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# AS-204 REENTRY GUIDANCE AND NAVIGATION EQUATIONS AND FLOW LOGIC

By Oliver Hill and Donald Beggs

Mission Analysis Branch

MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER

HOUSTON, TEXAS

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# PROJECT APOLLO

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Mission Planning and Analysis Division

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# AS-204 REENTRY GUIDANCE AND NAVIGATION EQUATIONS AND FLOW LOGIC

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## SUMMARY AND INTRODUCTION

The basic guidance and navigation equations and flow logic applicable to the AS-204 mission are presented in reference 1. In order to reflect the current reentry guidance logic, constants, and gains for mission AS-204, some portions of reference 1 require minor modifications, which are presented in this note.

A method of generating a reference matrix (REFSMAT), which relates the platform alignment to an earth centered inertial (ECI) reference, is presented in detail. The real-time computer program is to have the capability of accepting any REFSMAT which will be in the form of a $3 \times 3$ matrix. Subroutine DELBTA which requires a REFSMAT input compensates for the error in the lift vector attitude resulting from a skewed platform.

## REAL-TIME PROGRAM REQUIREMENTS

The current reentry guidance flow logic is presented in figures 1 through 8 of this document. The variable names and guidance gains and constants are presented in tables I and II, respectively. The final phase reference trajectory is presented in table III.

The navigation $\Delta V$ computation, figure 2, is included in order to have a more complete document. The navigation for the real-time computer program is to be obtained from the integration package. The total aerodynamic acceleration, D, used in the targeting phase, figure 4, is also to be obtained from the integration package.

## Initialization Phase

The unit target vector $\overline{\mathrm{URTO}}$ is defined by the longitude and geodetic latitude of the desired splash point. The time increment TN is a constant added to the current flight time in order to obtain a nominal time of flight.

In case of a RCS deorbit the command module will reenter with lift vector down. For a nominal reentry (SPS deorbit) the command module will reenter with lift vector up. The selection of the initial (entry) attitude of the lift vector for the real-time computer program is a flight controller function.

## Platform Alignment

The platform alignment for the reentry simulation must be the same as the actual platform alignment and should be obtained from the real-time routine that defines the actual alignment. The platform alignment (REFSMAT) must be input into the real-time computer program as X, Y, and Z components of the ECI reference frame. The REFSMAT will be in the form of a $3 \times 3$ matrix.

Prior to launch the platform is aligned with the Z-platform axis along the local vertical (down), the X-platform axis pointing down range at an azimuth of 71.9901°E of true north, and the Y-platform axis completing the orthogonal set. The geodetic latitude of the launch pad is 28.53077776° N. It should be noted that this latitude is not the local vertical along which the Z-platform axis is aligned. However, the geodetic latitude of the launch pad may be used for program development purposes until the data on the local vertical becomes available. The launch platform alignment will be used for all launch abort modes.

For SPS and RCS deorbits the platform will nominally be realigned prior to deorbit with the $\mathrm{X'sm}$ ($\mathrm{X'}$ - stable member) axis along the negative thrust vector ($-\overline{A}_T$), the $\mathrm{Y'sm}$ in the $-\overline{A}_T \times \overline{\mathrm{RG}}$ direction, and the $\mathrm{Z'sm}$ completing the orthogonal set. $\overline{\mathrm{RG}}$ is the position vector of the spacecraft at retrofire. The primed axes represent the stable members with zero compensation in pitch and yaw engine gimbal angles. The following transformation accounts for the effect of the engine gimbal angles:

$$\begin{bmatrix} \mathrm{Xsm} \\ \mathrm{Ysm} \\ \mathrm{Zsm} \end{bmatrix} = \begin{bmatrix} \cos P \cos Y & -\cos P \sin Y & \sin P \\ \sin Y & \cos Y & 0 \\ -\sin P \cos Y & \sin P \sin Y & \cos P \end{bmatrix} \begin{bmatrix} \mathrm{X'sm} \\ \mathrm{Y'sm} \\ \mathrm{Z'sm} \end{bmatrix}$$

Using the small-angle approximation, the transformation reduces to

$$\begin{bmatrix} Xsm \\ Ysm \\ Zsm \end{bmatrix} = \begin{bmatrix} 1 & -Y & P \\ Y & 1 & 0 \\ -P & 0 & 1 \end{bmatrix} \begin{bmatrix} X'sm \\ Y'sm \\ Z'sm \end{bmatrix}$$

Y and P are the yaw and pitch engine gimbal angles in radians. The small-angle approximation is used by MIT and is to be used in the real-time computer programs.

A more detailed description of the nominal reentry platform alignment is given in appendix A.

## Stabilization and Control System

The stabilization and control system presented in reference 2 for the AS-202 mission must be modified slightly for the AS-204 mission. The commanded roll position (ROLLCD) computed in the lateral logic section of the reentry guidance is treated as a bank angle by the AGC. The roll attitude error (RAE) which drives the stabilization and control system is the error between a commanded roll gimbal angle and the current roll gimbal angle. The roll gimbal angle corresponding to ROLLCD (commanded bank angle) can be obtained by entering ROLLCD into the transformation given in Appendix B at entry point "B" and exit at point "C" with the commanded roll gimbal angle.

## Subroutine DELBTA

The subroutine DELBTA which compensates for errors in the lift vector attitude resulting from a skewed platform is presented in appendix B. The subroutine is called from the reentry guidance lateral logic prior to calculating the commanded roll position ROLLCD.

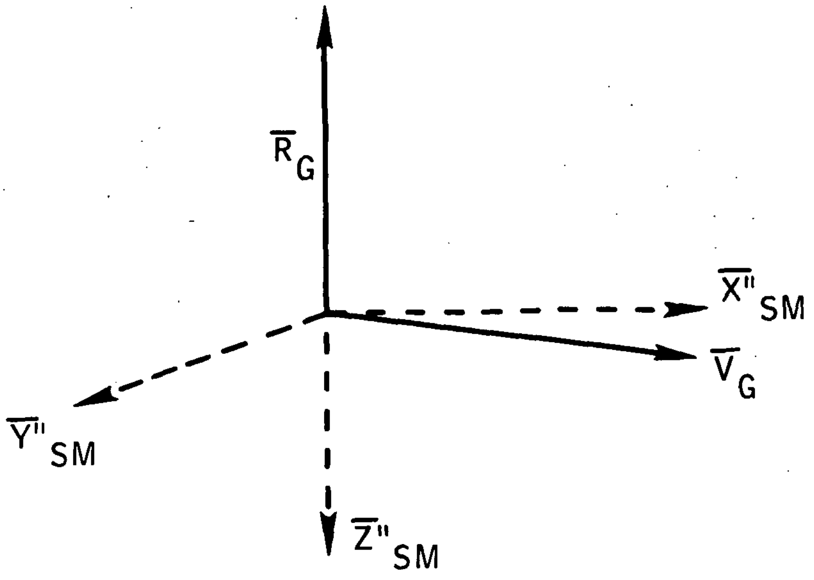
# APPENDIX A

## PLATFORM ALIGNMENT FOR SPS AND RCS DEORBIT

The AS-204 command module platform (stable member) is to be aligned such that

$$\begin{bmatrix} \bar{X}sm \\ \bar{Y}sm \\ \bar{Z}sm \end{bmatrix} = \begin{bmatrix} 1 & -Y & P \\ Y & 1 & 0 \\ -P & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{X}'sm \\ \bar{Y}'sm \\ \bar{Z}'sm \end{bmatrix}$$

