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LOGIC AND EQUATIONS FOR THE REAL-TIME COMPUTATION OF AS-207/208 INSERTION ELEMENTS AND SPECIALIZED ORBITAL MANEUVERS

By R. K. McDonough and W. A. Sullivan
Rendezvous Analysis Branch

MISSION PLANNING AND ANALYSIS DIVISION
MANNED SPACECRAFT CENTER
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PROJECT APOLLO

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SUMMARY AND INTRODUCTION

This internal note presents the logic and equations defining certain subroutines which are called by the real-time computer programs for the AS-207/208 mission. The subroutines compute the launch vehicle insertion elements for the launch targeting processor and compute several specialized maneuvers for the rendezvous planning processors. Detailed flow charts are included in the appendix.

SYMBOLS

Constants	
π	3.1415927
μ	earth gravitational constant
J_2	second harmonic of earth's potential
ω_e	rotational rate of the earth
ϵ_T	iteration tolerance on time
Flags	
I	option to execute maneuver $I = 0$, output maneuver ΔV only $I = 1$, execute maneuver

J	control flag on input to ENSERT J = 1, input R_{BO}, V_{BO}, γ_{BO} J = 0, input a_{BO}, e_{BO}, l_{BO}
M	differential nodal regression bias control M = 1, no bias wanted M = 0, bias wanted
L1	passive vehicle number
L2	active vehicle number

Variables

a	semi-major axis
e	eccentricity
i	inclination
g	argument of perigee
h	longitude of ascending node
l	mean anomaly
n	mean motion
$\dot{g}$	rate of change of argument of perigee
$\dot{h}$	rate of change of longitude of ascending node
R	radius of vehicle
u	argument of latitude
$\dot{R}$	rate of change of radius
h"	mean longitude of ascending node
C_D	coefficient of drag
A	frontal area of vehicle

W	vehicle weight
I''	mean inclination
$\bar{R}(X, Y, Z)$	position of vehicle in rectangular coordinates
$\bar{V}(X, Y, Z)$	velocity of vehicle in rectangular coordinates
$\bar{S}(V, \gamma, \psi, R, \lambda, \phi)$	position and velocity in spherical coordinates
R_{BO}	radius at burnout
V_{BO}	velocity at burnout
γ_{BO}	flight-path angle at burnout
a_{BO}	semi-major axis at burnout
e_{BO}	eccentricity at burnout
l_{BO}	mean anomaly at burnout
Δt_{PF}	powered flight time
θ_{PF}	powered flight arc
δ_{ys}	yaw steer capability of launch vehicle
R_{LS}, λ_{LS}, ϕ_{LS}	launch site position
δ_W	wedge angle
T	epoch time
T_{LO}	lift-off time
T_{REF}	time to begin searching for common node
T_{CN}	time of arrival at a common node

ΔN	number of node desired after T_{REF}
ΔV	magnitude of total delta V
ΔV_H	magnitude of horizontal delta V
ΔV_R	magnitude of radial delta V
ΔV_Z	magnitude of out-of-plane delta V
ΔV_{PC}	magnitude of delta V for plane-change maneuver
ΔH	coelliptic height difference desired

DISCUSSION

The subroutines presented here have been developed for the AS 207/208 real-time computer programs. The input and output, as well as the control options, are designed for maximum support capability. The Analytic Ephemeris Generator (AEG) of reference 1 is used.

A vehicle-centered coordinate system is adopted for internal use by the subroutines. Each of the coordinates, $\bar{J}$, $\bar{K}$, $\bar{H}$, is a three-dimensional unit vector in the AEG coordinate system. The coordinates $\bar{J}$, $\bar{K}$, and $\bar{H}$ are along the instantaneous vechicle local vertical, local horizontal, and angular momentum vectors, respectively. The maneuver ΔV components, as measured in this system, show ΔV_H and ΔV_Z positive in the positive $\bar{K}$ and $\bar{H}$ coordinates, but ΔV_R positive in the negative $\bar{J}$ coordinate direction.

COEDH

Subroutine COEDH computes the maneuver required to place the vehicle into an orbit coelliptic to the given target orbit. The vehicles are required to be at position match at the maneuver time. The criteria used for the coelliptic maneuver are that

$$a_{I2} = a_{L1} - \Delta H$$

and

$$\dot{R}_{I2} = \dot{R}_{L1} \left[\frac{n_{I2}}{n_{L1}}\right],$$

where

$$n_{I2} = \sqrt{\frac{\mu}{a_{I2}^{3}}} \quad .$$

COEDH then computes the velocity and flight-path angle which satisfy these criteria and, from these, it computes the ΔV_H, ΔV_R, and total ΔV. An option is provided to execute the maneuver and output the resulting orbit through the AEG.

PARDV

Subroutine PARDV computes the maneuver which has its thrust initiation vector parallel to the target orbital plane and which has input ΔV_H and ΔV_R components. The subroutine computes ΔV_Z, the total ΔV, and the attitude angles. An option is provided to execute the maneuver and output the resulting orbit through the AEG.

CNODE

This subroutine computes the vehicle time of arrival at a common node between the vehicle orbital planes. Any desired node can be singled out throught input of a threshold time and the number of the consecutive nodal crossings after that time. CNODE then computes the instantaneous plane change maneuver required to force the vehicle into the target plane. The ΔV_H, ΔV_Z, and attitude angles are included in the output.

EXMAN

This subroutine executes an input maneuver by adding the ΔV components (directed along the corresponding vehicle-centered coordinate vectors) to the instantaneous velocity vector. This resulting vector and the instantaneous radius vector are converted to classical elements for output through the AEG.

ENSERT

Subroutine ENSERT computes the launch vehicle insertion elements for a given lift-off time and a given target orbit. The logic considers an input yaw steer capability. The insertion plane is biased for differential nodal regression if directed by input.

The magnitude of the differential nodal regression is approximated by

$$\Delta h = \frac{7}{3} \theta J_2 \cos i_{LI}$$

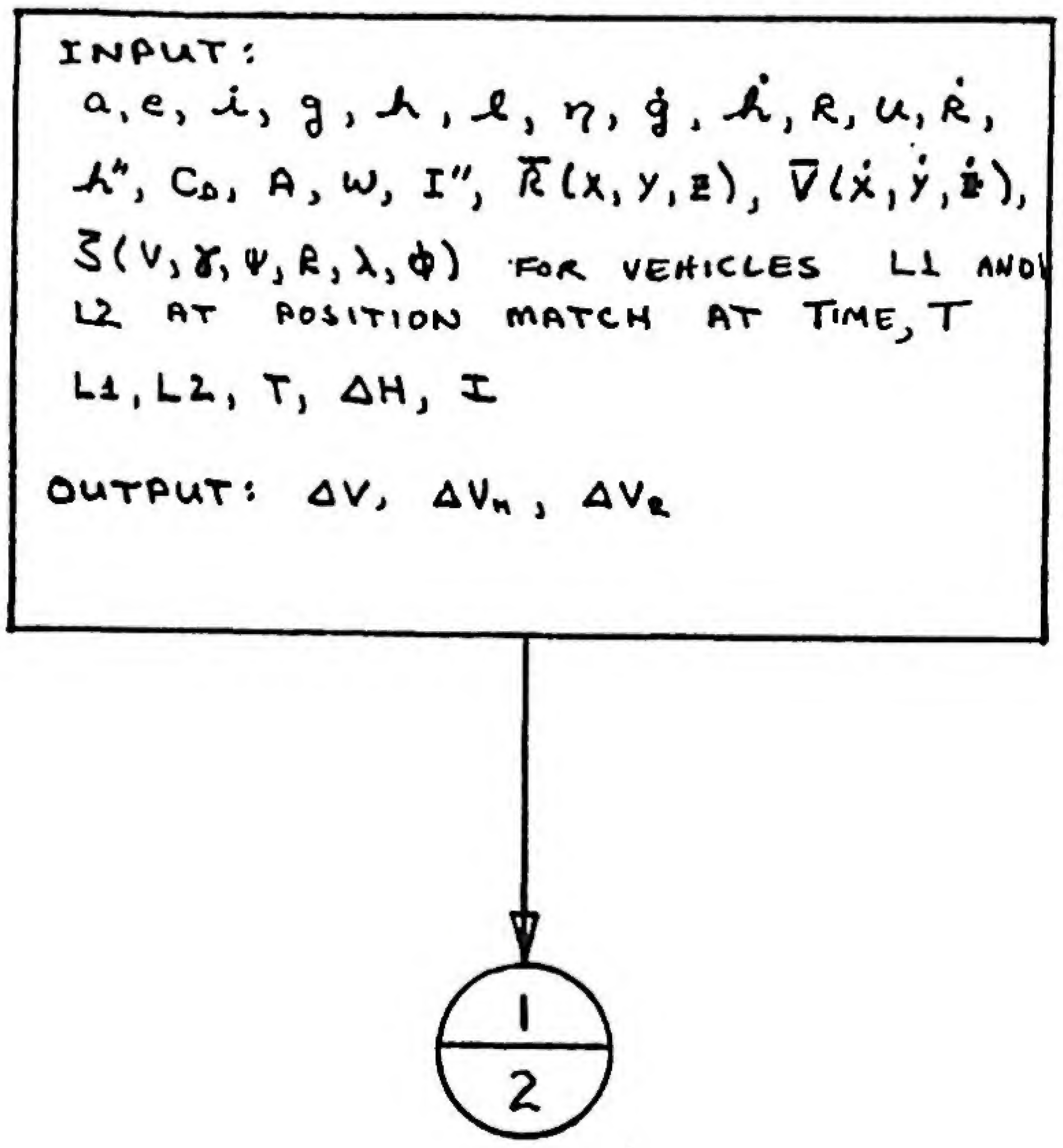
This relation is found in reference 2. The phase lag (θ) is computed by subroutine THETR (ref. 3). The basic approach used to compute the insertion position was drawn from reference 4. The logic was modified to fit the AEG and to include the effects of differential nodal regression.

