LANDMARK POSITION VECTOR (R_L)
S22.16A
PROCESS DATA FROM EACH SIGHTING MARK TO UPDATE CSM STATE VECTOR AND LANDMARK COORDINATES
S22B162
HAVE ALL MARKS BEEN PROCESSED
NO
DO NEXT MARK
YES
9DWTO6DW
CONVERT W MATRIX FROM 6X9 TO 6X6
S22-1PS
DISPLAY LANDMARK ID AND NEW LANDMARK COORDINATES
PEFORM ANOTHER SIGHTING
YES
NO
TERMINATE PROGRAM
MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.
DRAWN
7-12-67
FROM
USED ON
APPR'D
9-7
REV
APOLLO
GUIDANCE AND NAVIGATION
P22
ORBITAL NAVIGATION
SUNDISK 282
DOCUMENT NO.
GENERAL FLOW OF
FC - 0590
SHEET 1 OF 1



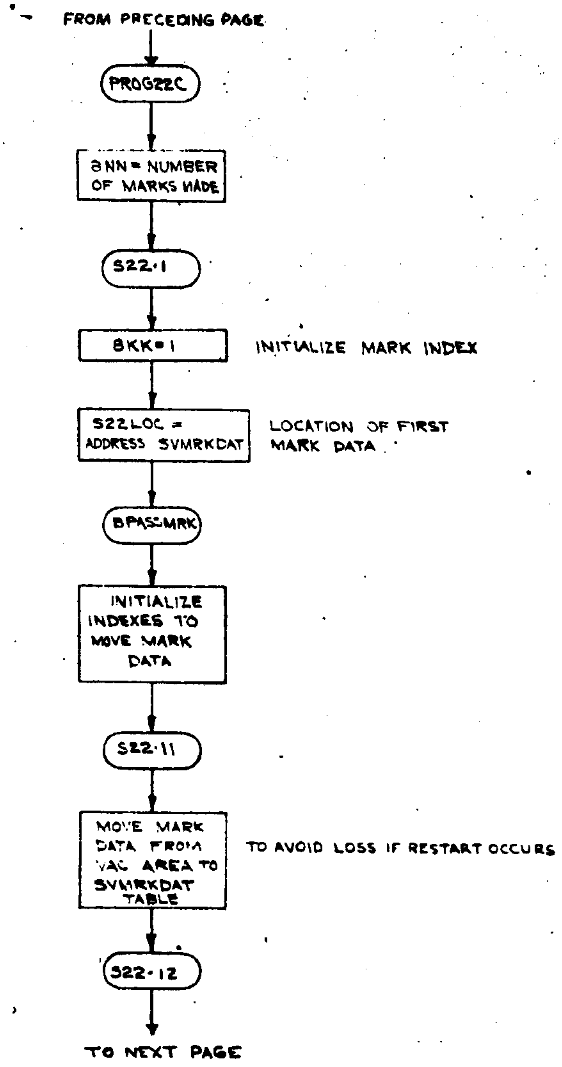
MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

DRAWN 7-7-67
PRGMR
ANALST 8-7-67
DOCMGR
APPR'D 8-2-67
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO.
FC 0590
REV 1 11 JAN 68
SHEET 1 OF 18



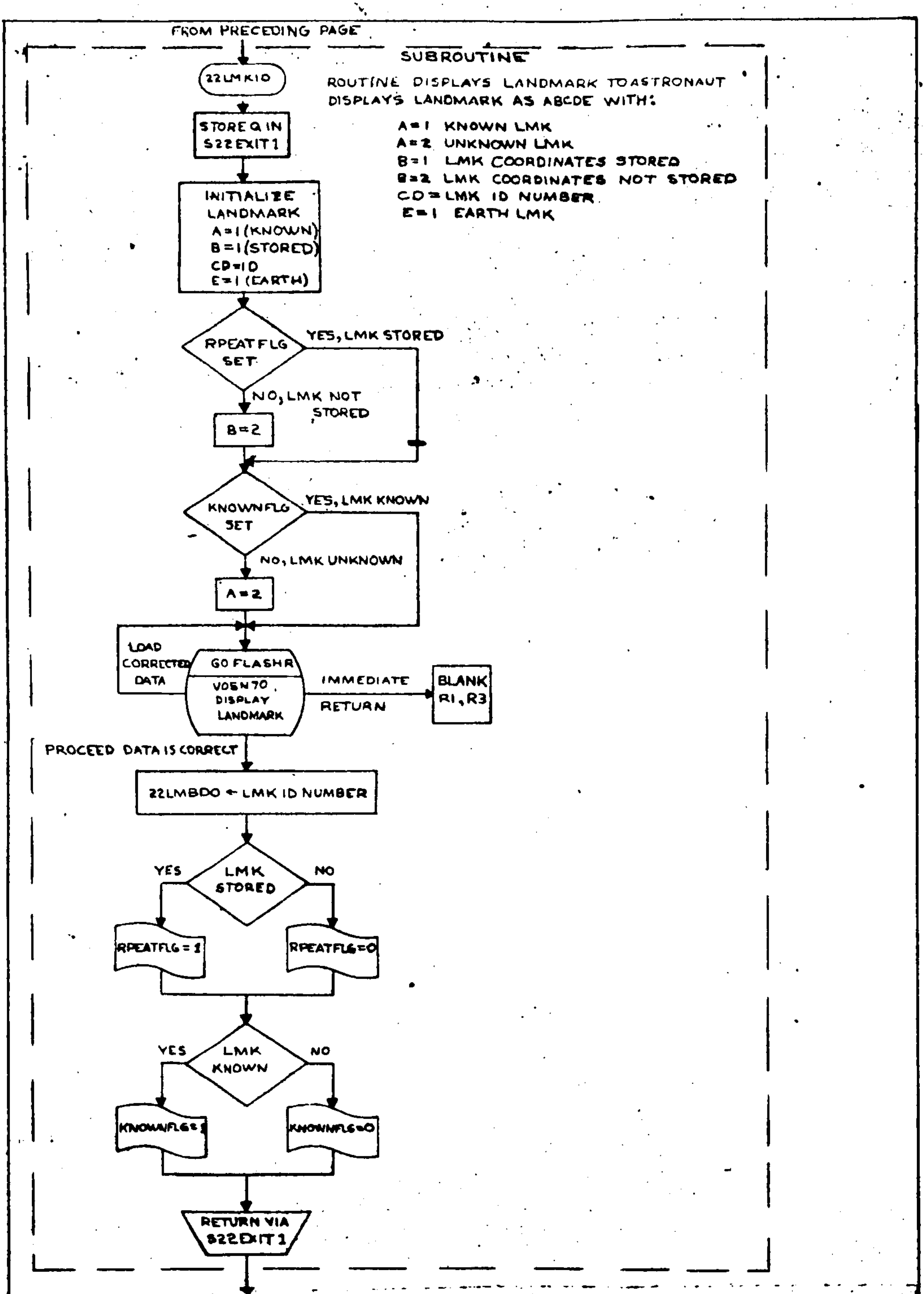
MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

DRAWN 7-10-67
PROG'R 8-1-67
ANALYST 8-7-67
DOC MGR
APPR'D 8-7-67
USED ON

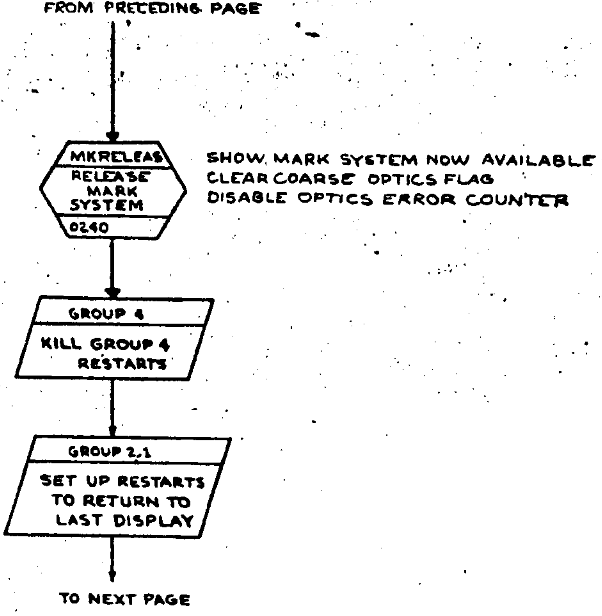
APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO. FC0590
REV 1 17 JAN 68
SHEET 2 OF 18



	MIT INSTRUMENTATION LAB CAMBRIDGE, MASS.	APOLLO GUIDANCE AND NAVIGATION
	DRAWN 7-12-67	P22 ORBITAL NAVIGATION
	PROG'D 8-1-67	
	ANALYST 8-2-67	SUNDISK 282 — DOCUMENT NO. FC0590
	DOC'MN	
USED ON	APPR'D 8-2-67	REV 1 17 JAN 68 — SHEET 3 OF 18



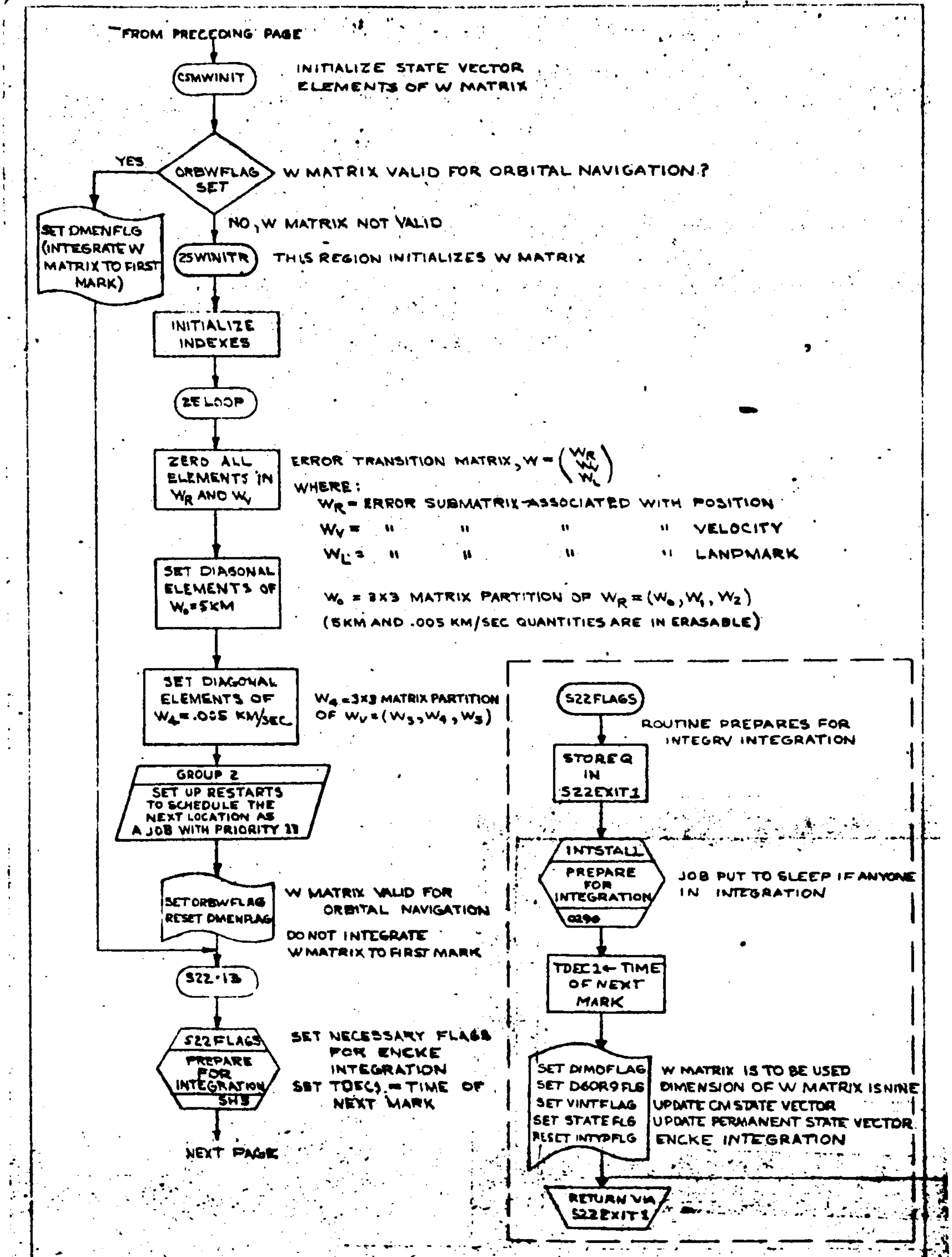
MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

DRAWN F. PEARSON 7/14/67
PROGR 8-1-67
ANALST 8-2-67
DOCHE
APPR'D 8-2-67
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO. FC0590
REV 1 17 JAN 68
SHEET 4 OF 18



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

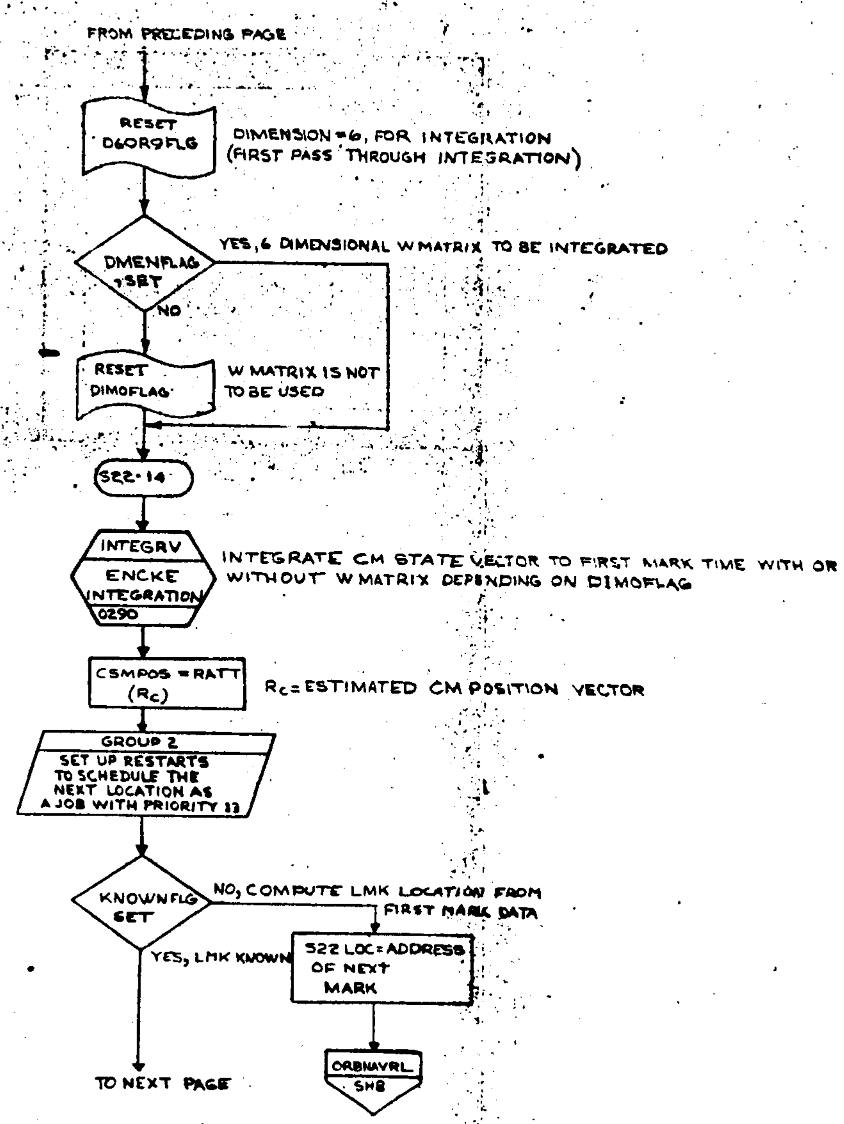
DRAWN O. Duclosson 7-10-67
PRGMR J. Bost 9-1-67
ANALST R. Lanna 8-7-67
DOCMA
APPR'D J. Heinze 8-7-67
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO.
FC 0590

