Oct. 27-29, 1965

THE ITERATIVE GUIDANCE LAW FOR SATURN

Helmut J. Horn

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Huntsville, Alabama

SUMMARY: Based on Lawden's equation, semi-explicit, "iterative" Saturn guidance equations are derived. They were successfully flight tested on Saturn I and analyzed for the main Apollo mission and other applications.

INTRODUCTION

The major mission of the Saturn is to serve as launch vehicle for the Apollo lunar landing. The mission profile is a three stage ascent into parking orbit, followed by a reignition and injection to lunar transit. The Saturn Guidance System is primarily designed to meet the requirements of this mission, but it will satisfy others, like two or three stage flights to various orbits or escape.

Various guidance modes were evaluated with regard to accuracy, stability, the flexibility to change missions prior to flight or in emergencies during flight, and compatibility with the guidance hardware.

The term "mode" or "scheme" is used to describe the mathematical model and equations of the system.

GUIDANCE FUNCTIONS DURING A SATURN MISSION

The Lift-off Phase

The relative motion between launch site and moon requires change of the parking orbit plane as a function of lift-off time and influences the energy requirements decisively. Optimum launch time, launch windows, and launch azimuth are computed prior to flight. The launch azimuth is presented as a polynomial of lift-off time, and the reference coordinate system in the on-board guidance computer rotated correspondingly during actual countdown.

Unguided Early Flight Phase

It is convenient to split the guidance equations into two parts. The "navigation equations" define the state vector as function of initial conditions (launch site location, azimuth, earth rotation), computed gravitational acceleration, and inertially measured acceleration.

The "steering equations" provide maneuver commands from a comparison of the current and the desired final state vector.

During most of the first stage flight, aerodynamic and control loads override other optimization factors. The vehicle follows a precalculated tilt program without feedback from the steering equations. The navigation equations are used during this flight phase to update the state vector for use during later guided flight.

The possibility of active guidance and the compatibility with load constraints is presently being analyzed.

Guided Flight to Orbit

The guided flight to orbit consists of the S-II burn. For performance reasons, the S-II stage starts out with maximum thrust and is later stepped down, making it two stages from the guidance

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics, October 27-29, 1965, in Baltimore, Md.

point-of-view. It is followed by the first burn of the S-IVB stage. During this phase both the navigation and the steering equations are actively used. Upon reaching orbital condition, the flight is terminated. Either velocity or predicted time-to-go may be used as cutoff criterion.

Stay in Parking Orbit and Reignition

During the stay in parking orbit, only the calculated gravitational acceleration is used to compute the state vector, because nongravitational forces are below the accelerometer drift. The steering equations will only determine the optimum thrust direction and time for reignition. During each one of up to three orbits, there is one ignition opportunity or launch window, and departure will occur at the first opportunity where all systems are clear.

Injection to Lunar Transit

The injection phase to lunar transit is designed to reach at a prescribed time a target point at the influence sphere of lunar gravitation, i.e. after about 50 hours coast. While this assures compatibility with the spacecraft guidance, which takes over after injection, it does not directly provide suitable end conditions for the Saturn guidance. A hypersurface, containing initial conditions for a coast flight to the target, was calculated and furnishes satisfactory end conditions for the S-IVB power flight.

Alternate Missions

In case of a single engine failure of the S-IC or the S-II stage or, if the S-II stage fails completely after a certain burning time, the launch vehicle can continue flight to the parking orbit. The guidance system will automatically guide it toward the original end conditions. The only additional information is a signal at the time of such a failure.

THE SATURN GUIDANCE HARDWARE

Discussion of the guidance hardware will be limited to those features which influence understanding, development, and analysis of the guidance law.

The inertial platform (ST-124M) is a three-gimbal platform. Three air bearing gyros stabilize the inner gimbal, servo torquers compensate the friction in the platform gimbal axes. Pick-ups on the gimbal axes provide attitude references to the control computer. The platform carries three pendulous gyro accelmeters. Each one consists of a single-degree-of-freedom gyro with a mass unbalance. It converts one nongravitational acceleration component into a moment. The gyro precesses at a rate proportional to the torque. The precession angles of the three gyros furnish the three inertial velocity components to the guidance computer. Prior to launch, the platform is leveled and optically aligned with launch azimuth. The launch vehicle digital computer has many functions beyond calculating the guidance commands: e. g., pre-launch support of the ground computer, timing signals to various vehicle components, orbital checkout, etc. Its guidance program will accept initial conditions prior to launch, attitude and velocity components from the inertial platform, facilitate in-flight updating of information and selection of alternate missions. It uses these data to compute the steering, cutoff, and ignition equations in real time. The computer is a serial, fixed-point, general purpose computer: It has a core memory expandable to 32 thousand words of 28 bits. Typical addition time is 82 micro-seconds. It uses triple modular redundancy in the control logic.

The guidance computer sends steering commands to the analog control computer. The control computer provides proper signal mixing, filtering and phasing for the commands to the control actuators.

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics, October 27-29, 1965, in Baltimore, Md.

DERIVATION OF THE ITERATIVE STEERING EQUATIONS

Comparison of Various Guidance Modes

Some requirements for an ideal guidance mode are: performance optimization, independence of past flight history, limited number of inputs to specify the basic and alternate missions, flexibility, generality and simplicity. As these requirements are mutually exclusive, a satisfactory compromise must be established.

