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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Analysis of the First Translunar Midcourse Correction for the Apollo C' Mission

TM-68-2011-2

DATE- December 9, 1968

FILING CASE NO(S)- 310

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ABSTRACT

A statistical analysis of the first translunar midcourse correction for the December 21, 1968 C' lunar mission disclosed that the magnitude of the required correction at TLI + 6 hours is less than 83.4 fps with a probability of 99.865%. Analysis of individual samples showed that the minimum required correction may occur anywhere between TLI and TLI + 11 hours. In order to minimize the expected correction, it should be planned at TLI + 4 hours, but the expected penalty for planning it at TLI + 6 hours is only 0.8 fps (at the 99.865% probability level). Considering the time desired for adequate tracking, preparation, etc., the six hour point looks like a good selection. Furthermore, it is unnecessary to consider changing the time of the midcourse during the flight in hopes of decreasing the required ΔV, since little or no savings will result.

The magnitude of the correction required at TLI + 6.5 hours in order to obtain the minimum ΔV free return is less than 51 fps with a probability of 99.865%.

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DATE: December 9, 1968

FROM: D. A. Corey
S. L. Levie, Jr.

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TECHNICAL MEMORANDUM

INTRODUCTION

Current planning for the Apollo C' lunar mission provides for the first translunar midcourse correction to be performed 6 hours after Translunar Injection (TLI + 6 hours). It is further planned that if the computed correction is greater than about 70 feet per second, the correction will be recomputed for an earlier execution time in hopes of saving fuel. Corrections as early as TLI + 3 hours would be considered.

In addition, a midcourse correction to achieve a minimum fuel free return trajectory (generally a lunar flyby) will be computed for the TLI + 6.5 hour point and sent to the spacecraft. This correction would be performed using the RCS in the event of an SPS failure for the six hour correction.

TLI + 6 hours was selected as the midcourse time primarily to allow sufficient time for MSFN tracking. Members of the Math Physics Branch of MPAD at MSC have stated that at least three hours of good tracking are required after TLI in order to reduce the uncertainty for the correction to about one foot per second. Furthermore, they have pointed out, the tracking data obtained during the first hour after TLI will be poor because of spacecraft activity during that period. Allowing one hour for post-TLI activity; 3 hours for tracking; one and one-half hours for data processing, computation of the correction, and uplinking the required data to the spacecraft; plus an additional one-half hour for the crew to prepare for the maneuver, yields the six hour figure. Perhaps the correction could be done reliably at TLI + 5 hours, but a maneuver time any earlier than that would certainly involve an uncertainty greater than one foot per second.

This memorandum will show that it is unnecessary to consider making the correction earlier than six hours. There is a substantial probability that the correction required at

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an earlier time is actually larger than the correction required at six hours. But, even in the cases where the required ΔV can be reduced by advancing the midcourse time, the improvement is generally insignificant, especially when the effects of the increased MSFN uncertainty are considered.

It has been previously shown (see, for example, Reference 1) that for fixed time of arrival* midcourse targeting the expected ΔV required as a function of time past TLI starts off high at TLI, drops fairly steeply to a minimum and then climbs slowly and monotonically. The minimum generally occurs somewhere between three and seven hours and is a function of the error sources involved and of the particular trajectory geometry. The results reported in this memorandum demonstrate that the same kind of behavior should be expected on the Apollo C' mission where Free Return - Best Adaptive Path (FRBAP) targeting will be used for the first translunar midcourse correction. FRBAP targeting essentially reoptimizes the mission to get to a desired lunar landing site. Total spacecraft fuel required for the mission is the minimized parameter. In addition, some results comparing the FRBAP corrections and the corrections required for a minimum fuel lunar flyby trajectory are presented.

DESCRIPTION OF METHODS USED

TLI Errors

A covariance matrix of actual state vector deviations after TLI was obtained from Reference 2. This matrix is presented in Table 1 and includes the effects of launch vehicle execution errors, earth orbit venting, and the post-TLI propulsive blowdown. The matrix is based on unofficial data from MSFC and should be considered preliminary. The nominal trajectory assumed was for the same day and launch azimuth as the reference trajectory used in this study. The matrix is valid for nominal TLI cutoff plus 15 minutes, second TLI opportunity. It is presented in the UVW coordinate system in which U is along the position vector, W is out of plane in the direction of the angular momentum vector, and V completes the right handed orthogonal coordinate system. The units are feet and feet per second.

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*This is basically equivalent to what MSC calls XYZT targeting.

Reference Trajectory

A patched conic free return reference trajectory for a December 1968 mission to Orbiter Site II-P-II was generated using BCMASP. The launch azimuth was 72° and the launch date was December 21, 1968. A Pacific injection was selected. The flight path azimuth over the pseudo-landing site was -78 degrees. Other significant parameters for the reference trajectory and vehicle are presented in Table 2.

Dispersions at the Midcourse Points

One hundred random samples were taken from the covariance matrix of errors at TLI + 15 minutes. These samples were added to the nominal conic state vector valid for the same time. These state vectors were saved and also conically propagated to the various midcourse times, namely TLI + 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 hours. The resulting state vectors were used in a FRBAP targeting program to determine the required ΔV.

An additional 400 samples were taken from the covariance matrix and propagated to TLI + 6 hours in order to obtain more precise midcourse correction statistics at that point.

FRBAP Targeting

A modified version of the BCMASP lunar trajectory targeting program was used to compute the required midcourse corrections. The principal modification was to the subroutine HITMSI which normally computes the earth launch time and TLI location to pass an earth centered conic through a specified point on the MSI with a specified geocentric energy. The modified routine solves the problem of passing a conic between the midcourse point and the desired MSI point with the specified energy.

The BCMASP program generally does a scan of several landing site approach azimuths in order to find the one which requires the least spacecraft fuel. Preliminary analysis showed, however, that within one degree, the nominal approach azimuth was optimum for the perturbed trajectories as well. Since the computer time required varies nearly linearly with the number of approach azimuths studied, the main production computer runs for this study considered only the nominal approach azimuth. Additionally, since crew training, lunar orbit sighting schedules, and on-board maps and charts are based on this approach azimuth, the first correction will undoubtedly be computed for this azimuth on the actual mission.

Conic trajectories were used throughout in order to consider a sufficient number of cases to be statistically meaningful within reasonable limits on the required computer time. The usual error analysis method of propagating state vectors and covariance matrices and targeting using free flight partials are, unfortunately, not applicable to a non-fixed time of arrival problem. The use of integrated trajectories would require a totally unreasonable amount of computer time. Previous studies and experience have shown, however, that the sensitivities about conic trajectories are very nearly the same as those about integrated trajectories so long as one stays well away from the MSI. The results obtained in this study, then, should be quite representative of the real world, since only the first midcourse correction was considered.

