APOLLO

R-415

APOLLO REENTRY GUIDANCE

by

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R-415

APOLLO REENTRY GUIDANCE

ABSTRACT

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The problem of designing an automatic, self-contained, inertial guidance system for the reentry phase of the Apollo mission is discussed in detail. The objectives, design criteria, and relationship between overall mission requirements and reentry range requirements are discussed and a system which achieves the desired performance is described in detail.

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APOLLO REENTRY GUIDANCE

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Summary

The problem of designing a system of steering logic for the reentry phase of the Apollo mission is discussed in detail. The guidance system is an automatic, self-contained inertial system using a digital computer. Its objective is to guide the spacecraft to a preselected landing site without violating certain prescribed conditions of safety. The relationship between overall mission requirements and reentry range requirements is discussed.

Performance, flexibility, and simplicity are put forth as design criteria. The critical measure of performance turns out to be the system's ability to steer properly in spite of initial navigation errors, especially the resulting error in indicated rate-of-climb. A system which achieves the desired performance is described in detail. Its principal features are approximate, analytical formulae for predicting the range of various trajectory segments and the use of a computed reference trajectory scheme for controlling during the critical, supercircular phase of reentry.

I. Introduction

The reentry phase of the Apollo mission is unique in several ways. It is the last link in a long sequence of steps; inaccuracies or mistakes committed during reentry cannot be compensated for in a subsequent phase. It is the only phase in which aerodynamic forces, rather than rocket thrust, are used to modify the spacecraft's trajectory. The period of communication - blackout during reentry is one of two periods (the other being flight behind the moon) during which important actions and computations within the spacecraft cannot be monitored and checked on the ground.

The accurate landings of Mercury flights MA-8 (Schirra) and MA-9 (Cooper) raise the question: Is reentry guidance necessary for lunar missions? The answer is, emphatically, yes. The main reason involves the dynamics of reentry at escape velocity. Although escape velocity is greater than orbital velocity by a factor of only $\sqrt{2}$, the sensitivity of reentry range to an error in the entry flight path angle is several orders of magnitude greater than in the orbital entry case. (see section II for detailed discussion) Thus a ballistic reentry or a constant L/D reentry flown open-loop fashion would impose unreasonable accuracy requirements on the preceeding phase. Furthermore, even if such an accuracy were possible, non-standard atmospheric conditions and non-standard spacecraft aerodynamic characteristics would cause large landing-point errors.

The two objectives of reentry guidance are, in order of importance:

1) safe return to the surface of the earth, and
2) landing-point control.

It is important to realize that these objectives must be achieved in spite of non-ideal equipment performance and non-standard environmental conditions.

For purposes of guidance system design, safe return means that the deceleration during reentry should never exceed some prescribed limit (perhaps 10 g's), nor should the spacecraft skip back out of the atmosphere at greater than orbital velocity. The reentry trajectory may include a free-fall, "ballistic lob" portion out of the atmosphere in order to reach a distant landing point, but this must be done at sub-circular velocity. A super-circular exit velocity would result in an extended elliptical flight, before return to the atmosphere, with limited supplies of power, oxygen, etc. The midcourse guidance phase has the first responsibility for a safe return in that the spacecraft must be steered into an acceptable "corridor" from which a safe return is possible. It is then the job of reentry guidance to achieve objective number two, range-control, without interfering with objective number one, safe-return.

Equipment is being designed to accomplish these objectives, as well as the objectives of the other mission phases, without the aid of external inputs (though capability exists to accept external inputs if available and useful). This equipment includes a space-sextant, an inertial measurement unit, IMU, and a general purpose type digital computer. During reentry the sextant is not useful. Figure 1 is a block diagram of the Apollo Guidance and Navigation system as used during reentry. The main sub-systems are the IMU, the navigation portion of the computer program, the steering portion of the computer program, the stabilization and control system and the spacecraft itself. The main subject of this paper is the development of the steering logic, the block in Fig. 1 whose inputs are navigation information and whose output is a roll command to the stabilization and control system.

The equipment available offers a unique design problem. First, the system must be tailored to operate successfully with the information available from the Inertial Measurement Unit. Secondly, the digital computer will strongly influence the steering logic. For example, logical branches are much faster on the digital machine than are the multiplications characteristic of linear feedback control systems.

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The primary reentry guidance system shown in Fig. 1 is capable of completely automatic operation. (This is not true of the other phases of the Apollo mission in which the astronaut's functions are vital and irreplaceable.) It is felt that the astronaut's most useful role during reentry is to monitor the operation of the automatic system and to be prepared at all times to override it in case of malfunction. In that event he would form the central link of a simple back-up system which could not match the automatic system's accuracy or ranging capability, but which could ensure a safe-return trajectory.

Listed below are basic ground rules and limitations which govern the reentry system design.

1) The system should be self-contained.
2) The system should be automatic.
3) No thrust is available during reentry, except for attitude control.
4) The reentry capsule is a low L/D vehicle.
5) There is only one control parameter, roll angle, which is used to point the lift vector in a desired direction.
6) Acceleration should not exceed some prescribed level.
7) Super-circular exit velocities are ruled out.
8) There is a limited supply of fuel for rolling maneuvers.
9) The computer speed and capacity are limited.
10) The accuracy of the initial navigation information is limited.
11) The accuracy of the inertial gyros and accelerometers are limited.

A basic ground rule not included above involves the ranging requirements. These are discussed in the following section.

II. Range Requirements and Capabilities

Requirements

The requirements imposed by the overall mission on the reentry phase are indicated below. There is no simple cause-and-effect relationship which clearly shows what the reentry range requirement should be. There is, rather, a complex system of trade-offs involving various operational considerations, the total impulse available to the spacecraft for the trans-earth injection and midcourse phases, the vehicle's aerodynamic characteristics, the midcourse guidance performance and the reentry guidance performance. A complete discussion of all of these matters is not attempted in this paper, but some of the overriding factors are discussed briefly.

An important operational requirement is that the system should be able to accomplish a complete lunar mission at any time during the lunar month in any year. This means that the moon's declination can be as large as 28.6 degrees, north or south, at the time of the moon-to-earth trajectory. (The maximum occurs in 1969.) The return trajectory, from the moon's sphere of influence to the point of entry into the earth's atmosphere, is approximately an ellipse with a transfer angle near 180 degrees. Therefore, when the moon has a substantial northerly declination, the reentry point must have a substantially southern latitude, and vice-versa. This concept is pictured in Fig. 2. Shown in Fig. 3 are ground tracks of paths leading to a landing site in southern United States. Also shown are lines of constant range to the landing site and loci of entry points corresponding to different lunar declinations. For a 28.6 degree north declination the minimum range from entry to landing is 4800 nautical miles and requires a 90 degree inclination return ellipse. Other operational factors, such as tracking station capabilities, may place a constraint on the allowable return ellipse inclination and, therefore, greatly increase the reentry range requirement.

Capabilities

The Apollo Command Module is a wingless, axially symmetric, reentry vehicle constructed so that its center of gravity is displaced from its axis of symmetry. When flying in the atmosphere it trims with a low, constant ratio of lift to drag. Its only means for modulating the trajectory is to roll about the wind axis, so that the lift vector may be pointed anywhere in the plane perpendicular to the velocity vector. Rolling is accomplished with reaction jets.

Shown in Fig. 4 are the range capabilities for a series of low L/D vehicles which hold lift up throughout the reentry trajectory. Range is plotted versus the initial flight path angle. The dotted line connects cases which have a maximum deceleration of 10 g's. This figure demonstrates several important points. It shows, first, that long range capability is due more to the initial energy (corresponding to escape velocity) than to the ability to use lift. Even the ballistic vehicle can achieve a very large reentry range if it enters at just the right flight path angle. A more important question, however, concerns sensitivity. The slopes of the solid lines in Fig. 4 are a measure of the sensitivity of reentry range to deviation in initial flight path angle. For example, the slope of the zero lift line at a range of 4000 n. m. is 10,000 n. m. /milliradian. For any given range, the sensitivity decreases for increasing L/D. For ranges greater than 1500 n. m., however, the sensitivities are still too great to permit an open-loop reentry quidance system.

The dotted line in Fig. 4 shows that lift pushes down the bottom of the acceptable reentry corridor. That is, due to the lifting capability a much wider range of initial flight path angles are permissible, relaxing the requirement on midcourse guidance. Furthermore, the figure shows that an L/D of approximately 0.4 or greater is required to maintain a 5000 n. m. range capability across the entire corridor.

The curves in Fig. 4 tell nothing about minimum range capability. Consider the case corresponding to an entry angle of 7.4 degrees and an L/D of 0.4. As soon as the peak (10 g) acceleration point is passed, the spacecraft can roll its lift vector downward to shorten range. In this particular case, a minimum range of 600 n.m. is achievable without exceeding 10 g's anywhere along the trajectory. Thus, lift serves to decrease sensitivity to errors, widen the corridor and extend the useful ranging capability (maximum and minimum) across the corridor.

The lateral range capability is quite small as shown in Fig. 5, which plots the entire reentry capability "footprint" for three values of L/D. The figure shows again that the ranging capability is due to initial velocity rather than aerodynamic characteristics. On the other hand, lateral range capability is increased significantly by increasing the lift to drag ratio.

III. Design Criteria and Philosophy

Three criteria for judging the design of a system of reentry steering logic are simplicity, performance and flexibility. These are discussed in turn below.