APPENDIX

FLOW CHARTS FOR SUBROUTINES

SUBROUTINE COEDH

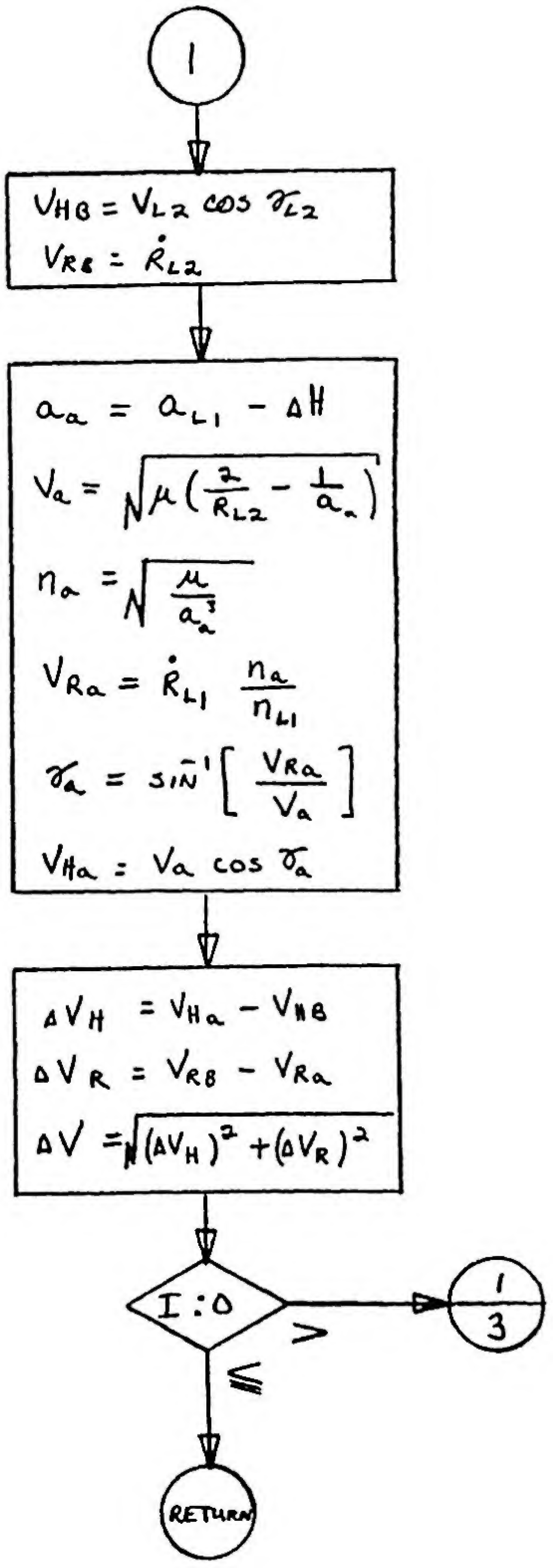
SUBROUTINE TO COMPUTE CDH MANEUVER

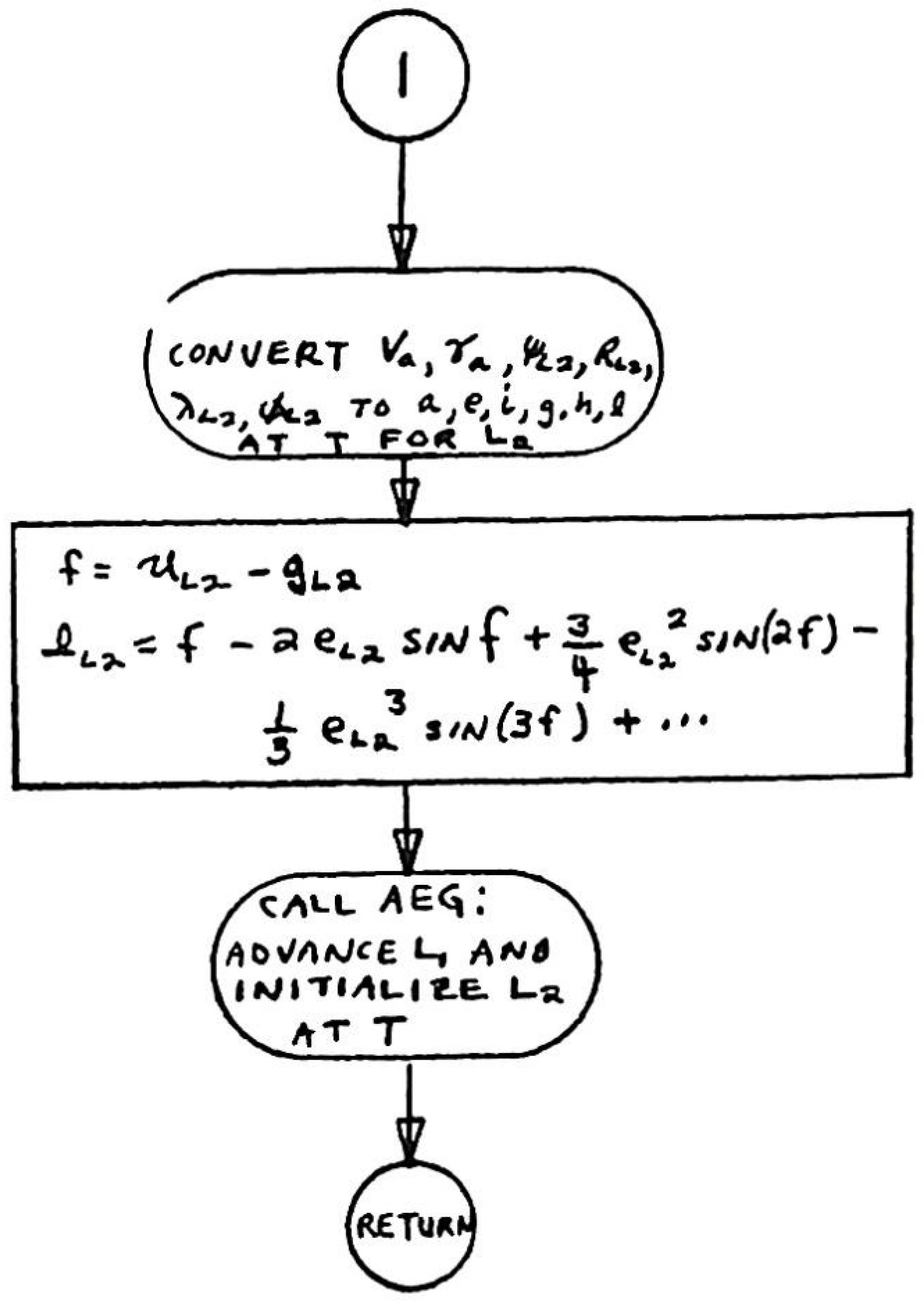


INPUT:

$a, e, i, g, h, \ell, \eta, \dot{g}, \dot{h}, R, u, \dot{R},$
$h'', C_D, A, \omega, I'', \bar{R}(x, y, z), \bar{V}(\dot{x}, \dot{y}, \dot{z}),$
$\zeta(V, \gamma, \psi, R, \lambda, \phi)$ FOR VEHICLES L1 AND
L2 AT POSITION MATCH AT TIME, T
L1, L2, T, ΔH, I

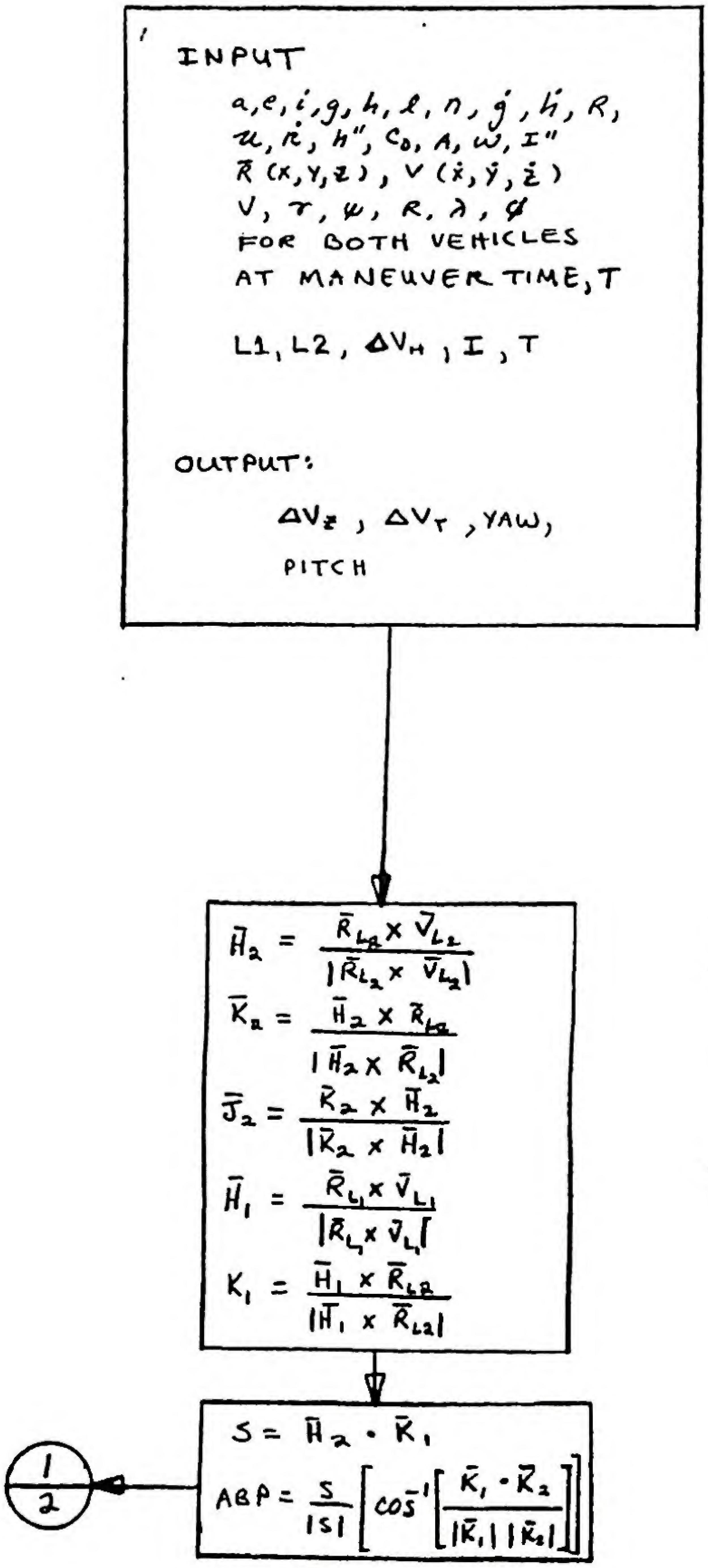
OUTPUT: ΔV, ΔV_H, ΔV_R


1
$V_{HB} = V_{L2} \cos \gamma_{L2}$
$V_{RB} = \dot{R}_{L2}$
$a_a = a_{L1} - \Delta H$
$V_a = \sqrt{\mu\left(\frac{2}{R_{L2}} - \frac{1}{a_a}\right)}$
$n_a = \sqrt{\frac{\mu}{a_a^3}}$
$V_{Ra} = \dot{R}_{L1} \frac{n_a}{n_{L1}}$
$\gamma_a = \sin^{-1}\left[\frac{V_{Ra}}{V_a}\right]$
$V_{Ha} = V_a \cos \gamma_a$
$\Delta V_H = V_{Ha} - V_{HB}$
$\Delta V_R = V_{RB} - V_{Ra}$
$\Delta V = \sqrt{(\Delta V_H)^2 + (\Delta V_R)^2}$
$I : 0$
$>$
$\leq$
1
3
RETURN