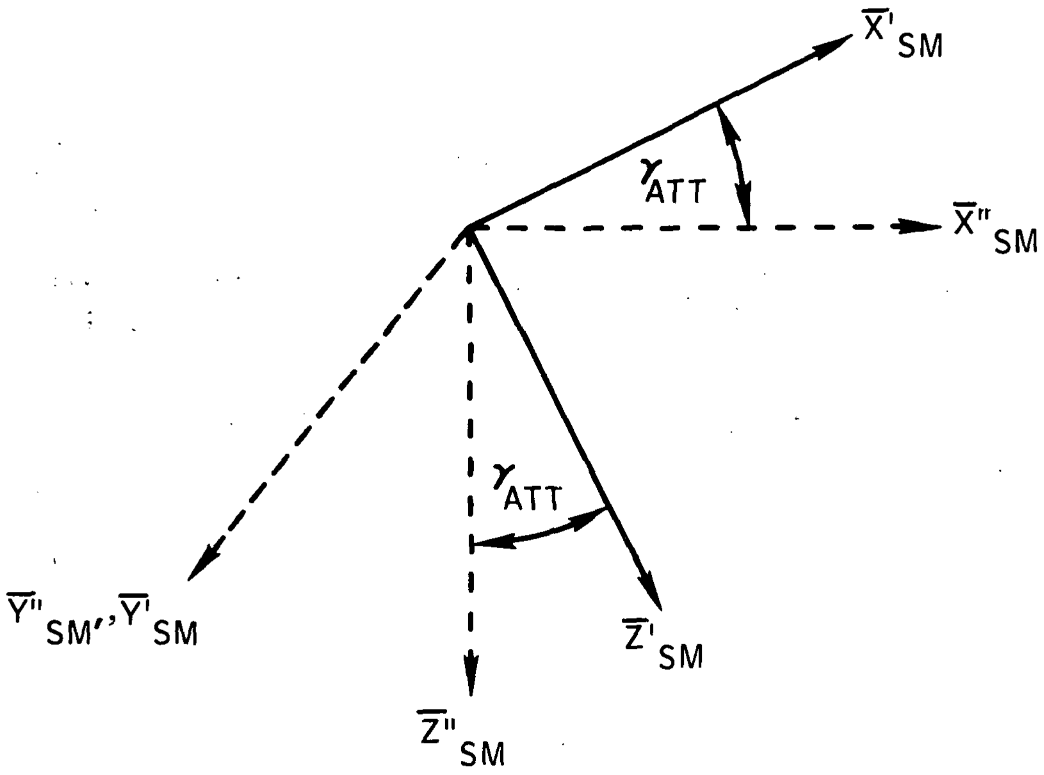
The engine gimbal angle matrix is defined in the text of this document. The components of the primed platform matrix are shown in the following figure.



$$\bar{Y}''_{SM} = \bar{V}_G \times \bar{R}_G$$
$$\bar{X}''_{SM} = \bar{R}''_G \times \bar{Y}''_{SM}$$
$$\bar{Z}''_{SM} = \bar{X}''_{SM} \times \bar{Y}''_{SM}$$

| | |
|---|---|
| $\bar{R}_G$ | Position vector at retrofire |
| $\bar{V}_G$ | Velocity vector at retrofire |

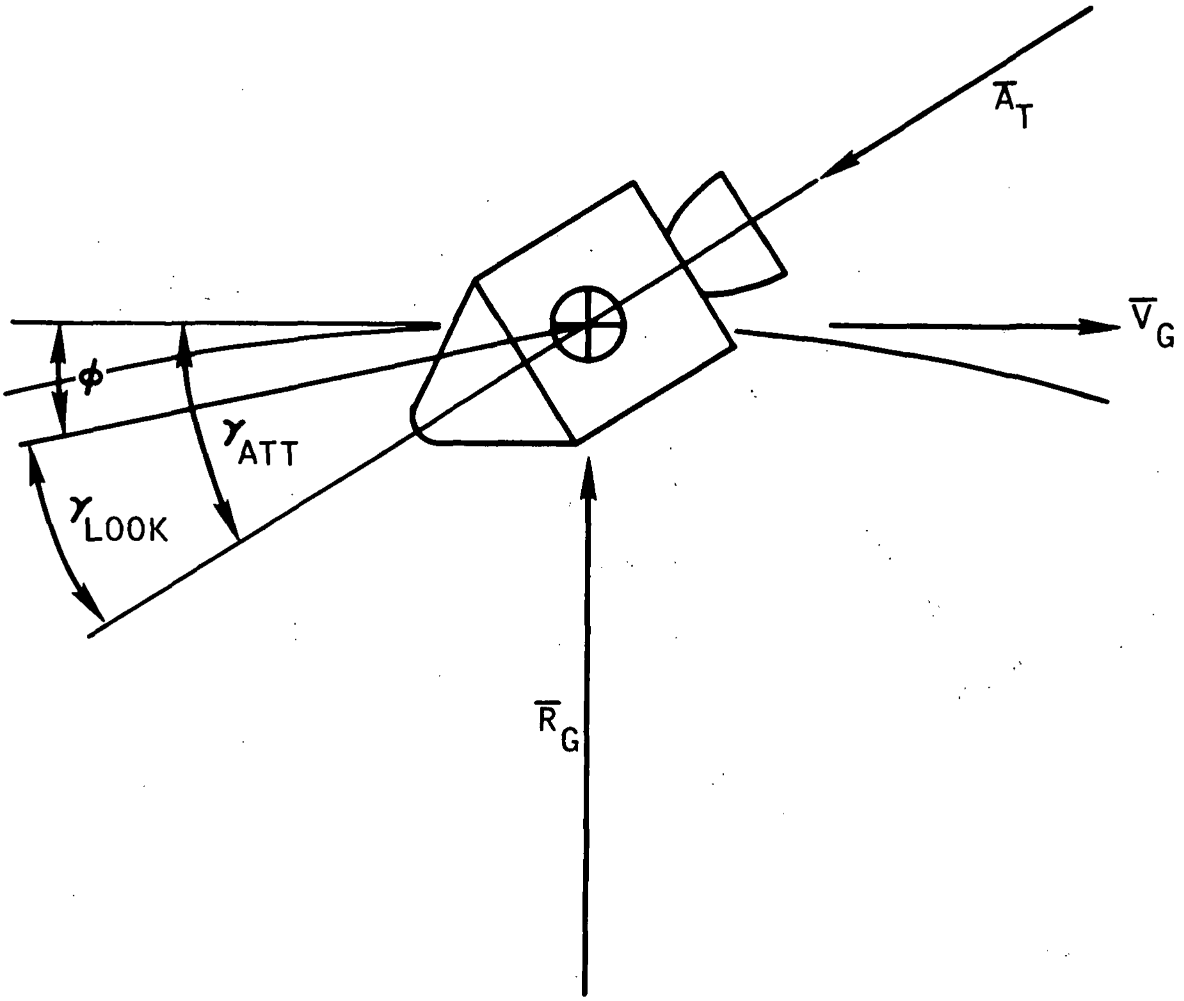
The platform attitude relative to the local horizontal is indicated by the double primed axes. The rotated attitude indicated by the primed quantities is shown in the following figure.



where

$$\overline{X}'_{sm} = \cos(\gamma_{ATT})\overline{X}''_{sm} + 0(\overline{Y}''_{sm}) - \sin(\gamma_{ATT})\overline{Z}''_{sm}$$
$$\overline{Y}'_{sm} = 0(\overline{X}''_{sm}) + 1(\overline{Y}''_{sm}) + 0(\overline{Z}''_{sm})$$
$$\overline{Z}'_{sm} = \sin(\gamma_{ATT})\overline{X}''_{sm} + 0(\overline{Y}''_{sm}) + \cos(\gamma_{ATT})\overline{Z}''_{sm}$$

The angle $\gamma_{ATT}$ is defined by this figure.



