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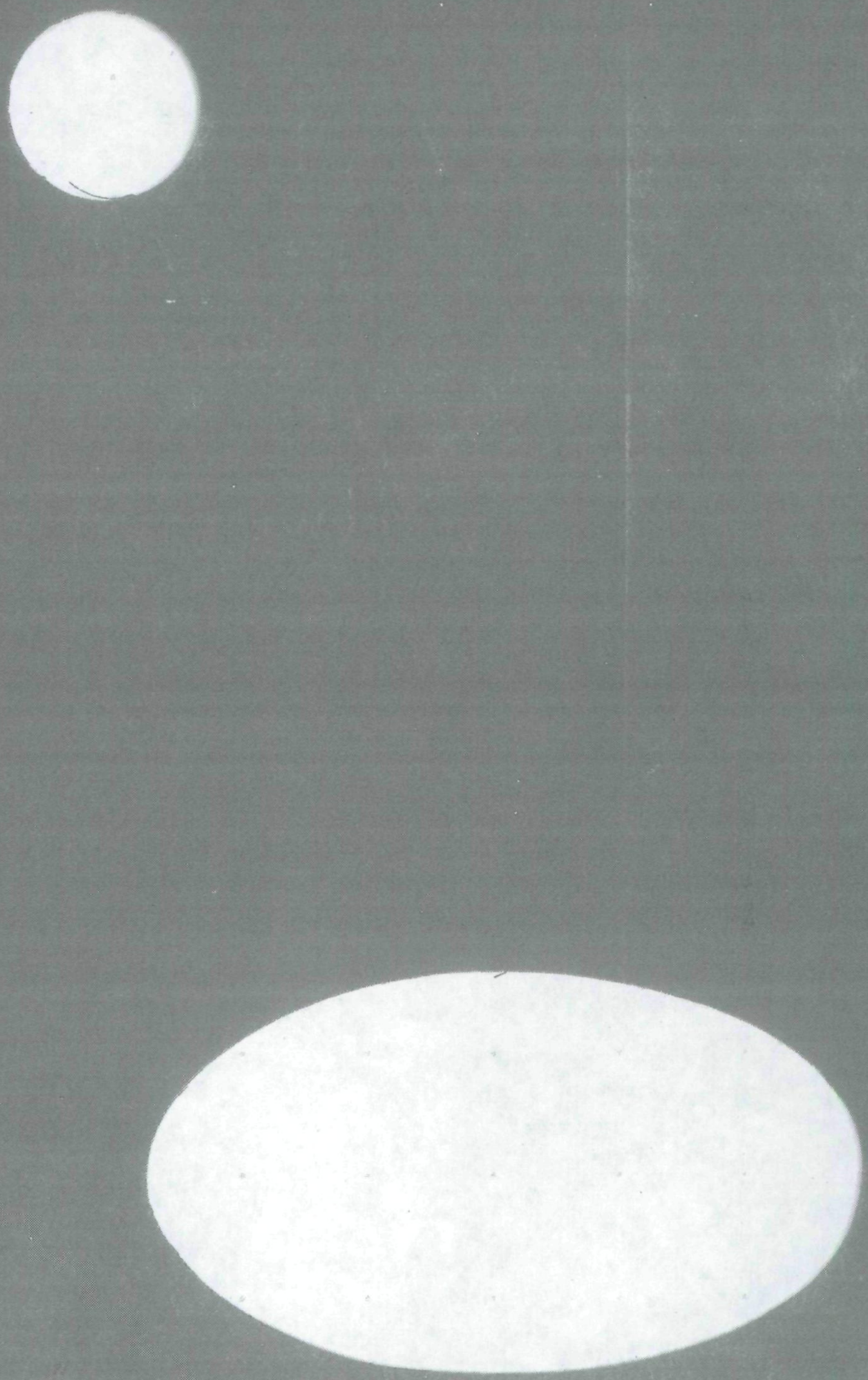

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Approved: Milton B. Trageser Date: 3/7/64
Milton B. Trageser, Director
Apollo Guidance & Navigation Program

Approved: Date: 3/7/64
Roger B. Woodbury, Deputy Director
Instrumentation Laboratory

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On the Perception of Flashes of Light at the Limit of their Perceptibility

Prepared by
George A. Davidson
and
Arthur C. Hardy
January 1964

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ON THE PERCEPTION OF FLASHES OF LIGHT AT THE LIMIT OF THEIR PERCEPTIBILITY(1)

1. PURPOSE

The laws of perception of flashes of light are of great interest, not only from the theoretical and philosophical point of view but also because of the important applications which flashing signals have in practice. In the past, numerous experimentalists like Helmholtz, Brucke (1866), Exner (1868), Allard (1872), Kunkel (1874), Swan (1849), Charpentier (1887 - 1890), Broca and Sulzer (1902), Martius (1902), McDougall (1904), etc. have studied the variations in the brightness sensation as a function of time. It is to Messrs. Andre Broca and Sulzer to whom we owe the most complete and scientific knowledge of that variation.

The curves (Fig. 1) which they obtained(2) by comparing a brief flash produced on a screen with the continuous effect produced by a standard source shows how apparent sensations vary as a function of time over a range of intensities measured in lux(3). These curves have, for the first time, established precisely that,

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(1) Translation of an Article by A. Blondel and J. Rey, Journal de Physique, Ser. 5, Vol. 1, 530 (1911). A short summary of this work was presented before the Academy of Sciences at their meeting on July 3, 1911.

(2) "The Luminous Sensation as a Function of Time" (Journal of General Physiology and Pathology, No. 4, July, 1902).

(3) 1 lux = 1/10 millilumen per square centimeter. It is the illuminance produced by one candle at a distance of one meter.

except for light of very low intensity, the luminous sensation reaches a maximum within a rather short time, and thereafter slowly approaches a somewhat lower steady-state value. (1)

These curves do not give, however, the solution to the problem that we are here considering, which is concerned with the limit of perception (that is to say, with the threshold of the sensation) for weak stimuli. The intersections of the curves with a horizontal line corresponding to the threshold of the sensation are uncertain. Moreover (and this is the primary objection), in the case of a threshold, the impression does not follow a curve but appears suddenly at a just-perceptible value when the action of the light is sufficient in intensity and duration. Otherwise, there is no sensation at all.

A simple analog allows this important difference to be clarified. In the experiments of Messrs. Broca and Sulzer, the eye appeared to behave like a galvanometer activated by a current during a part of its period of oscillation, where the damping varies inversely as the effective luminous intensity, because the sensation goes beyond the steady-state response in proportion to the magnitude of the exciting intensity.

