G. I. Silver

AC SPARK PLUG DIVISION **General Motors Corporation** **Milwaukee, Wisconsin**	**EXPERIMENTAL DESIGN EXHIBIT**	**XDE** 34-S-15		REV
	BY R. Picard	DATE May 28, 1965	TOTAL PAGES 34	PAGE 1

PRELIMINARY BLOCK II AND LEM PERFORMANCE SPECIFICATIONS - APOLLO STABILIZATION LOOPS

1. SCOPE

1.1 This specification establishes the performance required for the Block II and LEM APOLLO Guidance and Navigation Stabilization Loops.

2. APPLICABLE DOCUMENTS

2.1 The following documents shall form a part of this specification to the extent specified herein.

NASA PROCUREMENT SPECIFICATIONS

PS 1000031-1	(Resolver)
PS 2018623	(IG & MG TORQUE MOTORS)
PS 2018629	(OG TORQUE MOTOR)

NASA APOLLO GUIDANCE AND NAVIGATION SPECIFICATIONS

ND 1002037	(ENVIRONMENTAL QUALIFICATION SPECIFICATION)
ATP 2015497	(BLOCK II INERTIAL SUB-SYSTEM APOLLO TEST PROCEDURE)
ATP 6015497	(LEM INERTIAL SUB-SYSTEM APOLLO TEST PROCEDURE)

INTERFACE CONTROL DOCUMENTS

MH 01-01327-216	(G&N ELECTRICAL INPUT POWER)
MH 01-01349-416	(G&N THERMAL REQUIREMENTS)

XDE's

XDE 34-S-12	(ISS POWER SUPPLIES)

DRAWINGS

2015564	(STABILIZATION LOOPS)
2010040	(IRIG PRE-AMPLIFIER)
2010004	(GIMBAL SERVO AMPLIFIER)

ENGINEERING SPECIFICATIONS

ES-7925	(IRIG I&A)
ES-8153	(IRIG CALIBRATE & TESTING)

3. REQUIREMENTS

3.1 General - The purpose of the three stabilization loops is to maintain the orientation of the three 16-PIP accelerometers fixed with respect to inertial space within specified limits under external disturbances. External disturbances are discussed in section 3.2.1. A functional diagram of the loops is shown in Figure 1.

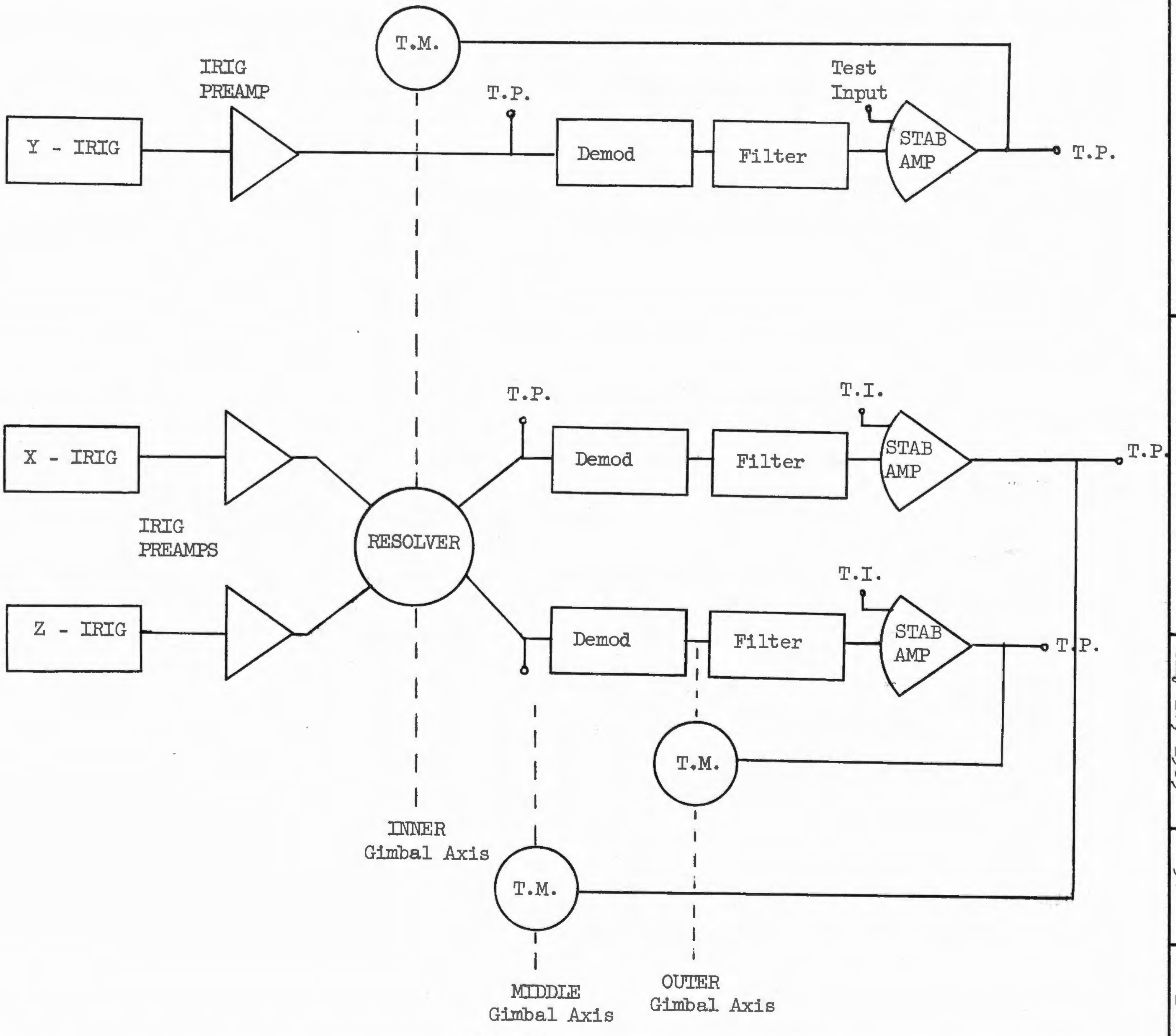

T.M.
IRIG PREAMP
Y - IRIG
T.P.
Test Input
Demod
Filter
STAB AMP
T.P.
T.P.
T.I.
X - IRIG
Demod
Filter
STAB AMP
T.P.
IRIG PREAMPS
RESOLVER
T.I.
Z - IRIG
Demod
Filter
STAB AMP
T.P.
T.M.
INNER Gimbal Axis
T.M.
MIDDLE Gimbal Axis
OUTER Gimbal Axis

FIGURE 1

3.2 Inputs

3.2.1 Disturbance Inputs - The stabilization loops must maintain adequate performance under external disturbances. Due to mass unbalance of the gimbals and gimbal bearing friction the following inputs to the system will cause disturbances to the orientation of the stable member. (Reference ND-1002037)

a) Acceleration -- 20 ± 1g maximum
b) Vibration -- Vibrations as stated in sections 4.9.1 and 4.9.2 of ND-1002037
c) Shock -- Sawtooth waveform of 6 millisecond duration, 15g peak.
d) Rotation -- 720 degree/sec about the outer gimbal axis. 60 degree/sec about any arbitrary axis in passing up to 10° of gimbal lock.

3.2.2 Guidance and Navigation Input Power - DC Power will be supplied to the APOLLO Guidance and Navigation equipment from the APOLLO spacecraft. The stabilization loops must maintain adequate performance under the variations in the power supply listed in MH 01-01327-216.

a) Steady State Voltage Limits -- 25.8 to 30.8 volts DC.
b) Transient Voltage Limits -- 24 to 32 volts DC, with recovery to steady state condition within 1.0 second.

3.2.3 Inertial Subsystem Generated Power - The stabilization loops shall maintain adequate performance under variations in the 3200 cps and -28VDC power supplies which are contained in the inertial subsystem. (Reference XDE 34-S-12)

3.2.4 Thermal Environment - The stabilization loop shall meet the performance requirements specified herein under the thermal environment specified in MH 01-01349-416.

3.3 Loop Performance

3.3.1 Steady State Errors - The gimbal angle steady state error due to static friction and mass unbalance in a gravitational field of 20 g's shall be as specified in Table 1.

TABLE 1

STEADY STATE GIMBAL ANGLE ERROR

	Low Gain	Nominal Gain	High Gain
Outer Gimbal Angle	45.40 $\overset{\frown}{\text{sec}}$	43.71 $\overset{\frown}{\text{sec}}$	41.62 $\overset{\frown}{\text{sec}}$
Middle Gimbal Angle	45.35 $\overset{\frown}{\text{sec}}$	43.69 $\overset{\frown}{\text{sec}}$	41.60 $\overset{\frown}{\text{sec}}$
Inner Gimbal Angle	44.90 $\overset{\frown}{\text{sec}}$	43.13 $\overset{\frown}{\text{sec}}$	40.96 $\overset{\frown}{\text{sec}}$

3.3.2 <u>Dynamic Error</u> - The dynamic error (gimbal angle/torque disturbance) for the three stabilization loops shall have a nominal frequency response as shown in Figure "2".

3.3.3 <u>Transient and Frequency Response</u> - The nominal transient and frequency responses of the torque developed by the torque motors for a torque disturbance input to the gimbals are shown in Figures 3 and 4 respectively.

3.4 <u>Components and Component Performance</u>

In order to meet the requirements specified in section 3.3, the following components shall be utilized in the Stabilization Loops:

a) APOLLO 25 IRIG
b) IRIG Pre-amplifier
c) Resolver
d) Demodulator
e) Filter
f) Stabilization Amplifier
g) Torque Motor
h) Gimbals

3.4.1 <u>APOLLO 25 IRIG</u> - The APOLLO 25 IRIG shall have the following characteristics:

a) $H = 0.434 \times 10^6$ GM-CM^2/ sec

b) Nominal damping (C) - 460,000 d-CM/rad/sec (4%/$^\circ$F change in C due to changes in temperature)

c) $I = 346.2$ GM-CM^2

d) Float Stops - 0.55° to 1.2° from null measured about input axis

e) Gyro Null Voltage - 6 mv maximum (at system level)

f) The net gain of IRIG shall be adjusted to a value of 7.0 ± 2.5% volts (rms)/rad by addition of a resistor in series with the signal generator.

