MASSACHUSETTS INSTITUTE OF TECHNOLOGY

APOLLO

GUIDANCE, NAVIGATION AND CONTROL

Submitted by: ______________ Date: 8 March 1971
G. M. LEVINE, DIRECTOR, GUIDANCE ANALYSIS
APOLLO GUIDANCE AND NAVIGATION PROGRAM

Approved: ______________ Date: 18 March 1971
S. L. COPPS, COLOSSUS PROJECT MANAGER
APOLLO GUIDANCE AND NAVIGATION PROGRAM

Approved: ______________ Date: 18 Mar 1971
R. H. BATTIN, DIRECTOR, MISSION DEVELOPMENT
APOLLO GUIDANCE AND NAVIGATION PROGRAM

Approved: ______________ Date: 19 Mar 71
D. G. HOAG, DIRECTOR
APOLLO GUIDANCE AND NAVIGATION PROGRAM

Approved: ______________ Date: 19 Mar 71
R. R. RAGAN, DEPUTY DIRECTOR
CHARLES STARK DRAPER LABORATORY

R-577

GUIDANCE SYSTEM OPERATIONS PLAN FOR MANNED CM EARTH ORBITAL AND LUNAR MISSIONS USING PROGRAM COLOSSUS 3

SECTION 5 GUIDANCE EQUATIONS (Rev. 14)

MARCH 1971

MIT CHARLES STARK DRAPER LABORATORY
CAMBRIDGE, MASSACHUSETTS, 02139

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R-577

GUIDANCE SYSTEM OPERATIONS PLAN FOR MANNED CM EARTH ORBITAL AND LUNAR MISSIONS USING PROGRAM COLOSSUS

SECTION 5 GUIDANCE EQUATIONS

Signatures appearing on this page designate approval of this document by NASA/MSC.

Approved: ______________________ Date: 7/21/68
Thomas F. Gibson
Asst. Chief, Flight Software Branch
Manned Spacecraft Center, NASA

Approved: ______________________ Date: 7/-1/68
James C. Stokes, Jr.
Chief, Flight Software Branch
Manned Spacecraft Center, NASA

Approved: ______________________ Date: 7-30-68
Lynwood C. Dunseith
Chief, Flight Support Division
Manned Spacecraft Center, NASA

Date: May 1968

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS

Section No. 5 Title: Guidance Equations (Revision 1)

PCR (PCN*)		TITLE
3		Attitude Error Display During First 10 Seconds of Launch (P-11)
60.1		30° - 76° Star Selection Capability in Auto Optics
63		Delete TLI Program
64		Add v, h and $\dot{h}$ Display to P-47
80.3	*	State Vector Synchronization
81		AUGE-KUGEL Coefficient Change
87	*	Initiation of Average-G Routine
88.1		Navigation Filter Variances Shifted to Erasable Memory
94	*	Entry Guidance Logic Modifications
108	*	P-37 Description Modification
109	*	Cross Product Steering Constant c Clarification
130		Backward Integration at Lunar Sphere of Influence
146.3	*	Emergency Termination of Integration Function
149	*	Low Altitude, High Inclination Correction in P-37
155		Lift-Vector-Up Roll Attitude When Drag Less Than 0.05 g's
160	*	Return-to-Earth Targeting Constants
166	*	Correct Setting of ORBW Flag (Partial)
168	*	IMU Error Variance Correction
171.1	*	Targeting Interface with P-40 and P-41
173.1	*	RMS Position and Velocity Error Display

PCR (PCN*)		
176	*	N-Second Lambert Cycle Updates
183.2	*	Section 5 Control Data Update
193	*	Low Thrust Detection Modification
199	*	Section 5 Update Based on MIT and MSC Reviews
†200	*	P-61 Title Change to Entry Preparation Program
404	*	Lunar Landmark Selection Routine Equation Modification

† Additional Material Added in Rev. 9.

Date: July 1968

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS

Section No. 5 Title: Guidance Equations (Revision 2)

PCR (PCN*)		TITLE
176	*	N-Second Lambert Updates (Reworked)
198	*	GSOP Update Section 4
220		Colossus Entry Logic Modifications
226.1		Change Priority Displays from 5 seconds to 2 seconds
439.1		Downgrade Preferred Orientation Flag
442	*	P-37 V-Gamma Polynomial Coefficients
462	*	Lambert Error in P-17
†465	*	Resetting of VHFRFLAG and STIKFLAG
468.1		Change R-32 to Program P-76
485	*	Increase Entry Steering DAP Damping
491	*	Longer wait periods in R-22
501		Low Thrust Threshold in Erasable
505	*	SPS Engine Mechanical Bias
506		Correction to Time of Flight Calculation in P-61

†Additional material added in Rev. 9.

Date: November 1968

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 1 (Rev. 237)

Section No. 5 Title: Guidance Equations (Revision 3)

PCR (PCN*)		TITLE
166	*	Correct Setting of ORBW Flag (Complete)
231		Redefine P-41 Preferred Orientation
507.1		Termination of Integration
509		Eliminate P11 Interlock in R30
510	*	Account for 7.96 sec. of Ullage
517	*	No 90^{o} Alarm for Trunnion Angle
519	*	Calculation of ϕ in R34
521		Change C Steering Constant in P37
522		Put COAS Variance in Erasable
524		Provide MAX Display for Perigee and Apogee in P30, P31, R30
†525.1		Increase Altitude above Earth for Disturbing Accelerations
†526		Variance for Integration Errors in Navigation
528		Display Change in P67
529		Inhibit Lateral Switch in P65
570	*	GSOP Change in R22
573.1	*	Definition of A_X and A_Y in GSOP
590	*	GSOP Error in Time-to-Go
597	*	Advanced Ground Track Error
621	*	Correct SPS Engine Data
626	*	Engine Flag in P37
628	*	COLOSSUS GSOP Section 5 Update

†Additional material added in Rev. 9.

Date: December 1968

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GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 1 and COLOSSUS 1A (Rev. 249)

Section No. 5 Title: Guidance Equations (Revision 4)

PCR (PCN*)		TITLE
250		MDOT in Erasable (Colossus IA)
544	*	Change Obsolete SPS Thrust in Section 5
602		Permanent LM State Vector Update with SURFFLAG Set (Colossus IA)
611		Use 6-Dimensional Matrix for V67 Computations (Colossus IA)

Date: March 1969

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2 (COMANCHE)

Section No. 5 Title: Guidance Equations (Revision 5)

PCR (PCN*)		TITLE
251		Spacecraft Backup for Saturn V Platform Failure
254.1		Modification of CDH Time Computation Logic
255		Option on Out-of-plane Display Computations
†256		Eliminate R35 and All Fixed Memory Lunar Landmarks
†261		Range Override of P37 Entry Range Calculations
572	*	IMU Pulse Torquing
576.1	*	Removal of Backward Updating Constraint on State Vector
638		KA Limit
655	*	Range Limit in Augekugel Program
659.1	*	Suppression of X Moduloing by Kepler
661	*	Test for State Vector Origin Change at the End of Integration
663		Δv Magnitude Display for P37
664	*	P23 Calculation of Secondary Body to CSM Vector
665		R31, R34 Improvement
668	*	Test for State Vector Origin Change Based on Total Position Vector Only

681	*	Concentric Rendezvous Maneuver Targeting, CSI/CDH
682	*	Add Code "00" to P23
684	*	P37 Alarm when Altitude < 400, 000 ft
686		Nominal Orientation for P52/P54 Using Precision Integration
690		Trans-Earth Coast Slow Down Capability for P37
691		P37 Δv_{LV} Display
692	*	Time-Theta and Time-Radius Alarm Abort
694	*	Abort Coasting Integration when in Infinite Acceleration Overflow Loop
724	*	Correct REFSMMAT Gravity Computation in P11
†728		Altitude Above Moon in N62
†729		Certain SPS Impulse Data in Erasable
730		H, V, γ Display with F21
†733		Increase Entry Speed Limit in P37
734	*	Change SPS Impulse Fixed Data

†Additional material added in Rev. 9.

Date: April 1969

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GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2 (MANCHE 45)

Section No. 5 Title: Guidance Equations (Revision 6)

PCR (PCN*)	TITLE
761.2	R-2 Lunar Potential Model

Date: April 1969

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577	Title:	For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2A (COMANCHE 55)
Section No. 5	Title:	Guidance Equations (Revision 7)

PCR (PCN*)	TITLE
†268.1	Reduction of P34 / P35 Run Time
†272	Display Change for Entry Programs 64 and 65
707.1	Change from 1968 / 1969 Ephemeris Data to 1969 / 1970 Ephemeris Data
†739	Modify V67 Loads and Display to ft and ft/sec
753	Impulse Values K_2, K_3 into Erasable

†Additional material added in Rev. 9.

Date: July 1969

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2C (COMANCHE 67)

Section No. 5 Title: Guidance Equations (Revision 8)

PCR (PCN*)		TITLE
250		MDOT in Erasable (Reincorporated)
468.1		Change in R-32 to Program P-76 (Reincorporated)
†509		Eliminate P11 Interlock in R30 (Reincorporated)
544	*	Change Obsolete SPS Thrust in Section 5 (Reincorporated)
602		Permanent LM State Vector Update with SURFFLAG Set (Reincorporated)
611		Use 6-Dimensional Matrix for V67 (Reincorporated)
659.1	*	Suppression of X Moduloing by Kepler (Reincorporated)
†707.1		Change from 1968/1969 Ephemeris Data to 1969/1970 Ephemeris Data (Reincorporated)
772.1		Libration Vector at Landing Time
773.1		Fix Constants for Planetary Inertial Orientation Subroutine
776.1		Improved R2 Model Timing
†787	*	Make N63 Count During P-61
799		Verb 82 Time Option
†801.1		Make BAILOUT Alarms Start with 3XXXX and POODOO Alarms Start with 2XXXX
803		Change Entry Final Phase Table
†809	*	Error Variance Computation for VHF Range and Alternate Line-of-Sight Measurements
831.1	*	Lambert Overflow Protection
†832.1		Remove Restriction of Running RO5 only in POO
837		Change Constant Drag Controller Gains
849	*	Fix for Com 16 Anomaly

†Additional material added in Rev. 9.

Date: November 1969

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577	Title:	For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2D (COMANCHE 72)
Section No. 5	Title:	Guidance Equations (Revision 9)

PCR (PCN*)	TITLE
936.1	Initialize V90 Time to TIG
960	Change Constant in R52
977*	Alignment Star Aberration Algorithm Change

Date: March 1970

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2E

Section No. 5 Title: Guidance Equations (Revision 10)

PCR (PCN*)	TITLE
288	Final Phase Reference Table Change (Program 67)
821.1	Move AZO To Fixed Memory
859	Modify Rate Drive in R-61
860	Modify VECPOINT During P-20
868	P-23 Changes
916	Delete P-17 and P-77
917	Delete P-31
954	Revised Manual Booster Backup Guidance
956	Time of Longitude
974	Reset RENDWFLAG During Pointing Vector Routine
985	Delete P-38, P-39 and P-78, P-79
986.1	Update Constants for 1970-1971 Ephemeris Year
987	Rate-Aided Optics (P-24)
1005*	GSOP Section 5, (Rev. 10) Editorial Changes

Date: June 1970

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577	Title:	For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2E
Section No. 5	Title:	Guidance Equations (Revision 11)

PCR (PCN*)	TITLE
1034	Deletion of Time of Longitude

Section 5.3.5 Earth Orbit Insertion Monitor Program (P-11), changed in the preliminary issue of COLOSSUS 2E (Revision 10) anticipating PCR 954, has been restored to its 2D form. PCR 954 is not being implemented for COLOSSUS 2E per decision made at the Software Development Plan Meeting No. 59 on May 27, 1970.

Date: October 1970

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577	Title:	For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2E
Section No. 5	Title:	Guidance Equations (Revision 12)

Revision 12 is published as change pages to Section 5 Revision 10, 11. With the pages substituted it becomes the control document for COLOSSUS 2E. The following NASA/MSC changes are included in this revision.

PCR (PCN*)	TITLE
993	P23 Auto Maneuver Change
1101*	GSOP Section 5 Revision 12 Changes

Date: December 1970

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577	Title:	For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 2E
Section No. 5	Title:	Guidance Equations (Revision 13)

Revision 13 is published as change pages to Section 5 Revision 10, 11, 12. With the pages substituted it becomes the control document for COLOSSUS 2E. The following NASA/MSC changes are included in this revision.

PCR (PCN*)	TITLE
1131*	GSOP Editorial Changes

Date: March 1971

REVISION INDEX COVER SHEET

GUIDANCE SYSTEM OPERATIONS PLAN

GSOP No. R-577 Title: For Manned CM Earth Orbital and Lunar Missions Using Program COLOSSUS 3

Section No. 5 Title: Guidance Equations

Revision 14 incorporates the following NASA/MSC approved changes and becomes the control document for COLOSSUS 3 (ARTEMIS Rev 71):

PCR (PCN*)	TITLE	PAGES AFFECTED
320	TLI Initiate/Cutoff Program	xxiv, 5.1-3, 9, 5.3-62, 62, 5.8-3
325	New Target ΔV Program	xxvii, 5.1-4, 9, 10, 16, 18, 21, 23, 5.6-84, 85
332	MINKEY Fixed Constants to E-Memory	5.8-2
342	Entry Gain and Constant Changes	5.7-9, 10
877*	New Impulsive Burn Logic	5.3-31, 32, 33, 5.8-3
878*	New CSMMASS Update Logic	5.3-7, 12, 13, 15, 16, 37, 38, 39, 5.8-3
1049	CSM Automatic Rendezvous Sequence (MINKEY)	xxiv, xxv, 5.1-3, 18, 20, 21, 31, 5.2-63–84, 5.4-1–30, 5.6-37, 38
1051	Universal Pointing	xxiv, 5.1-3, 5.2-63–84, 5.6-37
1054	Time of Longitude	xxvi, 5.1-8, 14, 15, 16, 17, 18, 19, 20, 22, 5.5-60–64, 5.6-30
1069.1	Delete Rendezvous and Orbit Navigation Tests for Earth Orbit	xxi
1082.1	Update Fixed Constants of 1971-1972 Ephemeris Year	5.1-26, 5.5-13, 17, 23, 5.6-78, 5.9-4, 5, 11
1123*	Put ΔH for P31 into Erasable	5.8-4
1136	Pinball Scale Factor Note for GSOP	5.6-50, 51, 52, 58, 59, 5.7-2, 8, 19
1144*	Section 5 Revision 14 GSOP Changes	(See Below)

NOTE: A solid bar, ▌, in the margin indicates a change in specification. The authorizing PCR (PCN*) is listed at the bottom of the page, except for Section 5.2.5 (PCR 1049, 1051) and Sections 5.4.1, 2 (PCR 1049) which have been extensively rewritten. A series of dots, ⁞, in the margin indicates a document improvement change authorized by PCN 1144*.

FOREWORD

SECTION 5 REVISION 14

The Guidance System Operations Plan (GSOP) for Program COLOSSUS 3 is published in six sections as separate volumes:

1. Prelaunch
2. Data Links
3. Digital Autopilots
4. Operational Modes
5. Guidance Equations
6. Control Data

With this issue, Section 5 is revised from the previous issue of COLOSSUS GSOP (Revision 13 Dec 1970 for COLOSSUS 2E) in order to reflect the NASA/MSC-approved changes listed on the "Revision Index Cover Sheet" at the beginning of this volume.

Although the GSOP specifies an earth-orbital capability for all programs—and this capability has been provided—verification testing shall not be accomplished for earth-orbit rendezvous and earth-orbit navigation with P22.

Technical writing was performed by Joseph Klawsnik.

This volume is published as a control document governing guidance and navigation computation programs for COLOSSUS 3. Revisions constituting changes to the COLOSSUS Program require NASA approval.

Jack C. Reed
APOLLO Documentation Group

TABLE OF CONTENTS

SECTION 5

GUIDANCE EQUATIONS

5.1 INTRODUCTION

5.1.1 GENERAL COMMENTS

The purpose of this section is to present the Guidance and Navigation Computation Routines associated with the CSM Apollo Lunar Landing Mission. These Routines are utilized by the Programs outlined in Section 4 where astronaut and other subsystem interface and operational requirements are described. The guidance and navigation equations and procedures presented in Section 5 are primarily concerned with mission type programs representing the current CSM GNCS Computer (CMC) capability. A restricted number of CMC service type program operations which involve computation requirements are also included.

The CSM GNCS Computer (CMC) guidance and navigation equations for the lunar landing mission are presented in the following six categories:

Section 5.2	Coasting Flight Navigation Routines
Section 5.3	Powered Flight Navigation and Guidance Routines
Section 5.4	Targeting Routines
Section 5.5	Basic Subroutines
Section 5.6	General Service Routines
Section 5.7	Entry Guidance

Guidance equation parameters required for program initialization and operation are listed in Section 5.8. These selected parameters are stored in the CMC erasable memory. General constants used in the equations of this volume are presented in Section 5.9.

A cross-reference between the CMC programs and routines of Section 4 that are described in Section 5 is listed in Section 5.1.2. In the Section 5 table of contents and text, missing section numbers correspond to COLOSSUS programs that have been deleted from the previous Section 5 GSOP's by MSC direction resulting from the CMC Fixed Memory Storage Review Meeting of August 28, 1967, and subsequently approved Program Change Requests (PCR) by the Software Change Control Board (SCB).

The principal authors making initial contributions to Section 5 were:

T. Brand	R. Morth	W. Robertson
P. Kachmar	R. Phaneuf	N. Sears
G. Levine	P. Philliou	W. Tempelman
W. Marscher	J. M. Reber	R. White
F. Martin		

5.1.2 Sections 4 and 5 Cross Reference

PROGRAM NUMBER	TITLE	PRINCIPAL SECTION 5 SUBSECTION NO.	PAGE
P-00	CMC Idling Program	5.6.12	5.6-78
P-11	Earth Orbit Injection (EOI) Monitor Program	5.3.5	5.3-47
P-15	TLI Initiate/Cutoff Program	5.3.6	5.3-62
P-20	Universal Tracking and Rendezvous Navigation Program	5.2.5	5.2-63
P-21	Ground Track Determination Program	5.6.5	5.6-30
P-22	Orbital Navigation Program	5.2.4	5.2-41
P-23	Cislunar Navigation Program	5.2.6	5.2-84
P-24	Rate-Aided Optics Tracking Program	5.6.17	5.6-88
P-29	Time-of-Longitude Crossing Program	5.5.15	5.5-60
P-30	External Δv Maneuver Guidance	5.3.3.3.1	5.3-17
P-31	Height Adjustment Maneuver	5.4.2	5.4-1
P-32	Coelliptic Sequence Initiation (CSI) Program	5.4.2	5.4-1
P-33	Constant Differential Altitude (CDH) Program	5.4.2	5.4-1
P-34	Transfer Phase Initiation (TPI) Guidance	5.4.2	5.4-1
P-35	Transfer Phase Midcourse (TPM) Guidance	5.4.2	5.4-1
P-36	Plane Change Maneuver Program	5.4.2	5.4-1
P-37	Return to Earth Maneuver Guidance	5.4.3	5.4-43

☒ Revised COLOSSUS 3
☐ Added GSOP # R-577 PCR # 325 Rev. 14 Date 2/71

5.1.3 GENERAL PROGRAM UTILIZATION

The following outline is a brief summary of the major CMC programs and callable routines that can be used in the principal phases of a lunar landing mission. This outline reflects the CMC capability for the nominal and abort cases of such a mission.

I Earth Orbit Injection (EOI) Phase

A) Nominal

P - 11 Earth Orbit Insertion (EOI) Monitor Program

B) Abort to Entry

P - 61 Entry Preparation Program

P - 62 CM/SM Separation Maneuver Program

P - 63 Entry Initialization Program

P - 64 Post 0.05 G Entry Program

P - 67 Final Entry Phase Program

II Earth Orbit Phase

A) Nominal

P-27 CMC Update Program (State vector update).

R-30 Orbit Parameter Display Routine

B) Aborts to Entry

1. RTCC Targeted Abort

P-27 CMC Update Program (De-orbit targeting).

P-30 External ΔV Maneuver Program (De-orbit)

P-40, P-41 — SPS or RCS Thrust Program

P-61, P-62, P-63, P-64, P-67 — Entry Programs

II Earth Orbit Phase (cont)

B. Aborts to Entry

2. CMC Targeted Abort

P - 22	Lunar Orbit Navigation Program (Earth Mode)
P - 21	Ground Track Program
P - 29	Time-of-Longitude Crossing Program
P - 37	Return to Earth Program (Targeting and Pre-Thrust)
P - 40, P - 41	SPS or RCS Thrust Program
P - 61, P - 62, P - 63, P - 64, P - 67	Entry Programs

C) Service Programs (Nominal and Abort Cases)

P - 52	IMU Realignment Program
P - 00	CMC Idling Program
R - 30	Orbit Parameter Display Routine

III Trans-Lunar Injection (TLI) Phase

A) Nominal

P - 47 Thrust Monitor Program (TLI Maneuver Monitoring)

P - 15 TLI Initiate/Cutoff Program

P - 77 Impulsive ΔV Program

B) Aborts to Earth Entry

1. RTCC Targeted Abort

P - 27 CMC Update Program (Return to earth maneuver targeting).

P - 30 External ΔV Pre-Thrust Program

P - 40 }
P - 41 } SPS or RCS Thrust Program

P - 61 to P - 67 } Entry Programs

III Trans-Lunar Injection (TLI) Phase (cont)

B) Aborts to Entry

2. CMC Targeted Abort

P - 23 Cislunar Navigation Program

P - 37 Return to Earth Program (Targeting and Pre-Thrust)

P - 40, P - 41 } SPS or RCS Thrust Programs

P - 61 to P - 67 } Entry Programs

C) Service Programs Used in Abort Cases

P - 51 IMU Orientation Determination

P - 52 IMU Realignment Program

P - 77 Impulsive ΔV Program

P - 06 GNCS Power Down

P - 00 CMC Idling

IV Trans-Lunar Phase

A) Nominal (Midcourse Correction Maneuvers)

P - 27 CMC Update Program (State vector update and cislunar MCC maneuver targeting).

P - 30 External ΔV Pre-Thrust Program

P - 40
P - 41 } SPS or RCS Thrust Program

P - 47 Thrust Monitor Program (Manual Transposition and Docking Maneuver)

B) Aborts to Earth Entry

1. RTCC Targeted Abort

P - 27 CMC Update Program (Return to earth maneuver targeting).

P - 30 External ΔV Pre-Thrust Program

P - 40
P - 41 } SPS or RCS Thrust Program

IV Trans-Lunar Phase (cont)

B) Aborts to Earth Entry

2. CMC Targeted Abort (Limited Capability)

P - 23	Cislunar Navigation Program
P - 37	Return to Earth Program Targeting and Pre-Thrust (Capability limited to outside lunar sphere of influence)
P - 40 P - 41	SPS or RCS Thrust Programs
P - 61 to P - 67	Entry Programs

C) Service Programs for Nominal and Abort Cases

P - 06	GNCS Power Down
P - 51	IMU Orientation Determination
P - 52	IMU Realignment Program
P - 00	CMC Idling Program
R - 05	S-Band Antenna Routine

V Lunar Orbit Insertion (LOI) Phase

A) Nominal

P - 27 CMC Update Program (State vector update and LOI maneuver targeting).

P - 30 External ΔV Maneuver Program (Second LOI Maneuver)

P - 40 SPS Thrust Program

B) Aborts to Return to Earth Trajectory

1. RTCC Targeted Abort

P - 27 CMC Update Program (Return to earth maneuver targeting).

P - 30 External ΔV Pre-Thrust Program (To establish safe lunar orbit if required)

P - 40 SPS Thrust Program

V Lunar Orbit Insertion (LOI) Phase (cont)

B) Aborts to Return to Earth Trajectory

2. CMC Semi-Controlled Abort (Limited Capability)

P - 22 Lunar Orbit Navigation Program

P - 21 Ground Track Program

P - 24 Rate-Aided Optics Tracking Program

P - 27 CMC Update Program (TEI targeting)

P - 29 Time-of-Longitude Crossing Program

P - 30 External Δ v Maneuver Guidance

P - 40 SPS Thrust Program

C) Service Programs for Nominal and Abort Cases

P - 52 IMU Realignment Program

R - 05 S-Band Antenna Routine

VI Lunar Orbit Phase Prior to Undocking and Separation (CSM Active)

A) Nominal

P - 22	Lunar Orbit Navigation Program
P - 21	Ground Track Program
P - 24	Rate-Aided Optics Tracking Program
P - 27	CMC Update Program (Lunar Landing timing and targeting parameters)
P - 29	Time-of-Longitude Crossing Program
P - 30	External Δv Maneuver Guidance
R - 33	CMC/LGC Clock Synchronization Routine
P - 40	SPS Thrust Program
P - 41	RCS Thrust Program
P - 52	IMU Realignment Program

B) Aborts to Return to Earth Trajectory

1. RTCC Targeted Abort

P - 27	CMC Update Program (TEI targeting and State vector update).
P - 30	External Δv Maneuver Guidance
P - 40	SPS Thrust Program

VI Lunar Orbit Phase Prior to Undocking and Separation (CSM Active) (cont.)

B) Aborts to Return to Earth Trajectory

2. CMC Semi-Controlled Abort (Limited Capability)

P - 22	Lunar Orbit Navigation Program
P - 21	Ground Track Program
P - 24	Rate-Aided Optics Tracking Program
P - 27	CMC Update Program (TEI targeting)
P - 29	Time-of-Longitude Program
P - 30	External Δv Maneuver Guidance
P - 40	SPS Thrust Program
P - 47	Thrust Monitor Program (TEI by DPS Case)
P - 77	Impulsive ΔV Program

C) Aborts to Safe Lunar Orbit (Bailout Burn)

P - 47 Thrust Monitor Program

D) Service Programs for Nominal and Abort Cases

P - 06	GNCS Power Down
P - 51	IMU Orientation Determination
P - 52	IMU Realignment Program
P - 00	CMC Idling Program
R - 05	S-Band Antenna Routine
R - 30	Orbit Parameter Display Routine

VII Descent Coast and Landing Phases

A) Nominal

P - 20 Universal Tracking and Rendezvous Navigation Program (Tracking attitude and monitoring only)

B) Aborts to Rendezvous Condition

1. LM Active Vehicle

P - 20 Universal Tracking and Rendezvous Navigation Program

P - 76 Target ΔV Program

P - 74 LM TPI Targeting Program (Monitoring or commands via voice link)

P - 75 LM TPM Targeting Program (Rendezvous MCC monitoring or commands via voice link)

P - 72 LM Co-elliptic Sequence Initiation (CSI)

P - 73 LM Constant Delta Altitude (CDH)

P - 21 Ground Track Program

P - 29 Time-of-Longitude Crossing Program

VII Descent Coast and Landing Phases (cont.)

B) Aborts to Rendezvous Condition

2. CSM Active Retrieval

P - 27	CMC Update Program (State vector update and phasing maneuver targeting if required)
P - 20	Universal Tracking and Rendezvous Navigation Program
P - 21	Ground Track Program
P - 29	Time-of-Longitude Crossing Program
P - 30	External ΔV Pre-Thrust Program
P - 31	Height Adjustment Maneuver Program
P - 32	CSM Co-Elliptic Sequence Initiation (CSI)
P - 33	CSM Constant Delta Altitude (CDH)
P - 34	TPI Pre-Thrust Program
P - 35	TPM Pre-Thrust Program
P - 36	Plane Change Maneuver Program
P - 40	SPS or RCS Thrust Programs
P - 41	
P - 47	Thrust Monitor Program (Manual terminal rendezvous maneuver)
P - 77	Impulsive ΔV Program
P - 79	Rendezvous Final Program

C) Service Programs

P - 52	IMU Realignment Program
R - 31	Rendezvous Parameter Display Routine No. 1
R - 34	Rendezvous Parameter Display Routine No. 2

R - 36	Out of Plane Rendezvous Display Routine
R - 63	Rendezvous Final Attitude Routine
P - 00	CMC Idling
R - 05	S-Band Antenna Routine
R - 30	Orbit Parameter Display Routine

VIII Lunar Stay Phase to LM Ascent Launch

A) Nominal

P - 22	Lunar Orbit Navigation Program (for landing site surveillance and CMC update if desired)
P - 21	Ground Track Program
P - 24	Rate-Aided Optics Tracking Program
P - 27	CMC Update Program (LM launch time update and Lunar Orbit Plane Change LOPC, targeting).
P - 29	Time-of-Longitude Crossing Program
P - 30	External ΔV Pre-Thrust Program (LOPC)
P - 40	SPS Thrust Program
P - 20	Universal Tracking and Rendezvous Navigation Program (Tracking attitude and monitoring only)

B) Service Programs

P - 51	IMU Orientation Determination
P - 52	IMU Realignment Program
P - 00	CMC Idling Program
R - 05	S-Band Antenna Routine
R - 30	Orbit Parameter Display Routine

IX LM Ascent and Rendezvous Phase

A) Nominal

P - 20 Universal Tracking and Rendezvous Navigation Program (Preferred Tracking Attitude during LM launch)

P - 27 CMC Update Program
(or)
P - 76 Target ΔV Program
} LM injection state vector

P - 20 Universal Trackind and Rendezvous Navigation Program (LM state vector updating)

P - 76 Target ΔV Program

P - 74 LM TPI Targeting Program (Monitoring)

P - 75 LM TPM Targeting Program (Monitoring)

P - 79 Rendezvous Final Program

P - 21 Ground Track Program

B) Aborts to Rendezvous Condition

1. LM Active Vehicle

P - 27 CMC Update Program (State vector updates)

P - 20 Universal Tracking and Rendezvous Navigation Program

P - 76 Target ΔV Program

P - 74 LM TPI Targeting Program (Monitoring)

P - 75 LM TPM Targeting Program (Monitoring)

P - 79 Rendezvous Final Program

P - 72 LM Co-elliptic Sequence Initiation (CSI)

P - 73 LM Constant Delta Altitude (CDH)

P - 21 Ground Track Program

P - 29 Time-of-Longitude Crossing Program

IX LM Ascent and Rendezvous Phase (cont)

B) Aborts to Rendezvous Condition

2. CSM Active Retrieval

P - 27	CMC Update Program (State vector updates and phasing maneuver targeting if required)
P - 20	Universal Tracking and Rendezvous Navigation Program
P - 30	External ΔV Pre-Thrust Program (Targeted from RTCC or LGC)
P - 31	Height Adjustment Maneuver Program
P - 32	CSM Co-Elliptic Sequence Initiation (CSI)
P - 33	CSM Constant Delta Altitude (CDH)
P - 36	Plane Change Maneuver Program
P - 21	Ground Track Program
P - 34	TPI Pre-Thrust Program
P - 35	TPM Pre-Thrust Program
P - 79	Rendezvous Final Program
P - 40, P - 41	SPS or RCS Thrust Programs
P - 47	Thrust Monitor Program (Manual terminal rendezvous maneuver)
P - 77	Impulsive ΔV Program

C) Service Programs for Nominal and Abort Cases

P - 52	IMU Realignment Program
R - 05	S-Band Antenna Routine
R - 31	Rendezvous Parameter Display Routine No. 1
R - 34	Rendezvous Parameter Display Routine No. 2
R - 36	Out of Plane Rendezvous Display Routine
R - 63	Rendezvous Final Attitude Routine

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R - 30 Orbit Parameter Display Routine

P - 00 CMC Idling Program

X Lunar Orbit Phase Prior to TEI

A) Nominal

P - 27 CMC Update Program (State vector update)
(or)
P - 22 Lunar Orbit Navigation Program

P - 21 Ground Track Program

P - 24 Rate-Aided Optics Tracking Program

P - 29 Time-of-Longitude Crossing Program

P - 52 IMU Realignment Program

P - 00 CMC Idling Program

P - 47 Thrust Monitor (Manual CSM-LM Separation Maneuver)

B) Aborts Prior to Nominal TEI

P - 27 CMC Update Program (State vector update)

XI Trans-Earth Injection (TEI) Phase

A) Nominal

P - 27 CMC Update Program

P - 30 External Δv Maneuver Guidance

P - 40 SPS Thrust Program

B) Aborts to Return to Earth Trajectory

No CMC targeting capability

P - 27 CMC Update Program

P - 30 External ΔV Maneuver Guidance

P - 40 SPS Thrust Program

P - 77 Impulsive ΔV Program

XII Trans-Earth Phase Midcourse Correction Maneuvers

A) Nominal

P - 27 CMC Update Program (Cislunar midcourse correction maneuver targeting)

P - 30 External ΔV Pre-Thrust Program

P - 40, P - 41 } SPS or RCS Thrust Program

B) Aborts to Maintain Earth Return Trajectory (CMC Limited Capability)

P - 23 Cislunar Navigation Program

P - 37 Return to Earth Program (Outside lunar sphere of influence only)

P - 40, P - 41 } SPS or RCS Thrust Programs

C) Service Programs for Nominal and Abort Cases

P - 06 GNCS Power Down

P - 51 IMU Orientation Determination

P - 52 IMU Realignment Program

R - 05 S-Band Antenna Routine

P - 00 CMC Idling Program

XIII Entry Phase

A) Nominal

P - 61	Entry Preparation Program
P - 62	CM / SM Separation Maneuver
P - 63	Entry Initialization
P - 64	Post 0.05 G Entry Phase
P - 65	Entry Up Control Phase
P - 66	Entry Ballistic Phase
P - 67	Final Entry Phase

5.1.4 COORDINATE SYSTEMS

There are six major coordinate systems used in the navigation and guidance programs. These six coordinate systems are defined individually in the following descriptions and referenced to control specifications of Section 5.9.2 where applicable. Any other coordinate system used in any particular CMC program is defined in the individual section describing that program.

5.1.4.1 Basic Reference Coordinate System

The Basic Reference Coordinate System is an orthogonal inertial coordinate system whose origin is located at either the moon or the earth center of mass. The orientation of this coordinate system is defined by the line of intersection of the mean earth equatorial plane and the mean orbit of the earth (the ecliptic) at the beginning of the Besselian year which starts January 1.2516251, 1972, et. The X-axis ($\underline{u}_{XI}$) is along this intersection with the positive sense in the direction of the ascending node of the ecliptic on the equator (the equinox), the Z-axis ($\underline{u}_{ZI}$) is along the mean earth north pole, and the Y-axis ($\underline{u}_{YI}$) completes the right-handed triad. The Basic Reference Coordinate System is presented in Ref. 1 of Section 5.9.2 as Standard Coordinate System 4, Geocentric Inertial in Fig. A-4.

This coordinate system is shifted from earth-centered to moon-centered when the estimated vehicle position from the moon first falls below a specified value r_{SPH}, and is likewise shifted from moon-centered to earth-centered when the estimated vehicle position from the moon first exceeds r_{SPH}. This procedure is described in Section 5.2.2.6 and Fig. 2.2-3. All navigation stars and lunar-solar ephemerides are referenced to this coordinate system. All vehicle state vectors are referenced to this system.

5.1.4.2 IMU Stable Member or Platform Coordinate System

The orthogonal inertial coordinate system defined by the GNCS inertial measurement unit (IMU) is dependent upon the current IMU alignment. There are many possible alignments during a mission, but the primary IMU alignment orientations described in Section 5.6.3.3 are summarized below and designated by the subscript SM:

1. Preferred Alignment

$$\underline{u}_{XSM} = \text{UNIT}(\underline{x}_B)$$

$$\underline{u}_{YSM} = \text{UNIT}(\underline{u}_{XSM} \times \underline{r})^{*} \qquad (1.4.1)$$

$$\underline{u}_{ZSM} = \underline{u}_{XSM} \times \underline{u}_{YSM}$$

where:

$\underline{u}_{XSM}$, $\underline{u}_{YSM}$, $\underline{u}_{ZSM}$ } IMU stable member coordinate unit vectors referenced to the Basic Reference Coordinate System

$\underline{x}_B$ = vehicle or body X-axis at the preferred vehicle attitude for ignition

$\underline{r}$ = position vector at ignition

2. Nominal Alignment (Local Vertical)

$$\underline{u}_{XSM} = (\underline{u}_{YSM} \times \underline{u}_{ZSM})$$

$$\underline{u}_{YSM} = \text{UNIT}(\underline{v} \times \underline{r}) \qquad (1.4.2)$$

$$\underline{u}_{ZSM} = \text{UNIT}(-\underline{r})$$

where $\underline{r}$ and $\underline{v}$ represent the vehicle state vector at the alignment time, t_{align}.

3. Earth Pre-launch Alignment

Prior to earth launch the IMU stable member is aligned to a local vertical axis system where

$\underline{u}_{ZSM}$ = UNIT ($-\underline{r}$) (local vertical)

$\underline{u}_{XSM}$ = UNIT ($\underline{A}$) where $\underline{A}$ is a horizontal vector pointed at the desired launch azimuth angle.

$\underline{u}_{YSM} = \underline{u}_{ZSM} \times \underline{u}_{XSM}$

* In P41 if the magnitude of the cross product is less than 2^{16} meters or in P40 if the magnitude of $\underline{u}_{XSM} \times \text{UNIT}\ \underline{r}$ is less than 2^{-12} radians, then $\underline{u}_{YSM}$ = UNIT $[\underline{u}_{XSM} \times (\text{UNIT}\ \underline{r} + 0.125\ \text{UNIT}\ \underline{v})]$ where $\underline{v}$ = velocity vector at ignition.

4. Lunar Landing Alignment

$$\underline{u}_{XSM} = \text{UNIT}\ (\underline{r}_{LS})\ \text{at}\ t_L$$
$$\underline{u}_{ZSM} = \text{UNIT}\left[(\underline{r}_C \times \underline{v}_C) \times \underline{u}_{XSM}\right] \qquad (1.4.3)$$
$$\underline{u}_{YSM} = \underline{u}_{ZSM} \times \underline{u}_{XSM}$$

where $\underline{r}_{LS}$ is the lunar landing site vector at the predicted landing time, t_L, and $\underline{r}_C$ and $\underline{v}_C$ are the CSM position and velocity vectors, as maintained in the CMC.

5. Lunar Launch Alignment

The same as that defined in Eq. (1.4.3) except that $\underline{r}_{LS}$ is the landing or launch site at the predicted launch time t_L.

The origin of the IMU Stable Member Coordinate System is the center of the IMU stable member.

5.1.4.3 Vehicle or Body Coordinate System

The Vehicle or Body Coordinate System is the orthogonal coordinate system used for the CSM structural body. The origin of this coordinate system is on the longitudinal axis of the CSM, 1000 inches below the mold line of the heat shield main structure ablator interface. The X-axis ($\underline{u}_{XB}$) lies along the longitudinal axis of the vehicle, positive in the nominal SPS thrust direction. The positive Z-axis ($\underline{u}_{ZB}$) is defined by an alignment target (labeled +Z) at the top of the service module and is normal to $\underline{u}_{XB}$. The Y-axis ($\underline{u}_{YB}$) completes the right-handed triad. This coordinate system is defined in Ref. 1 of Section 5.9.2 as the Standard Coordinate System 8c, CSM Structural Body Axes in Fig. A-8c.

5.1.4.4 Earth-Fixed Coordinate System

The Earth-Fixed Coordinate System is an orthogonal rotating coordinate system whose origin is at the center of mass of the earth. This coordinate system is shown in Ref. 1 of Section 5.9.2 as the Standard Coordinate System 1, Geographic Polar in Fig. A-1. The Z-axis of this coordinate system is defined to be along the earth's true rotational or polar axis. The X-axis is defined to be along the intersection of the prime (Greenwich) meridian and the equatorial plane of the earth, and the Y-axis is in the equatorial plane and completes the right-handed triad.

5.1.4.5 Moon-Fixed Coordinate System

The Moon-Fixed Coordinate System is an orthogonal rotating coordinate system whose origin is at the center of mass of the moon. This coordinate system is shown in Ref. 1 of Section 5.9.2 as the Standard Coordinate System 2, Selenographic Polar in Fig. A-2. The Z-axis is defined to be along the true polar or rotation axis of the moon, the X-axis is through the mean center of the apparent disk or along the intersection of the meridian of 0^o longitude and the equatorial plane of the moon, and the Y-axis is in the equatorial plane and completes the right-handed triad.

5.1.4.6 Navigation Base Coordinate System

The Navigation Base Coordinate System (subscript NB) is an orthogonal coordinate system whose orientation is essentially parallel to that of the CSM Vehicle Coordinate System. The Y_{NB} axis is defined by the centers of the two upper mounts between GNCS navigation base and the CM structure located at vehicle station points $X_C = 71.185$ and $Z_C = 35.735$ in Ref. 13 of Section 5.9.2. The positive Y_{NB} direction is in the same general direction as the CSM +Y axis. The Z_{NB} axis is defined as a line measured from the center line of the optics (shown in Section A-A of Ref. 1) through an angle of $32^o\ 31'\ 23.19''$ about the Y_{NB} axis towards the vehicle +Z axis, and located on the Y_{NB} axis half way between the mount points. The positive Z_{NB} axis is in the same general direction as the vehicle +Z axis. The X_{NB} axis is defined as $\underline{Y}_{NB} \times \underline{Z}_{NB}$ to complete the right-handed triad.

5.1.5 GENERAL DEFINITIONS AND CONVENTIONS

In this section the definitions of and the conventions for using a selected number of parameters are given. Although virtually all of the information in this section appears elsewhere in this document, this section provides a summary of facts which are distributed among various other sections.

5.1.5.1 Error Transition Matrix Maintenance

5.1.5.1.1 Definitions

The error transition matrix (W matrix) is defined in Section 5.2.2.4 and is used in processing navigation measurement data. Control of the W matrix is maintained by means of two flags, RENDWFLG (see Section 5.2.5.2.2) and ORBWFLAG (see Sections 5.2.4.5 and 5.2.6.4). If RENDWFLG is equal to one, then the W matrix is valid for processing rendezvous navigation data; while ORBWFLAG being equal to one indicates that the W matrix is valid for processing orbit or cislunar-midcourse navigation data. If both of these flags are equal to zero, then the W matrix is invalid. These two flags are mutually exclusive; that is, they cannot both be equal to one.

5.1.5.1.2 W Matrix Control Flags

The two W matrix control flags are maintained according to the following rules:

1. RENDWFLG and ORBWFLAG are both initially zero.

2. A CSM state vector update from the ground causes both flags to be zeroed.

3. A LM state vector update from the ground causes RENDWFLG to be zeroed.

4. There exists a special DSKY verb by which the astronaut can zero both flags.

5. Initialization of the W matrix for rendezvous navigation causes ORBWFLAG to be zeroed and RENDWFLG to be set to one.

6. Initialization of the W matrix for orbit or cislunar-midcourse navigation causes RENDWFLG to be zeroed and ORBWFLAG to be set to one.

7. If a computation overflow occurs during extrapolation of the W matrix, a program alarm will result, both flags will be zeroed, and the extrapolation of the state vector will continue without the W matrix.

8. Selection of P-24 causes both flags to be zeroed.

9. RENDWFLG is zeroed during the automatic W matrix reinitialization procedure as described in Section 5.2.5.3.

With regard to items 5 and 6 above, there exist in erasable memory three sets of initialization parameters for the W matrix; one for rendezvous navigation, one for orbit navigation, and one for midcourse navigation. Each of these sets contains two elements, a position element and a velocity element. At the time each item of navigation data is processed, the appropriate W matrix control flag is tested. If the flag is found to be zero, then the W matrix is initialized consistent with the appropriate erasable parameters, and the flags are set as indicated in items 5 and 6. See Sections 5.2.4.5, 5.2.5.2.2, and 5.2.6.4 for precise details of this initialization procedure.

5.1.5.1.3 W Matrix Extrapolation

Extrapolation of the W matrix is described in Section 5.2.2.4. Required in this extrapolation is the specification of the appropriate vehicle's state vector with which the W matrix is extrapolated. This extrapolation occurs during programs P-00, P-20, P-22, and P-23; and at the conclusion of programs P-40, P-41, and P-47. The conventions under which the extrapolation occurs during each of these programs are as follows:

P-00	:	The W matrix is extrapolated with the CSM (LM) state vector if ORBWFLAG (RENDWFLG) is equal to one. The W matrix is not extrapolated if both flags are equal to zero. (See Section 5.6.12.)
P-20	:	The W matrix is extrapolated with the state vector that is being updated if RENDWFLG is equal to one, and not extrapolated if RENDWFLG is equal to zero. (See Section 5.2.5.2.2.)
P-22, P-23	:	The W matrix is extrapolated with the CSM state vector if ORBWFLAG is equal to 1, and not extrapolated if ORBWFLAG is equal to zero. (See Sections 5.2.4.5 and 5.2.6.4.)
P-11, P-40, P-41, P-47	:	The result of the maneuver will be a final state vector at the end-of-maneuver time t_F. The CSM state vector that existed before the maneuver program will still exist; and, cotemporal with it, there will also be the LM state vector and the W matrix. The following steps are performed after the Average-G Routine (Sec. 5.3.2) is terminated:

1. If either W matrix control flag is equal to one, the old CSM state vector and the W matrix are extrapolated to time t_F.

2. The CSM state vector is initialized to the end-of-maneuver state vector.

3. If SURFFLAG is zero, the LM state vector is extrapolated to time t_F. The switch SURFFLAG indicates whether or not the LM is on the surface of the moon. This flag is set to one (zero) by means of a special DSKY verb by the astronaut when he receives voice confirmation that the LM has landed on (lifted off from) the lunar surface.

If a computation overflow occurs during any of the above W matrix extrapolations, a program alarm will result, both W matrix control flags will be zeroed, and the extrapolation of the state vector will continue without the W matrix.

5.1.5.2 Altitude Parameter Convention

In the following programs and routines the display parameter of the vehicle altitude or trajectory pericenter or apocenter altitude is measured with respect to the earth launch pad radius magnitude, r_{LP}, in earth orbit cases, or the lunar landing site radius magnitude, r_{LS}, in lunar orbit cases. The earth launch pad radius parameter, r_{LP}, is stored in fixed memory, and the lunar landing site radius, r_{LS}, is the magnitude of the landing site vector, $\underline{r}_{LS}$, available in erasable memory.

P-11	Earth Orbit Injection Monitor Program	Section 5.3.5
P-21	Ground Track Determination Program	Section 5.6.5
P-30	External ΔV Maneuver Guidance	Section 5.3.3.3.1
P-32 & 72	Pre-CSI Maneuver	Section 5.4.2
P-34 & 74	TPI Pre-Thrust Programs	Section 5.4.2
R-30	Orbit Parameter Display Routine	Section 5.6.10.1

The earth and lunar landmark coordinates required in programs P-22 (Orbit Navigation Program) P-23 (Cislunar Navigation Program) and P-24 (Rate-Aided Optics Tracking) involve an altitude parameter referenced to the Fischer ellipsoid for earth landmarks, and the mean lunar radius, r_M, for lunar landmarks. The altitude of the landing site displayed in the IMU Realignment Program (P-52) or the Backup IMU Realignment Program (P-54) is also referenced to the mean lunar radius.

The 400,000 foot and EMS altitude parameters used in the Entry program P-61 (Section 5.6.10) are referenced to the Fischer ellipsoid. The entry altitude of 400,000 feet used in the Return to Earth Guidance Program P-37 is likewise referenced to the Fischer ellipsoid.

5.1.5.3 Lunar Landing Site Definition

The lunar landing site is maintained in the CMC as a vector, $\underline{r}_{LS}$, in the Moon Fixed Coordinate System of Section 5.1.4.5. This landing site vector is stored in erasable memory prior to earth launch and can be changed in lunar orbit either by the Orbital Navigation Program P-22 (Section 5.2.4) or by the RTCC uplink program P-27. The final landing site vector existing in the CSM prior to CSM-LM separation in lunar orbit is used to initialize the LM Guidance Computer (LGC) for the LM descent and landing phases. This landing site initialization vector is referred to as the most recently designated landing site in Section 4.

5.1.5.5 Time Definitions

The time t_0 is the time interval between July 1.0 universal time of the year in question (i.e., the Greenwich midnight at the beginning of July 1) and the time the Guidance Computer clock is zeroed.

The time t (also known as mission time or Ground Elapsed Time GET) is the time maintained by the Guidance Computer clock, and hence is the time interval between when this clock was zeroed and current time.

Shortly before launch, the mission time t is zeroed and t_0 is established utilizing the ground check-out system ACE and the uplink (Verb 55) and down link. Within 0.5 sec of the time the computer receives the lift-off discrete, mission time t is again zeroed and t_0 is incremented by the elapsed time since the last zeroing of mission time (see program P-11, Section 5.3.5). During a flight, the mission time t may be adjusted as a convenience for the flight controllers in order that events may occur at the mission time scheduled in the Flight Plan. The adjustment is performed by the use of Extended Verb 70; an increment of time is added or subtracted from mission time t, and is subtracted or added respectively to t_0.

The time t_M is that time used to interrogate the Planetary Inertial Orientation Subroutine and is defined in Section 5.5.2.

5.2 COASTING FLIGHT NAVIGATION

5.2.1 GENERAL COMMENTS

The CMC Coasting Flight Navigation Routines which are presented in Sections 5.2.2 through 5.2.6 are used during free fall phases of the Apollo mission. The basic objective of these navigation routines is to maintain estimates of the position and velocity vectors of both the CSM and the LM. Let $\underline{r}$ and $\underline{v}$ be the estimates of a vehicle's position and velocity vectors, respectively. Then, the six-dimensional state vector, $\underline{x}$, of the spacecraft is defined by

$$\underline{x} = \begin{pmatrix} \underline{r} \\ \underline{v} \end{pmatrix}$$

Coasting Flight Navigation is accomplished by extrapolating the state vector, $\underline{x}$, by means of the Coasting Integration Routine (Section 5.2.2), and updating or modifying this estimated state using tracking data by the recursive method of navigation (Sections 5.2.3 - 5.2.6).

The Coasting Integration Routine (Section 5.2.2) is used by other navigation and targeting routines to extrapolate the following:

1) Present estimated CSM state vector
2) Present estimated LM state vector
3) An arbitrary specified state vector, such as the predicted result of a maneuver

State vector extrapolation is accomplished by means of Encke's method of differential accelerations. The motion of a spacecraft is dominated by the conic orbit which would result if the spacecraft were in a central force field. In Encke's method the differential equations for the deviations from conic motion are integrated numerically. This technique is in contrast to a numerical integration of the differential equations for the total motion, and it provides a more accurate orbit extrapolation. The numerical integration is accomplished by means of Nystrom's method which gives fourth-order accuracy while requiring only three computations of the derivatives per time step. The usual fourth-order Runge-Kutta integration methods require four derivative computations per time step.

Regardless of the accuracy of the state vector extrapolation, errors in the initial conditions will propagate and soon grow to intolerable size. Thus, it is necessary periodically to obtain additional data in the form of either new state vector estimates or modifications to the current state vector estimates. These state vector modifications are computed from navigation data obtained by means of navigation measurements.

The CSM GNCS uses optical angle data from the scanning telescope (SCT) and the sextant (SXT) and VHF range data to compute state vector changes, while the LM PGNCS uses rendezvous radar (RR) tracking data. Navigation measurement data are used to update state vector estimates during orbit navigation, rendezvous navigation, and cislunar-midcourse navigation procedures. These three navigation procedures will be used normally during the lunar-orbit navigation phase, all LM-CSM lunar-orbit rendezvous phases, and CSM return-to-earth aborts, respectively, in the lunar landing mission. However, in order to provide for alternate mission capability, the orbit and rendezvous navigation procedures can be used near the moon or the earth.

Although the state vector of the CSM is six-dimensional, it is not necessary that the quantities estimated during a particular navigation procedure be the position and velocity vectors of the CSM. A variety of "estimated state vectors", not necessarily of six-dimensions, are used.

In order to achieve desired landing objectives, it is necessary to expand the lunar-orbit navigation procedure to nine dimensions, and to include in the estimation the position vector of the landmark being tracked. The estimated state vector that is used in orbit navigation is given by

$$\underline{x} = \begin{pmatrix} \underline{r}_C \\ \underline{v}_C \\ \underline{r}_\ell \end{pmatrix}$$

where $\underline{r}_C$ and $\underline{v}_C$ are the estimated CSM position and velocity vectors and $\underline{r}_\ell$ is the estimated landmark position vector.

During the rendezvous phase, the six-dimensional state vector of either the CSM or the LM can be updated from the measurement data obtained with the CSM-based optics. Normally the LM state vector is updated, but the astronaut can select the CSM update mode. The selection of the update mode is based primarily upon which vehicle's state vector is most accurately known initially, and which vehicle is controlling the rendezvous maneuvers.

The standard six-dimensional CSM state vector is used during cislunar-midcourse navigation.

Navigation data is incorporated into the state vector estimates by means of the Measurement Incorporation Routine (Section 5. 2. 3) which has both six- and nine-dimensional modes. The Measurement Incorporation Routine is a subroutine of the following CMC navigation routines:

1) Orbit Navigation Routine (Section 5. 2. 4)

2) Rendezvous Navigation Routine (Section 5. 2. 5)

3) Cislunar-Midcourse Navigation Routine (Section 5. 2. 6)

Simplified functional diagrams of the navigation programs which use these routines are given in Figs. 2. 1-1, 2. 1-2, and 2. 1-3, respectively.

In all three navigation programs, estimated position and velocity vectors are obtained at required times by means of the Coasting Integration Routine (Section 5. 2. 2). The Measurement Incorporation Routine (Section 5. 2. 3) is used to incorporate the measurement data into the state vector estimates.

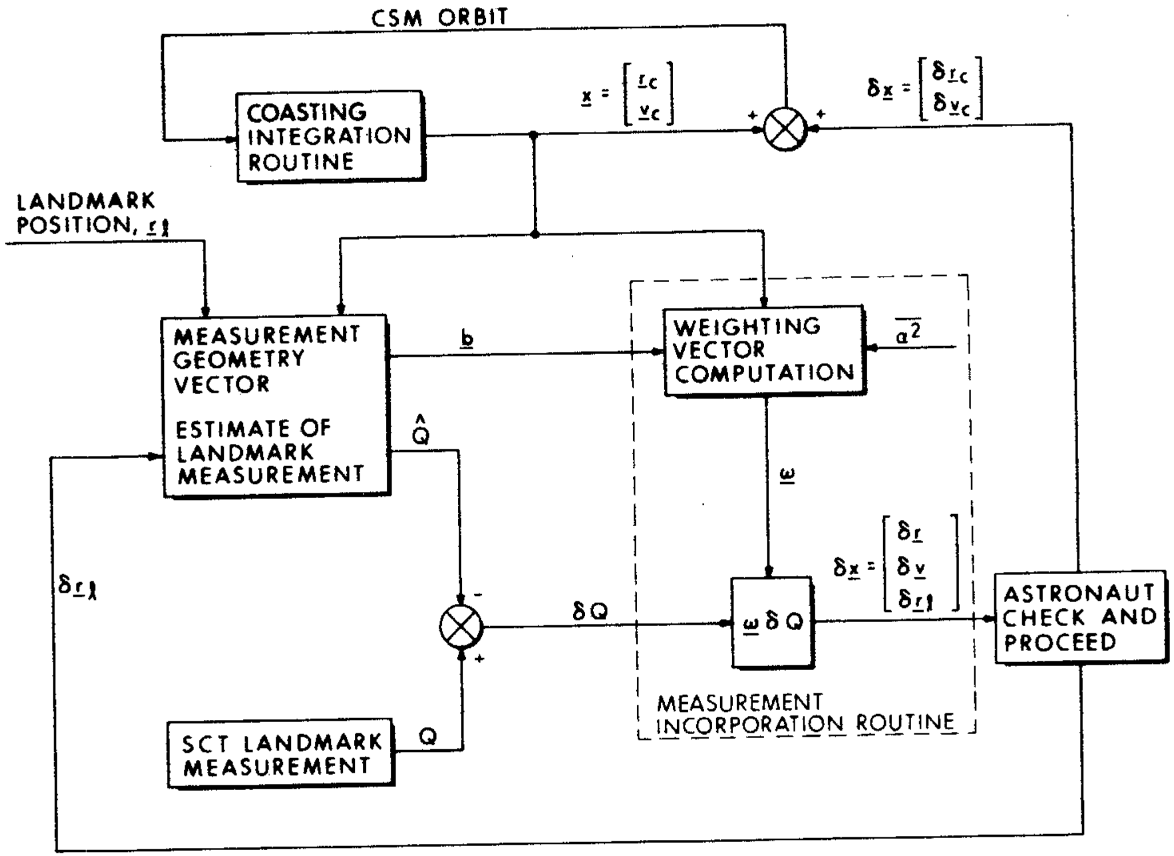

Fig. 2.1-1 Simplified Orbit Navigation Functional Diagram

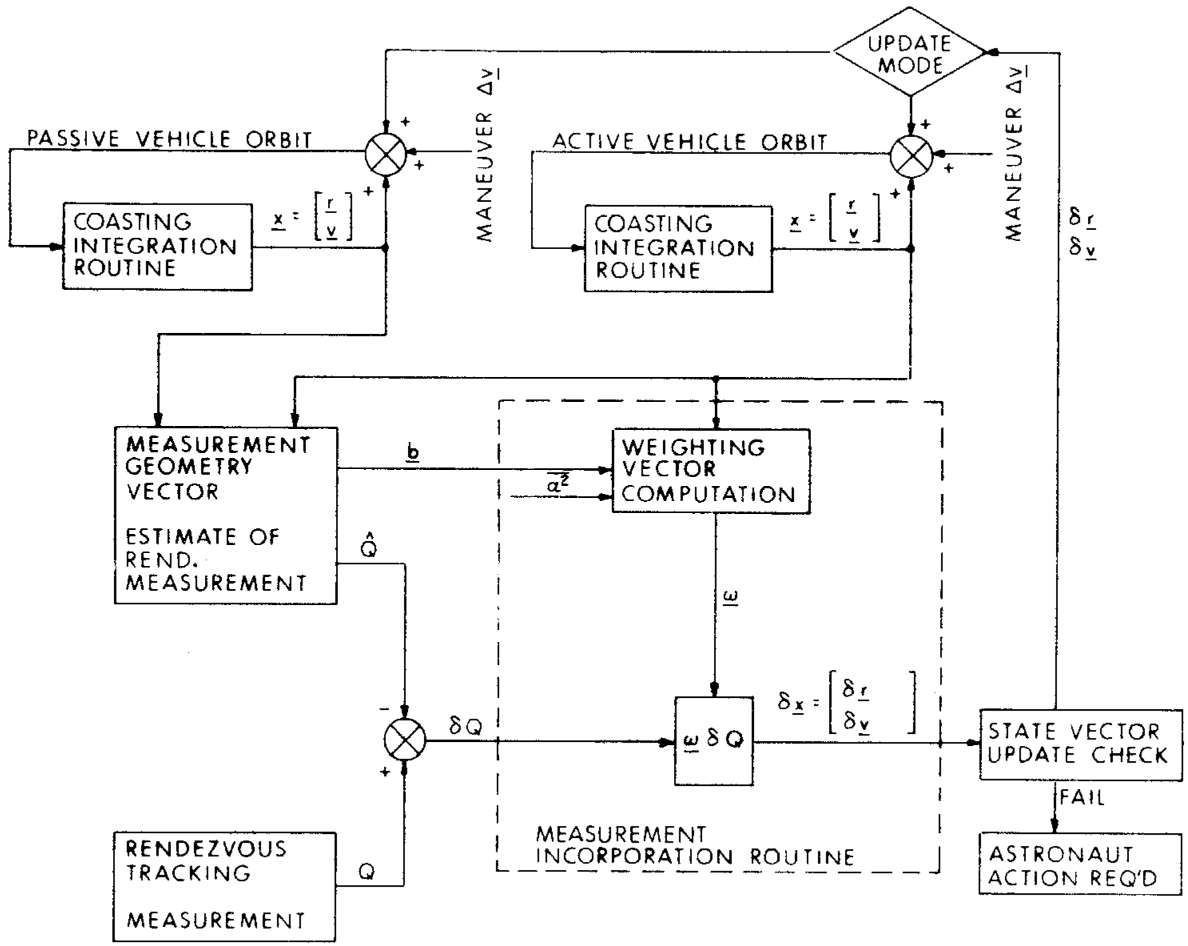

Fig. 2.1-2 Simplified CMC Rendezvous Navigation Functional Diagram

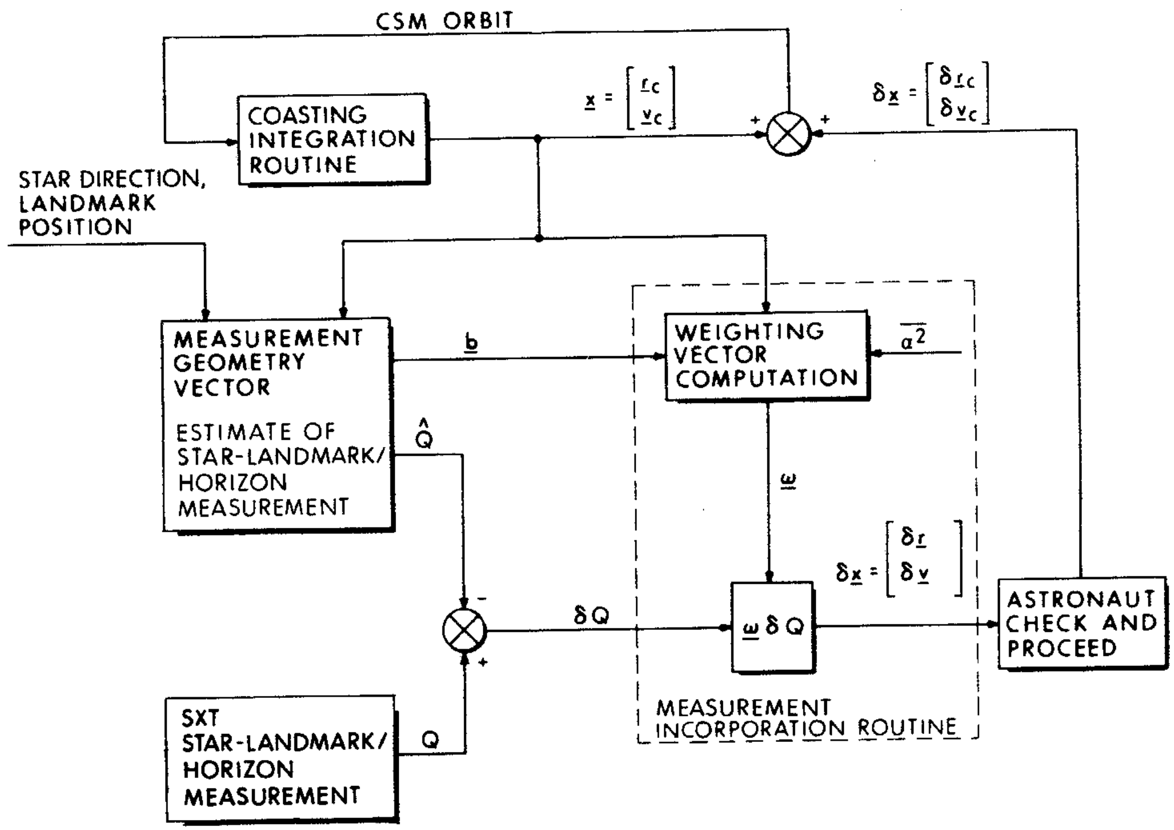

Fig. 2.1-3 Simplified Cislunar-Midcourse Navigation Functional Diagram

The navigation procedure, which is illustrated in simplified form in Figs. 2.1-1 to 2.1-3, involves computing an estimated tracking measurement, $\hat{Q}$, based on the current state vector estimates. This estimated measurement is then compared with the actual tracking measurement Q to form a measured deviation δQ. A statistical weighting vector, $\underline{\omega}$, is computed from statistical knowledge of state vector uncertainties and tracking performance, $\overline{\alpha^2}$, plus a geometry vector, $\underline{b}$, determined by the type of measurement being made. The weighting vector, $\underline{\omega}$, is defined such that a statistically optimum linear estimate of the deviation, $\delta \underline{x}$, from the estimated state vector is obtained when the weighting vector is multiplied by the measured deviation δQ. The vectors $\underline{\omega}$, $\underline{b}$ and $\delta \underline{x}$ are of six or nine dimensions depending upon the dimension of the state vector being estimated.

In an attempt to prevent unacceptably large incorrect state vector changes, certain validity tests have been included in the various CMC navigation routines.

In the Orbit Navigation Routine (Section 5.2.4) the astronaut tracks a landmark and acquires a number of sets of optical angle data before the state vector updating process begins. During the data processing procedure the landmark is out of sight, and it is not possible to repeat the tracking. Before the first set of data is used to update the estimated state vector, the magnitudes of the proposed changes in the estimated CSM position and velocity vectors, δr and δv, respectively, are displayed for astronaut approval. In general, successive accepted values of δr and δv will decrease during the processing of the tracking data associated with one landmark. Thus, if the MARK REJECT button has been used to erase all inaccurate marks, then all state vector updates should be either accepted or rejected. If the first displayed values of δr and δv are judged to be valid, then all data associated with that landmark will be accepted.

The actual values of the first displayed δr and δv will depend upon the statistical parameters stored in the CMC and upon the following types of errors:

Type 1: Errors in the current state vector estimates

Type 2: Errors in alignment of the IMU

Type 3: Reasonable tracking performance errors, including both hardware and astronaut errors

Type 4: A CSM GNCS failure

Type 5: Gross astronaut errors, such as incorrect identification of the landmark

The existence of Type 1 errors is precisely the reason that the landmark tracking is being done. It is the function of the navigation to decrease Type 1 errors in the presence of noise in the form of errors of Types 2 and 3. Since the landmark tracking should not be performed unless the IMU is well aligned and the GNCS is functioning properly, and since bad marks should be rejected, it follows that the purpose of the state vector change validity check is to discover a Type 5 error. This validity check cannot distinguish between a Type 4 error and a Type 5 error.

Based upon the last time that the state vector was updated, when the IMU last was realigned, and an estimate of the tracking performance for the first mark, very crude reasonable values for the first δr and δv can be generated by the astronaut. The CMC will provide no information to assist the astronaut in his estimates of reasonable values for δr and δv.

In the Rendezvous Navigation Routine (Section 5.2.5) measurement data is processed periodically, and it is desirable that the LM be tracked during the entire rendezvous phase up to the manual terminal maneuver. If the magnitudes of the changes in the estimated position and velocity vectors, δr and δv, respectively, are both less than preset tracking alarm levels, then the selected vehicle's state vector is automatically updated by the computed deviation, $\delta \underline{x}$, and no special display is presented, except that the tracking measurement counter is incremented by one. If either δr or δv exceeds its alarm level, then the state vector is not updated, and the astronaut is alerted to this condition by a special display of δr and δv. Included in this display is the source code which indicates whether optical or VHF range-link data caused the display.

If this display should occur because of optics data, then the astronaut should recheck the optical tracking and make sure that he is tracking the LM. Under certain conditions it is possible to mistake a star for LM reflected sunlight, and it may take a period of a few minutes to determine the LM target by watching relative motion of the target and star background. After the tracking has been verified, and navigation data has again been acquired, the astronaut has the option of commanding a state vector update if the tracking alarm is again exceeded, or of repeating further optical checks before incorporating the measurement data. If the astronaut cannot determine the LM target due to no positive acquisition (bright background, etc.) he can terminate the marking procedure and try to achieve tracking conditions at a later time.

The displayed values of δr and δv which have not passed the tracking alarm test will depend upon the statistical parameters stored in the CMC and upon the same five types of errors discussed previously in regard to orbit navigation. The tracking alarm criterion is incorporated in the Rendezvous Navigation Routine to alert the astronaut to the fact that the state vector update is larger than normally expected, and to prevent the estimated state vector from automatically being updated in such cases. The update occurs only by specific command of the astronaut. The tracking alarm level beyond which updating is suspended is primarily chosen to avoid false acquisition and tracking conditions. As previously mentioned, this condition is possible in the CSM if a star is optically tracked by mistake instead of the LM reflected sun light, and it is therefore possible for the alarm level to be exceeded in such cases even though the estimated state vectors are essentially correct. It is also possible for the state vector update alarm level to be exceeded after correct initial acquisition and tracking in the case where a poor estimate of either the CSM or LM state vector exists. In this case the astronaut would have to command the initial state vector update, after which the alarm level would seldom be exceeded during the remainder of the rendezvous phase. It should be noted that this statement is true only if the estimated state vector of the active vehicle performing a powered rendezvous maneuver is updated by the Average-G Routine in the case of the CSM being the active vehicle, or by a DSKY entry (P-76) of the maneuver $\Delta \underline{V}$ if the LM is the active vehicle (Section 5.6.14).

The previously discussed method which the astronaut can use to generate crude estimates of expected δr and δv values in the case of the Orbit Navigation Routine can also be applied to the Rendezvous Navigation Routine. The astronaut must decide whether or not he is tracking the LM. The CMC cannot make this decision.

In the Cislunar-Midcourse Navigation Routine (Section 5. 2. 6) the astronaut measures the angle between a star and a planetary landmark or horizon. The data from each angle measurement is processed immediately after it is made. The values of δr and δv are displayed for astronaut approval before the state vector is updated by the computed deviation $\delta \underline{x}$. Thus, it is a simple matter to repeat the measurement if the astronaut is uncertain as to the validity of the proposed state vector changes.

The parameters required to initialize the navigation routines (Sections 5. 2. 4 - 5. 2. 6) are the initial estimated CSM state vector, plus the initial estimated LM state vector for the Rendezvous Navigation Routine, initial state vector estimation error covariance matrices in the form of prestored diagonal error transition matrices (as defined in Section 5. 2. 2. 4), and a priori measurement error variances. The basic input to the navigation routines is SCT or SXT tracking angle data indicated to the CMC by the astronaut when he presses the MARK button signifying that he has centered the optical reticle on the tracking target (landmark or LM) or superimposed the two objects in the case of a star-landmark/horizon measurement,and automatically-acquired VHF range-link tracking data. The primary output of the navigation routines is the estimated CSM state vector, plus estimated landmark coordinates in the case of orbit navigation or the estimated LM state vector in the case of the Rendezvous Navigation Routine. The various guidance targeting modes outlined in Section5. 4 are based on the state vector estimates which result from these navigation routines.

5.2.2 COASTING INTEGRATION ROUTINE

5.2.2.1 General Comments

During all coasting phase navigation procedures, an extrapolation of position and velocity by numerical integration of the equations of motion is required. The basic equation may be written in the form

$$\frac{d^2}{dt^2} \underline{r}(t) + \frac{\mu_P^*}{r^3} \underline{r}(t) = \underline{a}_d(t) \tag{2.2.1}$$

where μ_P is the gravitational constant of the primary body, and $\underline{a}_d(t)$ is the vector acceleration which prevents the motion of the vehicle (CSM or LM) from being precisely a conic with focus at the center of the primary body. The Coasting Integration Routine is a precision integration routine in which all significant perturbation effects are included. The form of the disturbing acceleration $\underline{a}_d(t)$ depends on the phase of the mission.

An approximate extrapolation of a vehicle state vector in which the disturbing acceleration, $\underline{a}_d(t)$ of Eq. (2.2.1), is set to zero may be accomplished by means of the Kepler subroutine (Section 5.5.5).

5.2.2.2 Encke's Method

If $\underline{a}_d$ is small compared with the central force field, direct integration of Eq. (2.2.1) is inefficient. Therefore, the extrapolation will be accomplished using the technique of differential accelerations attributed to Encke.

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* In the remainder of Section 5.2 the subscripts P and Q will denote primary and secondary body, respectively. When the body is known, then the subscripts E, M, and S will be used for earth, moon, and sun, respectively. The vehicle will be indicated by the subscripts C for CSM and L for LM.

At time t_0 the position and velocity vectors, $\underline{r}_0$ and $\underline{v}_0$, define an osculating conic orbit. The position and velocity vectors in the conic orbit, $\underline{r}_{con}(t)$ and $\underline{v}_{con}(t)$, respectively, will deviate by a small amount from the actual position and velocity vectors.

The conic position and velocity at time t are computed as shown in Section 5.5.5. Required in this calculation is the variable x which is the root of Kepler's equation. In order to minimize the number of iterations required in solving Kepler's equation, an estimate of the correct solution for x is obtained as follows:

Let

$$\tau = t - t_0 \qquad (2.2.2)$$

During the previous computation cycle the values

$$\underline{r}' = \underline{r}_{con}\left(\tau - \frac{\Delta t}{2}\right)$$

$$\underline{v}' = \underline{v}_{con}\left(\tau - \frac{\Delta t}{2}\right) \qquad (2.2.3)$$

$$x' = x\left(\tau - \frac{\Delta t}{2}\right)$$

were computed. A trial value of $x(\tau)$ is obtained from

$$x_t = x' + s\left[1 - \gamma\, s\,(1 - 2\,\gamma\, s) - \frac{1}{6}\left(\frac{1}{r'} - \alpha\right) s^2\right] \qquad (2.2.4)$$

where

$$s = \frac{\sqrt{\mu_P}}{r'}\left(\frac{\Delta t}{2}\right)$$

$$\gamma = \frac{\underline{r}' \cdot \underline{v}'}{2r'\sqrt{\mu_P}} \qquad (2.2.5)$$

$$\alpha = \frac{2}{r'} - \frac{(v')^2}{\mu_P}$$

After specification of $\underline{r}_0$, $\underline{v}_0$, x_t and τ, the Kepler subroutine (Section 5.5.5) is used to compute $\underline{r}_{con}(\tau)$, $\underline{v}_{con}(\tau)$, and $x(\tau)$.

The true position and velocity vectors will deviate from the conic position and velocity since $\underline{a}_d$ is not zero. Let

$$\underline{r}(t) = \underline{\delta}(t) + \underline{r}_{con}(t)$$
$$\underline{v}(t) = \underline{\nu}(t) + \underline{v}_{con}(t) \qquad (2.2.6)$$

where $\underline{\delta}(t)$ and $\underline{\nu}(t)$ are the position and velocity deviations from the conic. The deviation vector $\underline{\delta}(t)$ satisfies the differential equation

$$\frac{d^2}{dt^2}\, \underline{\delta}(t) = - \frac{\mu_P}{r_{con}^3(t)} \left[f(q)\, \underline{r}(t) + \underline{\delta}(t) \right] + \underline{a}_d(t) \qquad (2.2.7)$$

subject to the initial conditions

$$\underline{\delta}(t_0) = \underline{0}, \quad \underline{\nu}(t_0) = \underline{0} \qquad (2.2.8)$$

where

$$q = \frac{(\underline{\delta} - 2\underline{r}) \cdot \underline{\delta}}{r^2} \qquad (2.2.9)$$

$$f(q) = q \frac{3 + 3q + q^2}{1 + (1 + q)^{3/2}} \qquad (2.2.10)$$

The first term on the right-hand side of Eq. (2.2.7) must remain small, i.e., of the same order as $\underline{a}_d(t)$, if the method is to be efficient. As the deviation vector $\underline{\delta}(t)$ grows in magnitude, this term will eventually increase in size. Therefore, in order to maintain the efficiency of the method, a new osculating conic orbit should be defined by the total position and velocity vectors $\underline{r}(t)$ and $\underline{v}(t)$. The process of selecting a new conic orbit from which to calculate deviations is called rectification. When rectification occurs, the initial conditions for the differential equation for $\underline{\delta}(t)$, as well as the variables τ and x, are again zero.

5.2.2.3 Disturbing Acceleration

The form of the disturbing acceleration $\underline{a}_d(t)$ that is used in Eq. (2.2.1) depends on the phase of the mission. In earth or lunar orbit, only the gravitational perturbations arising from the non-spherical shape of the primary body need be considered. Let $\underline{a}_{dP}$ be the acceleration due to the non-spherical gravitational perturbations of the primary body. Then, for the earth

$$\underline{a}_{dE} = \frac{\mu_E}{r^2} \sum_{i=2}^{4} J_{iE} \left(\frac{r_E}{r}\right)^i \left[P'_{i+1} (\cos\varphi)\, \underline{u}_r - P'_i (\cos\varphi)\, \underline{u}_z \right] \qquad (2.2.11)$$

where

$$P_2' (\cos\varphi) = 3 \cos\varphi$$

$$P_3' (\cos\varphi) = \frac{1}{2} (15 \cos^2\varphi - 3)$$

$$P_4' (\cos\varphi) = \frac{1}{3} (7 \cos\varphi \, P_3' - 4P_2') \qquad (2.2.12)$$

$$P_5' (\cos\varphi) = \frac{1}{4} (9 \cos\varphi \, P_4' - 5P_3')$$

are the derivatives of Legendre polynomials,

$$\cos\varphi = \underline{u}_r \cdot \underline{u}_z ,$$

$$\text{and} \quad \underline{u}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ as an approximation} \qquad (2.2.13)$$

and J_2, J_3, J_4 are the coefficients of the second, third, and fourth harmonics of the earth's potential function. The vectors $\underline{u}_r$ and $\underline{u}_z$ are unit vectors in the direction of $\underline{r}$ and the polar axis of the earth, respectively, and r_E is the equatorial radius of the earth.

In the case of the moon,

$$\underline{a}_{dM} = \frac{\mu_M}{r^2} \left\{ \sum_{i=2}^{4} J_{iM} \left(\frac{r_M}{r}\right)^i \left[P_{i+1}' (\cos\varphi)\underline{u}_r - P_i' (\cos\varphi)\underline{u}_z \right] \right.$$

$$+ 3J_{22} \left(\frac{r_M}{r}\right)^2 \left[\frac{-5(x_M^2 - y_M^2)}{r^2} \underline{u}_r + \frac{2x_M}{r} \underline{u}_x - \frac{2y_M}{r} \underline{u}_y \right]$$

$$\left. + \frac{3}{2} C_{31} \left(\frac{r_M}{r}\right)^3 \left[\frac{5x_M}{r} (1 - 7\cos^2\varphi)\underline{u}_r + (5\cos^2\varphi - 1)\underline{u}_x + \frac{10x_M z_M}{r^2} \underline{u}_z \right] \right\}$$

where:

$\underline{u}_r$ is the unit position vector in reference coordinates;

$\underline{u}_x$ is planetary $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ transformed to reference coordinates;

$\underline{u}_y$ is planetary $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ transformed to reference coordinates;

$\underline{u}_z$ is planetary $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ transformed to reference coordinates;

and x_M, y_M, z_M are the components of $\underline{r}$ in planetary coordinates, which are computed by the use of the Planetary Inertial Orientation Subroutine (Section 5.5.2). In addition, r_M is the mean lunar radius; and J_{22}, C_{31} are the coefficients of the terms describing the asymmetry about the pole of the moon's gravity; and the remaining symbols are defined as in Eq. (2.2.11).

During cislunar-midcourse flight (translunar and transearth) the gravitational attraction of the sun and the secondary body Q (earth or moon) are relevant forces. The accelerations due to the secondary body and the sun are

$$\underline{a}_{dQ} = - \frac{\mu_Q}{r_{QC}^3} \left[f(q_Q) \underline{r}_{PQ} + \underline{r} \right] \tag{2.2.15}$$

$$\underline{a}_{dS} = - \frac{\mu_S}{r_{SC}^3} \left[f(q_S) \underline{r}_{PS} + \underline{r} \right] \tag{2.2.16}$$

where $\underline{r}_{PQ}$ and $\underline{r}_{PS}$ are the position vectors of the secondary body and the sun with respect to the primary body, r_{QC} and r_{SC} are the distances of the CSM from the secondary body and the sun, and the arguments q_Q and q_S are computed from

$$q_Q = \frac{(\underline{r} - 2\underline{r}_{PQ}) \cdot \underline{r}}{r_{PQ}^2} \tag{2.2.17}$$

$$q_S = \frac{(\underline{r} - 2\underline{r}_{PS}) \cdot \underline{r}}{r_{PS}^2} \tag{2.2.18}$$

The functions $f(q_Q)$ and $f(q_S)$ are calculated from Eq. (2.2.10).

The position vectors of the moon relative to the earth, $\underline{r}_{EM}$, and the sun relative to the earth, $\underline{r}_{ES}$, are computed as described in Section (5.5.4). Then,

$$\underline{r}_{PQ} = \begin{cases} \underline{r}_{EM} & \text{if } P = E \\ -\underline{r}_{EM} & \text{if } P = M \end{cases} \tag{2.2.19}$$

and

$$\underline{r}_{PS} = \begin{cases} \underline{r}_{ES} & \text{if } P = E \\ \underline{r}_{ES} - \underline{r}_{EM} & \text{if } P = M \end{cases} \tag{2.2.20}$$

Finally,

$$\begin{aligned} \underline{r}_{QC} &= \underline{r} - \underline{r}_{PQ} \\ \underline{r}_{SC} &= \underline{r} - \underline{r}_{PS} \end{aligned} \tag{2.2.21}$$

5.2.2.4 Error Transition Matrix

The position and velocity vectors as maintained in the computer are only estimates of the true values. As part of the navigation technique it is necessary also to maintain statistical data in the computer to aid in the processing of navigation measurements.

If $\underline{\epsilon}(t)$ and $\underline{\eta}(t)$ are the errors in the estimates of the position and velocity vectors, respectively, then the six-dimensional correlation matrix E(t) is defined by

$$E_6(t) = \begin{pmatrix} \overline{\underline{\epsilon}(t)\ \underline{\epsilon}(t)^T} & \overline{\underline{\epsilon}(t)\ \underline{\eta}(t)^T} \\ \overline{\underline{\eta}(t)\ \underline{\epsilon}(t)^T} & \overline{\underline{\eta}(t)\ \underline{\eta}(t)^T} \end{pmatrix} \qquad (2.2.22)$$

In certain applications it becomes necessary to expand the state vector and the correlation matrix to more than six dimensions so as to include estimation of landmark locations in the CMC during orbit navigation, and rendezvous radar tracking biases in the LGC during the rendezvous navigation procedure. For this purpose a nine-dimensional correlation matrix is defined as follows :

$$E(t) = \begin{pmatrix} \multicolumn{2}{c}{E_6(t)} & \begin{matrix} \overline{\underline{\epsilon}(t)\ \underline{\beta}^T} \\ \overline{\underline{\eta}(t)\ \underline{\beta}^T} \end{matrix} \\ \overline{\underline{\beta}\ \underline{\epsilon}(t)^T} & \overline{\underline{\beta}\ \underline{\eta}(t)^T} & \overline{\underline{\beta}\ \underline{\beta}^T} \end{pmatrix} \qquad (2.2.23)$$

where the components of the three-dimensional vector $\underline{\beta}$ are the errors in the estimates of three variables which are estimated in addition to the components of the spacecraft state vector.

In order to take full advantage of the operations provided by the interpreter in the computer, the correlation matrix will be restricted to either six or nine dimensions. If, in some navigation procedure, only one or two additional items are to be estimated, then a sufficient number of dummy variables will be added to the desired seven-or eight-dimensional state vector to make it nine-dimensional.

Rather than use the correlation matrix in the navigation procedure, it is more convenient to utilize a matrix W(t), called the error transition matrix, and defined by

$$E(t) = W(t)\, W(t)^T \tag{2.2.24}$$

Extrapolation of the nine-dimensional matrix W(t) is made by direct numerical integration of the differential equation

$$\frac{d}{dt} W(t) = \begin{pmatrix} O & I & O \\ G(t) & O & O \\ O & O & O \end{pmatrix} W(t) \tag{2.2.25}$$

where G(t) is the three-dimensional gravity gradient matrix, and I and O are the three-dimensional identity and zero matrices, respectively. If the W matrix is partitioned as

$$W = \begin{pmatrix} \underline{w}_0 & \underline{w}_1 & \cdots & \underline{w}_8 \\ \underline{w}_9 & \underline{w}_{10} & \cdots & \underline{w}_{17} \\ \underline{w}_{18} & \underline{w}_{19} & \cdots & \underline{w}_{26} \end{pmatrix} \tag{2.2.26}$$

then,

$$\left.\begin{aligned} \frac{d}{dt}\,\underline{w}_i(t) &= \underline{w}_{i+9}(t) \\ \frac{d}{dt}\,\underline{w}_{i+9}(t) &= G(t)\,\underline{w}_i(t) \\ \frac{d}{dt}\,\underline{w}_{i+18}(t) &= \underline{0} \end{aligned}\right\} \quad i = 0,\ 1, \ldots, 8 \qquad (2.2.27)$$

The extrapolation may be accomplished by successively integrating the vector differential equations

$$\frac{d^2}{dt^2}\,\underline{w}_i(t) = G(t)\,\underline{w}_i(t) \qquad i = 0, 1, \ldots, 8 \qquad (2.2.28)$$

The gravity gradient matrix G(t) for earth or lunar orbit is given by

$$G(t) = \frac{\mu_P}{r^5(t)}\left[3\,\underline{r}(t)\,\underline{r}(t)^T - r^2(t)\,I\right] \qquad (2.2.29)$$

During cislunar-midcourse flight

$$G(t) = \frac{\mu_P}{r^5(t)}\left[3\,\underline{r}(t)\,\underline{r}(t)^T - r^2(t)\,I\right] + \frac{\mu_Q}{r^5{}_{QC}(t)}\left[3\,\underline{r}_{QC}(t)\,\underline{r}_{QC}(t)^T - r^2{}_{QC}(t)\,I\right] \qquad (2.2.30)$$

Thus, if D is the dimension of the matrix W(t) for the given navigation procedure, the differential equations for the $\underline{w}_i(t)$ vectors are

$$\frac{d^2}{dt^2}\underline{w}_i(t) = \frac{\mu_P}{r^3(t)}\left\{3\left[\underline{u}_r(t)\cdot\underline{w}_i(t)\right]\underline{u}_r(t) - \underline{w}_i(t)\right\} + M\frac{\mu_Q}{r^3_{QC}(t)}\left\{3\left[\underline{u}_{QC}(t)\cdot\underline{w}_i(t)\right]\underline{u}_{QC}(t) - \underline{w}_i(t)\right\} \qquad i = 0, 1, \ldots, D-1 \tag{2.2.31}$$

where $\underline{u}_r(t)$ and $\underline{u}_{QC}(t)$ are unit vectors in the directions of $\underline{r}(t)$ and $\underline{r}_{QC}(t)$, respectively, and

$$M = \begin{cases} 1 \text{ for cislunar-midcourse flight} \\ 0 \text{ for earth or lunar orbit} \end{cases} \tag{2.2.32}$$

It is possible for a computation overflow to occur during the W matrix integration if any element of the matrix exceeds its maximum value. This event is extremely unlikely because of the large scale factors chosen. The overflow occurs if

1) any element of the position part (upper third) of the W matrix becomes equal to or greater than 2^{19} m,

or

2) any element of the velocity part (middle third) of the W matrix becomes equal to or greater than one m/csec.

In addition, each element of the landmark part (lower third) of the matrix must remain less than 2^{19} m, but this part does not change during integration.

If overflow should occur, an alarm results, the two W matrix control flags are reset, and either new state vector estimates must be obtained from RTCC, or a sufficient number of navigation measurements must be made before the state vectors are used in any targeting or maneuver programs.

5.2.2.5 Numerical Integration Method

The extrapolation of navigational data requires the solution of a number of second-order vector differential equations, specifically Eqs. (2.2.7) and (2.2.31). These are all special cases of the form

$$\frac{d^2}{dt^2} \underline{y} = \underline{f}(\underline{y}, t) \tag{2.2.33}$$

Nystrom's method is particularly well suited to this form and gives an integration method of fourth-order accuracy. The second-order system is written

$$\frac{d}{dt} \underline{y} = \underline{z}$$

$$(2.2.34)$$

$$\frac{d}{dt} \underline{z} = \underline{f}(\underline{y}, t)$$

and the formulas are summarized below.

$$\underline{y}_{n+1} = \underline{y}_n + \underline{\phi}(\underline{y}_n) \Delta t$$

$$\underline{z}_{n+1} = \underline{z}_n + \underline{\psi}(\underline{y}_n) \Delta t$$

$$\underline{\phi}(\underline{y}_n) = \underline{z}_n + \frac{1}{6}(\underline{k}_1 + 2\underline{k}_2) \Delta t$$

$$(2.2.35)$$

$$\underline{\psi}(\underline{y}_n) = \frac{1}{6}(\underline{k}_1 + 4\underline{k}_2 + \underline{k}_3)$$

$$\underline{k}_1 = \underline{f}(\underline{y}_n, t_n)$$

$$\underline{k}_2 = \underline{f}(\underline{y}_n + \frac{1}{2} \underline{z}_n \Delta t + \frac{1}{8} \underline{k}_1 (\Delta t)^2, \quad t_n + \frac{1}{2} \Delta t)$$

$$\underline{k}_3 = \underline{f}(\underline{y}_n + \underline{z}_n \Delta t + \frac{1}{2} \underline{k}_2 (\Delta t)^2, \quad t_n + \Delta t)$$

For efficient use of computer storage as well as computing time the computations are performed in the following order:

1) Equation (2.2.7) is solved using the Nystrom formulas, Eq. (2.2.35). It is necessary to preserve the values of the vector $\underline{r}$ at times t_n, $t_n + \Delta t/2$, $t_n + \Delta t$ for use in the solution of Eqs. (2.2.31).

2) Equations (2.2.31) are solved one-at-a-time using Eqs. (2.2.35) together with the values of $\underline{r}$ which resulted from the first step.

The variable Δt is the integration time step and should not be confused with τ, the time since rectification. The maximum value for Δt which can be used for precision integration, Δt_{max}, is computed from

$$\Delta t_{max} = \text{minimum} \left(\Delta t_{\ell im}, \frac{K r_{con}^{3/2}}{\sqrt{\mu_P}} \right) \qquad (2.2.36)$$

where

$$\Delta t_{\ell im} = 4000 \text{ sec.}$$
$$K = 0.3 \qquad (2.2.37)$$

5.2.2.6 Coasting Integration Logic

Estimates of the state vectors of two vehicles (CSM and LM) will be maintained in the computer. In various phases of the mission it will be required to extrapolate a state vector either alone or with an associated W matrix of dimension six or nine.

To accomplish all of these possible procedures, as well as to solve the computer restart problem, three state vectors will be maintained in the computer. Let $\underline{x}_C$ and $\underline{x}_L$ be the estimated CSM and LM state vectors, respectively, and let $\underline{x}$ be a temporary state vector. The state vector $\underline{x}$ is a symbolic representation of the following set of variables:

$$
\begin{aligned}
\underline{r}_0 &= \text{rectification position vector} \\
\underline{v}_0 &= \text{rectification velocity vector} \\
\underline{r}_{con} &= \text{conic position vector} \\
\underline{v}_{con} &= \text{conic velocity vector} \\
\underline{\delta} &= \text{position deviation vector} \\
\underline{\nu} &= \text{velocity deviation vector} \\
t &= \text{time associated with } \underline{r}_{con}, \underline{v}_{con}, \underline{\delta} \text{ and } \underline{\nu} \\
\tau &= \text{time since rectification} \\
x &= \text{root of Kepler's equation} \\
P &= \text{primary body} = \begin{cases} 0 \text{ for earth} \\ 1 \text{ for moon} \end{cases}
\end{aligned}
\qquad (2.2.38)
$$

The state vectors $\underline{x}_C$ and $\underline{x}_L$ represent an analogous set of variables.

The Coasting Integration Routine is controlled by the calling program by means of the two indicators D and V. The variable D indicates the dimension of the W matrix with

$$D = 0 \qquad (2.2.39)$$

denoting that the state vector only is to be extrapolated. The variable V indicates the appropriate vehicle as follows:

$$V = \begin{cases} 1 \text{ for CSM} \\ 0 \text{ for LM} \\ -1 \text{ for state vector specified by calling program} \end{cases} \qquad (2.2.40)$$

In addition, the calling program must set the desired final time t_F; and, for V equal to -1, the desired state vector $\underline{x}$.

A simplified functional diagram of the Coasting Integration Routine is shown in Fig. 2.2-1. In the figure the indicated state vector is being integrated to time t_F. The value of Δt for each time step is Δt_{max} (Eq. (2.2.36)) or the total time-to-go whichever is smaller. The integration is terminated when the computed value of Δt is less than ϵ_t.

Figures 2.2-2, 2.2-3 and 2.2-4 illustrate in more detail the logic flow of this routine. In these figures certain items which have not been discussed fully in the text are explicitly illustrated. The following is a list of these items together with the number of the figure in which each occurs.

1) Saving of $\underline{r}$ values for W matrix integration: Fig. 2.2-2.

2) Change in origin of coordinates: Fig. 2.2-3.

3) Rectification procedure: Fig. 2.2-3.

4) Selection of disturbing acceleration: Fig. 2.2-4.

The logic flow shown in these figures is controlled by the three flags M, B, and F. Flag M is defined in Eq. (2.2.32), B prevents the recalculation of already available quantities ($\underline{r}_{PQ}$, $\underline{r}_{QC}$), and F is used to distinguish between state vector integration (F = 1) and W matrix integration (F = 0).

If the Coasting Integration Routine is requested to extrapolate the estimated LM state vector and the LM is on the lunar surface, the normal integration will not be performed. Instead, the LM position and velocity vectors ($\underline{r}_L$ and $\underline{v}_L$) are obtained by transforming the stored landing site position vector $\underline{r}_{LS}$ and the vector (0, 0, 1) from moon-fixed to basic reference coordinates with the Planetary Inertial Orientation Subroutine (Section 5.5.2) where $\underline{r}_L$ and $\underline{u}_Z$ are

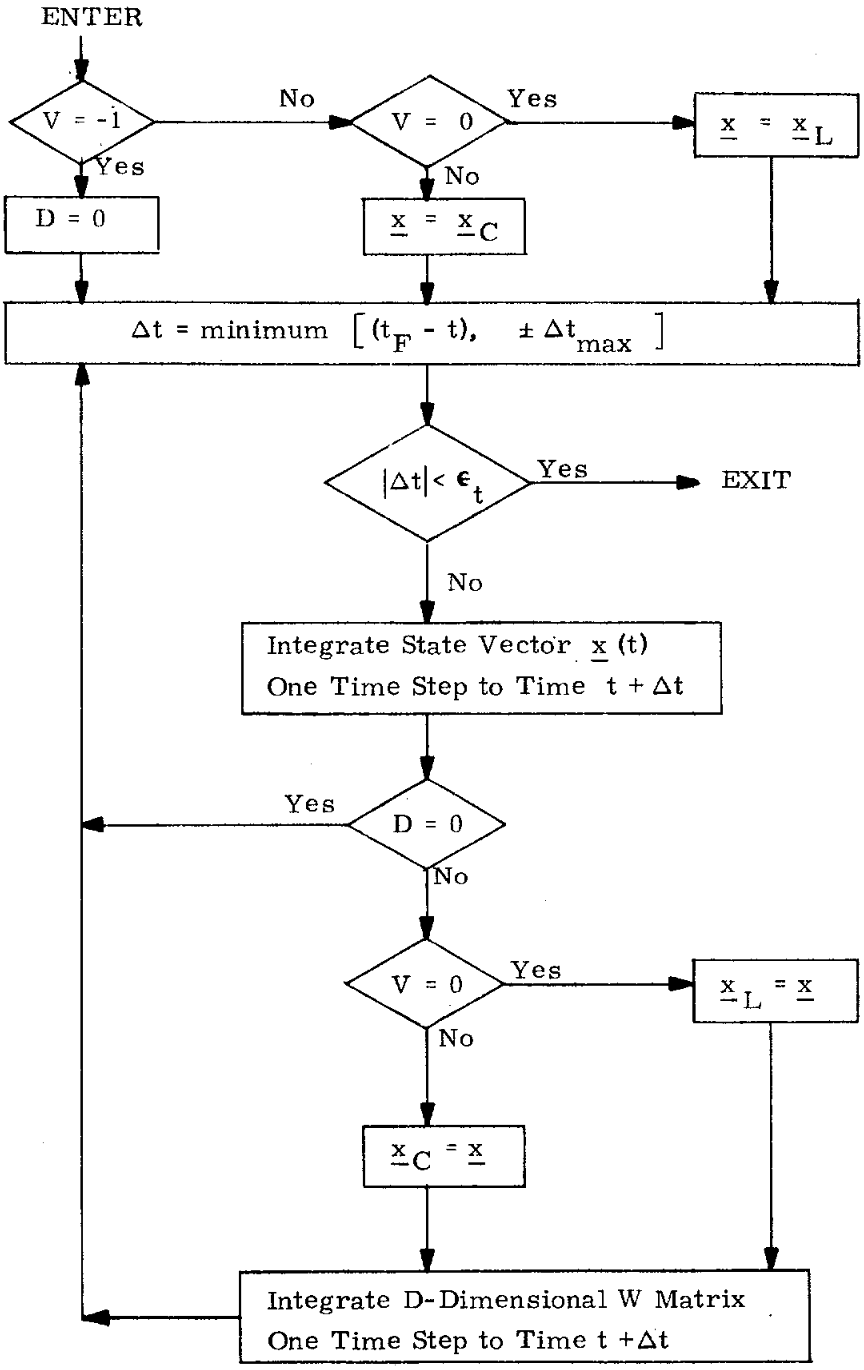


Figure 2. 2-1 Simplified Coasting Integration Routine Logic Diagram

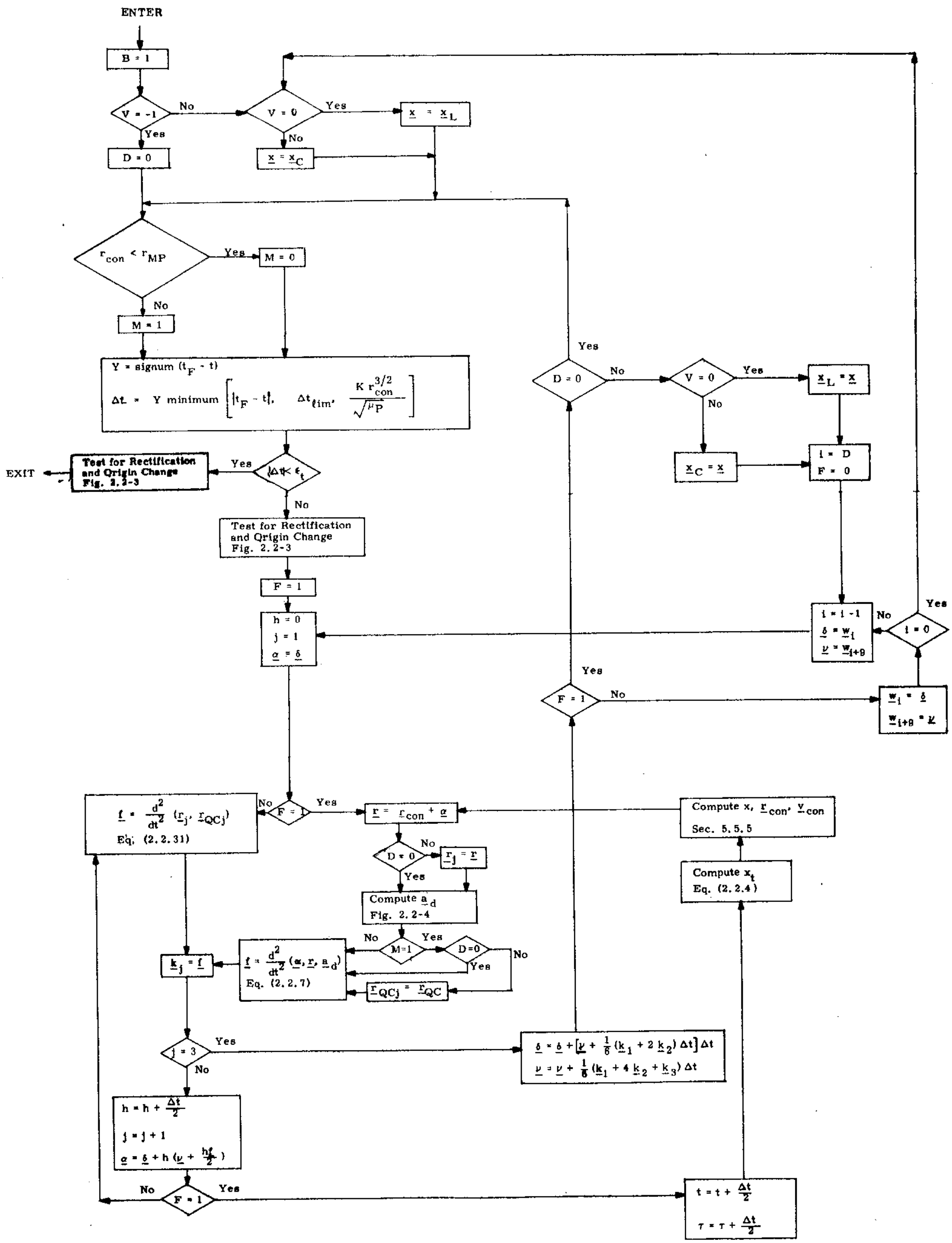


Figure 2.2-2 Coasting Integration Routine

Logic Diagram

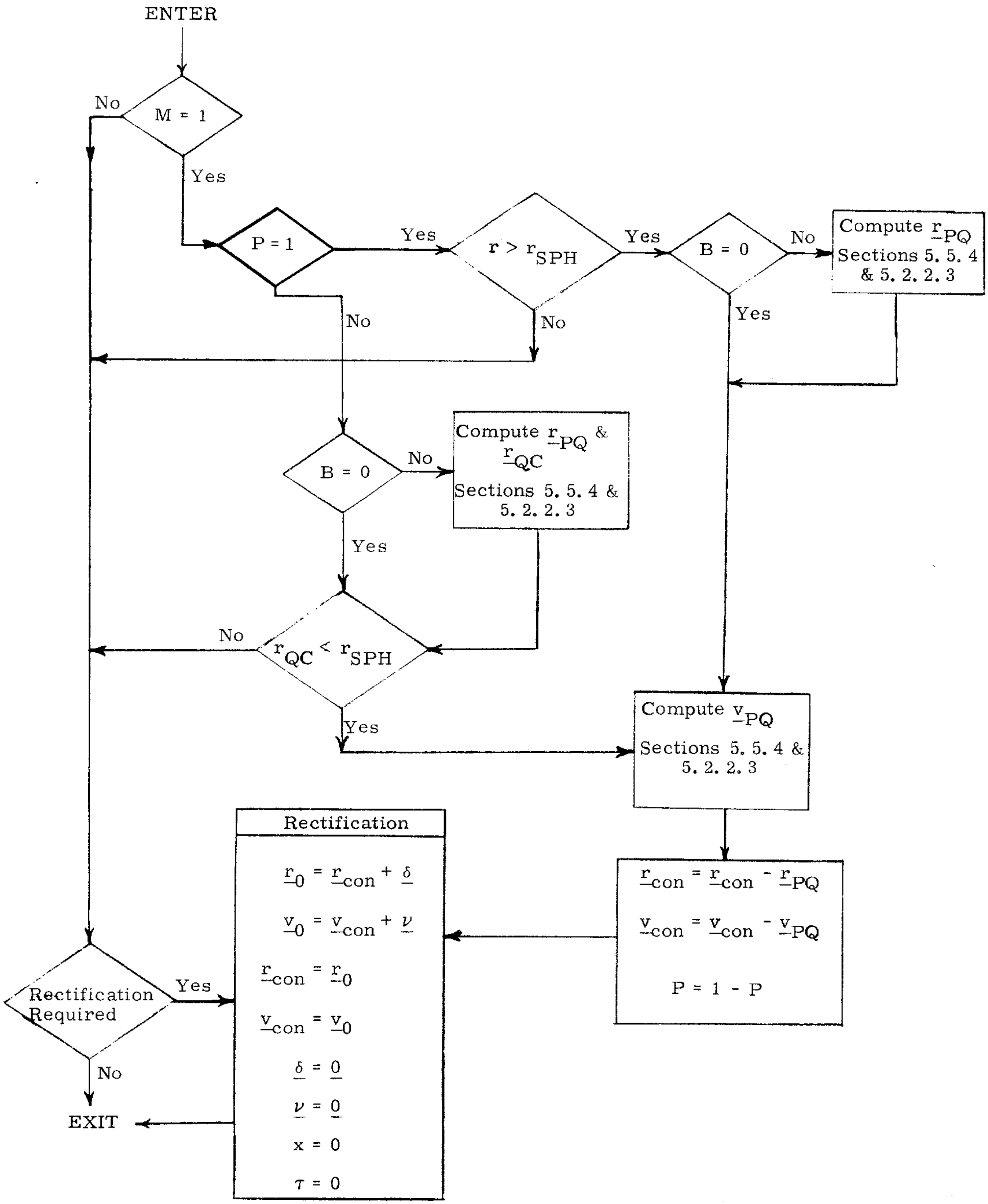


Figure 2.2-3 Rectification and Coordinate System Origin Change Logic Diagram

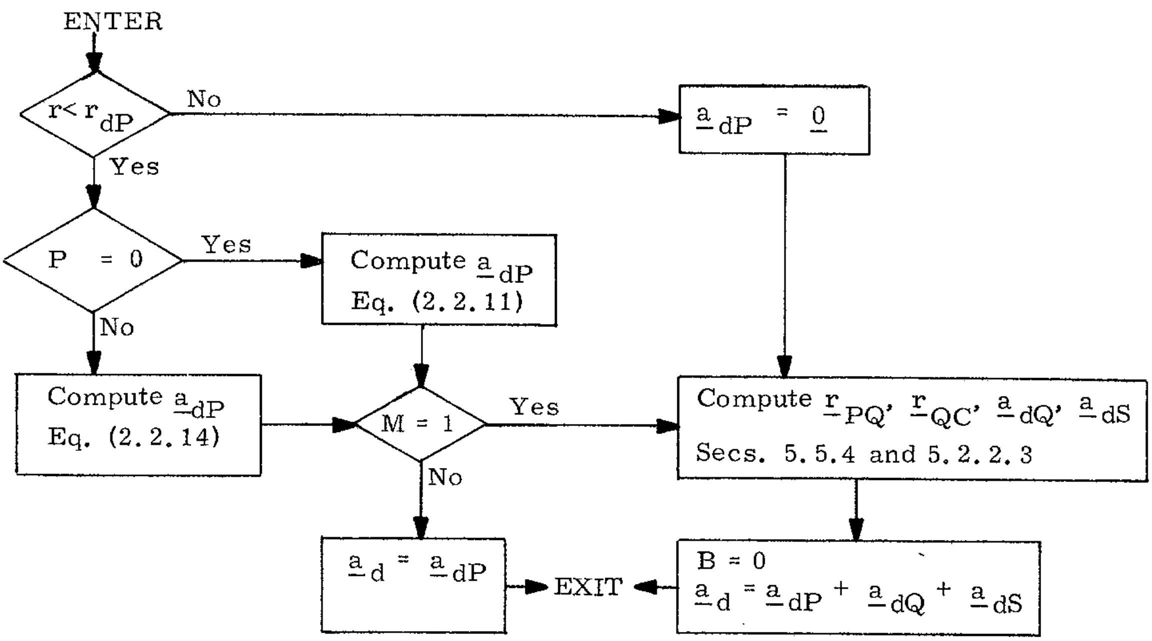


Figure 2.2-4 Disturbing Acceleration Selection Logic Diagram

the respective vectors in basic reference coordinates, and $\underline{v}_L$ is computed as follows:

$$\underline{v}_L = \omega_M \underline{u}_Z \times \underline{r}_L$$

where ω_M is the rotational rate of the moon with respect to inertial space. This procedure is not indicated in the figure.

There is a procedure for the emergency termination of the Coasting Integration Routine in order to permit correction of wrong erasable memory parameters. This emergency function is described in Section 5.6.12.

In addition to the general criterion discussed in Section 5.2.2.2, the requirements for rectification (which are not shown in Fig. 2.2-2) are functions of

1) the computer word length,
2) the fact that the computations are performed in fixed-point arithmetic,
3) the scale factors of the variables, and
4) the accuracy of the Kepler Subroutine (Section 5.5.5).

If

$$\frac{\delta}{r_{con}} > 0.01$$

or if

$$\delta > \begin{cases} 0.75 \times 2^{22} \text{m for } P = 0 \\ 0.75 \times 2^{18} \text{m for } P = 1 \end{cases}$$

or if

$$\nu > \begin{cases} 0.75 \times 2^{3} \text{ m/csec for } P = 0 \\ 0.75 \times 2^{-1} \text{m/csec for } P = 1 \end{cases}$$

then rectification occurs at the point indicated in Fig. 2.2-2. Also, if the calculation of the acceleration (Eq. (2.2.7)) results in overflow (i.e. any component is equal to or greater than 2^{-16} m/csec^2 for P = 0, or 2^{-20} m/csec^2 for P = 1), then the program is recycled to the beginning of the time step and rectification is performed, provided that δ is not identically zero (which may occur if an attempt is made to extrapolate a state vector below the surface). In this exceptional case, an abort occurs with alarm code 20430.

The definitions of the various control constants which appear in Figs. 2.2-1 to 2.2-4 are as follows:

ϵ_t = integration time step criterion

r_{SPH} = radius of lunar sphere of influence

r_{dE} = radius of relevance for earth non-spherical gravitational perturbations

r_{dM} = radius of relevance for moon non-spherical gravitational perturbations

r_{ME} = distance from the earth beyond which mid-course perturbations are relevant.

r_{MM} = distance from the moon beyond which mid-course perturbations are relevant.

5.2.3 MEASUREMENT INCORPORATION ROUTINE

Periodically it is necessary to update the estimated position and velocity vectors of the vehicle (CSM or LM) by means of navigation measurements. At the time a measurement is made, the best estimate of the state vector of the spacecraft is the extrapolated estimate denoted by $\underline{x}'$. The first six components of $\underline{x}'$ are the components of the estimated position and velocity vectors. In certain situations it becomes necessary to estimate more than six quantities. Then, the state vector will be of nine dimensions. From this state vector estimate it is possible to determine an estimate of the quantity measured. When the predicted value of this measurement is compared with the actual measured quantity, the difference is used to update the indicated state vector as well as its associated error transition matrix as described in Section 5.2.1. The error transition matrix, W, is defined in Section 5.2.2.4.

This routine is used to compute deviations to be added to the components of the estimated state vector, and to update the estimated state vector by these deviations provided the deviations pass a state vector update validity test as described in Section 5.2.1.

Let D be the dimension (six or nine) of the estimated state vector. Associated with each measurement are the following parameters which are to be specified by the program calling this routine:

$\underline{b}$ = Geometry vector of D dimensions

$\overline{\alpha^2}$ = A priori measurement error variance

δQ = Measured deviation, the difference between the quantity actually measured and the expected value based on the original value of the estimated state vector $\underline{x}'$.

The procedure for incorporating a measurement into the estimated state vector is as follows:

(1) Compute a D-dimensional $\underline{z}$ vector from

$$\underline{z} = W'^T \underline{b} \tag{2.3.1}$$

where W' is the error transition matrix associated with $\underline{x}'$.

(2) Compute the D-dimensional weighting vector, $\underline{\omega}$, from

$$\underline{\omega}^T = \frac{1}{z^2 + \overline{\alpha^2}} \underline{z}^T W'^T \tag{2.3.2}$$

(3) Compute the state vector deviation estimates from

$$\delta \underline{x} = \underline{\omega} \, \delta Q \tag{2.3.3}$$

(4) If the data pass the validity test, update the state vector and the W matrix by

$$\underline{x} = \underline{x}' + \delta \underline{x} \tag{2.3.4}$$

$$W = W' - \frac{\underline{\omega}\, \underline{z}^T}{1 + \sqrt{\dfrac{\overline{\alpha^2}}{z^2 + \overline{\alpha^2}}}} \tag{2.3.5}$$

In order to take full advantage of the three-dimensional vector and matrix operations provided by the interpreter in the computer, the nine-dimensional W matrix will be stored sequentially in the computer as follows:

$$\underline{w}_0, \underline{w}_1, \ldots, \underline{w}_{26}$$

Refer to Section 5. 2. 2. 4 for the definition of the W matrix. Define the three-dimensional matrices

$$W_0 = \begin{pmatrix} \underline{w}_0^T \\ \underline{w}_1^T \\ \underline{w}_2^T \end{pmatrix} \qquad W_1 = \begin{pmatrix} \underline{w}_3^T \\ \underline{w}_4^T \\ \underline{w}_5^T \end{pmatrix} \qquad \cdots\cdot \qquad W_8 = \begin{pmatrix} \underline{w}_{24}^T \\ \underline{w}_{25}^T \\ \underline{w}_{26}^T \end{pmatrix} \tag{2.3.6}$$

so that

$$W = \begin{pmatrix} W_0^T & W_1^T & W_2^T \\ W_3^T & W_4^T & W_5^T \\ W_6^T & W_7^T & W_8^T \end{pmatrix} \tag{2.3.7}$$

Let the nine-dimensional vectors $\delta\underline{x}$, $\underline{b}$, $\underline{\omega}$, and $\underline{z}$ be partitioned as follows:

$$\delta\underline{x} = \begin{pmatrix} \delta\underline{x}_0 \\ \delta\underline{x}_1 \\ \delta\underline{x}_2 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} \underline{b}_0 \\ \underline{b}_1 \\ \underline{b}_2 \end{pmatrix} \qquad \underline{\omega} = \begin{pmatrix} \underline{\omega}_0 \\ \underline{\omega}_1 \\ \underline{\omega}_2 \end{pmatrix} \qquad \underline{z} = \begin{pmatrix} z_0 \\ z_1 \\ \vdots \\ z_8 \end{pmatrix} = \begin{pmatrix} \underline{z}_0 \\ \underline{z}_1 \\ \underline{z}_2 \end{pmatrix}$$

(2. 3. 8)

Then, the computations shown in Eqs. (2.3.1) through (2.3.3) are performed as follows, using three-dimensional operations:

$$\underline{z}_i = \sum_{j=0}^{\frac{D}{3}-1} W'_{i+3j}\, \underline{b}_j$$

$$a = \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j \cdot \underline{z}_j + \overline{\alpha^2}$$

$$\underline{\omega}_i^T = \frac{1}{a} \sum_{j=0}^{\frac{D}{3}-1} \underline{z}_j^T\, W'_{3i+j} \qquad (2.3.9)$$

$$\delta \underline{x}_i = \delta Q\, \underline{\omega}_i \qquad \left(i = 0, 1, \ldots, \frac{D}{3} - 1\right)$$

Equation (2.3.5) is written

$$\gamma = \frac{1}{1 + \sqrt{\overline{\alpha^2}/a}} \qquad (2.3.10)$$

$$\underline{w}_{i+9j} = \underline{w}'_{i+9j} - \gamma\, z_i\, \underline{\omega}_j \qquad \left(\begin{matrix} i = 0, 1, \ldots, D - 1 \\ j = 0, 1, \ldots, \frac{D}{3} - 1 \end{matrix}\right)$$

The Measurement Incorporation Routine is divided into two subroutines, INCORP1 and INCORP2. The subroutine INCORP1 consists of Eqs. (2. 3. 9), while INCORP2 is composed of Eqs. (2. 3. 4) and (2. 3. 10). The method of using these subroutines is illustrated in Fig. 2. 3-1.