Minimum Free Return Delta V Targeting

The one hundred samples from the covariance matrix were also propagated to TLI + 6.5 hours and the required minimum ΔV midcourse correction was computed to achieve free return. In addition, the required ΔV for several other times after TLI were computed for two samples. This work was done by Mr. R. A. Bass of Bellcomm's Mission Analysis Department. Detailed results of this study will be reported separately, but some of the results will be presented here for completeness.

Statistical Processing

The required ΔV magnitudes obtained in this study are statistically distributed as the square root of the sum of squares of three gaussian distributed random variables. Furthermore, the three gaussian distributions have unequal variances so the magnitudes are not distributed as the square root of a usual (integer degrees of freedom) Chi squared distribution. Consequently, the standard deviation of the ΔV magnitudes represents some unknown probability and discussing three sigma points is not meaningful. Reference 3, however, presents a method for dealing with this situation by considering a Chi squared distribution with non-integer degrees of freedom. The method computes the degrees of freedom from the covariance matrix of the midcourse correction components and then computes the value of the magnitude for various probabilities. This method features very good accuracy in the tails of the distributions and so the primary results of this study were obtained using it.

In addition, plots of the sample cumulative distribution function for several parameters of interest are presented.

DISCUSSION OF RESULTS

ΔV Required vs. Time Past TLI

Figure 1 presents plots of four typical samples of the required first translunar midcourse ΔV magnitude as a function of time after TLI. The particular samples were selected from a set of 100 to demonstrate that the minimum required ΔV may occur at widely varying times after TLI.

Sample A is a case where the required correction at TLI + 6 hours is somewhat smaller than the correction at three hours while sample C shows the inverse situation. Sample B is a case where the required corrections are about the same at three and six hours. Samples A, B, and C have minimum required corrections at TLI + 9.46, 4.44, and 1.54 hours respectively.

The steep, negative slope shortly after TLI and the comparatively shallower, positive slope after the minimum was characteristic of most of the samples obtained. In 15% of the cases, however, the required correction increased monotonically from the TLI + .25 hour point.* Sample D is an example of this behavior.

In general, it is difficult to predict where the minimum will occur from studying the deviations in the UVW coordinate system. Instead, study of the perturbed conic elements is more fruitful. For errors in most of the orbital elements, the minimum correction will occur well after TLI, since the inertial velocity decreases very rapidly after TLI. This explains the steep, negative slope shortly after TLI. The major exceptions to this occur

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*It is possible that the corrections in these cases actually increase monotonically from TLI + 0 hours. However, TLI + .25 hours is the earliest time for which computations were made.

when there are large energy or eccentricity dispersions. These can be corrected more cheaply near TLI where the inertial velocity is large. Samples A and B had comparatively small energy and eccentricity dispersions while samples C and D had fairly large ones. In the case of sample D, both energy and eccentricity errors were large and dominated errors in the other elements. Consequently, the minimum required correction occurred early, at TLI + .25 hours. Sample C involved a smaller error in eccentricity and, in addition, a large out of plane error. The resulting minimum correction was a balance of the two effects and occurred at TLI + 1.5 hours.

The statistically expected behavior of the required midcourse correction as a function of time is presented in Figure 2 in the form of plots for various probability levels. They indicate that the required ΔV will be less than the plotted quantity with the specified probabilities. The probability levels of 84.134%, 97.725%, and 98.865% were selected to conform to the probabilities of the one, two, and three sigma points of a normal distribution -- they are not the one, two, and three sigma points for this distribution. In addition, the mean and the 50% (median) points are plotted. Figure 2 indicates that the best point at which to schedule the midcourse correction is at about TLI + 4 hours since the expected magnitude is at a minimum there. Additionally, note that the curves are very flat near the minimum and that the midcourse requirements vary by less than one foot per second over the region from TLI + 3 to TLI + 6 hours on all the curves. Consequently, it really makes little difference when, in that time span, the correction is scheduled.

For each of the 100 samples, the point with the smallest required correction and the two adjacent points were fitted with a parabola in order to interpolate for the time of the minimum correction and the corresponding magnitude. Checking revealed that this interpolation method was only valid to plus or minus about 15 minutes, but that was felt to be sufficiently accurate. Figure 3 presents the cumulative distribution function of the resulting distribution of the time of the minimum correction and Figure 4 presents the cumulative distribution function of the corresponding ΔV magnitudes. The median of the minimum midcourse time occurs at about TLI + 4 hours, the mean is at TLI + 4.5 hours, and the standard deviation is 3.2 hours. The median of the minimum midcourse correction was 12.2 fps, the mean was 19.8 fps and the standard deviation was 17.1 fps. The distributions of the time and the minimum ΔV magnitude are only weakly correlated, having a correlation coefficient of 0.1. No further statistical analysis of these distributions was performed so it is not possible to associate probability levels with the means and standard deviations of these distributions. They simply reiterate the presence of a substantial variation in the time of the minimum.

Figure 5 presents histograms of the distributions of the differences in the required corrections for TLI + 6 hours vs. TLI + 3, 4, 5, and 7 hours. These histograms give an indication of what improvements can be expected by advancing or delaying the time of the correction. In 52% of the samples, the three hour correction was smaller than the six hour correction. Out of 100 samples, the largest saving obtained by going to three hours was 7.2 fps. The five hour correction was better than the six hour correction in 63% of the cases, with the largest saving being 2.1 fps. The conclusion is that a correction at a time earlier than the planned six hours may be larger or smaller than the value at six hours, and when the earlier correction is smaller, it is only a few feet per second smaller.

Comparison of FRBAP and Minimum ΔV Flyby

Figure 6 presents plots of the required corrections as a function of the time past TLI for two additional samples. Also plotted are the ΔV magnitudes required for the corresponding minimum ΔV free return flyby corrections. Of the samples studied, there did not appear to be a strong correlation between the ΔV required for a minimum ΔV flyby and the FRBAP ΔV except, of course, that the former was always smaller. Note that the minimum ΔV flyby solution behaves generally the same as the FRBAP solution as a function of time past TLI. The time of the minimum required ΔV is generally not the same for both types of correction.

Required Corrections at TLI + 6 Hours

Figure 7 presents the cumulative distribution function of the FRBAP ΔV required at TLI + 6 hours. Five hundred samples were generated to produce the plot. In addition, the statistics produced from analysis of the correction covariance matrix are noted on Figure 8. The 99.865% point obtained from the 500 samples is 83.4 fps. This compares favorably with the 85.0 fps value obtained from the 100 samples used in Figures 1 and 2, indicating that the statistics presented in Figure 2 are representative of the underlying population.

Figure 8 presents the cumulative distribution function and the significant statistics for the minimum ΔV flyby correction at TLI + 6.5 hours. One hundred samples were used to obtain these statistics. The 99.865% point of this distribution fell at 50.8 fps. which is appreciably smaller than the corresponding FRBAP ΔV required at TLI + 6 hours.