1
CONVERT $V_a, \gamma_a, \psi_{L2}, R_{L2}, \lambda_{L2}, \phi_{L2}$ TO a, e, i, g, h, l AT T FOR L_2
$f = u_{L2} - g_{L2}$
$l_{L2} = f - 2 e_{L2} \sin f + \frac{3}{4} e_{L2}^2 \sin(2f) - \frac{1}{3} e_{L2}^3 \sin(3f) + \cdots$
CALL AEG: ADVANCE L_1 AND INITIALIZE L_2 AT T
RETURN

SUBROUTINE PARDV

SUBROUTINE TO INITIATE THRUST
PARALLEL TO TARGET ORBIT



INPUT

$a, e, i, g, h, \ell, n, \dot{g}, \dot{h}, R,$
$u, \dot{r}, h'', C_D, A, \omega, I''$
$\bar{R}(x, y, z), V(\dot{x}, \dot{y}, \dot{z})$
$V, \tau, \psi, R, \lambda, \phi$
FOR BOTH VEHICLES
AT MANEUVER TIME, T

L1, L2, ΔV_H, I, T

OUTPUT:

ΔV_Z, ΔV_T, YAW,
PITCH

$$\bar{H}_2 = \frac{\bar{R}_{L2} \times \bar{V}_{L2}}{|\bar{R}_{L2} \times \bar{V}_{L2}|}$$

$$\bar{K}_2 = \frac{\bar{H}_2 \times \bar{R}_{L2}}{|\bar{H}_2 \times \bar{R}_{L2}|}$$

$$\bar{J}_2 = \frac{\bar{K}_2 \times \bar{H}_2}{|\bar{K}_2 \times \bar{H}_2|}$$

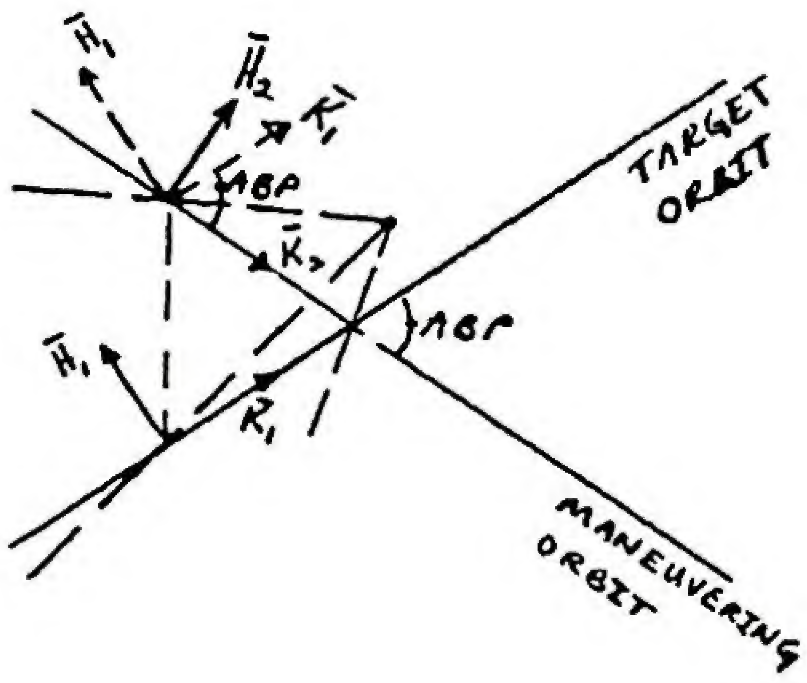
$$\bar{H}_1 = \frac{\bar{R}_{L1} \times \bar{V}_{L1}}{|\bar{R}_{L1} \times \bar{V}_{L1}|}$$

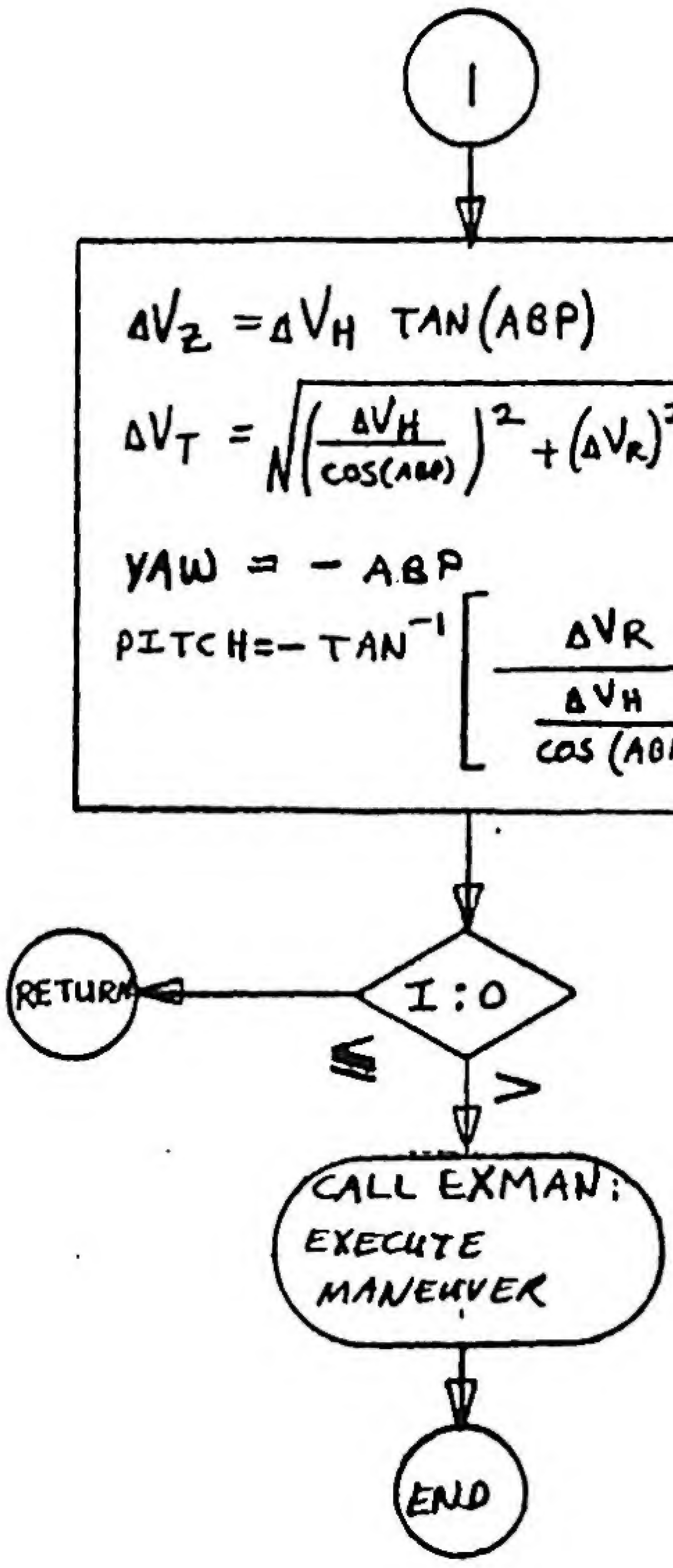
$$K_1 = \frac{\bar{H}_1 \times \bar{R}_{L2}}{|\bar{H}_1 \times \bar{R}_{L2}|}$$

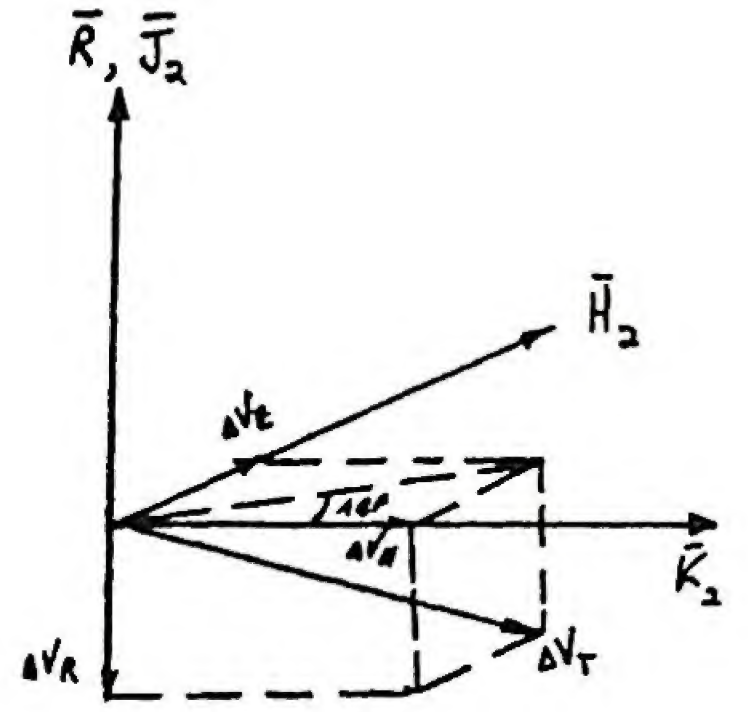
$$S = \bar{H}_2 \cdot \bar{K}_1$$

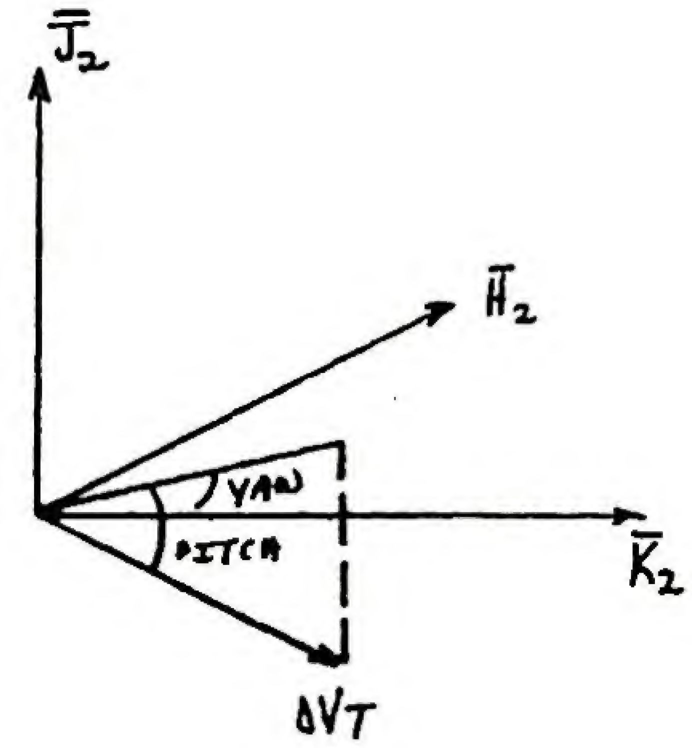
$$ABP = \frac{S}{|S|}\left[\cos^{-1}\left[\frac{\bar{K}_1 \cdot \bar{K}_2}{|\bar{K}_1|\,|\bar{K}_2|}\right]\right]$$