Since the estimated position and velocity vectors are maintained in two pieces, conic and deviation from the conic, Eq. (2. 3. 4) cannot be applied directly. The estimated position and velocity deviations resulting from the measurement, $\delta\underline{x}_0$ and $\delta\underline{x}_1$, are added to the vectors $\underline{\delta}$ and $\underline{\nu}$, the position and velocity deviations from the conics, respectively. Since $\underline{\delta}$ and $\underline{\nu}$ are maintained to much higher accuracy than the conic position and velocity vectors, a possible computation overflow situation exists whenever Eq. (2. 3. 4) is applied. If overflow does occur, then it is necessary to reinitialize the Coasting Integration Routine (Section 5. 2. 2) by the process of rectification as described in Section 5. 2. 2. 2. The logic flow of the subroutine INCORP2 is illustrated in detail in Fig. 2. 3-2.

Overflow occurs when

$$\delta \geq \begin{cases} 2^{22} \text{ m for } P = 0 \\ 2^{18} \text{ m for } P = 1 \end{cases}$$

or

$$\nu \geq \begin{cases} 2^{3} \text{ m/csec for } P = 0 \\ 2^{-1} \text{ m/csec for } P = 1 \end{cases}$$

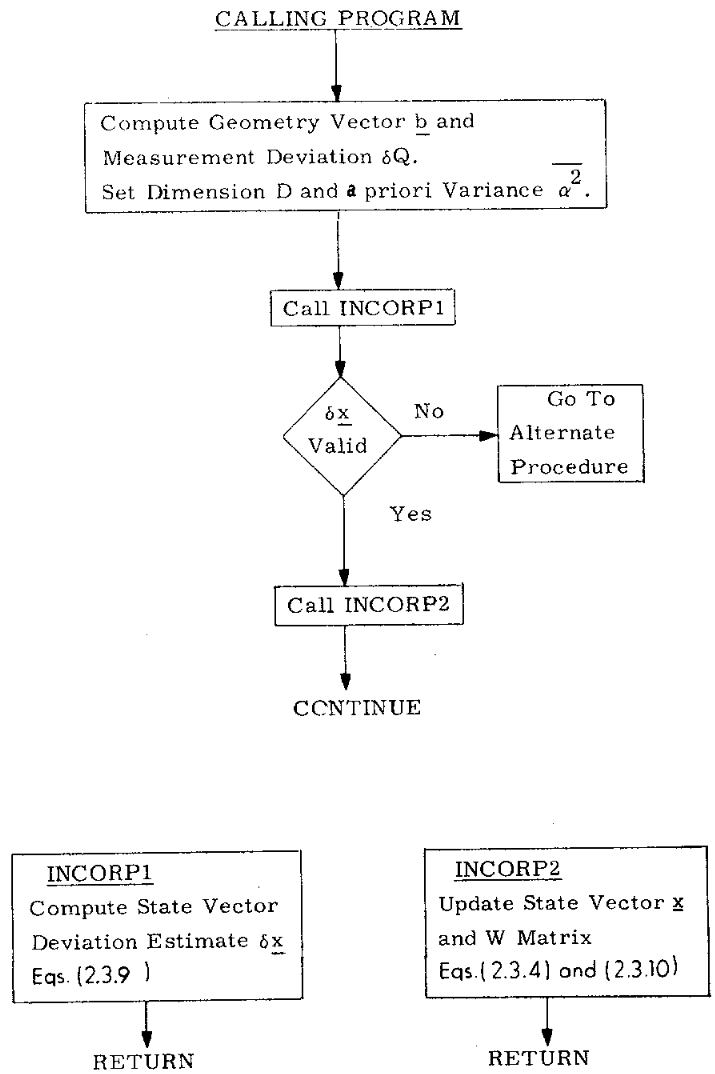


Fig. 2.3-1 Measurement Incorporation Procedure

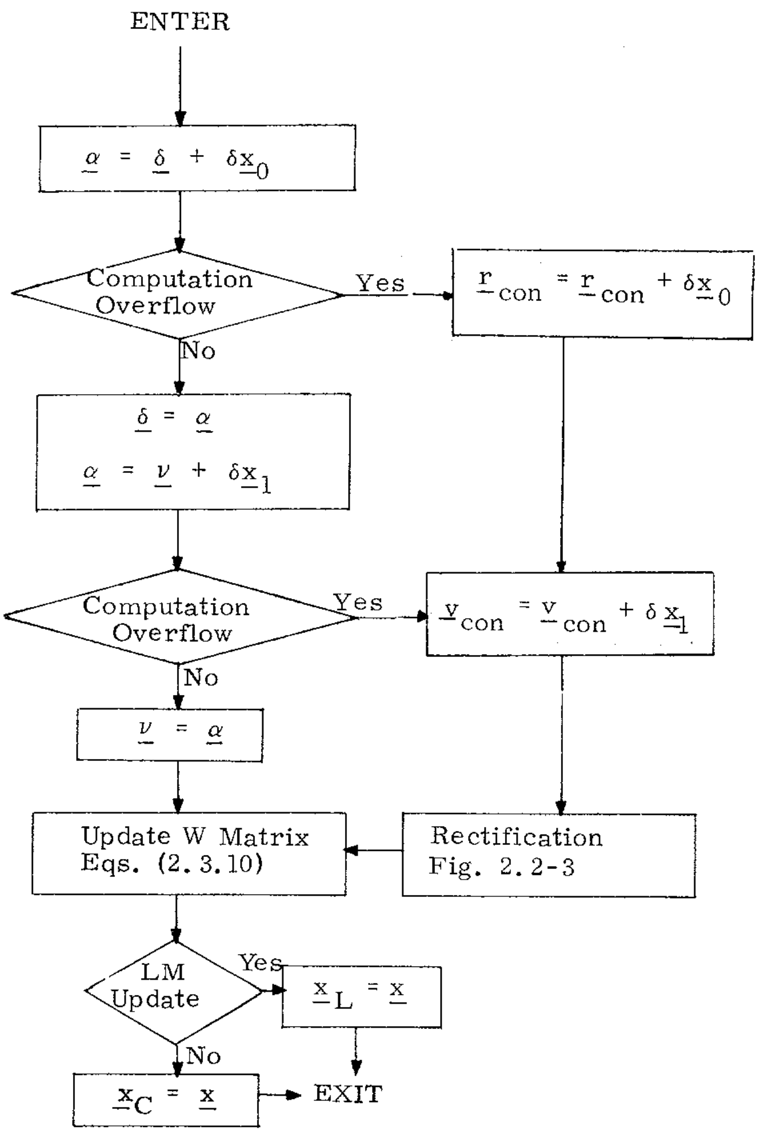


Fig. 2.3-2 INCORP2 Subroutine Logic Diagram

5.2.4 ORBIT NAVIGATION ROUTINE

5.2.4.1 Landmark Tracking Procedure

While the CSM is in lunar or earth orbit, landmark optical tracking data are used to update the estimated CSM state vector and the coordinates of the landmark that is being tracked, as described in Section 5. 2. 1. This routine is used to process the landmark-tracking measurement data, as shown in simplified form in Fig. 2. 1-1, and is used normally in lunar orbit in the lunar landing mission. The routine also can be used in earth orbit during abort situations or alternate missions.

In order to initially acquire and maintain optical tracking, the CSM attitude must be oriented such that the CSM-to-landmark line-of-sight falls within the SCT field of view. In the CSM GNCS there is no automatic vehicle attitude control during the landmark tracking procedure. Any desired attitude control must be accomplished manually by the astronaut or by use of the Barbecue Mode Routine (R64).

If the astronaut wishes, he may use the Automatic Optics Positioning Routine (Section 5. 6. 8) as an aid in the acquisition of the landmark. This routine has two modes which are relevant to orbit navigation. In the advanced ground track mode (which is useful in lunar orbit for surveillance, selection, and tracking of possible landing sites) the routine drives the CSM optics to the direction of the point on the ground track of the spacecraft at a time slightly more than a specified number of orbital revolutions ahead of current time.

In the landmark mode (which is useful for acquisition of a specified landmark) the routine drives the optics to the estimated direction of the specified landmark. Either the revolution number or the landmark must be specified by the astronaut. The computations and positioning commands in this routine are repeated periodically provided the optics mode switch is set to CMC. Thus, in the advanced ground track mode, the astronaut is shown continuously the ground track of the CSM for a future revolution. The reason for this mode is that it is desirable to select a landing site which is near the CSM orbital plane at the LM lunar landing time.

The Automatic Optics Positioning Routine is used in other routines to align the CSM optics to the directions of the following sighting targets:

1) The LM during the rendezvous phase

2) A specified star during IMU alignment procedures

After the astronaut has acquired the desired landmark (not necessarily the one specified to the Automatic Optics Positioning Routine), he switches the optics mode to MANUAL and centers the SCT or SXT reticle on the landmark. When accurate tracking is achieved, he presses the optics MARK button, causing the time of the measurement and all optics and IMU gimbal angles to be stored in the CMC. Up to five unrejected navigation sightings of the same landmark may be made during the tracking interval,and all sets of navigation data are acquired before processing of the data begins.

After the astronaut has completed the tracking of a landmark, he is asked by the CMC whether or not he wishes to identify the tracked landmark. If he does, then he enters into the CMC through the keyboard the identification number or the coordinates of the landmark, and the data are processed as described in Section 5. 2. 4. 2 thru 5. 2. 4. 5. The coordinates of the landing site will be stored in the CMC erasable memory and can be updated during the mission.

If the astronaut does not identify the landmark, then the Landing Site Designation procedure (Section 5.2.4.3) is used for the navigation data processing. In this process the landmark is considered to be unknown, and the first set of navigation data is used to compute an initial estimate of the landmark location. The remaining sets of data are then processed as described in Section 5. 2. 4. 2 to update the estimated nine-dimensional CSM-landmark state vector.

Whether the landmark is identified or not, one further option is available to the astronaut. He may specify that one of the navigation sightings is to be considered the designator for an offset landing site near the tracked landmark. In this case, the designated navigation data set is saved, the remaining sets of data are processed as described above, and then the estimated offset landing site location is determined from the saved data as described in Section 5. 2. 4. 4. This procedure offers the possibility of designating a landing site in a flat area of the moon near a landmark which is suitable for optical navigation tracking but not for landing.

Each set of navigation data which is used for state vector updating and not for landing site designation or offset produces two updates as described in Section 5. 2. 4. 2. For the first navigation data set the magnitudes of the first proposed changes in the estimated CSM position and velocity vectors, δr and δv, respectively, are displayed for astronaut approval. If the astronaut accepts these proposed changes, then all state vector updates will be performed, and all the information obtained during the tracking of this landmark will be incorporated into the state vector estimates. A detailed discussion of this state vector update validity check is given in Section 5. 2. 1.

After all of the sets of navigation data have been processed, the astronaut has the option of having the updated landmark coordinates (or the coordinates of the unknown landmark) stored in the erasable memory registers allocated to the landing site coordinates. In this manner the original coordinates of the landing site can be revised, or a new landing site can be selected.

The various functions for which the sets of navigation data acquired from the line-of-sight tracking measurements are used are presented in Sections 5.2.4.2 through 5.2.4.4. A detailed description in the form of logic diagrams of the entire Orbit Navigation Routine with all of its options is given in Section 5.2.4.5.

5.2.4.2 State Vector Update from Landmark Sighting

As mentioned in Section 5. 2. 1 the orbit navigation concept involves the nine-dimensional state vector

$$\underline{x} = \begin{pmatrix} \underline{r}_C \\ \underline{v}_C \\ \underline{r}_\ell \end{pmatrix} \qquad (2.4.1)$$

where $\underline{r}_C$ and $\underline{v}_C$ are the estimated CSM position and velocity vectors, respectively, and $\underline{r}_\ell$ is the estimated landmark position vector. Both the CSM state vector and the landmark position vector are estimated and updated through the processing of optical tracking data. A simplified functional diagram of the orbit navigation procedure is illustrated in Fig. 2. 1-1. In this section the method of updating the estimated nine-dimensional state vector from a landmark line-of-sight navigation measurement is given.

After the preferred CSM attitude is achieved and optical tracking acquisition is established (Section 5. 2. 4. 1), the astronaut enters tracking data into the CMC by pressing the optics MARK button when he has centered the SCT or SXT reticle on the landmark. As described in Section 5. 2. 4. 1, each set of navigation data contains the time of the measurement and the two optics and three IMU gimbal angles. From these five angles the measured unit vector, $\underline{u}_M$, along the CSM-to-landmark line-of-sight is computed in the Basic Reference Coordinate System from

$$\underline{u}_M = [\text{REFSMMAT}]^T \, [\text{NBSM}] \, \underline{u}_{NB} \qquad (2.\,4.\,2)$$

where [REFSMMAT] and [NBSM] are transformation matrices and $\underline{u}_{NB}$ is the measured line-of-sight vector in navigation base coordinates. All terms of Eq. (2. 4. 2) are defined in Section 5. 6. 3.

For the purpose of navigation it is convenient to consider the measured unit vector, $\underline{u}_M$, to be the basic navigation data. This navigation measurement of the line-of-sight vector, $\underline{u}_M$, is mathematically equivalent to the simultaneous measurement of the angles between the lines-of-sight to the landmark and two stars. The data are processed by selecting two convenient unit vectors (fictitious star directions), converting the vector $\underline{u}_M$ to an equivalent set of two artificial star-landmark measurements, and using the Measurement Incorporation Routine (Section 5. 2. 3) twice, once for each artificial measurement. These two unit vectors are chosen to be perpendicular to each other and to the current estimated line-of-sight vector so as to maximize the convenience and accuracy of the procedure.

Let $\underline{r}_C$ and $\underline{r}_\ell$ be the estimated CSM and landmark position vectors at the time of a given line-of-sight measurement. Then, the first state vector update for the measurement is performed as follows:

(1) Calculate the estimated CSM-to-landmark line-of-sight from

$$\underline{r}_{CL} = \underline{r}_\ell - \underline{r}_C \qquad (2.4.3)$$

$$\underline{u}_{CL} = \text{UNIT}\,(\underline{r}_{CL})$$

(2) Initialize the fictitious star direction to the vector

$$\underline{u}_s = \text{UNIT}\,(\underline{u}_{CL} \times \underline{u}_M) \qquad (2.4.4)$$

If the vectors $\underline{u}_{CL}$ and $\underline{u}_M$ are separated by a small enough angle, (possibly as much as $\sqrt{3} \times 2^{-19}$ rad.), then a computation overflow occurs in the execution of Eq. (2.4.4), and this set of measurement data is discarded because all of the components of the estimated state vector deviation, $\delta\underline{x}$, would be negligible for both state vector updates.

(3) Compute an artificial star direction from

$$\underline{u}_s = \text{UNIT}(\underline{u}_s \times \underline{u}_{CL}) \tag{2.4.5}$$

(4) Calculate the nine-dimensional geometry vector, $\underline{b}$, from

$$\underline{b}_0 = \frac{1}{r_{CL}} \underline{u}_s \tag{2.4.6}$$

$$\underline{b}_1 = \underline{0} \tag{2.4.7}$$

$$\underline{b}_2 = - \underline{b}_0 \tag{2.4.8}$$

(5) Determine the measured deviation, δQ, from

$$\delta Q = \cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \cos^{-1}(\underline{u}_s \cdot \underline{u}_{CL})$$

(2.4.9)

$$= \cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2}$$

(6) Incorporate the fictitious star-landmark measurement using the Measurement Incorporation Routine (Section 5.2.3).

Included in Step (6) is the state vector update validity check for the first proposed update.

It should be noted that the initialization of the star direction, $\underline{u}_s$, which is given by Eq. (2. 4. 4), is such that the first artificial star (computed from Eq. (2. 4. 5)) will yield the maximum value for the measured deviation, δQ, which is obtained from Eq. (2. 4. 9). The reason for selecting the first $\underline{u}_s$ vector in this manner is that there is only one state vector update validity check even though there are two updates.

Assuming that the first state vector update was accepted by the astronaut, the second update for this measurement data set is performed by first recomputing the estimated CSM-to-landmark line-of-sight vector from Eq. (2. 4. 3) using the updated values of the estimated CSM and landmark position vectors, $\underline{r}_C$ and $\underline{r}_\ell$, respectively. Then, Steps (3) - (6) are repeated, this time with no state vector update validity check.

If the astronaut rejects the first state vector update, then all of the navigation data is discarded, and no update occurs.

The results of the processing of the measured line-of-sight vector, $\underline{u}_M$, are updated values of the estimated position and velocity vectors of the CSM, $\underline{r}_C$ and $\underline{v}_C$, respectively, and an updated value of the estimated landmark position vector, $\underline{r}_\ell$.

5.2.4.3 Landing Site Designation

As mentioned in Section 5. 2. 4. 1 the nine-dimensional orbit navigation procedure provides the means of mapping on the surface of the planet a point which is designated only by a number of sets of optical tracking data. This process may be used to redesignate the landing site optically, or as an unknown landmark orbit navigation procedure.

Assume that an unmapped landmark has been tracked, and N sets of optical measurement data have been acquired as described in Section 5. 2. 4. 1. Let $\underline{u}_M$ be the measured unit CSM-to-landmark line-of-sight vector obtained from the first set of measurement data by means of Eq. (2. 4. 2). An estimate of the landmark position at the time of the first navigation sighting, t_M, is given by

$$\underline{r}_\ell = \underline{r}_C + r_C \left[\cos A - \left(\frac{r_0^{\,2}}{r_C^{\,2}} - \sin^2 A \right)^{1/2} \right] \underline{u}_M \quad (2.\,4.\,10)$$

where

$$\cos A = - \frac{\underline{u}_M \cdot \underline{r}_C}{r_C} \quad (2.\,4.\,11)$$

$\underline{r}_C$ is the estimated CSM position vector at time t_M, and r_0 is the estimated planetary radius. This initial estimated landmark position vector, $\underline{r}_\ell$, and the estimated CSM state vector are then updated by means of the standard Orbit Navigation Routine and the last N - 1 sets of tracking measurement data exactly as if the designated point were a mapped landmark.

The final results of this procedure are a location estimate for the designated point and an improvement in the estimated CSM state vector.

5.2.4.4 Landing Site Offset

During the landing site selection operation any visible landmark may be tracked that is in, or near, the desired landing area. In most cases this visible landmark will not be an acceptable touch down point, and it is desirable to offset the desired landing point away from the visible landmark used for tracking. This is accomplished by tracking the visible landmark and processing this data as previously described for either a mapped or an unknown landmark depending upon the type of landmark tracked. During this tracking operation a designated navigation data set can be taken by positioning the SXT to the desired actual landing point. This designated data set is saved, and, after the tracking data is processed for the visible landmark, the offset landing site location is computed from the saved data by means of Eq. (2.4.10). In this landing site offset calculation, the magnitude of the estimated position vector of the visible landmark is used for the estimated planetary radius r_0 in Eq. (2.4.10).

5.2.4.5 Orbit Navigation Logic

After all optical landmark tracking data have been acquired, the data processing procedure is initialized as illustrated in Fig. 2.4-1. It is assumed that the following items are stored in erasable memory at the start of the procedure shown in this figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5.2.2.6.

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ as defined in Section 5.2.2.4.

ORBWFLAG = $\begin{cases} 1 \text{ for valid W matrix} \\ 0 \text{ for invalid W matrix} \end{cases}$

This flag or switch is maintained by programs external to the Orbit.

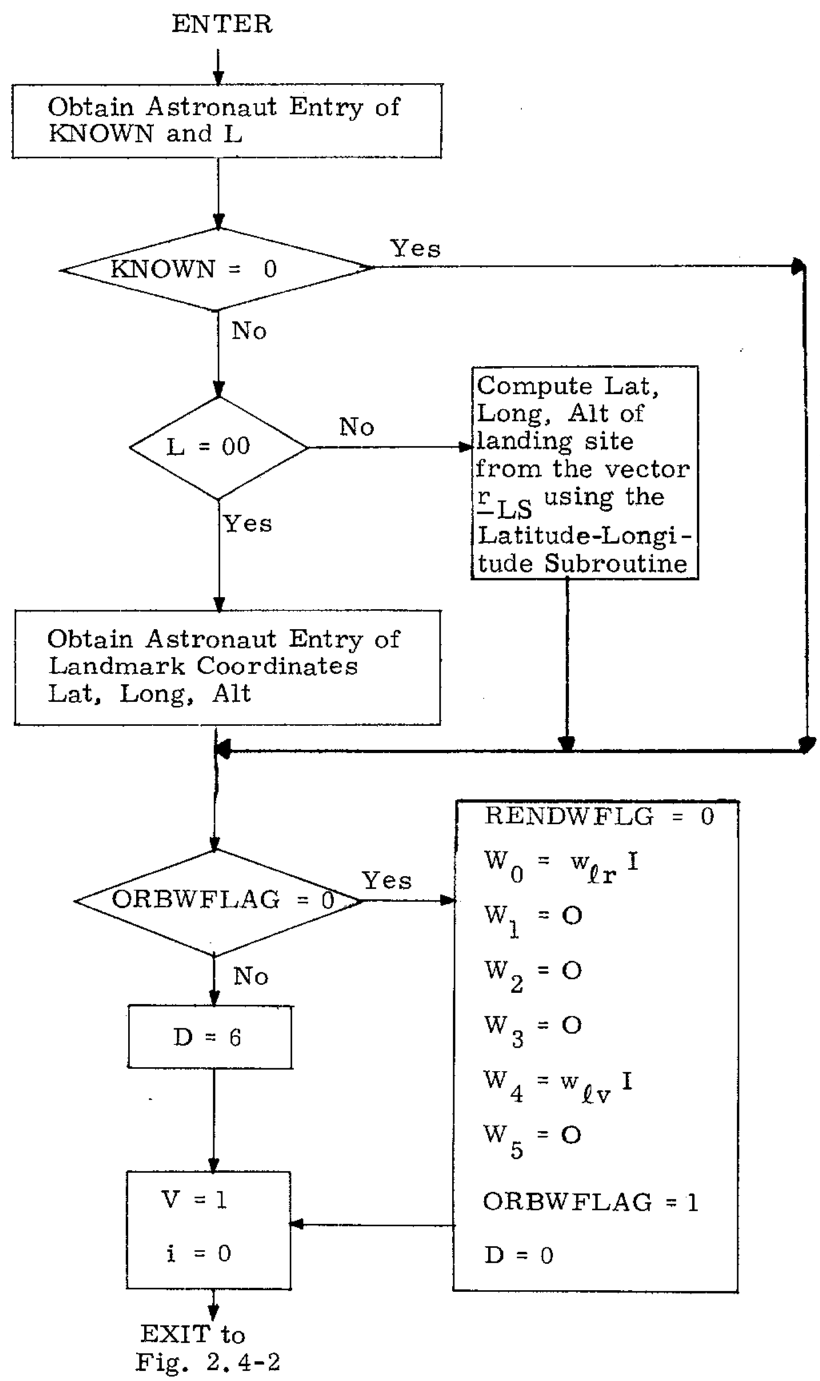


Fig. 2.4-1 Orbit Navigation Routine Initialization

Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing landmark tracking data. The flag is set to zero after each of the following procedures:

1) CSM state vector update from ground
2) Rendezvous navigation
3) Astronaut Command
4) Selection of P-24

RENDWFLG	=	Switch similar to ORBWFLAG but used for rendezvous navigation.
[REFSMMAT]	=	Transformation Matrix: Basic Reference Coordinate System to IMU Stable Member Coordinate System
N	=	Number of unrejected sets of navigation data acquired during the tracking of the landmark
t_{M1} to t_{MN}	=	The N measurement times associated with the N sets of navigation data

N sets of five optics and IMU gimbal angles each

$w_{\ell r}$, $w_{\ell v}$, w_{ℓ}	=	Preselected W matrix initial diagonal elements
var_{RP}	=	variance of the primary body radius error
OFF	=	1 through 5 for index of landing site offset designator; 0 for no landing site offset designator

The variables D and V are indicators which control the Coasting Integration Routine (Section 5.2.2) as described in Section 5.2.2.6, I and O are the three-dimensional identity and zero matrices, respectively, and i is an index which is used to count the navigation data sets.

In the initialization routine the astronaut enters into the CMC through the keyboard the following two items:

$$\text{KNOWN} = \begin{cases} 1 \text{ for mapped or known landmark} \\ 0 \text{ for unmapped landmark or landing site designation} \end{cases}$$

$$\text{L} = \begin{cases} 00 \text{ for a landmark whose coordinates are not stored in CMC memory} \\ 01 \text{ for the landing site} \end{cases}$$

If

$$\text{KNOWN} = 1 \quad \text{and} \quad \text{L} = 00$$

then the astronaut is further requested to enter the coordinates of the landmark; that is, latitude (Lat), longitude (Long) and altitude (Alt). Altitude is defined with respect to the mean lunar radius for lunar landmarks, and the Fischer ellipsoid for earth landmarks.

After completion of the initialization procedure the Orbit Navigation Routine begins processing the data.

For convenience of calculation in the CMC, Eqs. (2. 4. 5), (2. 4. 6), (2. 4. 7), and (2. 4. 9) are reformulated and regrouped as follows:

$$\underline{u}_s = \text{UNIT}(\underline{u}_s \times \underline{u}_{CL})$$

$$\underline{b}_0 = \underline{u}_s \qquad (2.4.12)$$

$$\underline{b}_1 = \underline{0}$$

$$\delta Q = r_{CL}\left[\cos^{-1}(\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2}\right]$$

This set of equations is used both by the Orbit Navigation Routine and the Rendezvous Navigation Routine (Section 5.2.5) in processing optical tracking data. To validate the use of Eqs. (2.4.12) it is necessary only to let

$$\overline{\alpha^2} = r_{CL}^{\ 2}\,(var_{SCT} + var_{IMU}) \qquad (2.4.13)$$

where var_{SCT} and var_{IMU} are the a priori estimates for the SCT and IMU angular error variances per axis, respectively.

The processing of the N sets of landmark-tracking navigation data is illustrated in Fig. 2.4-2. In the figure, F is the altitude flag as defined in Section 5.5.3.

As shown in the figure, the CSM state vector is integrated to the time of each measurement, and the measured line-of-sight vector $\underline{u}_M$ is computed. If this data set is an offset designator, then the vector $\underline{u}_M$ and the time are saved, and the program proceeds to the next measurement. If this is the first navigation data set for a known landmark, then the W matrix is initialized and the data are processed to obtain the two state vector updates. If this is the first measurement for an unknown landmark, then the landmark location is computed, and the W matrix is initialized (Fig. 2.4-3) using a procedure in which the geometry of the landmark mapping is explicitly accounted for. Included in this procedure are negative radicand and zero divisor checks. For this case, no state vector updating occurs. For all other sets of navigation data, two state vector updates are normally obtained.

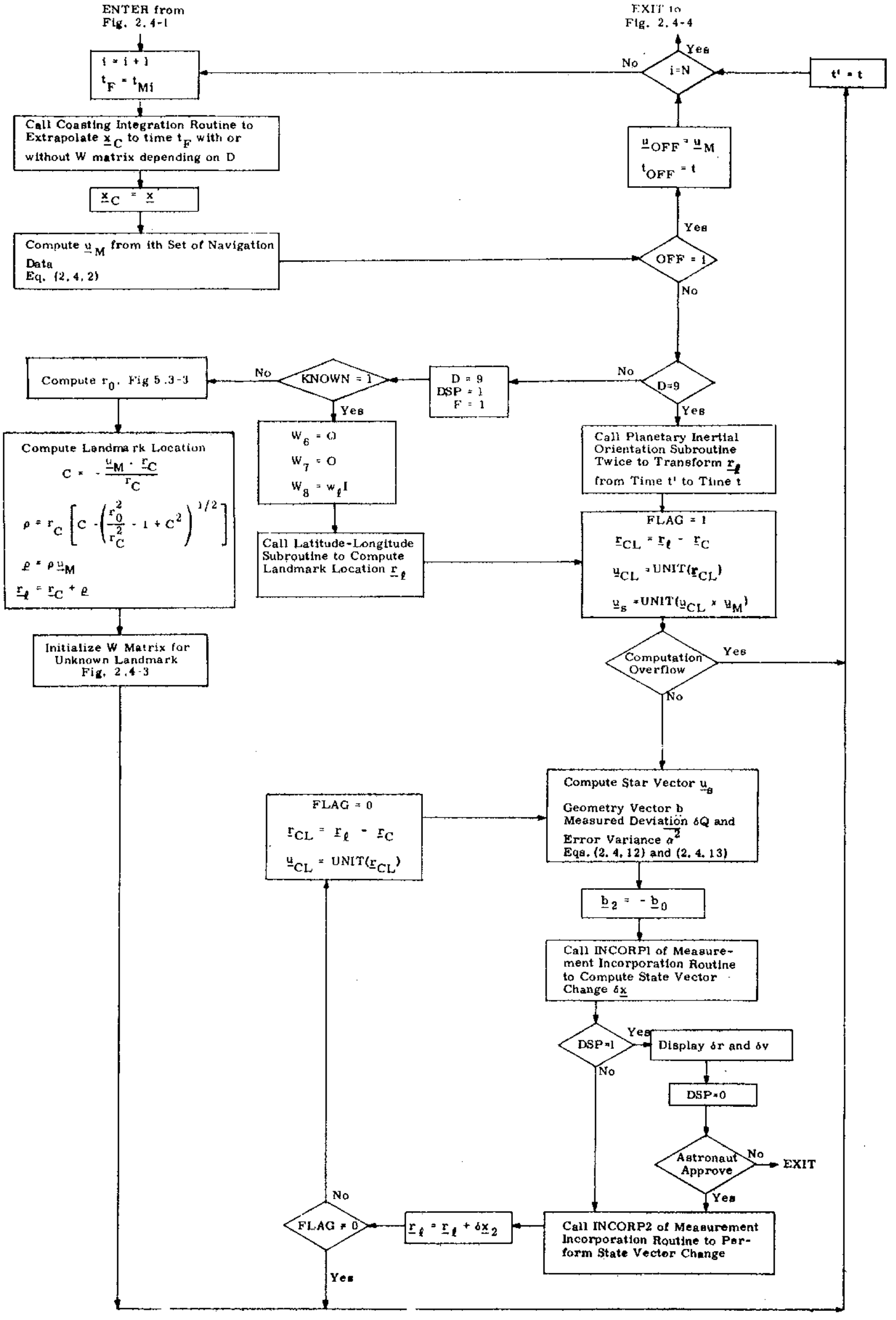


Figure 2.4-2 Orbit Navigation Routine Logic Diagram

ENTER

↓

Compute W_6 and W_7

$$d = \underline{u}_M \cdot \underline{r}_\ell$$

$$U = I - \frac{1}{d}\,\underline{u}_M\,\underline{r}_\ell^{\,T}$$

$$\underline{w}_{i+18} = U\,\underline{w}_i \qquad (i = 0, 1, \ldots, 5)$$

↓

Determine W_8

$$\begin{pmatrix} e_0 & & \\ e_1 & e_2 & \\ e_3 & e_4 & e_5 \end{pmatrix} = \frac{1}{2}\rho^2 \mathrm{var}_{SCT}\, UU^T + \mathrm{var}_{RP}\left(\frac{r_0}{d}\right)^2 \underline{u}_M \underline{u}_M^T$$

$$w_{74} = \sqrt{e_5}$$

$$w_{73} = e_4 / w_{74}$$

$$w_{72} = e_3 / w_{74}$$

$$w_{76} = \sqrt{e_2 - w_{73}^2}$$

$$w_{75} = (e_1 - w_{72} w_{73}) / w_{76}$$

$$w_{78} = \sqrt{e_0 - w_{72}^2 - w_{75}^2}$$

$$W_8 = \begin{pmatrix} w_{72} & w_{73} & w_{74} \\ w_{75} & w_{76} & 0 \\ w_{78} & 0 & 0 \end{pmatrix}$$

↓

EXIT

Fig. 2.4-3 W Matrix Initialization for Unknown Landmark

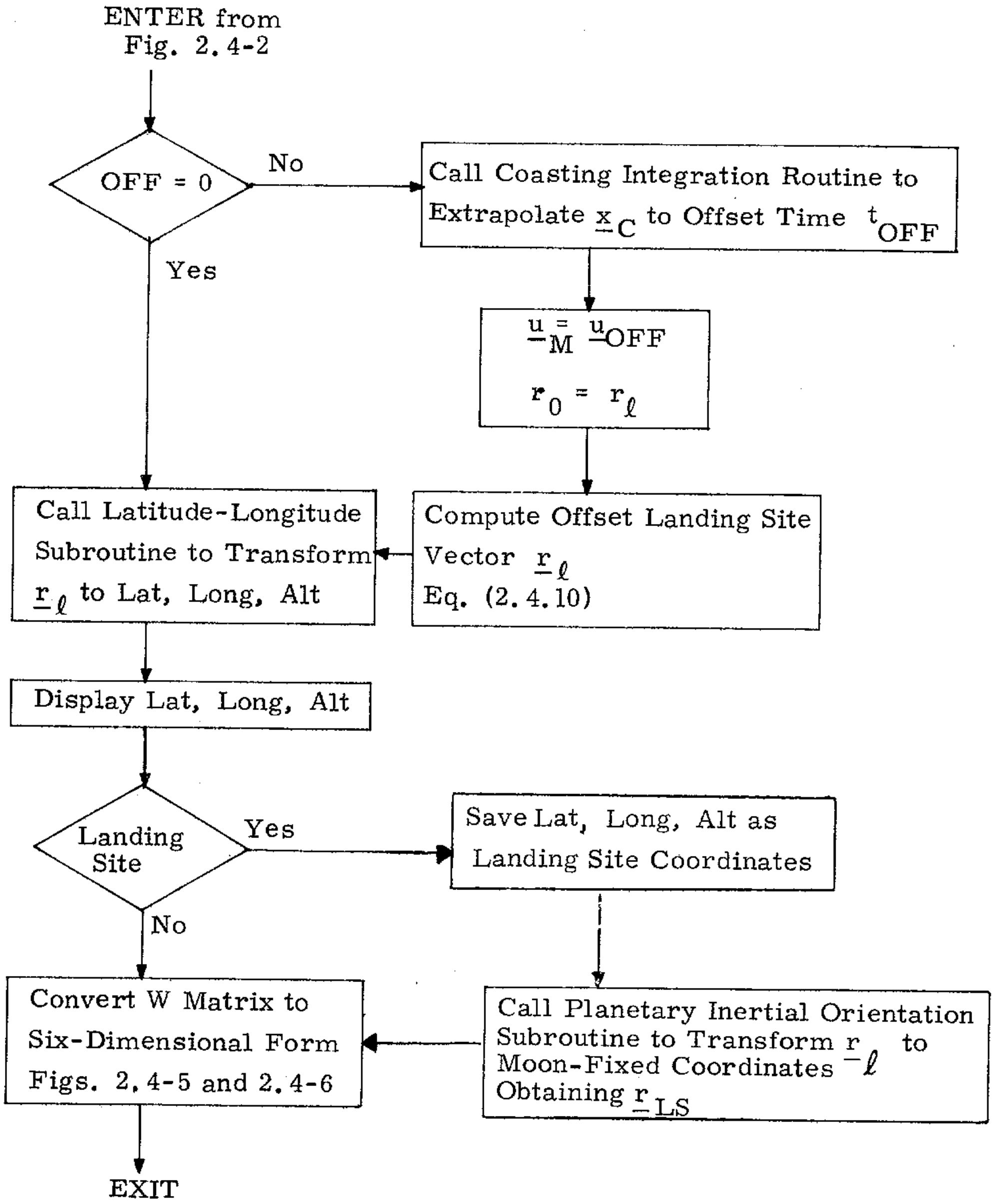


Fig. 2.4-4 Orbit Navigation Routine Termination

After the processing of the data is completed, the location of the offset landing site is computed from the saved data, if a data set was so designated, as shown in Fig. 2.4-4. Then, the final estimated landmark position vector is converted to latitude, longitude, altitude coordinates and these coordinates are displayed. If the tracked landmark is to be the landing site, then the landing site coordinates and the landing site vector ($\underline{r}_{LS}$) are saved in erasable memory.

The final operation of the Orbit Navigation Routine is to convert the nine-dimensional error transition matrix, W, to a six-dimensional matrix with the same CSM position and velocity estimation error variances and covariances. The reason for this procedure is that the W matrix, when it is initialized for processing the data associated with the next landmark, must reflect the fact that the initial landmark location errors are not correlated with the errors in the estimated CSM position and velocity vectors. Of course, after processing measurement data, these cross correlations become nonzero, and it is for this reason that the nine dimensional procedure works, and that it is necessary to convert the final W matrix to six-dimensional form.

The solution to the conversion problem is not unique. A convenient solution is obtained in the following manner.

The error transition matrix, W, has been defined in Section 5.2.3 in terms of the nine three-dimensional submatrices $W_0, W_1, \ldots, W_8$ as follows:

$$W = \begin{pmatrix} W_0^T & W_1^T & W_2^T \\ W_3^T & W_4^T & W_5^T \\ W_6^T & W_7^T & W_8^T \end{pmatrix} \qquad (2.4.14)$$

Let the elements of each of these submatrices be defined by

$$
W_i^T = \begin{pmatrix} w_{9i} & w_{9i+3} & w_{9i+6} \\ w_{9i+1} & w_{9i+4} & w_{9i+7} \\ w_{9i+2} & w_{9i+5} & w_{9i+8} \end{pmatrix} \quad (i = 0, 1, \ldots, 8) \tag{2.4.15}
$$

The W matrix is then converted to the following six-dimensional form:

$$
\begin{pmatrix} W_0^T & W_1^T \\ W_3^T & W_4^T \end{pmatrix} = \begin{pmatrix} w_0 & w_3 & w_6 & w_9 & w_{12} & w_{15} \\ w_1 & w_4 & w_7 & w_{10} & w_{13} & 0 \\ w_2 & w_5 & w_8 & w_{11} & 0 & 0 \\ w_{27} & w_{30} & w_{33} & 0 & 0 & 0 \\ w_{28} & w_{31} & 0 & 0 & 0 & 0 \\ w_{29} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.4.16}
$$

$$
W_2 = W_5 = O
$$

The twenty-one non-zero elements of the converted W matrix are computed by solving the following twenty-one equations:

$$
\sum_{k=0}^{5} w_{i+3k} \; w_{j+3k} = \sum_{k=0}^{8} w'_{i+3k} \; w'_{j+3k} = e_p \quad \begin{pmatrix} i,j = 0,1,2,27, \\ 28, 29 \\ i \leq j \\ p = 0,1,\ldots,20 \end{pmatrix} \tag{2.4.17}
$$

where primes refer to quantities before conversion, and the following table gives p as a function of i and j:

i	29	28	27	2	1	0	28	27	2	1	0	27	2	1	0	2	1	0	1	0	0
j	29	29	29	29	29	29	28	28	28	28	28	27	27	27	27	2	2	2	1	1	0
p	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

The twenty-one elements e_0 to e_{20} are computed as illustrated in Fig. 2.4-5. The converted W matrix is then calculated as shown in Fig. 2.4-6. Included in this procedure are negative radicand and zero divisor checks.

The Orbit Navigation Routine is now ready to process the data acquired in tracking the next landmark.

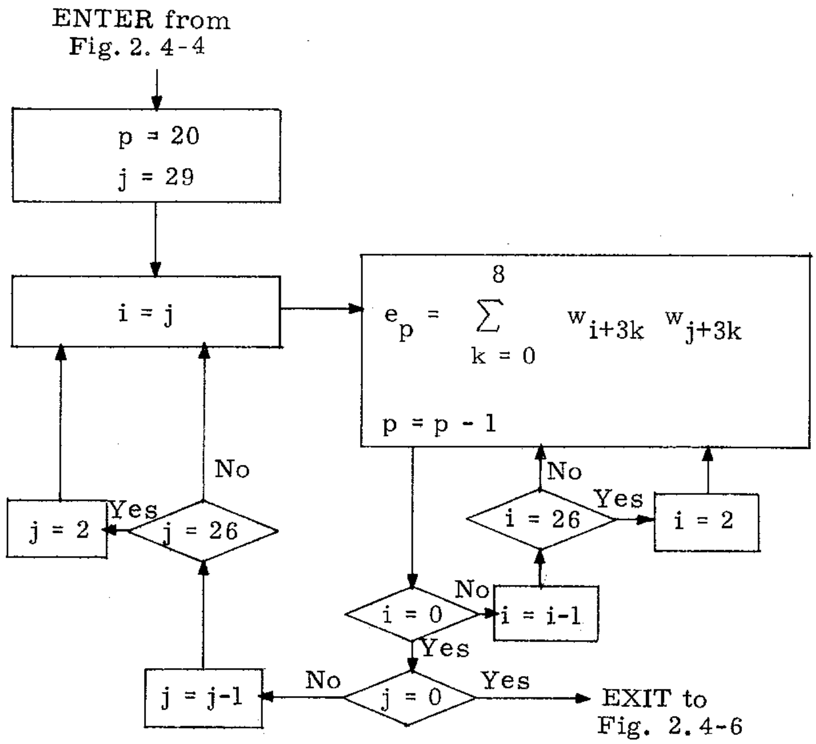


Fig. 2.4-5 Orbit Navigation Routine W Matrix Conversion, Part I

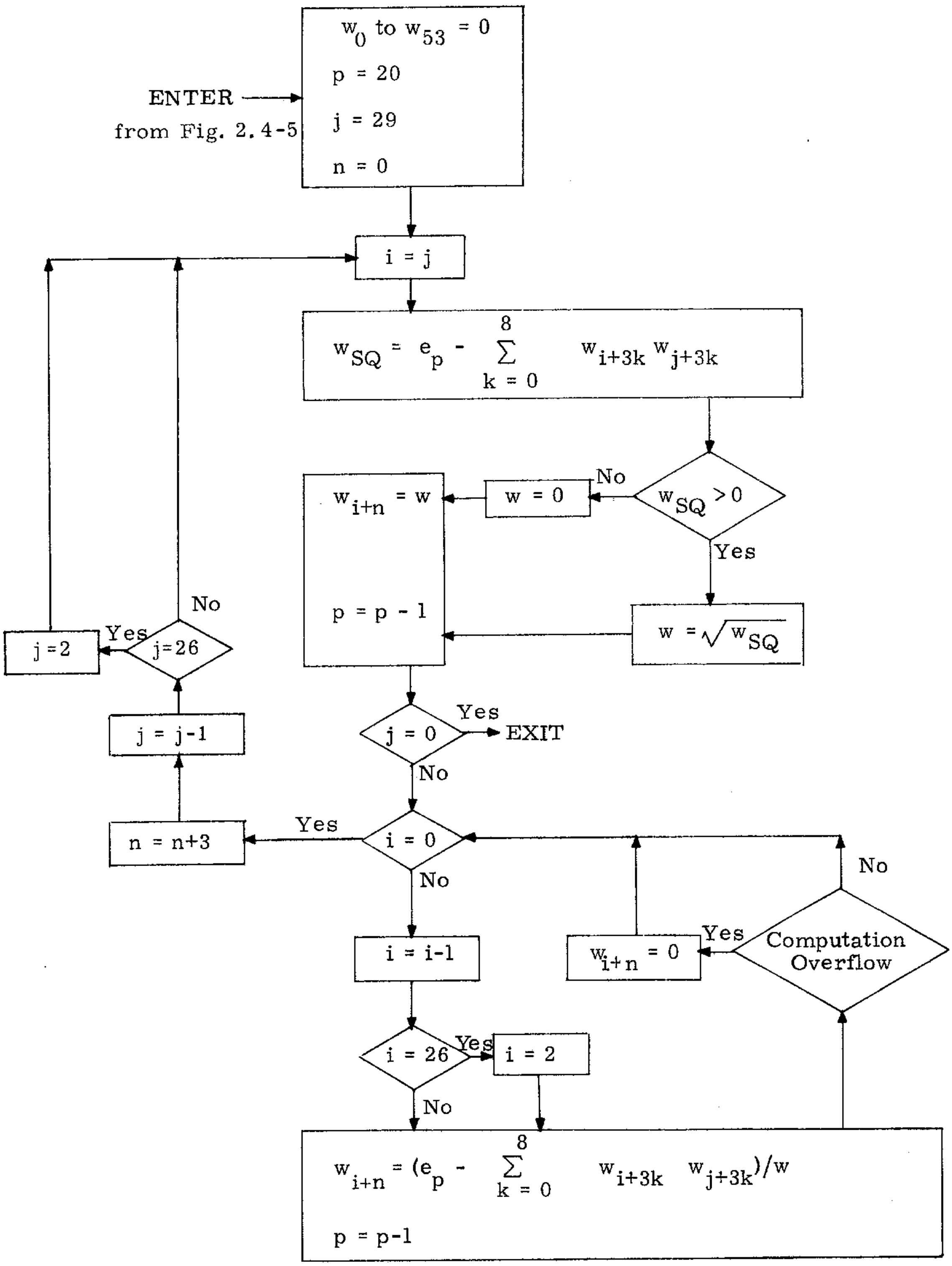


Fig. 2.4-6 Orbit Navigation Routine W Matrix Conversion Part II

5.2.5 Universal Tracking and Rendezvous Navigation Program (P20)

The CMC Universal Tracking and Rendezvous Navigation Program (P20) has two independent functions—tracking and navigation. The tracking function applies to any mission situation requiring the CSM to track a designated body with a specified pointing vector or to any mission situation requiring the CSM to rotate at a constant rate around a specified vector. The navigation function applies only to the Rendezvous Navigation Mode of P20.

5.2.5.1 Tracking

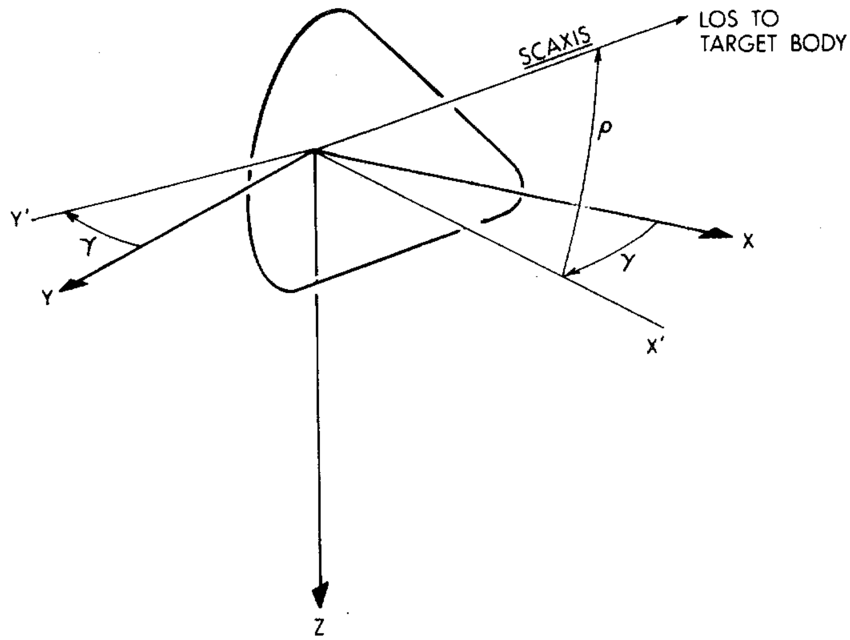
There are three nonrendezvous and two rendezvous tracking (or pointing) options:

Option	Purpose	Mode
0	Point a specified spacecraft vector at the LM. Do not constrain rotational attitude about this pointing vector. (Use VECPOINT)	Rendezvous
1	Point a specified spacecraft vector at a celestial body. Do not constrain attitude about this vector. (Use VECPOINT)	Nonrendezvous
2	Rotate at a specified rate about a specified spacecraft vector.	Nonrendezvous
4	Same as Option 0, but constraining attitude about the pointing vector to a specified value (OMICRON).	Rendezvous
5	Same as Option 1, but constraining attitude about pointing vector to a specified value (OMICRON).	Nonrendezvous

5.2.5.1.1 Option 0: Orient the Spacecraft Such that a Specified Body-Fixed Pointing Vector Aligns with the Estimated Line of Sight to the LM.—Option 0 is effected by the Tracking Attitude Routine (R61), which is called by P20 as prescribed for this option in GSOP Section 4. The pointing vector (SCAXIS) is defined by two angles:

1) A specified positive angle (γ) from the spacecraft +X-axis about the spacecraft Z axis ($0 \leq \gamma < 360$ deg)

2) A specified positive angle (ρ) about the Y'-axis produced in the XY plane as a result of γ ($-90 \leq \rho \leq 90$ deg).



The LOS is determined by advancing the estimated CSM and LM state vectors to the current time. (See Kepler Subroutine, Section 5.5.5.) Once the coincidence of SCAXIS and LOS has been established, R61 establishes a spacecraft angular rate to maintain the coincidence. For Option 0, R61 provides the RCS DAP with the following quantities:

1. $\underline{\omega}_{CA}$ ..... desired LOS rate plus LOS correction rate in control-axis coordinates

2. IGA_D, MGA_D, OGA_D ..... DAP reference angles, i.e., center of the deadband.

3. $\Delta\underline{G}A$..... a vector defining the desired incremental changes in the IMU gimbal angles every 0.1 second.

The desired LOS rate ($\underline{\omega}_{CA}$) is computed as follows:

$$\underline{u}_{LOS} = \underline{r}_L - \underline{r}_C$$

$$\underline{\omega}_{correction} = ([NBSM] \ \underline{SCAXIS} \times [REFSMMAT] \ UNIT \ (\underline{u}_{LOS}))/\Delta t_C$$

where [NBSM] uses the DAP reference angles THETADX, Y, and Z for the transformation from navigation-base to stable-member coordinates.

If $|\underline{\omega}_{correction}|$ exceeds MAXRATE, distribute as follows:

$$\underline{\omega}_{correction} = \text{MAXRATE [UNIT } (\underline{\omega}_{correction})]$$

$$\underline{\omega}_{LOS} = [\text{REFSMMAT}] \left\{ \frac{\text{UNIT } (\underline{u}_{LOS}) \times (\underline{v}_L - \underline{v}_C)}{|\underline{u}_{LOS}|} \right\} + \underline{\omega}_{correction}$$

$$\underline{\omega}_{CA} = [\text{NBCA}] [\text{SMNB}] \underline{\omega}_{LOS},$$

where $\underline{\omega}_{LOS}$ is the LOS rate in stable-member coordinates; [SMNB] and [NBSM] are transformation matrices defined in Section 5.6.3.2.1; [NBCA] is the matrix for transforming a vector from navigation-base to control-axis coordinates as defined below; $\underline{r}_L$, $\underline{v}_L$, $\underline{r}_C$, and $\underline{v}_C$ are the position and velocity vectors of the LM and CSM in basic-reference coordinates obtained by using the Kepler Subroutine of Section 5.5.5.; SCAXIS is the specified axis (γ, ρ); MAXRATE is the maximum correction rate; and Δt_C is a conservative estimate of the R61 computation interval (thus preventing overshoot in vehicle attitude by the time of the next mark).

$$[\text{NBCA}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 7.5 & -\sin 7.5 \\ 0 & \sin 7.5 & \cos 7.5 \end{bmatrix}$$

The vector $\Delta \underline{GA}$ is computed as follows:

$$\Delta \underline{GA} = \begin{bmatrix} \cos \text{IGA} \sec \text{MGA} & 0 & -\sin \text{IGA} \sec \text{MGA} \\ -\cos \text{IGA} \tan \text{MGA} & 1 & \sin \text{IGA} \tan \text{MGA} \\ \sin \text{IGA} & 0 & \cos \text{IGA} \end{bmatrix} (0.1) \; \underline{\omega}_{LOS}$$

where $\underline{\omega}_{LOS}$ is the LOS rate in stable-member coordinates computed previously.

5.2.5.1.2 Option 1: Orient the Spacecraft Such that SCAXIS Aligns with the LOS to a Specified Celestial Body.—Option 1 is effected by the Tracking Attitude Routine (R61), called by P20 as prescribed for this option in GSOP Section 4. SCAXIS is as defined in Section 5.2.5.1.1.

For Option 1, the LOS is determined by advancing the estimated CSM state vector to the current time. (See Kepler Subroutine, Section 5.5.5.) Once the coincidence of SCAXIS and LOS has been established, R61 establishes a spacecraft angular rate to maintain the coincidence. R61 quantities provided to the RCS DAP under tracking-option 1 are as described for Option 0 (Section 5.2.5.1.1), with the following exceptions:

$$\underline{u}_{LOS} = \text{line of sight direction,}$$

and,

for STARCODE = 0–45,

$$\underline{\omega}_{LOS} = \underline{\omega}_{correction};$$

for STARCODE = 46–50,

$$\underline{\omega}_{LOS} = [REFSMMAT] \left\{ \frac{UNIT\ (\underline{r}_C) \times (\underline{v}_C)}{|\underline{r}_C|} \right\} + \underline{\omega}_{correction}$$

5.2.5.1.3 Option 2: Rotate the Spacecraft About SCAXIS. —Option 2 is effected by the Automatic Rotation Routine (R67), called by P20 as prescribed for this option in GSOP Section 4. SCAXIS is as defined in Section 5.2.5.1.1. R67 initializes COF, BRATE, [DEL], [MIS], and ADB in KALCMANU (GSOP Section 3) and then executes the KALCMANU coding to issue the following quantities to the RCS DAP:

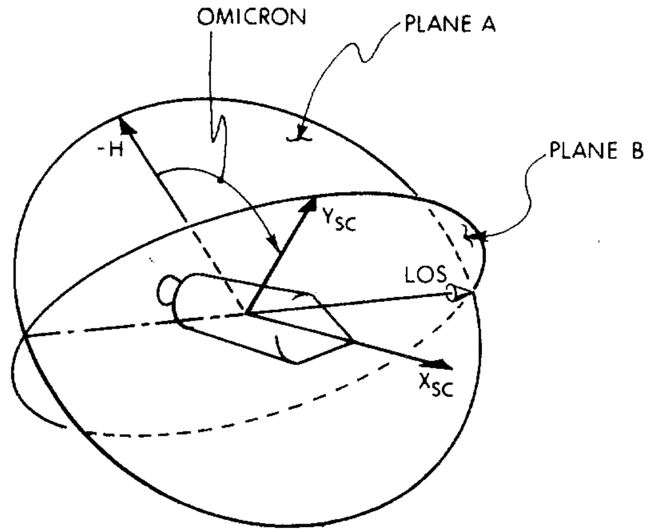
1. $\underline{\omega}_{CA}$ .... desired rotation rate
2. IGA_D, MGA_D, OGA_D, ... DAP reference angles, i.e., center of deadband.
3. $\Delta \underline{GA}$ .... a vector defining the desired incremental changes in the IMU gimbal angles every 0.1 second.

These calculations are repeated once per second.

5.2.5.1.4 Option 4: Orient the Spacecraft as in Option 0, but Further Specify an Angle (OMICRON) of Rotation About SCAXIS. —Option 4 is effected by the Tracking Attitude Routine (R61), called by P20 as prescribed for this option in GSOP Section 4.

SCAXIS is as defined in Section 5.2.5.1.1; OMICRON is the positive-sense angle between planes A and B—where plane A is defined by the negative momentum vector (-H) and the LOS, and plane B is defined by the spacecraft +Y-axis and the LOS.

Alignment of the LOS with either the spacecraft Y-axis or the momentum vector is proscribed since the necessary two-plane condition for specifying OMICRON would not exist.



For Option 4, R61 provides the following quantities to the RCS DAP:

1. $\underline{\omega}_{CA}$ .... desired LOS rate in control-axis coordinates
2. IGA_D, MGA_D, OGA_D, ... DAP reference angles, i.e., center of the deadband
3. $\Delta \underline{G}A$ .... a vector defining the desired incremental changes in the IMU gimbal angles every 0.1 second.

The desired LOS rate $\left(\underline{\omega}_{CA}\right)$ in control-axis coordinates is obtained as follows:

$$\underline{u}_{LOS} = \underline{r}_L - \underline{r}_C$$

$$\underline{\omega}_{LOS} = [REFSMMAT] \left\{ \frac{\text{UNIT}(\underline{u}_{LOS}) \times (\underline{v}_L - \underline{v}_C)}{|\underline{u}_{LOS}|} \right\}$$

$$\underline{\omega}_{CA} = [NBCA]\,[SMNB]\,\underline{\omega}_{LOS},$$

where $\underline{\omega}_{LOS}$ is the LOS rate in stable member coordinates; [SMNB] is a transformation matrix defined in Section 5.6.3.2.1; [NBCA] is the matrix for transforming a vector from navigation base to control axis coordinates as defined below; $\underline{r}_L$, $\underline{v}_L$, $\underline{r}_C$, and $\underline{v}_C$ are the position and velocity vectors of the LM and CSM in basic reference coordinates obtained by using the Kepler Subroutine of Section 5.5.5.

$$[NBCA] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 7.25 & -\sin 7.25 \\ 0 & \sin 7.25 & \cos 7.25 \end{bmatrix}$$

The vector ($\Delta \underline{GA}$) is computed as follows:

$$\Delta \underline{GA} = \begin{bmatrix} \cos IGA \sec MGA & 0 & -\sin IGA \sec MGA \\ -\cos IGA \tan MGA & 1 & \sin IGA \tan MGA \\ \sin IGA & 0 & \cos IGA \end{bmatrix} (0.1)\, \underline{\omega}_{LOS}$$

$$+ \underline{\omega}_{correction},$$

where

$$\underline{\omega}_{correction} = \begin{bmatrix} \theta_{DX} - OGA_D \\ \theta_{DY} - IGA_D \\ \theta_{DZ} - MGA_D \end{bmatrix} \frac{0.1}{\Delta t_C}$$

$\theta_{DX}, \theta_{DY}, \theta_{DZ}$ = desired gimbal angles

Δt_C = conservative estimate of R61 computation interval.

5.2.5.1.5 Option 5: Orient the Spacecraft as in Option 1, but Further Specify an Angle (OMICRON) of Rotation About SCAXIS. —Option 5 is effected by the Tracking Attitude Routine (R61), called by P20 as prescribed for this option in GSOP Section 4.

SCAXIS is as defined in Section 5.2.5.1.1; OMICRON is as defined in Section 5.2.5.1.4 (Option 4).

For Option 5, R61 quantities provided to the RCS DAP are as in Option 4, except as follows:

$\underline{u}_{LOS}$ = line of sight direction,

and,

for STARCODE = 0-45,

$\underline{\omega}_{LOS} = 0;$

for STARCODE = 46-50,

$$\underline{\omega}_{LOS} = [REFSMMAT] \left\{ \frac{UNIT\ (\underline{r}_C) \times (\underline{v}_C)}{|\underline{r}_C|} \right\}$$

5.2.5.2 Rendezvous Tracking Data Processing Routine (R22)

In the P20 Rendezvous Mode (Options 0 and 4), Routine R22 processes optics and VHF tracking data to update the state vector of either the CSM or LM. (See GSOP Section 4.)

5.2.5.2.1 Rendezvous Data-processing Logic. —Figure 2.5-3 is a flowchart of R22 logic. Observe that the routine alternates between checking the status of the VHF Range Flag and checking to see if optics mark data are in Position 1. Whenever optics mark data are found in Position 1, they are transferred to Position 2 and used to calculate the correction (or update) to the state vector. Whenever the VHF Range Flag is set, the routine reads the range from the VHF range-link if at least 60 seconds have expired since the last time range was read (and on-board range estimate is less than 327.67 n.mi.). Immediately after reading the range, a check is made to see if the Data Good discrete is being received from the VHF range-link, signifying that the range tracking network is tracking the target satisfactorily. If the Data Good discrete is present, the range data are used to calculate the correction to the state vector. The time t_{VHF} is used as the time of range measurement. If the Data Good discrete is not present, the Tracker Fail Light is turned on.

The manner in which the state vector correction is calculated for either optics mark or range-link data is given in Section 5.2.5.2.2. The connections labeled (E), (F), and (G) in Figure 2.5-3 correspond to those given for the logic flow of the rendezvous navigation computations given in Figure 2.5-4. To distinguish between the two types of data in Figure 2.5-3 use is made of a Source Code (SC), which is equal to one or two depending on whether it is optics mark or range-link data respectively.

After the state vector correction has been calculated it is seen in Figure 2.5-3 that a check is made to see if the magnitudes of the proposed corrections in position and velocity (δr and δv) exceed certain threshold limits (δr_{MAX} and δv_{MAX}) stored in erasable memory. The same threshold limits are used in this check whether the state vector correction is based on optics mark or range-link data. The purpose of this threshold check is to insure the validity of the proposed state vector correction (update). If the proposed correction exceeds either threshold limit, the magnitudes of the correction in position and velocity (δr and δv) and the Source Code (SC) are displayed to the astronaut. If the correction is the result of optical tracking data and the astronaut is sure that he is tracking the LM, he should command the update. Otherwise, he should reject the data and recheck the optical tracking. Additional details on the threshold check are given in Section 5.2.1. It should be noted that if optical data are being processed, there will actually be two separate corrections to the state vector, instead of one, due to the manner

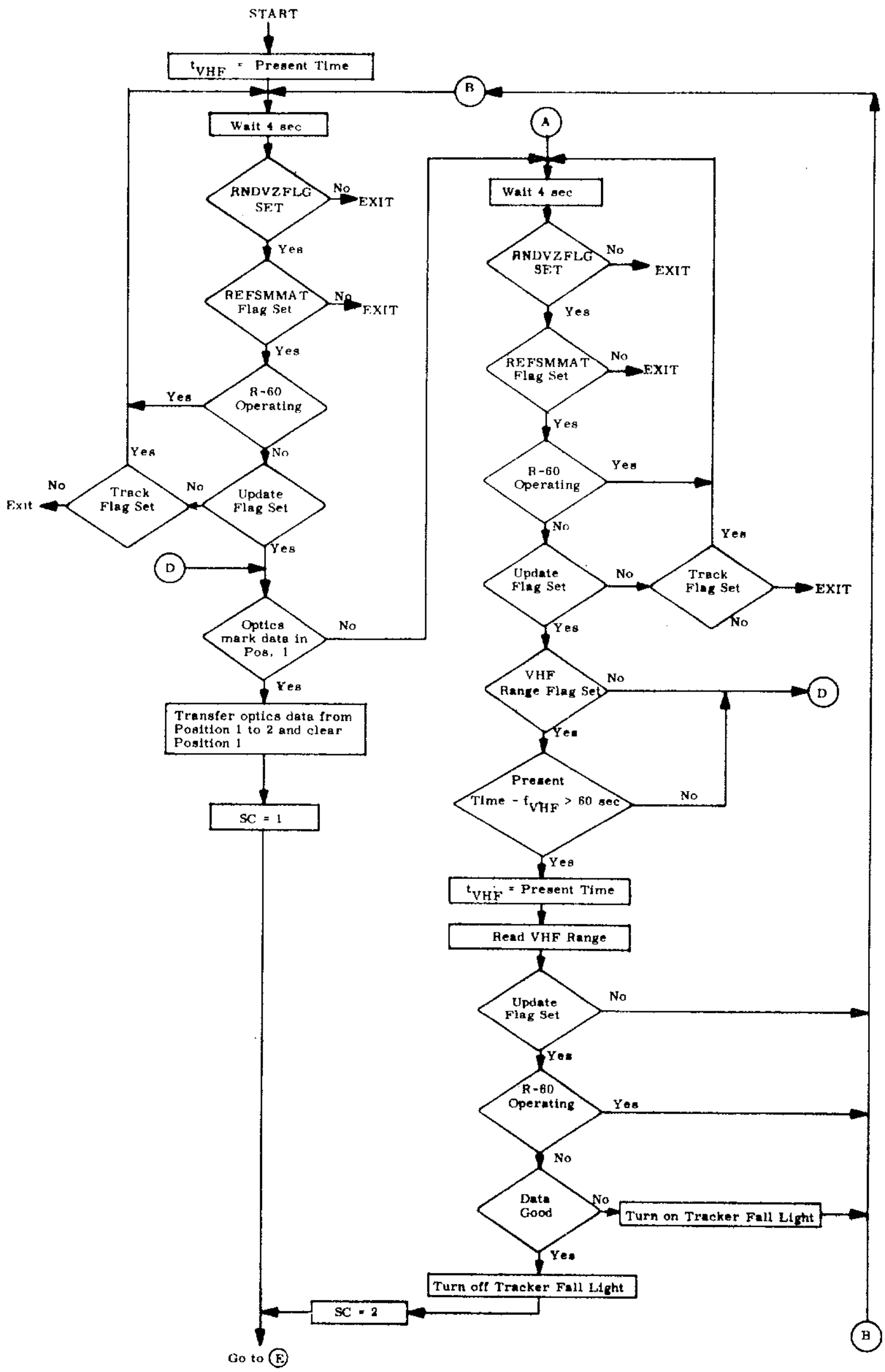


Figure 2.5-3. Rendezvous Tracking Data Processing (Sheet 1 of 2)

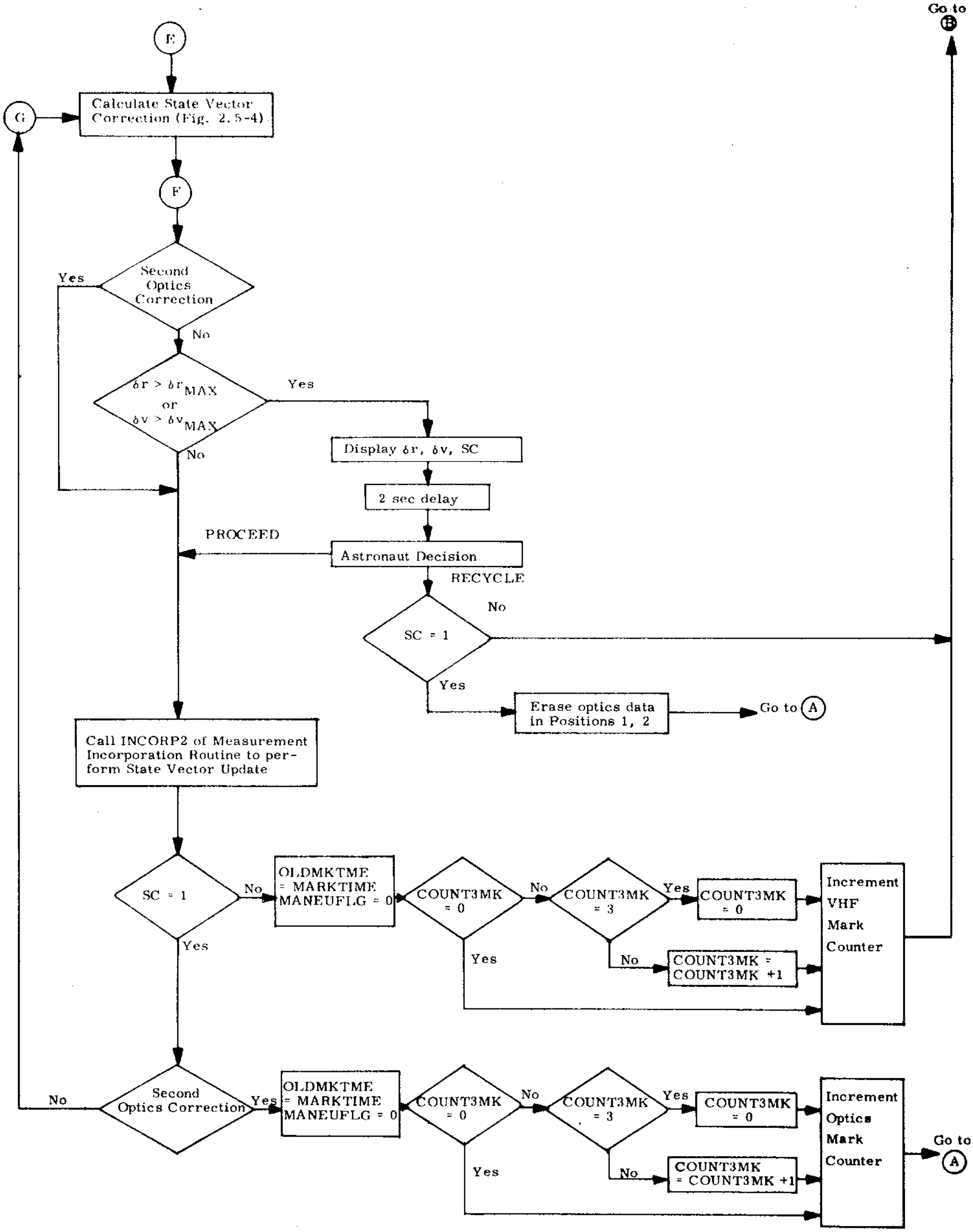


Figure 2.5-3. Rendezvous Tracking Data Processing (Sheet 2 of 2)

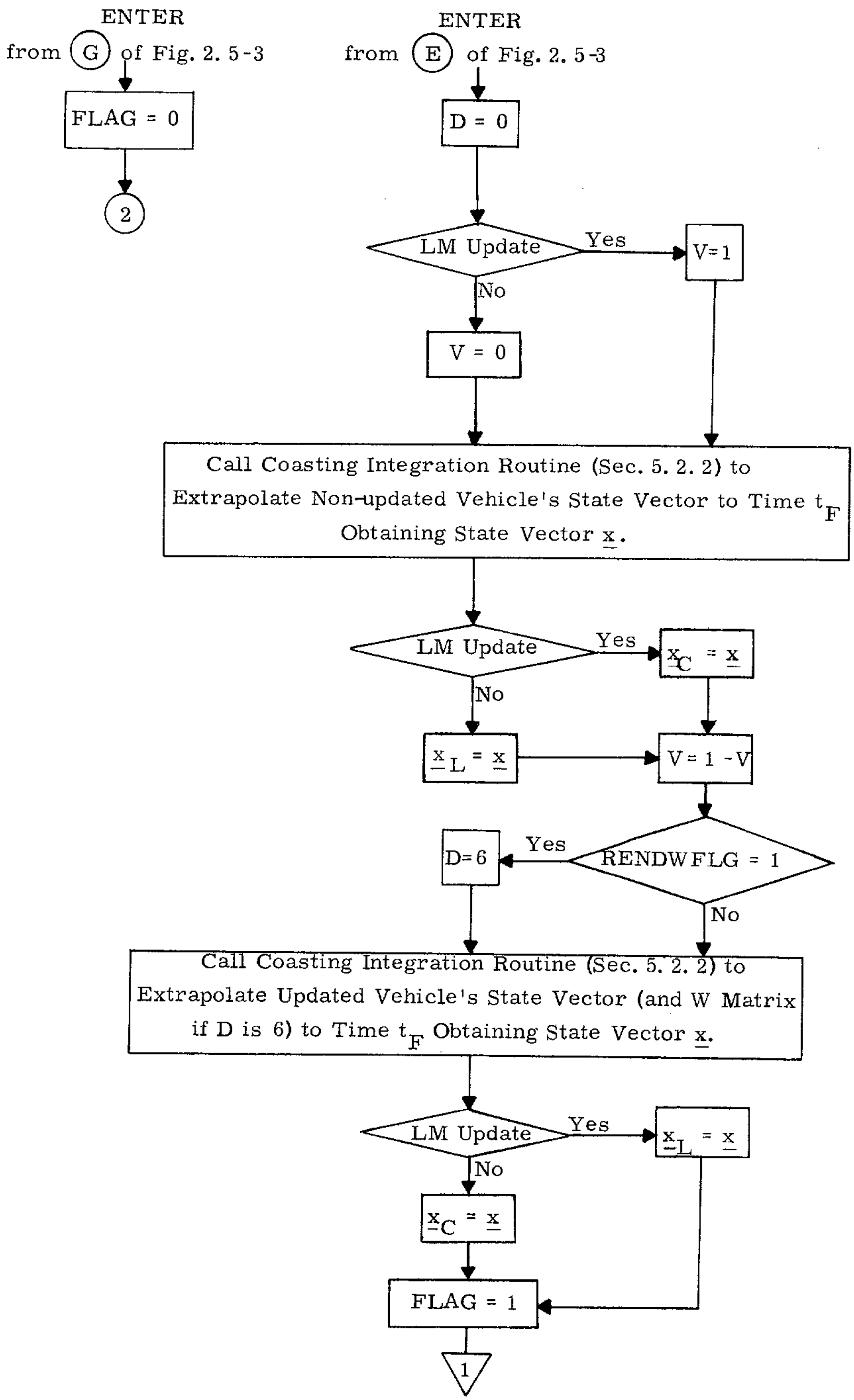


Figure 2.5-4 Rendezvous Navigation Computations (Sheet 1 of 4)

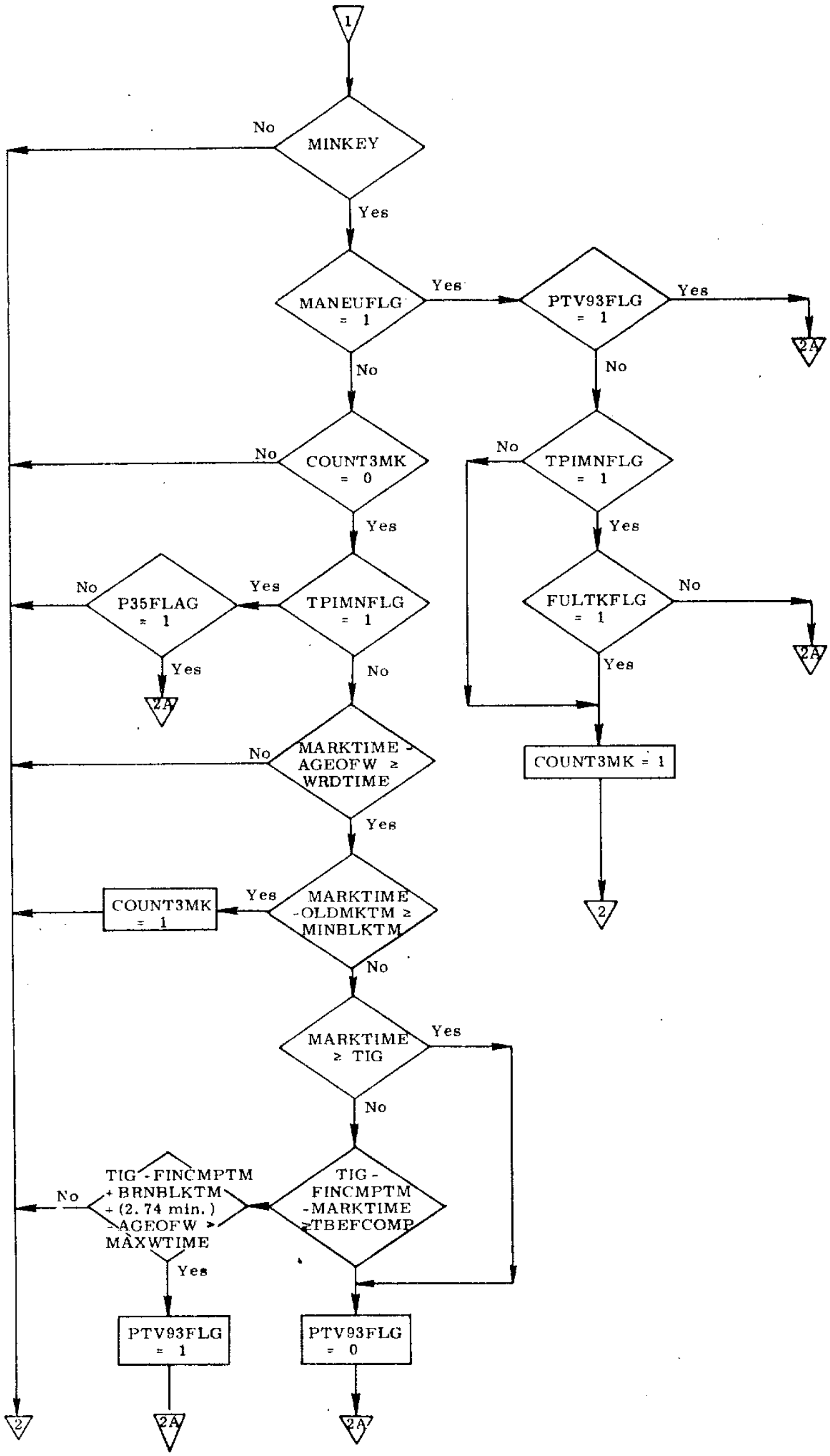


Figure 2.5-4. Rendezvous Navigation Computations (Sheet 2 of 4)

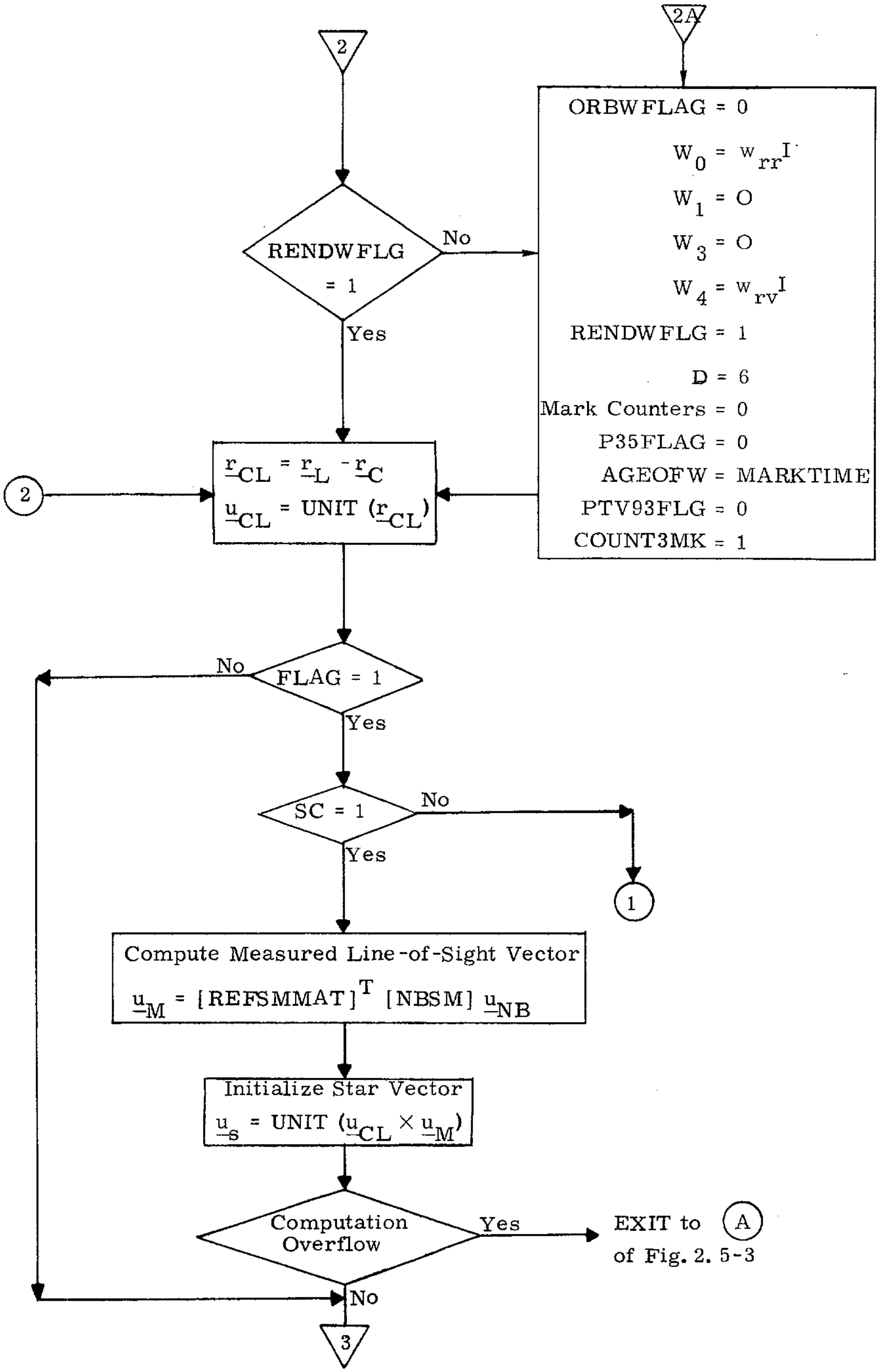


Figure 2. 5-4 Rendezvous Navigation Computations (Sheet 3 of 4)

(3)

Compute Star Vector, Geometry Vector, Measured Deviation, and Error Variance for Optics Measurement

$$\underline{u}_s = \text{UNIT}\ (\underline{u}_s \times \underline{u}_{CL})$$

$$\underline{b}_0 = \pm\ \underline{u}_s$$

$$\underline{b}_1 = \underline{0}$$

$$\delta Q = r_{CL} \left[\cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2} \right]$$

$$\overline{\alpha^2} = r_{CL}^{\ 2}\ (\text{var}_{SXT} + \text{var}_{IMU}) + \text{var}_{INT}$$

(1) →

Compute Geometry Vector, Measured Deviation, and Error Variance for Radar Measurement

$$\underline{b}_0 = -\underline{u}_{CL}$$

$$\underline{b}_1 = \underline{0}$$

$$\delta Q = R_M - r_{CL}$$

$$\overline{\alpha^2} = \text{maximum}\ (r_{CL}^{\ 2}\ \text{var}_R + \text{var}_{INT},\ \text{var}_{Rmin})$$

Call INCORP1 of Measurement Incorporation Routine to Compute State Vector Change $\delta \underline{x}$

EXIT to (F) of Fig. 2.5-3

Figure 2.5-4 Rendezvous Navigation Computations (Sheet 4 of 4) in which the optics data are used to calculate the state vector correction as explained in Section 5.2.5.2.2. Since the second optics correction is calculated using the state vector updated with the first optics correction, it is unlikely that the magnitude of the second correction will exceed that of the first. Consequently, a threshold check is made in Figure 2.5-3 only on the first optics correction.

At various points in Figure 2.5-3 it is seen that a check is made to see if the Update Flag is present (set). This flag is removed when there is no desire to process the optics and range-link data. It is removed by the CMC during CSM ΔV maneuvers and during certain time-consuming computations, and by the Target Delta V Program. If the Update Flag is not present, a check is made on the Track Flag. The Track Flag is removed when it is desired to temporarily terminate the rendezvous navigation process. If the Track Flag is present, it is seen in Figure 2.5-3 that the routine will continue to monitor the Track and Update Flags in a standby status until one of the flags changes state.

The range obtained from the VHF range-link by the Rendezvous Tracking Data Processing Routine is that measured by the range-link between the CSM and the LM. This data is sent to the CMC from the range-link as a 15 bit binary data word R_{RL}. In the CMC the range R_M in nautical miles is obtained as follows:

$$R_M = k_{RL} \, R_{RL}$$

where k_{RL} is the bit weight in nautical miles.

5.2.5.2.2 Rendezvous Navigation Computations. —Each set of optical navigation data contains the time of the measurement and the two optics and three IMU gimbal angles. From these five angles the measured unit vector $\underline{u}_M$ along the CSM-to-LM line-of-sight is computed in the Basic Reference Coordinate System from

$$\underline{u}_M = [\text{REFSMMAT}]^T \, [\text{NBSM}] \, \underline{u}_{NB} \qquad (2.5.1)$$

where [REFSMMAT] and [NBSM] are transformation matrices and $\underline{u}_{NB}$ is the measured line-of-sight vector in navigation base coordinates. All terms of Equation (2.5.1) are defined in Section 5.6.3.

For the purpose of navigation it is convenient to consider the measured unit vector $\underline{u}_M$ to be the basic navigation data. This navigation measurement of the line-of-sight vector $\underline{u}_M$ is mathematically equivalent to the simultaneous measurement of the angles between the lines-of-sight to the LM and two stars. The data are processed by selecting two convenient unit vectors (fictitious star directions), converting the vector $\underline{u}_M$ to an equivalent set of two artificial star-LM measurements, and using the Measurement Incorporation Routine (Section

5.2.3) twice, once for each artificial measurement. These two unit vectors are chosen to be perpendicular to each other and to the current estimated line-of-sight vector so as to maximize the convenience and accuracy of the procedure.

Let $\underline{r}_C$ and $\underline{r}_L$ be the estimated CSM and LM position vectors at the time of a given line-of-sight measurement. Then, the first state vector update for the measurement is performed as follows:

(1) Calculate the estimated CSM-to-LM line-of-sight from

$$\underline{r}_{CL} = \underline{r}_L - \underline{r}_C \tag{2.5.2}$$

$$\underline{u}_{CL} = \text{UNIT}\ (\underline{r}_{CL})$$

(2) Initialize the fictitious star direction to the vector

$$\underline{u}_s = \text{UNIT}\ (\underline{u}_{CL} \times \underline{u}_M) \tag{2.5.3}$$

If the vectors $\underline{u}_{CL}$ and $\underline{u}_M$ are separated by a small enough angle, (possibly as much as $\sqrt{3} \times 2^{-19}$ rad.), then a computation overflow occurs in the execution of Equation (2.5.3), and this set of measurement data is discarded because all of the components of the estimated state vector deviation, $\delta\underline{x}$, would be negligible for both state vector updates.

(3) Compute an artificial star direction from

$$\underline{u}_s = \text{UNIT}\ (\underline{u}_s \times \underline{u}_{CL}) \tag{2.5.4}$$

(4) Calculate the six-dimensional geometry vector, $\underline{b}$, from

$$\underline{b}_0 = \pm \frac{1}{r_{CL}} \underline{u}_s \tag{2.5.5}$$

$$\underline{b}_1 = \underline{0} \tag{2.5.6}$$

where the + (-) sign is selected if the CSM (LM) state vector is being updated, and $r_{CL} = |\underline{r}_{CL}|$

(5) Determine the measured deviation δQ from

$$\begin{aligned} \delta Q &= \cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \cos^{-1} (\underline{u}_s \cdot \underline{u}_{CL}) \\ &= \cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2} \end{aligned} \tag{2.5.7}$$

(6) Incorporate the fictitious star-LM measurement using the Measurement Incorporation Routine (Section 5.2.3).

Included in Step (6) is the state vector update validity check for the first proposed update, as described in Section 5.2.1.

It should be noted that the initialization of the star direction $\underline{u}_s$ which is given by Equation (2.5.3), is such that the first artificial star (computed from Equation (2.5.4)) will yield the maximum value for the measured deviation δQ which is obtained from Equation (2.5.7). The reason for selecting the first $\underline{u}_s$ vector in this manner is that there is only one state vector update validity check even though there are two updates.

Assuming that the first state vector update was valid, the second update for this measurement data set is performed by first recomputing the estimated CSM-to-LM line-of-sight vector from Equation (2.5.2) using the updated values of the estimated CSM and LM position vectors $\underline{r}_C$ and $\underline{r}_L$, respectively. Then, Steps (3) - (6) are repeated, this time with no state vector update validity check.

If the first proposed state vector update does not pass the validity check, then the magnitudes of the proposed changes in the estimated position and velocity vectors δr and δv, respectively, are displayed. If the astronaut is sure that he is tracking the LM, then he should command the update. Otherwise, he should reject the data and recheck the optical tracking. A detailed discussion of this state vector update validity check is given in Section 5.2.1.

The results of the processing of the measured line-of-sight vector $\underline{u}_M$ are updated values of the estimated position and velocity vectors of the CSM or the LM. These two estimated state vectors are used to compute required rendezvous targeting parameters as described in Section 5.4.2.

For convenience of calculation in the CMC, Equations (2.5.4)-(2.5.7) are reformulated and regrouped as follows:

$$
\begin{aligned}
\underline{u}_s &= \text{UNIT}\,(\underline{u}_s \times \underline{u}_{CL}) \\
\underline{b}_0 &= \pm\, \underline{u}_s \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= r_{CL} \left[\cos^{-1} (\underline{u}_s \cdot \underline{u}_M) - \frac{\pi}{2} \right]
\end{aligned}
\qquad (2.5.8)
$$

This set of equations is used both by the Rendezvous Navigation Routine and the Orbit Navigation Routine (Section 5.2.4) in processing optical tracking data. To validate the use of Equations (2.5.8) it is necessary only to let

$$\overline{\alpha^2} = r_{CL}^{\ 2}\, (\text{var}_{SXT} + \text{var}_{IMU}) + \text{var}_{INT} \qquad (2.5.9)$$

where var_{SXT} and var_{IMU} are the a priori estimates for the SXT and IMU angular error variances per axis, respectively. The variable var_{INT} is included in Equation (2.5.9) for the purpose of smoothing the effects of coasting integration inaccuracies.

After VHF range-link acquisition is established (Section 5.2.5.1) the measured CSM-to-LM range (R_M) is automatically acquired at approximately one-minute intervals (when on-board estimate of range is less than 327.67 n.mi.). The geometry vector and measured deviation for a VHF range measurement are given by

$$\underline{b}_0 = \pm \underline{u}_{CL}$$
$$\underline{b}_1 = \underline{0} \qquad (2.5.10)$$
$$\delta Q = R_M - r_{CL}$$

where the + (-) sign is selected if the LM (CSM) state vector is being updated. The measurement error variance is computed from

$$\overline{\alpha^2} = \text{maximum} \left(r_{CL}^2 \; var_R + var_{INT} , \; var_{Rmin} \right) \qquad (2.5.11)$$

where var_R is the range error variance corresponding to a percentage error and var_{Rmin} is the minimum range error variance.

The rendezvous navigation computations are illustrated in Figure 2.5-4. As shown in the figure, this set of computations is entered from two points (Ⓔ and Ⓖ) of Figure 2.5-3. It is assumed that the following items are stored in erasable memory at the start of the computation shown in the figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5.2.2.6.

$\underline{x}_L$ = Estimated LM state vector

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ or $\underline{x}_L$ as defined in Section 5.2.2.4.

WRDTIME = Specified minimum time between W-matrix reinitializations.

MINBLKTM (MAXBLKTM) = Specified maximum time after a mark that a W-matrix reinitialization can be performed.

TBEFCOMP = Specified minimum time before final targeting solution that W-matrix reinitializations will be allowed.

BRNBLKTM = Estimated total time without marking before and after a maneuver.

MAXWTIME = Specified maximum time between W-matrix reinitializations.

FINCMTM = Specified maximum time before a maneuver that a mark can be taken.

The following definitions apply:

$$\text{RENDWFLG} = \begin{cases} 1 & \text{for valid W-matrix} \\ 0 & \text{for invalid W-matrix} \end{cases}$$

This flag or switch is maintained by programs external to the Rendezvous Navigation Routine. It indicates whether or not the existing W-matrix is valid for use in processing LM tracking data. The flag is set to zero after each of the following procedures:

1) State vector update from ground
2) Orbit or Cislunar-Midcourse Navigation
3) Astronaut Command
4) Selection of P24

ORBWFLAG = Switch similar to RENDWFLG but used for orbit or cislunar-midcourse navigation.

[REFSMMAT] = Transformation Matrix: Basic Reference Coordinate System to IMU Stable Member Coordinate System.

t_F = Measurement time.

Five optics and IMU gimbal angles (or) R_M = Measured Range.

w_{rr} and w_{rv} = Preselected W-matrix initial diagonal elements. There is one value for each of these two initial diagonal W-matrix elements stored in the CMC erasable memory. These parameters nominally represent rendezvous injection conditions. They can be changed during the mission by the astronaut or by RTCC. During MINKEY, these parameters are changed automatically at the completion of each targeting program.

SC = Source Code: 1 Optics Measurement; 2 Range Measurement

MANEUFLG (maneuver has been performed)—set on final pass through any rendezvous targeting program except P36

TPIMNFLG (TPI has been performed)—set on final pass through P34

P35FLAG (MCC1 has been performed)—set on final pass through P35

MARKTIME —time of current mark

AGEOFW—time of last W-matrix reinitialization

OLDMKTME—time of last mark

PTV93FLG—causes W-matrix to be reinitialized on first mark after a maneuver

FULTKFLG—(single-sensor tracking denoted)—set by crew's VERB 57 ENTR.

t_{IG} —time of ignition

COUNT3MK—mark counter, counts up to three marks

The variables D and V are indicators which control the Coasting Integration Routine (Section 5.2.2) as described in Section 5.2.2.6, and I and O are the three-dimensional identity and zero matrices, respectively.

The optical measurement incorporation procedure outlined above should be repeated at about one minute intervals throughout the rendezvous phase except during powered maneuvers, as described in Section 5.2.5.1. As is indicated in Section 5.2.5.1, to achieve desired state-vector accuracy, it is important that SXT tracking data be taken over the largest possible angular sector in inertial space (as swept out by SXT line-of-sight). This is required since the SXT tracking provides information only in directions normal to the line-of-sight (as indicated by Equation 2.5.5), and thus the line-of-sight must be allowed to rotate in inertial space to achieve more complete update data. As mentioned previously, the VHF range data are obtained automatically, approximately every minute.

5.2.5.2.3 Rendezvous Navigation Computations (Alternate Line-of-sight). —During rendezvous, optical tracking data are normally obtained by means of SXT sightings of the LM from the CSM and processed as described in Section 5.2.5.2.2. Navigation data can also be obtained by means of a backup optical device. However, since data obtained in this manner are much less accurate than SXT sighting data, the backup device should be used for the sightings only if the SXT has failed, or if the astronaut cannot return to the lower equipment bay to make SXT sightings.

The processing of the data from a backup sighting is identical to the procedure described in Section 5.2.5.2 with the following two exceptions:

① The values of the shaft and trunnion angles associated with the particular device used are astronaut input items.

② The measurement error variance, Equation (2.5.9), is replaced by

$$\overline{\alpha^2} = r_{CL}^2 \left(var_{ALT} + var_{IMU}\right) + var_{INT} \qquad (2.5.12)$$

where var_{ALT} is the a priori estimate for the angular error variance of an alternate line-of-sight measurement per axis.

5.2.5.3 Automatic W-matrix Reinitialization (WRI)

The error-transition (W) matrix is defined in Section 5.2.2.4. In the P20 Rendezvous Mode, a capability has been provided for a minimum-keystroke-sequenced (MINKEY) rendezvous. (See GSOP Section 4 and "Users' Guide to APOLLO GNCS Major Modes and Routines," E-2448, paragraph 4.2.1.2.6.) A salient feature of MINKEY is automatic W-matrix reinitialization (WRI). The considerations underlying the automatic W-matrix reinitialization logic in Figure 2.5-4 are (1) to avoid premature reinitializations, which would degrade the error correction and correlation, (2) to avoid late reinitializations, which would allow the W-matrix to become superannuated, i. e., to shrink to the point that incoming measurements are downweighted excessively or to disintegrate in accuracy to the point that the matrix is no longer reliable.

Loss of filter gain is a function of the measurement geometry and the number of marks taken since the last WRI and is the result of incomplete error modeling; loss of reliability is a function of time since the last WRI and is the result of incorrect error modeling due to computer storage limitations. Since geometry is a factor in the loss of gain, determining when the loss will occur requires a bit-by-bit analysis of simulated computer operations for a particular mission. Determining how long the W-matrix can be extrapolated before it becomes unreliable is more difficult, since a major factor is the accuracy of the state vector which is not being updated. A generalized scheme that can be mechanized for all contingencies is possible, however, by following a minmax concept of reinitializing more often than may be necessary but less often than would significantly degrade performance.

Accordingly, the principal criterion for allowing a WRI after other than a first mark following a maneuver is whether a specified minimum time (WRDTIME) has elapsed since the last WRI. Typically, WRDTIME is about 25 minutes—long enough for effective correlation and smoothing, short enough to occur well before superannuation. The second criterion is whether no more than a specified maximum time (MAXBLKTM, or MINBLKTM, typically 5 minutes) has elapsed since the last mark. If more than MAXBLKTM has elapsed since the last mark (VHF or Optics), WRI will be inhibited for three marks in order to allow W-matrix correlation to reduce error buildup during the no-mark period. The third criterion for allowing WRI is whether more than a specified minimum time (TBEFCOMP) remains before the final targeting computation. For example, TBEFCOMP will have a preset value of about 10–15 minutes, allowing a sufficient period of post-WRI tracking to provide a correlated W-matrix and a best state-vector estimate for the targeting computation. If the other criteria have been satisfied and more than TBEFCOMP remains before final targeting, WRI is allowed and the mark is incorporated.

Before transfer-phase initiation (TPI), the nominal flow for a first mark following a maneuver is to inhibit WRI until three marks have been accumulated for correlation. The exception to this is when, as a result of the TBEFCOMP restriction preceding a maneuver, the W-matrix age would exceed MAXWTIME (~ 1 hour) should WRI be inhibited. For this contingency, WRI is specified to occur before the first mark is incorporated.

The other condition when WRI occurs before the first mark is incorporated is in the nominal-flow situation following TPI. The post-TPI geometry is such that filter gain deteriorates more rapidly than in the pre-TPI phase. Consequently, with both VHF and SXT operating, the nominal post-TPI procedure gives higher priority to restoring filter gain than to correlation: WRI occurs on the first mark, and is prohibited on subsequent marks.

Correlation in the unmeasured dimension becomes obligatory, however, when data are being incorporated from only one sensor. The post-TPI, single-sensor strategy is to reinitialize the W-matrix only once between TPI and the second midcourse correction (MCC2). The strategy is mechanized as follows. The first mark following TPI is incorporated without WRI. Since subsequent marks between TPI and the first midcourse correction (MCC1) will not pass the TBEFCOMP criterion, no WRI can occur during the TPI-MCC1 phase. Following MCC1, however, a WRI is specified to occur after the first three marks have been incorporated—regardless of TBEFCOMP. The single WRI, occurring three marks after MCC1, ensures the best post-TPI trade-off between the requirements of one-sensor correlation and filter gain.

5.2.6 CISLUNAR-MIDCOURSE NAVIGATION ROUTINE

5.2.6.1 General Comments

During the midcourse phase of the lunar mission, navigation data can be obtained by the measurement of the angle between the directions to a star and a planetary horizon or landmark, as described in Section 5.2.1. This routine is used to process the star-landmark/horizon measurement data, as illustrated in simplified form in Fig. 2.1-3, and is normally used only in an abort situation in conjunction with a return-to-earth targeting and maneuver procedure after the loss of ground communication. The Return-to-Earth Routine (Section 5.4.3) and this routine provide the CMC with the capability for guiding the CSM back to the earth and to safe entry conditions.

The acquisition of the star and landmark/horizon may be accomplished either automatically or manually. In the manual mode it is not necessary to have the IMU aligned or even on for this measurement since only the optics trunnion angle is used as measurement data. In the automatic acquisition mode, however, the IMU must be on and aligned prior to the initiation of this routine.

In the processing of the navigation data, it is necessary to distinguish between earth and lunar measurements, and between primary and secondary body measurements. This is accomplished by means of the variable Z, which denotes measurement planet, and which is part of the data loaded by the astronaut after the measurement. Also included in the data load are star and landmark or horizon identification.

5.2.6.2 Star-Landmark Measurement

Let $\underline{r}_C$ and $\underline{v}_C$ be the estimated CSM position and velocity vectors and W the error transition matrix. The SXT star-landmark angle measurement processing procedure is as follows:

1 Use the Coasting Integration Routine (Section 5.2.2) to extrapolate the estimated CSM state vector and the W matrix to the time of the measurement obtaining $\underline{r}'_C$, $\underline{v}'_C$, and W'.

2 Let $\underline{r}_{ZC}$ be the estimated CSM position vector relative to the measurement planet Z. Then

$$\underline{r}_{ZC} = \begin{cases} \underline{r}'_C & \text{if } Z = P \\ \underline{r}_{QC} & \text{if } Z = Q \end{cases} \tag{2.6.1}$$

where $\underline{r}_{QC}$ is the estimated position of the CSM relative to the secondary body Q, and is computed as described in Sec. 5.2.2.3.

3 Compute $\underline{r}_\ell$, the location of the landmark at the measurement time, by means of the Latitude-Longitude Subroutine (Section 5.5.3).

4 Compute the estimated pointing vector from

$$\begin{aligned} \underline{r}_{CL} &= \underline{r}_\ell - \underline{r}_{ZC} \\ \underline{u}_{CL} &= \text{UNIT}(\underline{r}_{CL}) \end{aligned} \tag{2.6.2}$$

(5) Let $\underline{u}_S$ be a unit vector in basic reference coordinates which defines the direction to the star whose coordinates are in the CMC fixed memory or were loaded by the astronaut. It is necessary to correct the star vector for aberration, i.e., the change in the observed star direction caused by velocity perpendicular to the direction. The observed star direction, $\underline{u}_S^*$, is given by

$$\underline{u}_S^* = \text{UNIT}\left(\underline{u}_S + \frac{\underline{v}'_C - \underline{v}_{ES}}{c}\right) \qquad (2.6.3)$$

where $\underline{v}_{ES}$ is the velocity of the sun relative to the earth, and c is the speed of light. The velocity vector of the sun relative to the earth, $\underline{v}_{ES}$, is assumed constant for the duration of the mission, as described in Section 5.5.4. The velocity of the moon relative to the earth, for the case in which the moon is the primary body, is negligible. The coordinates of a planet should not be loaded by the astronaut for cislunar navigation unless they correctly indicate the direction of the planet with respect to the CSM and allowance has been made for the difference in aberration between that indicated in Eq. 2.6.3 and that which truly exists for the planet.

(6) Correct $\underline{u}_{CL}$ for aberration as follows:

$$\underline{u}_{CL}^* = \text{UNIT}\left(\underline{u}_{CL} + \frac{\underline{v}'_C}{c}\right) \qquad (2.6.4)$$

(7) Then, if A is the measured angle, the six dimensional geometry vector, $\underline{b}$, and the measured deviation, δQ, are given by

$$COSQ = \underline{u}_s^* \cdot \underline{u}_{CL}^*$$

$$\underline{b}_0 = \frac{1}{r_{CL}} \text{ UNIT } (\underline{u}_s^* - COSQ\ \underline{u}_{CL}^*)$$

$$\underline{b}_1 = \underline{0} \qquad (2.6.5)$$

$$\delta Q = A - \cos^{-1} (COSQ)$$

(8) Incorporate the measurement into the CSM state vector estimate by means of the Measurement Incorporation Routine (Section 5.2.3) after astronaut approval.

5.2.6.3 Star-Horizon Measurement

The processing of a star-horizon measurement is the same as that of a star-landmark measurement except for Step (3) above. The estimated location of the landmark (horizon) $\underline{r}_\ell$ must be obtained from geometrical considerations.

Referring to Fig. 2.6-1, it is seen that the star unit vector, $\underline{u}_s$, and the estimated CSM position vector, $\underline{r}_{ZC}$, determine a plane. Assuming that the measurement planet is the earth and that the horizon of the earth is at a constant altitude, the intersection of this plane and the horizon of the earth is approximately an ellipse, called the horizon ellipse.

To determine the orientation of the horizon ellipse, define the following three mutually orthogonal unit vectors:

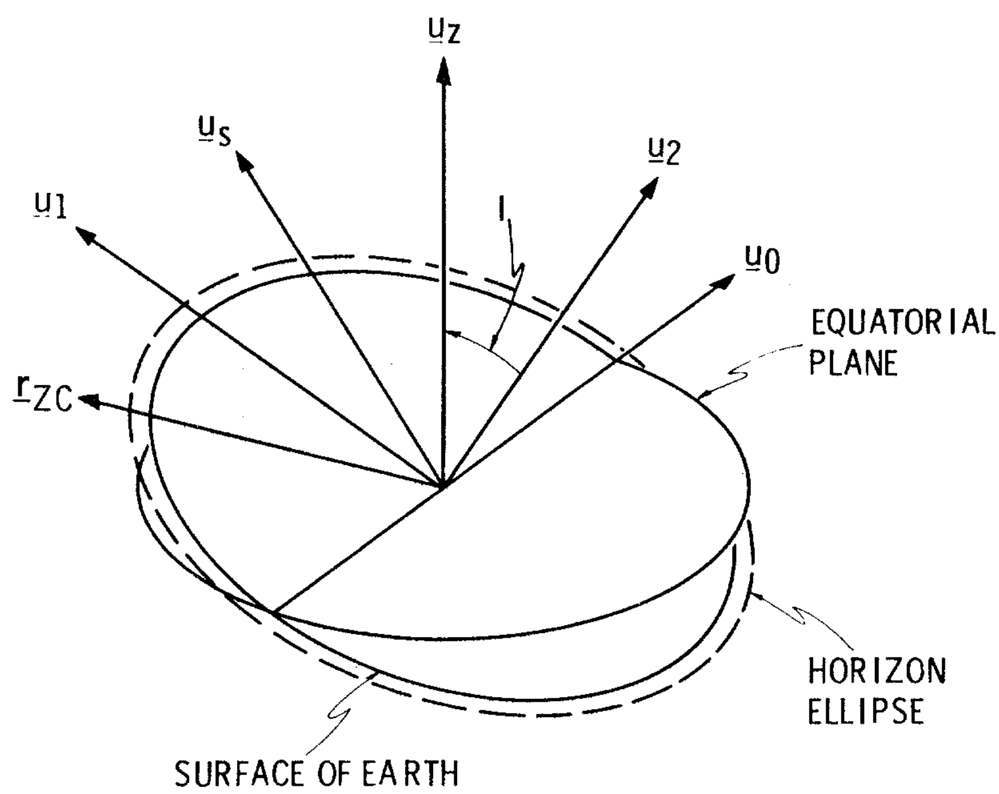


Figure 2. 6-1 Definition of Horizon Coordinate System

$$\underline{u}_2 = \text{UNIT}(\underline{u}_s \times \underline{r}_{ZC})$$

$$\underline{u}_0 = \text{UNIT}(\underline{u}_Z \times \underline{u}_2) \qquad (2.6.6)$$

$$\underline{u}_1 = \underline{u}_2 \times \underline{u}_0$$

where $\underline{u}_Z$ is a unit vector along the earth's polar axis and is given by

$$\underline{u}_Z = \begin{pmatrix} A_Y \\ -A_X \\ 1 \end{pmatrix} \qquad (2.6.7)$$

The angles A_X and A_Y are defined in Section 5.5.2. Then, as seen in Fig. 2.6-1, the vectors $\underline{u}_0$ and $\underline{u}_1$ are along the semi-major and semi-minor axes of the horizon ellipse, respectively; and $\underline{u}_2$ is perpendicular to the ellipse. The inclination angle I of the horizon ellipse with respect to the equatorial plane of the earth is obtained from

$$\sin I = \underline{u}_1 \cdot \underline{u}_Z \qquad (2.6.8)$$

The shape of the ellipse is defined by its major and minor axes. The semi-major axis a_H is given by

$$a_H = a + h \qquad (2.6.9)$$

$$h = h_0 - h_1 r_{ZC} \qquad (2.6.9a)$$

where a is the semi-major axis of the Fischer ellipsoid and h is the horizon altitude from Eq. (2.6.9a). This is an empirically derived horizon model. Quantities h_0 and h_1 are in erasable memory. Let r_F be the radius of the Fischer ellipsoid at the latitude equal to the inclination angle of the horizon ellipse computed from Eq. (5.3.1) of Section 5.5.3. Then, the semi-minor axis of the ellipse, b_H, is obtained from

$$b_H = r_F + h \tag{2.6.10}$$

The problem of determining the vector $\underline{r}_\ell$ can now be reduced to a two dimensional one. Define the Horizon Coordinate System to have its X- and Y-axes along $\underline{u}_0$ and $\underline{u}_1$, respectively, as illustrated in Fig. 2.6-2. Let

$$M = \begin{pmatrix} \underline{u}_0^T \\ \underline{u}_1^T \\ \underline{u}_2^T \end{pmatrix} \tag{2.6.11}$$

The matrix M is the transformation matrix from the Basic Reference Coordinate System to the Horizon Coordinate System. The vectors $\underline{r}_{ZC}$ and $\underline{u}_s$ are transformed to the Horizon Coordinate System as follows:

$$\underline{r}_H = M\,\underline{r}_{ZC}$$

$$\underline{u}_{sH} = M\,\underline{u}_s \tag{2.6.12}$$

Let x_H and y_H be the two non-zero components of $\underline{r}_H$, and let the two points of tangency from $\underline{r}_H$ to the horizon be $\underline{t}_0$ and $\underline{t}_1$.

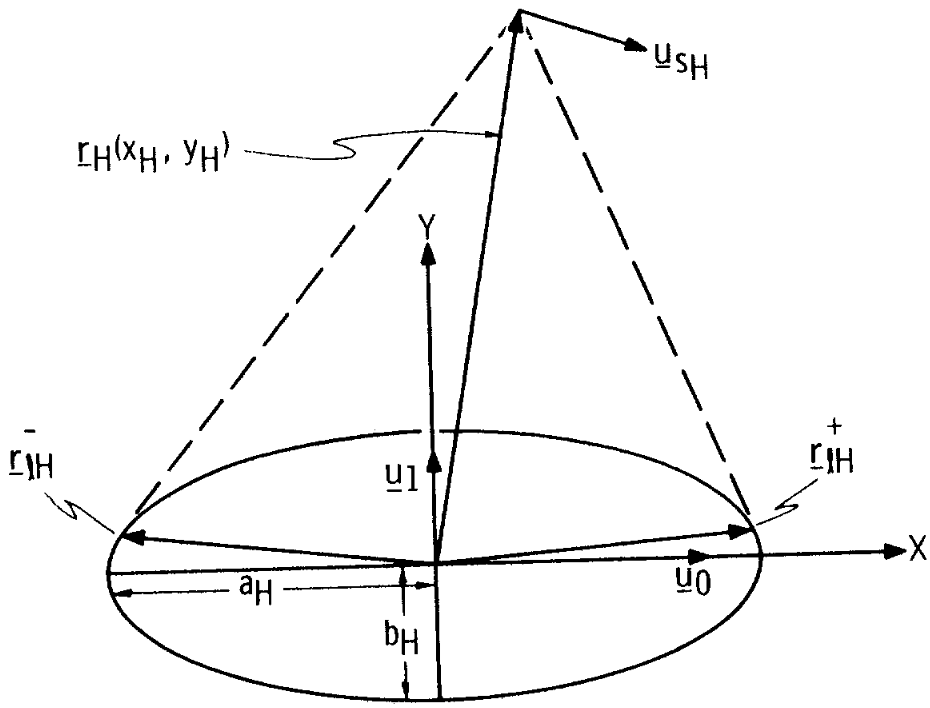


Figure 2.6-2 Geometry of Star-Horizon Measurement

The vectors $\underline{t}_0$ and $\underline{t}_1$ are obtained by solving simultaneously the equation of the horizon ellipse

$$\frac{x^2}{a_H^2} + \frac{y^2}{b_H^2} = 1 \qquad (2.6.13)$$

and the equation of the line which is tangent to the ellipse and which passes through the point (x_H, y_H)

$$\frac{xx_H}{a_H^2} + \frac{yy_H}{b_H^2} = 1 \qquad (2.6.14)$$

The following equations result:

$$\underline{t}_0 = \frac{1}{A} \begin{pmatrix} x_H + \frac{a_H}{b_H} y_H \sqrt{A - 1} \\ y_H - \frac{b_H}{a_H} x_H \sqrt{A - 1} \\ 0 \end{pmatrix} \qquad (2.6.15)$$

$$\underline{t}_1 = \frac{1}{A} \begin{pmatrix} x_H - \frac{a_H}{b_H} y_H \sqrt{A - 1} \\ y_H + \frac{b_H}{a_H} x_H \sqrt{A - 1} \\ 0 \end{pmatrix} \qquad (2.6.16)$$

where

$$A = \frac{x_H^2}{a_H^2} + \frac{y_H^2}{b_H^2} \qquad (2.6.17)$$

The two points of tangency, $\underline{t}_0$ and $\underline{t}_1$, correspond to the two horizon points, $\underline{r}_{\ell H}^{+}$ (near horizon) and $\underline{r}_{\ell H}^{-}$ (far horizon). To determine which $\underline{t}_i$ corresponds to which horizon, compute the two quantities

$$A_i = \underline{u}_{sH} \cdot \text{UNIT}\,(\underline{t}_i - \underline{r}_H) \quad (i = 0,\ 1) \qquad (2.6.18)$$

Then, the $\underline{t}_i$ which yielded the larger A_i is the near horizon and the other is the far horizon. Let $\underline{t}_k$ be the horizon which was used in the measurement. The horizon vector is then

$$\underline{r}_\ell = M^T\, \underline{t}_k \qquad (2.6.19)$$

The measurement processing is completed by following steps (4) - (8) of the star-landmark measurement procedure.

The star-horizon measurement processing procedure has been based upon the assumption that the planet involved in the measurement is the earth. If the moon is the measurement planet, and it is assumed that the moon is a sphere, then the entire procedure presented above is valid except for the computation of a_H and b_H. For a lunar-horizon measurement Eqs. (2. 6. 8) - (2. 6. 10) are replaced by

$$a_H = r_M$$
$$b_H = r_M \qquad (2.6.20)$$

where r_M is the mean radius of the moon.