$$\gamma_{ATT} = \phi + \gamma_{LOOK}$$

$$\gamma_{LOOK} = 31.7 \text{ deg}$$

$$\phi = \frac{\pi}{2} - \sin^{-1}\left(\frac{RF}{RG}\right)$$

RF Reference earth radius = 20 909 869.11 ft

RG Spacecraft position vector magnitude

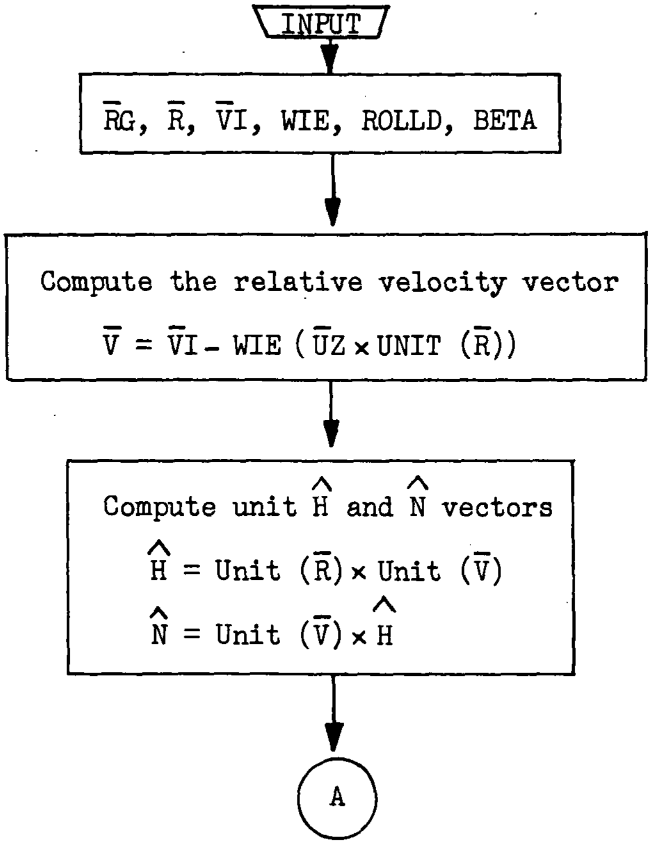
# APPENDIX B

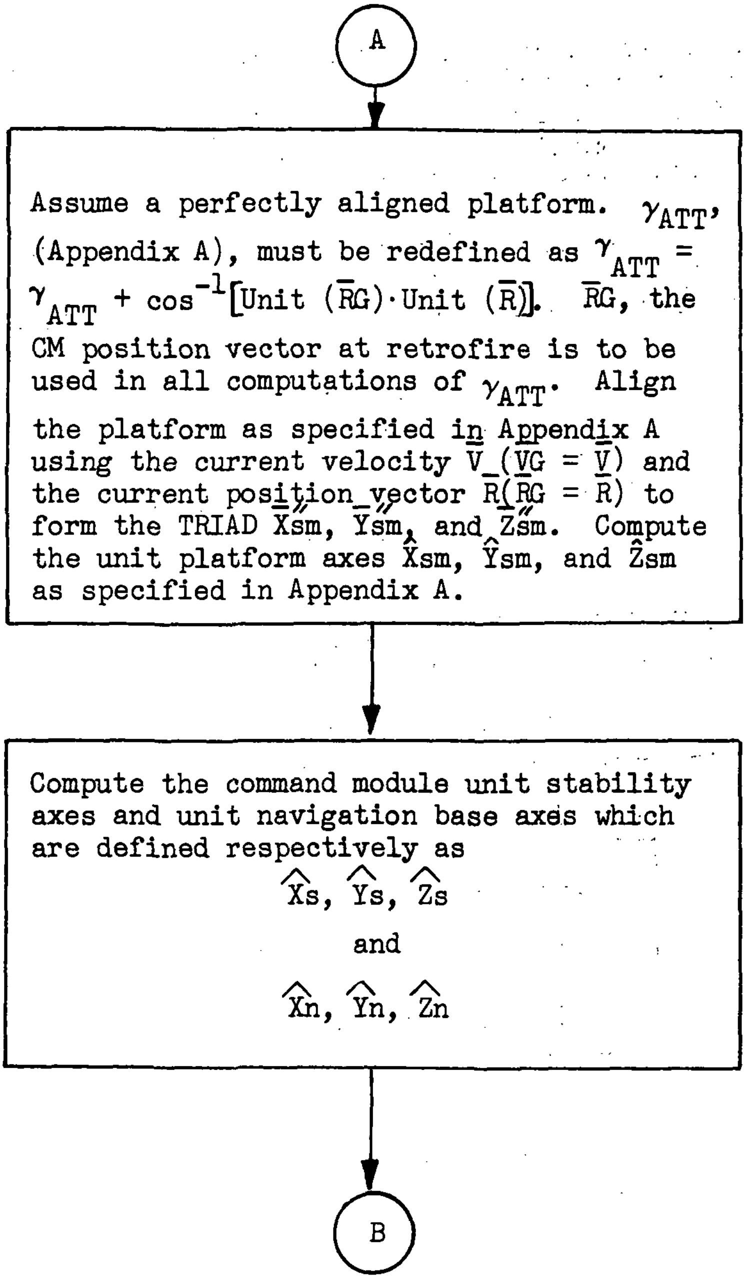
## SUBROUTINE DELBTA

The function of Subroutine DELBTA is to calculate a bias term which subsequently compensates for any roll attitude error resulting from an out-of-plane platform alignment. This correction is made by altering the roll command (ROLLCD) by the addition of a roll bias term (RBIAS). The roll command equation is given as

$$\text{ROLLCD} = \text{K2ROLL} \cos^{-1}\left(\frac{\text{L/D}}{\text{LAD}}\right) + 2\ \text{K1ROLL} + \text{RBIAS}\left(\frac{\pi}{180}\right)$$

The flow logic for the Subroutine DELBTA is as follows:





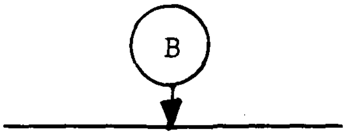
Assume a perfectly aligned platform. $\gamma_{ATT}$, (Appendix A), must be redefined as $\gamma_{ATT} = \gamma_{ATT} + \cos^{-1}[\mathrm{Unit}\ (\overline{RG})\cdot\mathrm{Unit}\ (\overline{R})]$. $\overline{RG}$, the CM position vector at retrofire is to be used in all computations of $\gamma_{ATT}$. Align the platform as specified in Appendix A using the current velocity $\underline{V}$ ($\underline{VG} = \underline{V}$) and the current position vector $\overline{R}$ ($\overline{RG} = \overline{R}$) to form the TRIAD Xsm, Ysm, and Zsm. Compute the unit platform axes $\hat{X}sm$, $\hat{Y}sm$, and $\hat{Z}sm$ as specified in Appendix A.

Compute the command module unit stability axes and unit navigation base axes which are defined respectively as

$$\widehat{Xs},\ \widehat{Ys},\ \widehat{Zs}$$

and

$$\widehat{Xn},\ \widehat{Yn},\ \widehat{Zn}$$



The following transformation matrices required to define the stability axes and the navigation base axes were developed for a right-hand system. The unit vectors $\hat{V}_R$, $\hat{H}$, and $\hat{N}$ are defined in figure 1 of this Appendix. The subscripts s and n indicate the stability and navigation base axes, respectively.

$$\begin{bmatrix} \hat{X}s \\ \hat{Y}s \\ \hat{Z}s \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\beta & \sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \hat{V}_R \\ \hat{H} \\ \hat{N} \end{bmatrix}$$

$$\begin{bmatrix} \hat{X}n \\ \hat{Y}n \\ \hat{Z}n \end{bmatrix} = \begin{bmatrix} \cos\zeta & 0 & -\sin\zeta \\ 0 & 1 & 0 \\ \sin\zeta & 0 & \cos\zeta \end{bmatrix} \begin{bmatrix} \hat{X}s \\ \hat{Y}s \\ \hat{Z}s \end{bmatrix}$$

$\beta$ = bank angle (roll attitude about the X-stability axis)

