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February 24, 1970

RTCC OFFLINE REQUIREMENTS FOR H-2: STAR-HORIZON OBSERVATION PROCESSOR

Mathematical Physics Branch
MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

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February 17, 1970

MEMORANDUM TO: See list below

FROM : FM/Mission Planning and Analysis Division

SUBJECT : Formulation for RTCC offline processing of star-horizon sightings

The attached internal note (69-FM-326) presents the equations and logic requirements for the RTCC offline star-horizon observation processor.

The offline processor determines the horizon altitude and trunnion bias using telemetered sextant data.

Emil R. Schiesser
Assistant Chief
Mathematical Physics Branch

The Flight Software Branch concurs with the above recommendation and requests IBM to proceed accordingly.

James C. Stokes, Jr., Chief
Flight Software Branch

APPROVED BY:

John P. Mayer
Chief, Mission Planning
and Analysis Division

Enclosure

Addressees: (See attached list)

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MSC INTERNAL NOTE NO. 69-FM-326

PROJECT APOLLO

RTCC OFFLINE REQUIREMENTS FOR H-2: STAR-HDRIZON OBSERVATION PRDCESSOR

By Paul Flanagan, Mathematical Physics Branch,
and Robert Kidd, TRW Systems Group

February 24, 1970

MISSION PLANNING AND ANALYSIS DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATIDN
MANNED SPACECRAFT CENTER
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Approved: James C. McPherson, Chief
Mathematical Physics Branch

Approved: John P. Mayer, Chief
Mission Planning and Analysis Division

RTCC OFFLINE REQUIREMENTS FOR H-2:

STAR-HORIZON OBSERVATION PROCESSOR

By P. Flanagan and R. Kidd

1.0 SUMMARY AND INTRODUCTION

During translunar and transearth coast, the ground will estimate the horizon altitude and trunnion bias from star-horizon optical measurements telemetered from the CSM. The purpose of this note is to present the formulation (basic requirements) for the offline RTCC program. Two program options are described. This program will provide the capability to process sextant trunnion angle data only and to include platform and vehicle orientation information and the sextant shaft angle measurements. The same prediction module used for the MSFN orbit determination processor will be used to determine the CSM ephemeris for the data span. The computations require the use of moon- and earth-fixed to basic reference (MNBY) transformations and access to the basic reference star catalogue. The flow chart for the effective horizon processor is presented in section 11.

2.0 SYMBOLS

CSM	command/service module
MNBY	basic reference coordinate system, mean of the nearest Besselian year
$\underline{A}^T_j$	partial derivative matrix of jth measurement
Δm_j	residual for jth measurement
ho	initial estimate of horizon altitude
bT_o	initial estimate of trunnion bias
w_j	observation weight for jth measurement

B_o	initial covariance matrix obtained from input standard deviations and correlation coefficient
a_o, b_o, c_o	semiaxes for the earth or moon reference ellipsoid
REFSMMAT	basic reference to stable member transformation matrix
$\underline{R}_v$	radius vector to vehicle
$\underline{U}$	unit vector to the star
$\underline{R}_s$	radius vector to the surface
$\underline{N}(\underline{R}_s)$	vector normal to the surface
m	star/horizon trunnion measurement
σ	standard deviation
K_j	weight scale factor
SC	trunnion standard deviation
LOS	line of sight
I_g, M_g, O_g	inner, middle, and outer gimbal angles, respectively
ξ	offset of the optical shaft axis from the z-body axis in the body x-z plane toward the x-body axis
Sh	optics shaft angle measurement
Tr	optics trunnion angle measurement
$\theta(t)$	transformation from the basic reference system to the moon- or earth-fixed system

3.0 PROCEDURE FOR PROCESSING STAR-HORIZON SEXTANT OBSERVATIONS

Ground processing of star-horizon observations is a job shop procedure (i.e., all input will be entered on cards which are similar to real-time MED entries; after the requested solutions have been obtained, the output will be printed). The data to be processed, the initialization parameters, and the CSM ephemeris vector are manually selected and are entered in card format.

The predictor is used to find the CSM position and velocity at the time of the first observation. Then a CSM ephemeris is generated across the time span of the data.