REV 1 17 JAN 68
SHEET 5 OF 18



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

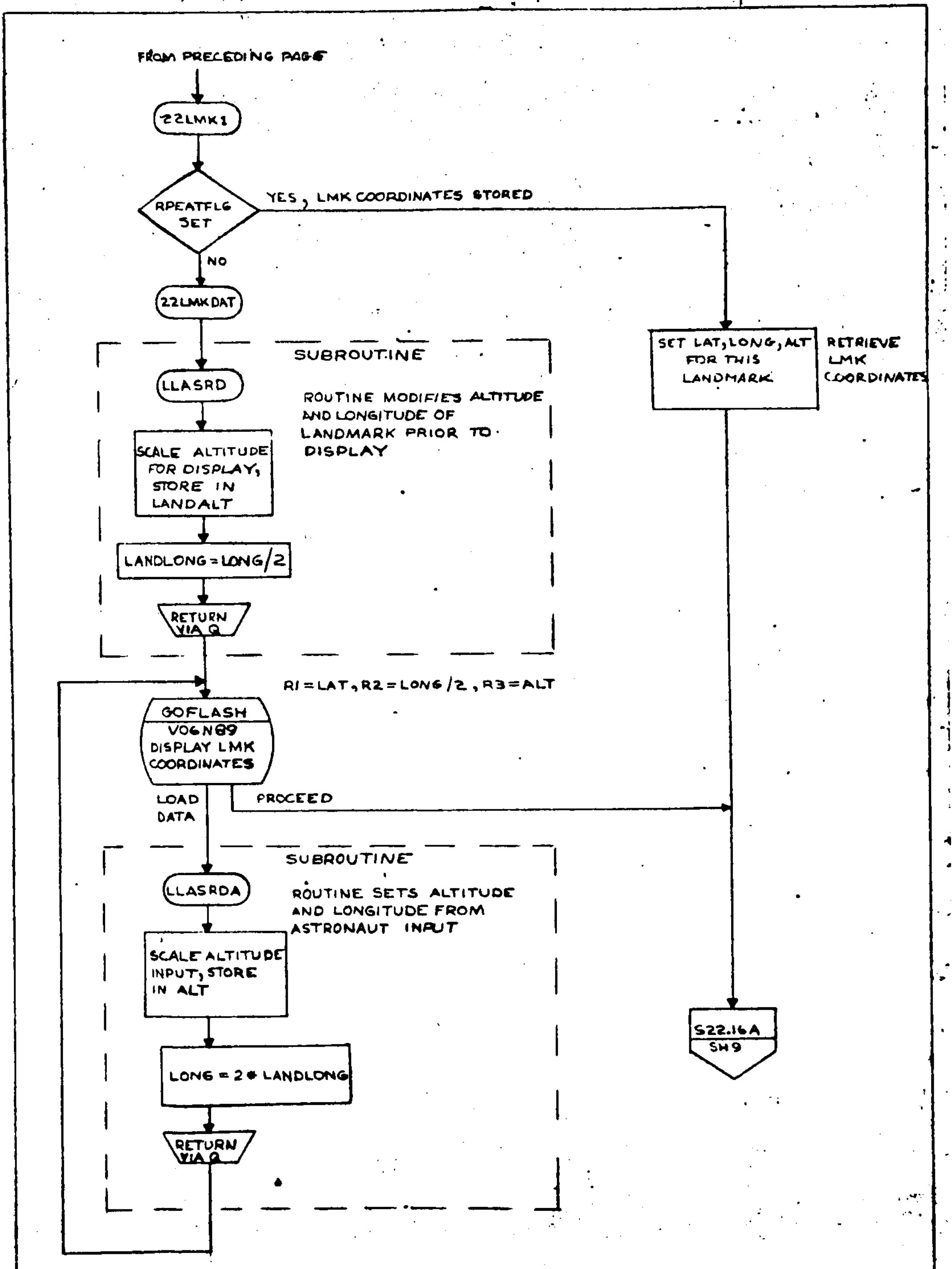
DRAWN 7-10-67
CHECKED
ANALYST 8-2-67
DOC MGR
APPR'D 8-2-67
USED ON

SUNDISK 282

DOCUMENT NO.
FC0590

REV 1 17 JAN 68

SHEET 6. OF 18



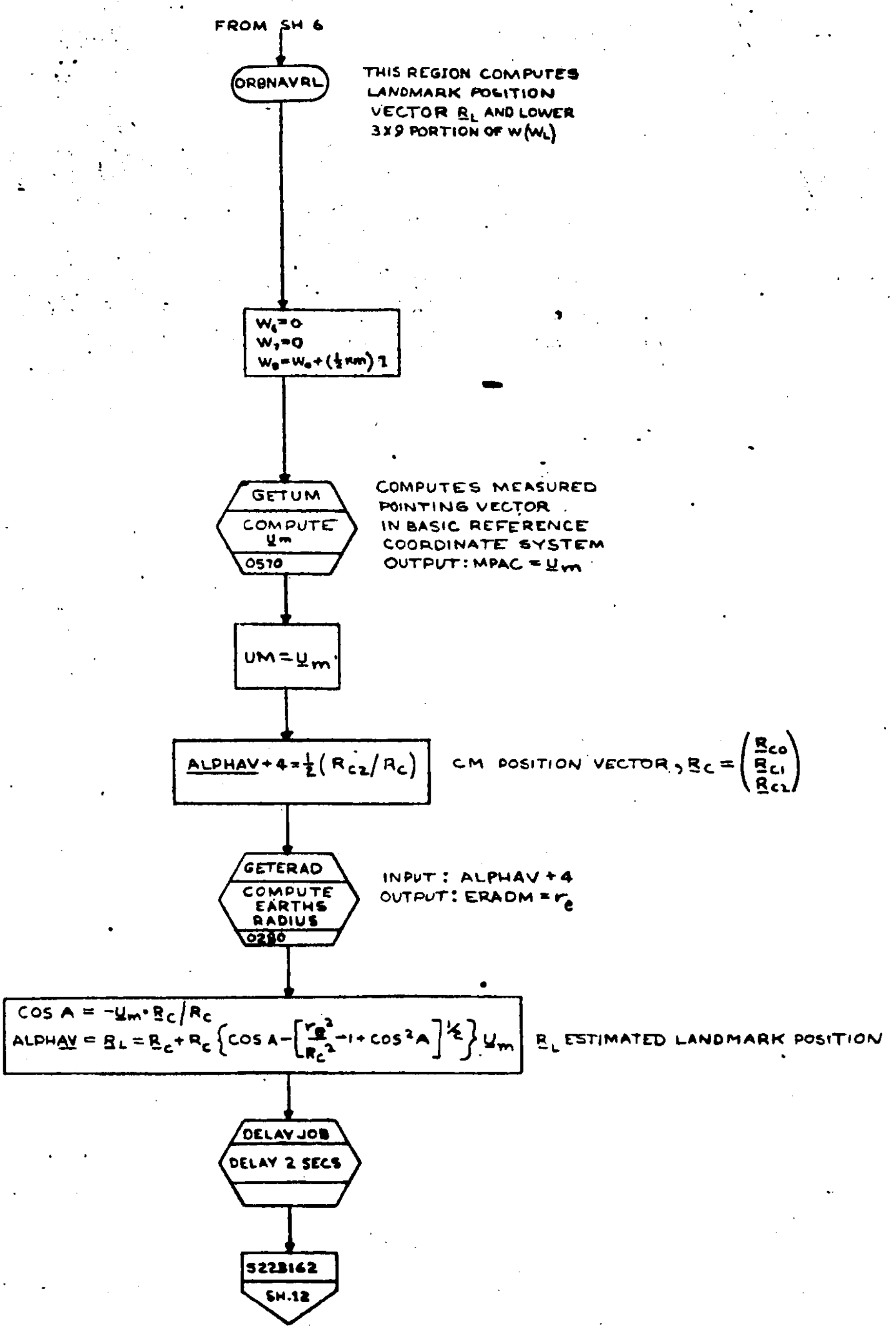
MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

APOLLO
GUIDANCE AND NAVIGATION

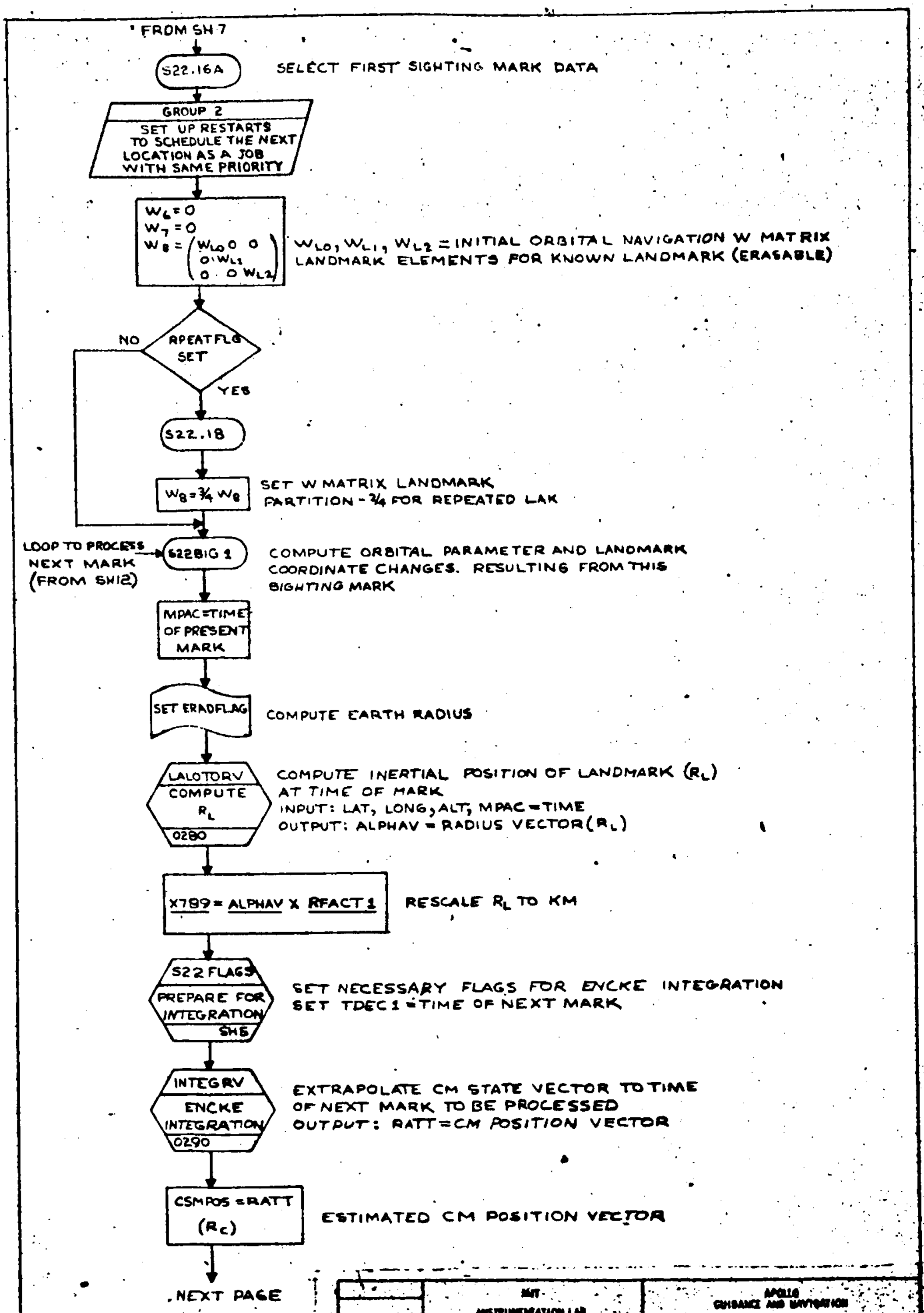
P22
ORBITAL NAVIGATION

DRAWN F PEARSON JR 7-10-67
PRGMR 9-1-67
ANALYST 8-7-67
DOC MR
APPR'D J Hoaire 8-7-67
USED ON

SUNDISK 282
DOCUMENT NO. FC0590
REV 1 17 JAN 68
SHEET 7 OF 18



	MIT INSTRUMENTATION LAB CAMBRIDGE, MASS.	APOLLO GUIDANCE AND NAVIGATION
	DRAWN F. PEARSON 7-17-6	P22 ORBITAL NAVIGATION
	PRGMR 6-2-67	SUNDISK 282
	ANALYST A-21	DOCUMENT NO. FC0590
	DOCMR	
USED ON	APPR'D 8-7-6	REV 1 17 JAN 68 SHEET [illegible] OF 18



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

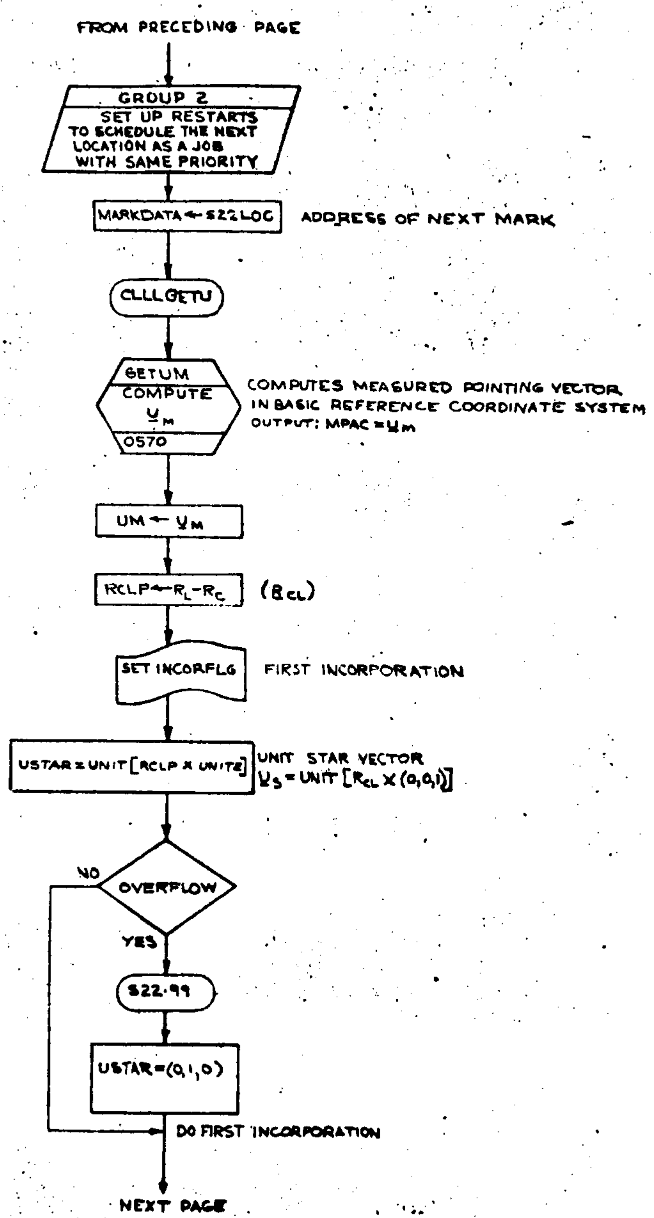
DRAWN F. PEARSON Jr. 7-10-67
PROGMR 8-1-67
ANALST 8-7-67
DOCMNR
APPR'D 8-7-67
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO.
FC 0590

REV 1 17 JAN 68
SHEET 9 OF [illegible]



