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RADAR REQUIREMENTS REPORT [U]

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Systems Analysis and
Dynamics & Performance Analysis

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Radar Requirements Report LED-540-1

1. INTRODUCTION

It has been apparent since the early days of the LEM pre-proposal studies that non-inertial sensors would play an essential part in the navigation and guidance of the LEM vehicle. At the time of the submission of the LEM proposal the types of sensors that could reasonably and usefully be employed had been narrowed to radars and lasers, and GAEC proposed a sensor system utilizing both. During the LEM negotiation period, discussion with MSC personnel revealed that MSC shared GAEC's opinion concerning the requirement for non-inertial sensors, but were not convinced that lasers would be required since they are intended primarily for slant range measurements. Preliminary study results indicate that lunar landing can be accomplished without slant range sensors and the sensor system presently recommended consists of radars only. GAEC's responsibilities in the design and procurement of the LEM radars were specified in Par. 2.1.1.6 of Exhibit A (Statement of Work) for the LEM spacecraft as follows:

> "The Contractor (GAEC) shall be responsible for the detail design of the LEM Range and/or Angle Tracking Sensor Equipment. The Navigation and Guidance System Associate Contractor (MIT) will be responsible for determination of the functional requirements of the equipment, insofar as they are related to primary guidance. The Contractor shall be responsible for determination of the functional requirements of the equipment that are independant of application to the primary guidance. The Contractor will be responsible for preparation of an overall specification for the equipment based on all functional requirements."

In order to discharge the responsibility thus assigned, GAEC proceeeded on a two-pronged effort. In a series of technical meetings with MIT, their radar utilization concept and approaches to radar design were discussed with them in considerable detail and the radar requirements, insofar as they pertained to primary Navigational and Guidance obtained. Concurrently, an internal study program was undertaken to define those radar requirements imposed by back-up guidance concepts, configuration and performance capabilities.

It is the purpose of the present report to discuss the radar utilization concepts in detail, delineate the proposed functional configuration of the radar sensor system and present a complete set of radar system performance specifications which encompasses the requirements of both the primary as well as the back-up N. & G. systems. The preliminary results of the internal studies to define radar requirements will be presented and will serve as the basis for the specification of radar performance capabilities.

The considerations and factors bearing upon the problem of hardware implementation of the radar system, a preliminary recommendation of system configuration and estimates of weight, power and reliability are presented in separate reports.

Particular attention has been paid to the terminal portion of the powered descent phase and to the rendezvous portion of the ascent phase, since these are the mission phases which involve the radar to the greatest extent. However, in the sections that follow, radar utilization in all mission phases will be discussed in some detail.

2. Basic Design Concepts

A. General

The primary guidance system configuration conceived by MIT requires several types of input information from sensors external to the IMU. The purpose of these sensors is to provide data that will permit a more accurate computation of guidance commands than can be made with IMU derived data alone (since the drift rate of the IMU tends to degrade the guidance accuracy) and thus allow the lunar landing and orbital rendezvous phases of the LEM mission to be completed within the design requirements of the nominal mission. The data required from external sensors is altitude above the lunar surface, and LEM velocity relative to local selenocentric coordinates both of which are obtained from the altimeter radar equipment. In addition, range, range rate, LOS angle and LOS angular rate of the CSM with respect to a LEM centered reference coordinate set are obtained from the rendezvous radar equipment. The dynamic range and accuracy requirements of this data, as far as they pertain to the primary N. & G. System, are derived from the MIT analysis of their guidance configuration as used in the various mission phases. In addition to design requirements placed upon the non-inertial sensors by the primary guidance system em- employing the IMU and AGC, requirements obtained from the use of the radar in a "manual alternate" primary mode as well as in the backup guidance mode must be considered in establishing overall radar design requirements. These overall requirements and the justification thereof are presented in Section 5.

B. Manual Alternate Mode

The manual alternate mode is defined as a mode in which the flight crew performs some of the tasks normally performed automatically by the primary guidance system without failure of any major functional element of the primary guidance having occurred. The following system design concepts were adoped in connection with these alternate manual modes:

1. Alternate manual modes will be utilized where their incorporation increases crew safety.

2. There will be a direct display of sensor data where possible for use in manual modes.

3. The automatic guidance modes will be compatible with manual monitoring and override whenever alternate manual modes are provided.

The use of the capabilities of the crew wherever possible to increase the probability of mission safety and success is one of the major design goals. The degree of accuracy to which a man can fly the nominal mission profile in an alternate manual mode is yet to be determined, but pending specific results from the required simulation studies, the primary equipment configuration will generally be expected to provide for this capability.

The second design concept directly affects the altimeter radar requirements, and requires that radar derived altitude rate and horizontal velocites must be displayed independently of the inertial guidance equipment. The reason for this is to insure, where possible, that information from the radar sensors may be monitored and/or used by the crew before degradation or loss of this information through failure of any processing equipment (such as the primary guidance computer). The alternate manual mode concept for descent and landing assumes that at some point in the descent trajectory (which will probably be below 15,000 ft. altitude to insure that good data from the altimeter radar is available), the crew can take over attitude and thrust control of the LEM and perform a safe terminal descent and landing. As previously mentioned, the degree of accuracy to which a man can fly a nominal descent trajectory must be determined through simulation studies. However, if it is assumed that a reasonable descent trajectory can be designed for optimal use of the crew and still remain safely within the fuel limitations, the requirements on the altimeter radar can be stated. Preliminary results from the fixed base terminal descent and landing simulation at MSC indicate that altitude, altitude rate, and horizontal velocities are required displays for the crew. Distance to go in range and cross range are also useful data. The altimeter radar can sense the former quantities and display them directly to the crew. The radar data can also be introduced into the AGC of the primary N. & G. system to update the IMU data from which distance to go can then be computed. Visual sightings of the landing site will serve to monitor this computed distance information and make the corrections necessary to land at the desired site.

The requirement for compatibility of the automatic guidance modes with manual monitoring and override is necessary to insure a smooth transition from automatic to manual or backup modes of operation. The major effect of this constraint upon the radar equipment is to require a continuous display of the important flight parameters to the crew.

C. Backup Guidance Modes

A backup guidance mode is defined as a mode in which the utilization of backup guidance equipment and/or extensive crew participation in the guidance operations is occasioned by a failure in the primary N & G system. The following design concepts were adopted in connection with backup guidance modes.

1. Crew safety is the primary design consideration.

2. The backup guidance system design must provide the ability to abort and return to the CSM safely at any time in the mission upon failure of any single major functional element in the primary system.

3. The backup guidance system design must provide the ability to abort to a clear pericynthion orbit wholly independent of the primary system.

4. The flight crew shall be utilized in such a way as to provide maximum crew safety.

5. The backup guidance system shall be simpler and more reliable than the primary system.

The concept that crew safety is the primary design consideration has a significant effect upon the choice of whether or not the backup guidance system should be configured to provide a capability for continuing the mission to a lunar landing despite a failure in the primary guidance system. Present thinking is that the crew have the option of aborting or of continuing descent to a lunar landing if: a) either the primary N & G system or the radar altimeter (but not both) have failed and b) if radar failure occurs subsequent to IMU updating. In the case of a failure in the inertial portion of the system, the backup guidance equipment in conjection with the radar altimeter can provide adequate information to permit a safe landing. The requirements on the radar altimeter in this case are no different from those obtained from consideration of the manual alternate mode.

The single failure of any major functional element in the primary N & G system may not disable the entire primary system, and the backup guidance equipment can possibly be designed to provide the functions necessary to supplement the remaining primary elements in the backup mode. (As described above for the case of an IMU failure in the latter portion of powered descent.) However, failure of the rendezvous radar will require a specific backup, such as, for instance, a CSM mounted tracking radar and a communication link between the LEM and CSM. The guidance computations for use by LEM will then be carried out on the CSM computer and transmitted to LEM via the communication link. The requirements on the rendezvous radar in the backup guidance mode are the same as those previously described for the manual alternate mode. If a tracking radar is placed on the CSM to backup the LEM rendezvous radar, it would appear very desirable for these equipments to be identically configured.

Use of the flight crew in the backup guidance scheme must take into account the crew task load and performance capability in determining the computation complexity and degree of automaticity of the backup system. A direct result of these considerations leads to the conclusion that the rendezvous radar antenna should be gimbaled. The use of a fixed radar antenna would make the task of obtaining sufficient, accurate radar data without the use of the AGC of the primary N & G system practically impossible.

The requirement that the backup guidance system be simpler and more reliable than the primary guidance system insures that the reliability goals for crew safety are met without paying a large weight penalty for carrying standby or redundant equipment. Thus, as far as the radar equipment is concerned, it appears very desireable to design the two radar equipments such that one is inherently capable of providing a backup for the other, rather than including a standby radar on the LEM expressly for an altimeter or a rendezvous backup function. Consideration of mission success and mission safety requirements lead to the conclusion that these requirements can be most readily satisfied by providing a backup altimetry function in the rendezvous radar equipment which will permit safe landing with the use of the inertial portion of the primary system despite the loss of the altimeter radar, and by backing up the LEM rendezvous radar with a similar radar on the CSM. The use of the rendezvous radar as an altimeter during power at descent has the additional advantage of providing a third source of altitude data to decide whether the IMU or the altimeter is in error. If there is a significant discrepancy in the two outputs, this information would then insure that if the crew decides to continue the mission to a lunar landing, they would do so with the properly functioning equipment, and thus increase the probability of survival as well as of mission success.

3. MISSION PROFILE

A - General

The primary function of the radar is to provide data for the LEM guidance system in both the primary and back-up modes. This data may be processed by the AGC for utilization in automatic guidance phases or it may be displayed to the crew to provide the capability of performing the guidance operations in manual, alternate or back-up modes. Thus, the radar requirements are directly related to the guidance schemes used in the various mission phases. In the discussion that follows, the mission profile will be described with respect to requirements for radar data, and the results of radar utilization studies pertinent to the particular phase under discussion will be presented. Excluded from this section is the rendezvous phase, which is discussed separately in Section 4.

B - Separation from CSM

Separation of the LEM from the CSM takes place in lunar orbit. The LEM RCS is utilized to apply the separation thrust, to achieve a separation distance of about 100 feet. There is no requirement for radar data during this phase, except possibly for the purpose of checking out those radar functions which operate in conjunction with a transponder on the CSM. Because of the close proximity of the two vehicles, it may also be possible to check the non-transponder functions by skin-tracking the CSM.

C - Injection into Transfer Orbit

It does not appear that radar data is required in this phase.

D - Coast to Pericynthion of the Transfer Orbit

In order to establish the orbit conditions resulting from the injection thrust, the range and range rate of the CSM relative to the LEM should be determined as a function of time. A study is presently underway to establish the accuracy requirements on range and range rate sensing to allow reliable prediction of pericynthion altitude. Preliminary results indicate that 1% range measurement error leads to approximately 5000 feet uncertainty in pericynthion, and that this is relatively independent of range from injection up to about 30° central angle traversed.

About 10 minutes prior to reaching apocynthion, the altitude, and possibly the velocity, measuring capability of the sensor system should be verified. This results in an altitude measurement range of about 70,000 feet, for a transfer orbit starting at 80 N.Mi and termination at 50,000 feet. For a sensor which provides total LEM velocity relative to local coordinates such as for example a three beam doppler, the velocity measurement range should be about 5700 fps, which is the horizontal component of LEM velocity near pericynthion.

E - Powered Descent

Powered descent is initiated at pericynthion (50,000 feet) of the transfer orbit. During the early portions of the descent, data for the guidance and control system is derived from inertial sensors. The errors in this data increase both in absolute value as well as in terms of percentage of measured values. Clearly, platform misalignment and drift results in cummulatively larger errors in velocity and position data. Errors in measurement of lunar orbit altitude translate directly into errors in knowledge of altitude relative to local terrain and becomes progressively more significant as altitude decreases. Terrain variations introduce additional perturbations into knowledge of altitude above the lunar surface, since inertial sensor cannot detect terrain changes.

For the above reasons, direct sensing of LEM position and velocity appears to be prerequisite to achieving successful descent and landings. The primary mode of sensor data utilization is in updating of inertial equipment. This technique generally leads to optimum knowledge of LEM dynamic parameters, since the good high frequency content of inertially derived data complements the good low frequency content of non-inertial sensor data. Furthermore, subsequent to the inertial updating of the inertial equipment, failure of the non-inertial sensors does not necessarily require that the mission be aborted. The concept proposed by MIT involves mixing of inertial and non-inertial data in the AGC according to the statistical variances of the error distributions. Since the quality of inertial data degrades with time and that of non-inertial data improves, progressively more weight is attached to non-inertial sensor data as the descent progresses.

It is anticipated that this data mixing will commence at an altitude of about 25,000 feet, and that at about 15,000 feet, altimeter data will be weighted at almost 100%. Estimates of relative weighing of velocity data have not been obtained as yet at GAEC.

The problem of determining the effect of terrain variations during the terminal portion of the powered descent has been the object of considerable attention and study. The preliminary results of this study are presented in Appendix 1. The following terrain situations were considered in the study. Starting at distance of 20 N.Mi from the nominal hover point, at an altitude of about 18,000 feet, constant slopes of $\pm 3^{o}$ and initial altitude and vertical velocity errors of 5000 feet and 3 fps were investigated. Single inflection points were then introduced at various points in the terminal portion of the trajectory and several combinations of slopes considered. These were $\pm 3^{o}$ and $\pm 6^{o}$, and $\pm 3^{o}$ and $\pm 15^{o}$. The effect of $\pm 3^{o}$ slopes and initial condition errors is shown in Figures A1-3 through A1-13. Essentially it was shown that a IMU updating altimetry data and two point prediction of landing site altitude always resulted in acceptable final conditions at hover at no significant Δ V penalty. Initial condition errors in altitude were of no consequence, but initial vertical velocity errors did lead to errors in final vertical and horizontal velocity. This indicates that vertical velocity updating of the IMU may be a requirement. Figure 5 A1-14 and 15 show the effect of a double slope surface. Again, for $\pm 3^{o}$ slopes there are no significant final condition errors or ΔV penalties. The effects of terrain variations including double slopes of $\pm 3^{o}$ and $\pm 6^{o}$ are shown in Figures A1-14 through A1-16, and those referring to double slopes of $\pm 3^{o}$ and $\pm 15^{o}$ are shown in Figures A1-17 through A1-20 . A general conclusion of these studies is that altitude updating of the IMU is generally satisfactory, except when the ground slopes become unreasonably steep. For such extreme cases, the attempt to accommodate the trajectory to these terrain variations was unsuccessful, in that intersection the lunar surface occurred prior to reading zero vertical velocity. Further studies of this problem are in progress to determine the exact limitations of the updating and prediction procedure desired, and to look into other techniques and methods.

F - Hover and Touchdown

The phase of the mission extending from hover altitude (approximately 1,000 feet) to touchdown has not yet been defined in great detail. Some general observations can be made however. If the primary Navigation and Guidance System is working properly the terminal descent from hover can be accomplished without altitude and vertical velocity data from external sensors, provided the IMU was updated at some previous point in the trajectory. Horizontal velocity data from non-inertial sensors may,however, be required since a) there is a tighter tolerance on horizontal than on vertical velocity at landing and b) horizontal velocity changes over a greater range of values during the final portion of the powered descent than does vertical velocity so that even updating early in that phase still leaves the possiblity of excessive errors being accumulated by the IMU.

In the event of failure in the inertial or computing portions of the primary Navigation and Guidance System occurring during or just prior to the hover phase, there may be increased mission safety if the landing is completed rather than the mission being aborted. Under these conditions, non-inertial data must be provided and in a form meaningful to the crew and suitable for direct display. Sufficient study and simulation has not yet been performed to indicate if LEM velocity data relative to body coordinates is adequate to permit successful landing or if resolution into local selenographic coordinates is required. In any event, it does not appear that the hover and touchdown phase requirements will impose any limiting tolerances on the non-inertial sensor capabilities.

G - Pre-Launch

On the lunar surface, the rendezvous radar can be used to track the CSM as it passes over the horizon and comes into view of the LEM. This tracking will be done only long enough to enable the LEM location relative to the CSM orbit to be determined. The data can be used as a backup to the optical tracker, and, by comparison with OMU measurement can be used to determine if the rendezvous radar is working properly.

The gimballed radar can also aid in aiming the OMU by locking on to the CSM and displaying the angles with respect to the LEM. Since the orbit of the CSM is known with respect to inertial space, angular data from the radar should be sufficient to calculate the LEM position with respect to the CSM.

H - Launch

Since the ascent is pre-programmed in both the primary and backup modes, non-inertial data is not required. If the launch is made at the proper time, the initial thrust will inject the LEM into the transfer orbit. However, if there is a timing error, the LEM may have to enter a parking orbit and await the proper injection conditions for transfer to the CSM. These conditions can be determined from data obtained by tracking the CSM. If the primary guidance system is functioning properly, the LEM orbit is accurately known and only the phasing or relative selenocentric angle between LEM and the CSM need be known. If the primary guidance system has failed and the LEM is on backup guidance, the errors at thrust termination will generally be larger and angle and range data may be needed for the computation of the proper phasing.

I - Ascent Coast and Mid-course Correction

The ascent coast phase carries the LEM from burnout altitude of the powered ascent to within homing rendezvous range of the CSM. During this phase, it is anticipated that non-inertial data referring to relative position and velocity of the LEM and the CSM will be required to reduce the effect of burnout errors on the ΔV penalties during homing rendezvous.

For powered ascent and coast phases performed under the direction of the primary Navigation and Guidance System, the errors at burnout will be comparatively small and the problem of performing a mid-course correction relatively easy. Since the AGC is available for computation and data smoothing, the type of data can be restricted, the number of measurements can be fairly large and the mathematical operations on the data can be extensive and sophisticated. Since MIT has the responsibility for devising the mid-course correction technique in the primary mode, no effort has been made at GAEC to study this problem.

For the case of a failure in the primary Navigation and Guidance, however, the situation is quite different. If the failure occurs prior to ascent, the powered ascent must be performed on back-up guidance equipment, and thus the burnout errors can be quite large. Furthermore, the computational facilities for mid-course correction determination will be quite limited, since it is highly desirable to keep the backup guidance equipment as simple and reliable as possible. Thus a mid-course correction technique requiring as few measurements as possible and involving a minimum of computation for deriving the corrective thrust from the measured data is essential.

A study was undertaken at GAEC to develop a correction technique meeting the above requirements and to evaluate its effectiveness. In order to minimize the number of measurements to be made, various combinations of relative data were investigated with regard to attainable accuracy. The time of performing the measurement and applying the correction was also studied. The computational regime was simplified to the extent of utilizing a simple linear combination of measured quantities, using non-time varying coefficients. Although this study, too, is not complete, some preliminary results can be cited.. Table 1 shows the distance of closest approach obtainable on a coasting trajectory with certain combinations of measurement errors. This value is obtained by multiplying the number of feet given in the Error column by the multiplying factor appearing in the column headed by the particular combination of measurement errors under consideration. The results are given for performing the measurement at 1750 and 2000 seconds from burnout, which for a nominal Hohmann transfer corresponds to about 35 N.Mi. and 25 N.Mi. distance from the CSM. For off nominal trajectories, these distances are, of course, considerably larger, but the resulting errors are approximately the same. Some general conclusions that can be drawn from the data presented are that a) better results are obtained if the corrections are performed later in time, b) range rate and angular measurements seem to yield smaller miss-distances for comparable measurement errors, c) even comparatively large measurement errors permit mid-course corrections adequate to place the LEM within efficient homing range of the CSM.

Appendices 2, 3 and 4 present a more detailed picture of the results obtained so far from the mid-course correction studies. In particular, Figures A2-1 and A2-2 give the ΔV penalty for making a mid-course correction from off-nominal trajectories corresponding to +1% and +3% thrust deviation during powered ascent. The burnout errors resulting from various values of off-nominal thrust are listed in Table 2. The significance of these results are discussed in some more detail in Section 4.

Guidance Scheme	Time of Meas.& Corr. (Sec.)	Case 1	2	3	4	5	6	7	8	9	
		R = .05%	.05	.5	.05	.05	.5	.5	1	2	
		$\dot{R}$ = 1%	1	.3	.3	.5	.5	.5	1	2	
		θ = 2 MR	6	2	6	2	4	6	12	24	Error Ft.
$R\dot{R}$	1750	3.26*	4.56	2.62	1.08	15.25	16.3	17.4	34.8	79.6	4,630
	2000	3.82	5.25	3.26	1.27	18.10	18.10	20.80	41.6	83.2	1,110
θR	1750	1	2.16	1	2.95	1.04	2.04	2.95	5.90	11.80	9,470
	2000	1	3.08	1	2.87	1.14	2.14	3.08	6.16	12.32	4,290
$\theta\dot{R}$	1750	1.01	3.10	1.07	3.10	1	2.06	3.10	6.20	12.4	3,800
	2000	1	3.00	1.04	3.00	1.01	2.01	3.00	6.00	12.00	2,690

TABLE 1 - ERRORS AT APOCYNTHION

* To get the error for any case, multiply error in same row by the factor shown. For example, in Case 1 of the $R\dot{R}$ scheme the error at apocynthion for correction at 1750 sec. is 3.26 x 4630 ft.

Thrust Variation, %	Ve Ft/Sec.	γ_e Deg.	he N.Mi.
-3%	+27.5	-.517	-2.31
-2%	+18.3	-.38	-1.55
-1%	+ 9.1	-.20	-.76
+1%	- 9.0	+.21	+ .8
+2%	-18.4	+.398	+1.58
+3%	-27.6	+.551	+2.36

Table 2

4. Rendezvous

A. General

One of the most significant factors in establishing certain aspects of the radar configurations proposed herein, has been the concept of an active homing phase during the terminal portion of the ascent to rendezvous flight. A terminal homing phase is recommended by GAEC as the basic rendezvous guidance mode primarily to allow the design of an alternate mode which provides for maximum crew participation in the operational procedure. It is then a logical extension of such a design philosophy to consider this "manual alternate" mode as the first tier back-up mode in the event of a non-radar-connected failure of the primary N. & G. system. As a consequence of this approach, it is recommended that the primary rendezvous guidance mode be fully compatible with the alternate and back-up modes to the extent of performing automatically (if the primary mode is automatic) exactly the same operational steps as the crew would perform in the alternate back-up modes. This assures that a) the crew is capable of monitoring the progress of the rendezvous phase and of determining if it proceeds properly, b) the dynamic and kinematic conditions existing at any instant are suitable for changeover to the alternate or back-up mode so that the crew is prepared to take over the operation and continue it to a successful conclusion without any mental reorientation, and d) astronauts need be trained for only one basic rendezvous maneuver to monitor and perform, and with respect to which to make operational decisions.

The effect of adopting the concept of a terminal homing phase which allows active crew participation in an alternate mode, and requires such participation in a back-up mode occasioned by failure of the primary N. & G. system, is that of requiring LOS rate measurement capability in the rendezvous radar and the capability of directly displaying radar data. All the modes of the rendezvous maneuver should be so fashioned as to take fullest advantage of the capability thus provided.

In the succeeding paragraphs, the rendezvous concept recommended by GAEC will be discussed in detail and it will be demonstrated that this technique has the effect of allowing a significant relaxation in the required tolerances of ascent trajectory control.

B. Rendezvous Maneuver

The basic feature of the terminal homing phase in the rendezvous maneuver is that the LEM essentially flies a collision course to the CSM. This characteristic is achieved by keeping the inertial rate of the LOS to the CSM below a given threshold value, which is established by the ability of a sensor to measure inertial rate.

As recommended by GAEC, the terminal homing rendezvous will be performed in a series of operational steps as follows:

1. Range to the CSM is measured continually during the coasting ascent flight. At a given range from the CSM, LOS rate to the CSM is measured and thrust is applied normal to the LOS to reduce this rate to the threshold level.

2. Following LOS rate reduction, range rate with respect to the CSM is measured and thrust applied along the LOS direction until range rate is reduced to a predetermined value appropriate to the range at which thrust was initiated.

3. The LEM is allowed to coast until the next range check point is reached. Steps 1 and 2 are then repeated.

4. This procedure is continued through a number of range check points until final docking range and near zero relative velocity are attained.

The attitude maneuvers required of the LEM as it proceeds through the above outlined operational sequence are as follows:

1. With the rendezvous radar locked on and tracking the CSM, the LEM attitude is adjusted to null the radar antenna gimbal angles. This results in the LEM Z-axis being directed along the LOS to the CSM and gives the crew direct CSM visibility through the forward cabin windows.

2. The direction of the normal component of the relative velocity vector is established from measurement of the inertial rates of the antenna. If the gimbal axes are aligned parallel to the LEM X and Y axes, then inertial gimbal rates are directly proportional to components of the relative inertial velocity vector along body X and Y axes. The LEM is rotated about the Z axis until one of the gimbal rates reaches the measurement threshold value. As a result, the body axis corresponding to the gimbal axis is now aligned with the net normal component of relative inertial velocity. Thrust along that axis is now applied to null the indicated LOS rate, and thus eliminate the normal velocity component. The RCS engines are used in this phase.

3. Range rate reduction to the value commensurate with the range at which the correction is made, is performed by the Z axis RCS engines, with the Z axis aligned to the LOS to the CSM.

The concept of multiple thrust phases rather than continuous control was adapted for several reasons. For one, the range versus range rate regime for multiple thrusts is a simpler one than for continuous thrust, and lends itself more readily toward a display presentation that the crew can follow in a manual mode. Furthermore, during some back-up modes involving LEM radar failure, several of the measurements required for successful rendezvous must be performed visually by the crew, and the coasting time between thrust applications allows this to be accomplished.

C. Rendezvous Study Results

A study was undertaken to analyze the various aspects of the terminal homing rendezvous maneuver described above. Of interest was the relationship between ΔV required to rendezvous and the range of initiation of the homing maneuver. Also the effect of LOS rate measurement accuracy on the capability of completing the rendezvous was investigated. The number of thrust phases, their range spacing and the range rate limits at these ranges were also studied.

