MSC INTERNAL NOTE NO. 64-FM-74

PROJECT GEMINI

LOGIC AND EQUATIONS FOR REAL-TIME COMPUTATION OF ARRIVAL TIME AT AN EQUATORIAL CROSSING, A SPECIFIED LONGITUDE, A SPECIFIED RADIUS, AND APSIS POINTS

Prepared by: Thomas Buckman

Prepared by: Stan Mann

Approved: Edgar C. Lineberry
Chief, Rendezvous Analysis Branch

Approved: John P. Mayer
Chief, Mission Planning and Analysis Division

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MANNED SPACECRAFT CENTER

HOUSTON, TEXAS
December 14, 1964

TABLE OF CONTENTS

1. Introduction

The purpose of this internal note is to discuss the development of four real-time computer program subroutines which are used in the General Purpose Maneuver Processor (GPMP) and the Agena Corrective Maneuver Processor (ACMP). The subroutines discussed herein are:

- STLO - Determines the next time of arrival of an orbiting vehicle at some specific input longitude
- STEQ - Determines the time of arrival of an orbiting vehicle at the next equatorial crossing point
- STCIR - Determines the next time of arrival of an orbiting vehicle at some specific input radius
- STAP - Determines the time of arrival of an orbiting vehicle at the next true apogee or perigee

These subroutines form a part of the ACM and GPM Processors and are utilized to establish the time at which certain maneuvers should be scheduled.

2. Discussion of Subroutine STLO

Subroutine STLO requires the input of a set of vehicle orbital elements for a given time and the desired longitude (λ). Since the longitude of ascending node (h_I) of the input vector will be based on an inertial reference, it must be changed to an earth fixed reference. This is accomplished by subtracting the earth rotation ($\omega_e t$). The distance along the equator (ν) between the present earth fixed ascending node (h_e) and the desired longitude (λ_x) is found by subtracting (h_e) from (λ_x). A negative distance along the equator would be meaningless here, so 2π must be added to the above result if ($\lambda_x - h_e$) is negative. The argument of latitude of the desired longitude crossing (U_L) is computed with the use of a spherical triangle that includes the inclination (I_I) and the proper equatorial distance (ν). An initial time of arrival estimate (Δt) is made by applying the vehicle mean motion (n) to the difference between the argument of latitude of the desired longitude (U_L) and the present argument of latitude of the vehicle (U_I). However, this time estimate will not be entirely correct because the earth rotational rate will affect the actual time of travel between the present position and the desired longitude. This discrepancy is initially corrected by adding the earth rotation ($\omega_e \Delta t$) to the desired longitude (λ). A more accurate time estimate is then computed using the "adjusted" longitude (λ_Z). When the longitude has been properly adjusted to account for earth rotation, the AEG Subroutine is called and the initial vector is updated to a time equal to the initial time plus the latest Δt estimate. The argument of latitude of the vehicle at this time is checked against the argument of latitude of the adjusted input longitude. If the difference between these two parameters indicates a time difference greater than an input tolerance, a new Δt is computed and the AEG is called again. This iteration continues until a convergence on U_L is attained. The time that permits this convergence will then be the desired time of arrival at the specific longitude (λ) and is denoted as t_{LON}.

3. Discussion of Subroutine STEQ

Subroutine STEQ begins the search for the next equatorial crossing with a set of vehicle orbital elements for a given time. The value of the vehicle argument of latitude (U_I) at this time is checked against π to determine if the next equatorial crossing will be the ascending or descending mode. If the upcoming node is ascending, the argument of latitude of the next crossing point will equal 2π. If the upcoming node is descending, the desired argument of latitude will equal π . A first estimate for the time of arrival is made by applying the vehicle mean motion (n_I) to the difference between the argument of latitude of the vehicle and the argument of latitude of the next crossing point. The later parameter is denoted by the value K_{110} in the flow charts. This time estimate is used as the initial estimate in an iteration loop using the AEG Subroutine. The AEG is called and the vector is updated to the time estimate. If the vehicle argument of latitude at this time is not within a pre-set tolerance of the value K_{110}, a new time estimate is computed and the iteration loop is entered again. The time corresponding to the argument of latitude that permits a convergence in the iteration loop will then be the desired time of equatorial crossing (t_{EQ}).

4. Discussion of Subroutine STCIR

Subroutine STCIR requires the input of a set of vehicle orbital elements for a given time and the desired radius of circularization (R_{CIR}). This subroutine begins its search for R_{CIR} at the time of the input elements. An initial calculation of the true anomaly of the vehicle at the desired radius is made using the following Keplerian relationship:

$$f_c = \cos^{-1} \left(\frac{a(1 - e^2) - R_{CIR}}{e \times R_{CIR}}\right) \tag{1}$$

This true anomaly (f_c) is compared to the true anomaly of the present position (f_I) and an initial time of arrival estimate is computed by applying the mean motion to the difference between these parameters. Since the computed true anomaly is not a unique solution due to the existence of two R_{CIR} positions in the orbit, a further test must be performed to insure the use of the next desired radius point. The AEG is updated to the initial time estimate, and the true anomaly of the resulting vector is noted. A more accurate computation of the desired true anomaly is made using equation (1) with the updated elements. This computed true anomaly is again compared with the actual true anomaly at the updated position and if the difference between them is greater than a pre-set tolerance, a new time estimate is calculated and entered in the iteration loop. In the course of iterating on the proper true anomaly, it is possible for a radius (R_{CIR}) that initially exists in the orbit to become non-existent in the present orbit due to drag. The presence of this situation is revealed when the value of cos (f_c) becomes greater than one. Should this situation occur the subroutine sets an error flag (K_{104}) and returns to the point where it was called. The time estimate that produces convergence on the computed true anomaly will be the proper time of arrival at the input radius and is denoted as t_{CIR}.

5. Discussion of Subroutine STAP

The STAP Subroutine requires the input of a set of vehicle orbital elements for some time and a K_{100} flag which determines which Apsis point is desired (K_{100} = 0, Apogee -- K_{100} = 1, Perigee). A first estimate of the time of arrival at the next desired apsis point is made by comparing the present mean anomaly to either π or 2π, depending on the K_{100} flag setting. If the apsis point desired is apogee, a check is made to determine if the apogee point has already been passed in the present orbit. If this is the case, then the present mean anomaly is compared to 3π rather than π to obtain the proper time estimate. The AEG is called and the present vector is updated to the initial time estimate. The mean anomaly at the updated position is then compared to the proper multiple of π (Apogee -- π, Perigee -- 2π). If the difference between these values is greater than a pre-set tolerance, than a new time estimate is calculated. This updated time estimate is then re-entered in the iteration loop. The time that permits convergence at the proper mean anomaly will be the correct time of arrival at the apsis and will be denoted as t_{AP}.

6. Required Inputs

The input that is required for each subroutine is divided into two major categories: constant input and variable input. The constant input is also subdivided further into three groups: 1) input that will not change after three weeks prior to launch, 2) input from three weeks prior until the program is loaded, and 3) input required after the launch.