On the other hand, in the case that we shall study, the stimulus is of very short duration; at most, no longer than the threshold of the sensation. The eye then behaves like a ballistic galvanometer, which integrates the stimulus. We have searched for the law connecting the integrated value with the intensity and time duration of the stimulus.

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(1) Helmholtz, Brucke and Exner had already pointed out the existence of a maximum, and recognized that the time required to reach the maximum becomes shorter when the intensity of the stimulus is greater; but their methods were very vulnerable to criticism. Charpentier, who had performed analogous experiments, had not found this maximum because he was experimenting with very low intensities. Moreover, his apparatus did not instantaneously uncover the observed area.

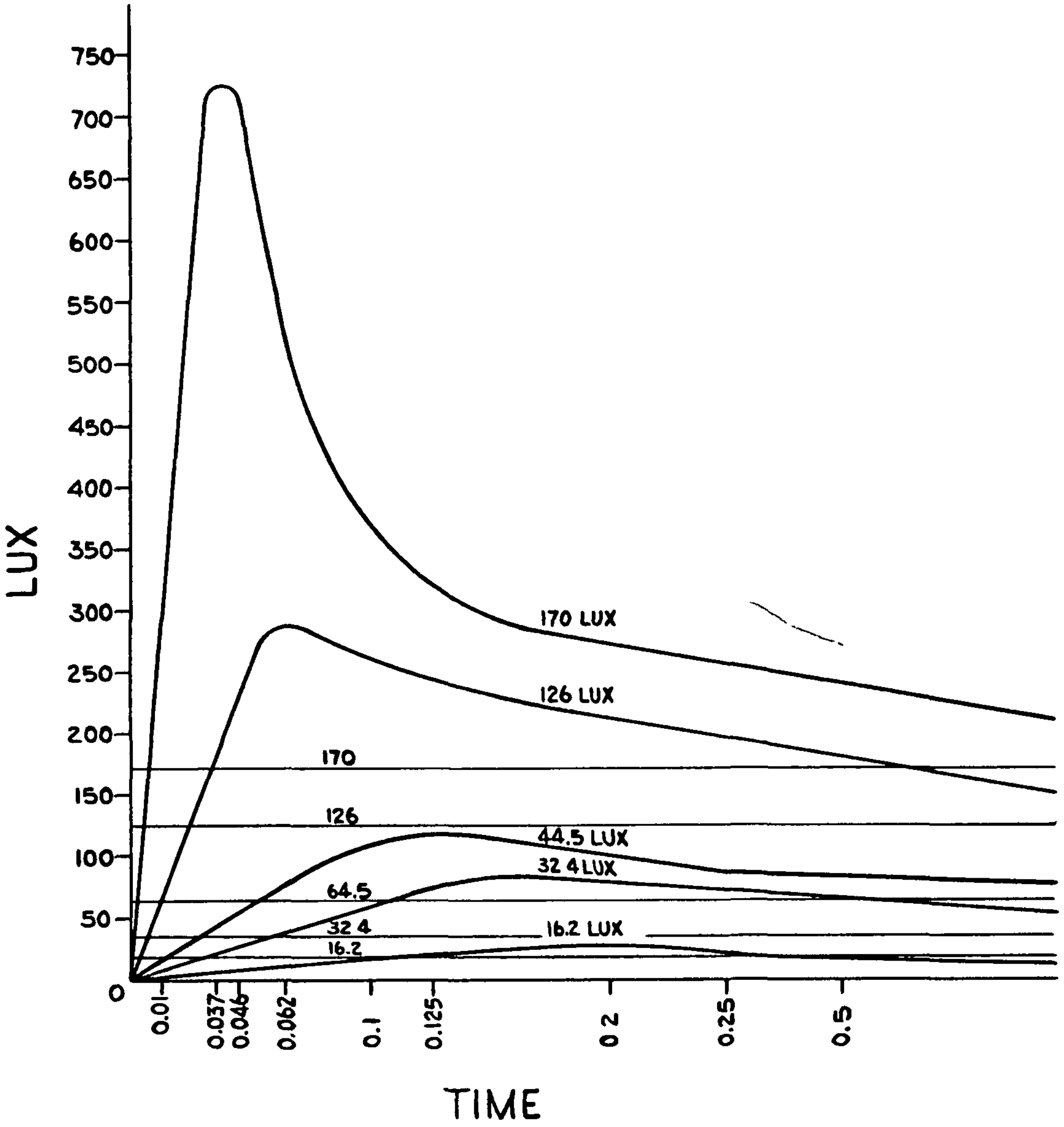

Fig. 1 Curves of Broca and Sulzer, representing the variation of apparent luminous impressions as a function of time for different intensities of area observed.

This problem has already been studied partially by different authors. As early as 1834, Talbot, (1) while studying the Talbot-Plateau law of persistence of vision based on the well known experiment of the revolving disk, concluded that the sensation should be proportional to the time duration of the stimulus. Swan(2) stated the same idea a little more clearly. The problem was not precisely defined, however, until the more recent investigations by Block(3) and Charpentier(4).

Bloch thought he had established that the stimulus required for the minimal sensation was substantially constant and proportional to the product of the intensity and time. Charpentier, who has verified this law within certain limits, agreed that the establishment of a retinal impression e produced by an intensity E during a short time interval t obeys the linear relationship $e = Et$.* It follows that light impressions produced on the retina by two sources of different intensities should appear equal if these sources are activated for periods inversely proportional to their intensities. Thence Et = a constant. Charpentier believed that the law of proportionality of impressions with time was true only when one had attained a steady state. He determined the times for different luminous intensities (unfortunately, all were notably higher than the perceptible limit of intensity), and thus

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(1) Philosophical Magazine, 1834, Ser. 3, Vol. V, p. 327.

(2) Transactions of the Royal Society of Edinburgh, 1849.

(3) Comptes Rendus de la Societe de Biologie, 1885, Ser. 8 Vol. 2, p. 495.

(4) Comptes Rendus de la Societe de Biologie, 1887, Vo. 2, p. 5.

*Translators' Note: Despite the immense number of English words employed in the field of photometry, there seems to be none in common use that represents the concept of conspicuity of a point source measured in terms of the number of lumens per unit area arriving from that source in the plane of the observer's eye. When Blondel and Rey use the symbol E to represent "eclairement," its literal translation in modern photometric parlance should be "illuminance." We have used the word "intensity."

found values varying between 1/8 and 1/10 of a second and appearing to be proportional to a power equal to 1/3 or 1/4 of the absolute flash of light. He admitted that all the lights were observed integrally for a considerable period of time, as if they were constant.