$\frac{1}{2}$



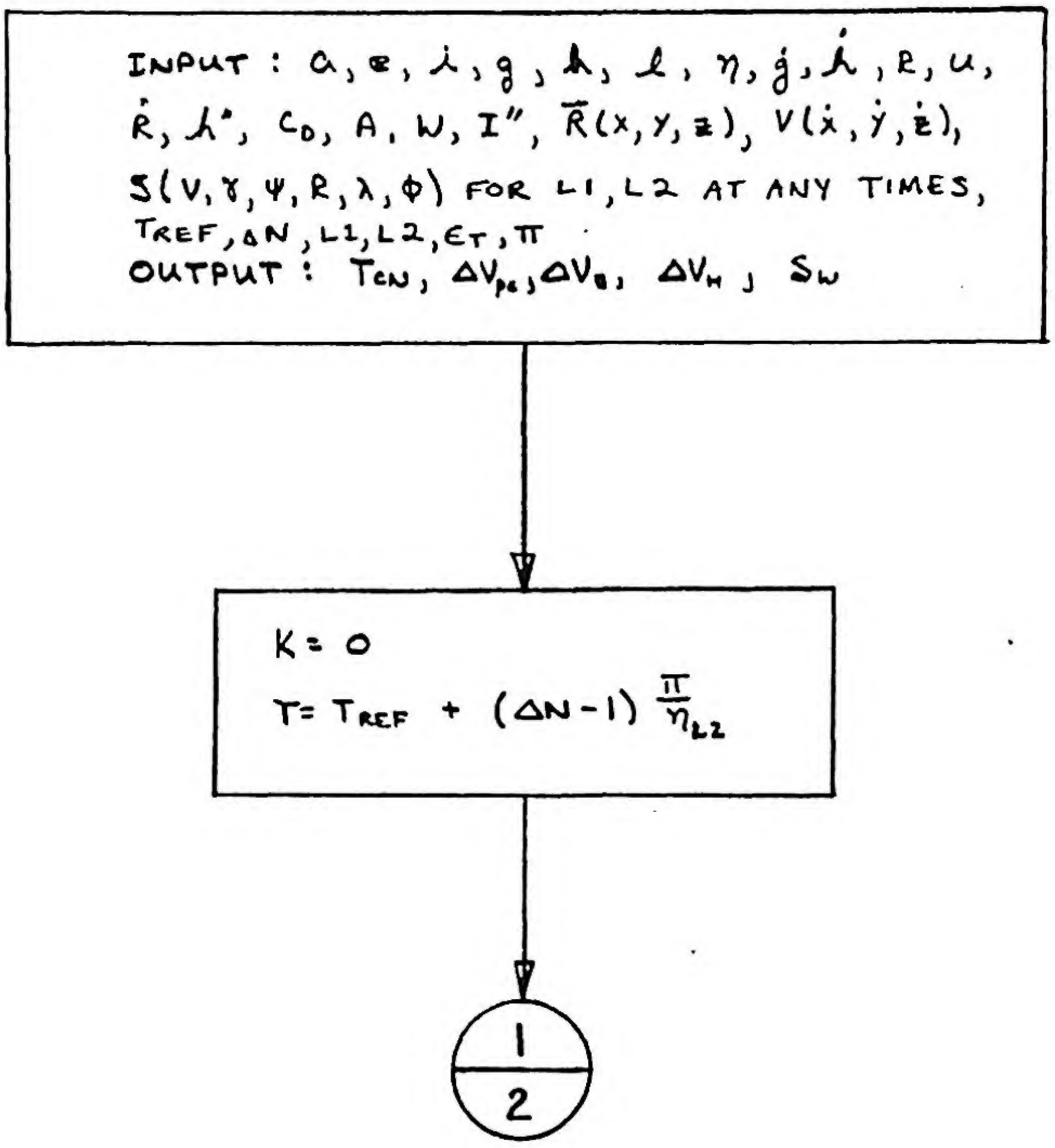

1
$\Delta V_Z = \Delta V_H \ \mathrm{TAN}(ABP)$
$\Delta V_T = \sqrt{\left(\frac{\Delta V_H}{\cos(ABP)}\right)^2 + (\Delta V_R)^2}$
$YAW = -ABP$
$PITCH = -\mathrm{TAN}^{-1}\left[\frac{\Delta V_R}{\frac{\Delta V_H}{\cos(ABP)}}\right]$
I : 0
RETURN
≤
>
CALL EXMAN: EXECUTE MANEUVER
END


$\bar{R}, \bar{J}_2$
$\bar{H}_2$
ΔV_Z
ABP
ΔV_H
$\bar{K}_2$
ΔV_R
ΔV_T


$\bar{J}_2$
$\bar{H}_2$
YAW
PITCH
$\bar{K}_2$
ΔV_T

SUBROUTINE CNODE

SUBROUTINE TO COMPUTE THE TIME OF ARRIVAL OF A VEHICLE AT A RELATIVE (COMMON) NODE



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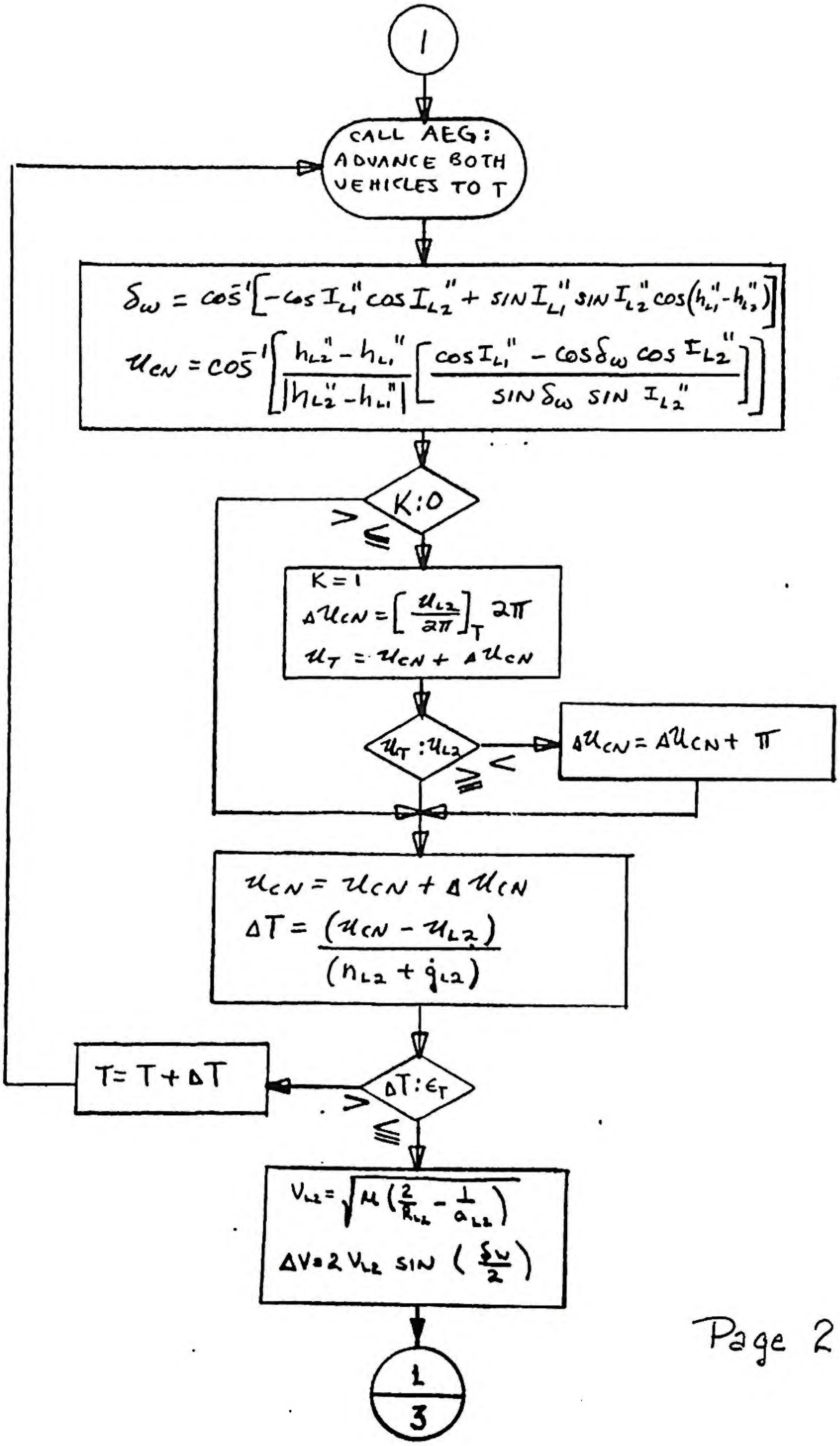
1
CALL AEG: ADVANCE BOTH VEHICLES TO T
$\delta_\omega = \cos^{-1}\left[-\cos I_{L1}'' \cos I_{L2}'' + \sin I_{L1}'' \sin I_{L2}'' \cos(h_{L1}'' - h_{L2}'')\right]$
$u_{CN} = \cos^{-1}\left[\frac{h_{L2}'' - h_{L1}''}{|h_{L2}'' - h_{L1}''|}\left[\frac{\cos I_{L1}'' - \cos\delta_\omega \cos I_{L2}''}{\sin\delta_\omega \sin I_{L2}''}\right]\right]$
K:0
>
≤
K = 1
$\Delta u_{CN} = \left[\frac{u_{L2}}{2\pi}\right]_T 2\pi$
$u_T = u_{CN} + \Delta u_{CN}$
$u_T : u_{L2}$
<
≥
$\Delta u_{CN} = \Delta u_{CN} + \pi$
$u_{CN} = u_{CN} + \Delta u_{CN}$
$\Delta T = \frac{(u_{CN} - u_{L2})}{(n_{L2} + \dot{g}_{L2})}$
$\Delta T : \epsilon_T$
>
≤
$T = T + \Delta T$
$V_{L2} = \sqrt{\mu\left(\frac{2}{R_{L2}} - \frac{1}{a_{L2}}\right)}$
$\Delta V = 2 V_{L2} \sin\left(\frac{\delta_\omega}{2}\right)$
1
3