This input is listed in the following sections:

3.1 STLO

3.2 STEQ

3.3 STCIR

3.4 STAP

3.5 AEG

3.1 Input for STLO

I. Constant Input

a. Category

1. π, ω_e, K_{101}, K_{102}, K_{103}

2. $\delta\theta$, δt

3. None

II. Variable Input

λ, t, h, U, $\dot{g}$, n

III. Output

t_{LON}, a, e, I, g, h, ℓ, R, U, $\dot{g}$, n

IV. Constant and Variable Input Required for:

AEG/DRAG

3.2 Input for STEQ

I. Constant Input

a. Category

1. π

2. δt

3. None

II. Variable Input

t, $\acute{g}$, u, n

III. Output

t_{EQ}, a, e, I, g, h, ℓ, R, I"

IV. Constant and Variable Input Required for:

AEG/DRAG

3.3 Input for STCIR

I. Constant Input

a. Category

1. π, K_{104}

2. δt

3. None

II. Variable Input

R_{CIR}, t, g, u, $\dot{g}$, n

III. Output

t_{CIR}, a, e, I, g, h, ℓ, R, u

IV. Constant and Variable Input Required for:

AEG/DRAG

3.4 Input for STAP

I. Constant Input

a. Category

1. π

2. δt

3. None

II. Variable Input

K_{100}, t, ℓ, $\dot{g}$, n

III. Output

t_{AP}, a, e, I, g, h, ℓ, R, u, $\hat{g}$, n

IV. Constant and Variable Input Required for:

AEG/DRAG

3.5 INPUT FOR AEG

I. CONSTANT INPUT

a. Category

1. π, u, ω_e, J_c, H_c, K_c, R_e, R_{POLE}, C_{50}, Δt_o, $\rho j (j = 1, 12)$, ϵ_1, K_9

2. A_L, C_D

3. None

II. VARIABLE INPUT

K_1, K_2, K_3, L_1, L_2, W_L, t, a, e, I, h, h, l

III. OUTPUT

a_f, e_f, I_f, g_f, h_f, l_f, n_f, $\dot{g}$, $\dot{h}$, r, u, $\dot{g}_D$, $\dot{r}$, h", I"

IV. CONSTANT AND VARIABLE INPUT REQUIRED FOR:

DRAG - π, u, ω_e, J_c, H_c, K_c, R_e, R_{POLE}, C_{50}, Δt_o, $\rho j (j = 1, 12)$, ϵ_1, K_9, A_L, C_D, a, e, g, (esing), (ecosg), L, u, W_L, n, g, γ_2', C_1, C_2, C_7, C_{30}, β, t_D, e", L", a"

7. Subroutine Outputs

The outputs for the subject subroutines are as follows:

STLO - t_{LON}, a, e, I, g, h, ℓ, R, U, $\dot{g}$, n

STEQ - t_{EQ}, a, e, I, g, h, ℓ, R, I"