The estimate of the horizon altitude and trunnion bias is updated with the assumption that the CSM position is known. A two by two initial covariance matrix for the state vector (horizon altitude and trunnion bias) is selected for use in the process.

A sequential batch solution may be obtained by selection of the initial estimate of the state vector, associated standard deviations, and correlation coefficient from a previous batch solution.

The horizon altitude and the trunnion bias estimates are obtained by an iterative Bayes' estimate given in equation (1).

$$\Delta\underline{X}_i = \left(\sum_j \underline{A}_j^T w_j \underline{A}_j + B_o^{-1}\right)^{-1} \left[\sum_j \underline{A}_j^T w_j \; \Delta m_j - B_o^{-1}(\underline{X}_{i-1} - \underline{X}_o)\right] \quad (1)$$

where $\underline{X}_i = \underline{X}_{i-1} + \Delta\underline{X}_i$

$$\underline{X}_o = \begin{bmatrix} ho \\ bTo \end{bmatrix}$$

$\underline{A}_j^T$ = partial derivative vector of jth measurement

$\Delta m_j = Tr_j - m_j$; residual for jth measurement

ho - initial estimate of horizon altitude

bT_o - initial estimate of trunnion bias

w_j - observation weight for the jth measurement

B_o - initial covariance matrix obtained from input standard deviations and correlation coefficient

The iteration of this equation will continue through a number specified by input. For each iteration, the current estimate for the horizon altitude and the trunnion bias are used to obtain the computed measurement and the partial derivatives. The trunnion bias is used only in the computation of the measurement m. The horizon altitude is used to change the semiaxes by use of equation (2).

$$a = a_o + h \quad (2a)$$

$$b = b_o + h \quad (2b)$$

$$c = c_o + h \quad (2c)$$

where a_o, b_o, c_o are the semiaxes of the earth or moon reference ellipsoid.

4.0 INITIALIZATION PREPROCESSING

A preprocessor is required to convert the input data to internal units of radians, earth radii, and earth radii/hour. The preprocessor stores the observation data into a batch with a maximum of 60 observations and a maximum data span of 24 hours. The preprocessor will store the following parameters in each observation frame: time, associated trunnion angle (deg), star ID (octal), near/far horizon flag, and a weight scale factor. Optional provision for shaft angles (deg) and gimbal angles (deg) and basic reference transformation matrix (REFSMMAT) is also required.

Additional input data required and the units are as follows.

a. CSM vector in basic reference coordinate system, e.r., e.r./hr

b. Earth/moon reference flag associated with observations (all observations in a single batch are referenced to only one body)

c. Initial estimate of horizon altitude, km

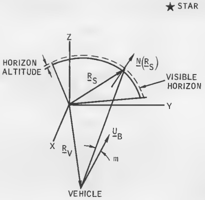
d. Initial estimate of trunnion bias, mrad

e. Initial standard deviations of the state vector (km, mrad) and associated correlation coefficient

f. Number of iterations

5.0 OBSERVATION MEASUREMENTS

A geometric description of the measurements is given in the following figure.



In this figure,

$\underline{R}_v$ is the radius vector to the vehicle

$\underline{U}_B$ is the unit vector to the star

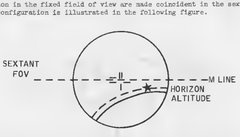
$\underline{R}_s$ is the radius vector to the surface

$\underline{N}(\underline{R}_s)$ is a vector normal to the surface

m is the star/horizon trunnion measurement

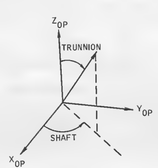
All vectors are in a moon- or earth-fixed coordinate system.

The star in the sextant movable field of view and the visible horizon in the fixed field of view are made coincident in the sextant. The configuration is illustrated in the following figure.



The observation is performed with the following restriction. The tangent to the horizon at the point sighted upon is parallel to the M line in the fixed field of view. With the sextant, this restriction implies that the observed trunnion motion of the star is normal to the surface at the point on the horizon sighted upon. Shaft, gimbal angles, and orientation angles are optionally used in the processor to evaluate and compensate for errors that occur while this procedure is being performed.