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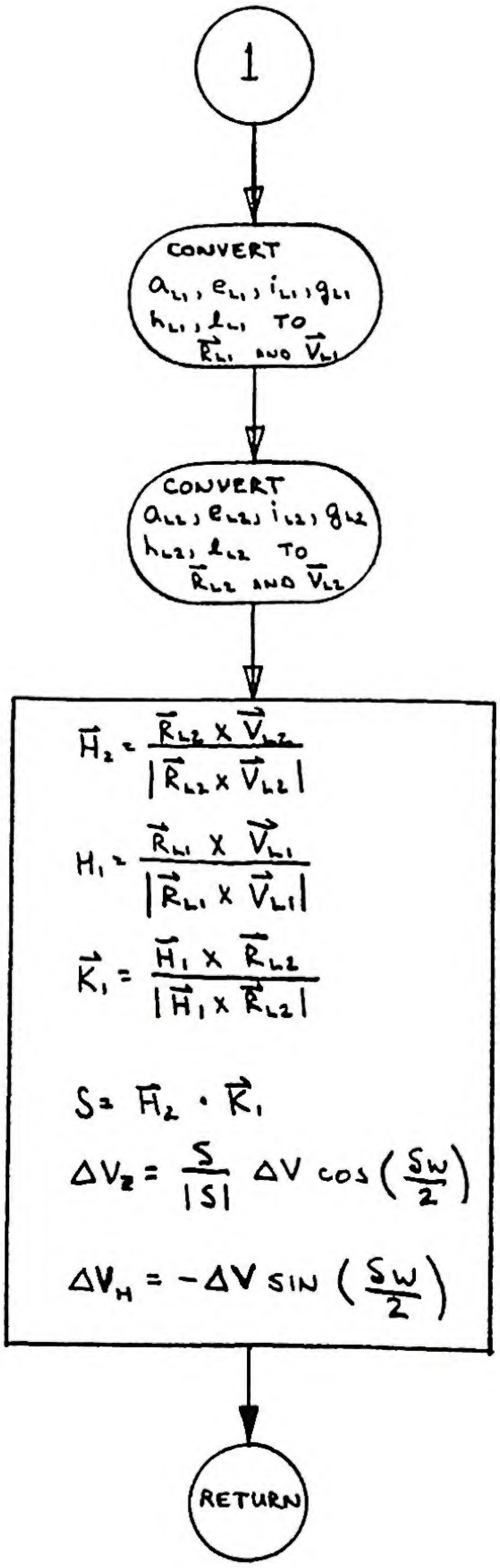
1
CONVERT
$a_{L1}, e_{L1}, i_{L1}, g_{L1}$
h_{L1}, ℓ_{L1} TO
$\vec{R}_{L1}$ AND $\vec{V}_{L1}$
CONVERT
$a_{L2}, e_{L2}, i_{L2}, g_{L2}$
h_{L2}, ℓ_{L2} TO
$\vec{R}_{L2}$ AND $\vec{V}_{L2}$
$\vec{H}_2 = \frac{\vec{R}_{L2} \times \vec{V}_{L2}}{|\vec{R}_{L2} \times \vec{V}_{L2}|}$
$H_1 = \frac{\vec{R}_{L1} \times \vec{V}_{L1}}{|\vec{R}_{L1} \times \vec{V}_{L1}|}$
$\vec{K}_1 = \frac{\vec{H}_1 \times \vec{R}_{L2}}{|\vec{H}_1 \times \vec{R}_{L2}|}$
$S = \vec{H}_2 \cdot \vec{K}_1$
$\Delta V_z = \frac{S}{|S|} \Delta V \cos\left(\frac{SW}{2}\right)$
$\Delta V_H = -\Delta V \sin\left(\frac{SW}{2}\right)$
RETURN

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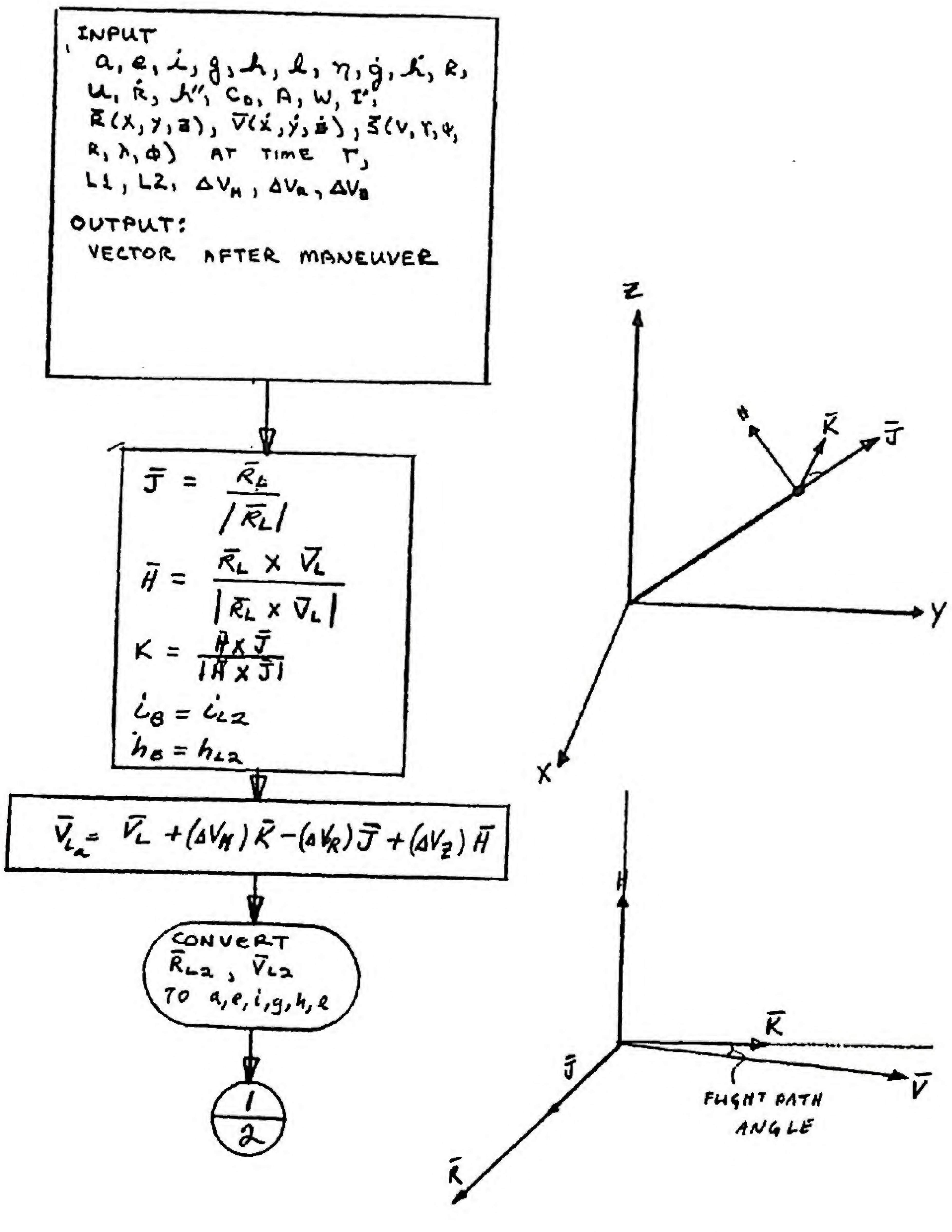
SUBROUTINE EXMAN