Simplicity

In this case simplicity is not really a measure of reliability. Since a general purpose type computer is used, the various arithmetic and logical steps of a complex program are carried out by the same hardware elements. A simple program does have an advantage over a complex one in that it is easier to "debug" it; that is, to make certain that it provides a proper course of action in every conceivable situation. The desire for simplicity is at times in conflict with the other criteria discussed below. These other criteria taken together might be termed "rationality". That is, based on the information available and the degree of confidence in that information, the system should do the most rational thing in all circumstances.

Performance

In evaluating the performance of a system of steering logic the only useful measure of effectiveness is the extent to which steering errors are minimized.

A distinction should be clearly made between navigation errors and steering errors. Navigation errors are inaccuracies in the determination of the spacecraft's own position and velocity. In considering the causes of missing a desired target point it is convenient to think of a navigation error as the error in where the spacecraft thinks the target is. Steering errors, on the other hand, represent the spacecraft's inability to reach the position where it thinks the target is.

The most obvious type of steering error involves the final phase of the reentry trajectory. Since there is only one control variable, roll angle, it is difficult to drive both the downrange and crossrange components of the indicated target displacement precisely to zero simultaneously even though the target was within the attainable range capability of the spacecraft shortly before the end of the flight. It is relatively easy, however, to make this steering error negligibly small. (See discussion of lateral control in section V). In a well designed system, therefore, the expected miss-distance should be approximately the same as the expected navigation error near the end of the reentry trajectory.

A more subtle, and potentially more serious, type of steering error involves control actions which are taken in a much earlier phase of reentry; specifically, while the velocity is still super-circular. The danger is that during this super-circular phase, while the sensitivities are high, improper control actions based on imperfect data (that is, navigation errors are present) may result in a large enough trajectory deviation such that later control actions are incapable of compensating sufficiently for the early mistakes. Thus, steering errors are a function of navigation errors, and the functional relationship is markedly dependent on the steering scheme. On the other hand, navigation errors are not a function of steering errors or the steering scheme, except in a second order way. To state the problem in another way, during the super-circular phase the range-capability "footprint" shrinks rapidly and there is a danger that the target will slip outside the shrinking footprint due to an improper control action. In this situation, a relatively small navigation error could cause a huge target miss if the steering logic does not diagnose the trend in time.

Anything which might cause the trajectory to deviate from a nominal trajectory (one resulting from perfect information and standard conditions) should be regarded as a possible source of a serious steering error. A list of these possible sources follows:

1) Errors in the indicated initial position and velocity (at the start of reentry).
2) Initial misalignment of the IMU.
3) IMU gyro and accelerometer errors.
4) Non-standard atmosphere.
5) Non-standard spacecraft aerodynamic characteristics.

The first three items in the above list are the causes of navigation errors.

A study of a variety of steering schemes has revealed that the chief troublemaker is the error in indicated rate-of-climb during the super-circular phase. This error, in turn, stems mainly from initial condition errors, item 1 above. In fact, the main theme of this paper is that the reentry steering scheme must be designed to keep negligible the steering error resulting from initial navigation errors, the navigation errors at the end of the midcourse phase.

Flexibility

There are two concepts of flexibility applicable to the Apollo reentry guidance problem; the first might be called mission flexibility, the second, trajectory shaping flexibility.

Although the primary mission is the lunar landing and return mission with reentry at approximately escape velocity, the system will also be used in some preliminary earth orbital missions and must also be capable of handling a continuous spectrum of conditions due to the possibility of aborts. Since there are a variety of possible reentry conditions and a variety of possible target ranges, it is desirable that the system easily adapt to these possibilities. This desire for mission flexibility points toward a steering scheme based on approximate analytical formulae which apply to a broad range of conditions rather than one based on a prestored reference trajectory which could apply accurately only to a narrow range of conditions.

The second concept of flexibility may be viewed in terms of a particular mission. Given a particular set of reentry conditions, there is some degree of choice in selecting a trajectory to reach a landing point at a particular range. There are a variety of factors, some unrelated to the guidance problem, which can influence this choice. For example, some trajectories are superior to others in the amount of heat to be absorbed by the reentering spacecraft. Another example is the fact that some trajectories make it easier for the astronauts to monitor the performance of the automatic system. The main point here is that a steering scheme which eliminates this choice is less desirable than one which leaves unspecified some "shaping parameters" which can be specified in a late stage of design or even in flight.

Two design principles which are sometimes ignored, but which are particularly important for missions of this type, may be stated as follows:

1) The steering must be consistent with the nature of the real input data.
2) The steering must be designed to dovetail with the general characteristics of the onboard digital computer.

These ideas are examined in some detail below.

Compatibility with Input Data

It is relatively simple to control a spacecraft if perfect information about present position and velocity is continuously available. In a real situation, however, the data is far from perfect due to measurement errors and other things that inject "noise" into the system. Therefore, as discussed previously, one of the most significant criteria for judging the merits of a set of steering laws is its ability to handle large errors in the input variables and still do an adequate job of guiding.

A corollary to the above is that the steering logic should not be over-designed. That is, the accuracy of the steering equations need not be apprecably better than the accuracy of the navigation. For example, if the knowledge of position is no better than say, 2 miles, it does not make sense to design steering equations that would guide the vehicle to within 10 or even 100 feet of where it thinks it should be. Of course, if this comes for free, i. e., no increase in complexity, nothing is wrong with it. But usually, a more accurate system will necessitate a more sophisticated and complicated set of steering equations.

Compatibility with Digital Computer

The use of a digital computer as a control element in a closed-loop type system is becoming more common every day. There are many obvious problems that must be investigated in programming a computer to perform the vital control and command function. Among these are the timing requirements, the storage allocation, the quantization effects, etc. For the most part, however, steering logic is designed considering an analog type system and the equations are then fitted to a digital computer. But a digital computer offers new fields and new ways to solve conventional problems. Tailor-making the steering equations for the characteristics of the digital computer used is a more subtle problem.

The ease with which a digital computer can introduce non-linearities into a control loop is an area that can be explored at length. In a conventional analog system, switching was costly and one usually relied on proportional type control. However, the decision-making and switching ability of a digital computer is cheap and fast. The conventional analog technique of adding and differencing signals and then weighting them with various gain constants can be replaced with switches and branch instructions in a digital computer. This often leads to much better performance since the effect of certain signals can be weighted in a highly non-linear manner to achieve desired results. It is almost always faster in terms of machine time since instructions such as branching and comparison are much faster than multiplication. However, a simple set of equations on a block diagram is now replaced by a complicated appearing flow-chart full of tests, branches, and multiple paths. Nevertheless, this may represent a system that is actually simpler, faster and more efficient for a digital computer to solve than the cleaner looking ones. In general, the field of tailoring control logic to accomodate a digital computer is a fertile one that remains largely unexploited.

IV. General Description of Steering Scheme

This section describes a specific set of steering laws to meet the objectives which were described previously. These steering laws are by no means in their final form, though as is shown in the next section, they will meet all mission objectives in the face of imperfect information. We describe them in detail now to point out the features which we feel must be incorporated in any guidance scheme for this mission.

Figure 6 shows that a typical entry trajectory is divided naturally into almost distinct parts. It is natural that the steering should be considered

In these same parts. Prior to entry, the IMU is aligned and the vehicle is oriented to reentry attitude. On entering the atmosphere, the phases are:

1) Ensure a safe capture avoiding excessive g's and heating at one extreme and an uncontrolled skip out of the atmosphere at the other extreme.
2) Steer to conditions at the edge of the atmosphere so that most of the desired range will be achieved in a ballistic phase.
3) Fly through this ballistic phase.
3') Or fly a constant altitude phase if the range is short.
4) Finally, steer to the landing site upon reentry.

Shown in Fig. 7 is a logic flow chart to accomplish each of these phases, much as will be programmed on the airborne computer. Updated navigation data appears (at the top of the figure) once in each computing cycle. The program flow proceeds (downward through the figure) through one of the four major sections, producing a roll angle command. This command is then the input to the stabilization and control system for one computing cycle period.

Range predictions are used in several places in the flow chart. These predictions are accomplished by dividing the trajectory into several segments and representing each by simple approximate equations. Limits and switching criteria are also represented in the same way. There are two reasons for this: first, more sophisticated formulations are not consistent with the accuracy of the input data; and secondly, the digital computer is especially adapted to this type of formulation. An alternate approach, for example, using fast time repetitive solutions to generate range predictions would put too severe a burden on the computer, and would generate a very accurate prediction only if it really had the indicated position, velocity, drag, etc.

The inputs to the steering logic are:

1) Vehicle acceleration
2) Total velocity
3) Altitude rate
4) Vehicle coordinates
5) Landing site coordinates

There are other possible input variables. Altitude is rejected in favor of acceleration. The acceleration provides a sort of pressure altitude. This input makes up for atmospheric variations in part, and it is the drag level rather than altitude which is a more significant factor in determining vehicle performance.

Altitude rate is selected over drag rate because of the possibility of noise in the drag rate signal, although, as will be shown, altitude rate signals have significant errors and there may be a possibility to correct them with drag rate information.

The navigation task of determining vehicle coordinates will not be discussed, and the procedure for determining the landing site coordinates on a rotating earth will be deferred.

Now consider each phase in detail.