CONCLUSIONS

The statistics produced by this study indicate that the optimum time (in terms of the minimum expected ΔV) to plan for the first translunar midcourse correction on a December 21, 1968 lunar orbit mission is at about TLI + 4 hours. However, the 99.865% point of the distribution of required ΔV magnitudes at TLI + 6 hours is only .8 fps. greater than the corresponding point at TLI + 4 hours. This difference is less than the expected uncertainty in the correction at TLI + 6 hours due to MSFN errors. Since the expected tracking errors will undoubtedly be larger for a correction at TLI + 4 hours, it can be concluded that there is no significant penalty involved in planning the correction for TLI + 6 hours.

As a function of time past TLI, the required ΔV magnitude generally starts off large after TLI, decreases fairly rapidly to a minimum which may be located anywhere from TLI + 1 to TLI + 11 hours, and then increases slowly. In 15% of the cases studied, however, the minimum occurred just after TLI and the required ΔV increased as a function of time after TLI. These cases involved dominant energy and/or eccentricity errors. The minimum ΔV magnitudes occurred later than TLI + 6 hours in 34% of the cases, later than TLI + 5 hours in 41% of the cases, and later than TLI + 4 hours in 53% of the cases. In 41% of the cases studied, the required ΔV was actually larger at TLI + 4 hours than at TLI + 6 hours. The curves of ΔV magnitude versus time past TLI are, however, generally quite flat in the region from three to six hours. Consequently, one can expect to gain very little, if anything, by advancing the time of the correction to earlier than TLI + 6 hours. In a rare case, one might decrease the required ΔV by as much as 4.5 fps by making the correction at TLI + 4 hours.

The required FRBAP midcourse correction at TLI + 6 hours is less than 85.4 fps with a probability of 99.865%. This number is based on processing of 500 Monte Carlo samples.

The minimum ΔV free return required ΔV at TLI + 6.5 hours is less than 50.8 fps with a probability of 99.865% based on 100 Monte Carlo samples.

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S. L. Levie Jr.

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Attachments
Figures
References
Tables

REFERENCES

1. B. G. Niedfeldt, "Midcourse Correction Penalties Due to Expected Translunar Injection Deviations as a Function of Earth Parking Orbit Stay Time", Bellcomm TM-67-2012-4, August 11, 1967.

2. M. R. Rother, "Preliminary launch vehicle covariance matrix for C' mission", U. S. Government Memorandum #68-FM73-489, FM7/ Guidance and Performance Branch/ Guidance Analysis Section, Manned Spacecraft Center, October 23, 1968.

3. H. J. Bixhorn and B. G. Niedfeldt, "A Simple Method for Approximating Quantiles of the Random Variables $X_1^2 + X_2^2 + X_3^2$", Bellcomm Memorandum for File, August 9, 1968.

TABLE 1

COVARIANCE MATRIX OF ERRORS AT TLI + 15 MINUTES

SECOND OPPORTUNITY DECEMBER 21, 1968 LAUNCH

	U	V	W	$\dot{U}$	$\dot{V}$	$\dot{W}$
U	.23271480+10	.46858996+10	-.50757886+08	-.26900468+07	.60531505+06	.74481547+05
V	.46858996+10	.16802540+11	.26078454+07	-.10386225+08	.46316702+07	.42998867+06
W	-.50757886+08	.26078454+07	.70831154+09	-.84250911+05	.22091040+04	.51900446+06
$\dot{U}$	-.26900468+07	-.10386225+08	-.84250911+05	.64777511+04	-.30017825+04	-.32726207+03
$\dot{V}$	.60531505+06	.46316702+07	.22091040+04	-.30017825+04	.17478498+04	.12088917+03
$\dot{W}$	.74481547+05	.42998867+06	.51900446+06	-.32726207+03	.12088917+03	.39875840+03

TABLE 2

SUMMARY OF REFERENCE TRAJECTORY CHARACTERISTICS

Launch Date	December 21, 1968
Launch Azimuth	72°
Lunar Landing Site	II-P-II, 2.725°N., 34.4°E
Translunar Flight Time	66.137 Hours
Transearth Flight Time	80.1098 Hours
Earth Landing Point	4.9897°S, 170.0° W.
Landing Site Approach Azimuth	-78°
Sun Elevation at Lunar Landing	8.4768°
Sun Azimuth at Lunar Landing	10.0553°
Entry Flight Path Angle	-6.4°
Total Spacecraft Weight at Injection	64540 lb.
Translunar Midcourse ΔV Budget	130 fps
Transearth Midcourse ΔV Budget	62 fps.
Spacecraft Weight Prior to CM/SM Separation	35199.4 lb.
SPS Thrust	20,000 lb.
SPS Specific Impulse	311 sec.