Optimization of the trajectory is based on calculus of variations. However, no general, explicit solution for a flight path optimization, a two point boundary condition problem, is known. For pre-flight optimization, a repetitive numerical integration with a computer programmed isolation of the required end conditions is available. This method is however too complex and time consuming to be used in real time on the on-board computer. One possible solution is to pre-compute an adequate number of reference trajectories, to curve-fit and store the results and use this "map" to find the proper command for any flight situation. The path-adaptive polynomial[1,2] and the minimax guidance mode[3] use this method. It has the advantage of very simple computer equations, and it can also readily handle cases where, because of other constraints, the trajectory will strongly deviate from a performance optimum. Another choice, which was selected for the Saturn guidance, is to find an approximate explicit solution to the trajectory optimization problem, which is simple and still yields adequate accuracy.

*Definition of symbols on page 11.

Two Dimensional, Single Stage Flight

The guidance equations are first derived for the restricted case of a two-dimensional trajectory and a single-stage vehicle. Then they are expanded to the general case.

The gravitation field of the earth for the range of a Saturn propelled flight is far from uniform. But, if the steering equation is solved periodically during flight, the gravitation field for the shrinking remaining part of trajectory approaches uniformity. Correspondingly, the errors caused by the initially wrong assumption will gradually disappear. Because of the similarity to a mathematical iteration, this method was called iterative[4-7].

Fried[8] and Lawden[9] gave an analytical solution for the flat earth trajectory optimization problem Equation (1)*.

$$\tan\varphi = \frac{\partial R/\partial \dot{y}_T + (T-t)\ \partial R/\partial y_T}{\partial R/\partial \dot{x}_T + (T-t)\ \partial R/\partial x_T} \quad \text{(1a)}$$

$$= \frac{a + bt}{1 + ct} \quad \text{(1b)}$$

The equation contains three free coefficients which are sufficient to satisfy three end constraints. Cutoff time provides one additional degree-of freedom, so it appears that four end conditions can be satisfied, e.g., both coordinates and both velocity components. However, it is obviously not very efficient to control range by lateral maneuvers. Actual trajectory calculation of a point landing verify this and lead to spiraling. The equation should therefore only be used with two end constraints, and the third one replaced by an orthogonality condition.

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics

By removing the x_T-constraint, optimization requires that

$$\partial R/\partial x_T = 0 \qquad (2)$$

and Equation (1) becomes

$$\tan \varphi = \frac{\partial R/\partial \dot{y}_T + (T-t)\ \partial R/\partial y_T}{\partial R/\partial \dot{x}_T} \qquad (3a)$$

$$= a' + b'\ t \qquad (3b)$$

If the additional x-constraint is required, programming of the thrust level can be used. It will also allow a limited control over a possible fifth constraint, the arrival time.

Even the simplified Equation (3) does not yield a truly explicit solution for the coefficients as function of the end constraints. Such a solution is, however, easily available if one more constraint is replaced by an orthogonality condition, e.g., by:

$$\partial R/\partial y_T = 0 \qquad (4)$$

A typical application for this equation would be an escape mission from orbit, with a specified end velocity vector, but no constraints on the injection coordinates. The solution can immediately be understood from Figure 1.

In order to maximize the inertially indicated velocity increase, v_i, resulting from a given ideal velocity, e.i., the integral of the absolute value of the thrust acceleration, the thrust direction has to be constant or, figuratively speaking, the "chain" of Δv_i's has to be stretched. The locus for the terminal velocity vector is a circle with the radius v_i and the center defined by the vector sum of the initial velocity v_1 and the gravitational velocity increase v_g. Figure 1 shows the optimum thrust direction φ'' for a specified final velocity direction θ_T, and the absolute optimum thrust direction φ'''.

The requirement for constant thrust direction results by substituting Equation (4) into Equation (3):

$$\tan \tilde{\varphi} = \frac{\partial R/\partial \dot{y}_T}{\partial R/\partial \dot{x}_T} \qquad (5)$$

The ideal velocity is determined by the rocket equation:

$$\dot{\tilde{v}}_i = F/m = c^* \frac{\dot{m}}{m_1+\dot{m}t} = \frac{c^*}{\tau-t} \qquad (6a,b,c)$$

$$\tilde{v}_i = c^* \ln \frac{m_1}{m_1+\dot{m}t} = c^* \ln \frac{\tau}{\tau-t} \qquad (7a,b)$$

$$\dot{\tilde{x}}_i = \dot{x}_T - \dot{x}_i = v_T \cos \theta_T - \dot{x}_1 \qquad (8a,b)$$

$$= c^* \cos \tilde{\varphi} \ln \frac{\tau}{\tau-t} \qquad (8c)$$

$$\dot{\tilde{y}}_i = \dot{y}_T - \dot{y}_1 + g\ T \qquad (9a)$$

$$= v_T \sin \theta_T - \dot{y}_1 + g\ T \qquad (9b)$$

$$= c^* \sin \tilde{\varphi} \ln \frac{\pi}{\tau-t} \qquad (9c)$$

$$\tan \tilde{\varphi} = \dot{\tilde{y}}_i/\dot{\tilde{x}}_i \qquad (10)$$

The simultaneous solution of Equations (7), (8), and (9) will provide T and $\tilde{\varphi}$ as required by the problem.

Three Constraints

For a given mission, e.g., injection in an orbit at a specified altitude, the optimum thrust-over-mass ratio can be determined. If this ratio cannot be implemented, e.g., because of existing hardware limitations, or if different missions call for orbits far removed from the optimum, Hohmann transfer or similar maneuvers are usually applied

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics, October 27-29, 1965, In Baltimore, Md.