The geometrical description of the shaft and trunnion in the body-fixed optical system is shown below.



6.0 STATE VECTOR WEIGHTS

The covariance matrix associated with the state vector can be eliminated by equating the inverse with zero ($B_o^{-1} = 0$). If non-zero initial standard deviations and correlation coefficients are entered, the initial covariance matrix is computed from the following matrix.

$$B_o = \begin{bmatrix} \sigma_{ho}^2 & \rho_o \sigma_{ho} \sigma_{bTo} \\ \rho_o \sigma_{ho} \sigma_{bTo} & \sigma_{bTo}^2 \end{bmatrix}$$

where σ_{ho} is the initial horizon altitude standard deviation

σ_{bTo} is the initial trunnion bias standard deviation

ρ_o is the initial correlation coefficient

7.0 OBSERVATION WEIGHTS

A weight scale factor K_j is entered with each observation. The observation weight w_j for the jth measurement is computed by use of equation (3).

$$w_j = \frac{K_j}{(SC)^2} \tag{3}$$

where SC is a scalar constant trunnion standard deviation.

8.0 COMPUTED OBSERVATIONS

The computed horizon location can be found by two methods. For method 1, in which only the trunnion angle is used, it is assumed that the surface normal vector is in the plane defined by the vehicle location, horizon location, and star position. By method 2, the horizon location is computed directly from the LOS defined by trunnion, shaft, gimbal angles, and REFSMMAT.

8.1 Method 1

The trunnion measurements can be used to estimate the horizon altitude and the trunnion bias if it is assumed that the star horizon measurements satisfy the conditions shown by equations (4) through (6).

On the surface of the ellipse,

$$F(\underline{R}_s) = 1 \tag{4}$$

The surface normal is orthogonal to the vehicle/horizon LOS; that is,

$$(\underline{R}_s - \underline{R}_v) \cdot \underline{N}(\underline{R}_s) = 0 \tag{5}$$

The surface normal is in the plane defined by $\underline{U}_B$ and the vehicle/horizon LOS; that is,

$$[(\underline{R}_s - \underline{R}_v) \times \underline{U}_B] \cdot \underline{N}(\underline{R}_s) = 0 \tag{6}$$

where $\underline{N}(\underline{R}) \triangleq \begin{bmatrix} R_x/a^2 \\ R_y/b^2 \\ R_z/c^2 \end{bmatrix}$

$$F(\underline{R}) \triangleq \frac{R_x^2}{a^2} + \frac{R_y^2}{b^2} + \frac{R_z^2}{c^2}$$

and a, b, c are the semiaxes of the ellipse.

The solution for $\underline{R}_s$ from these equations will be obtained by an iteration that forces the second and third equations to zero. An initial solution is obtained from equations (7) through (9).

$$\underline{R}_s = K_1\underline{U}_B + K_2\underline{R}_v \tag{7}$$

$$K_1 = \pm \left\{\frac{F(\underline{R}_v) - 1}{F(\underline{U}_B)F(\underline{R}_v) - [N(\underline{R}_v) \cdot \underline{U}_B]^2}\right\}^{1/2} \tag{8}$$

where K_1 is positive for the near horizon and negative for the far horizon.

$$K_2 = \frac{1 - K_1 \underline{N}(\underline{R}_v \cdot \underline{U}_B)}{F(\underline{R}_v)} \tag{9}$$

The equations and sequence for the iteration are given by equations (10) and (11).

$$\underline{\Delta R}_s = \begin{bmatrix} 2\underline{N}^T(\underline{R}_s) \\ \underline{N}^T(\underline{R}_v) - 2\,\underline{N}^T(\underline{R}_s) \\ \underline{N}^T[(\underline{R}_s - \underline{R}_v) \times \underline{U}_B] + [\underline{U}_B \times \underline{N}(\underline{R}_s)]^T \end{bmatrix}^{-1} \begin{bmatrix} 1 - F(\underline{R}_s) \\ \underline{N}(\underline{R}_s) \cdot (\underline{R}_s - \underline{R}_v) \\ [\underline{U}_B \times (\underline{R}_s - \underline{R}_v)] \cdot \underline{N}(\underline{R}_s) \end{bmatrix} \tag{10}$$

$$\underline{R}_s = \underline{R}_s + \underline{\Delta R}_s \tag{11}$$

The convergence test will be that $|\underline{\Delta R}_s|$ will decrease monotonically or that it will be less than 10^{-6} e.r. or that both conditions apply.

