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E-1429
LUNAR ORBIT DETERMINATION
BY STAR OCCULTATIONS AND
MSFN TRACKING

by
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LUNAR ORBIT DETERMINATION BY STAR
OCCULTATIONS AND MSFN TRACKING

ABSTRACT

This report summarizes analysis conducted to determine the accuracy that could be achieved in lunar orbit determination by star occultation measurements and Manned Space Flight Network (MSFN) tracking. Various performance accuracies were assumed for both star occultation measurements and MSFN tracking, and MSFN tracking for single station and two station tracking was compared. Combinations of star occultation and MSFN tracking were analyzed to determine the comparative effects of each type of measurement and the resultant performance.

The particular mission hypothesized for this analysis was a back-up navigation system for the Lunar Excursion Module (LEM). It was assumed the LEM had been injected into a lunar orbit by a relatively simple abort guidance system and the orbit parameters were to be determined by star occultations monitored by the astronaut and transmitted to earth in connection with MSFN tracking when the LEM was in the tracking zone of the MSFN. The objective of the LEM lunar orbit determination, was to calculate an intercept or transfer trajectory to the Command Service Module (CSM) so that terminal rendezvous could be performed by either the CSM or LEM. This report deals only with the LEM lunar orbit determination phase of this mission.

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SECTION I

INTRODUCTION

This report summarizes the results of a study conducted to determine the effectiveness of on-board star occultation measurements and earth tracking to determine lunar orbit parameters. The particular mission phase considered was long range mid-course correction computation during lunar rendezvous. The long range rendezvous mid-course correction phase would typically be at ranges of 200 nm to 10 nm between the Command Service Module (CSM) and Lunar Excursion Module (LEM) vehicles as contrasted to the terminal rendezvous phase involving ranges of less than 5 nm. The various computation networks which have been considered for the long range lunar rendezvous phase are summarized in Fig. 1. Virtually identical primary guidance and navigation (G & N) loops exist in both vehicles and under normal operation either vehicle could compute the required long range mid-course correction to establish an intercepting trajectory. The primary G & N units involved in this phase are the rendezvous radar, guidance computer (AGC) and inertial measurement unit (IMU). The scanning telescope (SCT) on the CSM or the fixed optical alignment telescope (OAT) on the LEM can be used as a back-up to the rendezvous radar during this phase if optical tracking can be achieved. The objective of this report is to determine the capability of the third loop shown in Fig. 1 consisting of on-board star occultation measurements and range and range rate measurements made by earth tracking networks.

The particular mission profile hypothesized for this study was as follows. The LEM had injected into a lunar orbit that had a clear perilune using a relatively simple on-board abort guidance system. After orbit injection no other navigation equipment was

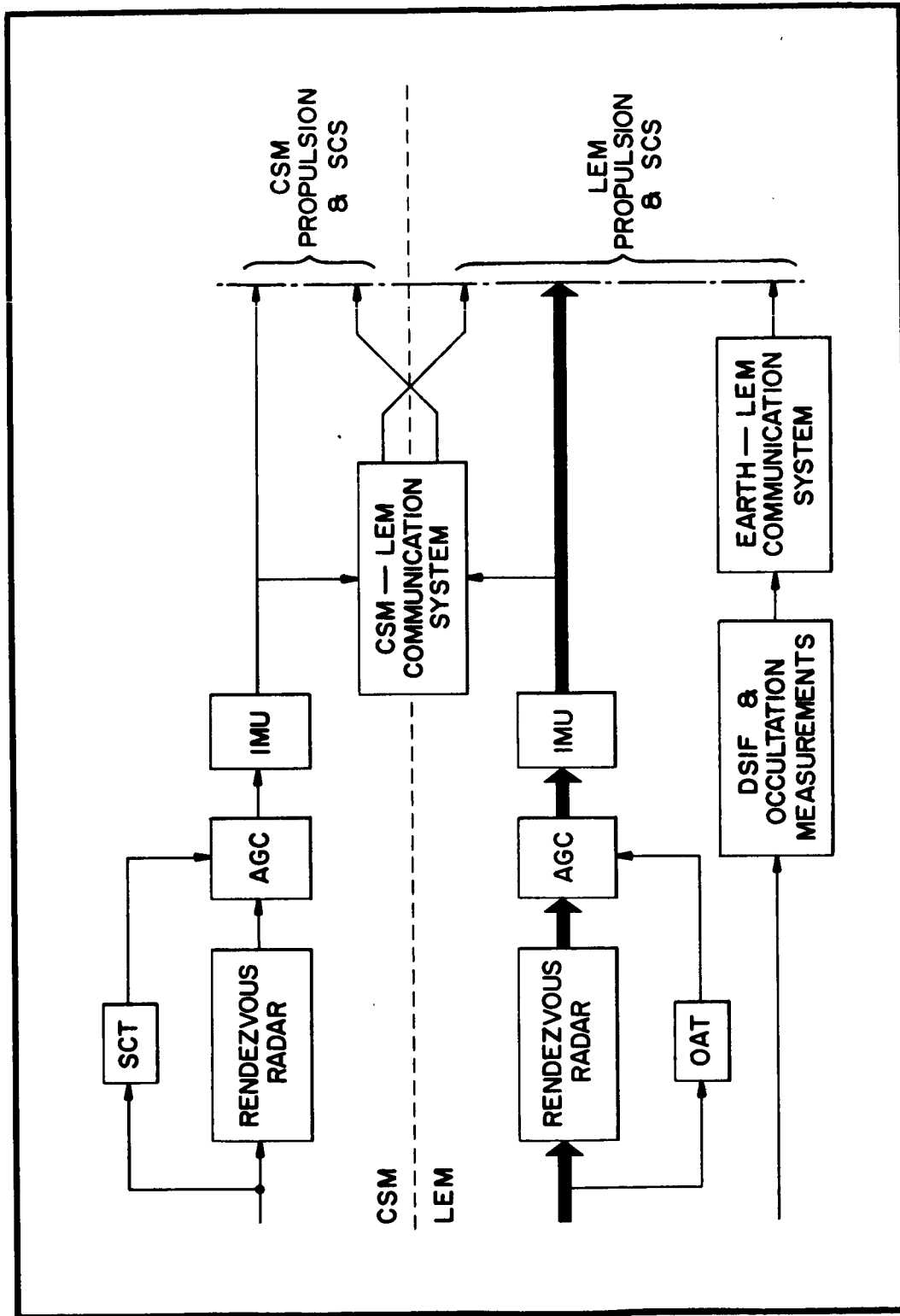


Fig. 1 Rendezvous long range mid-course correction computation loops.

available except a clock. The astronaut observed the occultation time of known navigation stars and recorded this data for transmission to earth over the voice communication system. During half the orbit it was assumed that earth based tracking systems comprising the Manned Space Flight Network (MSFN) such as DSIF or the Goddard Range and Range Rate systems could track the LEM. The LEM lunar orbit parameters would then be determined on earth by using various combinations of this data and an intercept trajectory would be computed. The required velocity correction and timing for this maneuver would be transmitted to the LEM (or the CSM in the case of LEM retrieval) over the voice or up-data link. The objective of the intercept or orbital transfer trajectory was to achieve conditions from which a terminal rendezvous could be made using the on-board equipment of either vehicle. The terminal rendezvous phase is not considered in this report other than to indicate typical closest approach or miss distances resulting from initial condition uncertainties for this type of mid-course back-up network.

In comparing the effectiveness of star occultation measurements and earth tracking network performance in determining lunar orbit parameters, realistic constraints were placed on star occultation accuracy assuming no compensation was attempted for lunar horizon uncertainty. In the case of earth tracking network capabilities, the overall performance figures currently estimated for future tracking systems were used realizing this performance should be considered preliminary and may be optimistic in some cases.

SECTION II

STAR OCCULTATION MEASUREMENTS

2.1 General

For operation of the primary guidance and navigation system, the astronauts can identify at least 28 major navigation stars from memory. These stars are essentially uniformly distributed over the celestial sphere. It was assumed that for the hypothetical back-up navigation case under consideration, the astronaut would monitor the occultation times of some of these stars by the lunar horizon. For the lunar orbit mission considered this would result in a star occultations approximately every 15 minutes. The occultations were assumed to be visually monitored through the LEM windows using no optical instrument and the times recorded with a stop watch on the vehicle master clock.

Assuming an accurate on-board clock, the accuracy of star occultation measurements in lunar orbit is a function of three major factors:

1. Lunar terrain or horizon uncertainties
2. Star occultation detection uncertainties
3. Human reaction time

The effect of horizon or lunar terrain uncertainties on the accuracy of occultation timing is illustrated in Fig. 2. If typical near circular orbital altitudes of 50,000 feet or 80 nm are considered the following occultation timing uncertainties result for assumed 1 sigma terrain altitude uncertainties:

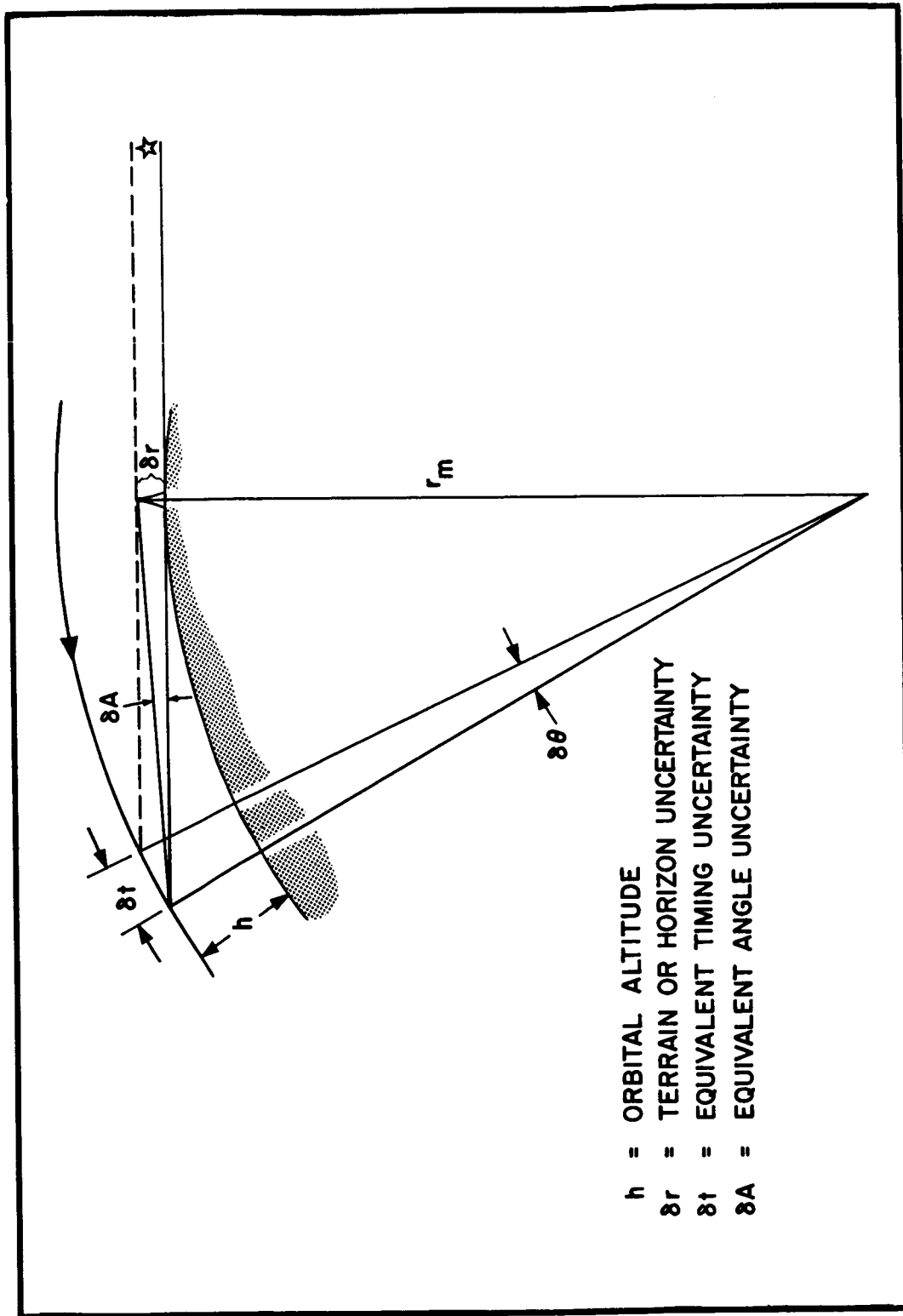


Fig. 2 Lunar orbit star occultation measurement uncertainty.

Orbital Altitude (h)	Terrain Altitude Uncertainty (δr) (in feet)	Resulting Timing Uncertainty (δt) (in seconds)	Effective Measuring Angle Uncertainty (δA) (in milliradians)
50,000ft	1500	2.1	2
50,000ft	4540	6.4	6
80nm	2400	1.2	1
80nm	4800	2.3	2

In this study it was assumed that no terrain uncertainty compensation during earth computation was attempted and therefore terrain uncertainties in altitude of approximately 5000 feet would result in occultation timing uncertainties of 6.5 seconds for low orbits (50,000ft) and 2 to 3 second uncertainties for high altitude orbits (80nm) if the occulted star was near the LEM orbital plane.

The second factor affecting occultation timing is occultation detection uncertainty. The rate at which stars located near the orbital plane descend toward the lunar horizon is about 1 mr/sec. The astronaut could be required to determine when the star is occulted by three types of lunar horizon lighting conditions; sun light, earth reflected light, or no light (star light). The relative uncertainty associated with detecting when a star occultation occurred for these three horizon lighting conditions was unknown at the time of this study, but it was assumed that detection uncertainty was less than that due to 5000 foot terrain uncertainties.

Human reaction time independent of detection uncertainty is estimated to be between 0.1 and 0.2 seconds and is therefore negligible compared to the other effects listed. Timing uncertainties due to vehicle attitude variations are also negligible.

2.2 Lunar Orbit Determination Technique

In this study it was assumed that star occultation measurements, earth tracking system data, or some combination of these two was used in the same statistical optimizing navigation

procedure that is normally carried out by the spacecraft guidance computer. This navigation technique has been presented in references 1 to 3 and is normally used for on-board control of the mid-course corrections for both the trans-lunar and trans-earth trajectories, earth orbit and lunar orbit navigation, and long range rendezvous corrections. Basically it is a linear statistical technique for estimating the spacecraft velocity and position vectors at any time. For the hypothetical back-up navigation case considered in this study, it was assumed that this navigation technique was done by the computers of the earth based (MSFN) tracking system. Other navigation techniques are possible, but it is felt that the results and accuracy for lunar orbit determination would be comparable. The performance presented in the following sections, 2.3, 3.3 and 4.1 is, therefore, the result of the various occultation or tracking measurements used with the primary Apollo navigation technique. It was assumed that in all calculations the motions of the earth and moon were accounted for in the navigation computation. Near circular orbits were used in this analysis, but the results would be applicable to the LEM orbits expected which have low eccentricity ($e < 0.09$).

2.3 Star Occultation Results

Tables 1 to 3 present the results of lunar orbit determination using only star occultations by the moon monitored by the astronaut and transmitted to the earth. It was assumed that the LEM was injected into lunar orbit with an RMS position and velocity uncertainty of 67, 115 feet and 60.5 fps, respectively. The one sigma values for this initial condition uncertainty in a LEM centered local vertical system are as follows:

$$\begin{array}{ll}
 \sigma_x = 51,000 \text{ ft} & \sigma_{\dot{x}} = 52 \text{ fps} \\
 \sigma_y = 30,300 \text{ ft} & \sigma_{\dot{y}} = 21 \text{ fps} \\
 \sigma_z = 30,300 \text{ ft} & \sigma_{\dot{z}} = 21 \text{ fps}
 \end{array}$$

where the x component is horizontal in the orbital plane in the general direction of the velocity vector, y is along the local vertical, and z is perpendicular to the orbital plane. These uncertainties are ten times those normally expected using the primary G & N system. These uncertainties were propagated in the form of a 6 by 6 covariance matrix with up-dating at the points where star occultation measurements were made. In this program, star occultations were made at approximately 15 minute intervals. If more than one of the stars identifiable by the astronaut was to be occulted at about this time, the navigation program chose that star which had the greatest effect in reducing the orbital uncertainty or covariance matrix. The star occultations, therefore, were not always near the LEM orbital plane. Those stars located off the orbital plane resulted in relative motions that were not perpendicular to the lunar horizon at occultation time. Terrain uncertainties in these cases would involve greater timing uncertainties than those listed in section 2.1 for perpendicular occultation motions. In the program used to simulate the star occultation navigation techniques, an equivalent angular uncertainty was used that can be related to an average terrain and timing uncertainty (Fig. 2 and section 2.1). In this respect it should be noted that the same equivalent angle uncertainty was used for all star occultations, where in actual practice star occultations occurring off the LEM orbital plane would not be as accurate as those near the plane if the same terrain uncertainty were assumed over the moon. Consequently, the results for the star occultation technique are slightly better than would normally be expected.

Table 1 presents the lunar orbit uncertainty resulting from occultations made with an equivalent angle uncertainty of 1 mr. This would be equivalent to a relatively high orbit (80nm) with a terrain or horizon one sigma uncertainty of 2400 ft. (1.2 second timing uncertainty for stars near the LEM orbital plane).

As indicated in Table 1, star occultation measurements were taken every 15 minutes (0.25 hr). The second and fourth columns indicate the RMS position and velocity uncertainties prior to the measurement, and the third and fifth columns indicate the RMS uncertainties after the occultation measurement was made. At the end of six hours (approximately 3 orbits) the LEM orbital RMS uncertainties had been reduced to 4000 ft in position and 3.3 fps in velocity. The one sigma components for the final uncertainty in each coordinate are also listed in Table 1. The uncertainties in the vertical component of position (δy) and range component of velocity ($\delta \dot{x}$) are seen to be considerably less than the other position and velocity uncertainties. This is because the geometry vector of the measurement (Ref. 1), defined as the direction along which most of the information is obtained, subtends an angle of only 22° with the vertical for an 80 nm altitude orbit so that a projection of this vector into a local vertical system has its largest component along the local vertical. For an occultation measurement, the geometry vector is in the plane defined by the LEM position vector and the vector from the LEM to the star, and is perpendicular to the latter vector. As the trajectory propagates in time, the accuracy in the vertical component of position (y) manifests itself in the reduction of the range component of velocity (\dot{x}) because of the strong correlation between these two in a near circular orbit.

Table 2 presents the results for a similar occultation measurement schedule with an equivalent angle uncertainty of 2 mr that would be applicable to a timing uncertainty of 2.1 seconds for an 80 nm orbit ($\delta r = 4800$ ft). Comparing the final results of Table 2 with Table 1, it can be seen that the final lunar orbit uncertainties at the end of two orbits are doubled when the equivalent angle uncertainty is doubled.

Table 3-a summarizes star occultation results for a 6 mr equivalent angle uncertainty. Tables 2 and 3-a illustrate the effect of a one sigma altitude terrain uncertainty of approximately 5000 feet

on high lunar orbits. With reference to Table 3-a, it can be seen that accurate determination of lunar orbits by star occultations is very sensitive to terrain or horizon uncertainties.

In Table 3-b, the results of star occultations are given for a 50,000 foot circular orbit with a 6 mr equivalent angle uncertainty. The uncertainties at the final time are 17,000 feet and 17 ft/sec compared to 23,000 feet and 20 ft/sec for the 80 nm orbit of Table 3-a. This improvement arises because the geometry vector (h vector) is more nearly vertical in a low altitude orbit. The magnitude of the geometry vector $= 1/\bar{\rho} \cdot \bar{V}$ where $\bar{\rho}$ = unit vector in direction of h vector and \bar{V} is the LEM velocity vector. The smaller the altitude, the closer to the vertical is the $\bar{\rho}$ vector. For a near circular orbit the angle between $\bar{\rho}$ and \bar{V} is nearly 90° . This makes the magnitude of the h vector large, thereby improving the effectiveness of the measurement. Even though the h vector for star occultations is more effective at low altitude orbits, this effect is more than offset by the decrease in accuracy caused by lunar terrain uncertainties at the lower orbits. The results of Table 2 (80 nm orbit) and Table 3-b (50,000 ft orbit) are applicable for roughly the same terrain uncertainty (4500 to 4800 ft) and a comparison of the final results of these two tables indicates the sensitivity of low altitude orbits to terrain uncertainty.