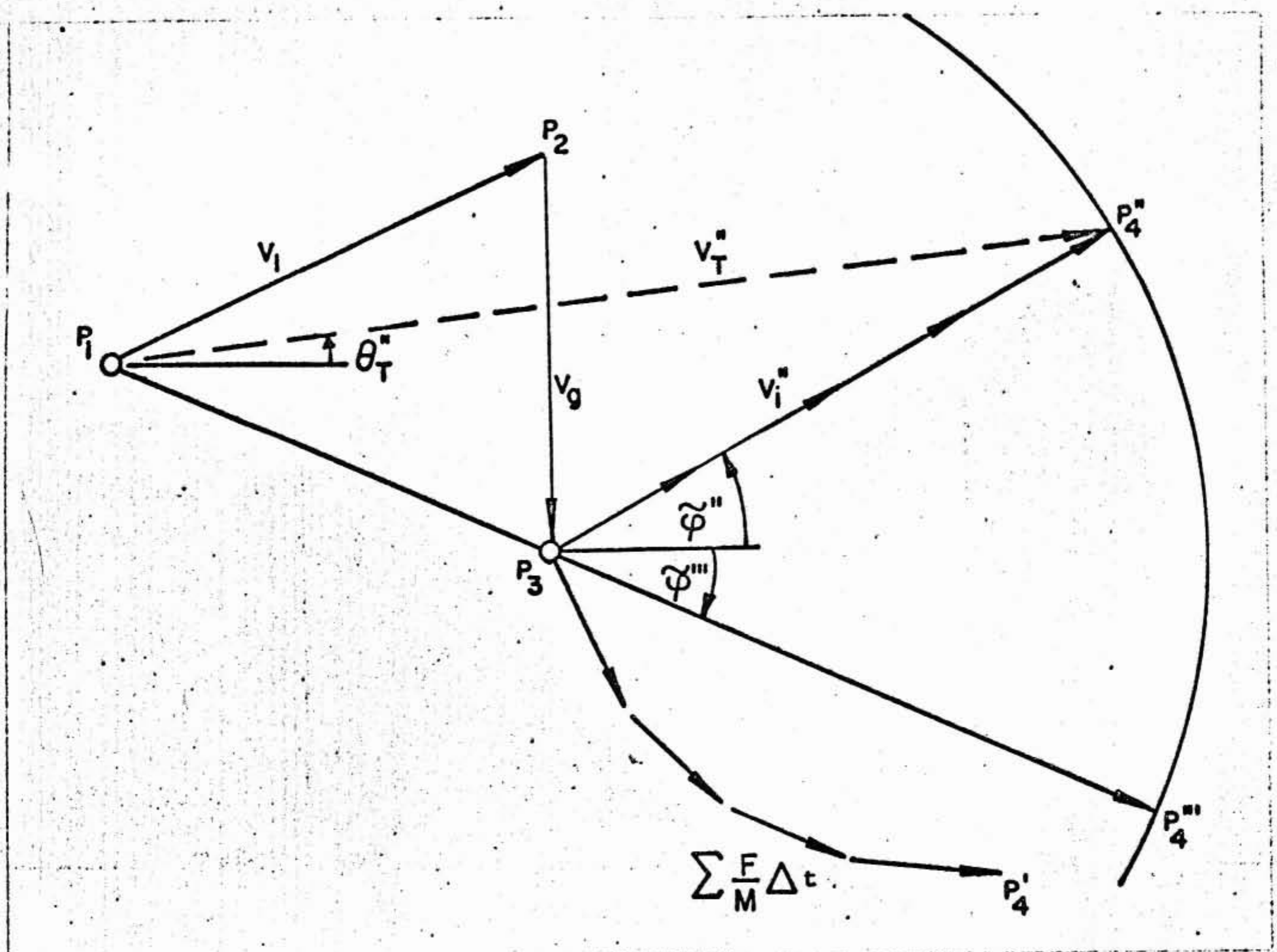

Figure 1: Optimization for Free Choice of End Location

after passing through an optimum intermediate (parking) orbit, to satisfy the mission and preserve payload. The powered trajectory into the optimum orbit (assuming mission gravity and disregarding constraints during early flight) follows the constant law of Equation (5). For trajectories close to optimum, the time varying attitude of Equation (3) can be treated as perturbation of Equation (5)

$$\tan \varphi = \tan \tilde{\varphi} - (k_1 - k_2 t) \qquad (11a)$$

which can be approximated by

$$\varphi = \tilde{\varphi} - (k_1 - k_2 t) \qquad (11b)$$

The normal acceleration for time-varying thrust attitude

$$\ddot{y}_c = c^* \sin [\tilde{\varphi} - (k_1 - k_2 t)]/(\tau - t) \qquad (12a)$$

can for small values of $(k_1 - k_2 t)$ be expressed as

$$\ddot{y}_i = \ddot{\tilde{y}}_i - c^* (k_1 - k_2 t) \cos \tilde{\varphi}/(\tau - t) \qquad (12b)$$

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics, October 27-29, 1965, in Baltimore, Md.

$$\dot{y}_i = \dot{\tilde{y}}_i - c^* \cos \tilde{\varphi} \left[k_1 \ln \frac{\tau}{\tau - t} - k_2 \left(\tau \ln \frac{\tau}{\tau - t} - t \right) \right] \quad (13)$$

$$y_i = \tilde{y}_i + c^* \cos \tilde{\varphi} \, (k_1 - k_2 \tau) \left[(\tau - t) \ln \left(\frac{\tau}{\tau - t} \right) - t \right] - \frac{1}{2} c^* \cos \tilde{\varphi} \, k_2 \, t^2 \quad (14a)$$

$$y_T(t) = y_1 + \dot{y}_1 t - c^* [\sin \tilde{\varphi} - (k_1 - k_2 \tau) \cos \tilde{\varphi}] \left[(\tau - t) \ln \frac{\tau}{\tau - t} - t \right] - \frac{1}{2} (c^* k_2 \cos \tilde{\varphi} + g) t^2 \quad (14b)$$

Equations (13), (14), and the equivalent equation for $\dot{x}_i$ are solved simultaneously for the end point (a,b) to provide k_1 and k_2 and to check or up-grade T.