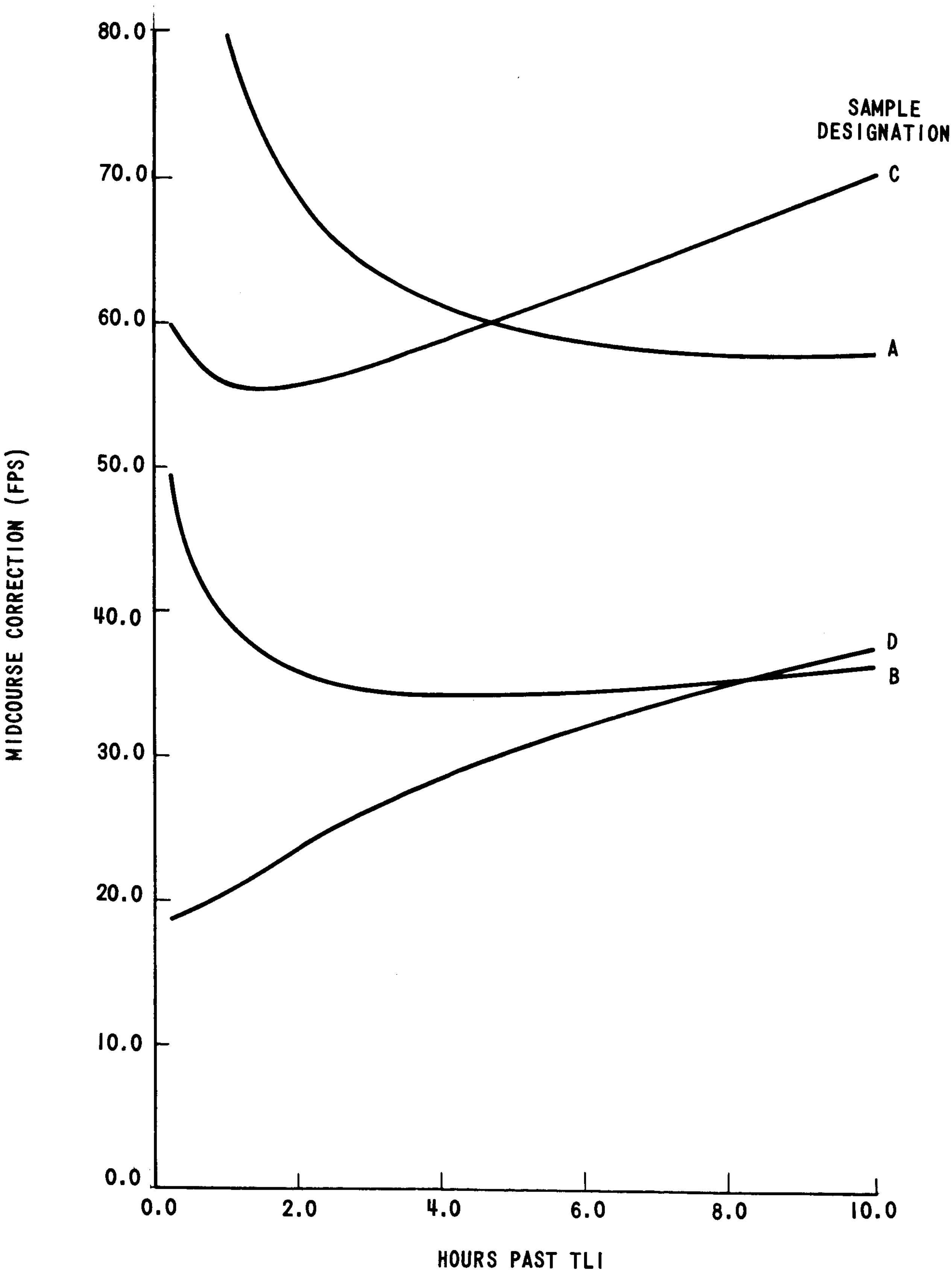

FIGURE 1 - MIDCOURSE CORRECTION vs. TIME PAST TLI, FOR TYPICAL TLI SAMPLES

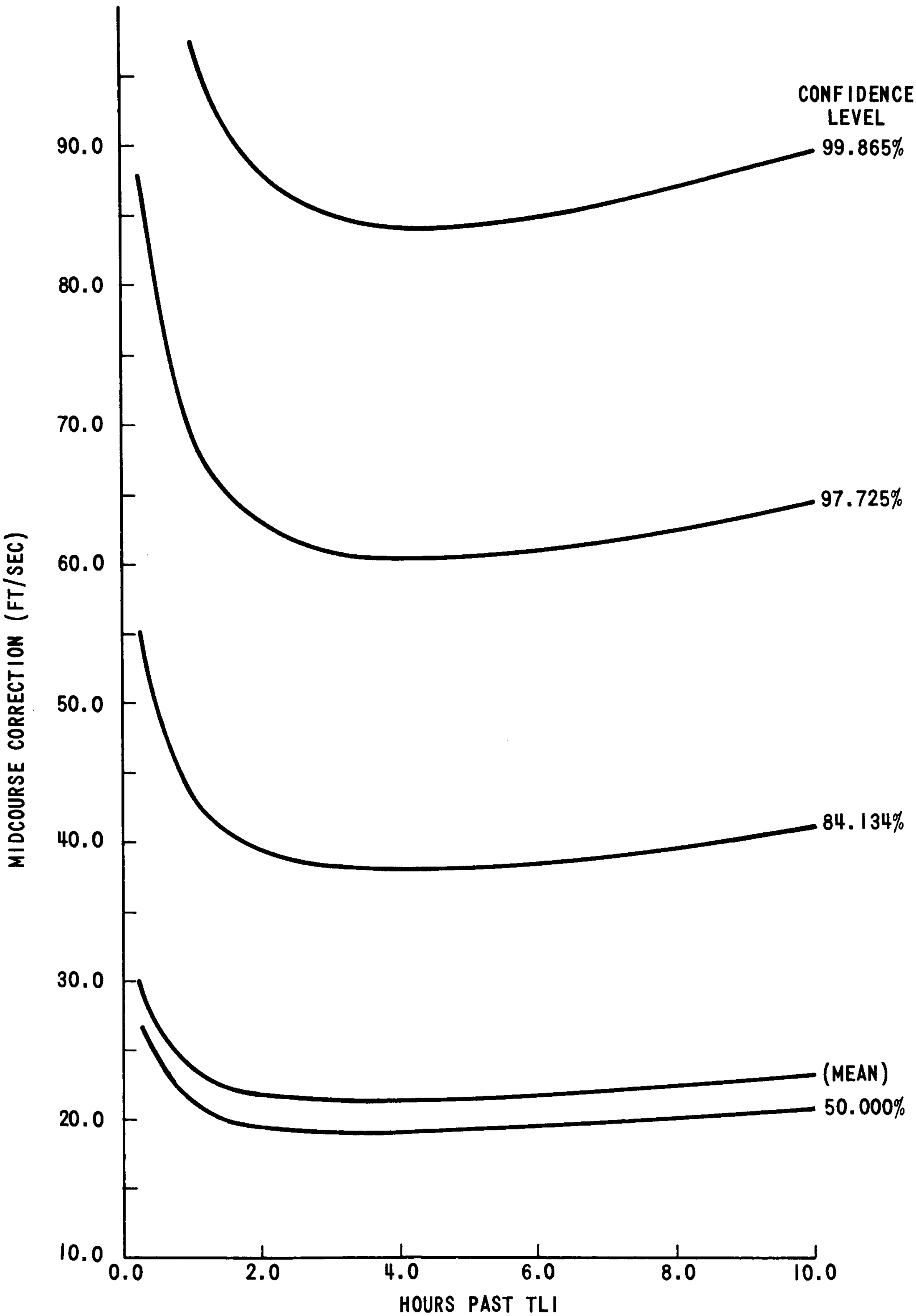

FIGURE 2 - MIDCOURSE CORRECTION vs. TIME PAST TLI, FOR VARIOUS CONFIDENCE LEVELS