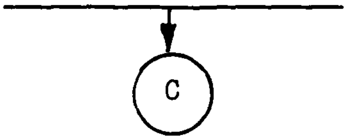
$\zeta = 33^\circ - \alpha$

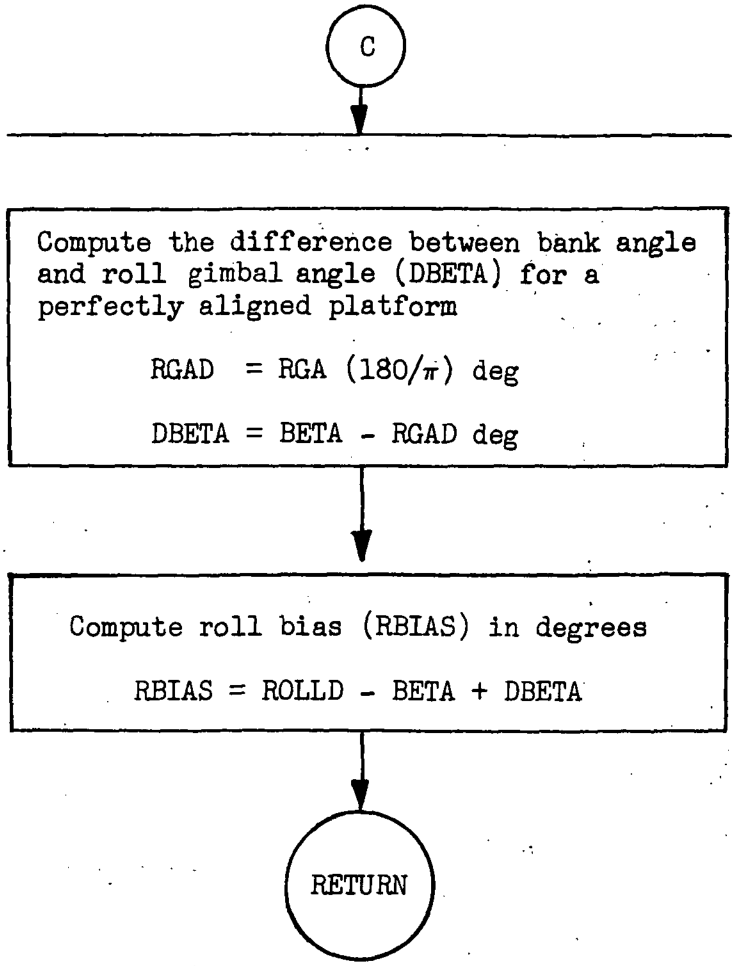
$\alpha$ = angle of attack (positive acute angle)

The roll gimbal angle RGA (in radians) is obtained from the equation

$$\text{RGA} = \tan^{-1}\left[\frac{-\hat{Z}n\cdot\hat{Y}p}{\hat{Y}n\cdot\hat{Y}p}\right]$$

where $\hat{Y}_p$ is the platform unit Y-axis.




C
Compute the difference between bank angle and roll gimbal angle (DBETA) for a perfectly aligned platform
RGAD = RGA (180/π) deg
DBETA = BETA - RGAD deg
Compute roll bias (RBIAS) in degrees
RBIAS = ROLLD - BETA + DBETA
RETURN

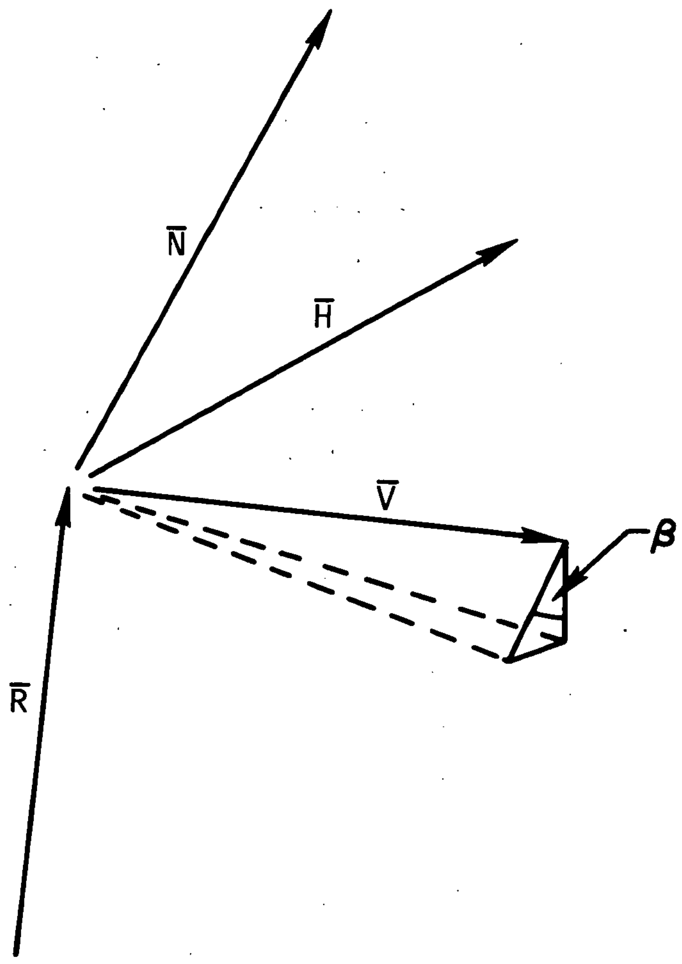


Figure 1. - Orthogonal set referenced to the osculating plane.

## TABLE I. - VARIABLES FOR REENTRY GUIDANCE

| | |
|---|---|
| BETA | Command module bank angle |
| BETA | Initial bank angle of CM at entry interface |
| $\overline{\text{DELV}}$ | Integrated acceleration vector |
| D | Total aerodynamic acceleration |
| $\overline{\text{G}}$ | Gravity vector |
| K1ROLL | Indicator for roll switch |
| K2ROLL | Indicator for roll switch |
| LATANG | Lateral range error (radians) |
| LEQ | Excess centrifugal force over gravity |
| L/D | Desired lift-to-drag ratio in the osculating plane |
| PREDANGL | Predicted range for the final phase |
| Q7 | Minimum drag for upcontrol |
| $\overline{\text{R}}$ | Position vector |
| RBIAS | Roll command bias |
| RDOT | Altitude rate |
| $\overline{\text{R}}_{\text{G}}$ | Position vector at platform alignment for deorbit |
| ROLLC | Roll command (radians) |
| $\overline{\text{RTE}}$ | Vector east at initial target $\overline{\text{URTO}}$ |
| $\overline{\text{RTINT}}$ | Target vector rotated through nominal flight time |
| RTOGO | Range-to-go parameter in final phase |
| SL | Sine of latitude |
| T | Time referenced to lift-off |
| THETA | Current great circle range-to-target (radians) |
| THETNM | Theta (nautical miles) |
| $\overline{\text{UNI}}$ | Unit vector normal to trajectory plane |
| $\overline{\text{URT}}$ | Current target vector |
| $\overline{\text{URTO}}$ | Initial target vector referenced to longitude and geodetic latitude of the desired splash point at lift-off |

## TABLE I. - VARIABLES FOR REENTRY GUIDANCE - Concluded

| | |
|---|---|
| $\overline{UTR}$ | Normal to $\overline{RTE}$ and $\overline{UZ}$ |
| $\overline{UZ}$ | Unit earth rate vector north |
| $\overline{V}$ | Relative velocity vector |
| $\overline{VGI}$ | Velocity vector at platform alignment for deorbit |
| $\overline{VI}$ | Inertial velocity vector |

## TABLE II. - GUIDANCE GAINS AND CONSTANTS

| | Symbol | Value | Units |
|---|---|---|---|
| G-limit | GMAX | 322. | fpss |
| Normalization factor, acceleration | GS | 32.2 | fpss |
| Lateral switch gain | KLAT | 0.0075 | n.d. |
| Factor in L/D computation for final phase | K13P | 4. | n.d. |
| Increment to Q7 to end Kepler | KDMIN | 0.5 | fpss |
| Time of flight calculation gain | KTETA | 1000 | sec |
| MAX L/D | LAD | 0.3 | n.d. |
| Lateral switch bias term | LATBIAS | .00012 | rad |
| LAD cos (15°) | L/DCMINR | 0.2895 | n.d. |
| Final phase L/D | LOD | 0.18 | n.d. |
| Minimum drag for upcontrol | Q7F | 6. | fpss |
| Time increment to obtain nominal reentry flight time | TN | 500. | sec |
| Minimum RDOT to initiate active guidance | VRCONTRL | 700 | fps |
| Velocity to switch to relative coordinates | VMIN | 12 833 | fps |
| Velocity to stop steering | VQUIT | 1000 | fps |
| Angle in rad to n. mi. | ATK | 3437.7468 | n.mi./rad |
| Nominal G for scaling | GS | 32.2 | fpss |
| Atmospheric scale height | HS | 28 500 | ft |
| Gravity harmonic coefficient | J | .00162345 | n.d. |
| Equatorial earth rate | KWE | 1546.70168 | fps |
| Earth gravitational constant | MUE | $3.986032233 \times 10^4$ | $m^3/sec^2$ |
| Earth radius | RE | 21 202 900 | ft |