To apply these equations to a non-uniform (Figure 2) central gravitational field, an "effective" gravitational acceleration g* and direction ϕ^* are introduced:

$$g^* = \frac{1}{2} (g_T + g_1) \quad (15)$$

$$\phi^* = \frac{1}{2} (\phi_T - \phi_1) \quad (16)$$

quation (2) defines the direction of the x-axis as normal to the specified end point coordinate. For the "flat" earth, with a specified altitude, this corresponds to the trajectory coordinate system, x being horizontal and y vertical. For other end conditions and for a spherical earth, the guidance coordinate system ξ, η has to be rotated against the space fixed trajectory coordinate system x, y. In the case of horizontal injection, e.g., at perigee into orbit, the rotation angle equals the range angle ϕ_T. Its calculation will be explained later. The attitude angle φ is measured against the ξ-axis. Equations (8a), (9a) and (10) become:

$$\dot{\tilde{\xi}}_i = \dot{\xi}_T - \dot{\xi}_1 + g^* T \sin \phi^* \quad (17)$$

$$\dot{\tilde{\eta}}_i = \dot{\eta}_T - \dot{\eta}_1 + g^* T \cos \phi^* \quad (18)$$

$$\tan \tilde{\varphi} = \dot{\tilde{\eta}}_i / \dot{\tilde{\xi}}_i \quad (19)$$

Following the process outlined for the uniform gravitational field, Equations (12), (13), and (14),

$$\ddot{\eta}_i = \ddot{\tilde{\eta}}_i - c^* (k_1 - k_2 t) \cos \tilde{\varphi} \, / \, (\tau - t) \quad (20)$$

$$\dot{\eta}_i = \dot{\tilde{\eta}} - c^* \cos \tilde{\varphi} \left[k_1 \ln \frac{\tau}{\tau - t} - k_2 \tau \left(\ln \frac{\tau}{\tau - t} - t \right) \right] \quad (21)$$

$$\eta_T(t) = \eta_1 + \dot{\eta}_1 t - c^* [\sin \tilde{\varphi} - (k_1 - k_2 t) \cos \tilde{\varphi}] \left[(\tau - t) \ln \frac{\tau}{\tau - t} - t \right] - \frac{1}{2} (c^* k_2 \cos \tilde{\varphi} + g \cos \phi^*) t^2 \quad (22)$$

Time-to-go, T, is predicted from Equation (7b), the final range angle ϕ_T by integrating Equation (8) and dividing esulting $\tilde{x}_T$ by the radius vector R_T.

k_1, k_2 and, if found necessary, an updated T can now be calculated by simultaneous solution of Equations (21), (22), and the corresponding Equation for T.