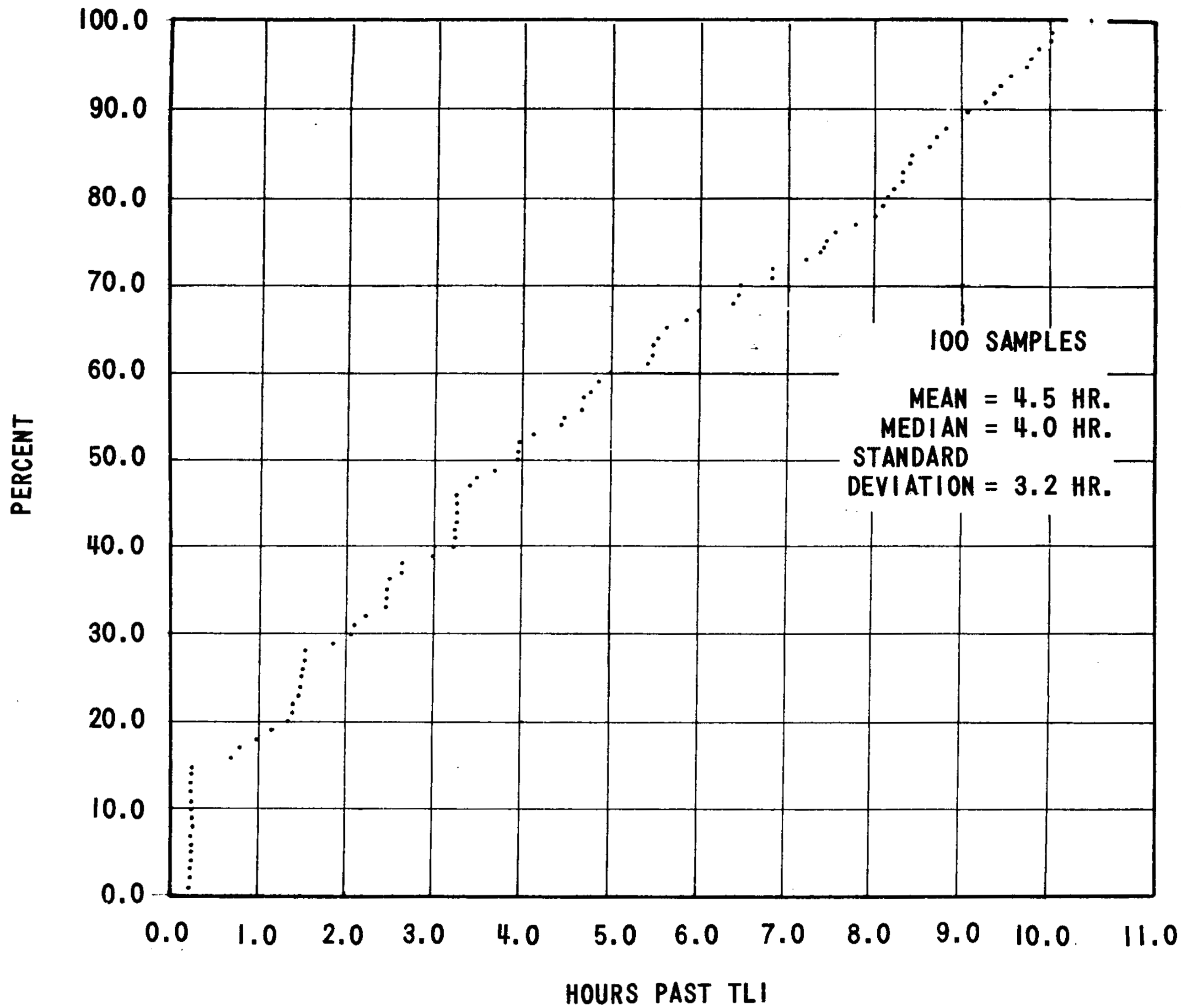

FIGURE 3 - CUMULATIVE DISTRIBUTION OF MIDCOURSE EXECUTION TIMES REQUIRING MINIMUM MIDCOURSE CORRECTION

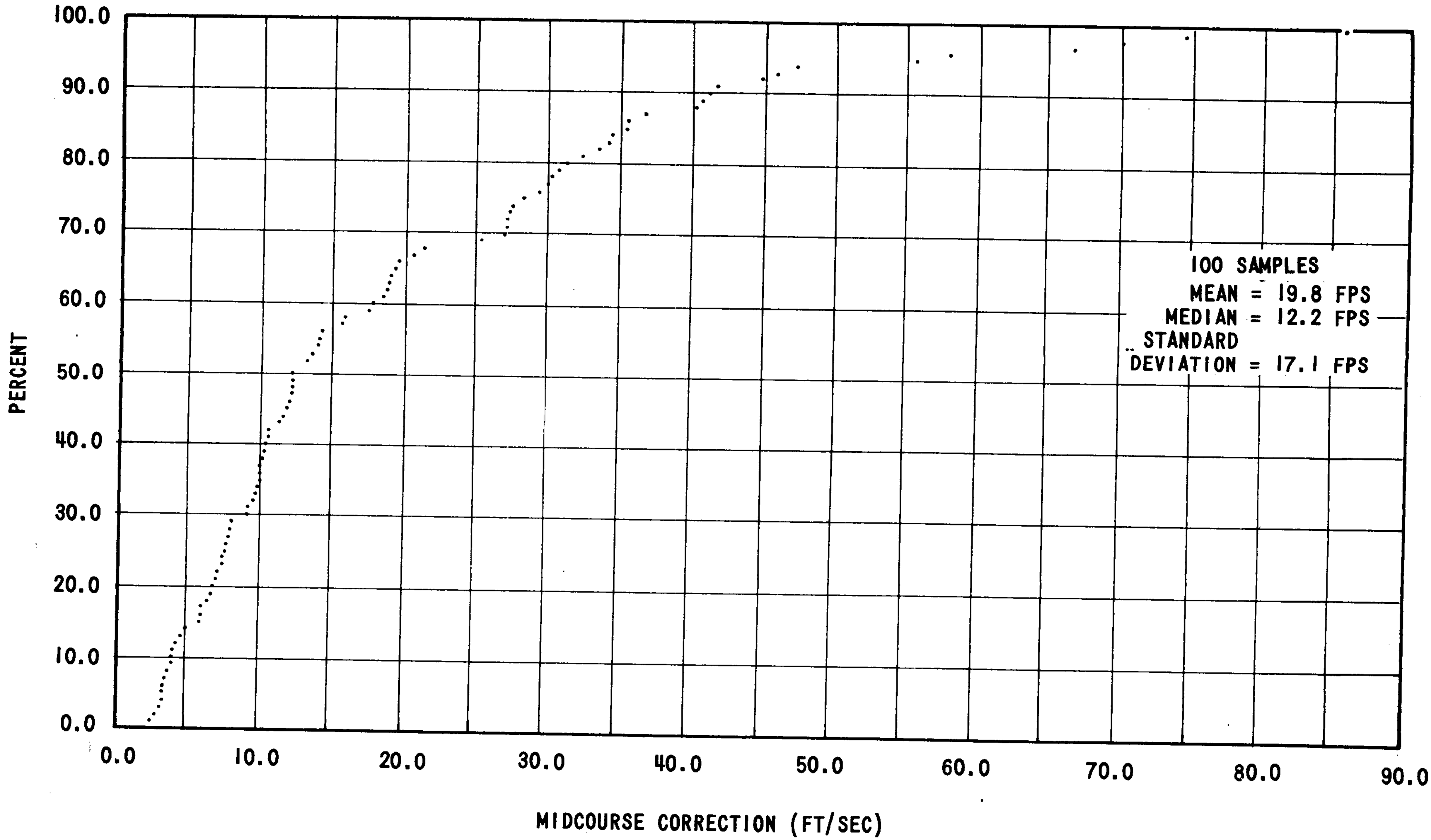