Tables 4, 5 and 6 again summarize star occultations results except that after the first orbit each star occultation measurement was used as both an occultation time and an orbit period measurement. Comparing the final orbital uncertainties of Tables 4, 5 and 6 with those of Tables 1, 2 and 3-a respectively, it can be seen that the addition of period measurements after the first orbit made a slight improvement in orbit determination using this navigation technique.

SECTION III

LUNAR ORBIT DETERMINATION BY MSFN TRACKING

3.1 The Manned Space Flight Network (MSFN)

The MSFN currently envisioned will consist of the following tracking networks:

- a. The Near Space Instrumentation Facility (NSIF). The NSIF will consist of the present Mercury Network modified to support Gemini and Apollo.
- b. The Goddard Space Flight Center Range and Range Rate Network. (GSFC/R & \dot{R}).
- c. The Jet Propulsion Laboratory Deep Space Instrumentation Facility (DSIF).
- d. The Department of Defense National Ranges. These are the Atlantic Missile Range (AMR), Air Proving Ground Center (APGC), White Sands Missile Range (WSMR), and the Pacific Missile Range (PMR). Present plans specify the use of the NSIF and the GSFC/R & \dot{R} tracking networks for primary support for Apollo Lunar Missions and DSIF and DOD Range as backup support.

The facilities of the MSFN that are of particular interest for this note are the GSFC/R & \dot{R} and the JPL/DSIF. Both of these networks are capable of measuring the range and range rate of a spacecraft at lunar distances. (Refs 4-6).

3.2 Accuracy of the MSFN Range and Range Rate Measurement

A number of information sources have been used to arrive at a "reasonable" estimate of measurement accuracy. From

available information the following 1 σ RMS measurement accuracies* were selected;

$$\delta R = \pm 50 \text{ feet}$$

$$\delta \dot{R} = \pm 0.5 \text{ feet per second}$$

In addition, a range measurement uncertainty of ± 3000 feet was rather arbitrarily selected to examine the effect of a "large" range tracking error.

It has been assumed that the R & \dot{R} errors are the total errors relative to the computing coordinate system which include:

- a. Tracking system errors, e. g. , bias and random errors
- b. Tracking system station position errors relative to the Earth center, and the uncertainty of the relative position of the moon and Earth.

- c. Propagation uncertainties

3.3 Results of Lunar Orbit Determination by MSFN Tracking

Table 7 summarizes the results of tracking the LEM in lunar orbit by a single MSFN station (DSIF or Goddard range and range rate system) over two orbital tracks on the earth side of the moon. In this example, it was assumed that the LEM was injected into orbit on the back side of the moon (point A of Fig. 3) with RMS uncertainties one-half of those used in the occultation examples. The position and velocity uncertainties propagated to values of 125,700 ft and 113.4 fps respectively at point B of Fig. 3 (0.55 hours in Table 7) when the first MSFN tracking measurement was made. In the single MSFN tracking station example summarized in Table 7, only one of two stations shown in Fig. 3 was used. As indicated in Fig. 3, the MSFN stations were positioned above and below the LEM lunar orbit plane by

*These accuracies associated with a sampling rate of 10 per second and a smoothing time of 1 minute were confirmed in a telephone conversation with Dr. F. VonBun, Chief, Systems Analysis, GSFC, 29 August 1963.

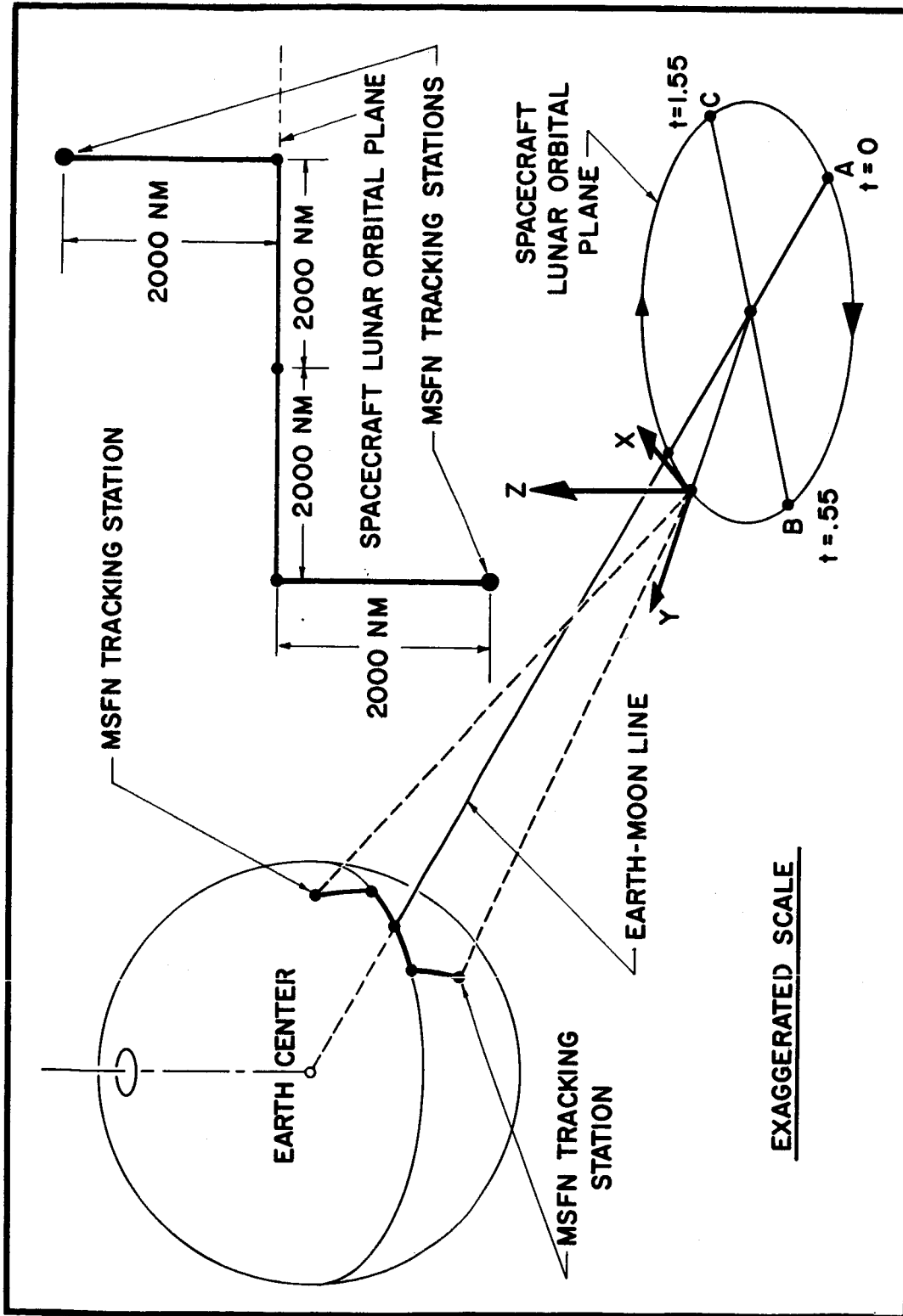


Fig. 3 Relation of MSFN tracking stations to the LEM lunar orbit plane.

2000 nm and also straddled the earth-moon line by 2000 nm. This location of the MSFN stations was considered to be very favorable for determining the LEM orbital parameters (3 velocity and 3 position components). The earth great circle separation distance between the two MSFN stations of Fig. 3 is in the order of 5300 nm. The MSFN tracking accuracy assumed for the results of Table 7 were one sigma uncertainties of 50 feet and 0.5 fps in range and range rate respectively. MSFN tracking was continued from point B to C of Fig. 3 with tracking sample rates of one each 5 minutes in the navigation technique used. At point C of Fig. 3 (1.55 hours in Table 7) the LEM position and velocity uncertainty had been reduced to 12,550 ft and 6.1 fps. As the LEM orbits the back side of the moon (C to B of Fig. 3) no tracking is possible. When the LEM reappears at point B, tracking is resumed to point C. At the final time (3.7 hours of Table 7) orbital uncertainties have been reduced to 8570 ft and 2.16 fps.

The components of this final uncertainty are listed in Table 7 and it can be seen that the final uncertainty in position and velocity is almost entirely in z or normal direction to the LEM orbital plane. This is primarily caused by the low angle (approximately 0.5 deg) the MSFN tracking line makes with the LEM orbital plane. This indicates that the MSFN tracking provides very good accuracy in the LEM orbital plane, but relatively weak accuracy for components normal to the orbital plane. This fact explains the behavior of the velocity and position uncertainties of Table 7 between the times 2.75 and 3.1. In this interval position uncertainty drops sharply while the velocity uncertainty increases in spite of MSFN measurements being taken. Since the total uncertainty is predominately in the z or normal direction, the z position and velocity uncertainties cycle through a maximum and minimum every half orbit and this variation due to orbital characteristics is greater than can be improved or reduced by MSFN tracking since it is in the weak direction for such improvement.

Comparing equivalent times (3.7 hours) in Tables, 4, 5 and 7, it can be seen that the star occultation examples equal or better overall position performance depending on the magnitude of terrain uncertainty, but MSFN provides better velocity information in all cases. The accuracy for the case of Table 6 is much worse than that which could be provided by single station MSFN tracking for the accuracies assumed in Table 7.

In order to ascertain if performance could be improved using a single MSFN station, but with twice the data sample rate, tracking data samples were increased from once every 5 minutes to once every 2.5 minutes. The results presented in Table 8 indicate that no material improvement was made over the slower rate performance of Table 7, and the same general characteristics described for the example of Table 7 persisted. In comparing Tables 7 and 8 at any given time, the slower data rate case (Table 7) may have a lower uncertainty than the higher data rate case (Table 8) because the cyclic variations in uncertainties which are due to a combination of measurement effects and orbital characteristics cross one another.