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Elec-

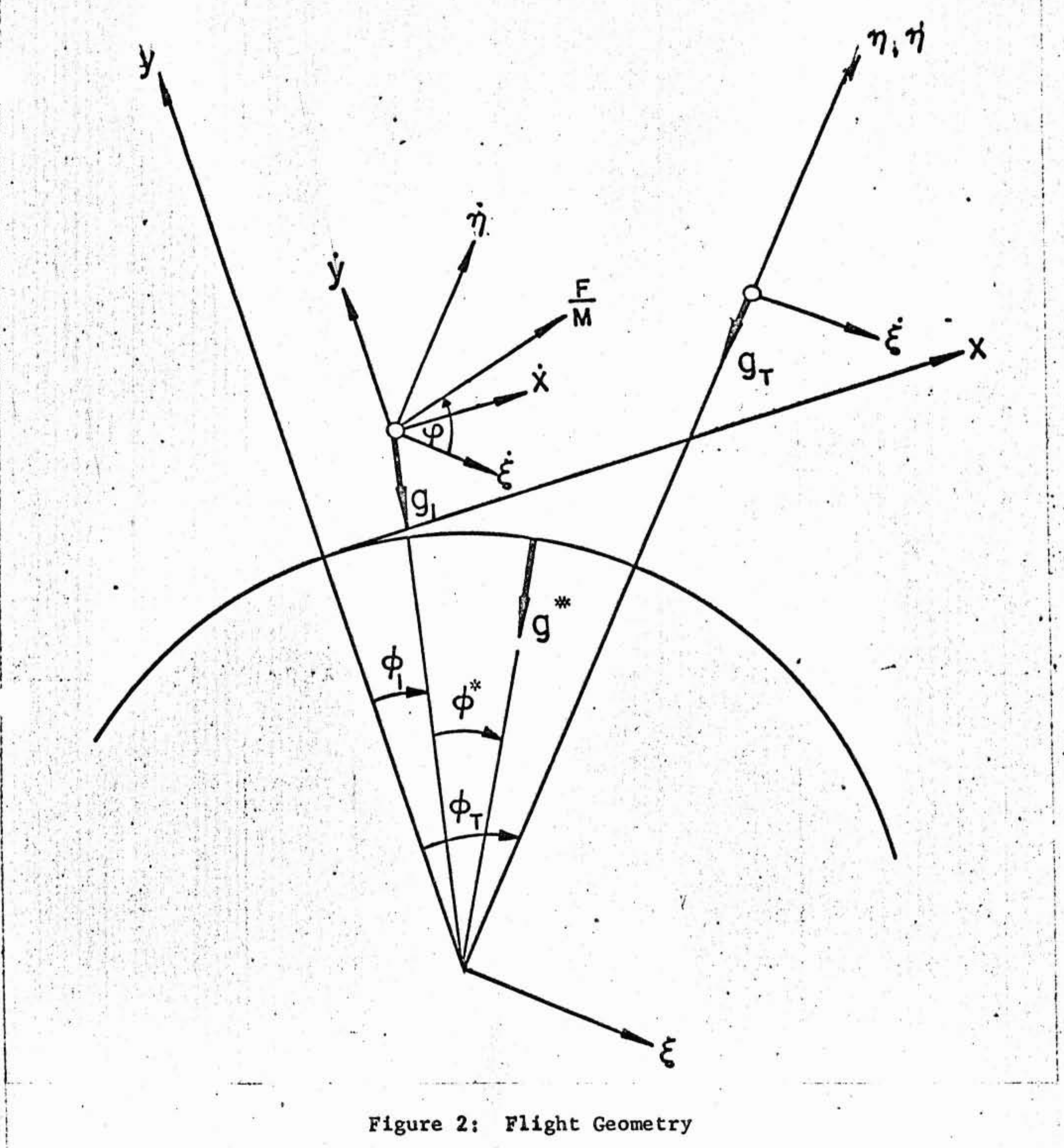

Figure 2: Flight Geometry

Multi-Stage, Three Dimensional Flight

The multi-stage Saturn requires optimization of trajectories with discontinuities. Fortunately tilt angle and rate remain continuous. As long as a constant or time linear attitude law is used, the same law can be used through all stations. If each stage, except the last one, is defined by c^*, τ , and T, the last stage by c^* and τ, it is possible to continue the equations for the individual stages into a set for the total vehicle. There is no point in writing these lengthy equations, but a flow

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics, October 27-29, 1965, in Baltimore, Md.

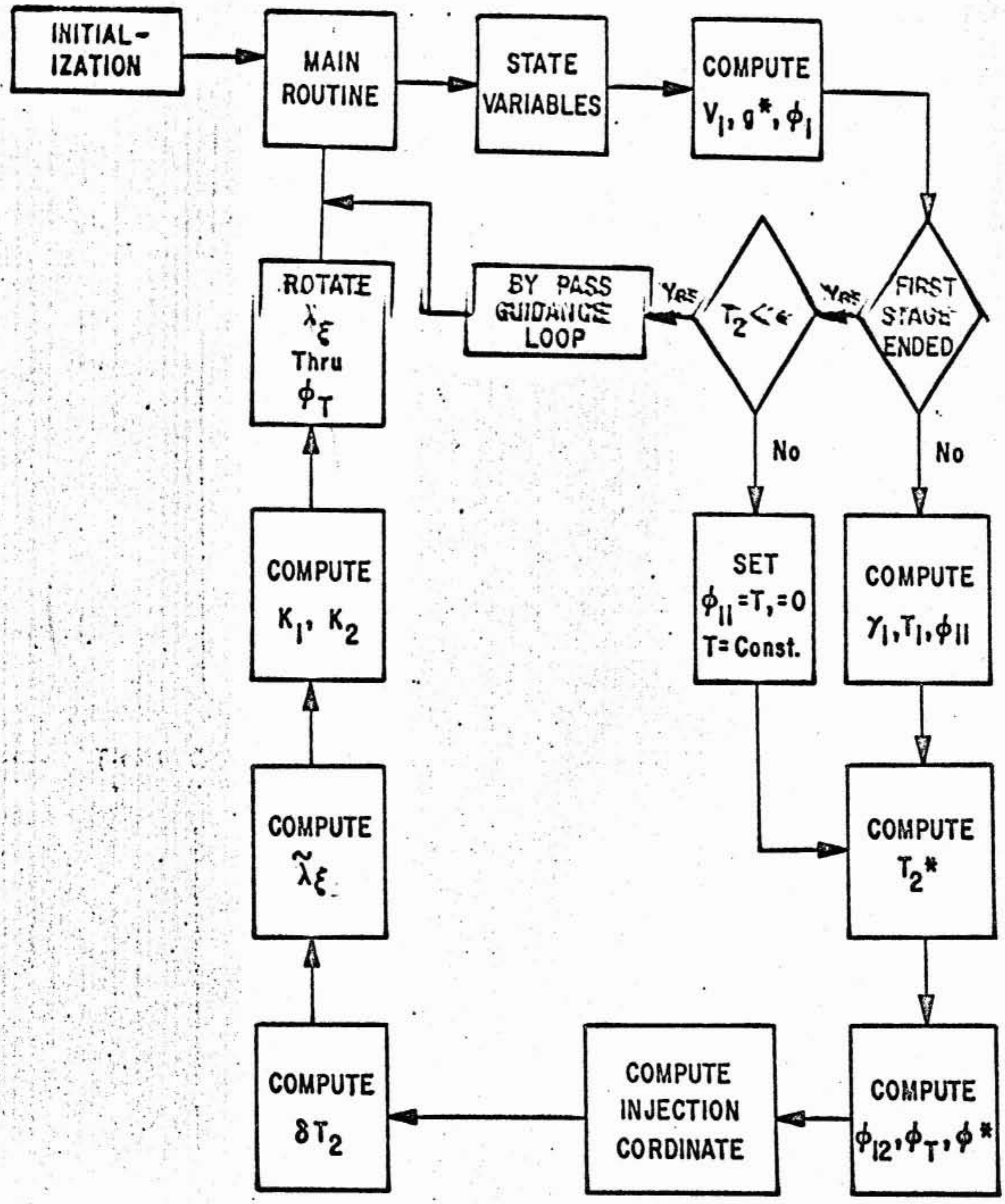

Figure 3: Flow Diagram

diagram will give some impression of the computation involved (Figure 3). Lateral guidance can easily be added, following the derivation for the in-plane guidance. The time-to-go, T, computation does not have to be repeated, and the gravitational terms are reduced in significance.

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics, October 27-29, 1965, in Baltimore, Md.