Initially the thrust spacing used in the LEM proposal phase was employed. This regime can be described by the following table:

Range N. Mi.	Range Rate fps
30	-
20	200
10	120
4	60
1	20
.08	0

At 30 N.Mi., only a LOS rate correction was made. At subsequent check-points, in addition to the LOS rate correction, the range rate was reduced to the value given in the table. The regime worked satisfactorily for some ascent trajectories, but failed to result in rendezvous for others.

The reason for such failure to complete the maneuver was due to characteristics of the particular ascent trajectory tried. What generally happened was that the range rate at the first check-point (20 N.Mi.) was already below the required value, and thus no correction was made. The coasting trajectory would then reach apocynthion and the LEM would start moving away from the CSM before the next checkpoint was reached. Thus rendezvous would never be achieved. In order to assure that the homing maneuver would always terminate properly, two modifications in the range - range rate regime had to be made. To assure that sufficient closing rate existed at all times, a minimum as well as a maximum range rate had to be specified at each checkpoint. Thus, if the range rate existing at a given range checkpoint is below the minimum, thrust is applied towards the CSM to increase it to the minimum level, whereas if range rate is above the maximum, thrust is applied away from the CSM to reduce range rate to the maximum level.

The second modification involved the checkpoint spacing. If this spacing is too large, the normal kinematics of the trajectory could reduce the relative range rate to zero and reverse the direction of relative motion between checkpoints, even if the range rates were within the limits specified. Thus, in general the checkpoints had to be spaced more closely during the early phases of the homing maneuver than had originally been proposed. The range - range rate regime used in the studies so far is given in Table 3 for the various conditions of range of initiation of the homing maneuver investigated.

LOS rate measurement capability turned out to be a significant factor in determining the ability to rendezvous successfully. Figures A5-2 and A5-5 show the effect of LOS rate threshold on miss-distance at rendezvous for various initiation ranges for the case of a Hohmann as well as a high energy (140^o) transfer. The designation of $\pm$ 2% thrust variation refers to the errors in initial conditions at burnout of the powered ascent phase. The burnout errors corresponding to various percentages of thrust variation during the powered phase are shown in Table 2. For the cases studied, it is clear that LOS rate errors of greater than 0.2 mr./sec. do not result in satisfactory rendezvous in all cases. However, for LOS rate errors of that magnitude, rendezvous was achieved for the case of initiation at all ranges up to 40 N.Mi.

The ΔV required for rendezvous as a function of initiation range is shown on Figures A5-3 and A5-6. For the case of coast trajectories which would nominally come to within 20 N.Mi. of the CSM, the range of initiation for most economical operation seems to be about 14-20 N.Mi. The $\pm$ 3% trajectories must be initiated at longer ranges since they would not otherwise come within 20 N.Mi. of the CSM, and a commensurately higher ΔV penalty results. Rendezvous from high energy transfer orbits requires about 40 fps more for the 1% case and about 150 fps more for the 2% case. The nominal, impulsive transfer requires about 40 fps more for the high energy transfer.

Although the radar requirements studies are far from complete, several interesting conclusions can be observed. As evidenced from ΔV penalities incurred from burnout errors corresponding to a $\pm$3% thrust variation (Figures A5-3,6) which are on the order of about 250-300 fps over the impulsive ΔV required for final velocity adaptation of the transfer orbit, mid-course corrections will very likely be required, particularly if ascent is performed with back-up guidance equipment. However, the requirements on the precision of such mid-course corrections are not very severe, since the miss-distance need be reduced to only about 10-15 N.Mi. to achieve the conditions for successful rendezvous. Thus, reference to Table 1 indicates, for example, that a radar

TABLE 3

RANGE RATE BOUNDS AT RENDEZVOUS RANGE TEST POINTS

Initial Rendezvous Range, N.Mi.	Range Test Point, N.Mi.	Range Rate Bounds, ft./sec.
40	40	350 - 250
	35	325 - 225
	30	300 - 200
	20	200 - 120
	10	120 - 70
	4	70 - 30
	1	30 - 10
30	30	300 - 200
	25	270 - 150
	20	200 - 120
	15	170 - 100
	10	120 - 70
	4	70 - 30
	1	30 - 10
20	20	200 - 120
	16	170 - 90
	12	130 - 75
	8	100 - 60
	4	70 - 30
	1	30 - 10
14	14	160 - 95
	10	120 - 70
	8	100 - 60
	4	70 - 30
	1	30 - 10

having a 3σ range measurement error of 2% and a 3σ angular measurement error of 24 mr (total of random plus bias error at time of measurement) can provide the data for a mid-course correction with a 3σ miss-distance of about 9 N.Mi., which is perfectly adequate for the homing maneuver. In particular, radar boresight accuracy requirements appear to be not very critical, slow variations in boresight direction resulting from thermal effects are of no consequence and in-flight calibration is not required.

The maximum ΔV penalty incurred through the utilization of the terminal homing concept can be estimated from the data generated so far and presented herein. Back-up guidance studies performed concurrently with the radar requirements studies indicate that burnout errors corresponding to $\pm 3\%$ thrust variation represent about the limits of errors that can be expected under the worst conditions, which would be abort from hover using back-up guidance equipment. From Figure A2-2 it can be seen that the ΔV required for mid-course correction from such a trajectory is about 50 fps. As a result of applying this correction, a trajectory similar to a $\pm 2\%$ thrust variation trajectory will be obtained. Initiating the homing phase at about 14 N.Mi. results in an additional ΔV for this phase of approximately 175 fps, for a total of 225 fps. This is 125 fps above the impulse ΔV for the Hohmann transfer at 80 N.Mi.

Further studies are already underway to refine these estimates for various other conditions, such as higher energy transfers, different radar error estimates, presence of radar errors not previously considered, etc.

D. Summary of Operational Modes

As presently conceived, there are four operational modes of performing the rendezvous maneuver, corresponding to successively greater degradation of the prime equipment. These modes, and their significant characteristics, are presented below:

1. Primary Mode

All equipment is operating normally. The rendezvous maneuver is performed as described above, with the AGC computing all attitude and thrust commands. Thrust is applied automatically and the duration of the impulse is computer controlled. Radar data is directly displayed to the crew, but used only to monitor the progress of the maneuver.

2. Manual Alternate

All equipments operate normally or the primary N. & G. system has failed, but the radar and the displays are functioning properly. From the range display, the crew determines when to perform the corrective maneuvers. Alignment of the Z-axis with the LOS is accomplished by reference to the gimbal angle display. The LEM is rotated about the Z-axis until the LOS rate display shows one component to be zero. Thrust is applied along the other axis until the display shows that component of LOS rate to be nulled. Thrust is then applied along either the positive or negative Z-axis until one of the range rate limits for that range is attained.

3. CSM Radar Utilization Mode

The LEM rendezvous radar has failed, but all other equipment is functioning normally. The LEM Z-axis is visually directed along the LOS to the CSM using the OMU and flashing lights on the CSM. Range and range rate data is communicated to the LEM crew from the CSM, as obtained from the CSM tracking radar. Prior to reaching the first checkpoint, the LEM crew determines the direction of the normal component of relative velocity by tracking the CSM at the center of the OMU crosshairs and rotating the LEM about the Z-axis until the relative motion of the CSM against its background (whether it be star background or lunar surface background) occurs along one of the coordinate axis of the OMU reticle. The impulse to be applied to null the normal component of velocity is obtained from the CSM via the communication link and applied along the body axis corresponding to the reticle axis aligned with the normal velocity vector component. The impulse to be applied in the direction of the LOS is likewise obtained from the CSM.

4. Manual - Visual Mode

Both radars or the LEM radar and the communication link have failed, but all other equipment is functioning normally. Approximate range and range rate data can be obtained by computation from the AGC. LOS rate direction is ascertained as in Mode 3 above, but in addition LOS rate magnitude must be determined. This can be performed visually only if the CSM is seen against a star background. Under those conditions, the rate at which the stars move relative to the CSM along the OMU reticle axis can be estimated and a thrust impulse applied to null it.

5. Preformance Requirements

A. Functional Description of the Radar Configuration

It is proposed that the LEM vehicle will have two independent radar sensors, a radar altimeter and a rendezvous radar. The radar altimeter will be a fixed antenna, three-beam doppler system and will provide data relative to the LEM body axes. The rendezvous radar will have a two degree of freedom gimbaled antenna and will provide space stabilized LOS rate data and body referenced LOS angle data. Data from both sensors will be displayed independent of both the primary and backup guidance systems.

B. Altimeter Parameters

The two position radar altimeter parameters required are altitude, altitude rate and horizontal velocity. Table I shows the range and accuracies required (3σ values).

	Quantity	Maximum	Minimum	Typical*	Accuracy
1.	altitude (h)	70,000 ft.	5 ft.	20,000 ft.	1% ± 5 ft.
2.	Altitude rate ($\dot{h}$)	500 fps.	1 fps.	-	1% ± 1 fps.
3.	horizontal vel.	+ 2,000 fps. - 100 fps.	1 fps.	-	1% ± 1 fps.
4.	position (Angle of axis of symmetry with respect to -X axis)	50°	0°	-	20

C. Justification of Parameter Values

1. Altitude

The maximum altitude requirements are obtained from the utilization of the radar altimeter during the synchronous coast phase to provide a check on the radar operation prior to initiation of powered descent. Since in the nominal mission, powered descent will be initiated at the 50,000 ft. pericynthion of the synchronous orbit, the 70,000 ft. requirement has been set as a desireable design goal. This allows approximately 10 minutes for radar checkout prior to reaching pericynthion. Minimum altitude requirements are obtained from consideration of radar altimeter useage during final let-down to the lunar surface. Without considering the degrading

* The accuracies stated apply up to range of the measured parameter given in the column headed "Typical" unless no typical value is stated.

effect of a dust cloud raised from the lunar surface during the final moments before touchdown, a 5 ft. minimum altitude resolution should be obtained to remain within vertical impact attenuation capabilities of the landing gear. If the primary system computation of altitude is updated just prior to final let-down, inertially computed altitude will be adequate for landing.

The altimeter radar accuracy requirements have been examined in the study of the use of the altimeter radar during the final portion of powered descent. (Appendix 1) The 1% requirement on altitude accuracy represents a reasonable value for this parameter and imposes no significant error in the nominal descent trajectory.

2. Altitude Rate

Maximum altitude rate and minimum altitude rate requirements are obtained from utilization of the radar altimeter in the powered descent and terminal let-down phases respectively. The values listed represent the largest and the smallest values encountered during radar utilization from 20,000 ft. to touchdown. The minimum value falls well within the impact capabilities of the landing gear and represents a design goal for the sensor rather than the safe minimum value permissable for the landing gear.

The altitude rate accuracy requirement of 1% ± 1 fps. is designed to provide sufficiently accurate data for the terminal portion of the descent and let-down. The descent trajectory is sufficiently sensitive to errors in altitude rate to require this accuracy, particularly in the manual alternate mode or when the inertial portion of the primary guidance system has failed.

3. Horizontal Velocity

The requirements for horizontal velocity data from the radar altimeter are obtained from the requirements of both the automatic and manual alternate modes during powered descent as well as from the hover and let-down phase. A velocity range to 2,000 fps. will include the horizontal velocities encountered below 20,000 ft. altitude with the nominal descent trajectory. 2,000 fps. is a typical value of V_H at which the specified accuracy is to be attained. 5,000 fps. is a desirable range which would allow complete checkout of all three beams of the radar prior to initiation of powered descent.

The minimum requirement is obtained from considerations of the landing gear capability at touchdown. A 1 fps. horizontal velocity resolution insures a measurement capability well within these limits.

While the three beam, doppler radar is relatively insensitive to altitude changes with respect to the local vertical, due to the large change in pitch orientation of the LEM with respect to the lunar surface during the final portion of the powered descent, the radar altimeter is required to have two nominal operating positions with respect to thrust axis. Errors in antenna orientation primarily appear as velocity errors in the horizontal and vertical outputs and should be kept to a minimum for the reasons discussed above.

D. Rendezvous Radar Parameters

The rendezvous radar must be able to provide the four parameters; range, range rate, angle and angle rate. Table 2 shows the ranges and accuracies required (3σ values).

Quantity	Maximum	Minimum	Typical **	Accuracy
1. Range (R)	400 N. Mi.	5 ft.	30-0.2 n.m.	(1% ± 5 ft.) 1.5% ± 30 ft.
2. Range rate ($\dot{R}$)	± 4800 fps.	1 fps	200-1000 fps	1.0% ± 1 fps.
3. Angle θ	-	-	-	15_{mr} bias 3_{mr} random
4. Angle Rate ($\dot{\theta}$)*	± 15 mr./sec.	0.2 mr/sec.	-	0.2 mr/second

* Not required for Primary Guidance

E. Justification of Parameters

1. Range

The maximum range requirement on the rendezvous radar is obtained from the utilization of the radar to track the CSM in its 80 N.M. orbit from the lunar surface during the pre-launch phase of the mission. A maximum range of 400 n. miles is obtained when the CSM appears over the lunar horizon. Minimum range measuring capability is required during the docking phase. The 5 ft. minimum range represents a design goal since this requirement must be finally established through docking simulation studies presently being carried out.

** Same footnote as for altimeter

The rendezvous radar accuracy requirements have been studied in some detail for backup guidance (See Appendix 4 and 5) to determine the ΔV penalties imposed by errors in the various radar parameters. The results of these studies in addition to the requirements of the primary guidance system have served to define the accuracies listed above. A range accuracy of 1.5% ± 30 ft. represents a reasonable value based on the mid-course correction error studies. The 1% ± 5 ft. accuracy is a design goal that is desired if it does not impose a significant penalty upon the radar design. The increased accuracy is desirable to supplement the crew capabilities in the docking mode. It also allows the use of the rendezvous radar as a backup to the altimeter and as third altitude sensor to decide whether the IMU or the altimeter are functioning properly in case of large discrepancy in the two outputs.

2. Range Rate

Maximum and minimum range rate values are determined by utilization of the rendezvous radar in the ascent and docking phases of the mission respectively. Relative rates between the LEM and the CSM will not exceed 1000 fps during ascent when it is desired to track the CSM for either monitoring or backup guidance measurements. Higher rates are obtained during lunar surface tracking of the CSM but are not required for guidance computations. Minimum rates are utilized solely for monitoring of the manual-visual docking phase and 1 fps. is chosen to insure performance within safe docking impact velocities. The range rate accuracy requirement is based upon the study results discussed above.

3. Angle

The minimum gimbal freedom of the rendezvous radar is chosen to insure the LEM of orientation flexibility so as to permit visual monitoring and/or thrusting capability during the rendezvous phase and landing with a beacon. Gimbal freedom is also required for tracking of the CSM while the LEM is on the lunar surface. Angular position accuracy requirements have also been obtained from studies of "mid-course" correction error penalties for both primary and back-up guidance schemes. The 15 mr. bias uncertainty represents a static error requirement over the period of the mid-course and rendezvous phase. The 3 mr. random uncertainty represents the maximum allowable value of short period variations in boresight accuracy to: a) attain the required measurement accuracy for midcourse correction if a non-homing rendezvous is used and b) to achieve the required LOS rate accuracy or the homing rendezvous.

4. Angle Rate

The maximum and minimum angular rates are obtained from the utilization of the rendezvous radar during the terminal rendezvous phase to measure line-of-sight rate to the CSM. The maximum and minimum values are obtained from the consideration of rendezvous from off-nominal trajectories studied in Appendix 5 . In order to measure the minimum line-of-sight rate accuracy during some of the off-nominal trajectories and to insure a successful rendezvous, the angle rate measurement accuracy must be within 0.2 mr/sec.

5. Antenna Gimbal Limits

A preliminary definition of the antenna gimbal axis orientation relative to LEM body axes and of the required angular freedom about these gimbal axes is shown in Figure 1. The gimbal limits presented are based on an analysis of possible tracking radar utilization during all mission phases as discussed in Section 3. The gimbal axis order and orientation are designed to be the same as those of the OMU, in order to permit digital readout of both OMU and radar position with a common set of digital shaft transducers.

6. Summary and Recommendations

The major conclusions derived from the study of the performance requirements of non-inertial sensors with respect to the LEM mission are summarized below:

- The primary utilization of non-inertial sensors occurs during a) the terminal portion of the powered descent phase and b) during ascent coast and rendezvous.

- These sensors are also useful during a) the coasting descent, for trajectory verification, b) stay on the lunar surface, for determination of CSM orbit, c) abort, to provide a non-inertial attitude reference and d) during powered descent towards a surface beacon, for providing LOS guidance data.

- During normal powered descent, altitude and velocity measurements are used for IMU updating.

- Studies of the effect of terrain slopes on the effectiveness of altitude updating of the IMU indicates that

  a) for reasonable slopes (up to about 6°) altitude updating can achieve successful landings.

  b) initial altitude offsets and reasonable measurement errors will not compromise the ability to land safely.

  c) the ΔV penalty for landing over terrain with reasonable slopes is insignificant.

  d) slant ranging to the landing point is required only for beacon guided descents or under conditions of very severe terrain slopes (15°).

- During coasting ascent, non-inertial measurements allow the performance of mid-course corrections and reduce the ΔV penalty for rendezvous.

- Mid-course correction studies indicate that

  a) a simple correction regime can reduce the ΔV penalty to reasonable levels even in the presence of large powered ascent burn-out errors.

  b) comparatively large measurement errors can be tolerated during mid-course, since even with large miss distances (5 N.Mi.) a homing rendezvous can be performed economically.

- Rendezvous study results indicate that

  a) manual operation of the homing rendezvous is feasible if the range rate is reduced gradually and attitude constraints for visibility are imposed.

  b) range rate should be a function of range.

  c) the thrust sequence for rendezvous should be performed by thrusting in separate orthogonal maneuvers to reduce range rate and LOS rate.

  d) since the stepwise reduction of range rate alters even a perfect intercept trajectory, LOS rate null must be maintained to assure rendezvous.

  e) the ΔV budget for the terminal homing phase is a function of the errors existing at rendezvous initiation.

  f) the ΔV penalty for adapting the nominal rendezvous maneuver to manual operation is negligible.

  g) in general, 20 N.Mi. is nearly the optimum distance for homing rendezvous initiation for all trajectories which are reasonably close to the nominal

  h) the x-axis impulses applied during the homing rendezvous approach or go below the estimated minimum impulse capability of the main ascent engine so that the RCS should be utilized for this phase.

As a result of the studies described and of the conclusions summarized above, the following recommendations are made:

- A radar system should be provided to furnish the non-inertial measurements required in the various mission phases.

- Two separate radars should be supplied - one for determination of altitude and velocity relative to the lunar surface, and one for tracking of the CSM and/or a surface beacon.

- The tracking radar should be implemented to be capable of backing up the altimeter and to provide the means of deciding between the altimeter and IMU if they provide significantly different indications of position or velocity.

- Data from these radars should be such as to be meaningful and useful to the crew if displayed directly.

- The tracking radar antenna should be mechanically gimbaled in a two-degree-of-freedom configuration to provide the orientation flexibility required for utilization in all those mission phases in which tracking data is utilized.

- Radar data display should be provided to allow maximum utilization of crew capabilities in performing the landing and rendezvous phases.

- Automatic modes of landing and rendezvous should be designed to be compatible with manual alternate or manual back-up modes in the sense of allowing efficient and successful completion of the maneuver in the event of failure of the automatic N. & G. system.

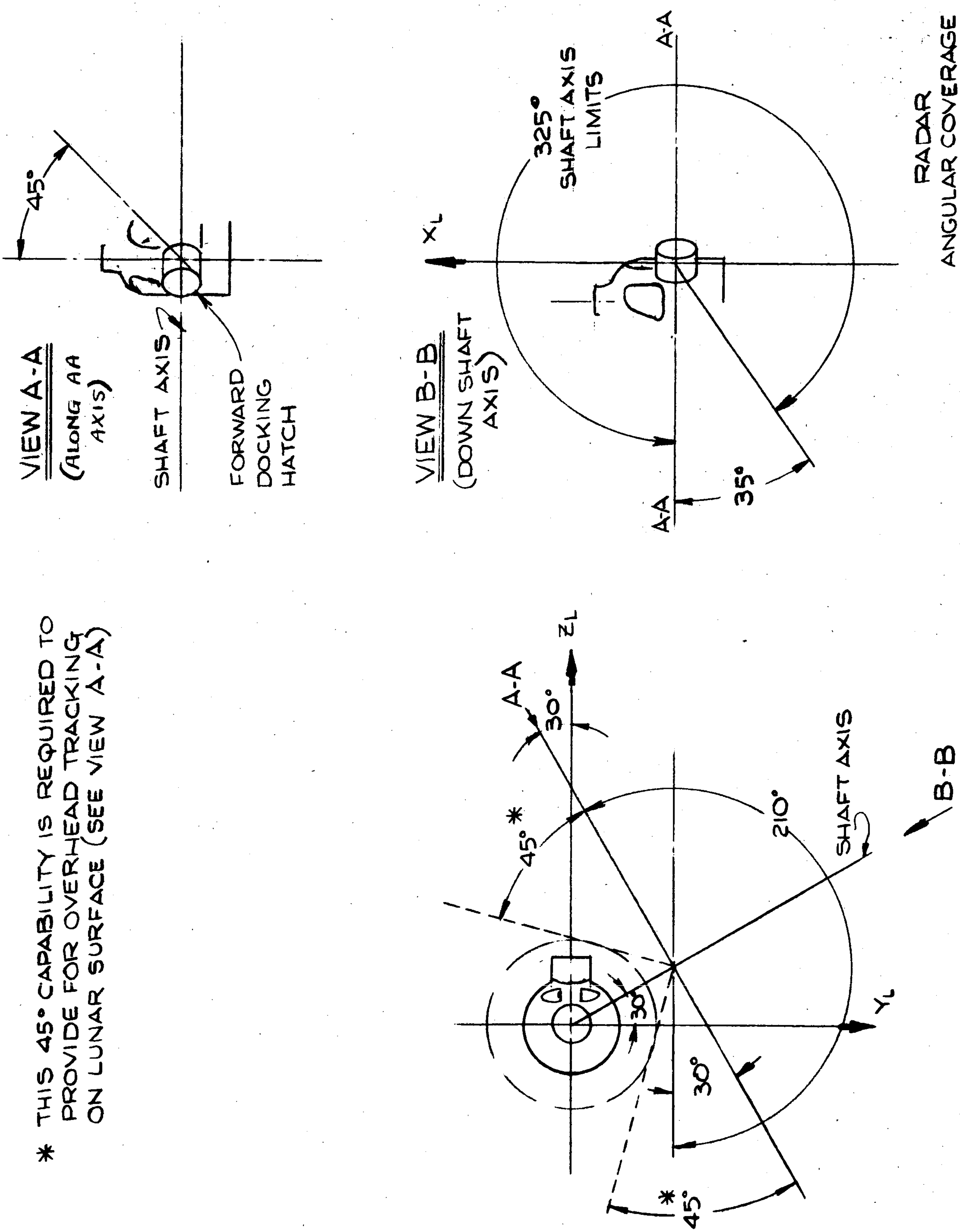

VIEW A-A
(ALONG AA AXIS)
X_L
45°
SHAFT AXIS
FORWARD DOCKING HATCH
VIEW B-B
(DOWN SHAFT AXIS)
325° SHAFT AXIS LIMITS
A-A
35°
RADAR ANGULAR COVERAGE
* THIS 45° CAPABILITY IS REQUIRED TO PROVIDE FOR OVERHEAD TRACKING ON LUNAR SURFACE (SEE VIEW A-A)
Z_L
30°
45° *
210°
SHAFT AXIS
B-B
Y_L
* 45°

FIG. 1

Appendix 1

Utilization of Altimeter Data During Terminal Portion of Powered Descent

Purpose -

It is the purpose of this analysis to study the utilization of a radar altimeter in conjunction with the LEM IMU and AGC to generate the navigational information required during the final 20 NM of descent - with termination at an altitude of 1000 ft. above the lunar surface. Since radar data provides essentially relative information, it is not impossible to correct the inertially computed LEM position on the basis of radar altimeter data unless the local surface is accurately known. For this study, radar altimeter information was used to determine the slope of the lunar surface and, with this information, to predict the coordinates of the desired target (hover point). This study considered the terminal effects of the following parameters: radar altitude errors, initial vertical position and velocity errors in the IMU and AGC variations in target altitude and various surface slopes.

Procedure -

The final descent motion was considered to be planar. For this analysis, the LEM was treated as a point mass. The system was analyzed and a powered flight simulation was performed on the IBM 7094. The simulation provided the following:

1. LEM Motion -

The actual trajectory flown by the LEM was determined by solution of the two-degrees-of-freedom-equations of translational motion for a point mass.

2. IMU -

The inertial measurement unit consisted of two integrating accelerometers orthogonally mounted on a gyro-stablized element. The outputs of the accelerometers were in the form of incremental velocity changes. The platform was aligned so that at the start of the final descent one accelerometer was oriented along the local vertical, while the other accelerometer was oriented along the local horizontal.

3. Radar Altimeter -

The altimeter was mounted such that it was always directed downward along the local vertical. The altitude (position) information was utilized for the AGC navigation computations.

4. Lunar Model -

The nominal lunar surface was considered to be the boundary of a uniformly dense spherical moon. The sloped surfaces were linear (i.e., infinite radius of curvature), where the magnitude of the slope was determined by the angle between the inclined surface and the local horizontal at the target (see Figure A1-1). The sign of the slope was defined by the following convention: A positive slope indicated that the lunar surface was decreasing in altitude as the LEM traveled toward the target.

5. Guidance Law -

The guidance law used for this analysis is based on the "Line of Sight" (LOS) proportional navigation technique developed by L.S. Cicolani, Ames Research Center, and presented in NASA TN-D-722. The form of the equations programmed for the simulation is the same as that described in GAEC Study Report PDM-323A-88. This guidance law utilizes slant range information and the angle between the velocity vector and the line-of-sight to determine thrust magnitude and direction (pitch attitude) commands. For the purpose of this analysis, the two required gains in the equation were fixed for all the simulated trajectories. This combination of gains appears to produce a thrust vector which does not follow the commands closely, but does not optimize the trajectory with respect to ΔV.

6. Thrust Vector Controller -

The thrust vector commands arising from the LOS proportional navigation guidance law were instantaneously transformed into engine thrusts and pitch attitude. No provision was made for response lags. A single throttleable descent engine with maximum thrust of 10,500 lbs. and a 10:1 throttling ratio was simulated.