Table 9 summarizes the orbit determination results when two MSFN stations were assumed making simultaneous measurements over the same time and mission profile of Table 7. The location of these two MSFN stations relative to the LEM orbital plane is shown in Fig. 3 and each station was assumed to have equal tracking performance; $\delta R = 50$ ft, $\delta \dot{R} = 0.5$ fps. Comparing the results of Tables 9 and 7, it can be seen that a substantial improvement is made when two MSFN stations are used. This point is illustrated in Fig. 4 where the minimum uncertainty points in velocity after measurements are plotted against time (column 5 of Tables 7 and 9). Comparing the uncertainties of Tables 7 and 9, it can be seen that at time 3.166 hours the position uncertainty in Table 7 reaches a minimum of about 3000 feet with a velocity uncertainty of about 10 fps, compared with 1400 feet

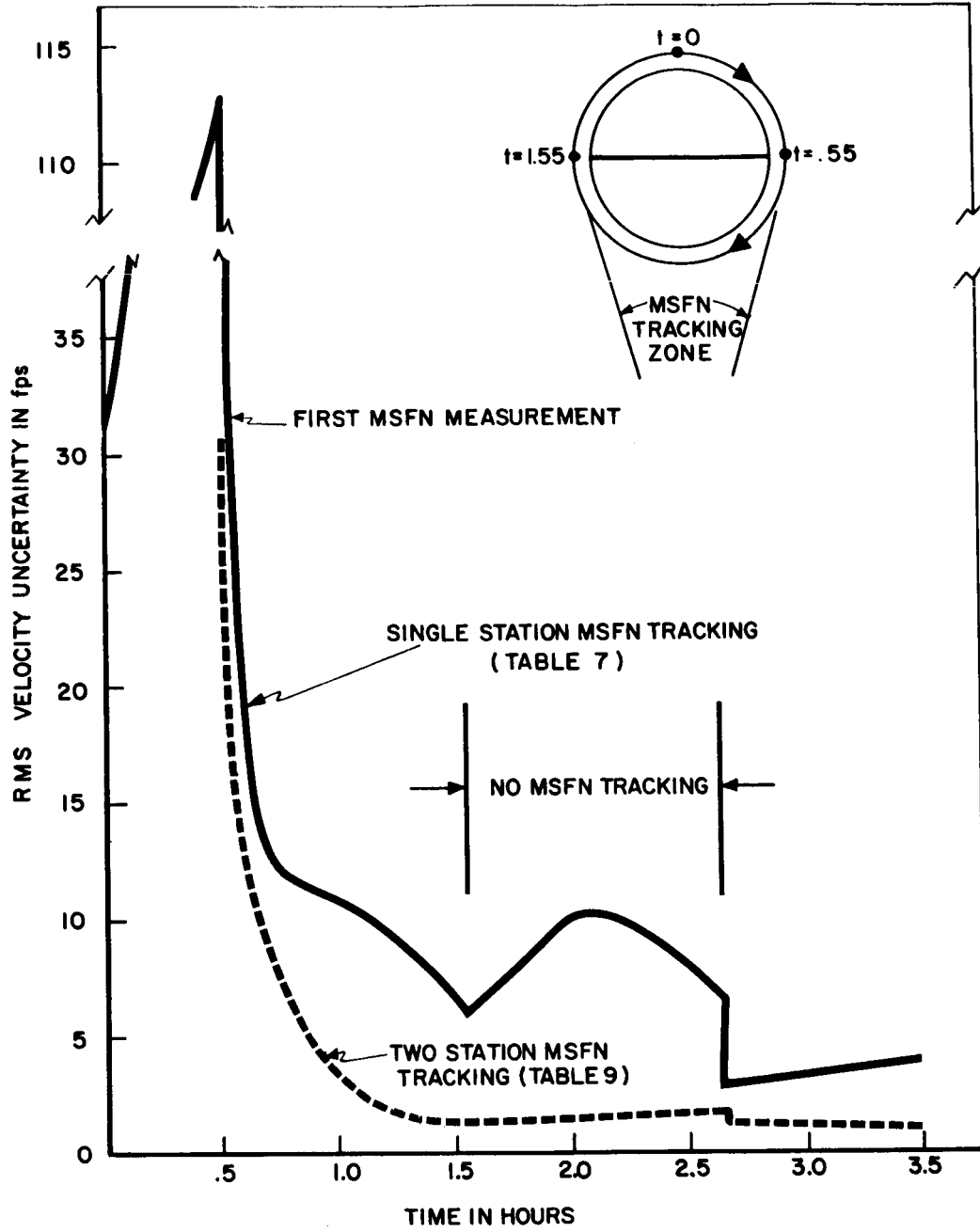


Fig. 4 Minimum lunar orbit velocity uncertainty resulting from MSFN tracking.

and 1.1 fps respectively for the same time in Table 9. The lowest velocity uncertainty in Table 7 occurs at the final time 3.666 hours at which time the position uncertainty is 8 times that of the two MSFN station tracking case of Table 9.

Performance of two MSFN station tracking is further evaluated in Table 10 in which the initial injection uncertainties are twice those of Table 9, (equal to those used in the star occultation examples). Comparing Tables 10 and 9 it can be seen that the performance of the 2 MSFN stations are equivalent at 1.383 hours (0.83 hours tracking time) even though the uncertainties at the start of tracking are twice as large.

Table 11 summarizes the results of using two MSFN stations but with a comparatively poor range tracking accuracy: one sigma values of $\delta R = 3000$ ft and $\delta \dot{R} = 0.5$ fps. Comparing the results of Table 11 with those of Table 9 it can be seen that orbit position and velocity determination is degraded by a factor of 8 when the poorer MSFN range accuracy is used. The overall uncertainty results of Table 11 are equivalent to the star occultation example of Table 5.

Table 12 lists the results of using only range rate ($\delta \dot{R} = 0.5$ fps) from two MSFN stations. The results of Table 12 are essentially the same as those of Table 11 indicating that when the range uncertainty is at the level of that in Table 11 ($\delta R = 3000$ ft), the range rate is the only effective tracking parameter.

Table 13 illustrates the effect of larger initial uncertainties when two MSFN stations are used with tracking accuracies of $\delta R = 3000$ ft and $\delta \dot{R} = 0.5$ fps. Comparing Tables 13 and 11 it can be seen that equivalent performance is not achieved as fast as in the similar initial condition examples of Tables 9 and 10 involving better range measuring accuracy.

SECTION IV
COMBINED MSFN TRACKING AND STAR OCCULTATION
LUNAR ORBIT DETERMINATION

4.1 Results of Combined Performance

From the results of Tables 7 and 9 in section 3.3, it was evident that simultaneous tracking from two MSFN stations provided the best lunar orbit determination for the tracking accuracies considered. Star occultation measurements were combined with both the most accurate single and two station MSFN tracking ($\delta R = 50$ ft, $\delta \dot{R} = 0.5$ fps) and the results are summarized in Tables 17 and 14, respectively. Comparing the results of Table 17 with Table 7 indicates that star occultation measurements with an effective angle uncertainty of 2 mr make a significant contribution to single station MSFN tracking. For example, at 3.67 hours in Tables 17 and 7, star occultation measurements have reduced the position uncertainty by at least a factor of 3 and the velocity uncertainty by 0.5 fps. At 2.83 hours both the position and velocity uncertainties are reduced by a factor of 2.

Comparing the results of Table 14 with Table 9 indicates that the star occultation measurements with an effective angle uncertainty of 2 mr resulted in little, if any, improvement in the two station MSFN performance for the high tracking accuracies considered.

Table 15 summarizes the results of combining star occultation measurements with less accurate ($\delta R = 3000$ ft, $\delta \dot{R} = 0.5$ fps) two station MSFN tracking. Comparing the results of Table 15 with Table 11, it can be seen that the star occultation measurements reduced the orbital uncertainties to essentially one-half of those resulting from MSFN tracking alone. This effect is illustrated in Fig. 5. The occultation timing accuracy used in Table 15 would be typical for 80 nm altitude orbits with

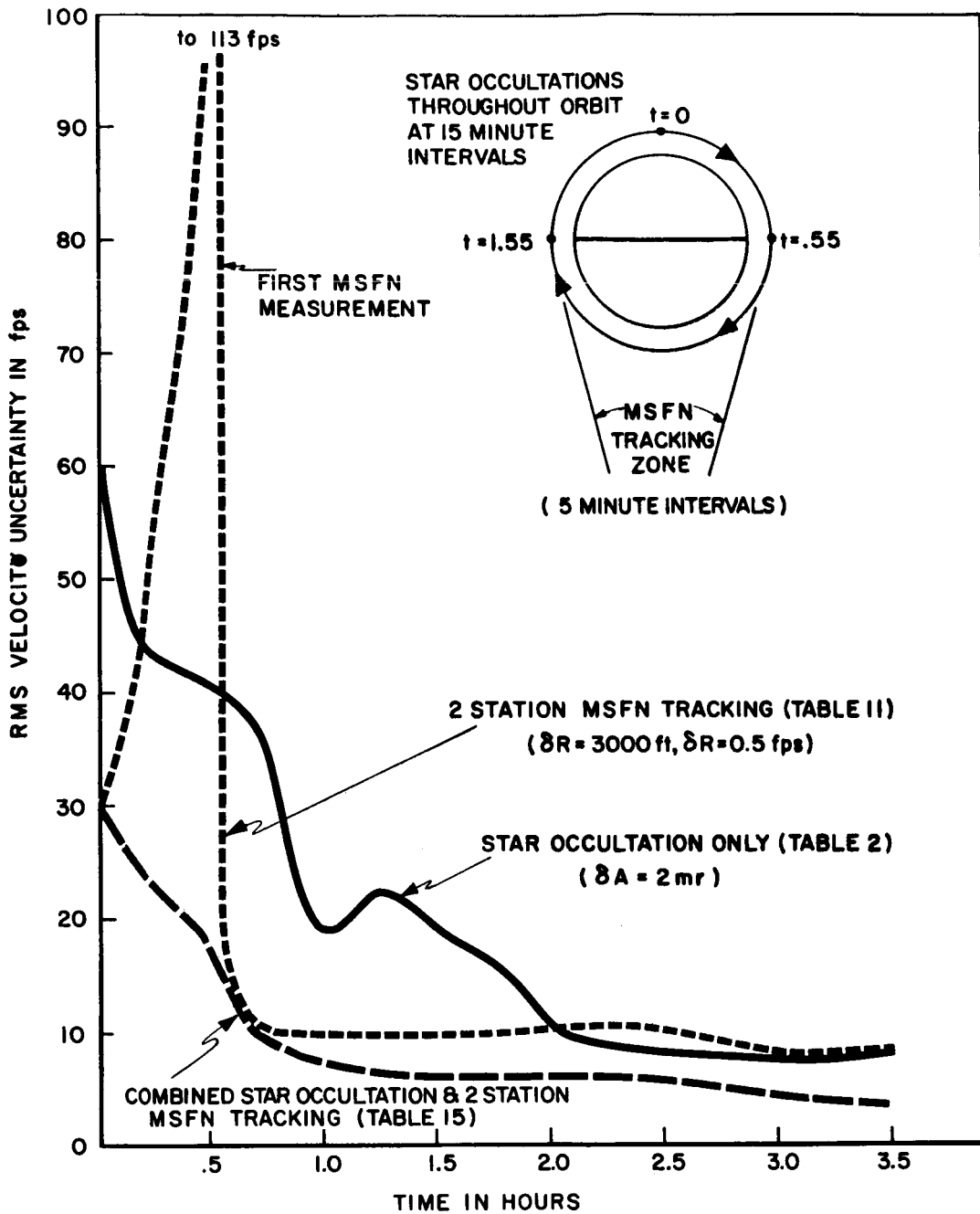


Fig. 5 Comparison of star occultation and MSFN tracking.

