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TITLE- Orbit Determination and Prediction for Apollo 11 LPO Using POLAR

TM- 69-2014-10

DATE- December 19, 1969

FILING CASE NO(S)- 310

AUTHOR(S)- M. V. Bullock
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FILING SUBJECT(S) (ASSIGNED BY AUTHOR(S))- Orbit Determination

ABSTRACT

The effectiveness of the POLAR version of the Osculating Lunar Elements Program (OLEP) is investigated using Doppler tracking data from the Apollo 11 mission. Free flight data and data containing propulsive maneuvers are utilized.

The processing of free flight data shows the effectiveness of the POLAR method. Two pass Doppler predictions based on two pass regressions exhibit essentially no error growth. Doppler errors associated with solutions derived from more than two passes of data are about the same size as those of the two pass solutions. These characteristics demonstrate the satisfactory nature of the linear approximation to the orbital parameters.

Processing of data containing propulsive maneuvers indicates the impossibility of performing meaningful orbit determination from such data. Residual growth characterizes such solutions.

A local vertical comparison of POLAR solutions shows a consistency in the size of the offsets in each direction. Out-of-plane dispersions exceed in-plane dispersions by an order of magnitude, and standard deviations of the planar elements I and Ω corroborate the out-of-plane insensitivity seen in the local vertical results.

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SUBJECT: Orbit Determination and Prediction for Apollo 11 LPO Using POLAR
Case 310

DATE: December 19, 1969

FROM: M. V. Bullock
A. J. Ferrari

TM- 69-2014-10

TECHNICAL MEMORANDUM

I. INTRODUCTION

The orbit determination and prediction difficulties experienced during the Lunar Parking Orbit (LPO) of the Apollo 11 mission give evidence that some problems in lunar navigation still do exist. Post flight analysis of Doppler tracking data from this period using unconventional techniques is of special interest since new insights may be gained.

The first twenty five passes (post LOI-2 circularization) of Command Service Module (CSM) tracking data acquired by the Manned Space Flight Network (MSFN) and Deep Space Network (DSN) were processed using the POLAR version of the Osculating Lunar Elements Program (OLEP).* Solutions were obtained by fitting (in a least squares sense) two consecutive passes of data. The prediction qualities of each solution were analyzed by propagation into the next two consecutive passes of data. Solutions were also obtained by fitting longer intervals of data, and error characteristics of longer prediction periods were considered.

The purposes of this analysis were twofold: to investigate the effectiveness and consistency of the POLAR approach to orbit determination and to study the effects of uncoupled propulsive maneuvers on the solutions obtained.

II. MATHEMATICAL BACKGROUND

In the POLAR concept the free-flight trajectory of a vehicle in lunar orbit is represented by linearly time-varying functions of the low-eccentricity elements,

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*Bullock, M. V. and Ferrari, A. J., "An Analysis of Apollo 10 Tracking Data Utilizing the POLAR Version of the Osculating Lunar Elements Program" TM-69-2014-9, September 30, 1969, Case 310.

a, $e_c = e \cos \omega$, $e_s = e \sin \omega$, I, Ω, $m = M + \omega$. The vehicle trajectory is modeled for this analysis by the following fundamental parameter set:

$$e_c(t) = e_{co} + e_{c1}t$$

$$e_s(t) = e_{so} + e_{s1}t$$

$$I(t) = I_o$$

$$\Omega(t) = \Omega_o$$

$$m(t) = m_o + m_1 t$$

In order to avoid the numerical singularities associated with near equatorial orbits, the low-eccentricity elements are defined in a special selenocentric frame which represents any orbit as a polar orbit. This transformation is accomplished by rotating the initial estimate of the selenocentric state at epoch through two of its associated Euler angles ($\hat{\Omega}$, $\hat{I}$).

During the orbit determination process, estimates are obtained for both the low-eccentricity elements and some of their corresponding linear time-dependent terms. The actual data reduction is accomplished using a classical least squares algorithm which minimizes the square of the error or residual obtained from processing k observations by differentially correcting the n solution parameters $\{\alpha_j\}$ as follows:

$$\Delta\alpha_j = \left[\sum_{i=1}^{k} J^T(t_i)\ W(t_i)\ J(t_i) \right]^{-1} \sum_{i=1}^{k} J^T(t_i)\ W(t_i)\ \Delta\lambda(t_i)$$

where $\Delta\lambda(t_i) = \lambda(t_i) - \hat{\lambda}(t_i)$.