7. AGC -

The function of the AGC is to read the output of the accelerometers and transform this information into current position and velocity information. It also processes the radar information and predicts the position of the target by a linear (two point) extrapolation technique. It processes the present inertial measurements and the target information to obtain the necessary inputs for the guidance law. Lastly, it implements the guidance law and issues the indicated thrust commands. All of these functions are performed at the same computer rate, which for this analysis is at 2 cycles per second.

The functional relationships between each of the above components can be seen in Figure A1-2.

8. <u>Cutoff Criteria</u> -

Ideally, it was desired to terminate the trajectory at an altitude of 1000 ft. above the lunar surface, with a zero total velocity. However, the particular guidance law that was utilized in this analysis does not provide uniform convergence of the position and velocity components. The lack of uniform convergence is most notable in the velocity components. For mission safety, and for need of a common reference, the radial velocity component was used to test for termination. The AGC computed value of radial velocity was used for cutoff rather than the actual velocity, to simulate the effect of automatic operation.

At the start of a typical trajectory the LEM is descending rapidly. As the LEM approaches the target, the downward velocity decreases monotonically. However, depending on several guidance parameters, when the LEM is within a short distance of the target (say, less than 100 feet) the downward velocity may suddenly start to increase again. Thus, two tests were used for termination: the first tested the AGC computed radial velocity, $\dot{r}$, to see whether it had passed through zero; the second, tested the two most recent values of $\dot{r}$ to see whether they were monotomially decreasing. If either test indicated that a termination condition had been exceeded, a "forced-halfing" subroutine was used to backtrack and locate the closest point to the termination point.

Assumptions -

The following assumptions were used throughout the analysis:

1. The IMU was last aligned 30 minutes before initiation of LOS navigation.

2. All IMU and AGC errors which have accrued since the last alignment appear only in the accelerometer loop which is vertical at initiation of LOS navigation.

3. Initial conditions for the final descent are determined by the final conditions of an optimum ΔV guided trajectory starting at pericynthion of 50,000 ft. These initial conditions are:

Initial LEM altitude above spherical lunar surface	= 18,828.8 ft.
Initial control angle	= 0.0 degrees
Initial radial velocity	= -248.68 ft./sec.

Initial tangential velocity	=	2140.68 ft./sec.
Central angle of target	=	1.1880 deg.
Specific impules of descent engine	=	310.0 sec.
Initial mass of LEM	=	455.44 slugs

4. The radar yields true vertical altitude information to within the radar accuracy. The radar accuracy is defined as a percentage of the altitude or by a constant "stand-off", whichever is greater.

Parameters Studied -

The following independent parameters were investigated.

1. Lunar Surface - The lunar surface was defined by two parameters: target altitude above the spherical moon, and the slope of the surface measured with respect to the local horizontal at the target. In the case where a slope change was simulated, the first incline was defined as previously described, while the second incline had a slope equal to the negative of the first slope.

   Surface altitudes beneath the Target altitudes above the spherical moon were varied from -4000 ft. to +5000 ft., while surface slopes of ± 3 degrees were considered.

2. <u>IMU Errors</u> - Pre-LEM studies indicated that the most critical sources of error for LOS guidance arise from errors in the vertical channel of the IMU and AGC. Thus, initial vertical position errors ranging between -5000 ft. and +5000 ft. were investigated. It was assumed that these errors were the result of integrating velocity errors. Further, it was assumed that these velocity errors were constant since the last IMU alignment. Since the last alignment occured 30 minutes earlier, the vertical velocity error, $\dot{EZ}_o$, corresponding to an error in vertical position, EZ_o, can be determined by the relationship

$$\dot{EZ}_o, = \frac{EZ_o}{30 \text{ min.}}$$

Therefore, the range of initial vertical velocity errors corresponding to the aforementioned initial vertical position errors is -2.78 ft./sec. to +2.78 ft./sec.

3. <u>Radar Errors</u> - Since the target is 1000 feet above the lunar surface, radar "stand-off" errors were neglected. Therefore, the only radar errors considered are those that are proportional to altitude - the range of the proportionality constant considered was between -1.5% to +1.5%.

<u>Outputs</u>

The following cut-off or hover parameters were studied:

1. <u>The final vertical velocity</u> - This should be nearly zero

for the ideal case. However, various error sources cause errors in the vertical velocity computation. Thus, when the IMU indicates a nearly nulled vertical velocity, the actual vertical velocity can be quite large. This "actual" velocity is the velocity plotted.

2. The final tangential velocity - Because of the non-uniform convergence of the state parameters, the tangential velocity does not approach zero as rapidly as the radial velocity. Thus, for the nominal trajectory (zero component error sources), although the final vertical velocity is less than 0.01 ft./sec., the tangential velocity is still 6.33 ft./sec.

3. ΔV - The ΔV plotted in the graphs is the sum of the ΔV used during the LOS navigation phase plus the tangential velocity remaining at cutoff.

4. Position - The position referred to in the appended figures is the actual slant-range distance between the LEM and the target at cutoff.

Following is a summary of the data displayed in Figures A1-3 - A1-15.

Figure A1-3 illustrates the trajectory of the LEM assuming no errors in the IMU-AGC or in the radar.

Figure A1-4 illustrates the effects of a 5000 ft. initial error in the computed vertical position. The actual trajectory flown is essentially identical to that in figure A1-3.

Figures A1-5 and A1-6 are plots of the cutoff parameters vs. initial error in the computer vertical velocity, with the radar errors 1.5% and 0.5% respectively.

Figure A1-7 illustrates the ΔV penalty vs. initial error in the computed vertical velocity. The upper plot corresponds to the case of figure A1-5, the lower plot to the case of figure A1-6.

Figures A1-8 and A1-9 are plots of cutoff parameters vs. positive and negative radar errors. The surface configuration and initial vertical errors are listed on the graph.

Figure A1-10 is a plot of cutoff parameters vs. negative radar errors where the initial vertical errors are the negative of those used for figures A1-8 and A1-9.

Figure A1-11 illustrates the ΔV penalty vs. radar errors, the upper middle, and lower graphs correspond to the conditions of figures A1-8, A1-9 and A1-10 respectively.

Figure A1-12 shows cutoff parameters vs. altitude of target above the spherical moon.

Figure A1-13 illustrates ΔV penalty vs. target altitude.

Figure A1-14 represents the trajectory flown by LEM when it traverses a surface which has a change in slope.

Figure A1-15 is a tabulation of the cutoff velocity parameters as the time at which the second slope is introduced is varied.

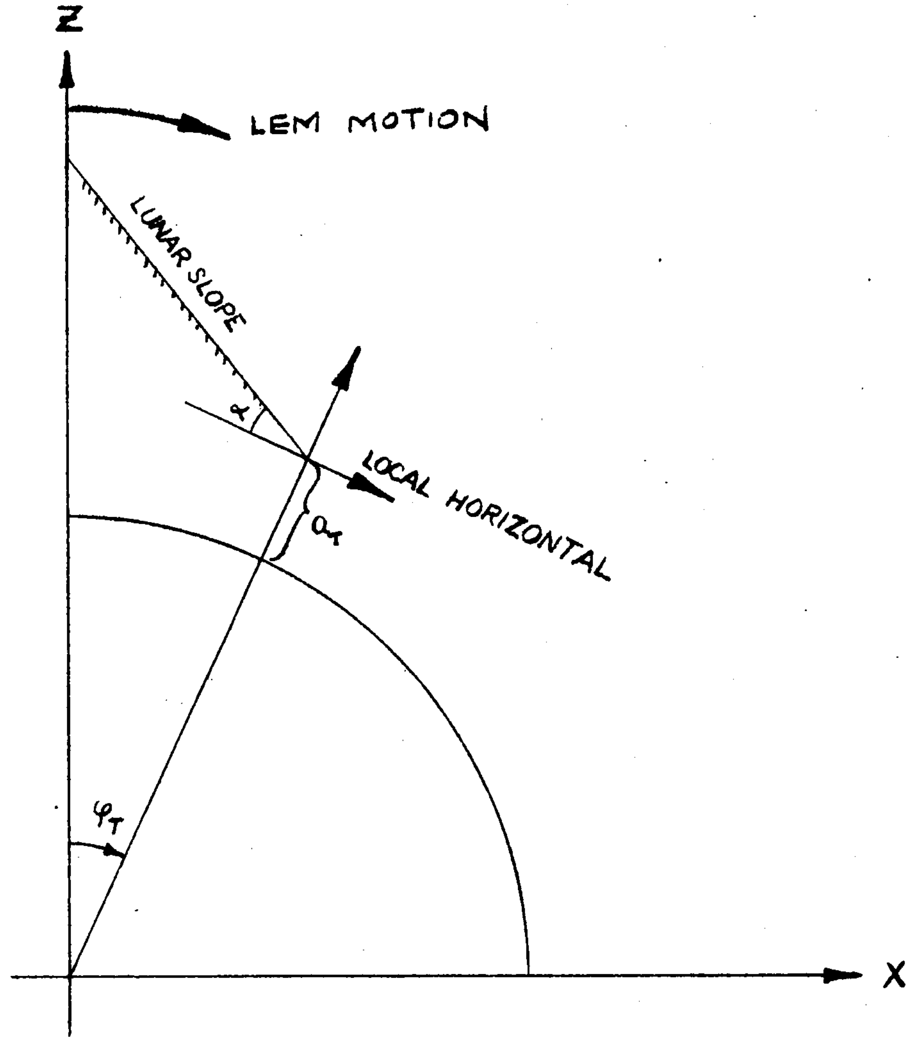
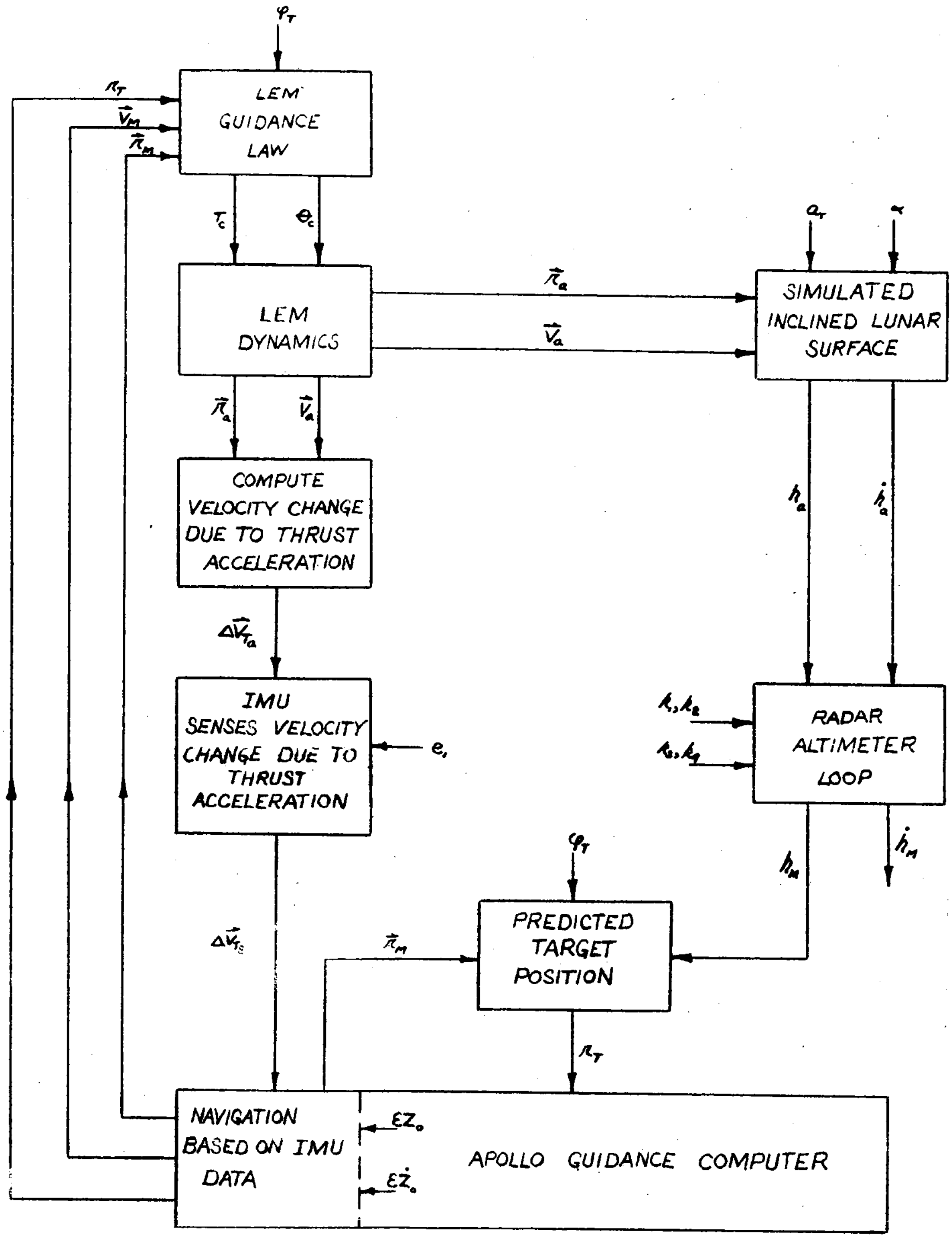


FIGURE A1-1 GEOMETRY AND NOMENCLATURE

FIGURE A1-2
GUIDANCE AND NAVIGATION BLOCK DIAGRAM



Nomenclature

A. Symbols

a	=	altitude of actual moon surface above the average surface
α	=	the angle of slope of the moon's surface
d	=	computational parameter required to determine the surface incline
e_1	=	error in incremental velocity due to Z-accelerometer bias error
g_z	=	acceleration due to lunar gravity along Z-axis
GM_m	=	gravitational constant for moon
h	=	altitude above actual lunar surface
H	=	altitude above average surface
EH	=	error in H
k_1, k_2	=	radar altimeter accuracy parameters to be read as an accuracy of $\pm 100k_1$ (%) or $\pm k_2$(ft), whichever is greater.
k_3, k_4	=	radar altimeter rate accuracy parameters to be read as an accuracy of $\pm 100k_3$ (%) or $\pm k_4$ (ft), whichever is greater
r, φ	=	inertial polar coordinates, where φ is measured positive clockwise from Z-axis.
R_o	=	radius of spherical moon
R_G	=	linear range-to-go along lunar surface
t	=	time
Δt_K	=	$t_K - t_{K-1}$
Δt_n	=	$t_n - t_{n-1}$
T, θ	=	thrust magnitude and orientation angle
X, Z	=	inertial rectangular Cartesian coordinates with origin at moon's center and Z-axis along position vector to LEM at time $t = 0$
(X, Z)	=	coordinates in XZ plane
Z_T	=	incremental velocity due to thrust

B. Subscripts

a	=	actual value
G	=	with respect to ground (lunar surface)
k	=	integration rate of equations of motion
M	=	measured value
M_c	=	corrected measured value
n	=	integration rate of IMU loop
S	=	sensed value
T	=	with respect to the target

C. Notation

$$(\dot{\ }) = \frac{d}{dt}$$

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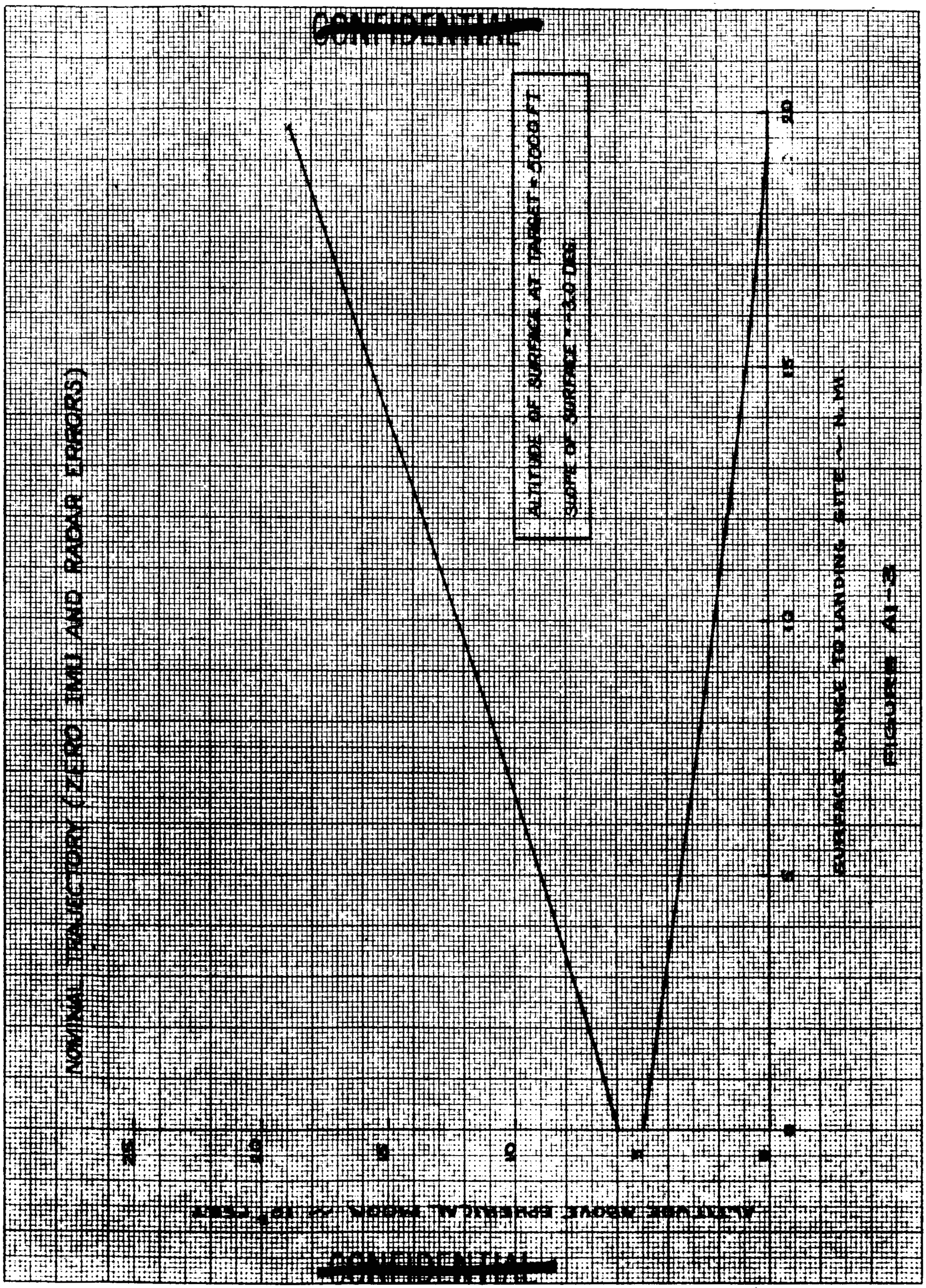

FIGURE A1-2

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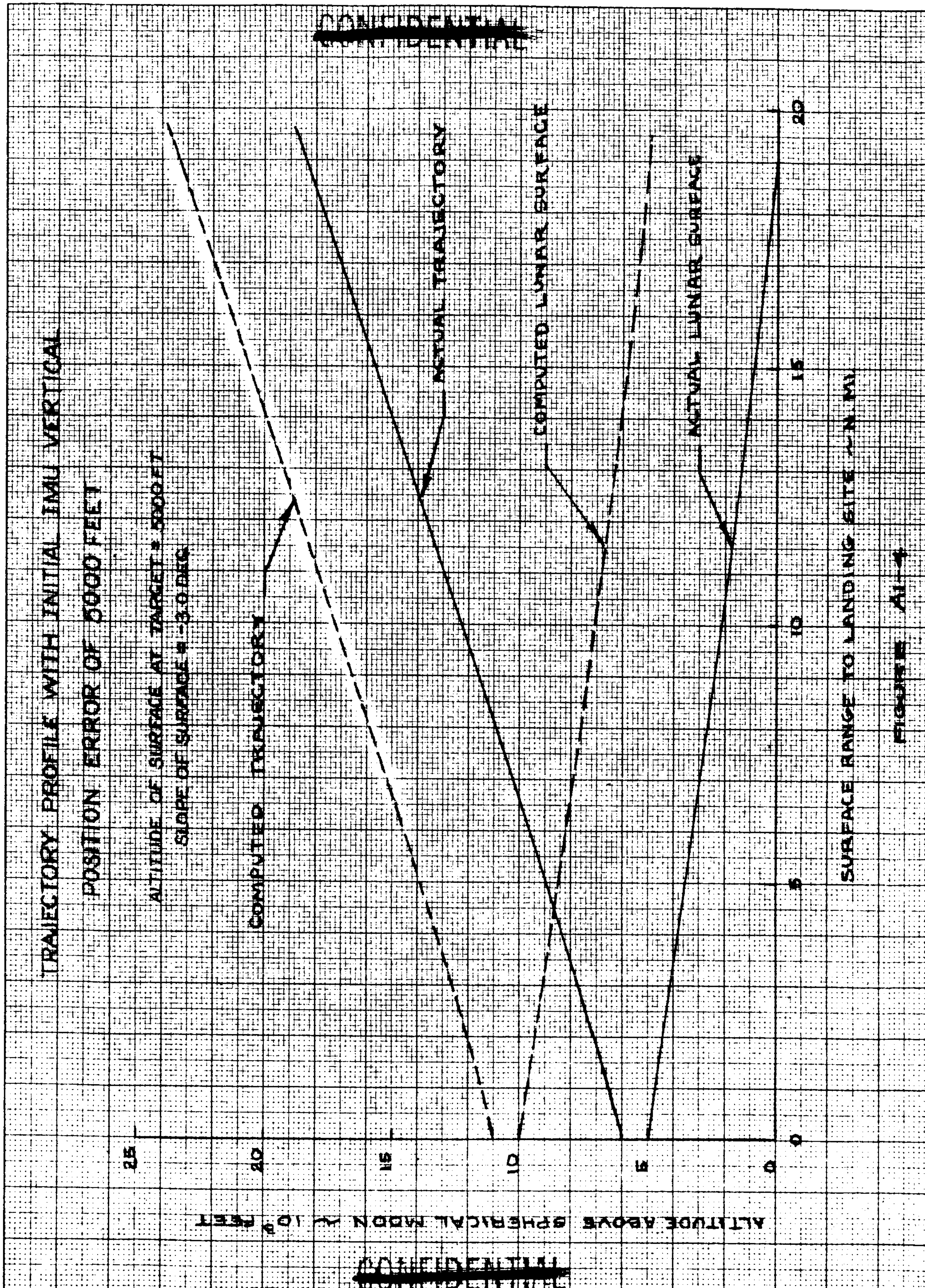

FIGURE A1-4

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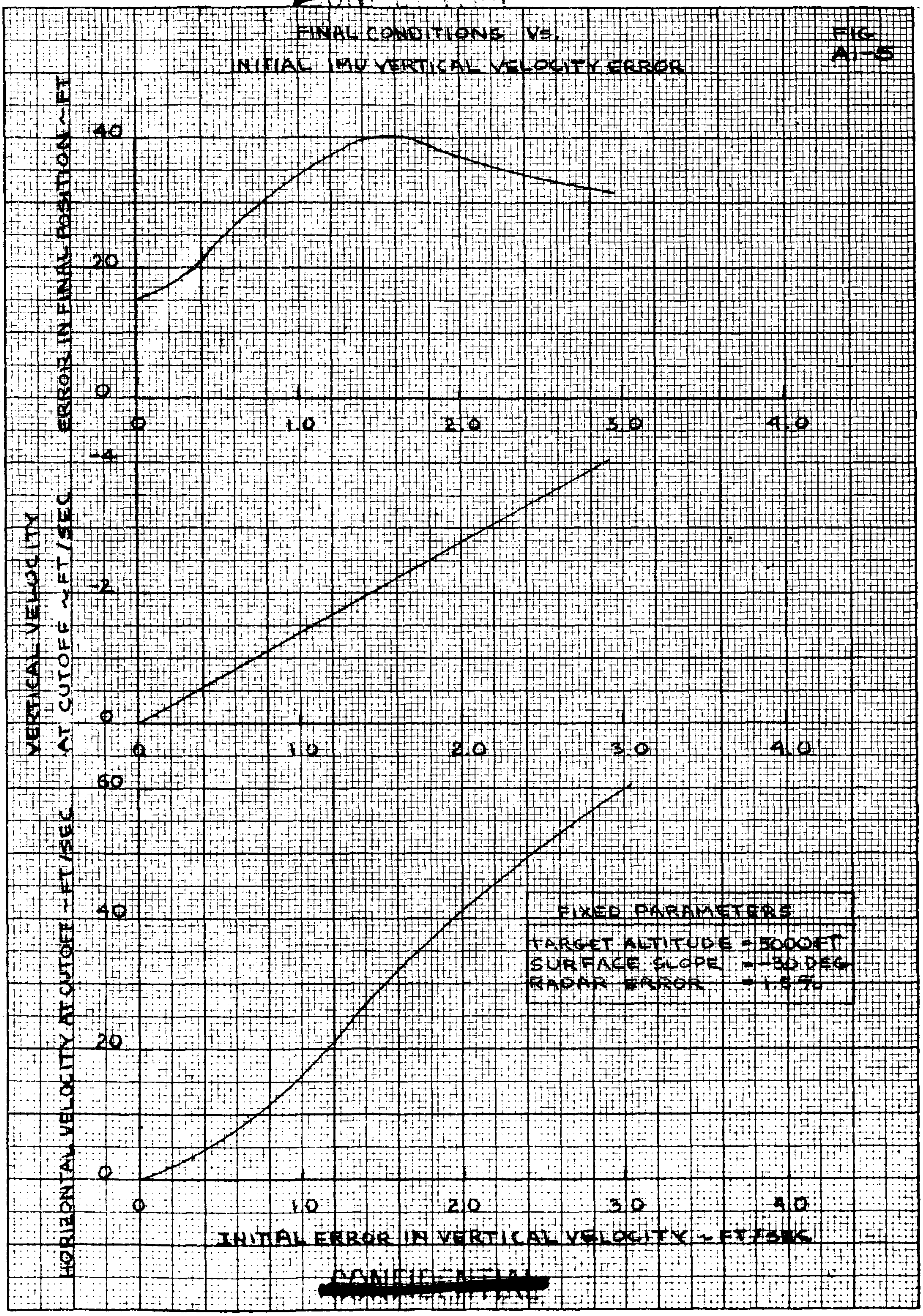