Phase I - Initial Descent

In most cases the effect of this portion of the steering program is to select a roll angle command and to hold it at that value until phase 2 begins. The governing factor in this selection is the performance of the previous phase (midcourse guidance). As can be seen in the logic flow diagram, the lift can be directed in one of three directions. Up lift, away from the earth, is called for in steep entries and down lift in shallow entries. This down lift is maintained until the drag has built up to a value, K_A, signifying capture by the atmosphere. There is also a possibility of directing lift to the side, should midcourse accuracies be such that the vehicle is in the center of the corridor, but with a large lateral error.

The specification of the dividing line between steep and shallow entries is somewhat subjective. Which is worse, a prolonged skip, or excessive g's and an extra high heat rate?

Phase 1 ends when the indicated rate of descent is reduced to a preselected level. (V_{R1} is that level on the chart). This level is one of the shaping parameters mentioned earlier. It could be computed, automatically, as a function of the initial range and entry angle or could be "keyed in" manually.

Phase 2 - Steer to Exit

A computed reference trajectory technique is the main feature of this phase. Fortunately, this reference trajectory computation can be accomplished by simple analytic formulae.

Errors in altitude rate information were the main reason for selecting this type of steering. The inexact knowledge of the vertical direction is the main source of this error. For example, a 3.5 n. m. error in the downrange component of the indicated spacecraft position at the start of entry means a one milliradian error in the knowledge of vertical and corresponding 36 feet per second error in altitude rate indication.

All other schemes studied were too sensitive to this error and uncontrolled skips often resulted. That is, the vehicle exited at super-circular velocity, flying roughly a full 360 degree orbit lasting several hours before re-entering. The inherent instability of the vehicle flight path in this super-circular region is the main reason for this phenomenon. Lift must be directed down to maintain equilibrium between centrifugal and gravity forces. If the vehicle is displaced below this equilibrium point, too much down lift results, and then divergence. If the vehicle is displaced up, there is too little down lift, and again divergence.

Many guidance schemes will overcome this divergence with good information, but special attention is required to prevent errors from causing the trajectory to diverge.

The reference trajectory gives a degree of stability to the actual trajectory even with bad information. That is, the vehicle is constrained near a preassigned drag history. Or said another way, the drag information is weighted more heavily than is the altitude rate information.

The reference trajectory we have studied is a constant L/D trajectory chosen so that exit conditions are proper to attain the desired range. In the equations below the symbol (L/D) refers to the vertical component of the lift to drag ratio, the predominant factor in determining range of gliding vehicles. The reference is calculated on the basis of drag level and velocity at the start of this phase.

An iteration is required to calculate exit velocity V_L and altitude rate $RDOT_L$ such that the predicted range angle A_P matches the desired range angle THETA. This predicted range angle is composed of four parts approximated by simple formulae. All derivations appear in Appendix A.

$$A_P = A_1 + A_2 + A_3 + A_4$$

where

A_1 = range angle to exit.

$$= \frac{1}{2} (L/D)_1 H_S \ln (D_0/D_{exit})$$

A_2 = range angle in ballistic phase.

$$= 2 \cos^{-1} ((1 - \bar{V}_L^2 \cos^2 \gamma) / \sqrt{1 + (-2\bar{V}_L^2 + \bar{V}_L^4) \cos^2 \gamma})$$

A_3 = range angle for equilibrium glide in final phase.

$$= -\frac{1}{2} (L/D)_2 \ln (1 - \bar{V}_L^2)$$

A_4 = range angle correction for improper flight path angle at start of equilibrium glide phase.

$$\left(\frac{2H_S G}{V_L (L/D)_2} - RDOT_L \right) \frac{V_L}{(1 - \bar{V}_L^2) G R}$$

where,

$(L/D)_1$ = reference L/D for reference path.

$(L/D)_2$ = reference L/D for equilibrium glide.

H_S = scale height of atmosphere

D_0 = drag at start of exit phase.

D_{exit} = drag at exit.

$\bar{V}_L^2 = V_L^2 / GR$

G = gravity

R = earth radius

$\cos \gamma$ = cos(flight path angle) $\approx \cos \frac{RDOT_L}{V_L}$

Note that this iteration is made but once, defining the reference trajectory V_L and $RDOT_L$.

The reference trajectory is a drag and altitude rate history as a function of velocity. The similarity in shape of constant L/D trajectories to equilibrium glide lines was noted. The equilibrium glide line is the locus of points where the lift force balances the centrifugal and gravity forces i. e.

$$\frac{V^2}{R} - G + \frac{1}{2} \rho V^2 \frac{SC_L}{m} = 0.$$

This suggested that the reference trajectory could be approximated by an equilibrium glide line at a reduced gravity, such that the glide line is displaced to the left. The reduction in gravity, G_{MAR}, for this approximation is

$$G_{MAR} = G - \frac{V_L^2}{R}$$

The reference trajectory is then a reference drag acceleration

$$D_{ref} = \left(\frac{V^2}{R} - G - G_{MAR} \right) / (L/D)_{ref}$$

The altitude rate reference, $RDOT_{ref}$, is chosen to correspond to constant L/D flight as described in Appendix B and is a linear function of velocity.

$$RDOT_{ref} = (L/D)_1 \; (V_0 - V)$$

where

V_0 = initial velocity.

The steering is then similar to that described in reference 2 among other places.

$$(L/D)_C = (L/D)_1 + K(\lambda_D^R (D - D_{ref}) + \lambda_{RDOT}^R (RDOT - RDOT_{ref}))$$

where

$(L/D)_C$ = commanded L/D

λ_D^R = influence function of drag on range

$$= \frac{\partial R}{\partial D}$$

λ_{RDOT}^R = influence function of RDOT on range

$$\frac{\partial R}{\partial RDOT}$$

K = gain chosen to over correct L/D

There is no use of reference range as in reference 2, since the reference trajectory was chosen to have the desired range.

The two influence coefficients are approximated by simple functions of L/D and range as described in the appendix

$$\lambda_D^R = K_1 R^2/(L/D)_1$$

and

$$\lambda_{RDOT}^R = \left((\lambda_{RDOT}^R)_0 - (\lambda_{RDOT}^R)_1\right) \frac{D}{D_0} + (\lambda_{RDOT}^R)_1$$

where

$$(\lambda_{RDOT}^R)_0 = K_2 R^2/(L/D)_1$$

$$(\lambda_{RDOT})_1 = \frac{\partial R}{\partial RDOT} \quad \text{for ballistic phase}$$

Since this reference trajectory is represented by a simple formulation, shaping of the path is possible. This shaping is desirable for several reasons.

1) Heating loads are affected by the path flown. In general, if convective heating is the predominant factor, a high heat pulse with the corresponding short time duration is desirable.

2) The sensitivity of the trajectory to deviations from desired values is affected by the path. The ballistic phase has sensitivity coefficients of roughly 2 n. m. /fps for both velocity and altitude rate deviations, see Fig. 8. The lower the exit velocity, the smaller are these sensitivities.

3) Some paths being further from uncontrolled skip, are more easily monitored by the astronaut and back-up equipment.

This particular scheme can be shaped by two means, somewhat independently. The exit velocity V_L can be changed somewhat by $(L/D)_1$ and the initial portion of the trajectory can be varied with a different $(L/D)_{ref}$. Also, with good knowledge of position and velocity prior to entry, some shaping is possible by modifying the selection of the initial roll angle.

Short Ranges

Short ranges (2000 n. m. or less) do not call for a ballistic phase. This condition is easily determined by a logical decision in the digital computer.

In this case, a constant altitude phase is initiated at or near the start of phase 2. Very short ranges call for negative lift limited by a g limiting logic.

As before, the constant altitude phase is amenable to simple formulae (derived in Appendix A). Altitude rate, $RDOT_C$, is commanded by the difference between predicted and desired range,

$$RDOT_C = K_3 (THETA - A_P)$$

The predicted range A_P has four components

$$A_P = A_{P1} + A_{P2} + A_{P3} + A_{P4}$$

where

A_{P1} = equilibrium glide range angle.

$$= -\frac{1}{2} (L/D)_2 \ln (1 - \bar{V}_{Eq}^2)$$

A_{P2} = constant altitude range angle

$$= \frac{V^2}{RD} \ln \left(\frac{V}{V_{Eq}}\right)$$

A_{P3} = correction for transition to equilibrium

$$= 2 H_S /R (L/D)_2 (1 - \bar{V}_{Eq}^2)$$

A_{P4} = range angle correction for transition to constant altitude

$$= V\, RDOT/R \left((L/D)_{MAX} D - \frac{V^2}{R} + G\right)$$

= 0 if phase started

where

V_{Eq} = lowest velocity for equilibrium at altitude

The altitude rate command is limited, if negative, by the following formulae derived in the appendix

$$VR_2 = \left(\frac{2H_S}{V} G_{MAX}\right)^2 + 2H_S \left(\frac{V^2}{R} - G\right) \ln \left(\frac{L}{L_{MAX}}\right) + 2H_S (L_{MAX} - L)$$

where

G_{MAX} = maximum allowable acceleration (ft/sec^2)

L = L/D D

$$L_{MAX} = (L/D)_{MAX} G_{MAX}$$

This last formula is surprisingly effective considering the assumption of constant velocity made in the derivation. With this limit, there is possible a minimum range trajectory which flies near a g limit all the way.