In this expression $\Delta\alpha_j$ is the differential correction vector (n x 1), $J(t_i)$ is a row vector (1 x n) containing the partial derivatives of the observable with respect to the parameters which are to be estimated (evaluated at time t_i), J^T is the transpose of the J vector, $W(t_i)$ is the weighting matrix (1 x 1), $\lambda(t_i)$ is the t_i^{th} observation, and $\hat{\lambda}(t_i)$ is the estimated observable at time t_i.

The processing of k observations and the resulting set of differential corrections $\{\Delta\alpha_j\}$ constitute one computing iteration. The convergence criterion for any two successive iterations is as follows:

$$\frac{\left.\sum_{i=1}^{k} [\Delta\lambda(t_i)]^2\right|_{(I-1)}}{\left.\sum_{i=1}^{k} [\Delta\lambda(t_i)]^2\right|_{(I)}} - 1 < \delta$$

where δ is a small positive number (10^{-4}) and (I - 1) and (I) designate the (I - 1) and (I) computing iterations.

The semi-major axis does not appear in POLAR as an explicit solution parameter. The estimate for the linear portion of the modified anomaly ($m(t) = m_o + m_1 t$) is used to imply a corresponding semi-major axis by using the classical Kepler relationship

$$m_1 = \sqrt{\frac{\mu}{a^3}}$$

or

$$a = \left[\frac{\mu}{m_1{}^2}\right]^{1/3}$$

where μ is the Newtonian constant times the lunar mass.

III. ANALYSIS OF RESULTS

Propulsive maneuvers are not modeled in POLAR, so the orbit determination results are a function of the free flight nature of the data. From a quality point of view the Doppler tracking data can be divided into three categories. The best data is that which is acquired during CSM free flight. Data acquired during uncoupled attitude maneuvers and during CSM water dumps is inferior to free flight data because these events change the orbit slightly, but it is still useable for analysis. Finally, data acquired during CSM ΔV Service Propulsion System (SPS) burns cannot be used in an analysis which does not model burns since the orbit is changed so drastically.

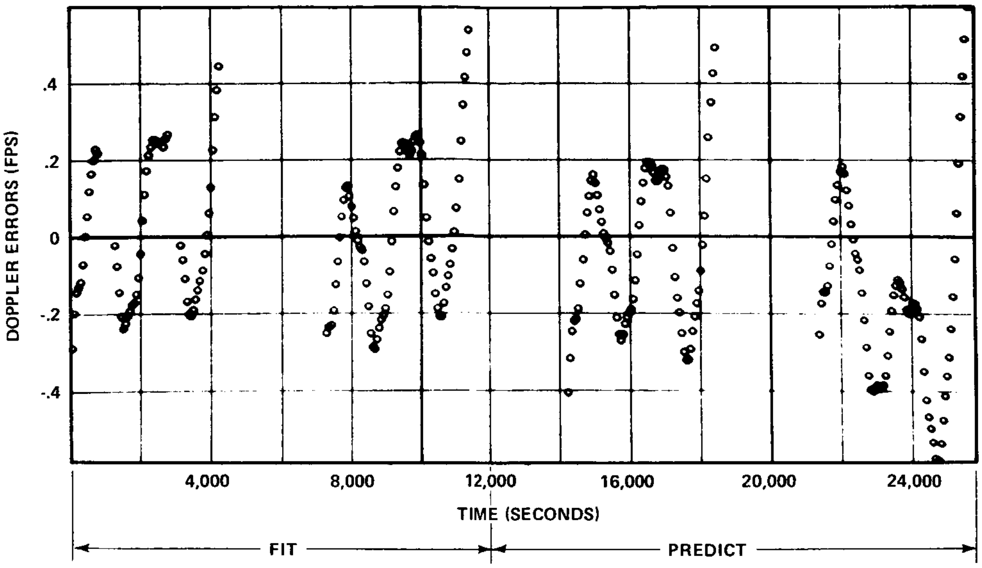
Since CSM uncoupled propulsive maneuvers occurred during almost every revolution in the lunar parking orbit of Apollo 11, no analysis can be made of purely free flight data. But there are some spans of time, with minimal maneuvers, in which the data is of nearly free flight quality. It is with these data that the POLAR approach to orbit determination will be evaluated. Some of the time periods in which the uncoupled maneuvers were more than minimal will be used to show the effects of these maneuvers on the orbit determination.