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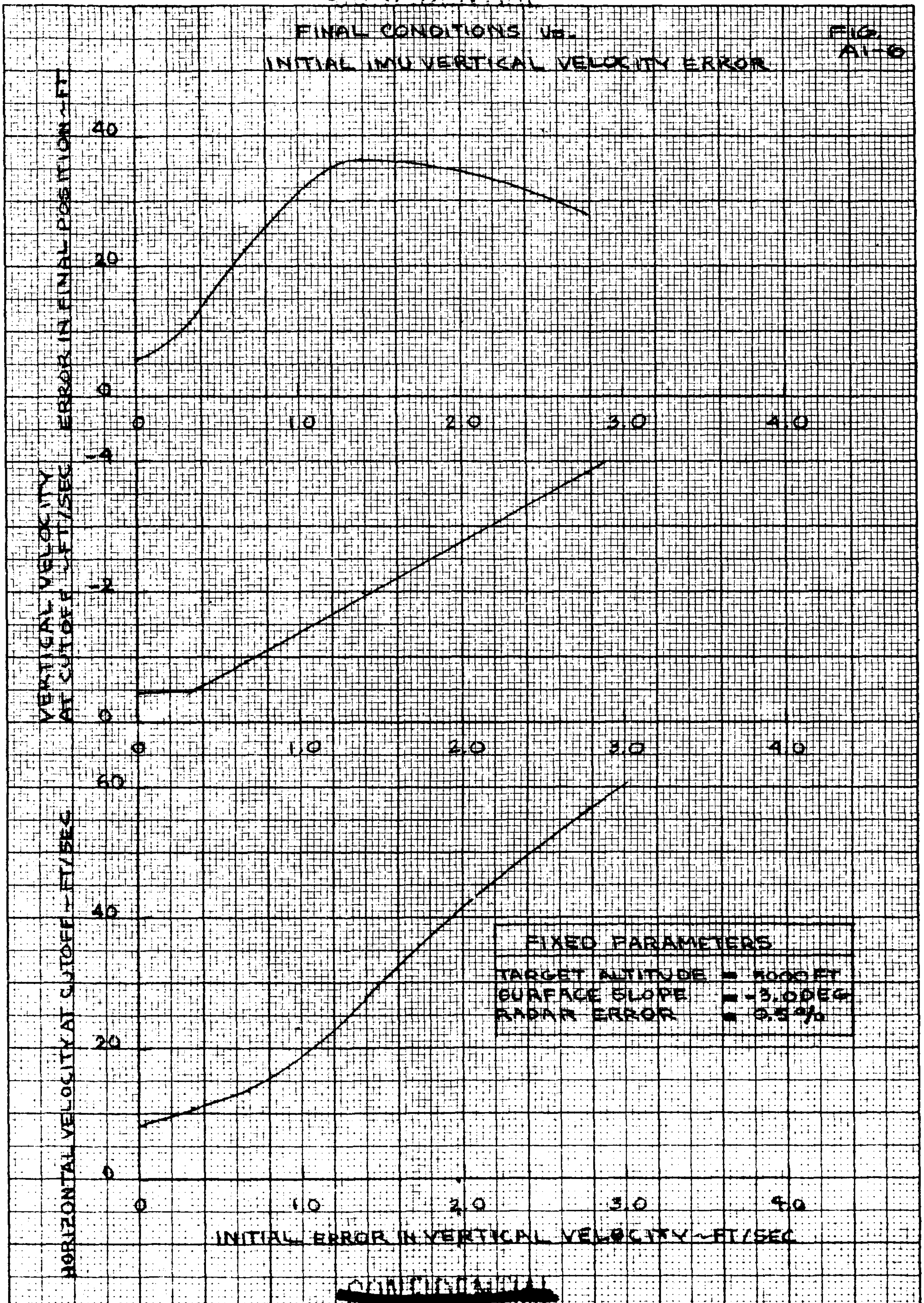

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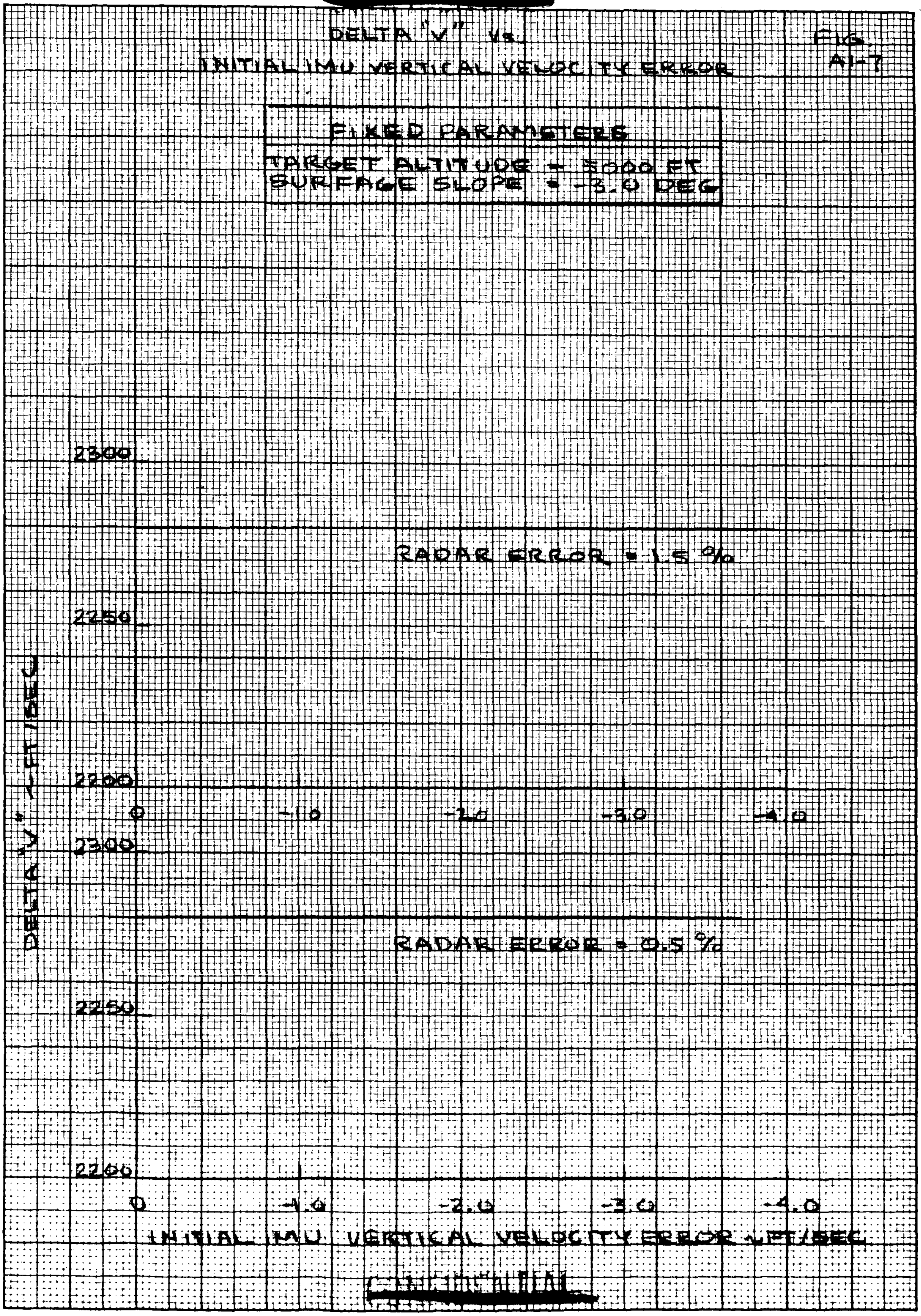

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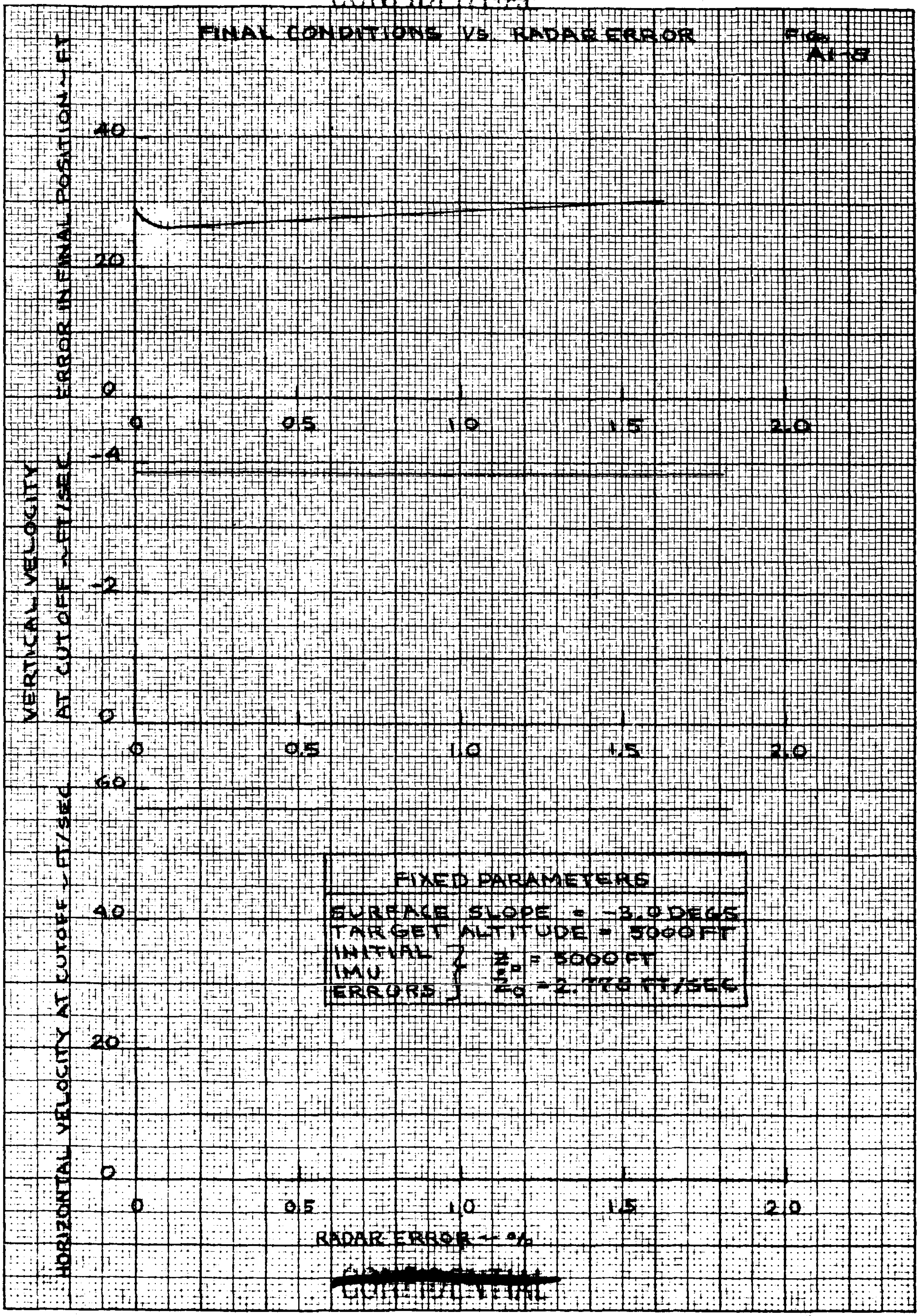

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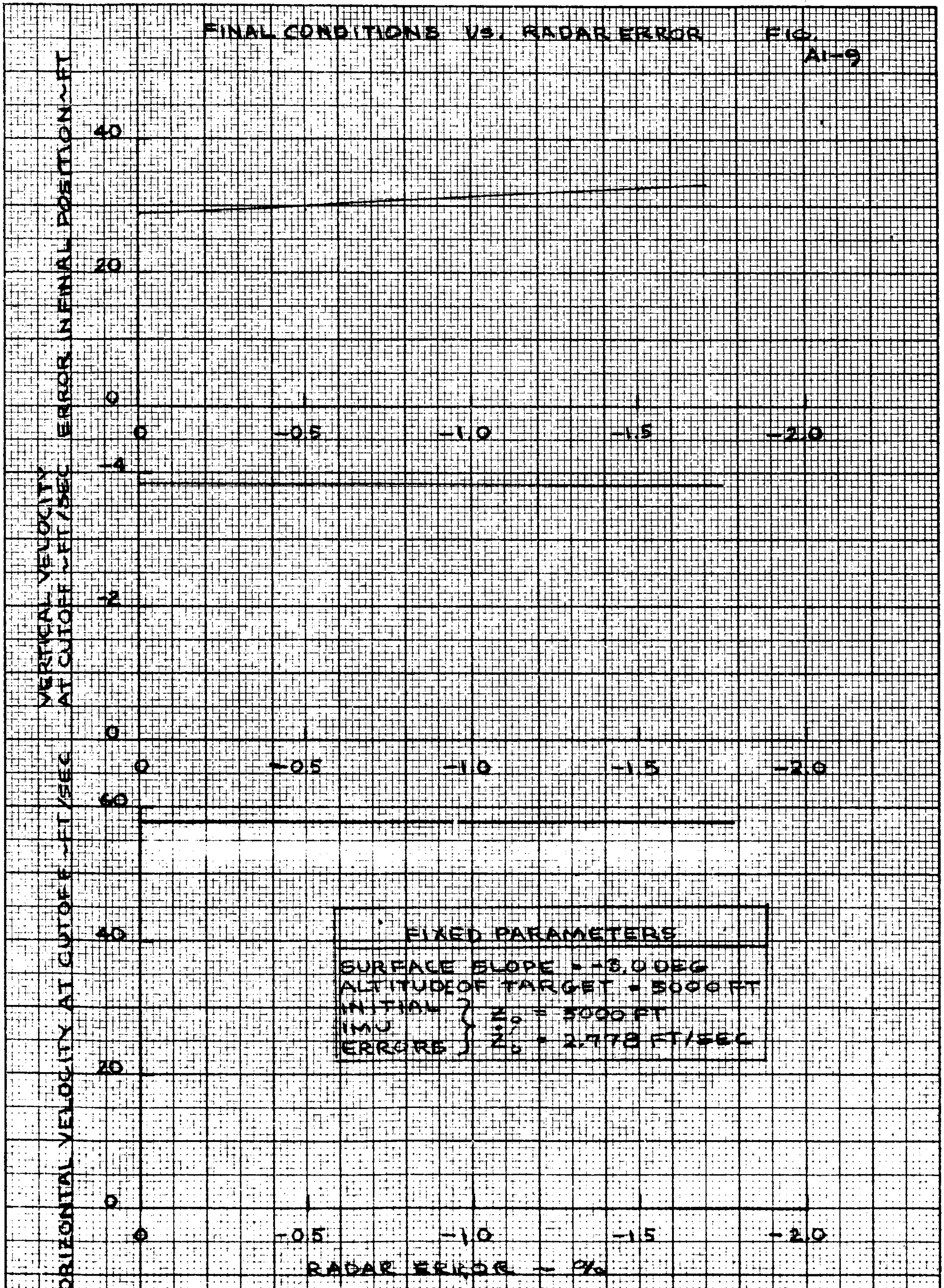

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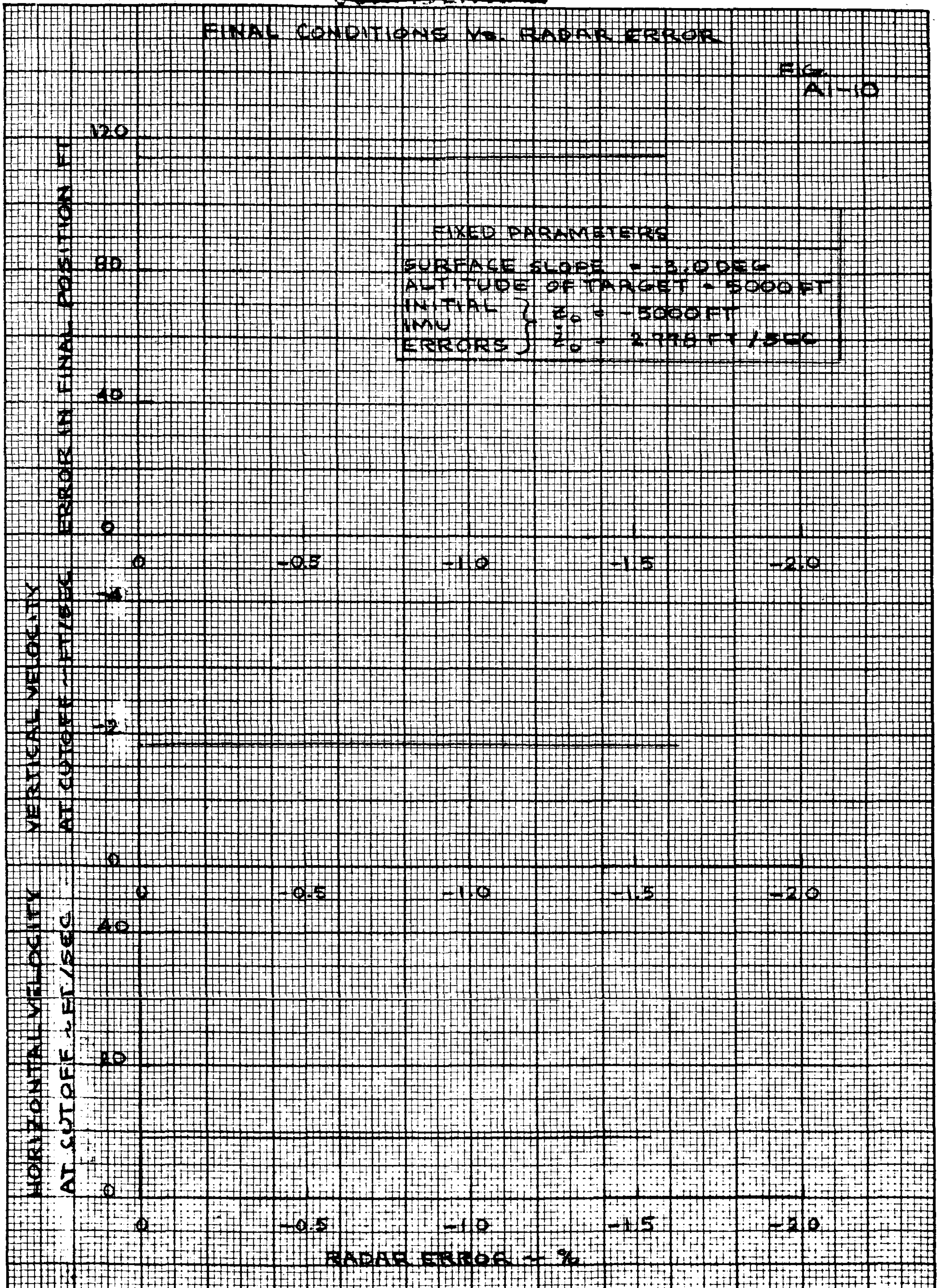
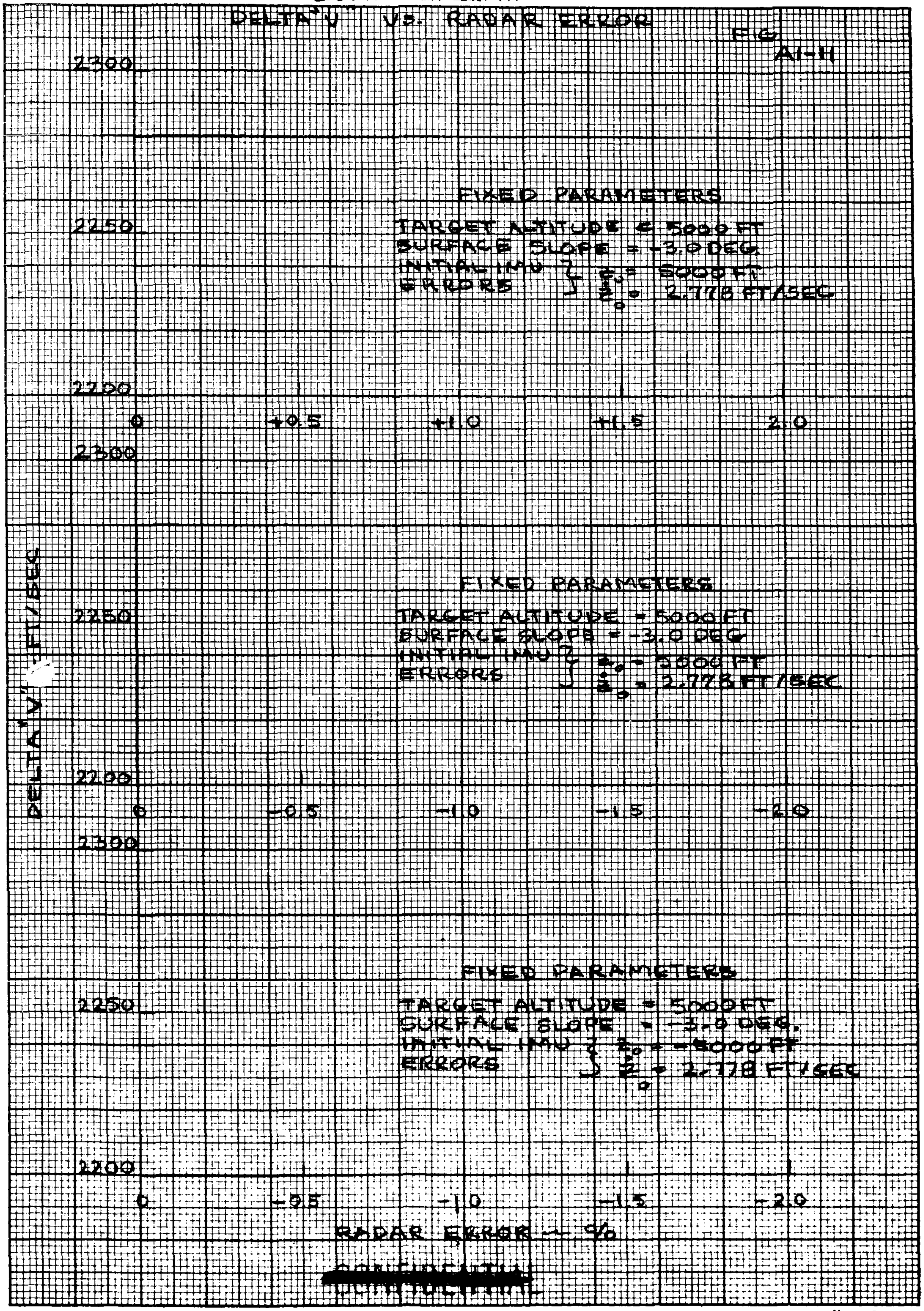
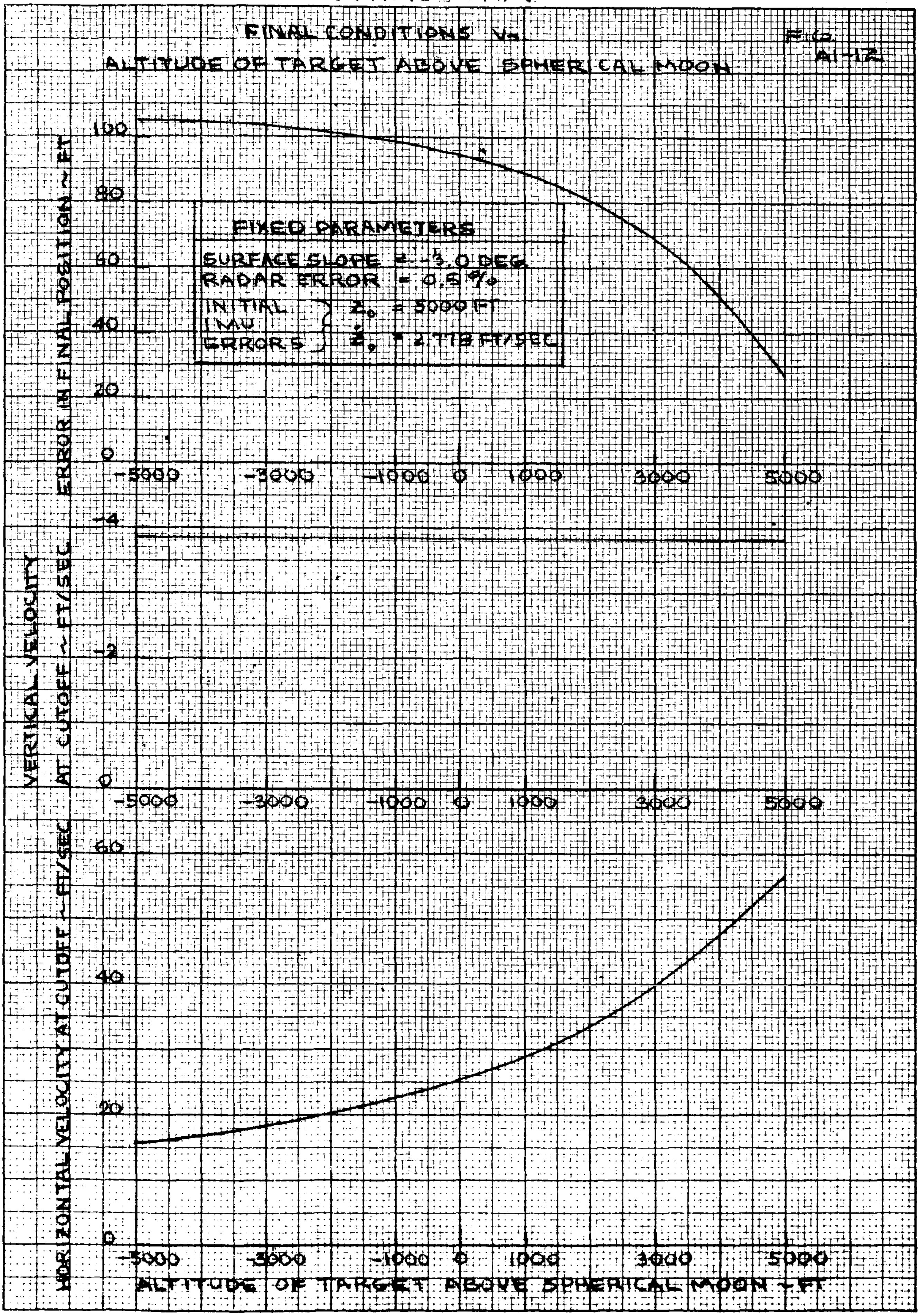