3.4.2 <u>IRIG Pre-amplifier</u> - The IRIG Pre-amplifier shall have the following characteristics:

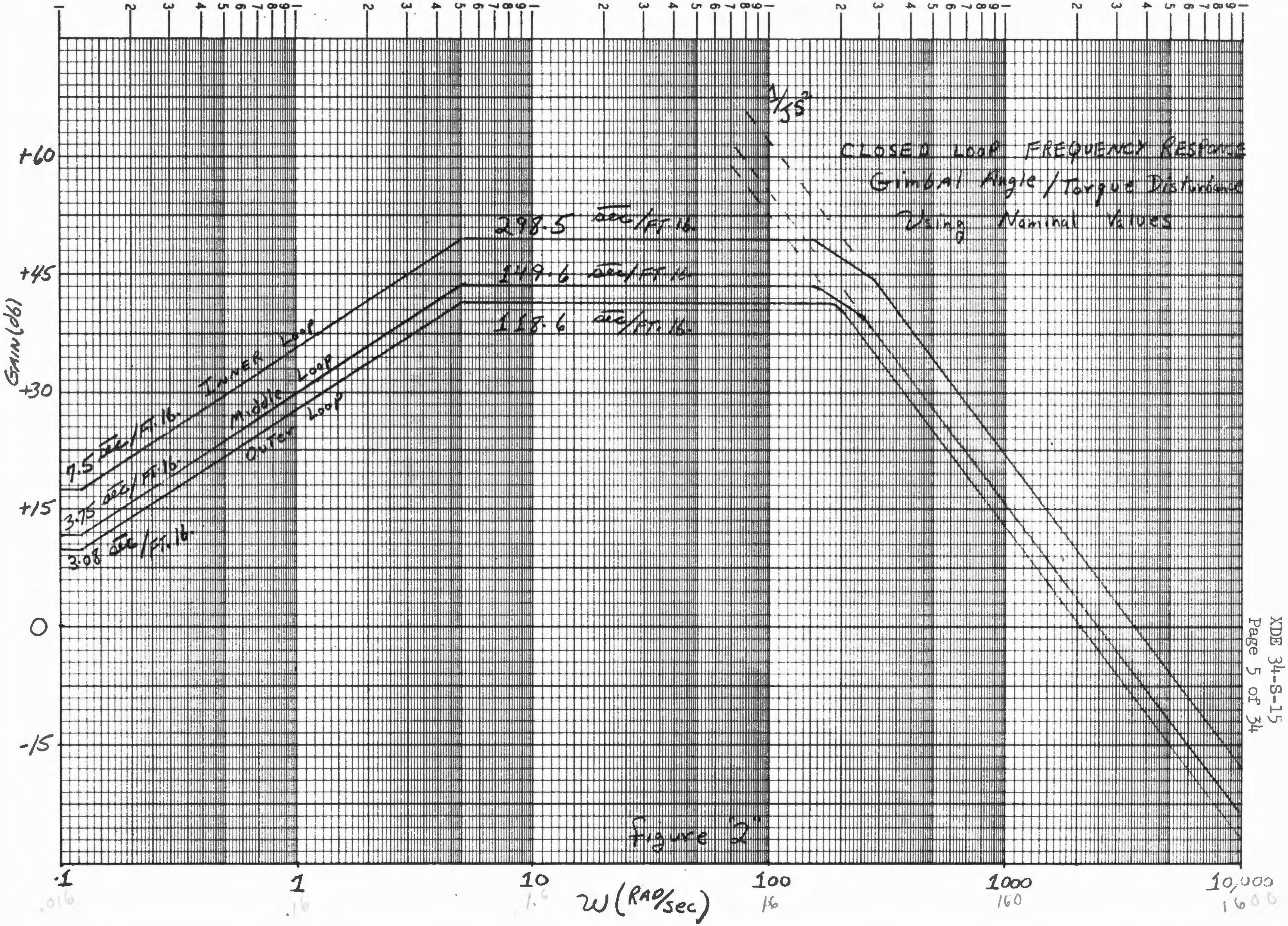
a) Gain - 214 ± 2 v/v

b) Phase Shift - 180 ± 10 degrees with a stability of ± 1°

c) Bandwidth - The minimum bandwidth (i.e., 3db from gain at 3200 cps) centered at 3200 cps shall be 1800 cps

d) Saturation Level - 10v rms minimum

e) Residual Noise - Less than 70 mv rms (input shorted)



CLOSED LOOP FREQUENCY RESPONSE
Gimbal Angle / Torque Disturbance
Using Nominal Values

figure "2"

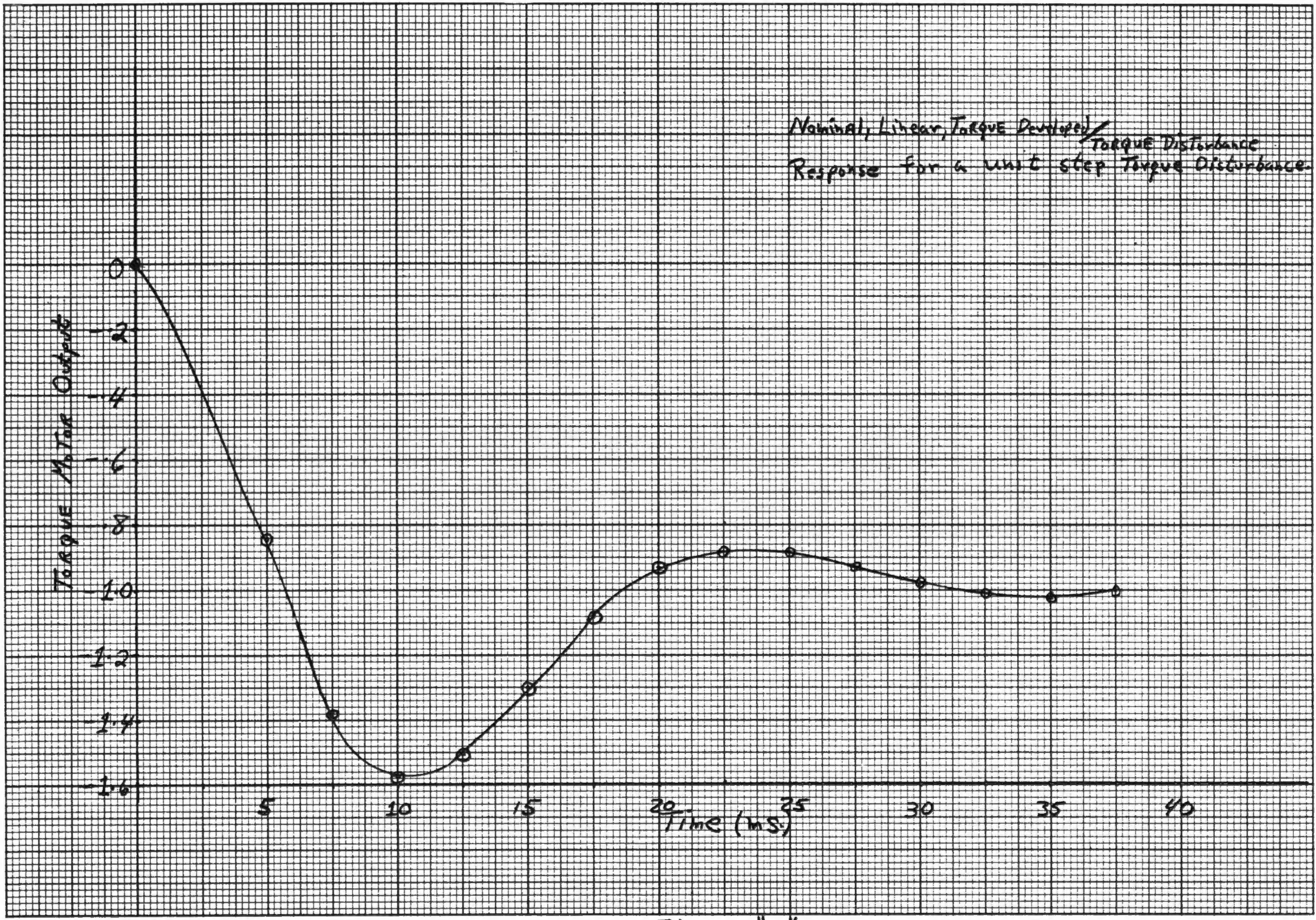

(Nominal, Linear, Torque Developed)/Torque Disturbance
Response for a unit step Torque Disturbance.
Torque Motor Output
0
-.2
-.4
-.6
-.8
-1.0
-1.2
-1.4
-1.6
5
10
15
20
25
30
35
40
Time (ms)

Figure "3"

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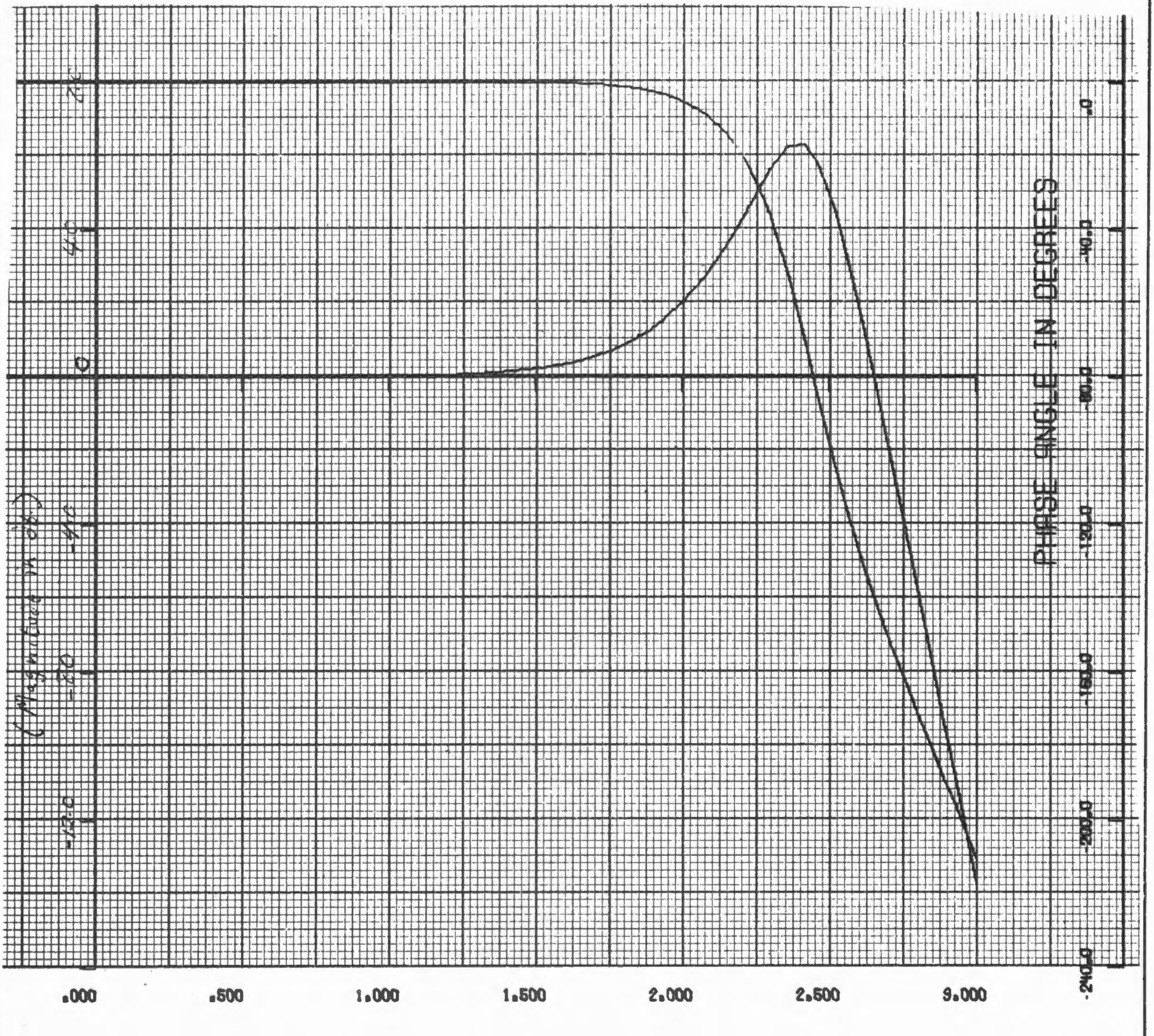


Torque Developed / Torque Disturbance closed loop frequency response

FIGURE 4

3.4.3 Resolver - The resolver used in the X and Z loops shall have the following characteristics:

a) Gain - 1.0 ± 3%

b) Angular Error - 10 arc minutes peak

c) Quadrature - The quadrature voltage introduced by the resolver shall be less than 50 mv.

d) Pick Up - The resolver when operating in the system shall pick up less than 5 mv in phase.