SUBROUTINE TO EXECUTE MANEUVER



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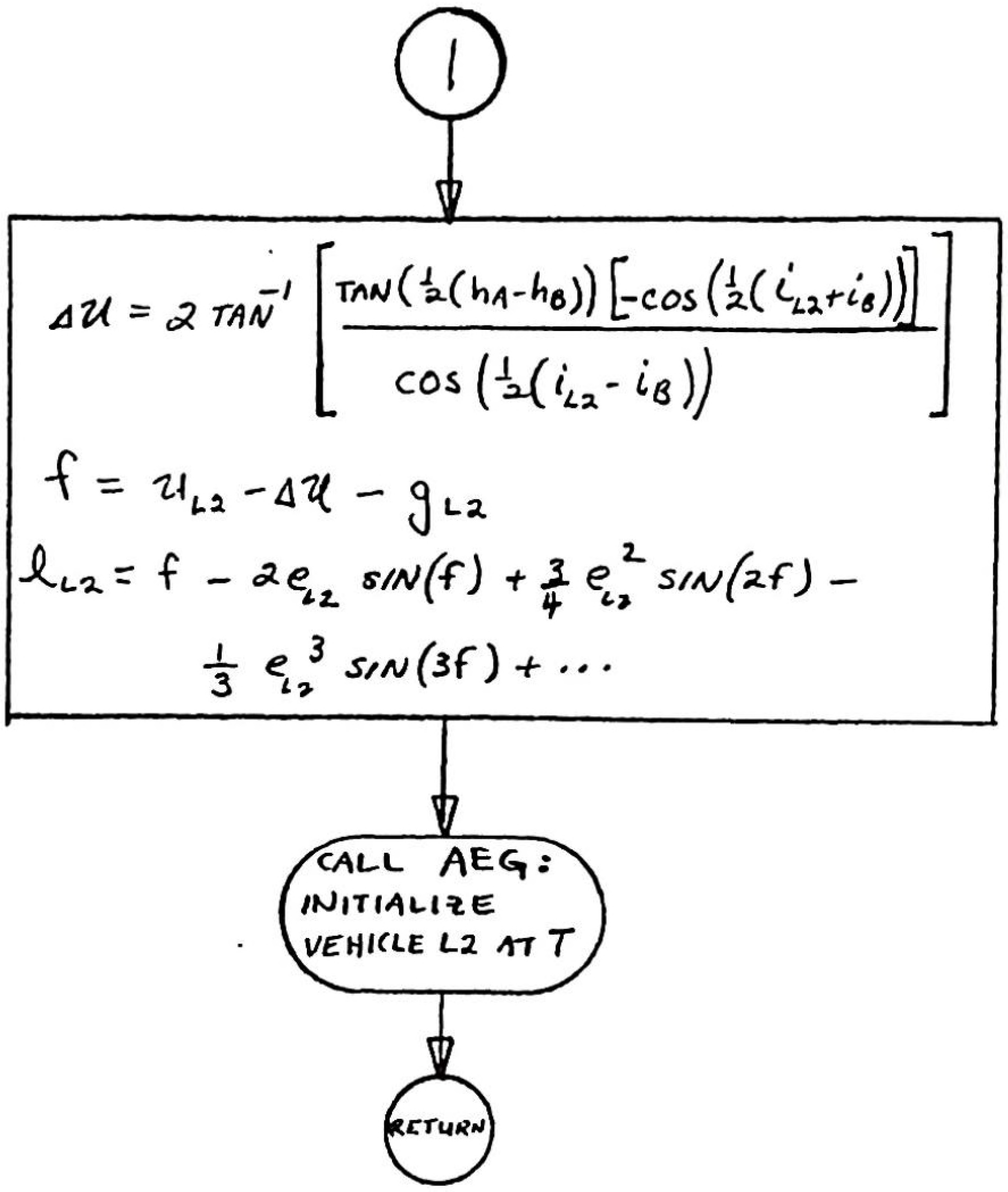


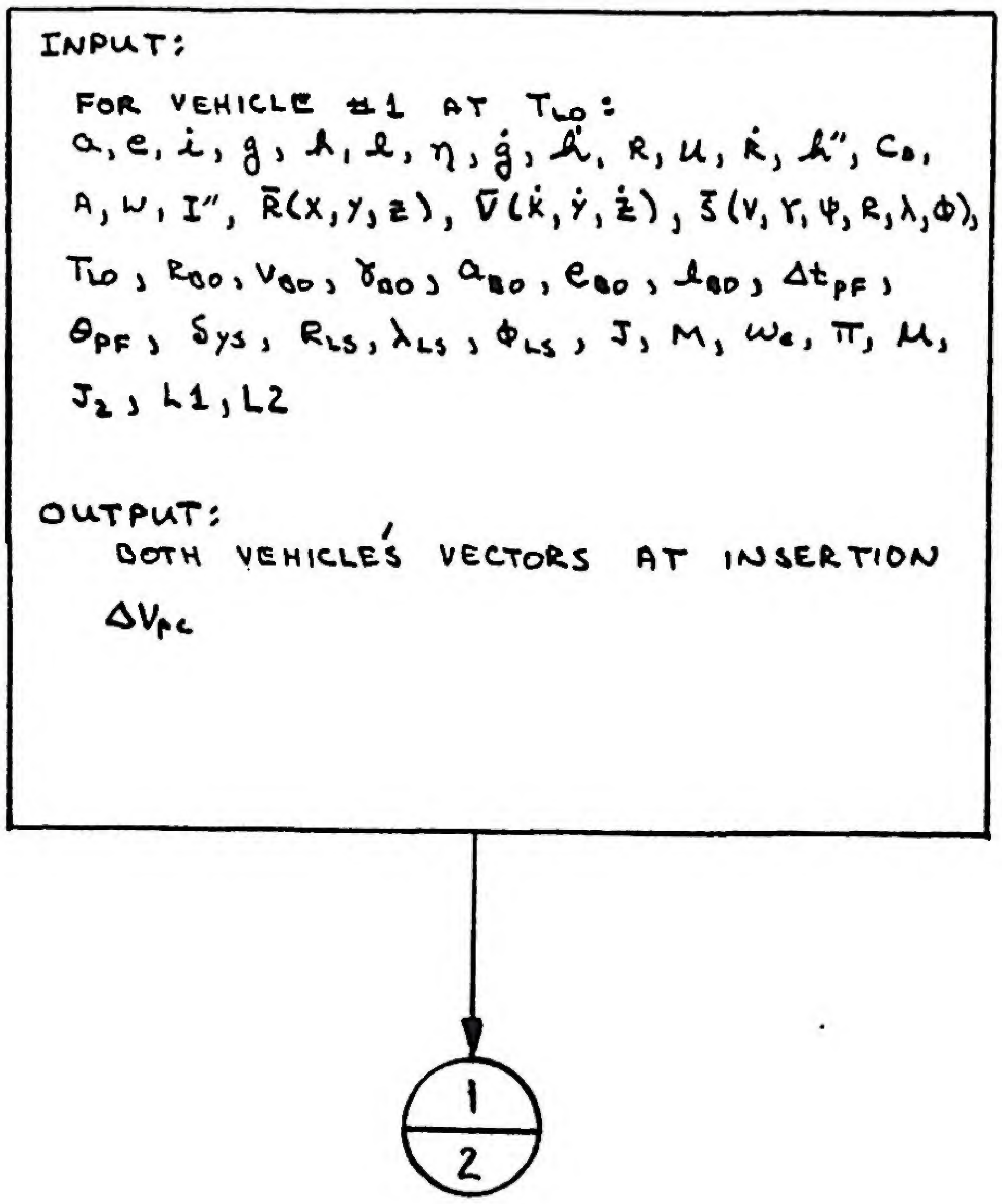
$$\Delta u = 2 \tan^{-1} \left[\frac{\tan\left(\frac{1}{2}(h_A - h_B)\right) \left[-\cos\left(\frac{1}{2}(i_{L2} + i_B)\right)\right]}{\cos\left(\frac{1}{2}(i_{L2} - i_B)\right)} \right]$$

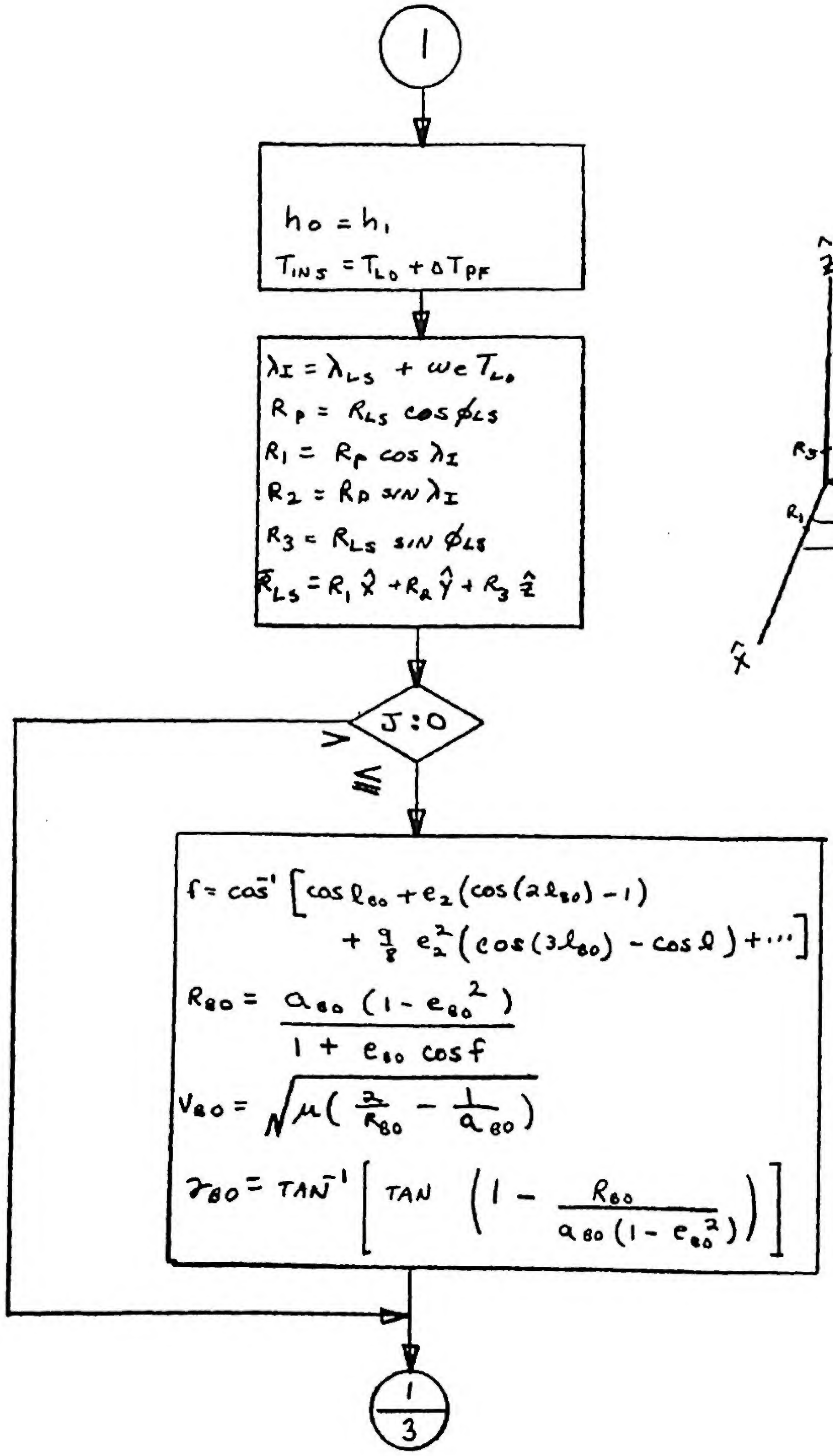
$$f = u_{L2} - \Delta u - g_{L2}$$

$$l_{L2} = f - 2e_{L2} \sin(f) + \frac{3}{4} e_{L2}^{2} \sin(2f) - \frac{1}{3} e_{L2}^{3} \sin(3f) + \cdots$$