5.2.6.4 Angle Measurement Processing Logic

The computational logic for the Cislunar-Midcourse Navigation Routine is illustrated in Fig. 2. 6-3. It is assumed that the following items are stored in erasable memory at the start of the computation shown in the figure:

$\underline{x}_C$ = Estimated CSM state vector as defined in Section 5. 2. 2. 6

W = Six-dimensional error transition matrix associated with $\underline{x}_C$ as defined in Section 5. 2. 2. 4

ORBWFLAG = { 1 for valid W matrix; 0 for invalid W matrix }

This flag or switch is maintained by programs external to the Cislunar-Midcourse Navigation Routine. It indicates whether or not the existing W matrix is valid for use in processing star-landmark/horizon angle measurement data. The flag is set to zero after each of the following procedures:

1) CSM state vector update from ground
2) Rendezvous navigation
3) Astronaut Command
4) Selection of P-24

RENDWFLG = Switch similar to ORBWFLAG but used for rendezvous navigation.

t_F = Measurement time

A = Measured angle

$\underline{u}_s$ = Measurement star

Z = Measurement planet = $\begin{cases} 0 \text{ for earth} \\ 1 \text{ for moon} \end{cases}$

L = Landmark switch or flag = $\begin{cases} 1 \text{ for landmark measurement} \\ 0 \text{ for horizon measurement} \end{cases}$

w_{mr} and w_{mv} = Preselected W matrix initial diagonal elements

For convenience of calculation in the CMC, Eqs. (2. 6. 5) are reformulated as follows:

$$
\begin{aligned}
COSQ &= \underline{u}_s^* \cdot \underline{u}_{CL}^* \\
\underline{b}_0 &= \text{UNIT} \left(\underline{u}_s^* - COSQ\, \underline{u}_{CL}^* \right) \\
\underline{b}_1 &= \underline{0} \\
\delta Q &= r_{CL} \left[A - \cos^{-1} (COSQ) \right]
\end{aligned}
\qquad (2.6.21)
$$

To make Eqs. (2. 6. 21) valid it is necessary only to let

$$\overline{\alpha^2} = r_{CL}^2 \, var_{TRUN} + var_L \qquad (2.6.22)$$

where var_{TRUN} and var_L are the a priori estimates of the SXT trunnion-angle and landmark or horizon error variances, respectively.

The variables D and V are indicators which control the Coasting Integration Routine (Section 5. 2. 2) as described in

Section 5.2.2.6, I and O are the three-dimensional identity and zero matrices, respectively, and F is the altitude flag as defined in Section 5.5.3.

The calculation of the estimated pointing vector, $\underline{u}_{CL}$, is illustrated in Fig. 2.6-4. This subroutine is used both in the processing of the navigation data and by Routine R-60 in the automatic acquisition mode to point the SXT landmark line-of-sight at the specified target. This subroutine will, at astronaut option, compute a CSM attitude for Routine R-60 where the shaft angle will be 180 degrees (e.g. so as to prevent an attached LM from occulting the SXT star line-of-sight when it is pointed at the star). This attitude is expressed by a set of IMU gimbal angles which are obtained by using the routine CALCGA of Section 5.6.3.2.2 where the inputs to this routine are the vectors $\underline{x}_{SM}$, $\underline{y}_{SM}$, $\underline{z}_{SM}$, $\underline{x}_{NB}$, $\underline{y}_{NB}$, and $\underline{z}_{NB}$ given in the following matrices:

$$\begin{bmatrix} \underline{x}^T_{SM} \\ \underline{y}^T_{SM} \\ \underline{z}^T_{SM} \end{bmatrix} = [REFSMMAT] \tag{2.6.23}$$

$$\begin{bmatrix} \underline{x}^T_{NB} \\ \underline{y}^T_{NB} \\ \underline{z}^T_{NB} \end{bmatrix} = [SBNB] \begin{bmatrix} \left[UNIT(UNIT(\underline{u}_s^* \times \underline{u}_{CL}) \times \underline{u}_{CL}) \right]^T \\ \left[UNIT(\underline{u}_s^* \times \underline{u}_{CL}) \right]^T \\ \underline{u}_{CL}^T \end{bmatrix} \tag{2.6.24}$$

where [SBNB] is given in Section 5.6.3.1.1, $\underline{u}^*_s$ is the vector to the star in basic reference coordinates and $\underline{u}_{CL}$ is the vector to the landmark or horizon in basic reference coordinates.

Finally, there is available either the landmark coordinates or the far horizon flag H defined by

$$H = \begin{cases} 1 & \text{for far horizon} \\ 0 & \text{for near horizon} \end{cases}$$

In the case of a horizon measurement the computational logic for determining the horizon vector $\underline{r}_\ell$ is shown in Fig. 2.6-5.

The landmark coordinates (latitude, longitude, altitude) are entered into the computer by the astronaut. The altitude is referenced to the Fischer ellipsoid for earth landmarks, and the mean lunar radius for lunar landmarks.

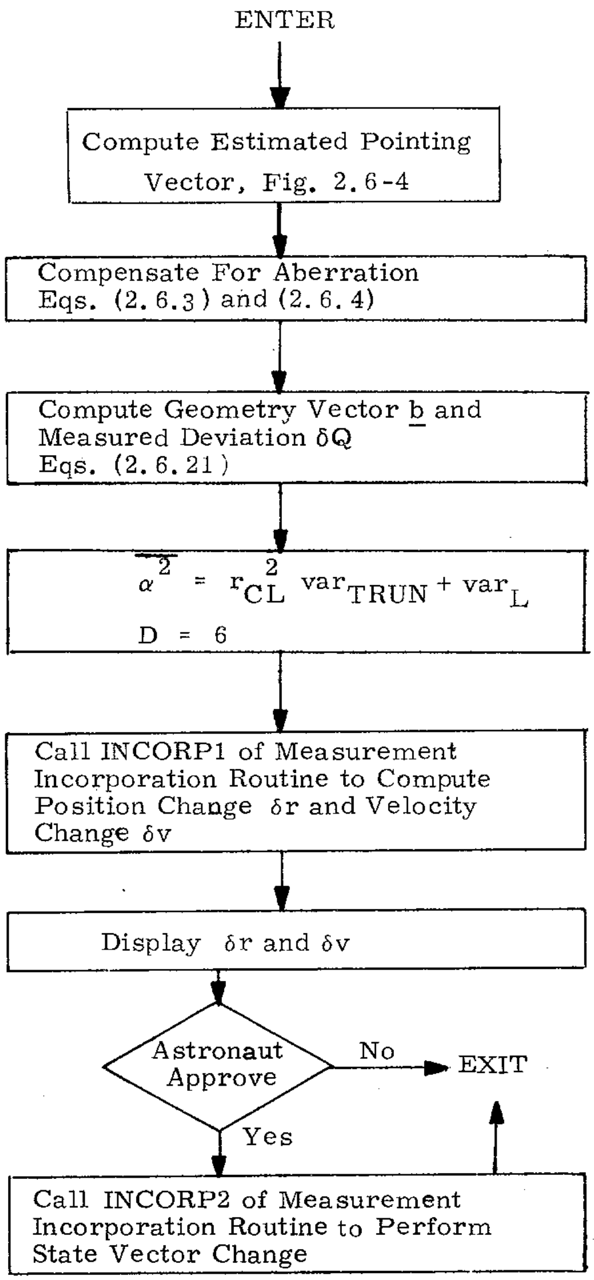


Figure 2.6-3 Cislunar-Midcourse Navigation Routine Logic Diagram

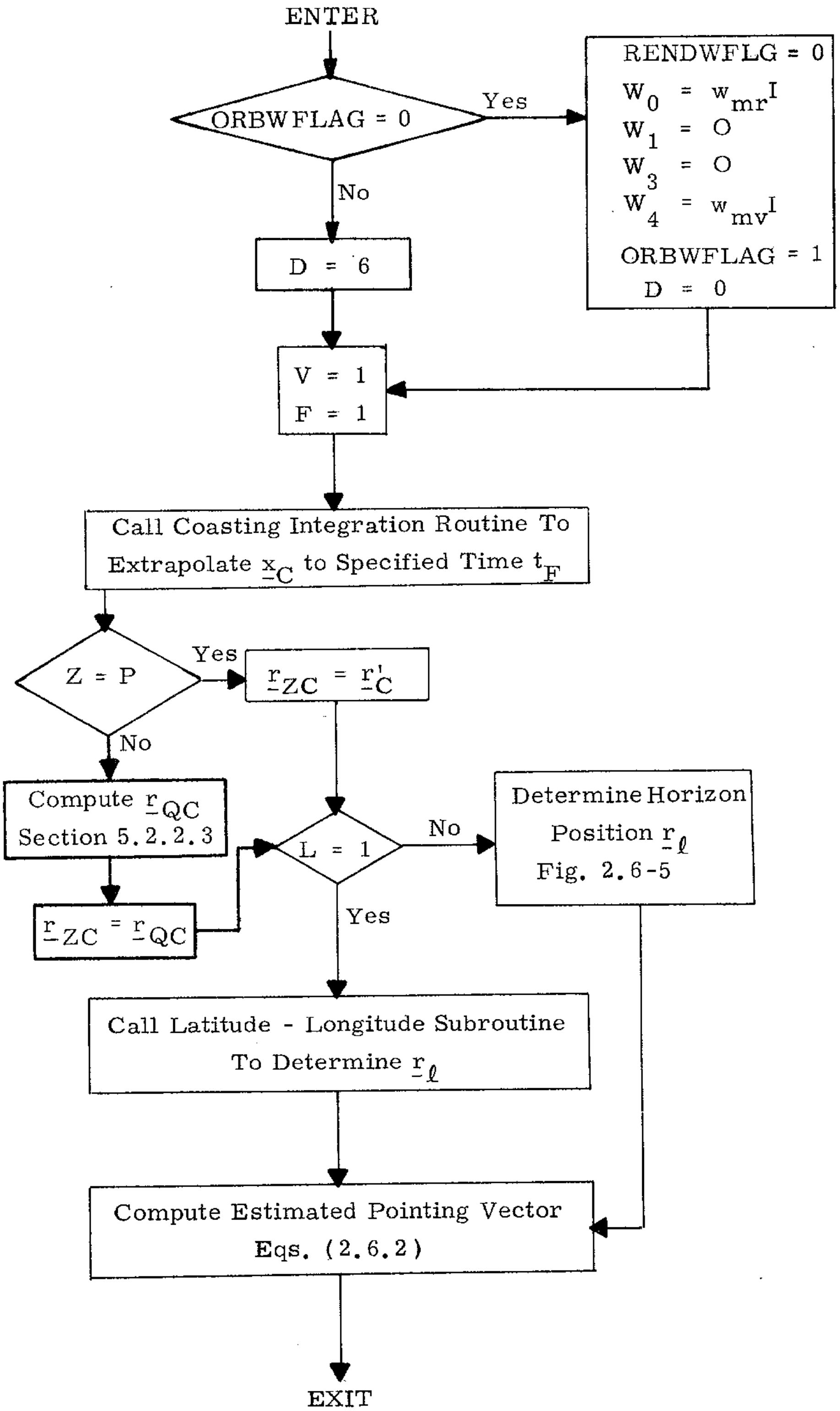


Fig. 2.6-4 Estimated Pointing Vector Computation Subroutine Logic Diagram

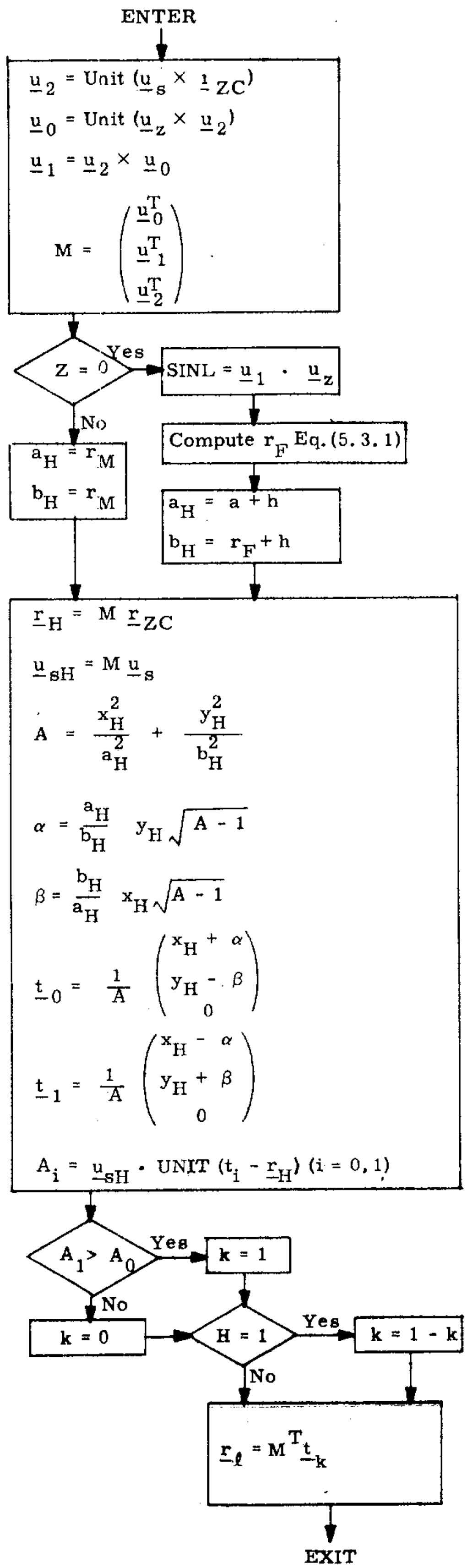


Figure 2.6-5 Horizon Vector Determination Logic Diagram

5.3 POWERED FLIGHT NAVIGATION AND GUIDANCE

5.3.1 GENERAL COMMENTS

The objective of the powered flight guidance routines is to maintain an estimate of the CSM state vector during the thrusting maneuvers, and to control the thrust direction such that the desired velocity cut-off conditions are achieved. The powered flight navigation program used to maintain an estimate of the vehicle state vector during all thrusting conditions is referred to as the Average-G Routine and is presented in Section 5.3.2.

For the Lunar Landing Mission the basic powered flight guidance concept used in the CMC is a velocity-to-be-gained concept with cross product steering (Section 5.3.3.4) which is used in each of the following two procedures:

1. Lambert Aim Point Maneuver Guidance (Section 5.3.3.3.2.).

2. External ΔV Maneuver Guidance (Section 5.3.3.3.1.).

These two procedures, based on the cross product steering concept, differ only in the unique generation of the desired velocity vector, $\underline{v}_R$ and are used to control all CMC guided powered maneuvers for the Lunar Landing Mission.

5.3.2 POWERED FLIGHT NAVIGATION - AVERAGE-G ROUTINE

The purpose of the Powered Flight Navigation Subroutine is to compute the vehicle state vector during periods of powered flight steering. During such periods the effects of gravity and thrusting are taken into account. In order to achieve a short computation time the integration of the effects of gravity is achieved by simple averaging of the gravity acceleration vector. The effect of thrust acceleration is measured by the IMU Pulsed Integrating Pendulous Accelerometers (PIPA) in the form of velocity increments ($\Delta \underline{v}$) over the computation time interval (Δt). The computations are, therefore, in terms of discrete increments of velocity rather than instantaneous accelerations. The repetitive computation cycle time Δt is set at 2 seconds to maintain accuracy and to be compatible with the basic powered flight cycle.

The Average-G Routine, in contrast to the Coasting Integration Routine, is used when a short computing time is required such as during powered flight. The Average-G Routine computations are illustrated in Figs. 3.2-1 and 3.2-2. The following defines the parameters used in these figures:

$\underline{r}(t)$	Vehicle position vector at time t.
$\underline{v}(t)$	Vehicle velocity vector at time t.
P_C	Planet designator $\begin{cases} 0 & \text{Earth} \\ 1 & \text{Moon} \end{cases}$
Δt	Computation cycle of 2 seconds.

$\Delta \underline{v}(\Delta t)$	The velocity vector change sensed by the IMU PIPA's over the time interval Δt. This velocity vector increment is initially sensed in IMU or Stable Member Coordinates and then transformed to the Basic Reference Coordinate System.
$\underline{g}_p(t)$	Previous gravity acceleration vector. This is a required initialization parameter and is supplied by the calling program.
$\underline{u}_r$	Unit vector in the direction of $\underline{r}$.
$\underline{u}_z$	Unit vector of earth's true polar axis in the Basic Reference Coordinate System.
μ_E	Earth gravitational constant.
μ_M	Moon gravitational constant.
r_E	Equatorial radius of the earth.
J	Gravity harmonic coefficient.
$\underline{g}_b(t)$	Component of the earth gravity acceleration vector representing earth oblateness effects.

With reference to Fig. 3.2-2 it can be seen that a single oblateness term is included in the earth gravity subroutine computation, but none for the lunar case.

The PIPA measured velocity $\Delta \underline{v}$ is compensated for instrument errors as described in Section 5.6.13 prior to being transformed into the Basic Reference Coordinate System and processed in the Average-G Subroutine of Fig. 3.2-1.

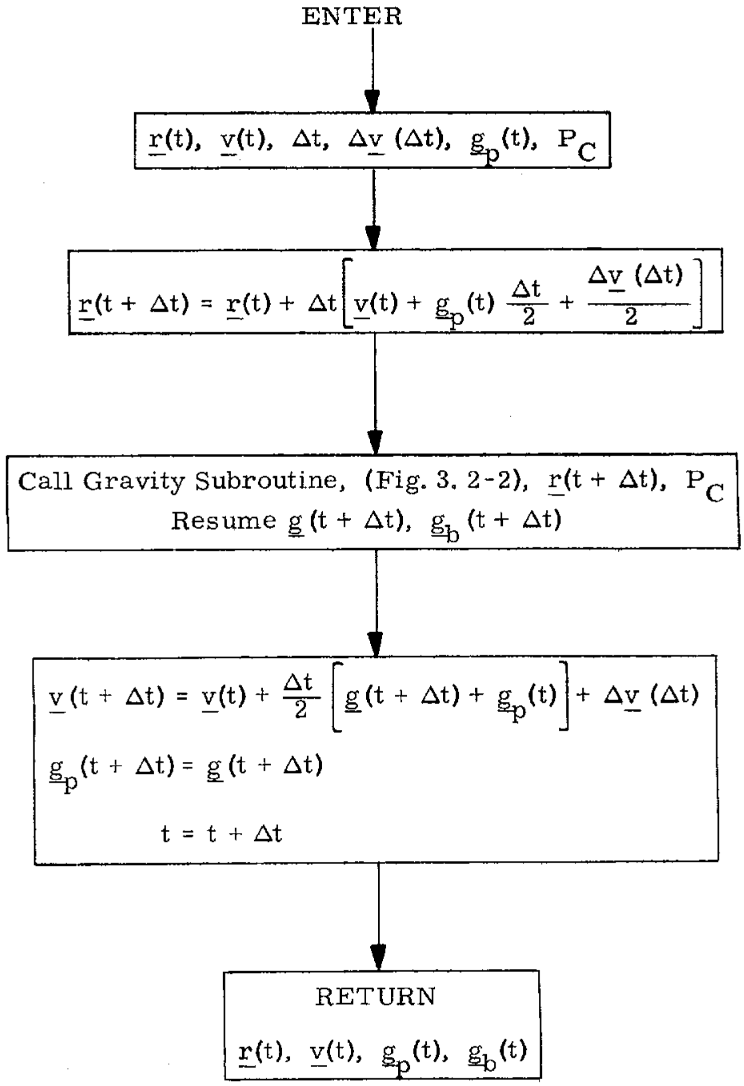


Figure 3. 2-1 Average-G Subroutine

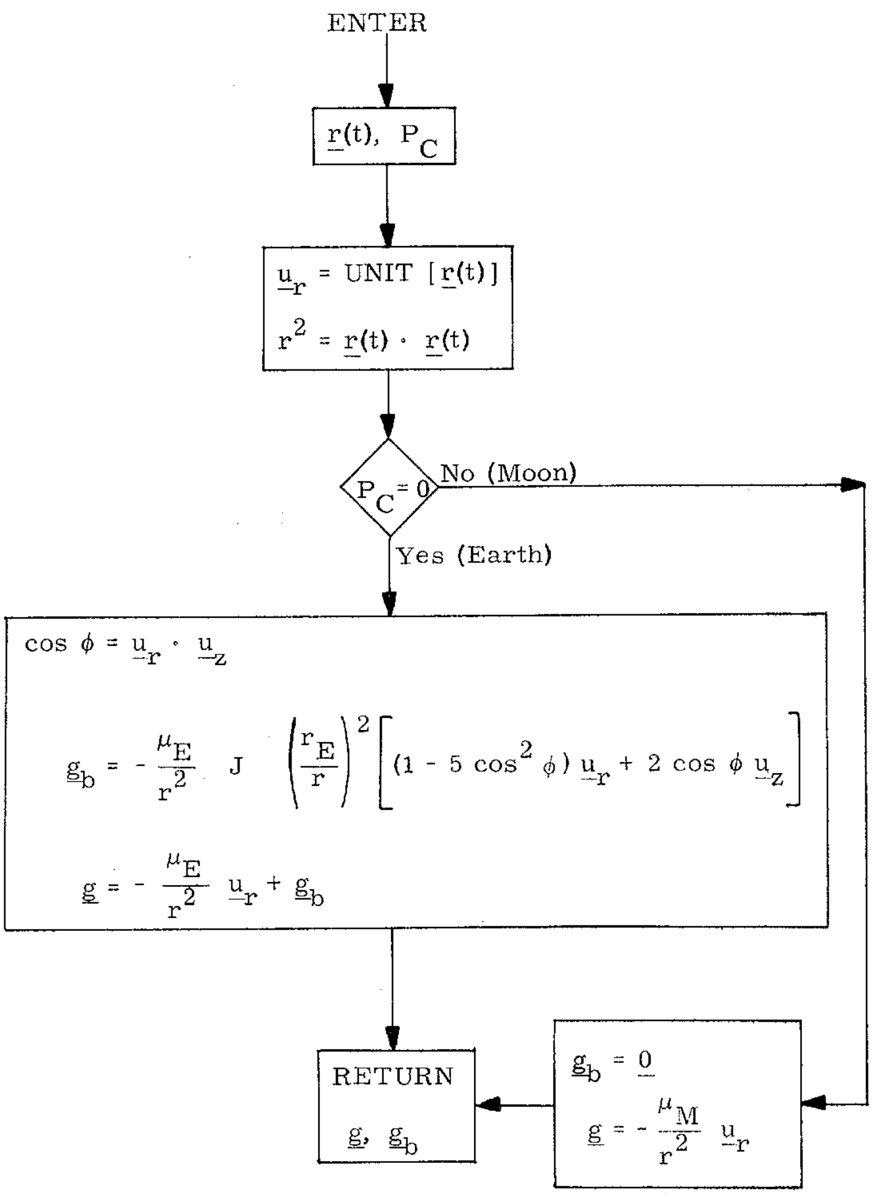


Figure 3. 2-2 Gravity Subroutine

5.3.3 POWERED FLIGHT GUIDANCE USING CROSS PRODUCT STEERING

5.3.3.1 Introduction

The cross product steering concept can be used to control the following maneuvers:

(a) Cislunar Midcourse Corrections, P-30 and RTCC targeted P-40/P-41

(b) Lunar Orbit Insertion (LOI), P-30 and RTCC targeted P-40

(c) Lunar Orbit Plane Change Maneuvers (LOPC), RTCC targeted P-40/P-41

(d) Rendezvous Intercept (P-34) and Midcourse Correction Maneuvers (P-35)

(e) Co-Elliptic Sequence Initiation (CSI) P-32; Constant Delta Altitude (CDH) P-33

(f) Return-to-Earth Maneuvers, P-37

(g) External ΔV and Orbital Phasing Maneuvers (RTCC or LGC Targeted Maneuver) P-30

(h) Transearth Injection (TEI), RTCC targeted P-40

The External ΔV Guidance mode (Section 5.3.3.3.1) is used for maneuvers in which the required velocity-to-be-gained in local vertical coordinates, $\Delta \underline{v}_{LV}$, is specified by P31, P32, P33, P36 or a source external to the CMC by use of program P-30. All other maneuvers are controlled by the Lambert Aim Point Maneuver Guidance Mode (Section 5.3.3.3.2) in which the required cut-off velocity, $\underline{v}_R$, is periodically computed by the Lambert subroutine during the maneuver to establish the desired intercept trajectory. Both External ΔV and Lambert Aim Point Guidance modes use the cross product steering concept to control the thrust direction along the velocity-to-be-gained vector, and terminate thrust when the desired velocity increment has been achieved.

Three subroutines are used repetitively in sequence (Section 5. 3. 3. 2) during cross product controlled maneuvers to accomplish this function. These are:

1. The Powered Flight Navigation Average-G Routine which computes the state vector accounting for the effects of thrust acceleration and gravity.

2. The Cross-Product Steering Subroutine which has 4 functions:

   a) incremental updating of the velocity-to-be-gained vector.

   b) generation of steering commands to the vehicle autopilot.

   c) computation of time-to-go before engine shut-off and the issuance of engine-off commands.

   d) Updating of the CSMMASS.

3. The Velocity-to-be-Gained Subroutine which repetitively solves the Lambert intercept problem when in the Lambert Aim Point guidance mode.

The Average-G Routine is described in Section 5. 3. 2. The other subroutines listed above are described in Sections 5. 3. 3. 4 to 5. 3. 3. 6. The Pre-Thrust Subroutines of Section 5. 3. 3. 3 initialize the powered maneuver programs for either the External ΔV or Lambert Aim Point guidance modes, and for the selected engine for the maneuver.

The CMC powered flight programs described by the computation subroutines presented in Section 5.3.3 are

P-40 Service Propulsion System (SPS) Thrust Prog.

P-41 Reaction Control System (RCS) Thrust Prog.

Active steering and engine-off commands are provided by program P-40. Maneuvers using RCS translation control (P-41) are manually controlled and terminated by the astronaut while the CMC displays the required velocity-to-be-gained in control coordinates (Section 5.3.3.3.3).

The functions of the External ΔV Pre-Thrust Program, P-30 are described in the pre-thrust subroutine description of Section 5.3.3.3.1.

5.3.3.2 Powered Flight Guidance Computation Sequencing

The time sequencing of SPS powered flight subroutines for External ΔV steering is illustrated in Figs. 3.3-1 thru 3.3-3, and for Lambert Aim Point steering in Figs. 3.3-4 thru 3.3-6. These figures represent the sequence of operations for P-40 SPS maneuvers lasting longer than 6 seconds. RCS controlled maneuvers (P-41) require manual steering control, and the general timing sequence is different from that described for SPS maneuvers. The general cross product steering concept is used to compute and display the required velocity-to-be-gained vector, but no engine-off computations are made for RCS maneuvers. The following description of sequence operations is restricted to SPS Maneuvers.

Figures 3.3-1 and 3.3-4 show the sequencing during the ignition count down which starts approximately 30 seconds before the nominal ignition time. Figures 3.3-2 and 3.3-5 show the normal sequencing for an engine-on time greater than 6 seconds as predicted by the pre-thrust subroutines. Figures 3.3-3 and 3.3-6 illustrate the sequencing during engine thrust termination.

The basic computation cycle time of the steering is 2 seconds and, as shown on the above figures, is initiated by the reading of the PIPA $\Delta \underline{v}$ registers. The various subroutines utilized during the 2 second cycle are sequenced in time as shown.

During Lambert Aim Point steering, however, the velocity-to-be-gained ($\underline{v}_G$) updating is anticipated to occur nominally every four seconds. However during SPS maneuvers the CMC computation occasionally prevents completion of the Lambert solution in 2 to 4 seconds. The program is capable of processing an N cycle update where N can be greater than 2. (In most cases, the time sequencing diagrams show only the nominal two and four second $\underline{v}_G$ updates.) This allows sequenced parallel computing functions, such as the demands of the autopilots and telemetry, to take place without interference with the basic two second cycle time. Should the $\underline{v}_G$ computation be completed in time for use during a given 2 second cycle, it will be utilized.

If the calculations conducted prior to the thrusting period (pre-thrust) indicate that the maneuver objective can be reached with a 6 second thrust period or less, provision is included (switches s_I and s_W) to preclude the generating of steering commands during the thrusting period with the engine-off signal set to be issued at the end of the estimated thrust period.

In addition to timing information, the sequence diagrams of Figs. 3.3-1 to 3.3-6 also show the basic information utilized by each subroutine and its source.

The guidance computer program which controls the various subroutines to create a powered flight sequence is called the Servicer Routine. The sequence diagrams of Figs. 3.3-1 to 3.3-6 define what the Servicer Routine does, but do not show the logic details of how these functions are accomplished.

The subroutines listed in Figs. 3.3-1 to 3.3-6 are described in Section 5.3.2 and the following Sections 5.3.3.3 to 5.3.3.6. These sections should be referenced in tracing the powered flight computation sequencing.

Ullage will be turned off at the fixed time of 2 seconds from engine-on command. Steering will be enabled (s_W set to 1) at the fixed time of 2 seconds from engine-on command.

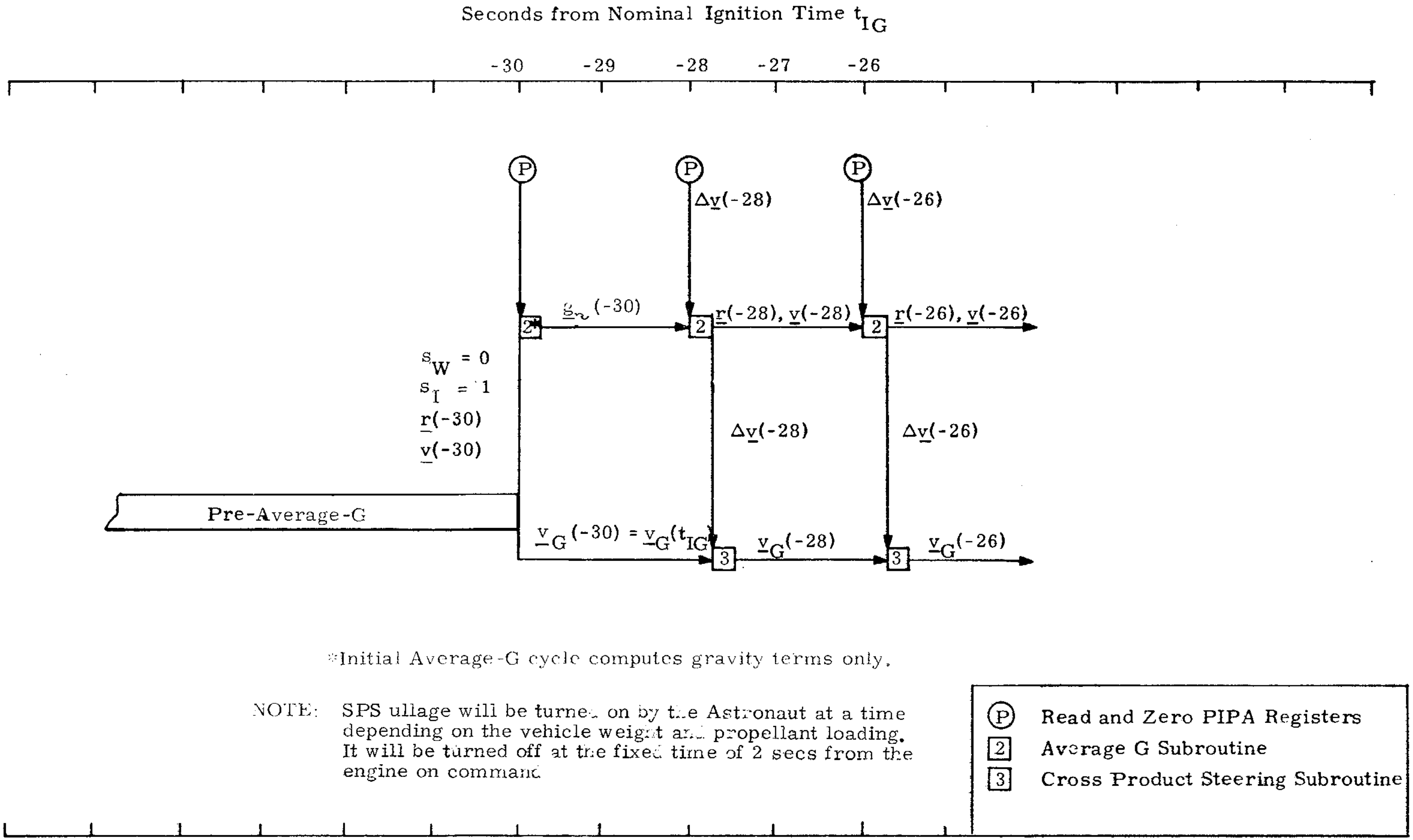


Fig. 3.3-1 Ignition Countdown - External ΔV Subroutine Sequencing for SPS Maneuvers

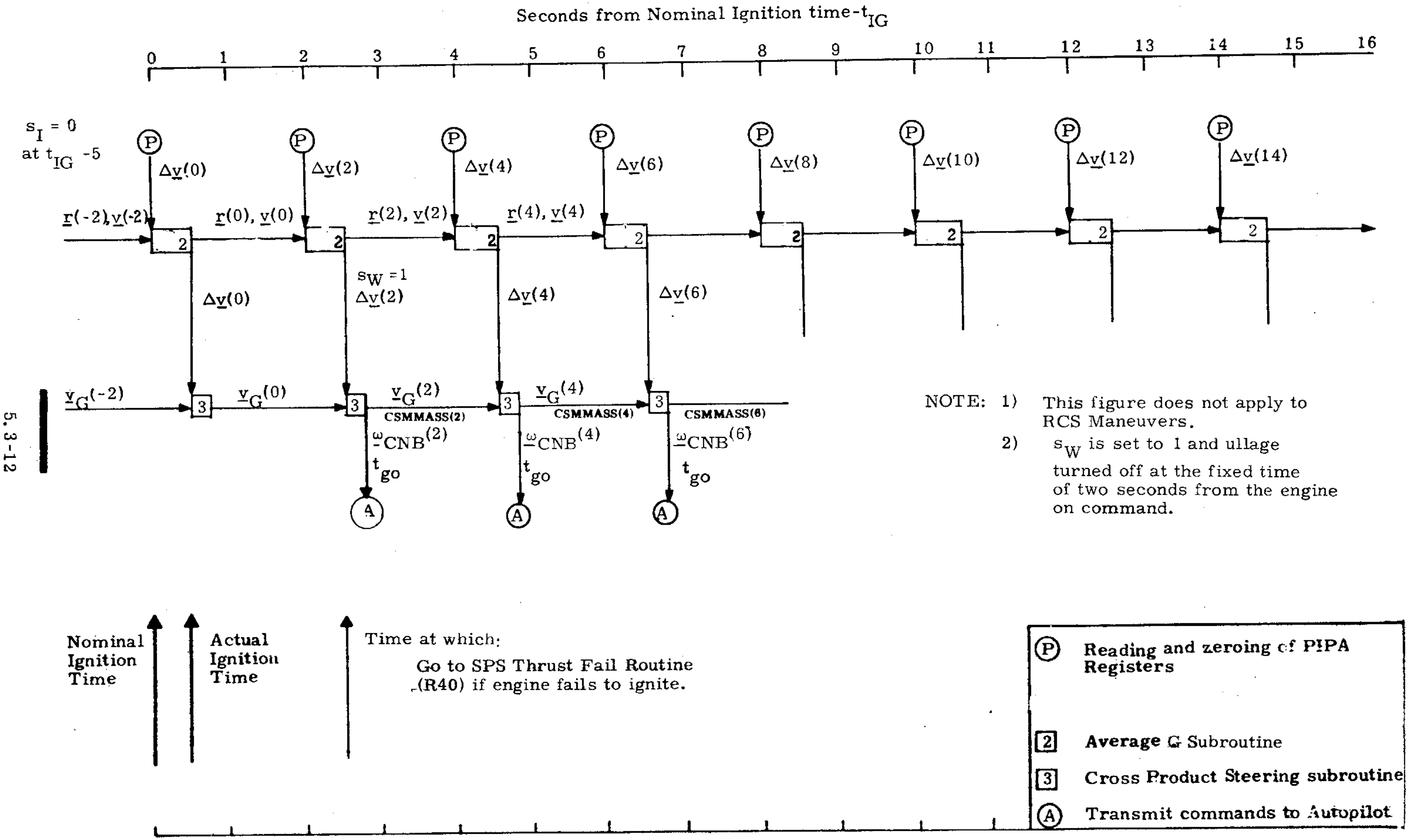


Fig. 3. 3-2 Normal External ΔV Subroutine Sequencing (Maneuver time greater than 6 sec) for SPS Maneuvers

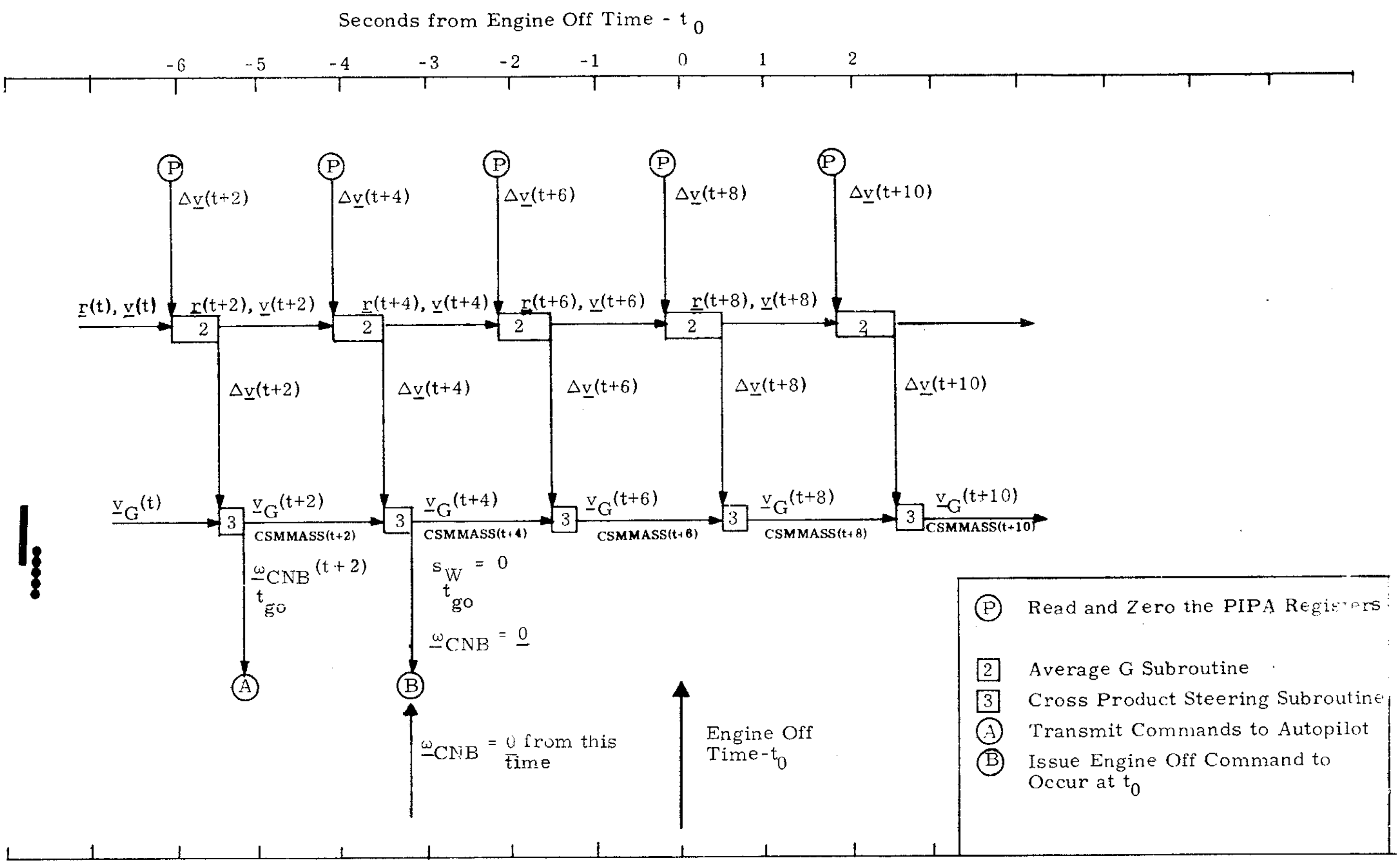


Fig. 3.3-3 Engine Off External ΔV Subroutine Sequencing for SPS Maneuvers

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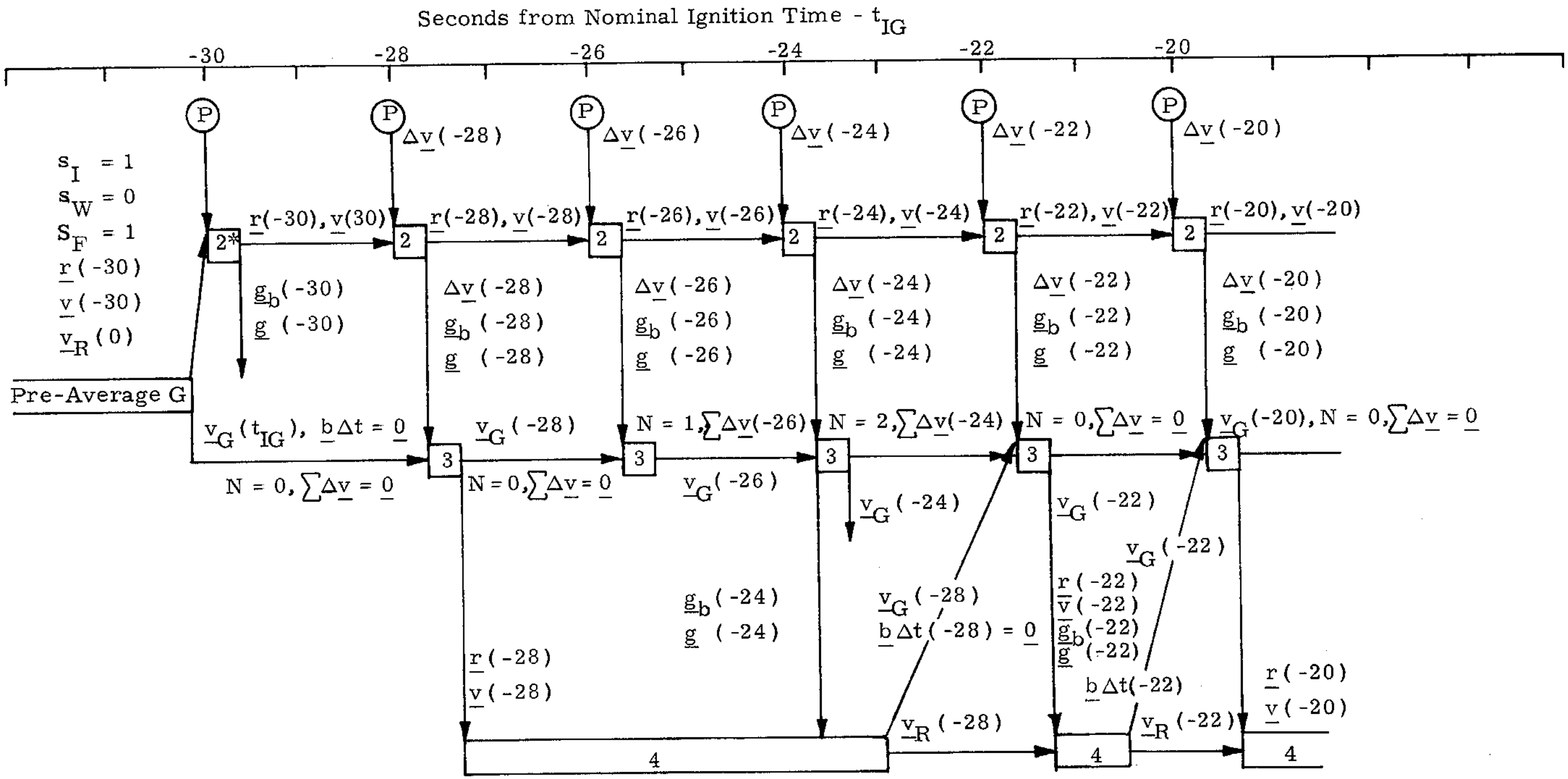


*Initial Average-G cycle computes gravity terms only.

NOTE: SPS ullage will be turned on by the Astronaut at a time depending on the vehicle weight and propellant loading. It will be turned off at the fixed time of 2 secs from the engine-on command.

(P)	Read and Zero Pipa Registers
2	Average G Subroutine
3	Cross Product Steering Subroutine
4	Velocity to be Gained Subroutine

Fig. 3.3-4 Ignition Countdown - Lambert Aim Point Subroutine Sequencing for SPS Maneuvers

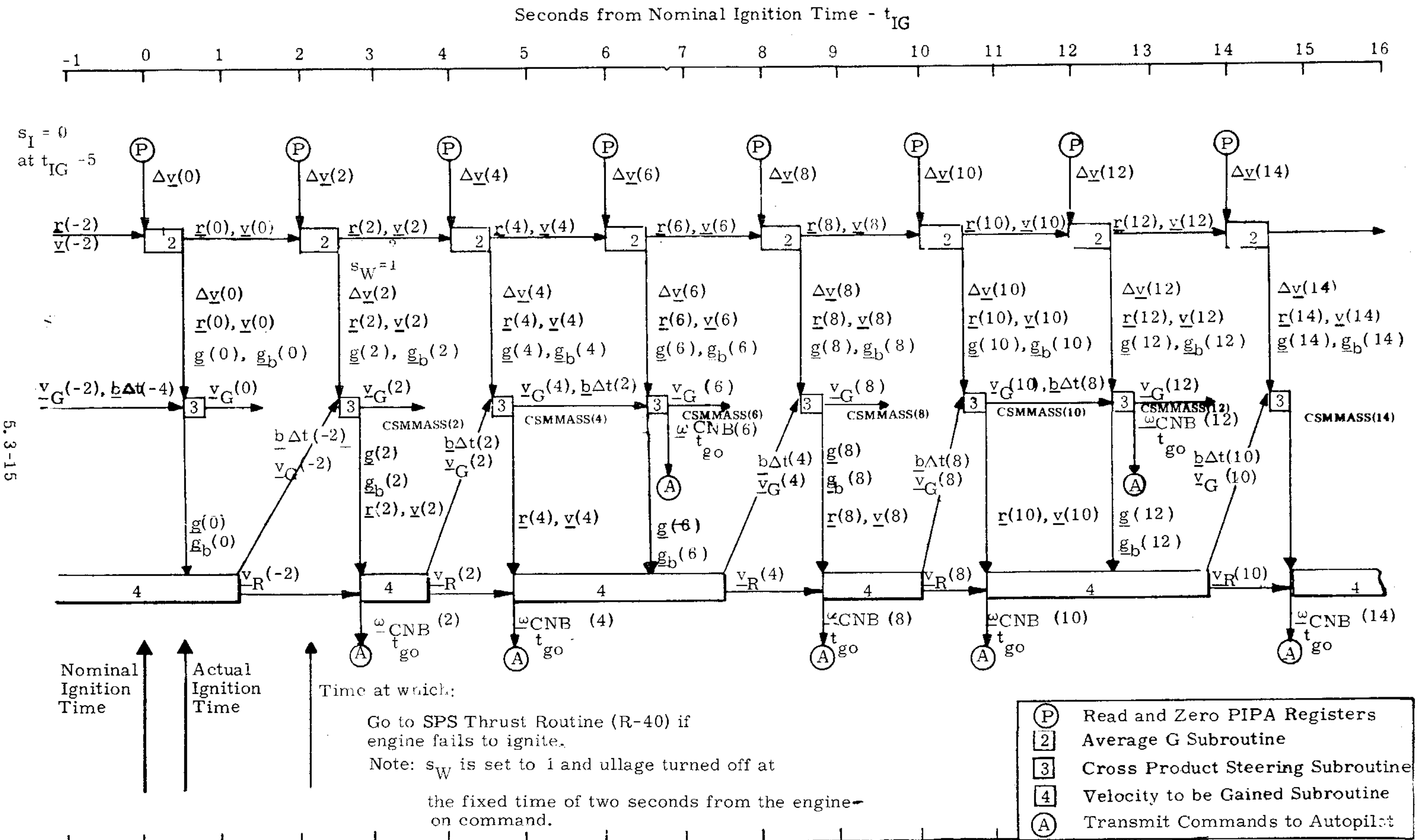


Fig. 3.3-5 Normal Lambert Aim Point Subroutine Sequencing (Maneuver time greater than 6 sec) for SPS Maneuvers

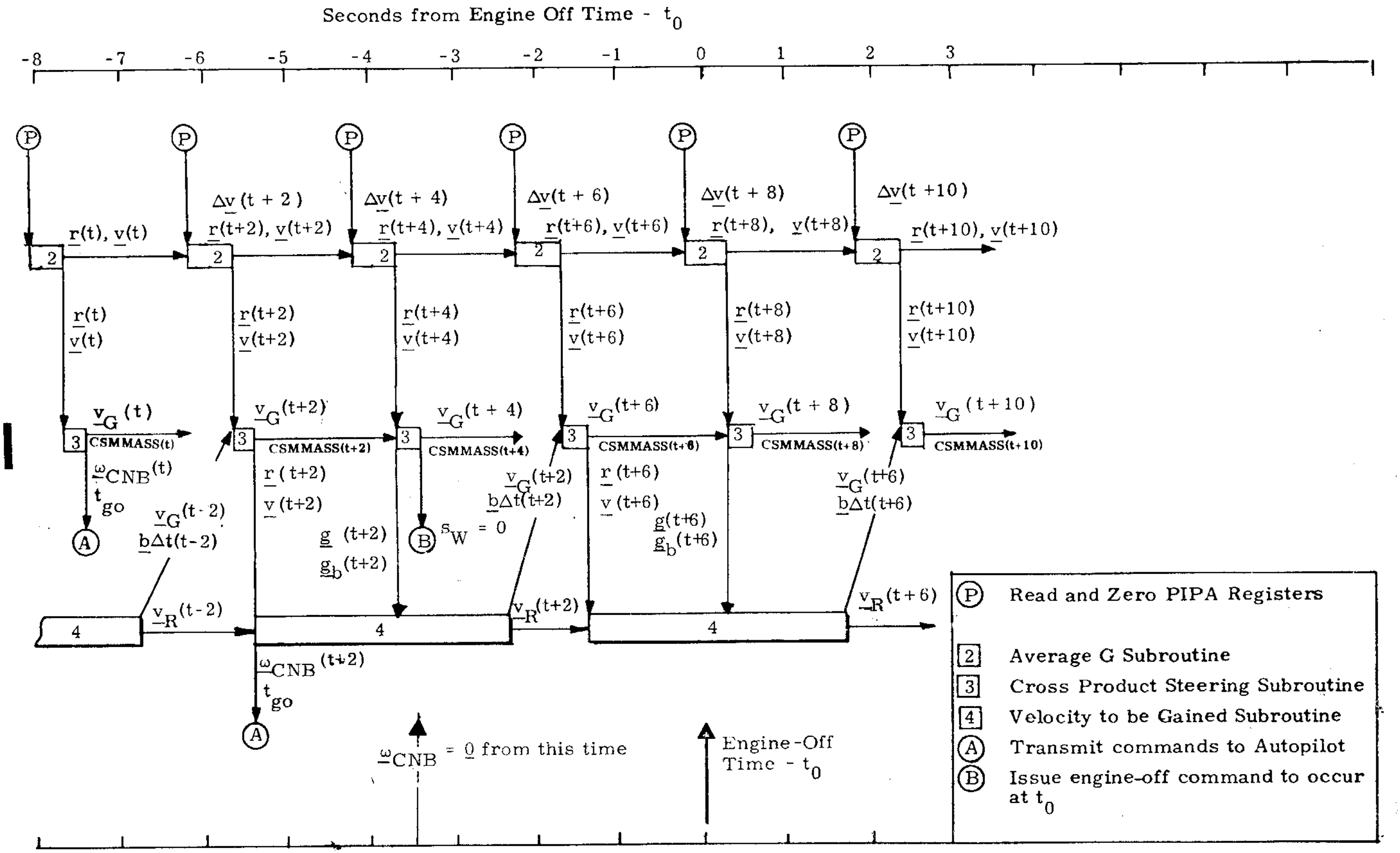


Fig. 3.3-6 Engine-off Lambert Aim Point Subroutine Sequencing for SPS Maneuvers

5.3.3.3 Pre-Thrust Computations

The objective of the computations required prior to thrusting maneuvers is to determine the following:

(1) The desired thrust direction at ignition.

(2) The duration of the powered maneuver estimated at time t_{IG} - 5 to determine if there will be enough time to allow active steering for the SPS.

(3) Whether an IMU realignment is required to avoid gimbal lock.

(4) Various parameters and variables required by subsequent powered flight routines.

The two major guidance modes using cross product steering are the External ΔV guidance mode and the Lambert Aim Point Maneuver guidance mode. The pre-thrust computations required for the External ΔV mode are presented in Section 5.3.3.3.1. Those required for the Lambert Aim Point Maneuver mode are described in Section 5.3.3.3.2. The initial IMU alignment computations and maneuver time logic is summarized in Section 5.3.3.3.3.

The following description of pre-thrust computations applies to both SPS (P-40) and RCS (P-41) maneuvers.

5.3.3.3.1 External ΔV Maneuver Pre-Thrust Computations

External ΔV maneuver guidance is normally used to control orbital phasing maneuvers or an externally targeted maneuver in which a constant thrust attitude is desired. The guidance program accepts input data via the DSKY (P30, P32 and P-33) or the telemetry uplink (P-27) in the form of 3 components of an impulsive $\Delta \underline{V}_{LV}$ expressed in a local vertical coordinate system of the active vehicle at the ignition time t_{IG}. An approximate compensation for the finite maneuver time is made within the program by rotating the $\Delta \underline{V}_{LV}$ vector, and the guidance program issues commands to the spacecraft control system so as to apply the compensated velocity increment along an inertially fixed direction. The active vehicle state vector is normally either available or can be extrapolated to the ignition time in the CMC.

The pre-thrust computations required for the External ΔV guidance mode are shown in Fig. 3. 3-7. The following parameter definitions refer to this figure.

$\Delta \underline{V}_{LV}$ — Specified velocity change in the local vertical coordinate system of the active vehicle at the time of ignition. This is an input parameter.

$$\Delta \underline{V}_{LV} = \begin{pmatrix} \Delta V_X \\ \Delta V_Y \\ \Delta V_Z \end{pmatrix}$$

$\Delta \underline{V}$ — Specified velocity change in Basic Reference Coordinates

t_{IG} — Ignition time, an input parameter.

$\Delta \underline{V}_P$ — The inplane velocity components of $\Delta \underline{V}_{LV}$ in the Basic Reference Coordinate System.

θ_T — The approximate central angle traveled during the maneuver.

MGA — Angle equivalent to the IMU middle gimbal angle when the vehicle X axis is aligned along $\Delta \underline{v}$. This angle is used to cneck for gimbal lock tolerance.

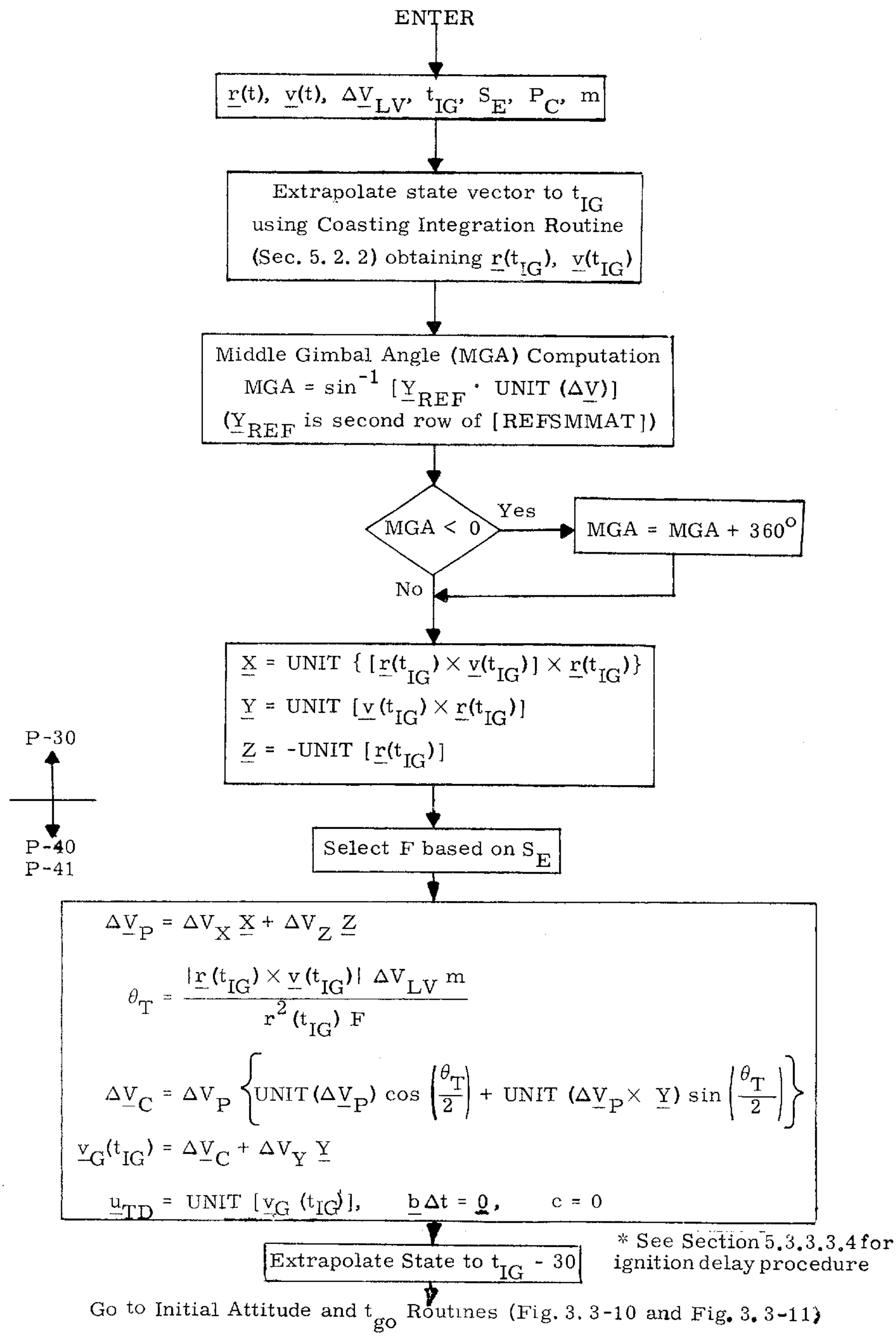


Figure 3. 3-7 External ΔV Prethrust Routine

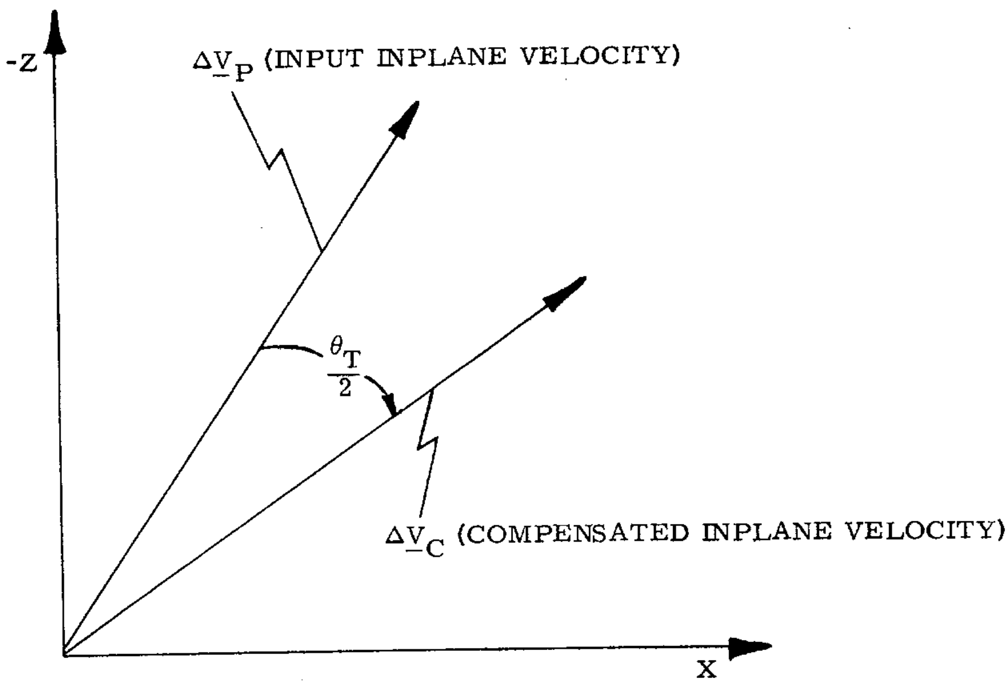


Figure 3.3-8 Inplane External ΔV Maneuver Angle Compensation

m	Vehicle mass.
S_E	Engine Select Switch $\begin{cases} 1 & \text{P-40 (SPS)} \\ 0 & \text{P-41 (RCS)} \end{cases}$
F	Prestored nominal thrust based on the engine selection switch S_E F_{SPS} for the SPS (P-40) F_{RCS} for the RCS (P-41) The values of the various engine thrust levels are listed in Section 5.9. In the case of P-41, RCS, the astronaut can select either a 2 or 4 jet translation maneuver.
$\Delta \underline{V}_C$	The compensated inplane velocity-to-be-gained vector.
$\underline{v}_G(t_{IG})$	Total velocity-to-be-gained at t_{IG}.
$\underline{u}_{TD}$	Unit vector in the desired initial thrust direction.
c	Cross Product Steering Constant This steering constant (Section 5.3.3.4) is set equal to zero for External ΔV maneuvers.

The inplane External ΔV maneuver angle compensation involving $\Delta \underline{V}_P$ and $\Delta \underline{V}_C$ is illustrated in Fig. 3.3-8.

5.3.3.3.2 Lambert Aim Point Maneuver Pre-Thrust Computations

The objective of the Lambert Aim Point Maneuver Guidance program is to control the cut-off velocity vector such that the resulting trajectory intercepts a specified target position vector at a given time. Targeting parameters are determined by CMC targeting programs P-34, P-35, or P-37.

The table below summarizes the maneuvers which can be performed by Lambert Aim Point Maneuver Guidance.

Maneuvers Controlled by Lambert Aim Point Maneuver Guidance

Maneuver	Program
Rendezvous Intercept	P-34
Rendezvous Midcourse Maneuver	P-35
Return-to-Earth Maneuvers	P-37

The targeting parameters which are computed by these programs are as follows:

1) Ignition time t_{IG}
2) Time of intercept of conic target aim vector (t_2)
3) Conic target aim vector $\underline{r}\,(t_2)$
4) c cross product steering constant

The Lambert Aim Point Maneuver guidance mode basically uses the Lambert Subroutine of Section 5.5.6 called through the Initial Velocity Subroutine of Section 5.5.11 during the pre-thrust phase to determine the required initial thrust direction, $\underline{u}_{TD}$. The pre-thrust computations required for the Lambert Aim Point guidance mode are shown in Fig. 3.3-9. The primary operation shown in this figure is the computation of the required intercept velocity vector $\underline{v}_R$ at the ignition time, t_{IG}. The following parameter definitions refer to Fig. 3.3-9.

$\underline{r}(t_2)$	Offset target intercept position vector at time t_2. This parameter is determined by the preceding targeting program and is an input to the Lambert Aim Point prethrust subroutine.
t_2	Intercept time of arrival associated with the offset target vector, $\underline{r}(t_2)$. This is an input parameter.
S_F	See discussion in paragraphs 5.3.3.4 and 5.3.3.5—controls first pass through cross-product steering routine and first pass through velocity-to-be-gained subroutine.
S_R	Target rotation switch set in P-34 and P-35, indicating that the target vector was rotated into the orbital plane due to proximity to 180^o transfer conditions.

$$S_R \begin{cases} 1 & \text{Target vector rotated} \\ 0 & \text{No rotation} \end{cases}$$

N_1	Number of target offset iterations desired in the Initial Velocity Subroutine of Section 5.5.11.
ϵ	Initial Velocity Subroutine parameter.
$\underline{v}_R(t_{IG})$	Required velocity vector at the ignition time t_{IG} to establish the intercept trajectory.
$\underline{g}$	Total gravity vector associated with $\underline{r}(t_{IG})$.
$\underline{b}\Delta t$	Δt times the $\underline{b}$-vector $(\dot{\underline{v}}_R - \underline{g})$, computed as the incremental change of the velocity-to-be-gained vector over one sample period minus current gravity.
MGA	The angle equivalent to the IMU middle gimbal angle when the vehicle X axis is aligned along $\underline{v}_G$.

c The cross product steering constant c is set equal to zero if the pre-thrust program was the External ΔV Program P-30 (Fig. 3.3-7). P-41 sets c = 0 (Fig. 3.3-9). P-40 sets c = 0 for all external Δv burns and sets c = ec for Lambert computations.

ec The pad loaded value for the cross product steering constant. The value of ec may range from -4 to +4. Programs P-32, P-33, P-34 and P-35 set ec = 1. P-37 (Section 5.4) sets ec = 0.5.

The rotation switch S_R is set in the Lambert Aim Point targeting programs P-34 and P-35 to indicate whether the target aim point vector was within a specified cone angle, ϵ, measured from the 180° transfer angle condition. If the initial target vector was within this cone angle, it is rotated into the active vehicle orbital plane in the Initial Velocity Subroutine of Section 5.5.11 so that excessive plane change and ΔV requirements are avoided about the 180° transfer angle condition. In the intercept targeting programs P-34 and P-35 the cone angle ϵ is set at 15°, and active vehicle transfer angles between 165° and 195° are normally avoided in the targeting procedure. If a transfer angle condition falling within this $180^\circ \pm 15^\circ$ sector (S_R = 1) is intentionally selected either during the TPI targeting (P-34), or results from a rendezvous midcourse correction maneuver (P-35) during an intercept trajectory targeted for more than 180°, the Lambert Aim Point Maneuver Pre-Thrust Routine of Fig. 3.3-9 increases the Initial Velocity Subroutine cone angle ϵ to 45° so that the active vehicle transfer angle will not change from inside to outside the cone angle during the powered maneuver. Such a condition is undesirable since the intercept trajectory would be retargeted during the powered maneuver. Likewise, if the initial transfer central angle falls outside the 15° cone angle ϵ of programs P-34 and P-35 (S_R = 0), ϵ is decreased to 10° in Fig. 3.3-9 to reduce the possibility of the transfer angle changing from outside to inside the cone angle during a powered maneuver. It should be noted that the program P-37 sets S_R equal to zero.

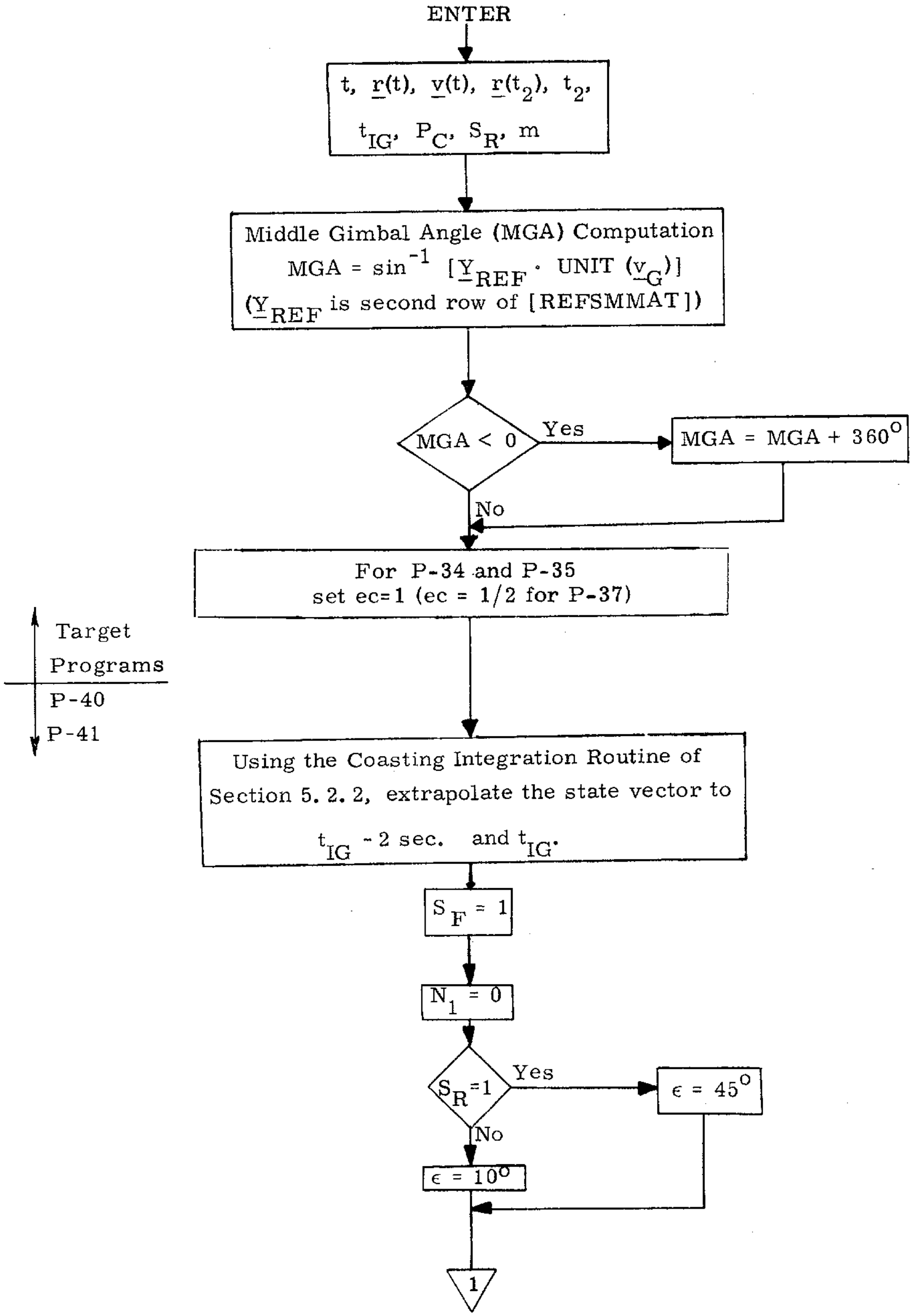
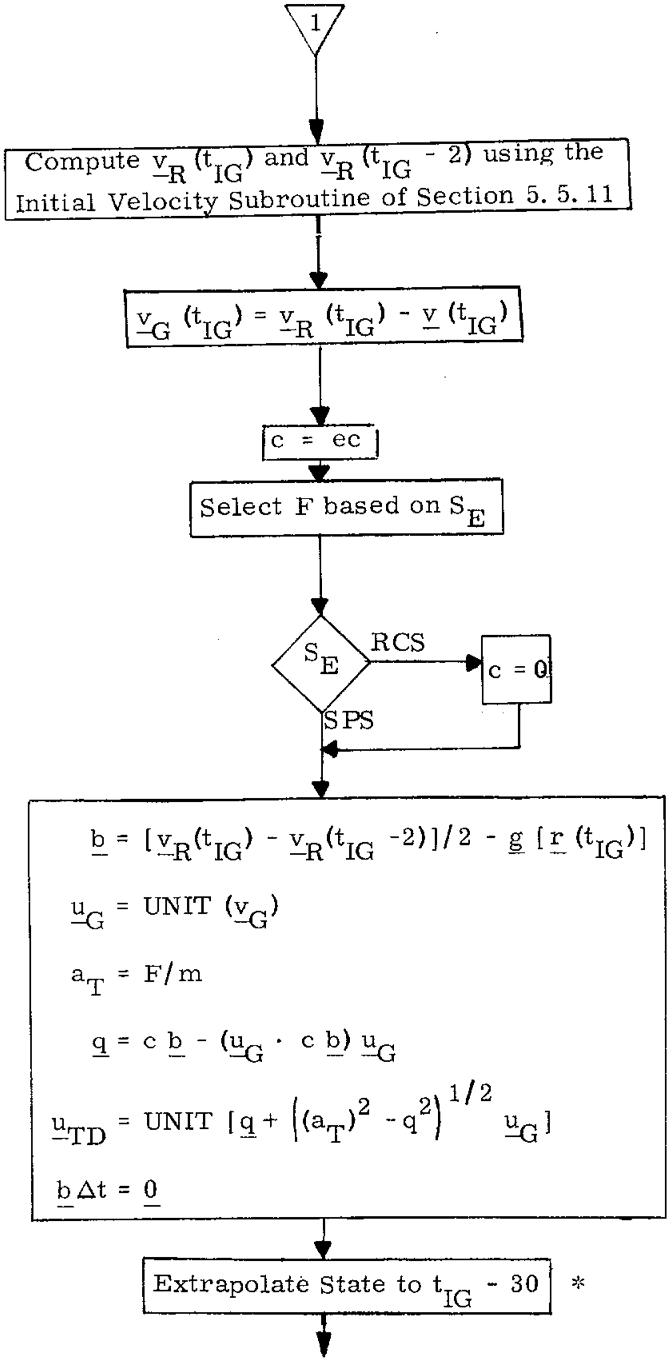


Figure 3. 3-9 Lambert Aim Point Maneuver Pre-Thrust Routine (page 1 of 2)



Go to Initial Attitude and t_{go} Routines (Fig. 3.3-10 and Fig. 3.3-11)

*See Section 5.3.3.3.4 for ignition delay procedure.

Figure 3.3-9 Lambert Aim Point Maneuver Pre-Thrust Routine

(page 2 of 2)

5.3.3.3.3 Initial IMU Alignment and Maneuver Time-to-Go Computations

Initial Attitude Routine

The following pre-thrust computations and functions are required for both the External ΔV and Lambert Aim Point guidance modes:

1. Determination of the preferred IMU alignment for the thrusting maneuver (Section 5.1.4.2.1).
2. Alignment of the vehicle thrust axis along the desired initial thrust direction, $\underline{u}_{TD}$.
3. Estimate the maneuver time, t_{go}, prior to engine ignition.

In Fig. 3.3-10 the following parameter definitions apply:

[REFSMMAT]	Transformation matrix from the BRC System to the IMU or Stable Member Coordinate System (Section 5.6.3.3).
[SMNB]	Transformation matrix from the Stable Member Coordinate System to the Navigation Base Coordinate System (Section 5.6.3.2)
$\underline{u}_A$	Unit vector of the assumed thrust acceleration in Navigation Base Coordinates. For the SPS, the unit vector $\underline{u}_A$ is along the engine axis (called engine bell).
$\underline{u}_D$	Unit vector of the desired thrust direction in Stable Member Coordinates.

$p_T = p + p_0$ Total pitch trim offset angle about the spacecraft Y axis.

$y_T = y + y_0$ Total yaw trim offset angle about the spacecraft Z axis.

where:

p and y Electrical pitch and yaw trim angles respectively.

p_0 Mechanical pitch trim angle equal to -2.15° about the spacecraft Y axis.

y_0 Mechanical yaw trim angle equal to $+0.95^{\circ}$ about the spacecraft Z axis.

For an SPS burn, because of pitch and yaw trim, $\underline{u}_A$, the unit vector of the assumed thrust acceleration vector in Navigation Base Coordinates is not a unit vector along the spacecraft X axis (1, 0, 0) but is a function of the pitch and yaw trim angles given above. In Fig. 3.3-10 it is called the Engine Axis unit vector, $\underline{u}_A$. The maneuver then places the engine axis unit vector ($\underline{u}_A$) along the desired unit thrust direction $\underline{u}_D$.

Time-to-Go Prediction Routine

Very short burns require special consideration since some interval of time elapses before effective steering is achieved. Not only must the autopilot react to the pointing commands, but the engine-off signal may be required before the $\Delta \underline{V}$ from the PIPA's can be measured. To this end an estimate of the burn time, t_{go}, is made on the basis of SPS engine data, prior to engine ignition as shown in Fig. 3.3-11.

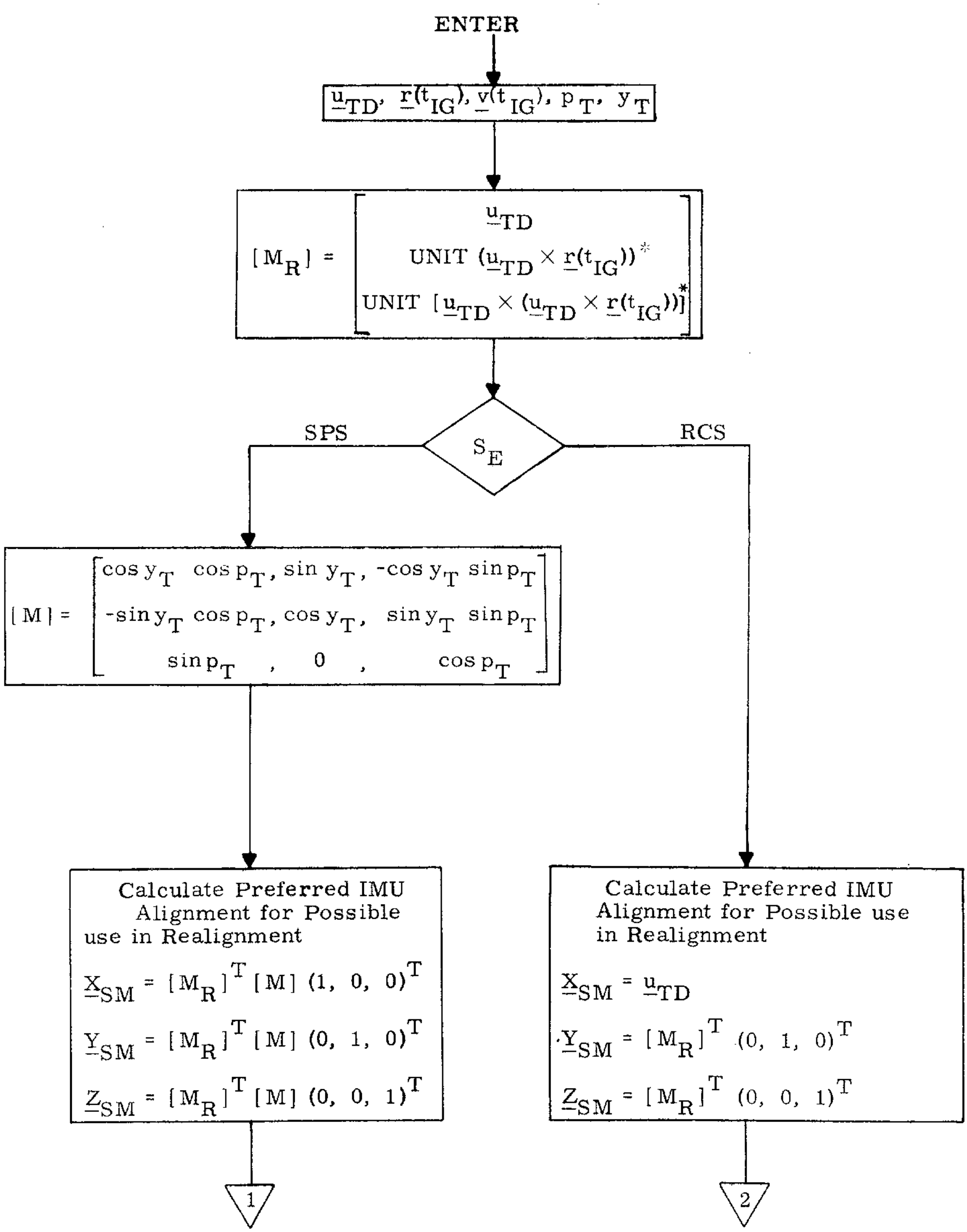


Figure 3.3-10 Initial Attitude Routine
(page 1 of 2)

*In P41 if $|\underline{u}_{TD} \times \underline{r}(t_{IG})| < 2^{16}$ meters, or in P40 if $(\underline{u}_{TD} \times \text{UNIT } \underline{r}(t_{IG})) < 2^{-12}$ radians, then replace this by

$$\text{UNIT}\,[\underline{u}_{TD} \times \{\text{UNIT } \underline{r}(t_{IG}) + 0.125 \text{ UNIT } \underline{v}(t_{IG})\}]$$

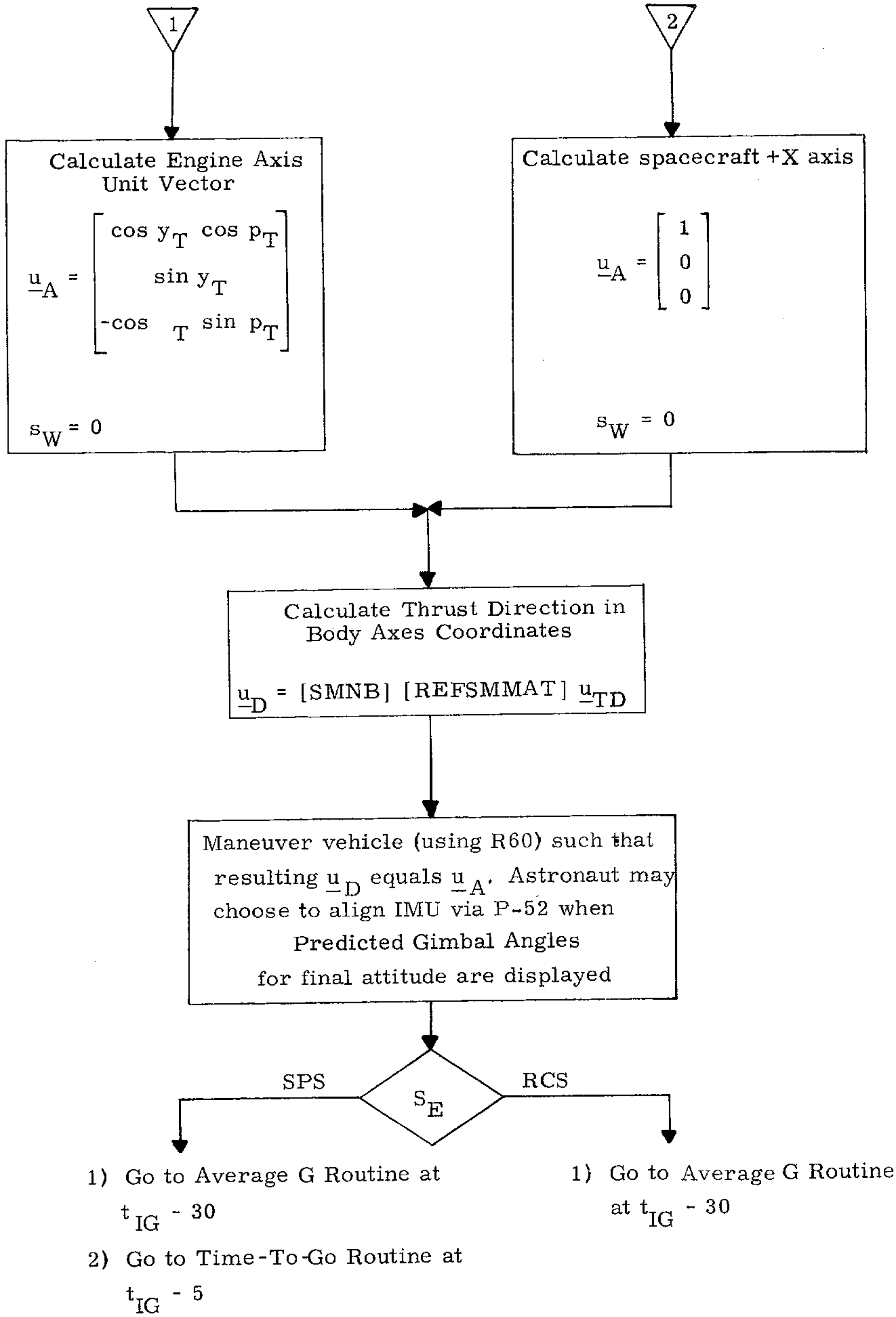


Figure 3.3-10 Initial Attitude Routine
(page 2 of 2)

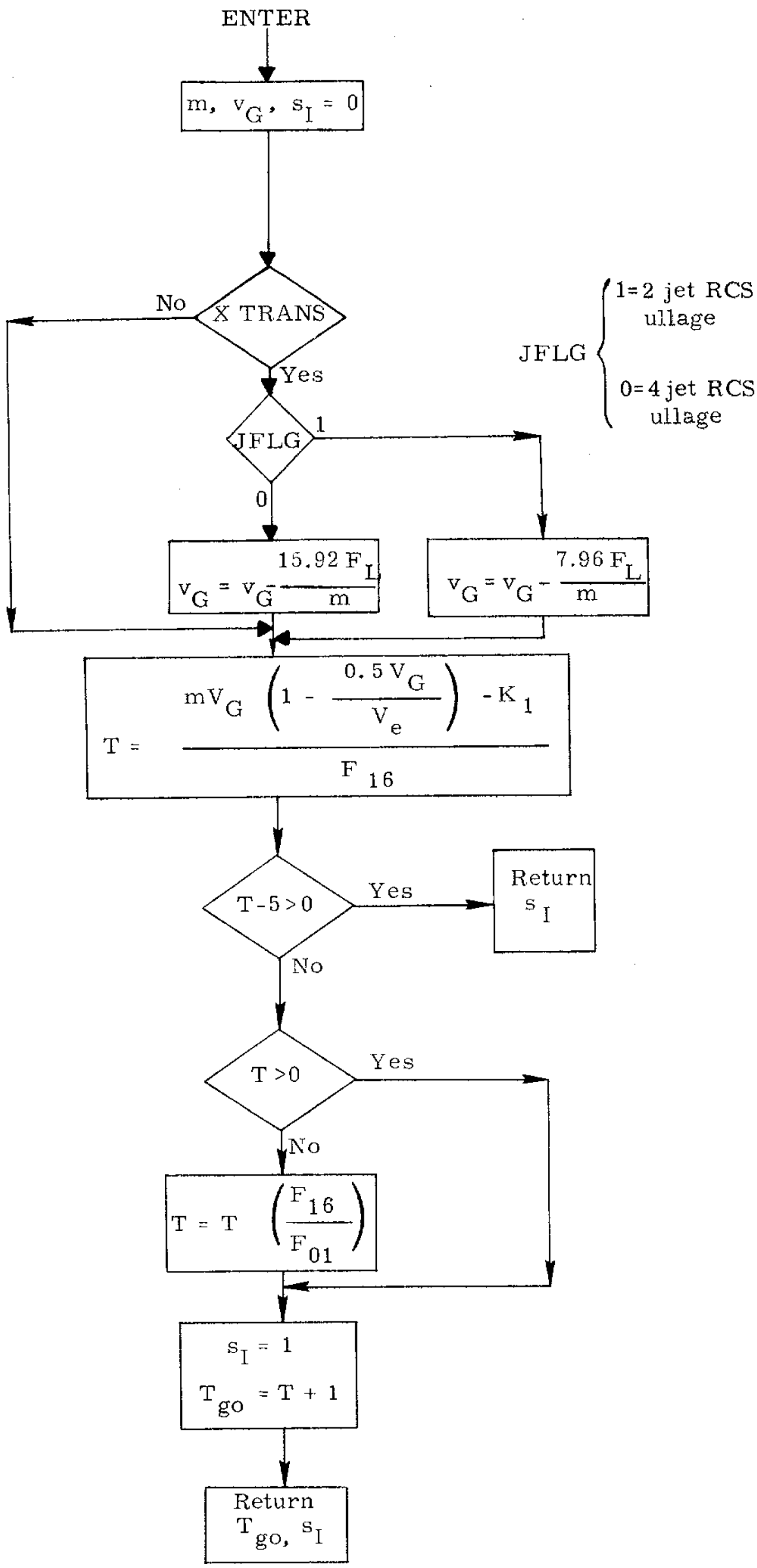


Figure 3.3-11. Time-to-go Prediction Routine

With reference to Fig. 3.3-11:

s_I — Impulse switch

$$s_I \begin{cases} 1 & \text{Indicates a maneuver less than 6 seconds and no steering is required} \\ 0 & \text{Indicates a maneuver duration of at least 6 seconds, and normal steering will occur} \end{cases}$$

F_L — Ullage thrust for 2 jets

K_1 — SPS impulse velocity acquired in a one-second maneuver for a unit mass vehicle. This value is erasable.

F_{01} — SPS minimum impulse constant equal to the slope of minimum impulse curve from 0 to 1 second, an erasable quantity.

F_{16} — Steady state thrust of the SPS engine. This value is used only in the short burn logic and is an erasable quantity.

V_e — Exhaust velocity.

The initial computation in Fig. 3.3-11 estimates the velocity-to-be-gained after ullage. There is an option of 2-jet or 4-jet RCS ullage, as indicated in Fig. 3.3-11. This subroutine is performed at approximately five seconds prior to ignition, the PIPAs are read at six seconds prior to ignition, and the ullage is terminated at two seconds after ignition; it is for this reason that about eight seconds of ullage are accounted for, as shown in Fig. 3.3-11.

The hand controller signals are observed at the initiation of this routine. The ullage compensation in the time-to-go routine will be made only if +X translation (XTRANS ON) is indicated.

If the SPS were chosen (P-40), a check is made, Fig. 3.3-11, to determine if the maneuver time is less than one second. If the maneuver time is less than one second, the t_{go} estimate is made on SPS minimum impulse test data represented by the constants K_1 and F_{01} (see Section 5.8). In this case, no active steering is attempted ($s_I = 1$). If the maneuver time is greater than one second but less than 6 seconds, t_{go} is computed as shown in Fig. 3.3-11, and again no active steering is attempted. If the estimated maneuver time is greater than 6 seconds, active steering is used, and t_{go} computations are performed during the maneuver.

With reference to Fig. 3.3-10, if the RCS were chosen for the maneuver, (P-41), the t_{go} prediction calculation is not made.

If the estimated maneuver time, t_{go}, for the SPS is less than 6 seconds, the Engine-Off signal is set for the actual ignition time plus t_{go}.

5.3.3.3.4 Ignition Delay Procedures Caused by Pre-Thrust Computations

The normal pre-thrust computations (Sections 5.3.3.3.1 and 5.3.3.3.2) require an extrapolation of the CSM state vector to thirty seconds prior to nominal ignition time, i.e., $t_{IG} - 30$. If the Coasting Integration Routine of Sec. 5.2.2 does not complete the extrapolation before $t_{IG} - 42.5$ occurs, then an ignition delay procedure occurs as follows:

1.) The astronaut is alerted to this condition by a program alarm.

2.) The integration continues one step at a time until the CSM state vector time minus the current time is greater than 12.5 seconds. This will permit a 5 second blanking interval (Section 4 State Vector Integration (MID to AVE) Routine (R-41). R-41 allows Average G initialization to occur at least 5.6 seconds after completion of last integration step.

3.) The maneuver ignition time is then redefined to be 30 seconds from the resulting CSM state vector time, and the normal pre-ignition sequence is started.

5.3.3.4 Cross Product Steering Routine

The cross product steering concept is the basic control concept for both External ΔV and Lambert Aim Point Guidance modes. The cross product steering logic is shown in Fig. 3.3-12. The following parameter definitions not previously described apply to this figure.

$\Delta \underline{v}_{SM}$	PIPA Measured velocity vector over the computation cycle Δt in IMU coordinates
$\Delta \underline{v}$	PIPA measured velocity over the computation cycle Δt transformed to the Basic Reference Coordinate System
Δv_p	A constant which establishes the Δv which must be sensed in a 2 second computation interval. It is 2 ft/sec (erasable) which is approximately equivalent to thrusting at 15% full SPS thrust with fully loaded vehicle for 2 seconds.
s_W	A logic switch in the cross product steering routine which when set to 1 allows steering commands and t_{go} calculations to be made. s_W is set to 1 at the fixed time of 2 seconds from engine on command if non-impulsive. s_W is set to 0 when the computed time-to-go first becomes less than four seconds.
$\Delta t_{tail\text{-}off}$	A constant representing the duration of a burn at maximum thrust equivalent to the tail-off impulse after the engine-off signal is issued.

$\Delta\underline{v}_G$	$\underline{b}\,\Delta t - \Delta\underline{v}$
V_e	Engine exhaust velocity = $g\ I_{SP}$
K	Guidance steering gain required for desired dynamic response of the combined powered guidance and thrust control (autopilot) loops
$\underline{\omega}_C$	Commanded attitude rate in the Basic Reference Coordinate System.
$\underline{\omega}_{CNB}$	Commanded attitude rate in Navigation Base Coordinates which is sent to the CSM attitude control system.
c	The cross product steering constant c, used by P-40/ P-41 in the equation $$\Delta\underline{m} = c\,\underline{b}\,\Delta t - \Delta\underline{v}$$ and certain pre-thrust computations. It will be zeroed by P-40 pre-thrust for external Δv burns and by any P-41 pre-thrust. It is set by P-40 pre-thrust to ec for Lambert burns.

$\Delta\underline{m}$	$c\,\underline{b}\,\Delta t - \Delta\underline{v}$
LOFLG	This flag prevents a premature engine fail indication after a manual engine start. LOFLG $\begin{cases} 1 & \text{steer} \\ 0 & \text{no steer} \end{cases}$
$\Sigma\Delta\underline{v}$	Accumulated PIPA readings used to generate an extrapolation of the Lambert soution. When a new solution becomes available $\Sigma\Delta\underline{v}$ is set to zero.

N — Number of cycles between Lambert solutions. The CMC computations during SPS maneuvers occasionally prevent completion of the Lambert solution in 2 to 4 seconds. The program is capable of processing an N cycle $\underline{v}_G$ update. When a new solution becomes available, N = 0.

S_F — Lambert first pass flag. The Lambert first pass starts at approximately t_{IG}-28. After the first pass is completed, it is set to zero.

$$S_F \begin{cases} 1 & \text{First Lambert pass. This results in bypass of the } \underline{b}\Delta t \text{ computation.} \\ 0 & \text{Not first Lambert pass.} \end{cases}$$

Switch s_W is set to zero for short duration thrust periods and during the first two seconds of long duration thrust periods. In both of these cases there is no active steering and the vehicle attitude is held at the pre-thrust alignment. When s_W is set to 1, active steering is performed. The time-to-go, t_{go} computation and steering commands $\underline{\omega}_{CNB}$ are performed as shown in Fig. 3.3-12. When the computed t_{go} becomes less than 4 seconds, then the engine-off signal is set and switch s_W is reset to zero. For the remainder of the maneuver, no further computations are made except for $\underline{v}_G$ updating.

With reference to Fig. 3.3-12, the logic switch s_W is set to zero by the sequencing routine. The only function of the cross product steering routine is to update the velocity-to-be-gained vector $\underline{v}_G$ with $\Delta \underline{v}$ and $\underline{b}\,\Delta t$, as long as s_W is zero.

The steering command generated by this routine is $\underline{\omega}_{CNB}$ which is in Navigation Base Coordinates. The objective of the cross product steering concept is to control the thrust acceleration vector such that the following condition is satisfied:

$$\underline{v}_G \times (c\,\underline{b} - \underline{a}_T) = \underline{0}$$

In general, however, there will be a directional error and the steering command $\underline{\omega}_{CNB}$ tends to align the vehicle such that the above equation is satisfied.

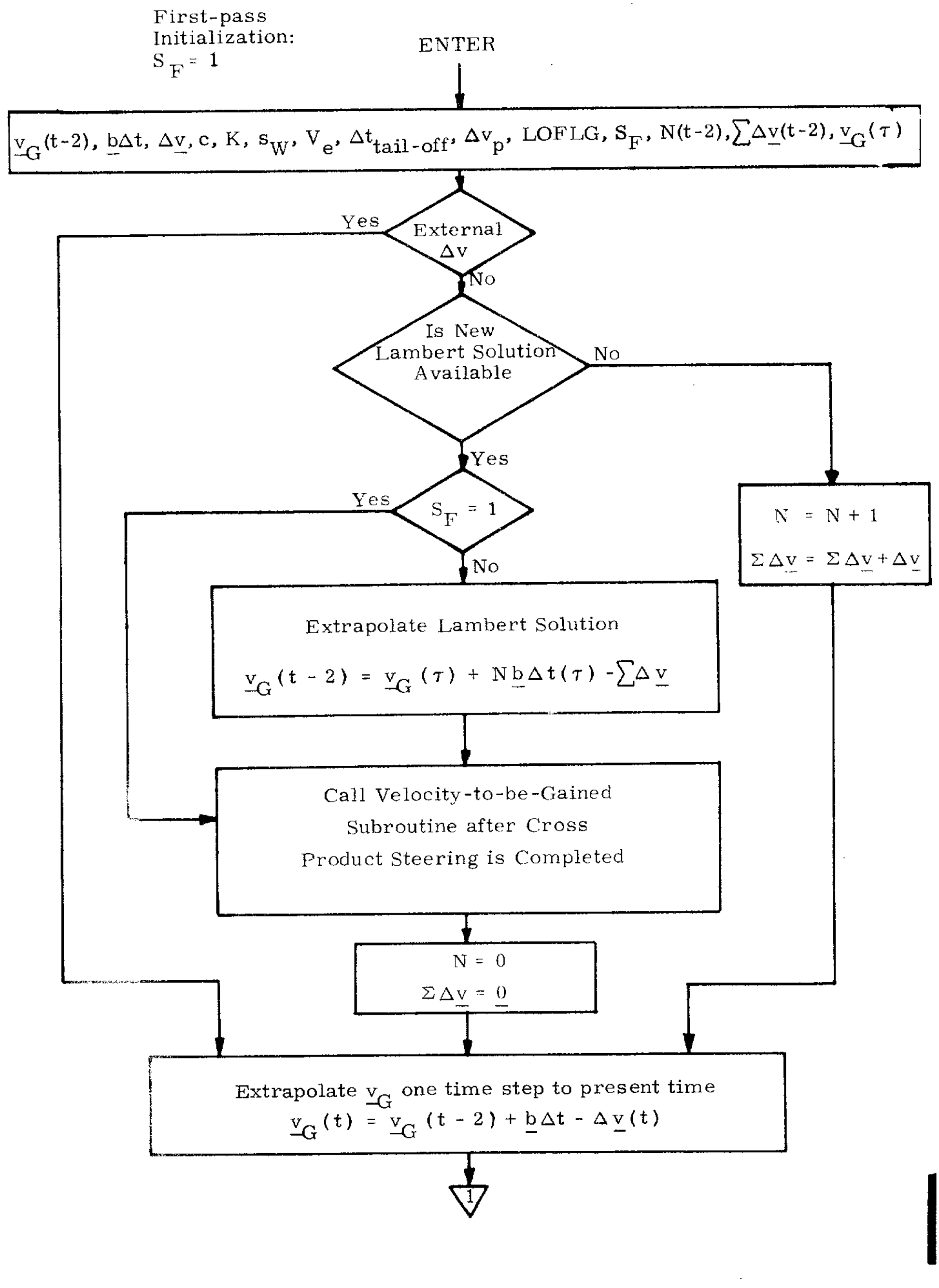


Figure 3.3-12 Cross Product Steering Routine (page 1 of 2): Update $\underline{v}_G$ Section

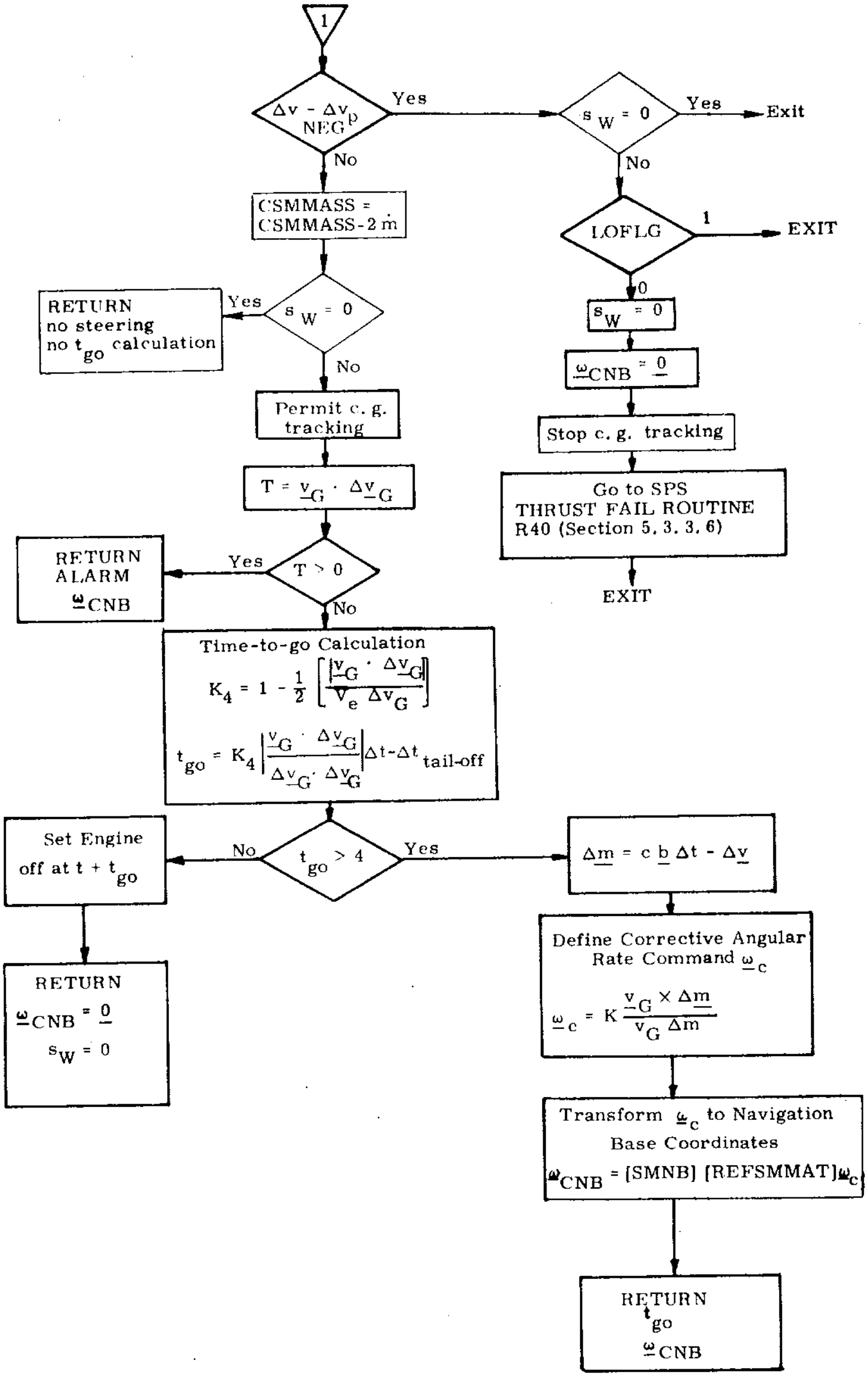


Figure 3.3-12. Cross Product Steering Subroutine: t_{go} and Steering Command Computations (page 2 of 2)

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[ ] Added GSOP # R-577 PCN # 878 Rev. 14 Date 3/71

The cross-product steering routine, as shown in Fig. 3.3-12 is divided into two sections.

(a) Update $\underline{v}_G$: This portion of the cross-product steering routine provides an up to date velocity-to-be-gained, $\underline{v}_G$, for use in the t_{go} and steering command computations. In the event a new velocity-to-be-gained is not available, the old $\underline{v}_G$ is extrapolated using the old $\underline{b}\Delta t$ value. If a new $\underline{v}_G$ and $\underline{b}\Delta t$ are available, they are used. N and $\sum \Delta \underline{v}$ are used when a new Lambert solution is available.

(b) t_{go} and steering command calculations: This portion of the cross product steering routine uses the $\underline{v}_G$ generated in the update $\underline{v}_G$ section to calculate t_{go} and steering commands to be sent to the DAP. In addition the following control options are provided:

(1) If the present Δv is less than Δv_p, where Δv_p is approximately equivalent to thrusting at 15% full SPS thrust with fully loaded vehicle for 2 secs., the steering rate commands are zeroed, c. g. tracking is stopped and the SPS Thrust Fail Routine (R-40) (Section 5.3.3.6) is entered. Δv_p is an erasable quantity. If the present Δv is greater than Δv_p, the CSMMASS is decremented by 2 $\dot{m}$.

(2) An alarm is issued if $\underline{v}_G \cdot \Delta \underline{v}_G$ is positive. This is equivalent to pointing the thrust vector in the wrong direction.

5.3.3.5 Velocity-to-be-Gained Routine

The velocity-to-be-gained computations shown in Fig. 3.3-13 are those carried out during the Lambert Aim Point powered flight guidance. The velocity-to-be-gained computation for the External ΔV guidance mode is simpler than that for the Lambert Aim Point guidance mode. The External ΔV velocity-to-be-gained computation is that shown in the cross product steering routine of Fig. 3.3-12 and is equivalent to

$$\underline{v}_G = \underline{v}_G - \Delta\underline{v}$$

since $\underline{b}\Delta t = \underline{0}$ for the External ΔV mode as shown in Fig. 3.3-7.

The velocity-to-be-gained computations for the Lambert Aim Point guidance mode involve the determination of a new $\underline{v}_G$ by processing the Lambert Subroutine via the Initial Velocity Subroutine. A second objective is the computation of a new $\underline{b}\Delta t$ parameter for use by the cross product steering routine. A new required velocity, $\underline{v}_R$, is also determined for use in the next computation cycle of the velocity-to-be-gained subroutine. The CMC computations during SPS maneuvers occasionally prevent completion of the Lambert Solution in 2 to 4 seconds. The program is capable of processing an N cycle $\underline{v}_G$ update (Fig. 3.3-13).

The following parameter definitions refer to Fig. 3.3-13.

$\underline{r}(\tau)$ $\underline{v}(\tau)$ τ	Active vehicle state vector at start of Lambert solution.
$\underline{r}(t_2)$	Offset target intercept position vector at time t_2. This parameter is determined by the preceding targeting program (P-34, P-35, or P-37).

t_2	Intercept time of arrival associated with the offset target vector $\underline{r}(t_2)$. This is an input target parameter.
t_{IG}	Nominal ignition time.
$(\tau - \Delta\tau)$	Required velocity vector from the preceding Lambert computation cycle.
$\Delta\tau$	Time interval between the current and previous Lambert computation cycle = $(N+1)\Delta t$.
S_R	Target rotation switch set in P-34 or P-35, indicating that the target vector was rotated into the orbital plane due to proximity to 180° transfer. $S_R \begin{cases} 1 & \text{Target vector rotated} \\ 0 & \text{No rotation} \end{cases}$
$\underline{g}(t)$	Most recent total gravity acceleration vector (Section 5.3.2).
$\underline{g}_b(t)$	Component of the most recent earth gravity acceleration vector representing earth oblateness effects (Section 5.3.2).
$\Delta\underline{v}(t)$	Measured PIPA velocity change in the Basic Reference Coordinate system.
N_1	Number of target offset iterations desired in the Initial Velocity Subroutine (Section 5.5.11).
ϵ	Initial Velocity Subroutine parameter.
$\underline{v}_R(\tau)$	Required velocity vector at time t. } Solution from most recent Lambert computation
$\underline{v}_G(\tau)$	Velocity-to-be-gained vector at time t. } Solution from most recent Lambert computation
$\underline{b}\Delta t(\tau)$	$\underline{b}$-vector solution from most recent Lambert computation.

N — Number of cycles between Lambert solutions. The CMC computations during SPS maneuvers occasionally prevent completion of the Lambert solution in 2 to 4 seconds. The program is capable of processing an N cycle $\underline{v}_G$ update.

S_F — Lambert first pass flag. The Lambert first pass begins at approximately t_{IG} - 28. After the first pass is completed, it is set to zero.

$$S_F \begin{cases} 1 & \text{First Lambert pass. This results in bypass of the } \underline{b}\Delta t \text{ computation.} \\ 0 & \text{Not first Lambert pass.} \end{cases}$$

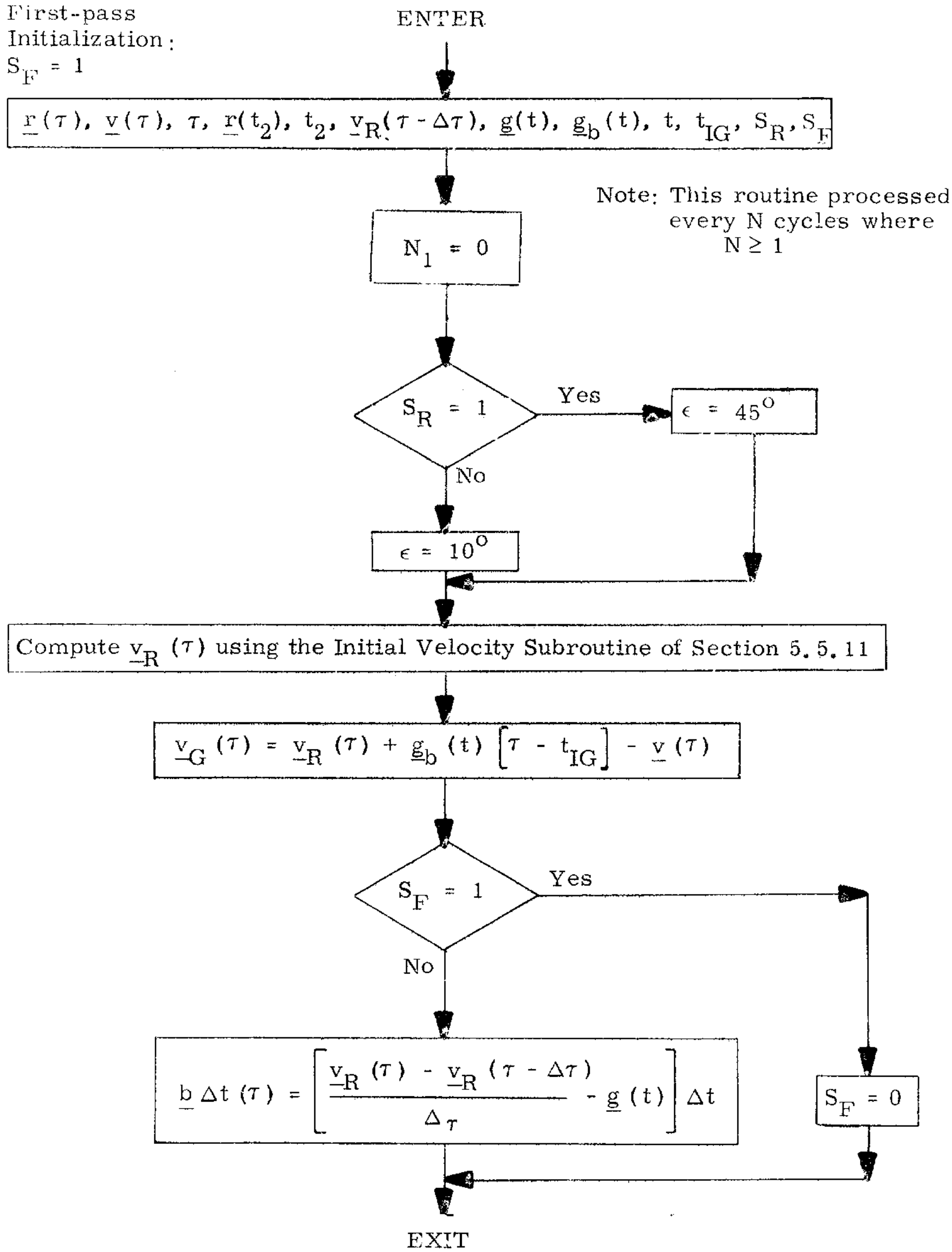


Figure 3.3-13 Velocity-to-be-Gained Subroutine for Lambert Aim Point Guidance

It should be noted that in Fig. 3.3-13 the velocity-to-be-gained, $\underline{v}_G$, derived from the Lambert solution using an offset target vector is modified by the term $\underline{g}_b(t)\ [\tau - t_{IG}]$. This term is an approximation to the velocity change contributed by the earth oblateness effect. The compensation used in this subroutine is computed as the current oblateness acceleration, $\underline{g}_b(t)$, multiplied by the time since nominal ignition $(\tau - t_{IG})$ where t_{IG} is the nominal ignition time. This correction is zero for lunar orbits. The objective of this correction is to reduce cut-off errors due to finite maneuver time effects, and to minimize commanded thrust attitude variations during the maneuver. These two effects occur during long maneuvers because in accounting for earth oblateness effects in the initial targeting programs (P-34 and P-35) it is assumed that an impulsive maneuver will be applied at ignition time. Since a finite maneuver time is required, the pre-computed target aim point becomes less accurate as the maneuver progresses. The $\underline{g}_b[\tau - t_{IG}]$ correction is an approximate substitute for a retargeting procedure which cannot be performed during a powered maneuver.

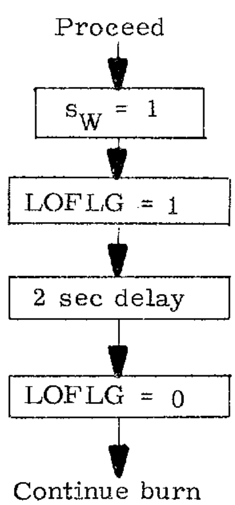
5.3.3.6 SPS Thrust Fail Routine (R-40)

The purpose of the SPS Engine Fail Routine (R-40) is to present the Astronaut with 3 options when a low SPS thrust is detected in the Cross Product Steering Subroutine (Section 5.3.3.4). The three options are:

(1) Command engine off and terminate P-40.

(2) Command engine off and return to the P-40 point where the impulse burn test is made.

(3) Proceed with the burn but with the thrust failure detection inhibited for 2 secs to prevent premature thrust fail indication as shown below:



See Section 4 for complete details of the R-40 routine.