a terrain uncertainty of 4800 feet resulting in an occultation timing uncertainty of 2.35 seconds for stars near the LEM orbital plane. It should be noted, however, in comparing Tables 15 and 11 that the major effect of adding star occultation data to the two station MSFN performance was to reduce the final uncertainty in the z direction. In order to do this, occultations must be taken for stars that are positioned off the LEM orbital plane. As a result the occultation will occur along a path that is more tangent to the lunar horizon than perpendicular to it, and the effective timing uncertainty is more than the 2.35 seconds for terrain uncertainties of 4800 ft previously mentioned. The results of adding star occultation to the two station MSFN tracking in Table 15 is therefore probably overly optimistic since both star occultations and MSFN tracking systems have their weakest effect on the z or normal direction to the LEM orbital plane.

The effect of occultation measurements for 6 mr effective angle uncertainty combined with two station MSFN tracking ($\delta R = 3000$ ft, $\delta \dot{R} = 0.5$ fps) is summarized in Table 16. Comparing the results of Table 16 with Table 11, it can be seen that occultation measurements of this accuracy result in only a slight improvement in orbit determination over that provided by two station MSFN tracking alone.

4.2 Resultant Terminal Rendezvous Conditions

The effects on the final position and velocity uncertainties of the various MSFN tracking and star occultation models upon the terminal rendezvous conditions were briefly investigated. Three orbit determination models were chosen, two station MSFN tracking (Table 9), two station MSFN tracking with 2 mr accuracy star occultation measurements (Table 15), and finally two station MSFN tracking with 6 mr accuracy star occultation measurements (Table 16). Coplanar CSM and LEM orbits were assumed along with a perfect LEM injection maneuver and

perfect knowledge of the CSM orbit. Under these assumptions, various orbital transfer trajectories were examined which traversed altitudes of 50,000 feet to 80 nm through central angles of 140° to 220° . Since the initial position and velocity uncertainties in all three orbit determination examples were essentially in the z or normal direction, the transfer trajectory miss distance at the CSM was also in the z direction assuming a perfect LEM injection maneuver. The point of closest approach was determined by propagating the covariance matrix on the transfer trajectory to the intercept or minimum range point and then generating a six dimensional 68% probability error ellipsoid. The maximum position dimension of this error ellipsoid was in the z direction and at least 10 times the magnitude of the other position error components for all transfer trajectories considered. In terms of terminal position error, the following summary presents a 95% probability of the point of closest approach being within the z direction value designated for each navigation example:

<u>Orbital Navigation Model</u>	<u>Final z Direction Error (in feet)</u>
1. Two MSFN stations with accurate tracking (Table 9)	3,100
2. Two MSFN stations ($\delta R = 3000$ ft) with 2 mr accuracy star occultations (Table 15)	12,000
3. Two MSFN station ($\delta R = 3000$ ft) with 6 mr accuracy star occultations (Table 16)	21,000

The above summary represents the terminal position errors due only to initial LEM orbit determination accuracy. From this summary it can be seen that two MSFN station high accuracy tracking ($\delta R = 50$ ft, $\delta \dot{R} = 0.5$ fps - Table 9) results in small miss distances. Even with relatively poor range accuracy ($\delta R = 3000$ ft, $\delta \dot{R} = 0.5$ fps - Table 16), the final miss distance is of a value that could probably be corrected with reasonable

terminal rendezvous techniques provided that the LEM orbital transfer injection error did not introduce significantly larger uncertainties.

SECTION V

SUMMARY AND CONCLUSIONS

For the lunar orbit determination models considered with the associated accuracies in star occultation and MSFN tracking measurements, the following summary can be made:

1. The dominant error in star occultation measurements is lunar horizon or terrain uncertainty. The effect of terrain uncertainty is most critical for low altitude orbits.

2. Accurate star occultation measurements (equivalent angle uncertainty = 2 mr) can reduce orbital RMS position uncertainties to 8000 ft and velocity uncertainties to 7 fps (Tables 2 and 5).

3. Single station MSFN tracking over two orbits can reduce orbital RMS position uncertainties to 8500 feet and velocity uncertainties to 2 fps (Table 7) or to 3000 feet and 10 fps one half an hour before (Table 7, time 3.17 hours). Both final position and velocity uncertainties are predominantly in the direction normal to the LEM orbital plane.

4. Two station MSFN tracking can reduce orbital RMS position and velocity uncertainties to levels of 1000 feet and 1 fps respectively (Table 9). The final uncertainty is still predominately in the direction normal to LEM orbital plane.

5. Reducing the range accuracy of the two MSFN stations ($\delta R = 3000$ feet) increased the final position and velocity uncertainties by a factor of 8 over the high accuracy ($\delta R = 50$ ft) case (Tables 9 and 11). Reduced MSFN range accuracy used with range rate accuracy of 0.5 fps (Table 11) has the same effect as using range rate data only (Table 12).

6. Star occultation measurements used in combination

with accurate range and range rate two station MSFN tracking made no significant contribution compared with MSFN tracking alone (Tables 14 and 9). Since star occultation measurements have their greatest angular accuracy for a given lunar terrain uncertainty in the LEM orbital plane, and MSFN tracking has its greatest accuracy in the same plane, star occultations do not effectively complement accurate two station MSFN tracking ($\delta R = 50$ ft, $\dot{\delta R} = 0.5$ fps) but can be effective for a MSFN tracking accuracy of $\delta R = 3000$ ft.

7. Star occultation measurements used in combination with accurate single station MSFN tracking ($\delta R = 50$ ft, $\dot{\delta R} = 0.5$ fps) did make a significant contribution as compared to single station MSFN tracking alone (Tables 17 and 7).

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APPENDIX

Table 1

Star Occultation Measurements for Lunar Orbit Determination

Data Rate Interval: 15 minutes (0.25 hours)
 Orbital Altitude 80 nautical miles
 Equivalent Angle Uncertainty (1σ): 1 milliradian

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.5	
0.25	105,300	41,100	107.9	43.2
0.50	54,300	36,900	55.3	41.2
0.75	57,800	41,500	45.0	35.8
1.00	58,500	32,000	40.1	16.8
1.25	31,700	14,800	21.9	18.7
1.50	18,400	12,600	20.5	12.1
1.75	16,700	9,500	10.6	9.3
2.00	13,200	7,900	7.6	6.2
2.25	9,200	7,100	6.5	5.5
2.50	7,700	6,000	5.8	5.2
2.75	6,400	5,700	5.4	5.2
3.00	6,500	5,700	5.1	4.6
3.25	6,300	5,500	4.5	4.4
3.50	5,900	5,100	4.8	4.2
3.75	5,400	4,900	4.4	4.0
4.00	5,300	4,800	4.2	3.9
4.25	5,200	4,800	4.2	3.8
4.50	5,100	4,700	3.9	3.7
4.75	4,900	4,600	3.9	3.8
5.00	4,900	4,500	3.9	3.6
5.25	4,800	4,500	3.6	3.5
5.50	4,600	4,300	3.6	3.5
5.75	4,400	4,100	3.6	3.4
6.00	4,300	4,000	3.4	3.3

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\dot{\delta x}$	$\dot{\delta y}$	$\dot{\delta z}$
2710	755	2920	0.6	2.2	2.4

Table 2

Star Occultation Measurements for Lunar Orbit Determination

Data Rate Interval: 15 minutes
 Orbital Altitude: 80 n. m.
 Equivalent Angle Uncertainty (1σ): 2 milliradians

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.0	
0.25	105,300	41,400	108.0	43.5
0.50	55,300	38,300	56.1	41.0
0.75	60,500	42,900	50.5	35.4
1.00	58,200	39,000	38.6	19.1
1.25	38,800	27,400	31.7	22.6
1.50	28,400	15,300	32.2	19.4
1.75	21,200	17,000	17.0	16.4
2.00	24,100	17,000	14.7	12.3
2.25	19,300	14,000	13.6	10.4
2.50	14,400	11,800	11.4	10.4
2.75	12,300	11,200	10.7	10.4
3.00	12,900	11,300	9.9	8.9
3.25	12,300	10,900	8.8	8.7
3.50	11,800	9,900	9.9	8.4
3.75	10,700	9,500	9.0	8.1
4.00	10,500	9,600	8.3	7.8
4.25	10,500	9,600	8.0	7.4
4.50	9,900	9,200	7.9	7.4
4.75	9,500	8,900	7.9	7.5
5.00	9,500	8,800	7.7	7.2
5.25	9,500	8,800	7.2	6.9
5.50	9,200	8,600	7.2	6.9
5.75	8,900	8,300	7.2	6.9
6.00	8,700	8,200	7.1	6.7

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
5,550	1,450	5,800	1.2	4.3	5.0

Table 3-a

Star Occultation Measurements for Lunar Orbit Determination

Data Rate Interval: 15 minutes
 Orbital Altitude: 80 nm
 Equivalent Angle Uncertainty (1σ): 6 milliradians