Phase 3 - Ballistic Lob

No steering is possible during this phase since the spacecraft has left the sensible atmosphere and there is no thrust available. There is a possibility of ground tracking and correcting, via uplink telemetry, of navigation information. Therefore, the navigation portion of the computer program must be written so that its normal sequence may be interrupted and the corrections properly read in.

Phase 4 - Final Glide

Equilibrium glide is the basis of this phase, but corrections are needed for flight path angle variation and potential energy correction.

These ranges are derived in the appendix

A_{P1} = equilibrium glide range.

$$= -\frac{1}{2}(L/D)_2 \ln(1 - \bar{V}^2)$$

A_{P2} = flight path angle correction

$$= \left(\frac{RDOT}{V} - 2H_S/R(L/D)_2\bar{V}^2\right)\frac{\bar{V}^2}{1 - \bar{V}^2}$$

A_{P3} = potential energy correction

$$= \frac{1}{R}(L/D)_2 H_S \ln\left(\frac{D_0 V^2}{D V_0^2}\right)$$

L/D is commanded on the difference in predicted and measured range

$$L/D_C = (L/D)_2 - \frac{K}{\ln(1 - \bar{V}^2)}(THETA - A_P)$$

We over correct by a factor of 2, K = 4, since the equilibrium glide range formula would give

$$\Delta L/D = \frac{-2}{\ln(1 - \bar{V}^2)}$$

Lateral Control Logic

The above discussion of the four phases of reentry emphasizes the problem of selecting the amount of lift to be applied in the vertical trajectory plane. This vertical component of L/D may take on any value between plus and minus $(L/D)_{MAX}$. If, for example, the commanded L/D is + 0.6$(L/D)_{MAX}$, this is achieved by rolling through 53 degrees either to the right or left, and results in a side component of L/D of 0.8 $(L/D)_{MAX}$. It is the function of the lateral logic to make the choice, at each computing interval, of either a right or left roll angle command.

Basically, the lateral logic simply directs the lift to the side where the target is. However, to avoid a large number of roll reversals, there is a dead zone built in the logic. That is, lift may be directed away from the target if the predicted impact point is within limits. This limit has been set at approximately one half the vehicle's lateral range capability and can be represented simply by the approximation

$$\psi_{\frac{1}{2}} = \frac{1}{2} \text{ vehicles lateral range capability}$$

$$= KV^2$$

When a roll reversal is called for, the logic insures that the vehicle will roll in the shortest direction; that is, less than 180 degrees. Typically, 4 or 5 roll reversals occur during an entire reentry trajectory.

Effect of Earth Rotation

Since navigation is performed in an inertial coordinate system, the indicated target point is continuously moving. In the early portions of reentry, a very rough time-of-flight prediction is made by dividing the range-to-go by orbital velocity. For purposes of steering, then, the target is assumed to be at a predicted future location given by

$$\bar{R}_{T_{pred}} = \bar{R}_T + R_{T1}(\bar{U}TR(\cos(W\,ETA) - 1) + \bar{U}TE \sin(W\,ETA))$$

where

$\bar{R}_T$ = target vector at start of reentry

$\bar{U}TR$ = unit vector perpendicular to north and $\bar{U}TE$

$\bar{U}TE$ = unit vector pointing east at start of entry

R_{T1} = R cos (latitude)

W = earth rate

ETA = estimated time of arrival measured from start of reentry

When the velocity is reduced below some preset value, say 15,000 fps, velocity relative to the earth is used as a steering input and the target is fixed at its instantaneous location at each calculation. That is, in the last half of the final glide phase, steering is performed in relative coordinates.

V. Performance

The principal measures of steering system performance, as indicated in section III, are the magnitudes of the various potential sources of steering error to which the system will adapt. A successful adaptation means that the steering error is kept smaller than the navigation error. It is impossible, in an unclassified paper, to present detailed, quantitative performance data. It is possible to state, however, that the system described meets the various quantitative performance requirements adopted. It is also possible to give a qualitative discussion of the manner in which this system does adapt to the error source found to be most troublesome.

The most severe requirement placed on the steering equations is to negotiate entry trajectories with a large error in the indicated altitude rate. For a number of reasons, this is the term with the greatest effect on various steering equations. Consequently, its effect will be discussed in some detail.

An important characteristic of the altitude rate error is that it is essentially constant during the critical, super-circular, phase. The inertial navigation system accurately monitors changes in velocity and its components, but has no knowledge of the errors in the initial navigation information fed into it. Thus, the indicated altitude rate signal is a smooth, "unnoisy" one with, possibly, a large bias error.

To illustrate the operation of the guidance equation, a set of examples will be analyzed. Figure 9 shows a plot of the reference trajectory given as altitude as a function of velocity. Also shown is the real trajectory flown. As shown these do not correspond exactly. However, the reference does serve as a guide-line to steer by. It's importance can be more easily understood by considering the case where a constant error of 100 ft/sec is introduced into the indicated altitude rate.

Two of these type trajectories are shown in Fig. 9, one for a +100 and the other for -100 ft/sec error. As shown they do not follow paths that are very close and do not exit from the atmosphere with a velocity near to the nominal value. What they do accomplish is to arrive at the right spot down-range. How they achieve this result is not a simple matter to understand in detail. However, some insight can be gained by examining Fig. 10. This shows the trade-off between velocity and altitude rate for constant range ballistic skips. As it points out, there is not a unique set of conditions that must be met in order to go a particular range during the skip portion. Indeed, there is a family of combinations of velocity and altitude rate that will fill the bill. It is this fact that is the nub of the steering logic that is presented in this paper. Although the nominal trajectory selects particular values of the exit condition variables, velocity and altitude rate, it is in general impossible to achieve them in the presence of noisy inputs and the limited response capability of the entry vehicle. However, it is not necessary to meet the nominal exit conditions exactly, or even very closely. What is attempted by the steering is to exit at some conditions that cause the vehicle to traverse the right amount of range.

The exit conditions for several flights, both with and without errors, are shown in Fig. 10. Although the points do not all agree precisely in their ranging characteristics, they are close enough so that the terminal phase can make up the difference quite easily. (This is, after all, all that is needed). Eventually, if the errors become large enough, the system just could not compensate well enough (especially, soon enough) for proper range control.

The important role performed by the reference trajectory in the successful augmentation of these steering equations is not readily apparent. It offers good firm bench marks upon which it is possible to steer in spite of rather significant noise sources. The trouble with the alternative approach, a predictive system, lies in its sensitivity to large input errors, especially since the dynamics of the equations of motion are basically divergent at this time and controlability is being lost at the same time. This does not mean that a predictive system could not be made to work, but to adequately desensitize it requires the addition of other equations to perform functions closely akin to the vital ones supplied by a reference trajectory.

Let us oversimplify their operation in order to present a better comparison. A predictive system computes, in each cycle, a predicted range based on the present indicated values of the input variables. It then selects a control action based on the difference between predicted and desired range. A reference trajectory approach selects control actions based on some relationship between certain input variables, such as velocity, drag and rate-of-climb; and a comparison with a "reference" set of relationships. In doing this, it pays more attention to developing trends. Although both approaches may work well with good data, it is this fundamental difference in concept that swings the balance to the reference trajectory appraoch when confronted with erroneous data.

To summarize, the chief criterion to judge the merit of a proposed set of guidance equations is their ability to steer to a predetermined landing site with the "least miss". However, it is felt that "least miss" should not mean the lowest possible figure for the "normal" or design case, but that it should imply something about holding down the miss distance to reasonable values for as broad a spectrum of off-design conditions as can be tolerated. In other words, the guidance system should handle input uncertainties as large as possible. The major contributor has been singled out to be errors in the indicated altitude rate. The ability of the system to withstand sizeable uncertainties in this variable has been demonstrated. And it has been mentioned that other noise sources have not degraded the system performance significantly. Thus, it is felt that a set of guidance equations with good design characteristics have been developed.

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2. Lessing, H. C., Tunnell, P. J. and Coate, R. E., Lunar Landing and Long Range Earth Reentry Guidance by Application of Perturbation Theory, Presented at Second Manned Space Flight Symposium, April 22-24, 1963.
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Appendix A

Derivation of Approximate Equations

For completeness the several equations used to calculate range and limiting velocities are assembled here. Most of these equations are well known. Some are unique. In all cases, approximations are made to simplify the analyses and yield formulas consistent with the digital computer used to implement them. It is realized that improvements in certain areas are possible to get more accurate formulas with little increase in complexity and work is continuing to this end. Also, no attempt is made here to reduce the number of time consuming multiplications and divisions as will be done before the equations are implemented.

The basic set of equations for a reentry vehicle analysis are in a rotating wind axis frame

$$\frac{dV}{dt} = -\frac{1}{2}\rho V^2 \frac{SC_D}{m} - G \sin\gamma \qquad \text{(A-1)}$$

$$\frac{d\gamma}{dt} = \frac{1}{V}\left(\frac{V^2}{R} - G\right)\cos\gamma + \frac{1}{2}\rho V \frac{SC_L}{m} \qquad \text{(A-2)}$$

$$\frac{dH}{dt} = V \sin\gamma \qquad \text{(A-3)}$$

$$\frac{d\,\text{Range}}{dt} = \frac{R_0}{R} V\cos\gamma \approx V\cos\gamma \qquad \text{(A-4)}$$

$$\rho = f(H) \qquad \text{(A-5)}$$

where

V = velocity

ρ = atmospheric density

$\frac{SC_D}{m}$ = ballistic parameter

$\frac{SC_L}{m}$ = lifting parameter

γ = flight path angle between velocity and local horizontal

G = gravity acceleration

H = altitude

R = radius of path

R_0 = earth radius

This set will be the basis of most of the derivations which follow.