5.3.4 THRUST MONITOR PROGRAM

The Thrust Monitor Program, P-47, is used during manual or non-GNCS controlled maneuvers to monitor and display the velocity change applied to the vehicle. The program first suspends state vector updating by the VHF Range-link (resets the update and track flags), and advances the vehicle state vector to the current time by the Coasting Integration Routine of Section 5.2.2. This operation is continued until the state vector is advanced several seconds ahead of the current time as described in Section 5.3.3.3.4. The Average-G Routine of Section 5.3.2 is then initiated to allow thrusting to be started as soon as possible. The Average-G Routine is left on until the program is terminated after completion of the maneuver. The primary output of P-47 is the measured maneuver $\Delta \underline{V}$ in vehicle coordinates as described in Section 4.

The two major maneuvers during which the Thrust Monitor Program is normally used are the Translunar Injection (TLI) maneuver controlled by the Saturn guidance system, and the manually controlled terminal rendezvous maneuvers required for a CSM retrieval of the LM. During the TLI maneuver a callable display of inertial velocity, altitude above the launch pad radius and altitude rate is available to the astronaut.

During active CSM terminal rendezvous maneuvers, the Rendezvous Display Routine R-31 is normally called to display relative range, range rate, and the vehicle X axis to the horizontal plane angle θ. The operation of R-31 with the Average-G Routine of P-47 is described in Section 5.6.7.1.

5.3.5 EARTH ORBIT INSERTION MONITOR PROGRAM - P-11

5.3.5.1 Introduction

The purpose of this section is to describe the operation and implementation of Program P-11, Earth Orbit Insertion Monitor.

This program is initiated by Program P-02, Gyro Compassing, when the liftoff discrete is detected or by the astronaut backup Verb 75 and it performs the following functions (in the order of occurrence)

(1) Zeroes the CMC clock at liftoff and updates the reference ephemeris time (see Time Definition, Section 5.1.5.5). (Time Subroutine)

(2) Computes the CMC state vector (in Basic Reference Coordinates) at liftoff and starts the Average-G computation (see Section 5.3.2) using this state vector. (State Subroutine)

(3) Finishes remainder of Pre-Launch torquing.

(4) Computes the matrix REFSMMAT which relates the IMU Stable Member orientation to the Basic Reference Coordinate System. (State Subroutine)

(5) Computes periodically the error between the nominal desired Saturn Launch Vehicle attitude (as determined by a time dependent polynomial) and the actual Saturn Launch Vehicle attitude (as determined by the CM IMU), and transmits the error to the CDU for display by the FDAI.*
(Attitude Error Subroutine)

*Flight Director Attitude Indicator

(6) Periodically computes Saturn Launch Vehicle inertial velocity magnitude, rate of change of the vehicle altitude above the launch pad radius, and the vehicle altitude above launch pad radius and displays this information via the DSKY. (Display Subroutine)

Program P-11, when activated by Program P-02 assumes

(1) The following information has been pad loaded

A_{ZP} – Azimuth of the launch vehicle on the pad measured from north to the +Z spacecraft axis positive in a right hand sense about the inward pad local vertical. See figure 3.5-1.

A_Z – launch azimuth measured from north to the X IMU Stable member axis positive in the same sense as A_{ZP}. See figure 3.5-1.

K_r – constant denoting the absolute value of the rate at which the Saturn will roll from the pad azimuth to the launch azimuth.

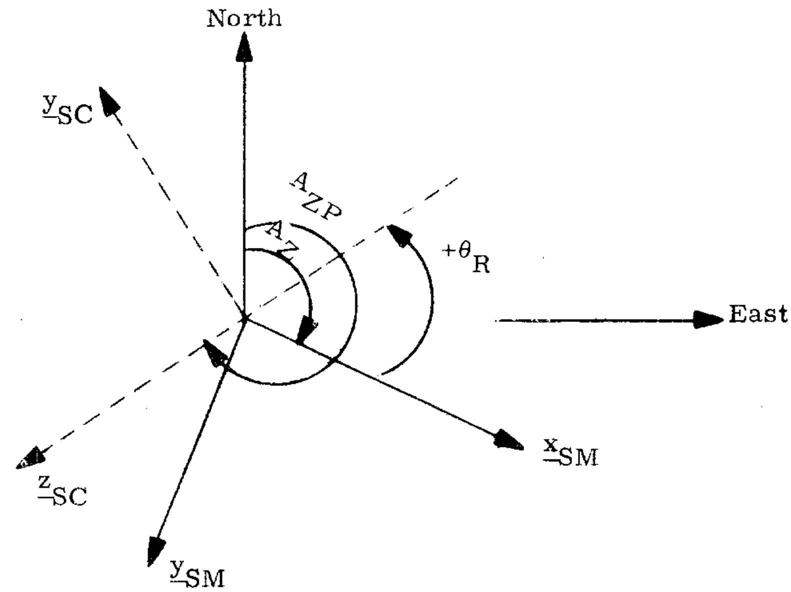
t_{E1} – the estimated time from liftoff at which the initial Saturn pitch and roll maneuvers will commence.

t_{E2} – time interval beginning at t_{E1} used by the Attitude Error subroutine. During t_{E2}, desired gimbal angles computed using polynomial functions.

$a_0, a_1, \ldots a_6$ – coefficients of a 6th order polynomial in time t, ($t_{E1} < t < (t_{E1} + t_{E2})$) describing the nominal Saturn pitch profile.

(2) The clock and reference ephemeris time were synchronized previously.

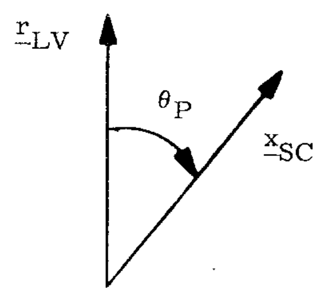
Azimuth and Roll Angle Definitions



- Plane of the paper is the pad local horizontal
- Up is out of the paper
- Spacecraft and IMU axis shown prior to launch
- After lift-off the Saturn rolls to align the $-\underline{z}_{SC}$ with $\underline{x}_{SM}$

Pitch Angle Definition

Inertial upward local vertical vector (established at liftoff)



- Plane of the paper is the instantaneous pitch plane
- $\underline{y}_{SC}$ is into the paper and northerly

Figure 3.5-1 Angle Definitions

5.3.5.2 Nomenclature for P-11

t	Computer clock reading at any time t
t_0	See Section 5.1.5.5.
h	Vehicle altitude above a sphere whose radius is that of the launch pad
$\dot{h}$	Rate of change of altitude measured in the direction of the vehicle radius vector
v	Inertial velocity magnitude of the vehicle
Lat_P	Launchpad geodetic latitude
Lon_P	Launchpad geodetic longitude
Alt_I	IMU altitude above the launch pad
$\underline{u}_z$	A vector in the Basic Reference Coordinate System in the direction of the earth's true pole
ω_E	The angular velocity of the earth
[REFSMMAT]	A matrix whose rows are the location of the IMU Stable Member axes in the Basic Reference Coordinate System
$\underline{CDU}$	The current value of the IMU Gimbal Angles
θ_P	The Saturn Vehicle nominal pitch angle measured from the Launch pad local vertical at liftoff to the Saturn X axis.(See Figure 3.5-1.)
θ_R	The Saturn nominal roll angle measured from the X IMU Stable Member axis to the negative Z spacecraft axis. (See Figure 3.5-1.)
$\underline{E}_a$	A vector representing the roll, pitch, and yaw errors about CM body axes (in a right hand sense).
$\underline{DGA}$	A vector representing the Saturn nominal desired IMU gimbal angles (x, y, z).

K_z — A constant computed in the State Subroutine which is utilized by the Attitude Error Display Subroutine.

Note that pad loaded variables are identified in Section 5.3.5.1.

5.3.5.3 Time Sequencing of P-11 Subroutines

Program P-11 is composed of four subroutines: Time Subroutine, State Subroutine, Attitude Error Subroutine, and Display Subroutine.

The Time Subroutine is selected by P-11 within 0.5 second of the lift-off discrete receipt. The State Subroutine is initiated immediately after the Time Subroutine and is through within 1 second after receipt of the lift-off discrete.

The cycling of the Attitude Error Subroutine is then started. This subroutine refreshes the attitude error FDAI display approximately every 1/2 second until P-11 is exited. Body axis attitude errors are computed from the differences between the desired gimbal angles ($\underline{DGA}$) and the actual gimbal angles ($\underline{CDU}$). $\underline{DGA}$ is obtained from a 3 part model of the Saturn SIC attitude profile. 1) Before pitchover, $t \leq t_{E1}$, $\underline{DGA}$ is the lift off attitude (from the Time Subroutine). Note that the Saturn yaw maneuver for tower clearance is not modeled. 2) During the maneuver, $t_{E1} < t \leq t_{E1} + t_{E2}$, $\underline{DGA}$ is obtained from a sixth order pitch polynomial and a first order roll polynomial (FDAI Display Subroutine No. 1). 3) When $t > t_{E1} + t_{E2}$ (termination of the pitch maneuver, i.e., tilt arrest) $\underline{DGA}$ is held constant at its last computed value. The values of t_{E1} and t_{E2} are chosen so as to provide a reasonably smooth model of expected Saturn performance.

At the same time the attitude error display is started, the cycling of the Display Subroutine is started. This subroutine displays v, h, and $\dot{h}$ via the DSKY. It is cycled every 2 seconds following the Average-G computations and is out of synchronization with the Attitude Error Subroutine.

5.3.5.4 Time Subroutine

The procedure for clock zeroing and presetting of reference time, t_0, is shown in Fig. 3.5-2. Refer to Time Definitions, Section 5.1.5.5, for a description of how the reference ephemeris time was originally synchronized with the AGC clock prior to lift-off.

This activity does not occur precisely at lift-off, but within 0.5 second maximum of the receipt of the lift-off discrete. In any event, the clock zeroing and the constant t_0 are not changed for the remainder of the mission unless P-27 intercedes.

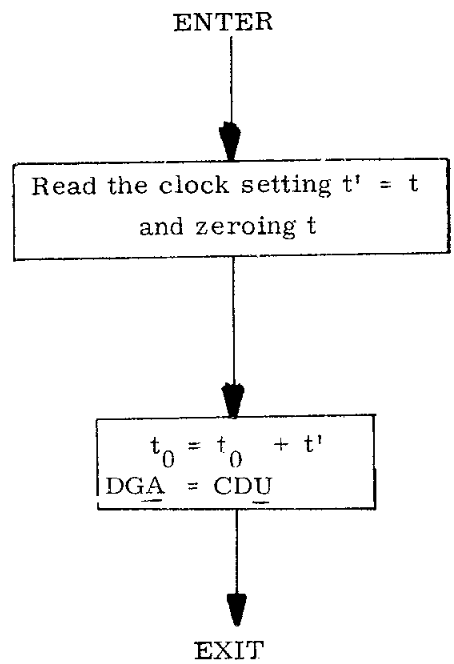


Figure 3.5-2 Time Subroutine

5.3.5.5 State Subroutine

The state vector of the vehicle at lift-off in Basic Reference Coordinates is that imparted by the earth. The first part of Fig. 3.5-3 shows this computation.

REFSMMAT, the matrix which relates the IMU stable member orientation at lift-off to the Basic Reference Coordinates, is determined as shown on page 2 of Fig. 3.5-3. This computation assumes that the stable member X and Y axes are normal to the pad local vertical and the X axis is aligned along the launch azimuth A_Z pointing down range and the Z axis toward the center of the earth. This computation determines the local vertical utilizing the characteristics of the earth's reference ellipsoid.

These computations take place within 0.75 second of the receipt of the lift-off discrete by Program P-11.

Attitude Error Subroutine

This subroutine computes and transmits to the FDAI the difference between a stored nominal Saturn Launch Vehicle attitude profile and the actual attitude profile as measured by the CM inertial measurement unit. Figures 3.5-4 and 3.5-5 present the details of these computations. This subroutine is cycled approximately every 0.5 seconds.

5.3.5.6 Display Subroutine

The computations of the display quantities v, h, $\dot{h}$ are shown on Fig. 3.5-6. Callable display parameters available during P-11 operation are described in Section 5.6.10, Orbital Parameter Display Computations.

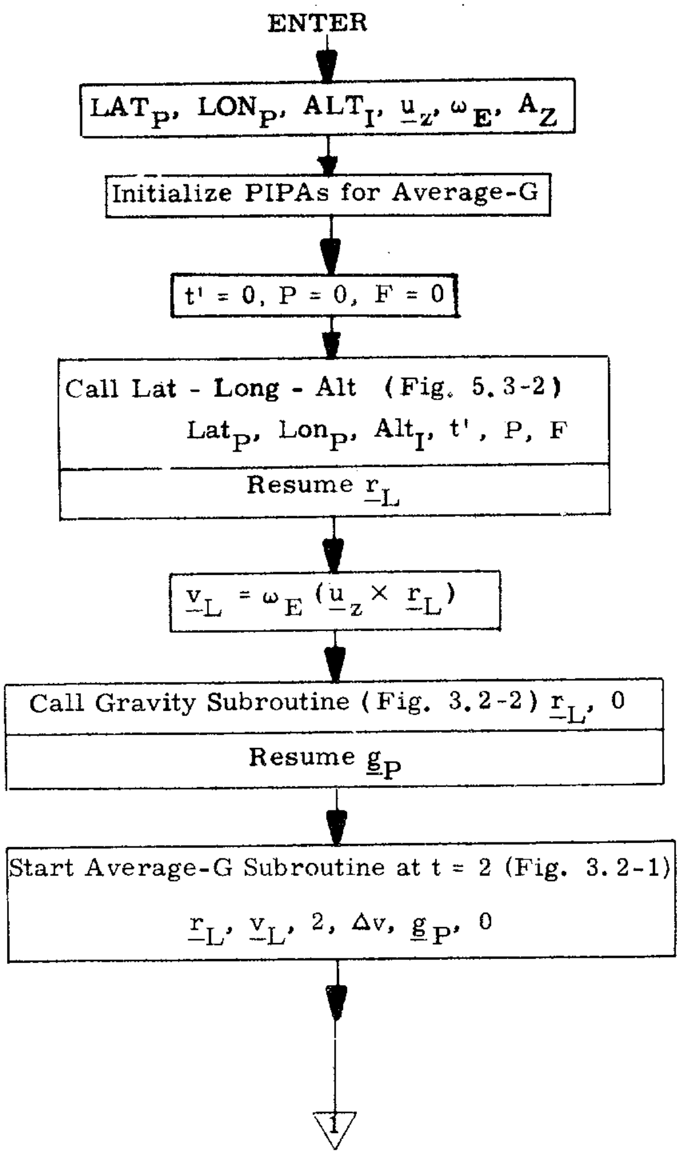


Figure 3.5-3 State Subroutine
(page 1 of 2)

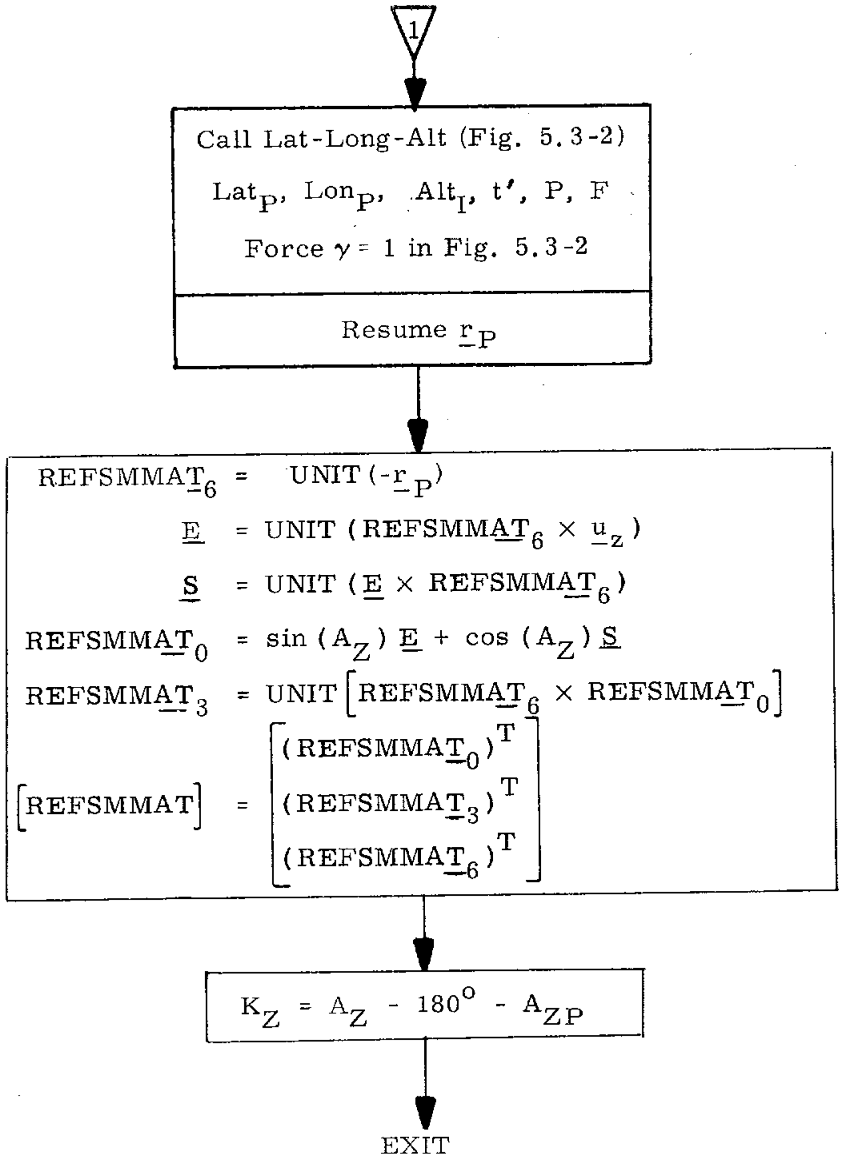


Figure 3.5-3 State Subroutine
(page 2 of 2)

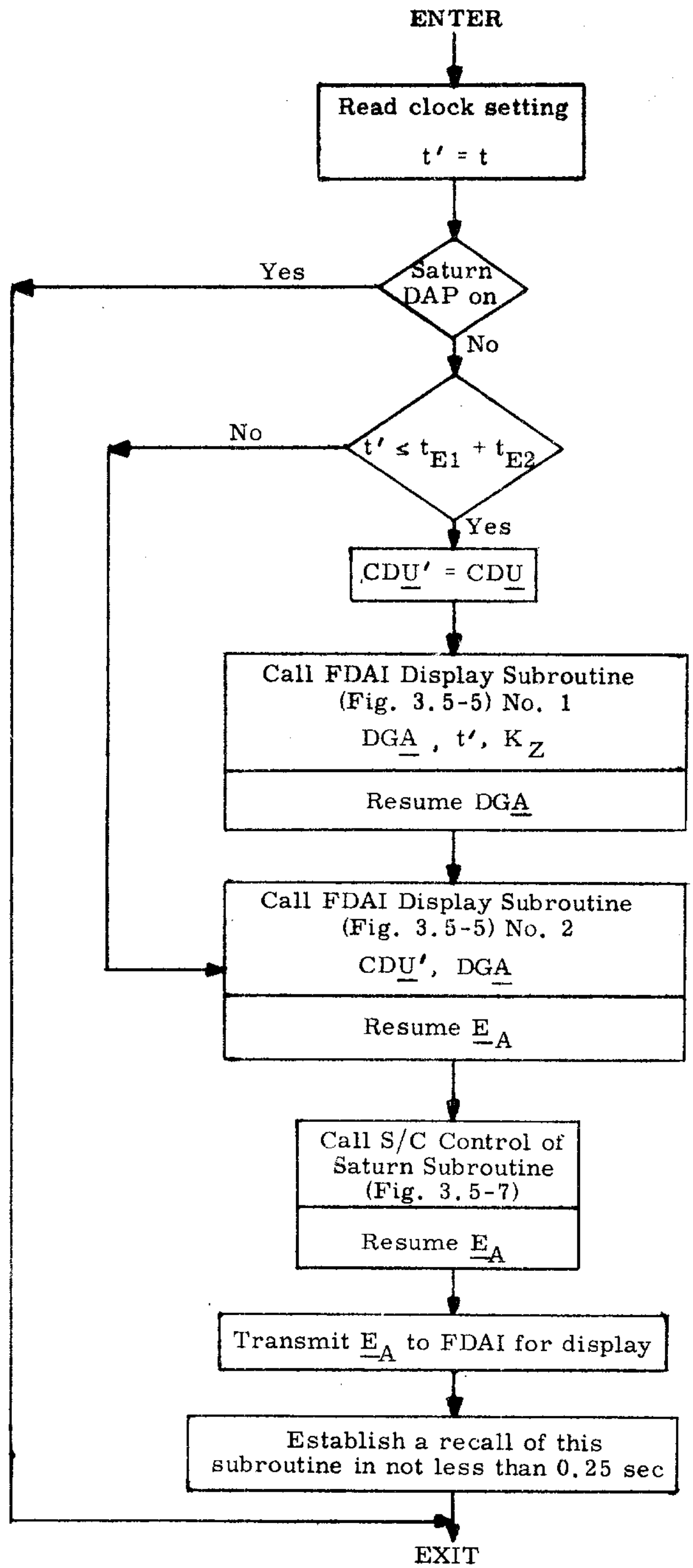


Figure 3.5-4 Attitude Error Subroutine

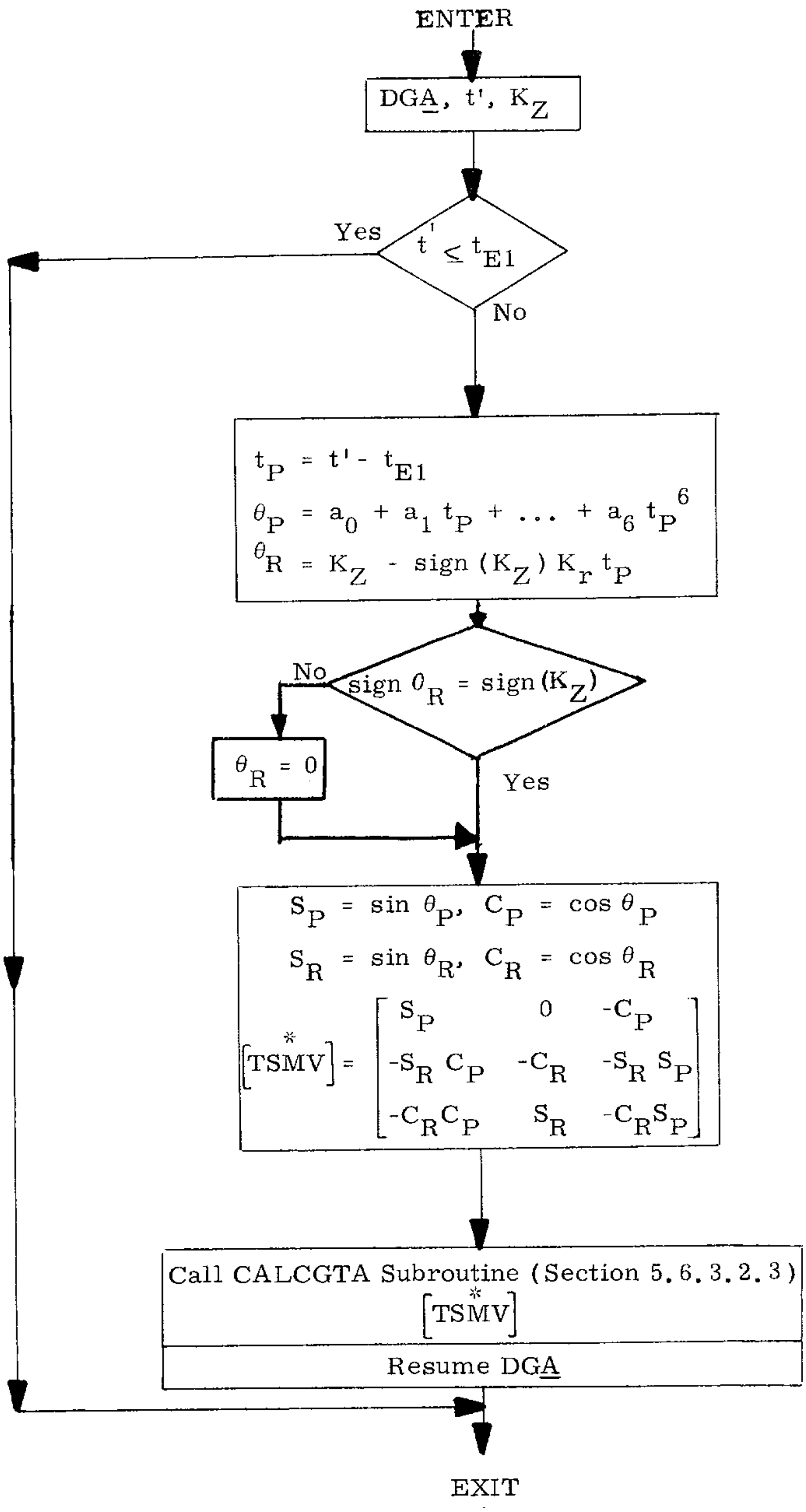


Figure 3.5-5 FDAI Display Subroutine No. 1
(page 1 of 2)

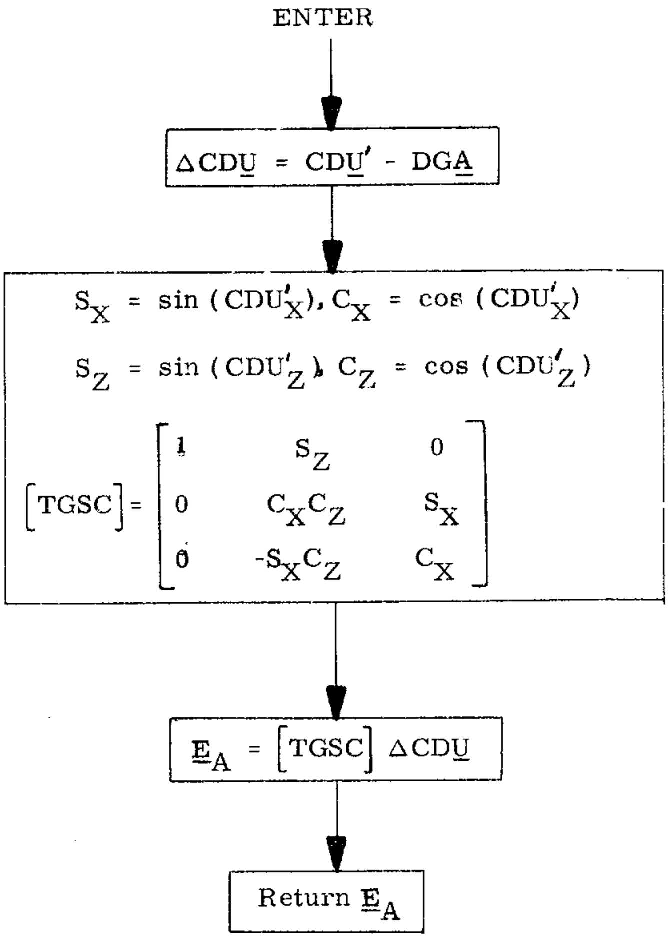


Figure 3.5-5 FDAI Display Subroutine No. 2
(page 2 of 2)

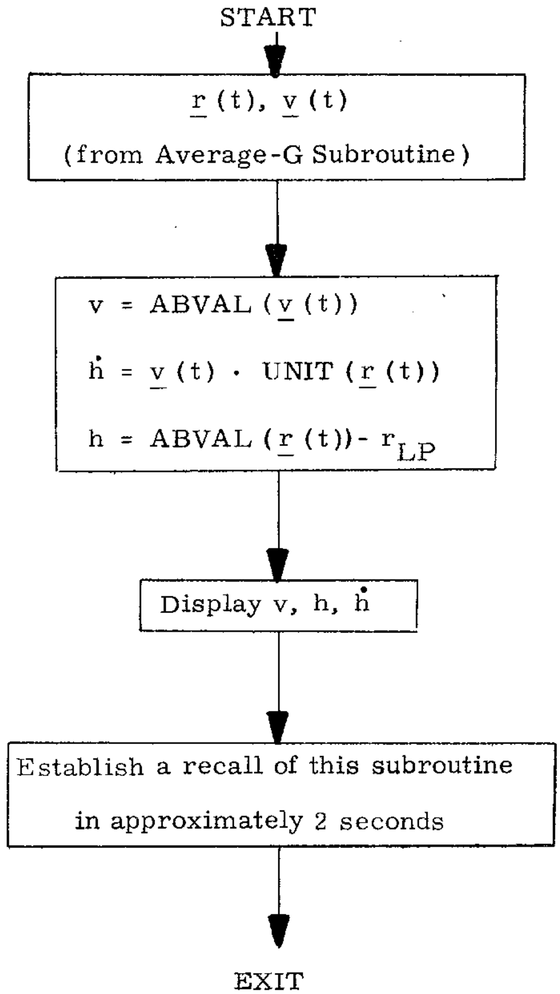


Figure 3.5-6 Display Subroutine

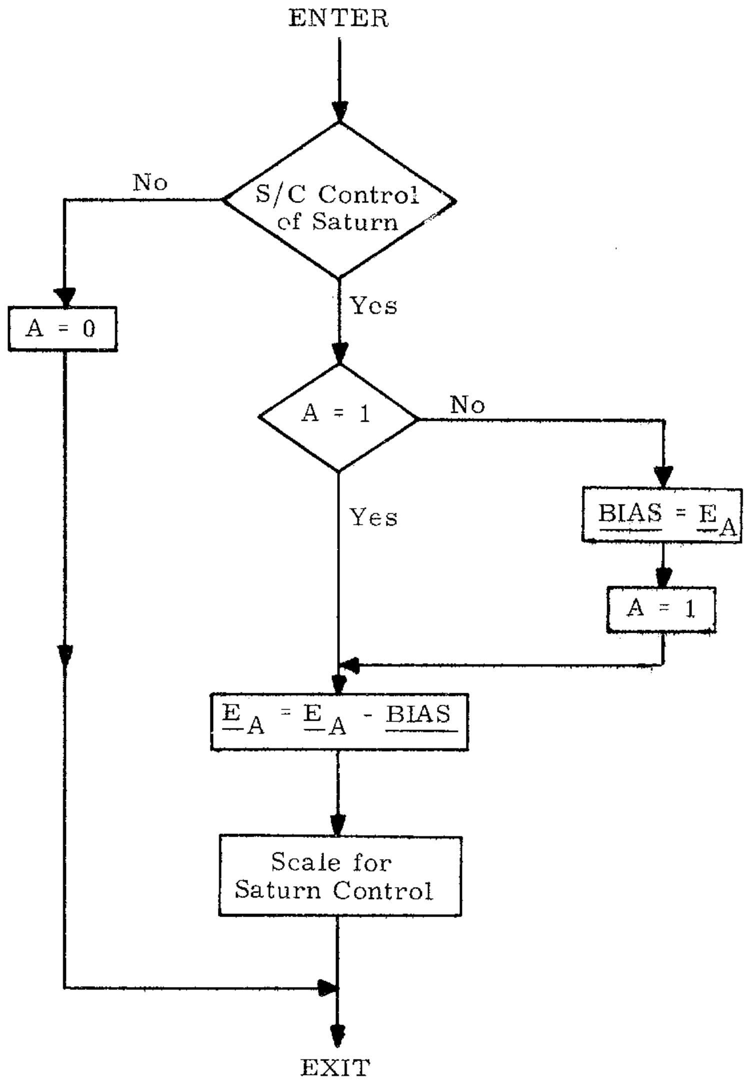


Figure 3.5-7 S/C Control of Saturn Subroutine

5.3.6 SATURN IVB SHUTDOWN COMPUTATIONS

This section describes the logic which monitors and shuts down the SIVB engine, during a GNCS backup of the TLI burn. It occurs as part of P15, the TLI Initiate/Cutoff Program, described in Section 4 GSOP.

The following nomenclature is used in the logic flow diagram (Figure 3.6-1).

t_{go}	Estimate of SIVB burn time	
t_{TGO}	Time-to-go	Before Ignition +10 sec: time from ignition After Ignition +10 sec: time from cutoff
$V_{C/O}$	Required TLI cutoff velocity magnitude (erasable padload)	
s_W	Steer-switch	Set 0 when P15 first called Set 1 at ignition +10 sec to enable shutdown computations Set 0 when shutdown command is issued, to prevent further t_{go} computations
$\underline{V}(t)$	Inertial velocity vector at time t	
V_G	Magnitude of velocity-to-be-gained	
D_{TF}	TLI tail-off constant (time delay for TLI tail-off)	

The logic is entered every $\simeq$ 2 seconds during P15 Average-G. When the time from ignition reaches ignition +10 seconds ($t_{TGO} \geq 10$), steer-switch is set 1 to allow t_{go} computations to take place. Note that this test will be passed (and s_W set 1) only once during the program, since t_{TGO} is then redefined to be time-from-cutoff.

The computation of t_{go} continues to take place until t_{go} is less than 4 seconds. At this time the engine off command is scheduled. Steer-switch is set 0 so that t_{go} computations will not be performed on the remaining passes.

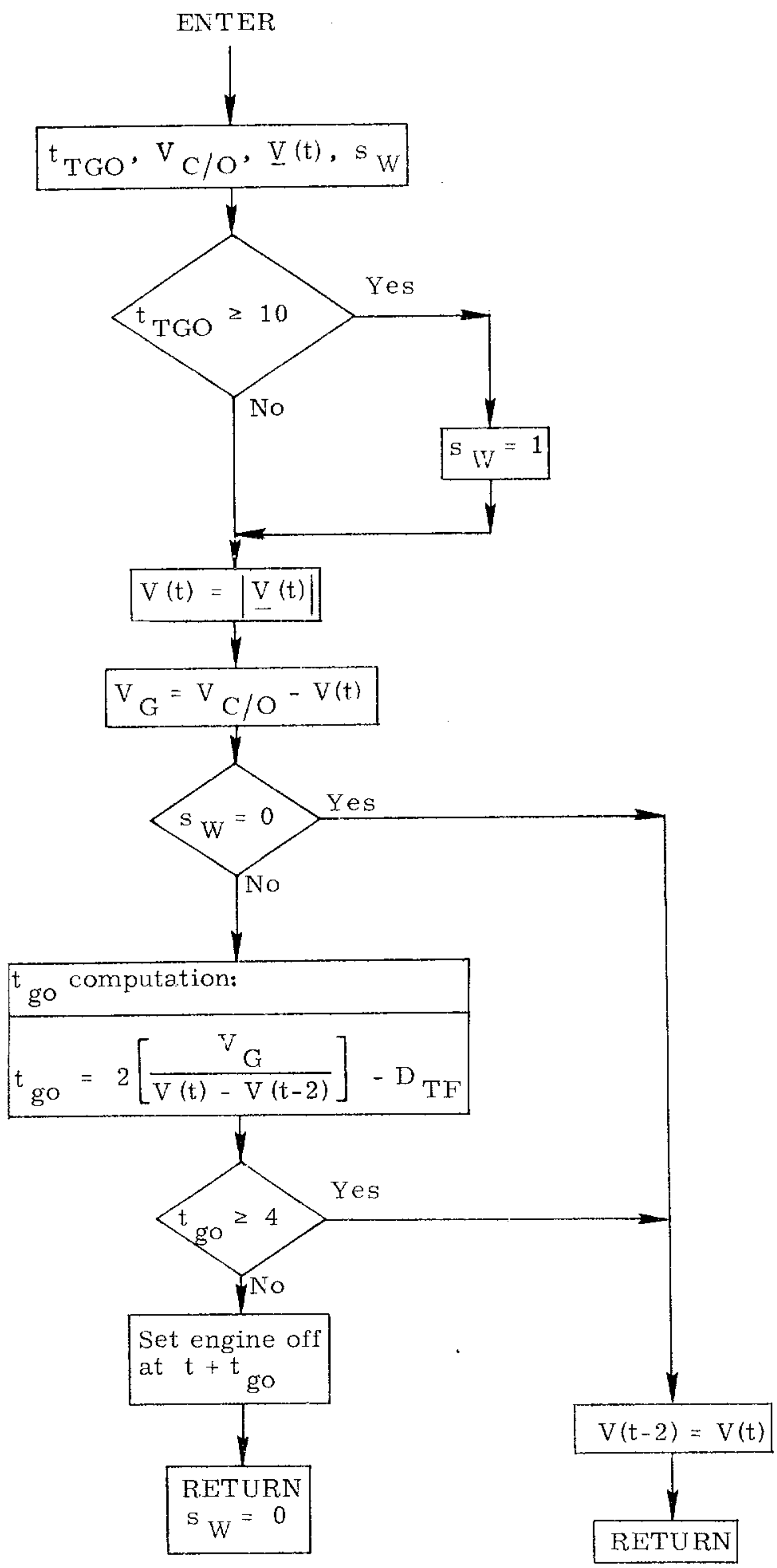


Figure 3.6-1. SIVB Shutdown Computations

5.4 TARGETING ROUTINES

5.4.1 General Comments

The objectives of the targeting routines presented in this section are to provide a CMC capability to determine the required input target parameters and set control modes for the various powered-flight guidance routines of Section 5.3. It should be noted that the CMC does not have a targeting capability for every phase of the lunar landing mission, and therefore, must rely on RTCC (ground) targeting for many of the nominal and abort phases of the lunar mission. The restricted targeting capability provided by the CMC is presented in the following subsections:

5.4.2 Rendezvous Retrieval Targeting Routines

These routines may operate in lunar or earth orbit.

5.4.3 Return-to-Earth Targeting Routines

These routines are restricted to operation outside the lunar sphere of influence for abort returns to earth.

All CMC targeting programs use the Basic Reference Coordinate System defined in Section 5.1.4.1. The Rendezvous Retrieval Targeting Routines of Section 5.4.2 can be used in either earth or lunar orbits. The basic input parameters required by the targeting routines of this section are the vehicle state vector estimates determined by the navigation programs of Section 5.2 and 5.3.2.

5.4.2 Rendezvous Retrieval Targeting

5.4.2.1 General

The nominal rendezvous maneuver sequence (direct rendezvous) uses the Transfer Phase Initiation Program and the Rendezvous Midcourse Maneuver Program. For backup rendezvous sequences, the Concentric Flight Plan Sequence, the Height Adjustment Maneuver Program, and the Plane Change Program are also available.

The Transfer Phase Initiation Program (P34) as used in the nominal rendezvous computes the parameters necessary to put the active vehicle on an intercept trajectory with the passive vehicle. This intercept is maintained by computing (and executing) midcourse corrections at pre-selected times after the Transfer Phase Initiation Maneuver using the Midcourse Maneuver Program (P35).

The Concentric Flight Plan Programs are used for those rendezvous situations where the phasing between the active and passive vehicles does not allow the direct rendezvous approach to be used.

The Concentric Flight Plan rendezvous scheme is illustrated in Figure 4. 2. 1. The following active vehicle impulsive maneuvers are used to establish a rendezvous intercept trajectory with the passive vehicle in an appropriately coplanar orbit:

1. Coelliptic Sequence Initiation (CSI) - (P32)
2. Constant Differential Altitude (CDH) - (P33)
3. Transfer Phase Initiation (TPI) - (P34)
4. Midcourse Maneuver Program (TPM) - (P35)

The CSI and CDH maneuvers satisfy the following constraints: specified times for the CSI and TPI maneuvers, a horizontal CSI maneuver, a constraint on either the number of apsidal crossings between CSI and CDH or the number of half periods between CSI and CDH, maneuver which results in "coelliptic" orbits following the maneuver and a specified geometry at the TPI point. The "coelliptic" or "concentric" orbits have approximately constant differential altitudes.

The Plane Change Maneuver (PC) is used in conjunction with the CSI maneuver of the Concentric Flight Plan to make the active and passive vehicle orbits coplanar prior to the CDH maneuver. The Plane Change Maneuver is performed ninety degrees after the CSI maneuver and is computed in the Plane Change Maneuver Program (P36).

Certain rendezvous situations (LM aborts at certain times in the powered descent, for example) require a Height Adjustment Maneuver (HAM) to be performed one-half revolution prior to the CSI maneuver in the Concentric Flight Plan Sequence so that the standard coelliptic differential altitude will be attained after the CDH maneuver. This maneuver is computed in the Height Adjustment Maneuver Program (P31).

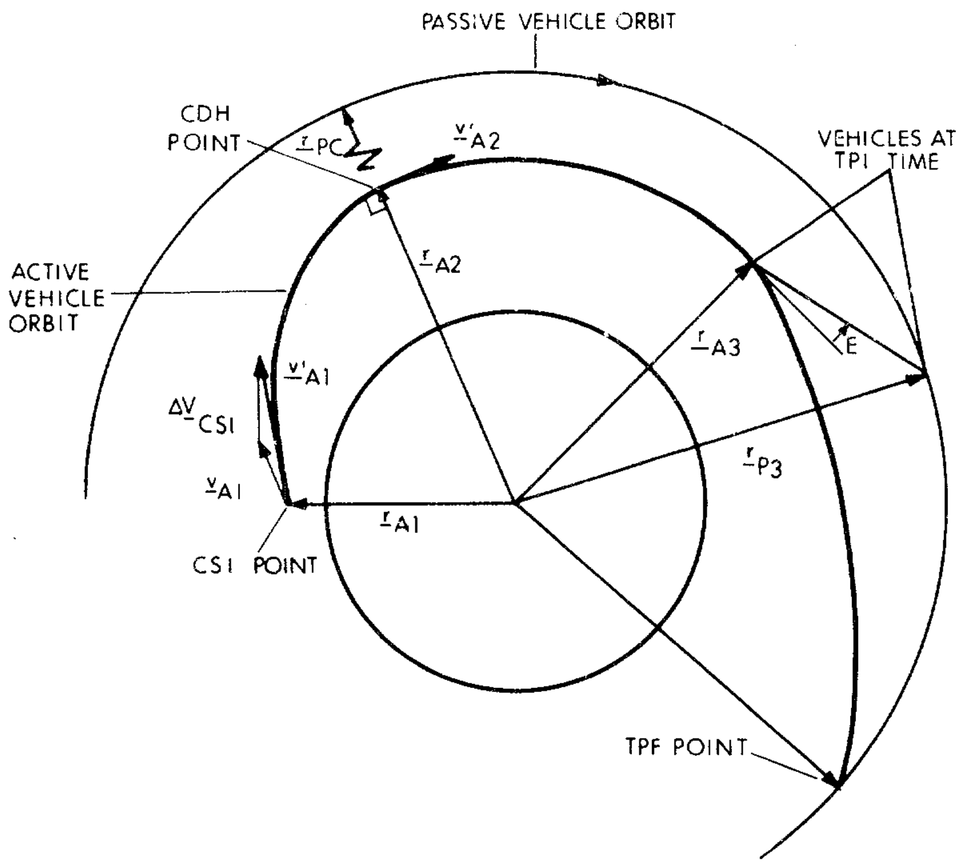


Figure 4.2-1 Concentric Rendezvous Profile

For CSI, CDH, TPI and TPM targeting there are two associated program numbers based on whether the CSM or the LM is the active vehicle. If the astronaut elects to make the LM active, the only change is the program number selected. All equations and program operations in Section 5.4.2 are identical for these two modes of operation. The Plane Change Program (P36) and the Height Adjustment Maneuver Program (P31) each have a single program number and consider the CSM as the active vehicle.

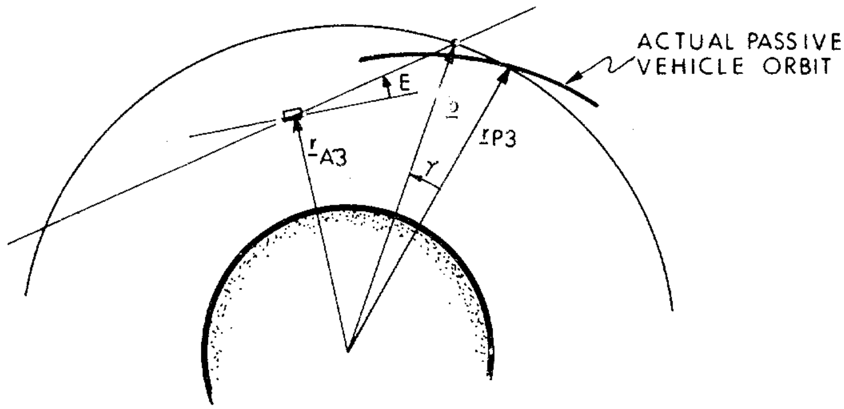
The pre-thrust programs, P31, P32, P33 and P36 use External ΔV guidance (Section 5.3.3.3.1) and P34 and P35 use the Lambert targeting concept. There are 2 options available for obtaining the aim point used in generating the Lambert ΔV solution. If the number of offsets (N) is set to zero, the target orbit is advanced through an ωt in P34, or the appropriate time of flight in P35, by use of the Kepler (conic) routine. Lambert solution is then generated to this aim point. The other option ($N > 0$) employs N precision offsets to generate the velocity correction required (N usually = 2). The target vehicle orbit is advanced the appropriate time of flight on a precision trajectory. The precision offset concept is then used to generate the ΔV solution. All six of the above programs use the Cross-Product Steering of Section 5.3.3.4.

The function SGN(X) used in the figures of Section 5.4.2 is set equal to +1 if X is zero or a positive number; otherwise, it is set equal to -1.

5.4.2.2 CSI Targeting

The CSI program, corresponding to P32 (CSM active) or P72(LM active) of Section 4, computes the parameters associated with the CSI and CDH maneuvers. The astronaut inputs are as follows:

1. Choice of active vehicle (P32 CSM, P72 LM).
2. Time t_1 of the CSI maneuver.
3. Number N of the apsidal crossing. (If N = 1 and CDHASW = 0, the CDH maneuver occurs when the active vehicle reaches its first apsidal point following the CSI maneuver, etc.)
4. Desired line of sight (LOS) angle E at the time of the TPI maneuver. (See Figure 4.2-2 and Figure 4.2-3.)
5. Time t_3 of the TPI maneuver
6. Selection of flag CDHASW to ensure that CDH will occur at a time, measured from the CSI time, equal to the post-CSI maneuver orbital period multiplied by N/2. If this option is desired, CDHASW is set equal to 1.



TPI geometry, active vehicle below

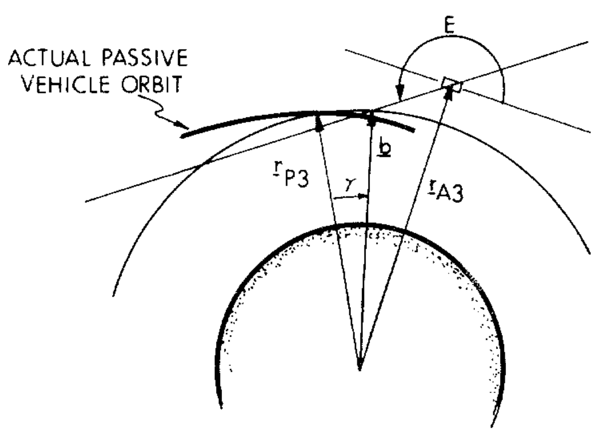
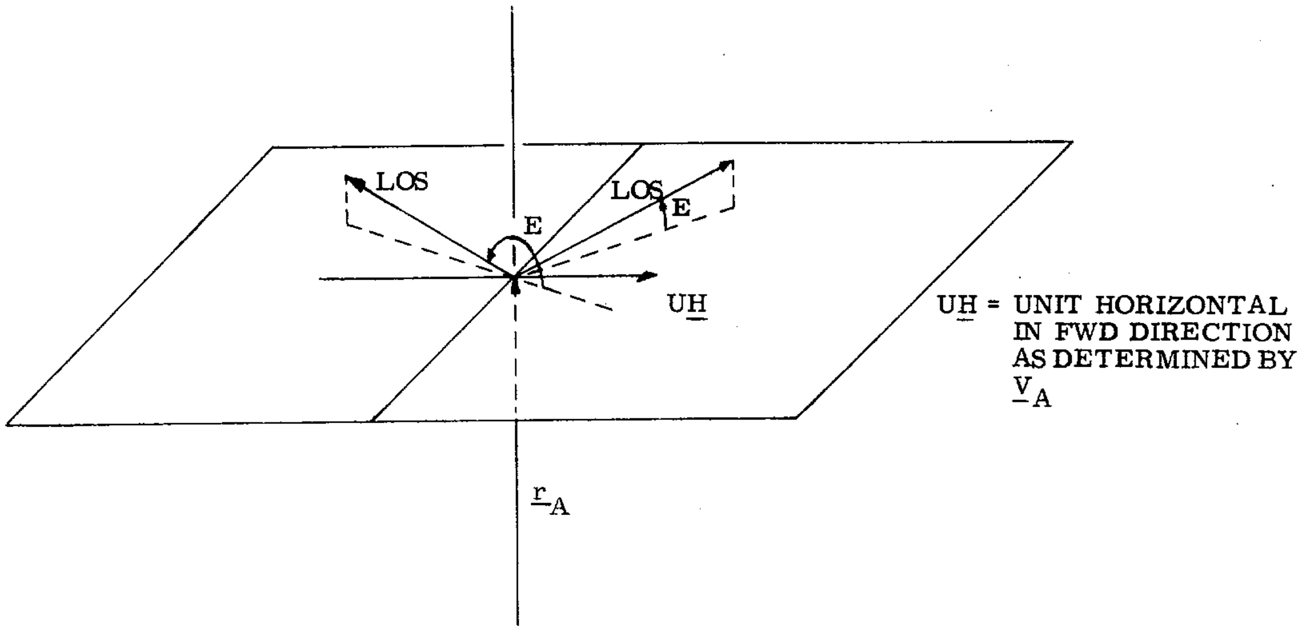


Figure 4.2-2 TPI geometry, active vehicle above



1) IF THE LOS PROJECTION ON $\underline{UH}$ IS POSITIVE:

   a) WHEN THE LOS IS ABOVE THE HORIZONTAL PLANE
   $0 < E < \pi/2$

   b) WHEN THE LOS IS BELOW THE HORIZONTAL PLANE
   $3\pi/2 < E < 2\pi$

2) IF THE LOS PROJECTION ON $\underline{UH}$ IS NEGATIVE

   a) WHEN THE LOS IS ABOVE THE HORIZONTAL PLANE
   $\pi/2 < E < \pi$

   b) WHEN THE LOS IS BELOW THE HORIZONTAL PLANE
   $\pi < E < 3\pi/2$

Figure 4.2-3 Definition of Elevation Angle, E

The active vehicle state vector $\underline{r}_A$, $\underline{v}_A$ and passive vehicle state vector $\underline{r}_P$, $\underline{v}_P$ are available in the guidance computer.

The following constraints must be satisfied (see Figure 4.2-1 and Figure 4.2-2).

1. The CSI ΔV is applied in the horizontal plane of the active vehicle at the CSI time.
2. The semimajor axis and radial component of velocity of the active vehicle are such that the active and passive vehicles are in coelliptic orbits following the CDH maneuver.
3. The line of sight between the active vehicle and the passive vehicle at the TPI time forms the angle E with the horizontal plane of the active vehicle.
4. The time intervals between the CSI-CDH and CDH-TPI maneuvers are 10 minutes or greater.
5. After both the CSI and CDH maneuvers, the perigee altitude of the active vehicle orbit is greater than 35,000 ft for lunar orbits and 85 n.mi. for earth orbits.

The program solution, as illustrated in the logic flow diagram in Figure 4.2-4, is based on conic trajectories and contains an iteration loop to select the CSI maneuver magnitude v_1.

After updating of the active and passive vehicle state vectors to the CSI time the following out-of-plane parameters are computed: out-of-plane velocity of the active vehicle relative to the passive vehicle orbital plane $\dot{Y}_A$, out-of-plane position of the active vehicle relative to the passive vehicle orbital plane Y_A and the out-of-plane velocity of the passive vehicle relative to the active vehicle orbital plane $\dot{Y}_P$.

The active vehicle state vector is then rotated into the passive vehicle orbital plane, resulting in coplanar orbits. The passive vehicle is updated through the time $t_3 - t_1$ to the TPI point obtaining $\underline{r}_{P3}$, $\underline{v}_{P3}$. The initial guess of v_1, when added to the velocity of the active vehicle, results in the active vehicle attaining a radius equal to r_{P3}, 180 degrees from the CSI point.

If CDHASW equals one, or if the eccentricity of the active vehicle orbit is less than 0.000488, or if the vertical velocity at the CSI point is less than 7 ft/sec, then the time t_2 of the CDH maneuver is set equal to the CSI time plus N times half the period t_P of the active vehicle. Otherwise, the angle ψ to the nearest perigee is computed and used in the Time-Theta Subroutine to find the corresponding time of flight Δt. This is then used to calculate t_2 based on the value of N. After verifying that t_3 is greater than t_2, both vehicles are updated to the CDH point.

The angle φ between the active and passive vehicles at the CDH point is calculated and used in the Time-Theta Routine to update the passive vehicle to a point radially above or below the active vehicle, obtaining $\underline{r}_{PC}$, $\underline{v}_{PC}$.

The semimajor axis a_A and the radial component of velocity v_{AV}, as well as the active vehicle position magnitude, uniquely define the orbit of the active vehicle immediately following the CDH maneuver. The equations for a_A (based on the semimajor axis a_P of the passive vehicle), and for v_{AV}, specify the "coelliptic" CDH maneuver, $\Delta \underline{V}_2$ (Figure 4.2-4).

The active vehicle state vector is next updated to the TPI time. The unit vector which passes through the active vehicle position and is coincident with the desired TPI line of sight is given by $\underline{u}_L$. The position vector of the two points of intersection between the line of sight and a circle (with origin at the center of the attracting body) as shown in Figure 4.2-2 is defined by:

$$\underline{b} = \underline{r}_{A3} + k\underline{u}_L \qquad (4.2.1)$$

where k normally assumes two values (see Figure 4.2-4). Equating the magnitude of $\underline{b}$ to $|\underline{r}_{P3}|$ results in a quadratic expression for k:

$$k^2 + 2k\underline{r}_{A3} \cdot \underline{u}_L + r_{A3}^2 - r_{P3}^2 = 0 \qquad (4.2.2)$$

If there are no real solutions to the above equation ($c_2 < 0$ in Figure 4.2-4), the line of sight does not intersect the circle. If there is a real solution to the above equation, the k of minimum absolute value is chosen as the desirable TPI solution. The error γ is defined as the central angle between the position vector of the passive vehicle at the TPI point and the position vector $\underline{b}$ representing where the passive vehicle should be based on the active vehicle location. This error is driven to zero using a Newton-Raphson iteration loop with v_1 as the independent variable. The two initial values of v_1 are 10 ft/sec apart with the succeeding step size ΔV_1 restricted to 200 ft/sec.

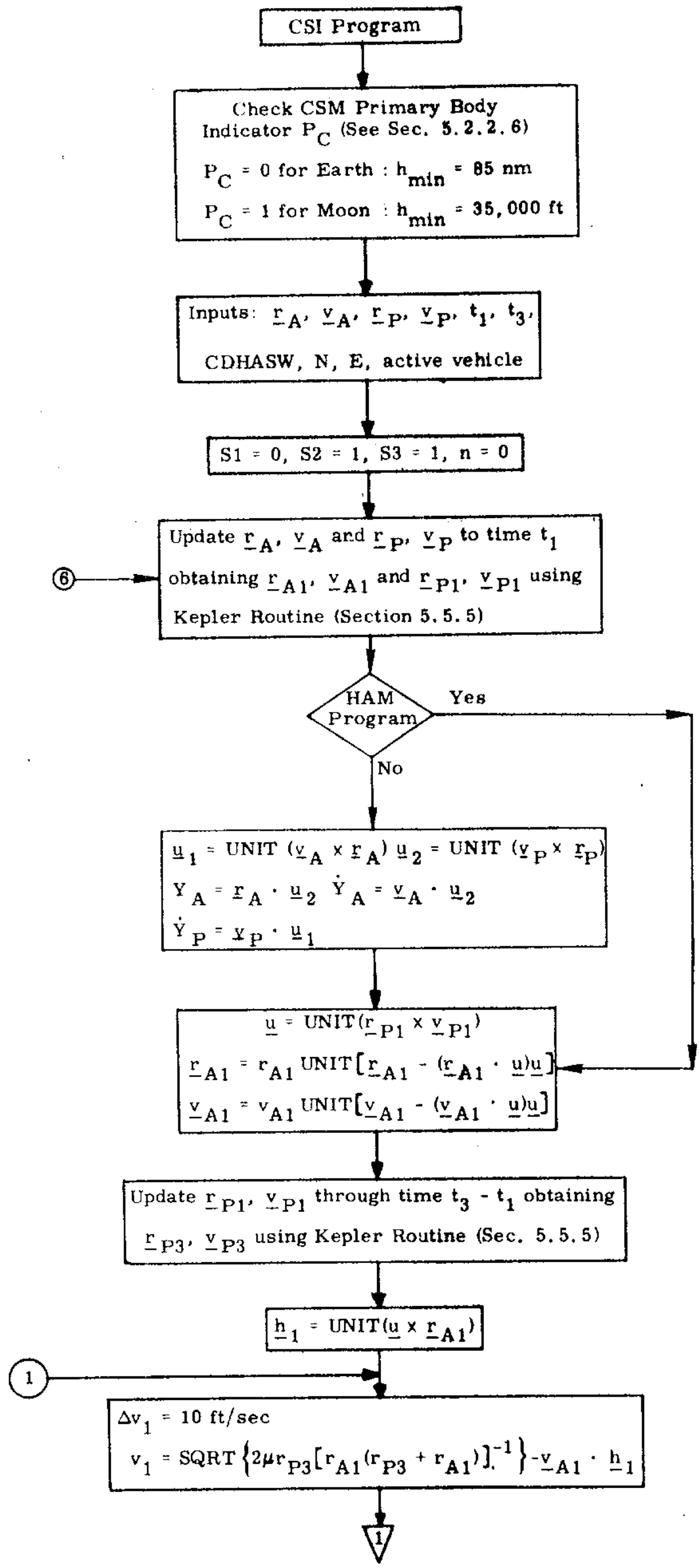


Figure 4.2-4. CSI-CDH Maneuver Program (Sheet 1 of 7)

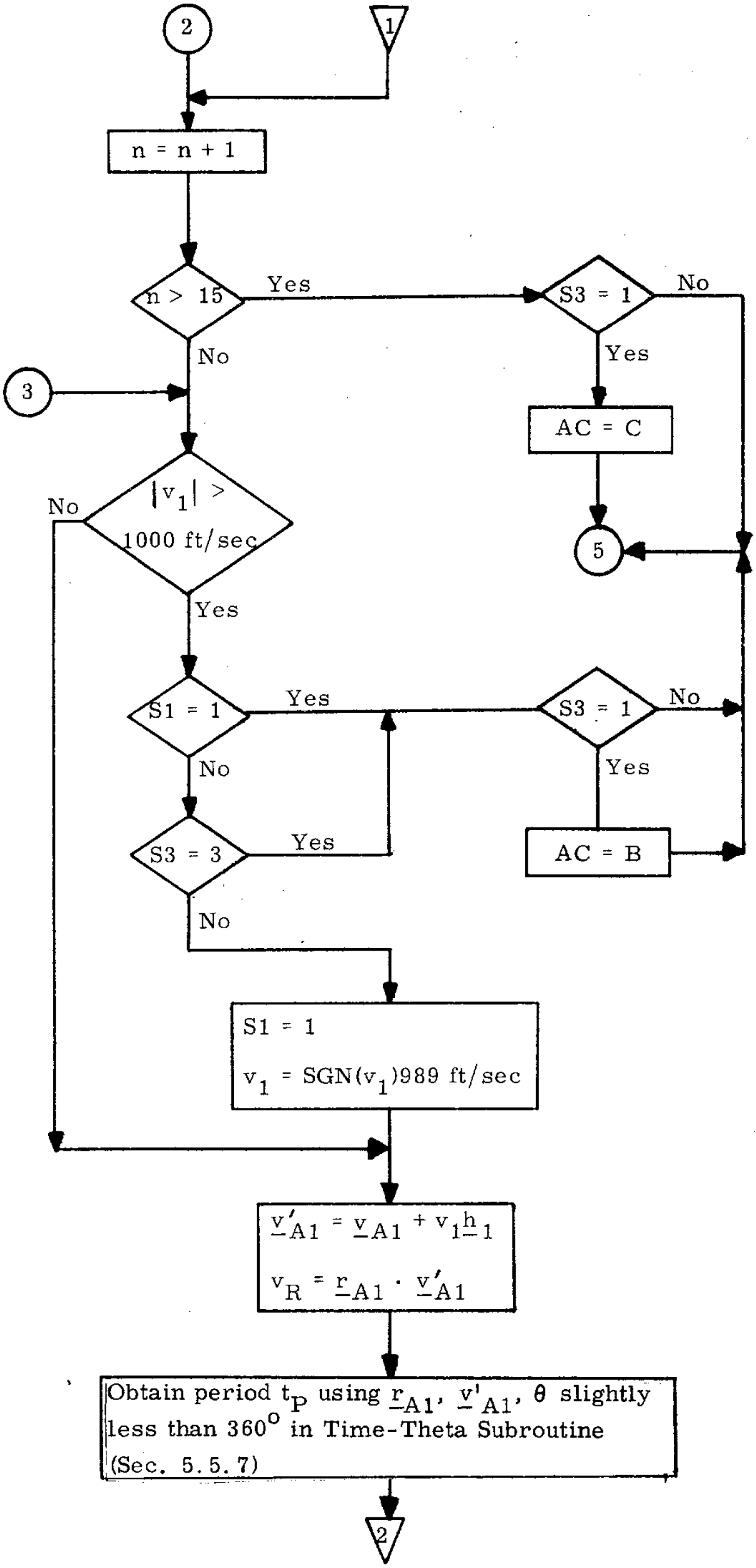


Figure 4.2-4. CSI-CDH Maneuver Program (Sheet 2 of 7)

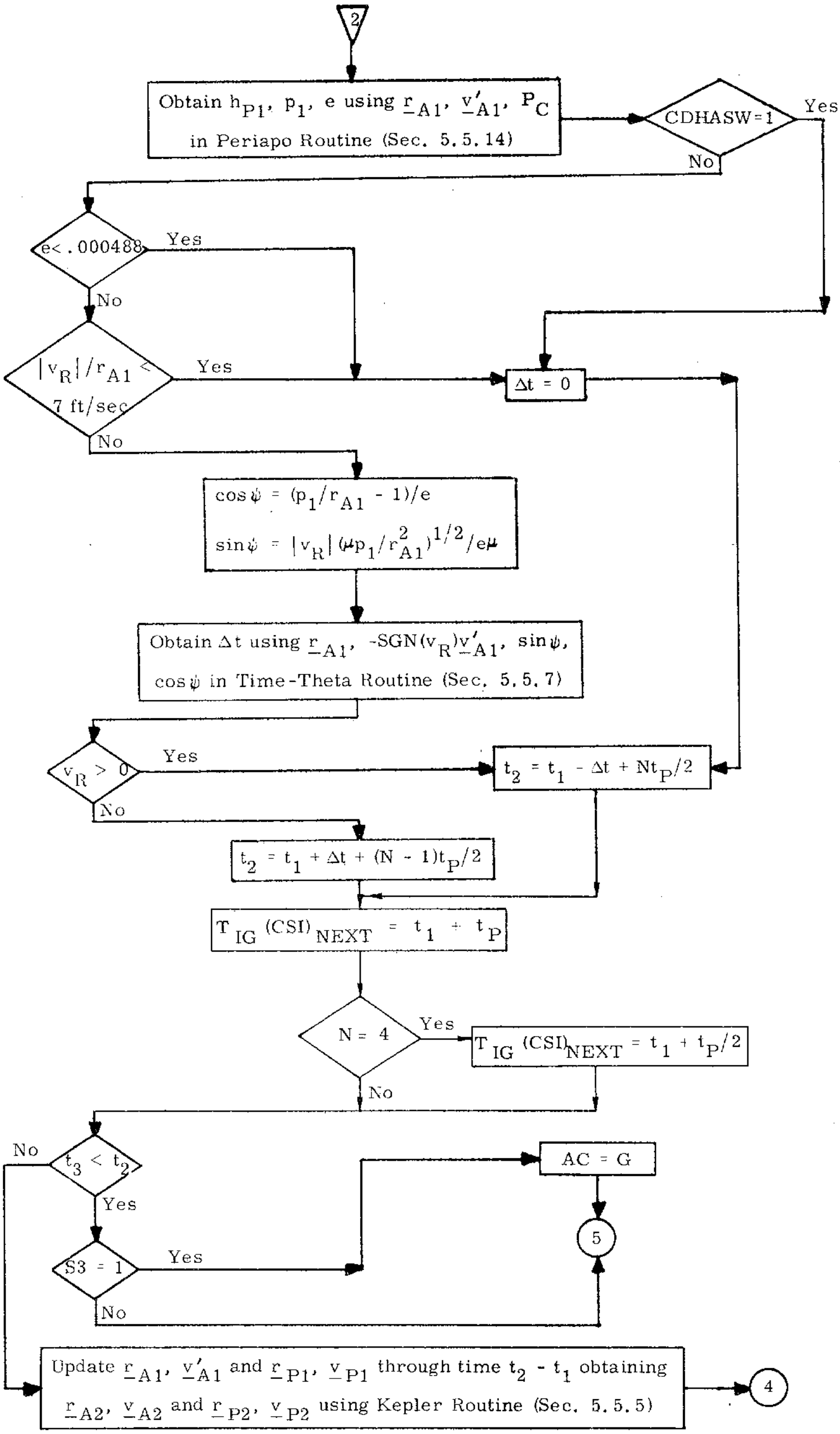


Figure 4.2-4. CSI-CDH Maneuver Program (Sheet 3 of 7)

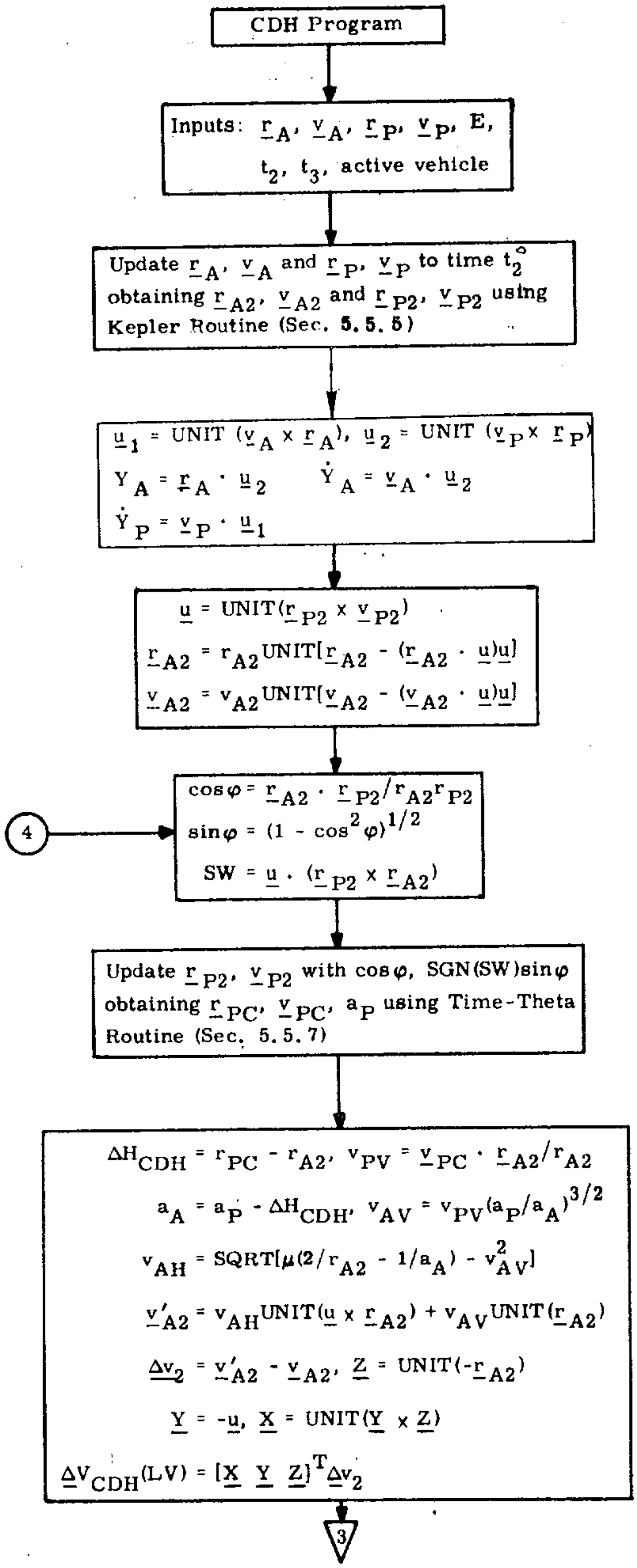


Figure 4.2-4. CSI-CDH Maneuver Program (Sheet 4 of 7)

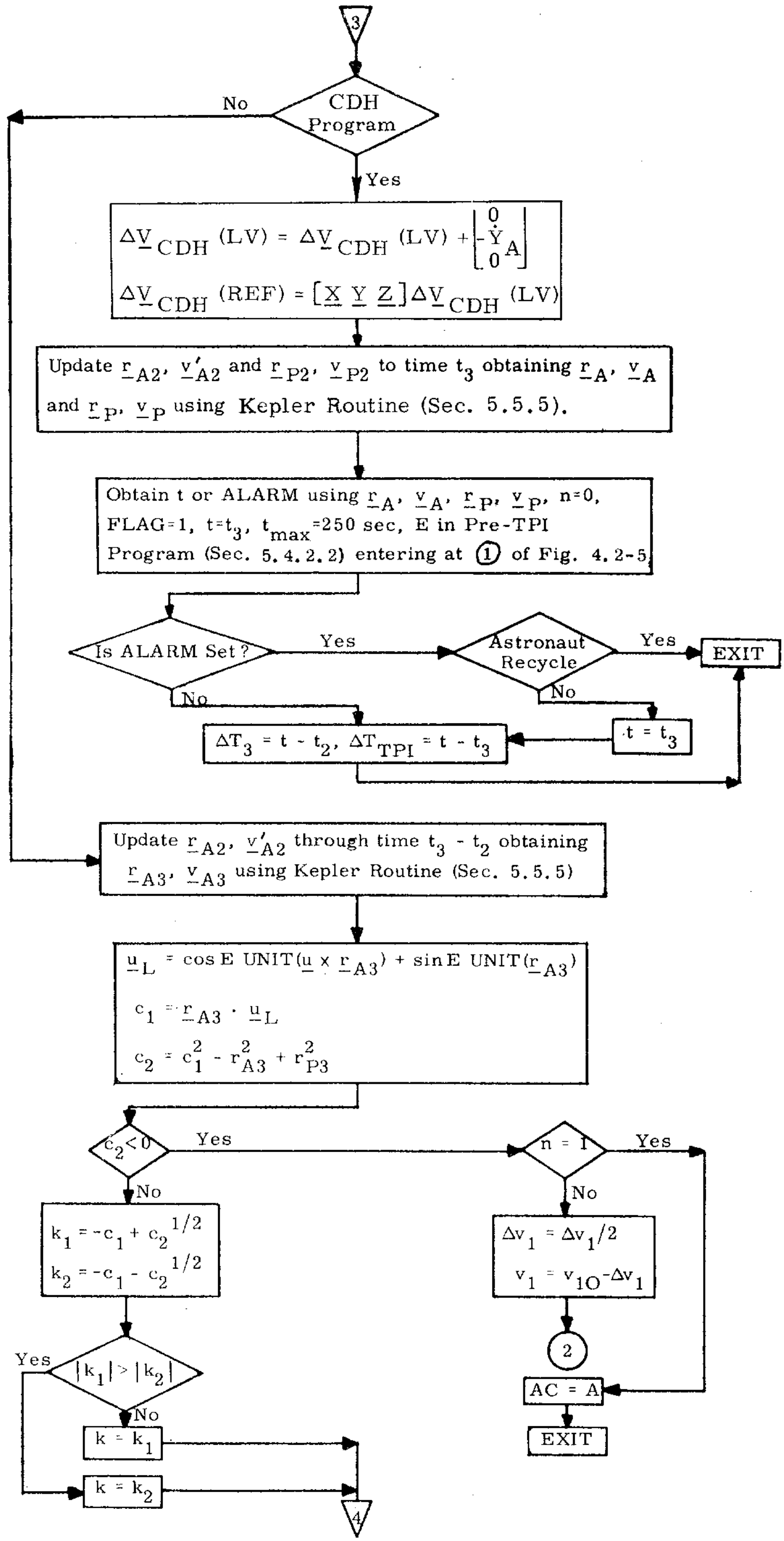


Figure 4.2-4. CSI-CDH Maneuver Program (Sheet 5 of 7)

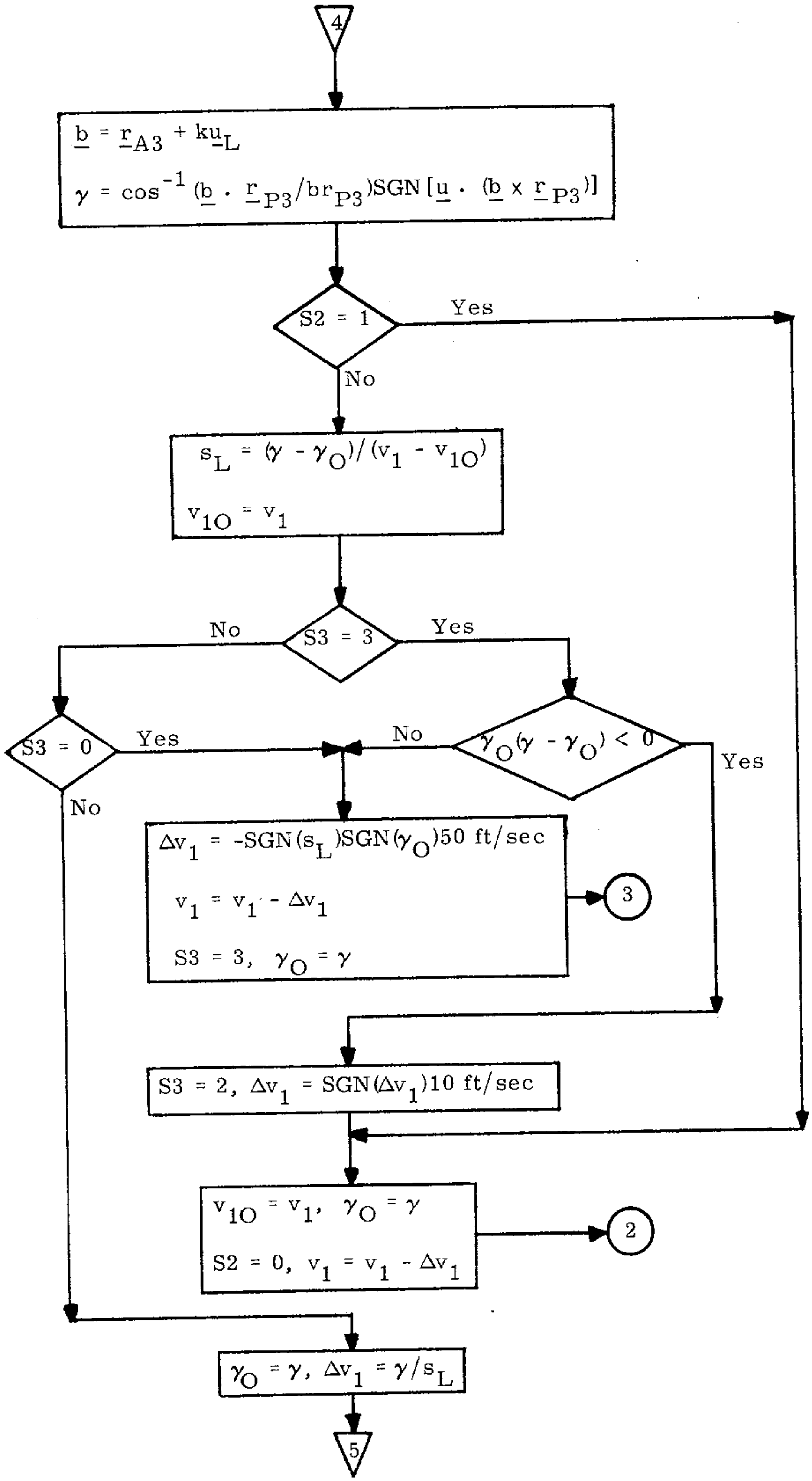


Figure 4.2-4. CSI-CDH Maneuver Program (Sheet 6 of 7)

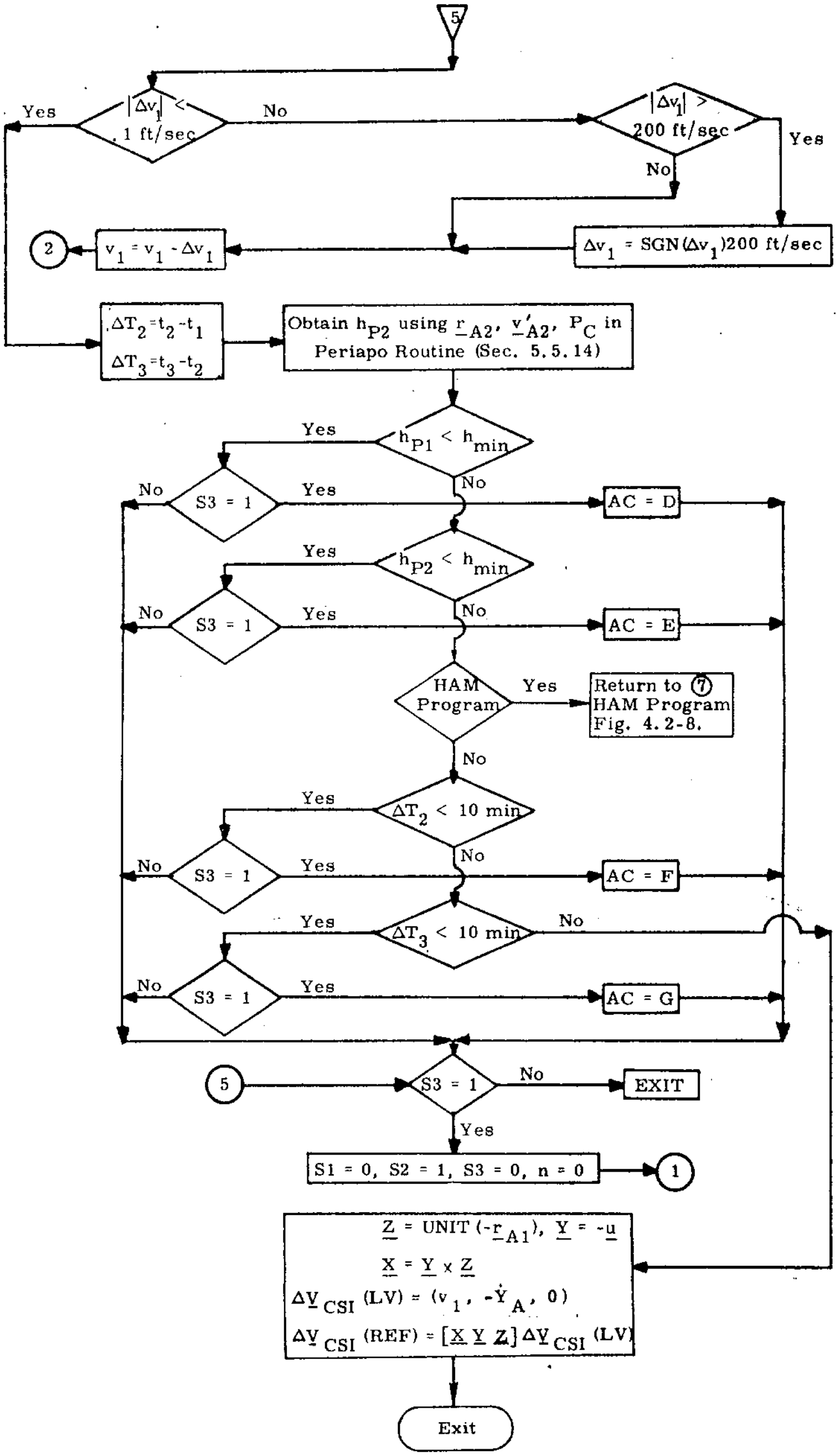


Figure 4.2-4. CSI-CDH Maneuver Program (Sheet 7 of 7)

A second computational attempt is made if the first iteration attempt fails. v_1 is incremented in steps of 50 ft/sec in the direction opposite to that initially taken in the first attempt until the error γ has undergone a sign change. The second attempt then starts with the latest value of v_1 and an adjacent point in the direction of the 50 ft/sec steps.

Included in the CSI program are the following seven program checks and the corresponding Alarm Codes (AC) (see Figure 4. 2-4).

1. If on the first iteration there is no solution to the TPI geometry, the iteration cannot be established. AC equals A in this case.
2. The iteration counter n exceeds 15. (AC = C)
3. Two succeeding iterations resulting in v_1 greater than 1000 ft/sec. (AC = B)
4. The altitude of perigee h_{P1} following the CSI maneuver is less than a minimum acceptable amount. (AC = D)
5. The altitude of perigee h_{P2} following the CDH maneuver is less than a minimum acceptable amount. (AC = E)
6. The time ΔT_2 between the CDH and CSI maneuvers is less than 10 minutes. (AC = F)
7. The time ΔT_3 between the TPI and CDH maneuvers is less than 10 minutes. (AC = G)

For the first check above, a check failure results in an immediate program exit. The last four checks are made only if the iteration has succeeded. The alarm codes are set (excluding the first check) only if the second iteration attempt has been made.

The displays for the CSI program are:

1. Differential altitude ΔH_{CDH} at the CDH point. ΔH_{CDH} is positive when the active vehicle's altitude is less than the passive vehicle's altitude when measured at the CDH point.
2. Time ΔT_2 between the CDH and CSI maneuvers displayed in minutes and seconds with hours deleted.

3. Time ΔT_3 between the TPI and CDH maneuvers displayed in minutes and seconds with hours deleted.

4. Out-of-plane parameters at t_{IG}: active vehicle out-of-plane position Y_A and velocity $\dot{Y}_A$ relative to the passive vehicle orbital plane; passive vehicle out-of-plane velocity $\dot{Y}_P$ relative to the active vehicle orbital plane.

5. $\Delta \underline{V}_{CSI}$ (LV) in local vertical coordinates at CSI time.

6. $\Delta \underline{V}_{CDH}$ (LV) in local vertical coordinates at CDH time.

7. The alarm codes.

In the CSI program,P32, the astronaut can change the CSI computed maneuver by overwriting the displayed maneuver velocity vector in local vertical coordinates, $\Delta \underline{V}_{CSI}$ (LV). In such a case the astronaut input then becomes the defined velocity-to-be-gained in the External ΔV Maneuver Guidance (Section 5. 3. 3. 3. 1) used to control the CSI maneuver.

Also displayed are M, the number of navigation measurements processed since the last W-matrix reinitialization; MGA, the middle gimbal angle at the start of the maneuver assuming plus X acceleration; and TFI, the time from ignition.

The middle gimbal angle is computed with the following equation:

$$MGA = \sin^{-1} \left[\underline{Y}_{REF} \cdot \text{UNIT} (\Delta \underline{V}) \right]$$

If $-90^\circ \leq MGA < 0^\circ$, MGA is redefined such that $270^\circ \leq MGA < 360^\circ$. $\Delta \underline{V}$ is the maneuver $\Delta \underline{V}$ in reference coordinates and $\underline{Y}_{REF}$ is the second row of the REFSMMAT matrix defined in Section 5. 6. 3. 3.

5. 4. 2. 3 CDH Targeting

The CDH program, corresponding to program P33 (CSM active) or P73 (LM active) of Section 4, computes the parameters associated with the CDH maneuver.

The astronaut inputs to this program are:

1. Choice of active vehicle (P33 CSM, P73 LM).

2. Time t_2 of CDH maneuver.

The active and passive vehicles' state vectors $\underline{r}_A$, $\underline{v}_A$, $\underline{r}_P$, $\underline{v}_P$ are available in the guidance computer. The TPI elevation angle E and the TPI maneuver time t_3 are also available from the previous CSI program computations.

Figure 4.2-4, sheet 4 of 7, illustrates the program logic. The state vectors of both the active and passive vehicles are advanced to the CDH time. The following out-of-plane parameters are then calculated: active vehicle out-of-plane position and velocity relative to the passive vehicle orbit plane, and the passive vehicle out of plane velocity relative to the active vehicle orbit plane.

After rotating the active vehicle state vector into the plane of the passive vehicle, the CDH maneuver is calculated as in the CSI program. Following a conic update of the state vectors to time t_3, the TPI program is used to calculate the time at which the specified elevation angle is attained. If the iteration was not successful, an alarm code is displayed. At this point, the astronaut can elect to recycle the program or to proceed with the calculation of the displays, which are:

1. Out-of-plane parameters at TIG:
   - active vehicle out-of-plane position and velocity
   - passive vehicle out-of-plane velocity
2. $\Delta\underline{V}_{CDH}$ (LV)
3. Differential altitude ΔH_{CDH} at the CDH point
4. Time ΔT_3 between the calculated TPI time and the CDH time displayed in minutes and seconds with hours deleted.
5. Time ΔT_{TPI} between the calculated TPI time and the specified TPI time. (This number is positive if the new TPI time is later than that previously used.)

If the iteration did not converge, the last two displays reflect an unchanged TPI time. In addition, M, MGA, and TFI are displayed as in the CSI program displays.

The astronaut can modify the CDH maneuver by overwriting the displayed maneuver velocity vector, $\Delta\underline{V}_{CDH}$ (LV), as in the CSI program.

5.4.2.4 TPI Targeting

The TPI program corresponds to program P34 (CSM active) or P74 (LM active) of Section 4. Its objective is to establish the terminal phase initiation (TPI) maneuver. The position of the TPI maneuver is determined by specifying either the TPI time or the elevation angle which specifies the relative geometry of the vehicles at the TPI point. The astronaut inputs are:

1. Choice of active vehicle (P34 CSM, P74 LM)
2. Time t of the TPI maneuver
3. Elevation angle E (set equal to zero if t is specified), defined in Figure 4.2-3
4. Central angle ωt of the passive vehicle between the TPF and TPI points
5. Number N_1 of precision offsets made in generating target vector for TPI maneuver. If set equal to zero, conic trajectories are used to generate the target vector.

The active $\underline{r}_{AI}$, $\underline{v}_{AI}$ and passive vehicle $\underline{r}_{PI}$, $\underline{v}_{PI}$ state vectors are available in the guidance computer. The program starts with a conic update of these vectors to the TPI time.

If the elevation angle is not specified, it is computed after the conic updating of both vehicles to the TPI maneuver time. If E is specified, an iteration procedure is initiated to find the TPI time at which E is attained. This procedure uses the input TPI time as the initial guess and is based on conic trajectories. The time correction δt is based on (1) the angular distance between $\underline{r}_P$ and the desired position of the passive vehicle, obtained by assuming the passive vehicle is in a circular orbit, and (2) assuming the vehicles are moving at a constant angular rate. The flow diagram for this procedure is shown in Figure 4.2-5.

The iteration is successful when the computed elevation angle E_A is sufficiently close to E.

To help insure convergence, the following steps are taken

a. The step size δt is restricted to 250 sec
b. If the solution has been passed ($\delta E \ \delta E_0 < 0$), the step size is halved and forced in the opposite direction of the last step.
c. If the iteration is converging ($|\delta E_0| - |\delta E| > 0$), the sign of δt is maintained.

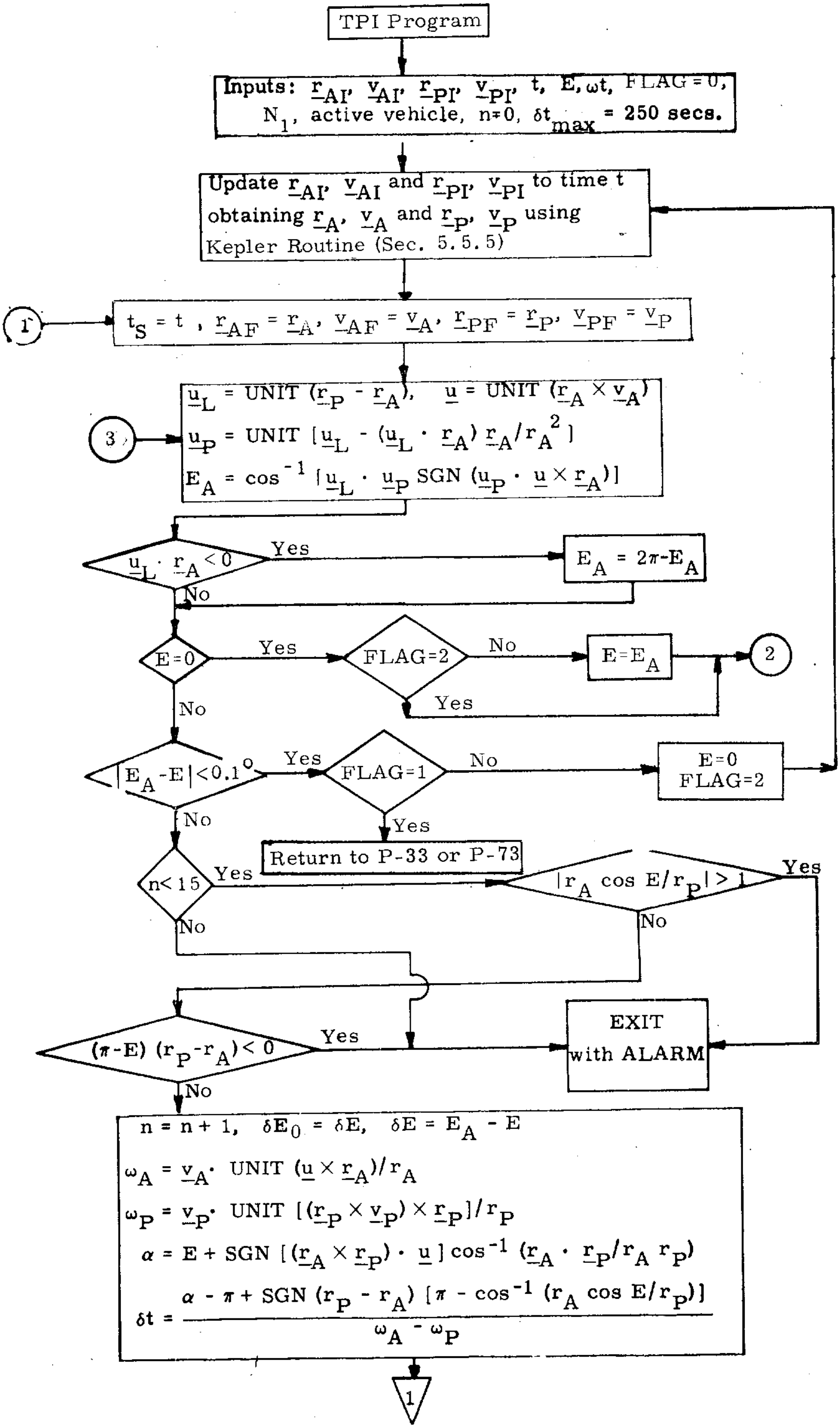


Figure 4.2-5. TPI Maneuver Program (Sheet 1 of 3)

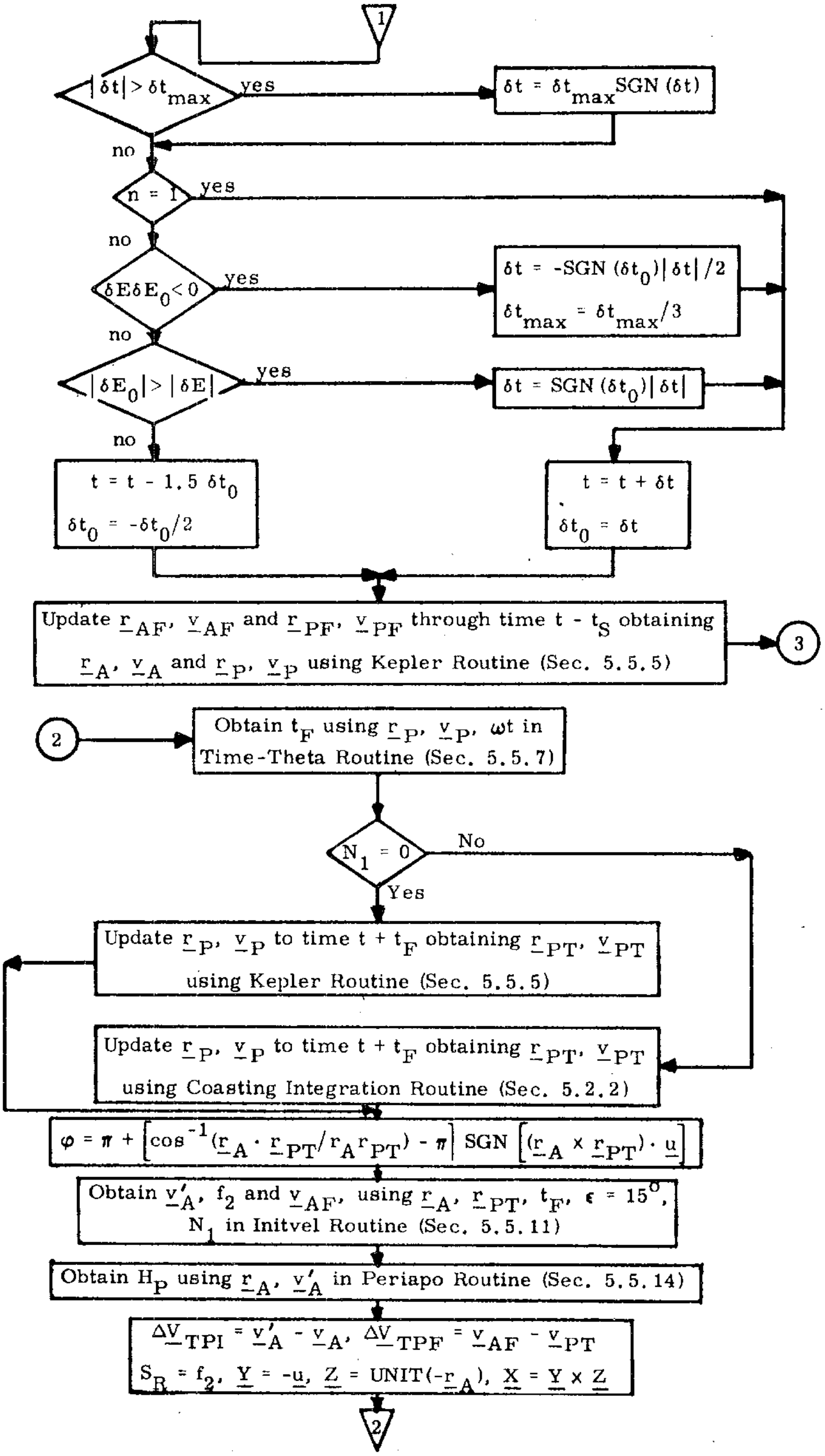


Figure 4.2-5. TPI Maneuver Program (Sheet 2 of 3)

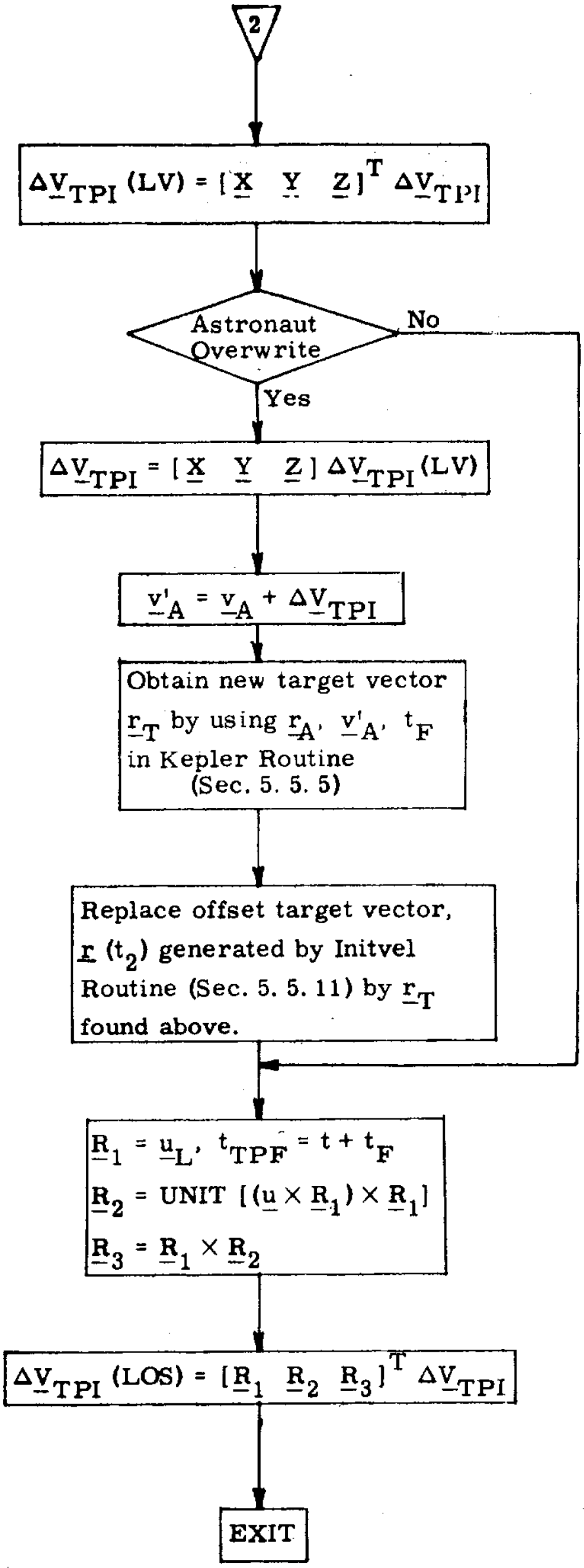


Figure 4. 2-5. TPI Maneuver Program (Sheet 3 of 3)

d. If the iteration is proceeding in the wrong direction, the step direction is reversed.

The iteration is terminated for any of the following reasons, and a single alarm as described in Section 4 is set:

1. The iteration counter has exceeded its maximum value of 15.

2. The line of sight emanating from the active vehicle does not intersect the circular orbit with radius equal to that of the passive vehicle.

3. The elevation angle is inconsistent with the relative altitudes of the two vehicles (e.g., if the elevation angle is less than 180° when the active vehicle is above the passive vehicle).

Upon convergence, the state vectors are conically updated to the TPI time.

The TPI-TPF phase of the program starts with the use of the angle ωt in the Time-Theta Subroutine (Section 5.5.7) to determine the corresponding transfer time t_F. The passive vehicle is next updated through t_F using either a conic or precision updating routine, depending on the value of N_1. The Initvel Routine (Section 5.5.11) is then called to compute a transfer trajectory between the TPI and TPF points with the cone angle ϵ set equal to 15°. The rotation projection switch f_2 is obtained from the Initvel Routine for use in the powered flight steering program as S_R.

The central angle φ traversed by the active vehicle from the TPI time to intercept is computed as shown in Figure 4.2-5 for display if requested by the astronaut. This display is used to avoid 180° transfer angle problems.

The displays for the TPI program are:

1. ΔV_{TPI}

2. ΔV_{TPF}

3. $\Delta \underline{V}_{TPI}$ (LV): local vertical coordinates

4. Perigee altitude (H_P) following the TPI maneuver

The $\Delta \underline{V}_{TPI}$ (LOS) (TPI ΔV in line-of-sight coordinates) is available but is not displayed automatically.

The display of $\Delta \underline{V}_{TPI}$ (LV) may be overwritten by the astronaut. If so, a new target vector is generated as shown in Figure 4.2-5. This new aimpoint is used for powered flight steering.

In addition, M, MGA, and TFI are displayed as in the CSI program displays.

5.4.2.5 Rendezvous Midcourse Maneuver Targeting (TPM)

This program, corresponding to program P35 (CSM active) or P75 (LM active) of Section 4, computes a midcourse correction maneuver. This maneuver insures that the active vehicle will intercept the passive vehicle at the time established in the previous TPI program. The astronaut may call this program any time after the TPI maneuver, but in general no later than 10 minutes before the intercept time. The flow diagram for this program is shown in Figure 4.2-6.

There is one astronaut input: choice of the active vehicle (P35 CSM, P75 LM). Available in the guidance computer are the intercept time t_{TPF} and the number N_1 of precision offsets (both available from the TPI program), the active ($\underline{r}_A$, $\underline{v}_A$) and passive ($\underline{r}_P$, $\underline{v}_P$) vehicle state vectors and a time delay. The time delay ($\delta\tau_3$ for P35 and $\delta\tau_7$ for P75) is the time required to prepare for the thrust maneuver and is stored in either of two erasable locations.

When the program is initiated, the number of navigation measurements since the last W-matrix reinitialization and the time since last maneuver or maneuver calculation are displayed. Based on this information and additional displays discussed in Section 5.6.7, the astronaut may elect to proceed with the midcourse maneuver at some time. When he does so, the program updates the state vectors to the present time plus the time delay using the Kepler Routine. After updating the passive vehicle to the time t_{TPF} to obtain the target vector $\underline{r}_{PT}$, the Initvel Routine is called with the cone angle ϵ set equal to 15^o, to obtain the velocity vector on the transfer ellipse. The rotation projection switch f_2 is obtained from the Initvel Routine for use in the powered flight steering program as S_R.

The central angle φ traversed by the active vehicle from the maneuver time to intercept is computed as shown in Figure 4.2-6 for display if requested by the astronaut. This display is used to avoid 180^o transfer angle problems.

After obtaining the maneuver $\Delta\underline{V}$, it is rotated into a local vertical (LV) and a line of sight (LOS) coordinate system obtaining $\Delta\underline{V}$ (LV) and $\Delta\underline{V}$ (LOS). $\Delta\underline{V}$ (LV) is displayed. Also displayed are M, MGA and TFI as in the CSI program displays. $\Delta\underline{V}$ (LOS) is available to the astronaut but not displayed.

The display of $\Delta\underline{V}$ (LV) may be overwritten by the astronaut. If so, a new target vector is generated as shown in Fig. 4.2-6. This new aim point is used for powered flight steering.

5.4.2.6 Plane Change Maneuver Targeting (PC)

This program, corresponding to program P36 of Section 4, computes the parameters associated with the plane change maneuver. It is normally used in conjunction with the Concentric Flight Plan Sequence.

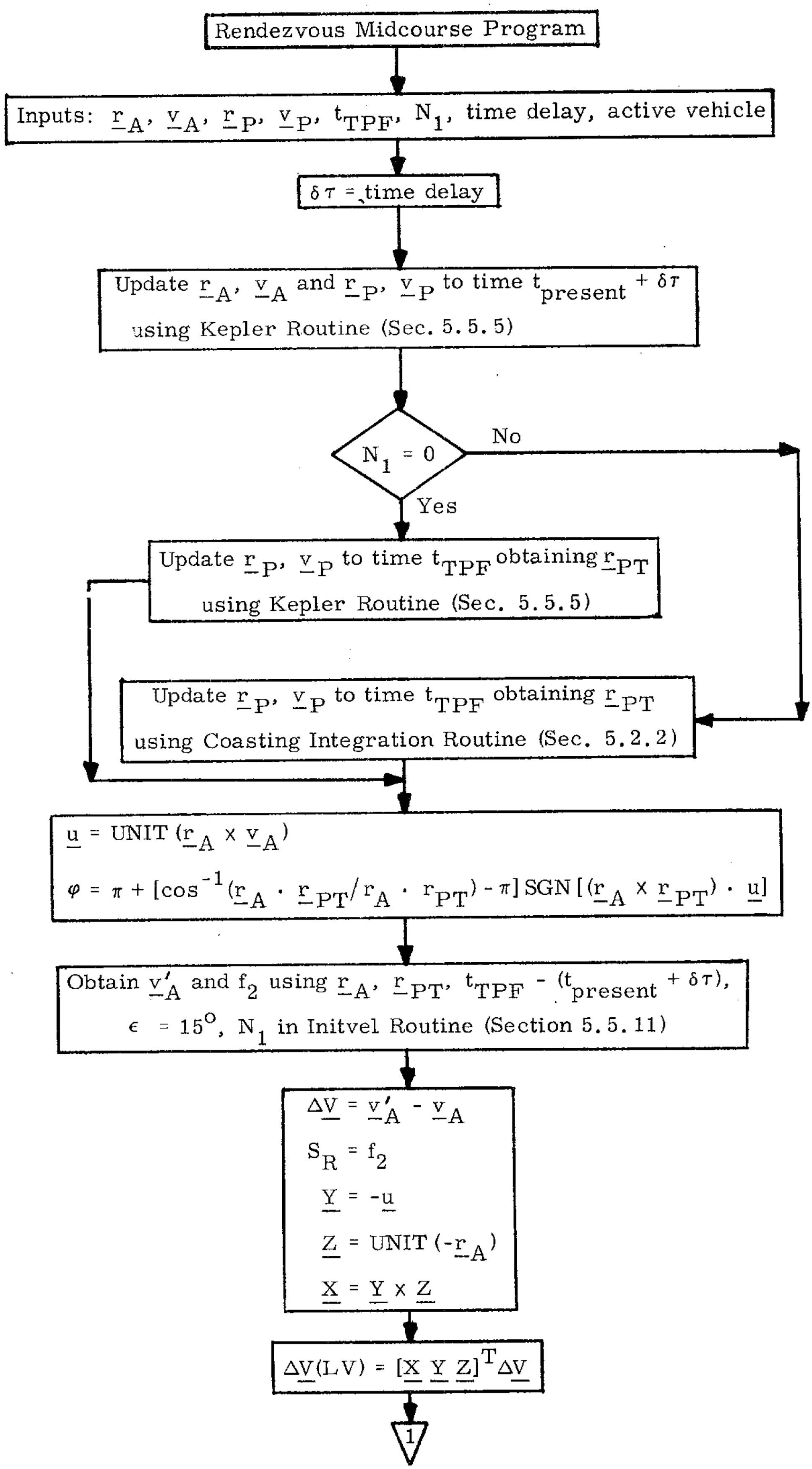


Figure 4.2-6. Rendezvous Midcourse Program (Sheet 1 of 2)

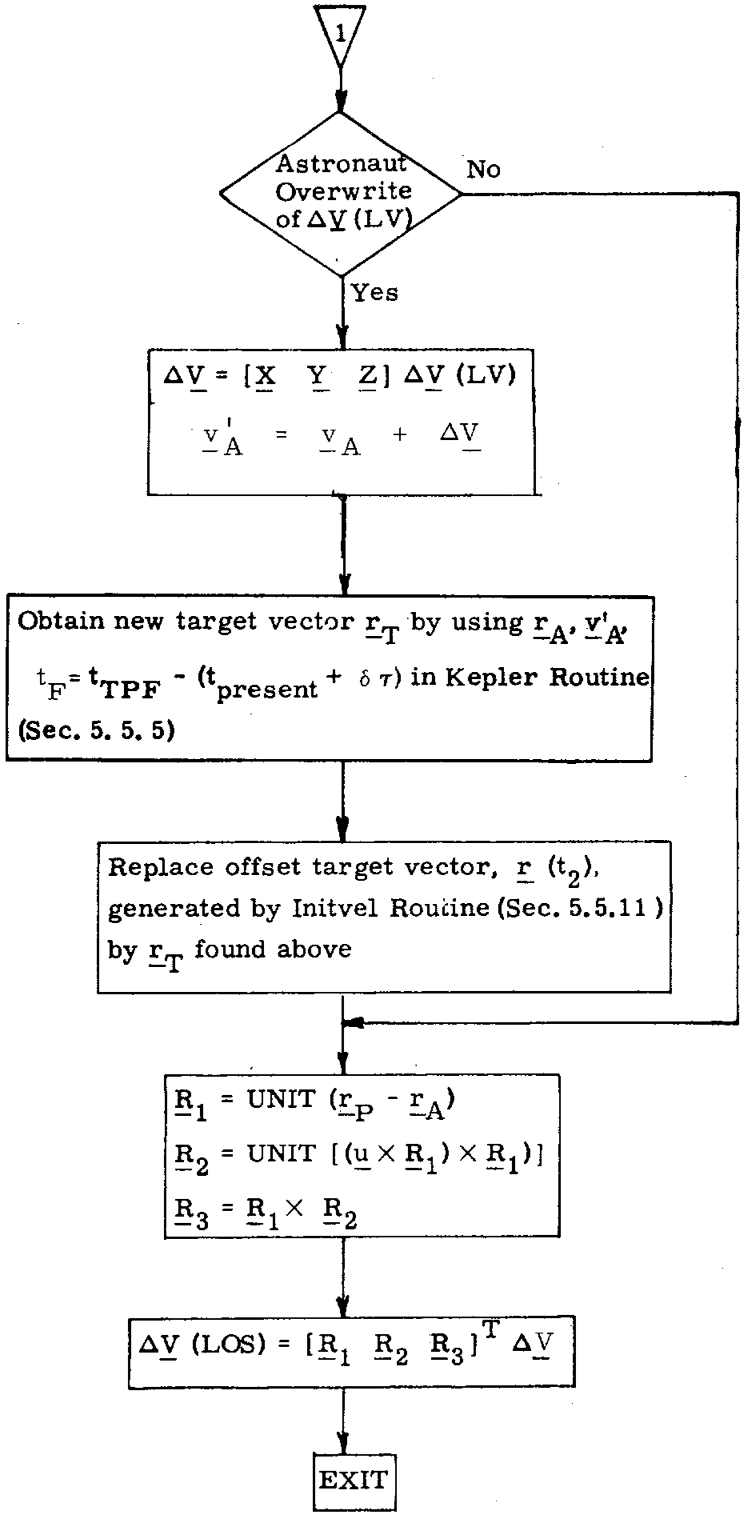


Figure 4. 2-6. Rendezvous Midcourse Program (Sheet 2 of 2)

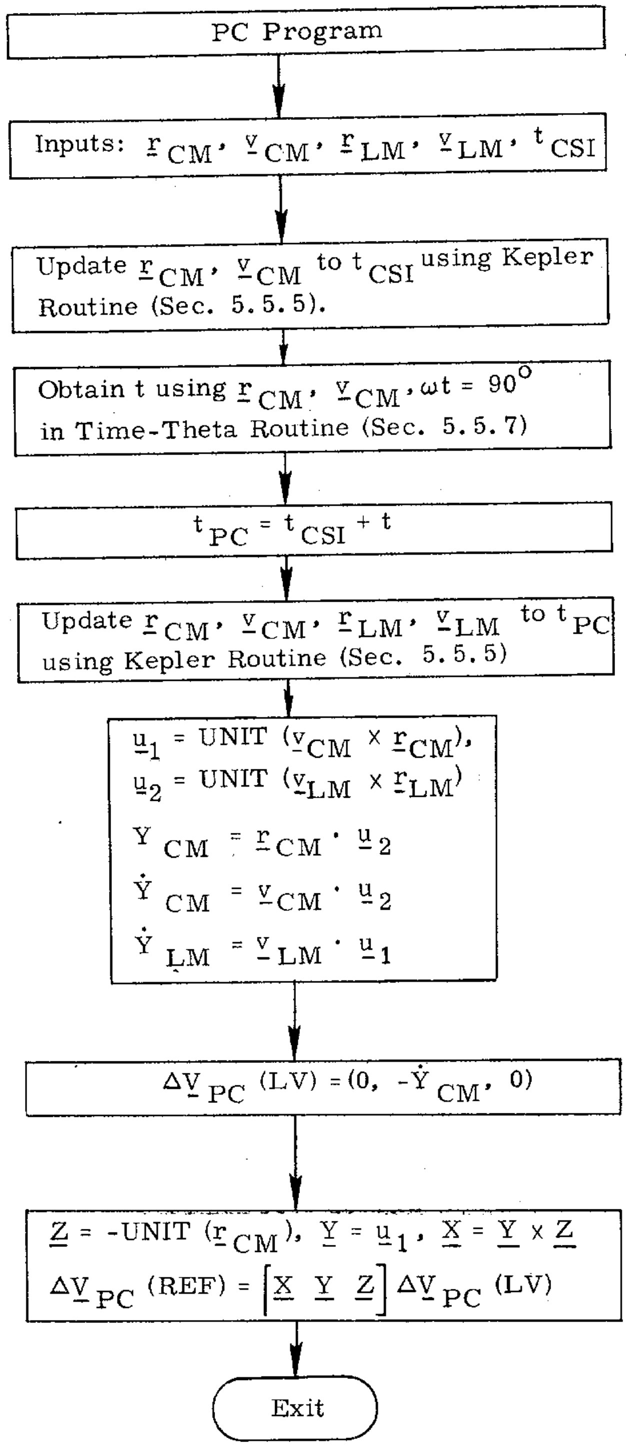


Figure 4.2-7. Plane Change Targeting

In normal operation, the CSI maneuver results in the active vehicle's velocity vector being parallel to the plane of the passive vehicle, causing a nodal point to occur 90 degrees from the active vehicle's CSI position. Program P36, in which the CSM is the active vehicle, is then used to compute the PC $\Delta \underline{V}$ at a nodal crossing 90 degrees after the CSM CSI position and the out-of-plane parameters at this point. If the CSI maneuver is done by the CSM, and if the Plane Change maneuver is then executed by the CSM, the active and passive vehicles' orbits will be coplanar.

Figure 4.2-7 illustrates the program flow. The CSM and LM post CSI state vectors $\underline{r}_{CM}$, $\underline{v}_{CM}$, $\underline{r}_{LM}$, $\underline{v}_{LM}$, are available in the guidance computer. The time of the previous CSI maneuver is also available from the previous CSI program computations.