3.4.4 Demodulator - The demodulator shall be a full wave diode bridge and have the following characteristics:

a) Frequency - 3200 cps

b) Output Saturation Level - 10 VDC minimum

c) Gain - 1.0 $^{vdc}/v$ (rms)

d) Quadrature Rejection - 20:1

3.4.5 Filter - The filter shall be a tuned fourth order butterworth and shall have the following transfer function:

$$\frac{e_{out}}{e_{in}} = \frac{.59\left[\frac{s^2}{(4613)^2} + 1\right]}{\left(\frac{s}{3378} + 1\right)\left[\frac{s^2}{(3604)^2} + \frac{2(.1786)}{(3604)} s + 1\right]\left[\frac{s^2}{(3787)^2} + \frac{2(.6192)}{3787} s + 1\right]}$$

3.4.6 Stabilization Amplifier - The three modules, each of which contain a filter, demodulator, and stabilization amplifier, are identical. The different gains for the three loops are accomplished by connections through the module connector. The stabilization amplifiers shall have the following characteristics:

a) DC Gain -

Inner Gimbal Loop:	16.4 ± 1.7 amps/volt
Middle Gimbal Loop:	32.8 ± 3.3 amps/volt
Outer Gimbal Loop:	49.2 ± 5.0 amps/volt

b) DC Offset - With a 3200 cps 0-phase signal of less than 20 mv into the demodulator, the output of the stabilization amplifier (using the inner loop connection) shall be 0.0 ± 0.1 vdc with respect to ground.

c) Output Saturation - ± 25.8 vdc minimum

d) Transfer Function - The stabilization amplifier shall have the following linear transfer function:

$$\frac{i_o}{e_{in}} = \frac{K\,(S/W_2 + 1)\,(S/W_3 + 1)}{(S/W_1 + 1)\,(S/W_4 + 1)}$$

K is the DC gain as listed in a)

$W_1 = .125 \pm .02 \text{ sec}^{-1}$

$W_2 = 5 \pm .75 \text{ sec}^{-1}$

$W_3 = 155 \pm 25 \text{ sec}^{-1}$

$W_4 = 2{,}000 \pm 350 \text{ sec}^{-1}$

3.4.7 Torque Motors - The torque motors shall have the following characteristics:

3.4.7.1 Inner & Middle Loop Torque Motors:

a) Sensitivity - 1.12 $\frac{\text{ft-lbs}}{\text{amp}}$ ± 10%

b) DC Resistance (25°C) - 60 ohms max.

c) Self Inductance - .04 henry nom.

d) Ripple Torque - 10 oz-in p-p max.

3.4.7.2 Outer Loop Torque Motor:

a) Sensitivity - .906 $\frac{\text{ft-lbs}}{\text{amp}}$ ± 10%

b) DC Resistance (25°C) - 33 ohms max.

c) Self Inductance - .028 henry nom.

d) Ripple Torque - 10 oz-in max p-p

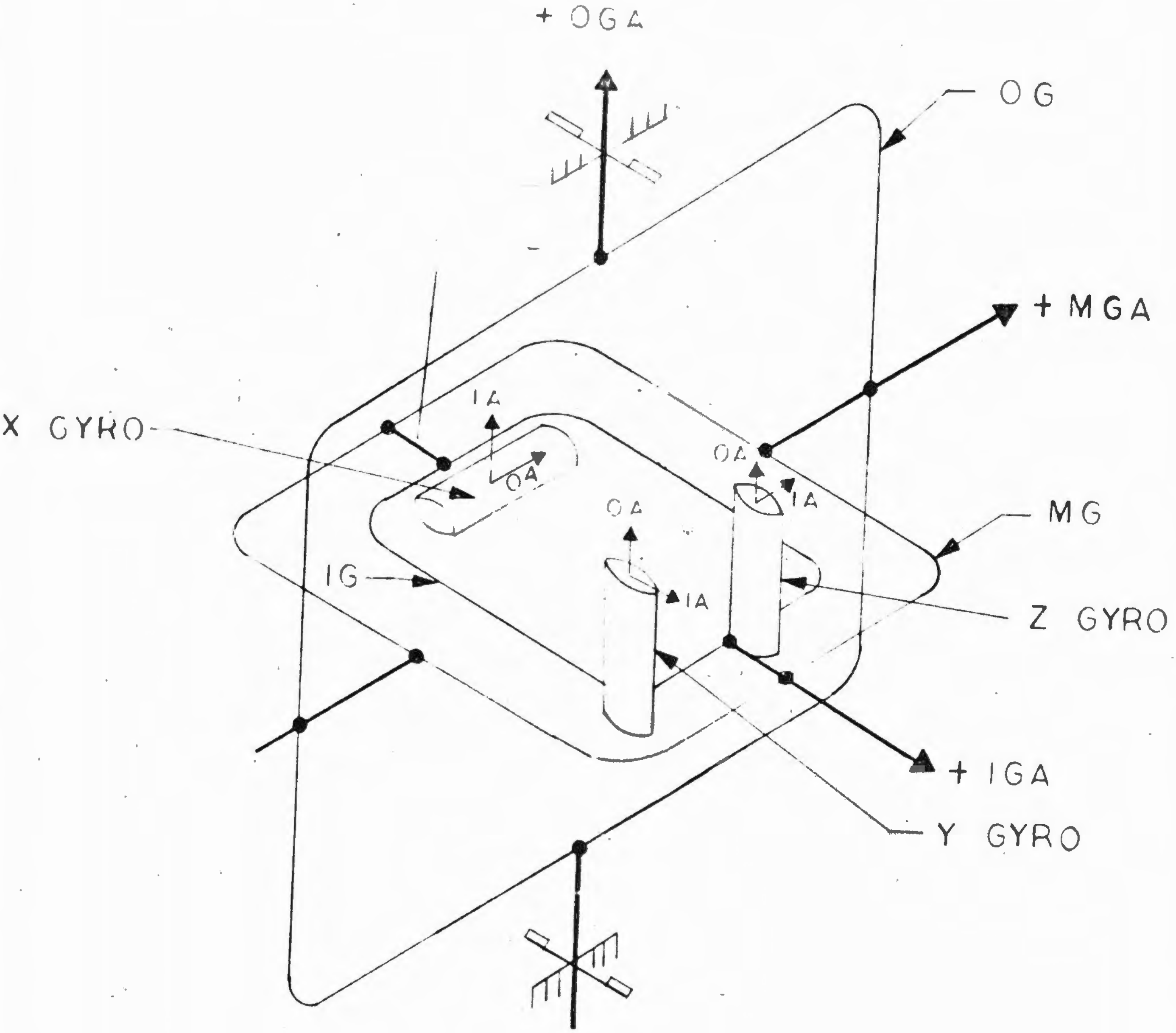


FIGURE 5

3.4.8 **Gimbals** - The orientation of the three gimbals and the three gyros is shown in Figure 5.

a) Static Friction - The maximum values for static friction in a 1.0 g field are:

Inner Axis	17.4 in-oz
Middle Axis	17.4 in-oz
Outer Axis	18.2 in-oz

b) Moment of Inertia - The gimbals shall have the following moments of inertia:

Inner Axis	$J \approx .017$ slug-ft^2
Middle Axis	$J \approx .033$ slug-ft^2
Outer Axis	$J \approx .050$ slug-ft^2

c) Mass Unbalance - The mass unbalance of the gimbals shall be less than 0.5 in-oz/g

4. QUALITY ASSURANCE PROVISIONS

4.1 General

In order to assure adequate performance of the Apollo Stabilization Loops the following tests shall be run.

4.1.1 **Steady State Error** - With the system in a steady state condition the following requirements shall be met:

a) Output of IRIG Preamplifiers - The output of the IRIG preamplifiers shall be less than 1.4v quadrature and the in phase voltage shall be less than

Inner Axis	75 mv
Middle Axis	80 mv
Outer Axis	80 mv

b) Stabilization Amplifier Outputs - The voltage output of the inner and middle loop stabilization amplifiers shall be between plus 5.6 and minus 5.6 volts with respect to ground.

The voltage output of the outer loop stabilization amplifier shall be between plus 4.1 and minus 4.1 volts with respect to ground.

c) Torque Motor Current - The torque motor current of the inner and middle loops shall be between + 95 ma and -95 ma.

The torque motor current of the outer loop shall be between +120 ma and -120 ma.

4.1.2 **Step Response** - With a d-c step input as per table (2) into the stabilization amplifier, the time for the output to reach its first overshoot peak shall not be greater than 25 ms. The preamplifier output shall not have more than six (6) peaks over a 5% tolerance band of the steady state value.

4.1.3 **Frequency Response** - With a variable frequency signal as per Table (2) applied to the stabilization amplifier, the IRIG Preamplifier shall have a 6db downpoint between 55 and 120 cps. The preamplifier output shall not have an Mp greater than 12 db.

TABLE 2

Stabilization Amplifier Test Voltages

Section	Gimbal Axis	Voltage
4.1.2	Inner	10.0 ± 0.5vdc
4.1.3	Inner	2.0 ± 0.1 v p-p
4.1.2	Middle	5.0 ± 0.25 vdc
4.1.3	Middle	1.0 ± 0.05 v p-p
4.1.2	Outer	5.0 ± 0.25vdc
4.1.3	Outer	1.0 ± 0.05 vp-p

5. PREPARATION FOR DELIVERY

Not applicable.

6. ANALYSIS

6.1 Introduction to Analysis

It is required that the stable member maintain its orientation with respect to inertial space so that the three mutually perpendicular PIPA's will measure the proper components of the spacecraft acceleration with respect to inertial space. The stable member must maintain this orientation in spite of disturbance torques and motions of the spacecraft with respect to inertial space.

The stable member is located within a gimbal system which provides three degrees of rotational freedom. Gyros are used to sense rotations of the stable member with respect to inertial space. Signals from these sensors are fed to a servo system which produces a corrective torque on the stable member.

The transfer functions for the individual components of the loop will be determined. These individual transfer functions will be combined to obtain the loop transfer functions. The performance of the system will then be determined from the loop transfer functions.

6.2 Loop Component Transfer Functions

The components of the three stabilization loops are identical with the exception of (1) the moments of inertia of the three gimbals, (2) the feedback connections used on the stabilization amplifiers, (3) the gyro output signals in the X and Z loops are passed through a resolver, and (4) the outer loop utilizes a different torque motor than the inner and middle loops.