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.6	
0.25	105,300	44,100	107.9	46.2
0.50	64,700	44,500	63.9	41.0
0.75	72,300	45,800	68.8	33.3
1.00	68,400	41,200	63.3	27.7
1.25	51,500	36,700	44.1	27.8
1.50	42,700	32,400	38.0	29.3
1.75	39,200	32,500	32.9	28.1
2.00	39,600	33,800	28.4	26.4
2.25	38,900	32,300	28.3	24.9
2.50	34,300	30,600	26.5	24.4
2.75	31,600	29,100	25.7	25.0
3.00	31,700	28,600	25.3	23.2
3.25	30,900	28,400	23.4	22.4
3.50	29,900	27,300	24.2	22.2
3.75	28,600	26,200	24.1	22.3
4.00	28,500	26,100	23.3	21.7
4.25	28,700	26,600	22.3	20.7
4.50	28,100	26,100	22.0	20.9
4.75	27,200	25,500	22.4	22.0
5.00	28,200	25,700	22.4	20.7
5.25	27,600	25,800	20.6	20.0
5.50	26,500	24,700	20.9	19.7
5.75	24,900	23,400	20.8	19.7
6.00	24,400	22,900	20.3	19.5

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
15,400	4,100	16,500	3.3	12.4	14.7

Table 3-b

Star Occultation Measurements for Lunar Orbit Determination

Data Rate Interval: 15 minutes
 Orbital Altitude: 50,000 ft
 Equivalent Angle Uncertainty (1σ): 6 milliradians

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.6	
0.25	112,500	41,700	129.2	46.8
0.50	49,000	34,200	55.8	45.0
0.75	53,000	42,100	55.1	37.1
1.00	49,500	43,900	41.4	31.1
1.25	35,400	29,500	34.5	32.4
1.50	31,300	28,100	34.1	31.2
1.75	34,500	28,200	25.9	24.8
2.00	28,500	25,700	25.2	24.8
2.25	26,200	23,600	25.9	25.5
2.50	27,000	24,000	24.3	23.2
2.75	26,200	24,100	22.4	20.9
3.00	23,300	22,100	22.3	21.3
3.25	21,400	20,400	22.2	22.1
3.50	22,900	21,500	20.8	19.8
3.75	22,800	21,100	19.3	18.6
4.00	20,200	19,000	20.4	19.6
4.25	19,600	18,700	19.8	19.5
4.50	20,900	19,800	18.1	17.6
4.75	20,200	19,100	18.0	17.7
5.00	18,800	17,900	18.8	18.1
5.25	18,800	17,900	17.8	17.6
5.50	19,100	18,200	17.0	16.6
5.75	18,300	17,500	17.2	16.8
6.00	17,550	16,800	17.5	16.9

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\dot{\delta x}$	$\dot{\delta y}$	$\dot{\delta z}$
11,600	1,350	12,100	1.22	11.1	12.7

Table 4

Star Occultation Measurements Combined with
Orbital Period Measurements After the First Orbit

Data Rate Interval: 15 minutes (0.25 hours)
Orbital Altitude: 80 nautical miles
Equivalent Angle Uncertainty (1σ): 1 milliradian

Compare with Table 1

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.6	
0.25	105,300	41,000	107.9	43.3
0.50	54,300	36,900	55.3	41.2
0.75	57,700	41,400	45.0	35.8
1.00	58,500	32,000	40.1	16.8
1.25	31,600	14,700	21.9	18.7
1.50	18,400	12,600	20.5	12.1
1.75	16,700	9,400	10.7	9.3
2.00	13,200	7,900	7.6	6.2
2.25	9,100	6,800	6.5	5.1
2.50	6,700	5,600	5.2	5.2
2.75	5,600	5,000	5.2	4.9
3.00	5,700	4,800	4.6	4.0
3.25	5,100	4,400	3.9	3.6
3.50	4,400	4,100	3.7	3.6
3.75	4,200	3,900	3.6	3.5
4.00	4,200	3,900	3.3	3.3
4.25	4,100	3,800	3.2	3.2
4.50	3,800	3,700	3.2	3.1
4.75	3,700	3,500	3.1	3.1
5.00	3,600	3,500	3.0	2.9
5.25	3,600	3,500	2.9	2.9
5.50	3,500	3,300	2.9	2.8
5.75	3,400	3,200	2.9	2.8
6.00	3,300	3,100	2.8	2,7

Final Uncertainty One Sigma Components
(feet)

δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
2,250	136	2,260	0.1	1.9	2.0

Table 5

Star Occultation Measurements Combined with
Orbital Period Measurements After the First Orbit

Data Rate Interval: 15 minutes
 Orbital Altitude: 80 n.m.
 Equivalent Angle Uncertainty (1σ): 2 milliradians

Compare with Table 2

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.6	
0.25	105,300	41,300	107.9	43.5
0.50	55,300	38,200	56.1	40.9
0.75	60,400	42,900	50.5	35.4
1.00	58,200	39,000	38.6	19.2
1.25	38,700	27,400	31.7	22.6
1.50	28,300	15,200	32.1	19.4
1.75	21,200	16,900	17.0	16.4
2.00	24,000	17,000	14.6	12.3
2.25	19,300	13,300	13.6	9.0
2.50	12,500	10,900	10.5	10.5
2.75	11,300	10,200	10.4	9.8
3.00	11,500	9,900	9.1	8.2
3.25	10,600	9,100	8.1	7.2
3.50	9,000	8,100	7.7	7.3
3.75	8,300	7,700	7.4	7.1
4.00	8,400	7,900	6.7	6.6
4.25	8,400	7,700	6.3	6.2
4.50	7,800	7,300	6.3	6.1
4.75	7,300	7,000	6.3	6.2
5.00	7,200	7,000	6.1	6.0
5.25	7,400	7,000	5.8	5.6
5.50	7,100	6,800	5.7	5.6
5.75	6,700	6,400	5.8	5.6
6.00	6,500	6,300	5.6	5.5
6.25	6,300		5.5	

Final Uncertainty One Sigma Components
(feet) (feet per sec)

δx	δy	δz	$\dot{\delta x}$	$\dot{\delta y}$	$\dot{\delta z}$
4,500	320	4,450	0.2	3.7	4.1

Table 6

Star Occultation Measurements Combined with
Orbital Period Measurements After the First Orbit

Data Rate Interval: 15 minutes
Orbital Altitude: 80 n. m.
Equivalent Angle Uncertainty (1σ): 6 milliradians

Compare with Table 3

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100	-	60.6	-
0.25	105,300	44,000	107.9	46.2
0.50	64,600	45,400	63.9	41.0
0.75	72,200	45,800	68.8	33.3
1.00	68,300	41,100	63.3	27.7
1.25	51,400	36,700	44.1	27.8
1.50	42,700	32,300	38.0	29.3
1.75	39,200	32,500	32.9	28.1
2.00	39,600	33,800	28.4	26.4
2.25	38,900	31,100	28.3	23.8
2.50	30,300	27,000	24.3	24.1
2.75	27,600	25,600	23.9	23.5
3.00	27,700	25,200	22.8	20.9
3.25	26,600	23,300	21.0	19.5
3.50	23,800	21,500	20.3	19.2
3.75	22,500	21,200	19.3	18.0
4.00	22,500	20,900	17.7	17.3
4.25	21,500	20,200	17.3	17.1
4.50	20,700	19,600	17.2	16.9
4.75	20,400	19,500	16.8	16.5
5.00	20,300	19,400	16.3	16.0
5.25	19,800	19,000	16.1	15.7
5.50	19,200	18,300	16.0	15.7
5.75	18,800	18,000	15.8	15.2
6.00	18,700	18,100	15.2	15.1

Final Uncertainty One Sigma Components
(feet) (feet per sec)

δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
12,600	1,000	13,000	0.7	10.1	10.1

Table 7

Lunar Orbit Determination by Single Station MSFN Tracking

Data Rate Interval: 5 minutes(0.0833 hours)
 Range Accuracy (1σ): 50 feet
 Range Rate Accuracy (1σ): 0.5 fps

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	33,600		30.3	
0.25	52,700		53.9	
0.50	110,300		102.1	
0.55	125,700	27,800	113.4	31.3
0.633	35,100	13,900	39.3	13.9
0.716	14,900	13,700	14.3	12.3
0.80	14,300	14,200	12.0	11.7
0.833	14,700	14,700	11.3	11.0
0.966	15,100	15,000	10.7	10.6
1.05	15,100	14,900	10.5	10.5
1.13	14,800	13,600	10.6	10.6
1.216	13,500	10,700	10.7	9.9
1.30	11,300	10,800	9.5	8.9
1.383	11,600	11,500	8.2	7.9
1.466	12,300	12,200	7.4	6.9
1.55	12,600	12,600	6.6	6.1
No Measurements				
2.666	12,900	12,500	6.6	3.1
2.750	12,000	11,800	4.6	4.5
2.833	10,700	10,300	6.4	6.2
2.916	8,800	8,300	8.0	8.0
3.000	6,500	5,800	9.4	9.3
3.083	4,000	3,400	10.1	10.0
3.166	3,000	2,900	10.1	9.7
3.250	4,600	4,500	9.3	8.6
3.333	6,500	6,400	7.8	6.9
3.417	7,900	7,700	5.9	5.2
3.500	8,600	8,300	4.1	3.7
3.583	8,800	8,600	2.7	2.5
3.666	8,800	8,600	2.1	2.1

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\dot{\delta x}$	$\dot{\delta y}$	$\dot{\delta z}$
86	56	8,570	0.04	0.05	2.2

Table 8

Lunar Orbit Determination by Single Station MSFNTracking with Increased Data Rate

Data Rate Interval: 2.5 minutes
 Range Accuracy (1σ): 50 feet
 Range Rate Accuracy (1σ): 0.5 fps

Compare with Table 7

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	33,500		30.3	
0.55	125,700	18,800	113.4	20.4
0.63	20,400	13,500	22.1	12.9
0.717	13,800	13,800	12.9	12.0
0.80	14,400	14,100	11.9	11.4
0.883	14,800	14,700	11.2	10.8
0.967	15,000	15,000	10.7	10.5
1.05	15,000	14,400	10.5	10.5
1.13	14,400	11,900	10.6	10.3
1.217	11,900	9,900	10.3	9.6
1.30	10,600	10,300	8.8	7.9
1.383	11,700	11,500	7.5	6.7
1.47	12,500	12,300	6.3	5.5
1.55	12,600	12,600	5.4	5.0
No Measurements				
2.67	12,800	12,200	5.4	3.6
2.75	11,800	10,900	5.2	5.0
2.83	10,400	9,200	7.4	7.2
2.92	8,400	6,900	8.7	8.5
3.00	5,900	4,300	10.1	9.8
3.083	3,370	2,300	10.4	10.3
3.17	3,200	2,800	10.4	9.8
3.25	5,300	4,800	9.6	8.5
3.33	6,500	6,200	8.0	6.8
3.416	7,900	7,700	6.2	5.0
3.50	8,700	8,600	4.4	3.4
3.583	9,100	9,100	2.8	2.5