The time t_{PC} of the Plane Change Maneuver is computed to be 90 degrees after the stored CSI time with the CSM as the active vehicle. The state vectors of both the CSM and LM vehicles are advanced to the PC time. The following parameters are then computed:

1. Out-of-plane position of the CSM relative to the LM orbital plane Y_{CM}
2. Out-of-plane velocity of the CSM relative to the LM orbital plane $\dot{Y}_{CM}$
3. Out-of-plane velocity of the LM relative to the CSM orbital plane $\dot{Y}_{LM}$
4. $\Delta \underline{V}_{PC}$ (LV)

The astronaut can modify the computed PC time or the PC maneuver $\Delta \underline{V}$. In the automatic rendezvous sequence, the astronaut will normally overwrite the $\Delta \underline{V}$ with a 0 since the CSM is the non-active vehicle. This will cause the automatic program controller to bypass the selection of a powered flight program.

The out-of-plane parameters and the local vertical $\Delta \underline{V}$ are displayed as well as M, MGA and TFI as in the CSI program displays.

5.4.2.7 Height Adjustment Maneuver Targeting (HAM)

The Height Adjustment Maneuver Program, P31, is used in conjunction with the Concentric Flight Plan sequence of maneuvers to perform a rendezvous from certain abort situations in which the CSM is the active vehicle. The HAM burn is performed one-half orbit prior to the final CSI maneuver. Its purpose is to adjust the altitude of the CSM at the CSI point so that the subsequent CSI, CDH maneuver sequence results in the desired altitude difference ΔH_{DES} between the CSM and LM from CDH to TPI.

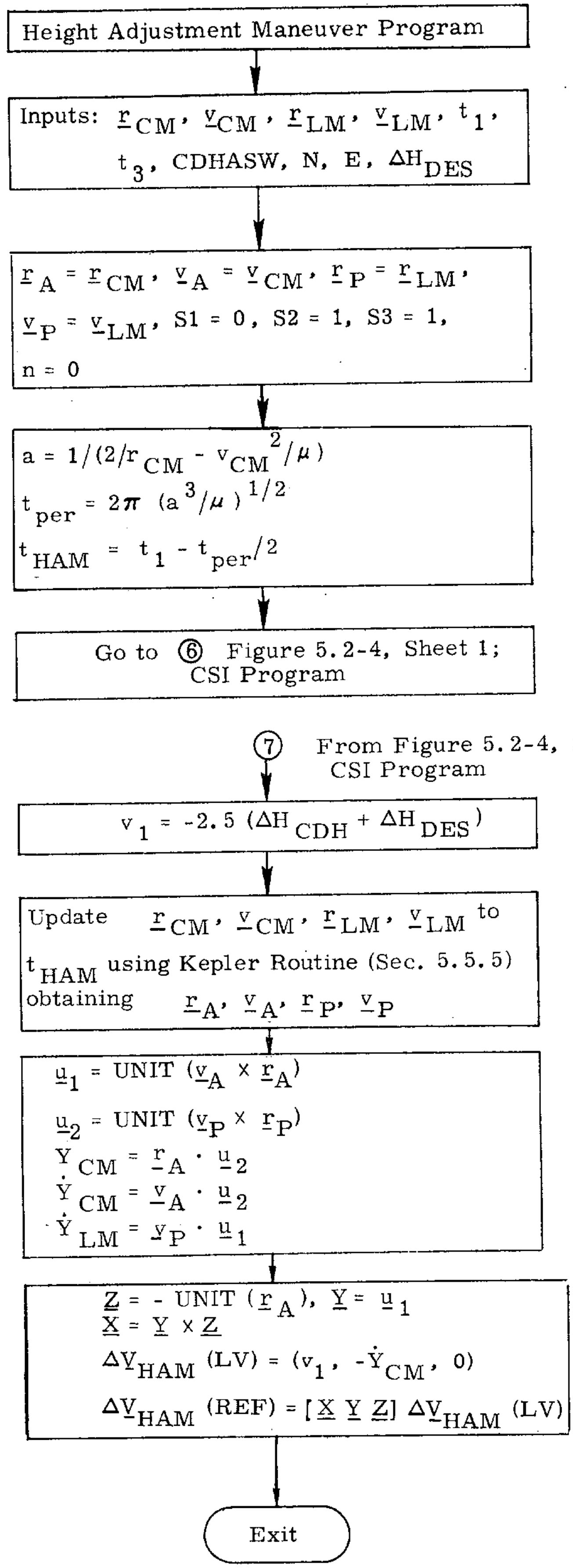


Figure 4.2-8. Height Adjustment Maneuver (Sheet 1 of 1)

The inputs to the program are the same as those for P32, with the addition of ΔH_{DES}. The active vehicle state vector, $\underline{r}_{CM}$, $\underline{v}_{CM}$ and passive vehicle state vector, $\underline{r}_{LM}$, $\underline{v}_{LM}$, are available in the guidance computer.

The desired altitude difference ΔH_{DES} between the vehicles from CDH to TPI which is used in the calculation of the Height Adjustment Maneuver, depends on the particular rendezvous being flown (i.e., the particular LM abort that has occurred). It is an erasable constant which can be changed by the crew. The program flow is illustrated in Figure 4.2-8.

The Height Adjustment Maneuver time t_{HAM} is computed to be one half of a CSM orbital period prior to the CSI maneuver. The CSI maneuver program is then entered and used to compute the differential altitude, ΔH_{CDH}, at the CDH point which would exist if no HAM were performed. When solving the Concentric Flight Plane Maneuver Sequence to obtain this ΔH_{CDH}, the same constraints must be satisfied as in the CSI program with the exception that there are no constraints the time intervals between the CSI-CDH and CDH-TPI maneuvers.

After the solution to the CSI maneuver is obtained (and the corresponding ΔH_{CDH}) the HAM $\Delta \underline{V}$ is computed using ΔH_{DES} and ΔH_{CDH} with an empirically derived formula. It is applied in the local vertical direction. In addition to displaying this $\Delta \underline{V}$, the out-of-plane parameters at the HAM ignition time are also computed and displayed.

As in the CSI program displays, M, MGA, and TFI are displayed.

NOTE: Pages 5.4-31 through 5.4-42 are deleted in Revision 14.

5.4.3 RETURN TO EARTH TARGETING

5.4.3.1 Introduction

The Return to Earth Targeting Program provides to the astronaut the ability to compute on board the spacecraft the targeting for a powered maneuver which returns the spacecraft to an earth reentry point with proper conditions for a safe reentry. This program is intended as a backup in the event of a communication or ground system failure and, thus, is completely independent of inputs from the ground system. The method of computation, however, does not compromise certain critical trajectory characteristics such as fuel usage and transit time for the expediency of minimum computer storage usage or computational speed.

Specifically this program provides powered maneuver targeting for :

(1) Returning from near earth orbits

(2) Returning from trajectories resulting from a translunar injection powered maneuver failure.

*(3) Returning from translunar midcourse

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* Outside moon's sphere of influence only

*(4) Returning from transearth midcourse

*(5) Midcourse corrections during transearth midcourse

*(6) Changes in transit time during transearth midcourse

Two modes of operation are provided: time critical and fuel critical. The former establishes trajectories which return to earth in a minimum of time, the latter, with a minimum of fuel.

The return trajectory is constrained to be in the plane which the spacecraft was in prior to the return so that optimum fuel utilization will be achieved.

Although explicit control of the spacecraft landing site is not a feature of this program, it is possible to achieve landing site selection through adjustment of the astronaut input to the program. When on translunar or transearth trajectories, the adjusted parameter is the desired fuel utilization. When in orbit near the earth, the designated return time is the adjusted parameter. The astronaut need only to note the earth track of the landing site as a function of the adjusted parameter to select the proper parameter value.

The computations take place in two steps. The first is the generation of a conic two body solution to the problem. This takes a relatively short computational time. The second step, the computation of a precise trajectory, takes a relatively long computational time.

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*Outside moon's sphere of influence only

The Return to Earth Program is described in increasing detail in the following subsections:

5.4.3.2 Program Input-Output

The Return to Earth Program may be called by the astronaut at any time the computer is operable. Should the designated time of return place the spacecraft at a point in space where the program is incapable of producing a successful solution, an alarm will occur. The current calling designation for this program is P-37 for both modes.

Return-to-Earth Targeting

After calling the program, the astronaut must load the following information into erasable memory via the keyboard.

(1) objective velocity change to be utilized during the maneuver

(2) time the maneuver is to be executed

(3) reentry angle (normally set to zero and internally calculated*)

Should a minimum fuel return be desired, the objective velocity change should be set to zero. Should the objective change be less than the minimum required, the program will select the minimum. When the objective velocity change is greater than the minimum, the return will require that amount of velocity change.

This program is not designed to operate from low altitude polar orbits unless the astronaut selects the reentry angle.

The program provides the following displays when both the conic trajectory computation and the precision trajectory computation are complete.

(1) latitude of the landing site

(2) longitude of the landing site

(3) velocity change required (vector)

*See 5.4.3.4.2 for detailed explanation of its use

(4) velocity magnitude at 400, 000 ft. entry altitude measured above the Fischer ellipsoid.

(5) flight path angle at 400, 000 ft. entry altitude

(6) transit time to 400, 000 ft. entry altitude from ignition.

The astronaut must select either the SPS or RCS engine after the precision trajectory calculation. The following quantities are displayed after the engine selection:

(7) middle gimbal angle at ignition

(8) time of ignition

(9) time from ignition

The precision trajectory displays can be significantly different from the conic displays, especially in the case of long transit time returns; therefore the astronaut should check the new displays before accepting the precise solution.

The cross product steering constant c (Section 5. 3. 3. 4) is set to ec of Section 5. 3. 3. 3. 2. Upon entering P-37 ec is set equal to 0. 5. In addition the rotation switch, S_R, which is used in the Lambert Aim Point pre-thrust computation of Section 5. 3. 3. 3. 2 is set to zero.

5.4.3.3 General Computational Procedure

This section contains a general description of the Return-to-Earth computational procedure as illustrated in Figures 4.3-1, 4.3-2, and 4.3-3.

The Return to Earth Targeting Program, as previously mentioned, may be used in either the fuel critical or time critical mode. When used in the fuel critical mode, the program generates a trajectory which meets specified constraints on reentry flight path angle and reentry altitude, and minimizes the impulsive velocity change required to achieve this trajectory. In the time critical mode, the program generates a trajectory which meets the specified reentry constraints and returns the spacecraft in the shortest possible time. The astronaut selects either mode by entering via the keyboard the objective velocity change (Δv_D). If Δv_D is set to zero, the routine will provide a fuel critical trajectory. If Δv_D is not set to zero, the routine will provide a time critical trajectory if possible, or select a fuel critical trajectory if Δv_D is less than the minimum required impulsive velocity change. In addition, during transearth coast, a negative Δv_D may be selected -- resulting in a slower return, i.e., slower than the minimum fuel return.

The Return to Earth Targeting Program has two major phases, an initial two body conic phase illustrated in Figure 4.3-1 and a numerically integrated precision trajectory phase illustrated in Figure 4.3-2. A conic routine which provides initial conditions for return trajectories for both phases is illustrated in Figure 4.3-3.

5.4.3.3.1 Nomenclature

The following is a list of the nomenclature used in Figs. 4.3-1, 4.3-2, and 4.3-3.

$\underline{y}(t_0) = (\underline{r}(t_0), \underline{v}(t_0))$	CSM state vector maintained in the CMC
$\underline{v}_1(t_1) = (\underline{r}(t_1), \underline{v}_1(t_1))$	pre-return state vector
$\underline{v}_2(t_1) = (\underline{r}(t_1), \underline{v}_2(t_1))$	post-return state vector
$\underline{y}(t_2) = (\underline{r}(t_2), \underline{v}(t_2))$	state vector at reentry
$\underline{y}(t_2)_{PRE}$	final state vector of a precision trajectory
R_{CON}	final radius of a conic trajectory
R_{PRE}	final radius of a precision trajectory
R_D	desired final radius (reentry radius)
$\gamma(t_2)_{CON}$	final flight path angle of a conic trajectory
$\gamma(t_2)_D$	desired final flight path angle
$\gamma(t_2)_{PRE}$	final flight path angle of a precision trajectory
$x(t_1)$	cotangent of the post-return flight path angle
Δv	impulsive velocity change
Δv_D	desired velocity change
ϵ_1 to ϵ_{10}	tolerances

t_{21}	time of flight $(t_2 - t_1)$
n_1	counter
f_1	flag set to $\begin{cases} 0 & \text{initial conic phase} \\ 1 & \text{precision trajectory phase} \end{cases}$

5.4.3.3.2 Discussion of the Conic Phase

The following discussion is intended to supplement the information contained in Fig. 4.3-1.

The pre-return CSM state vector ($\underline{y}_1(t_1)$) is obtained by numerical integration. An initial conic trajectory is then generated which meets the reentry and velocity change constraints of the problem. The reentry radius and flight path angle constraints, however, are known only as a function of the latitude and velocity magnitude at reentry. Therefore an iterative procedure is employed to generate this initial conic trajectory. Based on the pre-return radius magnitude, an estimate of the reentry constraints is made. This initial estimate is used to compute a conic trajectory which satisfies the Δv_D requirement. The final velocity magnitude and latitude are now available for use in a more precise calculation of the reentry constraints. An iterative process has now been started which quickly converges on a suitable initial conic trajectory.

During the above iterative process slight changes in the reentry flight path angle can produce significant changes in the minimum required velocity change. If the time critical mode is being used, it is possible that during a particular pass through the iteration loop the velocity change desired may be less than the minimum required. This does not necessarily preclude a time critical solution, and therefore, the iteration is continued using the minimum required velocity change.

A description, in general terms, of the procedure CONICRETURN, used to obtain the initial state vector of the conic return trajectory, will follow later in this section.

After an initial conic trajectory is computed, an estimate is made of the landing site, and the results are displayed. Based on this display the astronaut may elect to continue with the calculation of a numerically integrated trajectory.

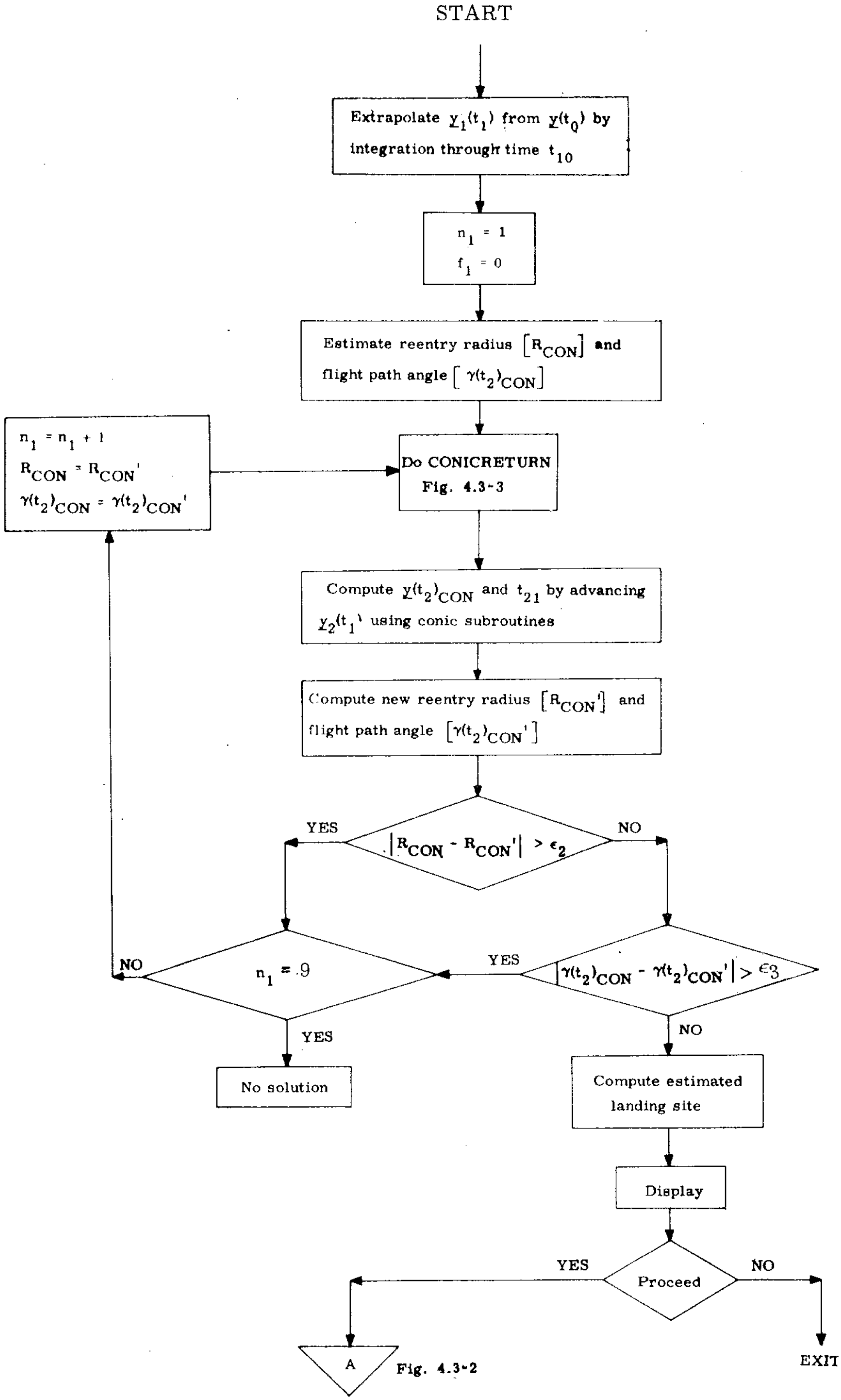

Figure 4.3-1 Simplified Logic Diagram of Return to Earth Targeting

5.4.3.3.3 Discussion of the Precision Phase

The following discussion is intended to supplement the information contained in Fig. 4.3-2.

The numerically integrated trajectory must satisfy the reentry constraints achieved by the initial conic trajectory and meet the Δv_D requirement as closely as possible. An iterative procedure is again employed to generate this trajectory. The post-return state vector of the previous conic trajectory is used to initialize the Coasting Integration Routine. The integration proceeds until the desired reentry flight path angle has been achieved. The reentry radius of the next conic trajectory is then offset to compensate for the error in the reentry radius of the numerically integrated trajectory. An iterative process has now been initiated which converges to the precise trajectory which meets the objective velocity change and the reentry constraints. Next a target is computed for the Lambert Aimpoint Powered Flight Routine, the landing site is calculated, and the results are displayed.

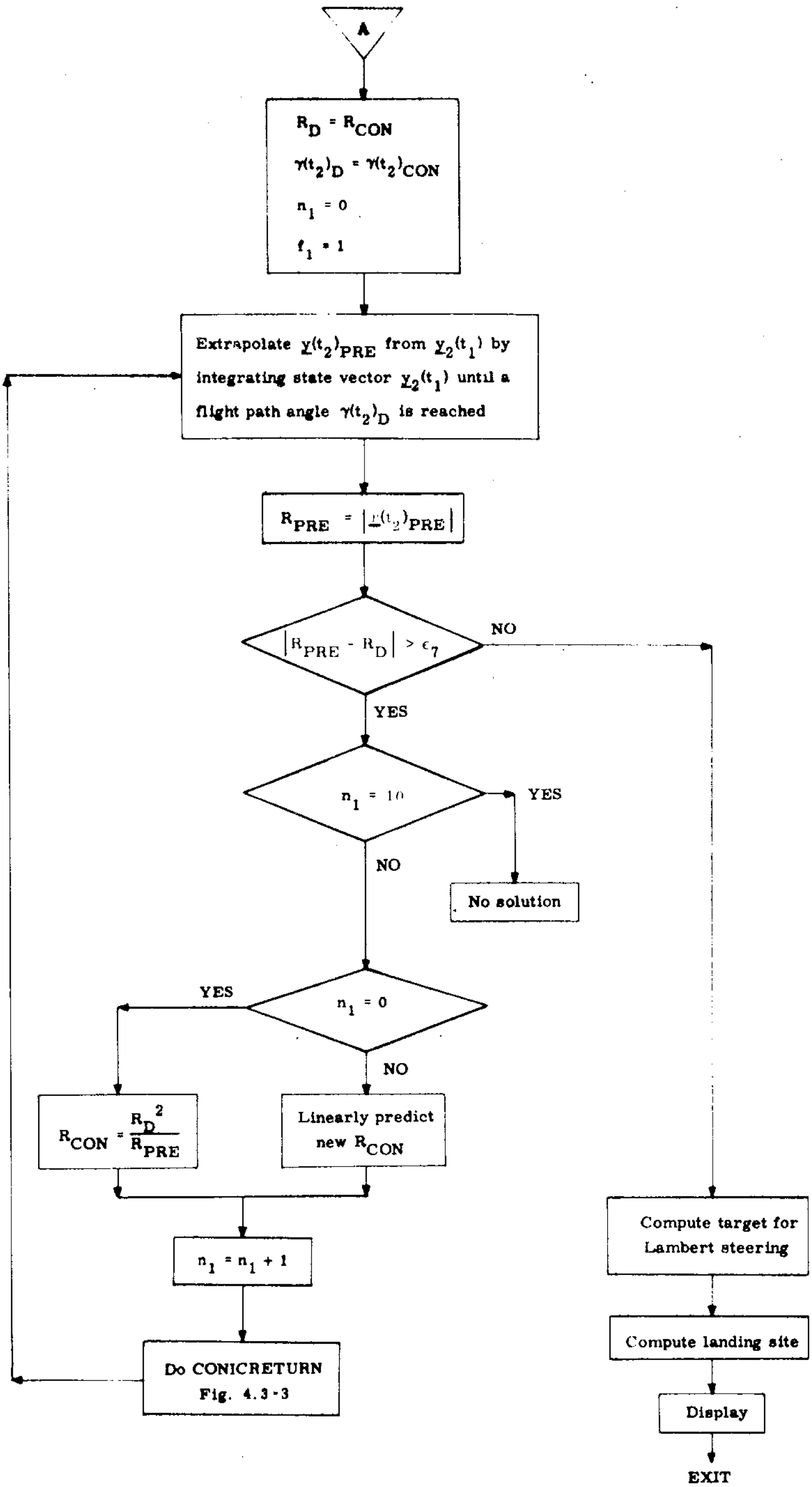

Figure 4.3-2 Simplified Logic Diagram of Return-to-Earth Targeting

5.4.3.3.4 Discussion of the Conic Routine

The following discussion is intended to supplement the information contained in Fig. 4.3-3.

The conic routine solves the following two conic problems.
(1) Given an initial position vector ($\underline{r}(t_1)$), a pre-return velocity vector ($\underline{v}_1(t_1)$), a final flight path angle ($\gamma(t_2)_{CON}$), and a final radius magnitude (R_{CON}), compute a post-return velocity vector ($\underline{v}_2(t_1)$) which minimizes the velocity change (Δv).
(2) Given the above quantities, compute a post-return velocity vector ($\underline{v}_2(t_1)$) which meets the objective velocity change (Δv_D).

An iterative procedure is again used. The independent variable of the procedure is the cotangent of the post-return flight path angle ($x(t_1)$), and the dependent variable is the velocity change (Δv).

Initial bounds on $x(t_1)$ are computed based on the maximum allowable semi-major axes. The upper bound constrains return trajectories which pass through apogee from going too near the moon, and the lower bound keeps the reentry velocity for direct return flights from exceeding the maximum allowable reentry velocity.

During the second phase of the Return to Earth Targeting Program, the calculation of a numerically integrated trajectory, additional bounds are imposed on the independent variable $x(t_1)$. In the case of a return from earth orbit, the resulting trajectory may be nearly circular, and will pass through the desired reentry flight path angle twice at nearly the same radius. Additional limits on $x(t_1)$ are computed to prevent conic trajectories generated during the search for a numerically integrated trajectory from oscillating between these two solutions.

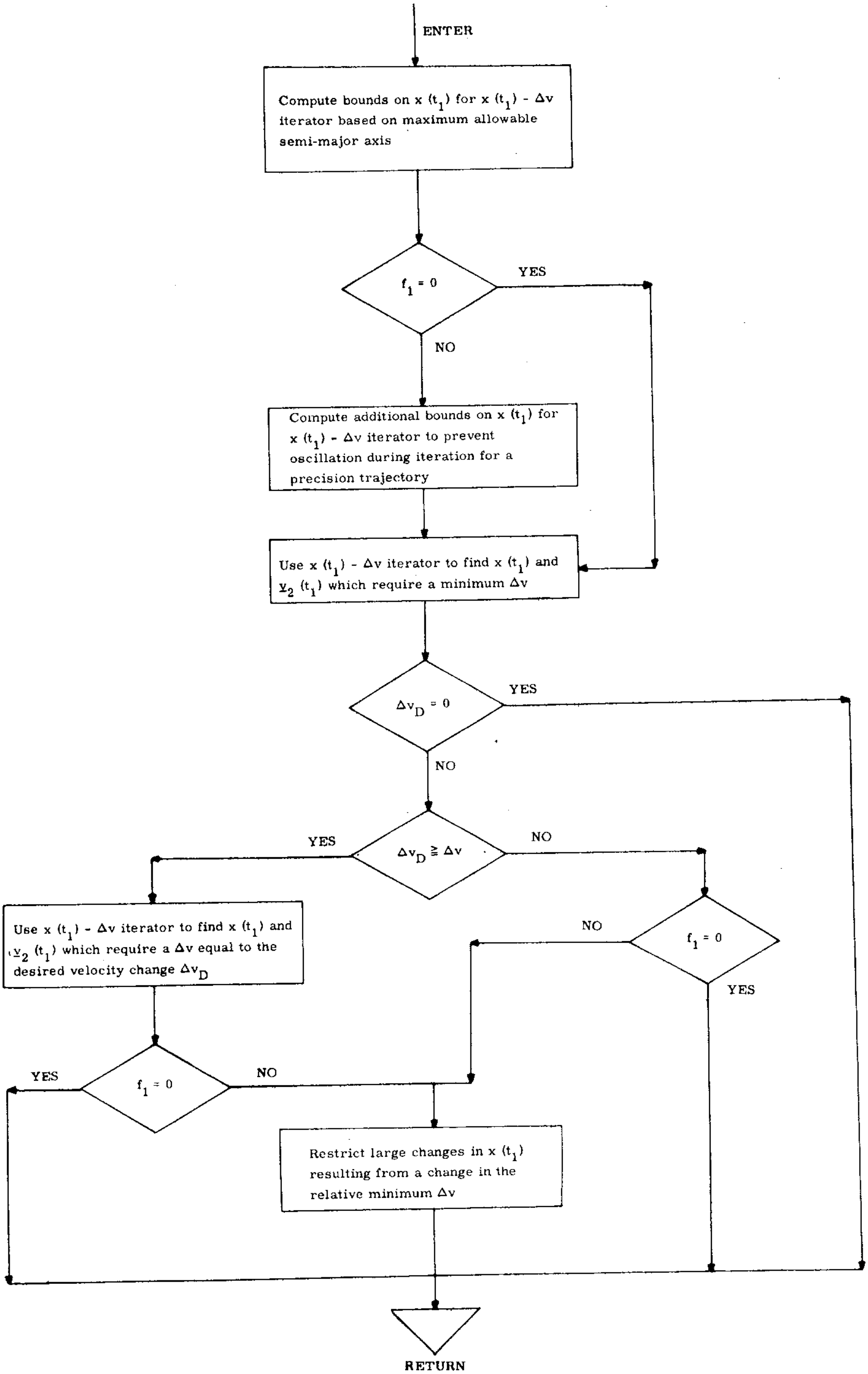

Figure 4.3-3 Simplified Logic Diagram of Return-to-Earth Targeting-Conic Return Routine

In all cases the $x(t_1)$ - Δv iterator is used to compute a post-abort velocity vector which requires a minimum Δv. In the event that a time critical answer is required, another iteration using the $x(t_1)$ - Δv iterator will be made to compute a post-return velocity vector which requires a Δv equal to Δv_D. If the velocity change desired is less than the minimum required, the second iteration will be omitted.

During the calculation of the numerically integrated trajectory, the intermediate conics are additionally restricted to prevent large changes in the post-return flight path angle. This additional restraint is necessary in the time critical mode for certain near earth returns because both absolute and relative minimum Δv solutions exist.

5.4.3.4 Logical Description

This section contains logical flow diagrams of the outside the sphere portion of the Return to Earth Targeting Program. Some items not fully discussed in the text are explicitly illustrated in the diagrams. Certain self contained procedures are detailed in the next section.

5.4.3.4.1 Nomenclature

A list of the nomenclature used in this section and not previously defined follows.

t_{IG}	desired ignition time
t_{21}	time of flight $(t_2 - t_1)$
$p(t)$	primary body $\begin{cases} 0 & \text{for earth} \\ 1 & \text{for moon} \end{cases}$
$x(t_1)_{LIM}$	limit on the cotangent of the post-return flight path angle
$x(t_1)_{MAX}$, $x(t_1)_{MIN}$	upper and lower bounds on the cotangent of the post-return flight path angle
$\Delta x(t_1)$	change in $x(t_1)$
$\Delta x(t_1)_{MAX}$	maximum allowed change in $x(t_1)$
$x(t_2)$	final cotangent of the flight path angle
$x(t_2)_{PRE}$	final cotangent of the flight path angle for a precision trajectory
p_{CON}	semi-latus rectum of a conic trajectory
R_{ERR}	precision trajectory reentry radius error

μ_E	earth's gravitation constant
MA_1, MA_2	maximum major axes
$\underline{n}$	normal vector
ϕ_2	$\begin{cases} +1 \text{ near apogee solution} \\ -1 \text{ near perigee solution} \end{cases}$
ϕ_4	precision trajectory direction switch
f_2	iterator mode flag
n_1, n_2	counters
β_1 to β_{14}	intermediate variables

5.4.3.4.2 Discussion of the Logical Flow Diagram

The following discussion is intended to supplement the information contained in the referenced diagrams.

Three inputs are required in addition to the CSM state vector (Fig. 4.3-4). They are the objective velocity change desired (Δv_D), the desired time of ignition (t_{IG}), and the reentry angle ($\gamma(t_2)_D$). The reentry angle should nominally be set to zero, allowing the program to compute a reentry angle in the center of the reentry corridor. The astronaut may select the reentry angle if a minimum Δv solution lying on the edge of the reentry corridor is required. This may be necessary in the event the reaction control system is used for the return. See Fig. 4.3-15 for a detailed explanation of the reentry angle calculations.

The Coasting Integration Routine is used to advance the state vector to the desired return time. A check is made to insure that the state vector is not inside the moon's sphere of influence. If the state vector is outside the sphere of influence, precomputations are executed. They are explained in detail later. A conic trajectory is then generated which satisfies the estimated reentry constraints and Δv_D requirements. The procedure CONICRETURN, which computes the initial state vector of this conic, will be discussed later. The Time Radius Subroutine, described in Section 5.5.8, is used to compute the time of flight to the desired radius and advance the state vector through that time of flight.

Based on this final state vector (Fig. 4.3-5), a new reentry radius (R_{CON}') and a new value for the cotangent of the reentry flight path angle ($x(t_2)'$) can be computed. If they are sufficiently close to the previously used values, the conic trajectory is accepted. Otherwise a new conic is computed which satisfies the new reentry constraints. A counter is employed to protect against an excessive number of iterations.

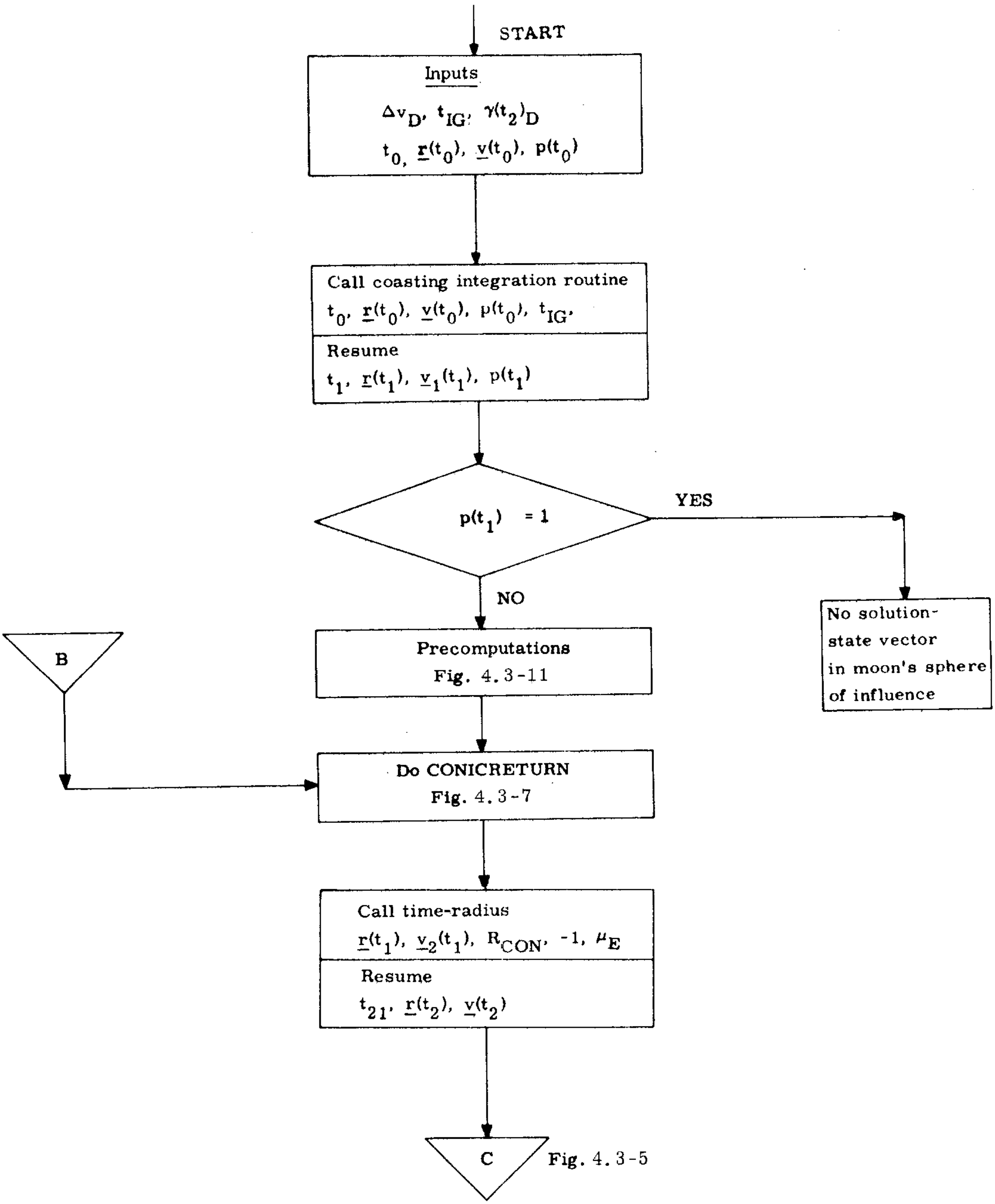

Figure 4.3-4 Logical Diagram of Return-to-Earth Targeting

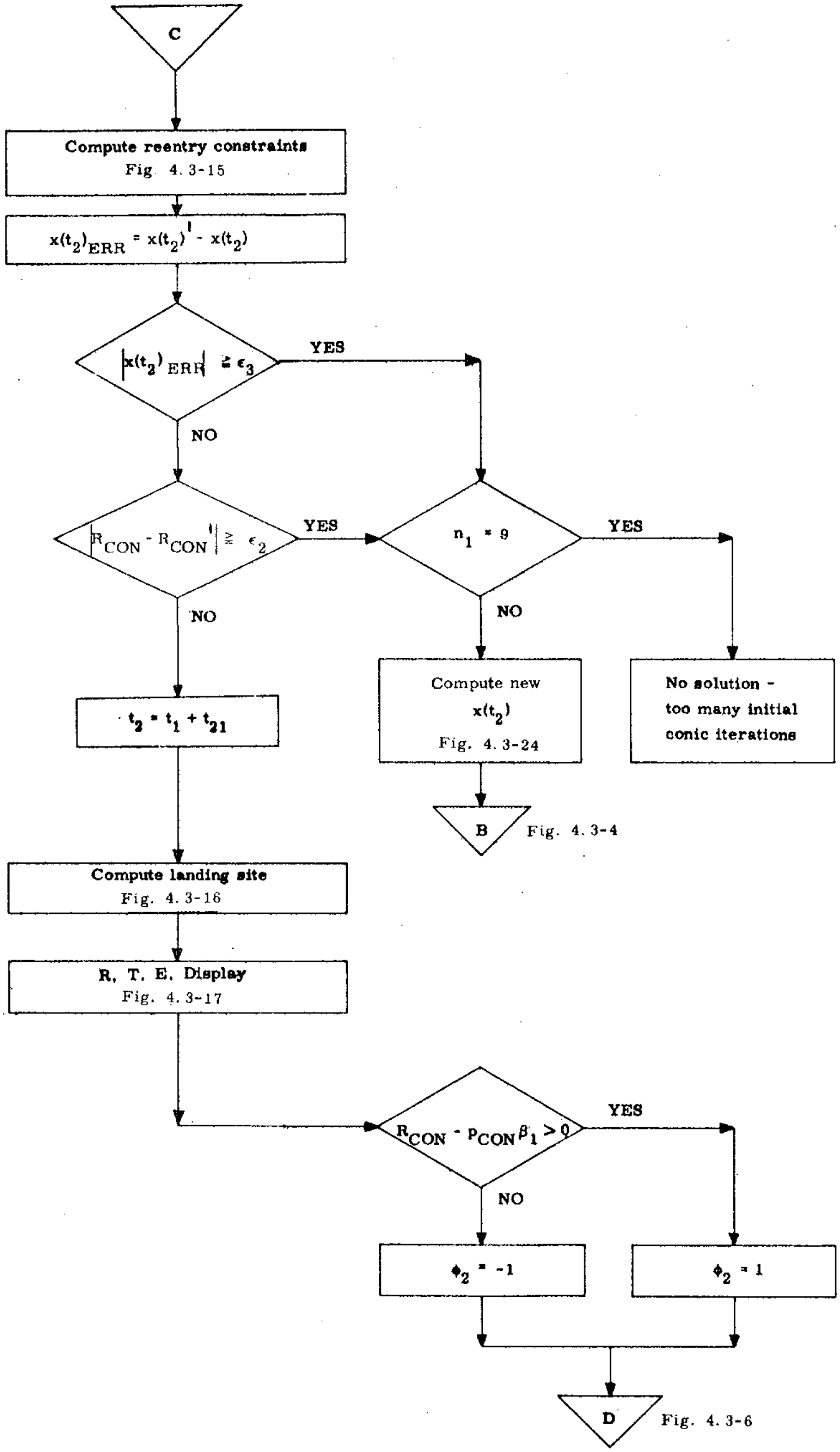

Figure 4.3-5 Logical Diagram of Return to Earth Targeting

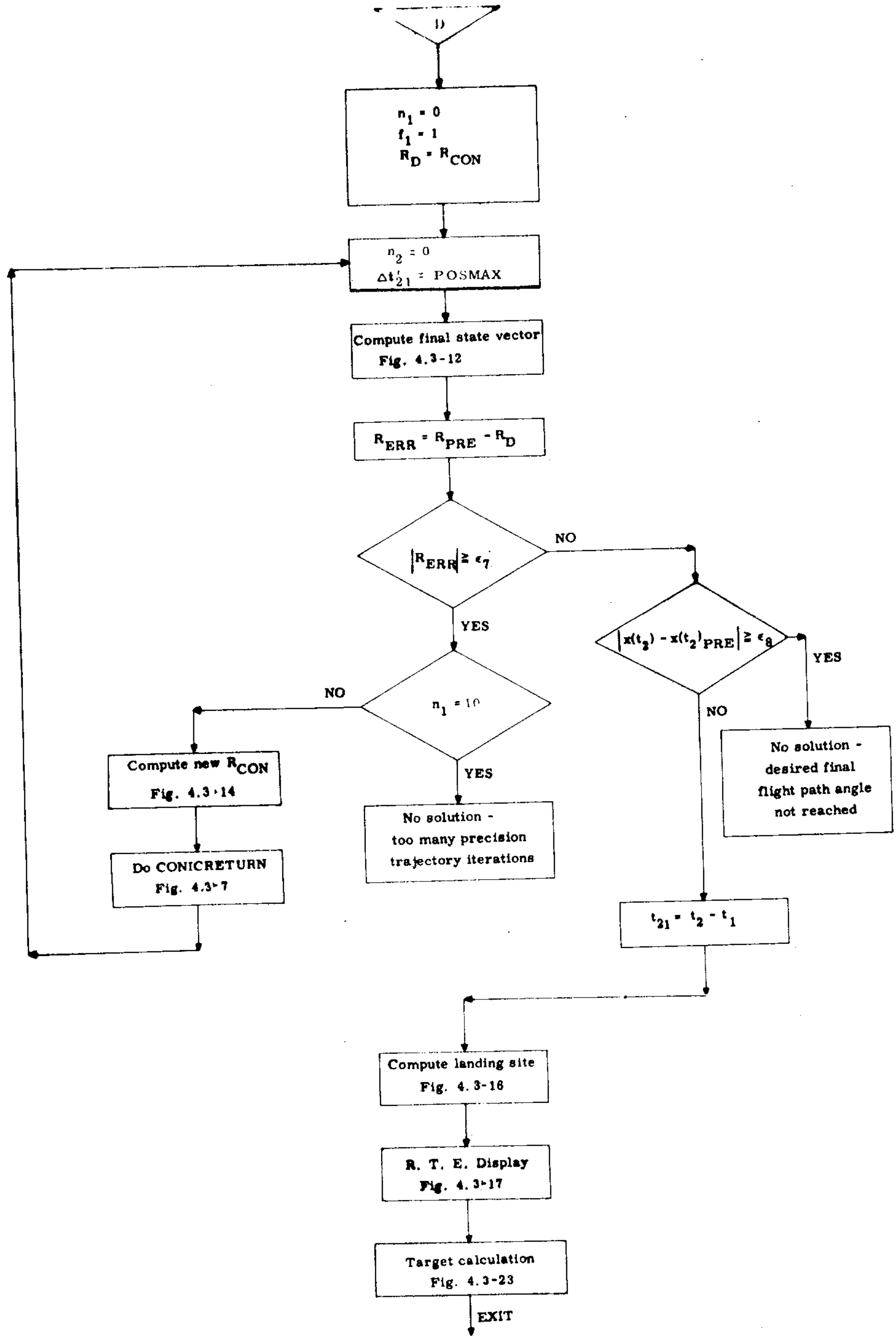

Figure 4.3-6 Logical Diagram of Return-to-Earth Targeting

If the initial conic is accepted, an estimate is made of the latitude and longitude of the landing site. For long flight times, however, differences in time of flight between the conic and numerically integrated trajectory will cause an error in the conically estimated landing site longitude. The results of the preceding calculations are displayed, and based on this information the astronaut may elect to proceed with the calculation of a numerically integrated trajectory or exit the program. As explained in Section 5.4.3.3, return trajectories from near earth orbits may be nearly circular, and therefore pass through the desired flight path angle twice with approximately the same radius. The unique solution reached by the initial conic is tested to determine whether the final state vector was achieved during the first or second pass through the desired reentry flight path angle. A switch ϕ_2 is set appropriately, and with one exception all succeeding precision and conic trajectories are required to behave in a similar manner. The exception will be explained in detail in Section 5.4.3.4.3.

The post-return state vector of the conic trajectory is now advanced until the desired flight path angle of reentry is reached (Section 5.4.3.5). A radius error will exist between the desired final radius (R_D) and the final radius magnitude of the numerically integrated trajectory. If this error is small, the precision trajectory is accepted. Otherwise a new conic trajectory is generated which uses an offset final radius magnitude. If the error in the final radius is small, a final check is made to insure that the reentry flight path angle is acceptable.

5.4.3.4.3 Discussion of Logical Flow Diagram - CONICRETURN

The procedure CONICRETURN, illustrated in Figures 4.3-7 and 4.3-8, is used to generate initial state vectors for the Return to Earth Targeting Program. Equations required to initialize CONICRETURN each time it is used are described in Section 5.4.3.5. The conic problems solved by the routine were described in Section 5.4.3.3.4.

The logical structure shown in Figs. 4.3-7 and 4.3-8 is based in the following facts.

(1) There is only one minimum if the pre-return velocity vector has a negative radial component.

(2) There may be two minima if the pre-return velocity vector has a positive radial component. In this case the absolute minimum post-return velocity vector has a positive radial component and the relative minimum has a negative component.

As explained in Section 5.4.3.3.4 and 5.4.3.4.2, it is necessary to compute additional bounds on $x(t_1)$ during the search for a precision trajectory. The logic used to do this is shown on the lower portion of Fig. 4.3-7.

A check is made to determine whether the locus of possible solutions includes trajectories which achieve the radius R_{CON} and the flight path angle $\gamma(t_2)$ near apogee, and if so the variable β_6 will be positive*.

*With one exception every conic solution generated by this routine achieves the desired reentry flight path angle ($\gamma(t_2)$) twice during the passage from apogee to perigee. The first time $\gamma(t_2)$ is reached during the passage from apogee to perigee will be referred to as near apogee, and the second time will be referred to as near perigee. The desired reentry angle will be achieved only once if $\gamma(t_2)$ equals the maximum flight path angle of the conic.

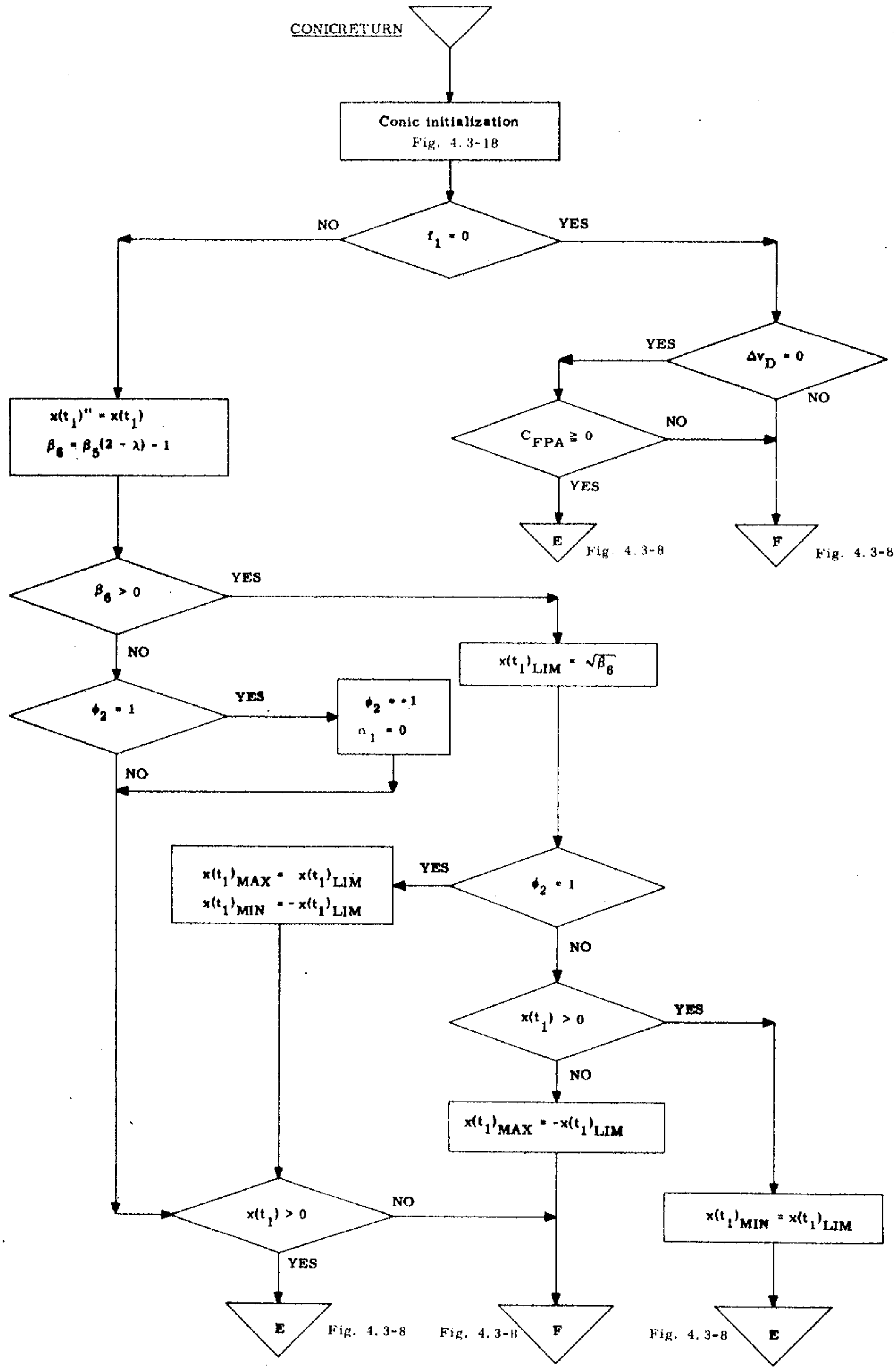

Figure 4.3-7 Logical Diagram of Return to Earth Targeting - Conic Return Routine

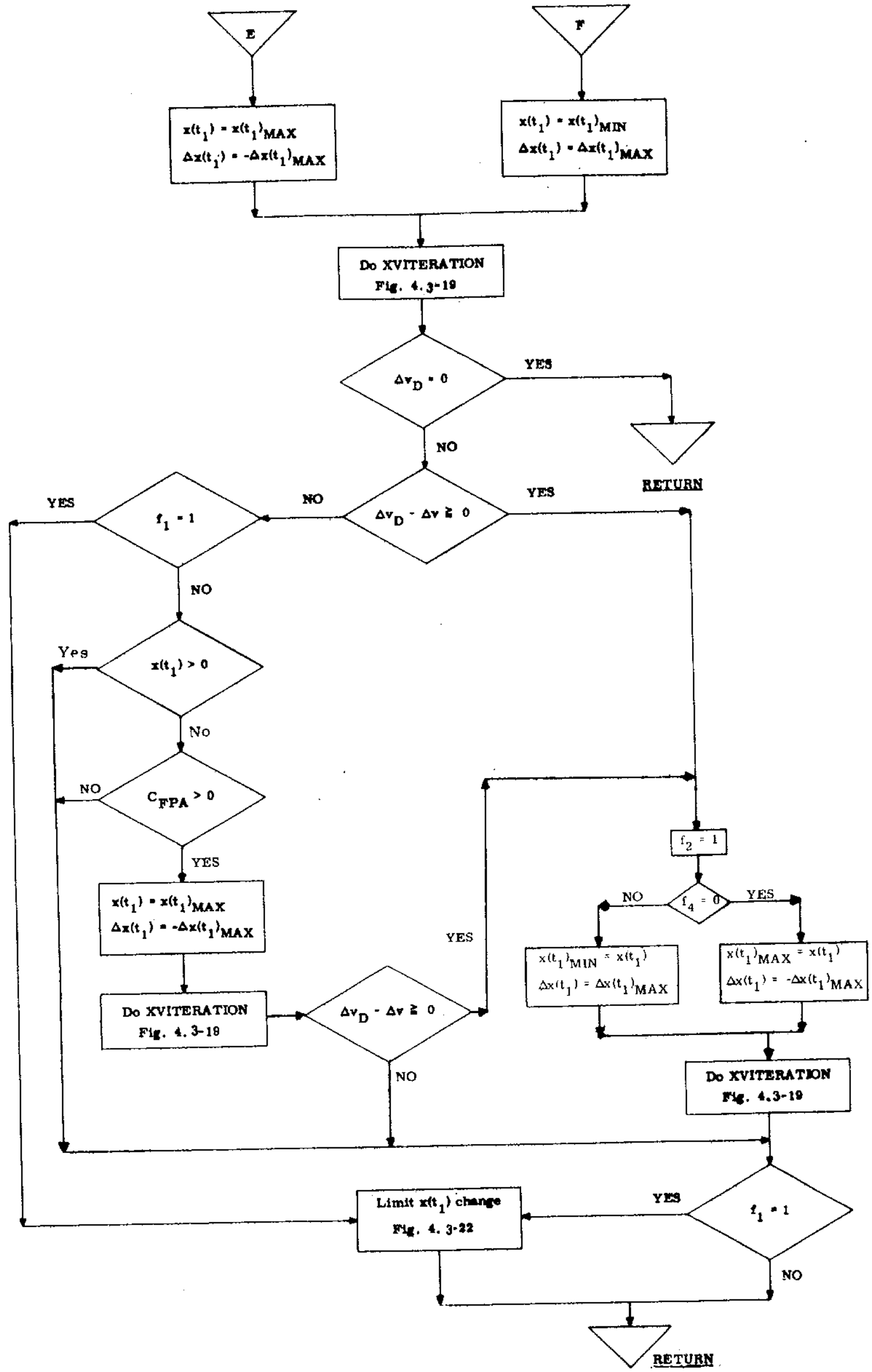

Figure 4.3-8 Logical Diagram of Return to Earth Targeting -- Conic Return Routine

If β_6 is negative, only near perigee solutions are possible and hence no additional bounds on $x(t_1)$ are computed. The initial conic trajectory may have achieved the desired reentry flight path angle near apogee, however, and this type of solution is no longer possible due to a change in R_{CON}. This is the one exception mentioned in Section 5.4.3.4.2. The value of ϕ_2 must be set to -1, forcing all subsequent conic solutions to reach the desired reentry flight path angle near perigee, and then the entire precision trajectory phase is recycled.

If β_6 is positive, it is used to calculate $x(t_1)_{LIM}$. Three types of solutions may exist.

(1) $-x(t_1)_{LIM} < x(t_1) < x(t_1)_{LIM}$

If $x(t_1)$ takes a value in this region, the desired reentry flight path angle ($\gamma(t_2)$) will be achieved near apogee.

(2) $x(t_1) < -x(t_1)_{LIM}$ or $x(t_1)_{LIM} < x(t_1)$

If $x(t_1)$ takes a value in these regions, the desired reentry flight path angle ($\gamma(t_2)$) will be achieved near perigee.

(3) $x(t_1) = x(t_1)_{LIM}$ or $x(t_1) = -x(t_1)_{LIM}$

If $x(t_1)$ takes on either of these values, the desired reentry flight path angle ($\gamma(t_2)$) will equal the maximum angle achieved by the conic.

The above information is used to set new bounds on $x(t_1)$ which will force the conic to reach a near apogee or near perigee solution as prescribed by the initial conic.

If the program is being operated in the fuel critical mode, the $x(t_1) - \Delta v$ iterator is used to compute an absolute minimum Δv solution.

If the program is being operated in the time critical mode, the $x(t_1) - \Delta v$ iterator will be used to compute a solution requiring a velocity change equal to the objective velocity change.

5.4.3.5 Detailed Flow Diagrams

Certain portions of the flow diagrams mentioned in the previous sections are illustrated in Figs. 4.3-11 through 4.3-24.

A list of nomenclature used in the following diagrams and not defined previously follows:

λ	radius ratio
p	semi-latus rectum
α	1/semi-major axis
$\underline{r}_2(t_2)$	target aim point
$\underline{r}_{LS}$	vector through landing site
θ_{LONG}	landing site longitude
θ_{LAT}	landing site latitude
$\dot{m}$	mass flow rate
F	thrust
f_3	flag set to {0 RCS jets; 1 SPS engine}
f_4	flag set for transearth coast slowdown ($-\Delta v_D$)
f_5	flag set to {+1 posigrade; -1 retrograde}
t_{LS}	time of landing
Δt_B	length of burn
$\Delta \underline{v}_{LV}$	velocity change vector in local vertical coordinates
a_{MG}	middle gimbal angle
$\theta_1, \theta_2, \theta_3$	intermediate variables
Δt_{21}	adjustment of t_{21}
ΔR_{CON}	change in the final radius of a conic trajectory
P37RANGE	{=0 AUGEKUGEL entry range used; ≠0 P37RANGE used}

The diagrams are essentially self-explanatory; however, a discussion of two of the diagrams will follow:

Precomputations

The equations illustrated in Figure 4.3-11 are used to compute basic variables required during the remaining portions of the program.

As mentioned in the introduction, the pre-return and post-return state vectors are required to lie in the same plane. If the pre-return position and velocity vectors are nearly colinear, the plane of the return trajectory will be defined to prevent reentry near the earth's poles.

Retrograde return to earth trajectories are not allowed.

Final State Vector Computation

The method illustrated in Figures 4.3-12 and 4.3-13 is used to extrapolate a post-return state vector to the desired reentry flight path angle. An iterative procedure is again used.

First the post-return state vector is advanced through the best estimate of the transit time to reentry using the Coasting Integration Routine. Since it is not necessary to explicitly compute the flight path angle of the resulting reentry state vector, the cotangent of the flight path angle is compared with the cotangent of the desired reentry flight path angle. If they are sufficiently close, the resulting reentry state vector is accepted.

If the desired cotangent of the reentry flight path angle is not achieved, an iterative procedure is started. This procedure uses Conic Subroutines to compute the time of flight (Δt_{21}) to the desired reentry flight path angle.

3

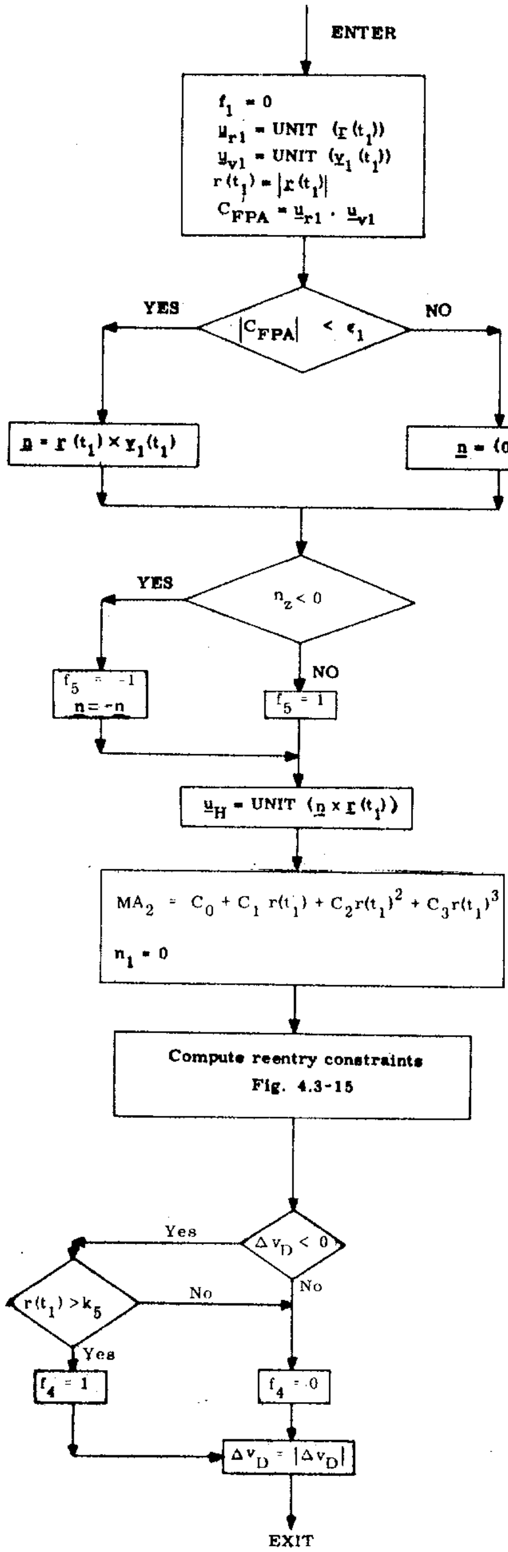

Figure 4.3-11 Precomputations

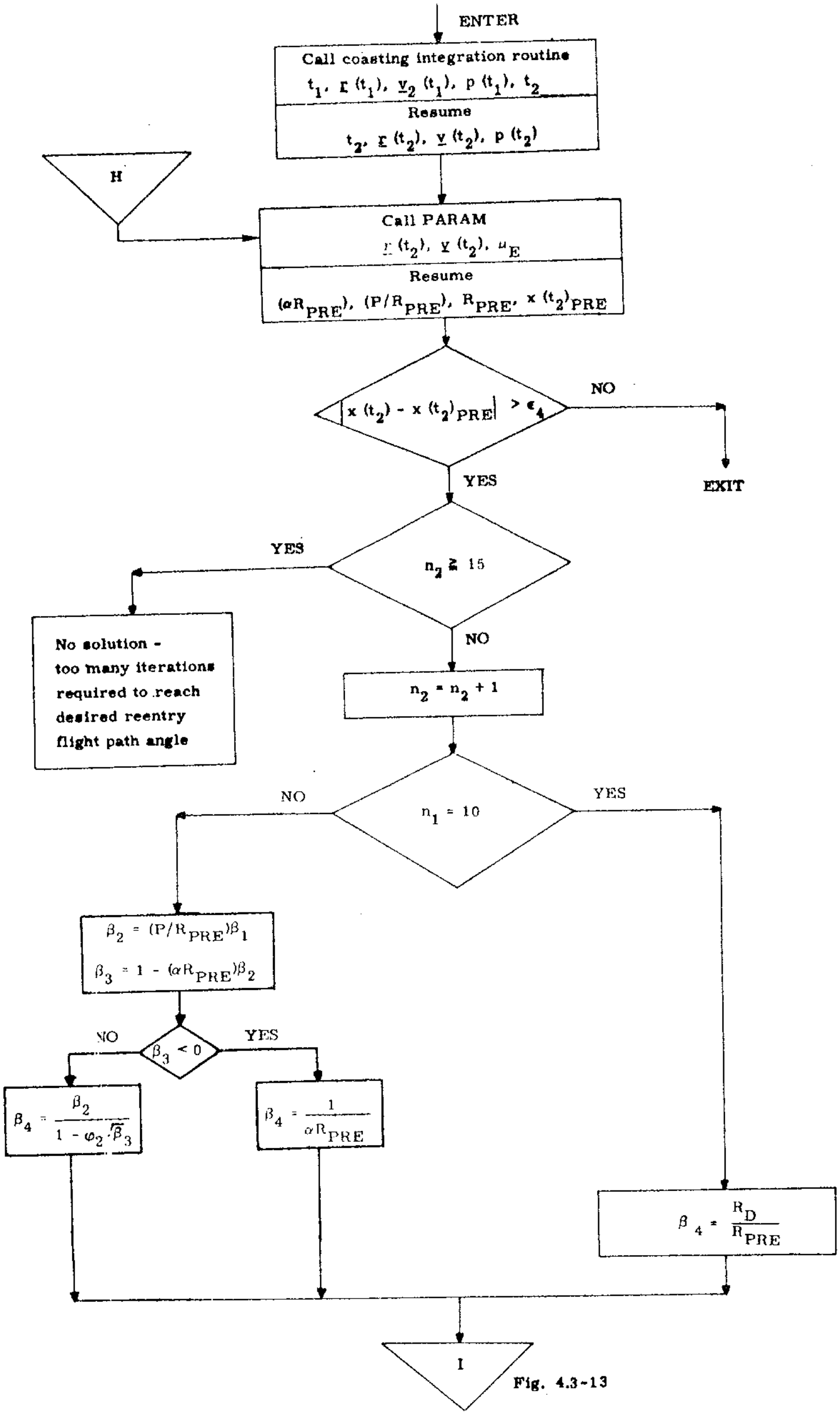

Figure 4.3-12 Final State Vector Computation

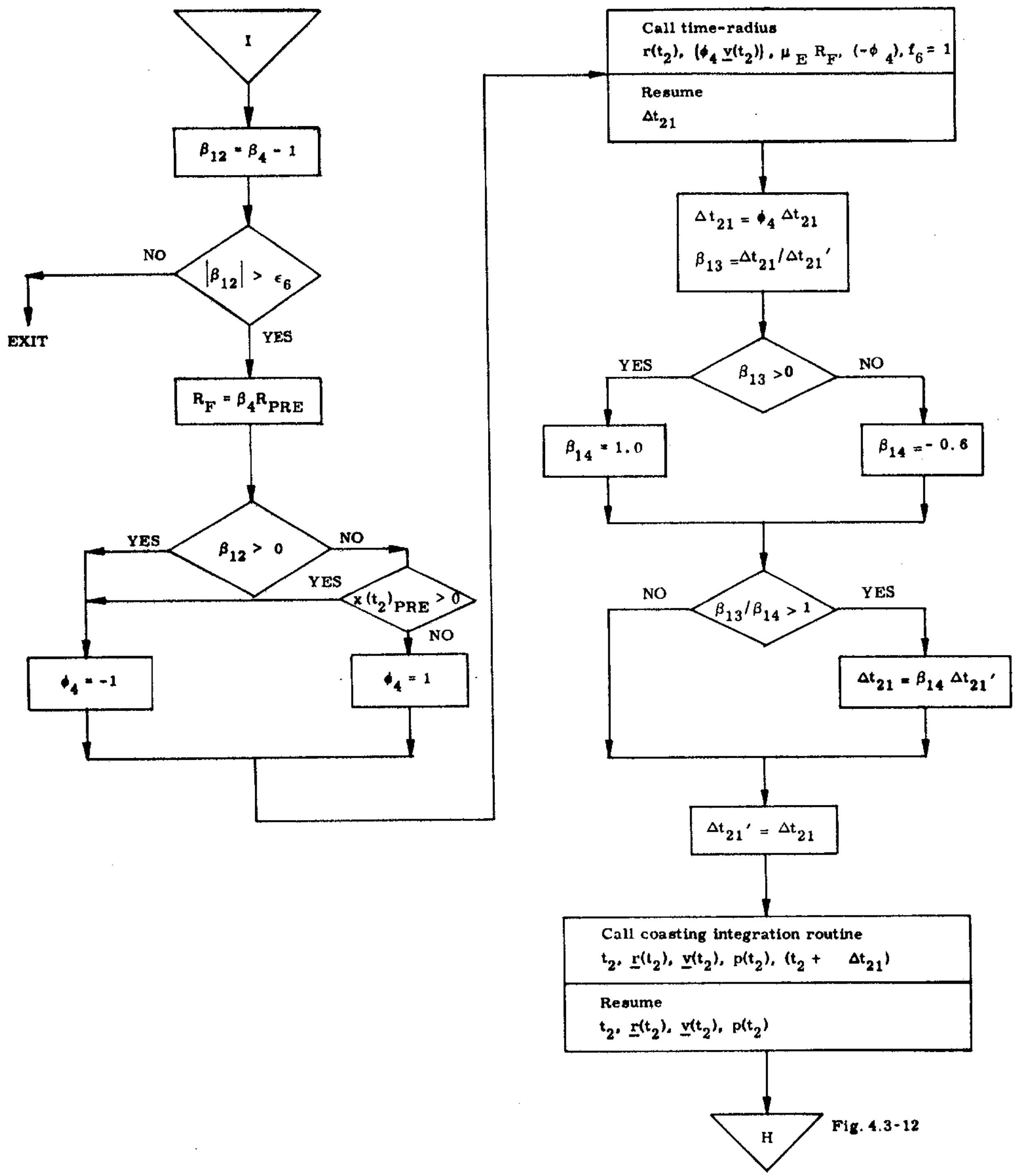

Figure 4.3-13 Final State Vector Computation

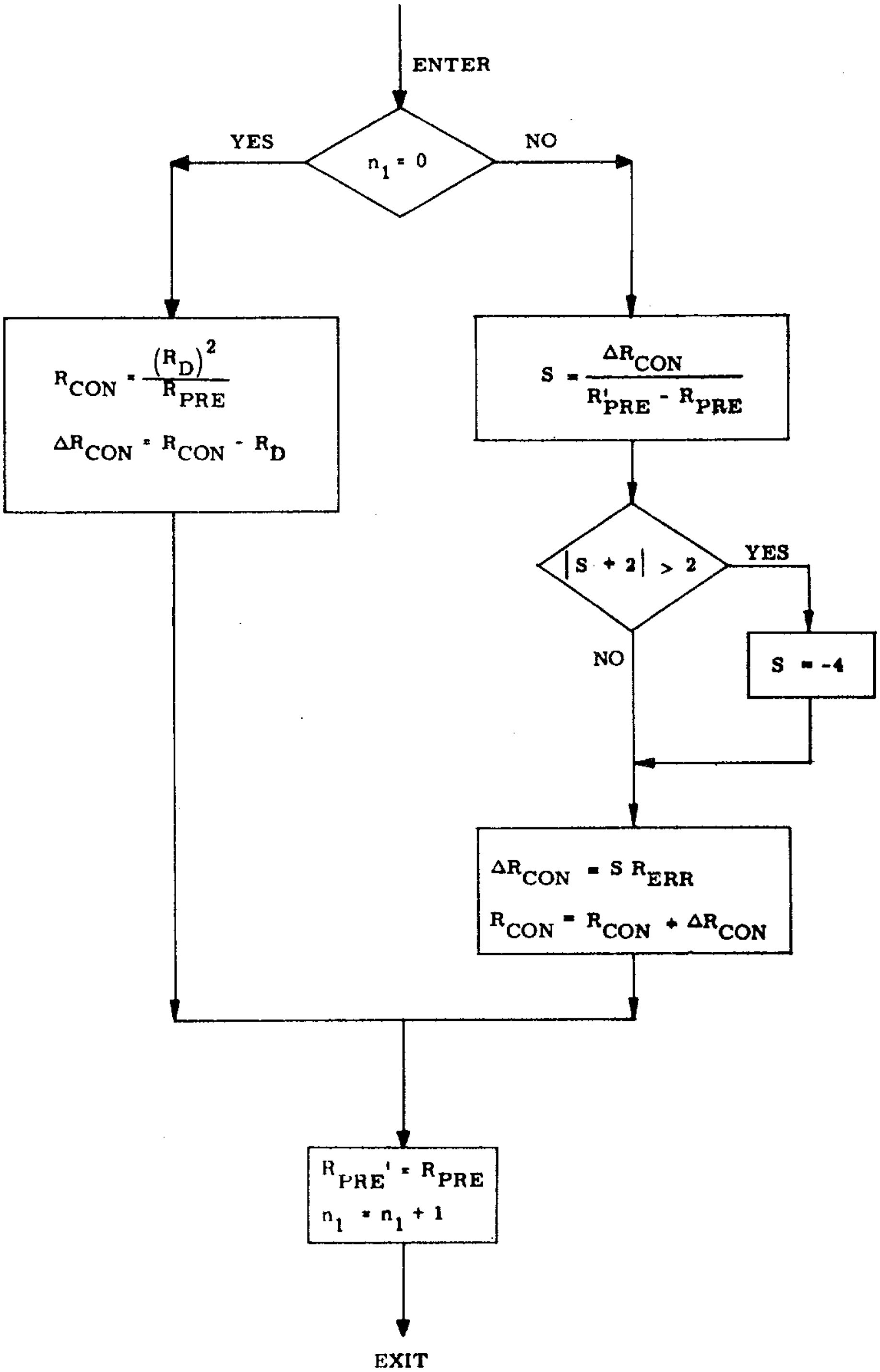

Figure 4.3-14 New R_{CON} Computation

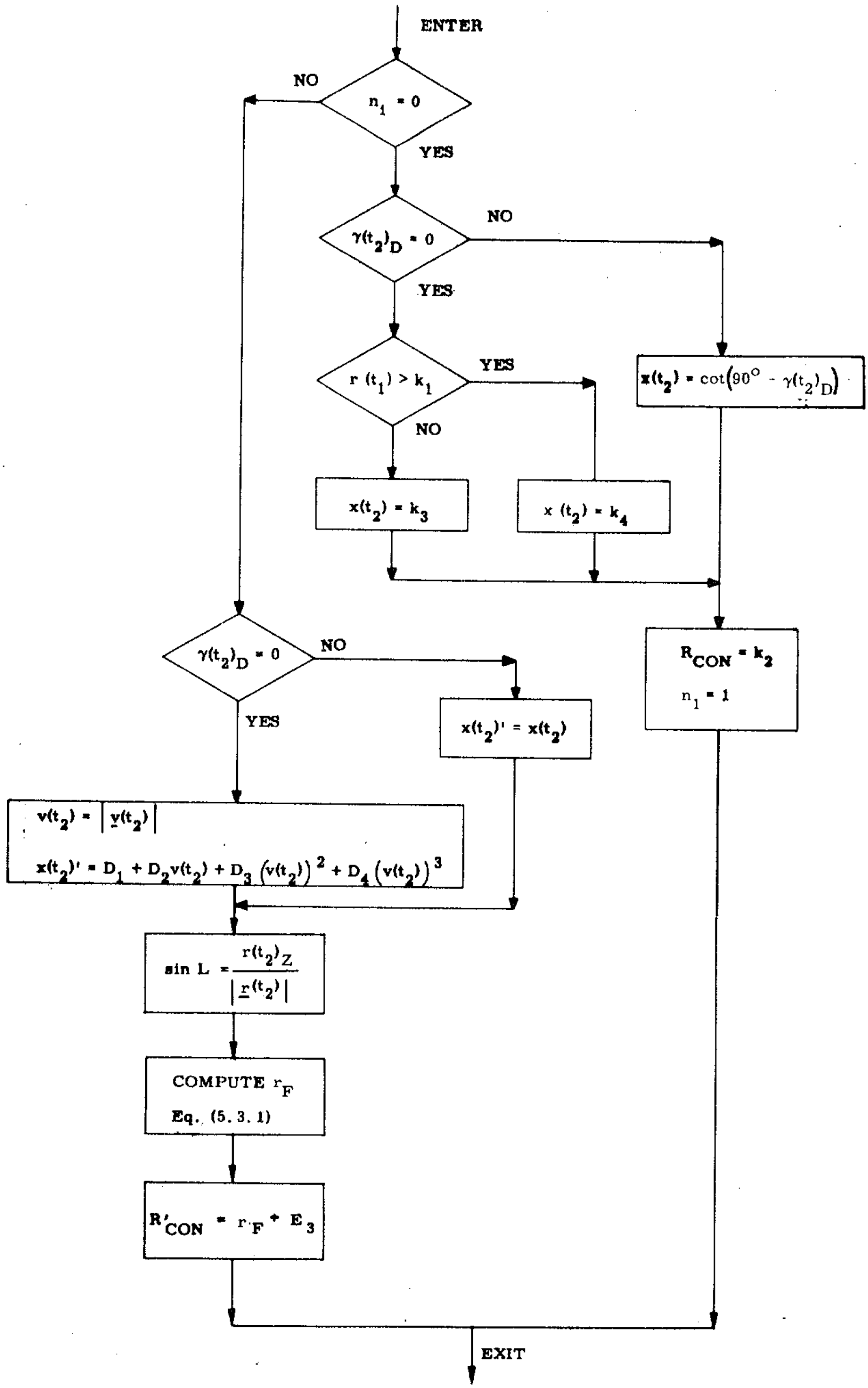

Figure 4.3-15 Reentry Constraint Computation

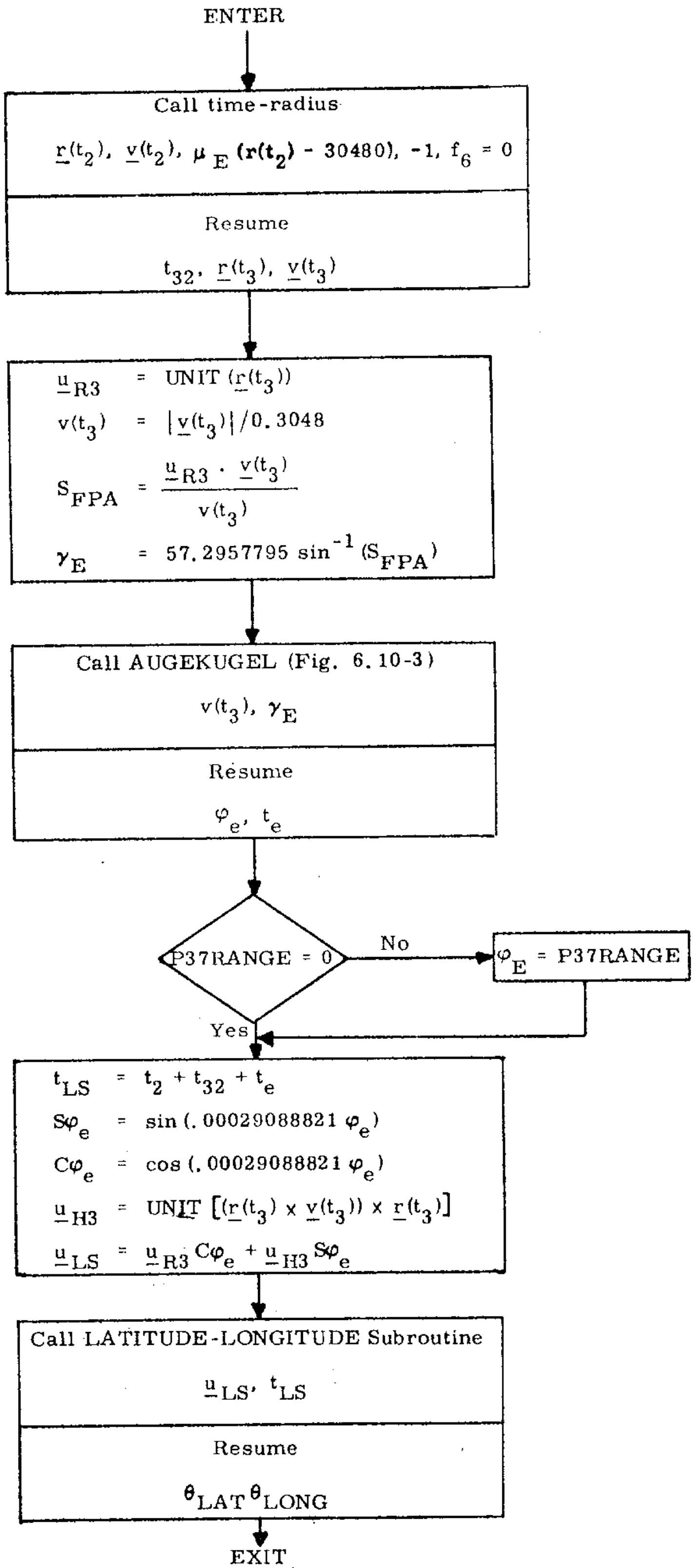

Figure 4.3-16 Landing Site Computation

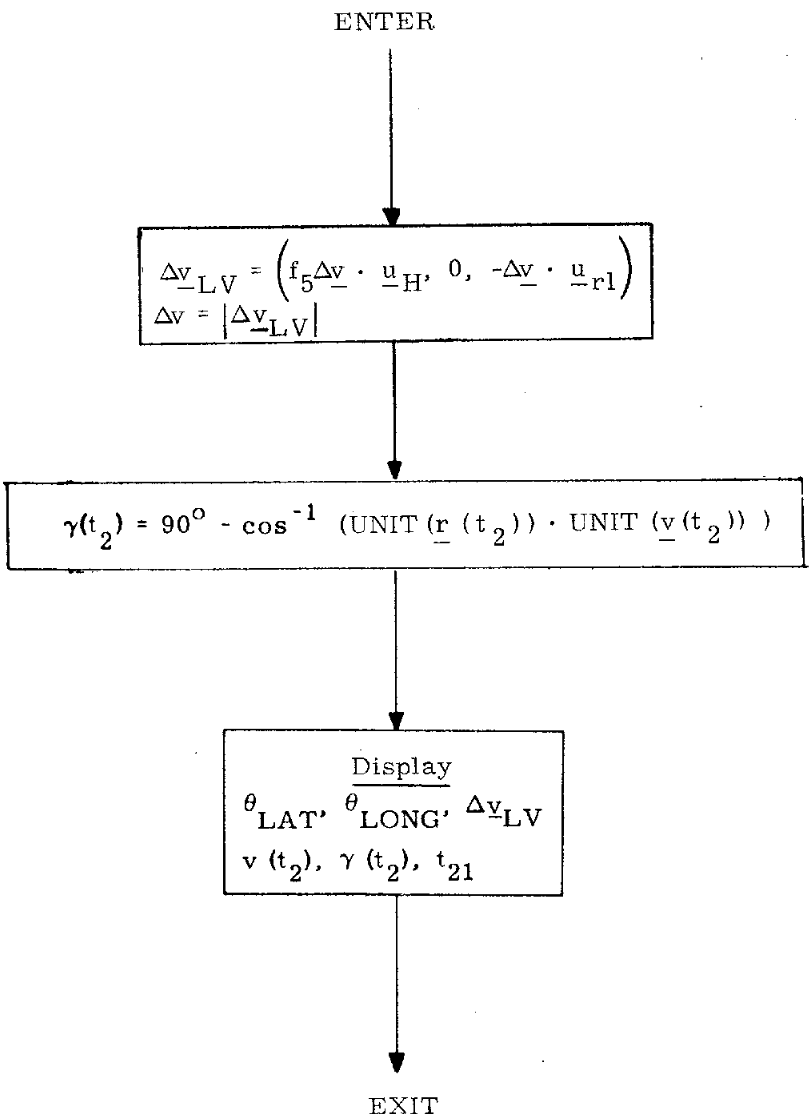

Figure 4. 3-17 Return-to-Earth Display

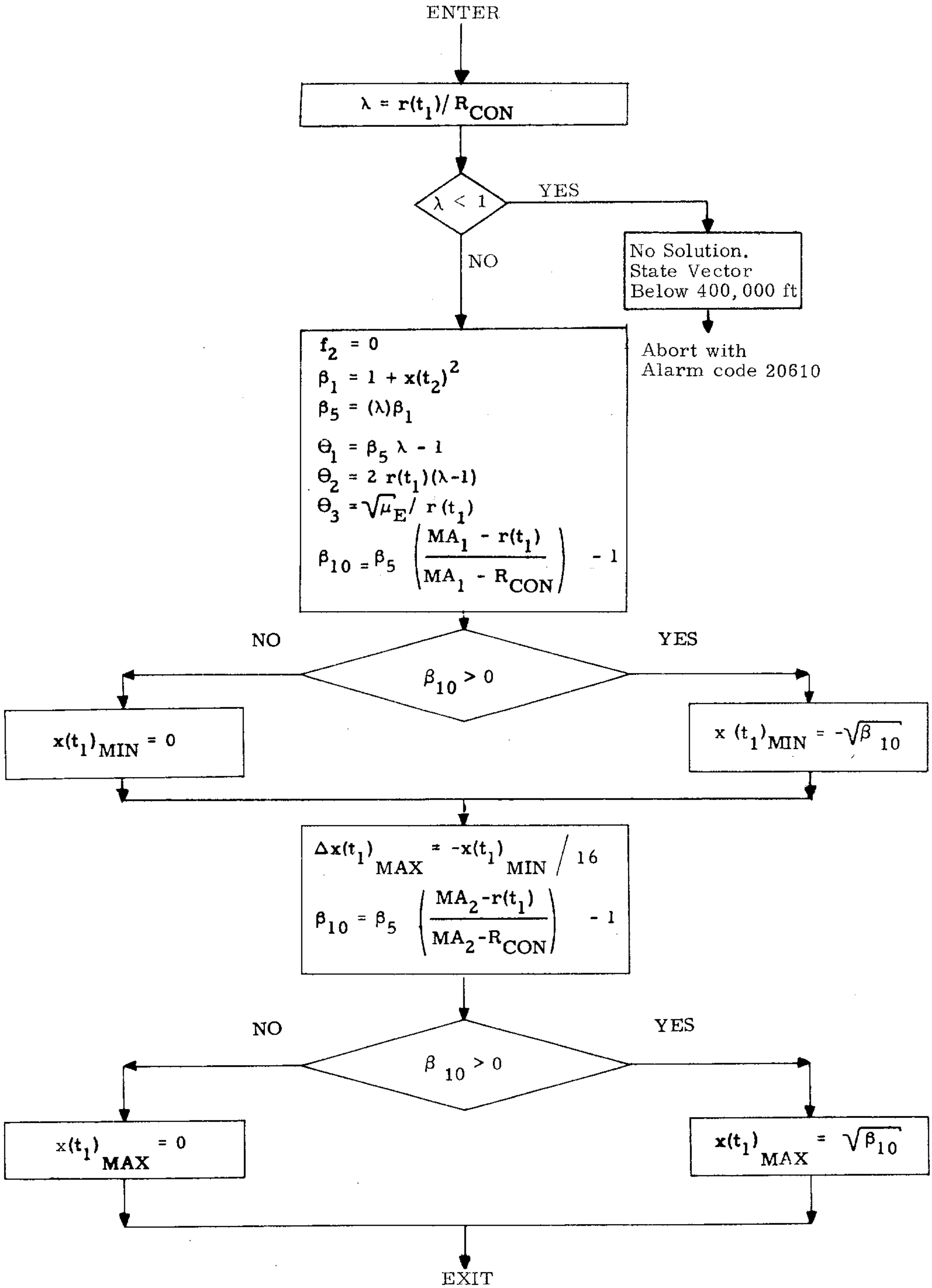

Figure 4.3-18 Conic Initialization

ENTER

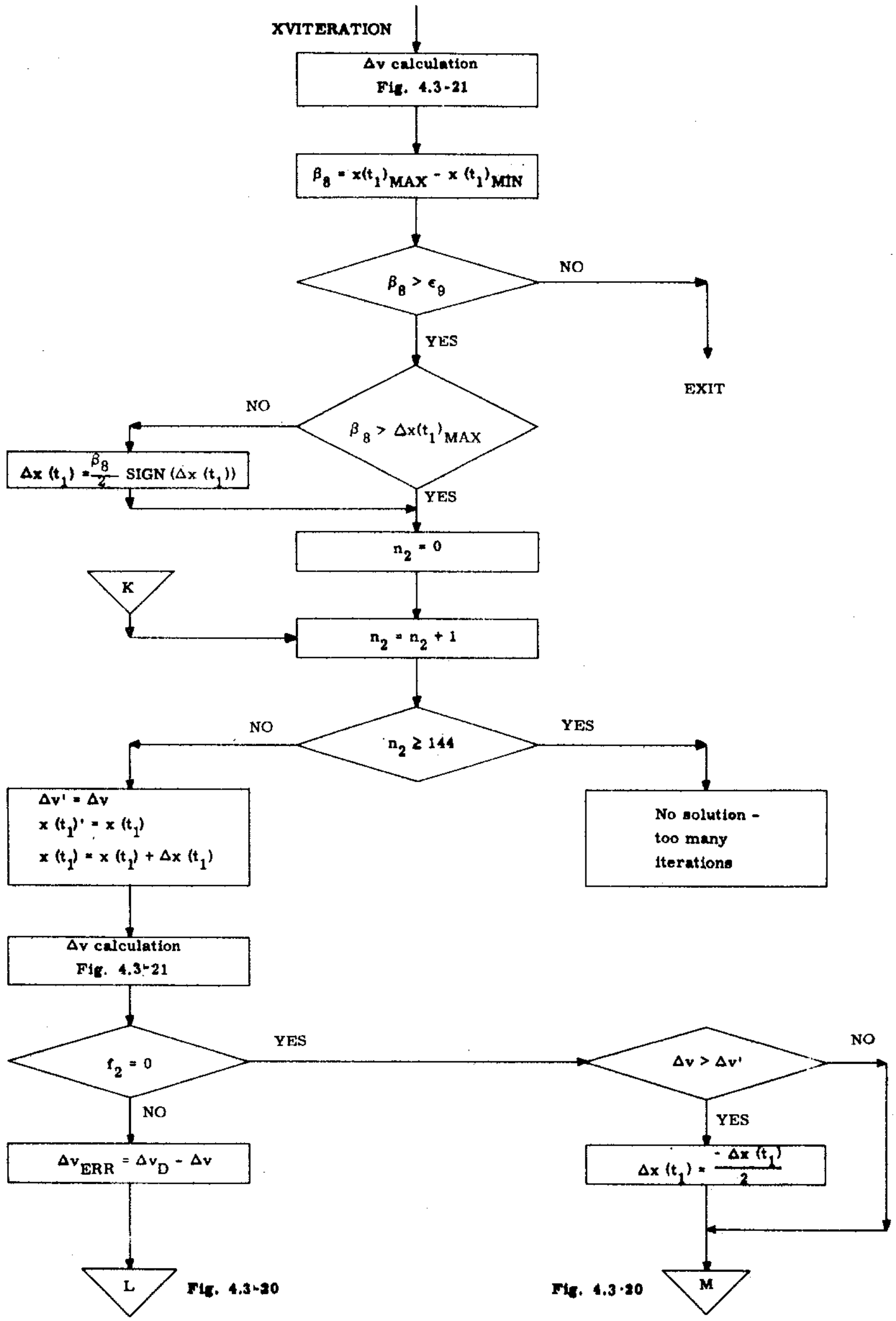

Figure 4.3-19 $x(t_1)$ - Δv Iterator

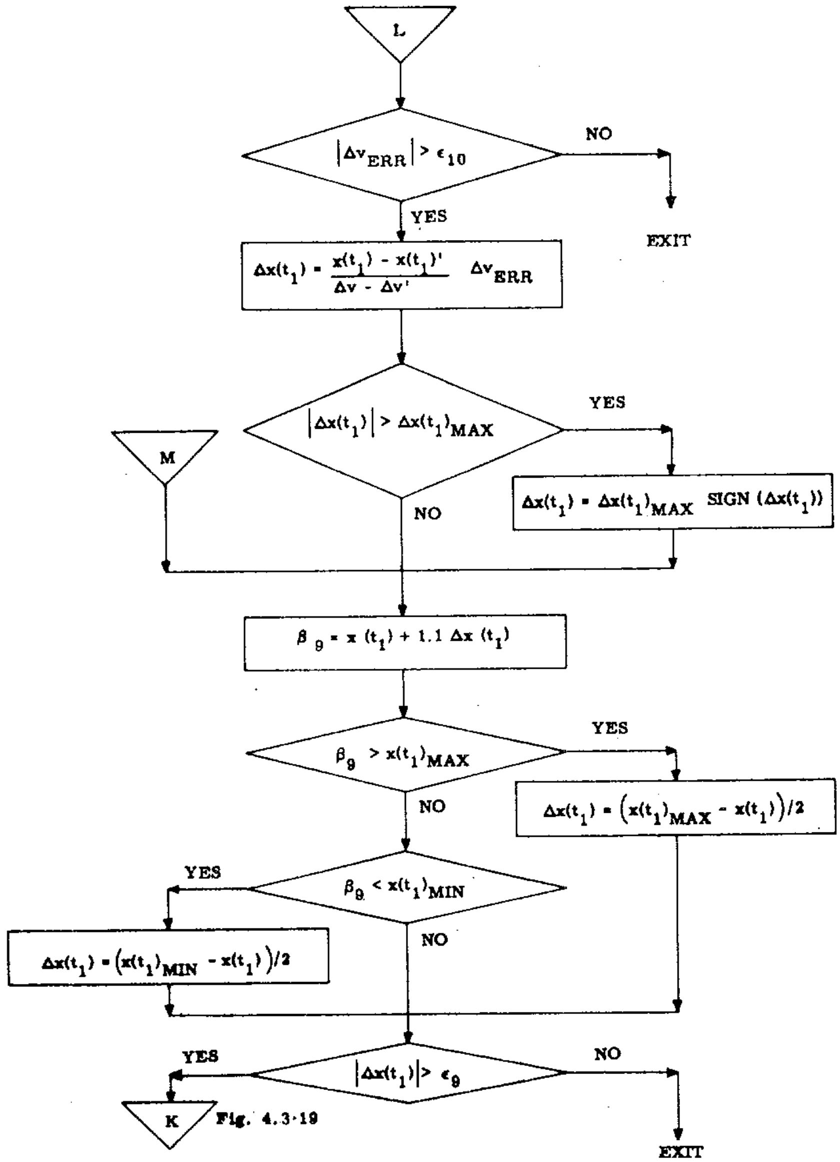

Figure 4.3-20 $x(t_1)$ - Δv Iterator

ENTER

$$p_{CON} = \frac{\theta_2}{\theta_1 - \left(x\,(t_1)\right)^2}$$

$$\underline{v}_2\,(t_1) = \theta_3 \sqrt{p_{CON}}\,\left(x\,(t_1)\,\underline{u}_{r1} + \underline{u}_H\right)$$

$$\Delta \underline{v} = \underline{v}_2\,(t_1) - \underline{v}_1\,(t_1)$$

$$\Delta v = |\,\Delta \underline{v}\,|$$

EXIT

Figure 4.3-21 Δv Calculation

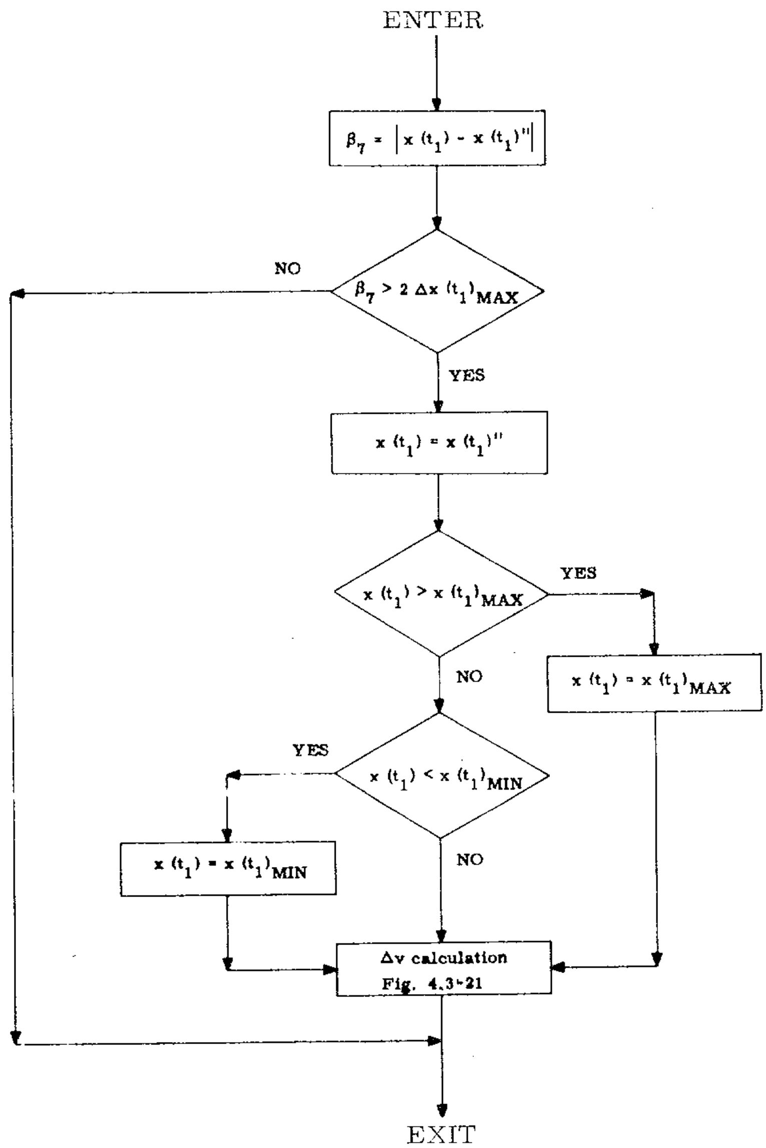

Figure 4.3-22 Limit $x(t_1)$ Change

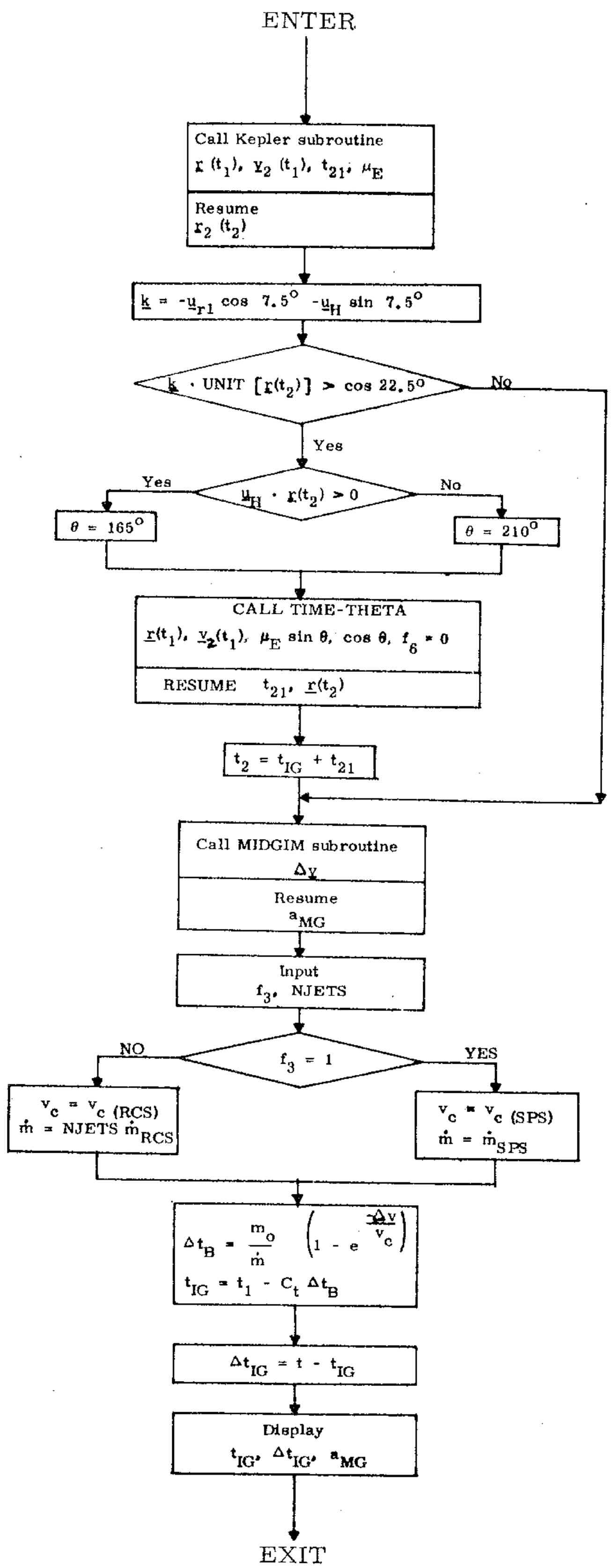

Figure 4.3-23 Target Computation

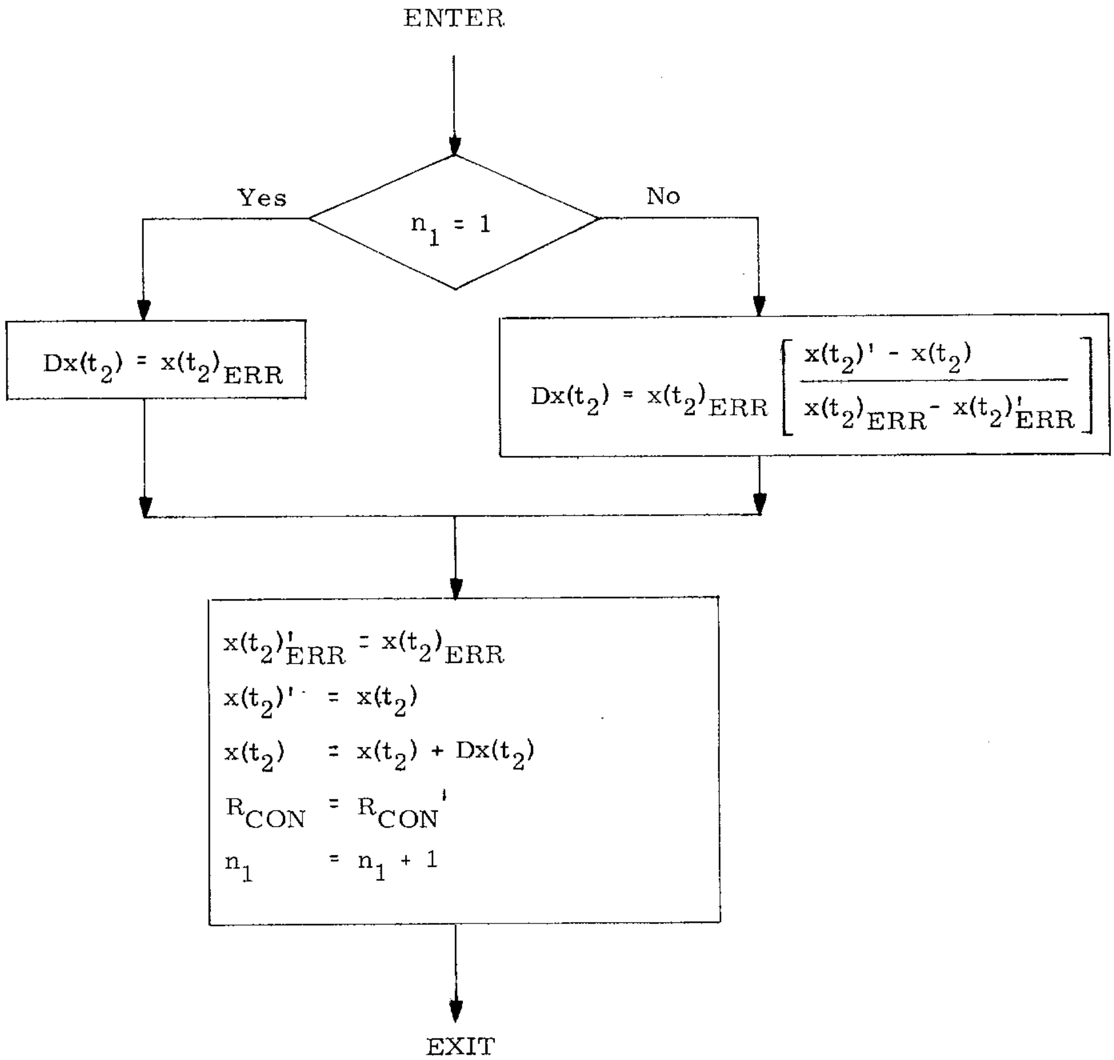

Figure 4.3-24 New $x(t_2)$ Computation

5.5 BASIC SUBROUTINES

5.5.1 GENERAL COMMENTS

The basic solar system and conic trajectory subroutines which are used by the various guidance and navigation routines are described in this section.

5.5.1.1 Solar System Subroutines

The subroutines used to determine the translation and rotation of the relevant solar system bodies (earth, moon and sun) are designed specifically for a fourteen day lunar landing mission. The method of computing the position and velocity of the moon and the sun relative to the earth is given in Section 5.5.4. The transformations between the Basic Reference Coordinate System and the Earth- and Moon-fixed Coordinate Systems are described in Section 5.5.2. The procedure for transforming between vectors in the Basic Reference Coordinate System and latitude, longitude, altitude coordinates is given in Section 5.5.3. Although these subroutines are normally used in the lunar landing mission, they are valid for use in any mission of not more than fourteen days duration in earth-moon space.

5.5.1.2 Conic Trajectory Subroutines

This is a description of a group of conic trajectory subroutines which are frequently used by higher level routines and programs in both the Command Module and the Lunar Module computers.

These subroutines, whose block diagrams are presented in Sections 5.5.5 to 5.5.10, provide solutions to the following conic problems. (See nomenclature which follows)

(1) Given $\underline{r}(t_1)$, $\underline{v}(t_1)$, t_D; solve for $\underline{r}(t_2)$, $\underline{v}(t_2)$

(Kepler Subroutine)

(2) Given $\underline{r}(t_1)$, $\underline{r}(t_2)$, t_{D21}, s_G; solve for $\underline{v}(t_1)$

(Lambert Subroutine)

(3) Given $\underline{r}(t_1)$, $\underline{v}(t_1)$, θ; solve for t_{21}, $\underline{r}(t_2)$, $\underline{v}(t_2)$

(Time-Theta Subroutine)

(4) Given $\underline{r}(t_1)$, $\underline{v}(t_1)$, $r(t_2)$, $s_{\dot{r}}$; solve for t_{21}, $\underline{r}(t_2)$, $\underline{v}(t_2)$

(Time-Radius Subroutine)

(5) Given $\underline{r}(t)$, $\underline{v}(t)$; solve for r_P, r_A, e

(Apsides Subroutine)

In addition, the following useful subroutines are provided.

(6) Conic Parameters Subroutine (See Fig. 5.10-1).

(7) Geometric Parameters Subroutine (See Fig. 5.10-2).

(8) Iterator Subroutine (See Fig. 5.10-3).

The solutions to the above set of conic problems have stringent accuracy requirements. Programming the fixed-point Apollo computer introduces two constraints which determine accuracy limitations: the 28 bit double precision word length, and the range of variables which is several orders of magnitude for the Apollo mission.

In order to maintain numerical accuracy when these subroutines are programmed into the Apollo computer, floating point programming techniques must be exercised. The effect is for even a simple equation to require a large number of computer instructions. The alternative to this is to separate the problem into phases, each with a different variable range. This, however, requires an even larger number of instructions. These considerations provide the incentive for efficiently organizing the conic equations as shown in the block diagrams.

In addition to the requirement for accuracy, the solution to the Kepler and Lambert Problems must be accomplished in a minimum of computation time in order that the guidance system operate satisfactorily in real time. This additional constraint dictates that a minimum of computer instructions be performed when solving the problem.

Method of Solution

To minimize the total number of computer instructions, the problems are solved in the "universal" form; i.e. only equations which are equally valid for the ellipse, parabola and hyperbola are used. Also these subroutines can be used with either the earth or the moon as the attracting body.

Kepler's equation, in the universal form, is utilized to relate transfer time to the conic parameters. All other necessary equations are also universal. The Kepler and Lambert problems are solved with a single iteration loop utilizing a simple first-order slope iterator. In the case of the Kepler problem a third order approximation is available to produce the initial guess for the independent variable (See Eq.(2. 2. 4) of Section 5. 2. 2. 2).

Sections 5. 5. 5 thru 5. 5. 10 provide block diagrams of the detailed computational procedures for solving the various problems. The equations are presented in block diagram form with the nomenclature defined below.

Range of Variables

As indicated previously, the programming of the conic subroutines requires a careful balance between accuracy, computational speed and number of instructions. This balance, in the Apollo Guidance Computer, leaves very little margin in any of these areas.

Since the values of problem variables are determined by the solution of the problem being solved and since the problem may originate from the ground system, it is essential that the variable range limitations be defined. The conic routines are incapable of handling problems when the solution lies outside of the range.

The following is a list of the maximum allowable numeric values of the variables. Note that, in addition to fundamental quantities such as position and velocity, there are limitations on intermediate variables and combinations of variables.

Scaling for Conic Subroutines (Sections 5.5.5 to 5.5.10)

Parameter	Maximum Value* Earth Primary Body	Maximum Value* Moon Primary Body
r	2^{29}	2^{27}
v	2^{7}	2^{5}
t	2^{28}	2^{28}
α**	2^{-22}	2^{-20}
α_N**	2^{6}	2^{6}
p_N	2^{4}	2^{4}
$\cot \gamma$	2^{5}	2^{5}
$\cot \frac{\theta}{2}$	2^{5}	2^{5}
x	2^{17}	2^{16}
$\xi = \alpha x^2$***	-50	-50
	$+4\pi^2$	$+4\pi^2$
$c_1 = \underline{r} \cdot \underline{v} / \sqrt{\mu}$	2^{17}	2^{16}
$c_2 = r v^2 / \mu - 1$	2^{6}	2^{6}
$\lambda = r(t_1)/r(t_2)$	2^{7}	2^{7}
$\cos \theta - \lambda$	2^{7}	2^{7}

* All dimensional values are in units of meters and centiseconds.

** The maximum absolute value occurs for negative values of this parameter.

***Both the maximum and minimum values are listed since neither may be exceeded.

Parameter	Maximum Value* Earth	Moon
e	2^3	2^3
x^2	2^{34}	2^{32}
$x^2 c(\xi)$	2^{33}	2^{31}
$x^3 s(\xi)/\sqrt{\mu}$	2^{28}	2^{28}
$c_1 x^2 c(\xi)$	2^{49}	2^{46}
$c_2 x^2 s(\xi)$	2^{35}	2^{33}
$x [c_2 x^2 s(\xi) + r(t_1)]$	2^{49}	2^{46}
$\xi s(\xi)$	2^7	2^7
$x^2 c(\xi)/r$	2^8	2^8
$\sqrt{\mu} x (\xi s(\xi) - 1)/r(t_2)$	2^{15}	2^{13}
$c(\xi)$	2^4	2^4
$s(\xi)$	2^1	2^1

* All dimensional values are in units of meters and centiseconds.

Nomenclature for Conic Subroutines (Sections 5.5.5 to 5.5.10)

$\underline{r}(t_1)$	initial position vector
$\underline{v}(t_1)$	initial velocity vector
$\underline{r}(t_2)$	terminal position vector
$\underline{v}(t_2)$	terminal velocity vector
$\underline{u}_N$	unit normal in the direction of the angular momentum vector
α	reciprocal of semi-major axis (negative for hyperbolas)
r_P	radius of pericenter
r_A	radius of apocenter
e	eccentricity
α_N	ratio of magnitude of initial position vector to semi-major axis
p_N	ratio of semi-latus rectum to initial position vector magnitude
γ	inertial flight path angle as measured from vertical
θ	true anomaly difference between $\underline{r}(t_1)$ and $\underline{r}(t_2)$
f	true anomaly of r (t_2)

x	a universal conic parameter equal to the ratio of eccentric anomaly difference to $\sqrt{+\alpha}$ for the ellipse, or the ratio of the hyperbolic analog of eccentric anomaly difference to $\sqrt{-\alpha}$ for the hyperbola
x'	value of x from the previous Kepler solution
t_{21}	computed transfer time from Kepler's equation $(t_2 - t_1)$
t'_{21}	transfer time corresponding to the previous solution of Kepler's equation
t_D	desired transfer time through which the conic update of the state vector is to be made
t_{D21}	desired transfer time to traverse from $\underline{r}(t_1)$ to $\underline{r}(t_2)$
t_{ERR}	error in transfer time
ϵ_t	fraction of desired transfer time to which t_{ERR} must converge
Δx	increment in x which will produce a smaller t_{ERR}
ϵ_x	value of Δx which will produce no significant change in t_{21}
$\Delta \cot \gamma$	increment in $\cot \gamma$ which will decrease the magnitude of t_{ERR}
ϵ_c	value of $\Delta \cot \gamma$ which will produce no significant change in t_{21}

μ	product of universal gravitational constant and mass of the primary attracting body
x_{MAX}	maximum value of x
x_{MIN}	minimum value of x
cot_{MAX}	maximum value of cot γ
cot_{MIN}	minimum value of cot γ
ℓ_{MAX}	upper bound of general independent variable
ℓ_{MIN}	lower bound of general independent variable
$x_{MAX\,1}$	absolute upper bound on x with respect to the moon
$x_{MAX\,0}$	absolute upper bound on x with respect to the earth
k	a fraction of the full range of the independent variable which determines the increment of the independent variable on the first pass through the iterator
y	general dependent variable
y'	previous value of y
y_{ERR}	error in y
z	general independent variable

Δz	increment in z which will produce a smaller y_{ERR}
s_G	a sign which is plus or minus according to whether the true anomaly difference between $\underline{r}(t_1)$ and $\underline{r}(t_2)$ is to be less than or greater than 180 degrees
$s_{\dot{r}}$	a sign which is plus or minus according to whether the desired radial velocity at $r(t_2)$ is plus or minus
$\underline{\eta}_1$	general vector # 1
$\underline{\eta}_2$	general vector # 2
ϕ	angle between $\underline{\eta}_1$ and $\underline{\eta}_2$
f_1	a switch set to 0 or 1 according to whether a guess of cot γ is available or not
f_2	a switch set to 0 or 1 according to whether Lambert should determine $\underline{u}_N$ from $\underline{r}(t_1)$ and $\underline{r}(t_2)$ or $\underline{u}_N$ is an input
f_3	a tag set to 0 or 1 according to whether the iterator should use the "Regula Falsi" or bias method
f_4	a flag set to 0 or 1 according to whether the iterator is to act as a first order or a second order iterator
f_5	a flag set to 0 or 1 according to whether Lambert converges to a solution or not

f_6	a switch set to 0 or 1 according to whether or not the new state vector is to be an additional output requirement of the Time-Theta or Time-Radius problems.
f_7	a flag set to 1 if the inputs require that the conic trajectory must close through infinity
f_8	a flag set to 1 if the Time-Radius problem was solved for pericenter or apocenter instead of $r(t_2)$
f_9	a flag set to 1 if the input to the Time-Radius Subroutine produced an e less than 2^{-18}.
t_p	period of the orbit
k_1	the minimal acceptance fraction of t_{D21} to which t_{ERR} must converge
n_1	a flag set to 0 or 1 according to whether or not the velocity vector at the terminal position is to be an additional output requirement of the Lambert Subroutine

5.5.2 PLANETARY INERTIAL ORIENTATION SUBROUTINE

This subroutine is used to transform vectors between the Basic Reference Coordinate System and a Planetary (Earth-fixed or Moon-fixed) Coordinate System at a specified time. These three coordinate systems are defined in Section 5.1.4.

Let $\underline{r}$ be a vector in the Basic Reference Coordinate System, $\underline{r}_P$ the same vector expressed in the Planetary Coordinate System, and t the specified ground elapsed time (GET). Then,

$$\underline{r}_P = M(t)\,(\underline{r} - \underline{\ell} \times \underline{r}) \qquad (5.2.1)$$

and

$$\underline{r} = M^T(t)\,(\underline{r}_P + \underline{\ell}_P \times \underline{r}_P) \qquad (5.2.2)$$

where M(t) is a time dependent orthogonal transformation matrix, $\underline{\ell}$ is a small rotation vector in the Basic Reference Coordinate System, and $\underline{\ell}_P$ is the same vector $\underline{\ell}$ expressed in the Planetary Coordinate System. The vector $\underline{\ell}$ is considered constant in one coordinate system for the duration of the mission. The method of computing M(t) and $\underline{\ell}$ depends on whether the relevant planet is the earth or the moon.