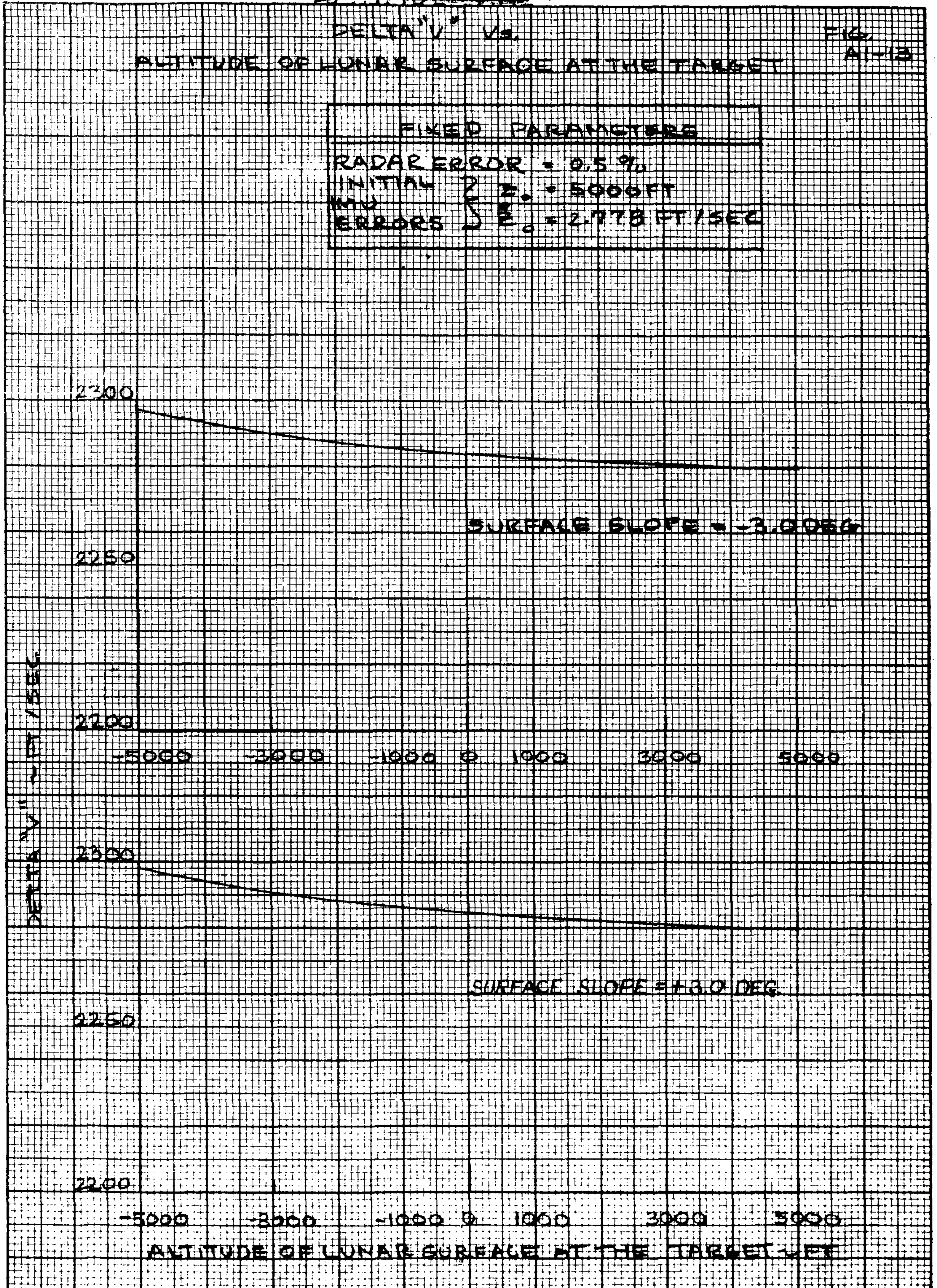
[redacted]


DELTA V VS. RADAR ERROR
FIG. A1-11
2300
2250
2200
FIXED PARAMETERS
TARGET ALTITUDE = 5000 FT
SURFACE SLOPE = -3.0 DEG.
INITIAL IMU ERRORS z_0 = 5000 FT
$\dot{z}_0$ = 2.778 FT/SEC
0
+0.5
+1.0
+1.5
2.0
2300
2250
2200
FIXED PARAMETERS
TARGET ALTITUDE = 5000 FT
SURFACE SLOPE = -3.0 DEG
INITIAL IMU ERRORS z_0 = 5000 FT
$\dot{z}_0$ = 2.778 FT/SEC
0
-0.5
-1.0
-1.5
-2.0
2300
2250
2200
FIXED PARAMETERS
TARGET ALTITUDE = 5000 FT
SURFACE SLOPE = -3.0 DEG.
INITIAL IMU ERRORS z_0 = -5000 FT
$\dot{z}_0$ = 2.778 FT/SEC
0
-0.5
-1.0
-1.5
-2.0
DELTA 'V' FT/SEC
RADAR ERROR ~ %
[redacted]

[redacted]


FINAL CONDITIONS Vs
ALTITUDE OF TARGET ABOVE SPHERICAL MOON
FIG. AI-12
FIXED PARAMETERS
SURFACE SLOPE = -3.0 DEG
RADAR ERROR = 0.5 %
INITIAL IMU ERRORS
Z_0 = 5000 FT
$\dot{Z}_0$ = 2.778 FT/SEC
ERROR IN FINAL POSITION ~ FT
VERTICAL VELOCITY AT CUTOFF ~ FT/SEC
HORIZONTAL VELOCITY AT CUTOFF ~ FT/SEC
100
80
60
40
20
0
-4
-2
0
60
40
20
0
-5000
-3000
-1000
0
1000
3000
5000
ALTITUDE OF TARGET ABOVE SPHERICAL MOON ~ FT

[redacted]



[redacted]

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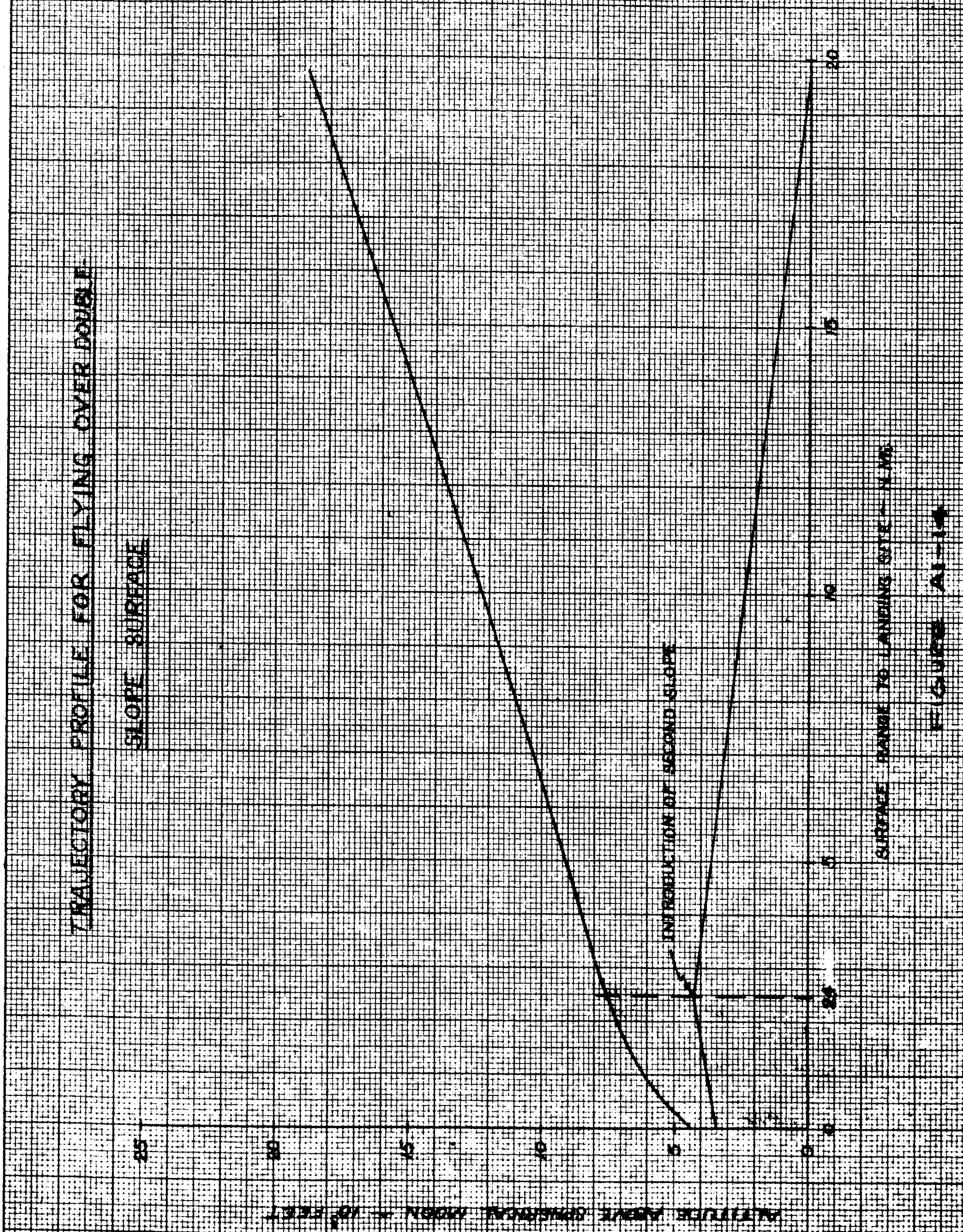

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FIG. A1-15

VARIATIONS IN FINAL VELOCITY PARAMETERS FOR DOUBLE-SLOPE SURFACES

*TIME AT WHICH SECOND SURFACE SLOPE APPEARS (SEC)	FINAL VERTICAL VELOCITY (FT/SEC)	FINAL HORIZONTAL VELOCITY (FT/SEC)	FINAL ΔV (+HORIZONTAL) (FT/SEC)
12	-1.26	8.63	2307.6
22	.02	2.38	2306.4
47	-1.09	7.23	2299.9
72	-.01	5.80	2300.4
82	-2.23	8.09	2288.1
92	-3.87	13.85	2295.2

* NOMINAL TIME OF FLIGHT FOR FINAL DESCENT = 112 SECS (APPROX)

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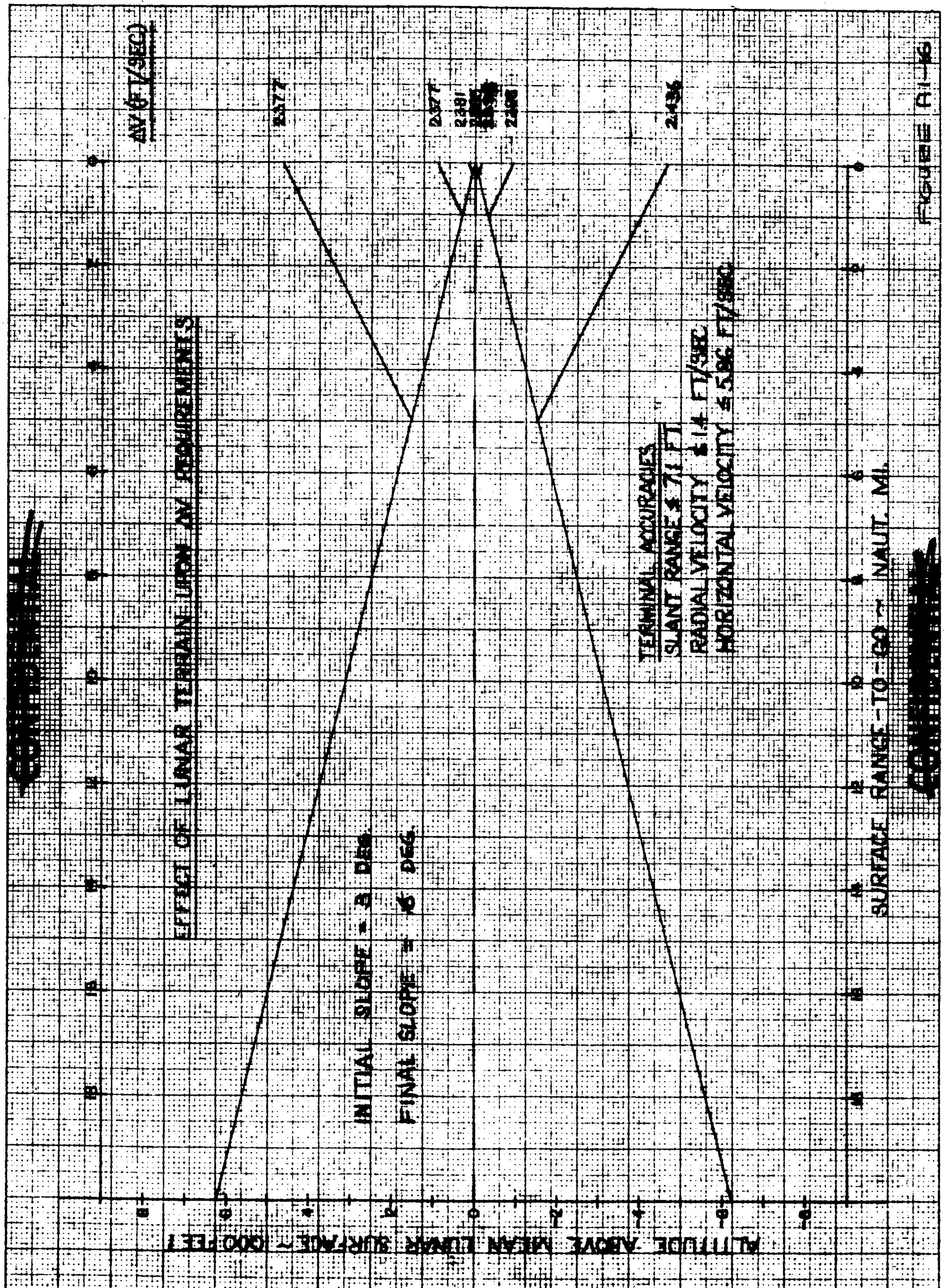
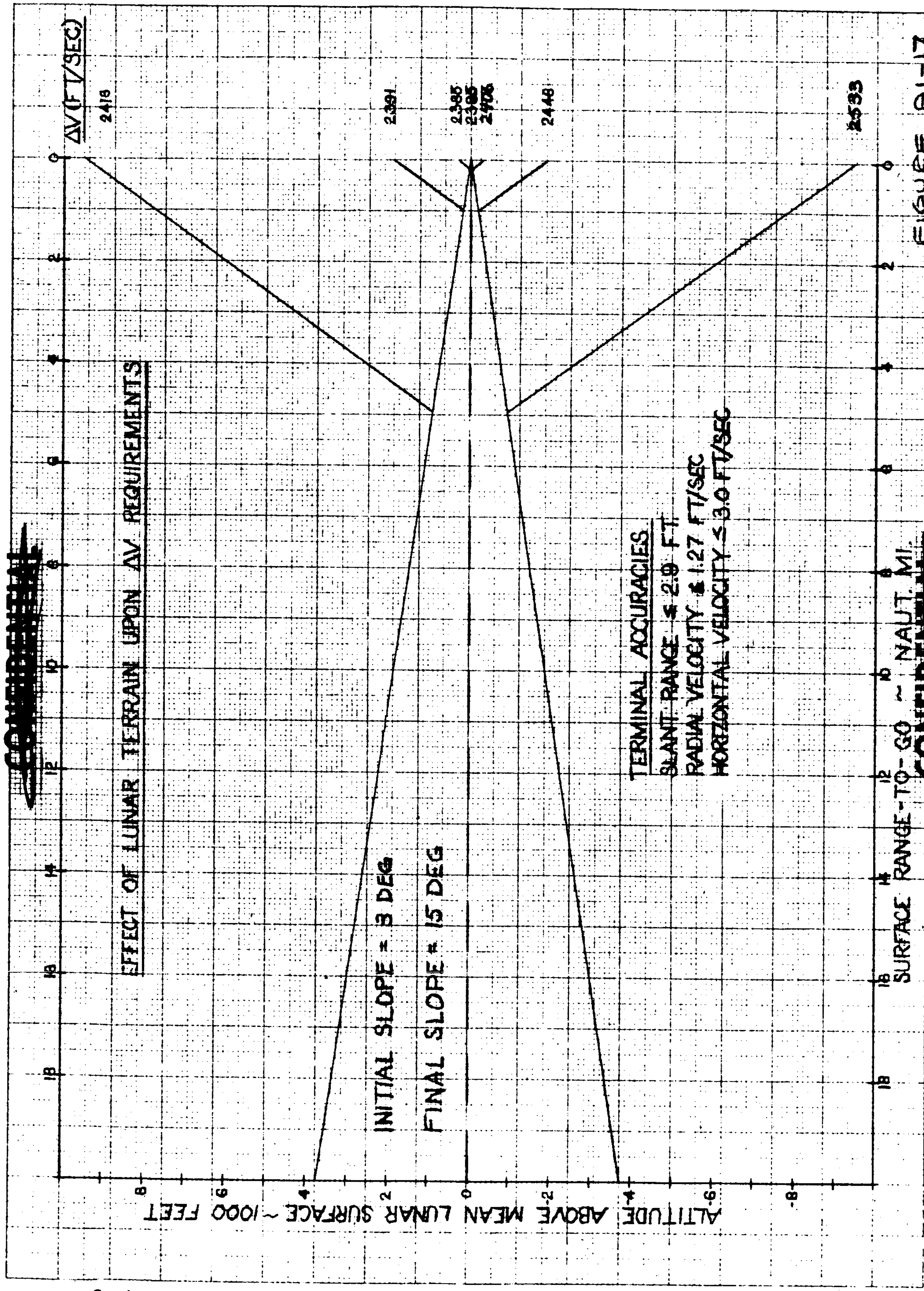

ΔV (FT/SEC)
EFFECT OF LUNAR TERRAIN UPON ΔV REQUIREMENTS
2377
2377
2381
2436
INITIAL SLOPE = 3 DEG.
FINAL SLOPE = 6 DEG.
TERMINAL ACCURACIES
SLANT RANGE ≤ 7.1 FT
RADIAL VELOCITY ≤ 1.4 FT/SEC
HORIZONTAL VELOCITY ≤ 5.96 FT/SEC
SURFACE RANGE-TO-GO ~ NAUT. MI.
ALTITUDE ABOVE MEAN LUNAR SURFACE ~ 1000 FEET
FIGURE A1-16


CONFIDENTIAL
ΔV (FT/SEC)
2418
2391
2448
2533
EFFECT OF LUNAR TERRAIN UPON ΔV REQUIREMENTS
INITIAL SLOPE = 3 DEG
FINAL SLOPE = 15 DEG
TERMINAL ACCURACIES
SLANT RANGE ≤ 2.9 FT.
RADIAL VELOCITY ≤ 1.27 FT/SEC
HORIZONTAL VELOCITY ≤ 3.0 FT/SEC
SURFACE RANGE-TO-GO ~ NAUT. MI.
ALTITUDE ABOVE MEAN LUNAR SURFACE ~1000 FEET
FIGURE B1-17
CONFIDENTIAL

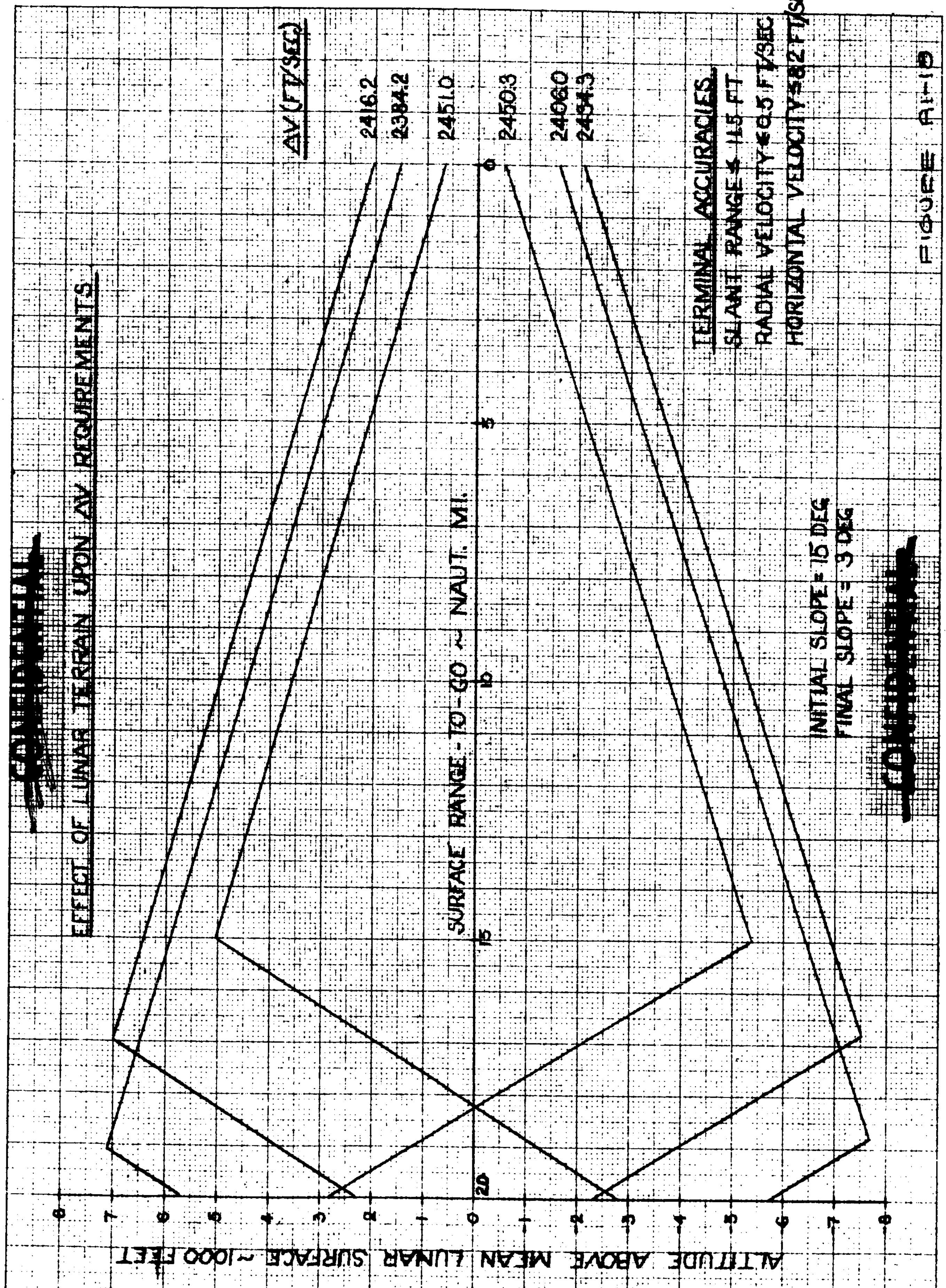

EFFECT OF LUNAR TERRAIN UPON ΔV REQUIREMENTS
ΔV (FT/SEC)
2416.2
2384.2
2451.0
2450.3
2406.0
2454.3
TERMINAL ACCURACIES
SLANT RANGE ≤ 11.5 FT
RADIAL VELOCITY ≤ 0.5 FT/SEC
HORIZONTAL VELOCITY ≤ 8.2 FT/SEC
SURFACE RANGE - TO - GO ~ NAUT. MI.
INITIAL SLOPE = 15 DEG
FINAL SLOPE = 3 DEG
ALTITUDE ABOVE MEAN LUNAR SURFACE ~ 1000 FEET

FIGURE AI-1[illegible]

