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A Possible Value of the Infinite Series $\sum_{n=1}^{\infty} \frac{1}{n^3}$

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ABSTRACT: *The value of the infinite series of the sum of reciprocals of the cubes has remained unknown so far. I have tried to find it by computation.*

***KEYWORD*:** Infinite series

***NOTATIONS*:** Notations are very simple.

I. INTRODUCTION

Although the values of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^4}$ are known to us, the value of $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is unknown to us. I have tried to find out by computation. ($\pi = 3.14$) has been taken. Only two digits after decimal have been used in this paper.

II. METHOD OF ANALYSIS

First of all we shall apply this method to find out the value of already known infinite series. Then we shall apply it to find out the value of $\sum_{n=1}^{\infty} \frac{1}{n^3}$.

Let $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{m}$ where we shall find the value of m. ----------- (1)

We know that terms after the third term contribute a little to the value of the infinite series. So we shall neglect the terms after third or fourth according to our need.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + - - - - +\infty$$

Now from (1)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + - - - - +\infty = \frac{\pi^2}{m}$$

Or, $1+0.25+0.11+0.06+0.04+0.03+0.02+----+\infty = \frac{\pi^2}{m}$

Or, $1.51 \approx \frac{\pi^2}{m}$ (terms after seventh have been neglected)

Or, $m \approx \frac{\pi^2}{1.51}$

Or, $m \approx \frac{9.86}{1.51}$

Or, $m \approx 6.53$

(actual value of m can not be greater than 6.53)

Now, if we neglect the digits after the decimal, we get

m=6.

Hence from (1) we get $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$. This is the actual value.

Now we apply this method on $\sum_{n=1}^{\infty}\frac{1}{n^4}$.

Let $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{m}$ ----- (2) where m is an unknown number.

$$\sum_{n=1}^{\infty}\frac{1}{n^4}=1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+----+\infty$$

Now, $1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+----+\infty=\frac{\pi^4}{m}$

Or, $1+0.06+0.01+----+\infty=\frac{\pi^4}{m}$

Or, $1.07\approx\frac{\pi^4}{m}$ (terms after third term have been neglected.)

Or, $m\approx\frac{\pi^4}{1.07}$

Or, $m\approx\frac{97.21}{1.07}$

$Or, m\approx 90.85$

(When we neglect the decimal fraction, we get)

$m=90$

Using (2) we get $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$. This is the actual value.

Now, we apply this method on $\sum_{n=1}^{\infty}\frac{1}{n^3}$.

Let $\sum_{n=1}^{\infty}\frac{1}{n^3}=\frac{\pi^3}{m}$ ------ (3) where m is an unknown number.

$$\sum_{n=1}^{\infty}\frac{1}{n^3}=1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\frac{1}{5^3}+\frac{1}{6^3}+\frac{1}{7^3}+----+\infty$$

$1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\frac{1}{5^3}+\frac{1}{6^3}+\frac{1}{7^3}+----+\infty=\frac{\pi^3}{m}$ (from 3)

Or, $1+0.13+0.04+0.02+0.01+0.00+----+\infty=\frac{\pi^3}{m}$

Or, $1.20\approx\frac{\pi^3}{m}$ (terms after sixth term have been neglected)

Or, $m\approx\frac{30.96}{1.20}$

$Or, m\approx 25.80$

(When we neglect the decimal fraction, we get)

$m=25$

Using (3), we get $\sum_{n=1}^{\infty}\frac{1}{n^3}=\frac{\pi^3}{25}$.

III. CONCLUSION

From the above method we find that $\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{\pi^3}{25}$

REFERENCE

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