Case I - Earth

For the earth, the matrix M(t) describes a rotation about the polar axis of the earth (the Z-axis of the Earth-fixed Coordinate

System), and the vector $\underline{\ell}$ accounts for the effects of precision and nutation by describing the small angle rotations about the X-axis (ℓ_X) and Y-axis (ℓ_Y) of the Basic Reference Coordinate System (BRCS) which are necessary to produce a coordinate system whose Z-axis is coincident with the true pole. Thus, if $\underline{u}_Z$, which is included in the erasable data load, is the unitized result of transforming into the BRCS the true pole unit vector in the Earth Fixed Coordinate System, then

$$\underline{u}_Z = \begin{pmatrix} \ell_Y \\ -\ell_X \\ \sqrt{1 - \ell_X^2 - \ell_Z^2} \end{pmatrix}$$

$$\underline{\ell} = \begin{pmatrix} \ell_X \\ \ell_Y \\ 0 \end{pmatrix}$$

$$A_Z = A_{Z0} + \omega_E (t + t_0) \qquad (5.2.3)$$

$$M(t) = \begin{pmatrix} \cos A_Z & \sin A_Z & 0 \\ -\sin A_Z & \cos A_Z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{\ell}_P = M(t)\, \underline{\ell}$$

where A_{Z0} is the angle between the X-axis of the Basic Reference Coordinate System and the X-axis of the Earth-fixed Coordinate System (the intersection of the Greenwich meridian and the equatorial plane of the earth) at July 1.0, 1971 universal time (i.e., the Greenwich midnight at the beginning of July 1, 1971), t_0 is the elapsed time between July 1.0, 1971 universal time and the time that the computer clock was zeroed, and ω_E is the angular velocity of the earth.

Case II - Moon

For the moon, the matrix M(t) accounts for the difference in orientation of the Basic Reference and Moon-fixed Coordinate Systems in exact accordance with Cassini's laws, and the rotation vector $\underline{\ell}$ corrects for deviations from the above orientation because of physical libration.

Define the following three angles which are functions of time:

B = the obliquity, the angle between the mean earth equatorial plane and the plane of the ecliptic.

Ω_I = the longitude of the node of the moon's orbit measured from the X-axis of the Basic Reference Coordinate System.

F = the angle from the mean ascending node of the moon's orbit to the mean moon.

Let I be the constant angle between the mean lunar equatorial plane and the plane of the ecliptic (5521.5''). Then, the sequence of rotations which brings the Basic Reference Coordinate System into coincidence with the Moon-fixed Coordinate System (neglecting libration) is as follows:

Rotation	Axis of Rotation	Angle of Rotation
1	X	B
2	Z	Ω_I
3	X	-I
4	Z	$\pi + F$

The transformation matrices for these rotations are, respectively,

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos B & \sin B \\ 0 & -\sin B & \cos B \end{pmatrix}$$

$$M_2 = \begin{pmatrix} \cos \Omega_I & \sin \Omega_I & 0 \\ -\sin \Omega_I & \cos \Omega_I & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos I & -\sin I \\ 0 & \sin I & \cos I \end{pmatrix} \qquad (5.2.4)$$

$$M_4 = \begin{pmatrix} -\cos F & -\sin F & 0 \\ \sin F & -\cos F & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The matrix M(t) is then given by

$$M(t) = M_4 \, M_3 \, M_2 \, M_1 \tag{5.2.5}$$

The following approximate method is used to determine the transformation between the Basic Reference and Moon-fixed Coordinate Systems.

The angles B, Ω_I and F are computed as linear functions of time. Let $\underline{\ell}_M$ be the value of the vector libration $\underline{\ell}_P$ (expressed in the Moon-fixed Coordinate System) at the time it will be used. The vector $\underline{\ell}_M$ is included in the pre-launch erasable data load and is considered constant throughout the flight. Then,

$$
\begin{aligned}
\underline{\ell}_P &= \underline{\ell}_M \\
t_M &= t + t_0 \\
B &= B_0 + \dot{B}\, t_M \\
\Omega_I &= \Omega_{I0} + \dot{\Omega}_I\, t_M \\
F &= F_0 + \dot{F}\, t_M \\
\underline{a} &= \begin{pmatrix} \cos \Omega_I \\ \cos B \sin \Omega_I \\ \sin B \sin \Omega_I \end{pmatrix} \\
\underline{b} &= \begin{pmatrix} -\sin \Omega_I \\ \cos B \cos \Omega_I \\ \sin B \cos \Omega_I \end{pmatrix}
\end{aligned}
\tag{5.2.6}
$$

$$\underline{c} = \begin{pmatrix} 0 \\ -\sin B \\ \cos B \end{pmatrix}$$

$$\underline{d} = \underline{b}\, C_I - \underline{c}\, S_I$$

$$\underline{m}_2 = \underline{b}\, S_I + \underline{c}\, C_I$$

$$\underline{m}_0 = -\underline{a} \cos F - \underline{d} \sin F \qquad (5.2.6)\ \text{(cont.)}$$

$$\underline{m}_1 = \underline{a} \sin F - \underline{d} \cos F$$

$$M(t) = \begin{pmatrix} \underline{m}_0^T \\ \underline{m}_1^T \\ \underline{m}_2^T \end{pmatrix}$$

$$\underline{\ell} = M^T(t)\, \underline{\ell}_P$$

where B_0, Ω_{I0}, and F_0 are the values of the angles B, Ω_I and F, respectively, at July 1.0, 1971 universal time; $\dot{B}$, $\dot{\Omega}_I$ and $\dot{F}$ are the rates of change of these angles; and C_I and S_I are the cosine and sine, respectively, of the angle I.

5.5.3 LATITUDE-LONGITUDE SUBROUTINE

For display and data load purposes, the latitude, longitude, and altitude of a point near the surface of the earth or the moon are more meaningful and more convenient to use than the components of a position vector. This subroutine is used to transform position vectors between the Basic Reference Coordinate System and Geographic or Selenographic latitude, longitude, altitude at a specified time.

In the case of the moon, the altitude is computed above either the landing site radius, r_{LS}, or the mean lunar radius, r_M. For the earth, the altitude is defined with respect to either the launch pad radius, r_{LP}, or the radius of the Fischer ellipsoid, r_F, which is computed from

$$r_F^2 = \frac{b^2}{1 - (1 - \frac{b^2}{a^2})(1 - \text{SINL}^2)} \qquad (5.3.1)$$

where a and b are the semi-major and semi-minor axes of the Fischer ellipsoid, respectively, and SINL is the sine of the geocentric latitude.

The computational procedures are illustrated in Figs. 5.3-1, 5.3-2, and 5.3-3. The calling program must specify either a vector $\underline{r}$ or latitude (Lat), longitude (Long), and altitude (Alt). In addition, the program must set the time t and the two indicators P and F where

$$P = \begin{cases} 0 & \text{for earth} \\ 1 & \text{for moon} \end{cases}$$

$$F = \begin{cases} 1 \text{ for Fischer ellipsoid or mean lunar radius} \\ 0 \text{ for launch pad or landing site radius} \end{cases}$$

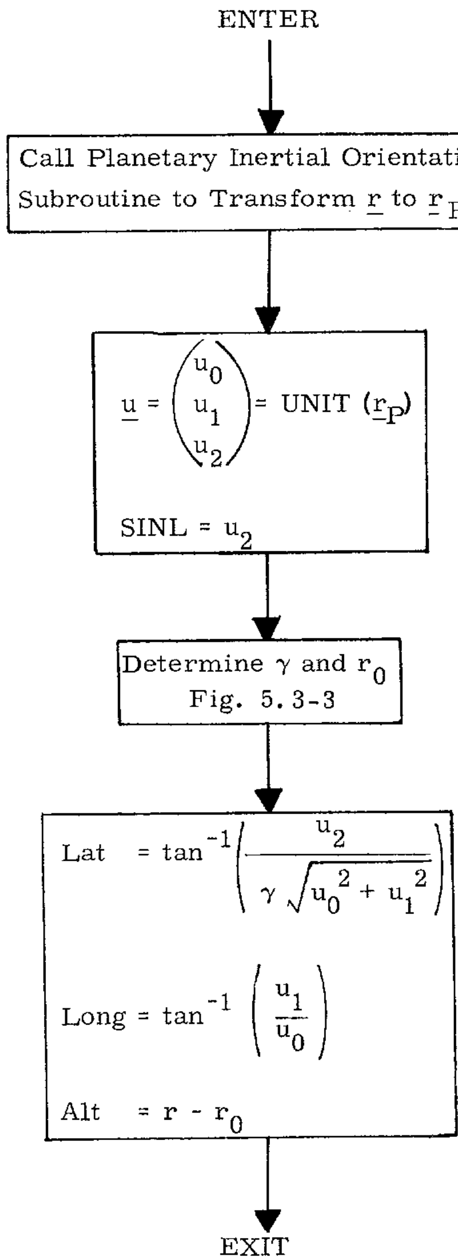

Fig. 5.3-1 Vector to Latitude, Longitude, Altitude Computation Logic Diagram

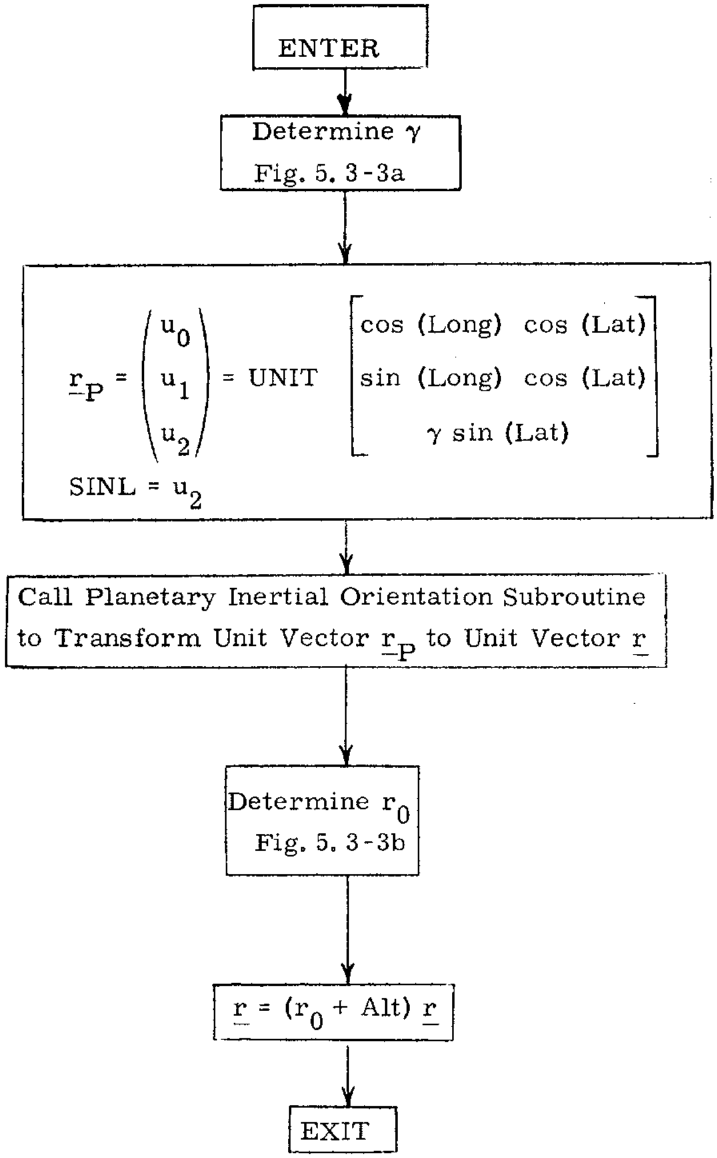

Fig. 5. 3-2 Latitude, Longitude, Altitude to Vector Computation Logic Diagram

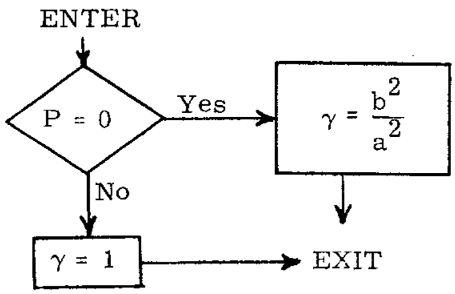

Figure 5.3-3a Determination of γ

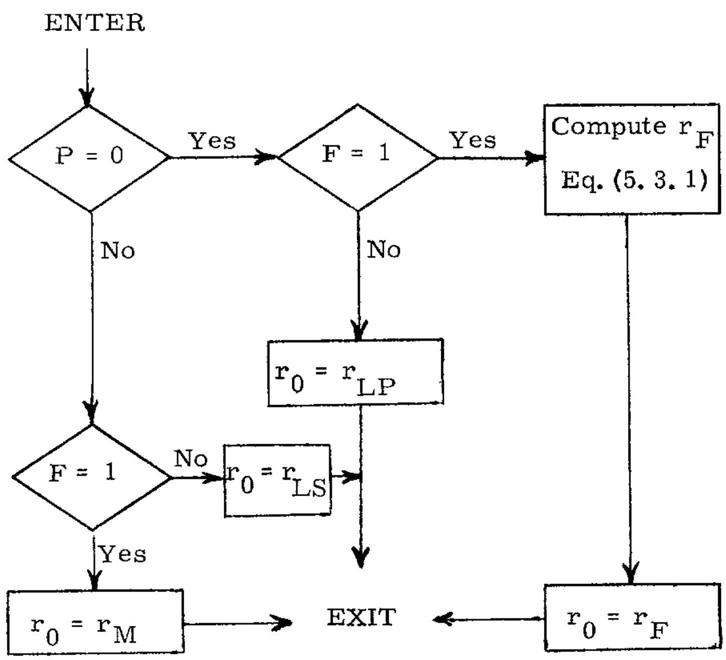

Figure 5.3-3b Determination of r_0

5.5.4 LUNAR AND SOLAR EPHEMERIDES

This subroutine is used to determine the position and velocity vectors of the sun and the moon relative to the earth. The position vectors of the moon and the sun are needed by the Coasting Integration Routine to compute gravity perturbations (Section 5.2.2.3). The velocity of the moon is used by the Coasting Integration Routine when a change in the origin of the coordinate system is performed at the sphere of influence of the moon (Fig. 2.2-3). The velocity of the sun is required,but not very accurately, to compute aberration corrections to optical sightings.

The position of the moon is stored in the computer in the form of a ninth-degree polynominal approximation which is valid over a 14.5 day interval beginning at noon ephemeris time on the day of the launch. The following parameters are included in the pre-launch erasable data load:

t_{M0} = the elapsed time between July 1.0, 1971 universal time and the time at the center of the range over which the lunar-position polynominal is valid. The value of t_{M0} will be an integral number of quarter days minus the difference between ephemeris time and universal time

$\underline{c}_0$ to $\underline{c}_9$ = vector coefficients

Let t be the specified ground elapsed time (GET), and t_0 be the elapsed time between July 1.0, 1971 universal time and the time that the computer clock was zeroed. Then, the approximate position and velocity of the moon are computed from

$$t_M = t + t_0 - t_{M0} \qquad (5.4.1)$$

$$\underline{r}_{EM} = \sum_{i=0}^{9} \underline{c}_i \, t_M^{\,i} \qquad (5.4.2)$$

$$\underline{v}_{EM} = \sum_{i=1}^{9} i \, \underline{c}_i \, t_M^{\,i-1} \qquad (5.4.3)$$

The approximate position and velocity of the sun are computed from the following items which are included in the pre-launch erasable data load:

$\underline{r}_{ES0}$, $\underline{v}_{ES0}$ = the position and velocity vectors of the sun relative to the earth at time t_{M0}.

ω_{ES} = the angular velocity of the vector $\underline{r}_{ES0}$ at time t_{M0}

Then,

$$\underline{r}_{ES} = \underline{r}_{ES0} \cos(\omega_{ES} t_M) + \left[\underline{r}_{ES0} \times \text{UNIT}(\underline{v}_{ES0} \times \underline{r}_{ES0})\right] \sin(\omega_{ES} t_M) \qquad (5.4.4)$$

$$\underline{v}_{ES} = \underline{v}_{ES0} \qquad (5.4.5)$$

5.5.5 KEPLER SUBROUTINE

The Kepler Subroutine solves for the two body position and velocity vectors at the terminal position given the initial position and velocity vectors and a transfer time to the terminal position.

This section contains information to aid the reader in understanding the less obvious aspects of the Kepler Subroutine block diagram depicted in Figs. 5.5-1 thru 5.5-3. The subroutines referred to in these figures are presented in Section 5.5.10. Nomenclature is found in Section 5.5.1.2.

Prior to entering the Kepler Subroutine an initial estimate of x can be generated via Eq. (2.2.4) of Section 5.2.2.2 with $\frac{\Delta t}{2} = t_D - t_{21}$ and $\tau = t_D$. However, x' and t_{21} are non-zero only if the subroutine is being used repetitively.

Although, theoretically, there is no upper bound on x, the practical bound is set to x_{MAX0} or x_{MAX1} to eliminate non-feasible trajectories and increase the accuracy to which x can be computed. In addition, αx^2 has a practical range of $-50 < \alpha x^2 < (2\pi)^2$ which determines an independent upper bound on x. The x_{MAX} used, then, corresponds to the smaller of the two values.

The transfer time convergence criterion is approximately the same as the granularity of the time input. Since, for some of the problems to be solved, the sensitivity of time to x is so large that the granularity in x, ϵ_x, produces a change in time which exceeds the granularity in time, it is necessary to introduce ϵ_x as a redundant convergence criterion.

The Kepler Subroutine, provided the parameter range constraints are satisfied, will always produce a solution.

A negative value of t_D will cause the subroutine to update the state vector backward in time (i. e. backdate the state vector). The subroutine may be called to update or backdate for any amount of time; there are no restrictions on whether the time t_D is less than a period.

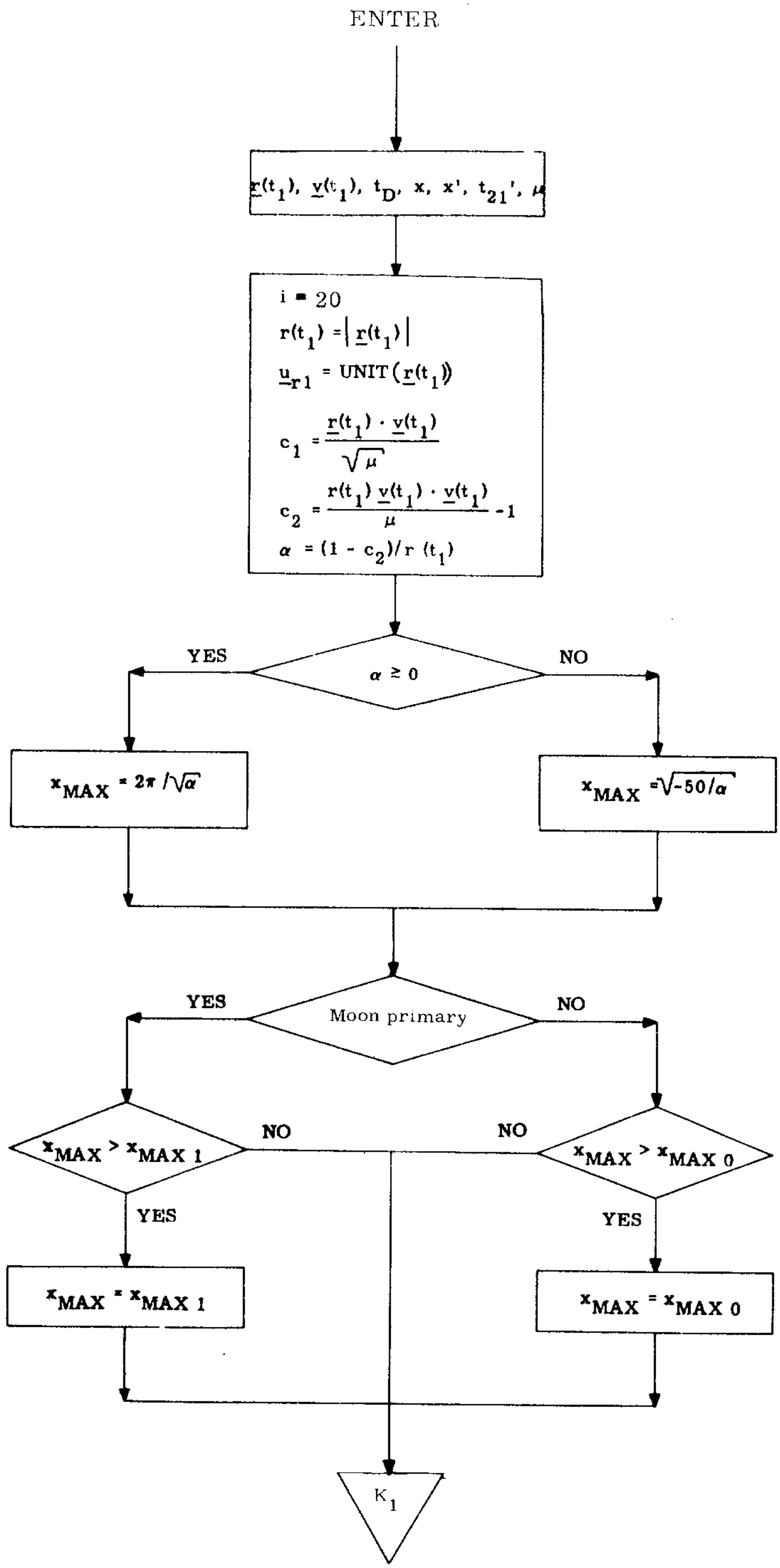

Figure 5. 5-1 Kepler Subroutine

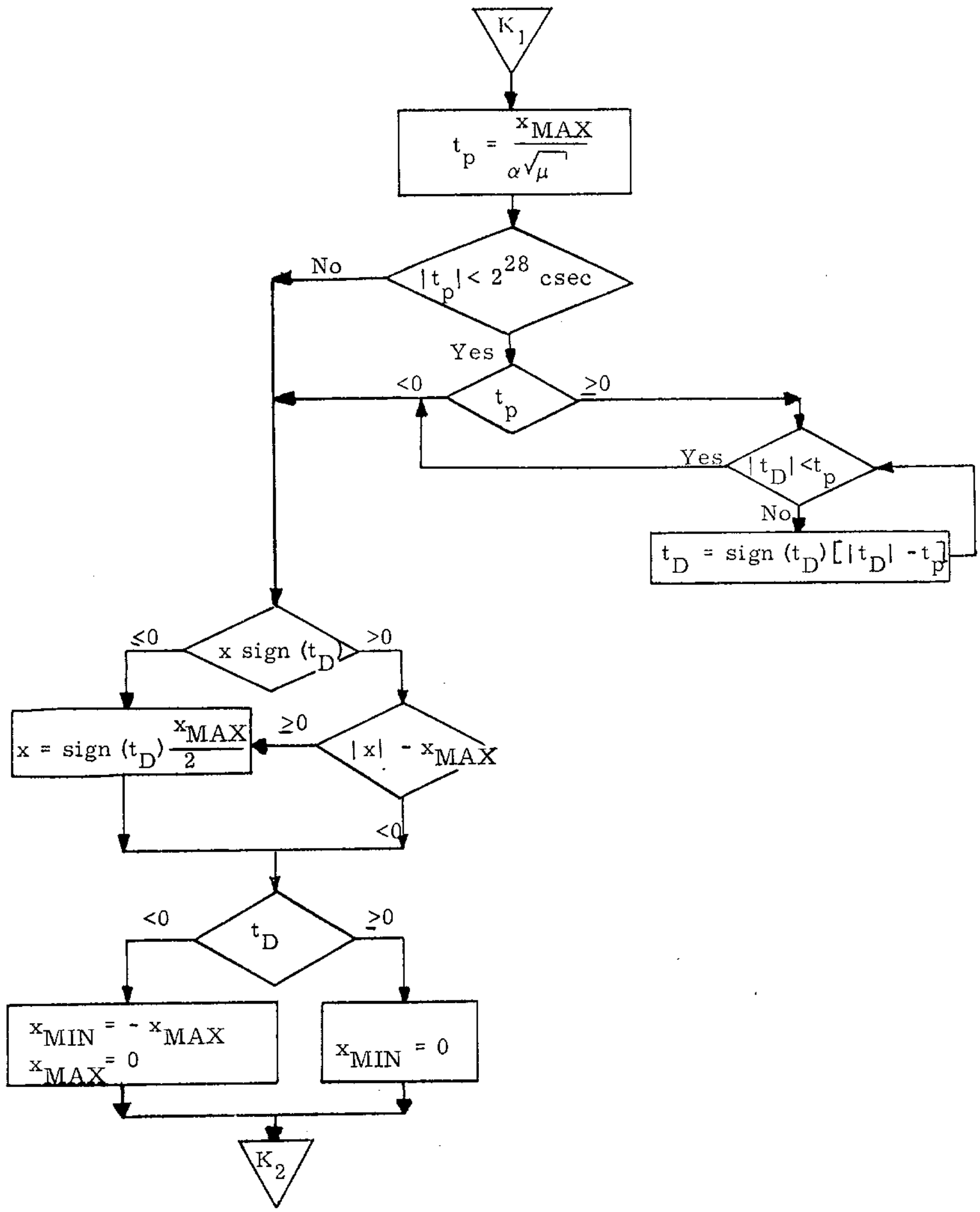

Fig. 5.5-2 Kepler Subroutine

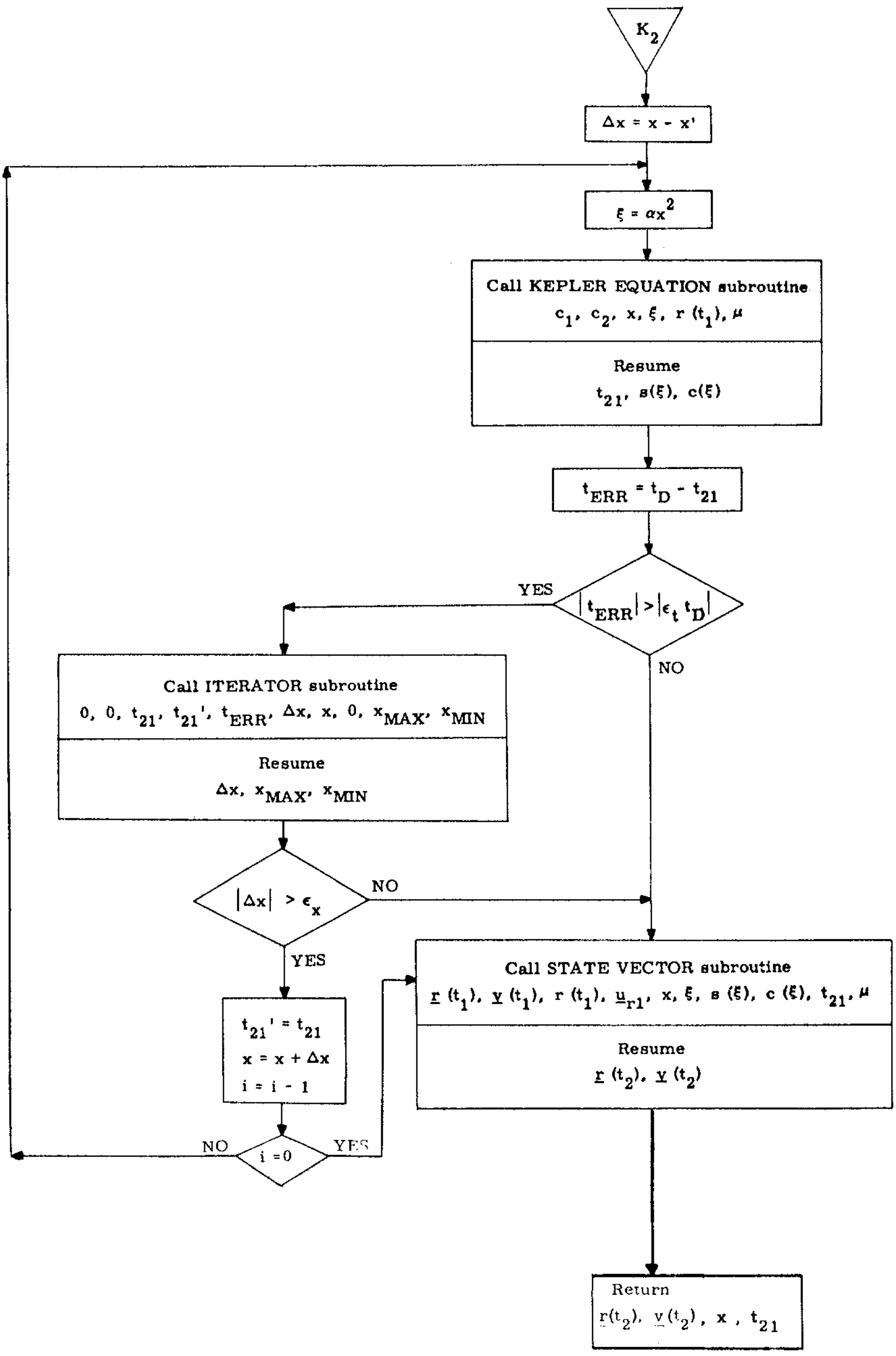

Figure 5.5-3 Kepler Subroutine

5.5.6 LAMBERT SUBROUTINE

The Lambert Subroutine solves for the two body initial velocity vector given the initial and terminal position vectors and a transfer time between the two.

This section contains information to aid the reader in understanding the less obvious aspects of the Lambert Subroutine block diagrams depicted in Figs. 5.6-1 and 5.6-2. The subroutines referred to in these figures are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

If the Lambert Subroutine is used repetitively and rapid computation is required, the previous value of the independent variable, $\cot \gamma$, can be used as a starting point for the new iteration. Flag f_1 provides this option.

The Lambert Subroutine computes the normal to the trajectory, $\underline{u}_N$, using the two input position vectors. If these vectors are nearly colinear, it is desirable to specify the normal as an input rather than rely on the ill-defined normal based on the two input position vectors. Flag f_2 provides this option. The presence of the inputs in parentheses, therefore, is contingent upon the setting of these flags.

The theoretical bounds on the independent variable, $\cot \gamma$, correspond to the infinite energy hyperbolic path and the parabolic path which closes through infinity. These bounds are dynamically reset by the iterator to provide a more efficient iteration scheme. In addition, if during the course of the iteration, $\cot \gamma$ causes a parameter of the problem to exceed its maximum as determined by its allowable range, the appropriate bound is reset and the iterator continues trying to find an acceptable solution. (This logic does not appear in Figs. 5.6-1 and 2 as it is pertinent only to fixed-point programming). If no acceptable solution is reached, the transfer time input was too small to produce a practical trajectory between the input position vectors. When this happens, $\Delta \cot \gamma$ approaches its granularity limit ϵ_c before time converges to within a fraction ϵ_t of the desired time. However, this same granularity condition exists when the sensitivity problem occurs as described in the Kepler Subroutine, Section 5.5.5. In this case an acceptable solution does exist. This dual situation is resolved via a third convergence criterion. If the error in transfer time is greater than the usual fraction ϵ_t of the desired transfer time, but still less than a slightly larger fraction k_1 of the desired transfer time and $\Delta \cot \gamma$ is less than ϵ_c, then the solution is deemed acceptable and the required velocity is computed.

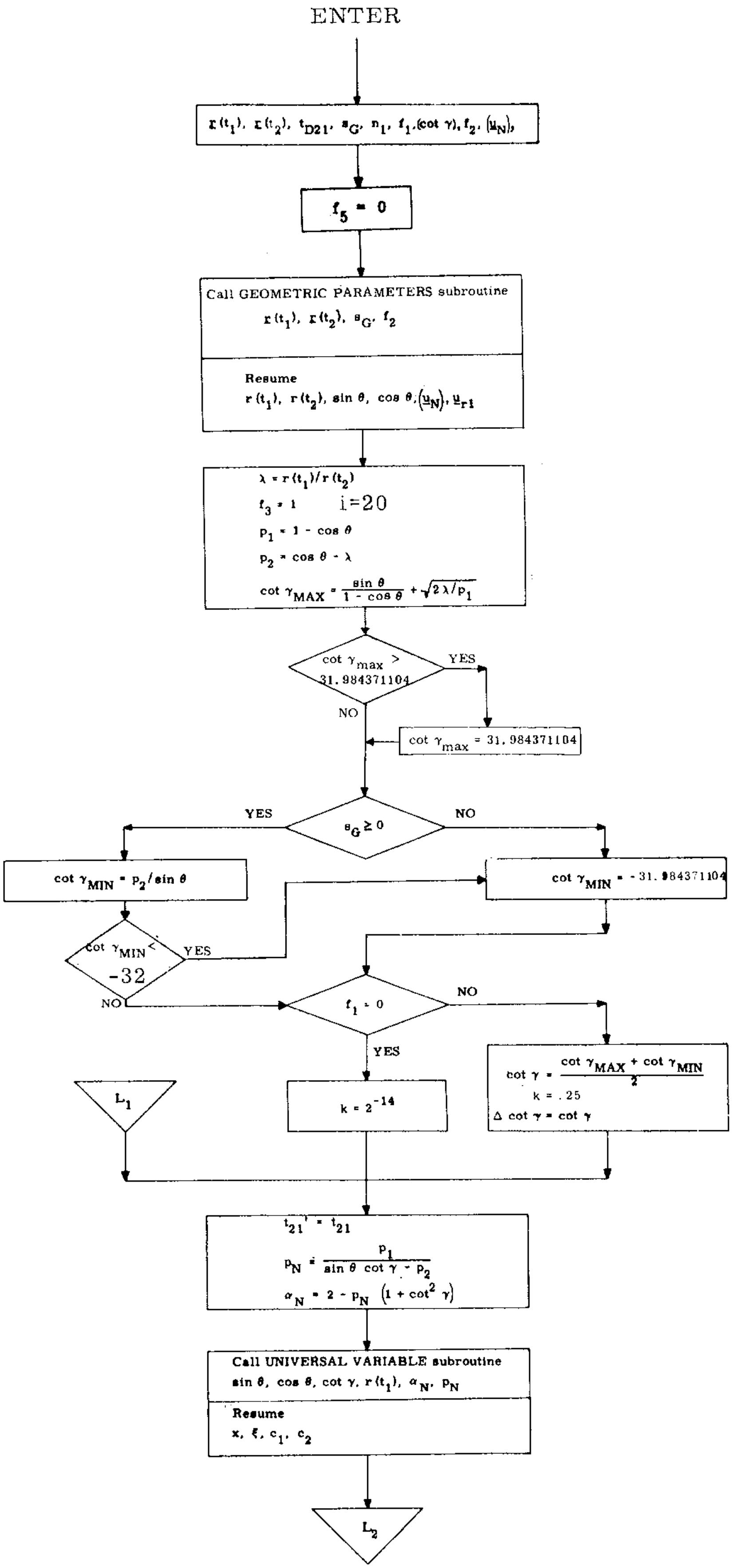

Figure 5.6-1 Lambert Subroutine

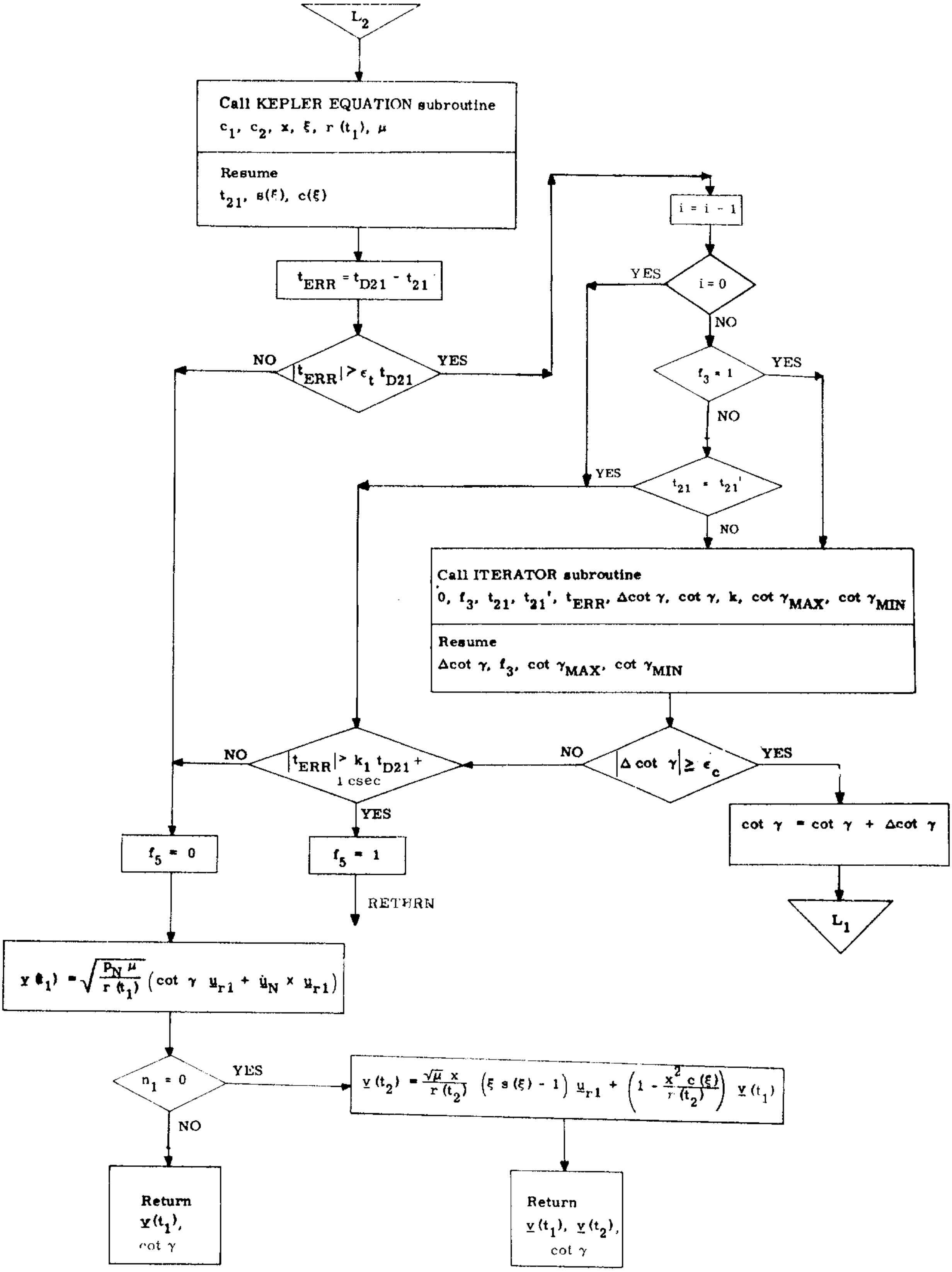

Figure 5.6-2 Lambert Subroutine

5.5.7 TIME-THETA SUBROUTINE

The Time-Theta Subroutine solves for the two body transfer time given the initial position and velocity vectors and the true anomaly difference (transfer angle) to the terminal position.

This section contains information to aid the reader in understanding the less obvious aspects of the Time-Theta Subroutine block diagram depicted in Fig. 5.7-1. The subroutines referred to in this figure are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

The flag f_6 must be zero if the user desires computation of the terminal state vector in addition to the transfer time.

If the conic trajectory is a parabola or hyperbola and the desired transfer angle, θ, lies beyond the asymptote of the conic, f_7 will be set indicating that no solution is possible.*

In addition to the parameter range constraints imposed on Kepler's equation, the additional restriction on Time-Theta that the trajectory must not be near rectilinear is indicated by the range of cot γ.*

The Time-Theta problem is not well defined for near rectilinear trajectories, i e. the transfer angle θ is no longer a meaningful problem parameter. This will not cause difficulties provided the input variables are within the specified ranges.

---

*If the Time-Theta Routine is called with inputs for which no solution is possible (for either or both of these two reasons), then the routine will abort with an alarm code of 20607.

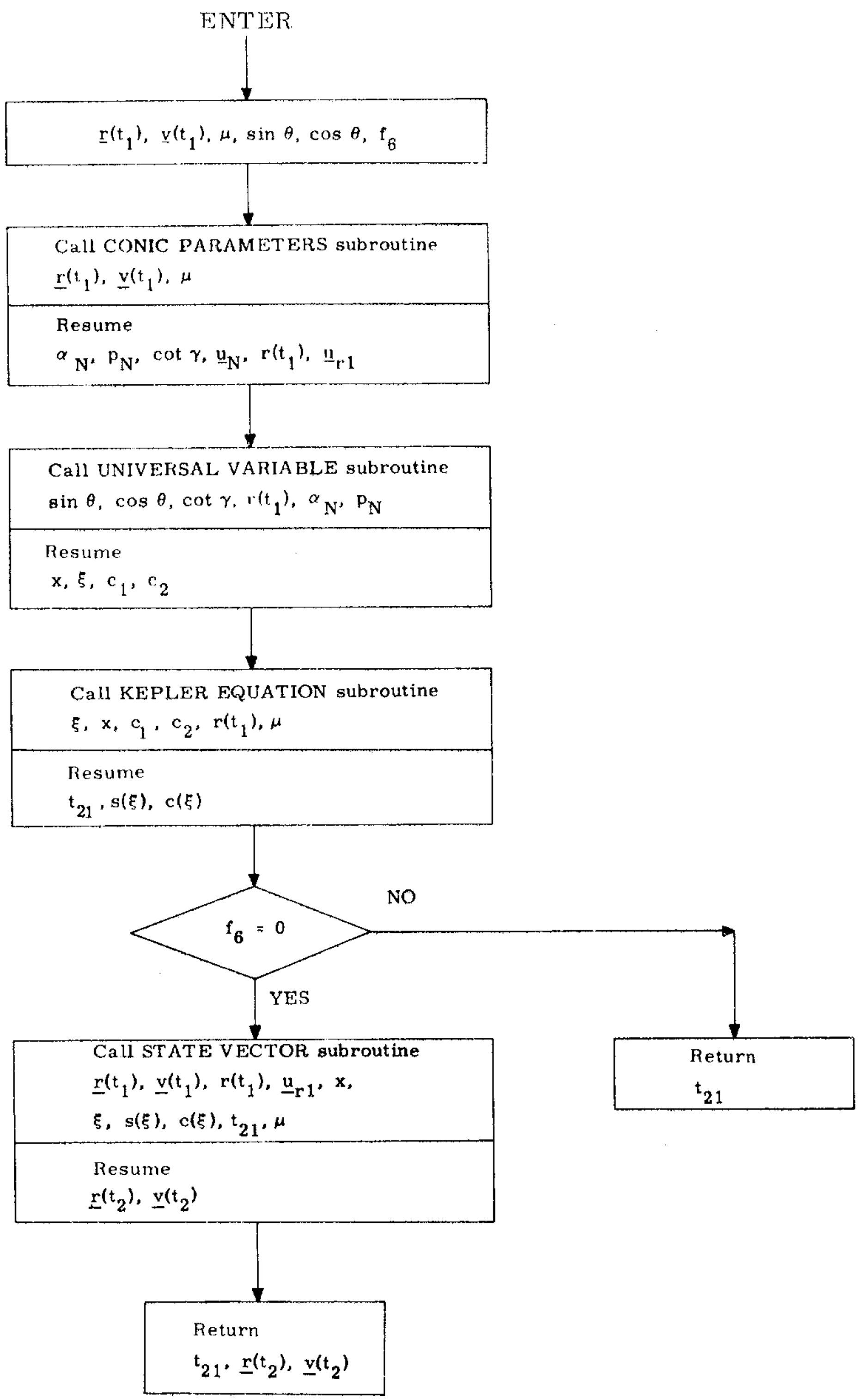

Figure 5.7-1 Time-Theta Subroutine

5.5.8 TIME-RADIUS SUBROUTINE

The Time-Radius Subroutine solves for the two body transfer time to a specified radius given the initial position and velocity vectors and the radius magnitude.

This section contains information to aid the reader in understanding the less obvious aspects of the Time-Radius Subroutine block diagrams depicted in Figs. 5.8-1 and 5.8-2. The subroutines referred to in this figure are presented in Section 5.5.10 and the nomenclature is found in Section 5.5.1.2.

Paragraphs 3, 4 and 5 of Section 5.5.7 apply to the Time-Radius Subroutine as well. *

Since an inherent singularity is present for the circular orbit case, near-circular orbits result in a loss of accuracy in computing both the transfer time, t_{21}, and the final state vector. This is caused by the increasing sensitivity of t_{21} to $r(t_2)$ as the circular orbit is approached. In the extreme case when the eccentricity is less than approximately 2^{-18}, the problem is undefined and the subroutine will exit without a solution, setting flag f_9 to indicate this.* (The precise conditions are given in Figure 5.8-1.)

If $r(t_2)$ is less than the radius of pericenter or greater than the radius of apocenter, then $r(t_2)$ will be ignored and the pericenter or apocenter solution, respectively, will be computed. A flag, f_8, will be set to indicate this.

*If the Time-Radius Routine is called with inputs for which no solution is possible (for any one or more of the reasons given in paragraphs 4 or 5 of Section 5.5.7 or paragraph 4 above), the routine will abort with an alarm code of 20607.

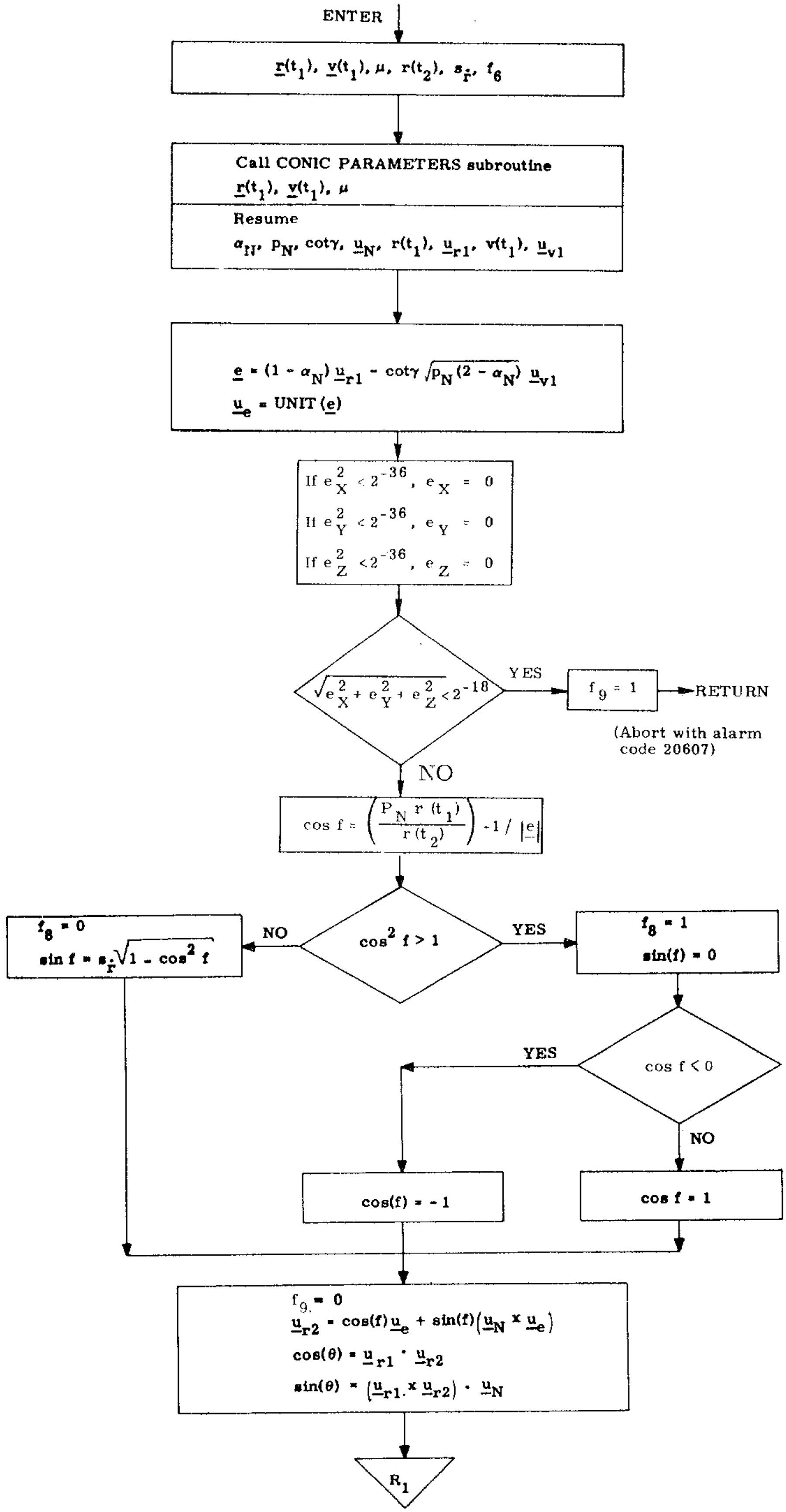

Figure 5. 8-1. Time Radius Subroutine

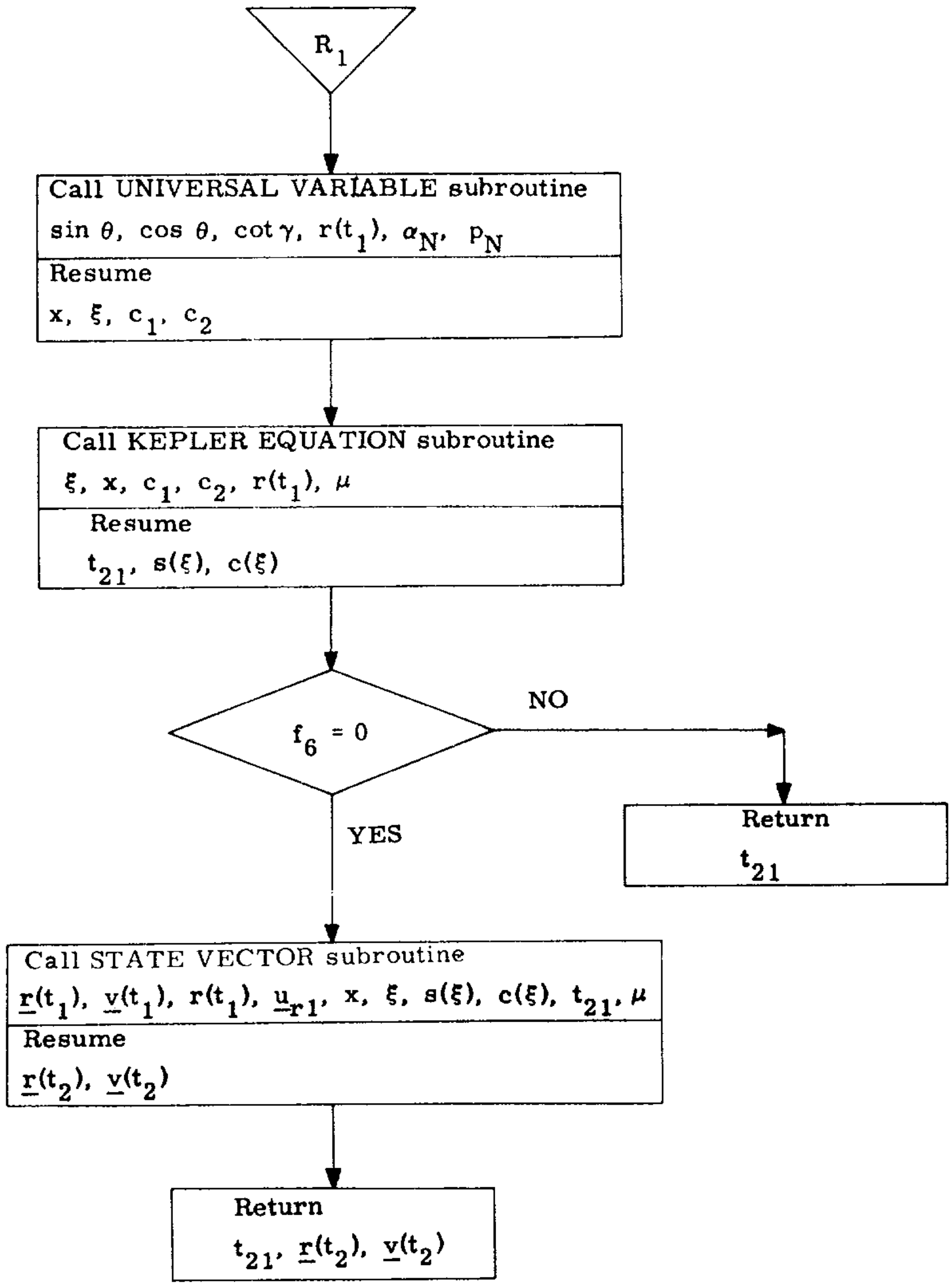

Figure 5. 8-2 Time-Radius Subroutine

5.5.9 APSIDES SUBROUTINE

The Apsides Subroutine solves for the two body radii of apocenter and pericenter and the eccentricity of the trajectory given the position and velocity vectors for a point on the trajectory.

This subroutine is depicted in Fig. 5.9-1. The subroutines referred to in this figure are presented in Section 5.5.10. Nomenclature is found in Section 5.5.1.2.

It is characteristic of this computation that the apsides become undefined as the conic approaches a circle. This is manifested by decreasing accuracy. When the conic is nearly parabolic, or hyperbolic, the radius of apocenter is not defined. In this event the radius of apocenter will be set to the maximum positive value allowed by the computer.

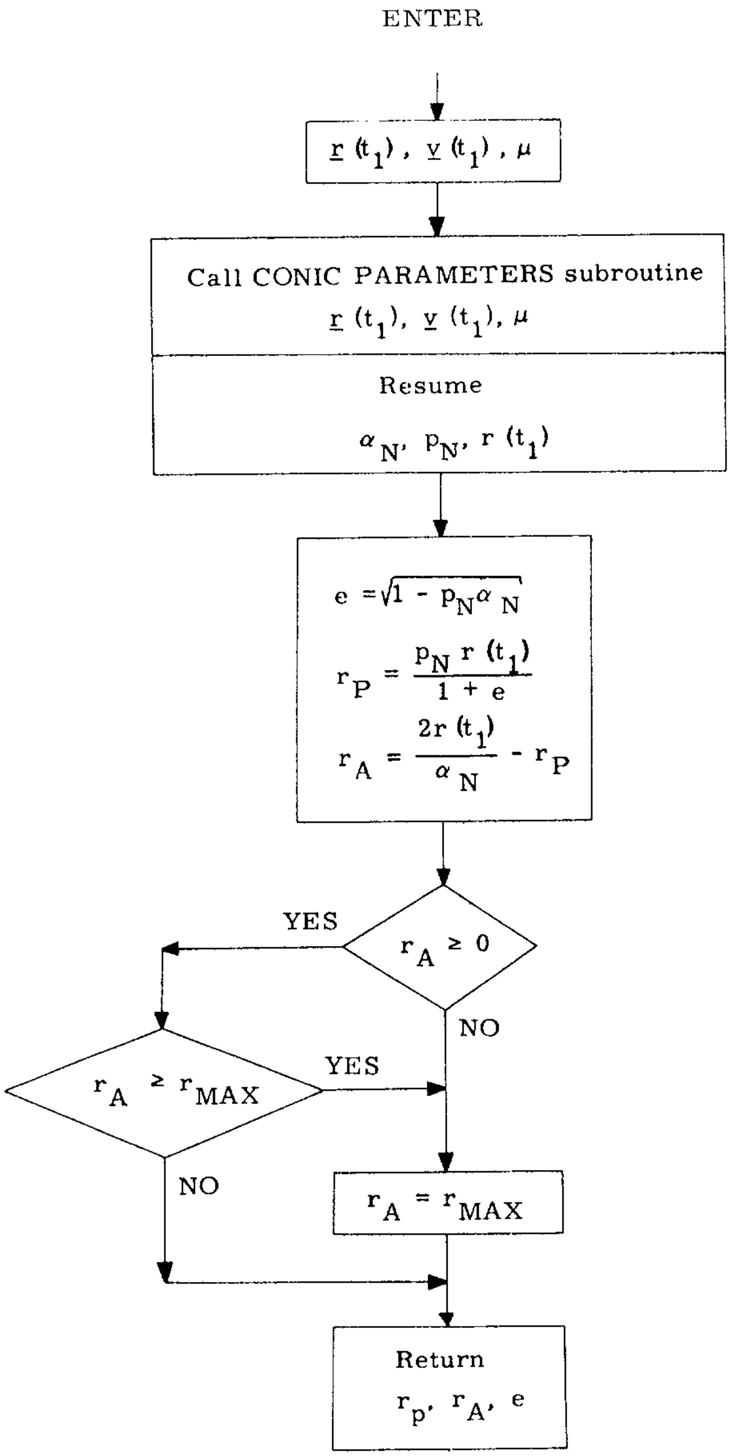

Figure 5.9-1 Apsides Subroutine

5.5.10 MISCELLANEOUS SUBROUTINES

There are, as part of the Conic Trajectory Subroutines, three subroutines which are useful in their own right. These are the Conic Parameters, the Geometric Parameters and the Iterator Subroutines which are depicted in Figs. 5.10-1, 5.10-2 and 5.10-3, respectively.

The Conic Parameters and Geometric Parameters Subroutines are self explanatory.

The Iterator Subroutine serves several purposes. It is used when flag f_4 is set to zero to solve for the value of the independent variable which drives the error in the dependent variable to zero, provided the function is monotonically increasing. To improve convergence for functions whose derivative changes rapidly, the limits are reset as shown in the block diagram.

With f_4 set to 1, the Iterator seeks a minimum of the function, provided the first derivative is single-valued between the limits. The inputs are redefined so that "y" is the derivative of the independent variable with respect to the dependent variable, and "x" is the value at which the derivative was computed or approximated. Since the desired value of y is zero, $y_{ERR} = -y$.

Since the Iterator uses the "Regula Falsi" technique, it requires two sets of variables to begin iteration. If only one set is available, flag f_3 must be set to 1, causing the iterator to generate the independent variable increment from a percentage of the full range.

In addition to the above subroutines there are three other subroutines of primary interest to the five basic conic subroutines described in Sections 5.5.5 to 5.5.9. These are the Universal Variable Subroutine, the Kepler Equation Subroutine and the State Vector Subroutine shown in Figs. 5.10-4, 5.10-5 and 5.10-6, respectively.

The Universal Variable Subroutine is utilized by the Lambert, the Time-Theta and the Time-Radius Subroutines to compute the universal parameter x required for the time equation. There are two different formulations required according to the size of the parameter w.

If the input to the subroutine requires the physically impossible solution that the trajectory "close through infinity", the problem will be aborted, setting flag f_7.

The Kepler Equation Subroutine computes the transfer time given the variable x and the conic parameters.

The State Vector subroutine computes the position and velocity vectors at a point along the trajectory given an initial state vector, the variable x and the transfer time.

The final miscellaneous subroutine, the SETMU Subroutine, is depicted in Fig. 5.10-7. It sets μ to the appropriate primary body gravitational constant consistent with the estimated CSM or LM state vector as defined in Section 5.2.2.6.

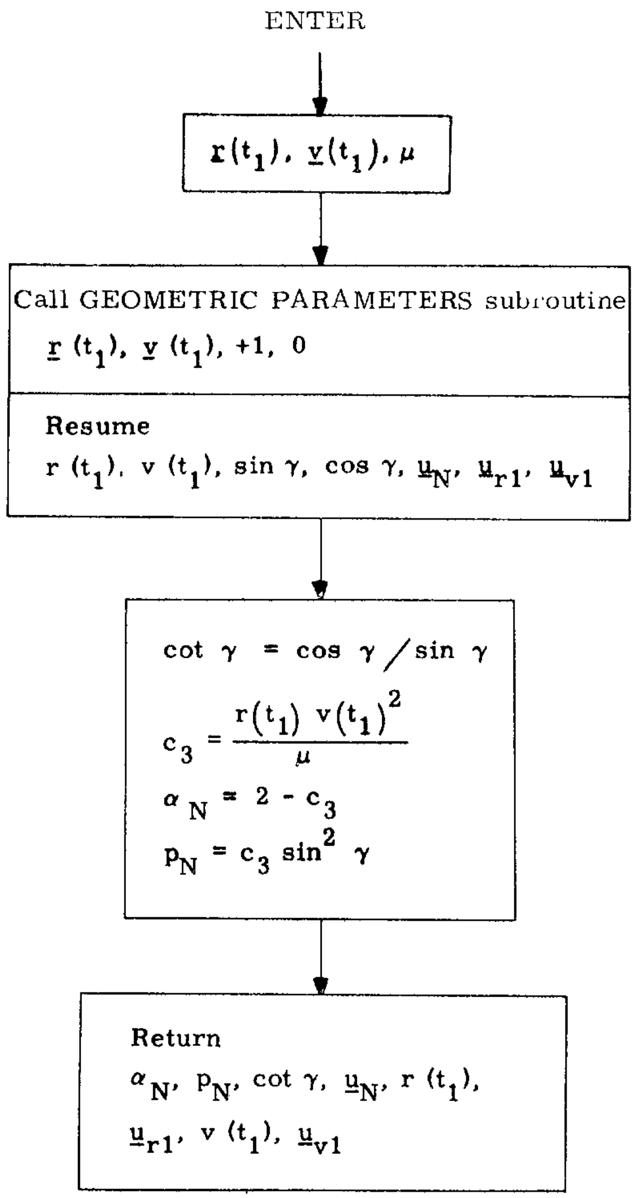

Figure 5.10-1 Conic Parameters Subroutine

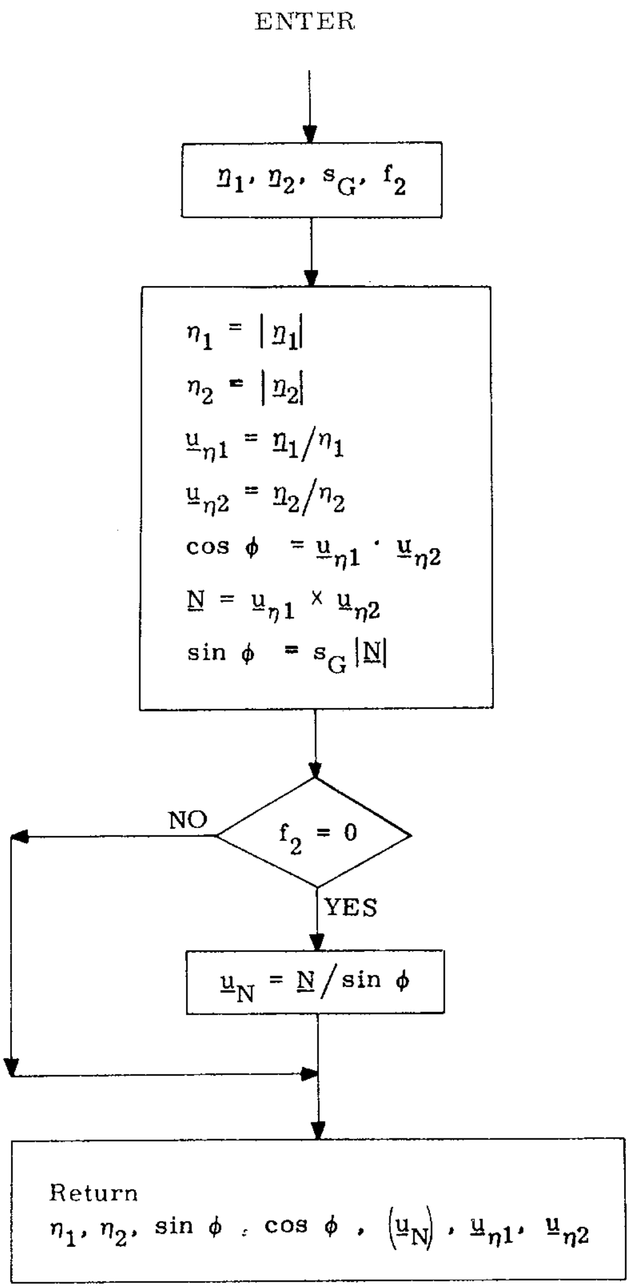

Figure 5.10-2 Geometric Parameters Subroutine

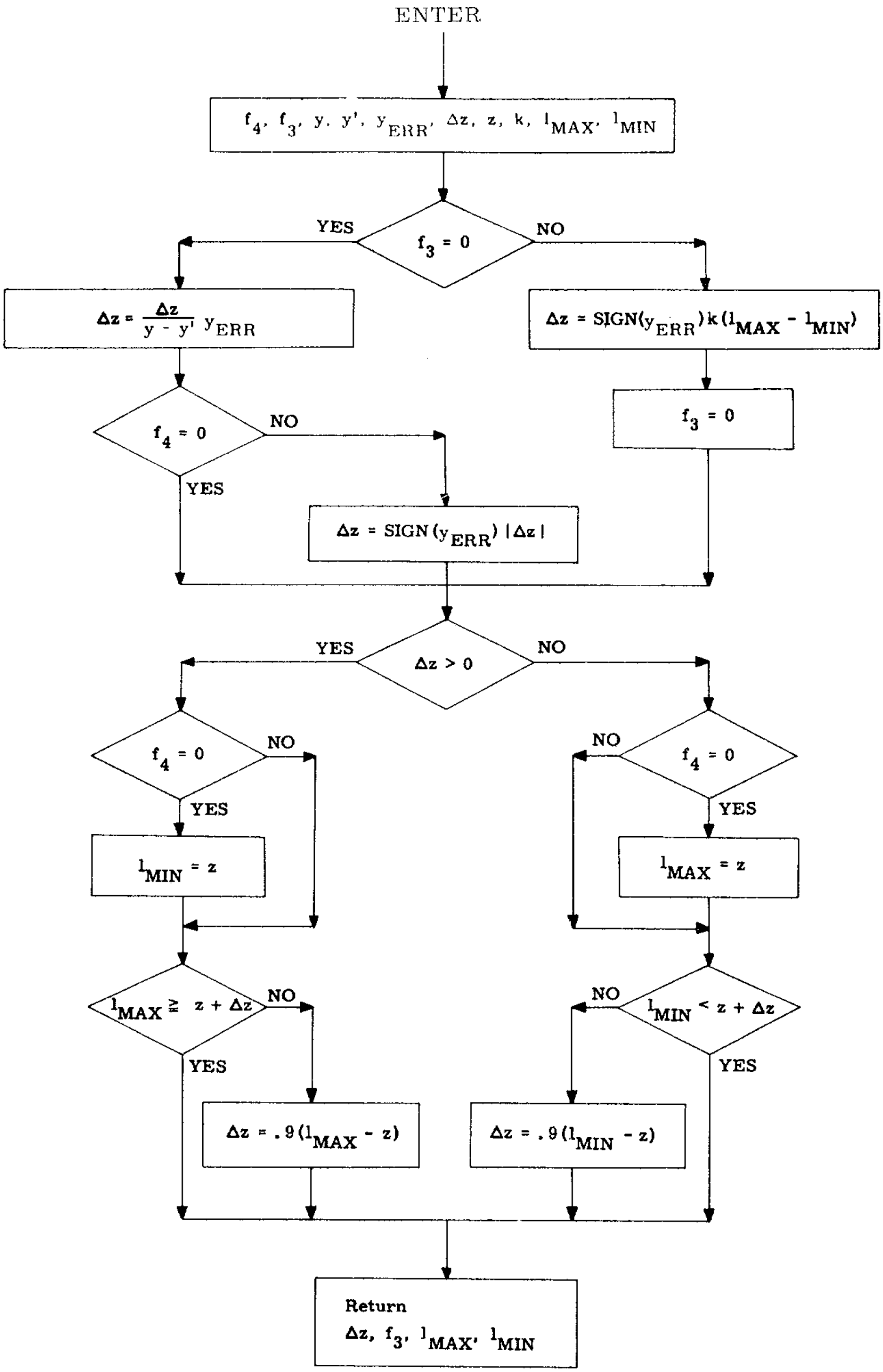

Figure 5.10-3 Iterator Subroutine

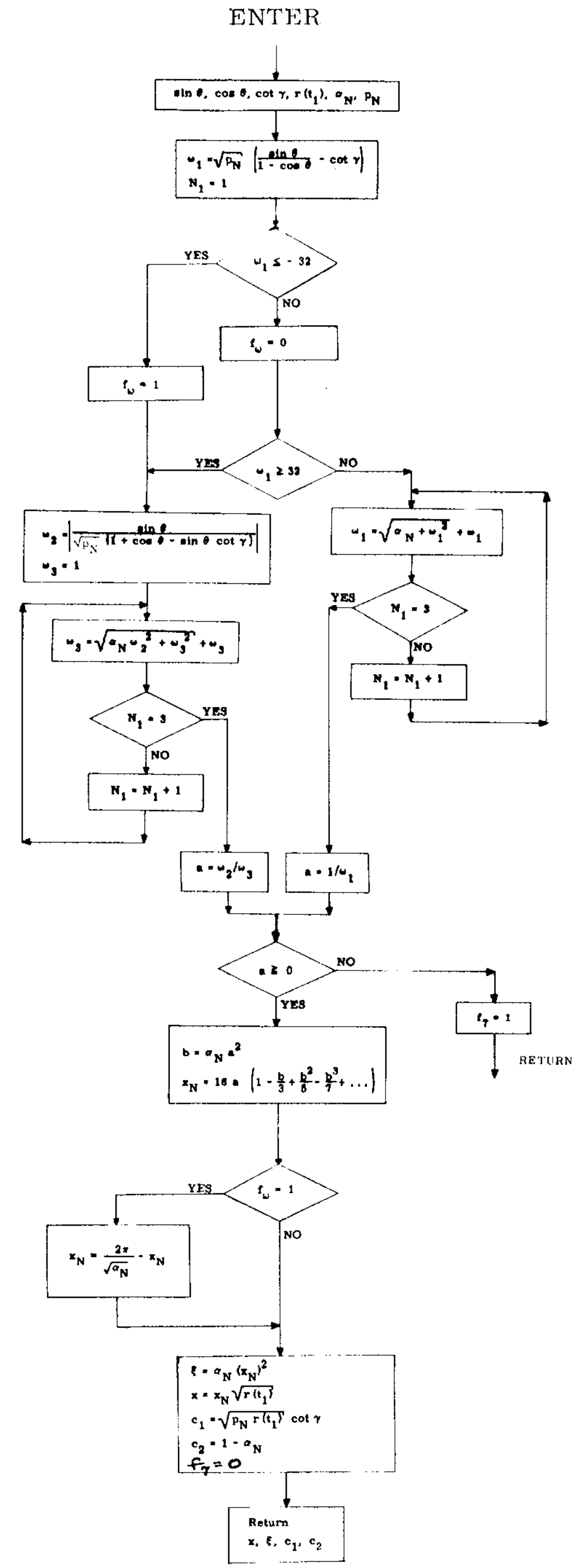

Figure 5.10-4 Universal Variable Subroutine

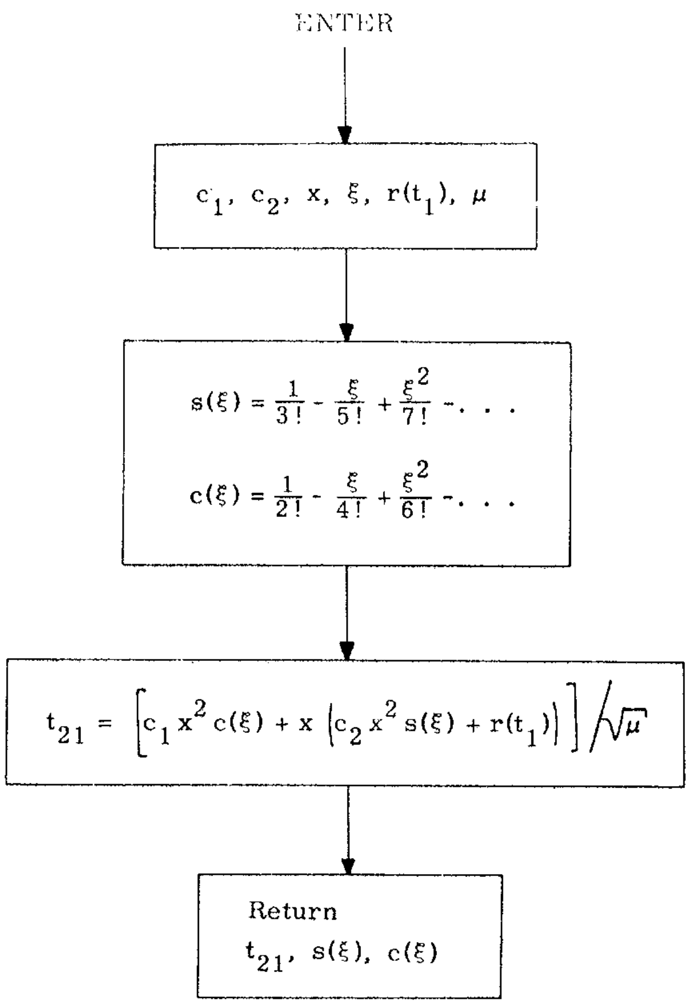

Figure 5.10-5 Kepler Equation Subroutine

ENTER

$\underline{r}(t_1),\ \underline{v}(t_1),\ r(t_1),\ \underline{u}_{r1},\ x,\ \xi,\ s(\xi),\ c(\xi),\ t_{21},\ \mu$

$$\underline{r}(t_2) = \left(1 - \frac{x^2}{r(t_1)}\, c(\xi)\right) \underline{r}(t_1) + \left(t_{21} - \frac{x^3}{\sqrt{\mu}}\, s(\xi)\right) \underline{v}(t_1)$$

$$r(t_2) = \left|\underline{r}(t_2)\right|$$

$$\underline{v}(t_2) = \frac{\sqrt{\mu}\, x}{r(t_2)} \left(\xi\, s(\xi) - 1\right) \underline{u}_{r1} + \left(1 - \frac{x^2 c(\xi)}{r(t_2)}\right) \underline{v}(t_1)$$

Return

$\underline{r}(t_2),\ \underline{v}(t_2)$

Figure 5.10-6 State Vector Subroutine

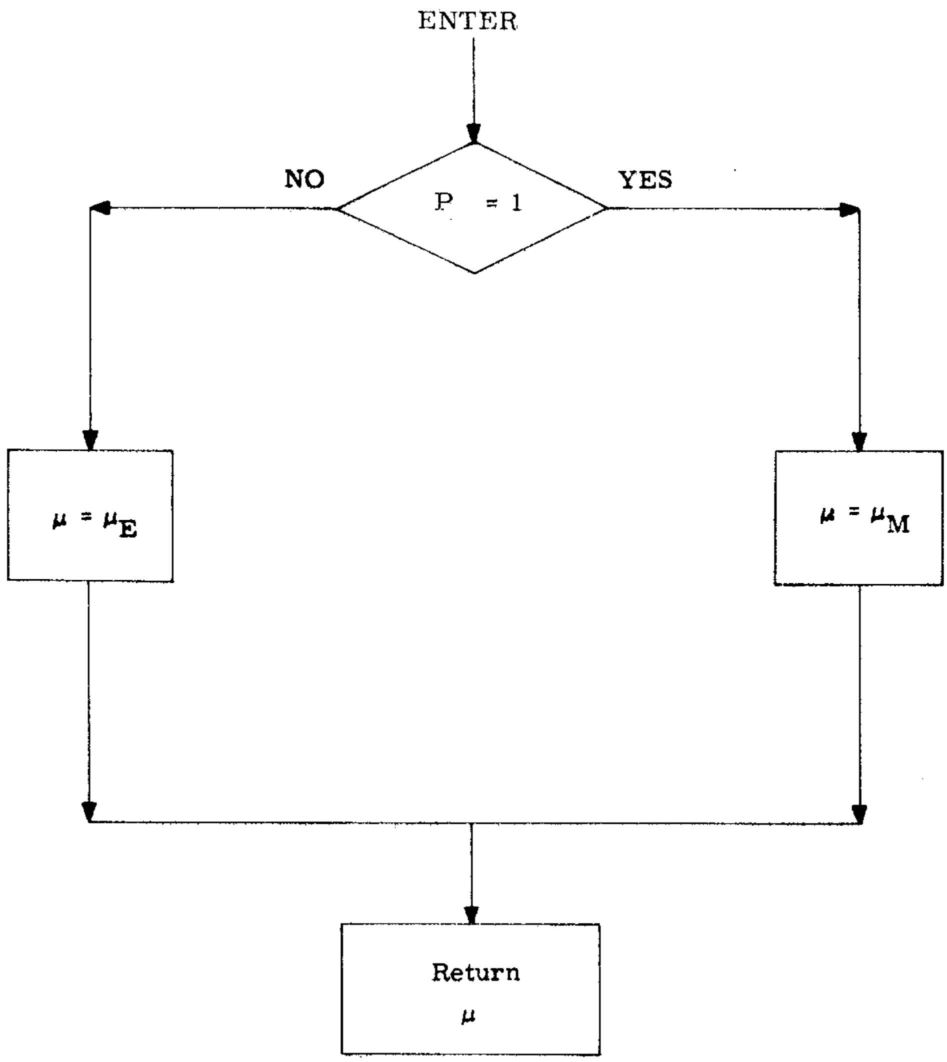

Figure 5.10-7 SETMU Subroutine

5.5.11 INITIAL VELOCITY SUBROUTINE

The Initial Velocity Subroutine computes the required initial velocity vector for a trajectory of specified transfer time between specified initial and target position vectors. The trajectory may be either conic or precision depending on an input parameter (namely, number of offsets). In addition, in the precision trajectory case, the subroutine also computes an "offset target vector", to be used during pure-conic cross-product steering. The offset target vector is the terminal position vector of a conic trajectory which has the same initial state as a precision trajectory whose terminal position vector is the specified target vector.

In order to avoid the inherent singularities in the 180° transfer case when the (true or offset) target vector may be slightly out of the orbital plane, the Initial Velocity Subroutine rotates this vector into a plane defined by the input initial position vector and another input vector (usually the initial velocity vector), whenever the input target vector lies inside a cone whose vertex is the origin of coordinates, whose axis is the 180° transfer direction, and whose cone angle is specified by the user.

The Initial Velocity Subroutine is depicted in Fig. 5.11-1. The Lambert Subroutine, Section 5.5.6, is utilized for the conic computations; and the Coasting Integration Subroutine, Section 5.2.2, is utilized for the precision trajectory computations.

Nomenclature for the Initial Velocity Subroutine

$\underline{r}(t_1)$	Initial position vector.
$\underline{v}(t_1)$	Vector (usually the actual initial velocity vector) used to determine whether the transfer from the initial position vector to the target vector is through a central angle of less or greater than 180°, and also used in certain cases to specify the transfer plane (see text).
$\underline{r}_T(t_2)$	Target Vector (True target vector if $N_1 > 0$, or Offset target vector if $N_1 = 0$).
t_D	Desired transfer time from initial position vector to target vector.
N_1	Number of offsets to be used in calculating the offset target vector from the true target vector. ($N_1 = 0$ implies conic calculations only with offset target vector input).
ϵ	Cone Angle of a cone whose vertex is the coordinate origin and whose axis is the 180° transfer direction (i. e., the negative initial position direction). The cone angle ϵ is measured from the axis to the side of the cone.
f_1	Switch set to 0 or 1 according to whether a guess of cot γ is input or not.
[cot γ]	Guess of cot γ.
$\underline{v}_T(t_1)$	Required initial velocity vector of a precision [a conic] trajectory which passes through the true [or offset] target vector, or the rotated true [or offset] target vector if the original target vector was in the cone, at the end of the desired transfer time, if $N_1 > 0$ [or $N_1 = 0$].
$\underline{r}(t_2)$	Computed offset target vector.
$\underline{v}_T(t_2)$	Final precision [conic] velocity vector resulting from a precision [conic] update of the initial position vector and the required initial velocity vector $\underline{v}_T(t_1)$, if $N_1 > 0$, [or $N_1 = 0$, respectively].
$\underline{r}_T'(t_2)$	Final precision position vector.
cot γ	Value to which the Lambert Subroutine converged (for later use as guess to minimize computation time).
f_2	Switch set to 0 or 1 according to whether the input (true or offset) target vector was not or was in the cone, and consequently was not or was rotated into the plane.

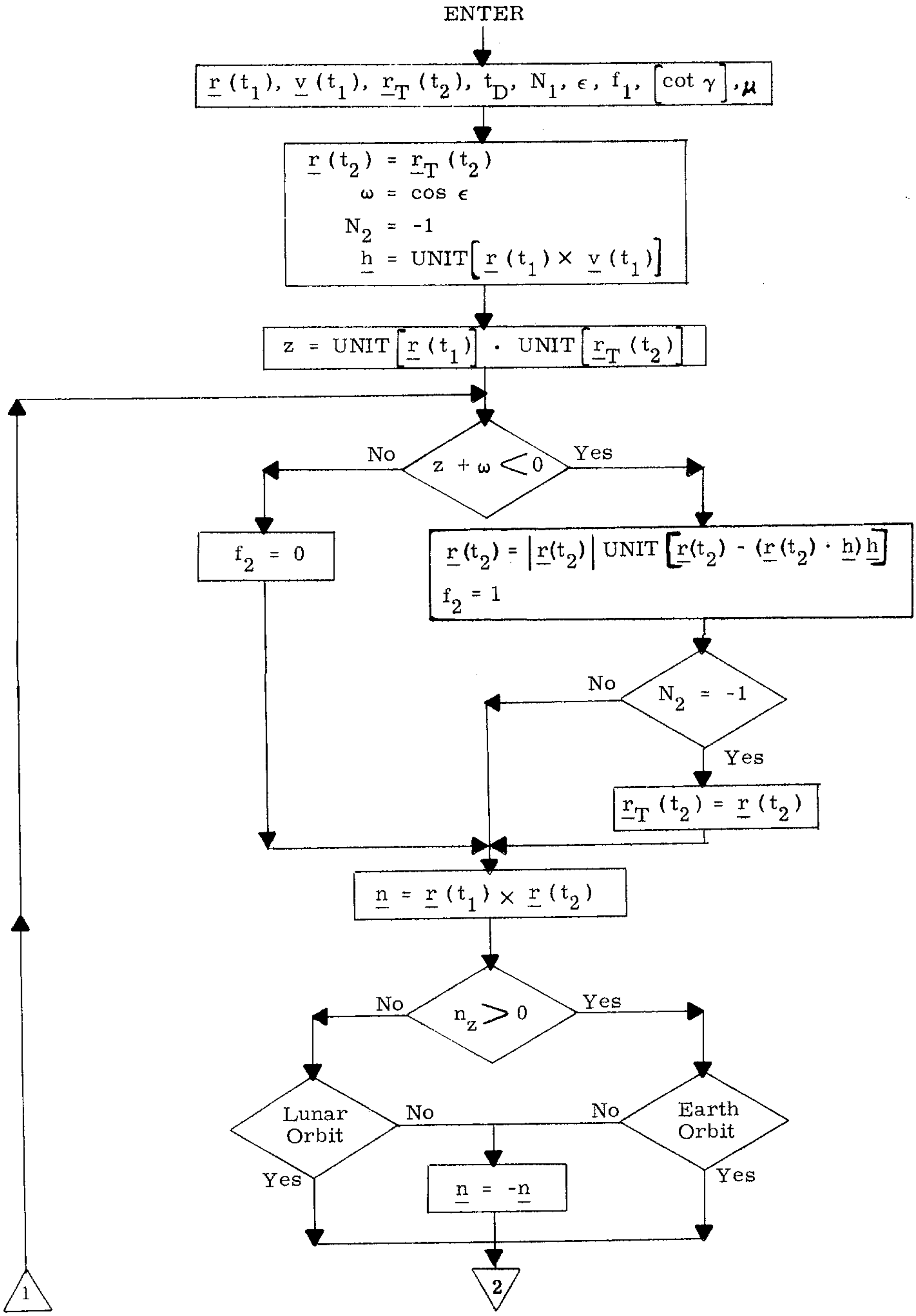

Figure 5.11-1 Initial Velocity Subroutine
(page 1 of 2)

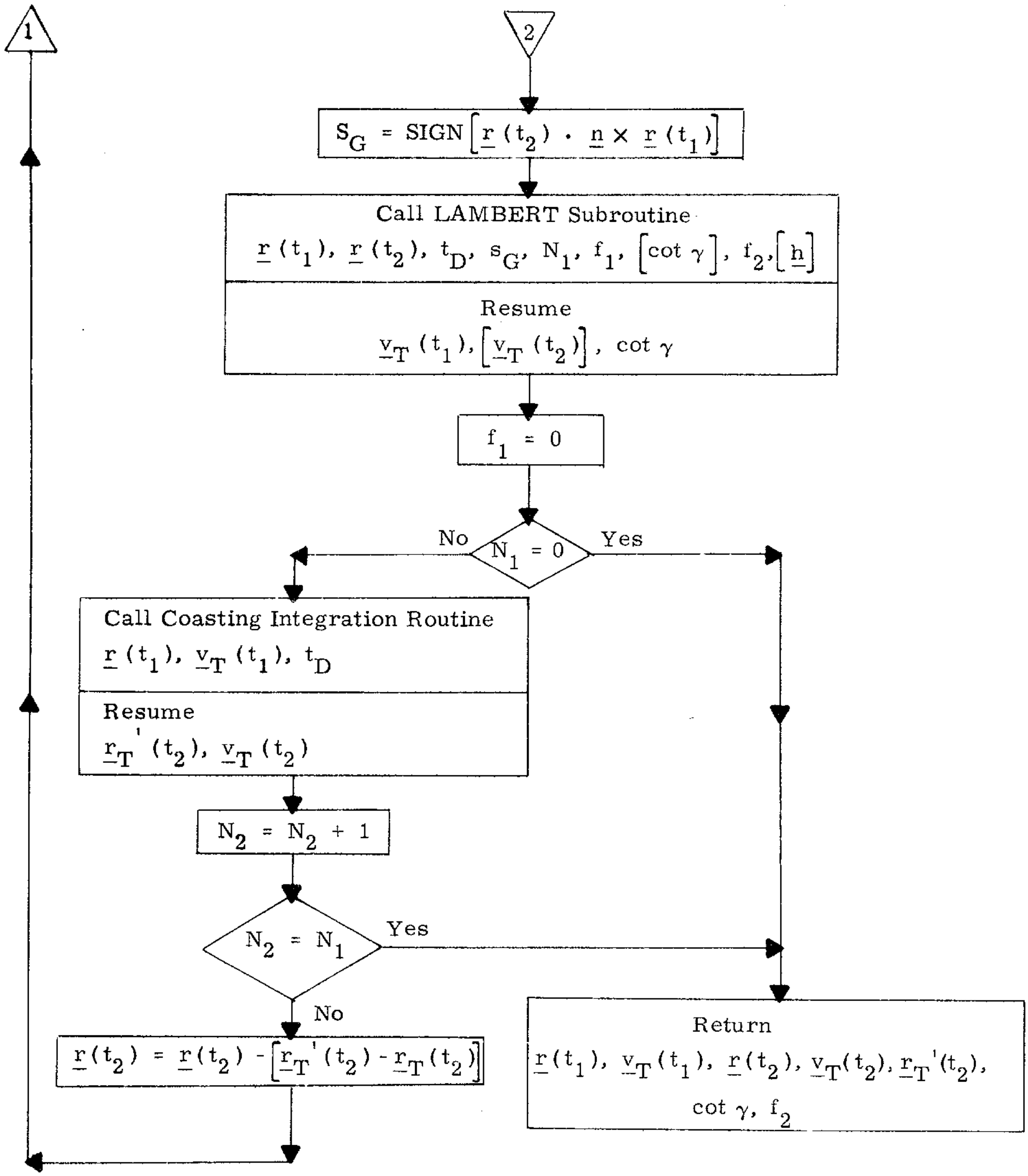

Figure 5.11-1 Initial Velocity Subroutine
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5.5.13 LOCSAM SUBROUTINE

The LOCSAM Subroutine computes the lines-of-sight of the Sun, Earth, and Moon with respect to the spacecraft in the Basic Reference Coordinate System. This data is used by the IMU alignment programs whenever the astronaut elects to sight on the Sun, Earth, or Moon instead of a star for purposes of IMU alignment. The data is also used by the Star Selection Routine (Section 5.6.4) when testing for star occultation. In addition, this subroutine computes the sizes of the occultation cones used in the Star Selection Routine and the correction for aberration of light which is applied in the IMU alignment programs to the line-of-sight unit vector of a star stored in basic reference coordinates.

The unit vectors $\underline{u}_S$, $\underline{u}_E$, and $\underline{u}_M$ specifying the lines-of-sight to the Sun, Earth, and Moon respectively, in the Basic Reference Coordinate System are computed as follows:

$$\underline{u}_S = \begin{cases} \text{UNIT } (\underline{r}_{ES}) & \text{if } P = E \\ \text{UNIT } (\underline{r}_{ES} - \underline{r}_{EM}) & \text{if } P = M \end{cases} \tag{5.13.1}$$

$$\underline{u}_E = \begin{cases} -\text{UNIT } (\underline{r}_C) & \text{if } P = E \\ -\text{UNIT } (\underline{r}_{EM} + \underline{r}_C) & \text{if } P = M \end{cases} \tag{5.13.2}$$

$$\underline{u}_M = \begin{cases} \text{UNIT } (\underline{r}_{EM} - \underline{r}_C) & \text{if } P = E \\ -\text{UNIT } (\underline{r}_C) & \text{if } P = M \end{cases} \tag{5.13.3}$$

where P, E, M, and S respectively denote the primary body, Earth, Moon, and Sun, $\underline{r}_C$ is the position vector of the CSM with respect to the primary body, and $\underline{r}_{EM}$ and $\underline{r}_{ES}$ are the position vectors of the Moon and Sun with respect to the Earth obtained from the Lunar and Solar Ephemerides Subroutine of Section 5.5.4. The line-of-sight vectors are determined for a time specified by the calling program or routine.

The occultation cones used in the Star Selection Routine for the Sun, Earth, and Moon are computed as follows:

$$c_S = \cos 15^\circ \qquad (5.13.4)$$

$$c_E = \begin{cases} \cos\left[5^\circ + \sin^{-1}\left(\frac{R_E}{r_C}\right)\right] & \text{if } P = E \\ \cos 5^\circ & \text{if } P = M \end{cases} \qquad (5.13.5)$$

$$c_M = \begin{cases} \cos 5^\circ & \text{if } P = E \\ \cos\left[5^\circ + \sin^{-1}\left(\frac{R_M}{r_C}\right)\right] & \text{if } P = M \end{cases} \qquad (5.13.6)$$

where c is the cosine of one half the total angular dimension of a cone and represents a more convenient way of treating the dimension of a cone in the Star Selection Routine, r_C is the magnitude of the CSM position vector, R_E is the equatorial radius (6378.166 km) of the Earth, and R_M is the mean radius (1738.09 km) of the Moon.

The vector $\underline{a}$ which is used by the IMU alignment programs to correct the stored star vectors for aberration of light is determined as follows:

$$\underline{a} = \underline{u}_S \times \underline{P}_{Ecl} + \frac{\underline{v}_C}{c} \qquad (5.13.7)$$

where $\underline{u}_S$ is as calculated by Eq. 5.13.1, $\underline{v}_C$ is the velocity of the CSM with respect to the primary body, c is the speed of light, and

$$\underline{P}_{Ecl} = c_A\,(0,\ -\sin B_0,\ \cos B_0).$$

5.5.14 PERICENTER–APOCENTER (PERIAPO) SUBROUTINE

The Pericenter - Apocenter Subroutine computes the two body apocenter and pericenter altitudes given the position and velocity vectors for a point on the trajectory and the primary body.

This subroutine is depicted in Fig. 5.14-1.

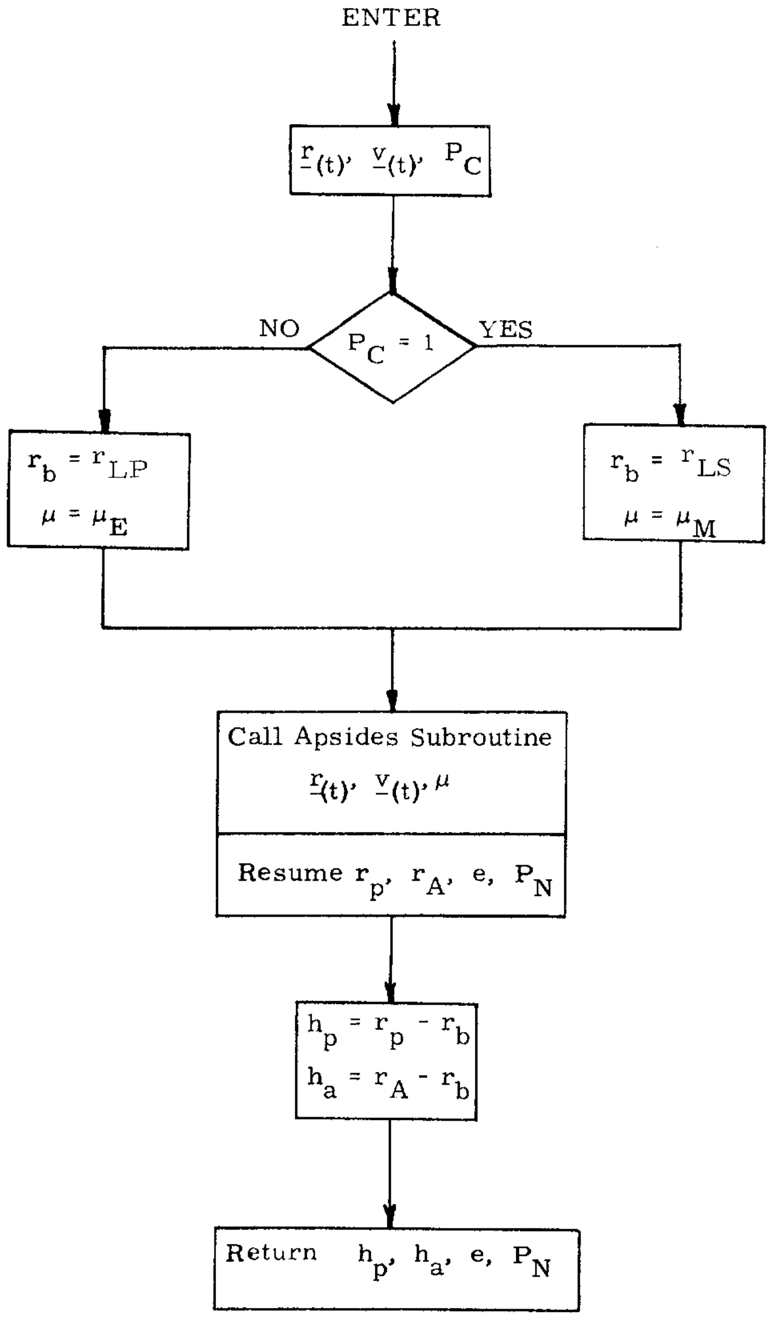


Figure 5.14-1 PERICENTER - APOCENTER SUBROUTINE

5.5.15 Time-of-Longitude Program (P29)

The Time-of-Longitude
time of the first crossing of a specified longitude <u>after</u> a specified base time, as well as the corresponding latitude, for either earth or lunar orbits. The program is depicted in Figure 5.15-1.

Inputs to the program consist of the base time t_0, the desired longitude ϕ_d, and a vehicle option (CSM or LM). The base time is measured in terms of Ground Elapsed Time, and is not to be confused with the t_0 defined in Section 5.1.5.5. The desired longitude is measured positive in an eastward direction from the conventional zero meridian of either the earth or moon; west longitudes may be specified negatively if desired.

The permanent state vector corresponding to specified vehicle is transferred to a temporary location and precision-integrated to the base time t_0 by the Coasting Integration Routine. Let $\underline{r}_0$ and $\underline{v}_0$ be the position and velocity vectors at the time t_0. It is re-emphasized that the program finds the Ground Elapsed Time of the first crossing after the base time t_0, regardless of where either vehicle is at the "present" time, provided that $\underline{r}_0$ and $\underline{v}_0$ define an elliptic orbit.

The unit angular momentum vector $\underline{\mu}_n$ is formed, and used to calculate a supplementary unit vector $\underline{\mu}_s$ which will be needed later to determine whether a transfer is through θ or 360-θ degrees. The direction of the vector $\underline{\mu}_n$ is then placed in the same hemisphere with the earth's or moon's north polar unit axis $\underline{\mu}_Z$. The unit local east vector $\underline{\mu}_E$ at the base position $\underline{r}_0$, and the unit vector $\underline{\mu}_c$ normal to both $\underline{\mu}_E$ and $\underline{\mu}_Z$ (and pointing to the center or $\underline{\mu}_Z$ axis) are calculated.

The iteration variables $\underline{r}$, t, and $\Delta\phi$ correspond to the guess on the latest iteration of the position, time, and longitude difference from the longitude of $\underline{r}_0$, of the spacecraft at the desired longitude; these variables are initialized to start the iteration process from the base point $\underline{r}_0$. The factors 16/15 and 327.8/328.8 reduce the number of iterations required for convergence of the procedure by approximating the ratio of the number of revolutions of the spacecraft with respect to inertial space to the number with respect to a fixed longitude in equal time periods. The values correspond to an earth-orbiting spacecraft with a period of 90 minutes and a lunar-orbiting spacecraft with a period of 120 minutes.

The iteration loop is entered with a call to the Latitude-Longitude Routine, which transforms the inertial position vector $\underline{r}$ at time t into body-fixed spherical coordinates so that its longitude may be compared with the desired longitude. If the two longitudes are within the tolerance ϵ_ϕ, the iteration process has converged and the corresponding time t and latitude λ are output for display purposes.

If the two longitudes are not within the tolerance ϵ_ϕ, a new current iterative approximation $\Delta\phi$ of the longitude difference between the longitude of $\underline{r}_0$ and the desired longitude is calculated. Because $\Delta\phi$ must always be positive for the earth and negative for the moon in order for the factor F to correctly compensate (reflecting the posigrade motion around the earth and the retrograde motion around the moon), and because of the moduling of the angles by 360°, a complicated series of tests is necessary to determine whether 360° should or should not be added or subtracted to $(\phi_d - \phi)$. After the first pass, the tests pick out the eight cases in which a 360° adjustment is necessary from the 24 possible arrangements of ϕ, ϕ_d, ϕ_0 and 0. These tests hold for all longitudes between -360° and $+360^\circ$.

After the new current guess of $\Delta\phi$ has been determined, the unit local east vector $\underline{\mu}'_E$ at the current iterative approximation of the desired longitude is calculated, and from this is obtained the current iterative unit position vector $\underline{\mu}_d$, which yields the current approximation of the transfer angle θ. Using the conic Time-Theta Routine, the base state vector $(\underline{r}_0, \underline{v}_0)$ is updated through θ, to a new terminal position $\underline{r}$, with which the iteration loop is re-entered.

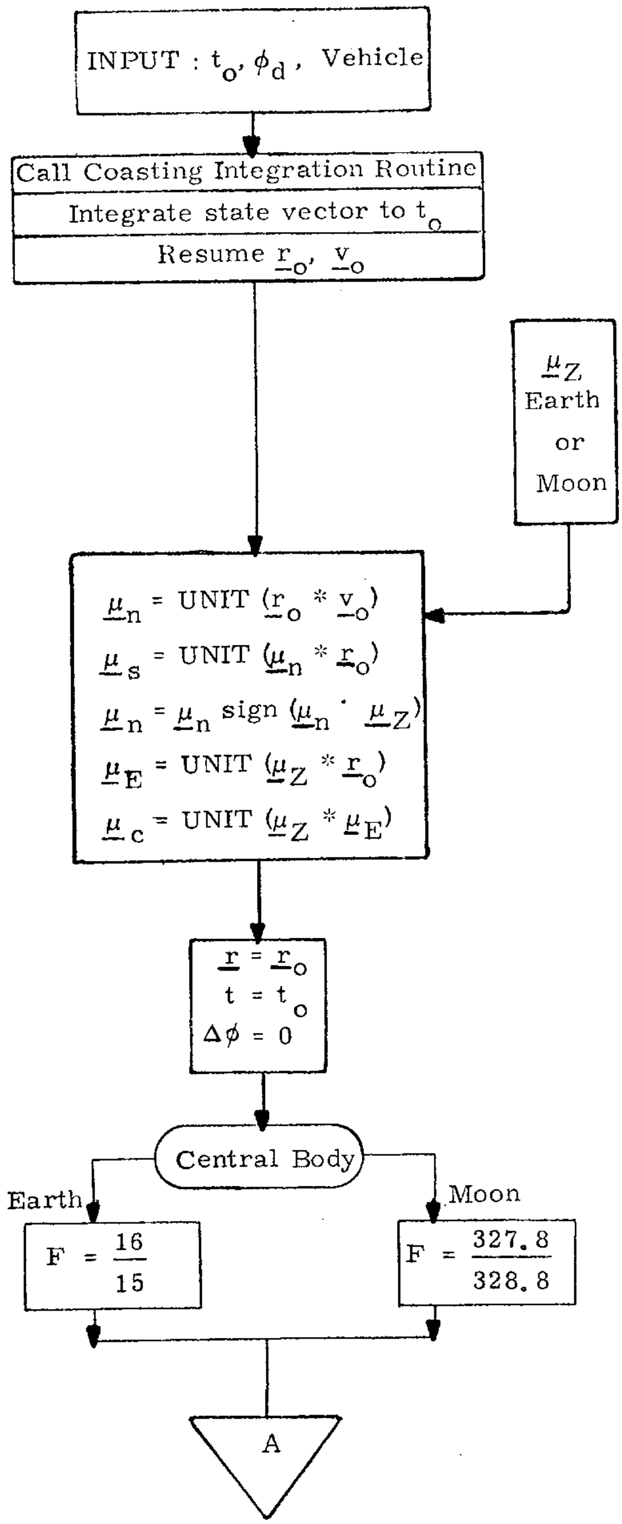


Figure 5.15-1 Time-of-Longitude Crossing Program (P29)
(Page 1 of 3)

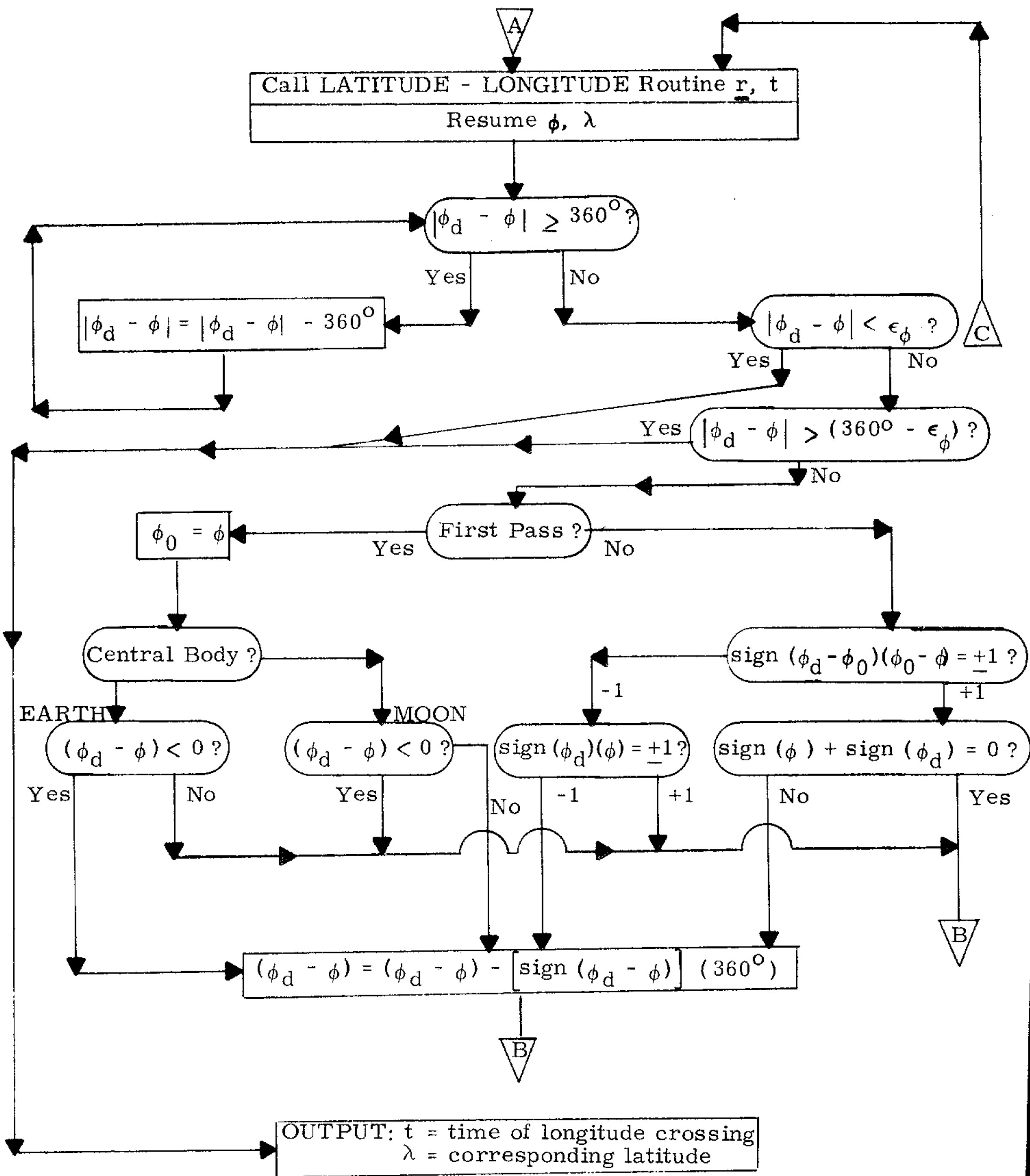


Figure 5.15-1 Time-of-Longitude Crossing Program (P29)
(Page 2 of 3)

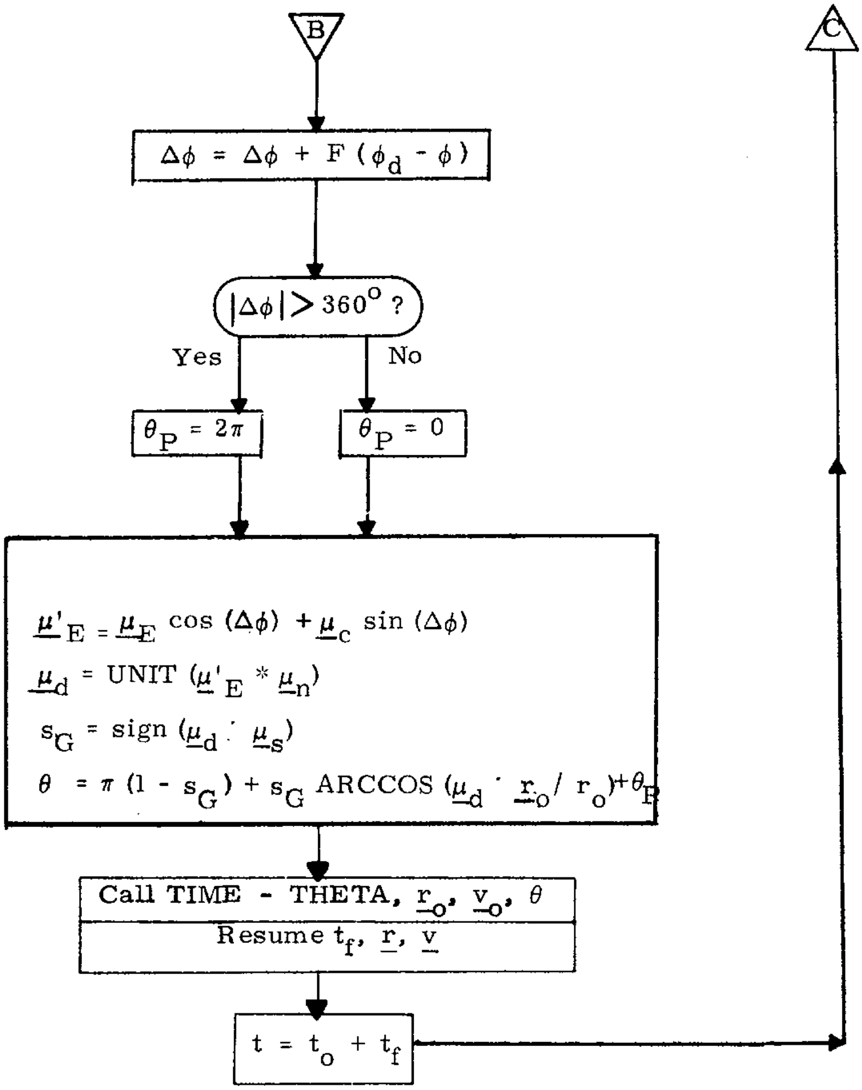


Figure 5.15-1 Time-of-Longitude Crossing Program (P29)
(Page 3 of 3)

5.6 GENERAL SERVICE ROUTINES

5.6.1 GENERAL COMMENTS

The routines presented in this section include the following general service functions:

1) IMU alignment modes

2) Basic Coordinate Transformations

3) Computer initialization procedures

4) Special display routines which can be called by the astronaut

5) Automatic optics positioning routine

5.6.2 IMU ALIGNMENT MODES

5.6.2.1 IMU Orientation Determination Program

The IMU Orientation Determination Program (P-51) is used during free-fall to determine the present IMU stable member orientation with respect to the Basic Reference Coordinate System by sighting on two navigation stars or known celestial bodies with the sextant (SXT) or the scanning telescope (SCT). At the start of program P-51 the astronaut acquires the desired celestial bodies by maneuvering the spacecraft until they are visible in one of the above optical devices. During this acquisition phase he monitors the FDAI Ball to avoid IMU gimbal lock. He may also have the IMU gimbal angles coarse aligned to zero if he so desires.

Afterwards, the following steps are performed on each of two celestial bodies:

1) The Sighting Mark Routine (R-53 of Section 4) is used to sight on one of the celestial bodies. The astronaut performs the sighting by centering the SXT or SCT on the celestial body and presses the MARK button. This mark causes the measurement time and the optics and IMU gimbal angles to be recorded by the computer.

2) The identity of the celestial body is indicated by the astronaut by use of a celestial body code. Separate codes are provided for the 37 navigation stars, Sun, Earth, Moon, and a general celestial body. The latter code is referred to as the Planet code.

3) The subroutine LOCSAM of Section 5.5.13 is used to compute the correction for aberration of light and the line-of-sight vectors of the Sun, Earth, and Moon in the Basic Reference Coordinate System for the time of the optical sighting (mark). If the Sun, Earth, or Moon code was selected, the vector for that body is set aside for use later.

4) If the Planet code was selected, the astronaut is requested to load the coordinates of the desired celestial body in the Basic Reference Coordinate System.

5) The unit vector $\underline{u}'_{CB}$ defining the direction of the celestial body as a result of the optical sighting is obtained in the IMU Stable Member Coordinate System as follows:

$$\underline{u}'_{CB} = \left[NBSM\right]\left[SBNB\right]\begin{pmatrix} \sin TA \ \cos SA \\ \sin TA \ \sin SA \\ \cos TA \end{pmatrix} \qquad (6.2.1)$$

where $\left[NBSM\right]$ is the transformation matrix in Section 5.6.3.2.1 using the IMU gimbal angles stored for the optical mark, $\left[SBNB\right]$ is the matrix in Section 5.6.3.1.1, and SA and TA are respectively, the shaft and trunnion angles of the SXT or SCT stored for the optical mark.

6) A correction for aberration of light is then applied to the unit vector $\underline{u}^*_{CB}$ defining the direction of the celestial body in the Basic Reference Coordinate System which was either taken from the computer's star catalog in fixed memory, computed by the subroutine LOCSAM in step 3, or loaded by the astronaut in step 4. The light aberration correction is made as follows:

$$\underline{u}_{CB} = \text{UNIT}\,(\underline{u}^*_{CB} + \underline{a}) \qquad (6.2.2)$$

where $\underline{a}$ is the aberration correction vector computed by the subroutine LOCSAM when this subroutine was called after the celestial body sighting, and $\underline{u}^*_{CB}$ and $\underline{u}_{CB}$ are, respectively, the uncorrected and corrected unit vectors to the celestial body in basic reference coordinates. Although this correction is correct only for distant objects such as stars, it is also applied, for reasons of program simplification, to the unit vectors of the Sun, Earth, Moon, or a planet, if any of these bodies are being used. It should be noted that the aberration correction is negligible in comparison to the errors in sighting on the Sun, Earth, or Moon.

When the above steps have been performed for two celestial bodies, the computer has the unit line-of-sight vectors for the two bodies in both IMU stable member and basic reference coordinates. Let $\underline{u}'_{CBA}$ and $\underline{u}'_{CBB}$ be the unit vectors for the two celestial bodies (A and B) which are obtained with Eq. (6.2.1) and let $\underline{u}_{CBA}$ and $\underline{u}_{CBB}$ be the unit vectors for the same bodies obtained with Eq. (6.2.2). At this point in the program the Sighting Data Display Routine (R-54 of Section 4) computes the angle between the unit line-of-sight vectors ($\underline{u}'_{CBA}$ and $\underline{u}'_{CBB}$) obtained for the two bodies in stable member coordinates and the angle between the corresponding unit line-of-sight vectors ($\underline{u}_{CBA}$ and $\underline{u}_{CBB}$) for the two bodies in basic reference coordinates. The magnitude of the difference between the two angles is displayed to the astronaut and he either accepts the results or repeats the IMU orientation determination process.

If he accepts the results of the Sighting Data Display Routine, the unit vectors $\underline{u}'_{CBA}$, $\underline{u}'_{CBB}$, $\underline{u}_{CBA}$, and $\underline{u}_{CBB}$ are used to determine the present stable member orientation and REFSMMAT, using the procedure given in Section 5.6.3.3.1.

5.6.2.2 IMU Realignment Program

The IMU Realignment Program (P-52) is used during free-fall to re-align the IMU to its presently assumed orientation or to align it from a known orientation to one of the desired orientations given in Section 5.6.3.3 and in P-52 of Section 4. This alignment is made by sighting on two navigation stars or known celestial bodies with the sextant (SXT) or the scanning telescope (SCT).