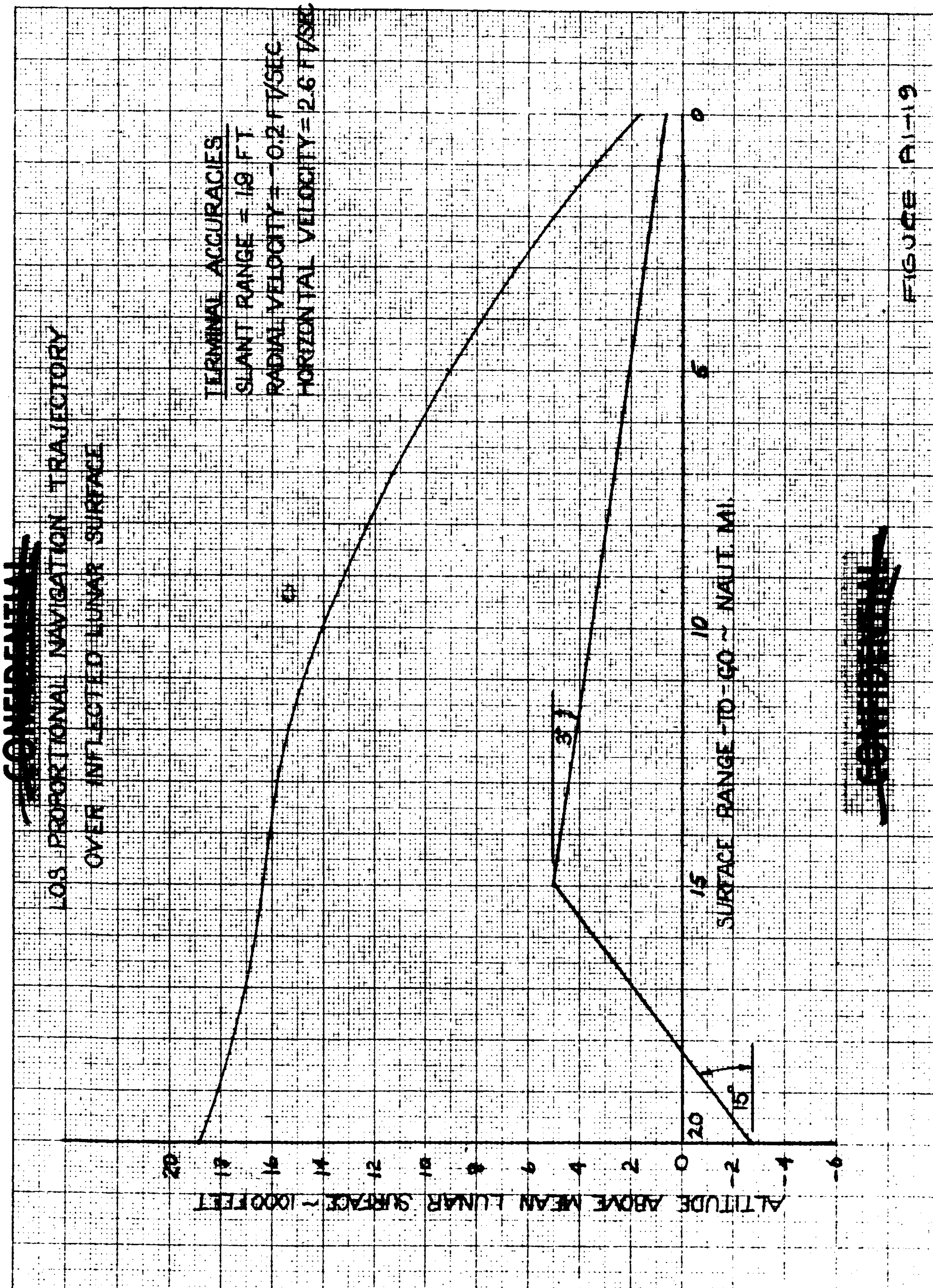

CONFIDENTIAL
LOS PROPORTIONAL NAVIGATION TRAJECTORY
OVER INFLECTED LUNAR SURFACE
TERMINAL ACCURACIES
SLANT RANGE = 1.9 FT
RADIAL VELOCITY = -0.2 FT/SEC
HORIZONTAL VELOCITY = 2.6 FT/SEC
3°
15°
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
ALTITUDE ABOVE MEAN LUNAR SURFACE ~ 1000 FEET
20
15
10
5
0
SURFACE RANGE-TO-GO ~ NAUT MI
CONFIDENTIAL

FIGURE A1-19

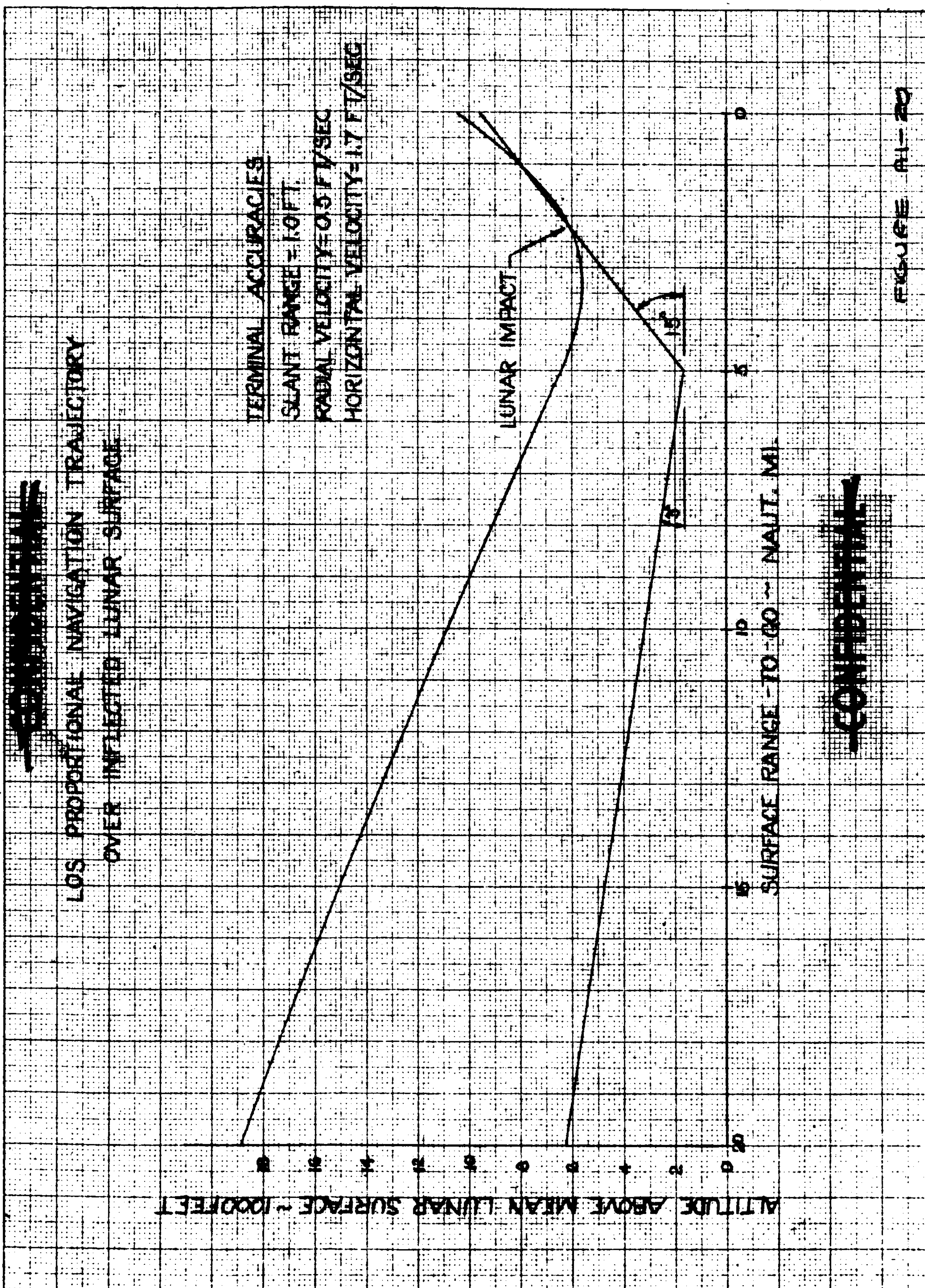

CONFIDENTIAL
LOS PROPORTIONAL NAVIGATION TRAJECTORY
OVER INFLECTED LUNAR SURFACE
TERMINAL ACCURACIES
SLANT RANGE = 1.0 FT.
RADIAL VELOCITY = 0.5 FT/SEC
HORIZONTAL VELOCITY = 1.7 FT/SEC
LUNAR IMPACT
15°
15°
SURFACE RANGE-TO-GO ~ NAUT. MI.
0
5
10
15
20
ALTITUDE ABOVE MEAN LUNAR SURFACE ~ 1000 FEET
0
2
4
6
8
10
12
14
16
18
CONFIDENTIAL

FIGURE A1-20

Appendix 2

Effect of Ascent Injection Errors and Time of Correction on ΔV Requirements for Midcourse Correction

Purpose:

The purpose of this study was to determine the effect of ascent injection errors on the total ΔV requirement for rendezvous when using a midcourse correction technique.

Procedures and Assumptions:

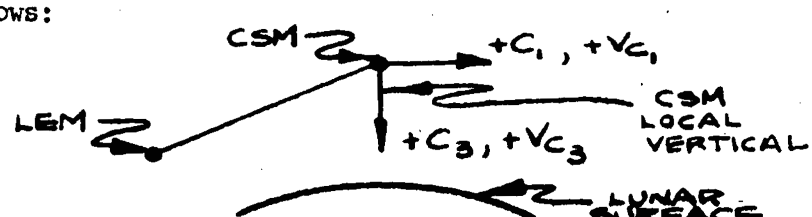
It was assumed that a perfect injection would result in a Hohmann Transfer orbit starting at an altitude of 50,000 ft. and ending at an apocynthion of 80 N. Mi. The total time of flight for this transfer is 3470 seconds.

For the purpose of this study, initial ascent injection errors were propagated to position and velocity errors at the time of midcourse correction. From these off-nominal conditions, the ΔV required for midcourse correction was determined. The ΔV correction maintained a constant total time of flight, which for the assumed Hohmann equals approximately 3470 seconds. The next step was a calculation of the ΔV required at intercept to go into a circular orbit. This impulse was nominally added to the ΔV midcourse impulse to obtain the total ΔV required.

The error propagation equations were based on a perfectly circular orbit of the CSM, and on a lunar expansion of gravity in the vicinity of the CSM. In addition, the equations assumed a point mass and a lunar transfer orbit.

Range of Parameters:

The injection errors were related to a CSM centered coordinate system defined as follows:



Two sets of injection errors were considered. They were obtained by assuming a pre-programmed pitch program and 1% and a 3% off-nominal thrust. When 1% excess thrust was considered, the injection errors were:

$C_1 = 6{,}804$ ft. $\qquad Vc_1 = 10.5$ fps.

$C_3 = 3{,}178$ ft. $\qquad Vc_3 = 13.1$ fps.

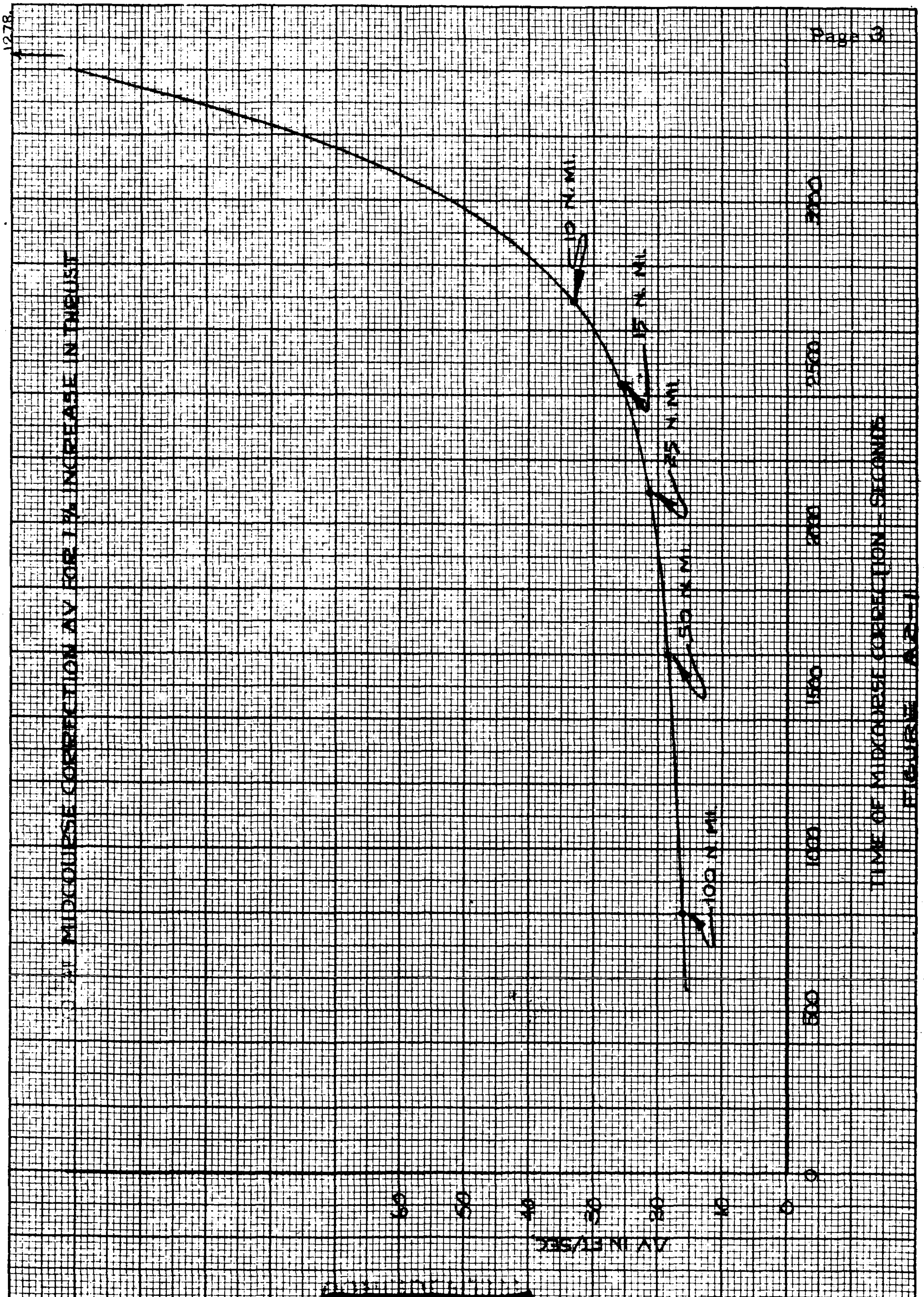
For 3% excess thrust,

$C_1 = 20{,}413$ ft. $\qquad Vc_1 = 31.4$ fps.

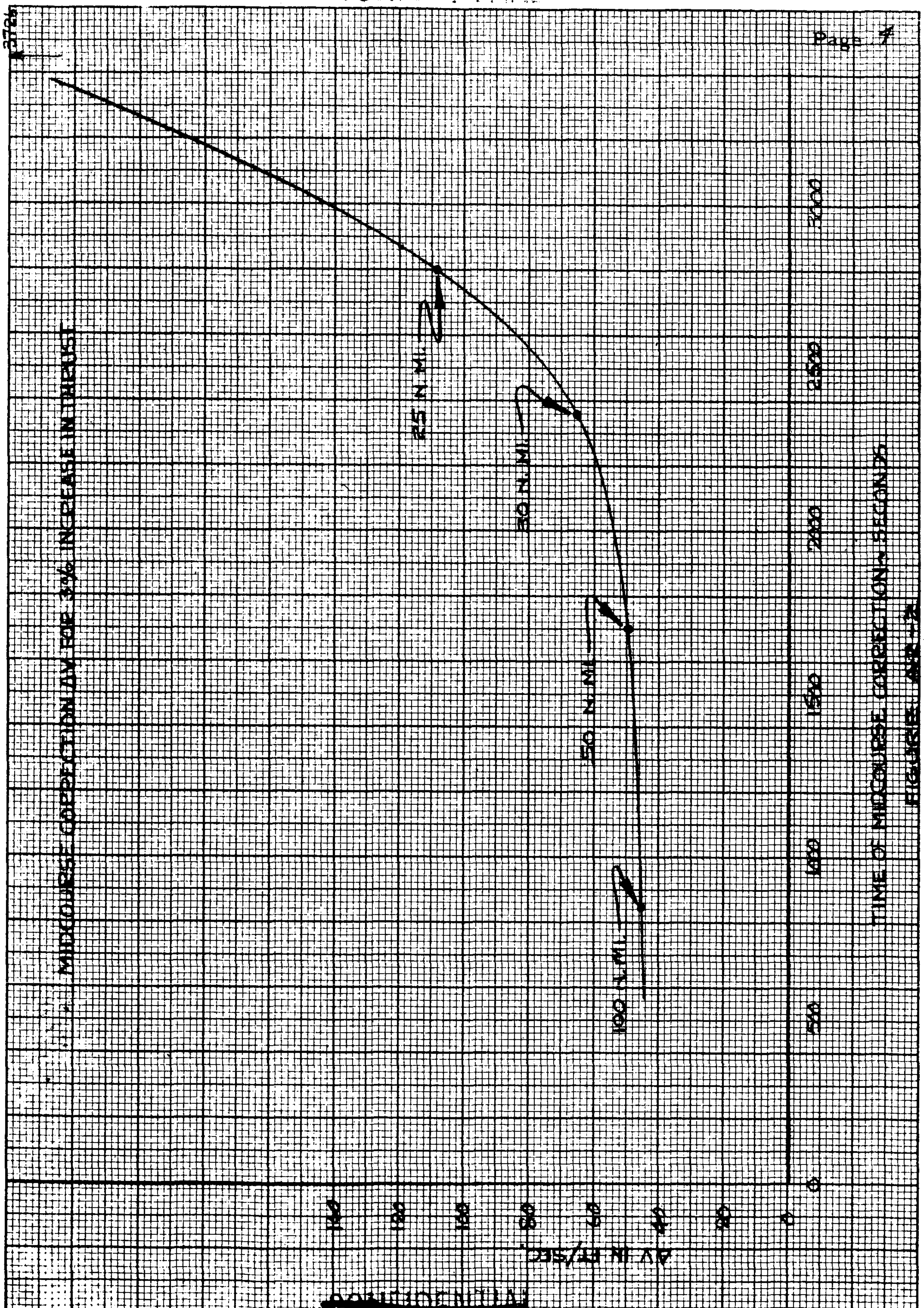
$C_3 = 9{,}533$ ft. $\qquad Vc_3 = 39.4$ fps.

The results are presented in Figures A2-1 - A2-4. Figures A2-1 & A2-2 represent the ΔV required for midcourse correction for the 1% and 3% cases, respectively, versus time of midcourse correction. The times of midcourse correction were all considered relative to engine burnout.

Figures A2-3 & A2-4 represent the total ΔV required for the 1% and 3% cases, respectively, versus the time of the midcourse correction.


MIDCOURSE CORRECTION ΔV FOR 1% INCREASE IN THRUST
TIME OF MIDCOURSE CORRECTION - SECONDS
ΔV IN FT/SEC
1278
10 N.MI.
15 N.MI.
25 N.MI.
50 N.MI.
100 N.MI.
0
500
1000
1500
2000
2500
3000
0
10
20
30
40
50
60
FIGURE A2-1

[redacted]



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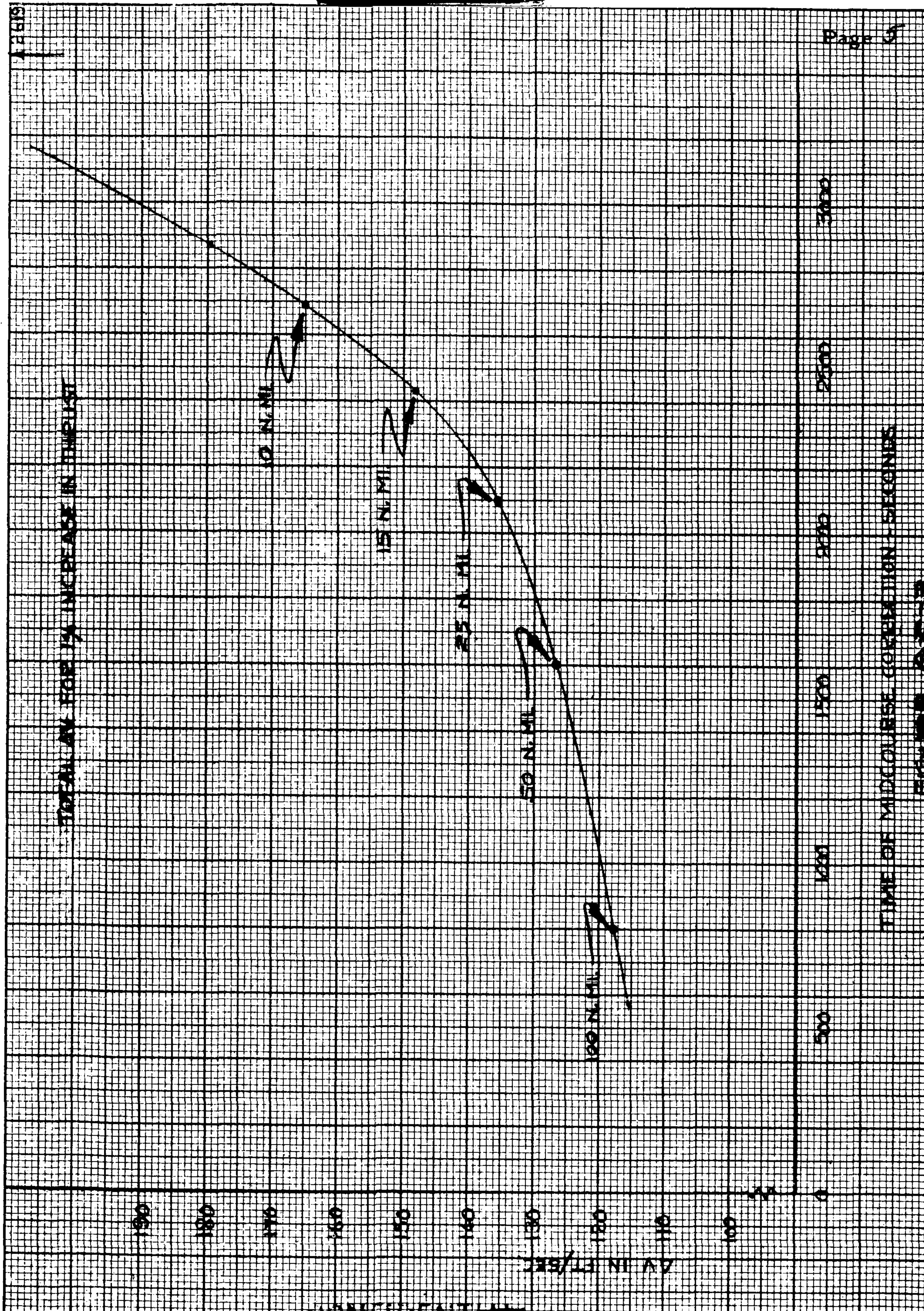

TOTAL ΔV FOR 1% INCREASE IN THRUST
10 N.MI
15 N.MI
25 N.MI
50 N.MI
100 N.MI
TIME OF MIDCOURSE CORRECTION - SECONDS
0
500
1000
1500
2000
2500
3000
ΔV IN FT/SEC
190
180
170
160
150
140
130
120
110
100

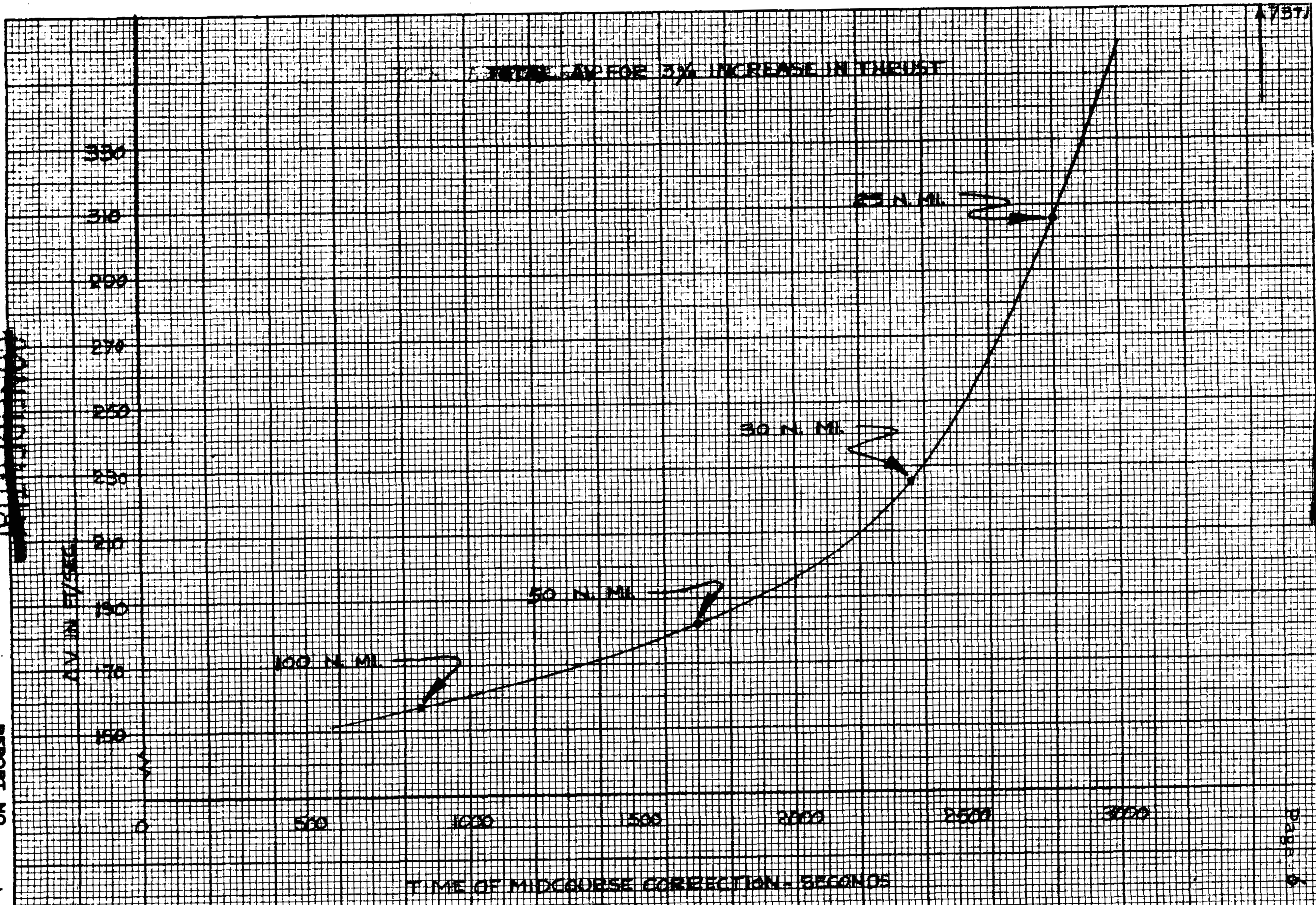

TOTAL ΔV FOR 3% INCREASE IN THRUST
ΔV IN FT/SEC
350
310
290
270
250
230
210
190
170
150
25 N. MI.
30 N. MI.
50 N. MI.
100 N. MI.
0
500
1000
1500
2000
2500
3000
TIME OF MIDCOURSE CORRECTION - SECONDS

FIGURE A

Appendix 3

Effect of Single Point Radar Measurement Errors On Accuracy of Midcourse Corrections

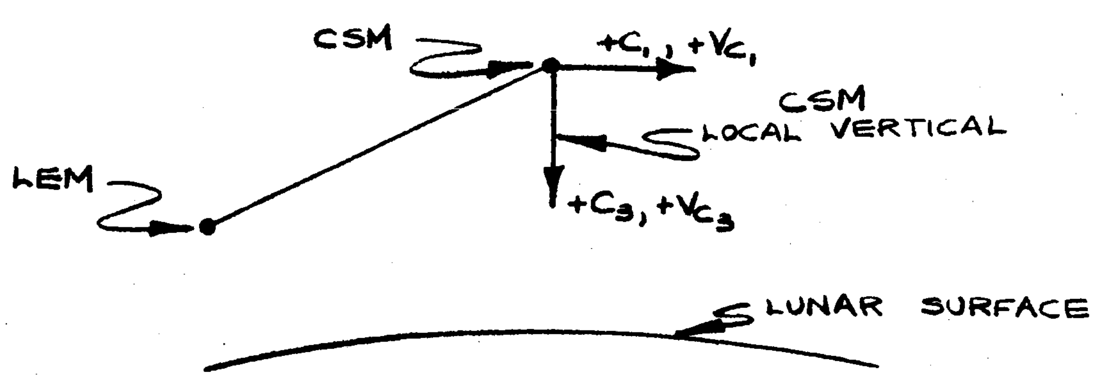
Purpose:

Using radar data, LEM position and velocity relative to the CSM can be determined from a single point measurement of range, range rate, angle and angle rate. Using navigation data, course corrections can be made to intercept the CSM. The purpose of this study is to determine the effect of the radar measurement error on the trajectory errors.