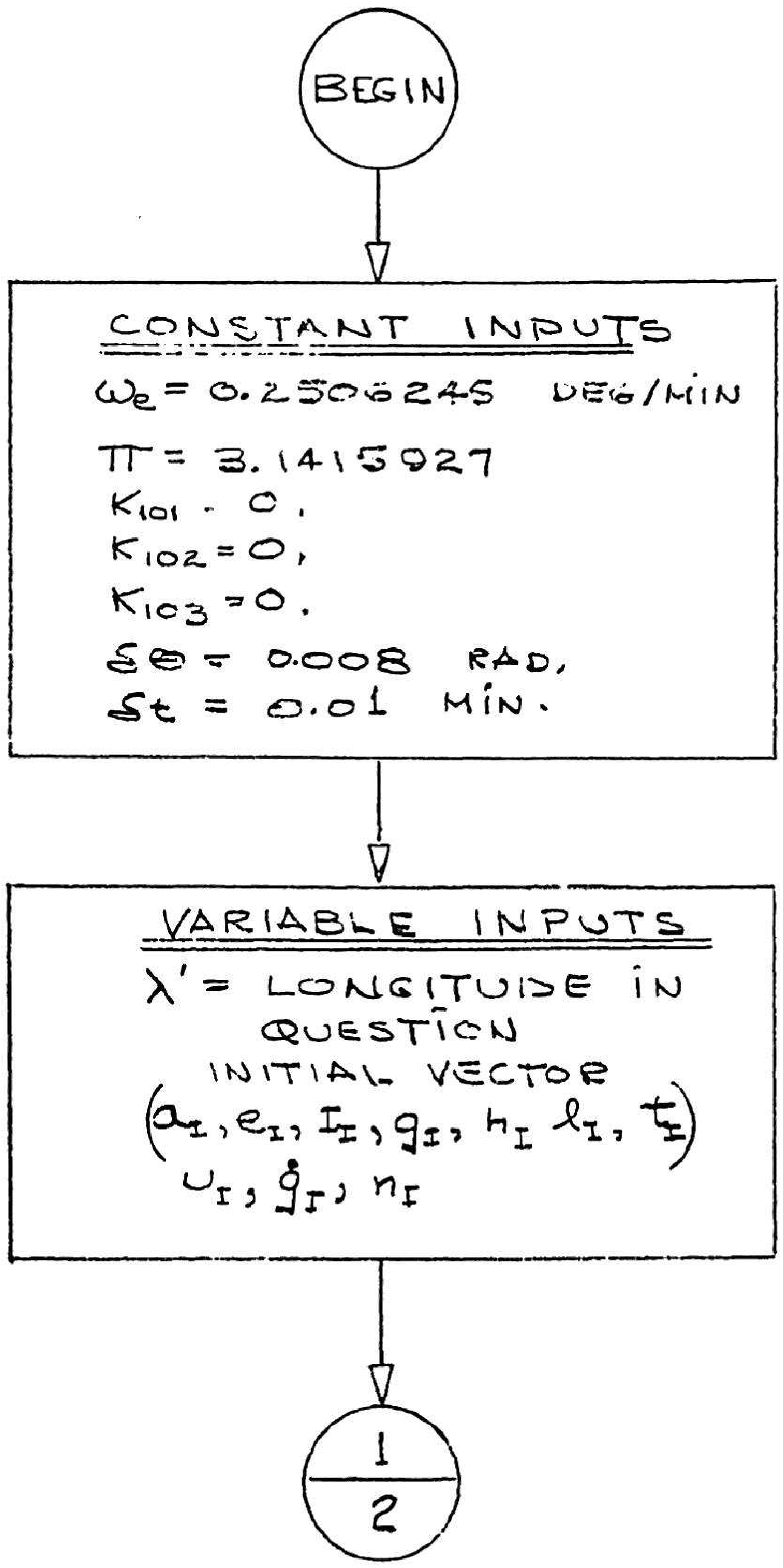
STCIR - t_{CIR}, a, e, I, g, h, ℓ, R, U

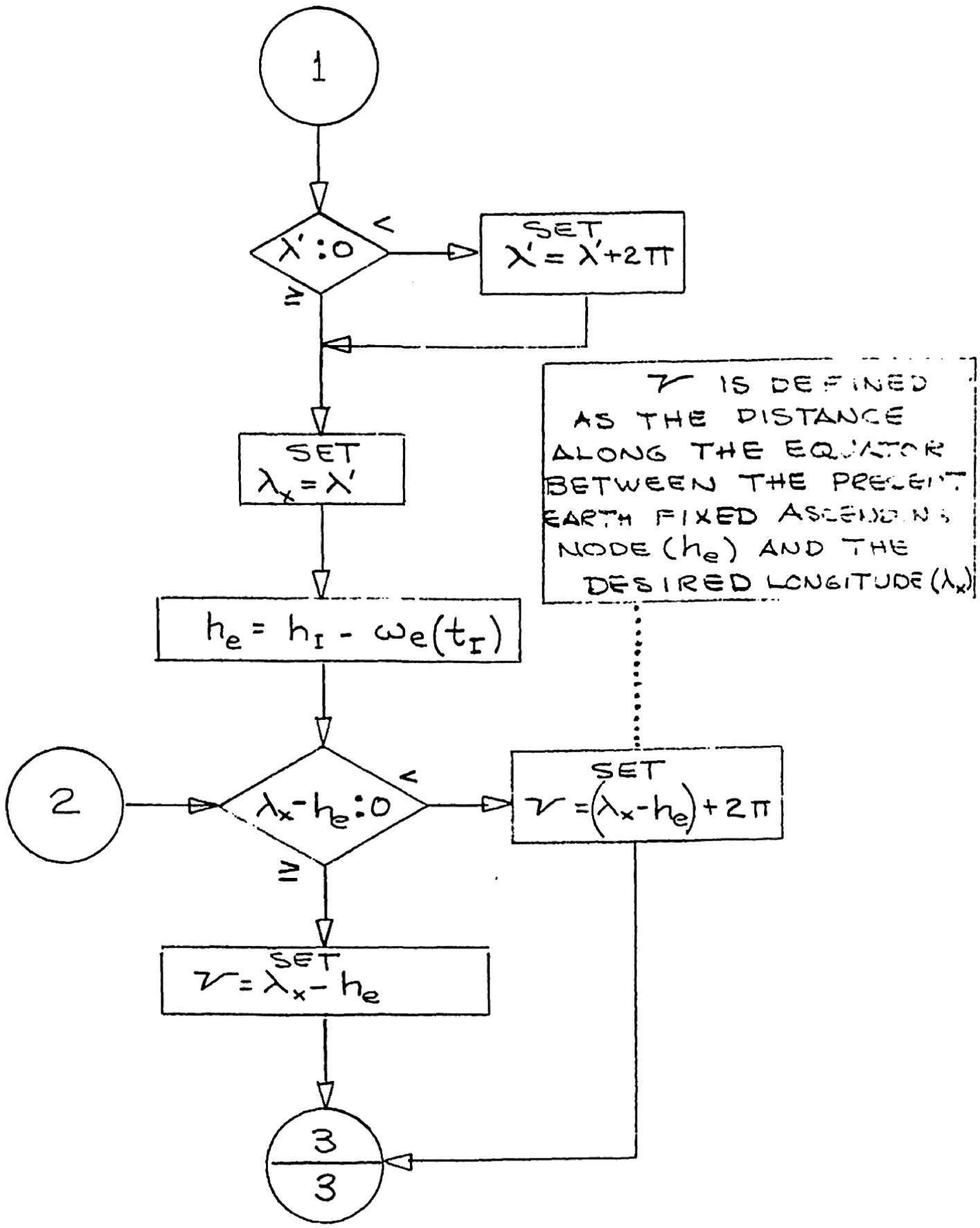
STAP - t_{AP}, a, e, I, g, h, ℓ, R, U, $\dot{g}$, n

