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SUBJECT: Summary of Fixed Point Orbital Navigation Using Modified Euler Integration and a Two Term Potential Function

The equations defining modified Euler integration and the logical sequence of computation are as follows:

Routine I

0) $\dot{y}(t)$ , $y(t)$ , $\ddot{y} = f(y)$ , h are given initially

1) $\ddot{y}(t) = f(\ y(t)\ )$

2) $y^p(t + h) = y(t) + h\,\dot{y}(t)$

3) $\ddot{y}^p(t + h) = f(y^p(t + h))$

4) $\dot{y}^c(t + h) = \dot{y}(t) + \frac{h}{2}\left[\ddot{y}(t) + \ddot{y}^p(t + h)\right]$

5) $y^c(t + h) = y(t) + h\,\dot{y}(t) + \frac{h^2}{2}\ddot{y}(t)$

During subsequent cycles through the above routine, the initial values of $\dot{y}(t)$ and $y(t)$ are replaced (beginning in Step 1) by $y^c(t+h)$ and $\dot{y}^c(t+h)$ from steps 4 and 5 respectively. Note that the second derivative is computed twice, first in step 1 and later in step 3. The superscripts p and c refer to predicted and corrected values respectively. This routine may also be referred to as trapezoidal integration if $\dot{y} = f(t)$ only (a simplified case) is to be integrated.

If the equation defining the $\ddot{y} = f(y)$ is complex, resulting in an unacceptable computation time delay, the sequence can be altered slightly as follows to reduce the delay:

Routine II

0) $\dot{y}(t)$ , $y(t)$ , $f(y)$ , h are given initially

1) $\ddot{y}(t) = f(\ y(t)\ )$

2) $y^c(t + h) = y(t) + h\,\dot{y}(t) + \frac{h^2}{2}\ddot{y}(t)$

3) $\ddot{y}^{pc}(t + h) = f(\ y^c(t + h)\ )$

4) $\dot{y}^c(t + h) = \dot{y}(t) + \frac{h}{2}\left[\ddot{y}(t) + \ddot{y}^{pc}(t + h)\right]$

During subsequent cycles through this routine, the initial values of $y(t)$, $\dot{y}(t)$ and $\ddot{y}(t)$ are replaced in step 1 by the corrected values just computed. Note that the second derivative is computed once each cycle in step 3 except for the initial pass through step 1.

This simplified routine appears to produce a truncation error of approximately twice the complete solution of routine I but will permit a faster integration cycle. Strictly, routine II is not modified Euler.

Routine I was implemented in 26 bit, fixed point with $f(y)$ describing an oblate Earth by a two term potential function. Comparison was made with a standard trajectory program implemented in 36 bit, floating point with $f(y)$ describing an oblate Earth by a six term potential function and integration done with a double precision Runge - Kutta - Gill technique. The error in radius caused by the fixed point, truncated implementation is shown on the attached graph. (Figure 1)

The small error produced during the one orbit shown arises from the careful fixed point scaling and data manipulation found necessary to preserve the accuracy of the computations.

Further work should be planned to include fixed point implementation of Routine II and a comparison of the errors for both Routines I and II should be expanded to include the individual components of the radius ( X, Y, and Z inertial) and the components of velocity ($\dot{X}$, $\dot{Y}$, and $\dot{Z}$ inertial). Also, the error curves should be extended to at least three orbits.

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Attachment