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

DRAWN 7-1-
PRGMR 8-1-67
ANALYST 8-2-67
DOCMN.
APP'D 8-7-67

USED ON

APOLLO
GUIDANCE AND NAVIGATION

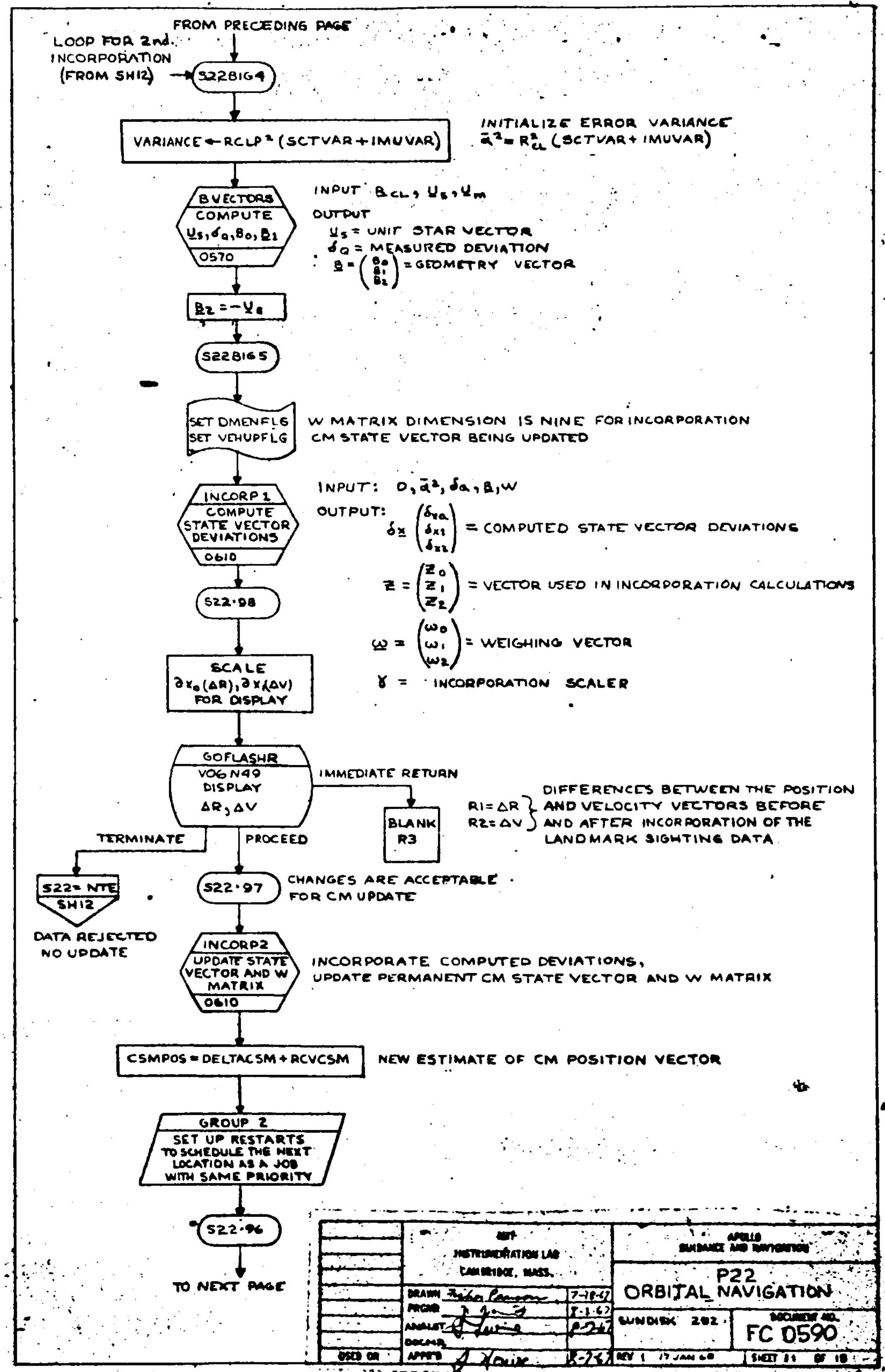
P22
ORBITAL NAVIGATION

SUNDISK 282

DOCUMENT NO.
FC 0590

REV 1 17 JAN 68

SHEET 10 OF 18



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

DRAWN	7-18-67
PRGMR	8-1-67
ANALST	8-2-67
DOCMGR	
APPR'D	8-7-67

USED ON

APOLLO
GUIDANCE AND NAVIGATION

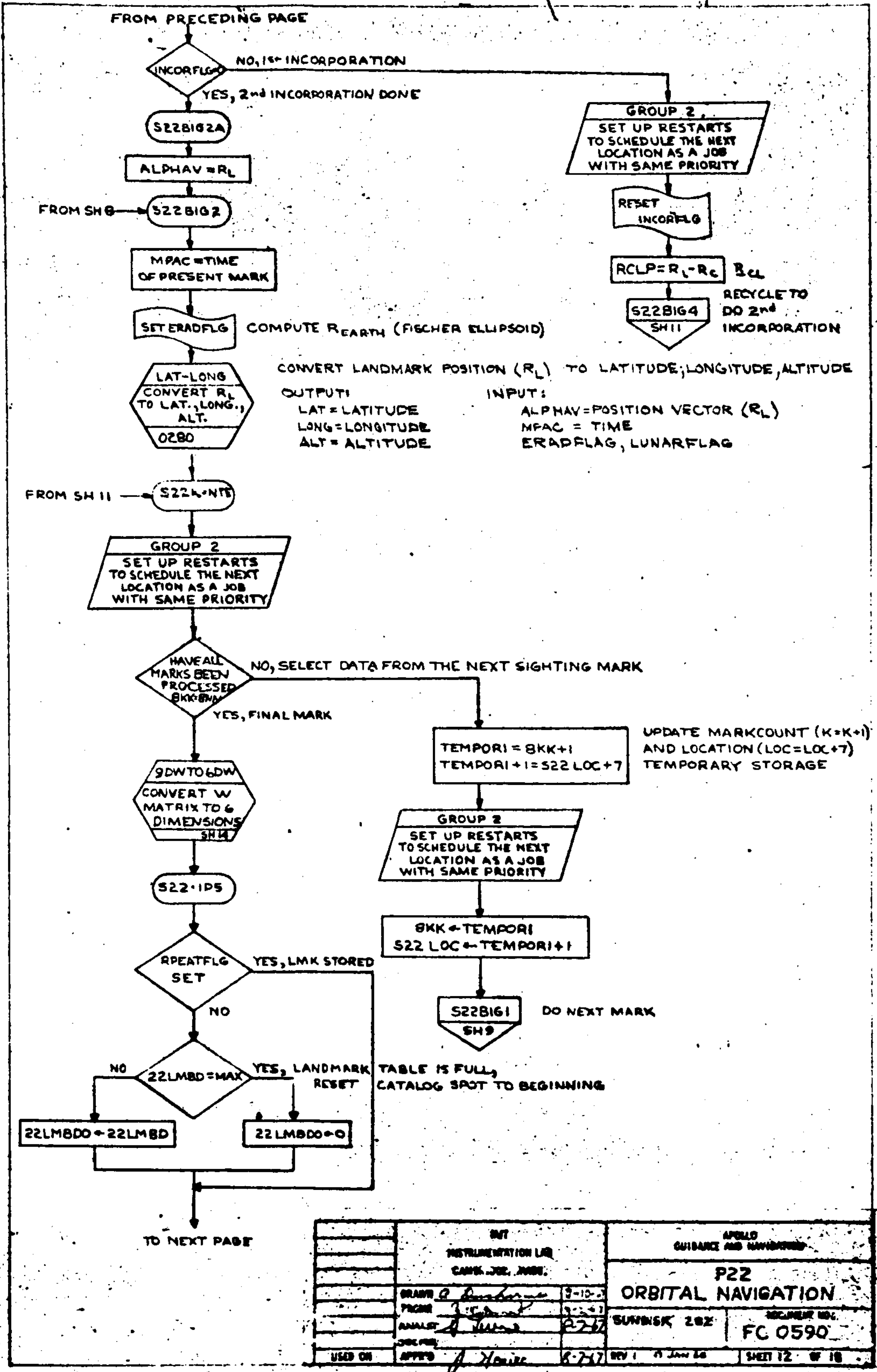
P22
ORBITAL NAVIGATION

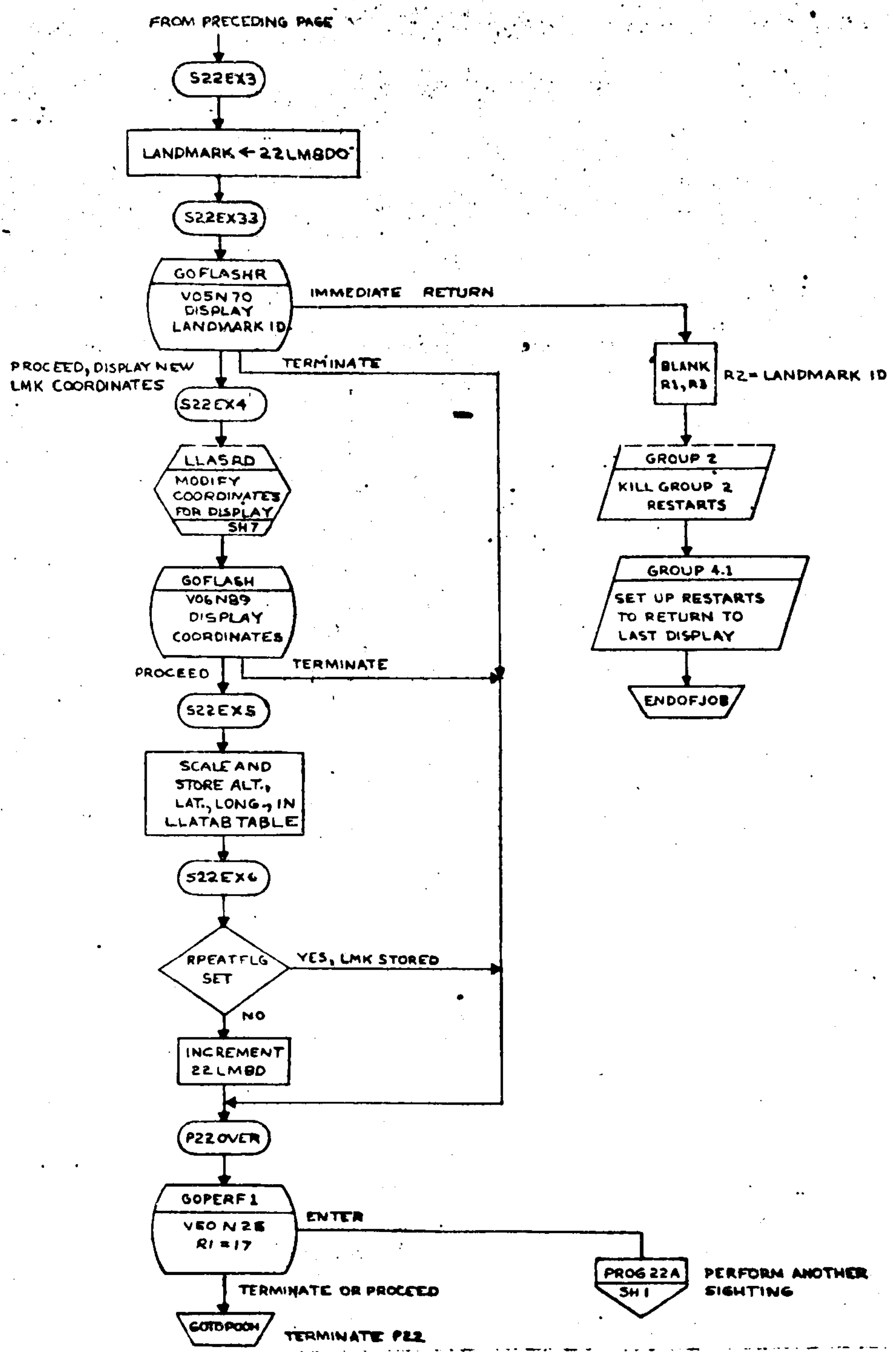
SUNDISK 282

DOCUMENT NO.
FC 0590

REV 1 17 JAN 68

SHEET 11 OF 18


FROM PRECEDING PAGE
INCORFLG
NO, 1st INCORPORATION
YES, 2nd INCORPORATION DONE
S22BIG2A
ALPHAV = R_L
FROM SH 8
S22BIG2
MPAC = TIME OF PRESENT MARK
SET ERADFLG
COMPUTE R_EARTH (FISCHER ELLIPSOID)
LAT-LONG
CONVERT R_L TO LAT., LONG., ALT.
0280
CONVERT LANDMARK POSITION (R_L) TO LATITUDE, LONGITUDE, ALTITUDE
OUTPUT:
LAT = LATITUDE
LONG = LONGITUDE
ALT = ALTITUDE
INPUT:
ALPHAV = POSITION VECTOR (R_L)
MPAC = TIME
ERADFLAG, LUNARFLAG
GROUP 2
SET UP RESTARTS TO SCHEDULE THE NEXT LOCATION AS A JOB WITH SAME PRIORITY
RESET INCORFLG
RCLP = R_L - R_C
R_CL
S22BIG4
SH 11
RECYCLE TO DO 2nd INCORPORATION
FROM SH 11
S22LNTS
GROUP 2
SET UP RESTARTS TO SCHEDULE THE NEXT LOCATION AS A JOB WITH SAME PRIORITY
HAVE ALL MARKS BEEN PROCESSED 8KK=8N
NO, SELECT DATA FROM THE NEXT SIGHTING MARK
YES, FINAL MARK
TEMPORI = 8KK+1
TEMPORI+1 = S22 LOC+7
UPDATE MARKCOUNT (K=K+1) AND LOCATION (LOC=LOC+7) TEMPORARY STORAGE
9DWTO6DW
CONVERT W MATRIX TO 6 DIMENSIONS
SH 14
GROUP 2
SET UP RESTARTS TO SCHEDULE THE NEXT LOCATION AS A JOB WITH SAME PRIORITY
8KK ← TEMPORI
S22 LOC ← TEMPORI+1
S22·IPS
RPEATFLG SET
YES, LMK STORED
NO
S22BIG1
SH 9
DO NEXT MARK
NO
22LMBD = MAX
YES, LANDMARK TABLE IS FULL, RESET CATALOG SPOT TO BEGINNING
22LMBDO ← 22LMBD
22LMBDO ← 0
TO NEXT PAGE
MIT INSTRUMENTATION LAB CAMB. 39, MASS.
DRAWN
CHECKED
ANALYST
APPR'D
USED ON
APOLLO GUIDANCE AND NAVIGATION
P22
ORBITAL NAVIGATION
SUNDISK 282
DOCUMENT NO.
FC 0590
REV 1
SHEET 12 OF 18



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

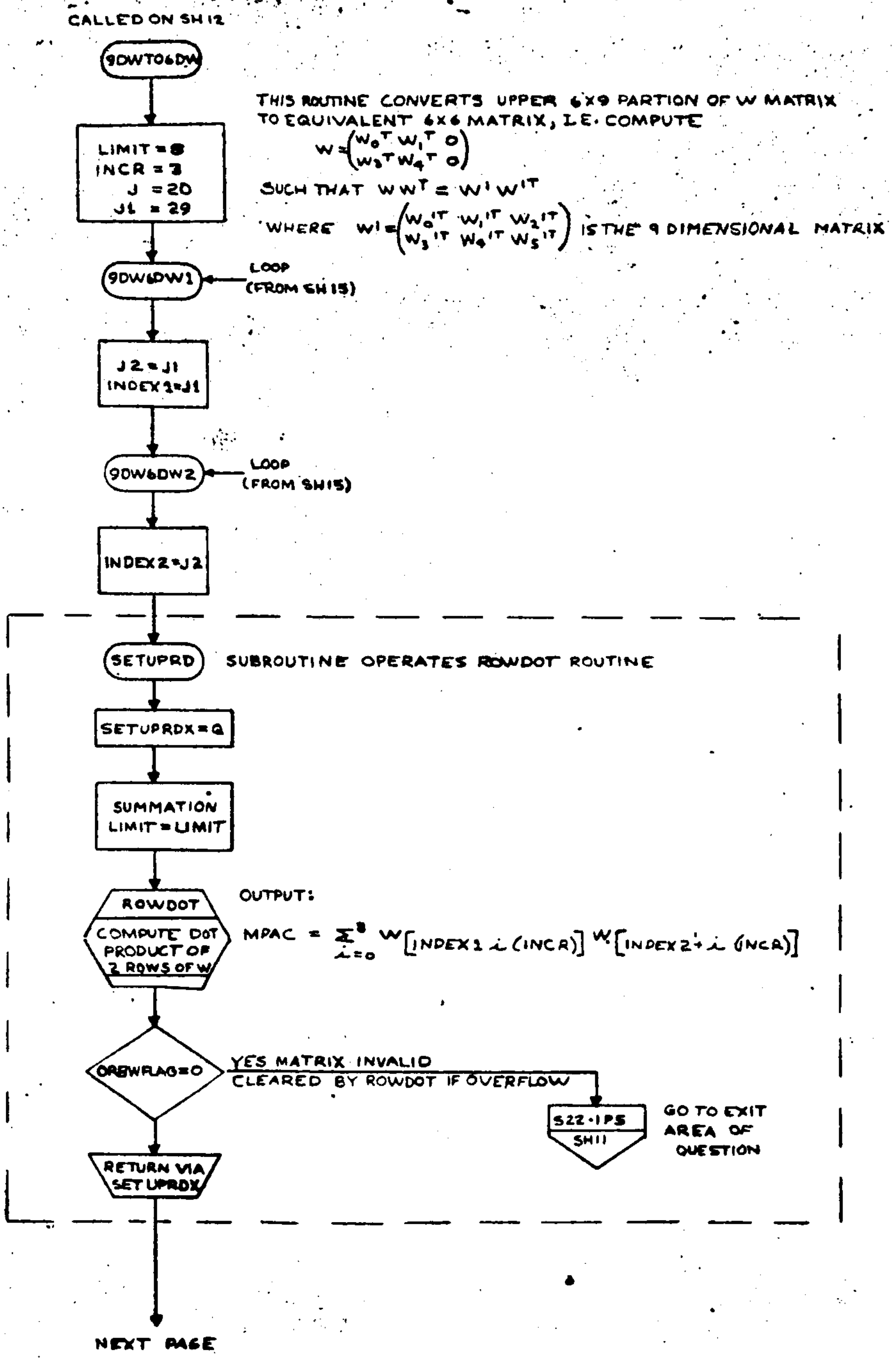
DRAWN P. PEARSON 7-12-67
PRGMR 8-1-67
ANALYST 8-7-67
CHKG
APPR'D 8-7-67
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO. FC 0590
REV 1 17 JAN 68
SHEET 13 OF 18

MATRIX CONVERSION ROUTINE



THIS ROUTINE CONVERTS UPPER 6X9 PARTION OF W MATRIX TO EQUIVALENT 6X6 MATRIX, I.E. COMPUTE

$$W = \begin{pmatrix} W_0^T & W_1^T & 0 \\ W_3^T & W_4^T & 0 \end{pmatrix}$$

SUCH THAT $WW^T = W'W'^T$

WHERE $W' = \begin{pmatrix} W_0'^T & W_1'^T & W_2'^T \\ W_3'^T & W_4'^T & W_5'^T \end{pmatrix}$ IS THE 9 DIMENSIONAL MATRIX

SUBROUTINE OPERATES ROWDOT ROUTINE

OUTPUT:

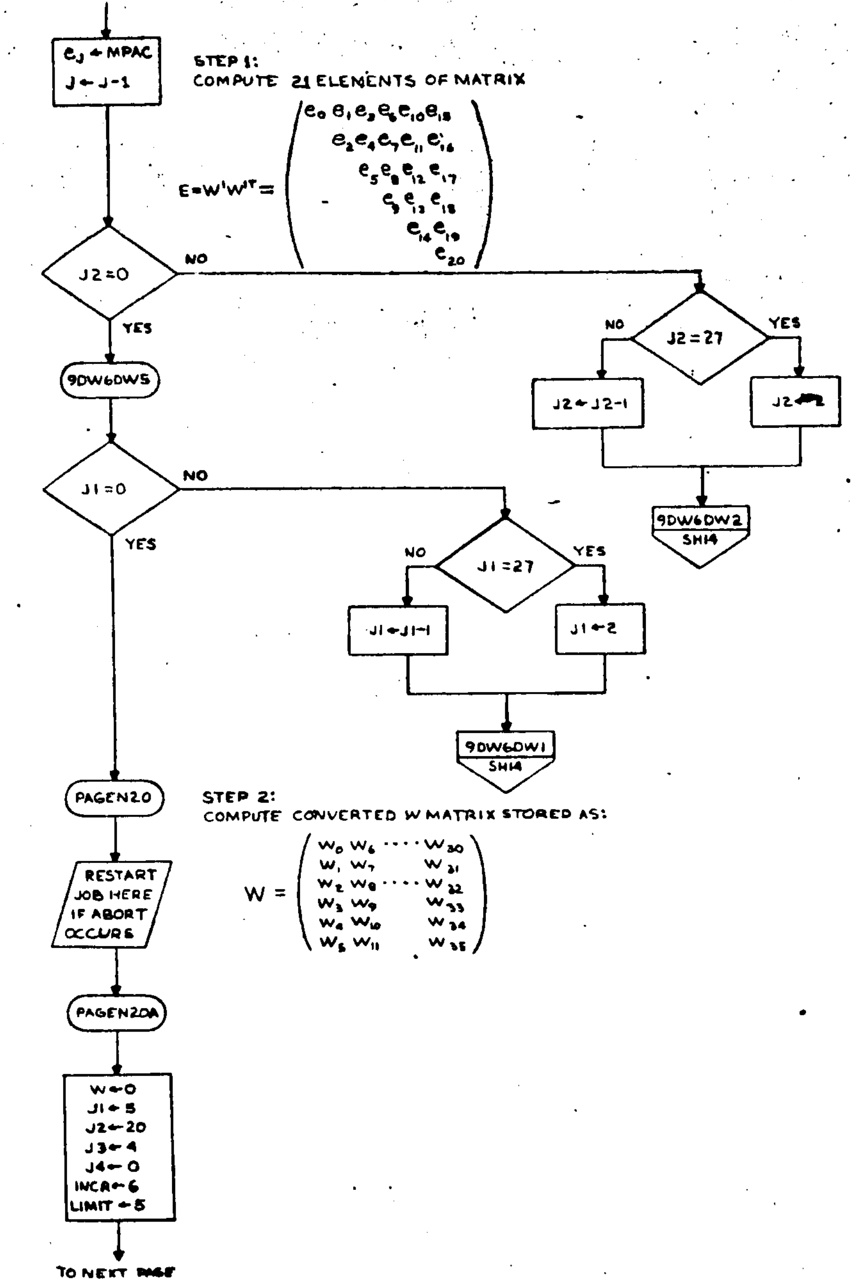
$$MPAC = \sum_{i=0}^{8} W\left[INDEX1\ i\ (INCR)\right] W\left[INDEX2 + i\ (INCR)\right]$$