a. Processing of Near Free Flight Data

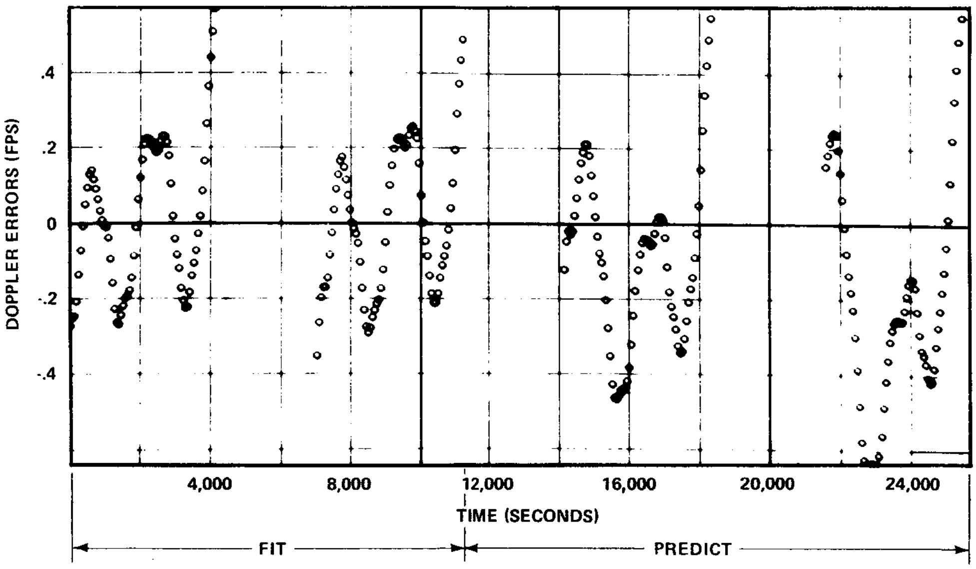
POLAR processing of two passes of near free flight data results in very little growth in the Doppler residuals when the solutions are propagated through two adjacent passes of data. The excellent quality of these solutions is apparent in the representative residual plots in Figure 1. From the point of view of the observable, a very good fit has been achieved. The lack of growth of the residuals in the propagation region indicates that the coefficients of the linear terms have been determined accurately. Short period terms are the only ones that have not been accounted for, and they make a consistent appearance in the residuals.

The POLAR solution process works well when the technique is extended to more than two passes of data. Examples of five and ten pass fits and of a four pass propagation from a two pass fit are shown in Figure 2. The values of the orbital state obtained from the longer arc fits differ very little from the two pass values. Although the peak-to-peak residual values are larger than those for the two pass results (.9 fps and 1.2 fps vs. .8 fps), the error growth is minimal. The statistical characteristics of the results change very

