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COLOSSUS Memo # 193

TO: Distribution  
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SUBJECT: P37 Ignition Time Bias

P37 Return to Earth biases the desired ignition time by half of the expected burn time. This will improve the performance of the resulting Lambert burn in those cases where a large central angle is traversed during the course of the burn, such as a return from earth orbit.

To compute the expected burn time the following equation is used:

$$\Delta t_B = \frac{m_0}{\dot{m}} (1 - e^{-\Delta v/v_c})$$

The quantity " $1 - e^{-\Delta v/v_c}$ " is approximated by a second order polynomial whose coefficients were chosen to minimize the absolute error in the computation over the expected range of  $\Delta v$ . The effect of minimizing absolute error rather than relative error results in the "over-biasing" of very short burns such as trans-earth coast midcourse corrections. This will have negligible effect on the accuracy of the midcourse correction and the resulting trajectory. This "over-biasing" may be seen in Mission F, where a 3.7 fps burn was biased by 17.48 seconds rather than the correct value of 7.46 seconds.

In future programs this error could be reduced for short burns by replacing the present coefficients with the coefficients of a Taylor's series, however this would reduce computation accuracy for long burns.

Present series:

$$1 - e^{-\Delta v/v_c} = 5.6681958 \times 10^{-4} + 0.97949284 (\Delta v/v_c) - 0.38829576 (\Delta v/v_c)^2$$

Taylor's series:

$$1 - e^{-\Delta v/v_c} = \Delta v/v_c - \frac{1}{2} (\Delta v/v_c)^2$$

Comparison of percent error

$\Delta v$ (fps)	present series	Taylor's series
2	280%	0%
4	139%	0%
6	92%	0%
8	68%	0%
10	54%	0%
100	3.2%	0%
1000	1.1%	0.2%
3000	0.5%	1.7%
5000	0.7%	4.7%
7000	2.2%	9.7%
9000	5.1%	16.6%