Massachusetts Institute of Technology
Instrumentation Laboratory
Cambridge, Massachusetts

COLOSSUS Memo #59

To: Distribution

From: J. A. Saponaro

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Subject: Problems Concerning the Lunar Sphere of Influence

I. Introduction

The purpose of this memo is to describe the present design of COLOSSUS with respect to program determination and events that occur in the program at a crossing of the lunar sphere of influence.

Actually, the events do not occur when the vehicle crosses the sphere, but when the integrated state vector does. The program as designed appears to be internally consistent in its processing; however, problems exist in external communications near the sphere.

An attempt is made to list some of the problems and constraints, specifically in ground/AGC communications near the sphere. To obtain a better understanding of these problems and potential ones, a description of the program logic and changes which occur at the lunar sphere is also included.

Since this memo's purpose is primarily to point out what the present design is and the problems which exist with it, section 6 proposes some changes to remove some of the restrictions if it is deemed necessary. (It appears to me that it is).

2. What is the Sphere?

When considering the motion of the spacecraft in the presence of both the moon and the earth it is important to select the body to which spacecraft motion is to be referred. That is, it is necessary to know at what distances the perturbing acceleration of the earth becomes equal to that of the moon.

These distances define a surface about the moon which is approximated by a sphere and is referred to as the "sphere of influence of the moon" with respect to the earth. Inside the lunar sphere the motion of the spacecraft is determined with the moon as the origin and primary body and earth secondary and outside the sphere the earth is the primary body and origin.

The AGC program approximates[1] the radius of the lunar sphere at 64, 373, 760 meters from the center of the moon.

3. Program Determination of the Sphere

In COLOSSUS the coasting flight integration routine determines when the spacecraft is in or out of the lunar sphere. It does this by examining the vehicle's state at the beginning of each time step during an integration. It essentially compares the distance of the state from the center of the moon and the radius of the lunar sphere and changes the origin to moon or earth accordingly.

The logic used is explained in more detail in Appendix A and is contained in R577 section 5 COLOSSUS Dec 1967 Pg 5. 2-30.

1 The actual values of the radius of the sphere of influence range from 32, 000 to 41, 000 miles.

It is important to note here that the integration routine has many modes of operation which essentially are as follows:

a) Precision (Encke) integration of the actual LM or CM state
b) Precision (Encke) integration of the actual (permanent) LM or CM state and update (replace) the LM or CM state.
   note: This mode is always used by P00, or navigation programs and is the mode which integrates the W-Matrix.
c) Conic (Kepler) "integration" of the actual LM or CM state.
d) Integration of a specified state vector Encke or Kepler.

In all modes the tests described in Appendix A for the origin change are made. Also in all cases Index register 2 is set to 0 or -2 for earth or moon respectively as output indicators to the calling program. However, only in b) above is the actual state origin changed and flags LMOONFLG and CMOONFLG set.

In all other modes index register 2 output from integration indicates which sphere the state vector just integrated is in.

4. What Happens When the Origin is Changed?

In summary, the integration routine is the only AGC routine which changes automatically the origin and further it only changes the permanent memory state on a precision update and replace type integration (mode b above).

When the permanent state vector is updated across the sphere (either way) the following occurs:

a) The permanent state vectors (RRECTCSM, VRECTCSM, TDELTCSM, etc.) are transformed to the new origin earth or moon and the scaling changes accordingly.

The scaling for earth and moon is shown in Appendix B. The integration routine starts outputting to the calling program the resulting state in 2 scalings. That is, RATT, VATT are always 2^{29} meters and 2^{7} m/cs. and RATT1 and VATT1 are 2^{27} meters and 2^{5} m/cs inside the lunar sphere. Outside the sphere, RATT1 and VATT1 are the same as RATT and VATT.

b) The origin of the coordinate system changes accordingly and LMOONFLG and CMOONFLAG are set or reset depending on which vehicle is being integrated, indicating the origin.

c) The origin of the downlink state vectors RN, VN and ROTHER VOTHER, PIPTIME are changed to earth or moon accordingly. However, the scaling remains at 2^{29} meters and 2^{7} m/cs always. It should be noted that there is a short time interval during which both states are not at the same origin since only one state is integrated at a time in P00.