6.2.1 IRIG - The performance equation of the gyro is

$$(Is^2 + Cs)\,Ag = HW_{IA} + M_{CMD} - HA_g\,W_{SRA} + U - IW_{OA}\,s \qquad (1)$$

where

I = effective output axis moment of inertia
C = Damping constant about the output axis
H = Angular momentum
W_{IA} = Angular rate about the input axis
W_{SRA} = Angular rate about the spin reference axis
W_{OA} = Angular rate about the output axis
M_{CMD} = Command torque about the output axis
U = Uncertainty torque about the output axis
s = Laplace operator
Ag = Gyro output angle

The last three terms in the right member of Eq (1) are all undesirable inputs to the gyro. The cross-coupling of the spin reference axis component of angular velocity is minimized by keeping the gyro output angle very small. The uncertainty torque is kept small by careful manufacture. Since H is over 1000 times larger than I, the effect of angular acceleration about the output axis is small.

With no command torque being applied to the gyro, the transfer function of the gyro can be expressed as:

$$\frac{E_o}{W_{IA}} = \frac{H/C \quad K}{S\,(S\tau+1)} \qquad (2)$$

where

$$K = \frac{E_o}{\theta_{OA}}$$

$$\tau = I/C$$

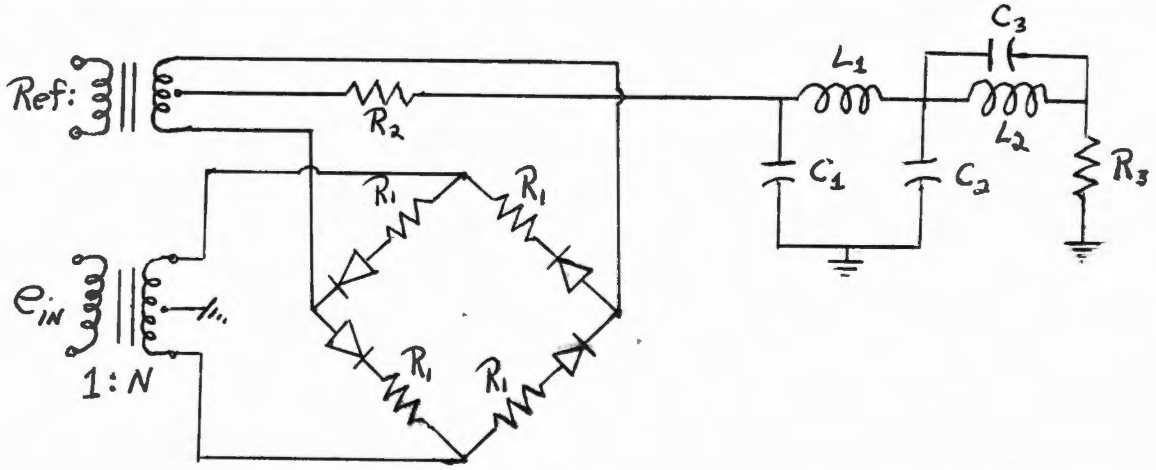
Inserting the values given in 3.4.1.

$$\frac{E_O}{W_{IA}} = \frac{7.0}{S\left(\frac{S}{1330} + 1\right)} \quad \frac{\text{volts (rms)}}{\text{rad.}} \qquad (3)$$

6.2.2 Gyro Preamplifier - The transfer function of the preamplifier is a pure gain over the frequency band of interest.

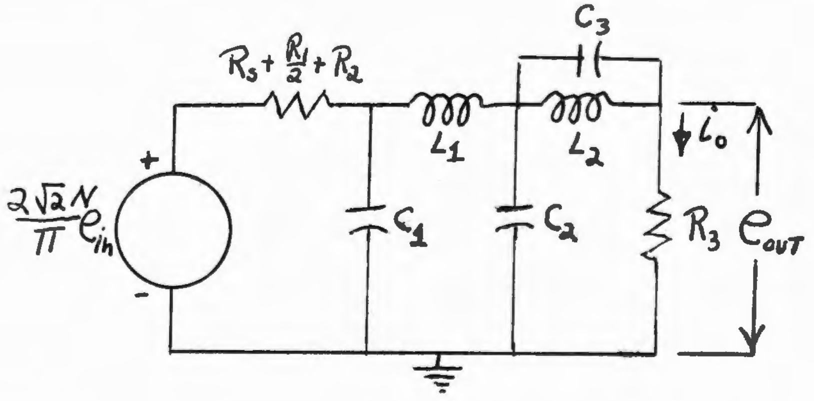
$$\text{Gain} = 214 \ {}^{v}/{}_{v} \qquad (4)$$

6.2.3 Demodulator and Filter - The demodulator is a full wave bridge type followed by a tuned fourth order butterworth filter as shown in Figure 6.



DEMODULATION

FILTER



EQUIVALENT CIRCUIT

FIGURE 6

The transfer admittance is:

$$\frac{i_o}{e_{in}} = \frac{\frac{2\sqrt{2}}{\pi}N\,(S^2L_2C_3 + 1)}{\begin{aligned}&S^5\,(L_1L_2C_1C_2C_3R_3(R_S + R_{1/2} + R_2) + S^4\,(L_1L_2)\Big[C_2C_3R_3 + C_1C_2(R_S + R_{1/2} + R_2)\\ &\qquad + C_1C_3(R_S + R_{1/2} + R_2)\Big]\\ &+ S^3\Big[L_2C_3R_3(R_S+R_2+R_{1/2})\,(C_1+C_2) + L_1L_2(C_2+C_3) + L_1C_1C_2R_3\,(R_S+R_{1/2}+R_2)\Big]\\ &+ S^2\Big[L_2C_3R_3 + (R_S+R_{1/2}+R_2)\,(L_2C_1+L_2C_2+L_2C_3+L_1C_1) + (L_1C_2R_3)\Big]\\ &+ S\Big[L_1+L_2+R_3\,(R_S+R_{1/2}+R_2)\,(C_1+C_2)\Big] + \Big[R_5 + (R_S+R_{1/2}+R_2)\Big]\end{aligned}} \tag{5}$$

where: R_S (reflected source impedance) $\approx$ 10K

$R_1 = 20K$

$R_2 = R_3 = 51K$ $\quad L_1 = 25H$, $L_2 = 10H$

$C_1 = .0047$ ufds, $C_2 = .01$ ufds

$C_3 = .0047$ ufds, $N = 1.5$

The transfer function becomes:

$$\frac{i_o}{e_{in}} = \frac{1.155\text{x}10^{-5}\left[\frac{S^2}{(4613)^2} + 1\right]}{\left[\frac{S}{3378} + 1\right]\left[\frac{S^2}{(3604)^2} + \frac{2(.1786)S}{3604} + 1\right]\left[\frac{S^2}{(3787)^2} + \frac{2(.6192)}{3787}S + 1\right]} \tag{6}$$

where $i_o = \frac{e_{out}}{R_3}$

$$\therefore \frac{e_{out}}{e_{in}} = \frac{.59\left[\frac{S^2}{(4613)^2} + 1\right]}{\left[\frac{S}{3378} + 1\right]\left[\frac{S^2}{(3604)^2} + \frac{2(.1786)S}{3604} + 1\right]\left[\frac{S^2}{(3787)^2} + \frac{2(.6192)}{3787}S + 1\right]}$$

6.2.4 Stabilization Amplifier - The stabilization amplifier uses a notch network in the feedback path to produce the necessary linear compensation. To allow the loop to settle from large angles, non-linear compensation is inserted in the forward path. The output of the amplifier, due to the current sampling resistor, acts as a constant current source driving the torque motors. The amplifier gain for each loop is selected to give each loop approximately the same open loop gain. A simplified diagram of the amplifier is shown below.

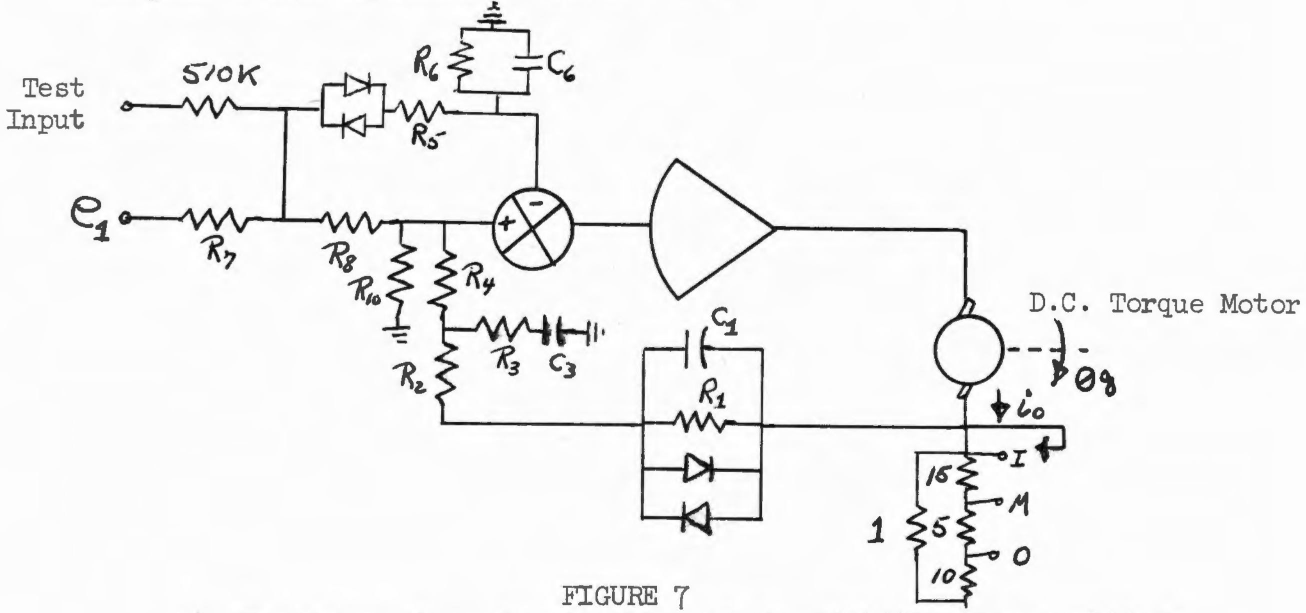


FIGURE 7

a) The linear transfer function of the stabilization amplifier is shown below:

$$\frac{i_o}{e_1} = \frac{16.4K\left[S^2 + S\,\frac{R_1C_1(R_2+R_4)+R_4C_3(R_1+R_2)+R_3C_3(R_1+R_2+R_4)}{R_1C_1C_3(R_2R_3+R_3R_4+R_2R_4)} + \frac{R_1+R_2+R_4}{R_1C_1C_3(R_2R_3+R_3R_4+R_2}\right.}{\left[SR_1C_1 + 1\right]\left[SR_3C_3 + 1\right]} \qquad (7)$$

for: R_1 = 2 Meg R_3 = 1K R_7 = 71K C_1 = 4.ufds

R_2 = 20K R_4 = 30K R_8 = 51K C_3 = .5 ufd

The transfer function becomes:

$$\frac{i_o}{e_i} \approx \frac{16.4K\ \left[S/5 + 1\right]\left[S/155 + 1\right]}{\left[S/.125 + 1\right]\left[S/2000 + 1\right]} \qquad (8)$$

where K = 1 for I.G.
K = 2 for M.G.
K = 3 for O.G.