MacDougall, (1) using a more refined procedure, found it necessary to carry the exposure to 1/5 of a second for very faint sources. One of us (2) had pointed out, as early as 1893, that one should not assume the discontinuity resulting from the sudden change in slope of the sensation curve (to be) the point of integrated perception, for "natura non fecit angulos." He had therefore proposed to replace the straight line by a curve, joining it to the horizontal line at the same point. A better explanation of Charpentier's results can be found today from curves in Fig. 1, where the portions near the origin rise virtually in a straight line right up to the horizontal lines representing steady sensation. It is likely that Messrs. Bloch and Charpentier thought that the curves stopped at these horizontals. Inasmuch as they extend much higher, the law of Bloch loses all philosophical basis.

Moreover, the law is contradicted by experiment, inasmuch as lights so weak as to be almost at the limit of sensation were being employed, beyond which no additional time will extend the limit. Thus, in the experiments of Messrs. Rocca and Sulzer, the time necessary for steady sensation varied between 1 second

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(1) Journal of Psychology, Vol. 1, Part 2, June, 1904. MacDougall described also the extraordinary speed with which the luminous sensation subsided once having reached its maximum, but did not specify the curve in a detailed manner.

(2) A. Blondel, "On Flash-Lights and the Physiological Perception of Instantaneous Flashes" (Proceedings of the International Maritime Congress, London, 1893, Section IV, p. 39).

(for strong light) and 2 1/2 seconds (for weak light), and Mr. Ribiere, experimenting by a different method and from a different point of view, found(1) that the range limit of a short flash increases almost indefinitely with its duration and remains always inferior to a flash of the same intensity from a steady source.

These facts allow us to conclude that the product Et should increase as the intensity of the light approaches the threshold of the sensation, and that Bloch's law should apply only to very short and intense flashes of light. Moreover, it seems obvious a priori that the time required to detect in space the location of a point source just at the threshold of visibility should be long enough so that the eye, in searching for that point, will not have to remain fixed in that direction too long before perceiving the light.

We have been led to infer that Bloch and Charpentier's faulty law ought to be replaced by one in which the duration of perception of light just capable of producing the threshold of sensation should be infinite. If one traces, in rectangular coordinates, a curve having time for abscissas and intensity E for ordinates, Bloch's law Et = a constant would be represented by an equilateral hyperbola (I) having for asymptotes the two axes of the coordinate system; whereas the law we wish to promulgate should approximate an equilateral hyperbola (II) having for its asymptote a higher horizontal $E = E_o$ (where E_o is the just-perceptible intensity). Then the equation takes the form:

$$(E - E_o)\, t = C^{te} .$$

Not wishing to be bound by a purely theoretical inference, we have investigated experimentally the relationship between E and t (when they produce the limiting sensation), taking care to use numerous observers who were not informed of previous results.

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(1) M. Ribiere, Beacons and Maritime Signals, 1908, p. 15.

2. METHODS AND APPARATUS USED

The observations were carried out with brief flashes of pin-point origin, - that is to say, flashes produced by an artificial luminous source having dimensions too small to be resolved by the human eye, because it is thus that flashes at a great distance are perceived.

All of our experiments were performed in the laboratory, in order to avoid all the perturbating effects that one encounters in the atmosphere, such as variations in atmospheric absorption, effect of ambient illumination, fatigue of observers, etc. Only in the laboratory can one study phenomena of this character with the precision required to establish a law, if only approximately.

One can, under these conditions, consider whether to compare the effect of a flash to that of a steady light just perceptible and serving as a reference, or whether to just compare two flashes, one of which is held constant. It is this latter method which our experience has shown to be preferable, for it alone allows that there be obtained impressions which can be compared one with the other. Furthermore, experience has shown us that the fatigue of the observers is very much greater when one takes as a measure of comparison a steady light instead of a flash. That is so, we think, because the limit of perception is much less definite in the case of a steady point of light than in the case of a short flash.

Certainly flashes produced by sources of short duration should be followed by intervals long enough so that one should have no fear of the mutual effect of successive impressions(1). That is why we adopted, in general, a periodicity of about

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(1) This condition is even more necessary when one measures impressions above the limit, for one then has to fear after-images, as described by Broca and Sulzer.

3 seconds between the successive appearance of flashes.

On this assumption, we employed the method of equalization. That is to say, we tried to equalize two brief flashes differing in their values of E and t. That equalization can be accomplished in two ways: by observing the two flashes alternately, each flash made to be seen once out of two times, or, conversely, by observing the two flashes simultaneously. The first method has already been recommended by MacDougall(1), who, moreover, did not use it to equalize the flashes, but to determine the differences that caused perceptible gradations in the sensation. The second method was already employed in 1893(2) by one of us, who rotated a black disk containing a radial slit of constant width in front of a black screen to which were attached circular bits of white paper all of the same size and arranged along a radius(3).

But that method seemed to present, at that time, the inconvenience that the sensations did not appear simultaneously. In the present experiments, it was recognized that it would suffice to obtain the necessary apparent simultaneity if the luminous excitations were (instead of commencing at the same time) to terminate at the same time.

One apparatus allowing the second method to be used was studied by Marsat, an engineer with Sautter-Harle, and was used in carrying out the greater portion of experiments, the results of which appear below. Another apparatus allowing use of either one or the other method was built by Camillerapp, a construction engineer, from a design by one of us. We shall describe each apparatus briefly.

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(1) Loc. cit.

(2) A. Blondel, loc. cit.

(3) The distance from the axis of rotation to the center of the first little circle represented a weak flash, giving a measure of the duration limit, which was thought to be possible of determination.

Apparatus of Mr Rey (Figs. 2 and 3)

The device employed is shown schematically in Fig. 2. It includes a luminous source S consisting of an incandescent lamp with a U-shaped filament, the two upright sections as parallel as possible.

The source actually used came from a portion of one of the upright sections of the filament, the length of which was limited by screens. The line of light falls on a small lens L_1, located at a distance of approximately 40 to 80 centimeters.

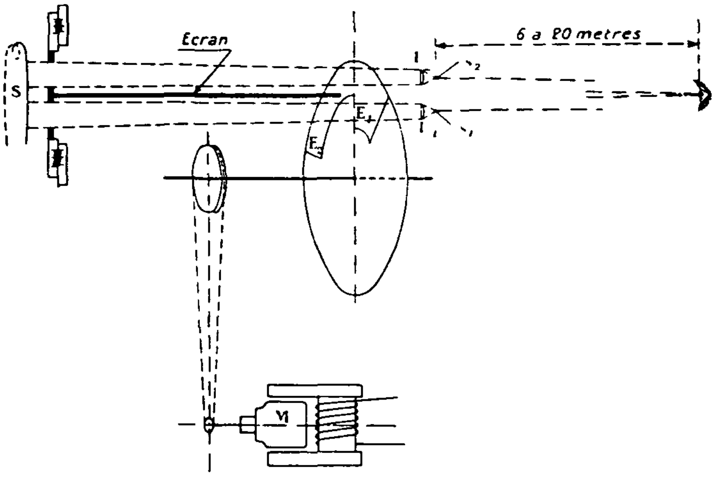


Fig. 2

The image formed by that lens, having a focal length of only 8 mm, constitutes a secondary source S_1, of such small dimensions as to be considered a point source for an observer whose eye is situated at a distance of 6 to 20 meters.

