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LOGIC FOR THE EARTH ORBITAL AEG IN THE APOLLO REAL-TIME RENDEZVOUS SUPPORT PROGRAM

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MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
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PROJECT APOLLO

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By Edward J. Kenyon

SUMMARY AND INTRODUCTION

This report presents the program logic and equations for the analytic ephemeris generator (AEG) that will be used in the Apollo real-time rendezvous support program (ARRS - ref. 1) for earth orbital mission planning and in the Real-Time Computer Complex (RTCC). The AEG was initially developed in 1964 (ref. 2); the drag computation has since been modified. The modified drag routine and the complete flow charts are presented herein.

The development of the AEG is thoroughly described in references 2 through 4, and the development of drag computations is presented in reference 3. Hence, this report only deals with changes incorporated into the computation of drag effects.

Changes to the Drag Computation

Basically, the changes to the drag computation can be broken down into three areas:

1. The method by which Simpson's rule is applied has been changed. Previously the effects of drag were multiplied by the proper Simpson's rule multiplier at the end of each drag step (usually 450 seconds) until the total update time had been reached. Presently each drag step is treated independently; it is integrated as if the step were the total update time. This method then divides the step into two equal parts, which produce the three points needed for the Simpson's rule integration. The process is repeated until the desired time is reached. This method has been found to be much more accurate than the one presented in the original flow charts (ref. 2).

2. In the earlier program the constants of integration, A1 and A2, found in the equations for the elements (e sin g) and (e cos g) in subroutine DRAG were computed only one time for each vector update. Now, to improve the calculation of the drag force, TD, these constants are

evaluated at the beginning of each drag step using the drag-modified osculating elements produced at the end of the previous integration step.

3. The last change is in the AEG program logic. The original version sent the DRAG subroutine a set of osculating and mean elements at the beginning of the vector update, and DRAG used these elements to integrate out to the desired time. This method is a good one as long as the update is not much greater than 1 day. Longer updates should have the opportunity of using the AEG's complete equations so that all perturbations due to the earth's potential can be accurately applied to the elements. The change allows this to be done; the elements are now recomputed every 24 hours by letting the AEG break the total update into 24-hour segments which are integrated separately by subroutine DRAG. Naturally, the last segment does not have to be equal to a full day. Using this improved method, the perturbations due to drag are added to a better set of mean elements, which, in turn, improves the overall prediction.

USE OF THE AEG AND DRAG SUBROUTINES

For the sake of brevity, the AEG and DRAG subroutines will be referred to collectively as "Subroutine AEG". The equations and logic have been programed in Fortran IV, and card decks are available for local usage. The argument of subroutine AEG is written (T, L_1, L_2) and the common, /AEG/ (A, E, AI, G, H, AL, AN, GN, HN, R, U, K_1, K_2, RD, HDP, CD, ARE, WHT, AIDP, RECT, SPHER, PERIOD) where

A_i	a, semimajor axis, ft
E_i	e, eccentricity
AI_i	I, inclination, rad
G_i	g, argument of perigee, rad
H_i	h, inertial longitude of ascending node, rad
AL_i	l, mean anomaly, rad
AN_i	n, mean motion
GN_i	g, secular rate of advance of argument of perigee, rad/sec
HN_i	h, secular rate of regression of node, rad/sec

R_i	r, radius, ft
U_i	u, argument of latitude
T	t, time, sec
L_1, L_2	vehicle indices, 1, 1 for the first vehicle coordinates only; 1, 2 for both vehicles; 2, 2 for the second vehicle only
K_{1i}	initialization control number: zero to initialize, 1 otherwise
K_{2i}	drag control number: 1 if drag effects are to be computed, zero if not
RD_i	r, radius rate, fps
HDP_i	h", secular node, rad
CD_i	C_D, drag coefficient
ARE_i	A_L, frontal area of vehicle, ft^2
WHT_i	W_L, weight of vehicle, lb
$AIDP_i$	I", mean inclination, rad
RECT	X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$
SPHER	V, γ, ψ, R, λ, ϕ
γ	flight path angle
ψ	azimuth
λ	longitude
ϕ	latitude

All quantities except T, L_1, and L_2 are dimensioned variables, with two locations reserved for each. Orbital elements are input to the program only when they are to be initialized ($K_{1i} = 0$). The drag constants CD_i, ARE_i, and WHT_i are always input, as are K_{2i}, K_{3i}, L_1, L_2, and T. The rest are always output.