1. Equilibrium Glide Range

Assume:

1. R, L/D, and G are constant
2. gravity term ($G \sin\gamma$) in drag equation is negligible
3. flight path angle is small, $\cos\gamma = 1$, $\sin\gamma = \gamma$

The equilibrium glide is the locus of points where the centrifugal, gravity and lift accelerations balance. In this case, Eq. (A-2) becomes

$$\frac{V^2}{R} - G = -\frac{1}{2}\rho V^2 \frac{SC_L}{m} \qquad \text{(A-6)}$$

The range angle is

$$A_{P1} = \frac{1}{R}\int_{t_1}^{t_2} V\,dt \qquad \text{(A-7)}$$

Substitute for dt from drag Eq. (A-1) to get

$$A_{P1} = \frac{1}{R}\int_{V_1}^{V_2} \frac{V\,dV}{-\frac{1}{2}\rho V^2 \frac{SC_D}{m}}$$

$$= \frac{1}{R}\int_{V_1}^{V_2} \frac{V\frac{SC_L}{m}}{\left(\frac{V^2}{R} - G\right)\frac{SC_D}{m}}\,dV \qquad \text{(A-8)}$$

$$= \frac{1}{2} L/D \ln\left(\frac{1-\bar{V}_2^2}{1-\bar{V}_1^2}\right), \text{ where } \bar{V}^2 = \frac{V^2}{RG}$$

The above form of the equation is good for below satellite speed. Another form is derivable for conditions above satellite speed. It is clear the equation is singular at satellite speed since zero lift is required for equilibrium at that condition.

2. Equilibrium Glide Flight Path Angle

Make same assumptions as for equilibrium glide range and assume also exponential variation of density with altitude.

$$\rho = \rho_0 e^{-H/H_S}$$

Divide Eq. (A-3) by (A-1) to eliminate time

$$\sin\gamma = -\frac{1}{2}\rho V \frac{SC_D}{m}\frac{dH}{dV} \qquad \text{(A-9)}$$

take differential of (A-6)

$$\left(\frac{2V}{R} + \rho V \frac{SC_L}{m}\right) dV = -\frac{1}{2}\frac{d\rho}{dH}\frac{SC_L}{m} V^2\,dH$$

$$\frac{d\rho}{dH} = -\frac{\rho}{H_S}$$

so

$$\frac{dH}{dV} = -\frac{H_S\left(\frac{2V}{R} + \rho V \frac{SC_L}{m}\right)}{\frac{1}{2}\rho V^2 \frac{SC_L}{m}} = H_S \frac{2G/V}{\frac{1}{2}\rho V^2 \frac{SC_L}{m}} \qquad \text{(A-10)}$$

Substitute (A-10) into (A-9) to get

$$\sin \gamma = -\frac{2G\,H_S}{V^2\,L/D} \qquad \text{(A-11)}$$

3. Range Angle Correction for Improper Flight Path Angle at Start of Equilibrium Glide Phase

From Rosenbaum's thesis, Ref. 3, we see the sensitivity of range to variations in flight path angle from equilibrium glide value

$$\frac{\partial \text{ Range}}{\partial \gamma} = \frac{R\,\overline{V}^2}{1 - \overline{V}^2}$$

The change in flight path angle from ballistic phase to equilibrium glide is

$$\Delta\gamma = \frac{2\,G\,H_S}{V^2\,L/D} + \frac{1}{V}\ \text{RDOT}$$

so the range correction is

$$A_1 = \frac{1}{R}\,\frac{\partial \text{ Range}}{\partial \gamma}\,\Delta\gamma$$

$$A_1 = \left(\frac{2\,H_S\,G}{V(L/D)} + \text{RDOT}\right)\frac{V}{(1 - \overline{V}^2)\,GR} \qquad \text{(A-12)}$$

For transition from ballistic phase to equilibrium glide RDOT = - RDOT_L. For transition from constant altitude to equilibrium glide, RDOT = 0.

4. Range Angle for Constant L/D Portion

Assume altitude rate constant and velocity constant for this calculation. The usual condition is a fairly rapid change in velocity and build up in altitude rate. In this case, the range angle is

$$A_1 = \frac{1}{R}\,V_L\,\Delta t$$

where

$$\Delta t = \Delta H/\text{RDOT}_L$$

$$= H_S \ln (D_0/D_{min})/\text{RDOT}_L$$

so we can write

$$A_1 = \frac{1}{R}\,\frac{V_L}{\text{RDOT}_L}\,H_S \ln\left(\frac{D_0}{D_{min}}\right) \qquad \text{(A-13)}$$

5. Constant Altitude Range

The range equation (A-7) is directly integrable since ρ is constant at constant altitude.

$$A_{P2} = -\frac{2}{R}\,\frac{1}{\rho}\int_{V_1}^{V_2} \frac{V\,dV}{V^2\,\frac{SC_D}{m}}$$

$$= \frac{V^2}{RD}\ln\left(\frac{V_1}{V_2}\right) \qquad \text{(A-14)}$$

6. Transition to Constant Altitude from Positive RDOT

Assume constant ρ, V and that full negative lift is applied to make the transition. The equation for RDOT is then assuming small flight path angles

$$\frac{d\,\text{RDOT}}{dt} = -L_1 + \frac{V^2}{R} - G$$

where

$$L_1 = \frac{1}{2}\,\rho\,V^2\left(-\frac{SC_L}{m}\right)$$

The time elapsed in the transition is

$$\Delta t = \frac{-\Delta\text{RDOT}}{-L_1 + \frac{V^2}{R} - G}$$

and the range angle is

$$A_{P4} = \frac{1}{R}\,V\,\Delta t = \frac{V\text{RDOT}}{R\left\{L/D\ D - \frac{V^2}{R} + G\right\}}$$

7. Ballistic Range

From polar form of conic we have

$$R = \frac{\ell}{1 - e\cos f}$$

R = radius
ℓ = semi latus rectum
e = eccentricity
f = true anomaly

we see that 2f = ballistic range

$$A_2 = 2\cos^{-1}\frac{(1 - \ell/R)}{e}$$

But

$$\ell/R = \overline{V}^2\cos^2\gamma$$

and

$$e^2 = 1 - 2\,\overline{V}^2\cos^2\gamma + \overline{V}^4\cos^2\gamma$$

so finally,

$$A_2 = 2\cos^{-1}\left\{\frac{1 - \overline{V}^2\cos^2\gamma}{\sqrt{1 + (\overline{V}^4 - 2\,\overline{V}^2)\cos^2\gamma}}\right\}$$

It is straight forward to show that the sensitivity coefficients of range to velocity and flight path angle are

$$\frac{\partial A_{P2}}{\partial V} = K_1 \cos^2 \gamma (2 + 2\, u (1 - \overline{V}^2)/e), \quad \text{RAD/FT/SEC}$$

$$\frac{\partial A_{P2}}{\partial \gamma} = - K_1 \cos \gamma \sin \gamma (2 + u (2 - \overline{V}^2)/e), \quad \text{RAD/RAD}$$

where

$$K_1 = 2\overline{V}^2 / \sqrt{1 - u^2}\; e$$

$$u = (\ell / R - 1)/e$$

8. Maximum Allowed Negative Altitude Rate

Assume

1. constant velocity
2. small flight path angle
3. exponential change of density with altitude
4. full up lift is applied to avoid acceleration

then

$$\frac{d\,RDOT}{dt} = \frac{V^2}{R} - G + \frac{1}{2} \rho V^2 \frac{SC_L}{m}$$

eliminate time by

$$dt = \frac{dH}{RDOT}$$

and set

$$CF = \frac{V^2}{R} - G, \qquad L = \frac{1}{2} \rho V^2 \frac{SC_L}{m}$$

This gives

$$R_{DOT}\, d R_{DOT} = (CF + L)\, dH$$

Integrate this

$$\int_1^2 RDOT\, d\, RDOT = \int_{H_1}^{H_2} CF + L_1\, e^{-\frac{H - H_1}{H_S}}\, dH$$

$$RDOT_2^2 - RDOT_1^2 = 2\, CF (H_2 - H_1) + 2 H_S (L_2 - L_1)$$

$$= 2\, CF\, H_S \ln\left(\frac{L_1}{L_2}\right) + 2 H_S (L_2 - L_1)$$

To find $RDOT_2$ consider drag acceleration only

$$D = \frac{1}{2} \rho V^2 \frac{SC_D}{m}$$

set differential of above equal to zero to find $\frac{dH}{dV}$ at g_{max}

$$\frac{dH}{dV} = \frac{2H_S}{V}$$

use equation (A-9) to get

$$\sin \gamma = - \frac{2H_S}{V^2}\; g_{max}$$

or

$$RDOT_2 = V \sin \gamma = - \frac{2H_S}{V}\; g_{max}$$

so we can write finally

$$(RDOT_1)^2 = \left(\frac{2H_S\, g_{max}}{V}\right)^2 + 2 H_S\, CF \ln\left(\frac{L}{L_{max}}\right)$$

$$+ 2 H_S (L_{max} - L)$$

9. Potential Energy Correction to Equilibrium Glide Range

The approximation is that the glide slope is proportional to L/D, a better approximation for high L/D vehicles. In this case the range is

$$A_{P3} = \frac{1}{R} (L/D)_2 (H - H_0)$$

In terms of measurable drag and velocity this is

$$A_{P3} = \frac{1}{R} (L/D)_2 H_S \ln\left(\frac{\rho_0}{\rho}\right)$$

$$= \frac{1}{R} (L/D)_2 H_S \ln\left(\frac{D_0 V^2}{D V_0^2}\right)$$

The constants D_0, V_0 are the final values that all trajectories with the same W/C_DA tend to.