FIGURE 4 - CUMULATIVE DISTRIBUTION OF MINIMUM MIDCOURSE CORRECTIONS

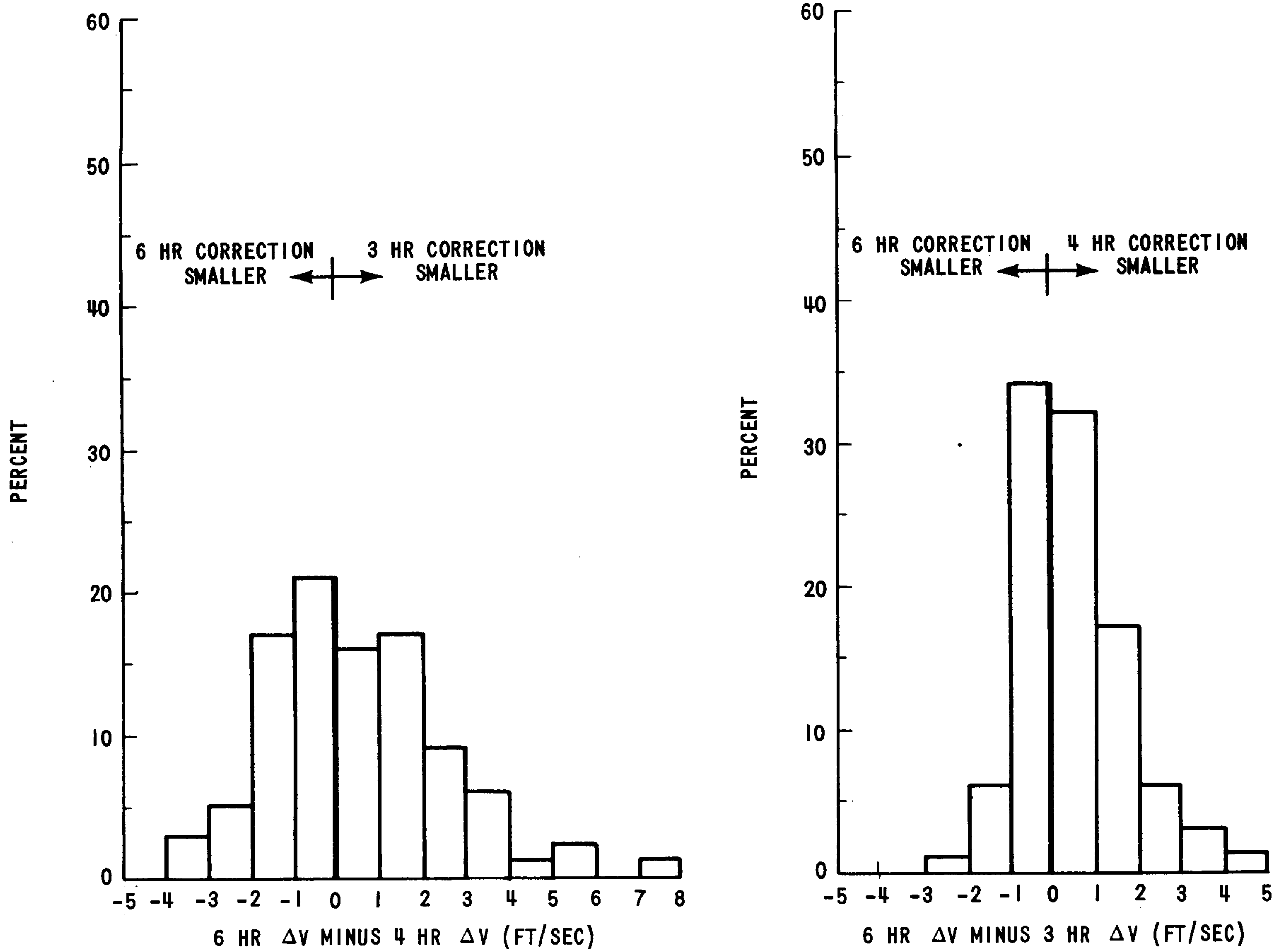

FIGURE 5 - HISTOGRAMS OF DIFFERENCE BETWEEN THE TLI + 6 HR MIDCOURSE MAGNITUDE AND MAGNITUDES FOR VARIOUS TIMES PAST TLI.