A second lens L_2 is lighted by another portion of the same upright section of the filament of source S, the length of this portion being adjustable The image produced by L_2 forms a second point source S_2 whose intensity can be varied at will and is found to be proportional to the length of filament used

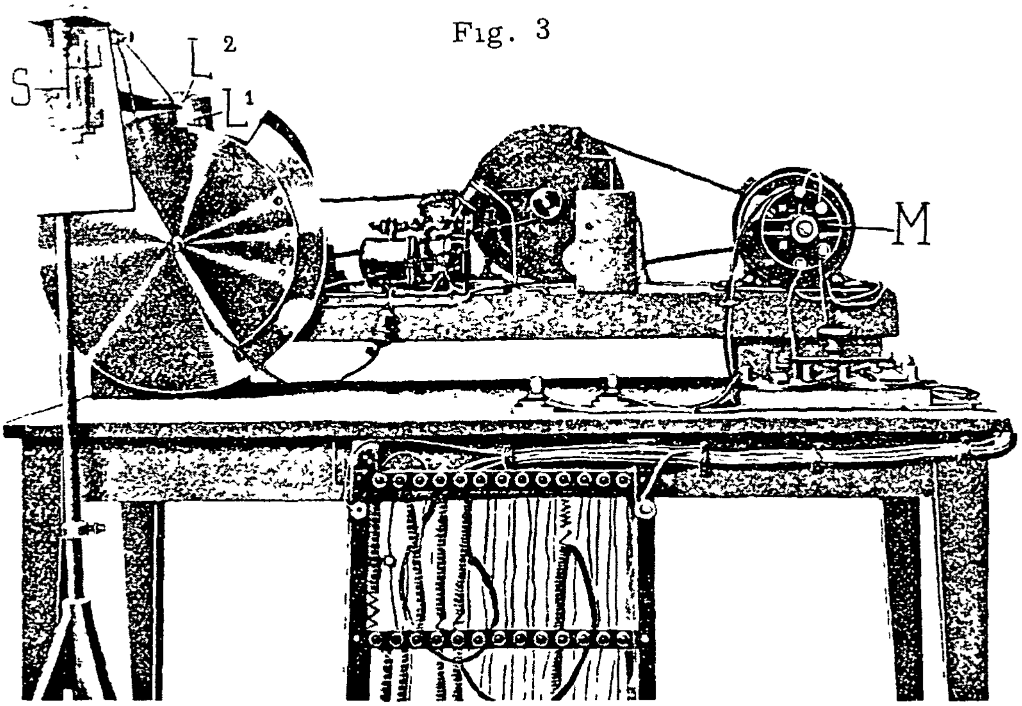
Between lenses L_1 and L_2 and the source S, a system of rotating screens is placed, made up of 2 solid disks slit radially and set in motion at a constant speed by an electric motor M.

The whole system is the equivalent of a single disk with two apertures, one of fixed dimensions, the other of variable size

The aperture of fixed dimensions E_1 passes in front of lens L_1 and produces a flash of constant intensity and duration.

The other opening E_2 passes in front of lens L_2 and produces a flash whose duration varies with the size of the aperture and whose intensity varies in accordance with the length of filament used. The openings are so arranged as to produce simultaneous flashes.

Fig. 3



The principle of the method for making measurements is as follows:

The observer compares the two flashes by, first, coming close enough to the apparatus for the flashes to be clearly visible. He then steps back until he can see only one flash. After receiving an appropriate signal, the operator modifies the intensity of source S_2 and the observer repeats the procedure. When he signals again, source S_2 is again modified (successively) until the observer can no longer see the two flashes from the same distance. The two flashes are then of equal visibility. That is to say, they have the same range limit for one particular observer. At that point, the length of the filament of source S_2 is measured and recorded

Experience has shown that, when the observer again approaches the instrument, the impression of equality ceases and the more rapid flash appears to be the more intense.

In order that the observations shall be correct, it is absolutely necessary that the interval between flashes be the same for both sources and that the duration of the interval be not less than 3 seconds.

With the aid of techniques described above, and illustrated in Fig. 3, it has been possible for us to produce flashes of any duration, from a thousandth of a second to 3 seconds; that is to say, variable in the extreme ratio of 1 to 3000. The narrow width of the filament source S, in the neighborhood of 1/10 mm, which is almost negligible in front of the 6 mm slit, allows instantaneous appearance and disappearance of the flashes, reducing to an almost insignificant value the conspicuous systematical errors mentioned in connection with Charpentier's method.

Apparatus of Blondel

This instrument is based on the use of diaphragmed lenses, introduced in photometry by Bouguer, later by Cornu, and considerably improved by the author(1). Figures 4, 5, and 6 show schematically the arrangement of the apparatus.

The apparatus comprises 3 photometric tubes T_1, T_2, and T_3, each equipped with an adjustable cat's-eye 0_1, 0_2, and 0_3 whose displacement is read from a little counter placed in front of it(2). At the entrance of each tube is to be found a diaphragm D_1, D_2, and D_3 with a hole about 10 mm in diameter in the center. The hole is covered with a little screen of opal glass. Each screen is strongly lighted by a filament f_1 of a Nernst lamp N, supported by a cylindrical member M, fitted tightly and held by friction in tube T_1. One can vary within certain limits the intensity of the light by moving the member M_1 in or out of T_1. Perforations in the member (concealed on the outside by protecting screens) permit cooling. The three Nernst lamps are supplied from a 110-volt circuit.

Screen E_3 produces an image at the conjugate focus on a small circular aperture a_3, supplied with a screen of ground glass several millimeters in diameter. Similarly, screens E_1 and E_2 produce images on screens a_1 and a_2 after being reflected by mirrors at 45°, M_1 and M_2.

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(1) He has replaced the old cat's-eye with a special cat's-eye formed by two curtains displaced parallel to one another where they meet on contact with a rectangular diaphragm which has for an aperture the whole width, and whose height varies from one diaphragm to another. This arrangement allows the maximum sensibility of the scale as well as maximum ratio of the readings to the apertures. Another refinement consisted of replacing the simple lenses of Bouguer and Cornu (which gave deformed images) by a system of double achromatic lenses, making it possible to obtain very short focal lengths with very little aberation.