Appendix B

Reference Trajectory

This section describes in more detail the reference trajectory control used in the second phase of the entry steering. The significant feature of this steering is an L/D command based on the differences from a reference drag level and altitude rate level

$$(L/D)_C = (L/D)_1 + K\left(\lambda_D^R (D - D_{ref}) + \lambda_{RDOT}^R (RDOT - RDOT_{ref})\right)$$

There are four functions of velocity in this reference control

$$D_{ref},\ RDOT_{ref},\ \lambda_D^R,\ \lambda_{RDOT}^R$$

as compared with five in other published reference trajectory schemes. (See Ref. 2.) The range of the reference trajectory was chosen to match the desired range so the reference range term is missing. It is the purpose of this section to describe the assumptions that led to the approximate representation of these four functions.

The reference drag is an approximation to that of a constant L/D trajectory. This type of trajectory was chosen for two reasons: first, the linear perturbation analysis is directly and simply applicable, secondly, the reference could match extreme cases where near full up lift is required thereby utilizing the full maneuver capability of the vehicle. It was noted that the constant L/D trajectory is similar in shape to equilibrium glide lines in the super circular region. These lines are asymptotic to satellite velocity at high altitudes. The reference trajectory is to be asymptotic to a velocity V_L which is less than satellite velocity. This desired asymptote could be viewed as satellite velocity in a reduced gravity field

$$V_L^2 = (G - G_{MAR}) R$$

where G_{MAR} is chosen to give the correct value of V_L. The equilibrium glide line is then

$$\left(\frac{V^2}{R} - G - G_{MAR} \right) = (L/D)_{ref} D_{ref}$$

The reference L/D positions the trajectories primarily at the start of reference control, since all glide lines approach the same line at high altitudes. This starting point need not coincide with the vehicle drag level. As a matter of fact, steep entries call for a starting reference drag less than the actual drag and shallow entries call for a larger reference drag.

The reference altitude rate ($RDOT_{ref}$) is calculated by assuming that aerodynamic forces predominate and the vehicle flight path does not change appreciably. In this case, the change in altitude rate from zero is related to the change in velocity by the vehicle L/D.

$$RDOT_{ref} = (L/D)_1 (V_0 - V)$$

The assumptions are borne out by computer runs of constant L/D trajectories. This is because the other two forces, centrifugal force and gravity force, oppose each other in the region of interest, and in fact, exactly cancel at satellite speed. A correction allowing for the difference between these two forces has been studied, but this refinement was found to have little effect.

There is a relation between velocity change, $V_0 - V_L$, and L/D that is needed before this analysis is complete. The clue to this analysis is the linear relation of altitude rate and velocity change. The altitude rate is assumed to be

$$\frac{dH}{dt} = K_2 (V_0 - V)$$

The drag equation is then, assuming an exponential atmosphere and neglecting the gravity component

$$\frac{dV}{dt} = - K_1 e^{-H/H_S} V^2$$

divide the second equation by the first to eliminate the time dependence, valid because

$$\frac{dH}{dt}$$

is always positive

$$\frac{dV}{dH} = - \frac{K_1}{K_2} \frac{e^{-H/H_S} V^2}{(V_0 - V)}$$

Separate the variables and integrate from initial velocity V_0 and altitude H_0 to final velocity V_L and altitude H_L

$$\int_{V_0}^{V_L} \frac{V_0 - V}{V^2} dV = - \int_{H_0}^{H_L} \frac{K_1}{K_2} e^{-H/H_S} dH$$

$$1 - \frac{V_0}{V_L} + \ln \frac{V_0}{V_L} = \frac{K_1}{K_2} H_S \left(e^{-H_L/H_S} - e^{-H_0/H_S} \right)$$

assume e^{-H_L/H_S} is negligible compared with e^{-H_0/H_S} so we can write finally

$$\frac{V_0}{V_L} - 1 - \ln \frac{V_0}{V_L} = \frac{H_S D_0}{V_0^2 K_2} = \frac{H_S D_0}{V_0^2 (L/D)_1}$$

This calculation yields L/D_1 in terms of V_0, V_L and D_0. L/D_1 in turn determines $RDOT_L$ and the combination of $RDOT_L$ and V_L are required for the proper range in the ballistic phase. It is apparent that an iteration is necessary to find the proper combination of variables. Also, the role of the initial drag level in the reference trajectory is clearly displayed. viz. Higher drag level will give a greater velocity change other things being constant.

The two influence functions λ_D^R and λ_{RDOT}^R were solved directly in a series of computer runs using perturbation ("adjoint") equations to a series of constant L/D reference trajectories. This type of technique is now becoming standard and will be found in Ref. 2.

It was noted that there was a relation among these influence functions for different trajectories at a particular point on the trajectory, namely at the bottom of the initial pullout. This relation is for related altitude and altitude rate influence functions.

$$\left(\lambda_H^R\right)_0 = \frac{.000018\ R^2}{L/D} \quad \text{Ft/Ft}$$

$$\left(\lambda_\gamma^R\right)_0 = \frac{15\ R^2}{L/D} \quad \text{Ft/Rad}$$

R = range in nautical miles

Shown in Figures 11 and 12 are these formulae compared with the computer results. It is seen that the functions are directly proportional to the range squared, and inversely proportional to L/D. This latter fact bears out the fact seen in Fig. 4 that the higher L/D trajectories appear to be less sensitive.

Further, it was noted that the altitude influence function decays exponentially with altitude

$$\lambda_H^R = \left(\lambda_H^R\right)_0 e^{-(H-H_0)/H_S}$$

where

H_0 = initial altitude

H_S = atmosphere scale height

Also, it was noted that the flight path angle influence coefficient decays exponentially with altitude to a constant value

$$\lambda_\gamma^R = \left\{\left(\lambda_\gamma^R\right)_0 - \left(\lambda_\gamma^R\right)_1\right\} e^{-(H-H_0)/H_S'} + \left(\lambda_\gamma^R\right)_1$$

where

H_S' = another scale height different from H_S

$\left(\lambda_\gamma^R\right)_1$ = constant value of influence function.

This constant value is related to the ballistic

$$\frac{\partial R}{\partial \gamma}$$

but is somewhat less because of the compensating effect of the subsequent final glide phase. The formulae are compared with computer results in Figures 13 and 14.

Summarizing, the influence functions can be written in terms of Drag and RDOT by noting a change in drag, thus

$$\lambda_H^R\ (H - H_{ref}) = \lambda_H^R\ H_S \ln\left(\frac{D}{D_{ref}}\right)$$

$$\approx \left(\lambda_H^R\right)_0 \frac{D_{ref}}{D_0} H_S \frac{D D_{ref}}{D_{ref}}$$

so

$$\lambda_D^R = -\left(\lambda_H^R\right)_0 \frac{H_S}{D_0}$$

similarly

$$\lambda_{RDOT}^R = \lambda_\gamma^R \frac{\partial RDOT}{\partial \gamma} = \lambda_\gamma^R \frac{1}{V}$$

so

$$\lambda_{RDOT}^R = K\left(\frac{D_{ref}}{D_0}\right)^{H_S/H_S'} + \left(\lambda_\gamma^R\right)_1 \frac{1}{V}$$

where

$$\frac{H_S}{H_S'} \approx 2$$

$$K = \left(\left(\lambda_\gamma^R\right)_0 - \left(\lambda_\gamma^R\right)_1\right) \frac{1}{V}$$

Other possibilities occur in relating these influence functions. For example, it was noted that the ratio of these influence functions is almost constant over the region of interest. This fact was incorporated in one version of steering equations which gave adequate, though somewhat inferior, performance.

Some effort has been made to find an analysis to support these empirical results. So far, no complete answers have been found. The perturbation equations have been reduced to a simpler form retaining only the more significant terms, and the desensitizing effect of L/D has been noted. But this simpler set of equations is a time varying linear set, and further analytical progress will be slow.