An example of the operations of subroutine AEG is as follows

Initialization

Define values (through input) for A_i, E_i, AI_i, G_i, H_i, AL_i, C_{Di}, ARE_i, WHT_i ($i = L_1, L_2$), and set $K_{1i} = 0$. Call the AEG subroutine (K_{1i} will be set to 1 after initialization is complete). If an error of input has been made (e.g., $G_i > \pi$), the message "AEG WILL NOT INITIALIZE" will be printed.

Orbit Prediction

To obtain a set of classical elements describing the vehicle's position and velocity at a time different from that at initialization, define new values for T, K_{2i}, reset L_1, and L_2, and call the AEG subroutine. Output will be the orbital elements and the quantities AN_i, GN_i, HN_i, R_i, U_i, GD_i, RD_i, HDP_i, and $AIDP_i$.

TABLE I.- DEFINITIONS OF AEG FLOW CHART SYMBOLS

Variable	Definition	Unit
a	Semimajor axis	ft
e	Eccentricity	
I	Inclination	rad
g	Argument of perigee	rad
h	Longitude of ascending node	rad
ℓ	Mean anomaly	rad
n	Mean motion	rad/sec
$\dot{g}$	Secular rate of change of argument of perigee	rad/sec
$\dot{h}$	Angular rate of change of the longitude of the ascending node	rad/sec
R_L	Radius to either vehicle	ft
U	Argument of latitude	rad
$\dot{g}_D$	Secular rate of change of argument of perigee due to drag	rad/sec
t	EPOC	sec
L_1	Refers to Agena	
L_2	Refers to spacecraft	
K_1	Initialization control for AEG (0 for Initialization, 1 for no initialization)	
K_2	Drag control for AEG	
$\dot{R}_L$	Radius rate of change	fps
h''	Mean longitude of the ascending node	rad
C_D	Drag coefficient	
A_L	Frontal area of vehicle	ft^2
W_L	Weight of vehicle	lb
K_3	Control number for reduced AEG	

TABLE I.- DEFINITIONS OF AEG FLOW CHART SYMBOLS - Concluded

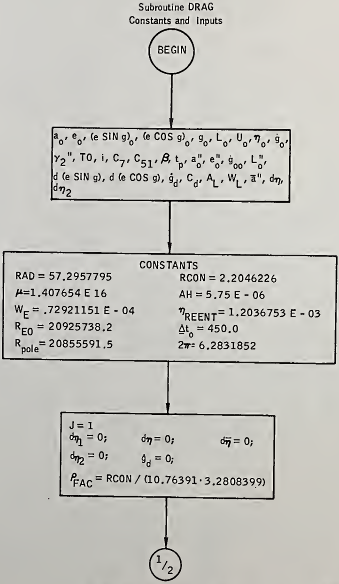
Variable	Definition	Unit
I''	Mean inclination	rad
	Subscripts	
I	Initial quantity	
i	Vehicle index	
j	Initialization iteration index	

TABLE II.- DEFINITION OF SUBROUTINE DRAG FLOW CHART SYMBOLS

Variable	Definition	Unit
a	Semimajor axis	ft
e	Eccentricity	
g	Argument of perigee	rad
e sin g	Eccentricity multiplied by the sine of argument of perigee	
e cos g	Eccentricity multiplied by the cosine of argument of perigee	
L	Argument of longitude	rad
U	Argument of latitude	rad
n	Mean motion number	
$\dot{g}$	Secular rate of change of argument of perigee	rad/sec
γ_2'	Term computed in AEG	
C_1	Constants computed in AEG	
C_2	Constants computed in AEG	
C_3	Constants computed in AEG	
β	Sine of mean inclination	
Δt_D	Interval over which drag computed	sec
a"	Mean semimajor axis	ft
e"	Mean eccentricity	
d_g	Change in "g" due to drag	
L"	Mean argument of longitude $(g + \ell)$	
d(e sin g)	Change in "e sin g" due to drag	
d(e cos g)	Change in "e cos g" due to drag	
$\dot{g}_i$	Secular rate of change of perigee due to drag	rad/sec