(2) Figure 4 gives a cross-section, assuming, for simplicity, that all three tubes T_1, T_2, and T_3 lie in the same plane. Actually, T_3 has its axis raised higher by 3 centimeters with respect to the other two.

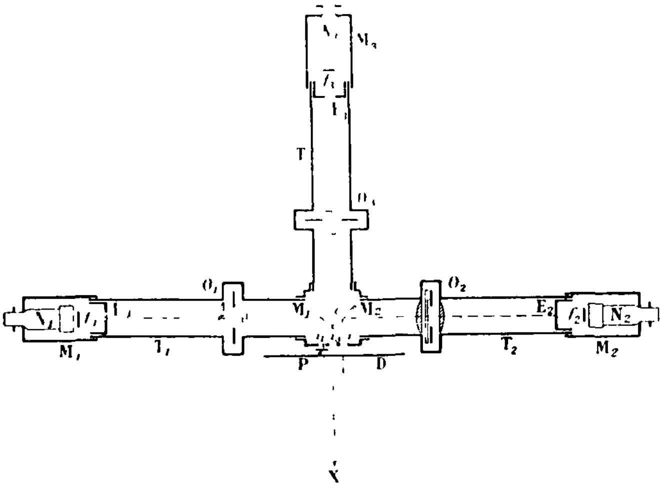


Fig. 4 Cross-section of the system.

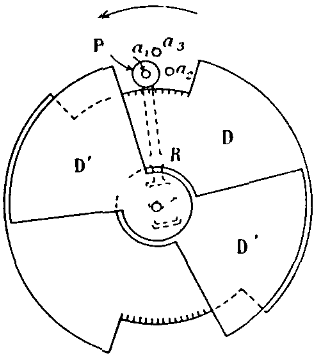
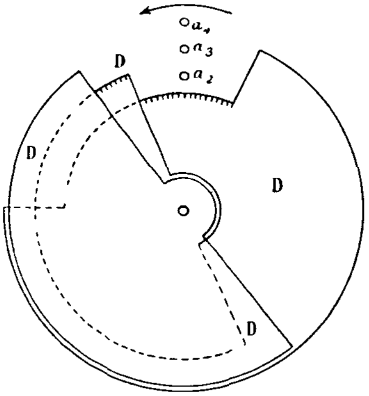


Fig. 5 Diagram of the disk with the two adjustable openings and of the oscillating paddle P, with relation to the 3 holes a_1, a_2 and a_3, which serve to produce the point sources.

Fig. 6 Diagram of the disk with the adjustable openings and of the holes producing the light sources.



Since tube T_3 is placed a little higher than the others, the three ground glasses a_1, a_2, and a_3 appear, as shown in Fig. 5, with a_3 at the summit of an equilateral triangle of about 30 mm on a side. In front of each ground glass is placed a small diaphragm (not shown) having a small hole 1 or 2 mm in diameter which allows pin-point images to be observed. One can vary the intensity of each one of these light sources by maneuvering one of the cat's-eyes.

It is these three light sources that the observer compares in the direction X at a distance of from 2 to 6 meters.

Between the observer and the light sources is placed an opaque disk D which is driven at a speed of several revolutions per second by an electric motor not shown, whose purpose it is to extinguish (during a variable period) certain of these points. Figures 4 and 5 represent two devices that one can use at will to produce brief flashes.

In the device in Fig. 5, the disk D has two graduated cut-outs, the size of the opening being regulated at will by two independent sectors D' and D'' (accomplished by adjusting knobs not shown).

Disk D has a radius a little smaller than the distance that separates its center from the small screen a_3, and therefore the point of light produced by the Nernst lamp N_3 is always visible and can serve as a reference to guide the eye and even eventually as a point of comparison. The intensity of this reference light is adjustable, in effect, by the cat's-eye 0_3. An opaque paddle P is driven by an eccentric which gives the paddle an alternating motion along the axis of oscillation R, thus alternately obscuring screen a_1 and screen a_3, respectively, during passage across one or the other of the two openings, in such a manner that the observer sees appear alternately the flashes produced by a_1 and a_2, which are adjusted respectively by the widths of the two graduated openings. One thus realizes the desideratum of the psychophysicist, which is to compare the two flashes appearing alternately. Their durations are measured by the angles and the speed of the disk, determined with the aid of a counter, and adjustable by the rheostat of the motor. One equalizes the apparent flashes by adjusting the cat's-eye 0_1 and 0_2.

One can also compare the sensations of these intermittent flashes with those of the fixed point a_3(1).

The arrangement in Fig. 6 allows simultaneous flashes to be produced also, as in the apparatus of Rey. In this case, one replaces the two holes a_1 and a_2 by a single light from a mirror placed at the point of symmetry C. The other hole is hole a_3, lighted by screen E_3. Under the central housing of the apparatus, there is placed another small reference hole a_4 lighted by a small supplementary incandescent lamp which serves only to fix direction. The two sectors D and D' serve to regulate the widths of the two slits located at different distances from the center so as to give to flashes a_2 and a_3 different durations but terminating at the same instant.

Thanks to the extremely small dimensions of point sources, the duration of flashes can be ascertained with great precision. The apparatus lends itself, as is evident, to a variety of combinations.

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(1) The triangular arrangement of the three points of light makes readily distinguishable the flash on the right from the flash on the left. That is why we used two independent mirrors M_1 and M_2. One can, however, in certain cases, replace these two mirrors by a single one passing through the center of symmetry C of the apparatus and by replacing the oscillating plate P by another eccentric drive that alternately displaces the single mirror in such a manner that it reflects)on one and the same aperture placed under a_3) alternately the rays coming from E_1 and E_2. J. of Physics, Ser. 5, Vol. 1 (July, 1911)

3 DISCUSSION AND CALCULATION OF EXPERIMENTAL RESULTS

The experiments of which we shall summarize the results were carried out, for the most part, in the laboratory of M. M Harle and Company by means of the original apparatus, with the collaboration of George Guy, a former student of the Ecole Polytechnique, who directed the observers and assisted us with the calculations The latter were verified elsewhere by Blondel, in his own laboratory, with the second apparatus.

Measurements

The observations comprised 25 series carried out by 17 observers of different ages and professions. workmen, engineers, foremen, employees, etc. These observers naturally had very different visual acuities, some being myopic, others presbyopic, others had astigmatism. In a word, one might say that all usual types of eyesight were represented.

To establish a basis of comparison, we took as a standard for each observer a measurement made by the observer himself under specified conditions, and considered, as a consequency, only relative values (1).