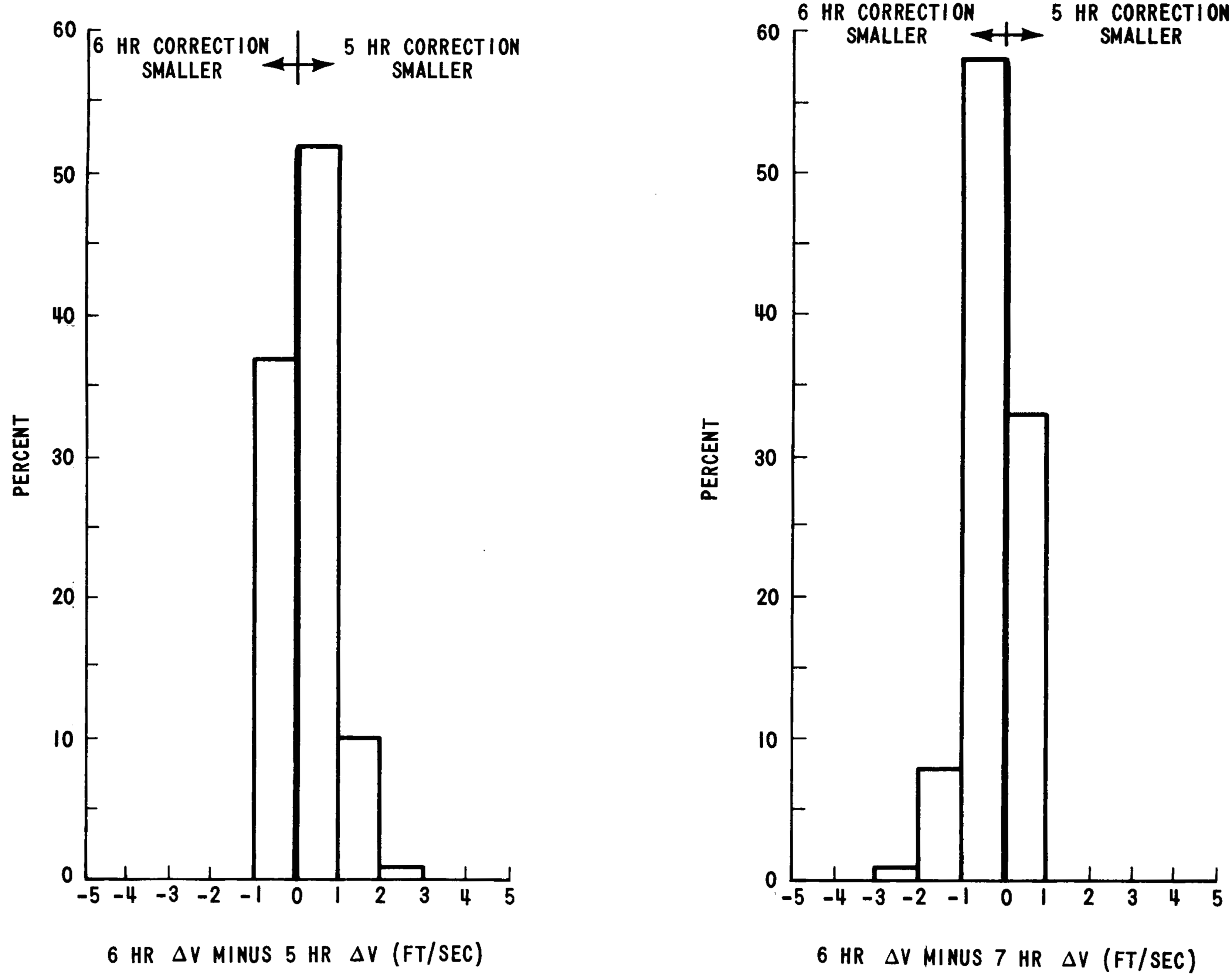

FIGURE 5 (CONTINUED) - HISTOGRAMS OF DIFFERENCE BETWEEN THE TLI + 6 HR. MIDCOURSE MAGNITUDE AND MAGNITUDES FOR VARIOUS TIMES PAST TLI

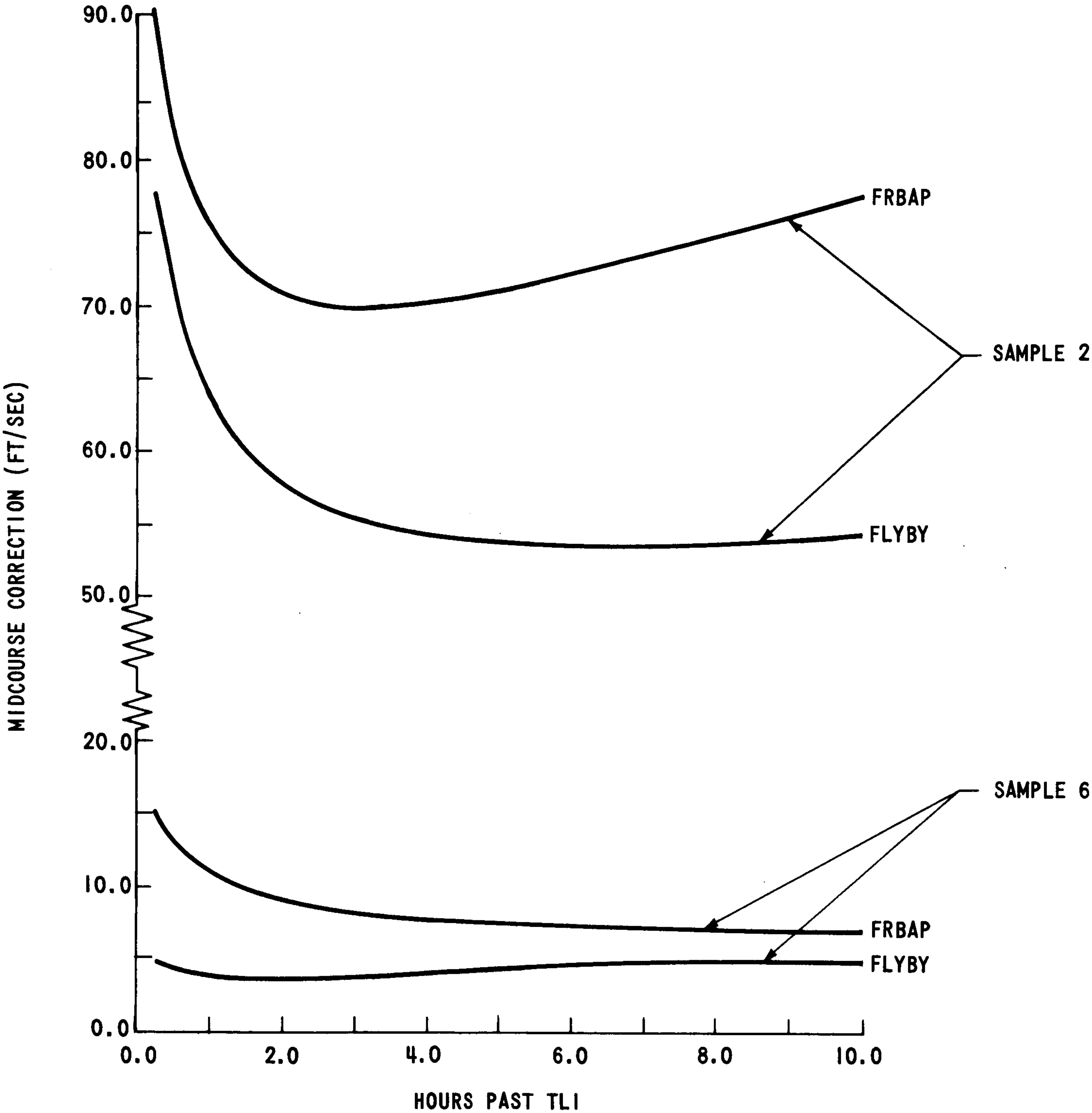

FIGURE 6 - MIDCOURSE CORRECTION VS. TIME PAST TLI FOR FREE RETURN BEST ADAPTIVE PATH TARGETING AND MINIMUM FUEL FLYBY TARGETING (TLI SAMPLES NO. 2 AND NO. 6)