CLEARED BY ROWDOT IF OVERFLOW

GO TO EXIT AREA OF QUESTION

	MIT INSTRUMENTATION LAB CAMBRIDGE, MASS.		APOLLO GUIDANCE AND NAVIGATION	
	DRAWN F. PEARSON	7-11-67	P22 ORBITAL NAVIGATION	
	PROGR. P. SMITH	8-3-67	SUNDISK 282	DOCUMENT NO. FC 0590
	ANALST	8-7-67		
	DOCMG			
USED ON	APP'D J. Henize	8-7-67	REV 1 17 JAN 68	SHEET 14 OF 18

MATRIX CONVERSION ROUTINE



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

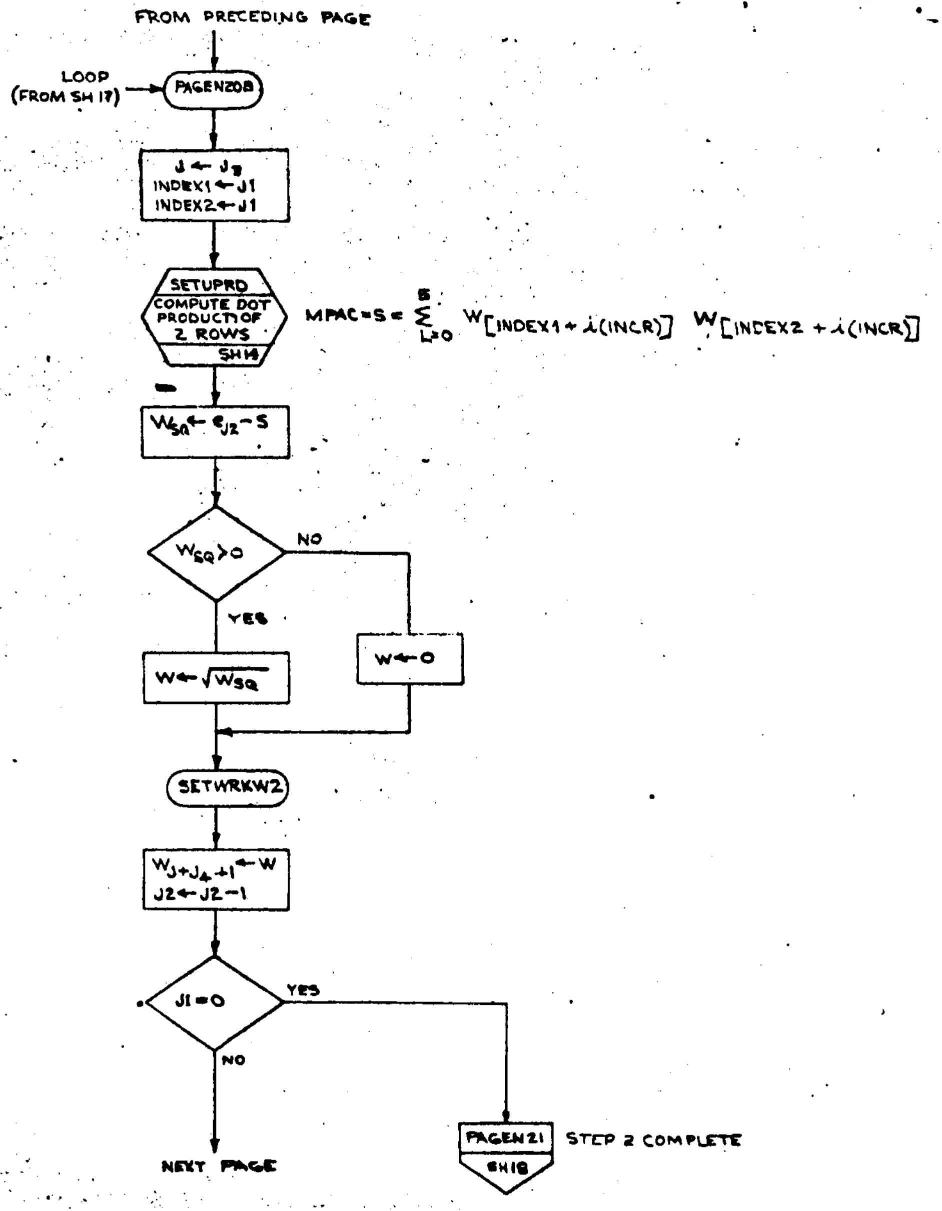
DRAWN Fika Pearson 7-19-67
PROGR R. Smith
ANALST 8-2-67
DOC MGR
APPR'D J. Hemize 8-2-67
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO.
FC 0590
REV 1 17 AUG 69
SHEET 15 OF 18

MATRIX CONVERSION ROUTINE



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

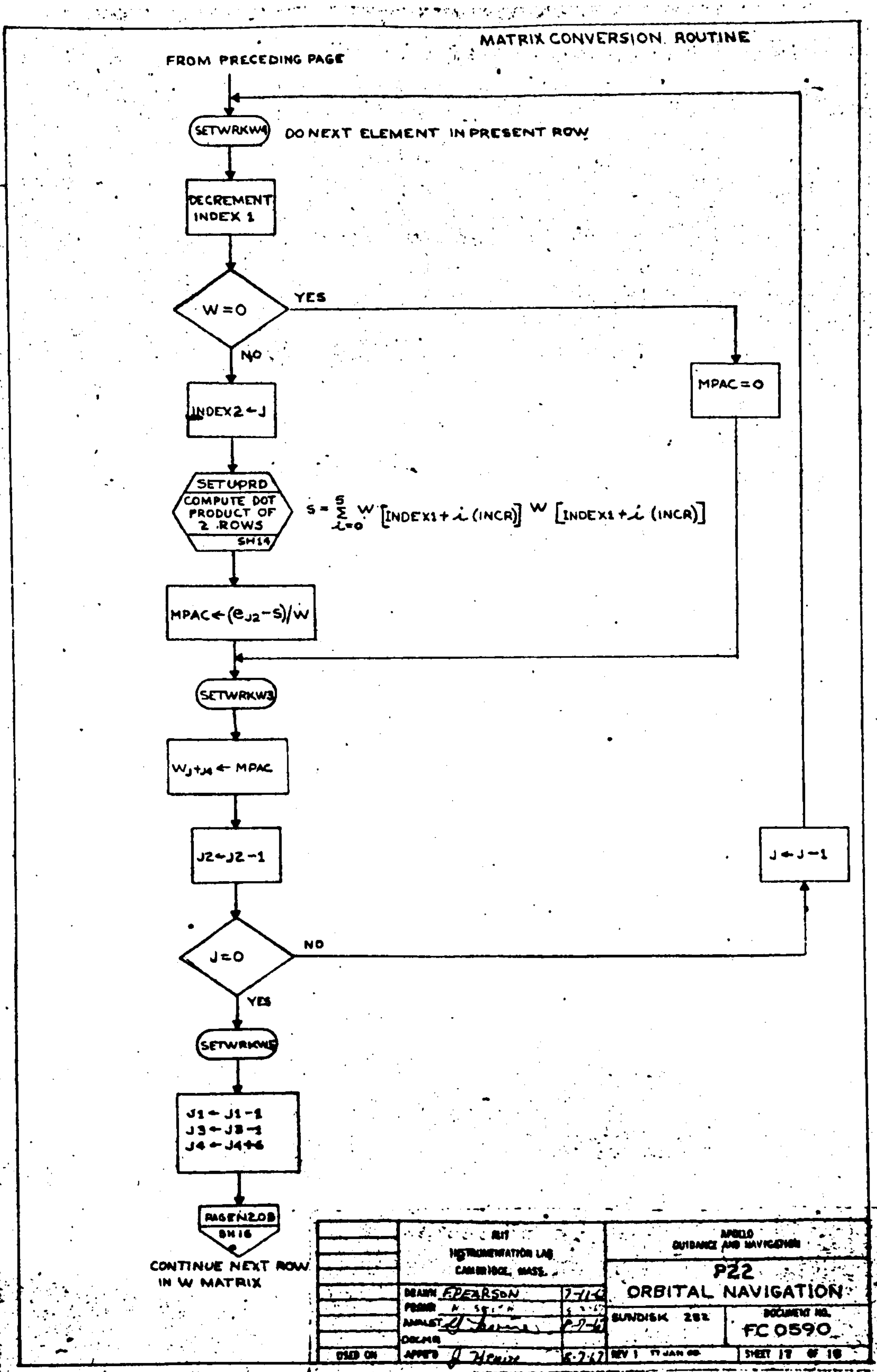
DRAWN [illegible] 7-12-6
PROGR. [illegible] 6-2-67
ANALYST [illegible]
DOCMEN
APP'D [illegible] 8-2-67
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282
DOCUMENT NO.
FC 0590
REV 1 17 JAN 68
SHEET 16 OF 18

MATRIX CONVERSION ROUTINE



MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

DRAWN	F.PEARSON	7-11-6
PROGR	N. SEARS	
ANALST		
DOC.MGR		
APP'D		6-7-67
USED ON		

SUNDISK 282

DOCUMENT NO. FC 0590

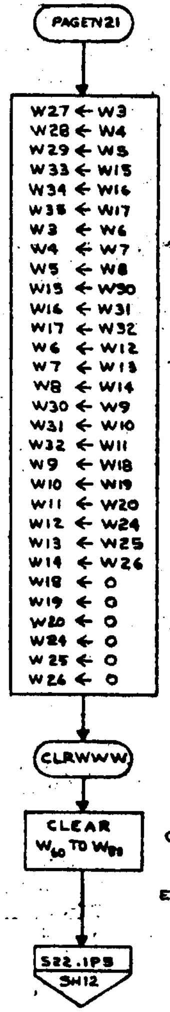
REV 1 JAN

SHEET 17 OF 18

MATRIX CONVERSION ROUTINE

STEP 3, ORDER MATRIX IN STANDARD MANNER. FINAL RESULT IS OF THE FORM

$$W = \begin{pmatrix} W_0^T & W_1^T \\ W_3^T & W_4^T \end{pmatrix} = \begin{pmatrix} W_0 & W_3 & W_6 & W_9 & W_{12} & W_{15} \\ W_1 & W_4 & W_7 & W_{10} & W_{13} & 0 \\ W_2 & W_5 & W_8 & W_{11} & 0 & 0 \\ W_{27} & W_{30} & W_{33} & 0 & 0 & 0 \\ W_{28} & W_{34} & 0 & 0 & 0 & 0 \\ W_{29} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



CLEAR MATRIX TEMPORARY STORAGE

END OF MATRIX CONVERSION ROUTINE

MIT
INSTRUMENTATION LAB
CAMBRIDGE, MASS.

DRAWN J. PEARSON 7-12-67
PROGRM
ANALYST
DOCMNR
APPR'D
USED ON

APOLLO
GUIDANCE AND NAVIGATION

P22
ORBITAL NAVIGATION

SUNDISK 282

DOCUMENT NO.
FC 0590

REV 1 11 JAN 68

SHEET 18 OF 18

APPENDIX B

MIT APOLLO COMPUTER LOGIC CHECKLIST INTERFACE

(GSOP IV)

APOLLO COMPUTER LOGIC CHECKLIST INTERFACE

ORBITAL NAVIGATION PROGRAM (P22)

LOGIC REV 28 03/07/68
CHECKLIST REV 29 06/06/68

PURPOSE

(1) TO LOCATE AND TRACK A LANDMARK SUITABLE FOR NAVIGATION PURPOSES. ACQUISITION MAY BE AIDED BY THE CMC WITH THE OPTICS AUTOMATICALLY POINTED AT A "KNOWN" (SEE ASSUMPTIONS BELOW) LANDMARK WHOSE LATITUDE, LONGITUDE, AND ALTITUDE HAVE BEEN KEYED INTO THE DSKY BY THE ASTRONAUT. IT MAY ALSO BE DONE BY THE ASTRONAUT WITH THE OPTICS MANUALLY POINTED AT A "KNOWN" LANDMARK PREDETERMINED BY THE ASTRONAUT OR A "KNOWN" OR "UNKNOWN" (SEE ASSUMPTION BELOW) LANDMARK SELECTED AT WILL DURING SEARCH FOR TARGETS OF OPPORTUNITY.

(2) TO OBTAIN SIGHTING MARKS ON THE CHOSEN LANDMARK.

(3) TO CALCULATE THE ORBITAL PARAMETER CHANGES GENERATED BY THE LANDMARK SIGHTING.

(4) TO DISPLAY THE ORBITAL PARAMETER CHANGES GENERATED, FOR DECISION AS TO THEIR VALIDITY BY THE ASTRONAUT/GROUND BEFORE THEIR INCORPORATION INTO THE CMC CALCULATION OF SC POSITION AND VELOCITY.

(5) TO PROVIDE UPDATED COORDINATES OF THE KNOWN LANDMARKS.

(6) TO PROVIDE COORDINATES OF UNKNOWN LANDMARKS (LANDMARK DESIGNATION ROUTINE).

ASSUMPTIONS:

(1) THERE ARE TWO TYPES OF LANDMARK TRACKING METHODS:

(A) "KNOWN" LANDMARK TRACKING-THE TRACKING OF A LANDMARK IDENTIFIED TO THE CMC BY LATITUDE, LONGITUDE,AND ALTITUDE AFTER THE MARKS ARE COMPLETE.

(B) "UNKNOWN" LANDMARK TRACKING-.THE TRACKING AND MARKING OF A LANDMARK OR SURFACE FEATURE IDENTIFIED TO THE CMC ONLY AS AN UNKNOWN LANDMARK.

(2) DURING THIS PROGRAM THE GNCS HAS THE CAPABILITY FOR ATTITUDE CONTROL OF THE VEHICLE. POSSIBLE ATTITUDE CONTROL METHODS MIGHT BE AS FOLLOWS (IN ALL CASES CARE MUST BE TAKEN TO MONITOR POSSIBLE IMPENDING IMU GIMBAL LOCK).

(A) MANUAL CONTROL BY THE PILOT OR NAVIGATOR WITH THE ROTATIONAL HAND CONTROLLER.

(B) MANUAL RATE CONTROL BY THE NAVIGATOR WITH THE MINIMUM IMPULSE CONTROL IN THE GNC FREE MODE.

(3) THE PROGRAM MAY BE PERFORMED WITH SIVB ATTACHED IF THE LAUNCH VEHICLE GUIDANCE SWITCH IS PLACED IN THE CMC POSITION THEREBY PERMITING SIVB ATTITUDE CONTROL WITH THE ROTATIONAL CONTROLLER. GNC A/P CONTROL IS REQUIRED IN THIS CASE

(4) THE GNCS STARTUP AND CHECKOUT PROGRAM (P05) AND THE INFLIGHT ALIGNMENT-IMU ORIENTATION DETERMINATION PROGRAM (P51) MUST BE COMPLETED BEFORE THE COMPLETION OF THIS PROGRAM.
THE ISS MAY BE:

(A) OFF (STANDBY).

(B) ON, AND NOT ALIGNED SINCE TURN ON.

(C) ON AND AT AN INERTIAL ORIENTATION KNOWN ONLY INACCURATELY BY THE CMC, I.E. HAVING BEEN ALIGNED AT LEAST ONCE SINCE TURN ON BUT NOT WITHIN THE LAST 3 HOURS.

(D) ON AND AT AN INERTIAL ORIENTATION KNOWN ACCURATELY BY THE CMC, I.E. HAVING BEEN ALIGNED WITHIN THE LAST 3 HOURS.

IF (A) IS TRUE AND P22 IS SELECTED, THE CMC WILL REQUEST SELECTION OF P05.
IF (B) IS TRUE AND P22 IS SELECTED, THE CMC WILL REQUEST SELECTION OF P51.
IF (C) IS TRUE THE CMC MAY OR MAY NOT HAVE A SATISFACTORY INERTIAL REFERENCE
IF (D) IS TRUE THE CMC HAS A SATISFACTORY INERTIAL REFERENCE.

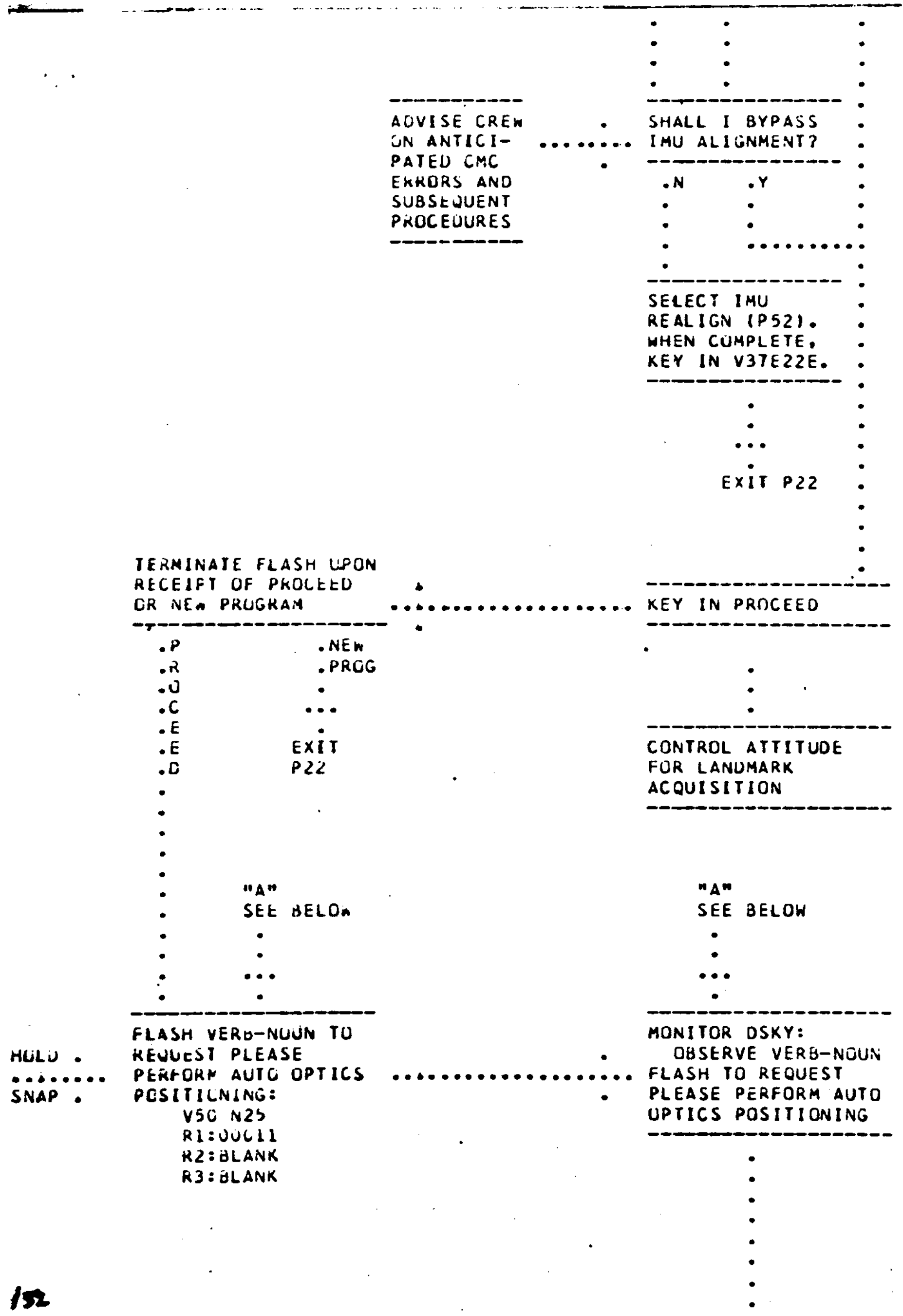
(5) THE GROUND TRACK DETERMINATION PROGRAM P21 IS AVAILABLE TO AID THE CREW IN THE SELECTION OF APPROPIATE LANDMARKS PRIOR TO THE SELECTION OF P22 AND TO PROVIDE UPDATED GROUND TRACK INFORMATION AFTER THE COMPLETION OF P22.