PASSES 5 - 8



PASSES 6 - 9



PASSES 7 - 10

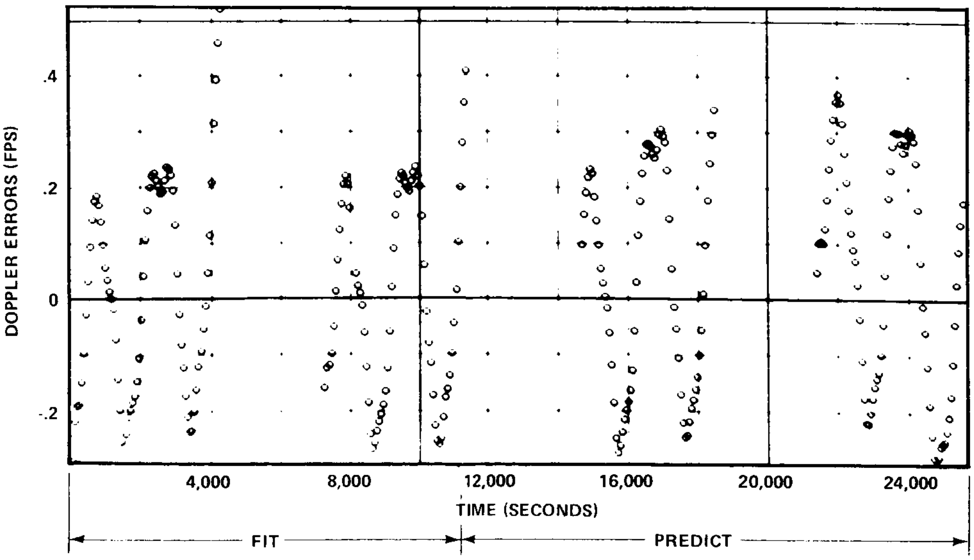


FIGURE 1 – TWO-PASS PROCESSING OF FREE FLIGHT DATA

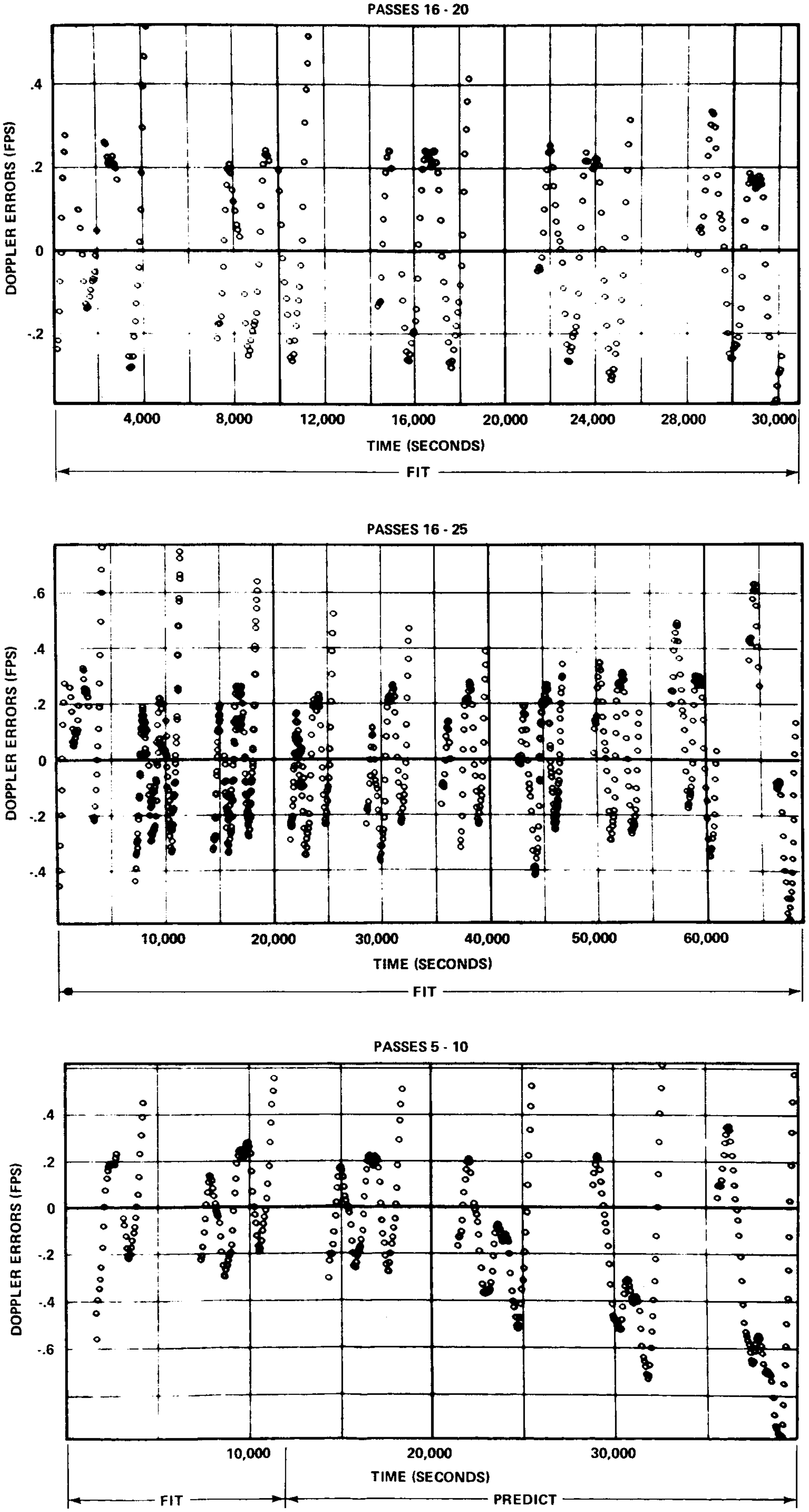


FIGURE 2 – MULTI-PASS PROCESSING OF FREE FLIGHT DATA

little. The two pass second moment and standard deviation are .0361 and .189, compared with .0373 and .193 for five passes and .0516 and .227 for ten. The fact that reasonable solutions are obtainable over longer periods of time indicates that the linear approximation to the osculating elements is a satisfactory one. If this model were not adequate, this fact would be manifested in residual growth and changes in the epoch values. Extending the propagation region to cover two more passes results in only a 30% growth in the peak-to-peak residual value. Some of this growth can be attributed to the propulsive vents that are present in the data.