5. What are the Problems?

An attempt was made to examine each of the mission programs to determine if problems occur near the sphere. The first step was to categorize the programs as to where it was assumed they operate. The categories used are:

CATEGORY	MEANING
1	Operates in earth or lunar sphere and across the sphere with no known problems.
2	Operates in earth or lunar sphere but has restricting problems concerned with the sphere.
3	Operates in earth or lunar <u>orbit</u> only. That is, it is generally assumed to not operate in translunar or transearth trajectories and therefore does not have any sphere problems.

4 Operates in earth sphere only.

5 Operates in lunar sphere only.

6 Operates on the earth's surface only.

This chart is included as Appendix C.

Some comments are made about various problem areas with the present design:

a) P30 External Delta V

The $\Delta \overline{V}$ input must be in a local vertical system at an origin which corresponds to the CM state at TIG. The problem here is that the ground must uplink the $\Delta \overline{V}$ local vertical whose origin is consistent with what the AGC will determine for CM state origin at TIG.

b) P31 General Lambert

The constraint in this program is that the target vector (RTARG) and the CM state, as determined by the AGC, must both be in the same sphere. Note this a general constraint which is not accounted for in program logic of the Lambert routine and applies to P40 Lambert burns as well.

c) P37 Return to Earth (RTE) Targetting

The RTE program will compute a return trajectory only if the CM state at TIG is outside the lunar sphere (i. e. it will alarm otherwise). It should be noted that the constraint is at TIG and that the actual return trajectory solution can cross the lunar sphere. The program computes a "direct return" trajectory.

One problem to note is that if the CM state at TIG is outside the lunar sphere but behind the moon in such a way that the return trajectory may intersect the moon itself. This situation occurs only in a limited set of conditions but the essence is that RTE does not know where the moon is nor does it constrain the return trajectory by the position of the moon.

d) P40, 41 SPS RCS Thrusting

As indicated above these programs have the same problem as P31 for all Lambert burns near the sphere. That is, the CM state when the Average G is initialized by AVETOMID must be in the same sphere as the target vector. Once AVERAGE G begins there is no center change. Presently there is a program discrepancy here in that AVERAGE G is initialized with a state in RN, VN which can have different origin than is indicated by CMOONFLG. This means an inconsistency in the Downlink of the state vector actual origin and CMOONFLG origin. This can occur because MIDTOAVE routines initialize RN, VN using a mode of the integration routine which does not do a permanent update (i.e. Mode a).

e) P17, P20, P34, P35, P38, P39, P74, P75, P78, P79, R31, R34, R36, R32

These are programs and routines which are used during rendezvous and assume that both vehicles states are in the same sphere and in earth or lunar orbits for rendezvous. The targeting programs are coded to assume that CMOONFLG defines which sphere. The programs initialize MU based on this flag. There is no coding in any of the above programs to examine sphere changes during the program operation. Both states must remain in that sphere for the entire operation of the routine. This is not considered a program problem.

f) Other Problems

There are a set of programs which are generally assumed to be used only in earth or lunar orbit but can be used for the entire range of the mission with some problems. For example P21 is limited in its altitude display to 10000 naut. miles which is also true of the perigee and apogee display in R30. Obviously, these programs, regardless of the above mentioned problems, would also be discontinuous in their output if operated through the lunar sphere. That is, a lat. long. for earth would suddenly change to moon.

This is also not considered a problem.

g) P27 CMC Update

The CMC update program P27 requires updating of the state vectors with an indication of origin and vehicle. This is done through a word UPSVFLG which contains the following:

UPSVFLG = -1 LM state earth origin
-2 LM state moon origin
+1 CM state earth origin
+2 CM state moon origin

These controls update the permanent memory state vector (RRECTCSM, RECTLM, etc.) and set CMOONFLG or LMOONFLG accordingly.

It is important to note here what P00 does with respect to an uplinked state.

P00 always integrates the LM state to the time of the CM state. Also, it will not integrate the CM state if the time of the CM state is ahead of current time. It will integrate the state when it becomes older than 4 time steps.

P00 also processes the LM and CM state independently even though the vehicles may be attached. If it can be assumed that both states are always uplinked together then an uplink of the states ahead of current time (for example in a different sphere than the present state) P00 will not integrate these states until the CM state is 4 time steps old.

However if only the LM state is uplinked in advance of current time, (e.g. in lunar sphere) then it will be integrated back to the time of the CM state.