At the beginning of program P-52 the astronaut indicates which of the following stable member orientations is desired:

1) Preferred - for thrusting maneuvers

2) Landing Site - for LM lunar landing or launch

3) Nominal - for alignment with respect to local vertical

4) REFSMMAT - for re-alignment to presently assumed orientation

The Preferred, Landing Site, and Nominal orientations are defined in Section 5.6.3.3. If the astronaut selects the Landing Site or Nominal orientation, it is computed by program P-52 in the manner shown in Section 5.6.3.3 and in P-52 of Section 4. The Preferred orientation must be computed prior to entering program P-52. Whenever the astronaut selects the Preferred, Landing Site, or Nominal orientation, the program also computes and displays the IMU gimbal angles for the desired stable member orientation using the present vehicle attitude. These angles are computed by the routine CALCGA of Section 5.6.3.2.2 where the inputs to this routine are the unit vectors ($\underline{u}_{XSM}$, $\underline{u}_{YSM}$, $\underline{u}_{ZSM}$) defining the desired stable member axes with respect to the Basic Reference Coordinate System, and the unit vectors ($\underline{x}_{NB}$, $\underline{y}_{NB}$, $\underline{z}_{NB}$) defining the present navigation base axes with respect to the Basic Reference Coordinate System, which are computed as follows:

$$\underline{x}_{NB} = \left[REFSMMAT\right]^T \underline{x}'_{NB}$$

$$\underline{y}_{NB} = \left[REFSMMAT\right]^T \underline{y}'_{NB} \qquad (6.2.3)$$

$$\underline{z}_{NB} = \left[REFSMMAT\right]^T \underline{z}'_{NB}$$

where $\left[REFSMMAT\right]^T$ is the transpose of the present $\left[REFSMMAT\right]$ and $\underline{x}'_{NB}$, $\underline{y}'_{NB}$, and $\underline{z}'_{NB}$ define the navigation base axes with respect to the present Stable Member Coordinate System and are computed by the routine CALCSMSC of Section 5.6.3.2.5.

If the computed IMU gimbal angles are unsatisfactory, the astronaut maneuvers the vehicle to a more suitable attitude and has the program re-compute and display the new gimbal angles. Once satisfactory angles have been obtained, the astronaut keys in a "PROCEED" and is then requested to indicate whether the IMU is to be aligned to the desired orientation by use of the Coarse Alignment Routine (R-50 of Section 4) or by torquing the gyros. If he elects to have the gyros torqued to the desired orientation, the gyro torquing angles are computed by the routine CALCGTA of Section 5.6.3.2.3 where the inputs to this routine are the unit vectors ($\underline{x}_D$, $\underline{y}_D$, $\underline{z}_D$) defining the desired stable member axes with respect to the present Stable Member Coordinate System which are computed as follows:

$$\underline{x}_D = [REFSMMAT]\ \underline{u}_{XSM}$$

$$\underline{y}_D = [REFSMMAT]\ \underline{u}_{YSM}$$

$$\underline{z}_D = [REFSMMAT]\ \underline{u}_{ZSM}$$

where $\underline{u}_{XSM}$, $\underline{u}_{YSM}$, and $\underline{u}_{ZSM}$ were previously defined. During the period when the gyros are being torqued, the IMU gimbal angles are displayed to the astronaut so that he may avoid gimbal lock by maneuvering the spacecraft. After the gyro torquing process, the astronaut either terminates program P-52 or performs a fine alignment with optical sightings on two celestial bodies.

If the astronaut elects to have the IMU aligned to the desired orientation by means of the Coarse Alignment Routine (R-50), this routine will command the IMU to the computed gimbal angles if any of the required gimbal angle changes is greater than one degree. If all of the required gimbal angle changes are less than or equal to one degree, routine R-50 will leave the IMU at its present orientation, and it will be assumed by program P-52 that it is at the desired orientation. It should be noted that there is no computation of gimbal angles or coarse alignment of the IMU if the astronaut selects the REFSMMAT orientation at the beginning of program P-52.

Afterwards, the astronaut maneuvers the vehicle to a desired attitude for celestial body acquisition and decides whether to select his own celestial bodies for sighting purposes or to use the Star Selection Routine of Section 5.6.4. If he selects the Star Selection Routine, program P-52 will call the subroutine LOCSAM of Section 5.5.13 prior to calling the Star Selection Routine since LOCSAM computes the directions and the occultation cones of the Sun, Earth, and Moon used in the occultation tests of the Star Selection Routine. When LOCSAM is called for this purpose, the specified input time is the present time plus an additional amount (T_{SS}) in order to insure that the time used in computing the LOS vectors for the Sun, Earth, and Moon is near the middle of the sighting mark process. The time increment T_{SS} is based upon estimates of the time required to do the Star Selection Routine and to perform the optical sightings. The primary purpose of using T_{SS} is to insure that the LOS vector of the primary body used in the occultation test is that which occurs during the sighting mark process.

If the Star Selection Routine is unable to find two satisfactory stars at the present vehicle attitude, the astronaut either repeats the above process of changing the vehicle attitude and using the Star Selection Routine, or decides to select his own celestial bodies.

Afterwards, the following steps are performed on each of two celestial bodies:

1) The celestial body code is established for one of the two bodies which are to be sighted upon (i.e. the astronaut either accepts the celestial body code for one of the stars found by the Star Selection Routine or loads his own celestial body code).

2) The subroutine LOCSAM of Section 5.5.13 is used to compute the line-of-sight vectors of the Sun, Earth, and Moon for the current time in case the Sun, Earth, or Moon code was selected. If one of these codes was selected, the vector computed for that body is set aside for use later.

3) If the Planet code was selected, the astronaut is requested to load the coordinates of the desired celestial body in the Basic Reference Coordinate System.

4) The program then calls the Auto Optics Positioning Routine (R-52) which in turn calls the Sighting Mark Routine (R-53). After the SXT and SCT have been driven to the LOS of the celestial body by routine R-52 the astronaut manually centers the SXT or SCT on the celestial body and presses the MARK button.

5) When a satisfactory optical sighting (mark) has been obtained, the same steps (2 through 6) are performed after the optical sighting as are given in Section 5.6.2.1 for the IMU Orientation Determination Program (P-51) except that the unit vector

$\underline{u}_{CB}$ (see Eq. (6.2.2) in step 6 of Section 5.6.2.1) is expressed in terms of the desired IMU Stable Member Coordinate System instead of the Basic Reference Coordinate System as follows:

$$\underline{u}_{CB} = \text{UNIT}\left\{\left[\text{REFSMMAT}\right]_D (\underline{u}^*_{CB} + \underline{a})\right\} \quad (6.2.4)$$

where $\underline{a}$ is the aberration correction vector computed by the subroutine LOCSAM, and $\left[\text{REFSMMAT}\right]_D$ is the matrix (Section 5.6.3.3) for transforming vectors from the Basic Reference Coordinate System to the desired IMU Stable Member Coordinate System.

After the above steps have been performed on two celestial bodies, the computer has the unit line-of-sight vectors for the two bodies in both present and desired IMU stable member coordinates. Let $\underline{u}'_{CBA}$ and $\underline{u}'_{CBB}$ be the unit vectors for the two bodies (A and B) which are obtained with Eq. (6.2.1) and let $\underline{u}_{CBA}$ and $\underline{u}_{CBB}$ be the unit vectors for the same bodies obtained with Eq. (6.2.4). At this point in the program the Sighting Data Display Routine (R-54 of Section 4) computes the angle between $\underline{u}_{CBA}$ and $\underline{u}_{CBB}$, and the angle between $\underline{u}'_{CBA}$ and $\underline{u}'_{CBB}$, and displays the magnitude of the difference between the two angles to the astronaut. If he accepts the results, the four vectors are used in the routine AXISGEN of Section 5.6.3.2.4 to compute the desired stable member axes with respect to the present Stable Member Coordinate System, which are used in the Gyro Torquing Routine

(R-55 of Section 4). Routine R-55 computes the gyro torquing angles required to drive the IMU stable member to the desired orientation by using the above vectors in the routine CALCGTA of Section 5.6.3.2.3. The gyro torquing angles are displayed to the astronaut so that he can decide whether to have the gyros torqued through these angles or not. If he is not satisfied with the results of the Sighting Data Display Routine or the Gyro Torquing Routine, he may repeat the optical sightings without having to terminate the program.

It should be noted that the ground can indicate to the CMC via uplink the present stable member orientation or a desired stable member orientation. If the present orientation is being indicated, this is done by transmitting a REFSMMAT to the CMC. Under normalcircumstances, however, it would not be desirable for the ground to indicate the present stable member orientation since this orientation should be determined by the CSM GNCS. However, if an orientation different from the present orientation is desired by the ground, this desired orientation can be transmitted as a Preferred orientation. By treating a desired stable member orientation in this manner, program P-52 will be able to correct for any large differences between the present and desired orientations by coarse alignment. In addition, this approach avoids the introduction of orientation errors which affect celestial body acquisition.

5.6.2.3 Backup IMU Orientation Determination Program

The Backup IMU Orientation Determination Program (P-53) is used during free-fall to determine the present IMU stable member orientation with respect to the Basic Reference Coordinate System by sighting on two navigation stars or known celestial bodies with some optical device other than the sextant or scanning telescope. An optical device which is considered to be a backup to the normal CSM optics is the Crew Optical Alignment Sight (COAS). This program is identical to program P-51 of Section 5.6.2.1 except that the Alternate LOS Sighting Mark Routine (R-56 of Section 4) is used in place of the Sighting Mark Routine (R-53) to sight on the celestial bodies. Whenever routine R-56 is called by program P-53, the astronaut is requested to load the coordinates of the backup optical device before performing the optical sighting. These coordinates are the equivalent SXT shaft and trunnion angles of the device which would be indicated for the SXT if it were possible to direct the SXT along the same direction as the optical device. When an optical sighting is performed using routine R-56, the measurement time, the IMU gimbal angles, and the equivalent shaft and trunnion angles of the backup device are stored by the computer. These angles are used just as in step 5 of Section 5.6.2.1 to obtain a unit vector $\underline{u}'_{CB}$ in the IMU Stable Member Coordinate System where SA and TA are now the equivalent shaft and trunnion angles of the backup device.

5.6.2.4 Backup IMU Realignment Program

The Backup IMU Realignment Program (P-54) is used during free-fall to re-align the IMU to its presently assumed orientation or to align it from a known orientation to one of the desired orientations given in Section 5.6.3.3 and in P-54 of Section 4 by sighting on two navigation stars or known celestial bodies with some optical device other than the sextant or scanning telescope. Such an optical device may be the Crew Optical Alignment Sight (COAS). This program is identical to program P-52 of Section 5.6.2.2 except that the Alternate LOS Sighting Mark Routine (R-56 of Section 4) is used in place of the Sighting Mark Routine (R-53) to sight on the celestial bodies. In addition, the Auto Optics Positioning Routine (R-52) is not used in this program. A description of Alternate LOS Sighting Mark Routine is given in Sections 4 and 5.6.2.3.

5.6.3 IMU ROUTINES

5.6.3.1 Sextant Transformations

5.6.3.1.1 Sextant Base - Navigation Base

To transform a vector from sextant base to navigation base coordinates, use is made of the following transformation matrix:

$$[SBNB] = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix} \qquad (6.3.1)$$

where $\alpha = -32^{\circ}\ 31'\ 23.19''$

The above matrix and its transpose $[NBSB]$ are both present in computer memory.

The sextant base coordinate system referred to above is frequently called the Block I navigation base coordinate system in deference to the present navigation base coordinate system (Block II). Any transformation of vectors between the sextant and the present navigation base requires use of the above transformations.

5.6.3.1.2 Sextant to Navigation Base (SXTNB)

To obtain a unit vector ($\underline{u}_{NB}$), which specifies the direction of the line-of-sight of the sextant in navigation base coordinates, use is made of the following:

$$\underline{u}_{NB} = \left[SBNB\right] \begin{pmatrix} \sin TA \ \cos SA \\ \sin TA \ \sin SA \\ \cos TA \end{pmatrix} \qquad (6.3.2)$$

where $\left[SBNB\right]$ is the transformation matrix given in Section 5.6.3.1.1, and TA and SA are the trunnion (precision) and shaft angles, respectively, of the sextant.

5.6.3.1.3 Calculation of Sextant Angles (CALCSXA)

Given a unit star vector $\underline{s}_{SM}$ in stable member coordinates, this routine computes the angles SA and TA required to position the optics such that the line of sight lies along the star vector.

The star vector is first transformed to the sextant base coordinate system as follows:

$$\underline{s}_{SB} = \left[NBSB\right] \left[SMNB\right] \underline{s}_{SM} \qquad (6.3.3)$$

where [NBSB] and [SMNB] are given in Sections 5.6.3.1.1 and 5.6.3.2.1, respectively.

Next, a unit vector $\underline{u}_{TDA}$, defining the direction of the trunnion (precision) drive axis in sextant base coordinates, is obtained:

$$\underline{u}_{TDA} = \text{UNIT}\left[\underline{z}_{SB} \times \underline{s}_{SB}\right] \qquad (6.3.4)$$

where

$$\underline{z}_{SB} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Using $\underline{s}_{SB}$ and $\underline{u}_{TDA}$, the angles SA and TA are determined as follows:

$$\begin{aligned}
\sin SA &= \underline{u}_{TDA} \cdot (-\underline{x}_{SB}) \\
\cos SA &= \underline{u}_{TDA} \cdot \underline{y}_{SB} \\
SA &= \text{ARCTRIG}(\sin SA, \cos SA) \\
TA &= \cos^{-1}\left[\underline{z}_{SB} \cdot \underline{s}_{SB}\right]
\end{aligned} \qquad (6.3.5)$$

where ARCTRIG implies computing the angle, choosing either $\sin^{-1}$ or $\cos^{-1}$ so as to yield maximum accuracy, and

$$\underline{x}_{SB} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \underline{y}_{SB} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

5.6.3.2 IMU Transformations

5.6.3.2.1 Stable Member-Navigation Base

Let IGA, MGA, OGA be the IMU inner, middle and outer gimbal angles, respectively. Define the following matrices:

$$Q_1 = \begin{pmatrix} \cos IGA & 0 & -\sin IGA \\ 0 & 1 & 0 \\ \sin IGA & 0 & \cos IGA \end{pmatrix} \qquad (6.3.6)$$

$$Q_2 = \begin{pmatrix} \cos MGA & \sin MGA & 0 \\ -\sin MGA & \cos MGA & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (6.3.7)$$

$$Q_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos OGA & \sin OGA \\ 0 & -\sin OGA & \cos OGA \end{pmatrix} \qquad (6.3.8)$$

Stable Member to Navigation Base Transformation

$$\underline{U}_{NB} = Q_3 \, Q_2 \, Q_1 \, \underline{U}_{SM}$$

$$\left[SMNB \right] = Q_3 \, Q_2 \, Q_1 \qquad (6.3.9)$$

Navigation Base to Stable Member Transformation

$$\underline{U}_{SM} = Q_1^T \, Q_2^T \, Q_3^T \, \underline{U}_{NB}, \quad \left[NBSM \right] = Q_1^T \, Q_2^T \, Q_3^T \qquad (6.3.10)$$

5.6.3.2.2 Calculation of Gimbal Angles (CALCGA)

Given a stable member orientation and a navigation base orientation both referred to the same coordinate system, the following procedure is used to compute the corresponding gimbal angles.

$$\underline{a}_{MG} = \text{UNIT}\,(\underline{x}_{NB} \times \underline{y}_{SM})$$

$$\cos OGA = \underline{a}_{MG} \cdot \underline{z}_{NB}$$

$$\sin OGA = \underline{a}_{MG} \cdot \underline{y}_{NB}$$

$$OGA = \text{ARCTRIG}\,(\sin OGA,\ \cos OGA)$$

$$\cos MGA = \underline{y}_{SM} \cdot (\underline{a}_{MG} \times \underline{x}_{NB})$$

$$\sin MGA = \underline{y}_{SM} \cdot \underline{x}_{NB} \qquad (6.3.11)$$

$$MGA = \text{ARCTRIG}\,(\sin MGA,\ \cos MGA)$$

$$\cos IGA = \underline{a}_{MG} \cdot \underline{z}_{SM}$$

$$\sin IGA = \underline{a}_{MG} \cdot \underline{x}_{SM}$$

$$IGA = \text{ARCTRIG}\,(\sin IGA,\ \cos IGA)$$

where the inputs are three vectors along the stable member axes and three vectors along the navigation base axes.

5.6.3.2.3 Calculation of Gyro Torquing Angles (CALCGTA)

In the fine align procedure, after the present platform orientation is determined, the torquing angles required to move the platform into the desired orientation must be computed. This is achieved as follows:

Let $\underline{x}_D$, $\underline{y}_D$, and $\underline{z}_D$ be the desired stable member axes referred to present stable member orientation. The rotations are performed in three steps: (1) rotating through θ_y about the y axis, yielding $\underline{x}'_D$, $\underline{y}_D$, $\underline{z}'_D$; (2) rotating through θ_z about the z' axis, yielding $\underline{x}''_D$, $\underline{y}'_D$, $\underline{z}'_D$; (3) and finally rotating through θ_x about the x'' axis, yielding $\underline{x}''_D$, $\underline{y}''_D$, $\underline{z}''_D$. The relevant equations are as follows:

$$\underline{z}'_D = \text{UNIT}\,(-x_{D,\,3},\ 0,\ x_{D,\,1})$$

$$\sin\theta_y = z'_{D,\,1}$$

$$\cos\theta_y = z'_{D,\,3} \qquad (6.3.12)$$

$$\theta_y = \text{ARCTRIG}\,(\sin\theta_y,\ \cos\theta_y)$$

$$\sin\theta_z = x_{D,\,2}$$

4

$$\cos \theta_Z = z'_{D,3}\, x_{D,1} - z'_{D,1}\, x_{D,3}$$

$$\theta_Z = \text{ARCTRIG}\,(\sin \theta_Z, \cos \theta_Z)$$

$$\cos \theta_X = \underline{z}'_D \cdot \underline{z}_D \qquad (6.3.12 \text{ cont.})$$

$$\sin \theta_X = \underline{z}'_D \cdot \underline{y}_D$$

$$\theta_X = \text{ARCTRIG}\,(\sin \theta_X, \cos \theta_X)$$

The required inputs are the three coordinate axes of the desired stable member orientation referred to the present stable member orientation.

5.6.3.2.4 Coordinate Axes Generator (AXISGEN)

Given two unit vectors (usually star vectors), $\underline{s}_A$ and $\underline{s}_B$, expressed in two coordinate systems, denoted by primed and unprimed characters, i.e., $\underline{s}'_A$, $\underline{s}'_B$, $\underline{s}_A$, $\underline{s}_B$, this routine computes the unit vectors $\underline{x}$, $\underline{y}$, $\underline{z}$ which are the primed coordinate system axes referred to the unprimed coordinate system. This is accomplished by defining two ortho-normal coordinate sets, one in each system, in the following manner:

$$
\begin{aligned}
\underline{u}'_X &= \underline{s}'_A \\
\underline{u}'_Y &= \text{UNIT}\,(\underline{s}'_A \times \underline{s}'_B) \\
\underline{u}'_Z &= \underline{u}'_X \times \underline{u}'_Y \\
\underline{u}_X &= \underline{s}_A \\
\underline{u}_Y &= \text{UNIT}\,(\underline{s}_A \times \underline{s}_B) \\
\underline{u}_Z &= \underline{u}_X \times \underline{u}_Y
\end{aligned}
\qquad (6.3\ 13)
$$

The primed coordinate system axes expressed in terms of the unprimed coordinate system axes are:

$$\underline{x} = u'_{X1}\underline{u}_X + u'_{Y1}\underline{u}_Y + u'_{Z1}\underline{u}_Z$$

$$\underline{y} = u'_{X2}\underline{u}_X + u'_{Y2}\underline{u}_Y + u'_{Z2}\underline{u}_Z \qquad (6.3.14)$$

$$\underline{z} = u'_{X3}\underline{u}_X + u'_{Y3}\underline{u}_Y + u'_{Z3}\underline{u}_Z$$

It should be noted that vectors can be transformed from the unprimed to the primed coordinate systems by using the following matrix constructed with the output (Eq. 6.3.14) of AXISGEN:

$$\begin{bmatrix} \underline{x}^T \\ \underline{y}^T \\ \underline{z}^T \end{bmatrix} \qquad (6.3.15)$$

5.6.3.2.5 Calculation of Stable Member Coordinates of the Spacecraft (CALCSMSC)

To determine the directions of the X, Y, and Z axes of the present vehicle coordinate system or the navigation base coordinate system with respect to the IMU Stable Member Coordinate System, use is made of the routine CALCSMSC.

The unit vectors $\underline{x}_{NB}$, $\underline{y}_{NB}$, and $\underline{z}_{NB}$ defining the directions of the navigation base coordinate system axes with respect to the IMU Stable Member Coordinate System are determined as follows:

$$\underline{x}_{NB} = \begin{pmatrix} \cos IGA \ \cos MGA \\ \sin MGA \\ -\sin IGA \ \cos MGA \end{pmatrix}$$

$$\underline{z}_{NB} = \begin{pmatrix} \cos IGA \ \sin OGA \ \sin MGA \\ + \cos OGA \ \sin IGA \\ -\sin OGA \ \cos MGA \\ \cos OGA \ \cos IGA \\ -\sin OGA \ \sin MGA \ \sin IGA \end{pmatrix}$$

$$\underline{y}_{NB} = \underline{z}_{NB} \times \underline{x}_{NB}$$

where IGA, MGA, and OGA are the inner, middle, and outer IMU gimbal angles, respectively. It should be noted that the rows of the transformation matrix [SMNB] also give the above vectors.

5.6.3.3 REFSMMAT Transformations

The matrix required to transform a vector from the Basic Reference Coordinate System to the IMU Stable Member Coordinate System is referred to as REFSMMAT. This matrix can be constructed as follows with the unit vectors $\underline{u}_{XSM}$, $\underline{u}_{YSM}$, and $\underline{u}_{ZSM}$ defining the orientations of the stable member axes with respect to reference coordinates.

$$\text{REFSMMAT} = \begin{bmatrix} \underline{u}_{XSM}^T \\ \underline{u}_{YSM}^T \\ \underline{u}_{ZSM}^T \end{bmatrix} \qquad (6.3.16)$$

5.6.3.3.1 Present REFSMMAT From Star Sightings

The present IMU stable member orientation with respect to the Basic Reference Coordinate System, and the associated REFSMMAT, can be determined by sighting on two navigation stars with the CSM optics. If $\underline{s}'_A$ and $\underline{s}'_B$ are the unit vectors defining the measured directions of the two stars in the present Stable Member Coordinate System, and $\underline{s}_A$ and $\underline{s}_B$ are the unit vectors to the corresponding stars as known in the Basic Reference Coordinate System, then these vectors can be used as the input to the routine AXISGEN (Section 5.6.3.2.4) to obtain the present IMU orientation and REFSMMAT (Eqs. (6.3.14) and (6.3.15)).

5.6.3.3.2 Alignment for Thrusting Maneuvers (Preferred Orientation)

During certain thrusting maneuvers the IMU will be aligned according to the following equations.

$$\begin{aligned} \underline{u}_{XSM} &= \text{UNIT}(\underline{x}_B) \\ \underline{u}_{YSM} &= \text{UNIT}(\underline{u}_{XSM} \times \underline{r})^{*} \qquad (6.3.17) \\ \underline{u}_{ZSM} &= \underline{u}_{XSM} \times \underline{u}_{YSM}^{*} \end{aligned}$$

where $\underline{x}_B$ is the vehicle or body X-axis at the preferred vehicle attitude for ignition and $\underline{r}$ is the CSM position vector.

The associated transformation matrix (REFSMMAT) is given by Eq. (6.3.16).

5.6.3.3.3 Alignment to Local Vertical in Orbit (Nominal Orientation)

The IMU stable member may be aligned to the local vertical at a specified time. For this type of orientation the stable member axes are found from the following.

$$\begin{aligned} \underline{u}_{XSM} &= (\underline{u}_{YSM} \times \underline{u}_{ZSM}) \\ \underline{u}_{YSM} &= \text{UNIT}(\underline{v} \times \underline{r}) \qquad (6.3.18) \\ \underline{u}_{ZSM} &= \text{UNIT}(-\underline{r}) \end{aligned}$$

where $\underline{r}$ and $\underline{v}$ are the position and velocity vectors of the CSM at the specified time. The vectors $\underline{r}$ and $\underline{v}$ are computed by the Coasting Integration Routine of Section 5.2.2.

The REFSMMAT associated with this IMU orientation is found from Eq. (6.3.16).

---

*In P41 if $|\underline{u}_{XSM} \times \underline{r}| < 2^{16}$ meters or in P40 if $(\underline{u}_{XSM} \times \text{UNIT } \underline{r}) < 2^{-12}$ radians, then $\underline{u}_{YSM} = \text{UNIT } [\underline{u}_{XSM} \times (\text{UNIT } \underline{r} + 0.125 \text{ UNIT } \underline{v})]$ where $\underline{v}$ is the CSM velocity vector

5.6.3.3.4 Lunar Landing or Launch Orientation

The proper IMU orientation for lunar landing or launch is defined by the following equations:

$$\underline{u}_{XSM} = \text{UNIT}\ (\underline{r}_{LS})\ \text{at}\ t_L$$

$$\underline{u}_{YSM} = \underline{u}_{ZSM} \times \underline{u}_{XSM} \qquad (6.3.19)$$

$$\underline{u}_{ZSM} = \text{UNIT}\ (\underline{h}_C \times \underline{u}_{XSM})$$

where

$$\underline{h}_C = \underline{r}_C \times \underline{v}_C$$

is the orbital angular momentum vector of the CSM, and $\underline{r}_{LS}$ is the landing site position vector at the nominal time of lunar landing or launch, t_L, depending on the alignment mode.

The REFSMMAT associated with this IMU orientation is found from Eq. (6.3.16).

5.6.3.3.5 Earth Pre-launch Alignment

Prior to earth launch the IMU Stable Member is aligned to a local vertical axis system.

$\underline{u}_{ZSM}$ = UNIT $(-\underline{r})$ (local vertical)

$\underline{u}_{XSM}$ = UNIT $(\underline{A})$ where $\underline{A}$ is a horizontal vector pointed at the desired launch azimuth angle.

$$\underline{u}_{YSM} = \underline{u}_{ZSM} \times \underline{u}_{XSM}$$

The REFSMMAT associated with this IMU orientation is given in Eq. (6.3.16).

5.6.4 STAR SELECTION ROUTINE

The Star Selection Routine is used by the IMU Realignment Program (P-52) and the Back-Up IMU Realignment Program (P-54) to select the best pair of stars in the viewing cone of the sextant for fine alignment of the IMU. The logic diagram for this routine is shown in Fig. 6.4-1.

Each pair from the computer catalog of 37 stars is tested to see if both stars are within a 76 degree viewing cone centered with respect to the shaft drive axis of the sextant (SXT).

Afterwards, the routine checks to see if the angle of separation between the stars is at least 30 degrees.

If a pair passes the above tests, a check is then made to see if either star is occulted by the Sun, Earth, or Moon. The sizes of the occultation cones about each of the three bodies are such as to not only account for true occultation but to also prevent the selection of stars too near the bodies because of visibility problems. The directions and the associated occultation cone sizes of the three bodies are actually computed by the subroutine LOCSAM (Section 5.5.13) which is called by the IMU Realignment Programs just prior to calling the Star Selection Routine.

The pair of stars passing the above tests and having the largest angular separation is chosen by this routine. If the routine is unable to find a satisfactory pair of stars after testing all combinations, it is seen in Program P-52 of Section 4 that an Alarm Code is displayed, whereupon the astronaut may either repeat the star selection process at a different spacecraft attitude or select his own stars later.

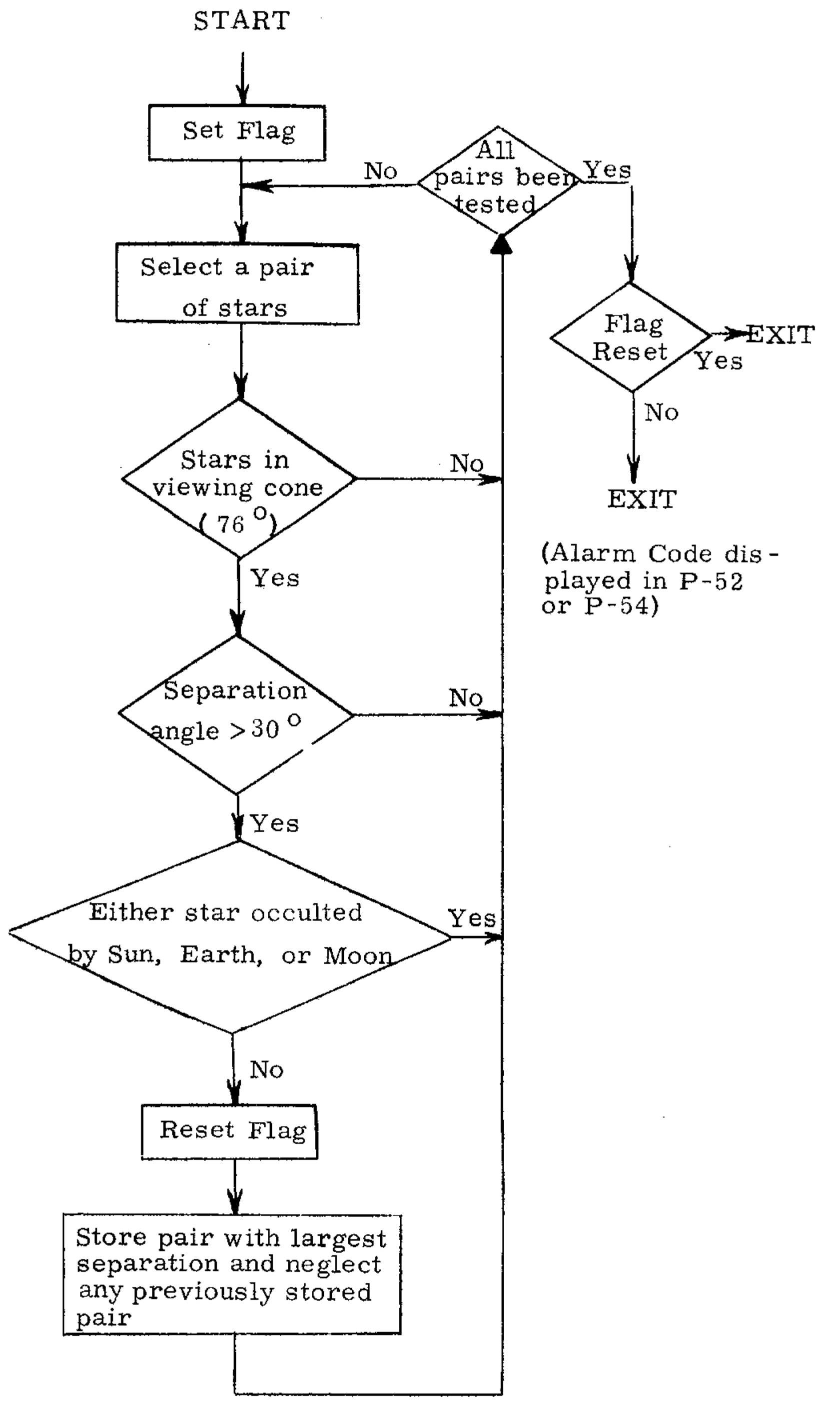


Figure 6.4-1 Star Selection Routine

5.6.5 Ground Track Routine

This routine is used by the astronaut in near-earth or near-moon orbit to obtain CSM or LM trajectory information. The astronaut specifies a time (GET) and a vehicle (CSM or LM). The routine uses the Coasting Integration Routine (Section 5.2.2) to extrapolate the desired vehicle's state vector to the specified time. The resulting estimated position vector is converted to latitude, longitude, altitude coordinates by means of the Latitude-Longitude Subroutine (Section 5.5.3), and these data are displayed. Altitude is defined with respect to the landing site radius for lunar orbit, and the launch pad radius for earth orbit. The astronaut can request the state vector extrapolation to continue in ten minute steps, or to another specified time, and obtain additional displays of the coordinates of points in the spacecraft's orbit. Alternatively, the astronaut can request a display of altitude (to 10 nm), inertial velocity magnitude (to 1 ft/sec), and flight path angle (in degrees) at an astronaut-specified time.

As an additional option, the astronaut may specify a longitude, a base time (GET) and a vehicle (CSM or LM), and request the computer to solve for and display the time (GET) of the first crossing of the specified longitude by the vehicle after the base time. The Time-of-Longitude Program, described in Section 5.5.15, is used for these calculations.

5.6.6 S-BAND ANTENNA ROUTINE

The S-Band Antenna Routine (R - 05) is used to compute and display 2 angles γ and ρ defined in Fig. 6.6-1. Once the program is initiated by the astronaut the computer will automatically update the display at a rate no greater than once per second, depending on other computer activity. This updating will continue until S-Band Lock-on is achieved and signaled to the CMC by the astronaut through the DSKY. If the S-Band Routine is interrupted the display of γ and ρ will be stopped until the astronaut reinitiates the routine.

The accuracy associated with the computation and display of γ and ρ does not indicate the pointing accuracy of the associated antenna alignment or the accuracy associated with the antenna gimbal angles.

The angles shown in Fig. 6.6-1 are computed without consideration of any constraints imposed by the actual antenna and therefore, if the present vehicle attitude results in antenna angles beyond the gimbal limit, the CMC will not indicate that the antenna line-of-sight cannot be pointed according to these angles.

The equations and logic flow used to compute γ and ρ are shown in Fig. 6.6-2. With reference to the first portion of this figure, the program will automatically determine whether the Basic Reference Coordinate System is earth or moon centered and will compute the proper values of γ and ρ for either case.

γ is defined as a positive rotation from the plus body X axis about the CSM body Z axis. $(0 \leq \gamma < 360^{\circ})$

ρ is defined as a positive rotation about the Y' axis. Y' is an axis in the CSM body XY plane rotated from the CSM Y body axis about the +Z axis direction an amount γ. $(-90^{\circ} \leq \rho \leq 90^{\circ})$

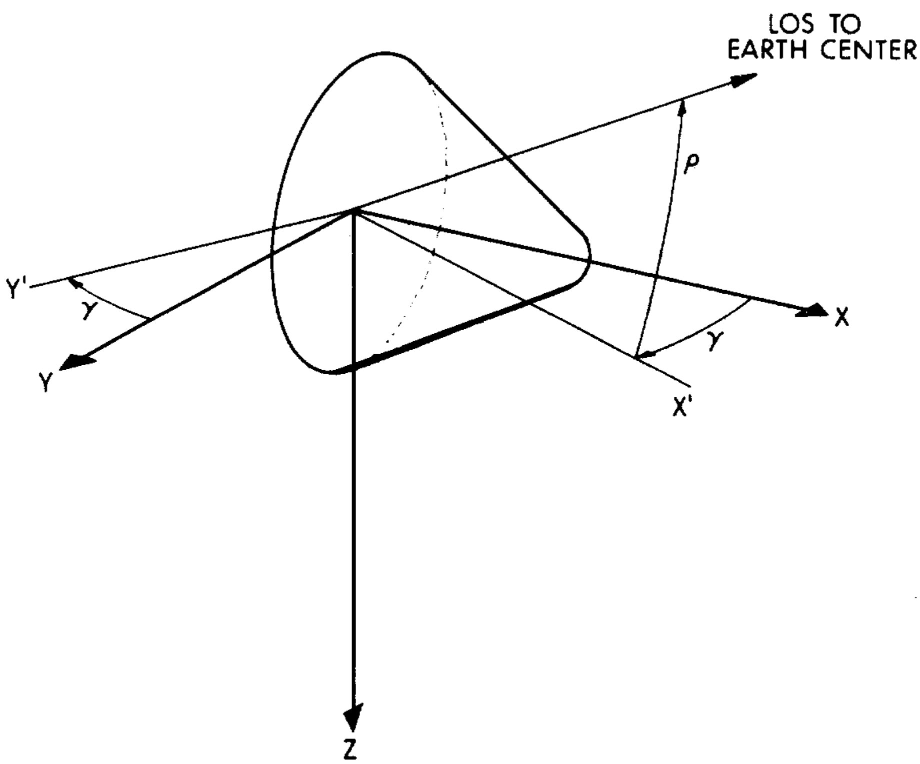


Figure 6.6-1 Definition of CSM S-Band Display Angles

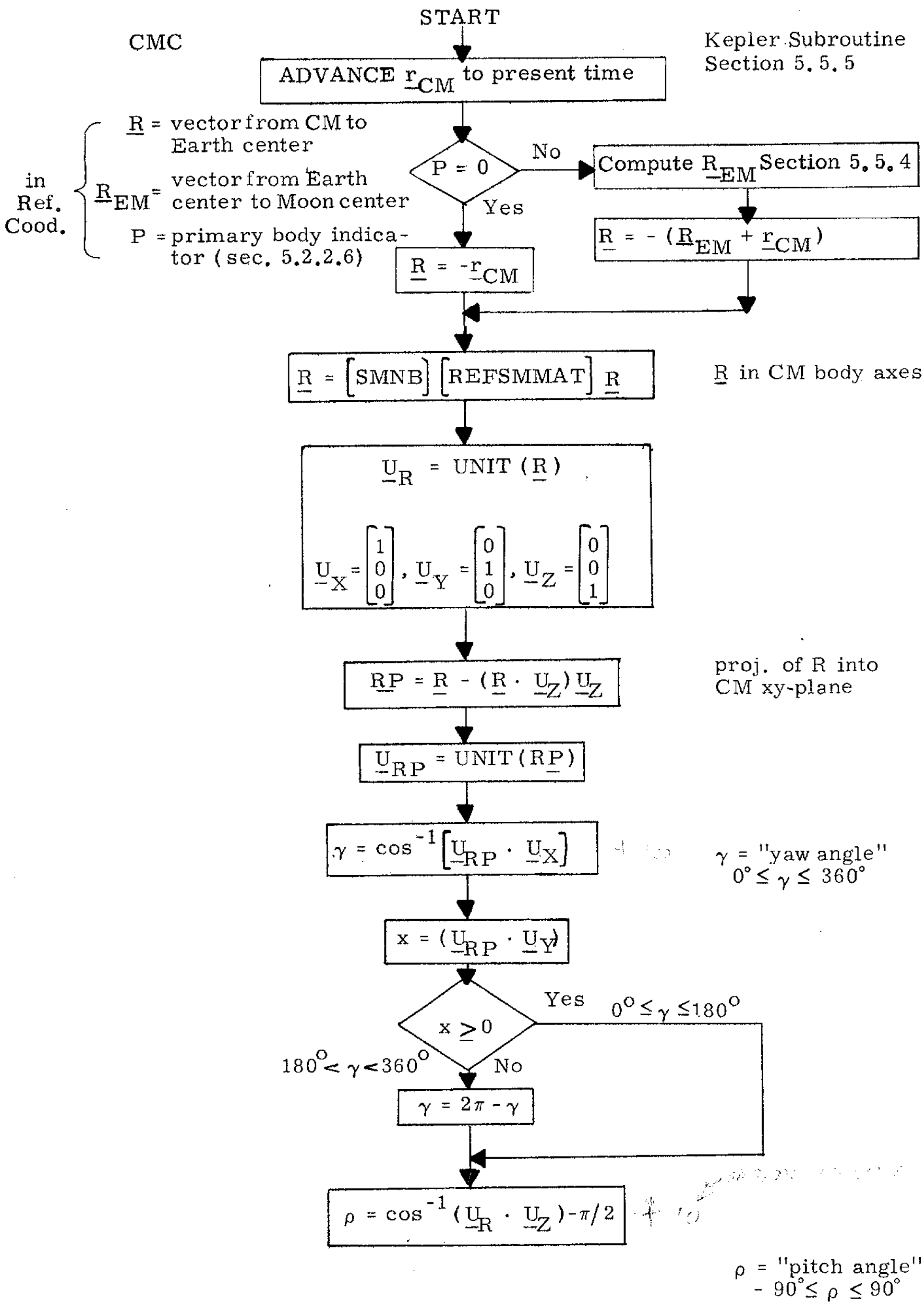


Figure 6.6-2 S-Band Angle Computations and Logic

5.6.7 ADDITIONAL RENDEZVOUS DISPLAYS

During the final phases of rendezvous the following four routines may be called by the astronaut for the purpose of computing and displaying special quantities related to the rendezvous geometry.

5.6.7.1 Range, Range Rate, Theta Display

Routine R-31 may be called upon to compute and display the range and range rate between the two vehicles and an angle θ shown in Fig. 6.7-1.

The angle θ represents the angle between the CSM X-body axis and the local horizontal plane. It is defined in a manner completely analogous to the definition of E in Section 5.4.2.2. Theta, therefore, can have values between 0 and 360°.

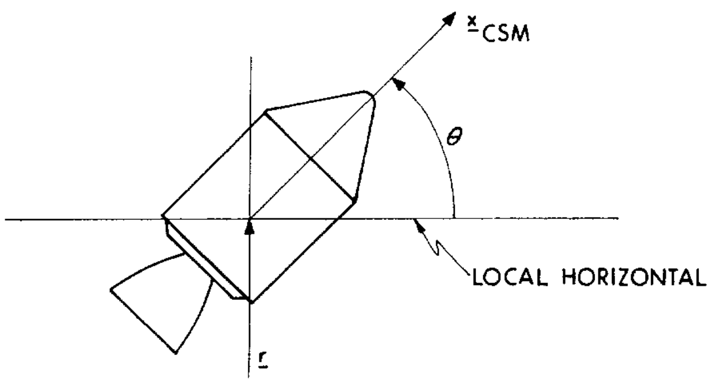


Figure 6.7-1 Definition of Theta

The equations used to compute the display parameters for R-31 are given below.

$$\underline{R} = \underline{r}_L - \underline{r}_C$$

$$\underline{u}_R = \text{UNIT}(\underline{R})$$

$$\text{RANGE} = |\underline{R}| \qquad (6.7.1)$$

$$\text{RANGE RATE} = (\underline{v}_L - \underline{v}_C) \cdot \underline{u}_R \qquad (6.7.2)$$

To compute θ the following vector is defined

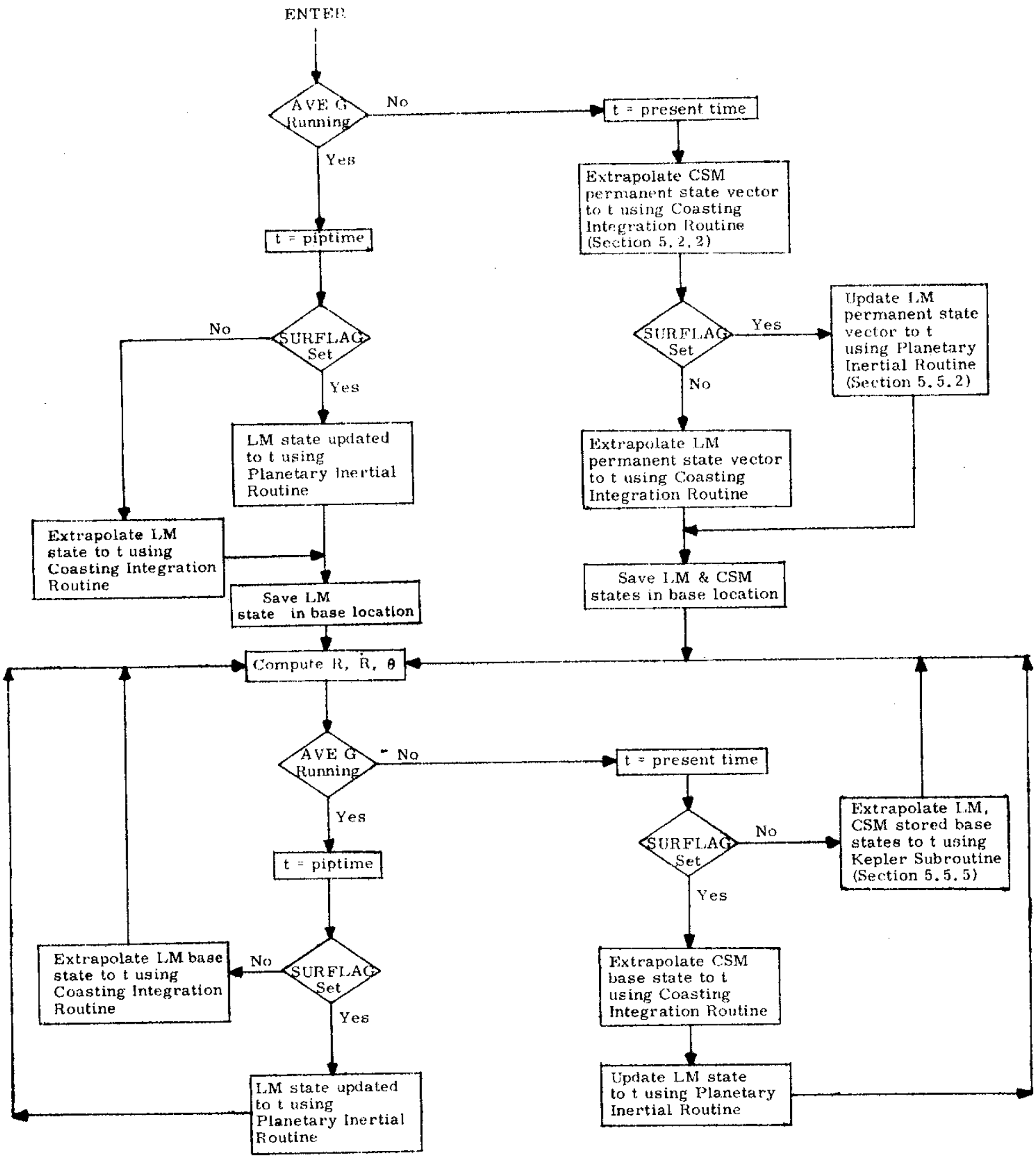


Figure 6.7-2 Range, Range Rate, θ

$$\underline{u}_X = [\text{REFSMMAT}]^T \, [\text{NBSM}] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where NBSM and REFSMMAT are defined in Section 5.6.3 and $\underline{u}_X$ is a unit vector along the X-body axis expressed in basic reference coordinates. The angle θ is then found as follows:

$$\underline{u} = \text{UNIT} \, (\underline{r}_C \times \underline{v}_C)$$

$$\underline{u}_P = \text{UNIT} \left[\underline{u}_X - \left(\frac{\underline{u}_X \cdot \underline{r}_C}{r_C^2} \right) \underline{r}_C \right]$$

$$\theta = \cos^{-1} [\underline{u}_X \cdot \underline{u}_P \, \text{SGN} \, (\underline{u}_P \cdot \underline{u} \times \underline{r}_C)] \qquad (6.7.3)$$

$$\text{If } \underline{u}_X \cdot \underline{r}_C < 0; \; \theta = 2\pi - \theta$$

The three displays of R-31 are automatically updated until R-31 is terminated by the astronaut. The logic flow required to accomplish this update is shown in Fig. 6.7-2.

5.6.7.2 Range, Range Rate, Phi Display

Routine R-34 may be initiated by the astronaut to display the computed range and range rate between the two vehicles as well as an angle ϕ shown in Fig. 6.7-3.

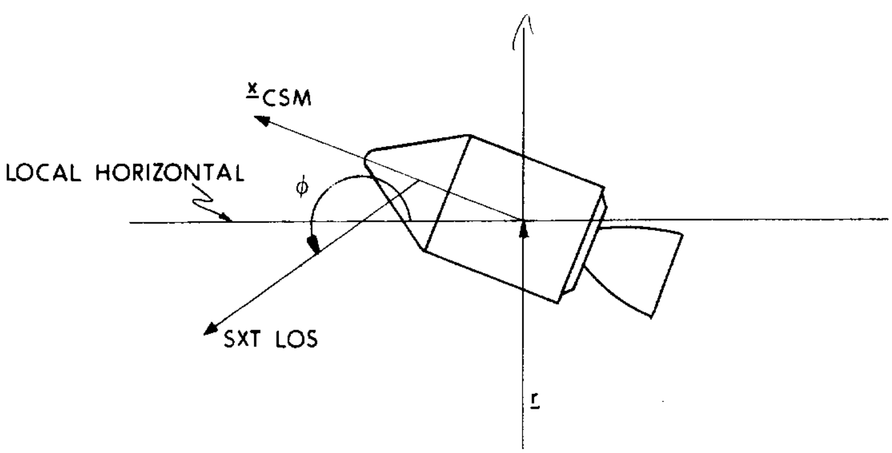


Figure 6.7-3 Definition of Phi

The angle ϕ represents the angle between the SXT line of sight and the local horizontal plane. It is determined in the same way as θ in Eq. (6. 7. 3) except that

$$\underline{u}_X = [\,REFSMMAT\,]^T\,[\,NBSM\,]\,\underline{u}_{NB}$$

where $\underline{u}_{NB}$ is the sextant line of sight in navigation base coordinates as given in Section 5. 6. 3. 1. 2.

The three displays of R-34 are automatically updated in a manner similar to the R-31 update.

5.6.7.3 Final Attitude Display

Routine R-63 may be used to compute and display the gimbal angles required to point a specified spacecraft vector, SCAXIS (as defined in Section 5. 2. 5. 1), at the LM. After initiation of this routine the state vectors of both vehicles are extrapolated to the present time plus one minute. Based on these new state vectors the required gimbal angles are computed and displayed.

There is no automatic display update. However, R-63 may easily be recycled to manually accomplish an update of the display.

5.6.7.4 Out-of-Plane Rendezvous Display

Routine R-36 may be used during any phase of the rendezvous sequence to provide information about the out-of-plane position and velocity of the CSM relative to the LM orbital plane and the out-of-plane velocity of the LM relative to the CSM orbital plane. These three quantities are computed for a given time. The present time of ignition is automatically selected in the absence of a time input by the astronaut.

The definition of the three quantities computed is given by the following set of equations where $\underline{r}_{CM}$, $\underline{v}_{CM}$ is the state vector of the CSM and $\underline{r}_{LM}$, $\underline{v}_{LM}$ is the state vector of the LM.

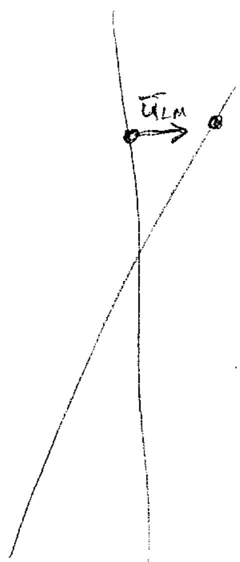
$$\underline{u}_{CM} = \text{UNIT}\,(\underline{v}_{CM} \times \underline{r}_{CM}) \qquad (6.7.4)$$

$$\underline{u}_{LM} = \text{UNIT}\,(\underline{v}_{LM} \times \underline{r}_{LM}) \qquad (6.7.5)$$

$$\dot{Y}_{CM} = \underline{v}_{CM} \cdot \underline{u}_{LM} \qquad (6.7.6)$$

$$Y_{CM} = \underline{r}_{CM} \cdot \underline{u}_{LM} \qquad (6.7.7)$$

$$\dot{Y}_{LM} = \underline{v}_{LM} \cdot \underline{u}_{CM} \qquad (6.7.8)$$



5.6.8 AUTOMATIC OPTICS POSITIONING ROUTINE

5.6.8.1 General

This routine is used during alignment and navigation procedures to automatically point the optics in the direction of a specified tracking target to aid the astronaut in the acquisition of the target. It is also used to provide optics shaft and trunnion rates from computed estimates of LOS rate. The four target modes available in this routine are as follows:

1) Star mode
2) LM mode
3) Landmark mode
4) Advanced ground track mode

The calculations in this routine are repeated approximately every two seconds; positioning commands are issued provided the optics mode switch is set to CMC. The rate aided optics feature is available only to P-24 when the optics switch is set to MANUAL.

The routine consists of two subroutines, Line-of-Sight and Command. Let t be the time (GET) at the beginning of a cycle and let

$$t_\delta = t + \delta t \qquad (6.8.1)$$

where δt is a lead term to compensate for system lags. During the first part of the cycle the desired line-of-sight vector at time t_δ, $\underline{u}_{LOS}$, is computed. The Command Subroutine then computes updated values of the desired optics angles from the desired line-of-sight vector, $\underline{u}_{LOS}$, and the current vehicle attitude. The entire procedure is then repeated. A basic CMC routine drives the optics to the orientation indicated by the desired angles.

5.6.8.2 Line-of-Sight Subroutine

5.6.8.2.1 Star Mode

This mode is used during IMU alignment procedures to point the optics in the direction of a specified star. The desired line-of-sight vector, $\underline{u}_{LOS}$, is the specified unit star vector stored in the CMC fixed memory. The optics may also be directed along lines-of-sight for stars and celestial bodies whose unit position vectors have been input via N88.

5.6.8.2.2 LM Mode

This mode is used during LM tracking phases to point the optics in the direction of the LM. The Kepler Subroutine (Section 5.5.5) is used to compute $\underline{r}_C$ and $\underline{r}_L$, the estimated conic position vectors of the CSM and the LM, respectively, at time t_δ. Then, the desired line-of-sight vector is given by

$$\underline{u}_{LOS} = \text{UNIT} (\underline{r}_L - \underline{r}_C) \qquad (6.8.2)$$

5.6.8.2.3 Landmark Mode

This mode is used during the orbit navigation phase to point the optics in the direction of a landmark specified either by code number for a landing site, or by latitude, longitude, altitude for a landmark whose coordinates are not stored in the CMC memory. The Kepler Subroutine (Section 5. 5. 5) is used to compute $\underline{r}_C$, the estimated conic position vector of the CSM at time t_δ. Then, $\underline{r}_\ell$, the estimated position of the landmark at time t_δ is computed by means of the Latitude-Longitude Subroutine (Section 5. 5. 3). Finally, the desired line-of-sight is given by

$$\underline{u}_{LOS} = \text{UNIT} (\underline{r}_\ell - \underline{r}_C) \qquad (6.8.3)$$

5. 6. 8. 2. 4 Advanced Ground Track Mode

This mode is used during the lunar-orbit navigation phase to aid in the surveillance, selection and tracking of possible landing sites by driving the optics to the direction of a point on the ground track of the CSM at a time slightly more than a specified number of orbital revolutions from current time. Let the specified revolution number be N, and assume that N is no greater than four.

Let $\underline{r}_C$ be the estimated CSM position vector at time t. The local vertical vector, $\underline{u}_{LV}$, at time t is given by

$$\underline{u}_{LV} = - \text{UNIT} (\underline{r}_C) \qquad (6.8.4)$$

The desired line-of-sight vector, $\underline{u}_{LOS}$, is determined such that the angle between the vectors $\underline{u}_{LOS}$ and $\underline{u}_{LV}$ is approximately 60°. Thus, the astronaut can survey the advanced ground track of the CSM, and select a landing site over which the CSM will pass in N orbital revolutions plus about 1/2 (5 – N) minutes. After the astronaut selects the landing site, he switches the optics mode to MANUAL in order to obtain navigation data as described in Section 5. 2. 4. 1.

The value of the angle between the vectors $\underline{u}_{LOS}$ and $\underline{u}_{LV}$ is chosen to be 60° in order to ensure that the astronaut will have sufficient time to center the selected landing site in the field of view of the optical instrument before the first navigation sighting should be made. After about one minute of manual tracking, the angle between the line-of-sight to the selected landing site and the current local vertical direction (not the vector $\underline{u}_{LV}$) will be approximately 45°. At this time it is desirable for accurate tracking conditions to have been attained, and for the MARK button to be pressed. Further navigation sightings of the landing site should then continue. The data obtained are processed by the Orbit Navigation Routine (Section 5.2.4).

The desired line-of-sight vector, $\underline{u}_{LOS}$, is computed by first rotating the local vertical vector about the polar axis of the moon to account for lunar rotation, and then rotating the resulting vector about the normal to the CSM orbital plane to attain the desired 60° angle. The computation sequence is as follows:

(1) Call the Kepler Subroutine (Section 5.5.5) to determine $\underline{r}_C$ and $\underline{v}_C$, the estimated CSM conic position and velocity vectors at time t.

(2) Let

$$\underline{u}_{LOS} = - \text{UNIT}(\underline{r}_C) \qquad (6.8.5)$$

(3) Call the Planetary Inertial Orientation Subroutine to transform the lunar polar vector (0, 0, 1) to the Basic Reference Coordinate System, obtaining the first rotation vector $\underline{u}_R$.

(4) Let the first rotation angle A be

$$A = A_M N \tag{6.8.6}$$

where A_M is the angle through which the moon rotates in one nominal lunar-orbital period.

(5) Rotate the vector $\underline{u}_{LOS}$ about $\underline{u}_R$ through the angle A by

$$\underline{u}_{LOS} = (1 - \cos A)(\underline{u}_R \cdot \underline{u}_{LOS})\,\underline{u}_R + \underline{u}_{LOS}\cos A + \underline{u}_R \times \underline{u}_{LOS}\sin A \tag{6.8.7}$$

(6) Compute the second rotation vector from

$$\underline{u}_R = \text{UNIT}\,(\underline{v}_C \times \underline{r}_C) \tag{6.8.8}$$

(7) Let the second rotation angle be

$$A = \frac{\pi}{3} - A \tag{6.8.9}$$

(8) Compute the final desired line-of-sight vector $\underline{u}_{LOS}$ by repeating Eq. (6.8.7)

5.6.8.3 Command Subroutine

In this subroutine the line-of-sight vector, $\underline{u}_{LOS}$, is transformed to stable member coordinates by means of

$$\underline{s}_{SM} = \left[REFSMMAT \right] \underline{u}_{LOS} \qquad (6.8.10)$$

The desired optics trunnion (precision) and shaft angles, TA and SA, respectively, are computed as described in Section 5.6.3.1.3.

5.6.8.4 Rate-Aided Optics Subroutine

This subroutine is used to assist the astronaut in landmark tracking at low altitudes, where line-of-sight rates are high. This is called only by the Rate-Aided Optics Tracking Program (P-24). In this subroutine, shaft and trunnion rates are supplied to the optics instead of shaft and trunnion angles. For each cycle these rates are computed by

$$V_{TA} = (TA_{NEW} - TA_{OLD}) / (t_{\delta NEW} - t_{\delta OLD}) \ (TRUNSF)$$

(6.8.11)

$$V_{SA} = (SA_{NEW} - SA_{OLD}) / (t_{\delta NEW} - t_{\delta OLD}) \ (SHAFTSF)$$

where: TA_{NEW}, SA_{NEW}, TA_{OLD}, SA_{OLD} are the desired shaft and trunnion angles computed in Section 5.6.8.3 during the present and previous cycle, respectively.
$t_{\delta NEW}$, $t_{\delta OLD}$ are the times computed in Eq. (6.8.1) for the present and previous cycles respectively.
TRUNSF and SHAFTSF are pad loaded scale factors for computing drive rate.

At every N^{th} cycle (where N is an erasable integer) if a new mark has been made and is valid (i.e., not rejected) the routine will compute a new latitude, longitude and altitude for the landmark to be used by the Landmark Line-of-Sight Routine (Section 5.6.8.2.3).

This is done as follows:

(1) Compute $\underline{r}_C$, the estimated conic position vector of the CSM at the time of the navigation sighting using the Kepler Subroutine (Section 5.5.5).

(2) Obtain the latest valid navigation sighting (time, three IMU angles and two optics angles). From these angles compute the measured unit vector $\underline{u}_M$ along the CSM-to-landmark line-of-sight in the Basic Reference Coordinate System from Eq. (2.4.2).

(3) Compute the estimated landmark position $\underline{r}_\ell$ at at the time of the sighting from Eq. (2.4.10) using $(r_0 + \text{landmark altitude})^2$ instead of $(r_0)^2$.

(4) Use the Latitude-Longitude Subroutine (Section 5.5.3) to convert $\underline{r}_\ell$ to latitude, longitude and altitude.

If no new mark has been made by the N^{th} cycle, the update will be made after the next mark.

5.6.10 ORBITAL PARAMETER AND ENTRY DISPLAY COMPUTATIONS

This section presents the CMC computations required for the displays in the following routines and programs:

a) The Orbital Parameter Display Routine R-30, which is callable by the astronaut via an extended verb.

b) The Splash Error Computation Subroutine DELRSPL which is automatically called by R-30 when either the Earth Orbital Injection Monitor Program P-11 or the CMC Idling Program P-00 is running.

c) The Entry Preparation Program P-61 which provides displays to initialize the EMS.

d) The CM/SM-Separation-and Pre-Entry-Maneuver Program (P62) and the Entry Ballistic Program (P-66), which display IMU Gimbal Angles corresponding to current CM hypersonic trim attitude.

e) The Entry Final Phase Program (P67) which displays current CM geographic location.

5.6.10.1 The Orbital Parameter Display Routine R-30

The Orbit Parameter Display Routine R-30 may be called by the astronaut via an extended verb in order to compute and display certain orbital parameters defined below. This display will be automatically updated only when Average G is running. Also the option to select the vehicle for which these orbit parameters are to be displayed and the time of the state vector to be used for these calculations will only be available when Average G is not running.

In the normal case the apocenter altitude, pericenter altitude and the time from a reference altitude (300,000 ft. for Earth orbit, 35,000 for Lunar orbit) is displayed. If the chosen orbit does not intercept this altitude, the third display is -59B59. Under certain circumstances explained in Section 4 an additional display of time from pericenter may also be requested by the astronaut. The details of each option and acceptable astronaut responses are discussed in Section 4.

The computational logic and equations used in R-30 are shown in Fig. 6.10-1. The following is a list of important parameter definitions which apply to these figures.

$\underline{r}$, $\underline{v}$	:	State vector of the selected vehicle
P_c	:	Primary body designator $\begin{cases} 0 & \text{Earth} \\ 1 & \text{Moon} \end{cases}$
h_a	:	Apocenter altitude
h_p	:	Pericenter altitude
r_{LP}	:	Earth launch pad radius
r_{LS}	:	Lunar landing site radius
t_{ff}	:	Time from a reference altitude
t_{PER}	:	Time from pericenter
SPLERROR	:	See Section 5.6.10.2

It may be added that t_{ff} and t_{PER} represent the transfer times between the specified state vector and the specified terminal radius (to the reference altitude or to pericenter). However, if Average-G is not running, the transfer time is corrected to account for the difference between the specified state vector time and the present time.

The TFF Subroutines, TFFCONIC, TFFRP/RA, CALCTPER, and CALCTFF, which are called by R-30 to do the mathematical computations, are described in Section 5.6.10.4.

The Splash Error Computation Subroutine DELRSPL which is also called by R-30 if P-11 or P-00 is running is described in Section 5.6.10.2.

It should be noted that R-30 and DELRSPL use the standard values of the gravitational constants. This is in contradistinction to the "adjusted" values used in the TFF subroutines for the computations involved in the P-61 displays, which are used for near earth trajectories.

It should be noted that the transfer times when displayed on the DSKY are the negative of the calculated values, as in count-down. In the event that the present position lies between perigee and terminal radius, a positive time representing time since terminal point is displayed. When perigee is passed, negative time to terminal radius is displayed once again. However, for outbound hyperbolas or parabolas, positive time since terminal point is displayed.

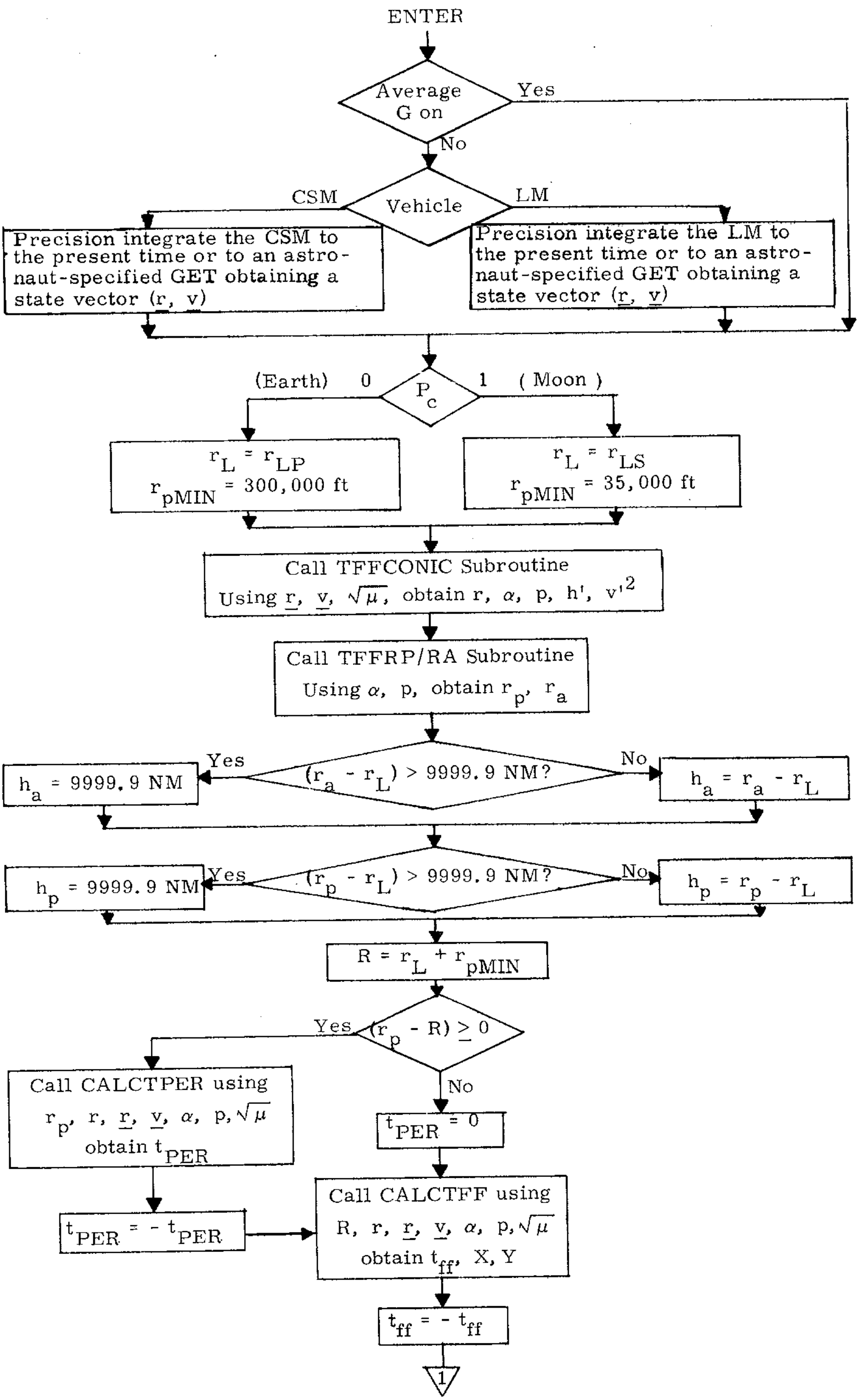


Figure 6.10-1 Orbital Parameter Display Routine
(page 1 of 2)

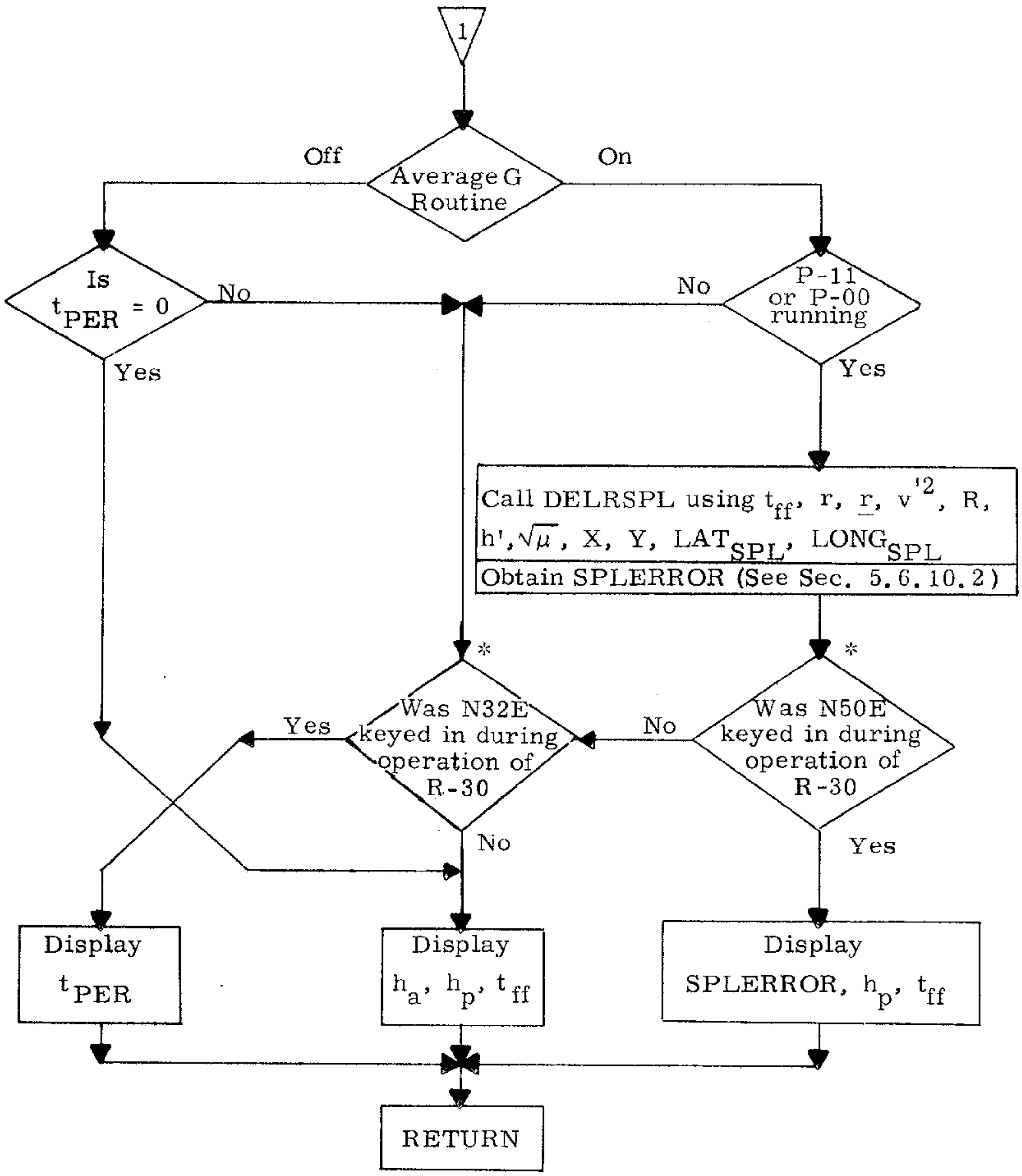


Figure 6.10-1 Orbital Parameter Display Routine
(page 2 of 2)

Note: The values of t_{ff} and t_{PER} as displayed on the DSKY are first corrected for the difference between the state vector time and the present time before being made to actually "count down" every second, i.e., the DSKY displays are automatically updated once a second, but only if Average-G is not on. This is not shown explicitly in the above figure.

*These blocks are indicative of use of routine. For complete specification of DSKY interface, see Section 4 of GSOP.

5.6.10.2 The Splash Error Computation Subroutine DELRSPL

The Splash Error Computation Subroutine DELRSPL is automatically called by the Orbital Parameter Display Routine R-30 if (and only if) the Earth Orbital Injection Monitor Program P-11 or P-00 is running.

DELRSPL provides an approximate indication of landing point miss distance, SPLERROR. The error is computed as the difference between an approximation to the range to be covered and the range to the predicted target point. This assumes that all range components lie in the same plane. The range to the predicted target point is the distance between the present position and the target, as specified by the target coordinates and an estimate of the time of impact. The flight time is the sum of the conic free-fall time to 300, 000 ft altitude and an estimate of the entry duration. The approximate range to be covered is the sum of the conic free-fall range to 300, 000 ft altitude and an estimate of the entry range. However, if the conic path from the present position does not intercept the 300, 000 ft altitude, or the vehicle's present location has already passed below the intercept altitude, then the miss distance is defined and displayed as minus the range to the target. In both cases, the sign convention is: negative ranges correspond to undershooting the target.

DELRSPL uses the VGAMCALC and TFF/TRIG Subroutines which are described in Section 5. 6.10. 4. It also uses one of the Latitude-Longitude Subroutines described in Section 5. 5. 3 for converting a known latitude and longitude into a position vector. Further DELRSPL uses the empirical formulas for the entry range and entry time given by the AUGEKUGEL Subroutine.

The coefficient 3437. 7468 which is used twice in the DELRSPL calculations is a conversion factor of the reentry calculations, and is defined in Section 5. 7 of this GSOP. (It is called "ATK" in that section.)*

The parameters LAT_{SPL} and $LONG_{SPL}$ are the geodetic latitude and longitude of the splash point. These quantities are either pad loaded, uplinked, or entered via the DSKY by the astronaut.

It should be noted that the Earth Orbital Insertion Monitor Program P-11 has automatic displays, which are not associated with R-30 or DELRSPL. These are described in Earth Orbital Insertion Monitor Program Section (5. 3. 5. 6).

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*See note on page 5. 7-2.

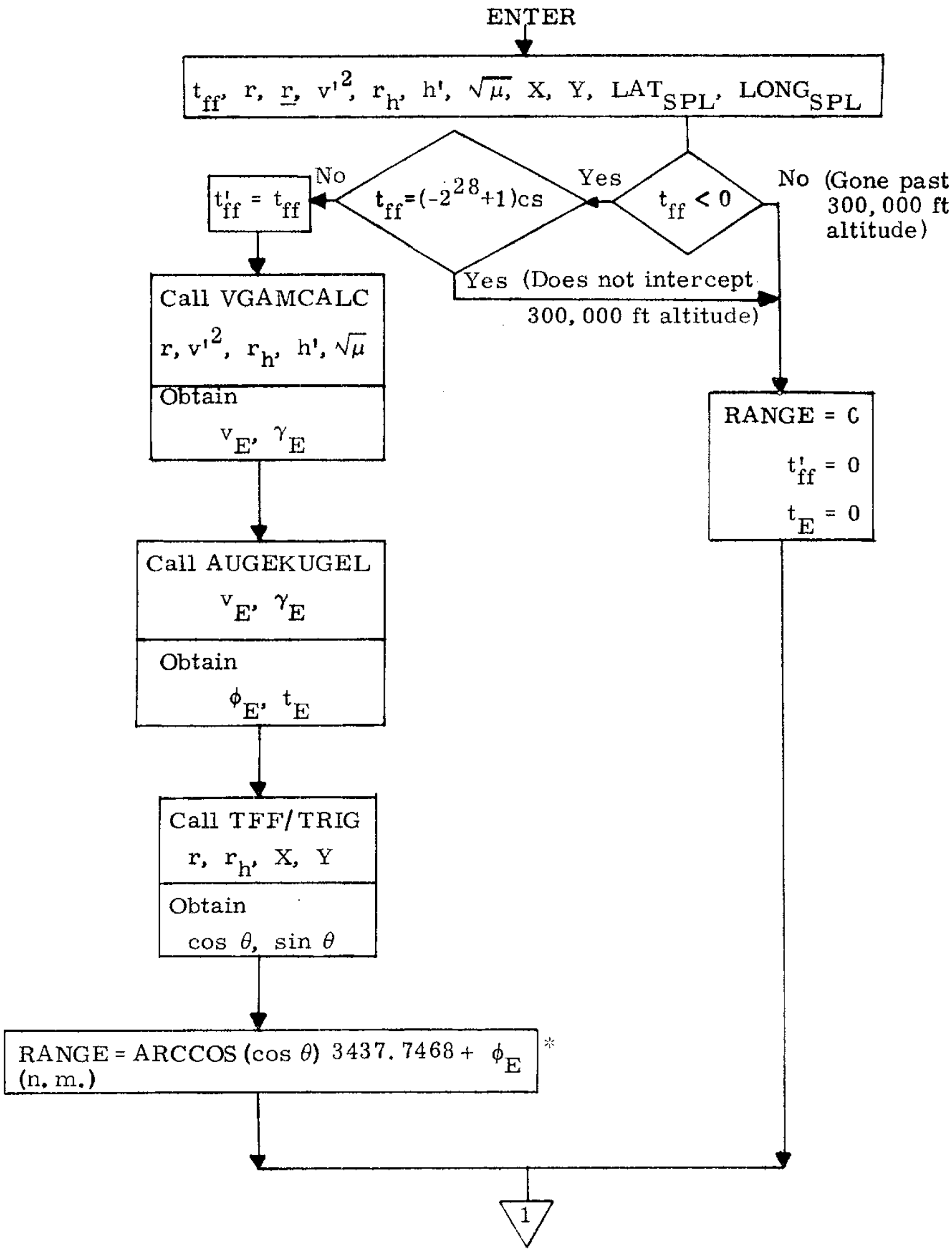

Figure 6. 10-2 Splash Error Computation Subroutine
(page 1 of 2)

*See note on page 5. 7-2.

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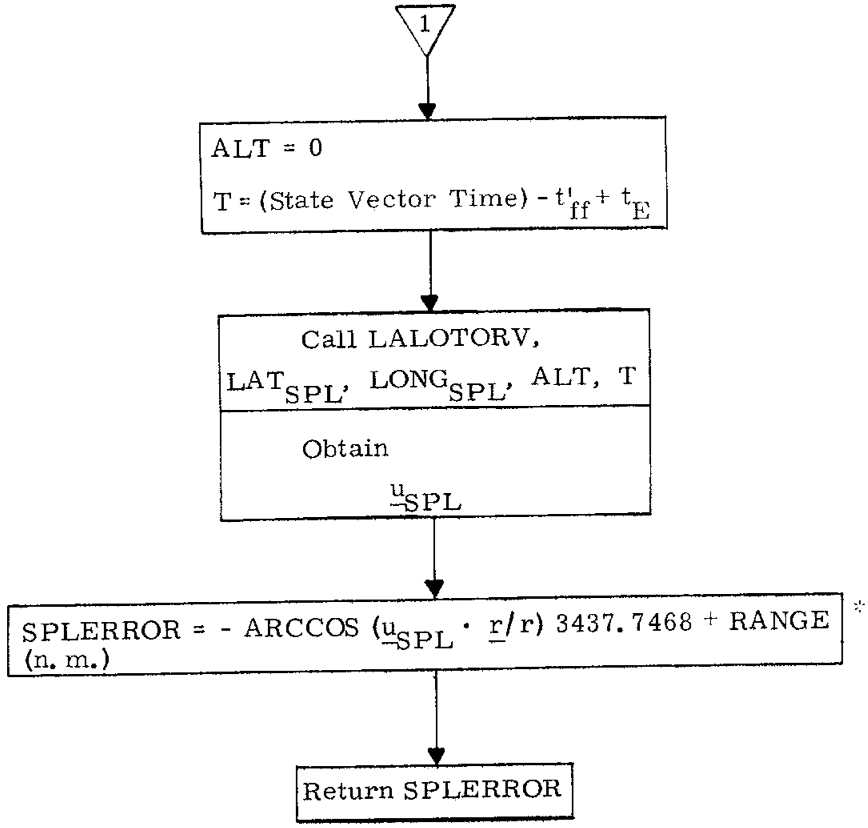


*See note on page 5.7-2.

Figure 6.10-2 Splash Error Computation Subroutine (page 2 of 2)

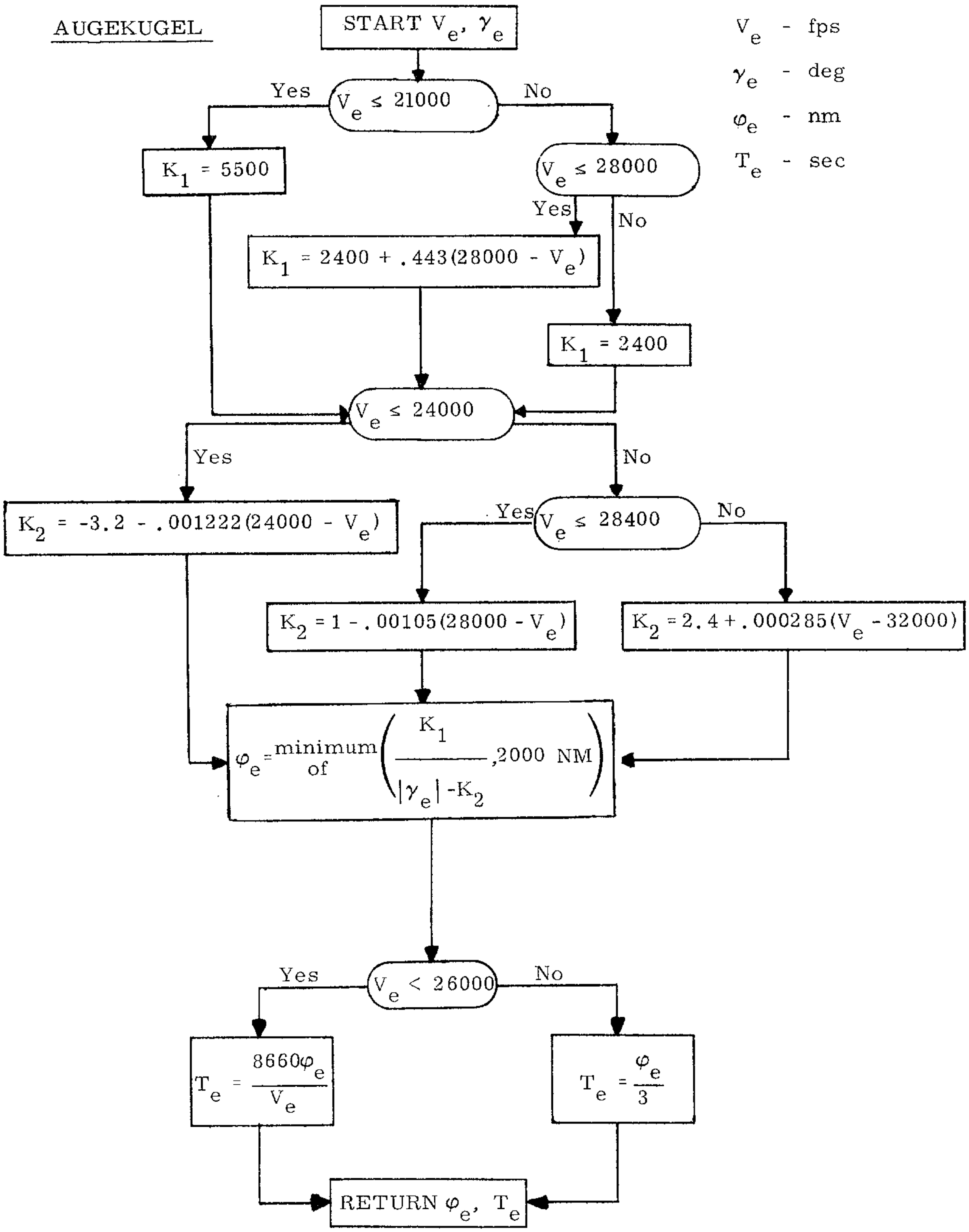


Figure 6.10-3 Empirical Relations for Entry Range

5.6.10.3 The Entry Preparation Program (P-61) Displays

The following display computations are made in Program P-61:

GMAX	:	Maximum predicted deceleration for entry at the nominal roll angle (L/D=0.3).
VPRED	:	Predicted inertial velocity at an altitude of 400,000 feet.
GAMMAEI	:	Predicted flight path angle between the inertial velocity vector and the local horizontal at an altitude of 400,000 feet.

The altitude of 400,000 feet is referenced to the Fischer ellipsoid, and a two step iteration is made to calculate this radial length.

The following three display parameters are required to initialize the Entry Monitor System, EMS:

RTGO	:	Range-to-go from the EMS altitude to the predicted splash point.
VIO	:	Predicted inertial velocity at the EMS altitude.
TTE	:	Time to go from the current time to the EMS altitude (negative and decreasing). TTE is decremented every two seconds until 0.05 g is sensed.

The EMS altitude is measured above the Fischer ellipsoid and is stored in erasable memory during the pre-launch erasable load. (EMSALT)

The principal computation required for these display parameters is the conic calculation of time to a radius of specified length. The intersection of a conic with an altitude above the Fischer ellipsoid is a two step iteration. An initial calculation is made using a guess of the radius length. The latitude of the conic with this assumed length is then used to calculate the radius of the Fischer ellipsoid. This is then used to calculate a second radial length, and display quantities are referred to this second radius. Additional iterations are not required to achieve the necessary display accuracy.

The computation logic for the P-61 displays is presented in Fig. 6.10-4. These computations use the TFF Subroutines which are described in Section 5.6.10.4.

The earth gravitation constant used in these calculations may be increased to recognize missions under consideration that are confined to low inclination angles from the equator where oblate earth effects can be significant.* The drag loss in reaching the EMS point (0.05G) is also approximated for these display computations.

Besides the TFF Subroutine symbols (which are defined in Section 5.6.10.4), and the symbols which are defined above, the following additional symbols are used in Fig. 6.10-4:

THETA	:	The angle between the present position and the initial target vector at the nominal time of arrival. (This is a reentry variable- See Section 5.7).
KTETA1	:	Time of flight constant = 1100
$\underline{r}$, $\underline{v}$	:	State vector of the vehicle at the present time

*The choice of whether the spherical earth gravitational constant or the adjusted constant is used depends on the value of the pad loaded EMS altitude.

r_G : first guess of the terminal radius

$= r_{PAD} + 284,643$ feet

$= 21,194,545$ feet

(since $r_{PAD} = 6,373,338$ m $= 20,909,902$ ft)

μ_1 : Gravitational constant used with an adjusted value that is 0.12% greater than the standard value of the gravitational constant, used if and only if the EMS altitude is less than a certain value.

μ_E : Gravitational constant for spherical earth.

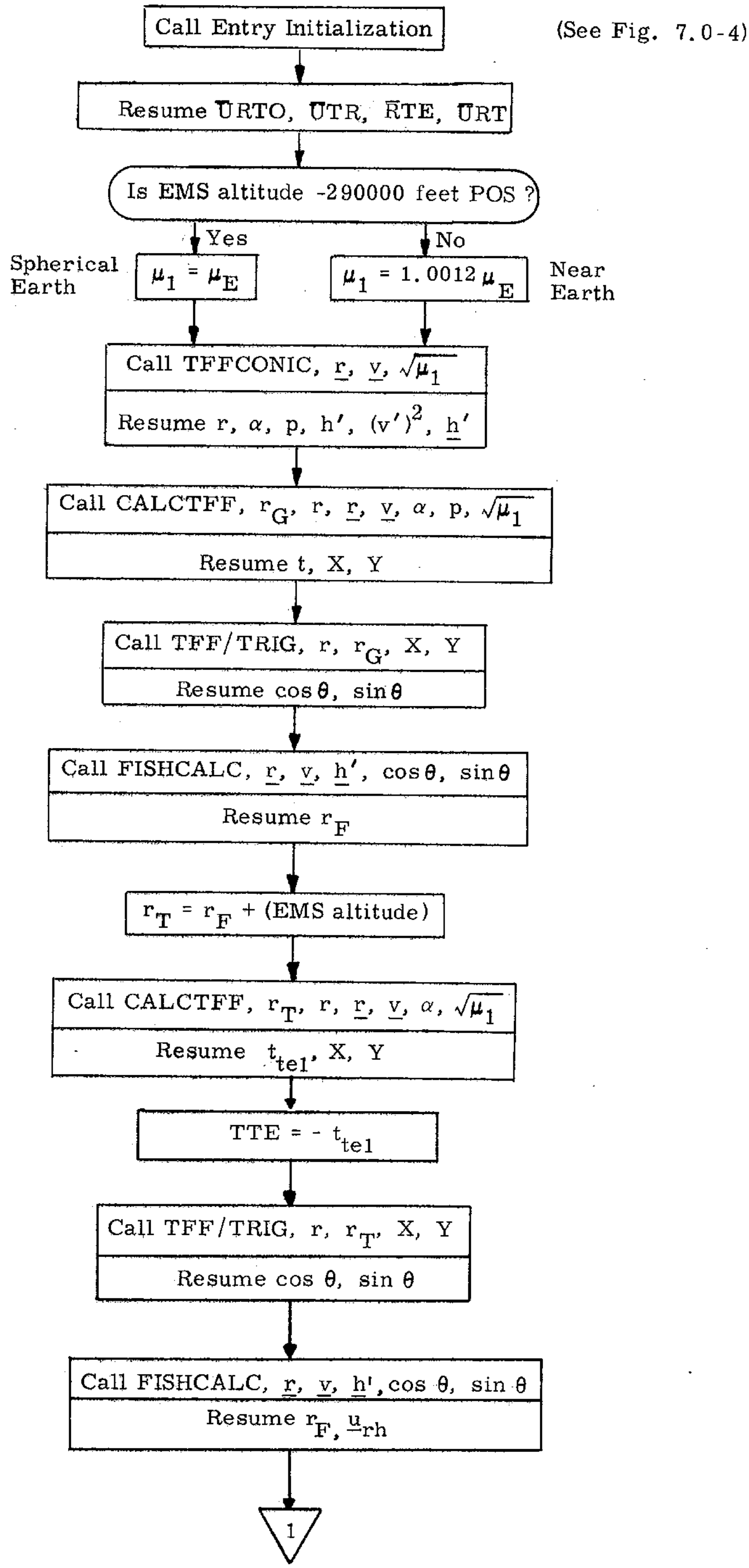


Figure 6.10-4 Display Calculations for P-61
(page 1 of 3)

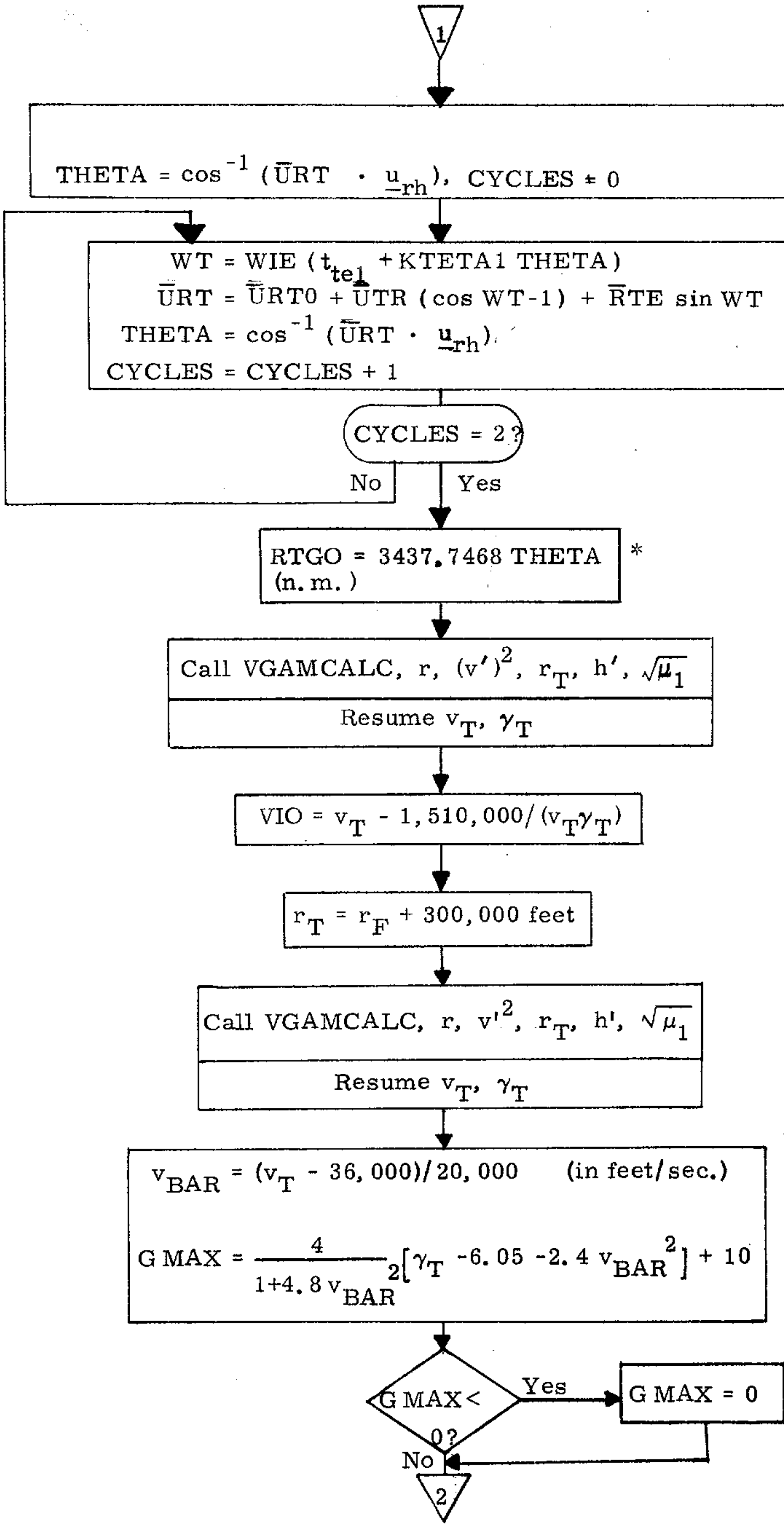
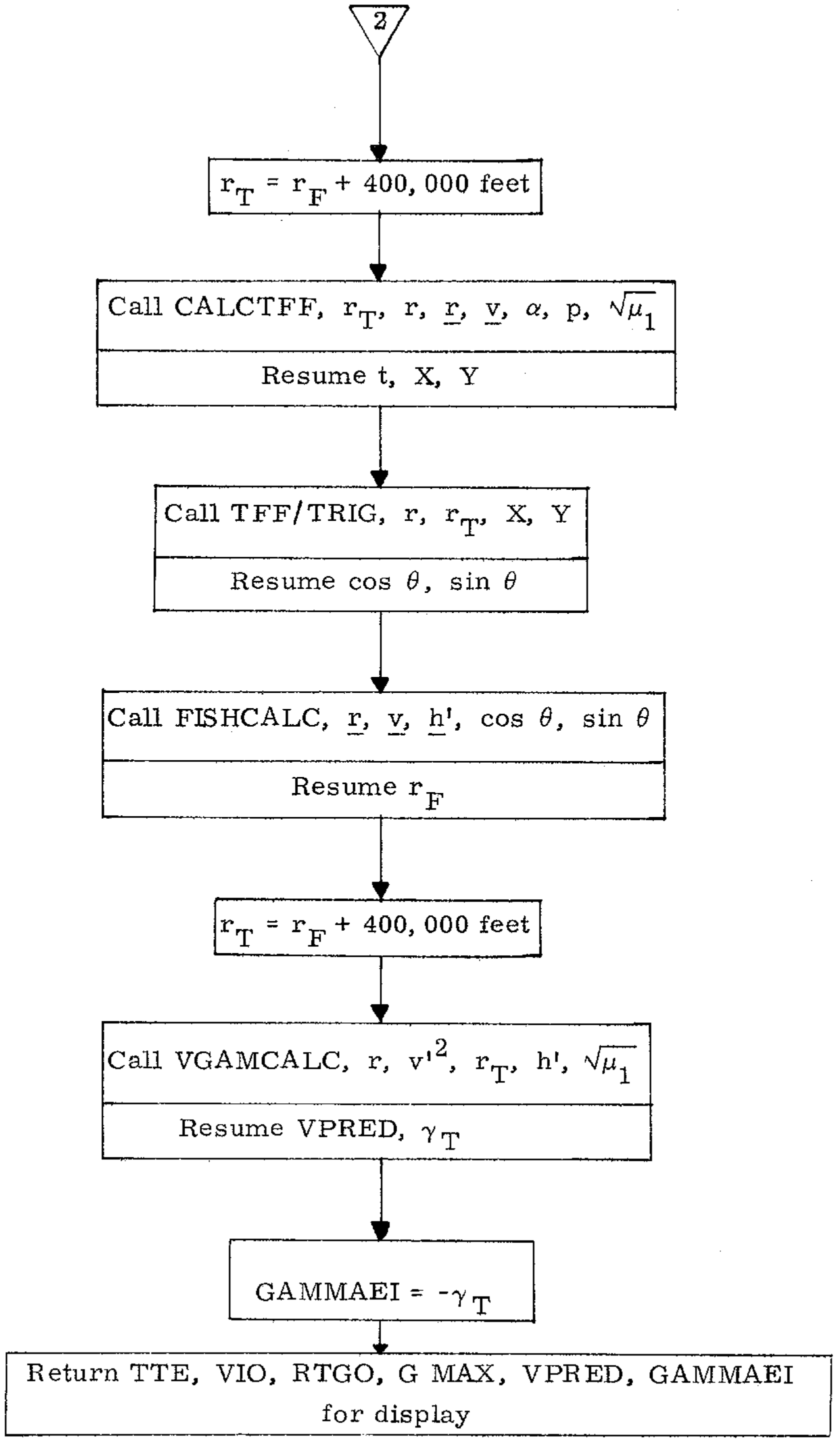


Figure 6.10-4 Display Calculations for P-61
(page 2 of 3)

*See note on page 5.7-2.



*

Figure 6.10-4 Display Calculations for P-61
(page 3 of 3)

*See note on page 5.7-2.

5.6.10.4 The TFF Subroutines

The TFF Subroutines are used to calculate various conic parameters, such as the semi-latus rectum, the semi-major axis, apocenter and pericenter radii, the transfer time from an initial or present position to specified terminal radii (which may include apocenter and pericenter), the transfer angle of such transfers, and the velocity magnitude and flight-path angle at the terminal position. For the subroutines, the terminal radius is defined to be on the inbound side of the trajectory.

It should be noted that the transfer time displayed on the DSKY is the negative of the calculated value, as in count-down, when the present position approaches the terminal radius. In the event that the present position lies between terminal radius and perigee, a positive time representing time since terminal point is displayed. When perigee is passed, negative time to terminal radius is displayed for ellipses. However, for outbound hyperbolas or parabolas, positive time since terminal point is displayed.

The TFFCONIC Subroutine (Fig. 6.10-5) calculates various conic parameters and stores them for short-term use. It also computes other intermediate quantities to avoid duplication by following subroutines.

The TFFRP/RA Subroutine (Fig. 6.10-6) calculates pericenter and apocenter radii for the general conic. For hyperbolic or parabolic orbits, or for those elliptic orbits where the apocenter radius exceeds the scaling, r_a is set equal to $r_{a\,MAX}$, which corresponds to the largest number expressible in the double precision CMC word.

The CALCTFF/CALCTPER Subroutine (Fig. 6.10-7) calculates the time of free-fall to a given radius or to pericenter depending on the entry point to the subroutine. The subroutine also calculates the quantity Y required by the TFF/TRIG routine in its computation of the transfer angle. In the event that the terminal radius does not lie on the present conic, t is set to the largest time expressible in double precision, $2^{28} - 1$ centiseconds, as a flag. After TFFCONIC has been called, CALCTFF/CALCTPER may be called as desired with different terminal radii to obtain the various times of free fall to these radii for the same conic.

The TFF/TRIG Subroutine (Fig. 6.10-8) calculates the sine and cosine of the transfer angle θ from the initial position vector to the terminal position vector. The computation of the intermediate parameter Y is done during the appropriate branch in CALCTFF/CALCTPER and saved temporarily. Thus the caller of TFF/TRIG may make only one call for each call to CALCTFF/CALCTPER.

FISHCALC Subroutine (Fig. 6.10-9) calculates the sine of the geocentric latitude of the terminal position vector, and then calls the Fischer Ellipsoid Radius Routine (part of the Latitude-Longitude Subroutines of Section 5.5.3) to obtain the radius of the Fischer ellipsoid, using Eq. (5.3.1).

VGAMCALC Subroutine (Fig. 6.10-10) calculates the velocity magnitude at the terminal position and the flight-path angle relative to the local horizontal at the terminal position. The user of the subroutine must supply the proper sign to the flight-path angle.

Nomenclature of the TFF Subroutines:

$\underline{r}$ or $\underline{r}_0$, $\underline{v}$ or $\underline{v}_0$	}	Present or initial state vector
μ	:	Gravitational constant
α	:	Reciprocal of the semi-major axis of conic (negative for hyperbolas)
p	:	Semi-latus rectum of conic
e	:	Eccentricity of conic
r_p	:	Pericenter radius
r_a	:	Apocenter radius
r_h	:	Terminal position radius
v_h	:	Velocity magnitude at terminal position
γ_E	:	Flight-path angle relative to local horizontal at the terminal position
S_{TFF}	:	Switch set to 0 or 1 according to whether the transfer time is calculated to the terminal position radius (CALCTFF) or to the pericenter (CALCTPER)
θ	:	Transfer angle
r_F	:	Radius of Fischer ellipsoid along the terminal position unit vector
t	:	Conic transfer time from the present or initial position to the terminal position (CALCTFF) or to pericenter (CALCTPER)
$r_{a_{MAX}}$	:	The largest apocenter radius expressible in the double precision CMC word at the relevant scaling.

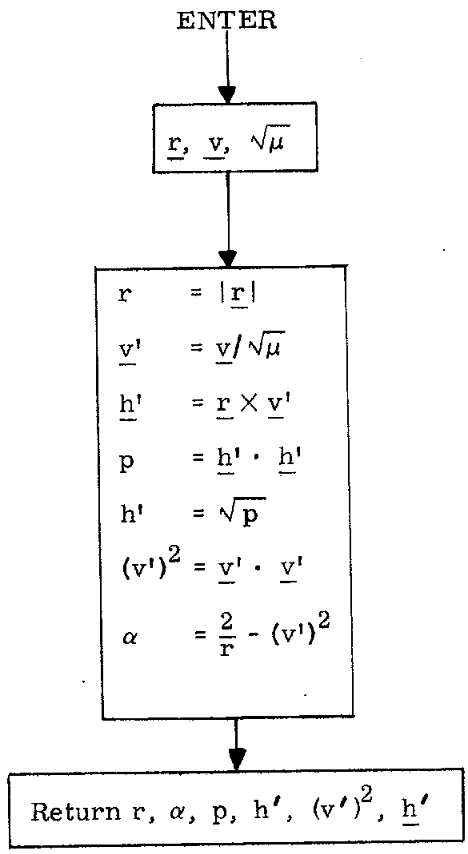


Figure 6.10-5 TFFCONIC Subroutine

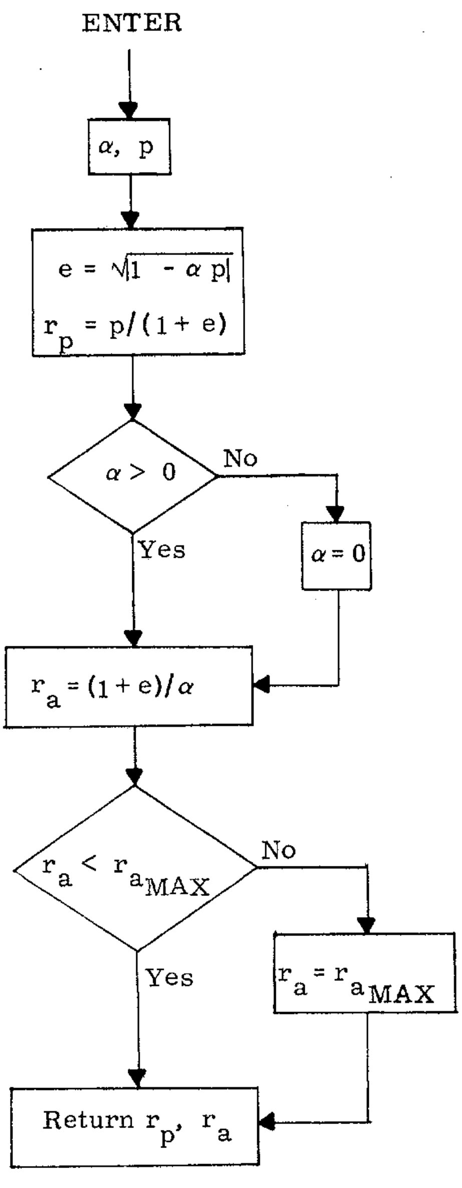
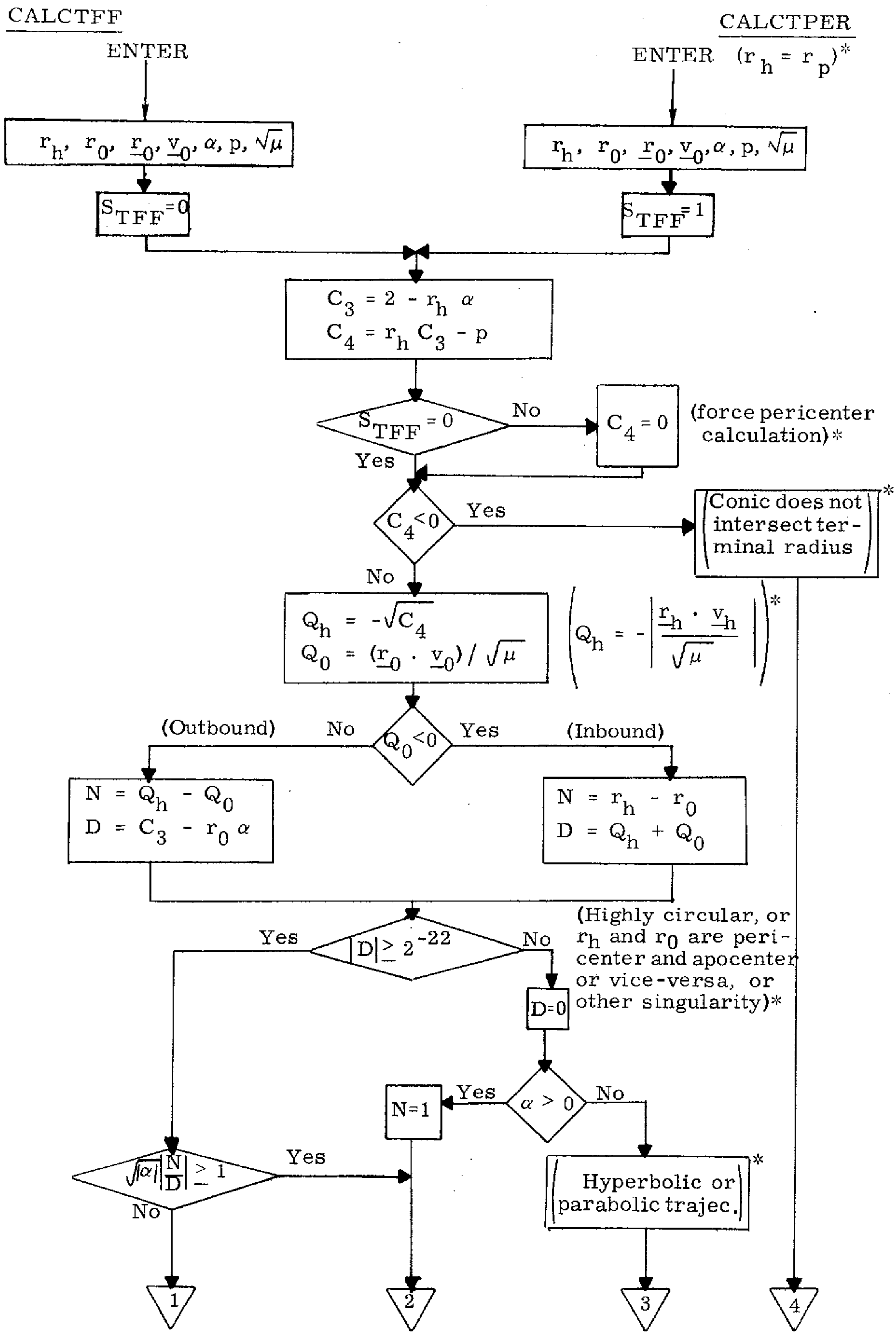
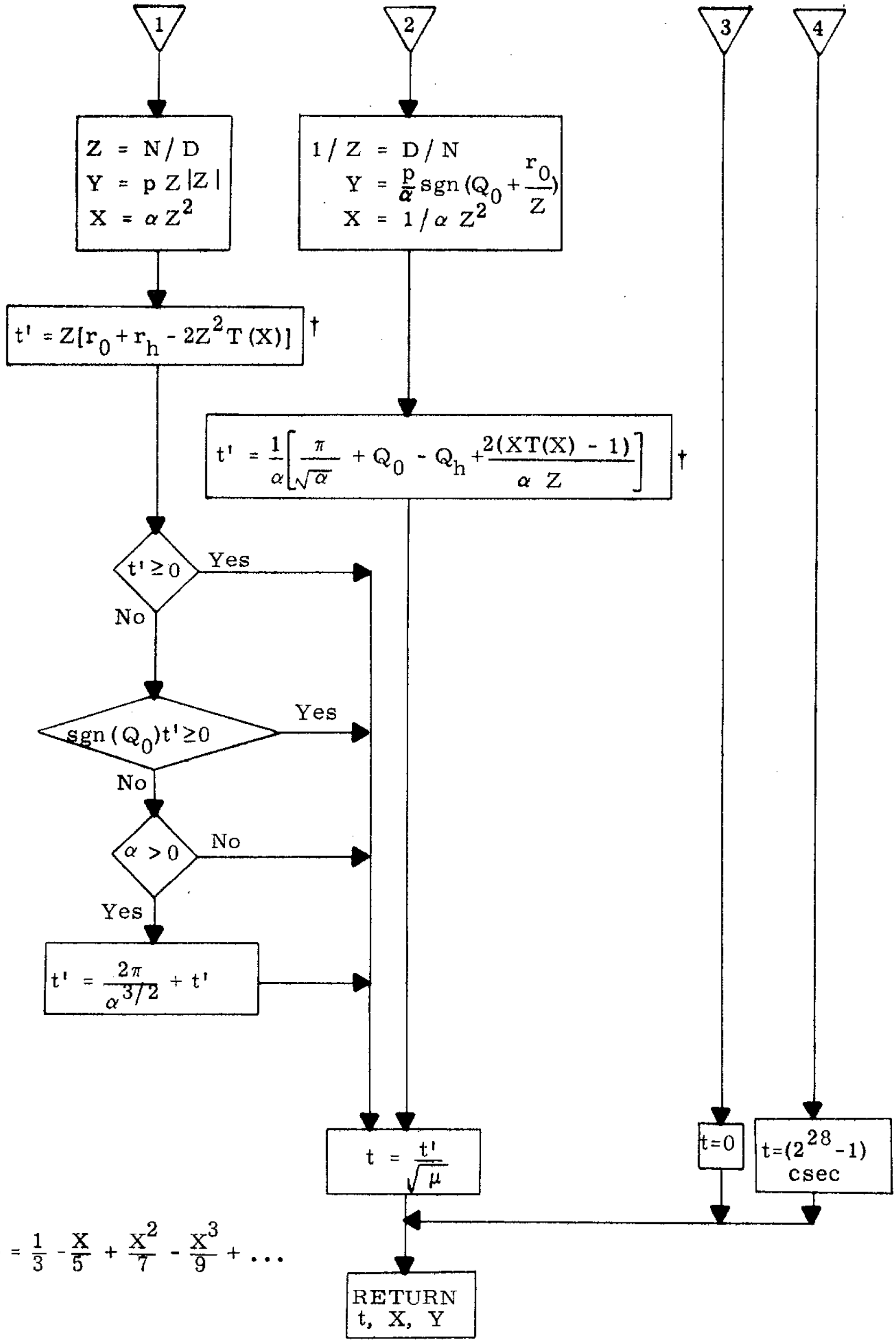


Figure 6.10-6 TFFRP/RA Subroutine



*Supplementary Information

Figure 6.10-7 CALCTFF/CALCTPER Subroutine (page 1 of 2)



$^{\dagger}T(X) = \frac{1}{3} - \frac{X}{5} + \frac{X^2}{7} - \frac{X^3}{9} + \ldots$

Figure 6.10-7 CALCTFF / CALCTPER Subroutine
(page 2 of 2)

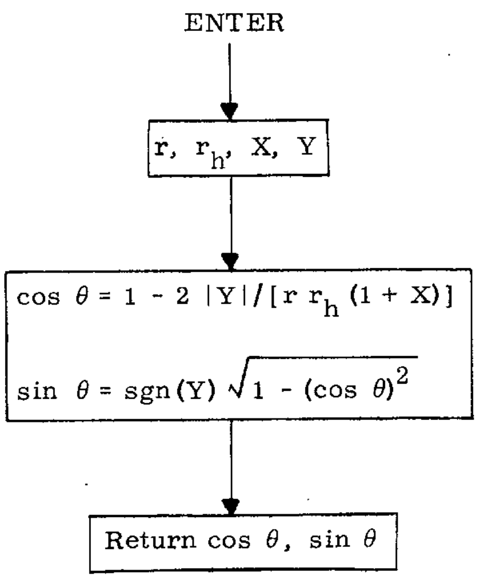


Figure 6.10-8 TFF/TRIG Subroutine

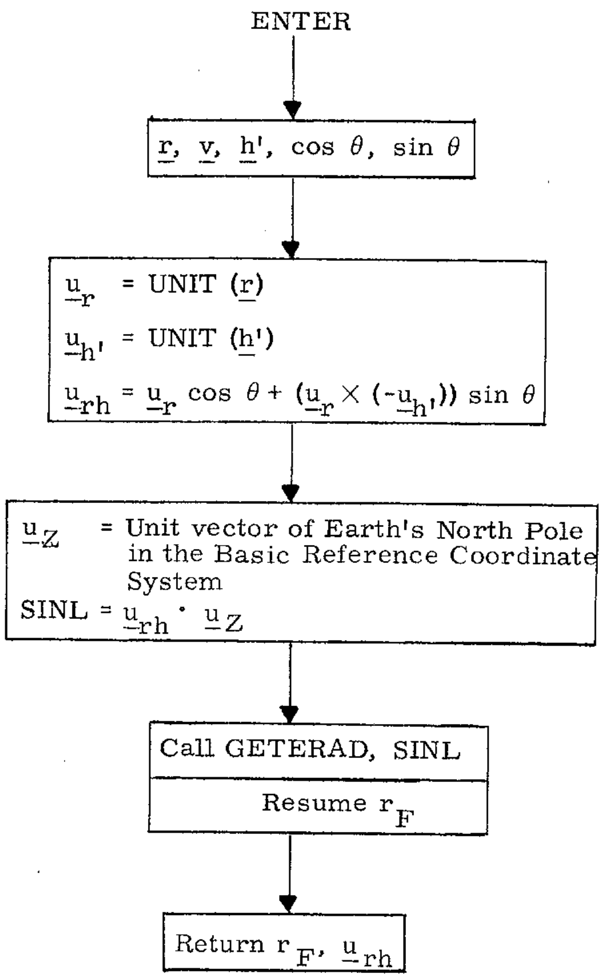


Figure 6.10-9 FISHCALC Subroutine

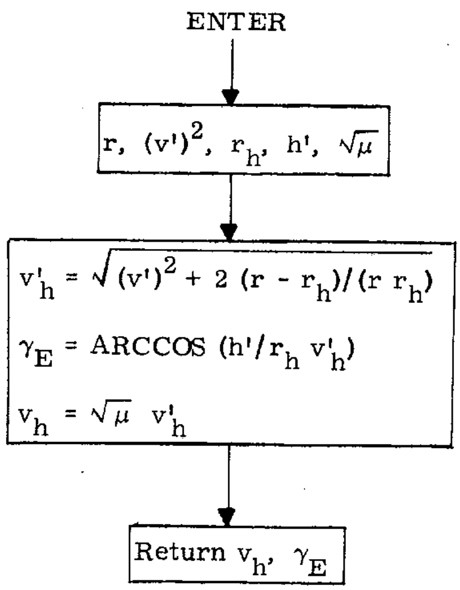


Figure 6.10-10 VGAMCALC Subroutine

5.6.10.5 Equations Used in Calculating the Conic Time of Flight

The time of flight along any conic trajectory proceeding from an arbitrary radius $\underline{r}_0$ in the direction of velocity $\underline{v}_0$ to a radius of specified length r_h is provided by Kepler's equation. The following formulation, while not quite universal, is explicit and direct. With a single equation, time of flight can be calculated for conic paths that are hyperbolic or parabolic and for those elliptic paths having $|\Delta E| \leq 90^\circ$. An alternate equation is needed for the elliptic paths having $|\Delta E| > 90^\circ$. Highly circular orbits are excluded by the constraint of length for the terminal radius. In the absence of a vector direction, highly circular cases are not treated.