b) When the feedback diodes conduct, the transfer function becomes:

$$\frac{i_o}{e_i} = \frac{.41K \left(S \dfrac{(R_3R_2 + R_3R_4 + R_2R_4)\, C_3}{R_2 + R_4} + 1\right)}{(S\, R_3C_3 + 1)} \qquad (9)$$

inserting values:

$$\frac{i_o}{e_i} = \frac{.41K \left[S/455 + 1\right]}{\left[S/2{,}000 + 1\right]} \qquad (10)$$

c) For large input signals, the forward diodes conduct and the transfer function becomes:

$$\frac{i_o}{e_i} \approx \frac{\left[\dfrac{R_{10}}{R_7+R_8+R_{10}} - \dfrac{R_6}{R_5+R_6+R_7}\right] \left[S \dfrac{R_6R_{10}(R_5+R_7)\, C_6}{(R_5+R_6+R_7)\, R_{10} - R_6(R_7+R_8+R_{10})} + 1\right]}{\left[S \dfrac{(R_5+R_7)R_6C_6}{R_5+R_6+R_7} + 1\right]} \qquad (11)$$

where:

$R_5 = 75K$ $R_6 = 33K$

$R_{10} = 33K$ $C_6 = .34$ ufds

$$\frac{i_o}{e_i} = \frac{.029\ (S/14.7 + 1)}{(S/109 + 1)} \qquad (12)$$

6.2.5 Torque Motor - The torque motor is driven by the stabilization amplifier. The constant current source makes the motor time constant negligibly small resulting in the transfer functions in eq's (13 & 14).

$$\frac{T}{I} = 1.12 \, \frac{\text{Ft.Lbs}}{\text{amp}} \quad \text{for Inner and Middle Gimbals} \qquad (13)$$

$$T/I = .906 \, \frac{\text{Ft.lbs}}{\text{amp}} \quad \text{for Outer Gimbal} \qquad (14)$$

6.2.6 Gimbals - The moments of inertia of the gimbals are:

Inner	$J \approx .017$ slug-ft^2
Middle	$J \approx .033$ slug-ft^2
Outer	$J \approx .05$ slug-ft^2

The transfer function for the gimbal is approximately given by:

$$\frac{\theta}{T} = \frac{60}{KS^2} \; {}^{\text{Rad}}/_{\text{ft-lb.}} \qquad (15)$$

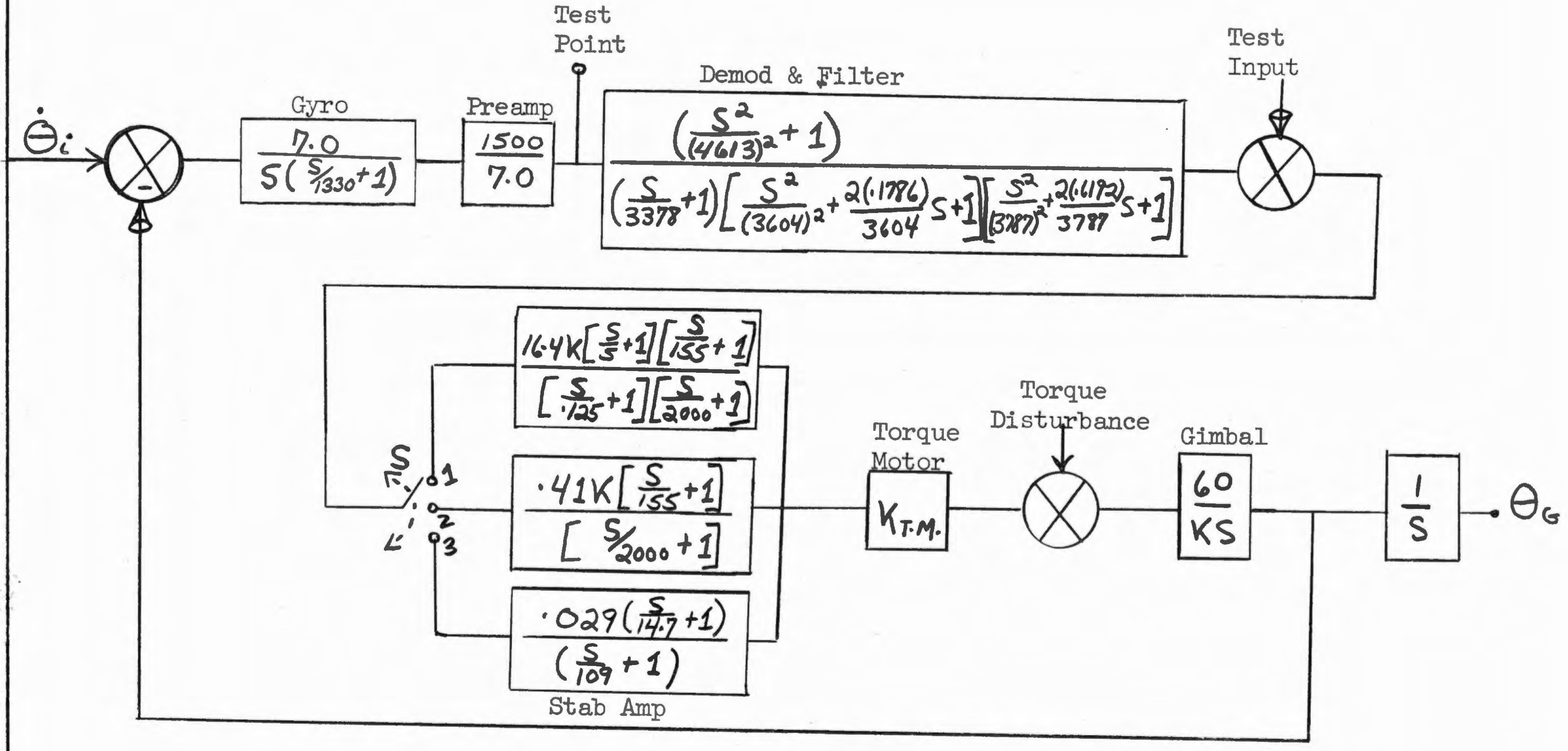


FIGURE 8

where: S is at position 1: for: small signal response:
2: feedback diodes on:
3: large signal response (forward diodes on):

$K_{T.M.} = 1.12 \frac{\text{ft-lbs}}{\text{amp}}$ - for Inner and Middle Loops

$K_{T.M.} = .906 \frac{\text{ft-lbs}}{\text{amp}}$ - for Outer Loop

K = 1 for Inner Loop
= 2 for Middle Loop
= 3 for Outer Loop

NOTE: The Demod - Filter Combination is represented with unity d.c. gain, the gain resistors are taken into account in the stab. amp transfer function.

6.3 Stabilization Loop Analysis

Using the transfer functions developed in Section 6.2, the block diagram of the loop is shown in Figure 8. Assuming linear operation of the stabilization amplifier (S in position 1), and nominal values for the loop components, the component transfer functions are combined to give the following loop analysis.

6.3.1 Gimbal Angle/Torque Disturbance - The appropriate block diagram is shown in Figure 9.

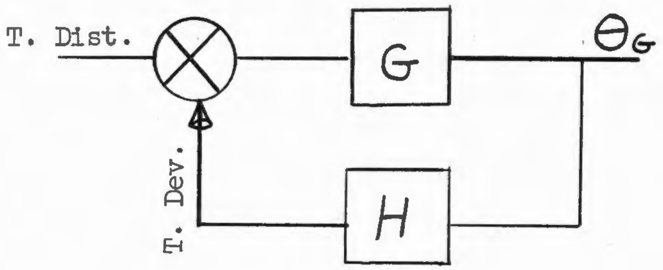


FIGURE 9

where: $G = \frac{60}{KS^2}$

$$H = \frac{24{,}600\, K_{T.M.}\, K \left(\frac{S}{5} + 1\right)\left(\frac{S}{155} + 1\right)\left(\frac{S^2}{(4613)^2} + 1\right)}{\left[\frac{S}{.125} + 1\right]\left[\frac{S}{1330} + 1\right]\left[\frac{S}{2000} + 1\right]\left[\frac{S}{3378} + 1\right]\left[\frac{S^2}{(3604)^2} + \frac{2(.1786)}{3604}S + 1\right]\left[\frac{S^2}{(3787)^2} + \frac{2(.6192)}{3787}S + 1\right]}$$

The open loop transfer function (GH) is given in eg. (16) and the open loop frequency plots are shown in figures (11 and 12).

$$GH = \frac{1.476 \times 10^{6} K_{TM} (S/5 + 1)(S/155 + 1)\left(\frac{S^2}{(4613)^2} + 1\right)}{S^2\left[\frac{S}{.125}+1\right]\left[\frac{S}{1330}+1\right]\left[\frac{S}{2000}+1\right]\left[\frac{S}{3378}+1\right]\left[\frac{S^2}{(3604)^2}+\frac{2(.1786)}{(3604)}S+1\right]\left[\frac{S^2}{(3787)^2}+\frac{2(.6192)}{3787}S+1\right]} \qquad (16)$$

The closed loop transfer function is given in eqs. (20 and 21) and the closed loop frequency response is shown in Fig. (2).

$$\frac{\Theta_G}{T.\ Dist} = \frac{G}{1 + HG} \qquad (17)$$

$$= \frac{1}{1/G + H} \qquad (18)$$

$$\frac{\Theta_G}{T.\ Dist.} = \cfrac{1}{\cfrac{KS^2}{60} + \cfrac{24{,}600K\, K_{TM} (S/5 + 1)(S/155 + 1)\left(\frac{S^2}{(4613)^2} + 1\right)}{\left[\frac{S}{.125}+1\right]\left[\frac{S}{1330}+1\right]\left[\frac{S}{2000}+1\right]\left[\frac{S}{3378}+1\right]\left[\frac{S^2}{(3604)^2}+\frac{2(.1786)}{3604}S+1\right]\left[\frac{S^2}{(3787)^2}+\frac{2(.6192)}{3787}S+1\right]}} \qquad (19)$$

Using a conversion factor of $2.0626 \times 10^5 \dfrac{\widehat{\text{sec}}}{\text{rad}}$

for S large: $$\frac{\Theta_G}{T.\ Dist.} \approx \frac{60\ \text{rad}}{KS^2\ \text{ft/lb}} \approx \frac{12.38 \times 10^6}{KS^2}\frac{\widehat{\text{sec}}}{\text{ft/lb}} \qquad (20)$$

for S small:

$$\frac{\Theta_G}{T.Dist}\left(\frac{\widehat{\text{sec}}}{\text{ft.lb.}}\right) = \frac{8.4\left[\frac{S}{.125}+1\right]\left[\frac{S}{1330}+1\right]\left[\frac{S}{2000}+1\right]\left[\frac{S}{3378}+1\right]\left[\frac{S^2}{(3604)^2}+\frac{2(.1786)}{(3604)}S+1\right]\left[\frac{S^2}{(3787)^2}+\frac{2(.6192)}{3787}S+1\right]}{K\, K_{TM}\left[\frac{S}{5}+1\right]\left[\frac{S}{155}+1\right]\left[\frac{S^2}{(4613)^2}+1\right]} \qquad (21)$$

6.3.2 Torque Developed/Torque Disturbance - From Figure 9, the torque developed by the torque motor for a given torque disturbance is given by:

$$\frac{T_{Dev}}{T_{Dist}} = \frac{GH}{1 + GH} \qquad (22)$$

$$\frac{T_{Dev}}{T_{Dist}} = \frac{(S/155 + 1)\left[\frac{S^2}{(4613)^2} + 1\right]}{\left[\frac{S}{477.5} + 1\right]\left[\frac{S^2}{(273)^2} + \frac{2(.43)}{(273)}S + 1\right]\left[\frac{S^2}{(3030)^2} + \frac{2(.994)}{3030}S + 1\right]\left[\frac{S^2}{(3610)^2} + \frac{2(.175)}{(3610)}S + 1\right]\left[\frac{S^2}{(3760)^2} + \frac{2(.62)}{(3760)}S + 1\right]} \qquad (23)$$

The closed loop frequency response from eq. 23 is shown in Figure 4. The torque motor output transient response for a unit step input is shown in Figure 3.