Procedures and Assumptions:

Four radar measurements (range, range rate, angle and angle rate) at a single point on the trajectory were considered. A Hohmann transfer orbit from 50,000 feet to an altitude of 80 N. Mi. was used to determine nominal values of range, range rate, angle and angular rate, and therefore numerical values of radar errors. Each radar measurement error was considered separately (statistically independent) and resolved into a CSM-centered coordinate system (see Figure A3-1). Finally each error was propagated to the trajectory point of interest. RSS computation was then made to describe the total effect of all the radar errors.

Figure A3-1



The error propagation equations were based on a perfectly circular orbit of the CSM, and on a linear expansion of gravity in the vicinity of the CSM. In addition, the equations assumed a point mass and a planar transfer orbit.

Range of Parameters:

The following points along the Hohmann transfer orbit were considered:

TIME FROM BURNOUT	RELATIVE RANGE TO CSM
624 secs.	115 N. Mi.
1904 secs.	30 N. Mi.
3470 secs.	0 N. Mi.

The following accuracies of radar systems were first considered:

R = Radar Range Error = 0.05%; 0.5%

$\dot{R}$ = Radar Range Rate Error = 0.3%; 1%

θ = Radar Angle Error = 0.002 radians; 0.006 radians

$\dot{\theta}$ = Radar Angle Rate Error = 0.001 radians/sec.

Results:

After examining the results (tables A3-1 & A3-2), it was obvious that the dominant error contribution was $\dot{\theta}$. In order to verify this another set of calculations were made using a $\dot{\theta}$ of one order of magnitude less (i.e. $\dot{\theta}$ = 0.00001 radians/sec). These results are summarized in Table A3-3. The results are presented in tabular form. Table A3-1 is the RSS of the component errors of position and velocity for the combination of the four radar error measurements.

Table A3-2 is the individual contribution of each type of measurement error to the RSS error.

Table A3-3 is a comparison of the RSS errors using a radar with $\dot{\theta} = 1 \times 10^{-4}$ rad/sec. or $\dot{\theta} = 1 \times 10^{-5}$ rad/sec.

Table A3-4 is a listing of initial radar measurement errors resolved into CSM coordinates.

TABLE A3-1

RSS OF THE PROPAGATED ERRORS

For All Cases Considered $\Delta\dot{\theta}$ = 0.0001 rad/sec

Radar Errors	Propagation Interval From	To	C_1(ft)	C_3(ft)	V_{C_1}(ft/sec)	V_{C_3}(ft/sec)
$\Delta\theta$=0.002 rad	625 secs	30 n.mi.	83,279	5,402	49.0	50.2
ΔR=0.05%	After Burnout	Apocynthion	18,545	133,697	185.7	97.5
$\Delta\dot{R}$=0.3%	30 n.mi.	Apocynthion	2,038	29,572	32.6	34.2
$\Delta\theta$=0.002 rad	625 secs	30 n.mi.	83,326	10,374	50.2	51.0
ΔR=C.05%	After Burnout	Apocynthion	41,554	135,219	188.1	97.5
$\Delta\dot{R}$=1%	30 n.mi.	Apocynthion	4,269	29,707	33.0	34.3
$\Delta\theta$=0.002 rad	625 secs	30 n.mi.	83,279	5,430	49.0	50.2
ΔR=0.5%	After Burnout	Apocynthion	18,561	133,704	185.7	97.5
$\Delta\dot{R}$=0.3%	30 n.mi.	Apocynthion	2,046	29,573	32.7	34.2
$\Delta\theta$=0.002 rad	625 sec	30 n.mi.	83,226	10,388	50.2	51.1
ΔR=0.5%	After Burnout	Apocynthion	41,602	135,225	188.1	97.5
$\Delta\dot{R}$=1%	30 n.mi.	Apocynthion	4,272	29,708	33.0	34.3
$\Delta\theta$=0.006 rad	625 secs	30 n.mi.	83,511	9,779	49.8	50.7
ΔR=0.05%	After Burnout	Apocynthion	40,192	135,123	187.8	97.6
$\Delta\dot{R}$=0.3%	30 n.mi.	Apocynthion	2,414	29,575	32.7	34.2
$\Delta\theta$=0.006 rad	625 secs	30 n.mi.	83,557	13,193	50.9	51.5
ΔR=0.05%	After Burnout	Apocynthion	54,787	136,629	190.2	97.7
$\Delta\dot{R}$=1%	30 n.mi.	Apocynthion	4,460	29,710	33.0	34.3
$\Delta\theta$=0.006 rad	625 secs	30 n.mi.	83,511	9,795	49.8	50.7
ΔR=0.5%	After Burnout	Apocynthion	40,241	135,129	187.8	97.6
$\Delta\dot{R}$=0.3%	30 n.mi.	Apocynthion	2,421	29,576	32.7	34.2
$\Delta\theta$=0.006 rad	625 secs	30 n.mi.	83,557	13,205	51.0	51.5
ΔR=0.5%	After Burnout	Apocynthion	54,823	136,635	190.2	97.7
$\Delta\dot{R}$=1%	30 n.mi.	Apocynthion	4,464	29,711	33.0	34.3

TABLE A3-2

PROPAGATION OF INDIVIDUAL ERRORS

Radar Error	Propagation Interval From	To	C_1(ft)	C_3(ft)	V_{C_1}(fps)	V_{C_3}(fps)
$\Delta\dot{\theta}$=0.0001 rad/sec	625 secs. After Burnout	30n n.mi.	83,245	3,619	48.8	50.1
$\Delta\theta$=0.002 rad	625 secs. After Burnout	30 n.mi.	2,198	2,882	3.0	2.5
$\Delta\theta$=0.006 rad	625 secs. After Burnout	30 n.mi.	6,595	8,646	9.1	7.5
ΔR=0.05%	625 secs. After Burnout	30 n.mi.	2	56	.06	.05
$\Delta\dot{R}$=0.5%	625 secs. After Burnout	30 n.mi.	16	557	.6	.5
$\Delta\dot{R}$=0.3%	625 secs. After Burnout	30 n.mi.	878	2,789	3.4	2.9
$\Delta\dot{R}$=1%	625 secs. After Burnout	30 n.mi.	2,921	9,285	11.4	9.6
$\Delta\dot{\theta}$ =0.0001 rad/sec.	625 secs. After Burnout	Apocynthion	6,617	133,366	185.1	97.5
$\Delta\theta$=0.002 rad	625 secs. After Burnout	Apocynthion	12,624	6,922	9.9	1.8
$\Delta\theta$=0.006 rad	625 secs. After Burnout	Apocynthion	37,871	20,766	29.8	5.5
ΔR=0.05%	625 secs. After Burnout	Apocynthion	200	134	.2	.04

TABLE A3-2 (CONT.)

Radar Error	Propagation Interval From	To	C_1(ft)	C_3(ft)	V_{C_1}(fps)	V_{C_3}(fps)
$\Delta R=0.5\%$	625 secs. After Burnout	Apocynthion	1,999	1,338	1.9	0.4
$\Delta \dot{R}=0.3\%$	625 secs. After Burnout	Apocynthion	11,720	6,368	9.5	1.0
$\Delta \dot{R}=1\%$	625 secs. After Burnout	Apocynthion	39,033	21,210	31.7	3.2
$\Delta \dot{\theta}=0.0001$ rad/sec	30 n.mi.	Apocynthion	1,598	29,558	32.6	34.2
$\Delta \theta=0.002$ rad	30 n.mi.	Apocynthion	457	151	.2	.1
$\Delta \theta=0.006$ rad	30 n.mi.	Apocynthion	1,372	454	.5	.3
$\Delta R=0.05\%$	30 n.mi.	Apocynthion	18.4	29.5	.04	.02
$\Delta R=0.5\%$	30 n.mi.	Apocynthion	184	295	.4	.2
$\Delta \dot{R}=0.3\%$	30 n.mi.	Apocynthion	1,180	889	1.4	.3
$\Delta \dot{R}=1\%$	30 n.mi.	Apocynthion	3,932	2,962	4.8	1.0

TABLE A3-3

COMPARISON OF ERRORS FOR $\Delta\dot{\theta} = 1 \times 10^{-4}$; 1×10^{-5} RAD/SEC

Propagation Interval From	To	Radar Errors Rad/Sec $\Delta\dot{\theta}$	Rad $\Delta\theta$	% ΔR	% $\Delta\dot{R}$	Propagated Errors C_1(Ft)	C_3(ft)	V_{C_1} (fps)	V_{C_3} (fps)
625 secs	30 n.mi.	$1x10^{-4}$	.002	.05	.3	83,279	5,402	49.0	50.2
		$1x10^{-5}$	.002	.05	.3	8,655	4,027	6.7	6.3
		$1x10^{-4}$	.006	.05	.3	83,511	9,779	49.8	50.7
		$1x10^{-5}$	.006	.05	.3	10,657	9,092	10.9	9.5
		$1x10^{-4}$	.002	.5	1	83,226	10,388	50.2	51.1
		$1x10^{-5}$	.002	.5	1	9,092	9,745	12.8	11.1
		$1x10^{-4}$	.006	.5	1	83,557	13,205	51.0	51.5
		$1x10^{-5}$	.006	.5	1	11,015	12,705	15.4	13.2
625 secs	Apocynthion	$1x10^{-4}$	.002	.05	.3	18,545	133,697	185.7	97.5
		$1x10^{-5}$	.002	.05	.3	17,240	16,321	23.0	10.0
		$1x10^{-4}$	.006	.05	.3	40,192	135,123	187.8	97.6
		$1x10^{-5}$	.006	.05	.3	39,649	25,490	36.3	11.3
		$1x10^{-4}$	.002	.5	1	41,602	135,225	188.1	97.5
		$1x10^{-5}$	.002	.5	1	41,078	26,028	38.1	10.5
		$1x10^{-4}$	.006	.5	1	54,823	136,635	190.2	97.7
		$1x10^{-5}$	.006	.5	1	54,426	32,569	47.3	11.7
30 n.mi.	Apocynthion	$1x10^{-4}$	.002	.05	.3	2,038	29,572	32.6	34.2
		$1x10^{-5}$	.002	.05	.3	1,276	3,091	3.6	3.4
		$1x10^{-4}$	.006	.05	.3	2,414	29,575	37.7	34.2
		$1x10^{-5}$	.006	.05	.3	1,817	3,120	3.6	3.4
		$1x10^{-4}$	.002	.5	1	4,272	29,708	33.0	34.3
		$1x10^{-5}$	.002	.5	1	3,966	4,198	5.8	3.6
		$1x10^{-4}$	.006	.5	1	4,464	29,711	33.0	34.3
		$1x10^{-5}$	.006	.5	1	4,172	4,220	5.9	3.6

TABLE A3-4

NAVIGATION ERRORS RESULTING FROM RADAR ERRORS

Radar Error	Test Point	C_1 (Ft)	C_3 (Ft)	V_{C_1} (fps)	V_{C_3} (fps)
$\Delta\dot{\theta} = 0.0001$ rad/sec	625 secs after burnout	-	-	42.6	55.2
$\Delta\theta = 0.002$ rad		853	1,103	-	-
$\Delta\theta = 0.006$ rad		2,559	3,310	-	-
$\Delta R = 0.05\%$		28	21	-	-
$\Delta R = 0.5\%$		276	213	-	-
$\Delta\dot{R} = 0.3\%$		-	-	1.4	1.0
$\Delta\dot{R} = 1\%$		-	-	4.5	3.5
$\Delta\dot{\theta} = 0.0001$ rad/sec	30 n.mi. range from CSM	-	-	17.9	2.3
$\Delta\theta = 0.002$ rad		36	46	-	-
$\Delta\theta = 0.006$ rad		1,072	137	-	-
$\Delta R = 0.05\%$		1	9	-	-
$\Delta R = 0.5\%$		11	89	-	-
$\Delta\dot{R} = 0.3\%$		-	-	0.1	0.6
$\Delta\dot{R} = 1\%$		-	-	0.3	2.2

Appendix 4

Effect of Radar Measurement Errors on the Midcourse Correction During Coasting Ascent

Purpose -

The purpose of this study was to determine the effect of radar measurement errors on coasting ascent midcourse corrections, using a two point measurement, differential correction technique.

Principle of Differential Correction Technique

The differential correction technique establishes differences between actual orbit and nominal orbit parameters. These differences result in terms called observational residuals. Observational residuals are differences between pre-computed and observed data. They can be expressed in terms of a first order Taylor's expansion of four orbital parameters. (Since this is a point mass, planar analysis, only four orbital parameters are involved.) This relationship is expressed as follows: *

$$\Delta p_1 = \frac{\partial p_1}{\partial \rho_1} \Delta \rho_1 + \cdots\cdots + \frac{\partial p_1}{\partial \rho_4} \Delta \rho_4 \qquad (1)$$

$$\vdots$$

$$\Delta p_4 = \frac{\partial p_4}{\partial \rho_1} \Delta \rho_1 + \qquad + \frac{\partial p_4}{\partial \rho_4} \Delta \rho_4$$

WHERE

Δp = observation residuals

$\frac{\partial p}{\partial \rho}$ = partials relating the observational residuals to the orbital parameter

ρ = orbital parameter

$\Delta \rho$ = differences in the orbital parameters of the actual and nominal orbits.

* Ref. S. Herrick, "Astrodynamics", D. Van Nostrand Co. Princeton, N.J., 1961. R.M. Baker and M. W. Makemson, "An Introduction to Astrodynamics", Academic Press, New York, 1960, pp. 142-152.

Principle of Differential Correction Technique (Continued)

Through inversion of the equations above, the observational residuals are used to determine the orbital parameters. Using these orbital parameters, the off-nominal position and velocity may be determined at the time of mid-course correction. This off-nominal position and velocity data will be used to determine the mid-course correction velocity impulse required for an intercept orbit. The steps previously described (observational residuals, to orbital parameters, to position and velocity, to velocity correction) can be combined so that the observed residuals are used directly to compute required mid-course correction at a specified time.

Procedures and Assumptions

The purpose of the study is to investigate the effectiveness of the differential correction technique when radar measurements are used to obtain the observational residuals. The method of analysis assumes that no injection errors have occurred and that the observational residuals are due to radar measurement errors. As a result, the midcourse correction which results from the observational residuals propagate trajectory position and velocity errors.

Since there are four orbital parameter differentials to be evaluated, a minimum of four observational residuals are required to solve for the unknowns. The necessary information has been obtained by observing two radar parameters at two separate times. This technique has the inherent advantage of eliminating some radar parameter performances requirements which are critical or difficult to achieve. For purpose of investigation, the radar parameters used for the measurement were combinations of range, ρ , range rate, $\dot{\rho}$, and line of sight, θ_R . Angle rate, $\dot{\theta}_R$, was not included because preliminary investigation indicated that the orbital parameter determination was highly sensitive to angular rate measurement errors, and the specified accuracies of angular rate being considered for the radar would not yield reasonable information.

Procedures and Assumptions (Continued)

The partials described above, which relate the observational residuals to the orbital parameters, are time dependent. Therefore, in order to achieve the best results with two measurements of the same parameter, it is desirable to separate the times of measurement as widely as possible. As a consequence, the effect of varying the time separation becomes a critical variable of the study.

The first measurement (t_1) was assumed to occur immediately after burnout and the time of the second measurement (t_2) was allowed to vary. The time of the midcourse correction (τ) was arbitrarily set at fifty seconds after t_2.

The midcourse correction errors are propagated to position and velocity errors at various points along the trajectory. These errors then form the basis of judging the effectiveness of the midcourse correction.

For this study a Hohmann transfer orbit from 50,000 ft. to an altitude of 80 n. miles was used to determine nominal values of range, range rate, and line-of-sight angle. The partials relating the observational residuals to orbital parameters, and the partials used to obtain the error propagation coefficients, were based on equations which assumed a perfectly circular orbit of the CSM, and a linear expansion of gravity in the vicinity of the CSM. In addition, the equations assumed a point mass and coplanar orbits.

Range of Parameters

The position and velocity errors were determined for the following points:

1. Apocynthion
2. 10 n. miles range from apocynthion
3. 20 n. miles range from apocynthion
4. 30 n. miles range from apocynthion
5. 40 n. miles range from apocynthion

Range of Parameters (Continued)

The range of accuracies for each radar parameter used in the study were:

K_1 = radar range (ρ) - 0.05% and 0.5%

K_2 = range rate ($\dot{\rho}$) - 0.3%; 1.0%, 5.0%

K_3 = line-of-sight angle (θ_R) - 2 mrad; 4 mrad; 6 mrad

Output

The data is presented in graph form. Position errors (ΔC_1 and ΔC_3) and velocity errors (ΔV_{c1} and ΔV_{c3}) are plotted as functions of time of second measurement t_2. Also plotted as a function of t_2 is the velocity impulse at midcourse which would result from the measurement errors in each of the cases studied.

Data for the apocynthion and 30 n. miles from apocynthion cases were hand plotted, and the automatic plotter was used for all other points under study.

It should be noted that the data presented consists of biased errors. Data is also available on the trajectory errors due to random radar errors.

Tables A4-1, A4-2 and A4-3 present a comparison of the effects of radar errors, when considered as biased and as random. The errors indicated are in position and velocity at apocynthion, when t_2 is 1675 seconds, for the combination of the radar parameters of range, range rate, and line-of-sight angle. The comparison shows the trajectory errors for both the biased and random cases are of the same order of magnitude, differing by no more than a factor of two in most cases.

TABLE 1

TRAJECTORY ERRORS FOR THE BIASED AND RANDOM
RADAR MEASUREMENT ERRORS

t_2 = 1675 sec.

	K_1	K_2	K_3	ΔC_1 (Ft.)	ΔC_3 (Ft.)	ΔV_{C_1} (Ft/Sec)	ΔV_{C_3} (Ft/Sec)	$\Delta V/M$ (Ft/Sec)
Biased	.0005	.010		2.02×10^4	2.35×110^3	2.48	-.374	11.6
Random				1.42×10^4	6.29×10^3	8.47	4.33	8.25
Initial				2.96×10^3	4.87×10^3	6.79	.181	-
Biased	.0005	.003		6.73×10^3	7.82×10^2	.858	-.127	3.86
Random				4.67×10^3	1.93×10^2	2.61	1.30	2.69
Initial				1.28×10^3	1.48×10^2	3.05	.074	-
Biased	.005	.010		2.89×10^4	3.35×10^3	3.95	-.561	16.6
Random				2.46×10^4	7.53×10^3	10.4	4.40	13.7
Initial				8.00×10^3	5.09×10^3	6.92	.424	-
Biased	.005	.003		1.54×10^4	1.79×10^3	7.33	-.314	8.84
Random				2.07×10^4	4.57×10^3	6.51	1.51	11.3
Initial				6.33×10^3	1.69×10^3	2.17	.321	-

Radar Parameter Combination

- Range & Range Rate ($\rho \dot{\rho}$) -

Table A4-1

TABLE 2

TRAJECTORY ERRORS FOR THE BIASED AND RANDOM

RADAR MEASUREMENT ERRORS

t_2 = 1675 sec.

	K_1	K_2	K_3	ΔC_1 (Ft.)	ΔC_3 (Ft.)	ΔV_{C_1} (Ft/Sec)	ΔV_{C_3} (Ft/Sec)	ΔV_M (Ft/Sec)
Biased		.010	.002	-1.10×10^4	-4.82×10^3	-7.96	.958	5.28
Random				2.83×10^4	1.61×10^4	22.9	2.81	13.1
Initial				-2.26×10^3	3.31×10^3	5.92	-1.58	-
Biased		.003	.004	-2.17×10^4	9.63×10^3	-14.3	1.81	10.5
Random				2.65×10	1.67×10^4	16.9	2.15	12.8
Initial				1.05×10^4	-788.1	.782	-2.48	-
Biased		.010	.005	-3.27×10^4	-1.44×10^4	-22.0	2.75	15.7
Random				4.57×10^4	2.22×10^4	31.9	3.99	21.8
Initial								
Biased		.003	.002	-1.09×10^4	4.82×10^3	-7.30	-.914	5.24
Random				1.48×10^4	7.05×10^3	10.2	1.27	7.07
Initial				1.09×10^4	1.22×10^3	4.75	-3.94	-
Biased		.010	.004	-2.18×10^4	-9.63×10^3	-14.9	1.85	10.5
Random				3.58×10^4	1.86×10^4	26.7	3.30	16.9
Initial								
Biased		.003	.006	-3.26×10^4	-1.44×10^4	-21.3	2.70	15.7
Random				3.88×10^4	1.68×10^4	24.4	3.10	18.8
Initial				3.91×10^3	260.1	1.37	-1.30	-

Radar Parameter Combination

- Los Angle & Range Rate $(\theta \dot{\rho})$ -

Table A4-2

TABLE 3

TRAJECTORY ERRORS FOR THE BIASED AND RANDOM RADAR MEASUREMENT ERRORS

	K_1	K_2	K_3	ΔC_1 (Ft)	ΔC_3 (Ft)	ΔV_{C_1} (Ft/Sec)	ΔV_{C_3} (Ft/Sec)	ΔV_M (Ft/Sec)
Biased	.0005		.002	2.82×10^3	3.39×10^3	4.64	2.32	1.45
Random				2.55×10^3	3.68×10^3	5.01	2.57	1.57
Initial				1.3×10^3	1.55×10^3	1.49	-2.18	-
Biased	.0005		.004	5.63×10^3	6.79×10^3	9.24	4.64	2.90
Random				4.93×10^3	7.31×10^3	9.96	5.13	3.08
Initial				2.19×10^3	3.31×10^3	3.29	-4.37	-
Biased	.005		.006	8.49×10^3	1.02×10^4	14.3	6.93	4.36
Random				1.03×10^4	1.18×10^4	16.4	7.73	5.67
Initial				7.03×10^3	3.11×10^3	2.18	-6.39	-
Biased	.005		.002	2.86×10^3	3.40×10^3	5.09	2.29	1.46
Random				7.67×10^3	5.66×10^3	8.40	2.66	3.66
Initial				5.28×10^3	-416.5	-1.44	-1.99	-
Biased	.005		.004	5.68×10^3	6.79×10^3	9.66	4.61	2.91
Random				8.76×10^3	8.48×10^3	12.0	5.18	4.52
Initial				6.15×10^3	1.35×10^3	36.8	-4.19	-
Biased	.0005		.006	8.45×10^3	1.02×10^4	13.8	6.96	4.36
Random				7.88×10^3	1.10×10^4	14.9	7.69	4.61
Initial				3.06×10^3	5.07×10^3	5.11	-6.57	-

Radar Parameter Combination

- Range & Los Angle ($\rho\theta$) -

Table A4-3

Appendix 5

Terminal Guidance Trajectory Analysis

Purpose -

This analysis was initiated with the following objectives:

1) Determine the radar LOS rate accuracy required to rendezvous successfully, using the GAEC - proposed LEM rendezvous guidance scheme.
2) Determine the ΔV required to perform the terminal guidance mission.

Procedure -

A two-degree of freedom, point mass, digital computer program was implemented with the following rendezvous procedures:

1) Before initiating rendezvous, the LEM is rotated so that the body Z_B-axis is pointed along the line of sight to the CSM.
2) The LEM is then rotated about the Z_B-axis until the body X_B-axis is parallel to the component of relative velocity perpendicular to the line of-sight. This attitude is now held constant throughout rendezvous, so that the Z_B-axis reaction jets may be used to adjust closing velocity, or range rate, and the X_B-axis reaction jets used to null LOS rate.
3) At a specified initial rendezvous range, the X_B-axis reaction jets ($\pm$ 400 pounds of thrust) are used to reduce the LOS rate to a minimum value.
4) After the LOS rate is nulled, the range rate is adjusted with $\pm$ 200 lbs of thrust to fall within the specified range rate limits corresponding to that range.
5) The LEM then coasts until the next range test point is reached, at which time steps (3) and (4) are repeated in the same sequence. This procedure is followed for a discrete number of range test points.

Parameters Studied

A. Ascent Trajectory Errors

The chosen nominal Trajectory has a pericynthion of 50,000 feet, and a CSM-LEM collision at an apocynthian of 80 N.MI. above the lunar surface, as shown by curve 2 in figure A5-1. The free-flight trajectories resulting from thrust magnitude variations of $\pm$ 2% from the nominal during boost were chosen as the largest expected off-nominal variations encountered by the LEM. These cutoff errors are based on a boost program in which boost ends when the integrated specific force has reached a prescribed value.

Parameters Studied (continued)

B. Initial Rendezvous Range

For the nominal (0% thrust variation) and the plus and minus 2% thrust variation ascent trajectories, rendezvous was initiated at 40,30,20, and 14 N.MI. The 14 n.mi. range was chosen because the ± 2% thrust free flight trajectories have a minimum miss distance which falls just within the 14 n.mi. range.