Final Uncertainty One Sigma Components

(feet)

(feet per sec)

δx	δy	δz	$\dot{\delta x}$	$\dot{\delta y}$	$\dot{\delta z}$
101	43	9,100	0.03	0.1	1.6

Table 9

Lunar Orbit Determination by Two Station MSFN Tracking

Data Rate Interval: 5 minutes
 Range Accuracy (1σ): 50 feet
 Range Rate Accuracy (1σ): 0.5 fps

Compare with Table 7

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	33,600		30.3	
0.55	125,700	16,000	113.4	18.5
0.633	17,400	5,300	19.9	11.1
0.716	6,200	4,200	11.1	8.0
0.80	4,900	3,800	7.8	5.7
0.883	4,400	3,500	5.5	4.1
0.966	3,900	3,200	3.9	2.9
1.05	3,400	2,900	2.8	2.3
1.13	3,100	2,800	2.1	1.9
1.216	2,900	2,600	1.8	1.6
1.30	2,600	2,200	1.6	1.5
1.383	2,200	1,900	1.5	1.4
1.466	1,900	1,800	1.4	1.4
1.55	1,800	1,600	1.4	1.4
No Measurements				
2.666	1,900	1,500	1.6	1.4
2.750	1,500	1,500	1.4	1.4
2.833	1,500	1,400	1.4	1.3
2.916	1,400	1,400	1.3	1.3
3.00	1,400	1,400	1.3	1.2
3.083	1,400	1,400	1.2	1.2
3.166	1,400	1,400	1.1	1.1
3.250	1,400	1,400	1.1	1.1
3.333	1,400	1,300	1.0	1.0
3.417	1,300	1,300	1.0	1.0
3.500	1,300	1,200	1.0	1.0
3.583	1,200	1,200	1.0	1.0
3.666	1,200	1,200	1.0	1.0

Final Uncertainty One Sigma Components

	(feet)			(feet per sec.)		
δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$	
25	35	1,180	0.03	0.02	0.98	

Table 10

Lunar Orbit Determination by Two Station MSFN
Tracking Under Large Initial Orbital Uncertainty Conditions

Date Rate Interval: 5 minutes

Range Accuracy (1σ): 50 feetRange Rate Accuracy (1σ): 0.5 fps

Compare with Table 9

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.6	
0.55	251,000	28,200	226.8	33.8
0.633	30,200	6,000	36.6	16.6
0.717	7,400	4,900	16.5	10.9
0.80	5,900	4,300	10.6	6.9
0.883	4,900	3,700	6.6	4.5
0.966	4,100	3,300	4.2	3.1
1.05	3,600	3,000	2.9	2.4
1.13	3,200	2,800	2.2	1.9
1.217	2,900	2,600	1.8	1.6
1.30	2,700	2,200	1.6	1.5
1.383	2,300	1,900	1.5	1.4
1.467	1,900	1,800	1.4	1.4
1.55	1,800	1,700	1.4	1.4
No Measurements				
2.67	1,900	1,500	1.6	1.4
2.75	1,500	1,500	1.4	1.4
2.83	1,500	1,400	1.4	1.3
2.917	1,400	1,400	1.3	1.3
3.00	1,400	1,400	1.3	1.2
3.083	1,400	1,400	1.2	1.2
3.17	1,400	1,400	1.1	1.1
3.25	1,400	1,400	1.1	1.1
3.33	1,400	1,300	1.0	1.0
3.417	1,400	1,300	1.0	1.0
3.50	1,300	1,300	1.0	1.0
3.583	1,300	1,200	1.0	1.0
3.67	1,200	1,200	1.0	1.0

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
25	34	1,140	0.03	0.02	0.98

Table 11

Two Station MSFN Tracking Under Reduced Range Accuracy

Data Rate Interval: 5 minutes
 Range Accuracy (1σ): 3000 feet
 Range Rate Accuracy (1σ): 0.5 fps

Compare with Table 9

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	33,500		30.3	
0.25	52,700		54.0	
0.50	110,200		102.1	
0.55	125,700	25,700	113.4	28.5
0.63	28,700	13,700	31.8	12.5
0.717	14,000	13,600	12.5	10.1
0.80	13,900	13,800	11.0	10.3
0.88	13,900	13,900	10.2	9.8
0.96	13,900	13,780	9.8	9.6
1.05	13,700	13,400	9.7	9.7
1.13	13,300	12,900	9.9	9.9
1.216	12,600	12,200	10.1	10.1
1.30	11,900	11,500	10.3	10.2
1.38	11,300	11,000	10.4	10.3
1.47	10,900	10,700	10.4	10.2
1.55	10,800	10,700	10.2	9.9
No Measurements				
2.67	11,700	10,900	10.5	9.7
2.75	10,900	10,900	9.6	9.2
2.83	11,200	11,100	9.2	8.8
2.917	11,200	11,100	8.8	8.5
3.00	11,300	11,200	8.5	8.3
3.08	11,300	11,200	8.3	8.2
3.17	11,100	10,900	8.3	8.3
3.25	10,800	10,500	8.4	8.4
3.33	10,400	10,100	8.6	8.5
3.417	9,900	9,800	8.7	8.7
3.50	9,600	9,500	8.7	8.6
3.583	9,400	9,400	8.7	8.5
3.67	9,400	9,300	8.5	8.3

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
288	137	9,300	0.1	0.2	8.3

Table 12

Two Station MSFN Tracking with Range Rate Only

Date Rate Interval: 5 minutes

Range Rate Accuracy (1σ): 0.5 fps

Compare with Table 11

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	33,500		30.3	
0.25	52,700		54.0	
0.50	110,000		102.0	
0.56	131,000	29,300	117.0	29.7
0.65	33,900	19,100	35.3	16.1
0.73	20,700	14,400	18.4	11.4
0.817	14,700	14,000	11.5	10.3
0.90	14,200	14,100	10.3	9.8
0.983	14,000	13,900	9.8	9.7
1.067	13,700	13,500	9.9	9.7
1.15	13,200	12,900	10.0	9.9
1.23	12,600	12,200	10.2	10.1
1.317	11,900	11,600	10.5	10.3
1.40	11,300	11,100	10.6	10.4
1.483	10,900	10,800	10.5	10.3
1.56	10,700	10,700	10.3	10.0
No Measurements				
2.683	11,800	11,100	10.6	9.8
2.76	11,300	11,100	9.8	9.3
2.85	11,200	11,200	9.2	8.9
2.93	11,400	11,300	8.8	8.6
3.016	11,400	11,400	8.5	8.4
3.10	11,400	11,300	8.4	8.3
3.183	11,200	11,000	8.4	8.4
3.27	10,800	10,700	8.6	8.5
3.35	10,400	10,300	8.7	8.7
3.43	10,000	9,900	8.8	8.8
3.50	9,800	9,600	8.9	8.9
3.60	9,500	9,500	8.8	8.7
3.683	9,500	9,500	8.7	8.5

Final Uncertainty One Sigma Componets

(feet)			(feet per sec.)		
δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
306	130	9,500	0.1	0.2	8.5

Table 13

Two Station MSFN Tracking Under Poor Range Accuracy and
Large Initial Orbit Uncertainty Conditions

Data Rate Interval 5 minutes
 Range Accuracy (1σ): 3,000 feet
 Range Rate Accuracy (1σ): 0.5 fps
 Compare with Table 11

Time (hrs)	RMS Position Uncertainty		RMS Velocity Uncertainty	
	Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0	67,100		60.6	
0.55	251,300	41,500	226.8	45.7
0.63	45,700	25,900	50.1	20.8
0.717	26,200	25,700	20.9	18.2
0.80	25,700	25,200	18.3	16.8
0.883	25,000	24,200	17.0	16.2
0.967	23,800	22,800	16.6	16.1
1.05	22,300	21,100	16.6	16.1
1.13	20,600	19,400	16.6	16.1
1.217	18,900	17,900	16.6	15.9
1.30	17,500	16,700	16.3	15.5
1.383	16,500	16,100	15.7	14.9
1.467	15,900	15,700	15.1	14.3
1.55	15,700	15,700	14.3	13.6
No Measurements				
2.67	16,400	15,700	13.9	12.9
2.75	15,700	15,700	12.9	12.2
2.83	15,700	15,600	12.2	11.7
2.917	15,600	15,400	11.8	11.4
3.00	15,300	15,100	11.5	11.2
3.083	14,900	14,600	11.3	11.2
3.167	14,300	13,900	11.4	11.2
3.25	13,700	13,100	11.4	11.3
3.33	12,900	12,400	11.5	11.3
3.417	12,300	11,900	11.4	11.1
3.50	11,900	11,800	11.1	10.7
3.583	11,800	11,800	10.7	10.1
3.67	12,000	11,900	10.1	9.7

Final Uncertainty One Sigma Components

(feet)			(feet per sec.)		
δx	δy	δz	$\dot{\delta x}$	$\dot{\delta y}$	$\dot{\delta z}$
296	148	11,900	0.1	0.2	9.7

Table 14

Combined Star Occultation and Two Station MSFN
Tracking for Lunar Orbit Determination

Data Rate Interval:

MSFN Tracking = 5 minutes

Star Occultations = 15 minutes

MSFN AccuracyRange (1σ) = 50 feetRange Rate (1σ) = 0.5 fpsStar Occultation Accuracy