(6) THE PROGRAM IS SELECTED BY THE ASTRONAUT BY DSKY ENTRY.

64

PROG CONT	CMC	GROUND	CREW	CHECKLIST	TIME	TOTAL TIME
			CREW PROG. SELECTION			
	START ORBITAL NAVIGATION PROGRAM (P22) DISPLAY PROGRAM 22		KEY IN ORBITAL NAVIGATION PROGRAM (P22) V 37 E 22 E	I THE FOLLOWING ARE REQUIRED: CMC - ON IMU - ON SCS - ON G/N PWR - AC1 G/N PWR OPTICS - ON OPTICS MODE - MAN THEN ZERO (15 SEC)		#10
			MONITOR DSKY: OBSERVE DISPLAY OF PROGRAM 22	MANEUVER CSM TO ATTITUDE DESIRED FOR LMK ACQUISITION: 1. USE RHC TO ALIGN SHAFT AXIS IN ORBITAL PLANE AND APPROX 10 DEGREES FORWARD OF LOCAL VERTICAL (MAINTAIN COAS ON HORIZON)		#20
				2. USE MIN IMPULSE CONT TO MAINTAIN SC PITCH RATE TO OFFSET ORBITAL RATE		
	DO IMU STATUS CHECK ROUTINE (R02)		DO IMU STATUS CHECK ROUTINE (R02)	3. ROLL CSM AS REQUIRED TO AVOID SHAFT AXIS PASSING WITHIN 10 DEGREES OF LMK		#30
				II KEY V37E 22E		

```
                       .
                       .
                       .
                       .
                       .
        -------------------
**      RESET RENDEZVOUS                                       III  PERFORM IMU STATUS CHECK
*28     FLAG                                                        ROUTINE (R02)
**      -------------------
                       .
                       .
                       .
        -------------------
        COMPUTE ANGLE BE-
        TWEEN Y AND V X R.
               -SM  -   -
        THIS ANGLE IS THE
        MAXIMUM MIDDLE GIM-
        BAL ANGLE POSSIBLE
        ASSUMING THE SC X
        AXIS IS KEPT IN THE
        ORBITAL PLANE.
        -------------------
                       .
                       .
                       .
        -------------------                      -------------------
        FLASH VERB-NOUN TO                       MONITOR DSKY:                     IV   CHECK CMC FLASHES VERB-NOUN TO
        REQUEST RESPONSE AND                       OBSERVE VERB-NOUN                    REQUEST RESPONSE AND DISPLAYS
HOLD .  DISPLAY MAX POSSIBLE                   . FLASH TO REQUEST RE-                   MAXIMUM MIDDLE GIMBAL ANGLE IF THE
........MIDDLE GIMBAL ANGLE  ................... SPONSE AND DISPLAY                     SC X AXIS IS KEPT IN THE ORBIT
SNAP .  IF THE X AXIS IS                       . OF MAX POSSIBLE                        PLANE
        HELD IN THE ORBIT                        MIDDLE GIMBAL ANGLE
        PLANE.                                   IF THE SC X AXIS IS                      FL V06 N45
            V06 N45                              KEPT IN THE ORBIT                          BLANK
            R1:BLANK                             PLANE                                      BLANK
            R2:BLANK                             -------------------                        MAX MGA       XXX.XX DEG
            R3:MGA                                          .
        IN DEG. TO NEAR-                                    .
        EST .01 DEG.                                        .
        -------------------                      -------------------
                      .                          IS MGA GREATER THAN
                      .                          60 DEG? IF SO, IT IS
                      .                          ADVISABLE TO REALIGN
                      .                          THE IMU IF TIME PER-
                      .                          MITS.
                      .                          -------------------
                      .                          .Y       .N
                      .                          .         .
                      .                          .         .
        -------------------                      .  ----------------
        WAIT FOR KEYBOARD                        .  HAS ISS BEEN
        ENTRY:                                   .  ALIGNED IN PAST
                                                 .  3 HOURS?
                                                 .  ----------------
                                                 .     .N          .Y
                                                 .     .           .
                                                 .     .           .
                                                 .     .           .
                                                 .     .           .
                                                 .     .           .
```



ACCEPT:
  (NO IMU REALIGNMENT NEEDED)
  PRO #90

  MANEUVER TO ACQUIRE LANDMARK

REJECT:
  (MAX MGA GREATER THAN 60 DEGREES
  OR IMU NOT ALIGNED IN PAST
  3 HOURS)

  SELECT IMU REALIGN PROGRAM
  P52/P54. #100

  WHEN ALIGNMENT IS COMPLETE
  RESELECT P22.

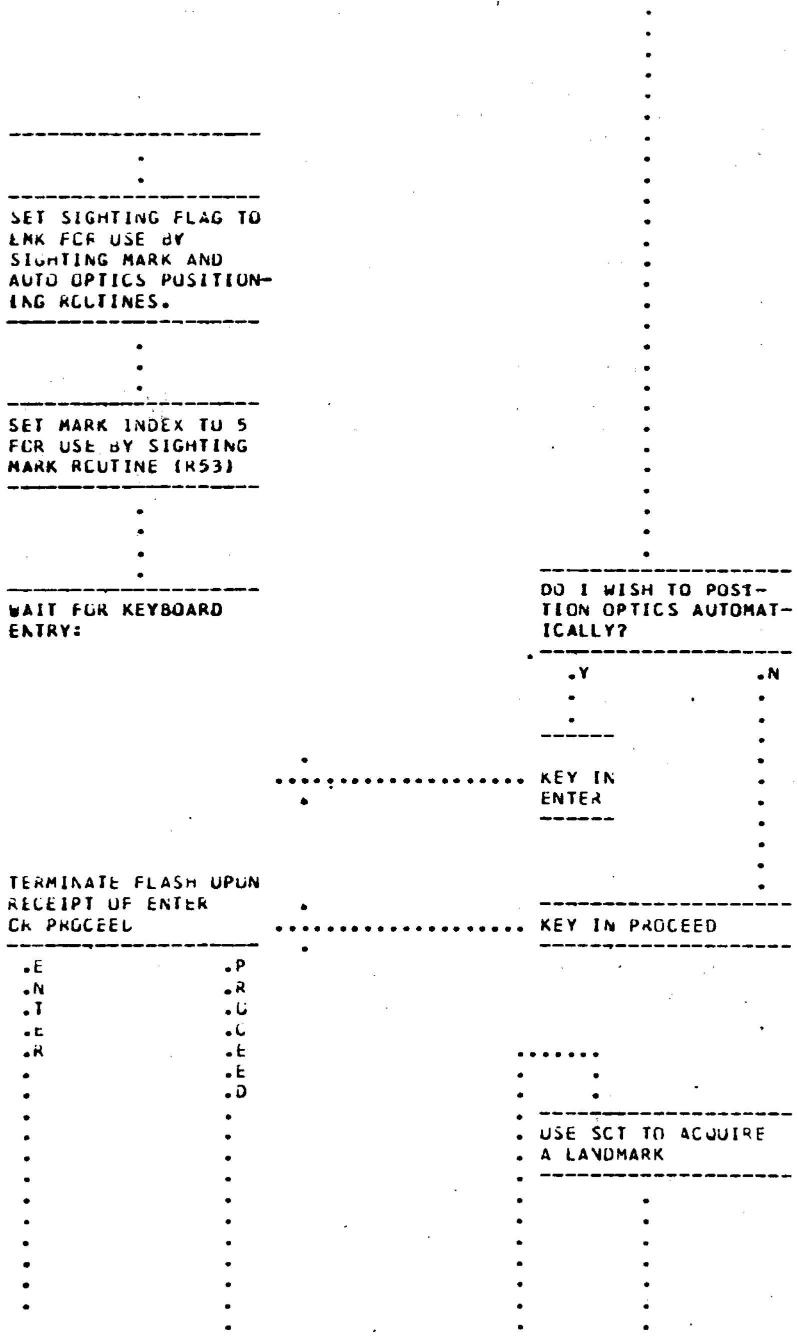
#110

#120

#130

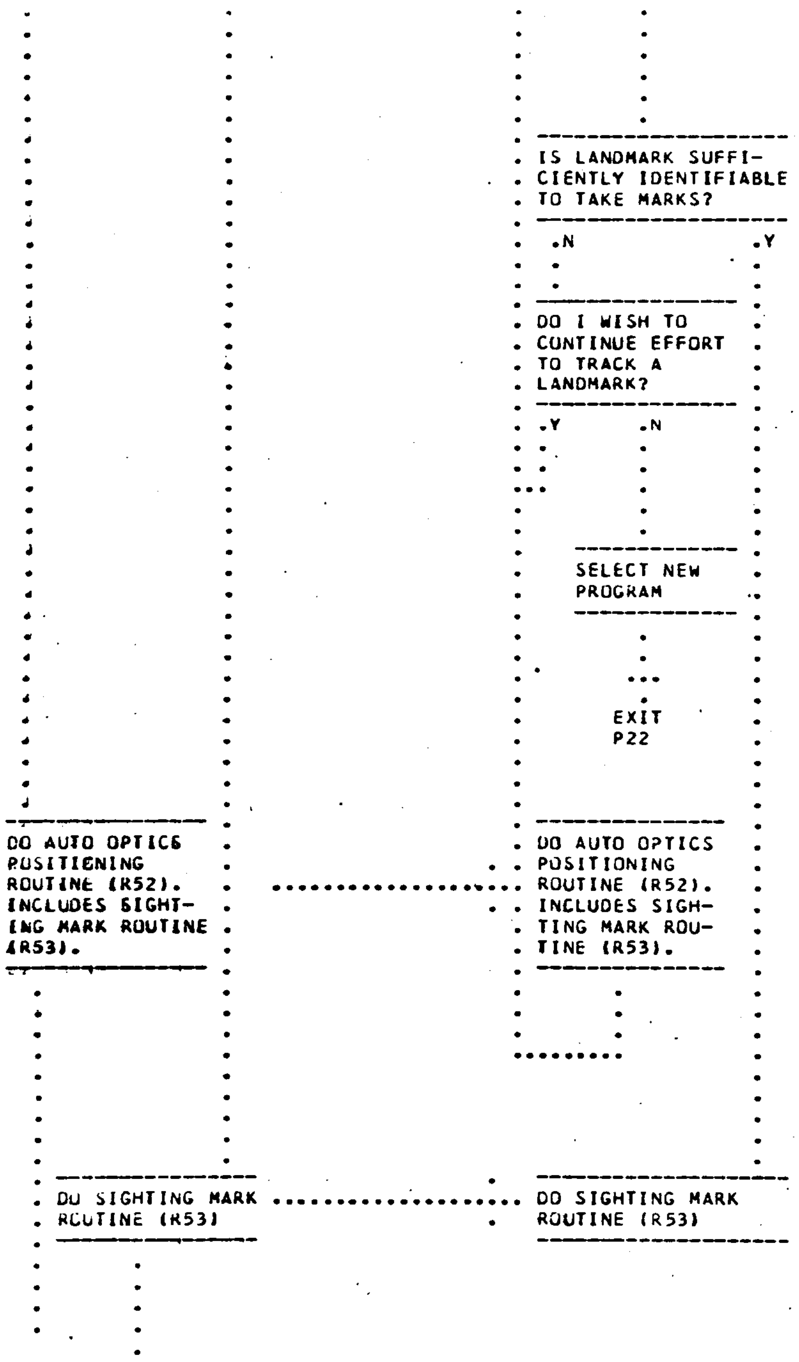
V CHECK CMC FLASHES VERB-NOUN TO
REQUEST AUTO OPTICS POSITIONING

  FL V50 N25
    00011
    BLANK
    BLANK



ACCEPT:
  OPTICS MODE - CMC
  ENTR
  PERFORM AUTO OPTICS POSITIONING
  ROUTINE (R52)

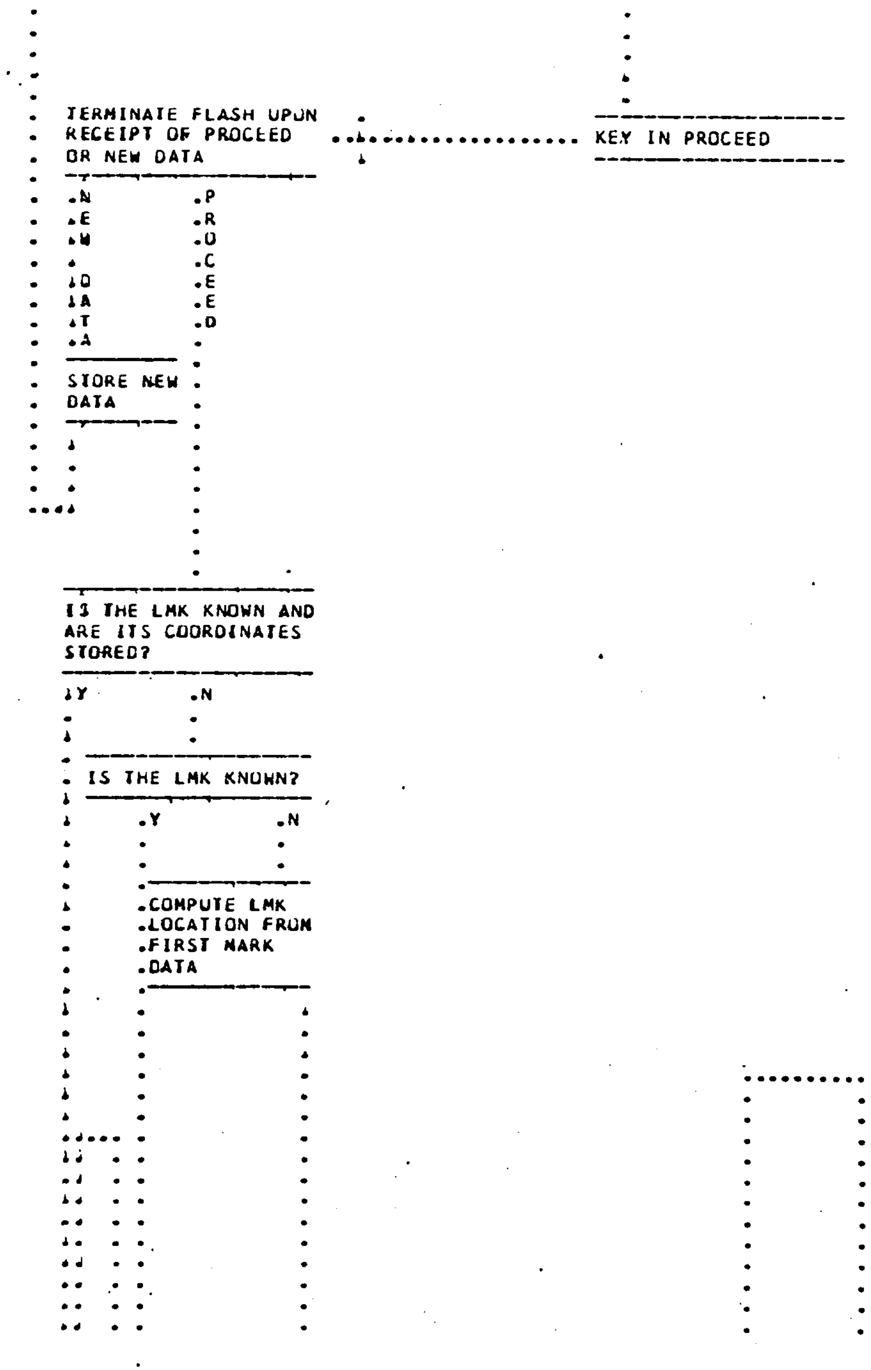
REJECT:
  PRO
  SET OPTICS MODE SW TO MAN
  ACQUIRE LANDMARK MANUALLY
  GO TO STEP VI