## TABLE II. - GUIDANCE GAINS AND CONSTANTS - Concluded

| | Symbol | Value | Units |
|---|---|---|---|
| Satellite velocity at RE | VSAT | 25 766.1973 | fps |
| Earth rate | WIE | $7.29211505 \times 10^{-5}$ | rad/sec |

### Targeting

| | | | |
|---|---|---|---|
| Pacific target: | | | |
| | Geodetic latitude | | |
| | Longitude | | |
| | Time increment to obtain nominal flight time | 500 | sec |
| Atlantic target (launch abort): | | | |
| | Geodetic latitude | 26.4° | N |
| | Longitude | 20.6° | W |
| | Nominal flight time | 1350 | sec |

| Switches | | Initial Value |
|---|---|---|
| RELVELSW | Relative velocity switch | 0 |
| EGSW | Final phase switch | 0 |
| GONEPAST | Indicates overshoot of target | 0 |

## TABLE III. - FINAL PHASE REFERENCE TRAJECTORY

| N | VREF, fps | RDTR RDOTREF, fps | AREF, fpss | FRDT F2 DR/DROT, n.mi./fps | FA F1 DR/DA, n.mi./fps | RTOGO, n.mi. | PP DR/DL/D, n.mi. |
|---|---|---|---|---|---|---|---|
| 1 | 0 | -331 | 34.1 | 0.0 | - .02695 | 0.0 | 1. |
| 2 | 337 | -331 | 34.1 | 0.0 | - .02695 | 0.0 | 1. |
| 3 | 1080 | -693 | 42.6 | .002591 | - .03629 | 2.7 | 12.88 |
| 4 | 2103 | -719 | 60.0 | .003582 | - .05551 | 8.9 | 21.82 |
| 5 | 3922 | -694 | 81.5 | .007039 | - .09034 | 22.1 | 43.28 |
| 6 | 6295 | -609 | 93.9 | .01446 | - .1410 | 46.3 | 96.70 |
| 7 | 8531 | -493 | 98.5 | .02479 | - .1978 | 75.4 | 187.44 |
| 8 | 10101 | -416 | 102.3 | .03391 | - .2372 | 99.9 | 282.2 |
| 9 | 14014 | -352 | 118.7 | .06139 | - .3305 | 170.9 | 329.4 |
| 10 | 15951 | -416 | 125.2 | .07683 | - .3605 | 210.3 | 465.5 |
| 11 | 18357 | -566 | 120.4 | .09982 | - .4956 | 266.8 | 682.7 |
| 12 | 20829 | -781 | 95.4 | .1335 | - .6483 | 344.3 | 980.5 |
| 13 | 23090 | -927 | 28.1 | .2175 | -2.021 | 504.8 | 1385. |
| 14 | 23500 | -820 | 6.4 | .3046 | -3.354 | 643.0 | 1508. |
| 15 | 35000 | -820 | 6.4 | .3046 | -3.354 | 643.0 | 1508. |

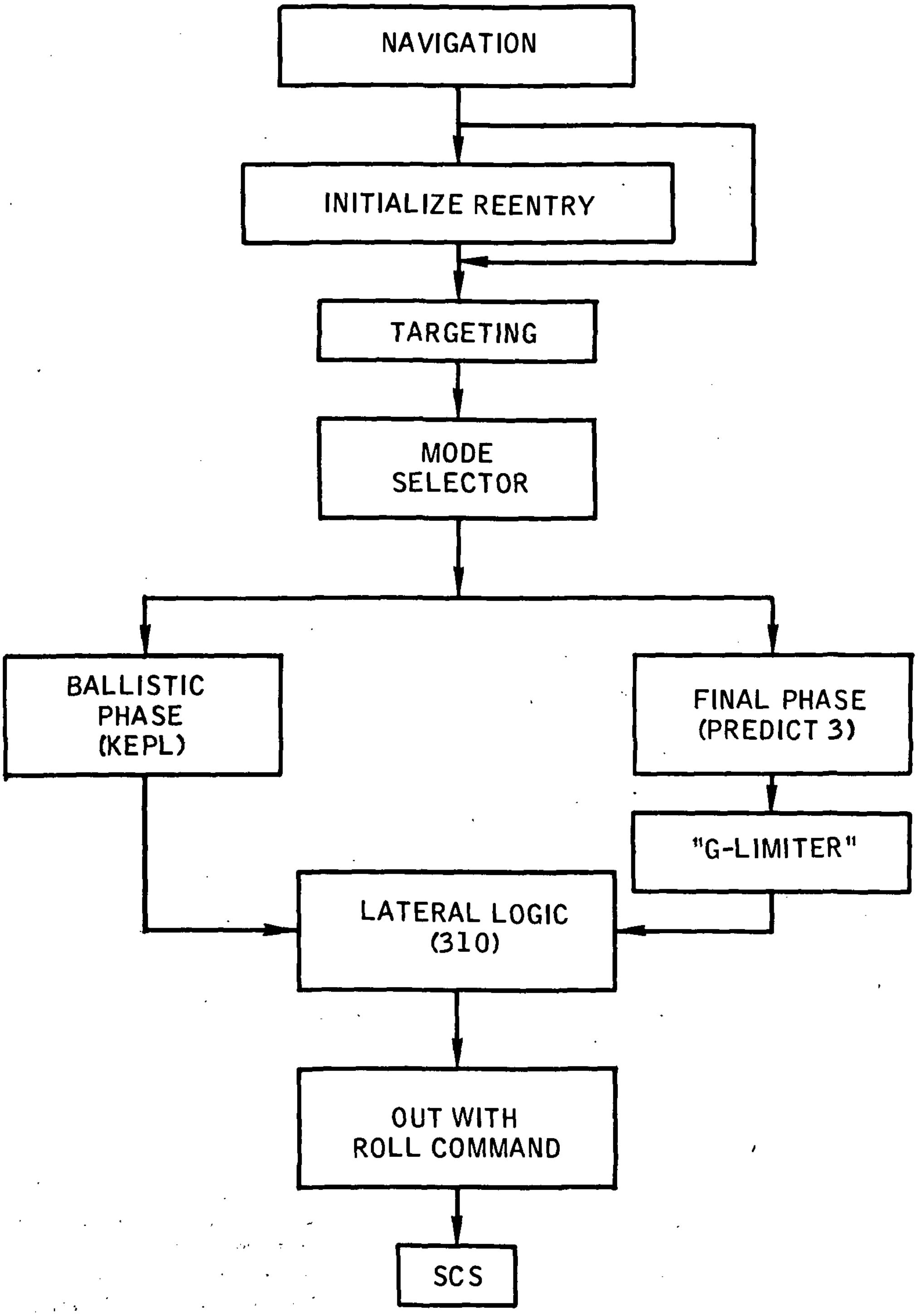


Figure 1.- Reentry steering.