b. Processing of Data Containing Propulsive Maneuvers

Figure 3 shows results using POLAR on data that has been corrupted by propulsive maneuvers. Propagation of the regression obtained from passes 8 and 9 is a striking example of the change that uncoupled maneuvers make in the orbit of the spacecraft. Pass 11 contains a platform alignment, and the solution obtained from the free-flight data of passes 8 and 9 is not an accurate description of the orbit after the maneuver.

An attempt to fit and propagate through maneuvers can be seen in the regression obtained from passes 11 and 12. Each of the passes (11, 12, 13, and 14) contains a maneuver, as indicated in the figure, and thus the orbit is different in every pass. In this case there are two problems: first, the data in the regression region is corrupted and does not provide an accurate estimate of the orbit; second, maneuvers in the propagation region make a good prediction based on previous data impossible.

IV. LOCAL SOLUTION COMPARISON

The consistency of POLAR solutions can be investigated by a comparison in a local vertical coordinate frame (U-radial, V-down-range, W-out-of-plane) among two pass propagation zone solutions and the corresponding two pass local solutions. Results are presented below from three pre-DOI and three post-DOI solution comparisons. (^ indicates propagation zone.) Passes with minimal maneuvers were chosen.

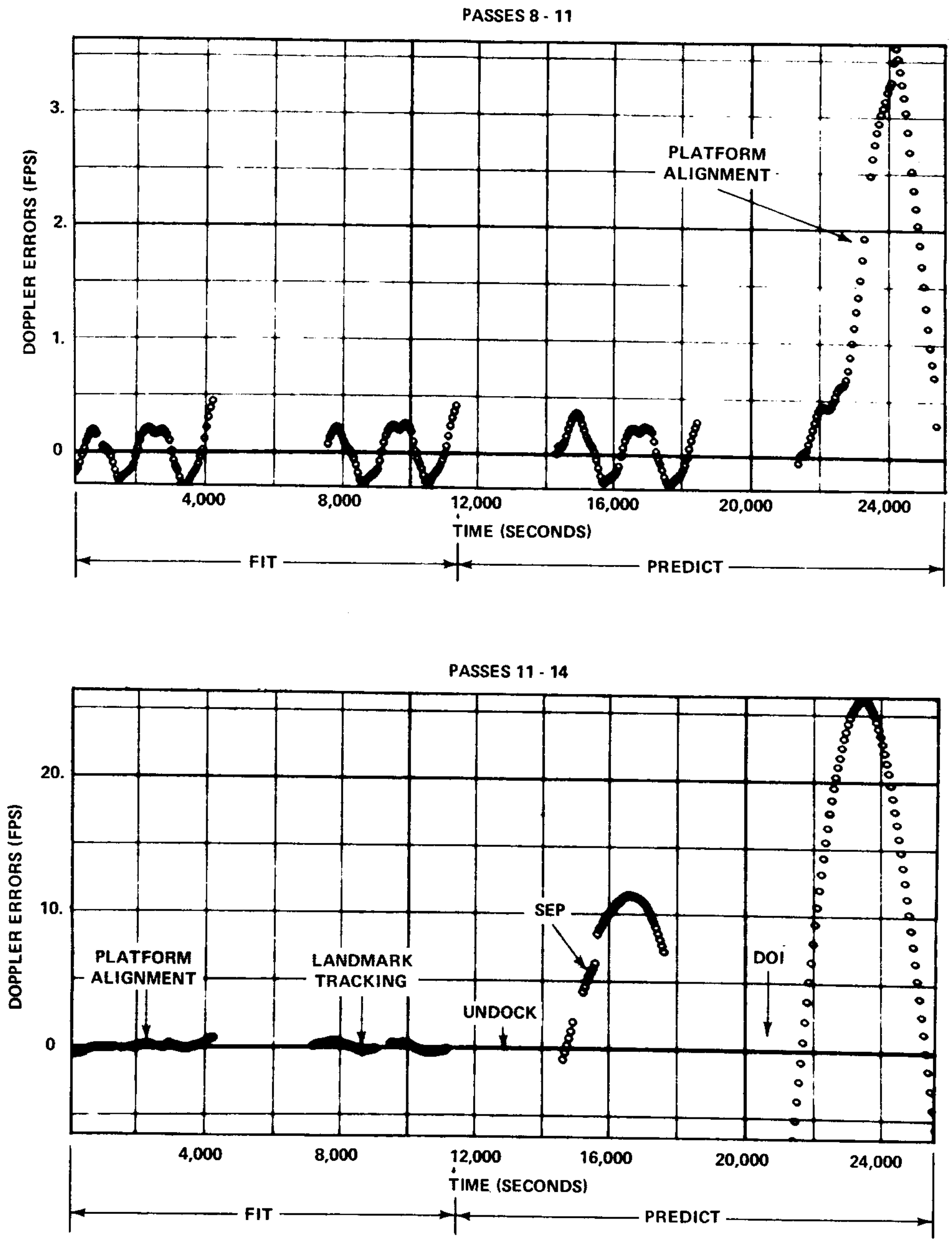


FIGURE 3 – PROCESSING OF DATA CONTAINING PROPULSIVE MANEUVERS

Passes	$\lvert \delta U_{max} \rvert$ (ft)	$\lvert \delta V_{max} \rvert$ (ft)	$\lvert \delta W_{max} \rvert$ (ft)
$\hat{5} - \hat{6}$ vs. $5 - 6$	175	750	3700
$\hat{6} - \hat{7}$ vs. $6 - 7$	160	750	6500
$\hat{7} - \hat{8}$ vs. $7 - 8$	105	390	6800
$\hat{19} - \hat{20}$ vs. $19 - 20$	150	400	4000
$\hat{20} - \hat{21}$ vs. $20 - 21$	180	350	3700
$\hat{21} - \hat{22}$ vs. $21 - 22$	200	350	3700

The similarity in size of the dispersions in each direction indicates the consistency of POLAR solutions. In all cases the in-plane errors are relatively small and the out-of-plane dispersions are an order of magnitude larger than the in-plane dispersions. The large out-of-plane errors could be a result of the uncoupled maneuvers or of a relative insensitivity of the Doppler data type to the planar elements Ω and I. In every case presented in the table a good fit, which extrapolated with very little growth in the residuals, was obtained.