The two conic parameters that appear in the time of flight equations are given by:

Angular momentum: $\underline{H} = \underline{r}_0 \times \underline{v}_0$

Semi-latus rectum: $$p = \frac{\underline{H} \cdot \underline{H}}{\mu} \qquad (6.10.1)$$

reciprocal semi-major axis: $$\alpha = \frac{2}{r_0} - \frac{\underline{v}_0 \cdot \underline{v}_0}{\mu}$$

The equation for reciprocal semi-major axis α has the usual sign convention: α is negative for hyperbolic orbits, zero for parabolic orbits, and positive for elliptic orbits.

Two intermediate quantities, given in two useful equivalent forms, are:

$$Q_0 = \text{sgn}(\dot{r}_0)\sqrt{r_0(2 - \alpha r_0) - p} = \underline{r}_0 \cdot \underline{v}_0/\sqrt{\mu}$$

$$Q_h = \text{sgn}(\dot{r}_h)\sqrt{r_h(2 - \alpha r_h) - p} = \underline{r}_h \cdot \underline{v}_h/\sqrt{\mu} \qquad (6.10.2)$$

It is these quantities that exclude highly circular orbits, since being proportional to radial velocity they become uselessly small.For TFF application to reentry, $\underline{r}_h$ is on the returning side, so the sign of $\dot{r}_h$ is chosen as negative. Excepting ellipses for which $|\Delta E| \geq 90^\circ$, the conic time of flight is given by

$$t = (Z/\sqrt{\mu})(r_0 + r_h - 2 Z^2 T(X)) \qquad (6.10.3)$$

where Z is the obtained by one of the following:

if Q_0 and Q_h have opposite signs:

$$Z = (Q_h - Q_0)/(2 - \alpha(r_0 + r_h)) \qquad (6.10.4a)$$

if Q_0 and Q_h have like signs:

$$Z = (r_h - r_0)/(Q_h + Q_0) \qquad (6.10.4b)$$

and where if

$$\alpha Z^2 < 1.0$$

then

$$X = \alpha Z^2 \qquad (6.10.5a)$$

For those elliptic cases having $|\Delta E| \geq 90^{\circ}$,

then

$$\alpha Z^2 \geq 1.0$$

and

$$X = 1/(\alpha Z^2) \qquad (6.10.5b)$$

The conic time of flight is given by

$$t = \left(\frac{1}{\alpha \sqrt{\mu}}\right) \left(\frac{\pi}{\sqrt{\alpha}} - Q_h + Q_0 + \frac{2\,(XT\,(X) - 1)}{\alpha Z}\right) \qquad (6.10.6)$$

The function T (X) is defined by the series

$$T(X) = \frac{1}{3} - \frac{X}{5} + \frac{X^2}{7} - \frac{X^3}{9} + \ldots$$

$$= \frac{\sqrt{X} - \arctan(\sqrt{X})}{X\sqrt{X}} \qquad \text{if } 0 \leq X \leq 1 \qquad (6.10.7)$$

$$= -\frac{\sqrt{-X} - \operatorname{arctanh}(\sqrt{-X})}{X\sqrt{-X}} \qquad \text{if } X \leq 0$$

The equations (6.10.4) are equivalent, except that each has a point of indeterminancy. The selection used in Eq. (6.10.4) excludes the indeterminant point of each from the region of application. The geometric significance of opposite signs for the Q's is that $\underline{r}_0$ and $\underline{r}_h$ lie on opposite sides of the conic axis of symmetry. For like signs, the radii lie on the same side. In application to spacecraft landing, $\dot{r}_h$ is negative and Eq. (4a) applies to outbound trajectories while Eq. (4b) to inbound ones.

The choice of Eq. (6.10.3) or Eq. (6.10.6) depends on whether αZ^2 is less than or greater than one. Since both Eq. (6.10.3) and Eq. (6.10.6) reduce to the same form when $X = 1.0$, the division made above is arbitrary. Equation (6.10.3) gives both positive and negative values, indicating time to terminal point, and time since terminal point passage. A positive value, time to terminal point, is always provided for the ellipse by replacing negative t by $2\pi/\alpha\sqrt{\alpha\mu} + t$. Equation (6.10.6) yields positive values only.

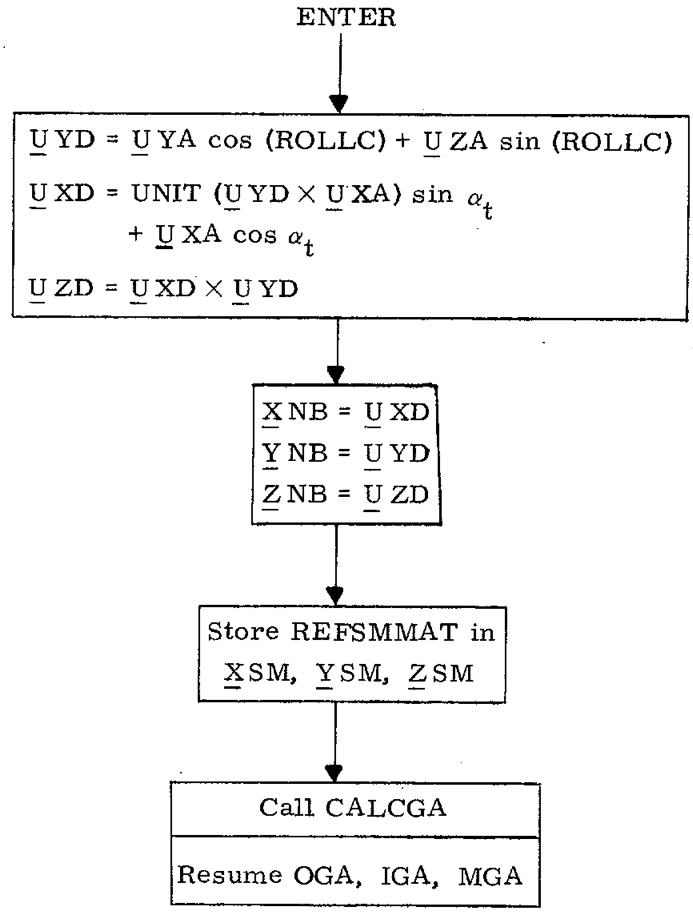
The time of flight equations are exact and involve approximations only to the extent that the function T (X) is represented by a polynomial. Furthermore, Eq. (6.10.3) is continuous as the conic changes from hyperbolic to elliptic.

5.6.10.6 Desired Gimbal Angle Display for Entry Attitude used during P62 and P66

In the CM/SM Separation and Pre-Entry Maneuver Program (P62) and in the Entry Ballistic Program (P66), the desired IMU Gimbal Angles corresponding to entry attitude (hypersonic trim with respect to the computed wind axis) at the present time are calculated and may be displayed.

The calculations are shown in Figure 6.10-11. The following nomenclature is used:

KWE	:	Equatorial Earth Rate = 1546.70168 ft/sec. (this is a reentry constant - See Section 5.7).
$\underline{u}_W$	:	Unit vector of Earth's North Pole direction, left by pad load.
$\underline{U}$XA, $\underline{U}$YA, $\underline{U}$ZA	:	The trajectory triad computed each two seconds in reference coordinates by CM / POSE (part of the DAP calculations).
$\underline{U}$XD, $\underline{U}$YD, $\underline{U}$ZD	:	Desired body triad for trimmed flight with respect to the relative velocity vector, using roll command and trim angle of attack.
OGA	:	Outer Gimbal Angle - Roll
IGA	:	Inner Gimbal Angle - Pitch
MGA	:	Middle Gimbal Angle - Yaw
ROLLC	:	Commanded Roll Angle
α_t	:	Hypersonic Trim Angle of Attack for CM, stored in erasable memory during the prelaunch memory load.



where:

$$\underline{U}XA = -\text{ UNIT }(\underline{V} + KWE\ (\text{UNIT }(\underline{r}) \times \underline{u}_W))$$

$$\underline{U}YA = -\text{ UNIT }(\underline{U}XA \times \text{UNIT }(\underline{r}))$$

$$\underline{U}ZA = (\underline{U}XA \times \underline{U}YA)$$

Figure 6.10-11 Calculation of Gimbal Angles Corresponding to Entry Attitude

5.6.10.7 Entry Terminal Display of CM Geographic Location

During the Entry Final Phase Program (P67) after the relative wind velocity becomes less than VQUIT (1000 fps), the Entry guidance ceases to compute a new roll command for specifying the CM lift vector direction. Instead the current value is kept constant and the DAP continues attitude control for the remainder of the flight. The guidance mode SELECTOR is changed to compute the CM geographic coordinates for display to the crew.

The computation is as follows:

Call LATITUDE-LONGITUDE Subroutine (Section 5.5.3) with the parameters as defined there having the following values:

$\underline{r}$ = Average G position vector
t = time of position vector
P = 0
F = 0

Obtain

Lat = present geodetic latitude
Long = present longitude

5.6.11 LGC INITIALIZATION

The LGC initialization procedure prior to LM separation is a manual operation which does not involve a numbered CMC program. After the LGC is activated the first requirement is to synchronize the LGC clock with that of the CMC. This is a count-down and mark procedure described in R-33, CMC/LGC Clock Synchronization Routine of Section 4, to obtain an average clock difference which is then used to increment the LGC clock. The CMC and LGC clock synchronization can also be checked by the Mission Ground Control Center using telemetry down-link data, which can provide a more precise difference to increment the LGC clock.

Next, the following parameters are voice-linked from the CSM or uplinked from the earth to the LM to be entered into the LGC:

1) $\underline{r}_C$: CSM position vector

2) $\underline{v}_C$: CSM velocity vector

3) t_C : CSM state vector time

4) $\underline{r}_{LS}$: lunar landing site vector in moon-fixed coordinates

5) t_0 : time difference between zero GET and July 1.0, 1971 universal time.

6) P_C : planet identifier

All of the above parameters are in octal, and all are double precision except item 5, t_0, which is triple precision and item 6, P_C, which is one bit. In the lunar landing mission the above items 1 through 4 are normally determined by the CMC Orbit Navigation Program P-22.

5.6.12 CMC IDLING PROGRAM

This program is used to maintain the CMC in a state of readiness for entry into any other program. While the idling program is in operation, the Coasting Integration Routine (Section 5.2.2) is used to advance the estimated CSM state vector (and the estimated LM state vector when the LM is not on the surface of the moon) to approximately current time. This procedure has the lowest priority of all programs, and is performed only when no other program is active. This periodic state vector extrapolation is not necessary from a theoretical point of view, but does have two practical purposes. First, it is advisable to maintain current (or at least nearly current) state vector estimates in case an emergency situation arises. Second, a significant amount of computation time is transferred from a period of high computer activity (navigation measurement processing, targeting, etc.) to a period of low activity.

The state vectors of both vehicles are extrapolated even if the vehicles are attached. There exists a special DSKY verb by means of which the LM state vector is made equal to the CSM state vector. This verb is used in conjunction with the LM/CSM separation maneuver in order to properly initialize the LM state vector. The use of this verb is followed by astronaut selection of the Target ΔV Program (Section 5.6.14) to incorporate the LM separation maneuver $\Delta \underline{V}$.

In order to use the Coasting Integration Routine in an efficient manner, the maximum value for the integration time step, Δt_{max}, is computed as described in Section 5.2.2.5. The value of Δt_{max} is a function of radial distance and varies from step to step. Let t_C be the time associated with the estimated CSM state vector and t_1 be the current time. The estimated CSM state vector is extrapolated ahead when

$$t_1 > t_C + 4 \Delta t_{max} \qquad (6.12.1)$$

The integration is terminated when Δt_{max} is more than the integration time-to-go. In this manner no extra and smaller-than-maximum integration time steps are performed, and the periodic integration is accomplished most efficiently.

The estimated LM state vector (if applicable) is then extrapolated to the CSM state vector time.

The error transition matrix W (see Section 5.2.2.4) is extrapolated with the estimated CSM (LM) state vector if ORBWFLAG (RENDWFLG) indicates that the W matrix is valid. ORBWFLAG is defined in Sections 5.2.4.5 and 5.2.6.4, and RENDWFLG in Section 5.2.5.2.

The logic for the periodic state vector extrapolation is illustrated in Fig. 6.12-1. The variables D and V are indicators which control the Coasting Integration Routine. The quantities $\underline{x}_C$ and $\underline{x}_L$ are the estimated CSM and LM state vectors, respectively, and $\underline{x}$ is a temporary state vector used for integration. Refer to Section 5.2.2.6 for precise definitions of these items. The switch SURFFLAG indicates whether or not the LM is on the surface of the moon. This flag is set to one (zero) by means of a special DSKY verb by the astronaut when he receives voice confirmation that the LM has landed on (lifted off from) the lunar surface.

As shown in the figure, time synchronization of the two state vectors is achieved and maintained by this program. The purpose of the state vector synchronization is to guarantee correct W matrix extrapolation during rendezvous navigation.

In order to permit correction of wrong erasable memory parameters which have caused or could cause an invalid and excessively lengthy integration process to begin, there is an emergency special DSKY verb to terminate or inhibit the Coasting Integration Routine. This special verb causes the following to occur:

1. If the Coasting Integration Routine is in operation, it is terminated at the end of the current time step.
2. The current program is terminated.
3. The CMC Idling Program (P-00) is activated.
4. The P-00 state vector test is bypassed so that no state vector integration test occurs until P-00 is reselected.

Note that this operation does not maintain state vector synchronization and can, therefore, cause incorrect W matrix extrapolation in rendezvous navigation.

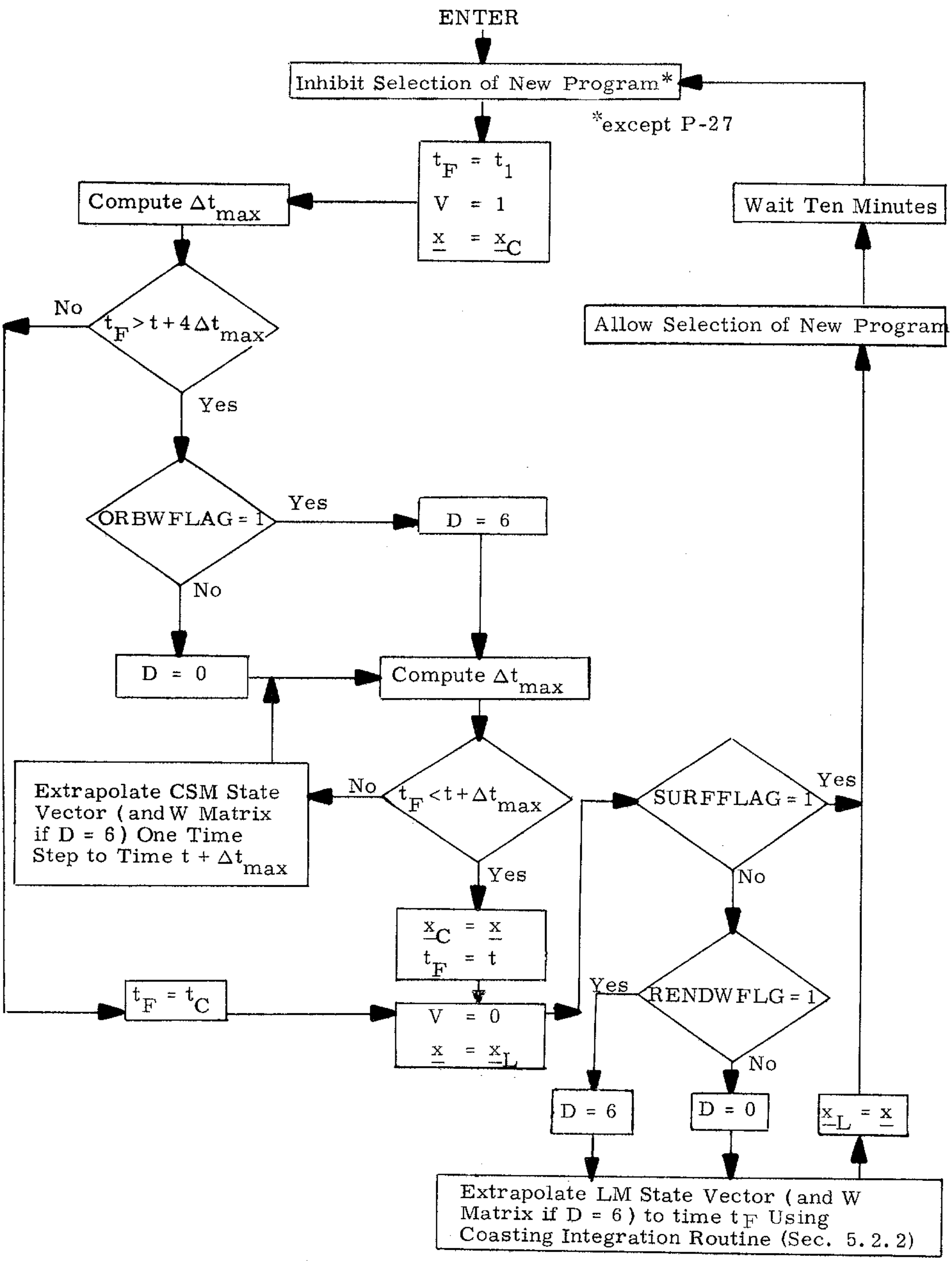


Figure 6.12-1 CMC Idling Program State Vector Extrapolation Logic Diagram

5.6.13 IMU COMPENSATION

The IMU Compensation is designed to compensate for PIPA bias and scale factor error and at the same time accumulate gyro torquing commands necessary to compensate for the associated bias and acceleration caused gyro drifts. The correction to the PIPA's is

$$PIPA_C = (1 + SFE_I)\ PIPA_I - BIAS_I\ \Delta t$$

where

$PIPA_C$ is the compensated data for the I^{th} PIPA denoted $PIPAX_C$, $PIPAY_C$, $PIPAZ_C$

$$SFE = \frac{SF - SF_{nom}}{SF_{nom}} \quad \text{(erasable load*)}$$

$$SF = \text{Scale-factor } \frac{CM/Sec}{Pulse}$$

$BIAS_I$ is the bias for the I^{th} PIPA (an erasable load)

The compensated data is then used to compute the IRIG torquing necessary to cancel the NBD, ADIA, and ADSRA gyro coefficients. The computations are

$$XIRIG = -ADIAX\ PIPAX_C + ADSRAX\ PIPAY_C - NBDX\ \Delta t$$

$$YIRIG = -ADIAY\ PIPAY_C + ADSRAY\ PIPAZ_C - NBDY\ \Delta t$$

$$ZIRIG = -ADIAZ\ PIPAZ_C - ADSRAZ\ PIPAY_C + NBDZ\ \Delta t$$

---

*The term "erasable load" refers to data entered into CMC erasable memory just prior to launch.

where

XIRIG, YIRIG, ZIRIG are gyro drift compensation

NBDX, NBDY, NBDZ are gyro bias drifts (an erasable load)

ADSRAX, ADSRAY, ADSRAZ are gyro drifts due to acceleration in spin reference axis (an erasable load)

ADIAX, ADIAY, ADIAZ are gyro drifts due to acceleration in the input axis (an erasable load)

When the magnitude of any IRIG command exceeds two pulses, the commands are sent to the gyros.

During free-fall only the NBDX, NBDY, NBDZ are the relevant coefficients and the routine is so ordered that only these terms are calculated for the gyro compensation.

5.6.14 ΔV Programs

The purpose of the Target ΔV Program (P76) is to update the estimated LM state vector in accordance with the maneuver $\Delta \underline{V}$ which is voice-linked to the CSM from the LM and then entered into the CMC as described in Section 5.2.1.

The purpose of the Impulsive ΔV Program (P77) is to update the estimated CSM state vector after the execution of a thrusting CSM-maneuver not monitored by the GNCS.

The logic for these programs is shown in Fig. 6.14-1. In the figure, $\Delta \underline{V}$ is the vehicle velocity change, expressed in that vehicle's local vertical coordinate system; and $t_{\Delta V}$ is the time of the maneuver.

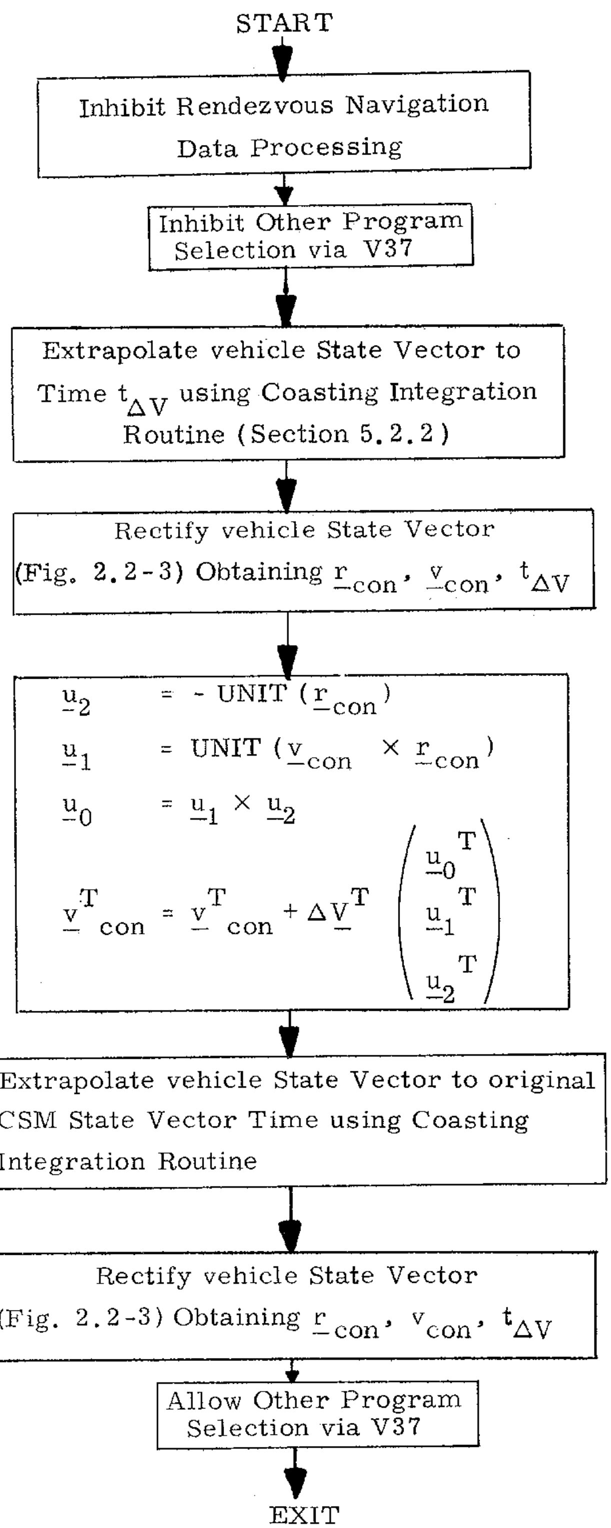


Figure 6.14-1. ΔV Programs: Target ΔV (P76), and Impulsive ΔV (P77).

5.6.16 RMS POSITION AND VELOCITY ERROR DISPLAY

In order to provide the capability for astronaut monitoring of the G & N system's estimate of state vector accuracy, there exists a special DSKY verb which causes the RMS position and velocity errors to be computed from the W matrix and to be displayed. Based upon the values in this display and the details of the particular mission, the astronaut will elect to stop the navigation that is in progress, to resume or continue with the current navigation procedure, or to reinitialize the W matrix and continue navigating. The capability of selecting the W matrix initialization parameters is also included in this process.

The logic for the RMS position error (Δr_{RMS}) and RMS velocity error (Δv_{RMS}) display is illustrated in Fig. 6.16-1. The vectors $\underline{w}_i$ are partitions of the W matrix as defined in Eq. (2.2.26) of Section 5.2.2.4. The variables w_{rr}, w_{rv}, $w_{\ell r}$, $w_{\ell v}$, w_{mr}, and w_{mv} are W matrix initialization parameters, and RENDWFLG and ORBWFLAG are the W matrix validity flags. See Sections 5.2.5.2, 5.2.4.5, and 5.2.6.4 for further definitions and usage of these terms.

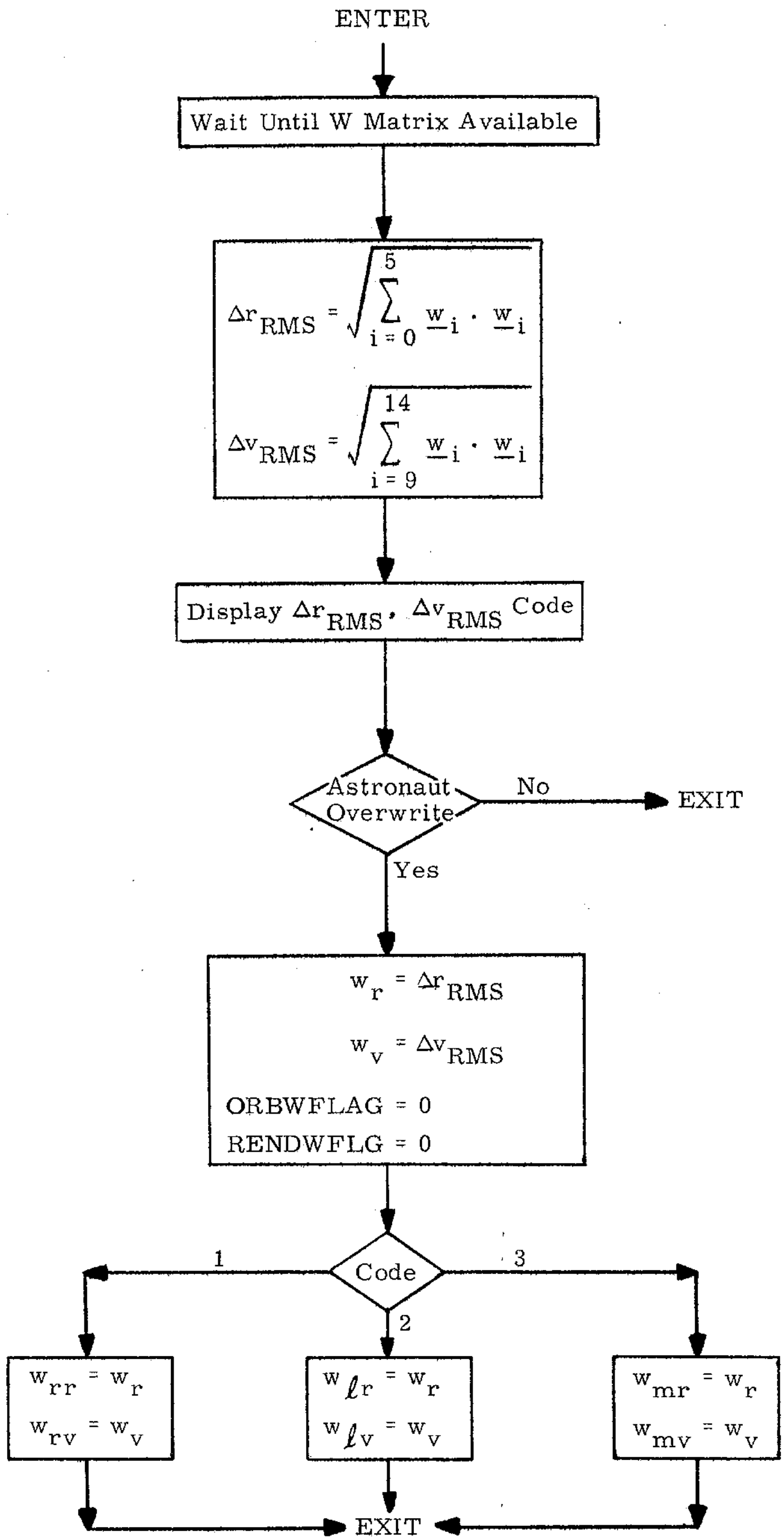


Figure 6.16-1 RMS Position and Velocity Error Display Logic Diagram

5.6.17 Rate-Aided Optics Tracking Program (P-24)

This program is used by the astronaut to assist him in tracking and marking on a landmark, especially at low altitudes. The program uses the Coasting Integration Routine (Section 5.2.2) to extrapolate the CSM's state vector to the present time. The astronaut then specifies the latitude, longitude and altitude of the desired landmark. The program then calls the Rate-Aided Optics Subroutine (Section 5.6.8.4) of the Automatic Optics Positioning Routine to assist him in tracking the landmark. This routine will supply a rate to the optics with the OPTICS MODE switch in MANUAL. While in this mode, the astronaut has the capability of taking an unlimited number of sighting marks on the landmark.

5.7 ENTRY GUIDANCE

The entry guidance is used for control of the CM entry vehicle and is described by the logic flow charts in Fig. 7.0-1 thru Fig. 7.0-16. A detailed description of the guidance and steering concept is presented in MIT/IL Report R-532, "Reentry Guidance for Apollo," R. Morth, January 1966.

Figure 7.0-0 illustrates the overall picture of operations during entry. Each block in Fig. 7.0-0 is described in detail in following figures. Figure 7.0-1 defines the symbols which represent computed variables stored in erasable memory. The display nouns, together with cross-reference information are tabulated in Fig. 7.0-2. The values and definitions of constants are given in Fig. 7.0-3.

The initialization routine shown in Fig. 7.0-4 is entered only once at the start of entry. Besides setting the appropriate variables to their initial values, this routine presets the variable SELECTOR to INITROLL.

Every pass through the entry equations (done once every 2 seconds) is begun with the section called navigation (see Fig. 7.0-5) and is the same as that presented in Section 5.3.2. This integrates to determine the vehicle's new position and velocity vectors. This navigation routine is started prior to encountering the entry interface at 400,000 feet altitude, and is continuously operated from that time to landing.

Next, the targeting section updates the desired landing site position vector and computes some quantities based on the vehicle's position and velocity and the position of the landing site. (See Fig. 7.0-6).

The mode selector chooses the next sequence of calculations depending upon the phase of entry trajectory that is currently being flown. The initial roll section maintains whatever roll angle was previously selected in the initialization routine and decides when to start the next phase, as shown in Fig. 7.0-7.

The next phase maintains a constant drag trajectory while testing to see if it is time to enter the up-control phase. The testing is presented in Figs. 7.0-8 and 7.0-9. The constant drag equations are given in Fig. 7.0-10. The other phases (up-control, ballistic, and final) are listed in Figs. 7.0-11, 7.0-12, and 7.0-13. (See Fig. 7.0-0.) The final phase is accomplished by a stored reference trajectory with linear interpolation between the stored points. Its characteristics as well as the steering gains are stored as shown in Fig. 7.0-14. The routine that prevents excessive acceleration build-up (G-limiter) is given in Fig. 7.0-15. Finally, the section that does the lateral logic calculations and computes the commanded roll angle is shown in Fig. 7.0-16.

The display nouns, together with cross-reference information are tabulated in Fig. 7.0-2. Those parameters that are computed for display purposes only during the entry phase are presented in Section 5.6.10.

For descriptive material about the interaction of Entry Guidance with the Entry DAP and with the mission control programs P61 - P67, see Section 3.

PINBALL (DSKY Display) NOTE: In the CMC, range is measured along a great circle and is expressed in revolutions for nouns 50, 63, 64, 66, and 67. (Noun 50, splash error, is discussed in 5.6.10.2; the other nouns are defined in Fig. 7.0-2 .) The DSKY display of the range components of these nouns uses the PINBALL scale factor 3441.327214 n.mi./rad. The conventional scaling is ATK of Fig. 7.0-3, corresponding to 60 n.mi. = 1 deg of arc. Since the flowcharts are drawn in terms of nautical miles, the DSKY-displayed range is larger than the flowchart range value by the factor 1.001041507.

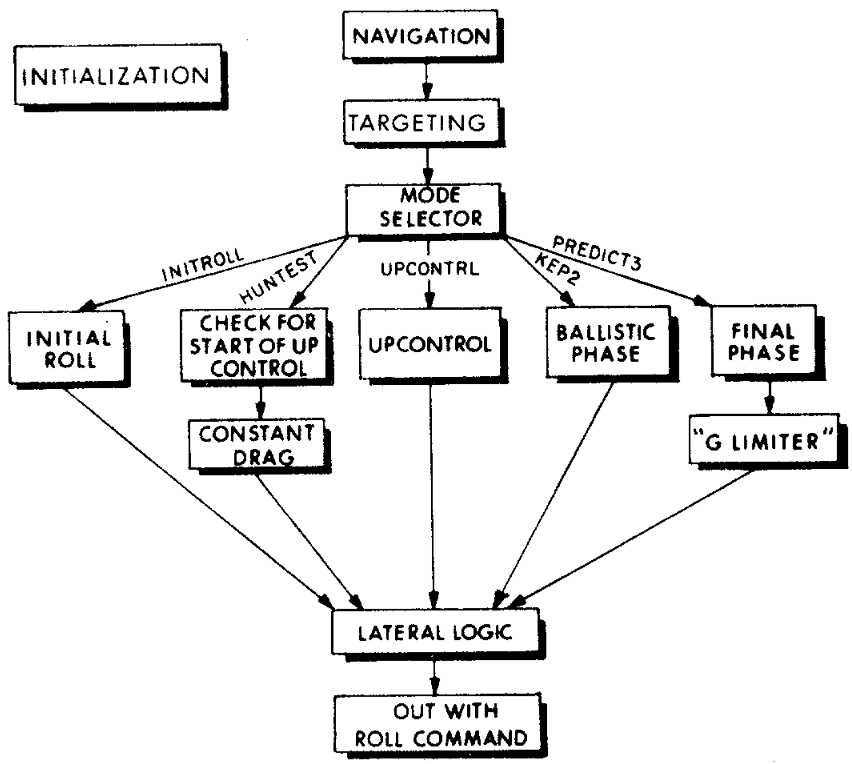


Fig. 7.0-0. Entry Computation

VARIABLE	DESCRIPTION	MAXIMUM VALUE *	COMPUTER NAME
URTO	INITIAL TARGET VECTOR	2 (UNIT VECTOR)	= RTINIT
UZ	UNIT VECTOR TRUE NORTH IN BASIC REFERENCE COORDINATE SYSTEM	1	= UNITW
V	VELOCITY VECTOR	2 VSAT	= VEL
R	POSITION VECTOR	2 EXP 29 METERS	= RN
VI	INERTIAL VELOCITY	128 M/CENTISEC	= VN
RTE	VECTOR EAST AT INITIAL TARGET	2	= RTEAST
UTR	NORMAL TO RTE AND UZ	2	= RTNORM
URT	TARGET VECTOR	2	= RT
UNI	UNIT NORMAL TO TRAJECTORY PLANE	2	
DELV	INTEGRATED ACCEL. FROM PIPAS	5.85 16384 CM/S	
G	GRAVITY VECTOR	128 M/CENTISEC	= GDT/2
AO	INITIAL DRAG FOR UPCONTRL	805 FPSS	FPSS=FT/SEC/SEC
AHOOKDV	TERM IN GAMMAL CALC. = AHOOK DVL	8	
A1	DRAG VALUE IN FACTOR CALCULATION	805 FPSS	
ALP	CONST FOR UPCONTRL	1	
ASKEP	KEPLER RANGE	21600 NM	NM = NAUTICAL MILE
ASP1	FINAL PHASE RANGE	21600 NM	
ASPUP	UP-RANGE	21600 NM	
ASP3	GAMMA CORRECTION	21600 NM	
ASPDWN	RANGE DOWN TO PULL-UP	21600 NM	
ASP	PREDICTED RANGE	21600 NM	NOT STORED
COSG	COSINE(GAMMAL)	2	= COSG/2
D	TOTAL ACCELERATION	805 FPSS	
DO	CONTROLLED CONSTANT D	805 FPSS	
DHOOK	TERM IN GAMMAL COMPUTATION	805 FPSS	
DIFF	THETA ATK-ASP (RANGE DIFFERENCE)	21600 NM	
DIFFOLD	PREVIOUS VALUE OF DIFF	21600 NM	
DLEWD	CHANGE IN LEWD	1	
DR	REFERENCE DRAG FOR DOWNCONTROL	805 FPSS	NOT STORED
DREFR( )	REFERENCE DRAG (FINAL PHASE TABLE)	805 FPSS	NOT SAVED
DVL	VS1-VL	2 VSAT	
E	ECCENTRICITY	4	NOT STORED
F1( )	DRANGE/D DRAG (FINAL PHASE TABLE)	2700/805	= FX +5
F2( )	DRANGE/D RDOT (FINAL PHASE TABLE)	2700/2VS NM/FPS	= FX +4 /8
F3( )	DRANGE/D (L/D) (FINAL PHASE TABLE)	2700 NM	= FX

* MAXIMUM VALUE DENOTES UNSCALED VARIABLE VALUE WHEN SCALED VARIABLE HAS MAXIMUM VALUE OF ONE.

Figure 7.0-1 Computer Variables

VARIABLE	DESCRIPTION	MAXIMUM VALUE	COMPUTER NAME
FACT1	CONST FOR UPCONTRL	2 VSAT	
FACT2	CONST FOR UPCONTRL	1/805 FPSS	
FACTOR	USED IN UPCONTRL	1	
GAMMAL	FLIGHT PATH ANGLE AT VL	1 RADIAN	
GAMMAL1	SIMPLE FORM OF GAMMAL	1 RADIAN	
HEADSUP	INDICATOR FOR INITIAL ROLL	1	
KA	DRAG TO LIFT UP IF DOWN	805 FPSS	= KAT
KLAT	LATERAL SWITCH GAIN	1 (NOM = .0125)	
K2ROLL	INDICATOR FOR ROLL SWITCH		
LAD	MAX L/D (MIN ACTUAL VEHICLE L/D)	1	
LADPAD	NOMINAL VEHICLE L/D, SP PAD LOAD	1 (NOM = 0.3)	
LATANG	LATERAL RANGE	4 RADIANS	
LAT(SPL)	GEODETIC LATITUDE OF TARGET	1 REVOLUTION	
LNG(SPL)	LONGITUDE OF TARGET	1 REVOLUTION	
LEQ	EXCESS C.F. OVER GRAV=(VSQ-1)GS	128.8 FPSS	
LEWD	UPCONTROL REFERENCE L/D	1	
LOD	FINAL PHASE L/D	1 (NOM = 0.18)	
LODPAD	FINAL PHASE L/D, SP PAD LOAD	1	
L/D	DESIRED LIFT TO DRAG RATIO (VERTICAL PLANE)	1	
L/D1	TEMP STORAGE FOR L/D IN LATERAL	1	
L/DCMINR	LAD COS(15DEG)	1 (NOM = 0.2895)	
PREDANGL	PREDICTED RANGE (FINAL PHASE)	2700 NM	= PREDANG
Q2	FINAL PHASE RANGE	21600 NM	
Q7	MINIMUM DRAG FOR UPCONTROL	805 FPSS	
RDOT	ALTITUDE RATE	2 VSAT	
RDOTREF	REFERENCE RDOT FOR UPCONTROL	2 VSAT	
RDOTREF( )	REFERENCE RDOT (FINAL PHASE TABLE)	2 VSAT	= FX +3 /8
RDTR	REFERENCE RDOT FOR DOWNCONT	2 VSAT	NOT SAVED
ROLLC	ROLL COMMAND	1 REVOLUTION	
RTOGO( )	RANGE TO GO (FINAL PHASE TABLE)	2700 NM	= FX +2
SL	SINE OF LATITUDE	1	NOT SAVED
T	TIME	8 28 CENTISEC	= PIPTIME-TIME/RTO
THETA	DESIRED RANGE (RADIANS)	2 PI RADIANS	= THETAH
V	VELOCITY MAGNITUDE	2 VSAT	
V1	INITIAL VELOCITY FOR UPCONTROL	2 VSAT	
VL	EXIT VELOCITY FOR UPCONTROL	2 VSAT	
VMAGI	INERTIAL VELOCITY MAGNITUDE,|VI|	128 M/CENTISEC	
VREF	REFERENCE VELOCITY FOR UPCONTROL	2 VSAT	
VREF( )	REFERENCE V (FINAL PHASE TABLE)	2 VSAT	NOT STORED
VS1	VSAT OR V1, WHICHEVER IS SMALLER	2 VSAT	
VBARS	$VL^2/VSAT^2$	4	
VSQ	NORMALISED VEL. SQUARED = $V^2/VSAT^2$	4	= VSQUARE
WT	EARTH RATE TIMES TIME	1 REVOLUTION	NOT SAVED
			= WIE (DTEAROT)
X	INTERMEDIATE VARIABLE IN G-LIMITER	2 VSAT	NOT SAVED
Y	LATERAL MISS LIMIT	4 RADIANS	NOT SAVED

Figure 7.0-1a Computer Variables (Continued)

EXTRA COMPUTER ERASABLE LOCATIONS NOT SHOWN ON FLOW CHARTS

VARIABLE	DESCRIPTION	MAXIMUM VALUE
GOTOADDR	ADDRESS SELECTED BY SEQUENCER	
XPIPBUF	BUFFER TO STORE X PIPA COUNTS	
YPIPBUF	BUFFER TO STORE Y PIPA COUNTS	
ZPIPBUF	BUFFER TO STORE Z PIPA COUNTS	
PIPCTR	COUNTS PASSES THRU PIPA READ ROUTINE	
JJ	INDEX IN FINAL PHASE TABLE LOOK-UP	
MM	INDEX IN FINAL PHASE TABLE LOOK-UP	
GRAD	INTERPOLATION FACTOR IN FINAL PHASE	
FX	DRANGE/D L/D = F3	2700 NM
FX + 1	AREF	805 FPSS
FX + 2	RTOGO	2700 NM
FX + 3	RDOTREF	VSAT/4
FX + 4	DRANGE/D RDOT = F2	21600/2VS NM/FPS
FX + 5	DRANGE/D DRAG = F1	2700/805 NM/FPSS
TEM1B	TEMPORARY LOCATION	
TIME/RTO	TIME OF INITIAL TARGET RTINIT	B 28 CENTISEC
DTEAROT	EST TIME BETWEEN RTINIT AND RT	B 28 CENTISEC
$\overline{\text{UNITV}}$	UNIT V VECTOR	2
$\overline{\text{UNITR}}$	UNIT R VECTOR	2
$-\overline{\text{VREL}}$	NEGATIVE VELOCITY REL TO ATMOSP	2 VSAT

COMPUTER SWITCHES		INITIAL STATE	CM/FLAGS = STATE +6
GONEPAST	INDICATES OVERSHOOT OF TARGET	BRANCH (1)	95D, BIT10
RELVELSW	RELATIVE VELOCITY SWITCH	NON-BRANCH (0)	96D, BIT 9
EGSW	FINAL PHASE SWITCH	NON-BRANCH (0)	97D, BIT 8
NOSWITCH	INHIBIT LATERAL SWITCH	NON-BRANCH (0)	98D, BIT 7
HIND	INDICATES ITERATION IN HUNTEST	NON-BRANCH (0)	99D, BIT 6
INRLSW	INDICATES INIT ROLL ATTITUDE SET	NON-BRANCH (0)	100D, BIT 5
LATSW	INHIBIT DOWNLIFT SWITCH IF NOT SET	BRANCH (1)	101D, BIT 4
.05GSW	INDICATES DRAG EXCEEDS .05 GS	NON-BRANCH (0)	102D, BIT 3
GONEBY	INDICATES GONE PAST TARGET (SET)	SELF-INITIALIZING	112D, BIT 8 IN STATE + 7

Figure 7.0-1b Computer Variables (Continued)

BODY ATTITUDE QUANTITIES (CM/POSE)

VARIABLE	DESCRIPTION	MAXIMUM VALUE
$-\overline{\text{VREL}}$	NEGATIVE VELOCITY REL TO ATMOS.	2 VSAT
$\overline{\text{OLDUYA}}$	USED FOR $\overline{\text{UYA}}$ BELOW 1000 FPS	2
$\overline{\text{UXA}}$/2	UNIT VECTOR TRIAD	2
$\overline{\text{UYA}}$/2	BASED ON	2
$\overline{\text{UZA}}$/2	THE TRAJECTORY.	2
$\overline{\text{UBX}}$/2	UNIT VECTOR	2
$\overline{\text{UBY}}$/2	BODY TRIAD	2
$\overline{\text{UBZ}}$/2	FOR CM.	2

Figure 7.0-1c Computer Variables (Continued)

DISPLAY QUANTITIES* (FOR USAGE,SEE SECTION 4: P 61 THROUGH P 67)

THIS TABLE PROVIDES THE READER WITH DISPLAY QUANTITY INFORMATION. THE VARIABLE NAMES REFLECT SECTION 5 USAGE. EXCEPTING IMBEDDED BLANKS, THE SECTION 4 NAMES ARE PARENTHESIZED,IF DIFFERENT. SIGN CONVENTIONS ARE FOUND IN SECTION 4. IF THE DISPLAY VARIABLE NAME DIFFERS FROM SECTION 5 USAGE, SAY BECAUSE OF SCALE FACTOR, THE ALTERNATE SECTION 5 NAME IS ENCLOSED IN ANGULAR BRACKETS: < >. DISPLAY QUANTITY INFORMATION APPEARING ELSEWHERE IN SECTION 5 OTHER THAN IN 5.7 MAY BE FOUND THUS:

N 60 & N 63 (ALL)	FIGURE 6.10-4 IN 5.6.10.3
N 22 (ALL)	FIGURE 6.10-11 IN 5.6.10.6
N 67 (LAT, LONG)	SECTION 5.6.13.7

VARIABLE	DESCRIPTION	MAXIMUM VALUE	DISPLAY NOUNS
GMAX	PREDICTED MAXIMUM ENTRY ACCEL	163.84 GS	N 60
VPRED	PREDICTED VELOCITY AT ALTITUDE 400K FT ABOVE FISCHER RADIUS.	128 M/CENTISEC	N 60
GAMMAEI	PREDICTED GAMMA AT ALTITUDE 400K FT ABOVE FISCHER RADIUS.	1 REVOLUTION	N 60
RTGO	RANGE ANGLE TO SPLASH (RTOGO) FROM EMSALT FT ABOVE FISCHER RADIUS. (IN NM)	1 REVOLUTION	N 63
VIO	PREDICTED VELOCITY AT ALTITUDE EMSALT FT ABOVE FISCHER RADIUS.	128 M/CENTISEC	N 63
TTE	TIME OF FREE FALL (TFE) TO ALTITUDE EMSALT FT ABOVE FISCHER RADIUS.	B 28 CENTISEC	N 63
OGA	OUTER GIMBAL ANGLE (OG ROLL)	1 REVOLUTION	N 22
IGA	INNER GIMBAL ANGLE (IG PITCH)	1 REVOLUTION	N 22
MGA	MIDDLE GIMBAL ANGLE (MG YAW) ALL ABOVE ANGLES FOR HYPERSONIC TRIM	1 REVOLUTION	N 22
D	DRAG ACCELERATION (G)	805 FPSS	N 64, N 74
VMAGI	INERTIAL VELOCITY MAGNITUDE (VI)	128 M/CENTISEC	N 64, N 68, N 74
ROLLC	ROLL COMMAND (BETA)	1 REVOLUTION	N 66, N 68, N 69, N 74
XRNGERR	CROSS-RANGE ERROR <LATANG IN NM>	1 REVOLUTION	N 66
DNRNGERR	DOWN RANGE ERROR ** <PREDANG - THETAH IN NM> ** <DNRNGERR DISPLAYS 9999.9 IF GONEPAST =1 IN P 67>	1 REVOLUTION	N 66
HDOT	ALTITUDE RATE <RDOT>	128 M/CENTISEC	N 68
Q7	EXIT DRAG LEVEL (DL)	805 FPSS	N 69
VL	EXIT VELOCITY FOR UP-CONTROL	2 VSAT	N 69
RTOGO	DESIRED RANGE ANGLE <THETAH> <-THETAH IF GONEBY =0, THETAH IF GONEBY =1>	1 REVOLUTION	N 67, N 64
LAT	GEODETIC LATITUDE OF VEHICLE	1 REVOLUTION	N 67
LONG	LONGITUDE OF VEHICLE	1 REVOLUTION	N 67

THE FOLLOWING IS A DATA INPUT NOUN (REFER TO SECTION 4,P 61 & P 62)

VARIABLE	DESCRIPTION	MAXIMUM VALUE	DISPLAY NOUNS
HEADSUP	INDICATOR FOR INITIAL ROLL	1 REVOLUTION	N 61
LAT(SPL)	GEODETIC LATITUDE OF TARGET (IMPACT LAT)	1 REVOLUTION	N 61
LNG(SPL)	LONGITUDE OF TARGET (IMPACT LONG)	1 REVOLUTION	N 61

*See note on page 5.7-2.

Figure 7.0-2 Display Information

CONSTANTS AND GAINS		VALUE		
C1	FACTOR IN ALP COMPUTATION	1.25		
C16	CONSTD GAIN ON DRAG	0.01		
C17	CONSTD GAIN ON RDOT	0.002		
C18	BIAS VEL. FOR FINAL PHASE START	500	FPS	
C20	MAX DRAG FOR DOWN-LIFT	210	FPSS	
C21	DRAG FOR NO LATERAL SWITCH	140	FPSS	
CHOOK	FACTOR IN AHOOK COMPUTATION	0.25		
CH1	FACTOR IN GAMMAL COMPUTATION	1.0		
COS15	COS(15 DEG)	0.965		
DLEWD0	INITIAL VARIATION IN LEWD	-0.05		
DT	COMPUTATION CYCLE TIME INTERVAL	2	SEC.	
GMAX	MAXIMUM ACCELERATION	257.6	FPSS	(8 G-S)
KA1	FACTOR IN KA CALC	1.3 GS		
KA2	FACTOR IN KA CALC	0.2 GS		
KA3	FACTOR IN DO CALC	90 FPSS		
KA4	FACTOR IN DO CALC	40 FPSS		
KB1	OPTIMIZED UPCONTROL GAIN	2.5		
KB2	OPTIMIZED UPCONTROL GAIN	0.0025		
KDMIN	INCREMENT ON Q7F TO DETECT END OF KEPLER PHASE	0.5	FPSS	
KTETA	TIME OF FLIGHT CONSTANT	1000		
KLAT1	FACTOR IN KLAT CALC	1/24		
K44	GAIN USED IN INITIAL ROLL SECTION	19749550 FPS		
LATBIAS	LATERAL SWITCH BIAS TERM	0.41252961 NM		
LEWD1	NOMINAL UPCONTROL L/D	0.15		
POINT1	FACTOR TO REDUCE UPCONTROL GAIN	0.1		
Q3	FINAL PHASE DRANGE/D V	0.07	NM/FPS	
Q5	FINAL PHASE DRANGE/ D GAMMA	7050	NM/RAD	
Q6	FINAL PHASE INITIAL FLIGHT PATH ANGLE	0.0349	RAD	
Q7F	MIN DRAG FOR UPCONTROL	6	FPSS	
Q7MIN	MIN VALUE FOR Q7 IN FACTOR CALCULATION	805	FPSS	(DISABLE FACTOR)
Q19	FACTOR IN GAMMAL1 CALCULATION	0.5		
VFINAL1	VELOCITY TO START FINAL PHASE ON INITIAL ENTRY	27000	FPS	
VFINAL	FACTOR IN INITIAL UP-DOWN CALC	26600	FPS	
VLMIN	MINIMUM VL	18000	FPS	
VMIN	VELOCITY TO SWITCH TO RELATIVE VEL	VSAT/2		
VRCONTRL	RDOT TO START INTO HUNTEST VRCONT = COMPUTER NAME	700	FPS	
25NM	TOLERANCE TO STOP RANGE ITERATION	25	NM	
VQUIT	VELOCITY TO STOP STEERING	1000	FPS	

---------- ------- --- ------- ---------

ATK	ANGLE IN RAD TO NM	3437.7468	NM/RAD
GS	NOMINAL G VALUE FOR SCALING	32.2	FPSS
HS	ATMOSPHERE SCALE HEIGHT	28500	FT
J	GRAVITY HARMONIC COEFFICIENT	0.00162346	
KWE	EQUATORIAL EARTH RATE	1546.70168	FPS
MUE	EARTH GRAVITATIONAL CONSTANT	3.986032233	E 14 CUBIC M/ SEC SEC
RE	EARTH RADIUS	21202900	FT
REQ	EARTH EQUATORIAL RADIUS	20925738.2	FT
VSAT	SATELLITE VELOCITY AT RE	25766.1973	FPS
WIE	EARTH RATE	0.0000729211505 RAD/SEC	

Figure 7.0-3 Constants, Gains and Conversion Factors

INITIALIZATION

Use N61 input data to obtain the initial target vector:
Call LATITUDE-LONGITUDE SUBROUTINE (Section 5.5.3) with the parameters as defined there having the following values:

Lat = LAT(SPL), Long = LNG(SPL), Alt = 0, F = 0, P = 0
t = TIME/RTO, where TIME/RTO = current state vector time

Obtain $\overline{URT0}$

$\overline{URT0}$ = UNIT($\overline{URT0}$)

$\overline{URTO}$ = TARGET VECTOR AT TIME OF INITIALIZATION

$\overline{RTE}$ = $\overline{UZ}$*$\overline{URTO}$

$\overline{UTR}$ = $\overline{RTE}$*$\overline{UZ}$

$\overline{URT}$ = INITIAL TARGET VECTOR AT NOMINAL TIME OF ARRIVAL (FOR THIS CALCULATION)

= $\overline{URTO}$ + $\overline{UTR}$(COS WT − 1) + $\overline{RTE}$ SIN WT
WHERE WT = W TNOM, TNOM = 500 SEC

$\overline{UNI}$ = UNIT ($\overline{V}$*UNIT($\overline{R}$))

LATANG = $\overline{URT}$.$\overline{UNI}$

THETA = ARCCOS($\overline{URT}$.UNIT($\overline{R}$))

K2ROLL = −SIGN(LATANG)

Q7 = Q7F

SELECTOR = INITROLL

FACTOR = 1

LOD = LODPAD

LAD = LADPAD

KLAT = KLAT1 LAD

L/DCMINR = LAD COS15

L/D = − LAD SGN(HEADSUP)

DIFFOLD = 0
DLEWD = DLEWD0
LEWD = LEWD1
} Initialized at start of HUNTEST

Q2 = −1992 + 3500 LAD

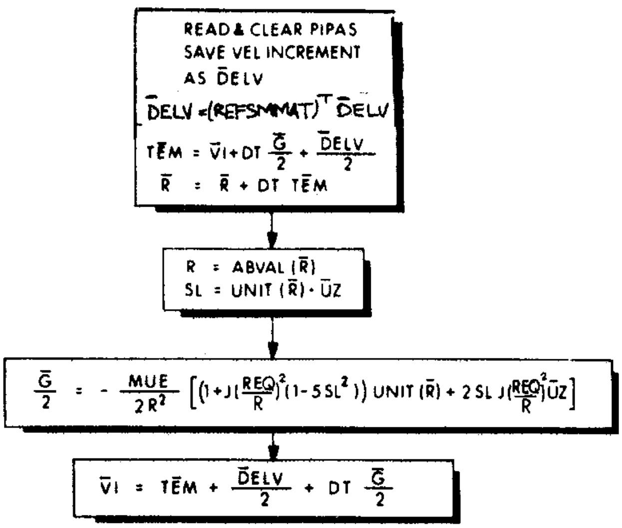
INITIALIZE SWITCHES

GONEPAST = 1
EGSW = 0
HIND = 0
LATSW = 1

RELVELSW = 0
.05GSW = 0
INRLSW = 0

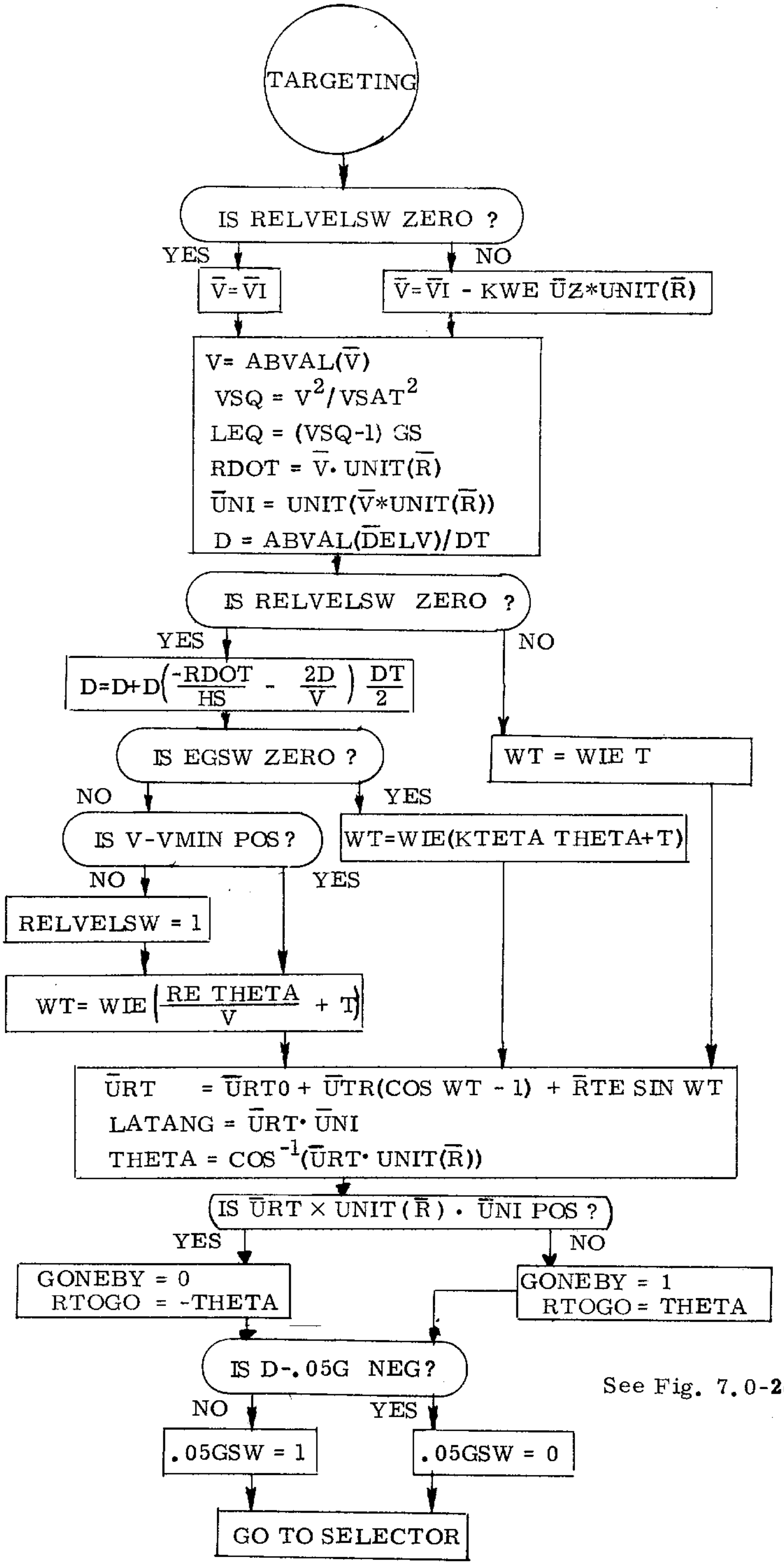
NOSWITCH = 0

Figure 7.0-4 Entry Program Initialization



Note:
This routine is identical to "Average-G" routine used elsewhere.

Fig. 7.0-5 "Average-G" Navigation Routine



See Fig. 7.0-2d

Figure 7.0-6 Targeting

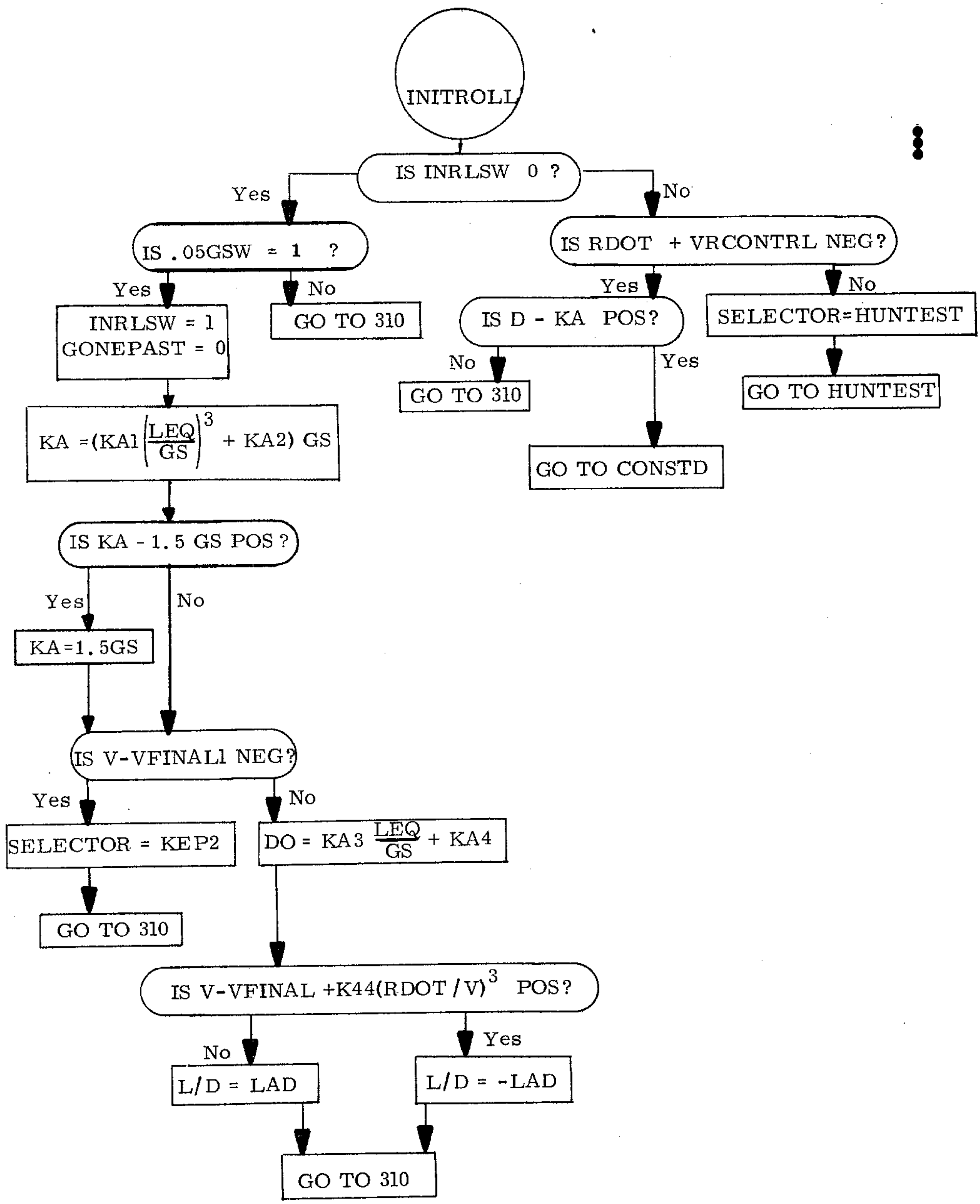


Figure 7.0-7 Initial Roll

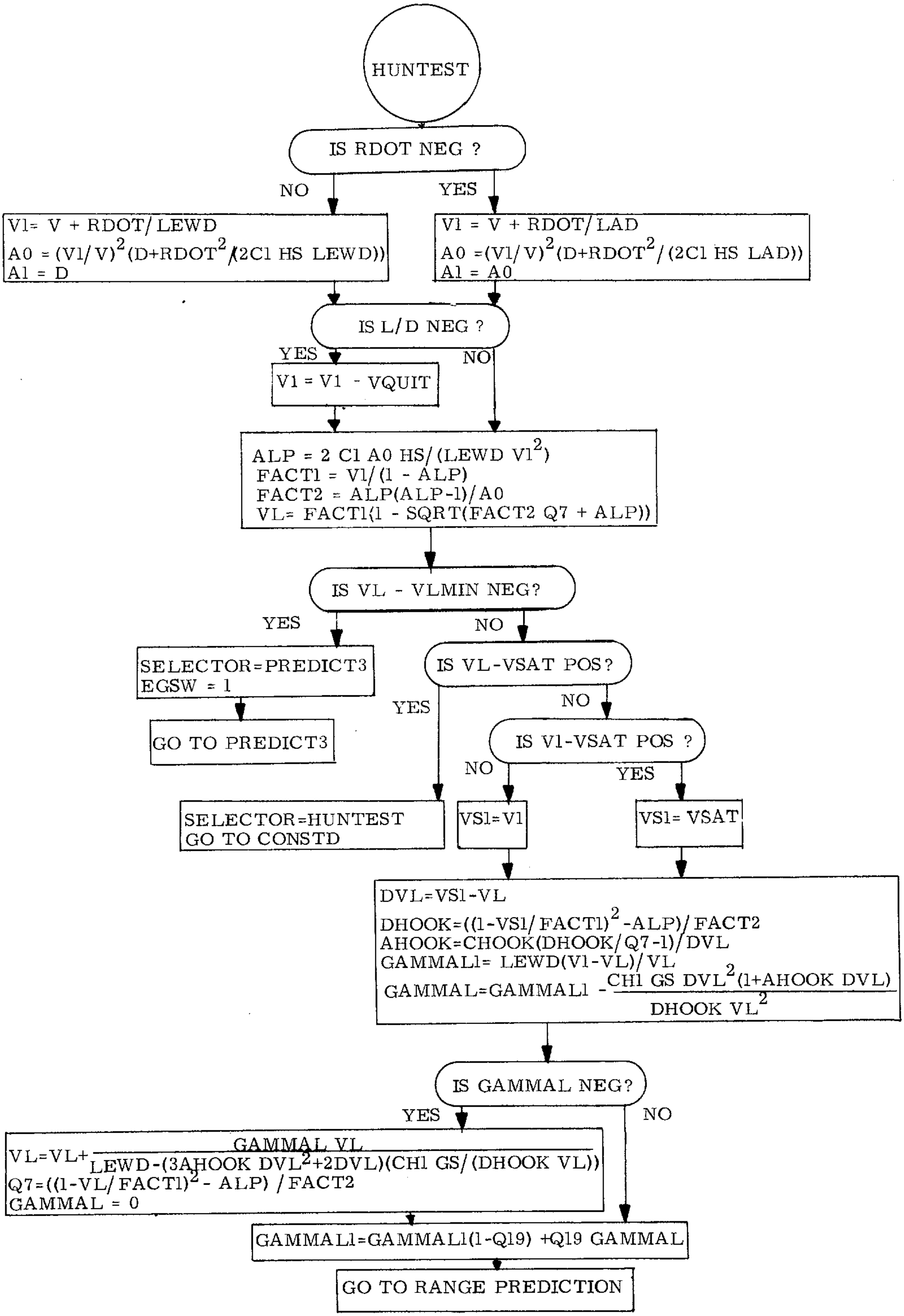


Figure 7.0-8 Huntest

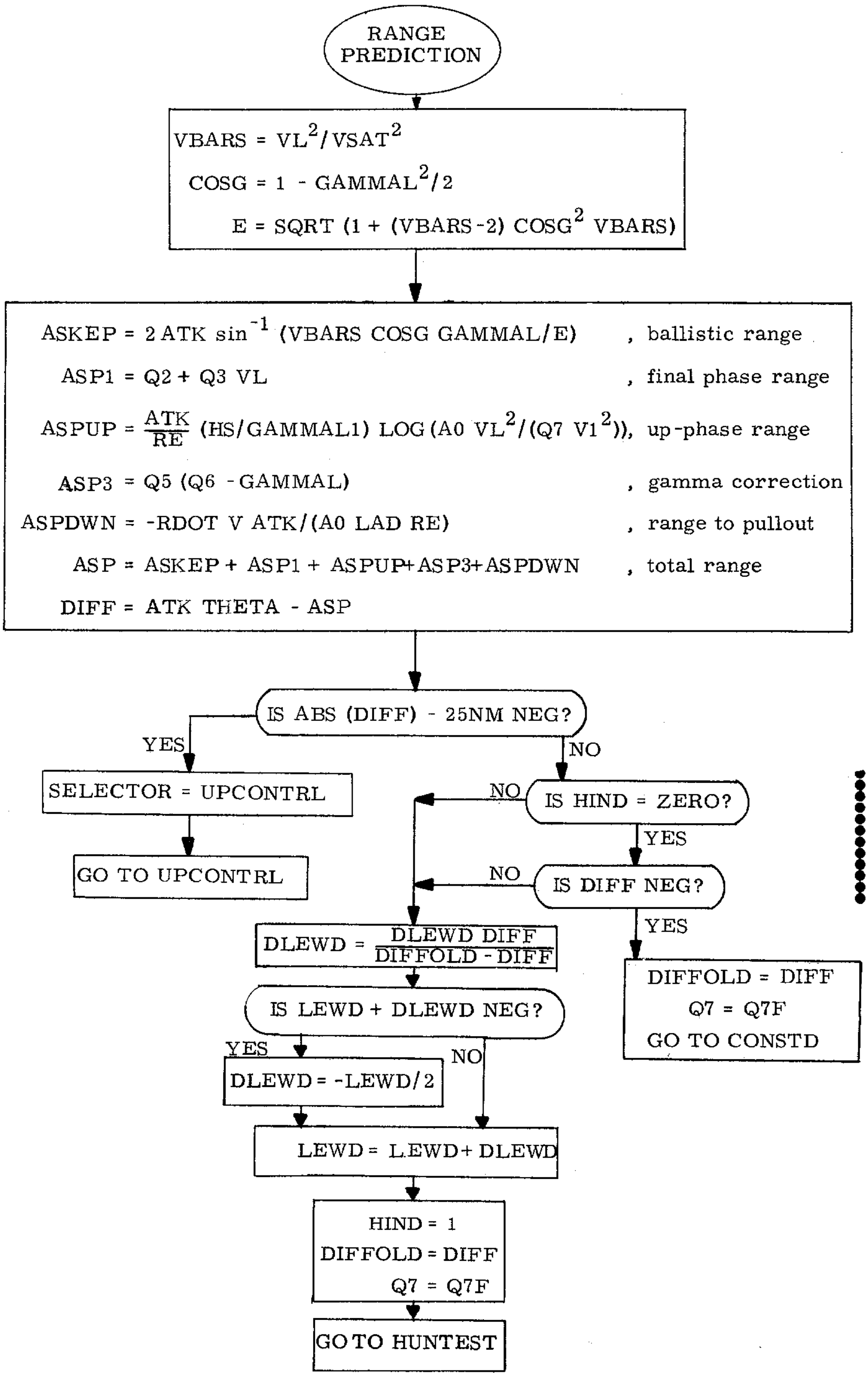


Figure 7.0-9 Range Prediction

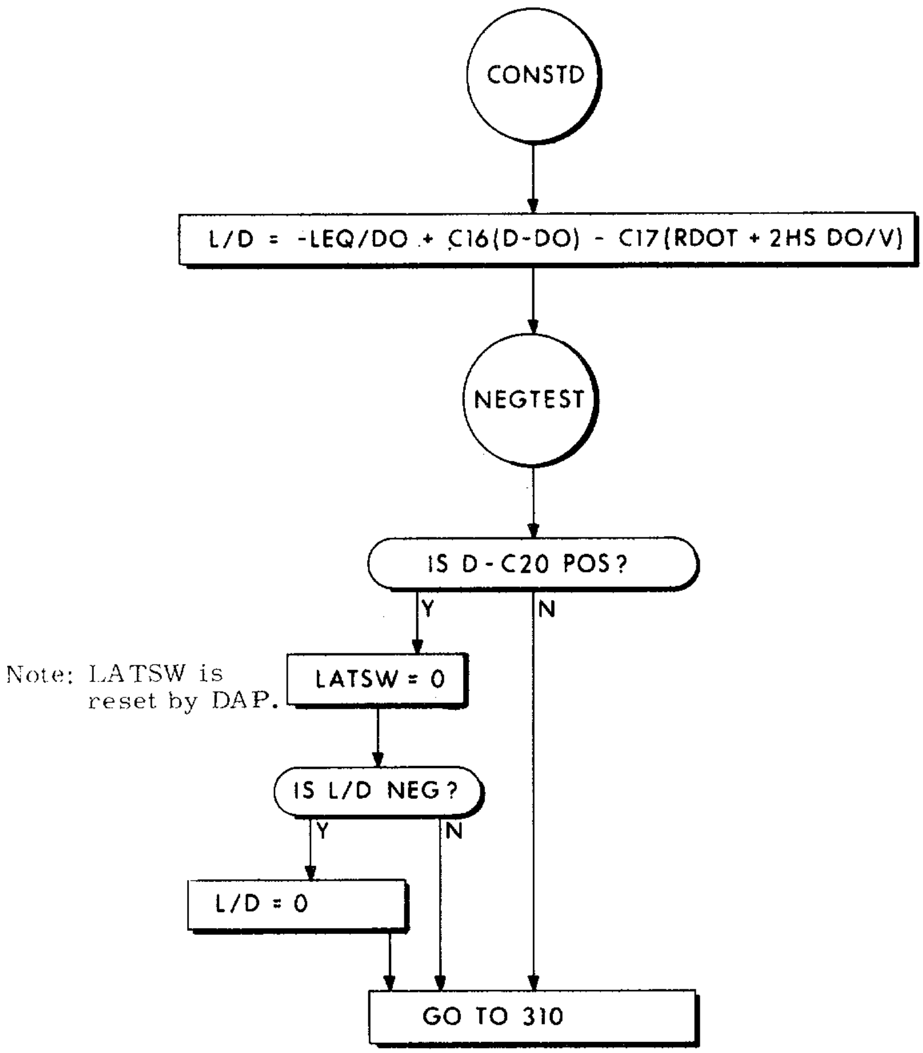


Fig. 7.0-10 Constant Drag Control

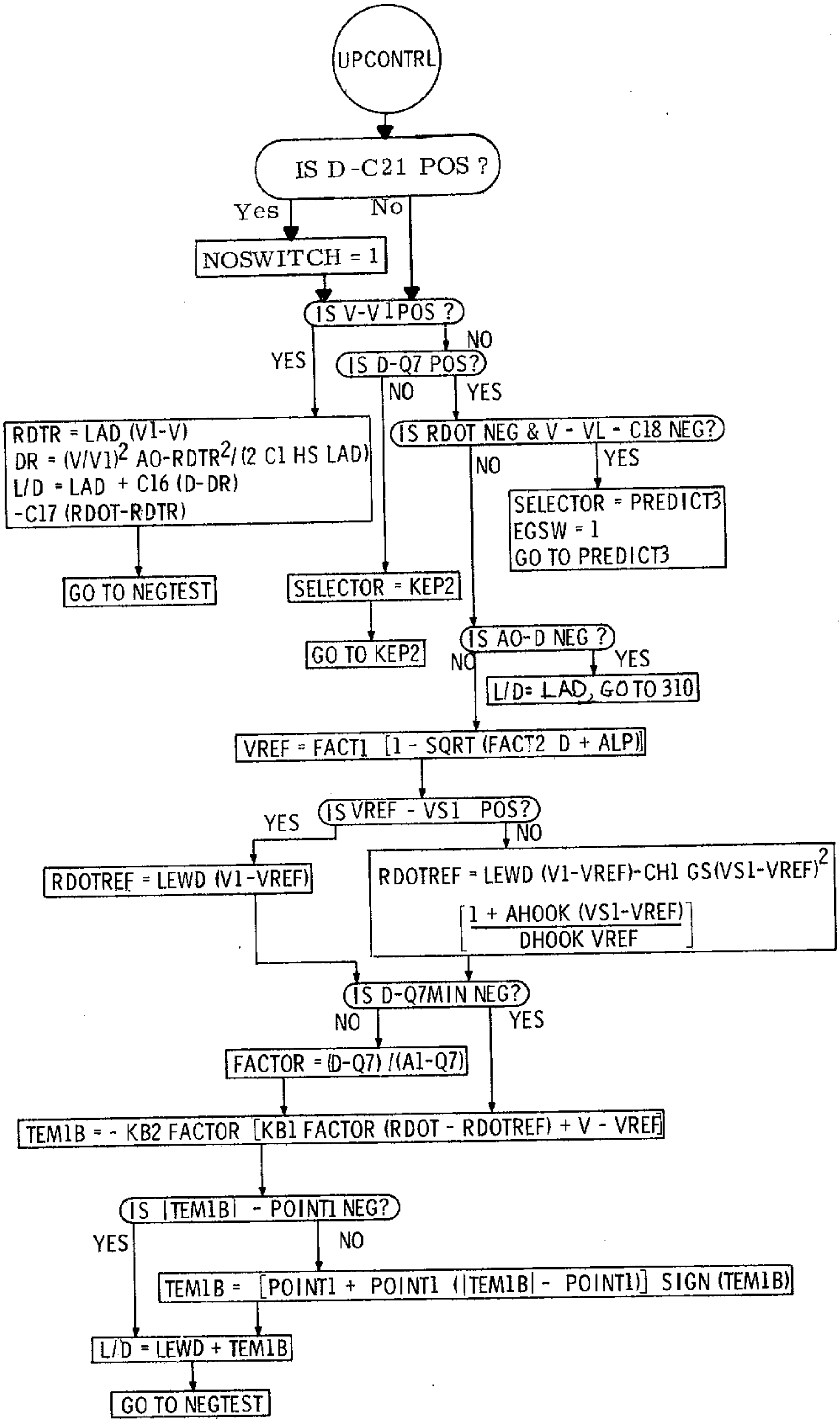
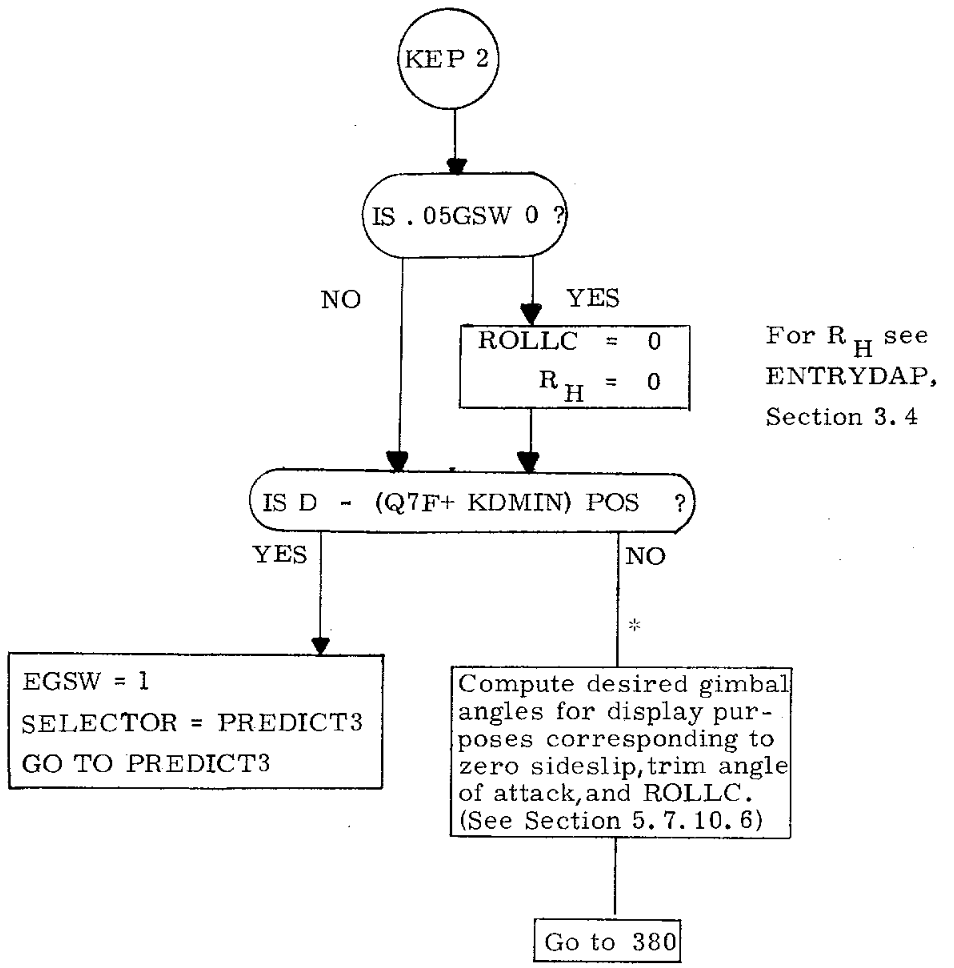


Fig. 7.0-11. Upcontrol



*During ballistic mode, DAP maintains attitude control to zero sideslip, trim angle of attack, and ROLLC (Section 3. 4).

Figure 7. 0-12 Ballistic Phase

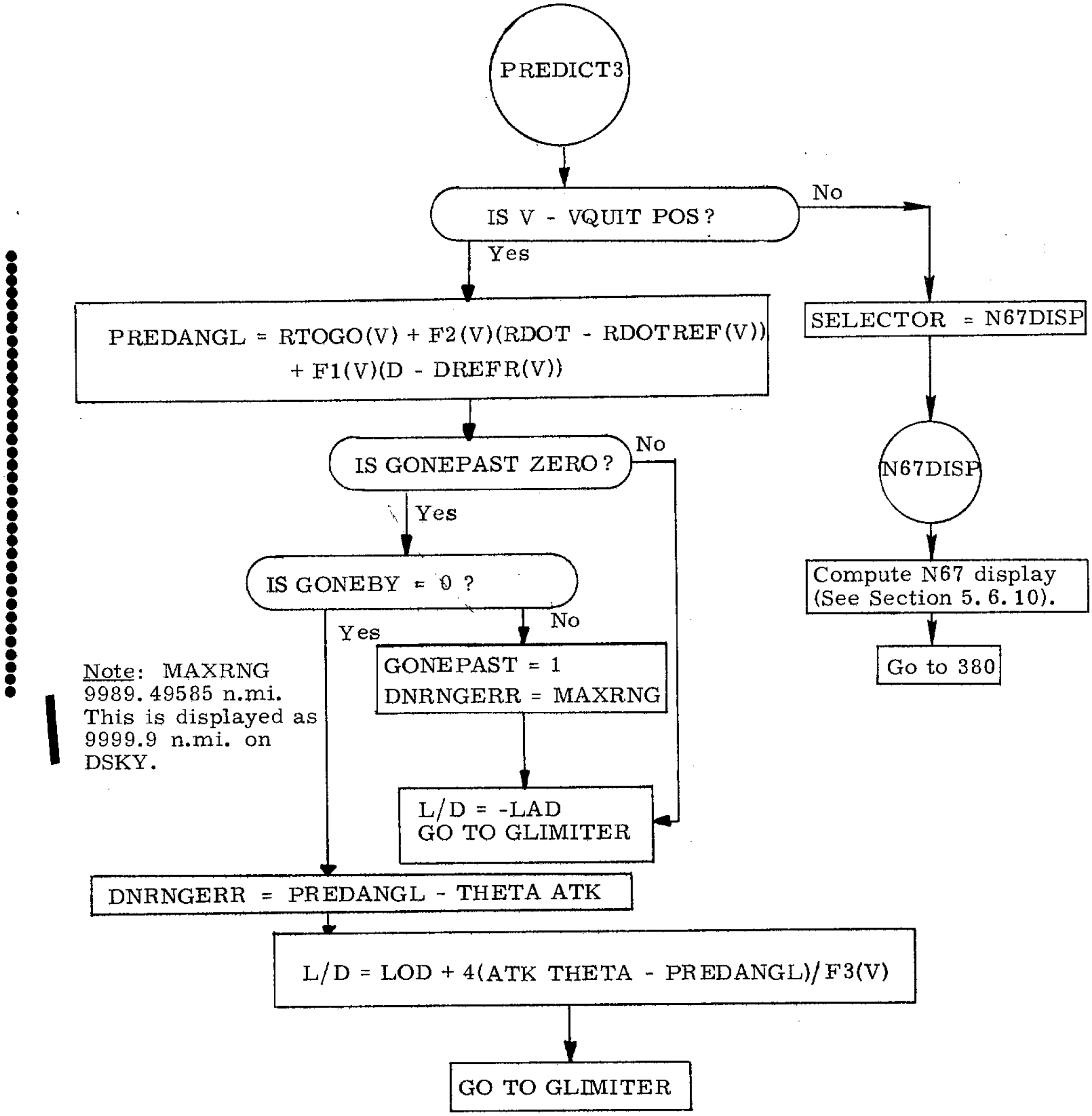


Figure 7.0-13 Final Phase

VREF	RDOTREF	DREFR	DR/DRDOT F2	DR/DA F1	RTOGO	DR/DL/D F3	
FPS	FPS	FPSS	NM/FPS	NM/FPSS	NM	NM	
994	-690.9	41.15	.002507	-.0346	3.7*	6.10	X2
2103	-719	60.	.003582	-.05551	10.4*	10.91	X2
3922	-694	81.5	.007039	-.09034	23.6*	21.64	X2
6295	-609	93.9	.01446	-.1410	46.3	48.35	X2
8531	-493	98.5	.02479	-.1978	75.4	93.72	X2
10101	-416	102.3	.03391	-.2372	99.9	141.1	X2
14014	-352	118.7	.06139	-.3305	170.9	329.4	
15951	-416	125.2	.07683	-.7500x	210.3	465.5	
18357	-566	120.4	.09982	-1.000x	266.8	682.7	
20829	-781	95.4	.1335	-1.000x	344.3	980.5	
23090	-927	28.1	.2175	-2.021	504.8	1385	
23500	-820	6.4	.3046	-3.354	643.0	1508	
35000	-820	6.4	.3046	-3.354	794.3	1508	

*Changed by PCR #803

xChanged by PCR #288

Figure 7.0-14a Final Phase Reference

Interpolation Procedure

Start with table entries. See Fig. 7.0-14a.

VREF(i) , i = 1, 13

RDOTREF(i), i = 1, 13

etc

Vary the index i until velocity magnitude lies between two values in the table. That is:

$$\text{VREF(i)} \leq \text{V} \leq \text{VREF(i+1)}.$$

Save the value i and form the linear interpolation factor

$$\text{GRAD} = \frac{\text{V - VREF(i)}}{\text{VREF(i+1) - VREF(i)}}$$

Then form interpolated values. For example

$$\text{RDOTREF(V)} = \text{RDOTREF(i)} + \text{GRAD}\ [\text{RDOTREF(i+1)} - \text{RDOTREF(i)}]$$

Figure 7.0-14b Final Phase Reference

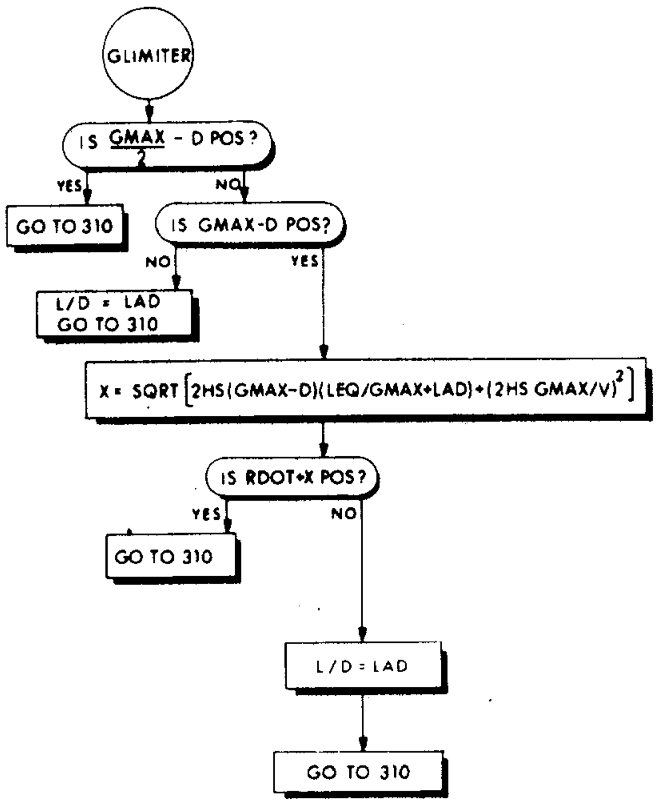


Fig. 7.0-15 G-Limiter

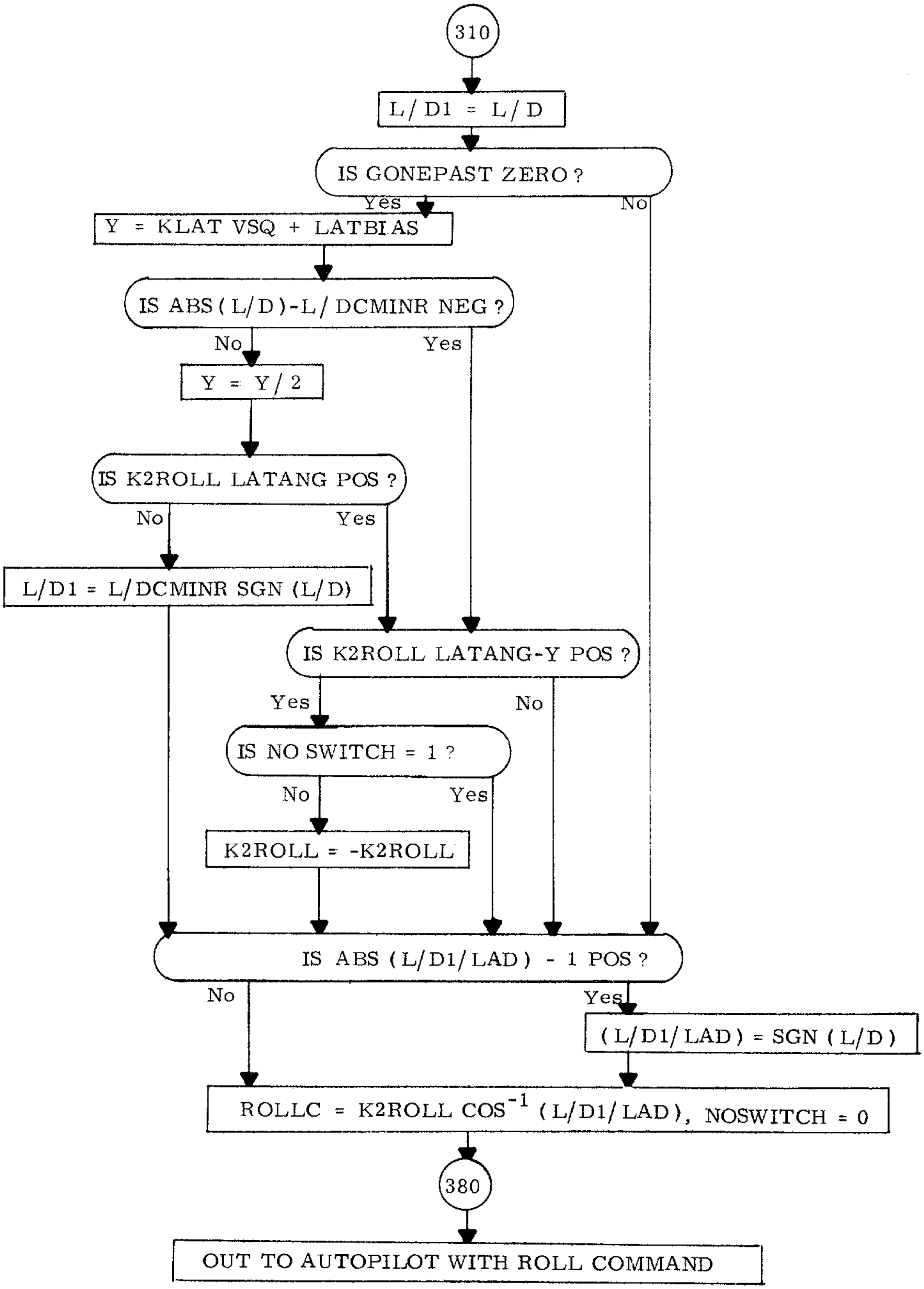


Fig. 7.0-16 Lateral Logic

5.8 ERASABLE MEMORY PARAMETER LIST

This section presents a list of selected parameters required for various program operations. It should be noted that this is a very limited CMC erasable parameter list. The objective of this selected list is to identify those parameters that would be stored in erasable memory and are required primarily to initialize the operation of programs and routines. In most cases these parameters cannot be originated within the CMC and must be stored prior to the mission. Thus, all of the listed parameters can be initialized before launch, and those that are not applicable will be changed during the mission prior to their usage. Some parameters will vary continually throughout the mission (e.g. vehicle state vectors), others are constant for any one mission phase, but may vary between different mission phases, and finally some may be constant for one mission, but be required to change for subsequent missions that use the same CMC program.

Page	Parameter
5.2-17	J_{22}
	C_{31}
5.2-26	$\underline{r}_{C0}$, $\underline{r}_{L0}$
	$\underline{v}_{C0}$, $\underline{v}_{L0}$
	$\underline{r}_{C\,con}$, $\underline{r}_{L\,con}$
	$\underline{v}_{C\,con}$, $\underline{v}_{L\,con}$
	$\underline{\delta}_C$, $\underline{\delta}_L$
	$\underline{\nu}_C$, $\underline{\nu}_L$
	t_C, t_L
	τ_C, τ_L
	x_C, x_L
	P_C, P_L

Page	Parameter
5.2-51	KNOWN
	L
	Lat
	Long
	Alt
	$w_{\ell r}$
	$w_{\ell v}$
5.2-52	OFF
	w_{ℓ}
	var_{RP}
5.2-57	$\underline{r}_{LS}$
5.2-69	SC
	δr_{MAX}
	δv_{MAX}
5.2-78	var_{INT}
5.2-79	var_R var_{Rmin}
	WRDTIME
5.2-80	MAXBLKTM TBEFCOMP BRNBLKTM MAXWTIME FINCMTM
5.2-81	w_{rr} w_{rv}
5.2-82	var_{ALT}
5.2-85	Z
5.2-90	h_0 h_1
5.2-96	L
	w_{mr}
	w_{mv}

Page	Parameter
5.2-97	H
5.3-3	$\Delta \underline{v}(\Delta t)$
5.3-18	$\Delta \underline{v}_{LV}$
	t_{IG}
5.3-19	S_E
	m
5.3-23	$\underline{r}(t_2)$
	t_2
5.3-24	c
	ec
5.3-28	p
	y
5.3-31	JFLG
5.3-32	F_{16}
	F_{01}
	K_1
5.3-34	Δv_p (SPS)
	$\Delta t_{tail-off}$ (SPS)
5.3-35	K
5.3-38	$\dot{m}$ (SPS)
5.3-48	A_{ZP}
	A_Z
	K_r
	t_{E1}
	t_{E2}
	$a_0, a_1, \ldots, a_6$
5.3-55	Lat_P
	Lon_P
	Alt_I
5.3-62	D_{TF}
	$V_{C/O}$

Page	Parameter
5.4-20	t
	E
	ωt
5.4-24	$\delta\tau_3$
	$\delta\tau_7$
5.4-28	ΔH_{DES}
5.4-49	t_1
	$\gamma (t_2)_D$
	Δv_D
5.4-71	P37 RANGE
5.4-77	D_1
5.5-13	ℓ_X
	ℓ_Y
	t_0
5.5-16	$\underline{\ell}_M$
5.5-23	t_{M0}
	$\underline{c}_0$ thru $\underline{c}_9$
5.5-24	$\underline{r}_{ES0}$
	$\underline{v}_{ES0}$
	ω_{ES}
5.6-44	TRUNSF SHAFTSF N
5.6-54	EMS Altitude

Page	Parameter
5.6-75	α_t (See Section 3.4)
5.6-82	SFE_1, SFE_2, SFE_3
	$BIAS_1$, $BIAS_2$, $BIAS_3$
	ADIAX, ADIAY, ADIAZ
	ADSRAX, ADSRAY, ADSRAZ
	NBDX, NBDY, NBDZ
5.7-10	LADPAD
	LODPAD

5.9 FIXED MEMORY CONSTANTS

Section 5.9.1 contains a list and the numerical values of those fixed memory constants in Section 5.2 through 5.6 which have not been specified previously. Those constants which are considered to be control type data are indicated by source references which are listed in Section 5.9.2. Explanatory comments are noted in Section 5.9.1 where applicable and listed in Section 5.9.3. It should be noted that only one page number is given for a constant in the list of Section 5.9.1 even though the constant may appear in other parts of Section 5. In these cases, the same value is used for the constant as reported in Section 5.9.1.

5.9.1 FIXED CONSTANTS

Page	Constant	Units	Value	Reference (Sec. 5.9.2)	Comments (Sec. 5.9.3)
5.2-13	μ_P μ_E	m^3/s^2	0.3986032×10^{15}	2, 3, 4	
	μ_P μ_M	m^3/s^2	0.4902778×10^{13}	2, 3, 4	
5.2-16	J_{2E}	–	0.10823×10^{-2}	2, 3, 4	
	J_{3E}	–	-0.23×10^{-5}	2, 3, 4	
	J_{4E}	–	-0.18×10^{-5}	2, 3, 4	
	r_E	m	6, 378, 165	2, 3, 4	
5.2-17	J_{2M}	–	0.207108×10^{-3}	2, 3, 4	
	J_{3M}	–	$-2.1(10)^{-5}$	2, 3, 4	
	J_{4M}	–	0	2, 3, 4	
	r_M	m	1, 738, 090	2, 3, 4	
5.2-18	μ_S	m^3/s^2	$0.132715445 \times 10^{21}$	2, 3, 4	
5.2-27	ϵ_t	csec	3.0		1
	ω_M	rad/sec	$2.66169948 \times 10^{-6}$		
5.2-29	r_{ME}	m	7, 178, 165		2
	r_{MM}	m	2, 538, 090		2
5.2-30	r_{SPH}	m	64, 373, 760	14	3
5.2-31	r_{dE}	m	80, 467, 200		4
	r_{dM}	m	16, 093, 440		4

Page	Constant	Units	Value	Reference (Sec. 5.9.2)	Comments (Sec. 5.9.3)
5.2-54	var_{SCT}	$(mr)^2$	1.0	4	
	var_{IMU}	$(mr)^2$	0.04	5	
5.2-64	Δt_C	sec	20		
5.2-65	MAXRATE	deg/sec	0.1		
5.2-76	k_{RL}	nm/bit	0.01	6	
5.2-78	var_{SXT}	$(mr)^2$	0.04	4	
5.2-87	c	ft/sec	9.835712×10^8	2, 3, 4	
5.2-96	var_{TRUN}	$(mr)^2$	0.0025	4	
	var_L	$(nm)^2$	1		11
5.3-3	J	–	1.62346×10^{-3}		
5.3-21	F_{SPS}	pounds	20,500		14
	F_{RCS}	pounds	196.568 or 393.136	9	
5.3-31	F_L	pounds	197	9	10
5.3-35	V_e (SPS)	m/sec	3151.0396	20	
5.4-52	ϵ_2	m	100.0		7
	ϵ_3	–	0.001		7
5.4-64	ϵ_7	m	1000		7
	ϵ_8	–	0.002		7

Page	Constant	Units	Value	Reference (Sec. 5.9.2)	Comments (Sec. 5.9.3)
5.4-73	ϵ_1	-	.99966		7
	C_0	m	1.81000432×10^8		7
	C_1	-	1.5078514		7
	C_2	1/m	$-6.49993054 \times 10^{-9}$		7
	C_3	$1/m^2$	$9.769389245 \times 10^{-18}$		7
	k_5	m	7.0×10^6		7
5.4-74	ϵ_4	-	0.00001		7
5.4-75	ϵ_6	-	0.000007		7
5.4-77	k_1	m	7.0×10^6		7
	k_2	m	6.495×10^6		7
	k_3	-	-.06105		7
	k_4	-	-.10453		7
	E_3	m	121920		7
	D_2	s/m	$-4.8760771 \times 10^{-4}$	19	7
	D_3	s^2/m^2	4.5419476×10^{-8}	19	7
	D_4	s^3/m^3	$-1.4317675 \times 10^{-12}$	19	7
5.4-80	MA_1	m	-6.986643×10^7		7
5.4-81	ϵ_9	-	2^{-20}		7
5.4-82	ϵ_{10}	m/s	0.01		7
5.4-85	v_c(SPS)	m/s	3151.0396	20	
	v_c(RCS)	m/s	2706.64	10	8
	$\dot{m}$(RCS)	kg/s	0.16375	10	

Page	Constant	Units	Value	Reference (Sec. 5.9.2)	Comments (Sec. 5.9.3)
5.4-85	C_t	–	0.5		12
5.5-8	(Kepler) ϵ_t	–	2^{-22}	12	
	(Lambert) ϵ_t	–	2^{-19}	12	
	ϵ_x (moon)	$m^{1/2}$	2^{-13}	12	
	ϵ_x (earth)	$m^{1/2}$	2^{-12}	12	
	ϵ_c	–	2^{-23}	12	
5.5-11	k_1	–	2^{-2}	12	
5.5-13	A_{Z0}	rad	4.85898502016		15
	$\dot{\omega}_E$	rad/ u-sec	7.29211514667 $\times 10^{-5}$		15
5.5-16	B_0	rad	4.09157363336 $\times 10^{-1}$		15, 5
	$\dot{B}$	rad/ u-sec	-7.19758599677 $\times 10^{-14}$		5
	Ω_{IO}	rad	5.52185714700		15, 5
	$\dot{\Omega}_I$	rad/ u-sec	-1.07047013100 $\times 10^{-8}$		15, 5
	F_0	rad	4.11720655556		15, 5
	$\dot{F}$	rad/ u-sec	2.67240425480 $\times 10^{-6}$		15, 5
5.5-17	C_I		$9.996417320(10)^{-1}$		5
	S_I		$2.676579050(10)^{-2}$		5
5.5-18	a	m	6,378,166	2,3,4	
	b	m	6,356,784	2,3,4	
5.5-22	r_{LP}	m	6,373,338	11	
5.5-57	c_A	arc-sec	20.496	8, 15	
5.6-8	T_{SS}	sec	240		6
5.6-39	δt	sec	1.9 2.4		16
5.9-5	Star Table		See Fig. 9.1-1		13

[x] Revised COLOSSUS 3
[ ] Added GSOP # R-577 PCR # 1082.1 Rev. 14 Date 3/71

Figure 9.1-1. Unit Vectors of the Navigational Stars (1971-1972)

Catalogue No. (octal)	Star Name	Vis. Mag	X Coordinate	Y Coordinate	Z Coordinate
1	α Andromedae (Alpheratz)	2.1	+.8747608555	+.0264803244	+.4838307948
2	β Ceti (Diphda)	2.2	+.9342466124	+.1739271769	-.31134[illegible]137
3	γ Cassiopeiae (Navi)	2.2	+.4773424940	+.1168141178	+.8709182540
4	α Eridani (Achernar)	0.6	+.4918322686	+.2207092653	-.8422520048
5	α Ursae Minoris (Polaris)	2.1	+.0128955818	+.0078096205	+.9998862988
6	θ Eridani (Acamar)	3.4	+.5448995598	+.5317389073	-.6483349473
7	α Ceti (Menkar)	2.8	+.7028937849	+.7178988678	+.0694227718
10	α Persei (Mirfak)	1.9	+.4101921571	+.4989947555	+.7633784305
11	α Tauri (Aldebaran)	1.1	+.3592769546	+.8927967521	+.2832502918
12	β Orionis (Rigel)	0.3	+.2[illegible]7345455	+.9691236271	-.1431958012
13	α Aurigae (Capella)	0.2	+.1367280274	+.6814364033	+.7189922632
14	α Carinae (Canopus)	-0.9	-.0616[illegible]80	+.6031289258	-.7952608559
15	α Canis Majoris (Sirius)	-1.6	-.1824340636	+.9404462129	-.2869738089
16	α_2 Canis Minoris (Procyon)	0.5	-.4122762536	+.9063680102	+.0923326629
17	γ Velorum (Regor)	1.9	-.3613649792	+.5745656444	-.7343634472
20	ι Ursae Majoris (Dnoces)	3.1	-.4661511162	+.4772744503	+.7449243154
21	α Hydrae (Alphard)	2.2	-.7745052221	+.6149043688	-.1484394758
22	α Leonis (Regulus)	1.3	-.8610673209	+.4632386329	+.2096974909
23	β Leonis (Denebola)	2.2	-.9657334280	+.0521664164	+.2542392790
24	γ Corvi (Gienah)	2.8	-.9524366380	-.0597677121	-.2988181236
25	α Crucis (Acrux)	1.6	-.4521486548	-.0495728431	-.8905639377
26	α Virginis (Spica)	1.2	-.9168160791	-.3506241694	-.1910784362
27	η Ursae Majoris (Alkaid)	1.9	-.5812217481	-.2911759648	+.7598669864
30	θ Centauri (Menkent)	2.3	-.6895375091	-.4185354938	-.5910719617
31	α Bootis (Arcturus)	0.2	-.786[illegible]186221	-.5221457573	+.33[illegible]965[illegible]611
32	α Coronae Borealis (Alphecca)	2.3	-.5324345035	-.7163035719	+.451[illegible]4372
33	α Scorpii (Antares)	1.2	-.3511952476	-.8242322268	-.4441681743
34	α Trianguli Austr. (Atria)	1.9	-.1142900725	-.34[illegible]201762	-.9334474056
35	α Ophiuchi (Rasalhague)	2.1	-.1120382967	-.9695442116	+.2177876068
36	α Lyrae (Vega)	0.1	+.1219537054	-.7702168243	+.6260138474
37	σ Sagittarii (Nunki)	2.1	+.2074286490	-.8718956797	-.4435890882
40	α Aquilae (Altair)	0.9	+.4540867784	-.8777459759	+.1528685933
41	β Capricorni (Dabih)	3.2	+.5524232365	-.7930716636	-.2566435348
42	α Pavonis (Peacock)	2.1	+.3205423120	-.4434583652	-.8370169081
43	α Cygni (Deneb)	1.3	+.4542117996	-.5390337930	+.7093195408
44	ε Pegasi (Enif)	2.5	+.8141988673	-.5553601830	+.1692786800
45	α Piscis Austr. (Fomalhaut)	1.3	+.8345006316	-.2386718657	-.4965369357

[x] Revised COLOSSUS 3
[ ] Added GSOP # R-577 PCR # 1082.1 Rev. 14 Date 3/71

5.9.2 REFERENCES FOR FIXED CONSTANTS

1. Project Apollo Coordinate System Standards, SE008-001-1, Office of Manned Space Flight - Apollo Program, National Aeronautics and Space Administration, Washington, D.C., June 1965.

2. Natural Environment and Physical Standards for the Apollo Program, NASA M-DE-8020-008B, April, 1965.

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16. Apollo Spacecraft Weight and Mission Performance Definition, Manned Spacecraft Center, NASA, December 12, 1967.

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5.9.3 COMMENTS ON FIXED CONSTANTS

1. The quantity ϵ_t is the limit used for the minimum time step in the Coasting Integration Routine. When the time step drops below ϵ_t, the routine is terminated.

2. The quantities r_{ME} and r_{MM} are the radii used by the Coasting Integration Routine to define a cislunar midcourse orbit inside of which the only gravitational force considered is that of the primary body. Outside these radii the gravitational attraction of the Earth, Moon, and Sun are all taken into account.

3. The quantity r_{SPH} is the radius of a sphere that approximates the surface of influence of the Moon as given in Fig. 1.3 of Reference 14.

4. The quantities r_{dE} and r_{dM} are the radii outside of which only the central force term of the gravitational attraction of the primary body is used.

5. The values given for the fixed constants used in the Planetary Inertial Orientation Subroutine (Section 5.5.2) are obtained by using Chapter 4 of Reference 7 and Sections 4C and 11D of Reference 8.

6. The quantity T_{SS} is a rough estimate of the time between the initiation of the Star Selection Routine in the IMU Realignment Program (P-52) or the Back-up IMU Realignment Program (P-54) and the midpoint of the optical sightings on the two celestial bodies.

7. The following comments are made with respect to the indicated quantities:

ϵ_1 criterion used to determine whether the pre-return position and velocity vectors are nearly collinear ($\epsilon_1 = \cos 1.5^\circ$).

ϵ_2 criterion used to determine whether the conic portion of Return to Earth has converged upon a suitable reentry radius.

ϵ_3 criterion used to determine whether the conic portion of Return to Earth has converged upon a suitable reentry angle (although the test is made on the cotangent of the angle it is equivalent to $.058^\circ$).

ϵ_4 criterion used to determine whether the precision portion of Return to Earth final state vector computation has reached the desired reentry angle (although the test is made on the cotangent of the angle it is equivalent to $.00058^\circ$).

ϵ_6 criterion used to determine whether the final state vector computation has already reached the best possible reentry angle.

ϵ_7 criterion used to determine whether the precision portion of Return to Earth has converged upon the reentry radius selected in the conic portion.

ϵ_8 criterion used to make a final check on the reentry angle reached by the precision portion of Return to Earth (although the test is made on the cotangent of the angle it is equivalent to $.116^\circ$).

ϵ_9	criterion used to determine whether the $x(t_1) - \Delta v$ iterator has reached a minimum value.
ϵ_{10}	criterion used to determine whether the $x(t_1) - \Delta v$ iterator has reached the desired Δv.
C_0, C_1, C_2, C_3	polynomial coefficients used to determine the maximum allowable major axis of return trajectories with a positive radial component based on the radius magnitude.
MA_1	Maximum allowable major axis of return trajectories with a negative radial component
k_1	radius used to determine which estimate of reentry angle should be used.
k_2	initial estimate of reentry radius.
k_3	initial estimate of the cotangent of the reentry angle used when the initial radius is less than k_1 (equivalent to -3° 29.5').
k_4	initial estimate of the cotangent of the reentry angle used when the initial radius is greater than k_1 (equivalent to -5° 58').
k_5	radius below which $-\Delta v$ option not available
D_2, D_3, D_4	polynomial coefficients used to determine the cotangent of the reentry angle based on reentry velocity magnitude
E_3	entry altitude above the Fischer Ellipsoid (equivalent to 400,000 ft).

8. The RCS exhaust velocity was computed by using the nominal specific impulse of the reference document and an acceleration of gravity of 32.174048 ft/sec^2.

10. The quantity F_L is ullage thrust using two RCS jets where consideration has been made for the 10^o offset of these jets.

11. The quantity var_L is a rough estimate of the landmark or horizon error variance during cislunar midcourse navigation.

12. The constant C_t when multiplied with the estimated length of the burn gives the adjustment made to compute the actual ignition time from the impulsive maneuver time. The presently adopted value is 0.5.

13. The direction of each of the 37 navigation stars in Fig. 9-1 is expressed as the components of a unit vector in the Basic Reference Coordinate System. These star directions are the mean places of the stars at the beginning of the Besselian year 1971. The term "mean place" is defined in Reference 8. The star directions are computed by MIT using essentially the same technique employed by the Nautical Almanac Office. Although there are slight differences between the directions of some of the stars in Fig. 9-1 and those given in the American Ephemeris and Nautical Almanac for 1971 (Reference 15), they are considered negligible in comparison to the other errors associated with a star sighting.

14. The precise value of thrust is not available. This value is a good working number.

15. Calculated using ΔT (1.0 July 1971) = 41.5 sec.

$$\Delta T \ (1972.0) = 42.0 \text{ sec.}$$

16. δt is 1.9 seconds for P-24, 2.4 seconds otherwise.

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