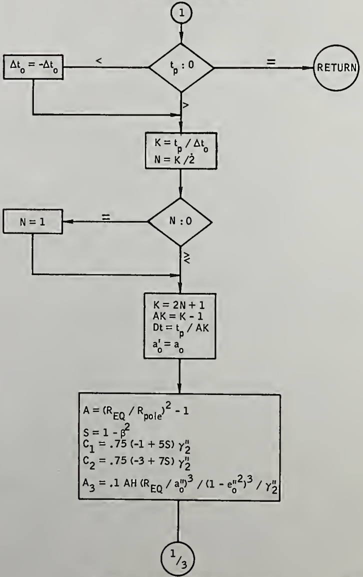
TABLE II.- DEFINITION OF SUBROUTINE DRAG FLOW CHART SYMBOLS - Concluded

Variable	Definition	Unit
C_L	Drag coefficient	
A_L	Frontal area of vehicle	ft^2
W_L	Weight of vehicle	lb
$\overline{a}''$	Average mean "A" between two points	ft
dN	Average rate of change of mean motion number due to drag	sec^{-1}
C_{30}	Constant computed in AEG	



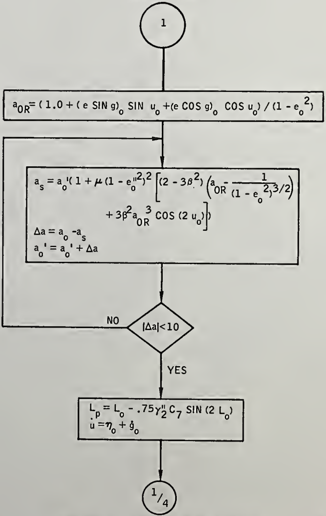
Flow chart 2.- Subroutine DRAG.

Time step and increment setup

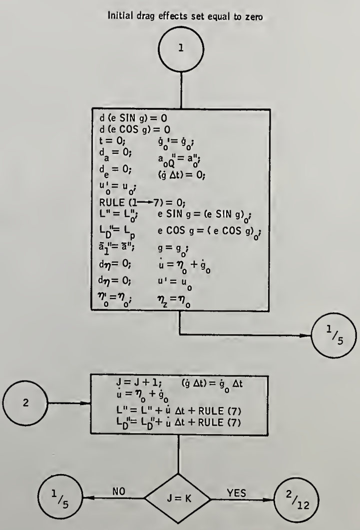


Flow chart 2.- Subroutine DRAG - Continued.

Mean a calculation



Flow chart 2.- Subroutine DRAG - Continued.



Flow chart 2.- Subroutine DRAG - Continued.

Little AEG section

1

$$L = L_D'' + .75\ \gamma_2'' C_7 \ \mathrm{SIN}\ (2\ L_D'')$$

$$g = g + \mathrm{RULE}\ (4)$$

$$A_1 = \mathrm{SIN}\ g\ (e\ \mathrm{SIN}\ g) + \mathrm{COS}\ g\ (e\ \mathrm{COS}\ g) - C_2\ (\mathrm{SIN}\ u') - C_1\ (\mathrm{COS}\ u') - \gamma_2'' C_{51}\ \mathrm{COS}\ (3\ u' - g)$$

$$A_2 = \mathrm{COS}\ g\ (e\ \mathrm{SIN}\ g) - \mathrm{SIN}\ g\ (e\ \mathrm{SIN}\ g) - C_2 (\mathrm{COS}\ u') + C_1\ (\mathrm{COS}\ u') - \gamma_2'' C_{51}\ \mathrm{SIN}\ (3\ u' - g)$$

$$C = g + (\dot{g}\ \Delta t)$$

$$(e\ \mathrm{SIN}\ g) = A_1\ \mathrm{SIN}\ C + A_2\ \mathrm{COS}\ C + C_2\ \mathrm{SIN}\ L + \gamma_2'' C_{51}\ \mathrm{SIN}\ (3\ L) + A_3 [\mathrm{COS}\ (\dot{g}\ \Delta t) - 1] + \mathrm{RULE} (5)$$

$$(e\ \mathrm{COS}\ g) = A_1\ \mathrm{COS}\ C - A_2\ \mathrm{SIN}\ C + C_1\ \mathrm{COS}\ L + \gamma_2'' C_{51}\ \mathrm{COS}\ (3\ L) + A_3 [\mathrm{SIN}\ (\dot{g}\ \Delta t)] + \mathrm{RULE}\ (6)$$

$$g = \mathrm{TAN}^{-1} \left[(e\ \mathrm{SIN}\ g) / (e\ \mathrm{COS}\ g) \right]$$