The standard deviations of the solution parameters corroborate the planar insensitivity that is seen in the local vertical dispersions. The table below presents the converged values of the angular parameters and their associated standard deviations for a typical two pass fit.

Parameter	Converged Value	Standard Deviation
I_o	$89\overset{\circ}{.}986$	$0\overset{\circ}{.}1601$
Ω_o	$179\overset{\circ}{.}888$	$0\overset{\circ}{.}0665$
m_o	$251\overset{\circ}{.}509$	$0\overset{\circ}{.}3584 \times 10^{-2}$
m_1	0.05058 deg/sec	$.3331 \times 10^{-6}$ deg/sec

The in-plane quantities m_o and m_1 are determined with much more certainty than the out-of-plane angles I_o and Ω_o. Tests have shown that σ_I is reduced when more passes of data are included in the fit, when more stations are used, and when optimum tracking station geometry (good north-south and east-west coverage) is available.

V. CONCLUSIONS

Orbit determination and prediction for the Apollo Lunar Parking Orbit have been investigated from the vantage point of an unconventional technique. Apollo 11 Doppler tracking data was analyzed using the POLAR orbit determination program with the following fundamental parameter set: e_{so}, e_{s1}, e_{co}, e_{c1}, I_o, Ω_o, m_o, m_1.

Processing of free flight data demonstrated the effectiveness of the POLAR approach. In every case propagation of two pass regressions resulted in very little growth in the Doppler residuals. Extending the fit region to cover more than two passes did not cause any substantial increase in the peak-to-peak residual values. The good prediction attained and the ability to achieve longer arc fits indicated that the linear approximation to the osculating orbital elements is a satisfactory assumption.

Processing of data containing propulsive maneuvers showed the impossibility of obtaining meaningful solutions from corrupted data. Such solutions gave very large propagation residuals. Difficulties were also experienced when a good solution was propagated through a maneuver, since the orbit was changed by the maneuver.

A local vertical coordinate comparison of representative POLAR solutions established the consistency of results obtained from free-flight data. Out-of-plane dispersions exceeded in-plane dispersions by an order of magnitude. Standard deviations of solution parameters substantiated the planar insensitivity shown by the local vertical analysis.

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