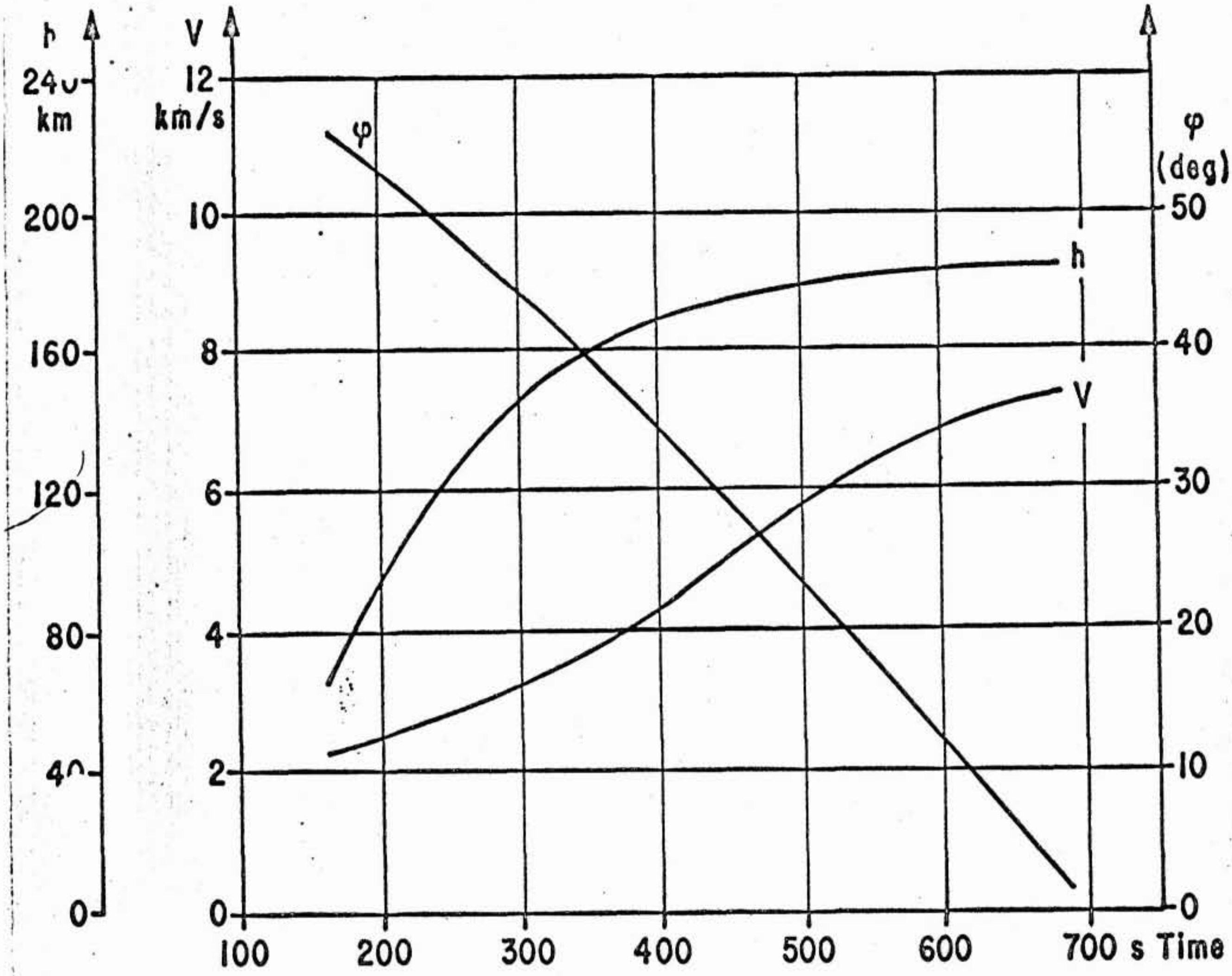

Figure 4: Typical Saturn V Trajectory

A typical Saturn V trajectory starts vertically for about 12 seconds. Tilt is initiated by an angle-of-attack "kick" between 12 and 35 seconds; which is optimized for maximum payload in orbit, followed by a gravity-turn, zero-lift arc for the remainder of the first stage. Guidance is used through the second (S-II) and third (S-IVB) stage. Figure 4 shows altitude, velocity, and attitude angle, as they result from use of the iterative guidance equations. Attitude changes almost linearly from 54° to 0°. The curvature at the early part is caused by the violation of the assumption of uniform gravity and the sma˜ angle approximation. The payload loss is about 0.02%, compared against a C.O.V. optimization.

The flight from orbit to injection shows an interesting aspect. Only two end constraints are required to start the correct transit coast: total energy and velocity direction. Consequently, a constant attitude should give the best performance (Equation 5). However, a 2% payload loss indicates that the basis assumptions are severely violated. By optimizing this flight arc with C.O.V and adding the resulting radius vector for the injection point as additional constraint, performance improves, and the loss is reduced to 0.03%. This solution is adequate for the basic Saturn missions.

STABILITY AND ERROR ANALYSIS

The partial derivatives of attitude with respect to the state variables are the most significant criteria for stability and accuracy. The F/m derivative is small during the entire flight, eliminating this usually rather noisy measurement as trouble source. However, as the trajectory optimization is based on a predicted relation of the future thrust profile for a stage to the instantaneously measured value, any major thrust change will cause a performance loss.

The other derivatives start at low values and increase approximately inversely proportional to the time-to-go (for velocity errors) or its square (for displacement). The tightening of the guidance loop toward the end of flight is very desirable as it keeps residual errors small. However, it creates a potential stability problem. This problem was eliminated without causing a significant error, by stopping computation of the steering equations at a given time-to-go (e.g., T = 20 seconds) and flying open loop. A better method is to freeze the time-to-go at a minimum value and continue guidance.

The low guidance gains at early flight make the system very tolerant to major disturbances, noise and time lags during this phase.