C. LOS RATE ACCURACIES

The effects of varying of the minimum LOS rate attainable by the LEM radar on completing rendezvous were investigated. LOS rates of 0.2, 0.5, and 1.0 milliradians/sec were studied as the minimum LOS rate that are attainable by the radar.

Results

Figure A5-1 shows the nominal and plus and minus 2% thrust free-flight trajectories as solid lines, and the rendezvous trajectories initiated from a range of 40 N.MI. as dashed lines. The steps at which LOS rate and range rate corrections were performed are indicated by triangles on the rendezvous trajectories.

The relationship between maximum allowable LOS rate and rendezvous miss distances are shown in figure A5-2 for the + and -2% thrust rendezvous trajectories. The study indicates that for the errors at cutoff resulting from variations in thrust magnitude during boost, the LOS rate must be reduced to within 0.2 milliradians/sec. to assure rendezvous.

The ΔV required for rendezvous as a function of initial rendezvous range is shown in figure A5-3. The plus and minus 3% thrust rendezvous trajectories have been included here as a loss for comparison with the ±2% thrust trajectories. For the trajectories indicated in figure A5-3, the LOS rate was nulled to a resolution of 0.2 milliradians/sec during rendezvous LOS rate adjustments.

Figures A5-1 through 3 present the results pertaining to a nominal Hohmann transfer. Corresponding results for a higher energy transfer (140° nominal central angle to rendezvous) are shown in Figs. A5-4-6.

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FIGURE 15-1
PAGE 3

2° OF FREEDOM RENDEZVOUS STUDY

±2% THRUST VARIATION - 180° FREE-FLIGHT INTERCEPTION

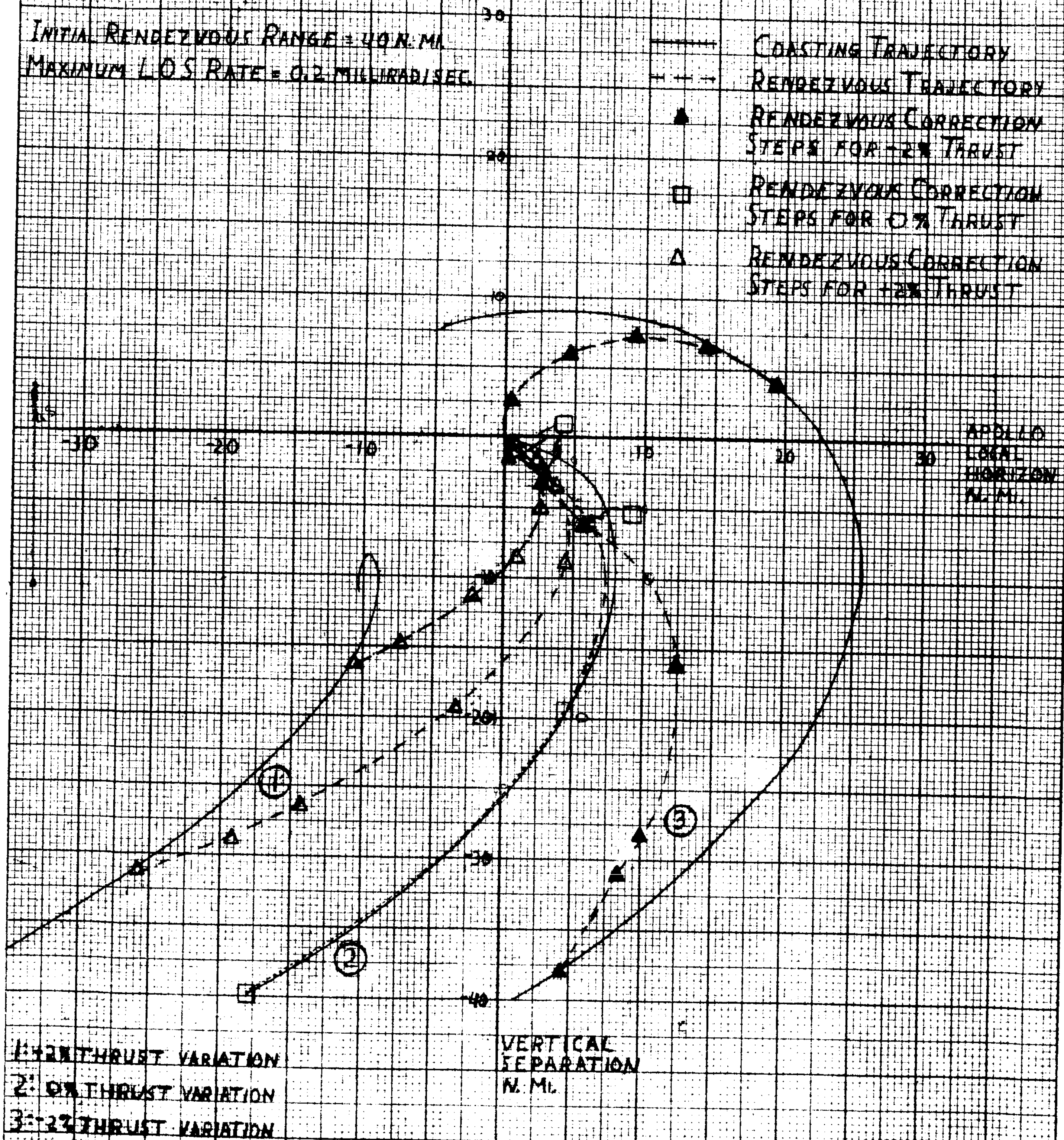

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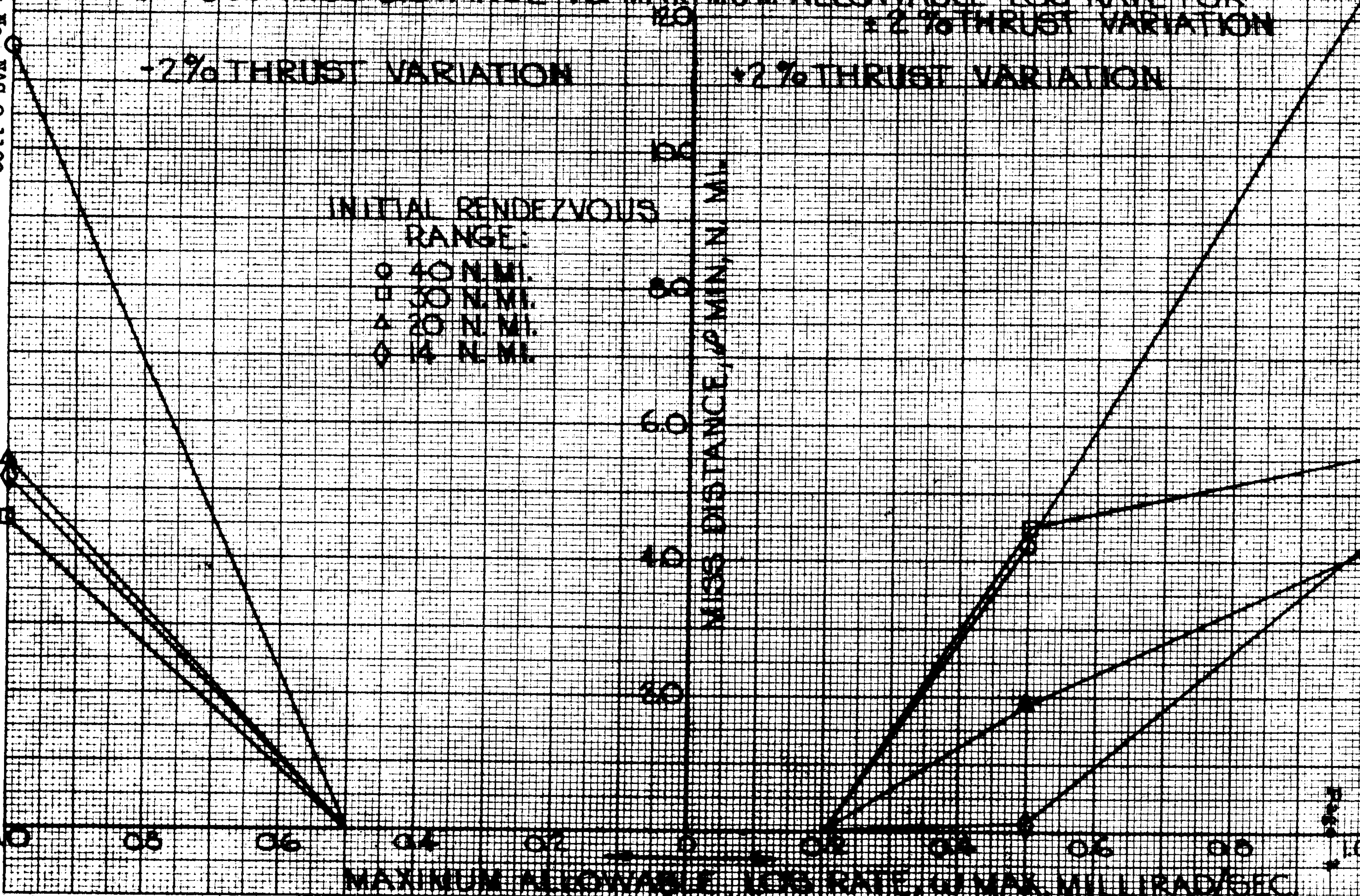
FIGURE A5-2

LEM 2°-OF-FREEDOM RENDEZVOUS STUDY

RENDEZVOUS MISS DISTANCE VS. MAXIMUM ALLOWABLE LOS RATE FOR ±2% THRUST VARIATION



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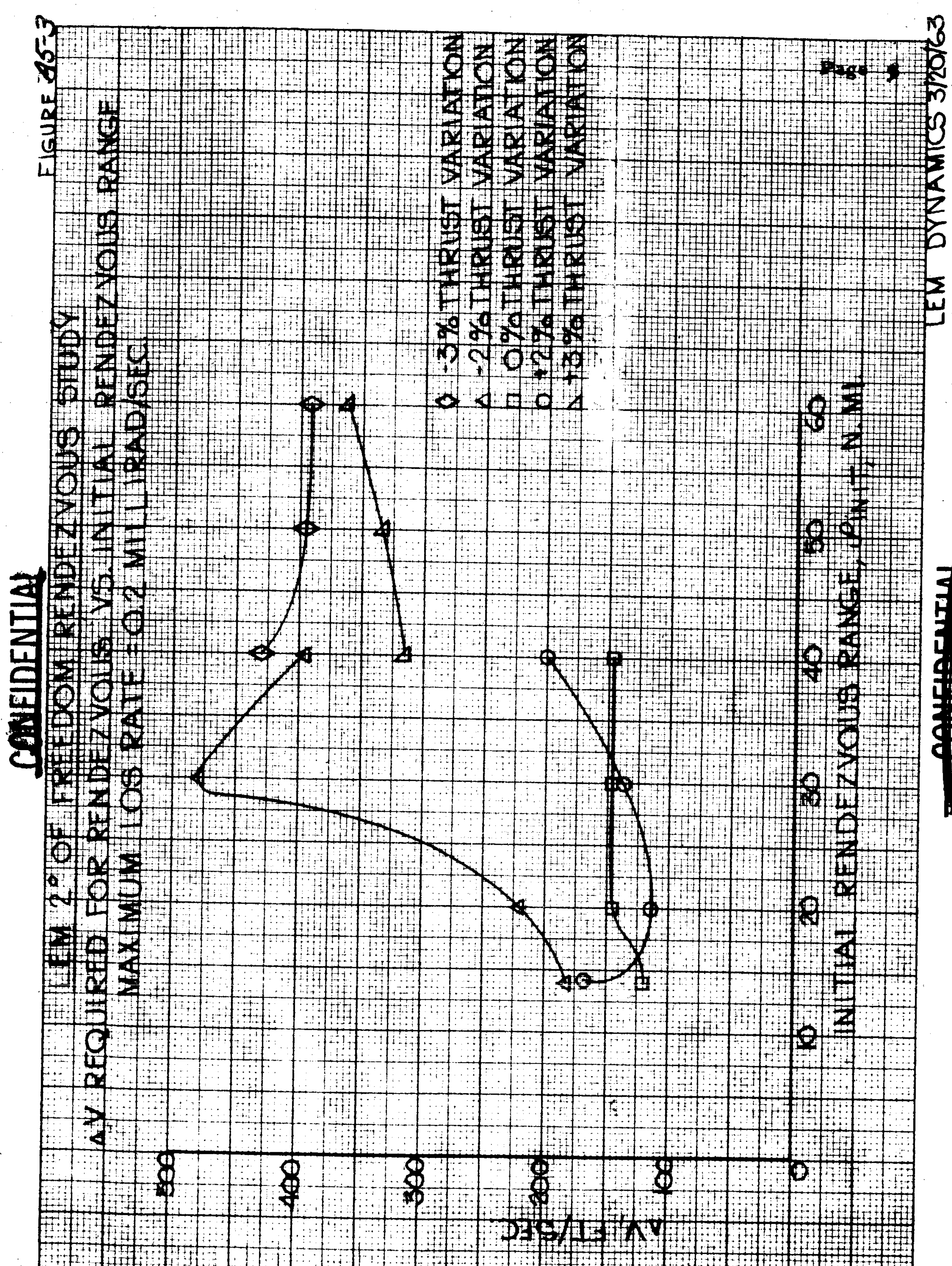
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FIGURE 15-3
LEM 2° OF FREEDOM RENDEZVOUS STUDY
ΔV REQUIRED FOR RENDEZVOUS VS. INITIAL RENDEZVOUS RANGE
MAXIMUM LOS RATE = 0.2 MILLIRAD/SEC
-3% THRUST VARIATION
-2% THRUST VARIATION
0% THRUST VARIATION
+2% THRUST VARIATION
+3% THRUST VARIATION
500
400
300
200
100
0
ΔV, FT/SEC
10
20
30
40
50
60
INITIAL RENDEZVOUS RANGE, ρINIT, N. MI
LEM DYNAMICS 3/20/63
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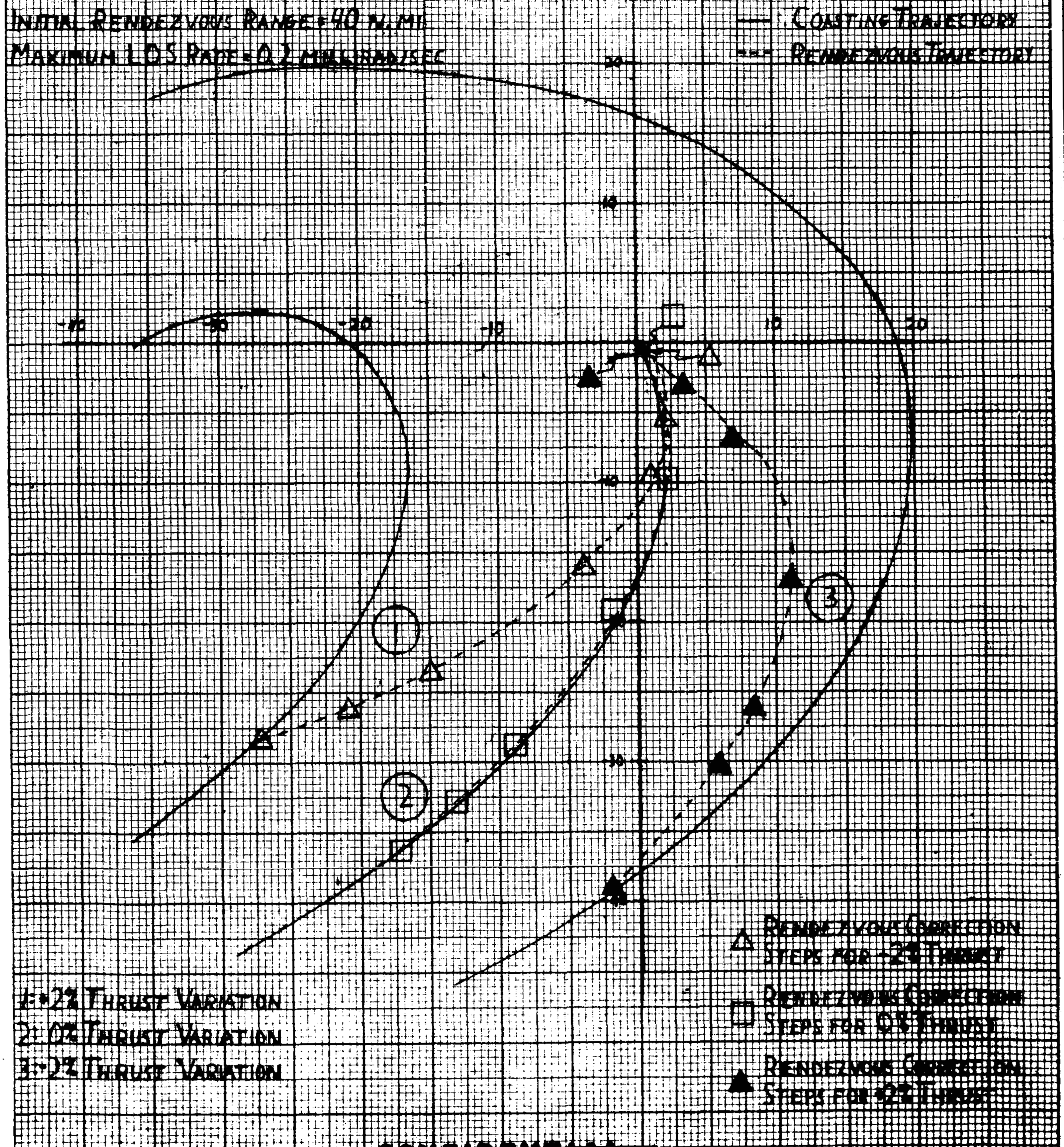
FIGURE 5-4
PAGE 8

2° OF FREEDOM RENDEZVOUS STUDY

140° FREE FLIGHT INTERCEPTION

±2% THRUST VARIATION



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3 April 1963

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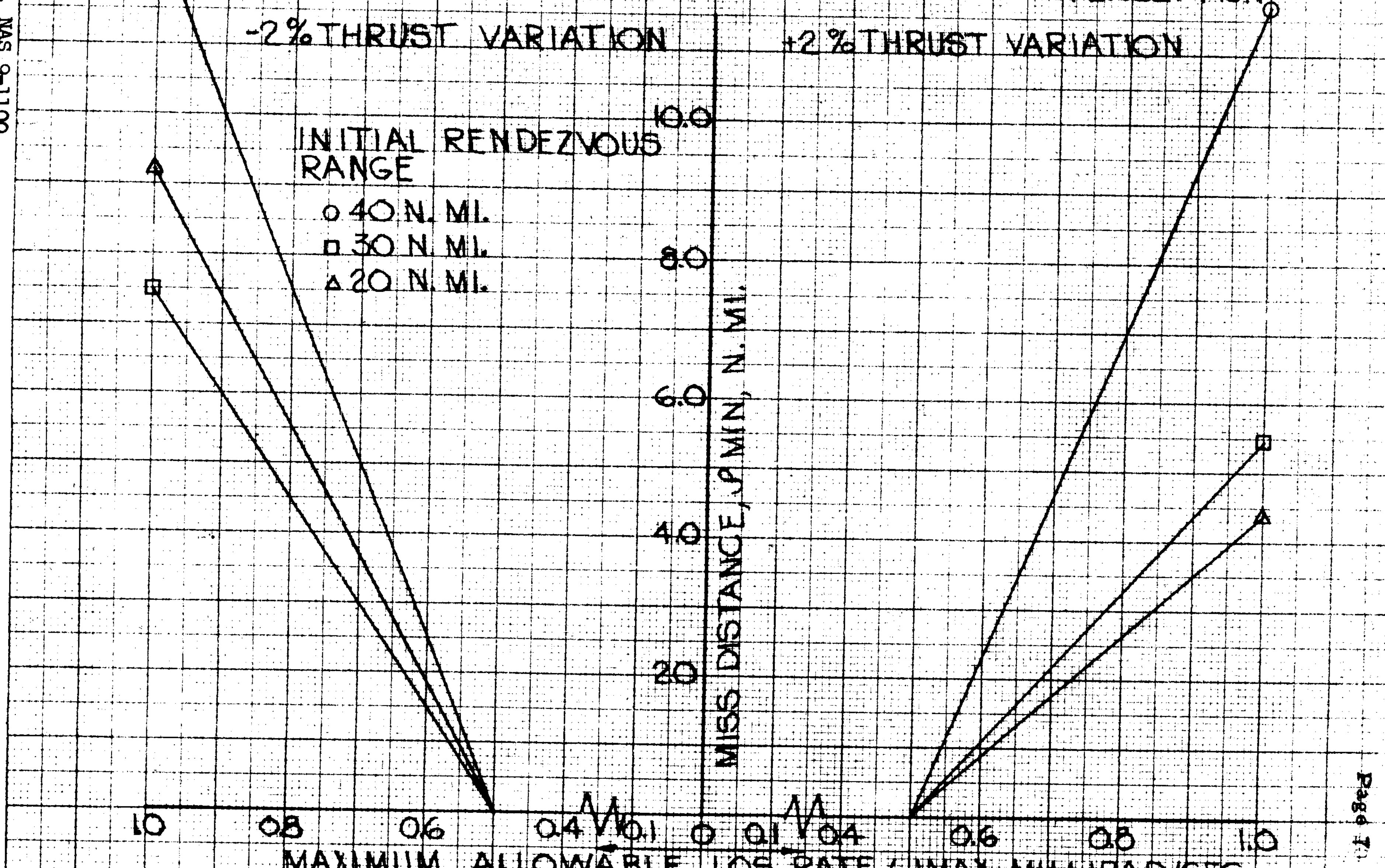
FIGURE M5-5

LEM 2° OF FREEDOM RENDEZVOUS STUDY

RENDEZVOUS MISS DISTANCE VS. MAXIMUM ALLOWABLE LOS RATE FOR ±2% THRUST VARIATION. 140° FREE-FLIGHT INTERCEPTION



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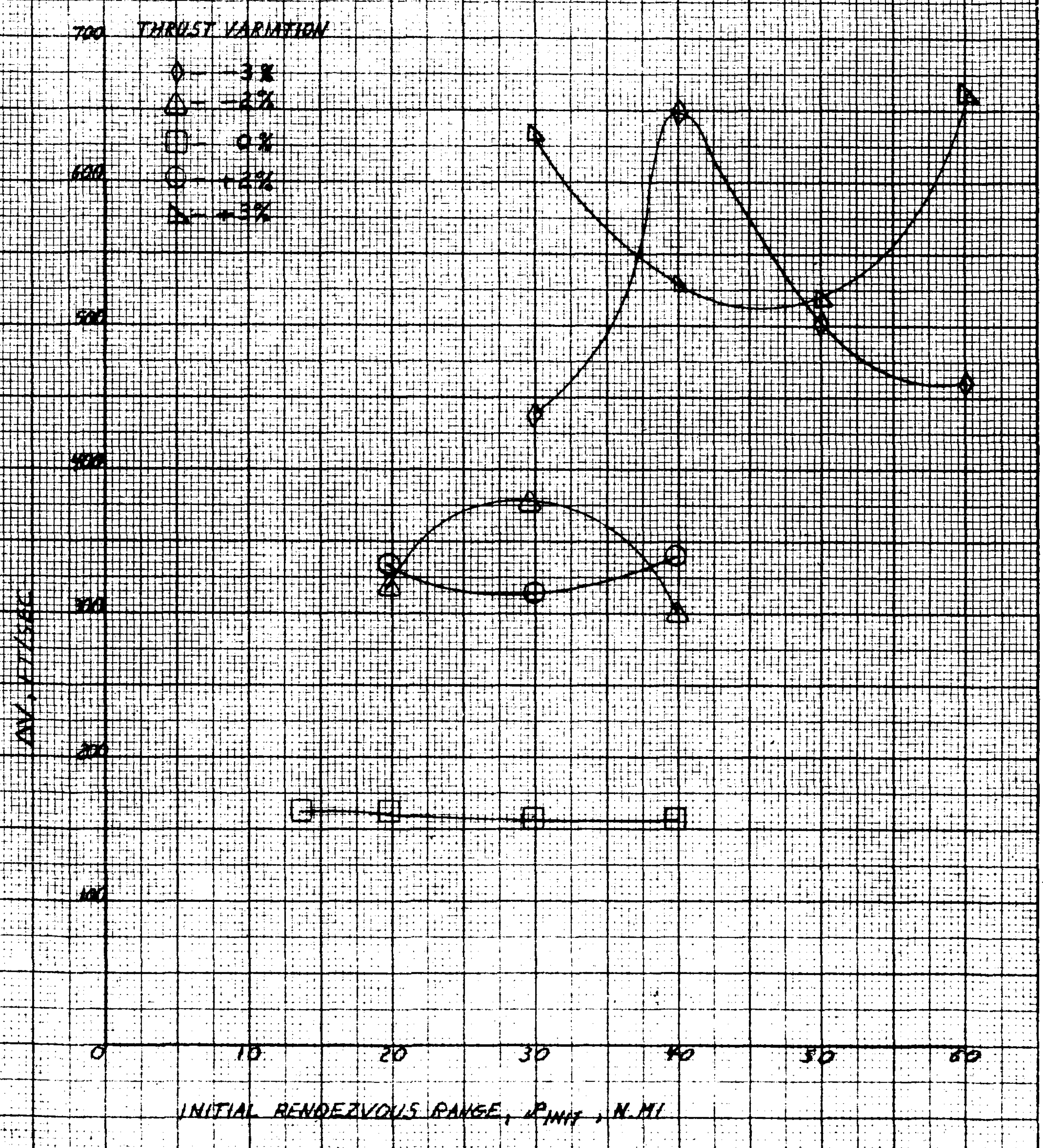
[redacted]

LEM 2° OF FREEDOM RENDEZVOUS STUDY

ΔV REQUIRED FOR RENDEZVOUS VS INITIAL RENDEZVOUS RANGE

140° FREE-FLIGHT INTERCEPTION

MAXIMUM ALLOWABLE LOS RATE = 0.2 MILLIRAD/SEC



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