MEMORANDUM

TO: J. Barker

FROM: R. Cushing

DATE: January 9, 1968

SUBJECT: Limits of Gimbal Rates

The following analysis is pursuant to establishing the values of "rate" limit for the gimbals to prevent the gas-bearing gyros from seeing a tumbling rate in excess of their allowable limit, now quoted at 6 rad/sec.

Summary

Under conditions of "caging", a gyro may experience an angular velocity of $\sqrt{5}\ W_L$ orthogonal to its spin axis, where W_L is the limit rate of the gimbal servo drive.

In "coarse align" a vehicular angular velocity must be added to the above number in a worst case sense, i.e. colinear and adding. If this velocity is specified to be 0.35 rad/sec (re-entry rate), then using this value of vehicular rotation, a maximum allowed rate of 6 rad/sec orthogonal to a gyro SA, and a safety factor of 2:1, the limit velocity of the gimbals would be slightly in excess of 1 rad/sec.

Development

1.0 With reference to Fig. 1, the following is stated or assumed.

1.1 Two gyros have their spin axis (SA's) in the plane of the platform, these axes being mutually orthogonal. It is assumed that either of these SA's may be rotated about the inner gimbal axis I I' arbitrarily, to position them in their most vulnerable position. The third gyro has its SA orthogonal to the plane of the platform. The vehicular velocity in this analysis is assumed to be zero.

1.2 The platform may be rotated about the middle gimbal axis M M' to any arbitrary angle P to position an SA to its most vulnerable position. M M' is orthogonal to O O', and lies in the plane of the platform.

1.3 The platform and middle gimbal are rotating about the O O' outer gimbal axis at the rate $\dot{O}$ and may be at any initial reference angle relative to space or vehicle. The outer gimbal vector velocity $\dot{O}$ has a component in the platform and a component normal to the platform. These are:

$\dot{O}$ cos P, in the plane of the platform

$\dot{O}$ sin P, normal to the platform

1.4 The rate of rotation of the middle gimbal M is always coplanar with the platform. Therefore in the plane of the platform there is a velocity vector which is the sum of two vectors, $\dot{M}$ and $\dot{O}$ cos P. Because these vectors are mutually perpendicular, the magnitude of their sum is:

Eq. 1
$$|W_P| = \left[\dot{M}^2 + \dot{O}^2 \cos^2 P\right]^{1/2}$$

where $|W_P|$ is the magnitude of the sum of velocity components in the plane of the platform. As stated in par. 1.1 one of the SA's in the plane of the platform is assumed to be positioned relative to W_P in the maximally vulnerable manner, i.e. exactly orthogonal to W_P.

 Page 3

This same SA is simultaneously orthogonal to the velocity vector $\dot{I} + \dot{O} \sin P$ that lies along the axis I I' of the inner gimbal. The total magnitude of the vector velocity seen by this SA is thus:

Eq. 2

$$W_M (P) = \left[\dot{M}^2 + \dot{O}^2 \cos^2 P + \dot{I}^2 + \dot{O}^2 \sin^2 P + 2\,\dot{I}\,\dot{O} \sin P\right]^{1/2}$$

$$= \left[\dot{M}^2 + \dot{O}^2 + \dot{I}^2 + 2\,\dot{I}\,\dot{O} \sin P\right]^{1/2}$$

where W_M (P) is the maximum velocity orthogonal to the SA positioned in the most vulnerable manner in the plane of the platform. $\dot{I}$ is the velocity of rotation of the inner gimbal.

2.0 Discussion

2.1 Using Eq. 2 and assuming that:

$\dot{O} = \dot{I} = \dot{M} = W_L$ = rotational velocity limit of the gimbal

$$W_M (P) = \left[3\, W_L + 2\, W_L \sin P\right]^{1/2}$$

at $P = 0^o$, $W_M (0) = W_L \sqrt{3}$

at $P = 90^o$, $W_M (90) = W_L \sqrt{5}$ (gimbal lock)

2.2 The gyro that has its SA along the I I' axis sees only that velocity component that lies in the plane of the platform. This is expressed in Eq. 1:

$$|W_P| = \left[\dot{M}^2 + \dot{O}^2 \cos^2 P\right]^{1/2}$$

in terms of W_L and $P = 0^o$:

W_P (max) = $W_L^2 \sqrt{2}$

2.3 In par. 1.1 the vehicle angular velocity, W_V, was arbitrarily assumed to be zero. It now may be assumed to be any reasonable value and to lie colinear and adding to the worst case velocity derived from Eq. 2. Since the specified re-entry velocity is 20^{o}/sec (.35 rad/sec), the maximum velocity seen by a gyro SA under worst case assumptions will be:

$$W_{SA\ (gimbals)} = \left[\sqrt{5}\ W_L + .35\right] \text{ rad/sec}$$

If K is a desired safety factor defining the ration

$$\frac{W_{SA\ (max)}}{W_{SA\ (gimbals)}}, \text{ i.e. } K > 1,$$

$$\text{then } W_{SA\ (max)} = K\left[\sqrt{5}\ W_L + .35\right]$$

As a working example, let $K = 2$; $W_{SA\ (max)} =$ 6 rad/sec; then:

$$W_L = \frac{1}{\sqrt{5}}\left[\frac{6}{2} - .35\right] \approx 1.17 \text{ rad/sec}$$

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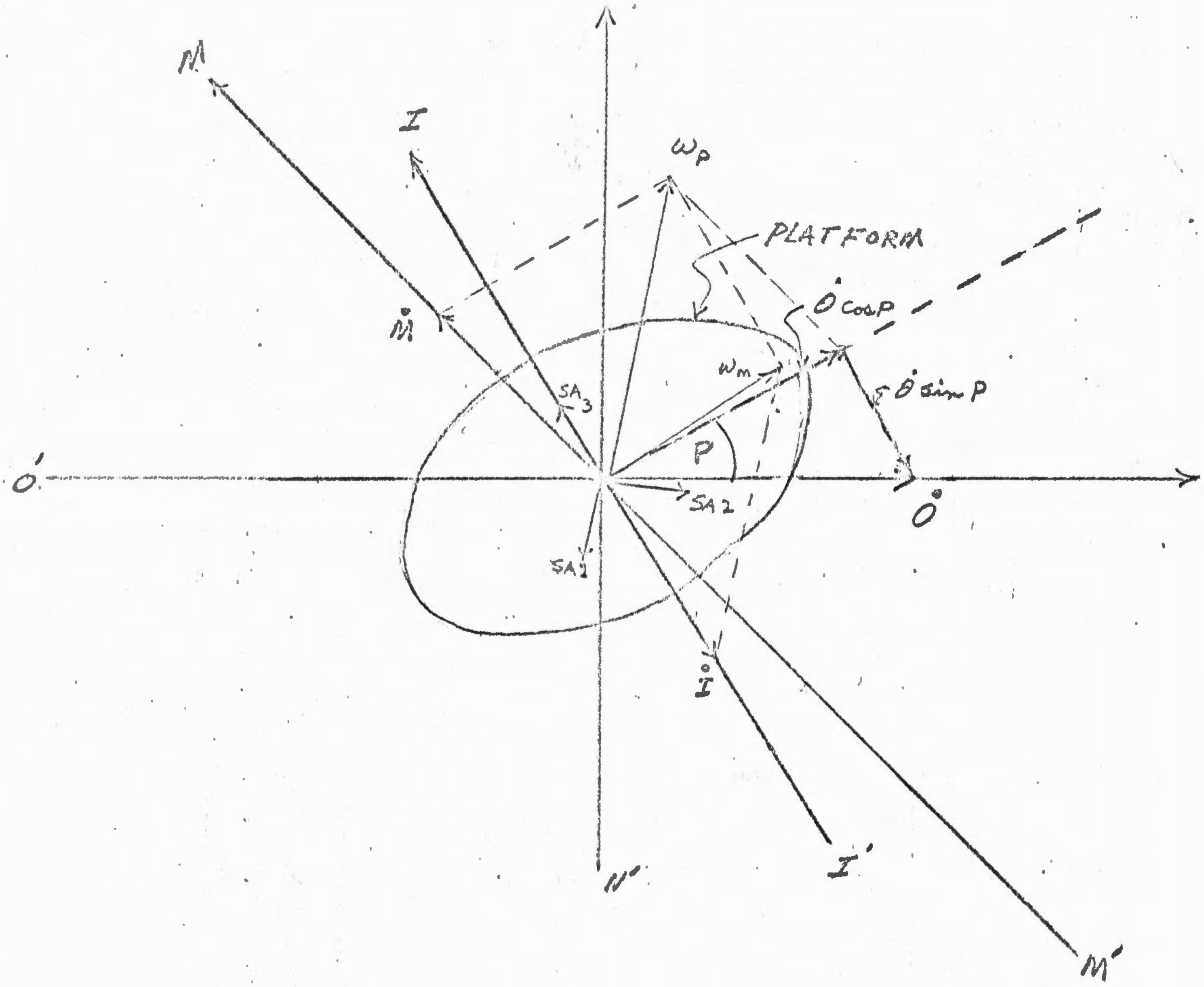

OO' is outer gimbal axis of rotation - $\dot{O}$ is rotational velocity about O
MM' " middle " " " " $\dot{M}$ " " " " MM
II' " inner " " " " $\dot{I}$ " " " " I

SA_1, SA_2 lie in the plane of the platform

SA_3 is orthogonal to the plane of the platform

Figure 1