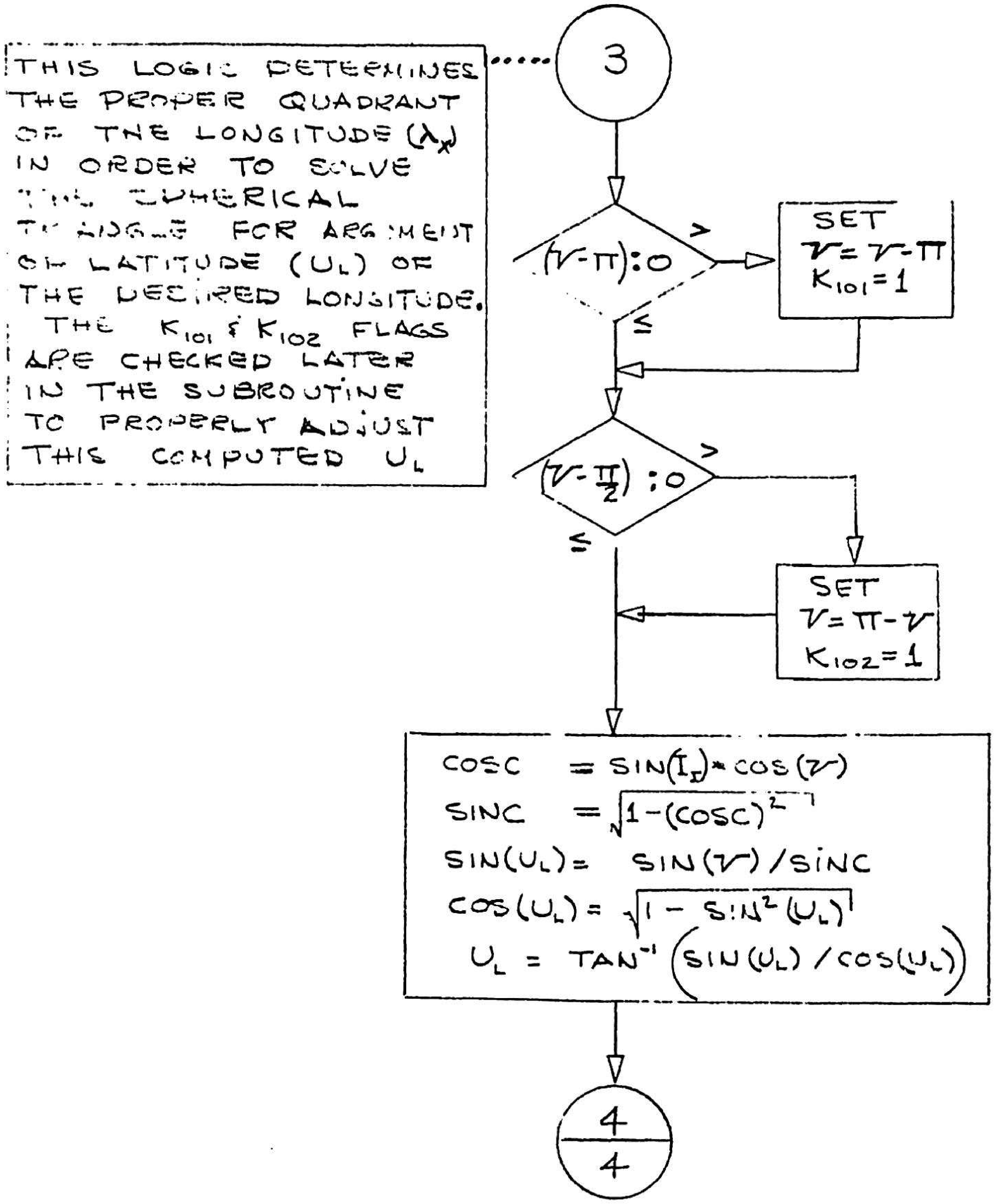
APPENDIX I

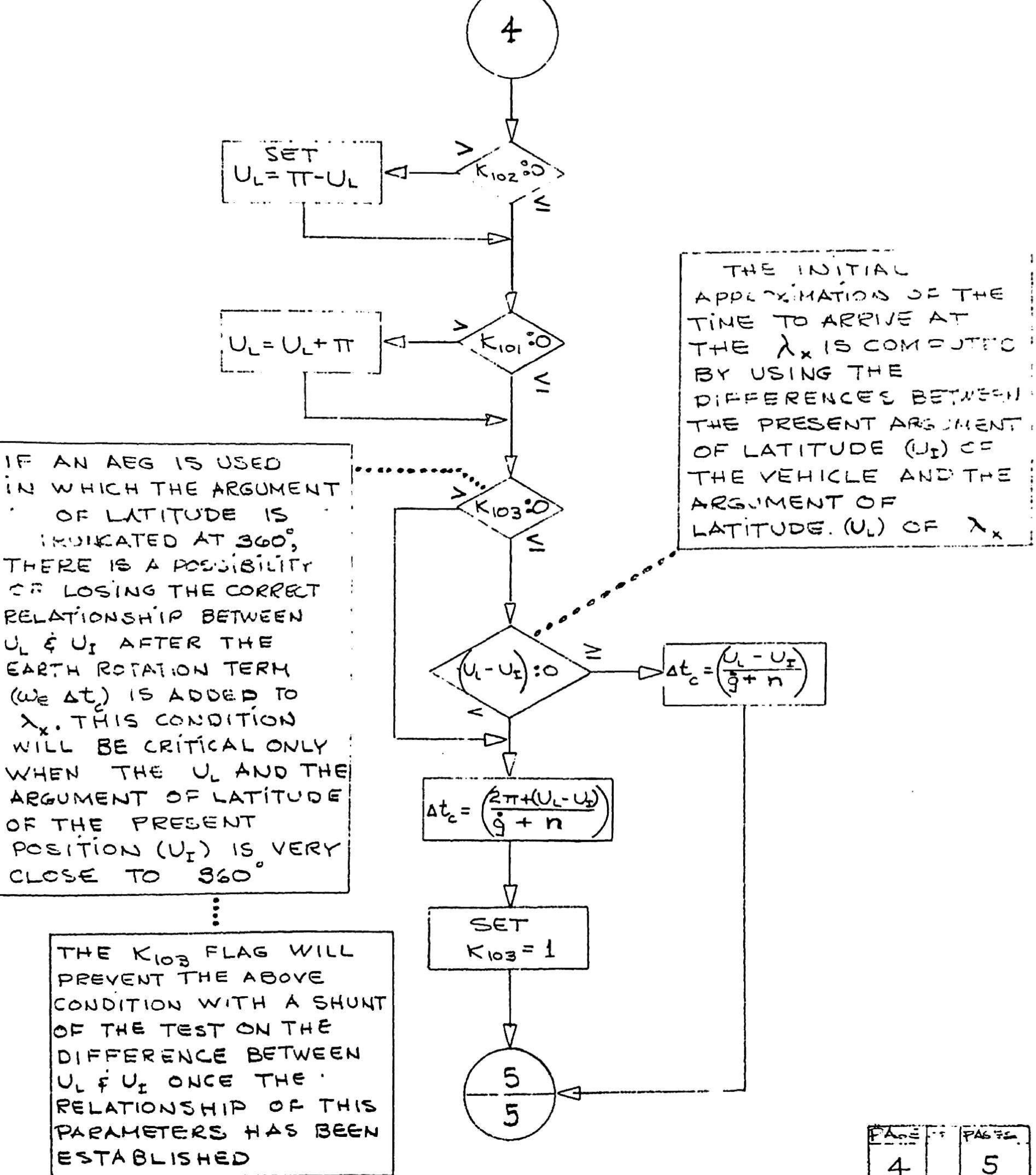
Detailed Flow Charts

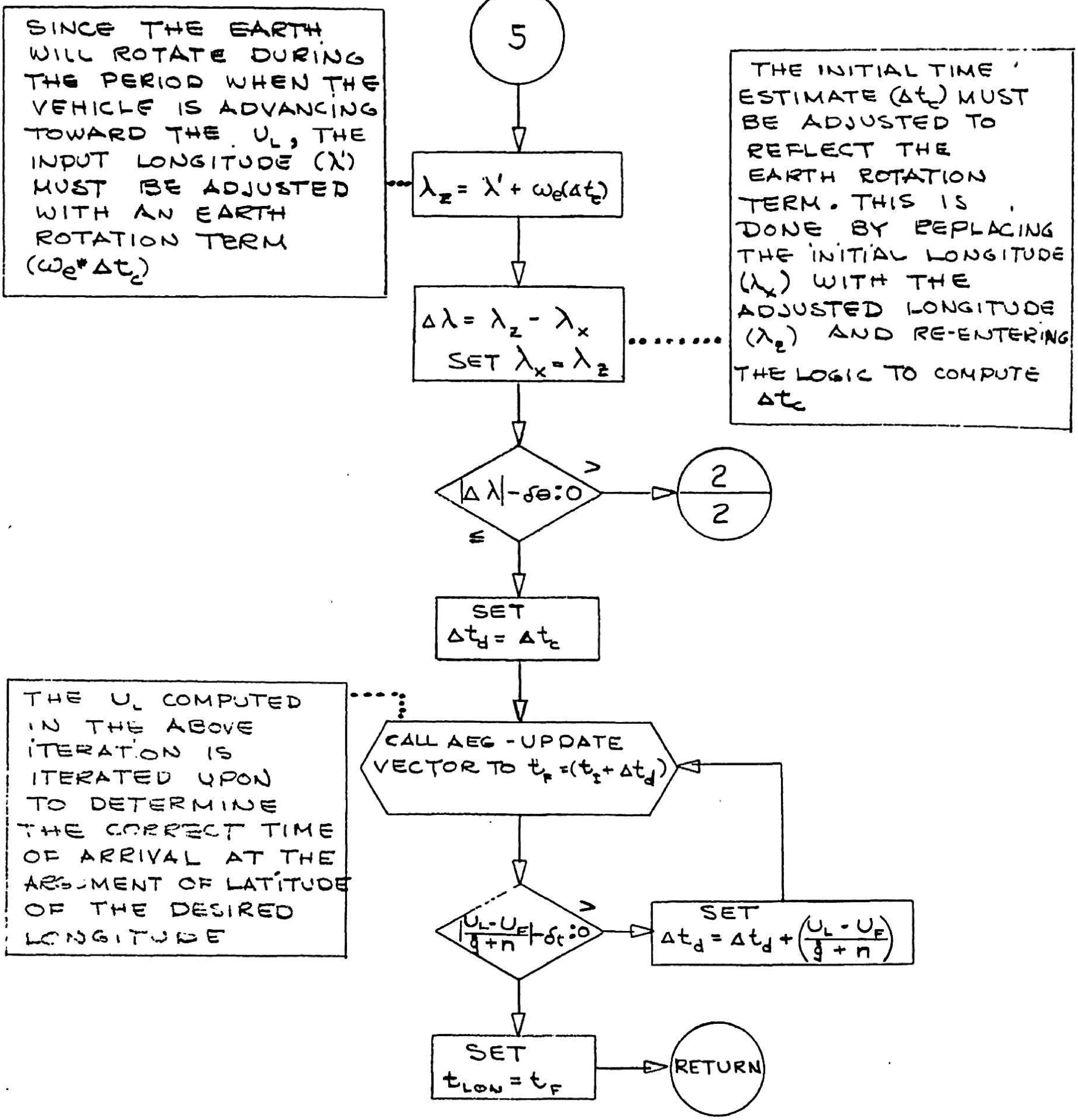
SUBROUTINE STLO




1
λ' : 0
<
≥
SET λ' = λ' + 2π
SET λ_x = λ'
ν IS DEFINED AS THE DISTANCE ALONG THE EQUATOR BETWEEN THE PRESENT EARTH FIXED ASCENDING NODE (h_e) AND THE DESIRED LONGITUDE (λ_x)
h_e = h_I - ω_e(t_I)
2
λ_x - h_e : 0
<
≥
SET ν = (λ_x - h_e) + 2π
SET ν = λ_x - h_e
3
3