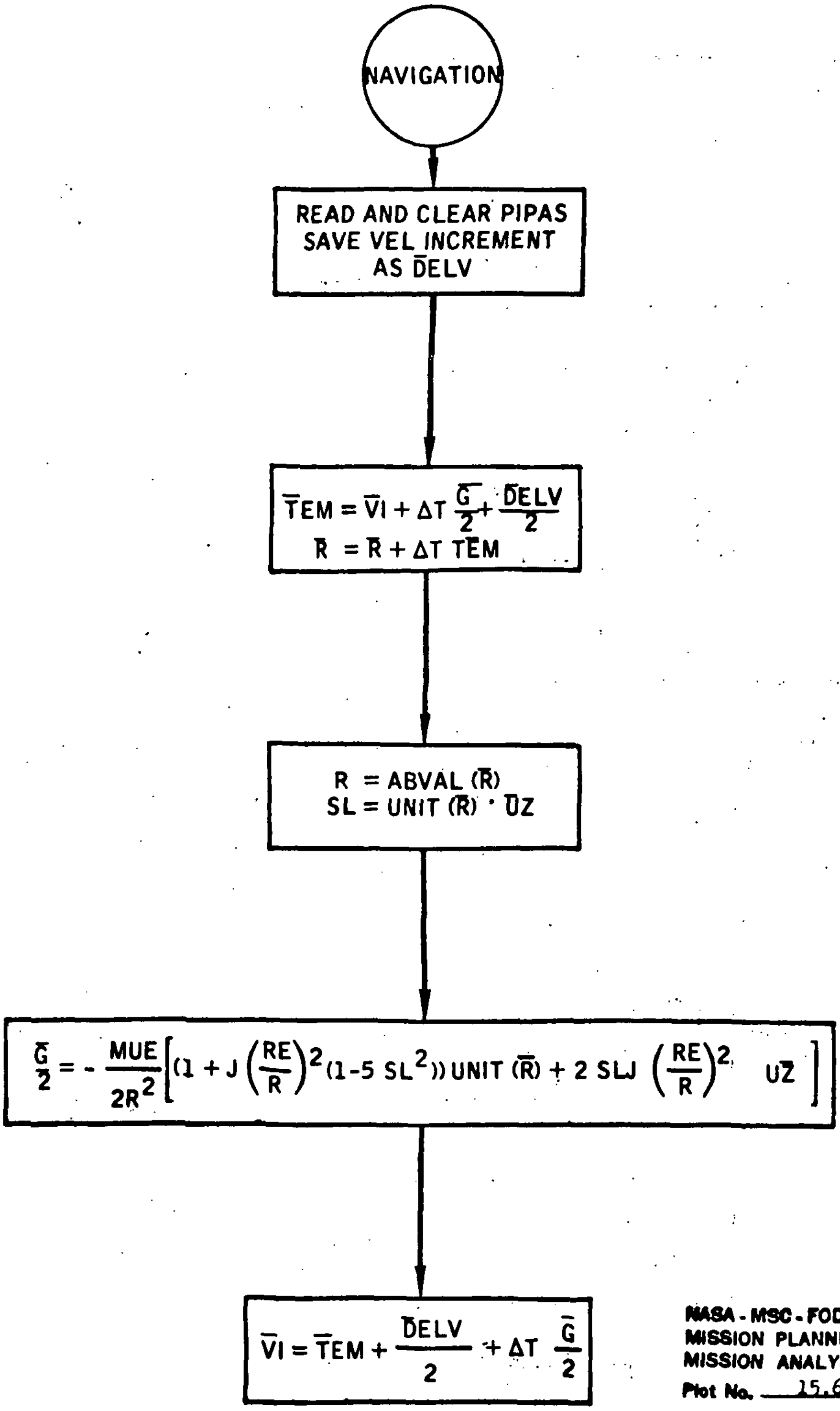


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FIGURE 2.- NAVIGATION ΔV COMPUTATION.

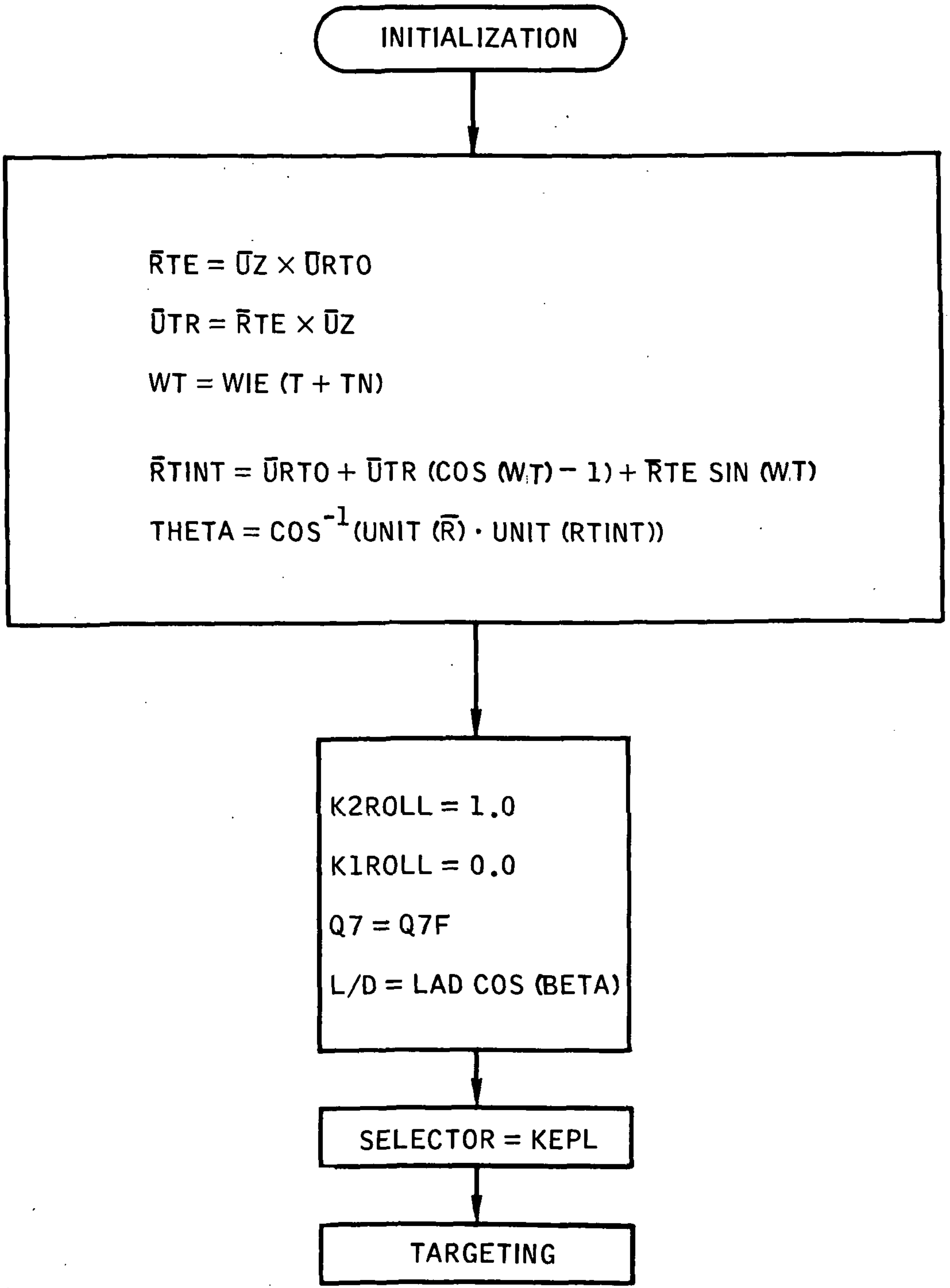

Figure 3.- Initialization.