Reducing the thrust during second S-IVB burn by 46% resulted in a error at periselenum of one moon radius, if no midcourse correction was used.

Guidance scheme errors for realistic variations of initial conditions (Table I) are very small. The effects of performance variations, changes in air density, and winds are equally insignificant.

A time lag of 5 seconds from measurement to steering command causes no error and no loss of weight in orbit. A 40 seconds lag caused 3 km altitude error and 11% payload load loss.

Periodic thrust fluctuations with a maximum amplitude of 65% of nominal and periods of 5 - 100 seconds create no serious stability problem.

TABLE I

Initial Stage Variable				Payload	Injection Errors		
Δx_1	$\Delta \dot{x}_1$	Δy_1	$\Delta \dot{y}_1$	Loss	Altitude	Velocity	Path Angle
km	m/s	km	m/s		m	m/s	degrees
+2.7	0	0	0	.11%	.1	0	+.001
0	+143	0	0	.32%	.13	-.04	0
0	- 57	0	0	.07%	.1	+.01	+.001
0	0	1.0	0	.11%	.1	0	+.001
0	0	0	+78	.14%	.1	-.01	0
0	0	0	-80	.11%	.1	0	+.001

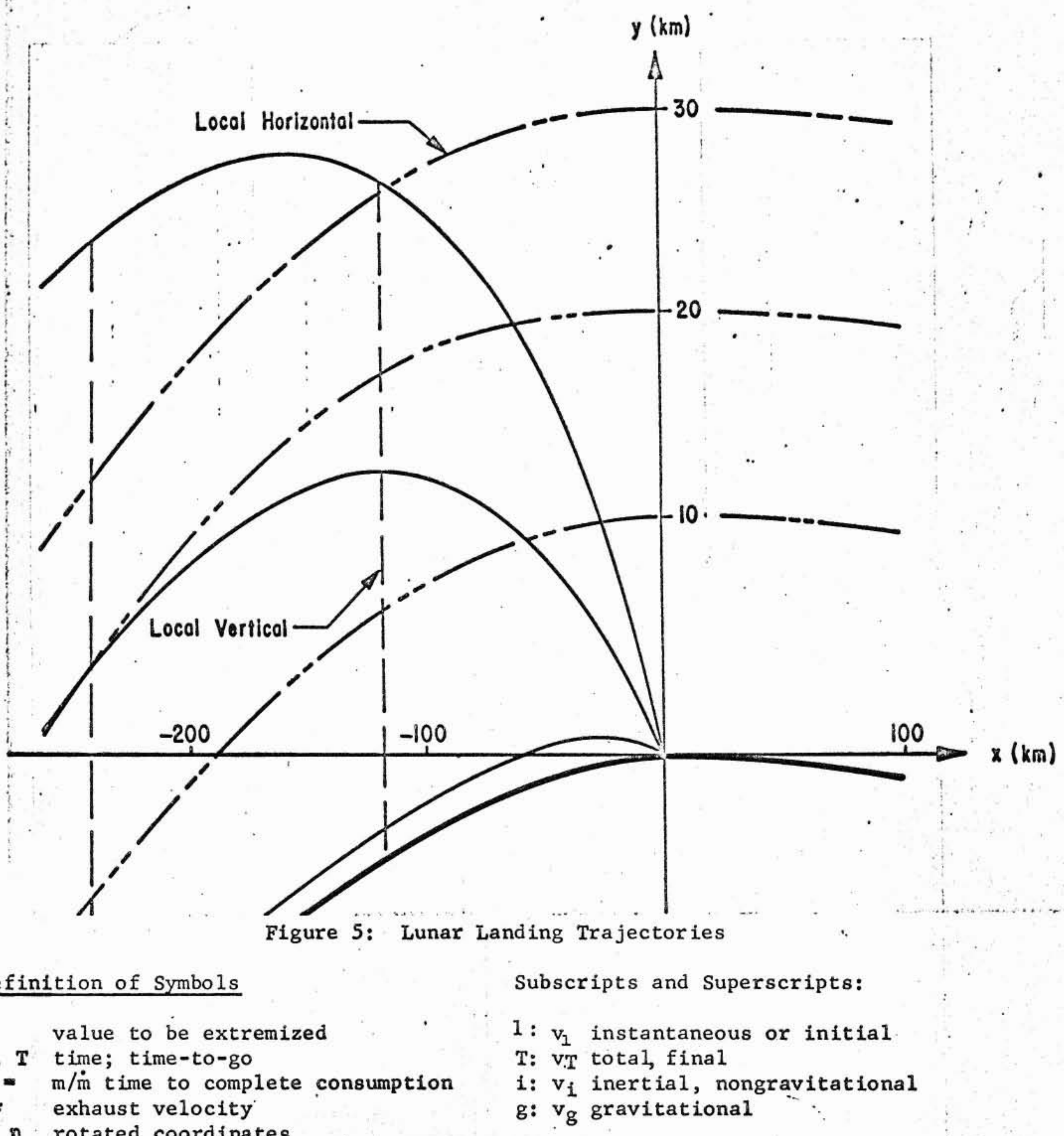

Figure 5: Lunar Landing Trajectories

Definition of Symbols

R	value to be extremized
t, T	time; time-to-go
τ =	$m/\dot{m}$ time to complete consumption
C*	exhaust velocity
ξ, η	rotated coordinates
φ	attitude angle
θ	path tangent
$\emptyset$	central or range angle

Subscripts and Superscripts:

1: v_1	instantaneous or initial
T: v_T	total, final
i: v_i	inertial, nongravitational
g: v_g	gravitational
$\sim$: $\tilde{\varphi}$	value for restricted case
*: $\emptyset^*$	effective

An engine failure presents another severe test for the guidance system. Table II shows effects of an S-II engine failure as function of fail time for nominal and nonnominal propellant loading.