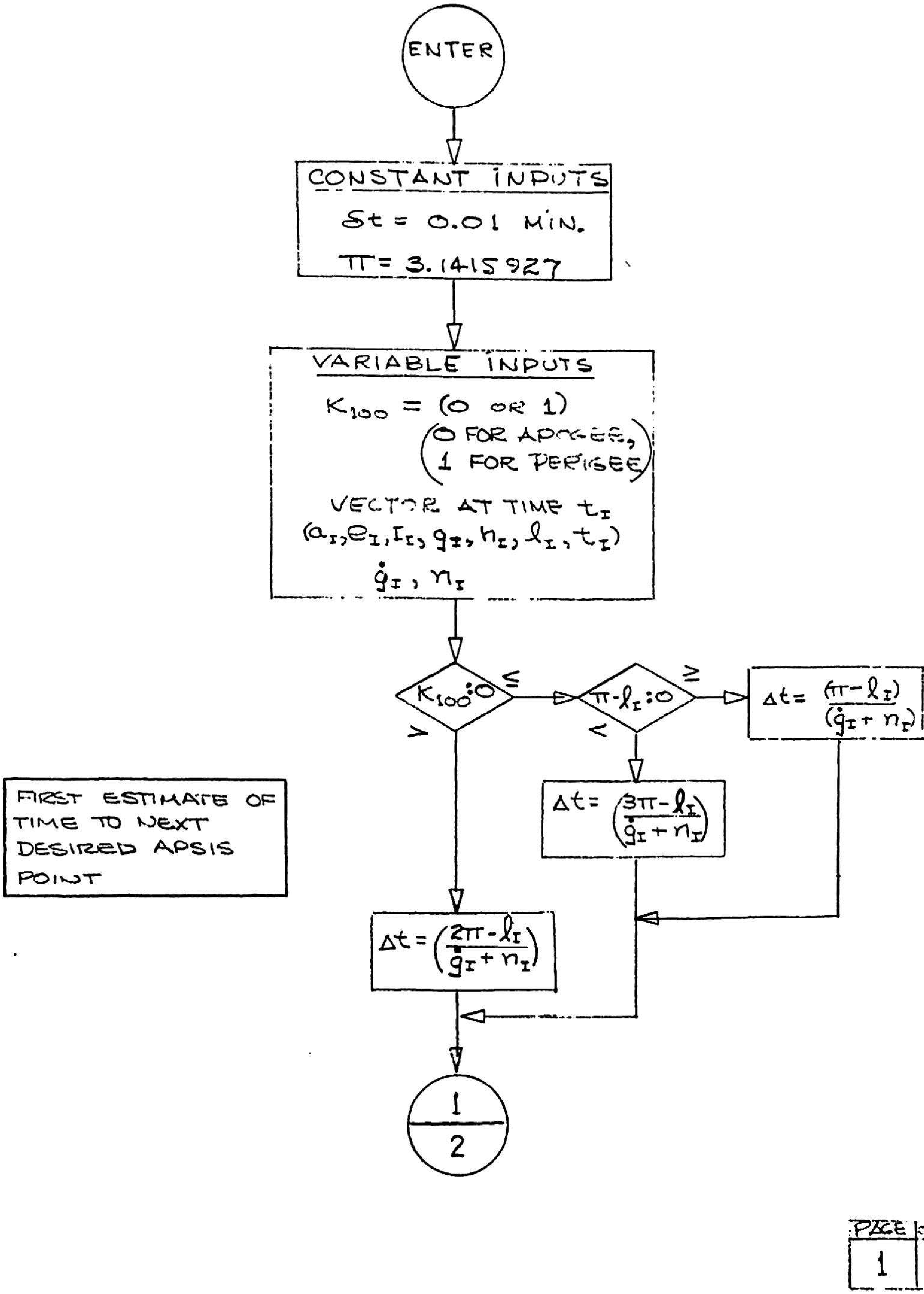

THIS LOGIC DETERMINES THE PROPER QUADRANT OF THE LONGITUDE (λx) IN ORDER TO SOLVE THE SPHERICAL TRIANGLE FOR ARGUMENT OF LATITUDE (UL) OF THE DESIRED LONGITUDE. THE K101 & K102 FLAGS ARE CHECKED LATER IN THE SUBROUTINE TO PROPERLY ADJUST THIS COMPUTED UL
3
(ν-π):0
>
≤
SET ν=ν-π K101=1
(ν-π/2):0
>
≤
SET ν=π-ν K102=1
COSC = SIN(I_I)*COS(ν)
SINC = √(1-(COSC)²)
SIN(UL) = SIN(ν)/SINC
COS(UL) = √(1 - SIN²(UL))
UL = TAN⁻¹(SIN(UL)/COS(UL))
4
4