6.3.3 IRIG Test Point/Test Input - When considering the voltage output of the IRIG Preamplifier for a test input to the stabilization amplifier, the block diagram shown in Figure 8 has the form shown in Figure 10.

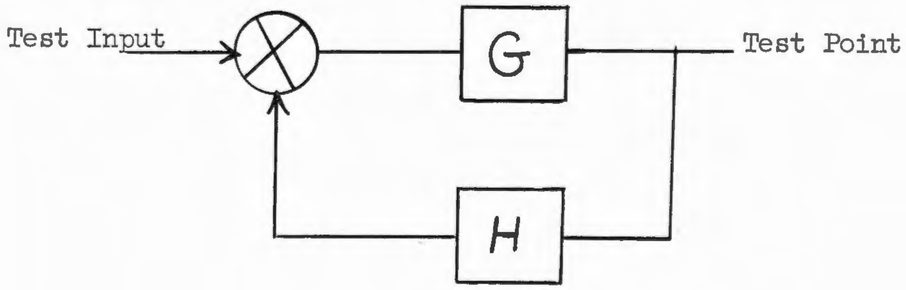


FIGURE 10

where:

$$G = \frac{1.476 \times 10^6 K_{TM} (S/5 + 1)(S/155 + 1)}{S^2 \left[\frac{S}{.125} + 1\right] \left[\frac{S}{1330} + 1\right] \left[\frac{S}{2000} + 1\right]}$$

$$H = \frac{\left[\frac{S^2}{(4613)^2} + 1\right]}{\left[\frac{S}{3378} + 1\right] \left[\frac{S^2}{(3604)^2} + \frac{2(.1786)}{3604} S + 1\right] \left[\frac{S^2}{(3787)^2} + \frac{2(.6192)}{3787} S + 1\right]}$$

The open loop transfer function (GH) is given in eq. (24) and the open loop frequency plots are shown in figures (11 and 12).

$$GH = \frac{1.476 \times 10^6 K_{TM} (S/5 + 1)(S/155 + 1)\left(\frac{S^2}{(4613)^2} + 1\right)}{S^2 \left[\frac{S}{.125}+1\right]\left[\frac{S}{1330}+1\right]\left[\frac{S}{2000}+1\right]\left[\frac{S}{3378}+1\right]\left[\frac{S^2}{(3604)^2}+\frac{2(.1786)}{3604}S+1\right]\left[\frac{S^2}{(3787)^2}+\frac{2(.6192)}{3787}S+1\right]} \qquad (24)$$

The closed loop transfer functions are given in eqs (26 and 27) and the closed loop frequency responses are shown in Figures (13 and 14).

$$\frac{\text{Test Point}}{\text{Test Input}} = \frac{G}{1 + GH} \qquad (25)$$

$$\frac{T.P.}{T.I.} = \frac{\left[\frac{S}{3378} + 1\right]\left[\frac{S}{155} + 1\right]}{\left[\frac{S}{547.6} + 1\right]\left[\frac{S^2}{(246)^2} + \frac{2(.421)}{(246)} S + 1\right]\left[\frac{S^2}{(3120)^2} + \frac{2(.96)}{(3120)} S + 1\right]} \qquad (26)$$

for the outer gimbal:

$$\frac{T.P.}{T.I.} = \frac{\left[\frac{S}{3378} + 1\right]\left[\frac{S}{155} + 1\right]}{\left[\frac{S}{409.4} + 1\right]\left[\frac{S^2}{(311)^2} + \frac{2(.459)}{(311)} S + 1\right]\left[\frac{S^2}{(3090)^2} + \frac{2(.988)}{3090} S + 1\right]} \qquad (27)$$

for the inner and middle gimbal:

The transient response is shown in Figure 15.

6.4 Steady State Errors

The steady state gimbal angle errors in a 20g field is analyzed in Section 6.4.1. The steady state values (in a 1.0 g field) for torque motor current, stabilization amplifier output and IRIG preamplifier output are derived in Section 6.4.2.

6.4.1 **Steady State Gimbal Angle Errors in a 20g Field** - The steady state gimbal error is a function of 1) static torque disturbance and the IRIG to torque motor gain, 2) the DC offset of the stabilization amplifier and 3) the quadrature 3200 cps voltage introduced by the IRIG and quadrature and in phase voltage introduced by the resolver.

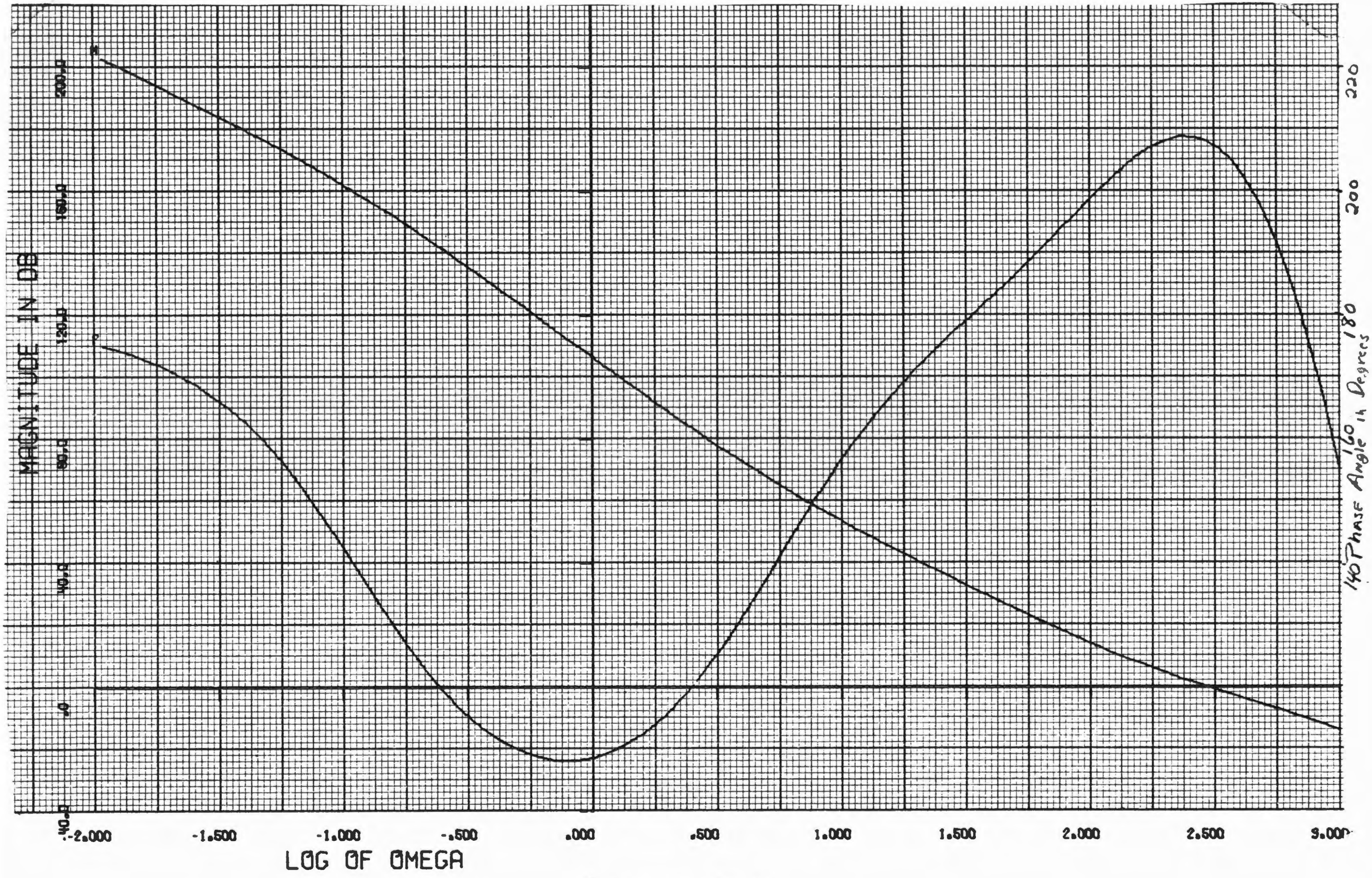
a) IRIG to Torque Motor Gain

	Low Gain	Nominal Gain	High Gain
IRIG	6.8	7.0	7.2
IRIG Preamp & Demod	212	214	216
Stabilization Amp.	14.8K	16.4K	18K
(IG and MG) Torque Motor	1.01	1.12	1.23
(OG) Torque Motor	.816	.906	.996
Resolver	.97	1.00	1.03