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(1) Inasmuch as our measurements were reported on the basis of a standard which varied from observer to observer, and which represented the minimum perceptibility of each observer, we have not expressed this law in absolute photometric units. Obviously, it is easy to have some idea of the mean order of magnitude of these measurements by noting that, according to early calculations of Messrs. Leonce Reynaud and Allard, the just-perceptible illuminance produced by a point source is much lower than the commonly used figure of 10^{-7} lux (illuminance produced by 1/10 of an international candle at a distance of 1 kilometer) The mean value is more nearly 0.50 to 0.60 10^{-7} when the observer is placed in a laboratory sheltered from extraneous light. The figure 10^{-7} more nearly approaches normal conditions of observation out of doors.

The instruments that we have described can be readily used for standardization of intensity of the point sources employed. All that is necessary is to standardize the strongest intensities by making them illuminate one portion of the field of a photometer (from a short known distance) while the other portion of the field is illuminated by a standard equal to one international candle.

Table I is a summary, for each observer, of the luminous intensities as a function of the intensity that is just perceptible by that observer.

Further, because it is very difficult to make direct comparisons between very long and very short flashes, comparisons of successive flashes were performed. In particular, brief flashes of less than 1 second were compared with flashes of 0. 3 or 0. 03 second; those of 1 second were then compared with longer ones up to 3 seconds. The results are summarized in the two tables that follow.

The first eight series (Table I) refer to measurements made on flashes of various durations, from 3/100 second to 3 seconds.

The other seventeen series (Table II) refer to flashes of durations varying between 1/1000 and 3/100 second.

The different points of observation correspond to eight different durations, in fractions of a second, as follows: 0. 001; 0. 003; 0. 01; 0. 03; 0. 1; 0. 3; 1; 3.

Calculations

In spite of all the precautions taken, the measurements varied within such wide limits that it did not seem justifiable to take arithmetical means. It is proper, on the basis of probability theory, to employ geometrical means. We obtained these easily by introducing, instead of the numbers themselves, their logarithms, since the mean of the logarithms of several numbers is the logarithm of the geometrical mean of those numbers(1).

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(1) In all that follows, we have compared the products Et with the corresponding products obtained for an exposure time of 3 seconds.

One should not conclude that we place much importance on this particular value, which is the most uncertain. We have wished simply to translate everything into relative values, and we have chosen 3 seconds because it was the extreme limit of our observations. Actually, they have no practical use except between 0 and 1 second, because, in all modern signals, one does not employ instantaneous flashes of a duration longer than or even approaching 1 second.

TABLE I

	t = 0.03 sec.				t = 0.10 sec.			
Number of the Measurement	E	$-\log\frac{Et}{3E_3}$	d	d^2	E	$-\log\frac{Et}{3E_3}$	d	d^2
1	16.4	1.274	0.148	0.0219	8	1.062	0.047	0.0022
2	18	1.067	-0.059	0.0035	7	0.954	-0.061	0.0037
3	20	1.217	0.091	0.0083	10.4	0.978	-0.037	0.0014
4	6	1.176	0.050	0.0025	3.6	0.875	-0.140	0.0196
5	16.7	1.180	0.054	0.0029	4	1.278	0.263	0.0692
6	12	1.263	0.137	0.0187	5.3	1.095	0.080	0.0064
7	13	0.947	-0.179	0.0320	4.24	0.911	-0.104	0.0108
8	10.7	1.104	-0.022	0.0005	4.9	0.920	-0.095	0.0090
Total		9.228	0.740	0.0903		8.073	0.827	0.1223
Mean		1.153	0.0925	0.1062		1.009	0.1034	0.1236

TABLE I (CONT'D)

	t = 0.30 sec.				t = 1.00 sec.				t = 3.00 sec.	
Number of the Measurement	E	$-\log \frac{Et}{3E_3}$	d	d^2	E	$-\log \frac{Et}{3E_3}$	d	d^2	E	$-\log \frac{Et}{3E_3}$
1	3.7	0.920	0.121	0.0146	3.4	0.434	0.010	0.0001	3.08	0
2	3.8	0.742	-0.057	0.0032	3	0.322	-0.102	0.0104	2.1	0
3	5.4	0.786	0.013	0.0002	3.5	0.451	0.027	0.0007	3.3	0
4	1.7	0.724	-0.075	0.0056	1.2	0.352	-0.072	0.0052	0.9	0
5	2.7	0.972	0.173	0.0299	2.1	0.558	0.134	0.0180	2.53	0
6	3.1	0.851	0.052	0.0027	2	0.518	0.094	0.0088	2.2	0
7	2.24	0.711	-0.088	0.0077	1.34	0.411	-0.013	0.0002	1.15	0
8	2.36	0.761	-0.038	0.0014	1.67	0.388	0.036	0.0013	1.36	0
Total		6.467	0.617	0.0653		3.434	0.488	0.0447		
Mean		0.808	0.0772	0.0904		0.429	0.061	0.0747		

In these two Tables, the first column gives the observation number. The second column indicates the relative intensity E of source S_2, expressed in millimeters of length of the filament used. The third column gives the value of the logarithm of the quotient of Et divided by the comparison value of Et (corresponding to an exposure time of 3 seconds). The fourth column gives the difference d between the logarithmic value of the actual observation (preceding column) and the value figured from the general law that we derived from the whole, and which we indicate in Table III. The fifth column gives the square of that difference. We have emphasized, in the fourth column, the probable difference, defined by the well-known condition that, in a series of observations, for the exposure time considered, there should be as many differences larger than the probable difference as there are differences smaller than the probable difference.

At the end of each Table, we have indicated the totals and the means of the values included in the respective columns. The mean of the third column is the logarithm of the geometrical mean.

The mean value in the fourth column represents what is called the mean difference; that is to say, the arithmetical mean of the differences all taken in absolute values.

The mean value in the fifth column is the quadratic mean difference.

From these Tables, we have prepared in Figure 7 a curve representing graphically the results of the observations. In that curve, the abscissas represent the duration of the flashes in seconds, the unit chosen in our calculations. The ordinates represent the value of the product Et (that is to say, the product of the intensity and its exposure time) divided by the value of that same product when the flash is of 3 seconds duration(1).

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(1) E is expressed here only in arbitrary or relative units and not in lux, but that has no effect on the result.

TABLEAU II.