In P00 the LM permanent state is not integrated if SURFLAG is on. That is, when the LM is on the moon's surface.

h) General

The most significant problem, as I see it, is that communication between the ground and the GNCS becomes difficult (especially) for the ground's point of view) with respect to uplink of data near the sphere. The problem is that the ground must predict when the AGC will determine it is in or out of the sphere and uplink the data accordingly. For example, if the ground is attempting to uplink data near the sphere for a P31 Lambert or P30 External ΔV burn the TIG which is uplinked must be such that CM state integrated to that time (TIG) will result in a state which is at the corresponding origin as the $\Delta \overline{V}$ or target vector uplinked.

As pointed out previously, the determination of origin is tested only at the beginning of each time step in orbital integration and therefore the origin change made by the integration routine will probably not occur exactly at the sphere radius. That is, at the beginning of a time step the state can be slightly greater than constant R_{SPH} radius of the sphere in which case it will be integrated through that entire step without changing origin. The size of a time step of a state near the sphere is approximately 4000 secs, which corresponds to approximately 2 - 3, 000 miles.

This defines a spherical shell 2 - 3, 000 miles thick around the moon which when the ground attempts to uplink information to the computer for an event to occur inside that shell it will be difficult for the ground to "psyche out" the AGC as to which origin the AGC will be using.

6. Possible Changes

There are several approaches which can be taken to improve the design. However the changes recommended here are primarily influenced by schedule constraints with a minimum impact to the program structure as it exists now. (Of course, one alternative is to "live with what exists now" and do nothing.)

The following changes are proposed:

a) P30 External ΔV

Add a display DSKY input (or uplink) to allow input of the origin of the ΔV which is input. P30 would require a change to compute the proper rotation matrix for the state vector origin at TIG and change the ΔV origin accordingly to maintain internal consistency.

b) P31 General Lambert

Add a flag to indicate the origin of RTARG and modify P31 logic to change RTARG to the origin corresponding to CM state vector at TIG. The RTARG flag will be changed by the program if RTARG is changed. This also implies slight changes to P34, 5, 8, 9 and 70's to set this flag based on CMOONFLG.

c) P40, 41 SPS, RCS Thrust

Change the initialization of AVERAGE G and initialization of RN, VN and PIPTIME to set CMOONFLG to the origin of RN VN to ensure downlink of consistent data.

Also a change will be required in the pre-thrust calculations of the P40/41 programs to ensure that origin of the state at AVERAGE G initialization is the same as RTARG origin and if not to change RTARG. Note that AVERAGE G initialization and TIG are 2 different times and, of course, the origin could change.

d) P37 Return to Earth Targeting

There are a number of changes which could be made to this program all of which require further evaluation and investigation of the problem to more fully determine the scope and of trajectories that cause problems.

One solution might be to add some logic in the end of the conic phase which takes the position of the CM at TIG and the computed return velocity and using time Theta computes the time to the earth moon line (i. e. $|\bar{R}_{EM}|$) then using the position at that time (output from the time Theta Sub) compare it to $|R_{EM}| \pm K$ where K is a distance from the moon and alarm if the position is that close to the moon.

After discussion of these problems and proposed changes it can be evaluated as to what, if any, changes should be made to the COLOSSUS programs to eliminate some of these problems.