SUBROUTINE ENSERT

SUBROUTINE TO COMPUTE AN INSERTION VECTOR.





$$h_0 = h_1$$

$$T_{INS} = T_{LO} + \Delta T_{PF}$$

$$\lambda_I = \lambda_{LS} + \omega_e T_{LO}$$

$$R_p = R_{LS} \cos \phi_{LS}$$

$$R_1 = R_p \cos \lambda_I$$

$$R_2 = R_p \sin \lambda_I$$

$$R_3 = R_{LS} \sin \phi_{LS}$$

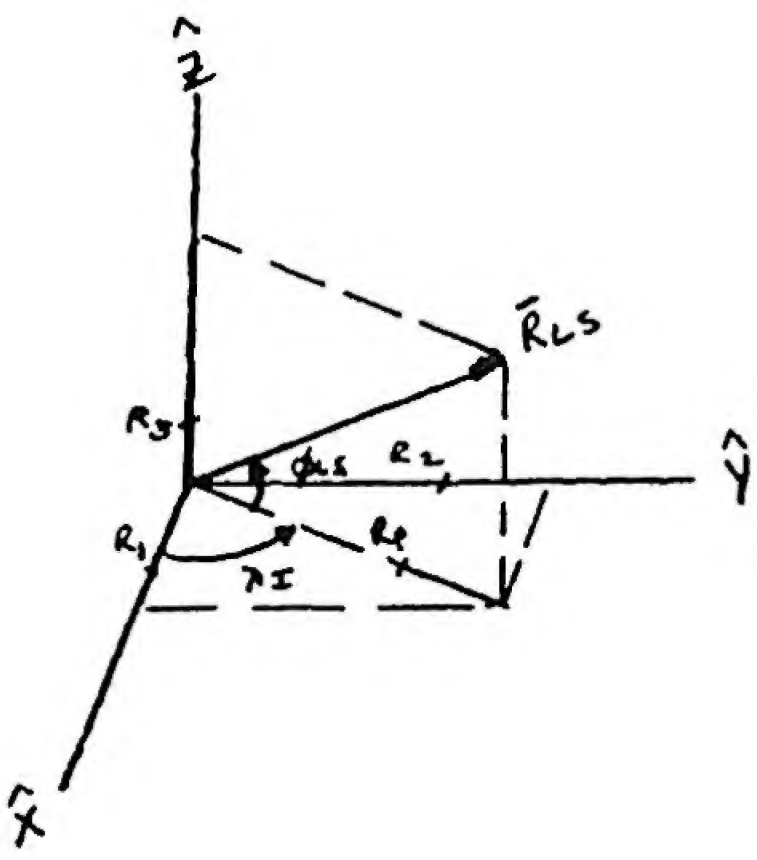
$$\bar{R}_{LS} = R_1 \hat{x} + R_2 \hat{y} + R_3 \hat{z}$$

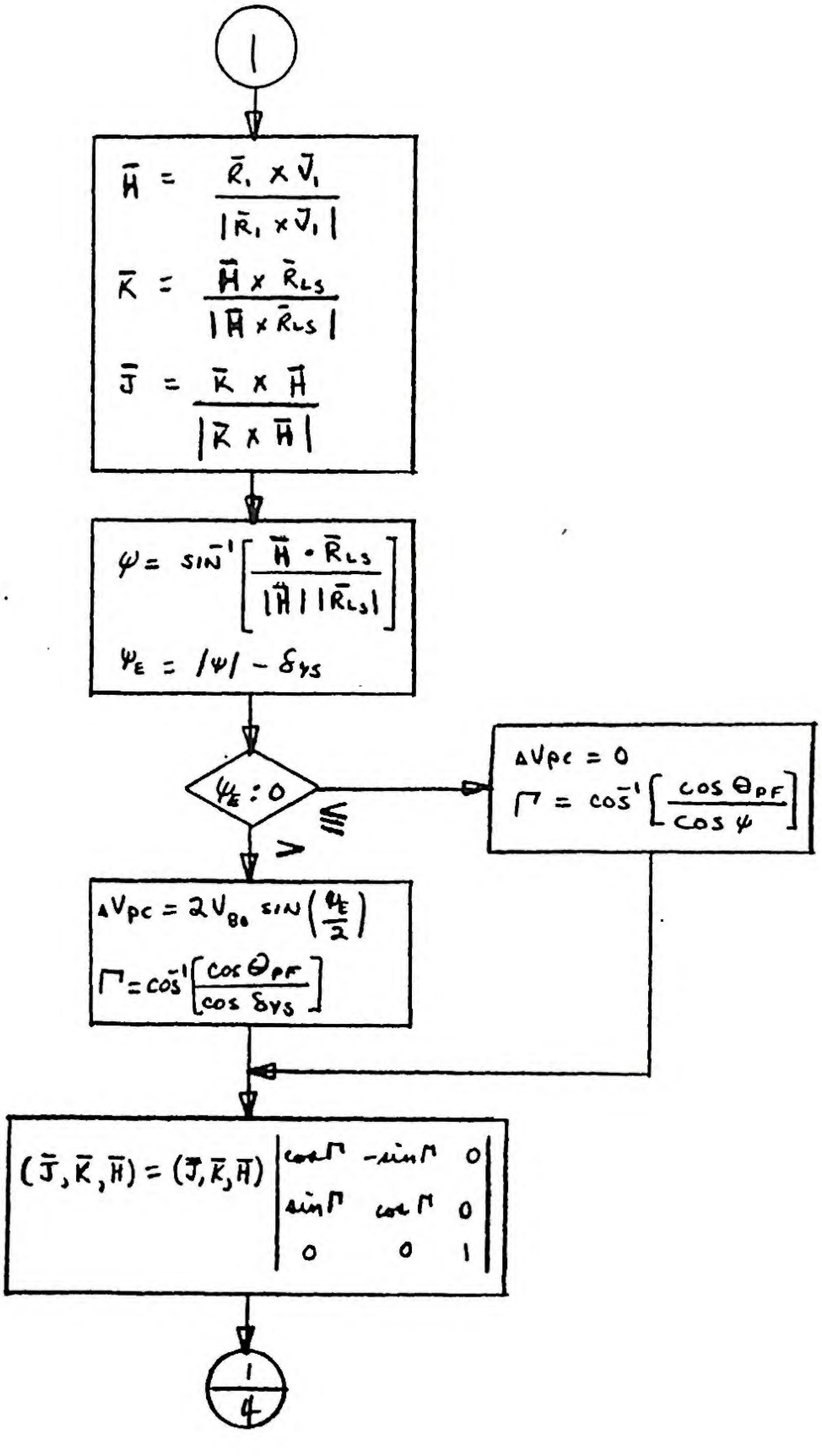
$$f = \cos^{-1}\left[\cos \ell_{BO} + e_2(\cos(2\ell_{BO}) - 1) + \frac{9}{8} e_2^2(\cos(3\ell_{BO}) - \cos \ell) + \cdots\right]$$

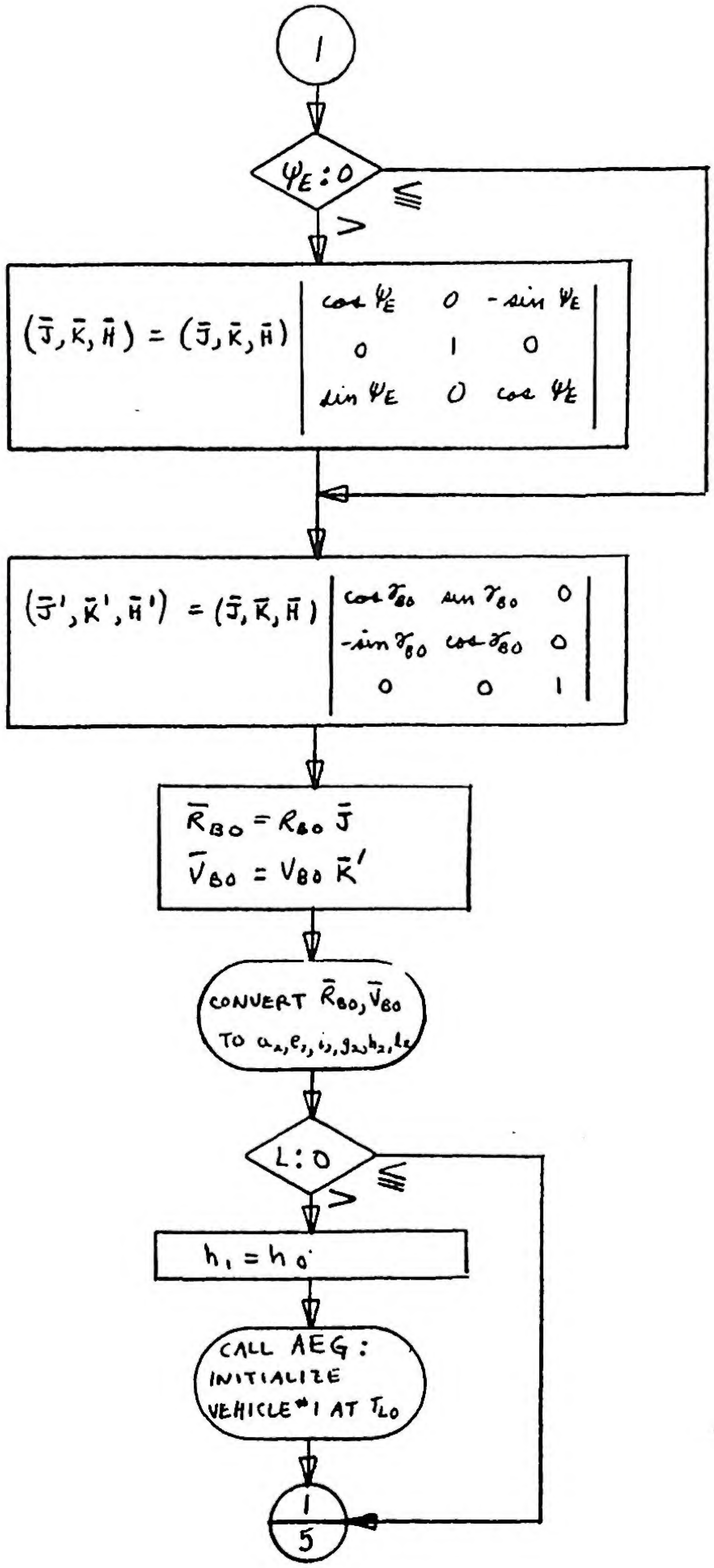
$$R_{BO} = \frac{a_{BO}(1 - e_{BO}^2)}{1 + e_{BO} \cos f}$$

$$V_{BO} = \sqrt{\mu\left(\frac{2}{R_{BO}} - \frac{1}{a_{BO}}\right)}$$

$$\gamma_{BO} = \tan^{-1}\left[\tan\left(1 - \frac{R_{BO}}{a_{BO}(1 - e_{BO}^2)}\right)\right]$$