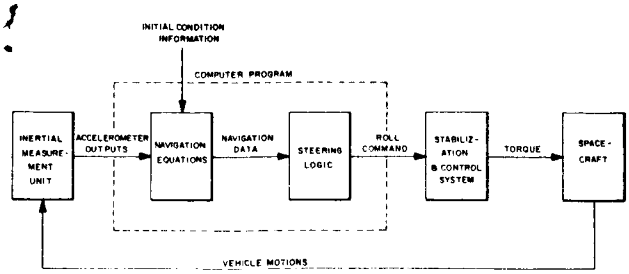

Fig. 1 System block diagram

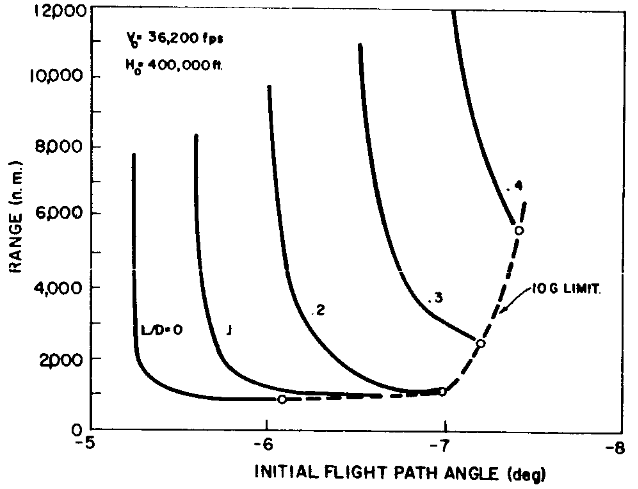

Fig. 4 Range capabilities

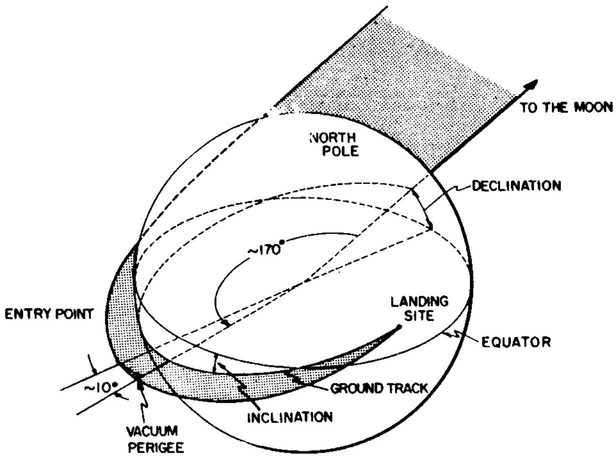

Fig. 2 Geometry of return trajectories

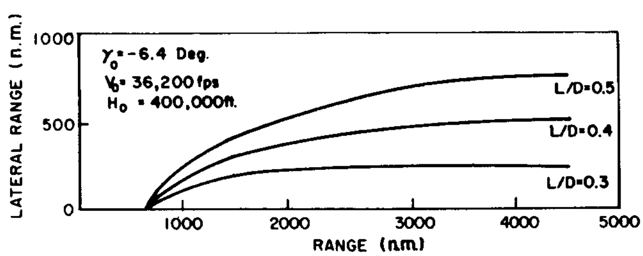

Fig. 5 Lateral range capability

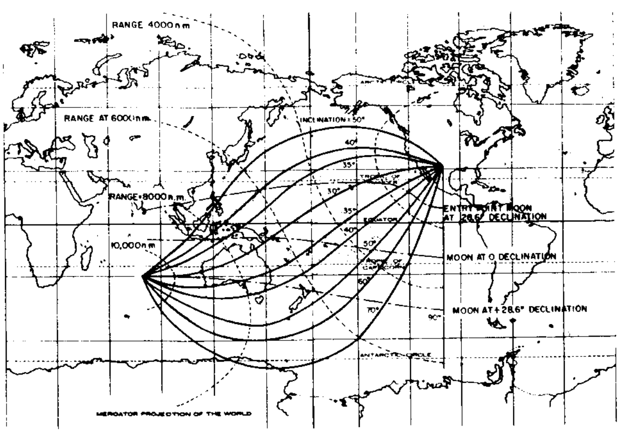

Fig. 3 Re-entry ranges to a landing site in Southern United States

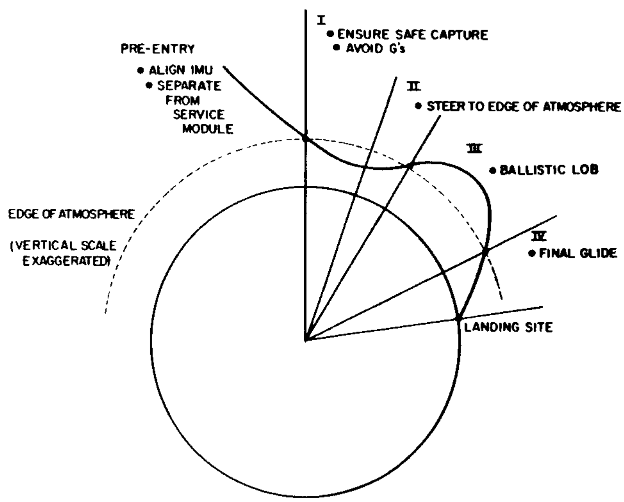

Fig. 6 Typical re-entry trajectory

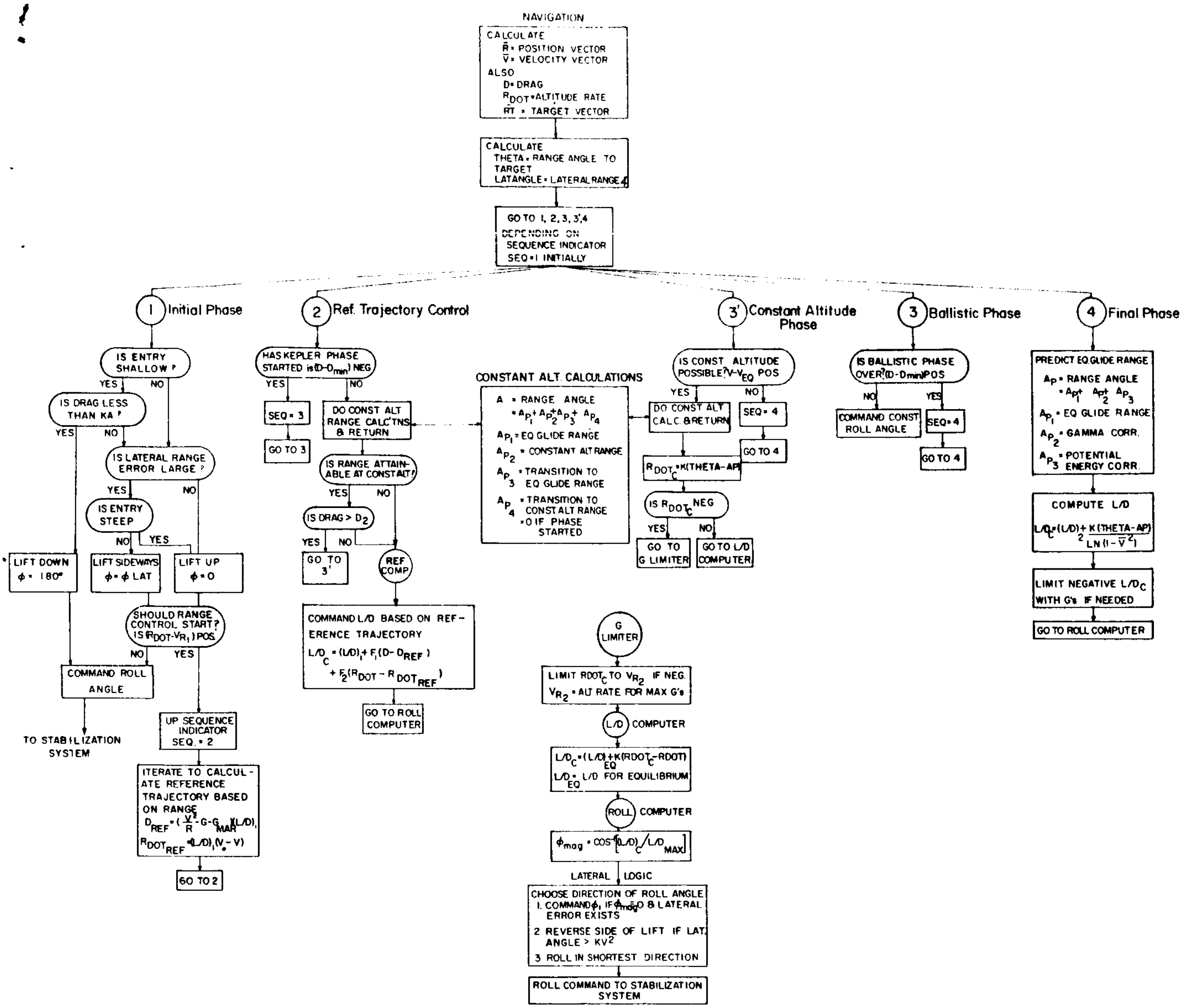

Fig. 7 Ballistic range sensitivity to velocity

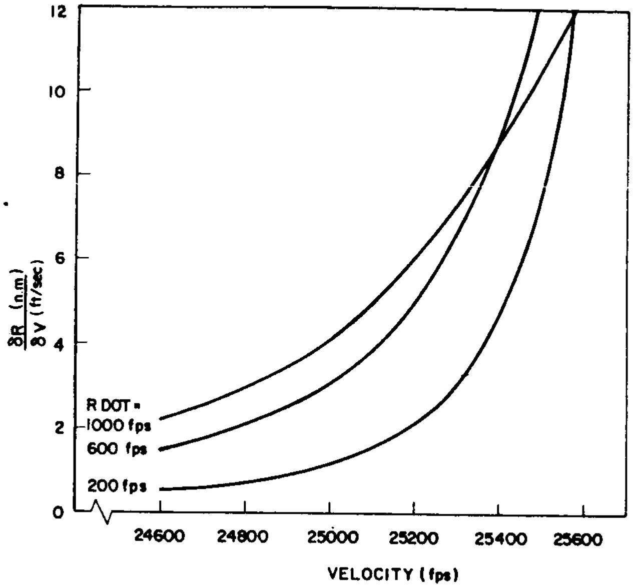

Fig. 8a Ballistic range sensitivity to velocity

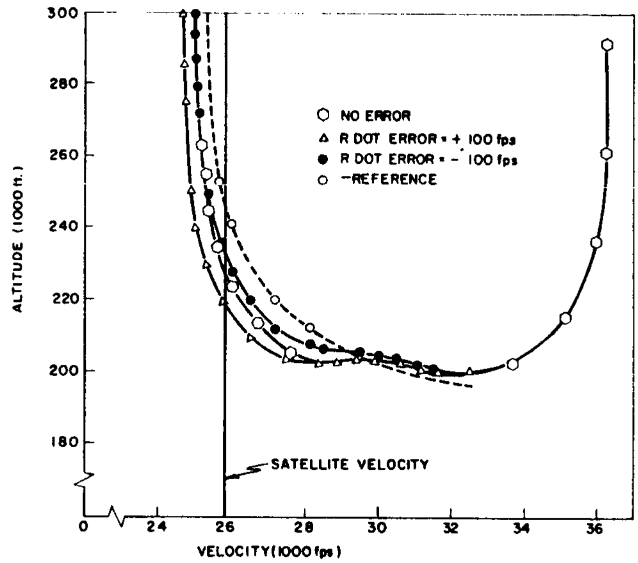