NUMÉROS de la mesure	$t = 0{,}001$				$t = 0{,}003$				$t = 0{,}01$				$t = 0{,}03$	
	E	$-\log \frac{Ft}{3E_3}$	d	d^2	E	$-\log \frac{Et}{3E_3}$	d	d^2	B	$-\log \frac{Et}{3E_3}$	d	d^2	E	$-\log \frac{Et}{3E_3}$
1	40	1,232	0,050	0,0025	12,4	1,263	0,085	0,0072	5	1,136	—0,028	0,0008	1,60	1,153
2	40	1,324	0,142	202	14,3	1,294	0,116	134	4,9	1,236	—0,072	52	1,98	—
3	40	1,350	0,168	282	—	—	—	—	5,7	1,196	0,032	10	2,1	—
4	40	1,070	—0,112	125	8,4	1,270	0,092	85	3	1,194	0,030	9	1,1	—
5	40	1,329	0,147	216	26	1,039	—0,139	193	7	1,086	—0,078	61	2	—
6	40	1,174	—0,008	1	17	1,069	—0,109	119	7	0,931	—0,233	543	1,4	—
7	40	1,204	0,022	5	19,3	1,044	— ,134	180	5,6	1,058	—0,106	112	1,5	—
8	40	1,108	—0,074	55	7,5	1,357	0,179	320	2,5	1,311	0,147	216	1,2	—
9	40	1,174	—0,008	0	18,4	1,035	—0,143	204	4,9	1,086	—0,078	61	1,4	—
10	40	1,174	—0,008	1	17	1,069	—0,109	119	4,6	1,114	—0,050	25	1,4	—
11	40	1,174	—0,008	1	19,8	1,003	—0,175	306	5,3	1,052	—0,112	125	1,4	—
12	40	1,142	—0,040	16	8,5	1,338	0,160	256	2,1	1,421	0,257	661	1,3	—
13	40	1,028	—0,154	237	10	1,153	—0,025	6	2,5	1,232	0,068	46	1,0	—
14	40	1,174	—0,008	0	21,8	0,961	—0,217	471	4,3	1,143	—0,021	4	1,4	—
15	40	1,159	—0,023	5	14	1,137	—0,041	17	4,25	1,132	—0,032	10	1,35	—
16	40	1,174	—0,008	1	17	1,069	—0,109	119	2,5	1,368	0,204	416	1,4	—
17	40	1,283	0,101	102	17	1,178	0 0	0	5	1,186	0,022	5	1,8	—
Totaux....		20,273	1,081	0,1274		18,279	1,833	0,2601		19,888	1,570	0,2364		
Moyenne..		1,193	0,0635	0,0866		1,142	0,1145	0,1275		1,1695	0,0923	0,1180		1,153

TABLE III

t	Geometric Mean	$\frac{t + 0.21}{3.21}$	Difference	$-\log\frac{t+0.21}{3.21}$	n	$\frac{e}{1.2}$	$\frac{2}{3}$
3.00 sec.	1	1	0	0	—	—	—
1.00	0.372	0.377	-0.005	0.424	0.054	0.051	0.050
0.30	0.1555	0.1588	-0.0033	0.799	0.066	0.064	0.060
0.10	0.1021	0.0965	+0.0056	1.015	0.087	0.085	0.082
0.03	0.0703	0.0748	-0.0045	1.126	0.075	0.077	0.071
0.01	0.0677	0.0685	-0.0008	1.164	0.072	0.077	0.079
0.003	0.0721	0.0664	+0.0057	1.178	0.112	0.095	0.085
0.001	0.0641	0.0657	-0.0016	1.182	0.040	0.053	0.058

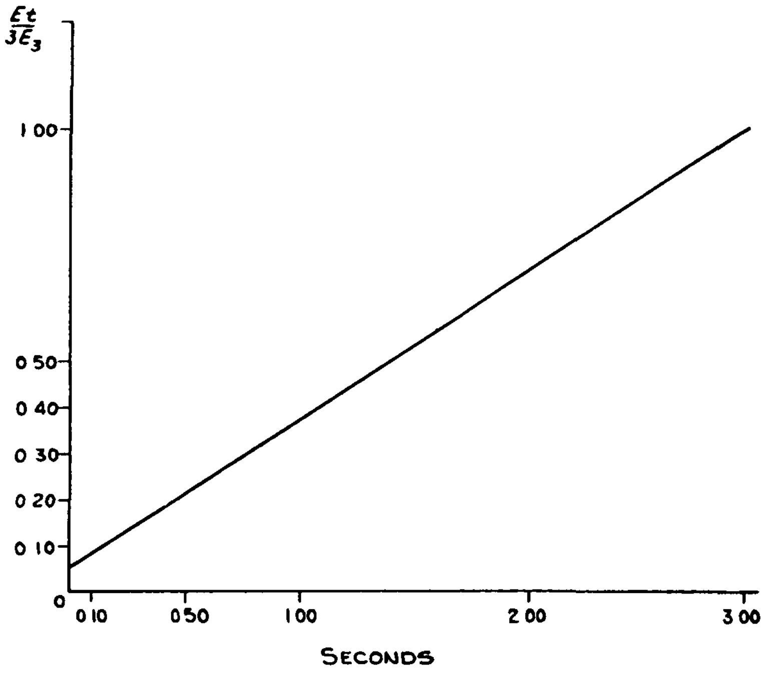

Fig. 7 Law of variation of the product Et as a function of time, plotted from experimental results.

It is remarkable to note that the mean values of the observations for each of the eight flashes considered line up almost exactly in a straight line. This result does not imply, as we shall see, exceptional precision, but is, in fact, rather fortuitous. It is nevertheless interesting to have obtained such good alignment for observations within such wide limits of exposure time. The result obtained for 3 seconds has no great value, we think, but the linear law seemed very definitely proved between 0 and 1 second, that is to say, within the limits ordinarily encountered.

Precision

We have thought it useful, in all cases, to investigate the degree of precision of the measurements, and Table III summarizes the calculations made to that end. (1)

The first column of that Table gives the time in seconds.

The second column lists the geometrical mean Et of all the observations and for each point observed; that is the number of the mean logarithm shown at the end of Tables I and II, in the third column.

The third column gives the mean value of Et, calculated from the straight line of Fig. 6, derived from the mean value of all the observations.

The fourth column is the difference between these two results.

The fifth column gives the logarithm of values in the third column. It is this value that has served as a basis to calculate the differences d included in the first two Tables.

The sixth column gives the probable difference, defined as previously.

The seventh column gives the mean difference divided by 1.2, the mean difference defined as previously.

The eight column represents two-thirds of the quadratic mean difference.

In accordance with the general theory of probability, the numbers included in columns 6, 7, and 8 should be roughly of

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(1) The variations in these measurements are attributable in large part to variations in the state of adaptation of the observer. Ideally, from a theoretical standpoint, it would be desirable to control the variation in the size of the pupil by placing before the eye of each observer an artificial pupil of 2 mm or 2 1/2 mm in diameter. But we have not introduced this complication inasmuch as most of the observers lack the training necessary to make use of it. And, what is more, we would be departing even farther from the conditions that obtain when an observer looks at a directional signal.

the same order of magnitude and should represent the logarithm of the probable error in the results. In this case, the logarithmic error falls between 0.05 and 0.10, and one can deduce from it that the relative probable error in the results falls between 12% and 25%. In a subject as poorly defined as physiological optics, and with observers of different capabilities, these error limits are very acceptable, and one can consider the linear law represented by Fig. 7 as established in a satisfactory manner.