8.2 Method 2

The position of the sighted horizon location satisfies equations (12a) through (12c).

On the surface of an ellipse,

$$F(\underline{R}_s) = 1 \tag{12a}$$

The surface normal is orthogonal to the vehicle/horizon LOS; that is,

$$(\underline{R}_s - \underline{R}_v) \cdot \underline{N}(\underline{R}_s) = 0 \tag{12b}$$

The surface position in the plane defined by vehicle and $\underline{U}'$, where $\underline{U}'$ is the approximate LOS from the spacecraft to the horizon.

$$(\underline{R}_v \times \underline{U}') \cdot \underline{R}_s = 0 \tag{12c}$$

The fixed LOS and the movable LOS of the sextant are used to define the position on the horizon that was used for the star horizon measurement. The unit vector $\underline{U}'$ to be used with the vehicle radius vector to define a plane that contains the position vector to the horizon is obtained from equations (13) through (15).

$$G = \begin{bmatrix} \cos Ig & 0 & \sin Ig \\ 0 & 1 & 0 \\ -\sin Ig & 0 & \cos Ig \end{bmatrix} \begin{bmatrix} \cos Mg & -\sin Mg & 0 \\ \sin Mg & \cos Mg & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos Og & -\sin Og \\ 0 & \sin Og & \cos Og \end{bmatrix} \tag{13}$$

$$\underline{w} = \theta(t)[\text{REFSMMAT}]^T G \begin{bmatrix} \cos \xi & 0 & \sin \xi \\ 0 & 1 & 0 \\ -\sin \xi & 0 & \cos \xi \end{bmatrix} \begin{bmatrix} \sin Sh \\ -\cos Sh \\ 0 \end{bmatrix} \tag{14}$$

$$\underline{U}' = \underline{U}_B + (1 - \cos Tr)\underline{w} \times (\underline{w} \times \underline{U}_B) + \sin Tr \, (\underline{w} \times \underline{U}_B) \tag{15}$$

where Ig, Mg, Og are the inner, middle, and outer gimbal angles, respectively

REFSMMAT is the transformation from the basic reference system to the spacecraft stable member system

$\xi = 32°31'23.19''$; the offset of the optical shaft axis from the Z-body axis in the body X-Z plane toward the X-body axis

Sh, Tr are the optics shaft and trunnion angle measurements, respectively

$\underline{U}_B$ is the unit star vector

$\theta(t)$ is the transformation from the basic reference system to the planet fixed system

The solution for $\underline{R}_s$ is given by equation (16).

$$\underline{R}_s = K_1\underline{U}' + K_2\underline{R}_v \tag{16}$$

where

$$K_1 = +\left[\frac{F(\underline{R}_v) - 1}{F(\underline{U}')F(\underline{R}_v) - [\underline{N}(\underline{R}_v) \cdot \underline{U}']^2}\right]^{1/2}$$

$$K_2 = \frac{1 - K_1\underline{N}(\underline{R}_v) \cdot \underline{U}'}{F(\underline{R}_v)}$$

8.3 Equations for Computed Observation

For both options, the computed LOS from the vehicle to the horizon is changed to an apparent LOS by equation (17).

$$\underline{U}_c = \text{unit}\left[\text{unit }(\underline{R}_s - \underline{R}_v) + \frac{\underline{V}_v}{c}\right] \tag{17}$$

where $\underline{V}_v$ is the velocity of the vehicle relative to the earth or moon

c is the speed of light

The computed observation is given by equation (18).

$$m = b_T + \cos^{-1}(\underline{U}_c \cdot \underline{U}) \tag{18}$$

where b_T is the estimate of the trunnion bias.