APPENDIX A

FLOW OF ORIGIN CENTER CHANGE LOGIC, COLOSSUS

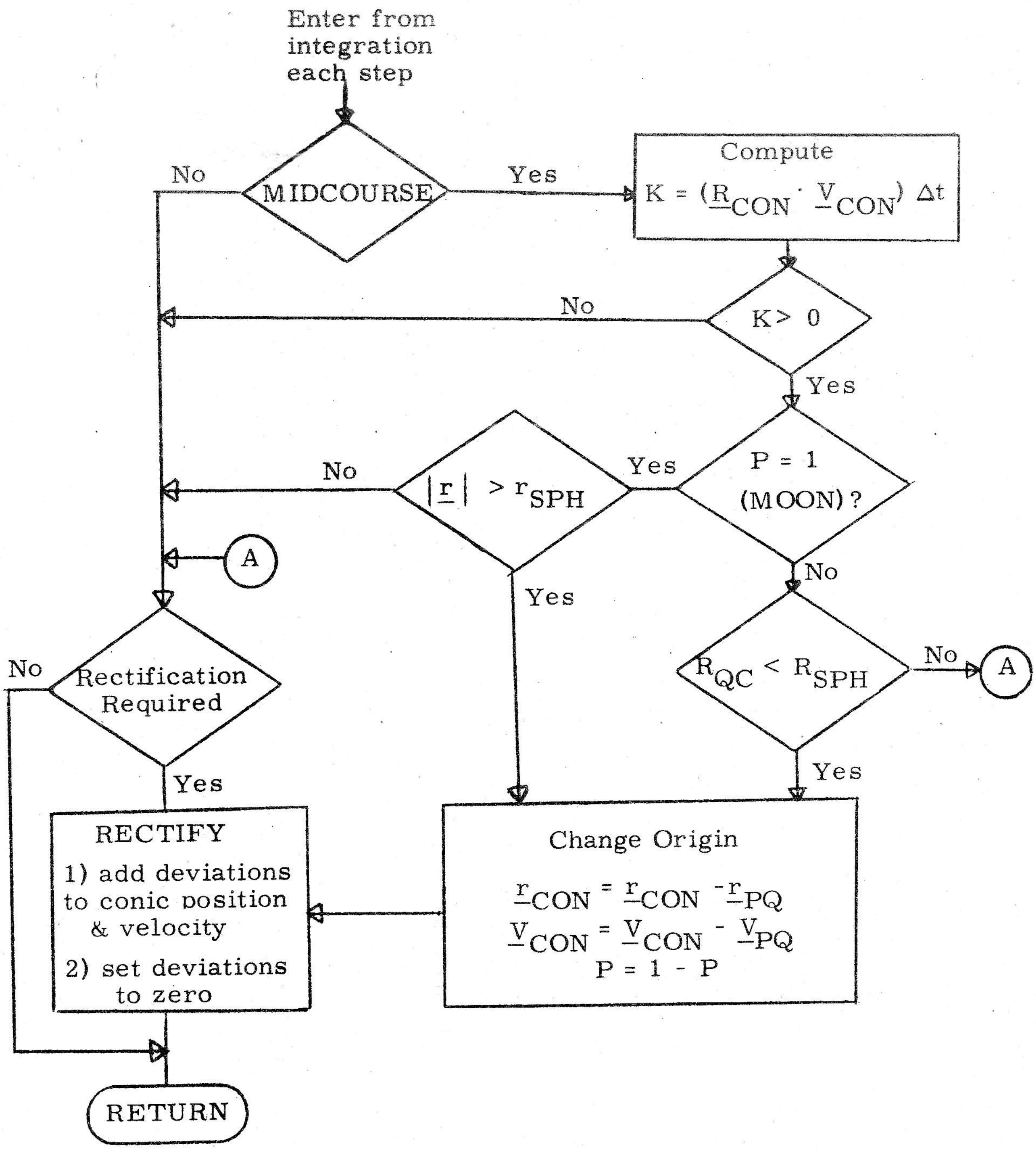
Below is a flow of the logic used in orbital integration for determining origin change.

The first test made determines if the state vector is in "midcourse" (M = 1). (i.e. magnitude of $\overline{R}$ greather than 6, 778, 165 meters for earth primary and 2, 138, 090 meters for moon primary. If the state is not in midcourse no coordinate center change is necessary. If it is, then program tests if the present state is heading toward or away from the lunar sphere. It computes and tests $K = (\overline{R}_{CON} \cdot \overline{V}_{CON}) \Delta t$ where $\overline{R}_{CON}$ and $\overline{V}_{CON}$ are the conic position and velocity and Δt is the time step and accounts for integrating a state vector forward or backward in time. If K is ≥ 0 then a state vector centered at either body is not approaching the sphere. If $K < 0$ then the routine essentially tests the distance from the moon (of this state) and determines if the primary body should be changed. If the origin center is changed the state is rectified and the integration continues with the state at the new origin.

See attached flow chart.