Numerical Expression of the Law

The straight line in Fig. 7 indicates that the product Et is a linear function of time, Et = A + Bt, where E is the intensity of illumination received at the pupil of the eye and supposedly constant during the duration of the flash, and A and B are two constants.

To determine the latter, let us note that when the flash lasts indefinitely ($t = \infty$), the intensity observed at the limit of the visual range is equal precisely to the threshold of perception, corresponding to just-perceptible intensity E_0; from which $B = E_0$. On the other hand, the straight line representing Et intercepts the axis of abscissas at a distance representing 21/100 of a second to the left of the origin; whence $A = 0.21\,E_0$, or, more generally, $A = a\,E_0$, where a is independent of time.

Finally,

$$Et = E_0(0.21 + t) = E_0(a + t), \qquad (1)$$

which can be written also

$$(E - E_0)t = 0.21\,E_0, \qquad (2)$$

and which is the kind of equation we visualized a priori (1).

The same numerical law can be rewritten as follows:

$$\frac{E}{E_o} = \frac{0.21 + t}{t} = 1 + \frac{0.21}{t}, \tag{3}$$

or also

$$t = \frac{0.21\, E_o}{E - E_o}. \tag{4}$$

The different variants can be used in accordance with the applications.

All these formuli are based on the threshold of visibility of brief flashes. The coefficient of atmospheric transmission has been assumed constant throughout the experiment.

Permanent Light Equivalent to a Brief Light

If one supposes that a steady source is substituted for a series of rapid flashes, that it is located at the same point, and that the intensity of that steady source is so regulated that the observer perceives it at the same range-limit (without changing his position), the preceding law gives us the relationship between the horizontal photometric intensity I_h of the light from the rapid flashes and the intensity I'_h of the substitute fixed light. Assuming that the duration of the flash is known, one sees from Eq. (3) that the apparent intensity of the light which produced the flashes is reduced in the proportion

$$\frac{t}{a + t}$$

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(1) Perhaps other investigators might find that the constant 0.21 requires correction, and that the constant E_o is perhaps a little different from the intensity that corresponds to the threshold of sensation, but that will not alter the general form of the equation, A and B being constants.

It will equal a fixed light I'_h when

$$\frac{I'_h}{I_h} = \frac{a + t}{t} . \qquad (5)$$

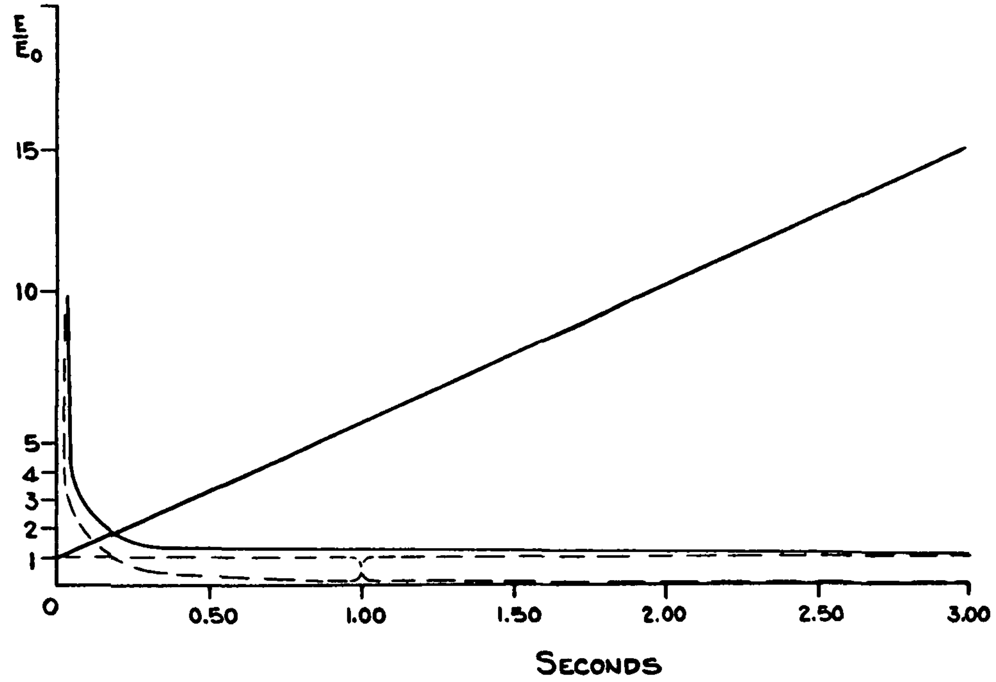

Fig. 8 Graphical comparison of the new law with Bloch's law.

Comparison of the New Law With That of Bloch

If on two rectangular axes (Fig. 8) one defines the abscissas in terms of time t and the ordinates in terms of intensity E, Eq. (1) becomes an equilateral hyperbola (II) having for asymptotes the axis of the E's and a horizontal E = E_0. Bloch's law, which should give for very short exposure time essentially the same value of E, will have for an equation $\frac{E'}{E_0} = \frac{0.21}{t}$ and will be represented by an equilateral hyperbola (I), superposable but everywhere lowered by a distance of E_0.

Table IV gives the ratio of ordinates $\frac{E}{E'}$ and shows clearly in what proportion the intensities should be increased according to our law, by comparison with what would be required for the same flash durations if one were to assume Bloch's law.

For very short exposures of the order of one-one-hundredth of a second, the difference is negligible. For a flash duration of one-tenth of a second, the increment of the ratio becomes about 50% and increases proportionately for longer exposures. If one assumes that most modern beacons employ flashes longer than a tenth of a second, one will understand how important it is to substitute for Bloch's law the more precise law that we have established.

TABLE IV

Duration t	Bloch's Law $\frac{E'}{E_o} = \frac{0.21}{t} + 1$	New Law $\frac{E}{E_o} = \frac{0.21}{t} + 1$	Ratio $\frac{E}{E'}$
0.01	21.00	22.00	1.047
0.025	8.40	9.40	1.12
0.05	4.20	5.20	1.24
0.10	2.10	3.10	1.40
0.20	1.05	2.05	1.95
0.30	0.70	1.70	2.43
0.40	0.525	1.525	2.90
0.50	0.42	1.42	3.40
0.60	0.35	1.35	3.86
0.70	0.30	1.30	4.33
0.80	0.2625	1.2625	4.81
1.00	0.21	1.21	5.76
2.00	0.105	1.105	10.52
3.00	0.07	1.07	12.95

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