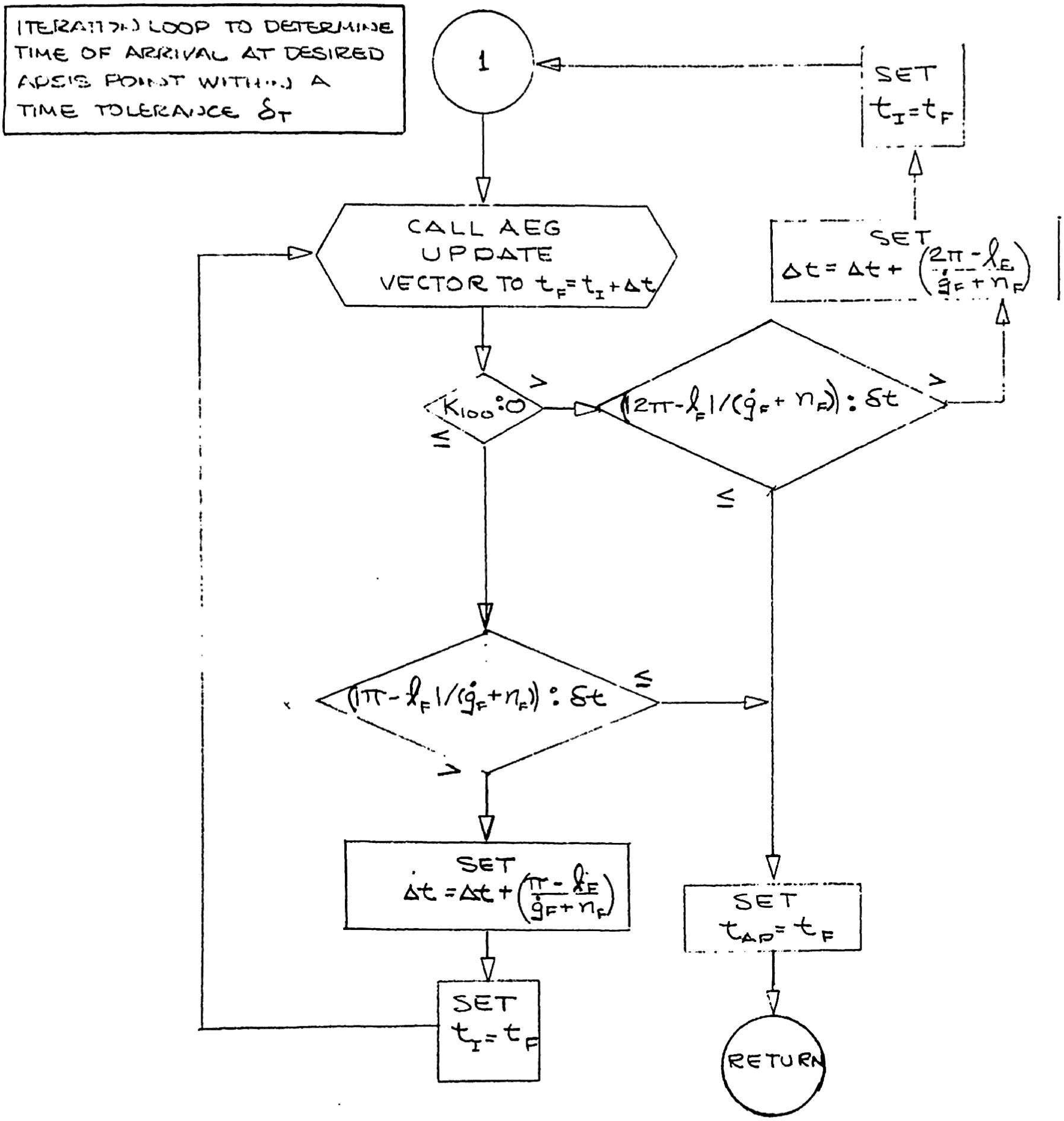

4
SET
$U_L = \pi - U_L$
$K_{102} : 0$
>
≤
$U_L = U_L + \pi$
$K_{101} : 0$
THE INITIAL APPROXIMATION OF THE TIME TO ARRIVE AT THE λ_x IS COMPUTED BY USING THE DIFFERENCES BETWEEN THE PRESENT ARGUMENT OF LATITUDE (U_I) OF THE VEHICLE AND THE ARGUMENT OF LATITUDE. (U_L) OF λ_x
IF AN AEG IS USED IN WHICH THE ARGUMENT OF LATITUDE IS TRUNCATED AT 360°, THERE IS A POSSIBILITY OF LOSING THE CORRECT RELATIONSHIP BETWEEN U_L & U_I AFTER THE EARTH ROTATION TERM ($\omega_E \Delta t_c$) IS ADDED TO λ_x. THIS CONDITION WILL BE CRITICAL ONLY WHEN THE U_L AND THE ARGUMENT OF LATITUDE OF THE PRESENT POSITION (U_I) IS VERY CLOSE TO 360°
$K_{103} : 0$
$(U_L - U_I) : 0$
≥
<
$\Delta t_c = \left(\frac{U_L - U_I}{\dot{g} + n}\right)$
$\Delta t_c = \left(\frac{2\pi + (U_L - U_I)}{\dot{g} + n}\right)$
SET
$K_{103} = 1$
THE K_{103} FLAG WILL PREVENT THE ABOVE CONDITION WITH A SHUNT OF THE TEST ON THE DIFFERENCE BETWEEN U_L & U_I ONCE THE RELATIONSHIP OF THIS PARAMETERS HAS BEEN ESTABLISHED
5
5



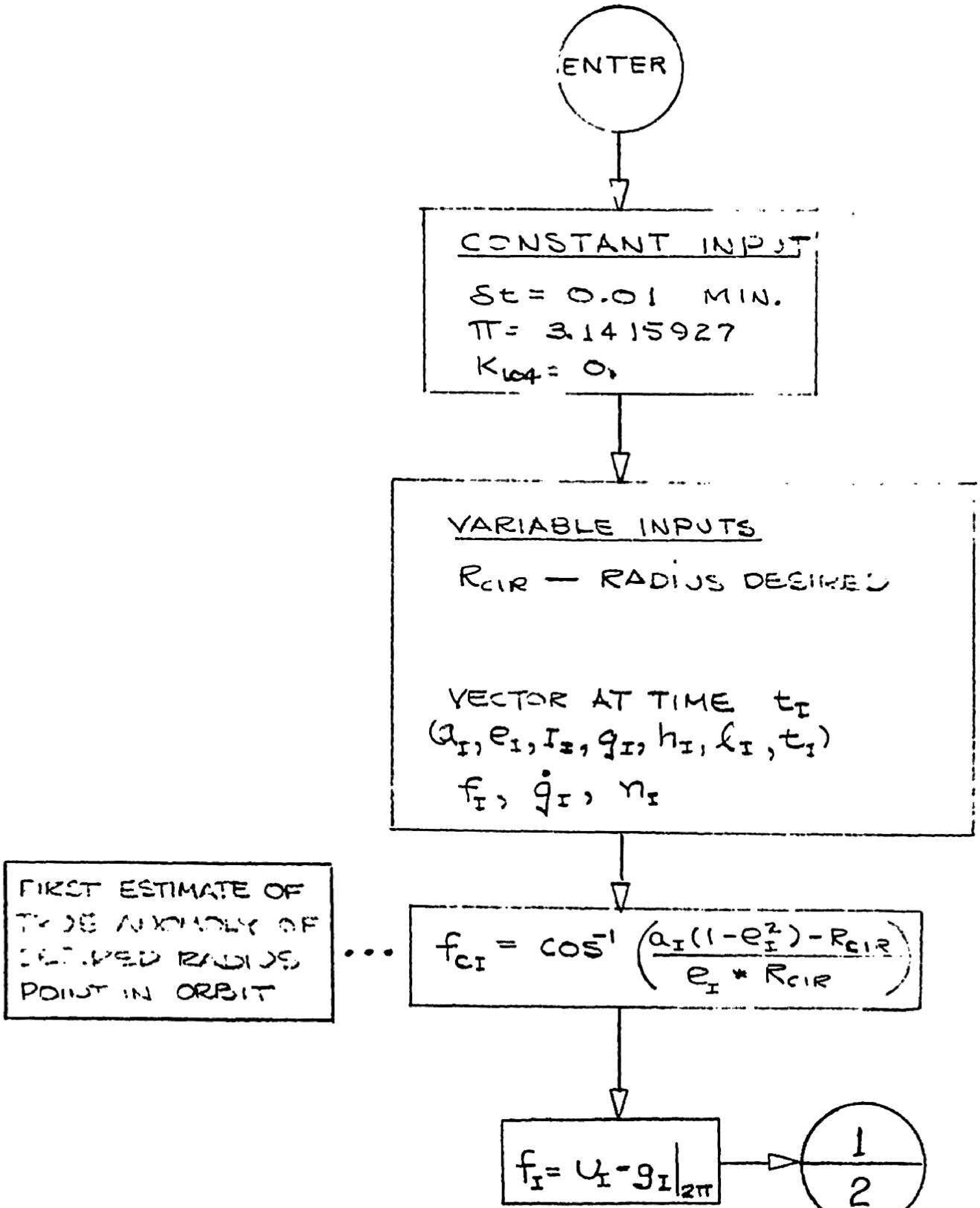
5
SINCE THE EARTH WILL ROTATE DURING THE PERIOD WHEN THE VEHICLE IS ADVANCING TOWARD THE U_L, THE INPUT LONGITUDE (λ') MUST BE ADJUSTED WITH AN EARTH ROTATION TERM ($\omega_e * \Delta t_c$)
$\lambda_z = \lambda' + \omega_e(\Delta t_c)$
THE INITIAL TIME ESTIMATE (Δt_c) MUST BE ADJUSTED TO REFLECT THE EARTH ROTATION TERM. THIS IS DONE BY REPLACING THE INITIAL LONGITUDE (λ_x) WITH THE ADJUSTED LONGITUDE (λ_z) AND RE-ENTERING THE LOGIC TO COMPUTE Δt_c
$\Delta\lambda = \lambda_z - \lambda_x$
SET $\lambda_x = \lambda_z$
$|\Delta\lambda| - \delta\theta : 0$
>
2
2
≤
SET $\Delta t_d = \Delta t_c$
THE U_L COMPUTED IN THE ABOVE ITERATION IS ITERATED UPON TO DETERMINE THE CORRECT TIME OF ARRIVAL AT THE ARGUMENT OF LATITUDE OF THE DESIRED LONGITUDE
CALL AEG - UPDATE VECTOR TO $t_F = (t_I + \Delta t_d)$
$\left|\frac{U_L - U_F}{\dot{g} + n}\right| - \delta_t : 0$
>
SET $\Delta t_d = \Delta t_d + \left(\frac{U_L - U_F}{\dot{g} + n}\right)$
SET $t_{LON} = t_F$
RETURN