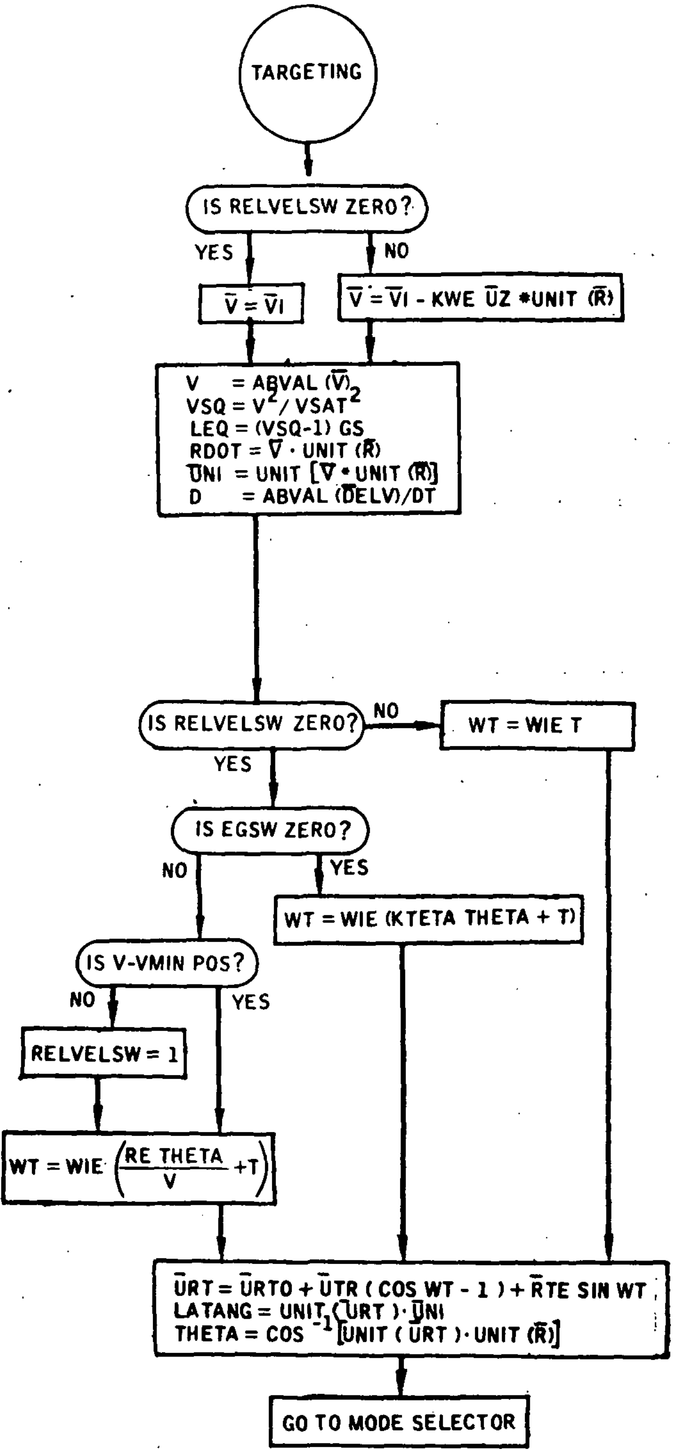

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FIGURE 4.- TARGETING

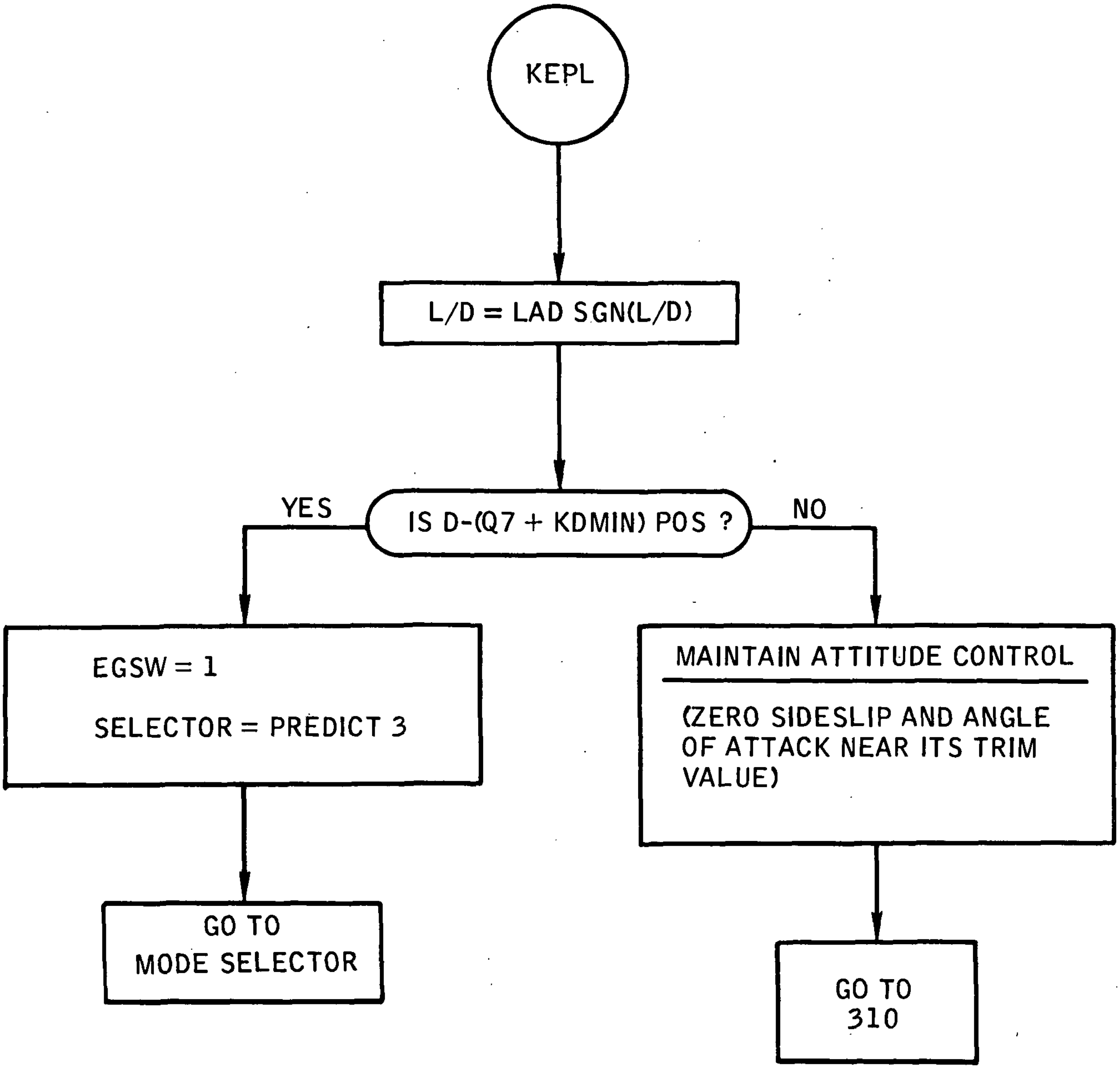

Figure 5.- Kepler.

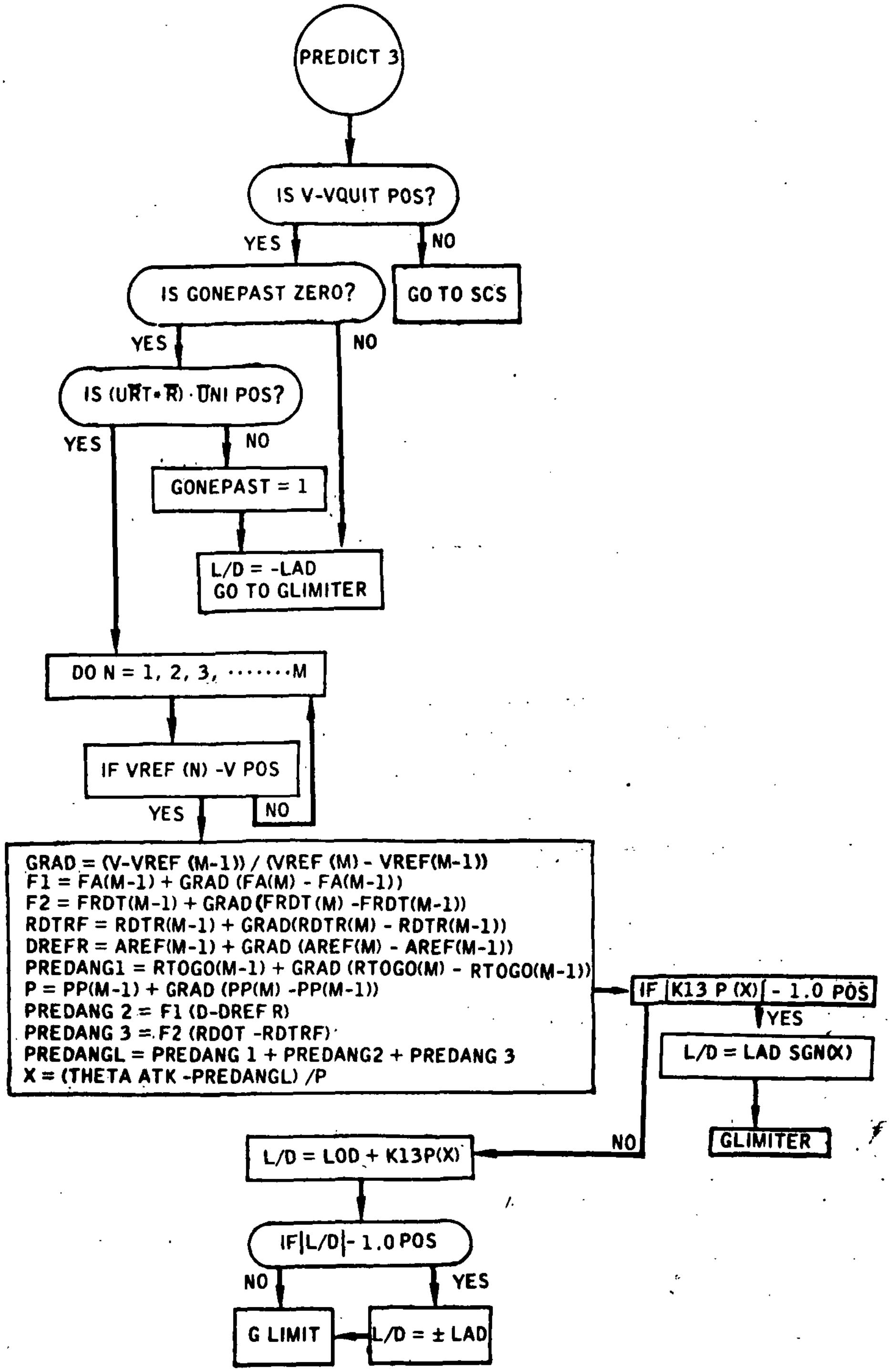

Figure 6.- Predict 3.