$\overline{R}_{CON}$, $\overline{V}_{CON}$	Conic position & velocity vectors of the state being integrated.
Δt	Time step computed by integration $\Delta t = .3 \frac{r^{3/2}}{\sqrt{\mu_p}}$ where it is signed + or - integrating forward or backward in time.
P	Primary body flag 0 = earth 1 = moon
$\overline{r}_{PQ}$	Vector from primary to secondary body $= \begin{cases} \overline{r}_{EM} & \text{when } P = 0 \\ -\overline{r}_{EM} & \text{when } P = 1 \end{cases}$ where $\overline{r}_{EM}$ is the position of the moon w.r.t. the earth generated by the lunar solar ephemeris routine.
$\overline{r}_{QC}$	Vector from the secondary body to the state (spacecraft)
$\overline{V}_{PQ}$	Velocity of the secondary body W.R.T. the primary body
R_{SPH}	Radius of sphere 64, 373, 760 meters

APPENDIX A



Enter from
integration
each step
MIDCOURSE
No
Yes
Compute
K = (R_CON · V_CON) Δt
K > 0
No
Yes
P = 1
(MOON)?
Yes
|r| > r_SPH
No
Yes
A
No
R_QC < R_SPH
No
A
Yes
Rectification
Required
No
Yes
RECTIFY
1) add deviations
to conic position
& velocity
2) set deviations
to zero
Change Origin
r_CON = r_CON - r_PQ
V_CON = V_CON - V_PQ
P = 1 - P
RETURN

APPENDIX B

PERMANENT STATE VECTORS

These register contain the data for the CSM and LM which the integration routine uses to initialize the temporary registers prior to the start of the integration procedure (i.e., it is the permanent record of position and velocity for the CSM and LM).

NAME/TAG	UNITS	MEANING	SCALING EARTH	SCALING MOON
RRECTCSM	meters	Rectified position vector for CSM	2^{29}	2^{27}
VRECTCSM	m/cs	Rectified velocity vector for CSM	2^{7}	2^{5}
TETCSM	cs	Time of the CM state vector	2^{28}	2^{28}
DELTACSM	meters	Position deviation vector δ (1)	2^{22}	2^{18}
NUVCSM	m/cs	Velocity deviation vector ν (1)	2^{3}	2^{-1}
RCVCSM	meters	Position conic vector CSM	2^{29}	2^{27}
VCVCSM	m/cs	Velocity conic vector CSM	2^{7}	2^{5}
TCCSM	cs	Time since rectification	2^{28}	2^{28}
XKEPCSM	(meters)1/2	Root of Keplers equation	2^{17}	2^{16}
RRECTLEM				
VRECTLEM				
TETLEM				
DELTALEM				
NUVLEM		Similarly for the LM, 42 permanent memory registers will be maintained.		
RCVLEM				
VCVLEM				
TCLEM				
SKEPLEM				

1) The position vector at time TETCSM = RCVCSM + TDELTACSM and velocity vector at time TETCSM is VCVCSM + TNUVCSM.

APPENDIX C

PGM #	NAME	CATEGORY
P00	CMC Idling	1
P01	Initialization	6
P02	Gyro Compassing	6
P03	Optical Verification	6
P06	Power Down	1
P07	System Test	6
P11	EOI Monitor	4
P17	TPI Search	3
P20	Rendezvous Navigation	3
P21	Ground Track	3
P22	Orbital Navigation	3
P23	CIS Lunar Midcourse Navigation	1
P27	Uplink	1
P30	External Delta V	2
P31	General Lambert	2
P34	TPI Targeting	3
P35	TPM Targeting	3
P37	RTE Targeting	4 (with problems)
P38	SOR Targeting	3
P39	SOM Targeting	3
P40	SPS Thrust	2
P41	RCS Thrust	2
P47	Thrust Monitor	2
P51	IMU Orientation	1
P52	IMU Realign	1
P53	Backup Orientation	1
P54	Backup Align	1
P61-67	Entry Programs	4
P74-79	Backup Rendezvous Targeting	Same as corresponding P3X's

ROUTINES (EXTENDED VB CALLABLE ONLY*)

R03	DAP Data Load	1
R05	S Band Antenna	1
R21, 23	Sighting Mark Routines & COAS	Same as P20
R30	Perigee, apogee, TFF	3(can be a 2 with problems)
R31	Range, Rate, Theta	3
R32	Target ΔV	3(can be a 2 with problems)
R34	Rendezvous Par	3
R35	Lunar Landmark Selection	5
R36	Out of Plane Display	3

* It is assumed that all other routines are in the same category as the program they are called by.