1
$\bar{H} = \frac{\bar{R}_1 \times \bar{V}_1}{|\bar{R}_1 \times \bar{V}_1|}$
$\bar{K} = \frac{\bar{H} \times \bar{R}_{LS}}{|\bar{H} \times \bar{R}_{LS}|}$
$\bar{J} = \frac{\bar{K} \times \bar{H}}{|\bar{K} \times \bar{H}|}$
$\psi = \sin^{-1}\left[\frac{\bar{H} \cdot \bar{R}_{LS}}{|\bar{H}||\bar{R}_{LS}|}\right]$
$\psi_E = |\psi| - \delta_{YS}$
$\psi_E : 0$
$\leq$
$>$
$\Delta V_{PC} = 0$
$\Gamma = \cos^{-1}\left[\frac{\cos\Theta_{PF}}{\cos\psi}\right]$
$\Delta V_{PC} = 2V_{B0}\sin\left(\frac{\psi_E}{2}\right)$
$\Gamma = \cos^{-1}\left[\frac{\cos\Theta_{PF}}{\cos\delta_{YS}}\right]$
$(\bar{J},\bar{K},\bar{H}) = (\bar{J},\bar{K},\bar{H})\begin{vmatrix}\cos\Gamma & -\sin\Gamma & 0\\ \sin\Gamma & \cos\Gamma & 0\\ 0 & 0 & 1\end{vmatrix}$
$\frac{1}{4}$


1
$\psi_E : 0$
$\leqq$
$>$
$(\bar{J}, \bar{K}, \bar{H}) = (\bar{J}, \bar{K}, \bar{H}) \begin{vmatrix} \cos\psi_E & 0 & -\sin\psi_E \\ 0 & 1 & 0 \\ \sin\psi_E & 0 & \cos\psi_E \end{vmatrix}$
$(\bar{J}', \bar{K}', \bar{H}') = (\bar{J}, \bar{K}, \bar{H}) \begin{vmatrix} \cos\gamma_{BO} & \sin\gamma_{BO} & 0 \\ -\sin\gamma_{BO} & \cos\gamma_{BO} & 0 \\ 0 & 0 & 1 \end{vmatrix}$
$\bar{R}_{BO} = R_{BO}\,\bar{J}$
$\bar{V}_{BO} = V_{BO}\,\bar{K}'$
CONVERT $\bar{R}_{BO}, \bar{V}_{BO}$ TO $a_2, e_2, i_2, g_2, h_2, l_2$
$L : 0$
$\leqq$
$>$
$h_1 = h_0$
CALL AEG: INITIALIZE VEHICLE #1 AT T_{LO}
1
5

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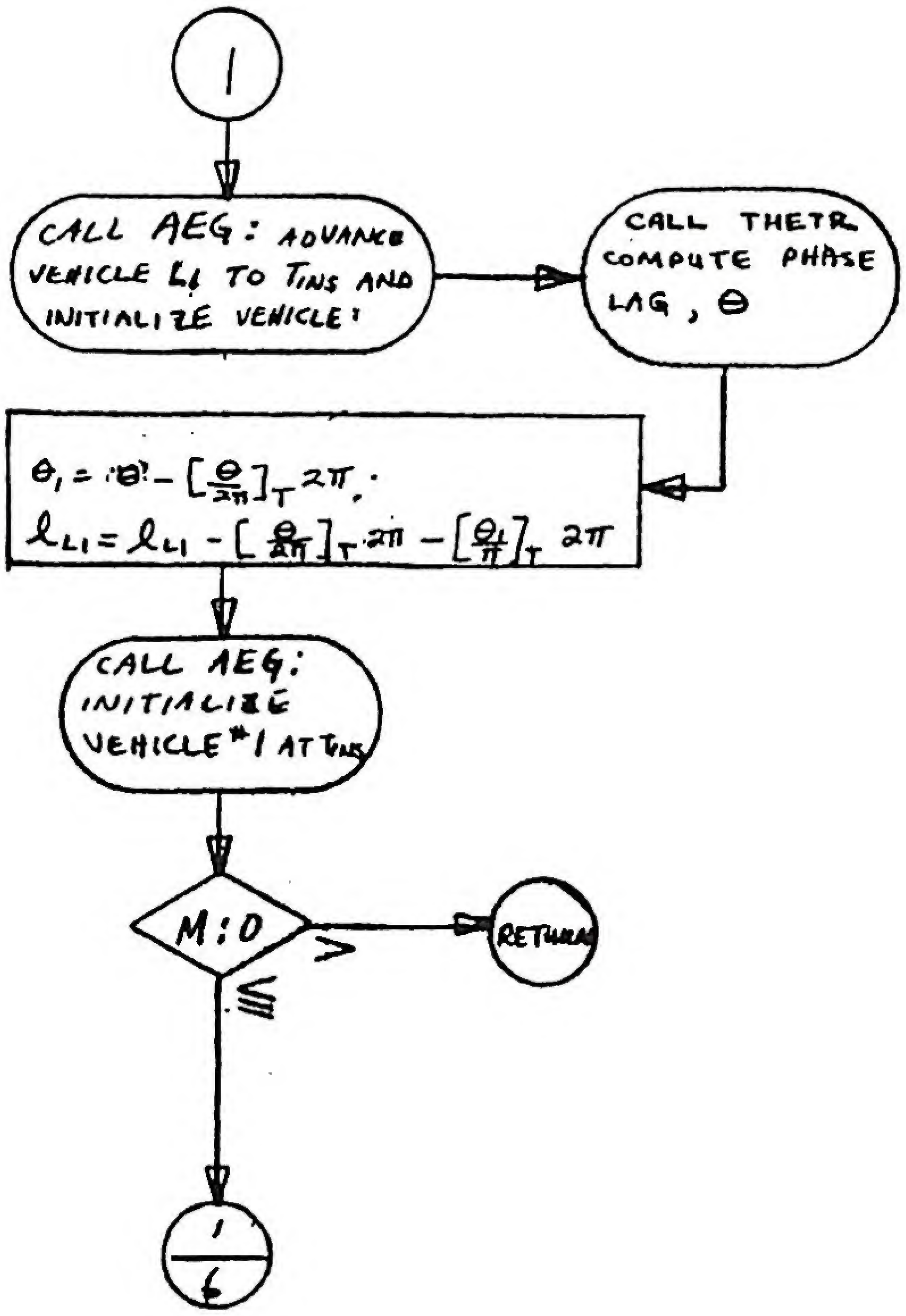
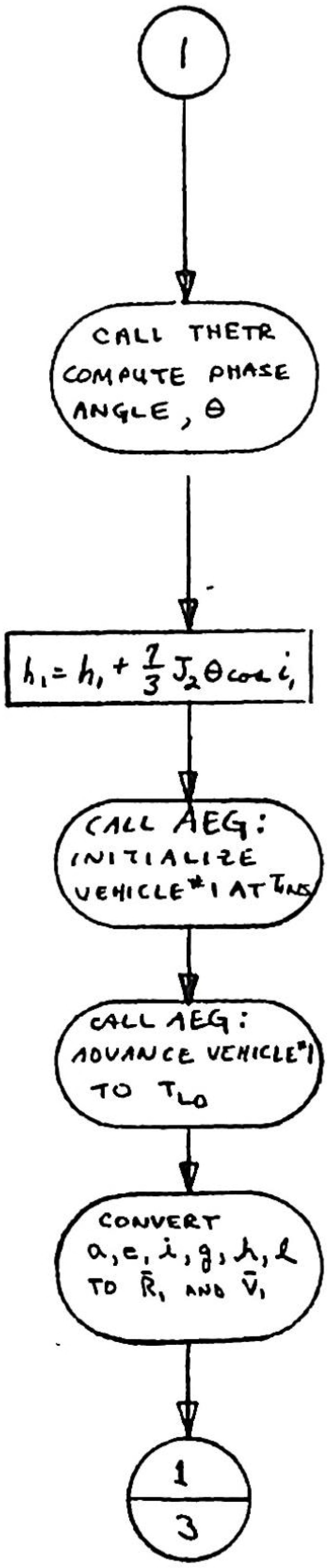

1
CALL AEG: ADVANCE VEHICLE L_1 TO T_{INS} AND INITIALIZE VEHICLE 1
CALL THETR COMPUTE PHASE LAG, Θ
$\theta_1 = \Theta - \left[\frac{\Theta}{2\pi}\right]_T 2\pi$
$\ell_{L1} = \ell_{L1} - \left[\frac{\Theta}{2\pi}\right]_T 2\pi - \left[\frac{\Theta_L}{\pi}\right]_T 2\pi$
CALL AEG: INITIALIZE VEHICLE #1 AT T_{INS}
M : 0
>
RETURN
≦
1
6


1
CALL THETR
COMPUTE PHASE
ANGLE, θ
$h_1 = h_1 + \frac{2}{3} J_2 \theta \cos i_1$
CALL AEG:
INITIALIZE
VEHICLE #1 AT T_{INS}
CALL AEG:
ADVANCE VEHICLE #1
TO T_{LO}
CONVERT
a, e, i, g, h, l
TO $\bar{R}_1$ AND $\bar{V}_1$
1
3

REFERENCES

1. Moore, A. J. and Reini, W. A.: Logic and Equations for Analytic Ephemeris Generation, MSC Internal Note No. 64-FM-50, Nov. 23, 1964.

2. Dugge, P.: Effect of Nodal Regression During Catchup Upon Out-of-Plane Angle, MAC Gemini Design Note No. 58, Dec. 7, 1962.

3. Moore, A. J. and Reini, W. A.: Determination of Orbital Arrival Times and the Relative Conditions Between the Agena and Gemini Spacecraft Vehicles at an Orbital Comparison Point, MSC Internal Note No. 64-FM-75, Nov. 30, 1964.

4. Knoedler, J. T.: LEM Lunar Launch Planning Display Processor Logic (Revision 1), TRW 3838-H004-R000, April 29, 1966.

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