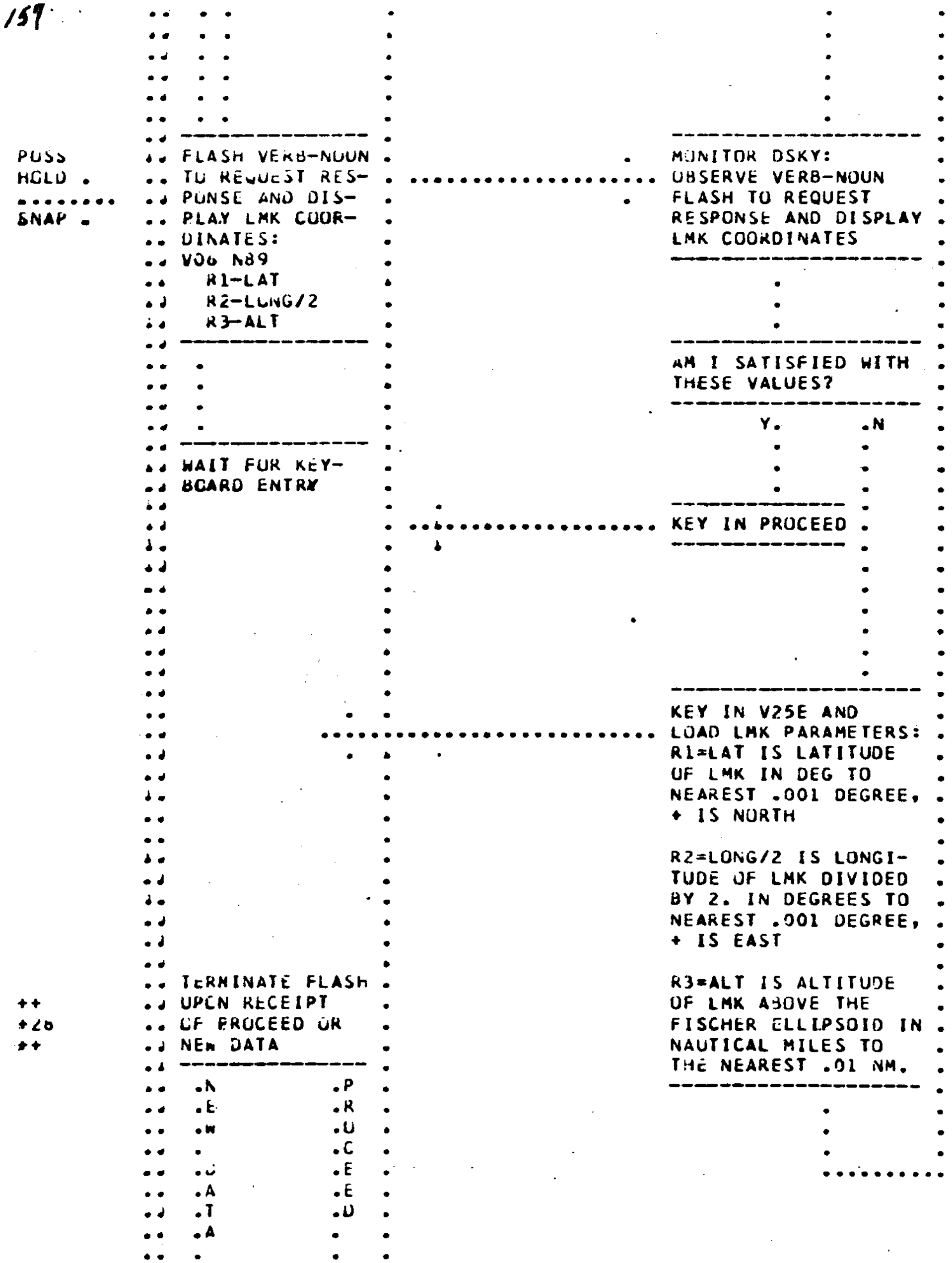
VI PERFORM SIGHTING MARK ROUTINE (R53)

```
     .  .    .                                                        ........
     .  .    .                                                        .      .                                      #250
     .  .    .                                                        .      .
     .  -------------------                           -------------------.
     .  FLASH VERB-NOUN                             . MONITOR DSKY:       .    VII  CHECK CMC FLASHES VERB-NOUN TO
HOLD . .TO REQUEST RESPONSE ......................... OBSERVE FLASHING    .         REQUEST RESPONSE AND DISPLAYS
.......AND DISPLAY LMK DATA                         . VERB-NOUN TO REQUEST.         LMK DATA
SNAP . .  V05 N70                                     RESPONSE AND DISPLAY.
     .    R1-BLANK                                    LMK DATA            .           FL V05 N70
     .    R2-ABCDE                                    R2-A=1 IF KNOWN LMK .             BLANK
     .    R3-BLANK                                       A=2 IF UNKNOWN   .             ABCDE
     .  R2-A=1 IF KNOWN LMK                              LMK              .             BLANK                       #260
     .     A=2 IF UNKNOWN                                B=1 IF LMK COOR- .
     .     LMK                                           DINATES ARE STOR-.
     .     B=1 IF LMK COOR-                              ED               .
     .     DINATES ARE STOR-                             B=2 IF LMK COOR- .              A=1 IF KNOWN LMK
     .     ED                                            DINATES ARE NOT  .              A=2 IF UNKNOWN LMK
     .     B=2 IF LMK COOR-                              STORED           .              B=1 IF LMK COORDINATES ARE
**   .     DINATES ARE NOT                               CD= LMK ID NO. N .                  STORED.
*28  .     STORED                                        NOTE: CD ALWAYS  .              B=2 IF LMK COORDINATES ARE
*    .     CD= LMK ID NO. N                              SET TO 0         .                  NOT STORED
*28  .     E=1 IF EARTH LMK                              E=1 IF EARTH LMK .              CD= LMK ID NO. N             #270
**   .     E=2 IF LUNAR LMK                              E=2 IF LUNAR LMK .              E=1 IF EARTH LMK
     .  -------------------                           -------------------.              E=2 IF LUNAR LMK
     .              .                                          .          .
     .              .                                          .          .
     .              .                                          .          .
     .              .                                 -------------------.         ACCEPT:
     .              .                                 IS THE DATA IN R2   .           PRO
     .              .                                 CORRECT FOR THIS    .
     .              .                                 SIGHTING?           .           IF UNKNOWN LANDMARK AND ONLY
     .              .                                 -------------------.           1 MARK ACCEPTED, GO TO STEP   #280
     .              .                                   .Y             .N  .           X BELOW.
     .              .                                   .              .   .
     .              .                                   .              .   .         REJECT:
     .  -------------------                             .  ---------------.           LOAD DESIRED DATA.
     .  WAIT FOR KEYBOARD          .                    .  KEY IN V22E AND.
     .  ENTRY                   ........................ . LOAD CORRECT    .
     .                             .                    .  DATA.          .
     .                                                  .  ---------------.
     .                                                  .              .   .
     .                                                  .              .   .                                        #290
     .                                                  .              .....
     .                                                  .
     .                                                  .
     .                                                  .
     .                                                  .
```

8



VIII IF LANDMARK KNOWN BUT COORDINATES



NOT STORED CHECK CMC FLASHES VERB-NOUN TO REQUEST RESPONSE AND DISPLAYS LANDMARK COORDINATES.

FL V06 N89

LAT	XX.XXX DEG
LONG/2	XX.XXX DEG
ALT	XXX.XX NM

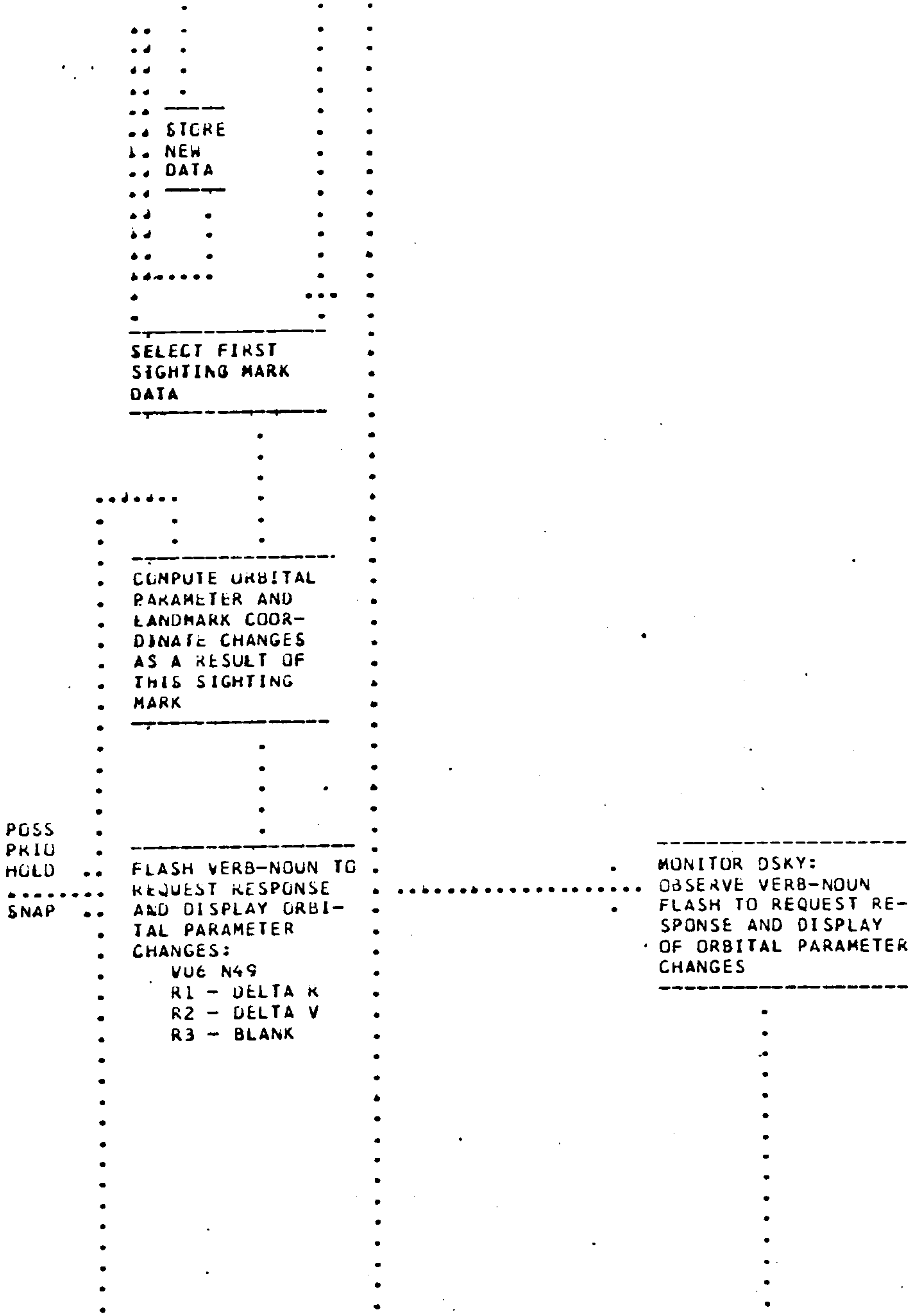
LAT-LATITUDE OF LANDMARK + IS NORTH

LONG/2-LONGITUDE OF LANDMARK DIVIDED BY TWO + IS EAST

ALT-ALTITUDE OF LANDMARK ABOVE FISCHER ELLIPSOID

ACCEPT:
PRO

REJECT:
LOAD LANDMARK COORDINATES



#410

#420

#43

IX CHECK CMC FLASHES VERB-NOUN TO REQUEST RESPONSE AND DISPLAYS ORBITAL PARAMETER CHANGES (2 DISPLAYS FOR EACH MARK) #440

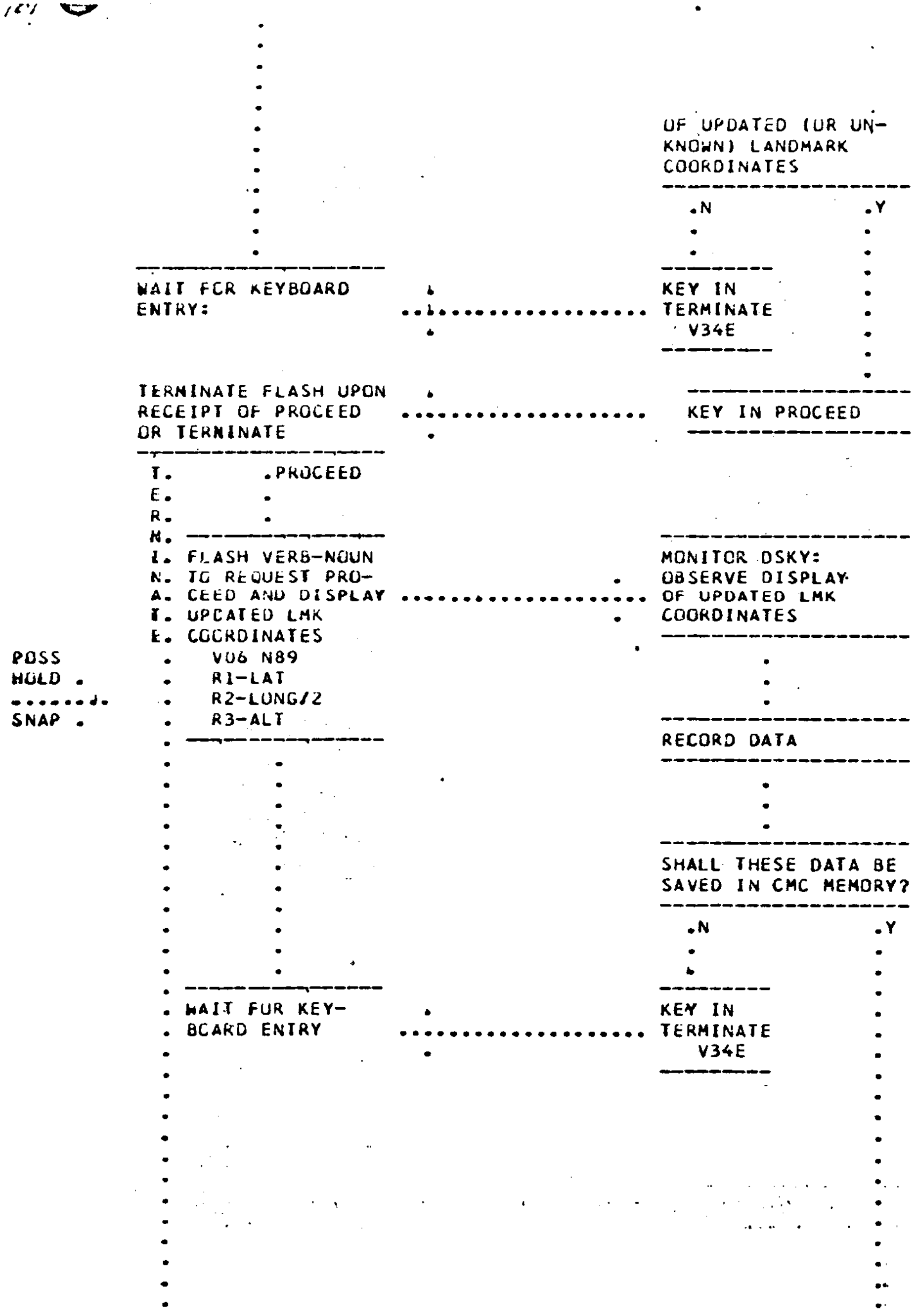
FL V06 N49

DELTA R	XXXX.X NM
DELTA V	XXXX.X FPS
BLANK	

```
.                                 .                           .
.                                 .                           .
.                                 .                           .
.                                 .                           .
.                                 .                           .
.  DELTA R-MAGNITUDE              .                           .             DELTA R - MAGNITUDE OF THE
.     OF THE DIFFER-              .                           .               DIFFERENCE BETWEEN THE POSI-         #450
.     ENCE BETWEEN                .                           .               TION VECTOR BEFORE AND AFTER
.     THE POSITION                .                           .               INCORPORATION OF THE LANDMARK
.     VECTOR BEFORE               .                           .               SIGHTING DATA.
.     AND AFTER IN-               .                           .
.     CORPORATION OF              .                           .
.     THE LANDMARK                .                           .
.     SIGHTING DATA.              .                           .
.     IN N.M. TO                  .                           .
.     NEAREST 0.1 N.M..           .                           .
.                                 .                           .             DELTA V - MAGNITUDE OF THE DIFF-
.  DELTA V-MAGNITUDE              .                           .               ERENCE BETWEEN THE VELOCITY          #460
.     OF THE DIFFER-              .                           .               VECTOR BEFORE AND AFTER INCOR-
.     ENCE BETWEEN                .                           .               PORATION OF THE LANDMARK
.     THE VELOCITY                .                           .               SIGHTING DATA.
.     VECTOR BEFORE               .                           .
.     AND AFTER IN-               .                           .
.     CORPORATION OF              .                           .
.     THE LANDMARK                .                           .
.     SIGHTING DATA.              .                           .
.     IN FPS TO NEAR-             .                           .
.     EST 0.1 FPS                 .                           .
.  ------------------             .                           .                                                    #470
.             .                   .                           .
.             .                   .                           .
.             .                   .                           .
.             .                   .                   ------------------
.             .                   .                   ARE ORBITAL
.             .                   .                   PARAMETER CHANGES
.             .                   .                   ACCEPTABLE FOR IN-
.             .                   .                   SERTION INTO CMC
.             .                   .                   CALCULATION OF                                               #480
.             .                   .                   POSITION AND
.             .                   .                   VELOCITY
.             .                   .                   ------------------
.             .                   .                      .Y           N.
.             .                   .                       .            .
.             .                   .                       .            .
.             .                   .                       .            .
.  ------------------             .   .               --------------   .
.  WAIT FOR KEYBOARD              . ..................... KEY IN PROCEED   .          ACCEPT: PRO
.  ENTRY                          .   .               --------------   .                                               #490
.                                 .                                    .
.  TERMINATE FLASH                .                                    .
.  UPON RECEIPT OF                .   .               ------------------
.  PROCEED OR TERMI-              . ..................... KEY IN TERMINATE          REJECT: V34E
.  NATE                           .   .                  V34 E
.  ------------------             .                   ------------------
.  .P               T.            .
.  .R               E.            .
.  .O               R.            .
.  .C               M.            .
.  .E                .            .                                                                                #500
.  .                 .            .
.  .                 .            .
```

74

```
     .  .                     . .
     .  .                     . .
     .  .E                    . .
     .  .D                    . .
     .  ----------------      . .
     .  UPDATE CMC STATE      . .
     .  VECTOR AND COM-       . .
     .  PUTE REVISED          . .
     .  LANDMARK COOR-        . .
     .  DINATES               . .                                                     #510
     .  ----------------      . .
     .            .           . .
     .            .           . .
     .  ----------------      . .
     .  HAVE THE ORBITAL      . .
     .  PARAMETER             . .
     .  CHANGES BEEN          . .
     .  COMPUTED TWICE        . .
     .  FOR THIS MARK         . .
     .  ----------------      . .                                                     #520
     .   .N   .Y              . .
     .    .    .              . .
     .    .    ----------------
     .....     HAVE ALL THE
     .         MARKS BEEN PRO-
     .         CESSED
     .         ----------------
     .          .N        .Y                                                          #530
     .          .         .
     .  -------------     .
     .  SELECT THE        .
     .  DATA FROM THE     .
     .  NEXT SIGHTING     .
     .  MARK              .
     .  -------------     .
     .          .         .
     .          .         .                                                           #540
     ...........          .
                          .
                          .
        ------------------            --------------------
HOLD .  FLASH VERB-NOUN           .   MONITOR DSKY:           X  CHECK CMC FLASHES VERB-NOUN TO
........TO REQUEST PROCEED ............OBSERVE DISPLAY OF         REQUEST RESPONSE AND DISPLAYS
SNAP .  AND DISPLAY LANDMARK      .   LMK ID NO. AT LOCA-        LANDMARK ID NO.
        ID NO.                        TIONS CD OF R2
          V05 N70                     --------------------       FL V05 N70                          #550
          R1-BLANK                            .                     BLANK
          R2-COCOO                            .                     LANDMARK ID     00C00
          R3-BLANK                            .                     BLANK
        ------------------            --------------------
                  .                   DO I DESIRE DISPLAY        IF DISPLAY OF LANDMARK COORDINATES
                  .
                  .
                  .
```