Fig. 9 Error performance

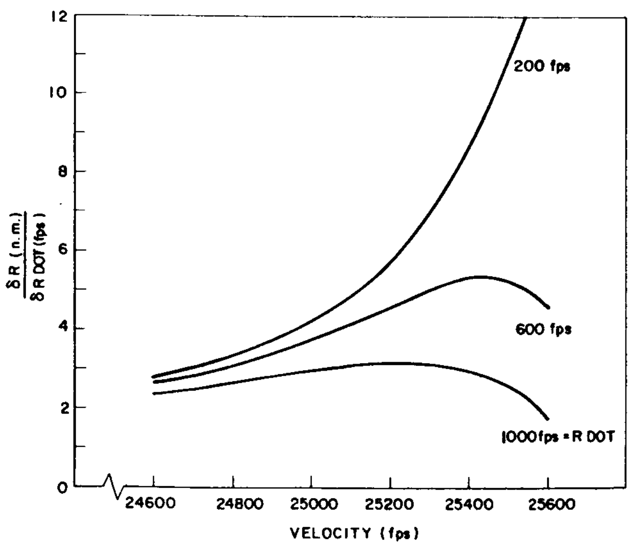

Fig. 8b Ballistic range sensitivity to altitude rate

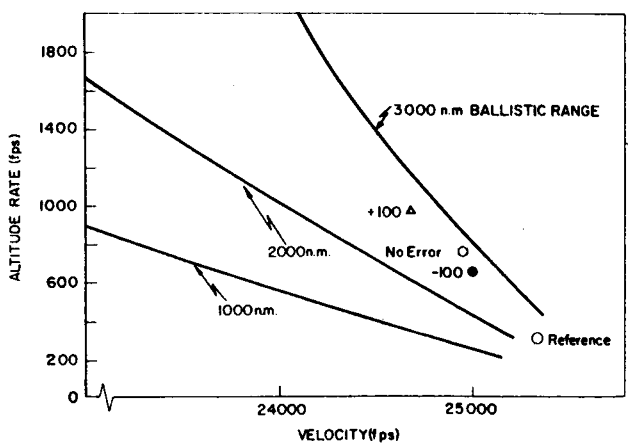

Fig. 10 Ballistic ranges

[redacted]

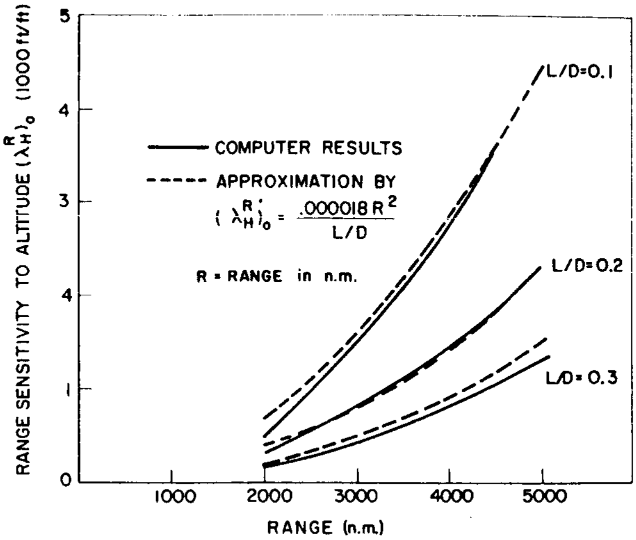

Fig. 11 Range sensitivity to altitude at bottom of initial pull-out

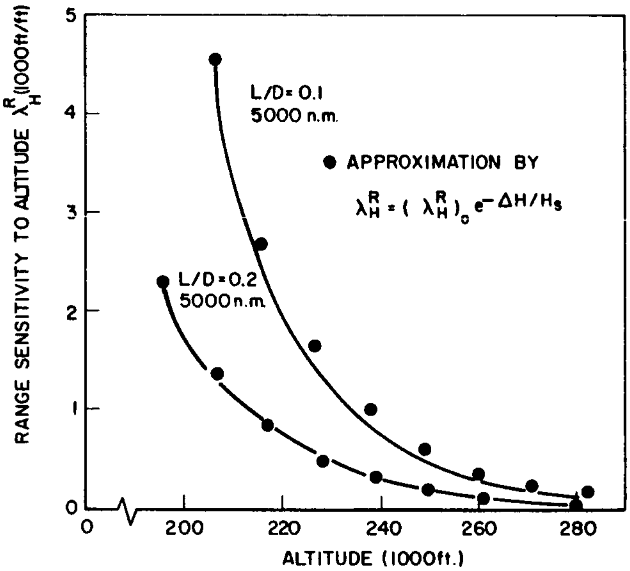

Fig. 13 Range sensitivity to altitude in super-circular region

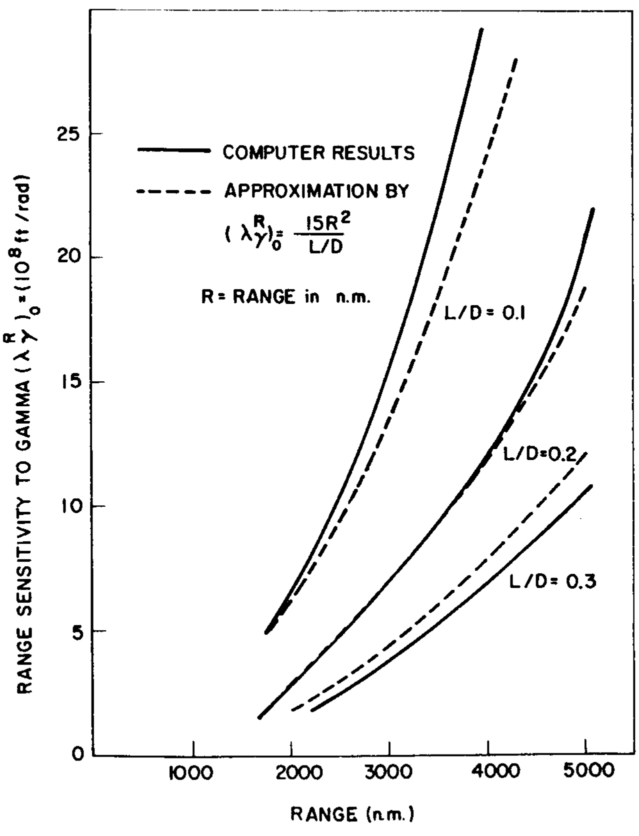

Fig. 12 Range sensitivity to flight path angle at bottom of initial pull-out

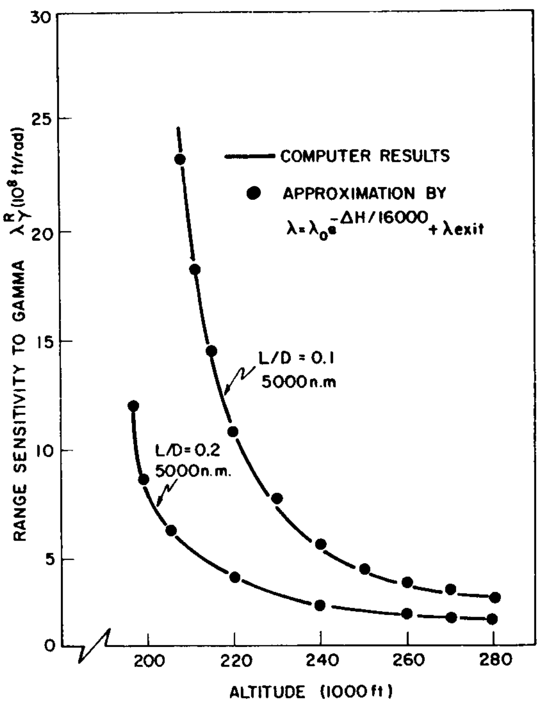

Fig. 14 Range sensitivity to flight path angle in super-circular region

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