TABLE II

Failure Time	Propellant Loading	Orbital Weight Loss	
		No Signal	Signal
Sec		%	%
10	nominal	4.5	0.08
10	off-nominal	1.8	0.11
120	nominal	0.3	0.03
no fail	nominal	0.01	0.01
no fail	off-nominal	0.03	0.03

If the guidance computer receives a signal at the time of engine out, the scheme losses are negligible. If this signal is eliminated, the losses, while still small, increase by a factor of more than 10. The reason is the fixed presetting of the time-to-go for the multi-stage vehicle, which is based on all-engine-working assumption. If this should become a problem, a possible solution would be to replace T_j by v_{ij} per stage. The cost would be the additional computation, of T_j from v_{ij} and the measured (F/m).

FLIGHT TESTS

A two-dimensional iterative guidance was successfully tested on Saturn flight 8, 9, and 10. Scheme errors were inside the telemeter accuracy and, as expected, well below the hardware errors.

OUTLOOK AND CONCLUSION

The iterative guidance mode was orginally developed as an economic trajectory optimization method. It has been used successfully for calculation of orbits involving extreme maneuvers, where the existing C.O.V. programs were difficult to establish. Its simplicity led to its use as a guidance scheme. It was first exercised on the problem of a lunar landing[6,7], e.g., from an elliptic intermediate orbit with a nominal periselenum of 20 km above ground. The guidance computes ignition time, thrust level and thrust direction. Figure 5 shows the landing trajectories for nominal altitude and a plus and minus 100% variation. Touchdown was within 0.4 meters of the designated point, the velocity within 0.1 m/s and payload losses compared to optimum within the computing accuracy.

One of the future problems under investigation are the extension of the system to lower thrust values. Better approximation of the trigonometric functions and the adaptation of Fried's more elaborate Equation[10] are under study.

References 11 through 15 list work along similar lines, conducted independently at various places. Lack of communication has, in the early phase, stimulated orginality in methods and approaches; however, it would be wasteful if continued.

In conclusion, the Iterative Guidance Mode for the Saturn shows promise to satisfy all requirements. While analysis and flight tests have shown no serious difficulties, some modifications and improvements will probably be made during the final implementation.

This paper was presented at the Twelfth East Coast Conference on Aerospace and Navigational Electronics, October 27-29, 1965, in Baltimore, Md.

REFERENCES

1. Hoelker, R.F., "The Evolution of Guidance Theory and Trajectory Analysis into a Single Scientific Discipline," prepared for presentation at the "Meeting of the Institute of Navigation," in Williamsburg, Va., June 1961. Marshall Space Flight Center, Huntsville, Alabama.
2. Miner, W.E., Schmieder, D.H., and Braud, N.J., "The Path-Adaptive Mode for Guiding Space Flight Vehicles," ARS Guidance, Control and Navigation Conference, Stanford, California, August 1961. Revised March 1962, Reprint from Progress in Astronautics and Rocketry, 1962 Academic Press, New York, New York.
3. Baker, C.D., "A Minimum Fuel Reserve Guidance System," Aero-Internal Note 3-64 (Limited Distribution) January 14, 1964.
4. Smith, I.E., Hart, J.J., and Chandler, D.C., "Procedure for Implementing A Simplified Path Adaptive Scheme," G. C. Marshall Space Flight Center, Aeroballistics Internal Note #25-62, July 31, 1962.
5. Horn, H.J., Martin, D.T., and Chandler, D.C., "An Iterative Guidance Scheme and Its Application to Lunar Landing," G. C. Marshall Space Flight Center report, MTP-AERO-63-11 (Limited Distribution) February 6, 1963.
6. Horn, H.J., "Application of An "Iterative Guidance Mode" To A Lunar Landing," Paper No. 1504 (63) presented at IIIrd European Space Flight Symposium and 15th Annual Meeting of the DGRR in Stuttgart, Germany, May 22-24, 1963. Reprinted in: Raumfahrtforschung, Heft 2, April-Juni 1964, pp 49-54.
7. Smith, I.E., and Deaton, Jr., E. T., "An Iterative Guidance Scheme For Ascent To Orbit (Suborbital Start of The Third Stage)," G. C. Marshall Space Flight Center report, MTP-AERO-63-44, May 29, 1963.
8. Fried, B.D., "On the Powered Flight Trajectory of an Earth Satellite," Jet Propulsion, Vol. 27, June 1957, pp 641-643.
9. Lawden, D.F., "Optimal Rocket Trajectories," Jet Propulsion, Vol. 27, December 1957, p. 1263.
10. Fried, Burton D., "Trajectory Optimization for Powered Flight in Two or Three Dimensions," in Space Technology, Edited by H. Seifert, 1959, p. 4-9.
11. Perkins, F.M., "Flight Mechanics of Ascending Satellite Vehicles," Jet Propulsion, Vol. 26, pp 352-358, May 1956.
12. MacPherson, D., "An Explicit Method of Guiding A Vehicle From An Arbitrary Initial Position and Velocity To A Prescribed Orbit," Aerospace Corporation, TDR-594 (1565-01)TN-1, February 13, 1961.
13. Cherry, G.W., "Orbit Insertion Guidance Technique, "Space Guidance Analysis Memo #30, MIT, Instrumentation Laboratory, November 29, 1962.
14. Cherry, G.W., "A Unified Explicit Technique for Performing Orbital Insertion, Soft Landing, and Rendezvous with a Throttleable Rocket-Propelled Space Vehicle," AIAA Guidance and Control Conference; August 12-14, 1963.
15. Perkins, F.M., "Optimum Guided Ascent," Aerospace Corporation, SSD-TDR-64-120, June 1964.