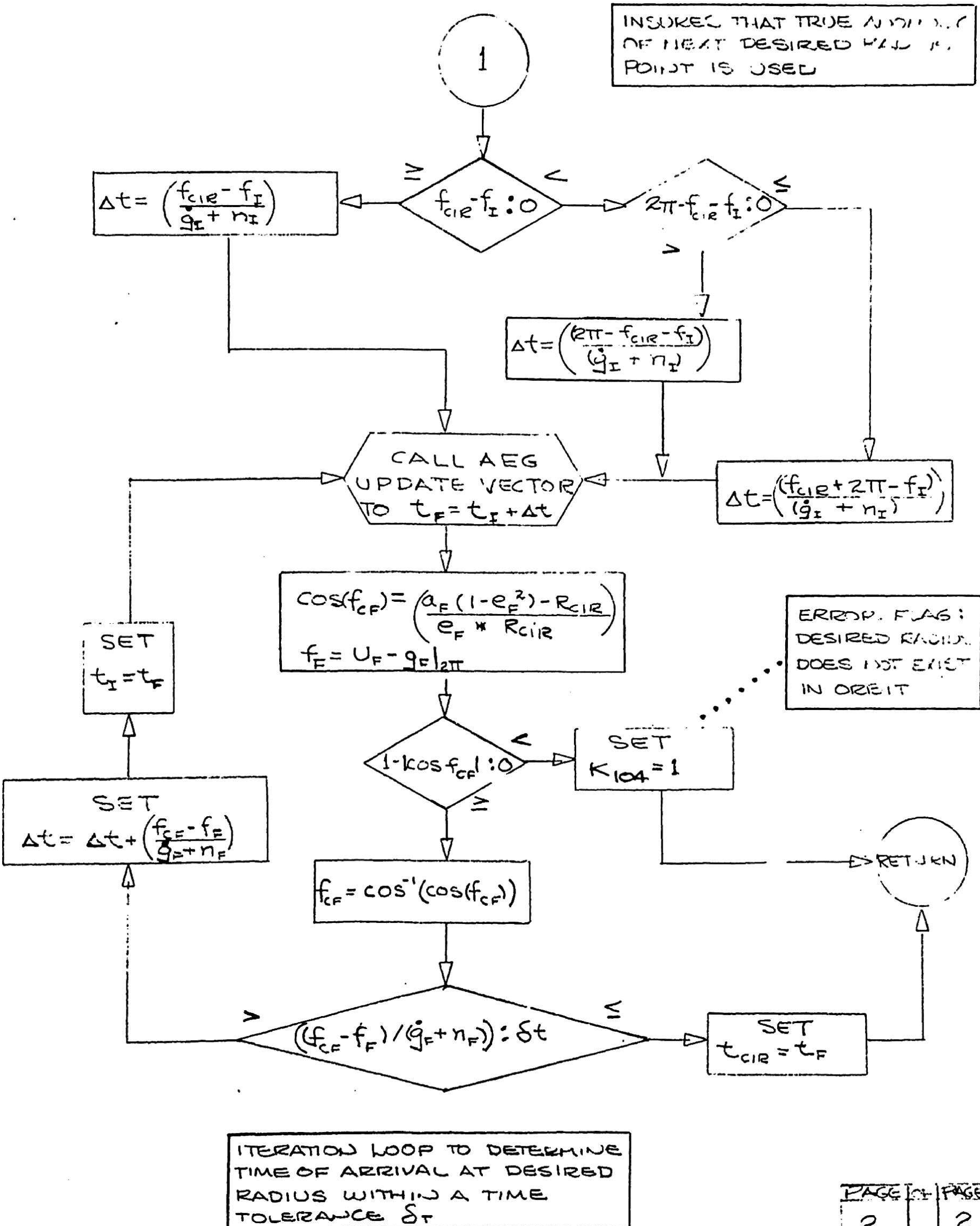
STAP SUBROUTINE



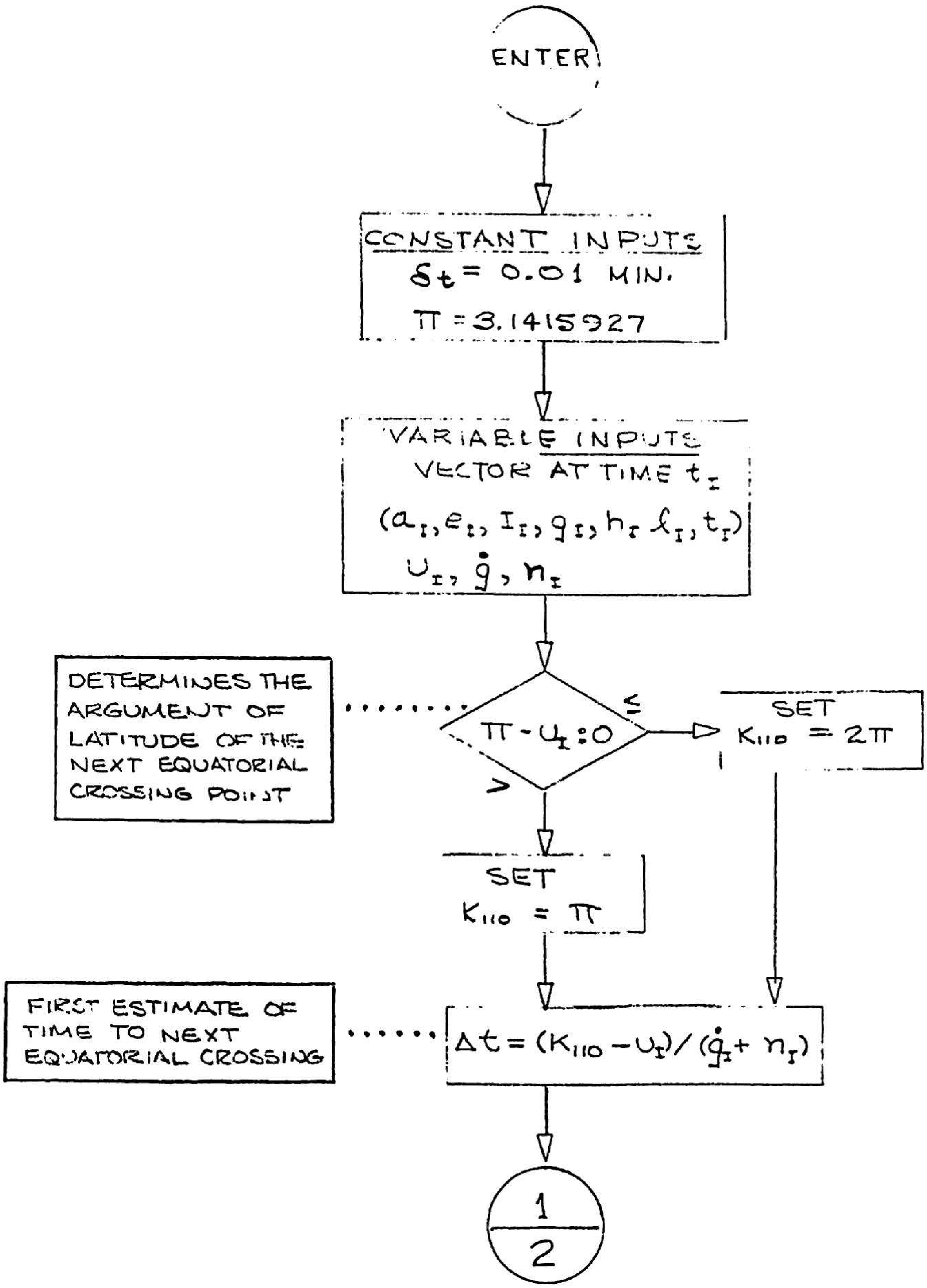

ITERATION LOOP TO DETERMINE TIME OF ARRIVAL AT DESIRED APSIS POINT WITHIN A TIME TOLERANCE δ_T
1
SET $t_I = t_F$
CALL AEG UPDATE VECTOR TO $t_F = t_I + \Delta t$
SET $\Delta t = \Delta t + \left(\frac{2\pi - \ell_E}{\dot{g}_F + n_F}\right)$
$K_{100} : 0$
>
≤
$(|2\pi - \ell_F| / (\dot{g}_F + n_F)) : \delta t$
>
≤
$(|\pi - \ell_F| / (\dot{g}_F + n_F)) : \delta t$
≤
>
SET $\Delta t = \Delta t + \left(\frac{\pi - \ell_E}{\dot{g}_F + n_F}\right)$
SET $t_{AP} = t_F$
SET $t_I = t_F$
RETURN