The apparent star location $\underline{U}$ is computed from equation (19).

$$\underline{U} = \theta(t)\ \text{unit}\left(\underline{U}_M + \frac{\underline{V}_{sp} + \underline{V}}{c}\right) \tag{19}$$

where $\underline{U}_M$ is the unit star vector without the change for aberration (MNBY)

$\underline{V}_{sp}$ is the velocity of the earth or moon relative to the sun (MNBY)

$\underline{V}$ is the velocity of the vehicle relative to the earth or moon (MNBY)

9.0 PARTIAL DERIVATIVES

The partial derivative of the trunnion observation with respect to the state vector (horizon altitude and trunnion bias) is defined by equations (20) through (22).

$$\underline{A}_j^T = \begin{bmatrix} \dfrac{\partial m_j}{\partial h} \\ \\ \dfrac{\partial m_j}{\partial b_T} \end{bmatrix} \tag{20}$$

$$\frac{\partial m}{\partial h} = \pm \frac{1}{\left|\left|\underline{R}s - \underline{R}v\right|\right|} \tag{21}$$

where the partial is negative for near horizon measurements and positive for far horizon measurements.

$$\frac{\partial m}{\partial b_T} = 1 \tag{22}$$

10.0 PROGRAM OUTPUT

All input data should appear as output. For both methods 1 and 2, the following block of information will be printed. The line preceding this information will indicate whether the solution is presented for the trunnion measurement (method 1) or the LOS measurement (method 2).

A Priori $bTo(MRD)$, $bTo(counts)$, $h_o(KM)$

$\sigma_{bTo}(MRD)$, $\sigma_{bTo}(counts)$, $\sigma h_o(KM)$,

ρ_o(correlation coefficient)

For each iteration:

Solution $bT(MRD)$, $bT(counts)$, $h(KM)$

$\sigma_{bT}(MRD)$ $\sigma_{bT}(counts)$, $\sigma_h(KM)$,

ρ_o(correlation coefficient)

Iteration number, μ, σ, N

For final iteration:

Time j, $\Delta m_j(MRD)$, H_j (KM), w_j

For each observation processed by method 2, two additional quantities will be computed to isolate sources of sighting errors.

SFOV (DEG)
NROT (DEG)

Equations required to define this output are as follows.

The star FOV error is given by

$$SFOV = \sin^{-1} (\underline{w} \cdot \underline{U}_B)$$

The normal rotation error is given by

$$NROT = \sin^{-1} \{\underline{w} \cdot [\underline{U}_B \times \text{unit } (\underline{R}_V \times \underline{U}_B)]\}$$

The effective height for each observation is given by

$$H_j = h + \Delta m_j \Big/ \frac{\partial m_j}{\partial h}$$

The weighted residual mean is given by

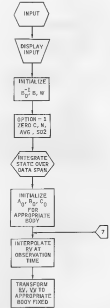
$$\mu = \frac{1}{N} \sum_j w_j \Delta m_j (SC)^2$$

The weighted residual standard deviation is given by

$$\sigma = \left(\frac{\Sigma_j [w_j \Delta m_j (SC)^2]^2 - N(\mu)^2}{N-1} \right)^{1/2}$$

where N is the number of measurements with $w_j \neq 0$.

11.0 FLOWCHARTS FOR THE EFFECTIVE HORIZON PROCESSOR



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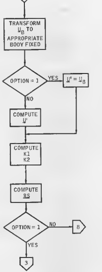
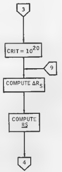
Flow chart 1.- RTCC offline effective horizon processor.



Flow chart 1 - Continued.



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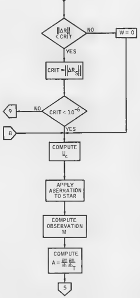
Flow chart 1.- Continued.



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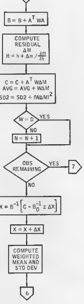
Flow chart 1.- Continued



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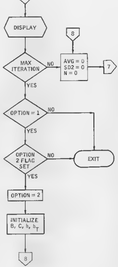
Flow chart 1.- Continued.



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Flow chart 1.- Concluded.

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