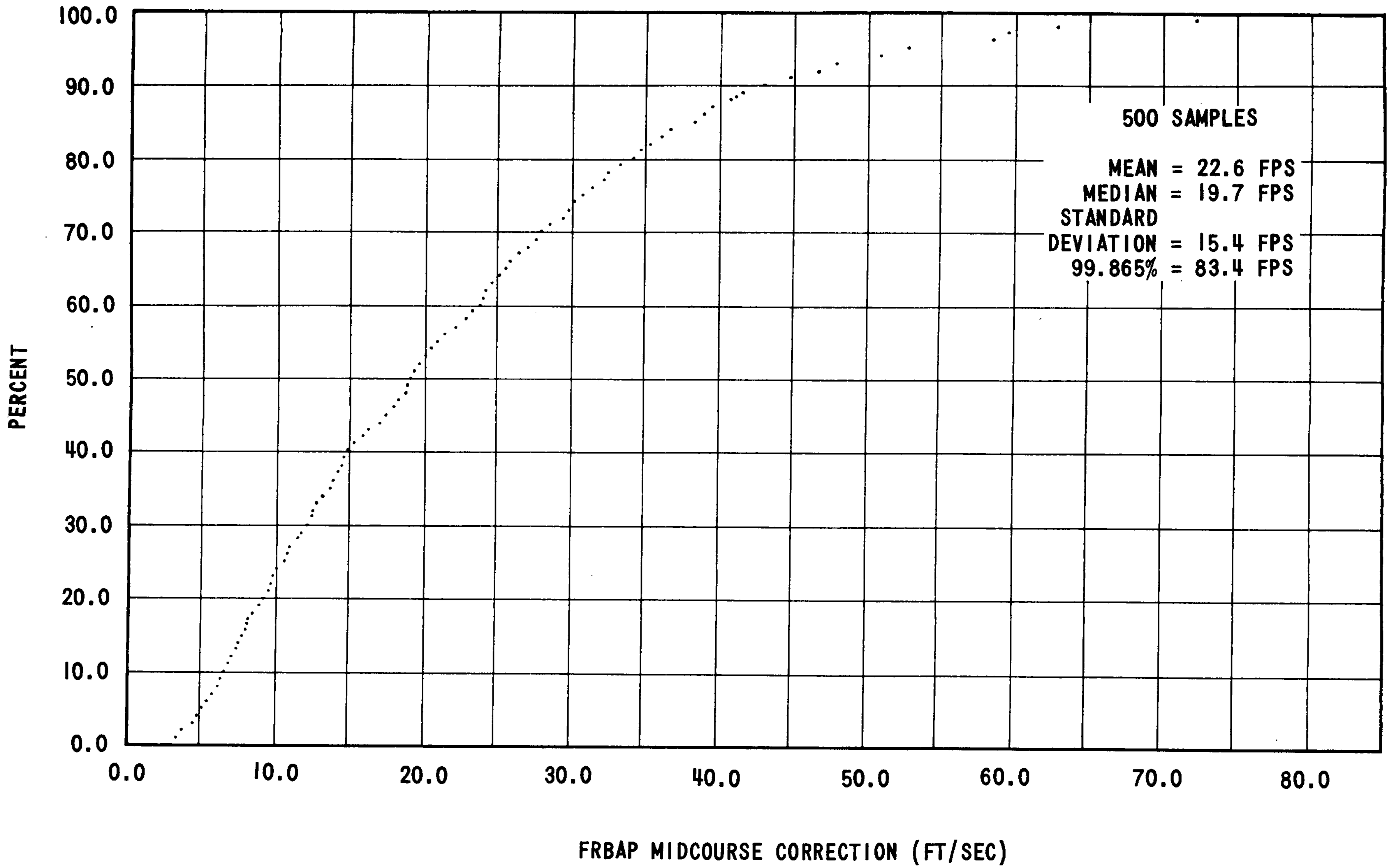

FIGURE 7 - CUMULATIVE DISTRIBUTION OF MIDCOURSE CORRECTIONS EXPECTED AT TLI = 6 HR. - FRBAP TARGETING

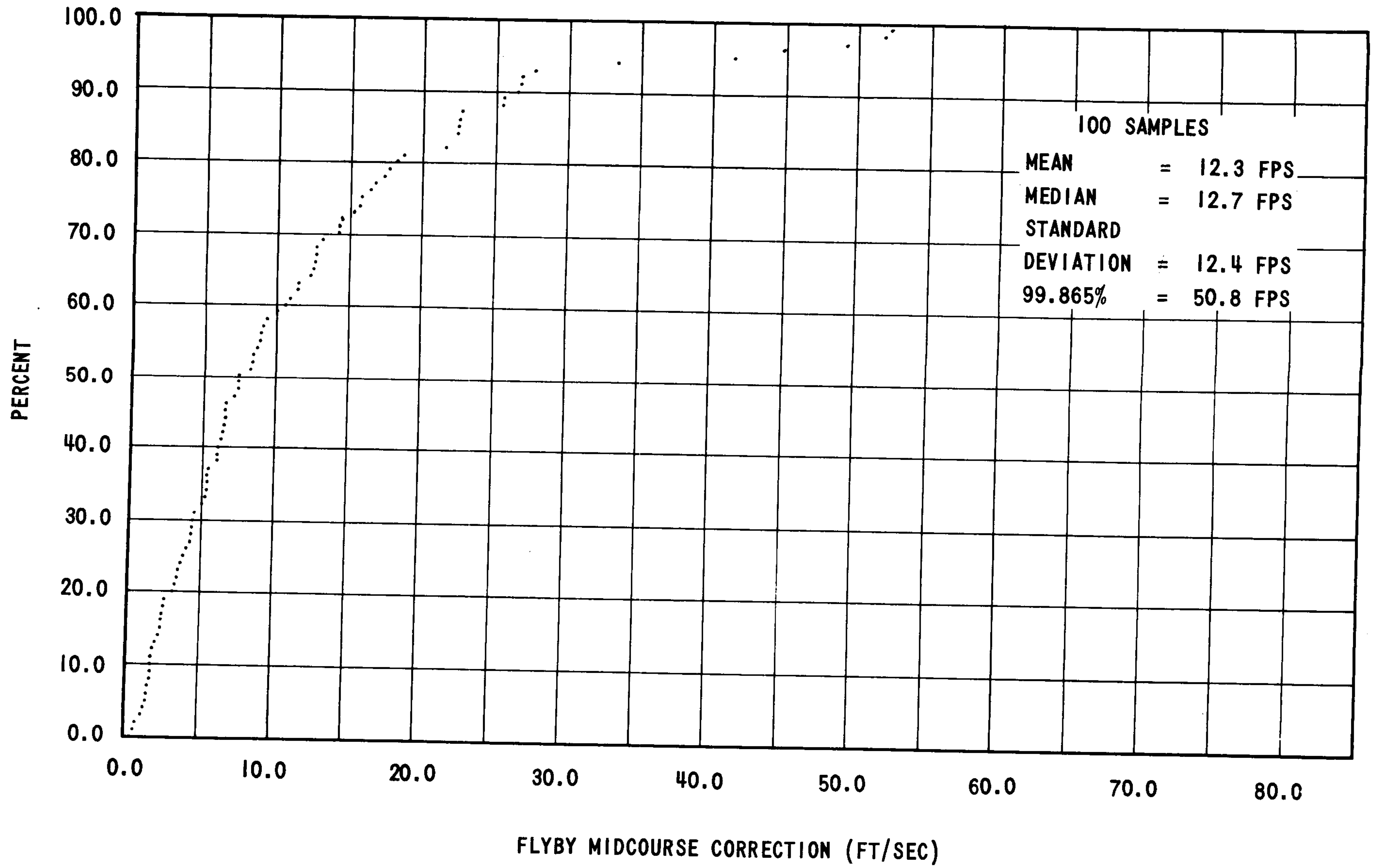

FIGURE 8 - CUMULATIVE DISTRIBUTION OF MIDCOURSE CORRECTIONS EXPECTED AT TLI + 6.5 HR. - MINIMUM FUEL FREE RETURN TARGETING