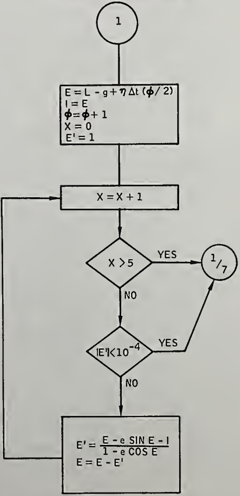
$$e = \sqrt{(e\ \mathrm{SIN}\ g)^2 + (e\ \mathrm{COS}\ g)^2}$$

$$\phi = 0$$

1/6

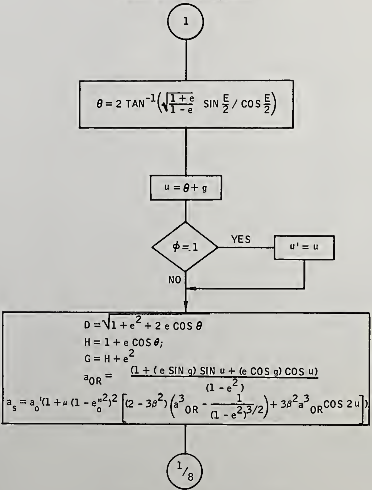
Flow chart 2.- Subroutine DRAG - Continued.

Calculation of eccentric anomaly



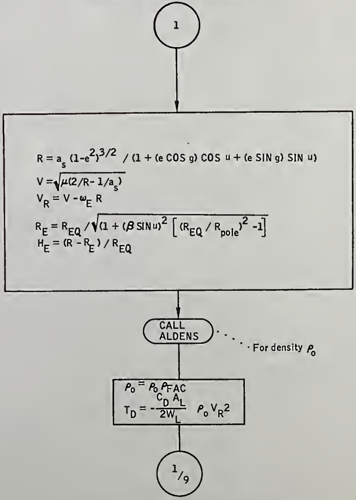
Flow chart 2.- Subroutine DRAG - Continued.

Factors for variation equations



Flow chart 2.- Subroutine DRAG - Continued.

Calculation of density and drag force T_D



Flow chart 2.- Subroutine DRAG - Continued.

Change in orbital elements due to drag

(1)

$$a_D = 2\ T_D\ D\ /\ (\eta_o\ (1 - e^2)^{1/2})$$

$$X = 2\ (1 - e^2)^{1/2}\ T_D\ /\ (a_s\ \eta_o\ D)$$

$$e_D = X\ (e + \text{COS}\,\theta)$$

$$(e\dot{g})_D = X\ \text{SIN}\ (\theta)$$

$$\dot{g}_D = (e\dot{g})_D\ /\ e$$

$$(e\ \text{SIN}\ g)_D = X\ (\ (e\ \text{SIN}\ g) + \text{SIN}\ u)$$

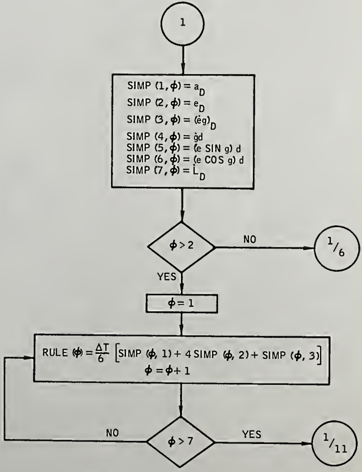
$$(e\ \text{COS}\ g)_D = X\ (\ (e\ \text{COS}\ g) + \text{COS}\ u)$$

$$\dot{L}_D = \frac{(e\dot{g})_D D^2}{H^2\ (H + (1 + e^2)^{1/2}\ G)} \left[1 - e^2\ (1 + \text{COS}^2\theta - 2\ e^3\ \text{COS}\,\theta - e^4\right]$$

(1/10)

Flow chart 2.- Subroutine DRAG - Continued.

Simpson's rule section



Flow chart 2.- Subroutine DRAG - Continued.