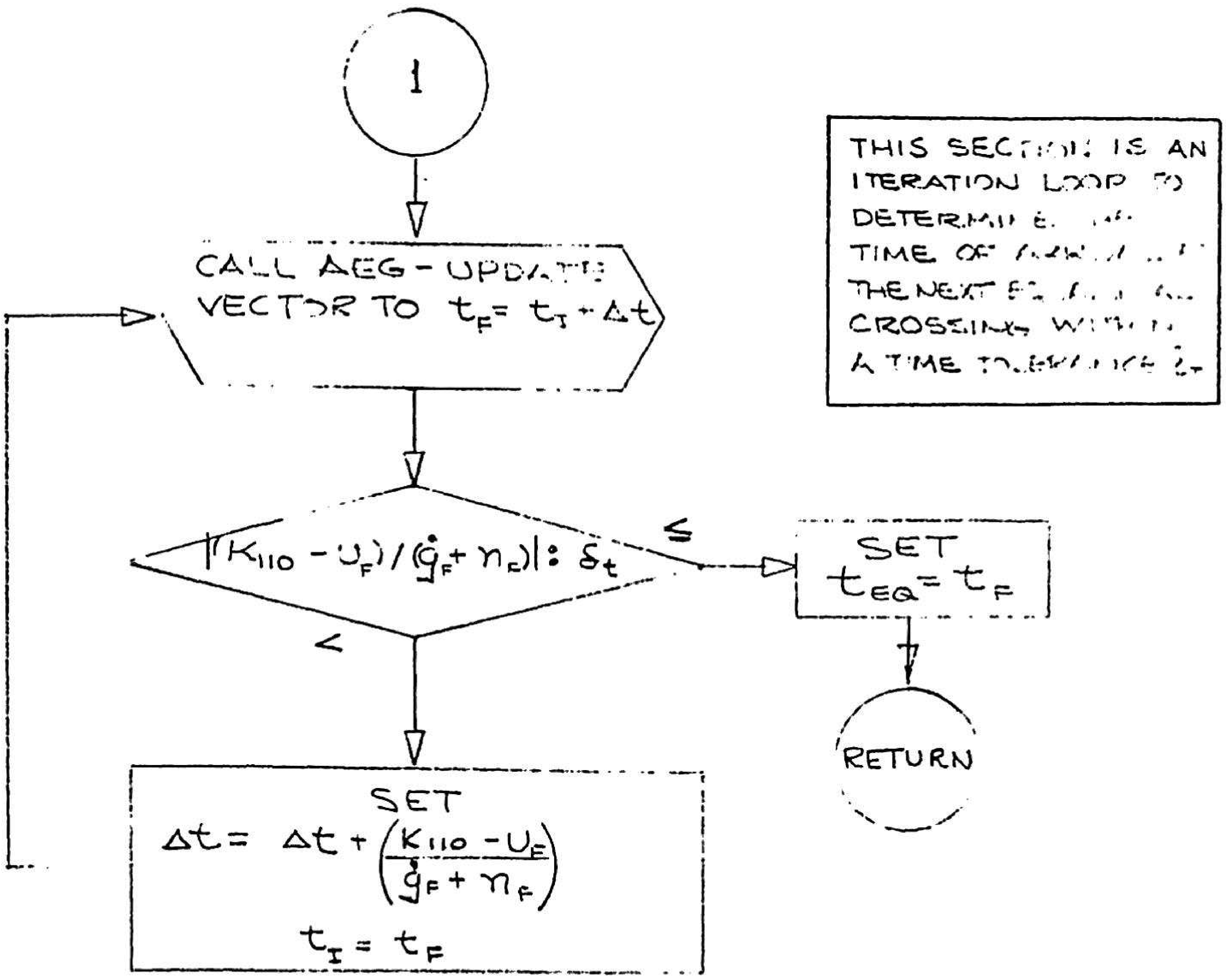
STCIR SUBROUTINE




INSURES THAT TRUE ANOMALY OF NEXT DESIRED RADIUS POINT IS USED
1
≥
<
$f_{cIR} - f_I : 0$
$\Delta t = \left(\frac{f_{cIR} - f_I}{\dot{g}_I + n_I}\right)$
≤
$2\pi - f_{cIR} - f_I : 0$
>
$\Delta t = \left(\frac{(2\pi - f_{cIR} - f_I)}{(\dot{g}_I + n_I)}\right)$
CALL AEG UPDATE VECTOR TO $t_F = t_I + \Delta t$
$\Delta t = \left(\frac{(f_{cIR} + 2\pi - f_I)}{(\dot{g}_I + n_I)}\right)$
$\cos(f_{cF}) = \left(\frac{a_F(1 - e_F^2) - R_{cIR}}{e_F * R_{cIR}}\right)$
$f_F = U_F - g_F|_{2\pi}$
SET $t_I = t_F$
ERROR FLAG: DESIRED RADIUS DOES NOT EXIST IN ORBIT
$1 - |\cos f_{cF}| : 0$
<
≥
SET $K_{104} = 1$
SET $\Delta t = \Delta t + \left(\frac{f_{cF} - f_F}{\dot{g}_F + n_F}\right)$
$f_{cF} = \cos^{-1}(\cos(f_{cF}))$
RETURN
>
≤
$((f_{cF} - f_F)/(\dot{g}_F + n_F)) : \delta t$
SET $t_{cIR} = t_F$
ITERATION LOOP TO DETERMINE TIME OF ARRIVAL AT DESIRED RADIUS WITHIN A TIME TOLERANCE δ_T

STEP SUBROUTINE




1
CALL AEG - UPDATE VECTOR TO $t_F = t_I + \Delta t$
$|(K_{110} - U_F)/(\dot{g}_F + \eta_F)| : \delta_t$
≤
<
SET $t_{EQ} = t_F$
RETURN
SET
$\Delta t = \Delta t + \left(\frac{K_{110} - U_F}{\dot{g}_F + \eta_F}\right)$
$t_I = t_F$
THIS SECTION IS AN
ITERATION LOOP
TIME OF
CROSSING
A TIME

APPENDIX II

Definition of Flow Chart Symbols

a - semimajor axis

e - eccentricity

I - inclination

g - argument of perigee

h - longitude of ascending node

ℓ - mean anomaly

t - time of vector

U - argument of latitude

$\dot{g}$ - secular rate of change of argument of perigee

n - mean motion constant

f - true anomaly

λ - input longitude

ν - distance along the equator between the present earth-fixed ascending node and the desired longitude

h_e - earth-fixed longitude of ascending node

λ_x - initial setting of λ in iteration loop

U_L - computed argument of latitude of the desired longitudinal crossing

COSC - cosine of wedge angle between longitude meridian and vehicle path

K_{101}, K_{102} - flags to establish correct quadrant of longitude (λ)

Δt - time interval used in iteration loop

K_{103} - flag to set proper relationship between the present argument of latitude and argument of latitude of desired longitude

$\Delta\lambda$ - change in input longitude due to earth rotation factor

$\delta\theta$ - angular tolerance in iteration loop

λ_Z - "adjusted" desired longitude due to earth rotation factor

K_{100} - option to search for either apogee or perigee

f_{CIR} - true anomaly of point of desired radius

K_{110} - flag to determine next node (ascending or descending)

I'' - mean inclination

K_{104} - error flag in STCIR - desired point does not still exist in orbit

t_{LON} - time of arrival at specific longitude

t_{EQ} - time of arrival at next equatorial crossing

t_{CIR} - next time of arrival at specific radius

t_{AP} - time of arrival at next apogee or perigee

Subscripts

I - initial

F - vector at desired time