IRIG to Torque Motor Gain

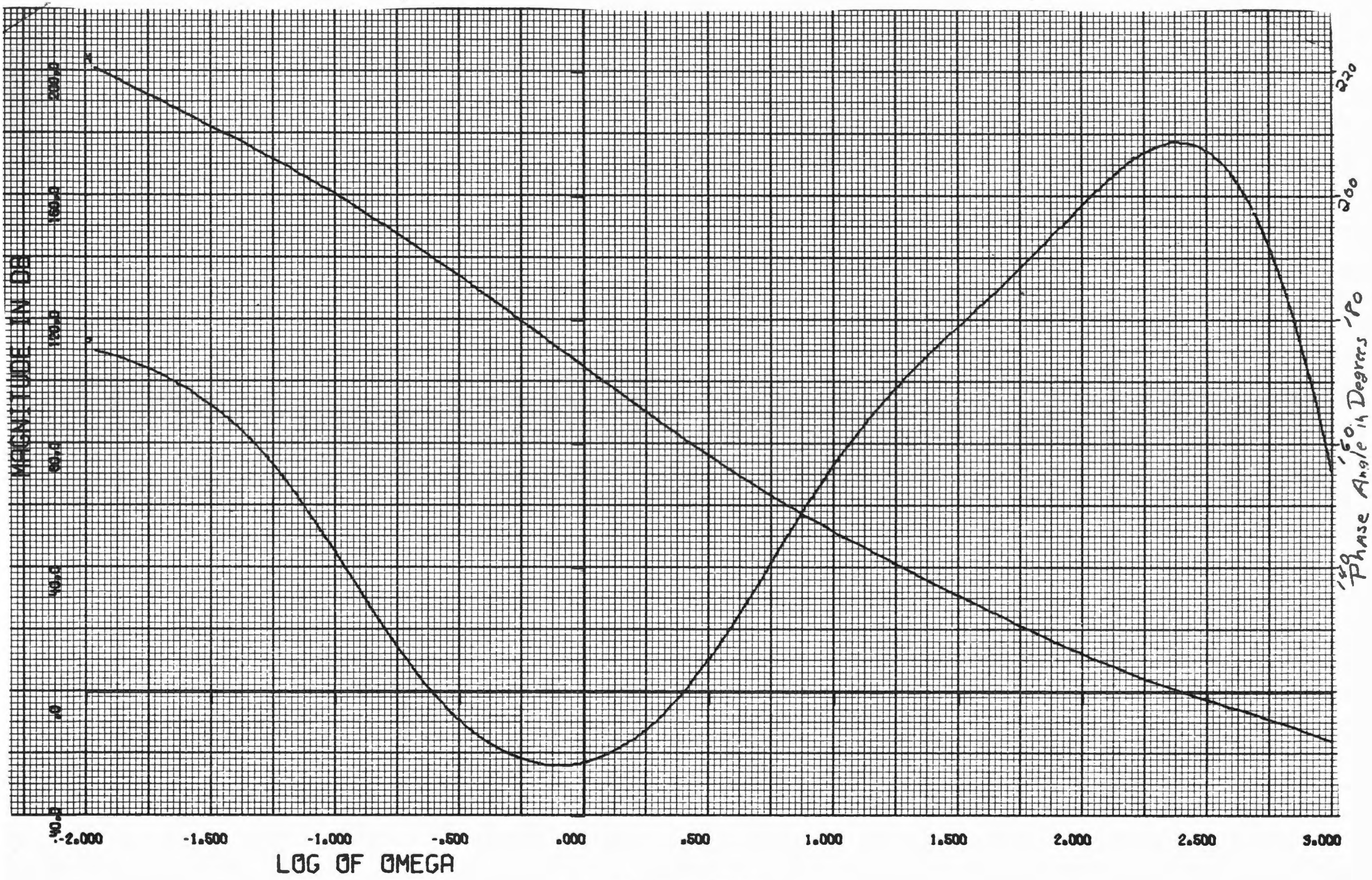
	Low Gain	Nom. Gain	High Gain
Inner Loop	$21{,}600 \frac{\text{ft-lb}}{\text{rad}}$	$27{,}300 \frac{\text{ft-lb}}{\text{rad}}$	$34{,}400 \frac{\text{ft-lb}}{\text{rad}}$
Middle Loop	$42{,}000 \frac{\text{ft-lb}}{\text{rad}}$	$55{,}000 \frac{\text{ft-lb}}{\text{rad}}$	$71{,}000 \frac{\text{ft-lb}}{\text{rad}}$
Outer Loop	$50{,}600 \frac{\text{ft-lb}}{\text{rad}}$	$66{,}600 \frac{\text{ft-lb}}{\text{rad}}$	$86{,}100 \frac{\text{ft-lb}}{\text{rad}}$

Using a conversion factor of 1074.296 ft-lb-$\widehat{\text{sec}}$/rad.in-oz, the reciprocals of the IRIG to the torque gain is shown in Table 3.

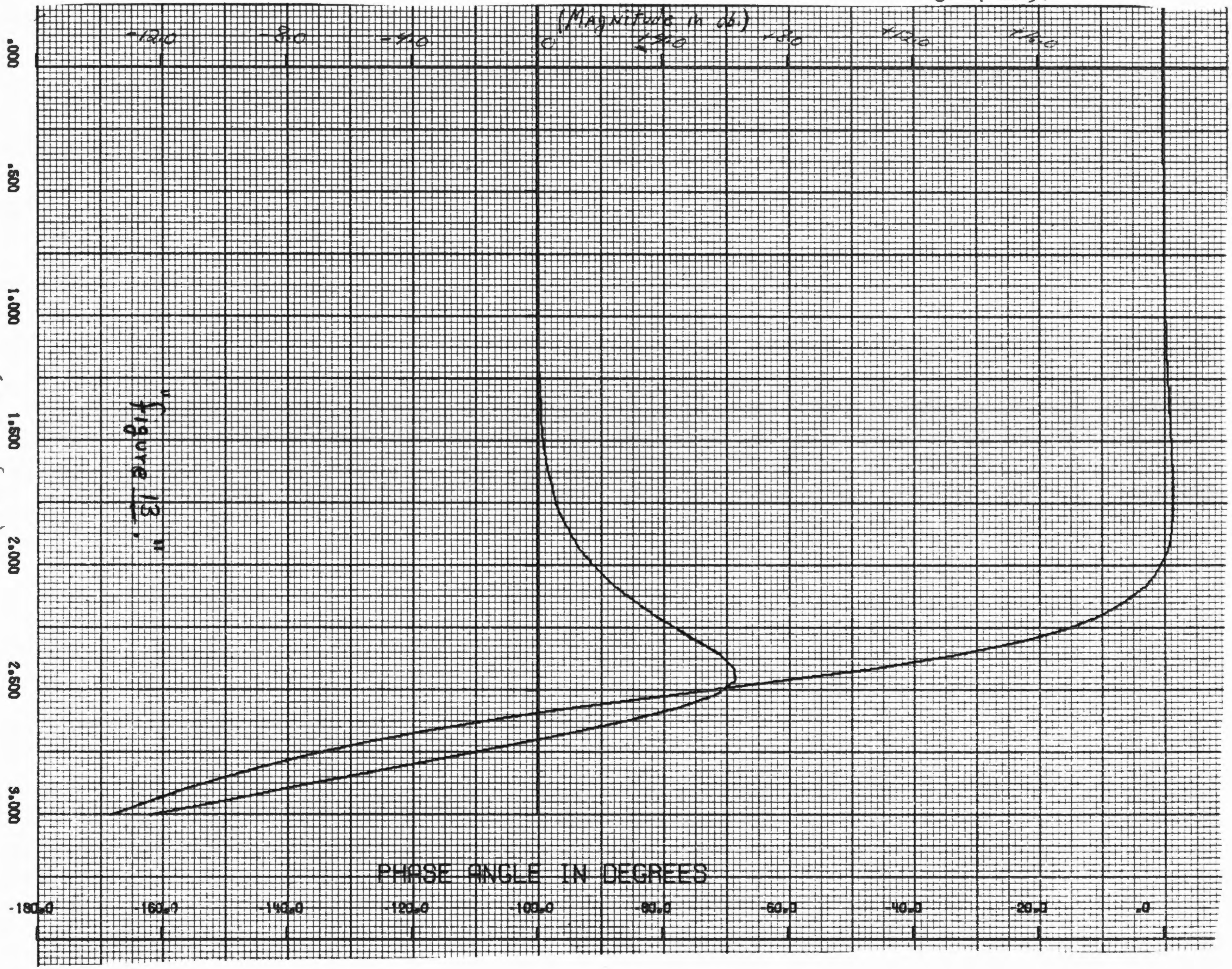


"Figure 11"

Open Loop Bode Plot for the inner and middle loops :

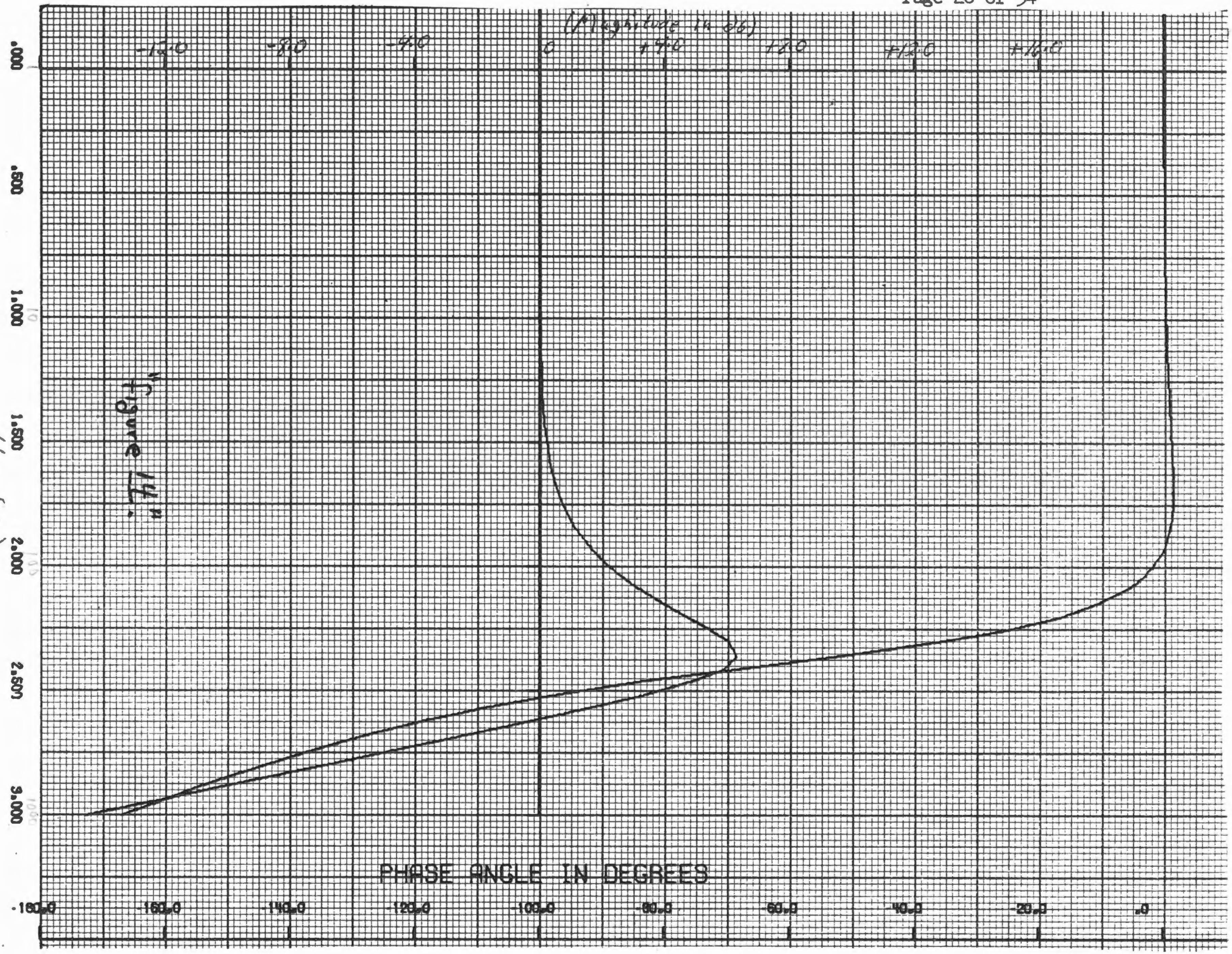


"figure 12."
Open Loop Bode Plot for the outer loop:



"Figure 13"

$\frac{\text{IRIG Test Point}}{\text{Test Input}}$ closed loop frequency response for the Inner and Middle Gimbals.