IS DESIRED:
  PRO

IF NOT: #560
  V34E
  GO TO STEP XII

#570

XI CHECK CMC FLASHES VERB-NOUN TO
REQUEST RESPONSE AND DISPLAYS
UPDATED LANDMARK COORDINATES

#580

75

FL V06 N89

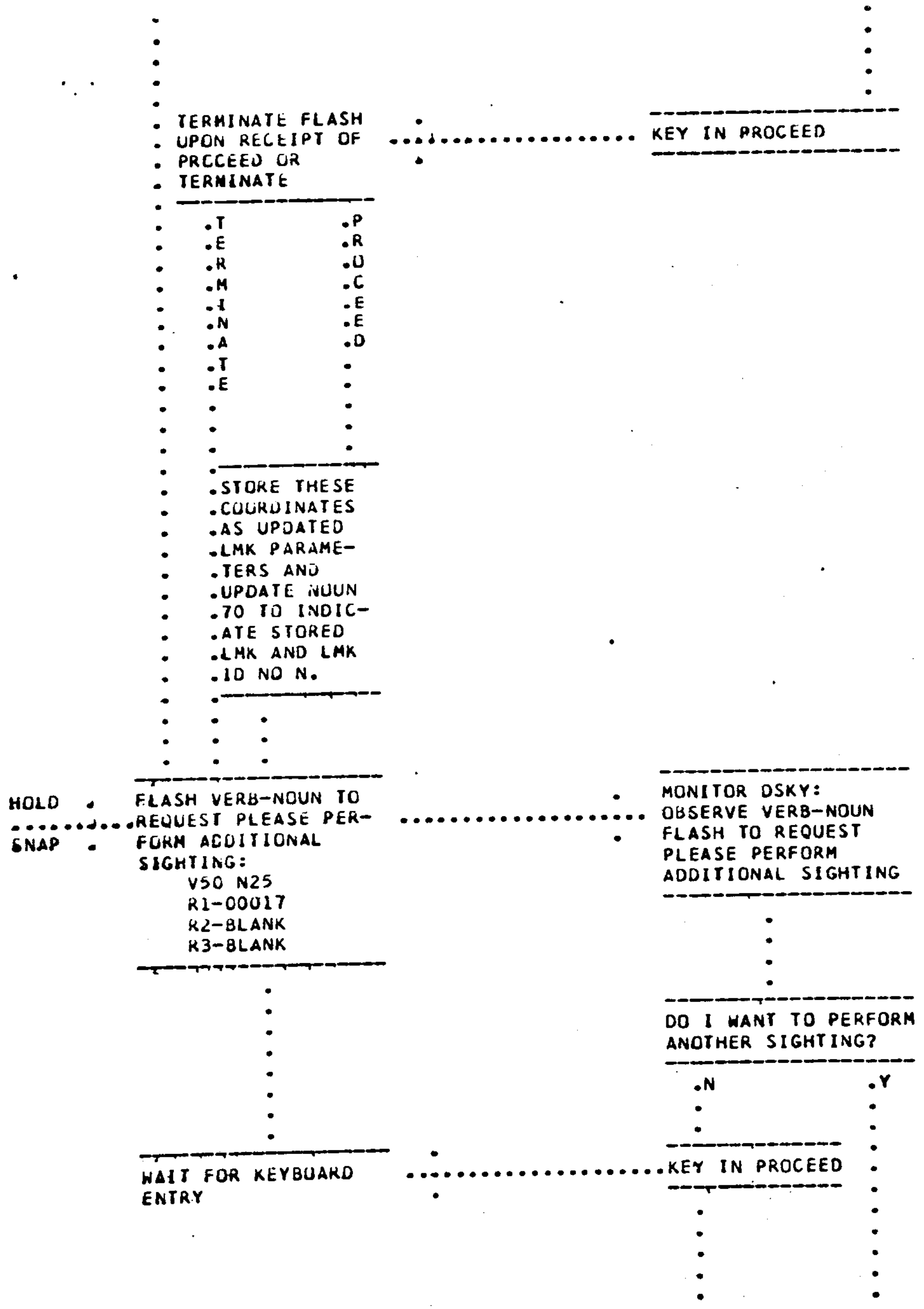
LAT	XX.XXX	DEG
LONG/2	XX.XXX	DEG
ALT	XXX.XX	NM

#590

RECORD DATA

REJECT (DATA NOT TO BE STORED
IN CMC): V34E

#600



ACCEPT (DATA TO BE STORED IN CMC):
  PRO

#610

#620

#630

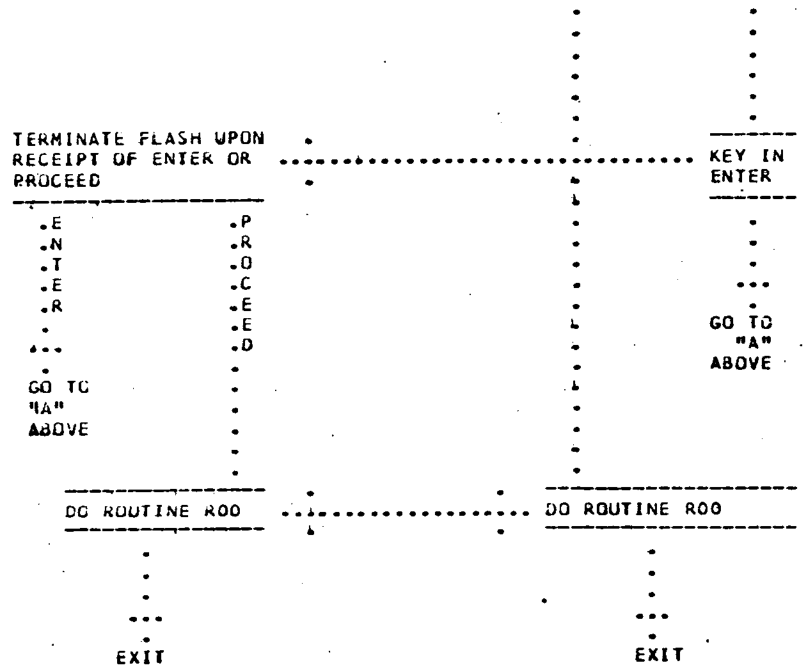
XII CHECK CMC FLASHES VERB-NOUN TO REQUEST PLEASE PERFORM ADDITIONAL SIGHTINGS

FL V50 N25
   00017
   BLANK
   BLANK #640

ACCEPT: ENTR
  RETURN TO STEP V

REJECT: PRO #650
  PERFORM PROGRAM TERMINATION ROUTINE (R00).

XIII SET:
  OPTICS MODE - ZERO



G/N OPTICS PWR – OFF
G/N PWR – OFF

#660

#670

#680

CHANGE CONTROL NOTES

APPENDIX C

APOLLO 7 CREW CHECKLIST FOR P22

SECTION 4. COAST

P21 - CSM GRND TRK DETERMINATION

CMC - ON(Req) pg 0/2-1

1 KEY V37E 21E

2 FL V06 N34 — TLAT, LONG
HRS 00XXX.
MIN 000XX.
SEC 0XX.XX

PRO   LDD

3 FL V06 N43
LAT XXX.XX DEG (+NORTH)
LONG XXX.XX DEG (+EAST)
ALT XXXX.X NM

V34E   V32E → Same parameters 10 min later

4 FL V50 N07

P22-CSM-ORBITAL NAV LANDMARK TRACKING

CMC - ON(Req) pg 0/2-1
IMU - ON(Req) pg 0/2-1
SCS - ON(Req) pg 0/3-1
CMC ATT-IMU
.05G sw - OFF
SCS LOGIC - BUS 3
G&N PWR OPTICS - ON
BMAG MODE (3) - RATE 2
OPTICS MODE - MAN
OPTICS MODE - ZERO (15 secs)

1 Key V37E 22E

2 FL V06 N45 — MAX MGA
R3 XXX.XX DEG

R3<60°
IMU ALIGNED
PRO → STEP 6

R3>60°
V37E52E → P52 (nominal opt.)

3 Select LMK

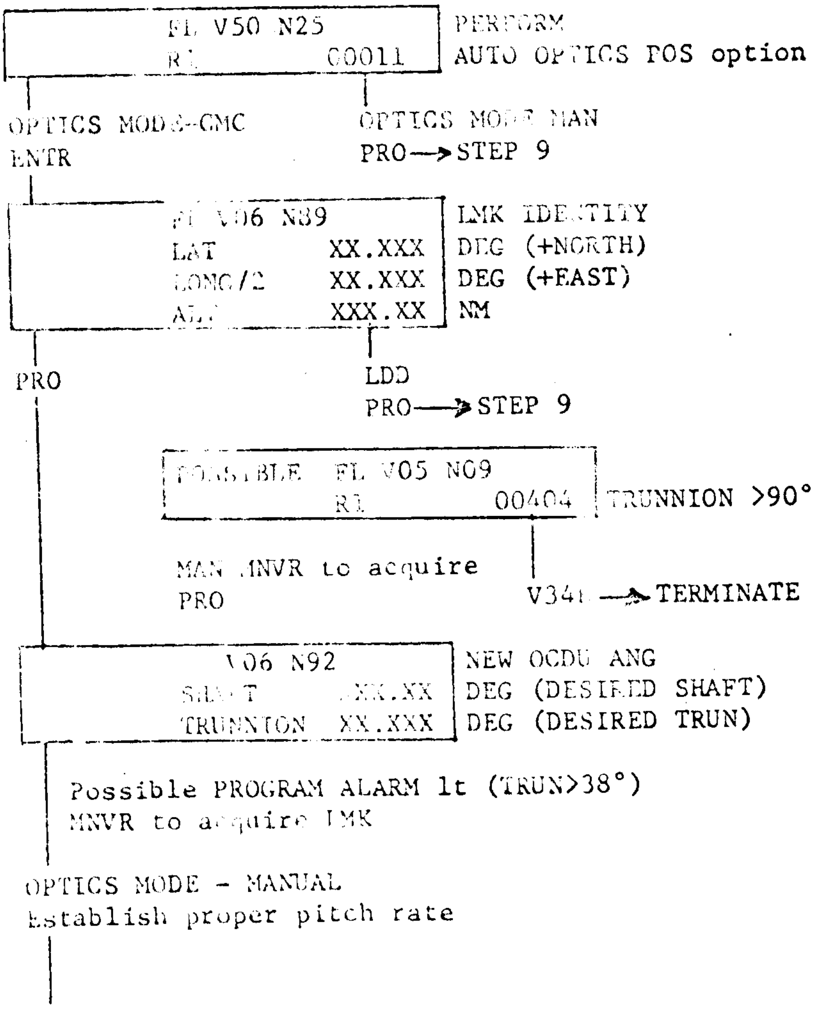
4 Perform ORDEAL INITIALIZATION pg 0/3-5

5 Mnvr to sighting attitude
ESTABLISH ORBITAL RATE
RHC - ALIGN X_{SC} 23° BELOW LOC HOR, SEF
HEADS UP (SHAFT AXIS IN ORBITAL PLANE
AND 10° FWD OF LOCAL VERTICAL)
RHC or MIC PITCH TO OFFSET ORB RATE,
ROLL TO AVOID SHAFT AXIS <10° OFF LMK

6

7

8



9 FL V51 N89 MARK
LAT XX.XXX DEG (+NORTH)
LONG/2 XX.XXX DEG (+EAST)
ALT XXX.XX NM

MARK
(KNOWN - 1 MARK MIN)SCT
(UNKNOWN - 2 MARK MIN)SXT
After suff marks
OPTICS MODE-CMC
ENTR

MARK REJECT

After 5 MARKS FL V50 N25
R1 00016 TERM MARK

ENTR
OPTICS MODE - CMC

10 FL V05 N70 LMK DATA
R2 ABCDE

LDD

	1	2
A	KNOWN	UNKNOWN
B	STORED	NOT STRD
CD	(I.D. NUMBER)	
E	EARTH	LUNAR

PRO

11 UNSTORED KNOWN R2=12XX1 FL V06 N89 LMK IDENTITY
LAT XX.XXX DEG
LONG/2 XX.XXX DEG
ALT XXX.XX NM

PRO LDD

12 KWN STRD or UNKWN/ UNSTRD FL V06 N49 ORB PARAMETER
ΔR XXXX.X NM
ΔV XXXX.X FPS

PRO → UPDATE STATE VECTOR (Till all marks used)

V34E → REJECT MARK DATA

AFTER LAST MARK USED

13 FL V05 N70 LMK ID
R2 XXCDX

PRO

V34E → STEP 15

Basic Date — Aug. 18, 1967, Rev. Dec 22, 1967
Changed — May 1, 1968

CSM 101

P22,23

14 FL V06 N89 — LMK IDENTITY
LAT XX.XXX DEG (+NORTH)
LONG/2 XX.XXX DEG (+EAST)
ALT XXX.XX NM

RECORD
PRO (stores in CMC)
V34E (Don't store)

15 FL V50 N25 — PERFORM ADDITIONAL SIGHTINGS
R1 00017

PRO
ENTR ▶ RET STEP 6

16 FL V50 N07

17 G/N PWR OPTICS - OFF

P23-CSM-CISLUNAR MIDCOURSE NAV MEASUREMENT PROGRAM

CMC - ON(Req) pg 0/2-1
G/N PWR OPTICS - ON (up)
CMC ATT - IMU
.05G sw - OFF
SCS LOGIC - BUS 3
OPTICS MODE - MAN
OPTICS MODE - ZERO (15 secs)
Move RHC to LEB
Select SC control
PERF STAR - LMK ACQ

1 Key V37E23E

2 FL V59 — PERFORM OPTICS CALIB

Ctr moon in SCT
ACQ LUNAR LMK IN SXT
SUPERIMPOSE SLOS & LLOS
MARK
PRO ▶ STEP 4

3 FL V06 N87 — BIAS ANGLE (TRUN)
R2 XX.XXX DEG

PRO
V32E ▶ RET STEP 2

APPENDIX D

SUNDISK P22 VERIFICATION TEST - TWO KNOWN LANDMARKS CASE

TABLE D-I.- DIFFERENCE BETWEEN THE BIT-BY-BIT AND THE ENGINEERING-SIMULATION UNIT b-VECTOR

Mark no.	Incorporation no.	b_1	b_2	b_3
1	First	.00010925	.00039536	-.00002127
	Second	-.00002955	.00012107	.000000174
2	First	.00625410	.000008850	-.00000745
	Second	.00000434	-.00002697	-.00000037
3	First	.00001870	.00008554	.00001657
	Second	-.00001613	-.00000897	.00000034
4	First	.00005400	.00006314	-.00001012
	Second	.00004329	.00035168	-.00000038
5	First	.00003496	.00006639	-.00000435
	Second	-.00021332	.00015567	.00000072
1	First	.00006993	.00029096	-.00012903
	Second	.000003	-.00002593	-.000012
2	First	-.00000597	-.00016595	-.00026864
	Second	-.00001594	.00004936	.00000018
3	First	.00004537	-.0000474	-.00001425
	Second	.01065872	.00019926	-.00005230

TABLE D-II.- THE BIT-BY-BIT UNIT b-VECTOR

Mark no.	Incorporation no.	b_1	b_2	b_3
1	First	.01722365	.06601096	-.99767023
	Second	.96760445	-.25247107	-.00000018
2	First	.01692410	.08906350	-.99596855
	Second	.99287134	-.11919103	.00000063
3	First	-.01325370	.11024054	-.99381657
	Second	.99284787	.11938646	.00000234
4	First	-.06199400	.11216514	-.99175388
	Second	.87520529	.48375168	.00000262
5	First	-.09830496	.07236639	-.99252165
	Second	.59285168	.80531167	-.00000277
1	First	-.02082393	.41681896	-.90875097
	Second	.99875302	.04992407	.00001247
2	First	-.10489403	.44763405	-.88804336
	Second	.97362556	.22815184	.00000115
3	First	-.39230337	.09168866	-.91525475
	Second	.23289772	.97249826	-.00240317

TABLE D-III.- THE BIT-BY-BIT VECTOR UPDATES

Mark no.	$\underline{r}_c$, ft			$\underline{v}_c$, fps			$\underline{r}_\ell$, ft		
	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Y}$	X	Y	Z
1	-16.0 658.4	-62.1 -171.8	940.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	115.3 4682.9	442.1 1222.0	-6684.6 0.0
2	-3.0 106.9	-12.2 149.3	25.8 13.2	-.0005 .0125	-.0005 .003	.006 .0009	20.2 -761.3	86.3 -1062.9	-184.0 55.6
3	-1.8 98.6	-10.0 360.0	9.4 33.6	.0005 .009	-.0009 .007	.0082 .0144	10.4 -697.7	72.5 -2562.8	-65.5 -239.5
4	0.0 19.6	-5.2 205.5	12.4 20.4	.0019 -.0125	.0029 .021	.0260 .016	-4.4 -145.8	42.9 -1457.4	-82.5 -143.4
5	-1.6 -4.4	5.6 42.7	19.2 4.8	.0019 -.0125	.0039 .0154	.0675 .004	-8.1 21.6	16.8 -297.	-116.1 -27.8
1	-25.0 803.4	-231.3 -72.1	661.6 -90.1	-.152 .404	.136 -.348	.073 -.342	-110.3 -5718.3	2210.7 -271.9	-4819.7 -30.4
2	.4 25.6	-2.6 314.8	.8 158.6	.0010 -.148	-.0010 .116	-.0005 .123	-2.0 -559.9	19.6 -2294.8	-7.6 -1038.8
3	-227.5 -56.1	642.4 152.4	425.3 69.1	-.515 -.138	.324 -.105	.511 -.038	352.4 160.8	-3991.3 -999.8	-2356.5 -302.0