Equivalent Angle Uncertainty: 2mr

Compare with Table 9

Time (hrs)	Type of Meas.	RMS Position Uncertainty		RMS Velocity Uncertainty	
		Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0		33,500		30.3	
0.25	S. O.	52,600	21,200	53.9	22.3
0.50	S. O.	29,500	20,800	29.6	19.5
0.55	MSFN	22,300	12,800	19.8	16.5
0.63	MSFN	13,900	5,300	17.5	10.8
0.716	Both	6,130	3,950	10.8	7.8
0.80	MSFN	4,700	3,800	7.4	5.6
0.883	MSFN	4,300	3,500	5.3	4.0
0.96	Both	3,800	3,000	3.8	2.9
1.05	MSFN	3,300	2,900	2.7	2.3
1.13	MSFN	3,000	2,700	2.1	1.9
1.216	Both	2,800	2,500	1.8	1.6
1.30	MSFN	2,600	2,200	1.6	1.5
1.383	MSFN	2,200	1,900	1.4	1.4
1.47	Both	1,900	1,800	1.4	1.4
1.55	MSFN	1,800	1,600	1.4	1.4
1.75	S. O.	1,600	1,600	1.4	1.4
2.00	S. O.	1,700	1,700	1.4	1.4
2.25	S. O.	1,800	1,800	1.4	1.4
2.50	S. O.	1,800	1,800	1.5	1.5
2.67	MSFN	1,900	1,500	1.5	1.3
2.75	Both	1,500	1,400	1.3	1.3
2.83	MSFN	1,400	1,400	1.3	1.3
2.916	MSFN	1,400	1,400	1.3	1.2
3.00	Both	1,400	1,300	1.2	1.2
3.083	MSFN	1,400	1,400	1.2	1.1
3.17	MSFN	1,400	1,300	1.1	1.1
3.25	Both	1,400	1,300	1.1	1.0
3.33	MSFN	1,300	1,300	1.0	1.0
3.41	MSFN	1,300	1,300	1.0	1.0
3.50	Both	1,300	1,200	1.0	1.0
3.583	MSFN	1,200	1,200	1.0	1.0
3.67	MSFN	1,200	1,100	1.0	1.0

Final Uncertainty One Sigma Components

(feet)

(feet per sec.)

δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
25	34	1,100	0.03	0.02	0.95

Table 15

Combined Star Occultation and Two Station MSFN TrackingUnder Poor Range Accuracy Conditions

Data Rate Interval:

MSFN Tracking: 5 minutes

Star Occultations: 15 minutes

MSFN AccuracyRange (1σ) = 3000 feetRange Rate (1σ) = 0.5 fpsStar Occultation Accuracy

Equivalent Angle Uncertainty 2mr

Compare with Table 14

Time (hrs)	Type of Meas.	RMS Position Uncertainty		RMS Velocity Uncertainty	
		Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0		33,500		30.3	
0.25	S. O.	52,700	21,200	53.9	22.3
0.50	S. O.	29,500	20,800	29.6	19.5
0.55	MSFN	22,200	15,800	19.8	17.1
0.63	MSFN	16,900	12,800	18.2	12.1
0.716	Both	13,300	9,800	12.1	10.4
0.80	MSFN	10,100	10,100	10.3	9.5
0.883	MSFN	10,800	10,400	9.3	8.7
0.96	Both	11,100	8,600	8.4	7.8
1.05	MSFN	9,000	8,900	7.7	7.4
1.13	MSFN	9,400	9,400	7.2	6.9
1.216	Both	9,500	7,700	6.8	6.5
1.30	MSFN	7,900	7,800	6.5	6.3
1.383	MSFN	7,900	7,900	6.3	6.1
1.46	Both	7,900	6,900	6.1	6.0
1.55	MSFN	6,900	6,900	6.0	5.9
1.75	S. O.	7,100	6,300	5.8	5.8
2.00	S. O.	6,900	6,300	5.6	5.6
2.25	S. O.	7,200	6,600	5.4	5.4
2.50	S. O.	7,300	6,800	5.5	5.5
2.67	MSFN	7,100	5,800	5.6	4.7
2.75	Both	5,900	5,200	4.7	4.5
2.83	MSFN	5,300	5,200	4.5	4.4
2.916	MSFN	5,300	5,300	4.4	4.3
3.00	Both	5,300	4,900	4.3	4.3
3.083	MSFN	5,000	5,000	4.2	4.2
3.167	MSFN	5,100	5,100	4.1	4.0
3.25	Both	5,100	4,800	4.1	4.0
3.33	MSFN	4,800	4,800	4.0	3.9
3.416	MSFN	4,900	4,800	3.9	3.9
3.50	Both	4,900	4,600	3.9	3.9
3.583	MSFN	4,600	4,600	3.8	3.8
3.66	MSFN	4,600	4,600	3.8	3.8

Final Uncertainty One Sigma Components

(feet)

(feet per sec.)

δx	δy	δz	$\delta \dot{x}$	$\delta \dot{y}$	$\delta \dot{z}$
284	130	4,600	0.1	0.2	3.8

Table 16

Combined Star Occultation and Two Station MSFN
Tracking with Poor Range and Occultation Accuracies

Data Rate Interval:

MSFN Tracking: 5 minutes
Star Occultations: 15 minutes

MSFN Accuracy

Range (1σ) = 3000 ft
Range Rate (1σ) = 0.5 fps

Star Occultation Accuracy

Equivalent Angle Uncertainty: 6mr

Compare with Table 14

Time (hrs)	Type of Meas.	RMS Position Uncertainty		RMS Velocity Uncertainty	
		Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0		33,500		30.2	
0.25	S.O.	52,600	25,700	53.9	26.7
0.50	S.O.	43,600	28,300	41.8	24.8
0.55	MSFN	30,600	21,700	26.4	23.9
0.63	MSFN	24,000	13,600	26.3	12.4
0.716	Both	13,900	12,900	12.5	10.9
0.80	MSFN	13,100	13,100	10.9	10.1
0.883	MSFN	13,400	13,300	10.1	9.6
0.967	Both	13,400	12,600	9.6	9.3
1.05	MSFN	12,600	12,460	9.4	9.2
1.13	MSFN	12,400	12,100	9.3	9.3
1.216	Both	12,000	11,200	9.4	9.3
1.30	MSFN	11,000	10,700	9.5	9.4
1.383	MSFN	10,600	10,400	9.5	9.4
1.46	Both	10,900	9,900	9.4	9.3
1.55	MSFN	9,900	9,900	9.3	9.0
1.75	S.O.	10,300	9,900	8.9	8.9
2.00	S.O.	10,800	10,500	8.3	8.3
2.25	S.O.	10,900	10,600	8.4	8.4
2.50	S.O.	10,600	10,300	8.9	8.9
2.67	MSFN	10,500	9,600	8.9	8.2
2.75	Both	9,600	9,300	8.2	7.9
2.83	MSFN	9,400	9,400	7.9	7.7
2.916	MSFN	9,500	9,500	7.6	7.4
3.00	Both	9,600	9,300	7.4	7.3
3.083	MSFN	9,300	9,300	7.3	7.2
3.167	MSFN	9,300	9,200	7.2	7.1
3.25	Both	9,200	8,800	7.2	7.2
3.33	MSFN	8,700	8,600	7.2	7.2
3.417	MSFN	8,500	8,400	7.2	7.2
3.50	Both	8,400	8,100	7.2	7.1
3.583	MSFN	8,100	8,100	7.2	7.1
3.67	MSFN	8,100	8,100	7.0	6.9

Final Uncertainty One Sigma Components
(feet) (feet per sec)

δx	δy	δz	$\dot{\delta x}$	$\dot{\delta y}$	$\dot{\delta z}$
286	134	8,100	0.1	0.2	6.9

Table 17.

Combined Star Occultation and Single Station MSFN

Tracking for Lunar Orbit Determination

Data Rate Interval:

MSFN Tracking = 5 minutes
 Star Occultations = 15 minutes

MSFN Accuracy

Range (1σ) = 50 feet
 Range Rate (1σ) = 0.5 fps

Star Occultation Accuracy

Equivalent Angle Uncertainty: 2 mr

Compare with Tables 7 and 14

Time (hrs)	Type of Meas.	RMS Position Uncertainty		RMS Velocity Uncertainty	
		Prior to Meas. (ft)	After Meas. (ft)	Prior to Meas. (fps)	After Meas. (fps)
0		33,500		30.3	
0.25	S. O.	52,600	21,200	53.9	22.3
0.50	S. O.	29,500	20,800	29.6	19.5
0.55	MSFN	22,300	14,900	19.8	16.7
0.63	MSFN	17,000	12,900	18.6	13.5
0.716	MSFN	14,200	13,000	13.8	11.8
0.75	S. O.	13,200	9,800	11.6	11.4
0.80	MSFN	10,400	10,200	11.2	10.8
0.883	MSFN	11,200	11,300	10.2	9.9
0.967	MSFN	12,200	12,300	9.2	8.9
1.00	S. O.	12,400	9,000	8.7	8.5
1.05	MSFN	9,400	9,400	8.2	8.0
1.13	MSFN	9,900	9,600	7.6	7.5
1.217	MSFN	10,000	8,700	7.1	6.8
1.25	S. O.	8,900	7,400	6.7	6.6
1.30	MSFN	7,700	7,500	6.3	5.9
1.383	MSFN	8,000	8,100	5.4	5.3
1.47	MSFN	8,500	8,500	4.9	4.7
1.50	S. O.	8,600	7,200	4.6	4.6
1.55	MSFN	7,300	7,300	4.5	4.3
1.75	S. O.	7,000	6,100	4.7	4.7
2.00	S. O.	5,100	4,700	5.5	5.5
2.25	S. O.	5,600	5,100	5.1	5.1
2.50	S. O.	6,600	6,000	4.0	4.0
2.67	MSFN	6,200	5,800	3.9	2.2
2.75	Both	5,600	5,000	2.6	2.6
2.83	MSFN	4,600	4,500	3.1	3.2
2.917	MSFN	4,000	3,800	3.7	3.8
3.00	Both	3,300	3,000	4.1	4.1
3.083	MSFN	2,600	2,400	4.3	4.2
3.17	MSFN	2,400	2,400	4.2	3.9
3.25	Both	2,800	2,900	3.7	3.1
3.33	MSFN	3,400	3,400	2.8	2.3
3.417	MSFN	3,700	3,500	2.0	1.7
3.50	Both	3,700	3,300	1.5	1.4
3.58	MSFN	3,300	3,000	1.4	1.4
3.67	MSFN	2,800	2,600	1.6	1.6

Final Uncertainty One Sigma Components

$\delta x = 48$ $\delta y = 49$ $\delta z = 2,600$ (feet) $\delta \dot{x} = 0.04$ $\delta \dot{y} = .04$ $\delta \dot{z} = 1.6$ (feet per sec.)

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