"figure 14."

$\frac{\text{IRIG test point}}{\text{Test input}}$ closed loop frequency response for the outer loop.

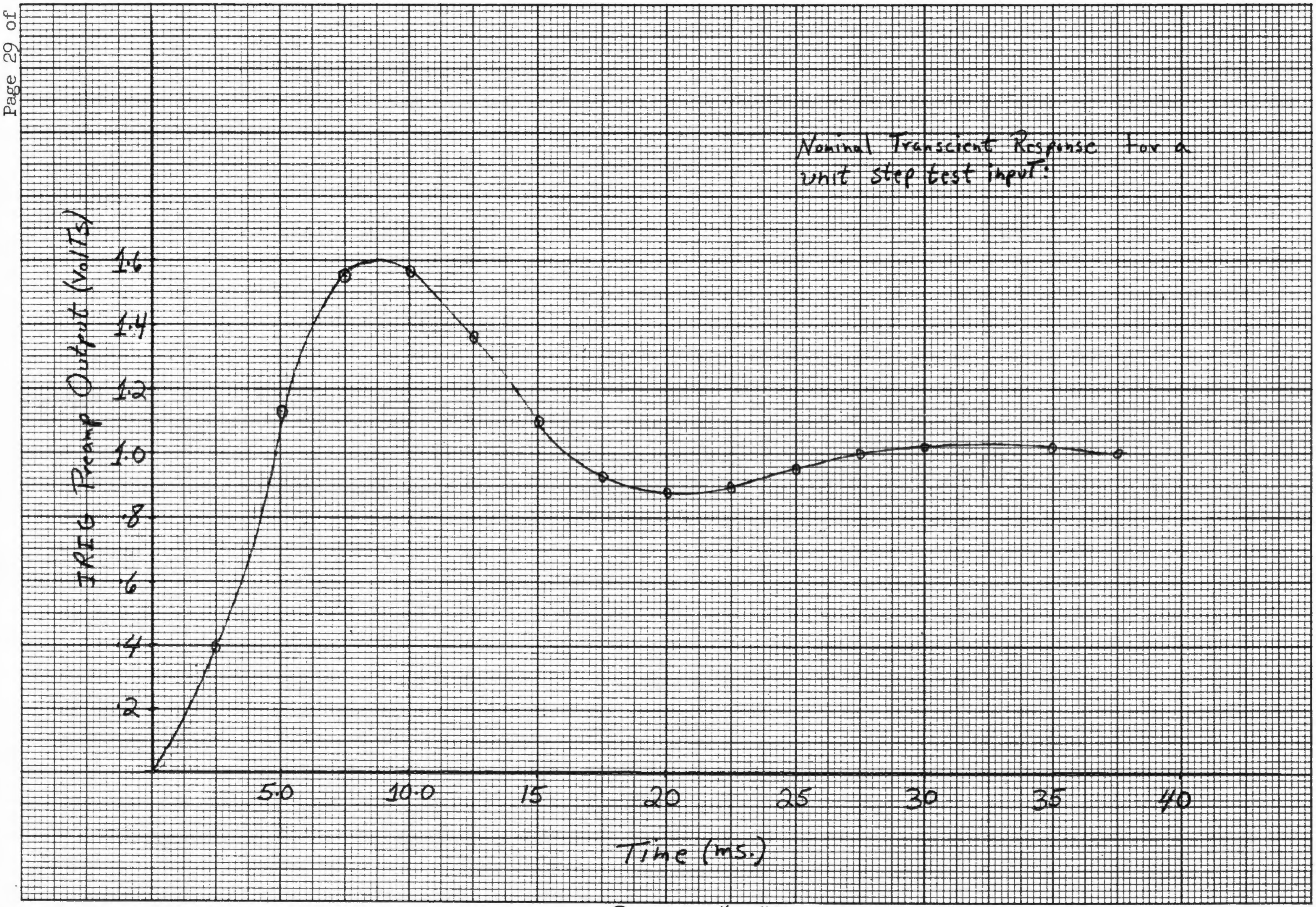

Nominal Transcient Response for a unit step test input:
IRIG Preamp Output (VolTs)
1.6
1.4
1.2
1.0
.8
.6
.4
.2
5.0
10.0
15
20
25
30
35
40
Time (ms.)

figure "15".

TABLE 3

RECIPROCAL OF IRIG TO TORQUE MOTOR GAIN IN $\overset{\frown}{\text{SEC}}$/IN.OZ

	LOW GAIN	NOMINAL GAIN	HIGH GAIN
Inner Axis	.050	.0394	.0312
Middle Axis	.0256	.0196	.0152
Outer Axis	.0213	.0161	.0125

b) Static Torque Disturbance at 20g

Inner Axis:

Slip Rings (2 sets)	3.4 in-oz
Torque Motor	8.0
Ball Bearing	12.0
Mass Unbalance	10.0
	33.4 in-oz
Effect of Stab. Amp. DC offset	0.5
	33.9 in-oz

Middle Axis:

Slip Rings (2 sets)	3.4 in-oz
Torque Motor	8.0
Ball Bearing	22.0
Mass Unbalance	10.0
	43.4 in-oz
Effect of Stab. Amp. DC Offset	0.5
	43.9 in-oz

Outer Axis:

Slip Rings (2 sets)	4.2 in-oz
Torque Motor	8.0
Ball Bearing	32.0
Mass Unbalance	10.0
	54.2 in-oz
Effect of Stab. Amp DC Offset	0.5
	54.7 in-oz

c) Quadrature and Unwanted In-Phase Signal

Inner Axis: The IRIG and its wiring introduce 6 mv of quadrature at the input of the IRIG preamplifier. This produces an unwanted signal at the output of the demodulator. The torque motors will drive the gimbal to an additional displacement to produce an additional in-phase IRIG output signal to cancel the effect of the quadrature signal. The additional gimbal displacement is listed below:

Low Gain	43.2 $\widehat{\text{sec}}$
Nominal Gain	41.8
High Gain	39.9

Middle and Outer Axis: The resolver introduces 50 mv of quadrature and 5 mv of in-phase signal at the input of demodulator. This produces at the demodulator output a 7.5 mv dc (unwanted) signal which is in addition to the unwanted signal caused by the IRIG and its wiring. The total gimbal angle errors introduced by quadrature and unwanted in-phase signals for the Middle and Outer Axis are listed below.

Low Gain	44.23 $\widehat{\text{sec}}$
Nominal Gain	42.83
High Gain	40.93

d) Gimbal Angle Error - Multiplying the reciprocals of the IRIG to torque motor gain by the static torque disturbance and adding in errors due to quadrature and unwanted in-phase signals, the maximum errors (listed below) result.

	Low Gain	Nom. Gain	High Gain
Outer Gimbal	45.40 $\widehat{\text{sec}}$	43.71 $\widehat{\text{sec}}$	41.62 $\widehat{\text{sec}}$
Middle Gimbal	45.35	43.69	41.60
Inner Gimbal	44.90	43.13	40.96

6.4.2 Quality Assurance Provisions - The expected maximum values for torque motor current, stabilization amplifier output and IRIG preamplifier output, for steady state stabilization loop performance in a 1.0 g field, are calculated below.

a) Torque Motor Current - Using the maximum value for static friction and mass unbalance and the minimum torque motor gain, the maximum expected values of torque motor current are given by:

$$I = \frac{17.9 \text{ in-oz}}{193.5 \ \frac{\text{in-oz}}{\text{amp}}} = 92.5 \text{ ma for the Inner and Middle Loops}$$

$$I = \frac{18.7 \text{ in-oz}}{156.6 \ \frac{\text{in-oz}}{\text{amp}}} = 119 \text{ ma for the Outer Loop}$$

b) Stabilization Amplifier Output - Using the maximum value of torque motor resistance, the maximum expected values of the stabilization amplifier outputs are given by:

$$E = (.092 \text{ amp})(61 \text{ ohms}) = 5.60^{v}$$

for the Inner and Middle loops.

$$E = (.119 \text{ amp})(34 \text{ ohms}) = 4.05^{v}$$

for the outer loop.

c) The in-phase and quadrature preamplifier outputs were calculated from the equations shown below and are listed in section 4.1.1.

$$e_i = \frac{e_q}{20} + e_s$$

$$e_q = e_g \text{ X } G \text{ X } \sin 10^{o} + e_n \text{ X } G \text{ X } \cos 10^{o}$$

$$e_i = e_g \text{ X } G \text{ X } \cos 10^{o} - e_n \text{ X } G \text{ X } \sin 10^{o}$$

where:

e_i = in-phase output voltage of IRIG preamplifier

e_q = quadrature output voltage of IRIG preamplifier

e_s = voltage required to overcome stiction

$e_s \approx$ 5 mv inner gimbal
2.5mv middle gimbal
2.0mv outer gimbal

e_n = IRIG null voltage

G = IRIG preamplifier gain

the 10^{o} phase shift is the maximum phase shift of the IRIG preamplifier

6.5 Nonlinear and Parameter Variation Analysis

The frequency and step responses previously described assumed that all components have nominal values. However, this will not be the case in actual operation as components will vary from their nominal value within certain specified tolerances.

The two extreme conditions considered are:

1. maximum bandwidth condition

2. minimum bandwidth condition

These conditions are obtained by arranging the stabilization loop parameters in the form shown in Table 4.

TABLE 4

	Irig Gain	Preamp Gain	Inertia	Torque Mtr Gain	Stab. Amp. Breaks
Maximum Bandwidth	high	high	low	high	high
Minimum Bandwidth	low	low	high	low	low

The step responses previously described assumed that the stabilization loop was linear. However, the following nonlinearities are present in the loop.

1. Preamp Saturation
2. Stabilization Amplifier Saturation
3. Stab. Amp. Nonlinear Compensation
4. Gimbal Friction
5. Demod Saturation

The objective of this study was to determine by an analog simulation how the performance of the stabilization loops could vary from the nominal linear case situation.

6.5.1 Closed Loop Frequency Response - The nominal closed loop frequency responses of the stabilization loop are shown in Figures 13 and 14. The two extreme responses which can be obtained if the parameters are arranged as shown in Table 4 are as follows:

1. With the parameters arranged to produce a minimum bandwidth:

   a) The -6 db point will be at a frequency greater than 55 cps.

2. With the parameters arranged to produce a maximum bandwidth:

   a) The -6 db point will be at a frequency less than 120 cps.

b) The resonant peak will not rise more than 12 db above relative zero.

Relative Zero is the gain at .1 cps

6.5.2 Step Response - The nominal linear step response of the stabilization loop is shown in Figure 13. The worst case step response will occur when the parameters of Table 4 are arranged to produce a maximum bandwidth condition, and the above nonlinearities are considered.

Using the test voltages shown in Table 2,

a) The time for the output to reach its first overshoot peak shall not be greater than 25 ms.

b) The preamplifier output shall not have more than 6 peaks over a 5% tolerance band of the steady state value.

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