TABLE D-IV.- ENGINEERING-SIMULATION VECTOR UPDATES

Mark no.	Simulation time	$\underline{r}_c$, ft			$\underline{v}_c$, fps			$\underline{r}_\ell$, ft		
		X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	X	Y	Z
1	4485.2604	-17.21 668.83	-66.01 -174.45	1003.82 0.00	0.00000 0.0	0.00000 0.0	0.00000 0.0	122.44 -4756.17	469.46 1240.54	-7138.31 .00
2	4485.4561	-3.19 108.81	-13.46 151.17	28.58 13.08	-.00132 0.01277	0.00131 -.00633	0.00611 -.00205	22.43 -772.61	96.17 -1076.28	-202.84 -93.87
3	4485.6890	-1.60 89.26	-10.06 326.41	9.36 30.34	-.0007 0.00661	0.00061 0.00515	0.00771 0.01168	10.52 -663.38	72.73 -2323.60	-65.45 -216.76
4	4485.9463	.0414 17.94	-5.67 188.69	13.51 19.13	-.00069 -.01164	0.00076 0.02543	0.02644 0.02136	-4.98 -133.86	46.39 -1336.89	-90.45 -131.24
5	4486.1872	-1.08 -4.05	4.68 39.34	16.60 4.51	-.00307 -.01395	0.00340 0.0188	0.05655 0.0087	-7.27 20.01	-15.87 -271.7	-102.28 -25.60
1	4490.4128	-25.58 790.47	-252.6 -68.80	664.56 -86.33	-.1614 0.3764	0.1426 -.3358	0.0803 -.3321	-119.3 -5623.7	2394.25 -267.07	-5224.78 -29.03
2	4490.6157	-.4303 28.38	2.43 307.5	-.6701 153.9	0.00090 -.1398	0.00074 0.108	0.00029 0.1134	1.94 -553.01	-18.70 -2256.2	7.15 -1019.1
3	4491.2170	-212.5 57.48	601.87 156.40	398.37 70.17	-.496 -.1399	0.312 0.1035	0.470 0.0452	333.28 165.5	-3726.5 -1024.5	-2201.7 -310.23

TABLE D-V.- BIT-BY-BIT ESTIMATED

LANDMARK POSITION

Mark no.	$\underline{r}_\ell$, ft		
	X	Y	Z
1 Before inc. 1	-11 915 844.	12 998 932.	11 228 431.
After inc. 2	-11 920 412.	13 000 596.	11 221 747.
2 After inc. 2	-11 932 272.	12 989 420.	11 221 465.
3 After inc. 2	-11 946 187.	12 974 770.	11 221 155.
4 After inc. 2	-11 960 939.	12 959 903.	11 220 926.
5 After inc. 2	-11 974 579.	12 946 970.	11 220 790.
1 Before inc. 1	-16 714 688.	7 377 763.3	10 167 490.
After inc. 2	-16 720 517.	7 379 702.1	10 163 640.
2 After inc. 2	-16 727 617.	7 362 582.6	10 161 587.
3 After inc. 2	-16 746 419.	7 313 567.1	10 158 936.

TABLE D-VI.- ENGINEERING-SIMULATION ESTIMATED LANDMARK POSITION

Mark no.	$\underline{r}_\ell$, ft		
	X	Y	Z
1	-11 915 826. -11 920 460.	12 998 456. 13 000 166.000	11 228 979. 11 221 841.33681
2	-11 932 336.	12 988 976.	11 221 542.
3	-11 946 184.	12 974 563.	11 221 259.
4	-11 960 923.	12 959 814.	11 221 037.
5	-11 974 566.	12 946 910.	11 220 907.
1	-16 714 671. -16 720 415.	7 377 346.9 7 379 474.1	10 167 805. 10 162 551.
2	-16 727 508.	7 362 355.6	10 161 539.
3	-16 746 321.	7 313 570.4	10 159 026.

TABLE D-VII.- DIFFERENCE BETWEEN THE BIT-BY-BIT AND THE ENGINEERING-SIMULATION W-MATRIX AFTER THE SECOND INCORPORATION

	Mark 1			Mark 5		
Component	X	Y	Z	X	Y	Z
ΔW_0, ft	-.01520	-.03490	-.06710	-.04470	.40780	-.26240
ΔW_4, fps	-.0000001	-.0000001	-.0000001	-.00015	.00069	-.00068
ΔW_8, ft	-1.48140	-.57540	-1.98560	-.78880	1.57070	.11410
	Mark 2			Mark 1		
ΔW_0, ft	.03320	-.10390	-.01080	-16.11995	3.30407	-15.87292
ΔW_4, fps	.0000035	-.0000022	.0000077	.00293018	-.00008940	.00247909
ΔW_8, ft	-1.97350	-1.70300	-2.01810	-2.2003	-2.3050	-1.5308
	Mark 3			Mark 2		
ΔW_0, ft	.02600	-.34920	.08300	-16.1051	3.7657	-15.823
ΔW_4, fps	-.0000478	-.0000446	-.00018039	.002760	-.0013531	.0005372
ΔW_8, ft	-1.35330	-6.20020	-2.10720	-1.2660	-3.6684	-4.7142
	Mark 4			Mark 3		
ΔW_0, ft	.03800	-.05430	-.33020	-16.0613	2.9635	-14.9107
ΔW_4, fps	-.0001806	.0000720	-.0004136	.0000667	-.003838	-.0005748
ΔW_8, ft	-1.2098080	-2.5446	-1.4757	.6733000	-3.7243	-.7403

TABLE D-VIII.- DIFFERENCE BETWEEN THE BIT-BY-BIT AND THE ENGINEERING-SIMULATION W-MATRIX AFTER MARK 5, SECOND INCORPORATION

Row	Column								
	1	2	3	4	5	6	7	8	9
1	0	-.24	-.0554	0	-.01179	.030013	.61	.264	1.749569
2	.224	.4	.283	.2826	.08	-.05447	-1.53	-.78	7.07284
3	.407	.2547	-.3	.0431	-.02617	-.11	-.564	-.0043	1.7
4	-.000617	.00438	.008588	.000113	.00032675	.001974	.02344	.015624	.04937078
5	.00399	.006111	.008874	.00011	.00069	.0020455	.057646	-.00589	.04312110
6	.00427	.009286	.0013978	.00030783	.001077	.00068	.040694	.05726	-.01098
7	.29	.057	-.47158	-5.988	.0024	.04023	-.7406	.0551	-.343
8	-.05725	.7704	.22	-.004	.059	.19746	.21	-1.56	.6417
9	-.042	-.354436	-.0886	-.00034	.020711	-.192	.251	.500764	.114

TABLE D-IX.- BIT-BY-BIT W-MATRIX AFTER MARK 5, SECOND INCORPORATION

Row	Column								
	1	2	3	4	5	6	7	8	9
1	2662.5	6.6128	-2.1871	162.95	-.45368	.17522	895.36	-36.374	-8.8359
2	-53.463	2667.9	9.8950	-13.912	154.16	.14236	125.55	887.57	-8.1131
3	-10.515	4.5900	2652.6	-1.8288	-.46151	159.03	13.556	4.3663	919.60
4	.0024359	-.22319	.18501	3.0244	-.0046193	-.0065560	.02662	-.07097	-.05119
5	-.21635	.02072	.20895	-.0065974	3.0233	.0070080	-.08832	.09278	.06768
6	-.18657	.21081	-.09389	-.0077541	.0067707	3.0070	-.04338	.06229	.14247
7	2381.05	-85.424	-7.3575	37.170	3.6388	1.9180	1646.97	226.379	19.236
8	339.98	2357.77	-40.991	98.035	99.660	-5.1141	-916.87	1702.25	110.855
9	39.639	4.5619	2434.49	11.807	3.5919	59.185	-110.117	-9.2364	1503.301

APPENDIX E

SUNDISK P22 VERIFICATION TEST - ONE KNOWN, ONE UNKNOWN LANDMARK CASE

TABLE E-I.- BIT-BY-BIT VECTOR UPDATES

	$\underline{r}_c$, ft			$\underline{v}_c$, fps			$\underline{r}_\ell$, ft		
Mark no.	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	X	Y	Z
Known									
2	292.6	-35.2	0.0	.0034	-.0059	-.0048	-700.3	84.1	0.0
3	307.6	622.4	60.9	.0174	.0077	.0154	-736.1	-1492.2	-145.0
4	84.9	1222.3	126.6	-.0405	.0810	.0733	-209.1	-2922.8	-302.0
5	-56.1	377.3	40.9	-.0482	.0637	.0328	124.2	-893.9	-90.5
Unknown									
1	686.4	-68.5	-77.7	.3887	-.2923	-.2932	5880.8	-283.1	-24.0
2	20.8	264.3	131.4	-.1157	.1100	.1100	-568.7	-2296.4	-1039.7
3	-32.4	85.3	36.2	-.0781	.0367	.0183	106.5	-655.6	-201.8

TABLE E-II.- ENGINEERING-SIMULATION VECTOR UPDATES

Mark no.	Simulation time	$\underline{r}_c$, ft			$\underline{v}_c$, fps			$\underline{r}_\ell$, ft		
		X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	X	Y	Z
Unknown										
2	1485.4561	.2003 309.48	1.643 -37.21	-18.24 -.0436	.000227 .003705	-.000285 -.00602	-.00014 -.0049	-.473 740.56	-3.94 88.89	43.899 -.032
3	1485.6890	.1953 305.48	4.767 616.59	-15.65 59.56	.000667 .0163	-.000698 .000397	-.00241 .00717	.3498 -730.07	-11.58 -1477.81	37.34 -143.728
4	4485.9463	.7398 85.69	-3.897 1223.6	13.23 127.44	-.00083 -.0443	.00085 .08038	.0073 .0757	-2.2567 209.7	10.077577 -2924.9	-31.165889 -300.73
5	4486.1872	-.1690 -56.88	7.716 381.08	33.82 41.35	-.0465 -.055	.00499 .07529	.0409 .0474	3.39 126.6	-13.77 -902.23	77.34 -90.90
Known										
1	4490.4128	22.27 688.65	-262.29 -65.52	678.05 -75.05	-.157 .389	.1199 -.285	.1282 -.288	-147.74 -5896.8	2955.4 -284.3	-647.0 -23.28
2	4490.6157	1.0377 21.46	-5.5764 228.9	1.5418 113.06	.002156 -.096	-.00211 .092	-.0038 .093	-5.255 -498.8	50.2 -2001.7	-19.12 -904.6
3	4491.2170	-162.62 -41.15	456.61 109.2	305.6 45.84	-.374 -.099	.254 .079	.387 .029	311.36 136.7	-3559.3 -837.4	-1992.8 -258.4

TABLE E-III.- BIT-BY-BIT ESTIMATED LANDMARK POSITION

Mark no.	$\underline{r}_\ell$, ft		
	X	Y	Z
Unknown			
2 Before Inc. 1	-11 931 967.	12 992 613.	11 220 524.
After Inc. 2	-11 932 667.	12 992 693.	11 220 577.
3 After Inc. 2	-11 946 634.	12 979 028.	11 220 454.
4 After Inc. 2	-11 961 449.	12 962 645.	11 220 120.
5 After Inc. 2	-11 974 998.	12 949 119.	11 219 936.
Known			
1 Before Inc. 1	-16 714 688.	7 377 776.1	10 167 490.
After Inc. 2	-16 720 707.	7 380 254.0	10 161 428.
2 After Inc. 2	-16 727 834.	7 363 162.1	10 160 369.
3 After Inc. 2	-16 746 721.	7 315 078.5	10 158 148.

TABLE E-IV.- ENGINEERING-SIMULATION

ESTIMATED LANDMARK POSITION

Mark no.	$\underline{r}_\ell$, ft		
	X	Y	Z
Unknown			
2	-11 931 917. -11 932 656.	12 992 613. 12 992 689.	11 220 566. 11 220 609.
3	-11 946 618.	12 979 045.	11 220 501.
4	-11 961 434.	12 962 673.	11 220 168.
5	-11 974 969.	12 949 141.	11 219 999.
Known			
1	-16 714 671. -16 720 716.	7 377 346.9 7 380 018.0	10 167 805. 10 161 313.
2	-16 727 763.	7 363 222.7	10 160 388.
3	-16 746 629.	7 314 991.9	10 158 136.

TABLE E-V.- DIFFERENCE BETWEEN THE BIT-BY-BIT AND THE ENGINEERING-SIMULATION ESTIMATED POSITION AND VELOCITY VECTOR

Mark no.	Time of state vector computation	$\Delta \underline{r}_c$, ft			$\Delta \underline{v}_c$, fps		
		ΔX	ΔY	ΔZ	$\Delta \dot{X}$	$\Delta \dot{Y}$	$\Delta \dot{Z}$
	Initialization	0.0	0.0	0.0	0.0	0.0	0.0
Unknown							
2	Before incorp.	2.0	2.0	-6.0	-.002	.005	.008
3	After second incorp.	6.0	24.0	-2.0	-.024	.029	.024
5	After second incorp.	-44.0	26.0	-3.0	.046	-.042	-.028
Known							
1	Before incorp.	11.6	2.0	2.0	-.016	-.004	.002
3	After second incorp.	17.0	78.7	-30.	-.022	.005	.040

TABLE E-VI.- DIFFERENCE IN THE RESIDUALS (ACTUAL MINUS ESTIMATED) OF THE BIT-BY-BIT AND THE ENGINEERING-SIMULATED POSITION AND VELOCITY VECTOR

Mark no.	Time of state vector computation	$\delta \underline{r}_c$, ft			$\delta \underline{v}_c$, fps		
		ΔX	ΔY	ΔZ	$\Delta \dot{X}$	$\Delta \dot{Y}$	$\Delta \dot{Z}$
	Initialization	0.0	0.0	0.0	0.0	0.0	0.0
	Unknown						
2	Before incorp.	-2.6	-2.0	7.0	-.232	-.005	.0006
	After second incorp.	13.0	-4.7	7.0	.002	-.005	-.0006
5	After second incorp.	43.4	-31.4	41.6	-.095	.010	.006
	Known						
1	Before incorp.	10.0	-1.7	2.0	-.017	.004	-.017
	After second incorp.	14.0	20.0	47.0	.021	.011	.015
3	After second incorp.	17.0	-76.	31.	-.019	-.007	-.040

TABLE E-VII.- DIFFERENCE IN THE DIAGONAL OF THE BIT-BY-BIT AND THE ENGINEERING-SIMULATION W-MATRIX

Mark no	Time of state vector computation	Δr_c, ft			Δv_c, fps			Δr_ℓ, ft		
		ΔX	ΔY	ΔZ	$\Delta \dot{X}$	$\Delta \dot{Y}$	$\Delta \dot{Z}$	ΔX	ΔY	ΔZ
	Initilization	0	0	0	0	0	0	0.0	0.0	0.0
Unknown										
1		0	0	0	0	0	0	0.0	0.0	0.0
2		-.112	.005	-.139	.86 E-4	.14 E-4	.03 E-4	2283.	32.89	-.802
3		-.012	-.027	.198	.66 E-4	.51 E-4	.29 E-4	-1.18	-.531	-.582
4		.028	-.204	-.609	1.9 E-4	1.4 E-4	3.6 E-4	-1.05	-2.01	-.956
5										
Known										
1		-17.9	-.357	-19.7	28.2 E-4	2.1 E-4	16.2 E-4	-2.28	-2.26	-1.47
2		-17.8	.324	-19.7	26.6 E-4	-6.01 E-4	10.7 E-4	-.629	-4.50	-3.86
3		-17.7	-.174	-18.7	4.07 E-4	21.2 E-4	.76 E-4	1.18	-3.74	.902

TABLE E-VIII.- DIAGONAL VALUES AT SPECIFIED

POINTS IN THE BIT-BY-BIT W-MATRIX

Mark no.	Time state vector computation	$\underline{r}_c$, ft			$\underline{v}_c$, fps			$\underline{r}_\ell$, ft		
		X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	X	Y	Z
	Initilization	2998.9	2998.9	2998.9	3.0248	3.0248	3.0248	4639.3	4639.3	4639.3
Unknown										
3		2283.9	2853.9	2289.3	3.0242	3.0252	3.02372	1992.6	4099.7	2012.8
Known										
1 Before inc.		-180.	949.	-170.	-.9446	-.1558	-.9459	7997.	7997.	7997.
1 After inc.		-164.	942.	-158.	-.9037	-.1436	-.8555	2303.	6986.	3227.
3 After inc.		-162.	991.	-164.	-1.021	-.3573	-1.038	1811.	1628.	1527.

REFERENCES

1. MIT: Guidance System Operations Plan for Manned CM Earth Orbital Missions Using Program Sundisk, Section 5, Guidance Equations (Rev. 2). MIT Instrumentation Laboratory, March 1968.

2. TRW: Programmed Guidance Equations for Sundisk Command Module Earth Orbital Program, Revision 1, based on DISK 282 Program. TRW/Houston, NAS 9-4816, February 25, 1968.

3. Clifford, J. B., Jr.: Apollo Coasting Flight Navigation Simulation - OBSIM/NAVSIM Program Formulation. TRW note no. 67-FMT-531, July 25, 1967.

4. Phillips, Laurel A.: Apollo Onboard Orbit Determination Equations. MSC memorandum no. 66-FM42-228, August 4, 1966.

5. Guidance System Operations Plan for Manned CM Earth Orbital Missions Using Program SUNDISK, Section 4, GNCS Operational Modes (Rev. 2). June 1968.

6. Kimball, Garner R.: Additional Sundisk Program Notes. MSC Draft for Corrections to Sundisk Program Notes, July 8, 1968.

7. Fox, M.: Discrepancy Report Status (June 28, 1968). TRW memorandum no. 68:7252.1-99, July 9, 1968.

8. Kimball, Garner R.: Updated Sundisk Program Notes and Discrepancies. MSC memorandum, May 18, 1968.

9. Flight Crew Support Division, Spacecraft Systems Branch: Crew Check List, Apollo 7. May 1, 1968.

10. Olah, G. T.: Sundisk P22 (Orbital Navigation) Verification. TRW memorandum draft, September 1968.