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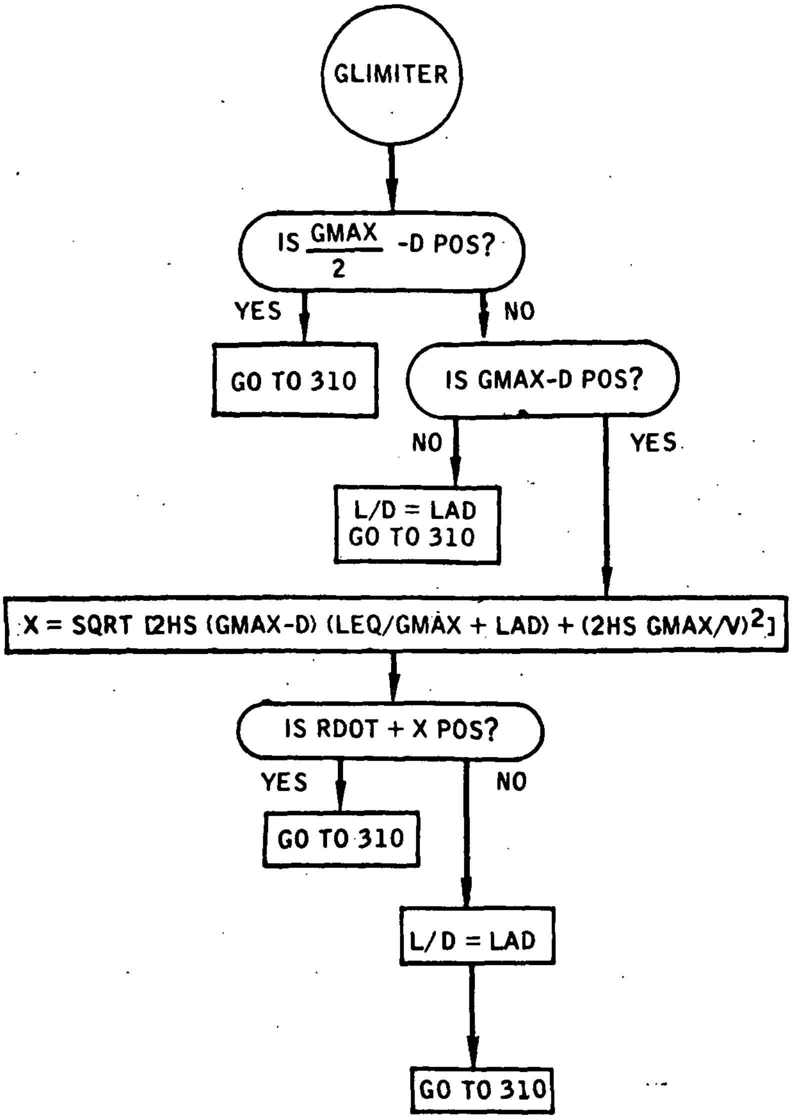

Figure 7.- G-Limiter

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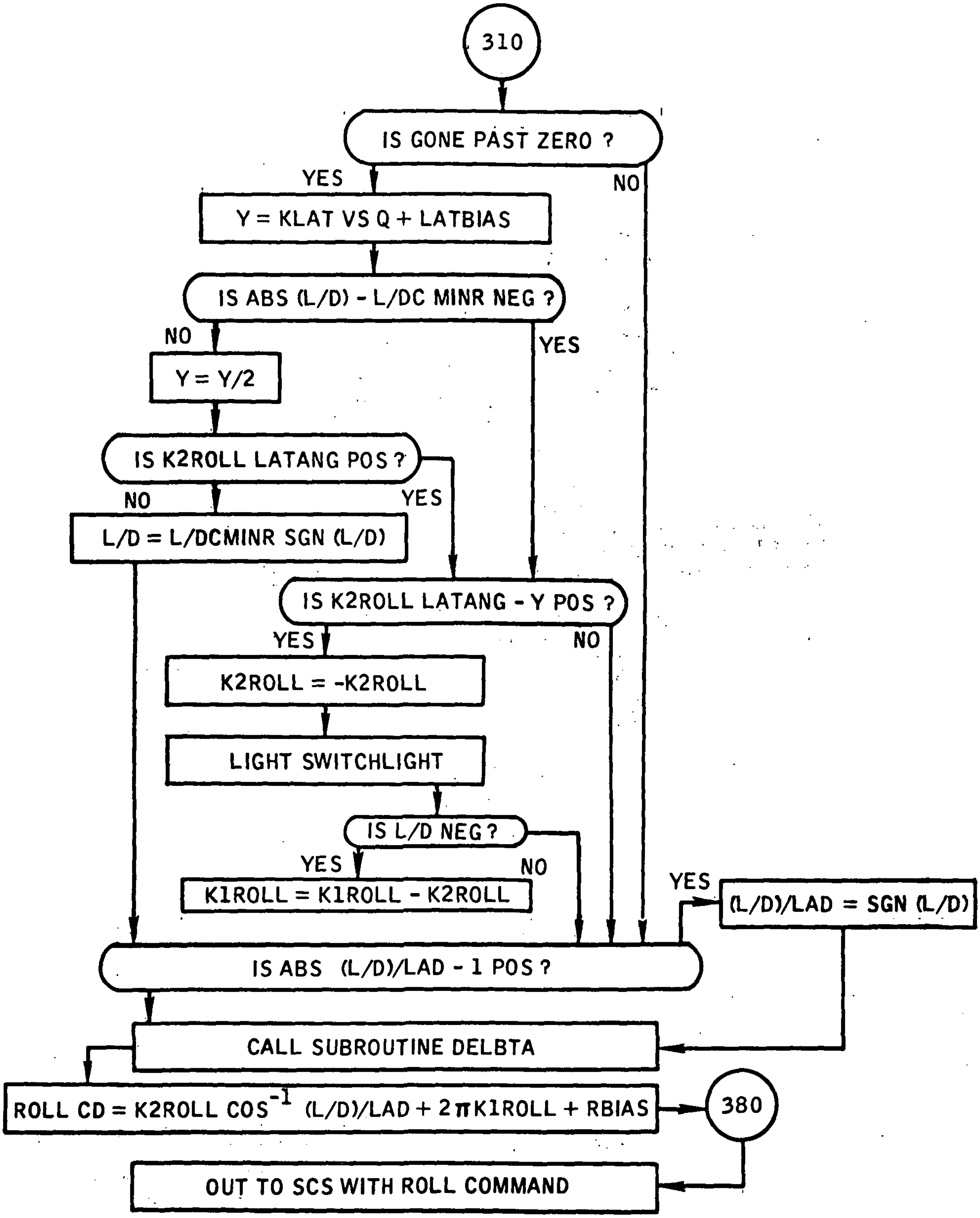

Figure 8.- Lateral control.

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1. MIT Instrumentation Laboratory: Preliminary Guidance and Navigation System Operations Plan, Apollo Mission 204, R-507, June 1966.

2. Hill, Oliver: AS-202 Reentry Guidance and Navigation Equations and Flow Logic, IN 66-FM-26.