Summation of drag effects

(1)

$d_a = d_a + \text{RULE (1)}$

$d_e = d_e + \text{RULE (2)}$

$d\,(e\,\text{SIN}\,g) = d\,(e\,\text{SIN}\,g) + \text{RULE (5)}$

$d\,(e\,\text{COS}\,g) = d\,(e\,\text{COS}\,g) + \text{RULE (6)}$

$d\,(\dot{g}d) = d\,(\dot{g}d) + \text{RULE (4)}$

$\bar{a}'' = \bar{a}'' + d_{a/2}$

$\eta_o = \eta_o + d\eta$

$\dot{g}_o = \dot{g}_o\,(\eta_o)$

$d\eta = -3/4\, d_a\, \eta_o / \bar{a}''$

$\eta_o = \eta_o + d\eta$

$\dot{g}_o = \dot{g}_o / \eta_o$

$d\bar{\eta} = d\bar{\eta} + 2d\eta$

$\eta_z = \eta_o' + d\bar{\eta}$

$\dot{g}_z = \dot{g}_o' \eta_z / \eta_o'$

$d\eta' = d\eta' + 2d\eta$

$\eta_Q = \eta_Q + 2d\eta$

$a_o' = a_o' + \text{RULE (1)}$

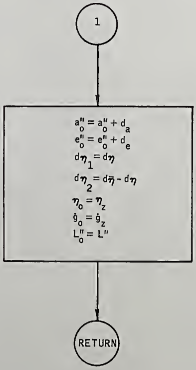
$a_{oQ}'' = a_{oQ}'' + \text{RULE (1)}$

$\bar{a}_1'' = a_{oQ}''$

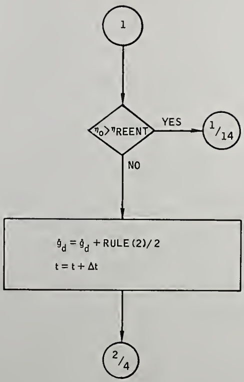
(1/13)

Flow chart 2.- Subroutine DRAG - Continued.

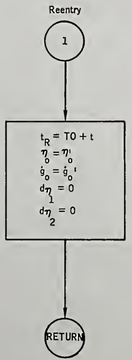
Modifying and setting quantities to be returned in subroutines argument



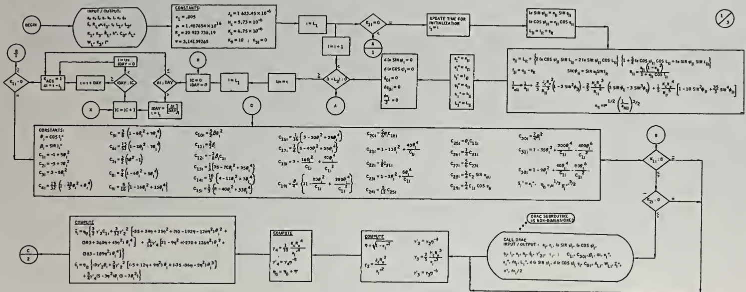
Flow chart 2.- Subroutine DRAG - Continued.



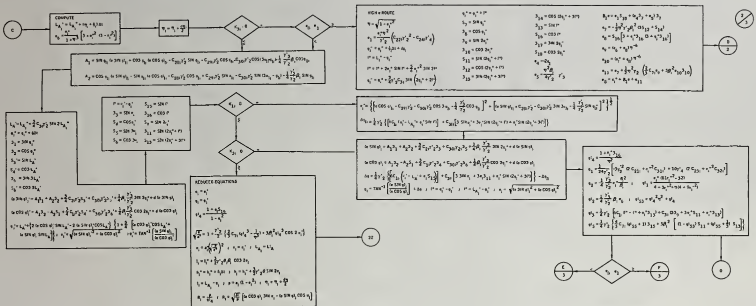
Flow chart 2.- Subroutine DRAG - Continued.



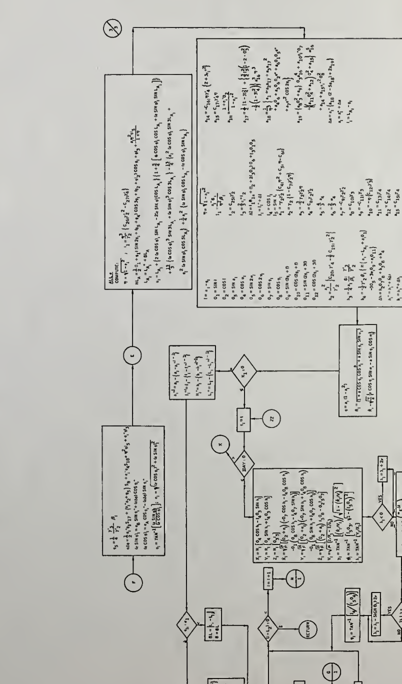
Flow chart 2.- Subroutine DRAG - Concluded.



Flow chart 1. - Subroutine AEC.



Flow chart 1. - Subroutine AEG - Continued.



Flow chart 1.- Subroutine AEG - Concluded.

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