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DISTURBANCE OBSERVER BASED CONTROL FOR ACTIVE SUSPENSION SYSTEMS

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Abstract. *This study is aimed at improving the ride quality of the driving passenger with the proposed procedure of a nonlinear disturbance observer-based backstepping-like control for an active quarter car suspension system. The design procedure involves transforming the quarter-car suspension model to a lower triangular nonlinear system, which can be used with backstepping-like scheme directly. The proposed control law with some auxiliary terms is synthesized via backstepping-like method. Such terms obtained from the disturbance observer design are included to eliminate the adverse effect arising from the inescapable road disturbance. Then, the overall closed-loop stability analysis with the result of the road disturbance is investigated. The developed control design is validated in the MATLAB environment. The efficacy of the proposed control is verified through simulation and compared with uncontrolled system (passive suspensions) and backstepping-like approach. Despite the presence of a road disturbance, the results indicate that the presented strategy offers better dynamic performances and a satisfactory disturbance rejection ability as compared with other two controllers above.*
Keywords: Backstepping-like control, Nonlinear disturbance observer, Active suspensions, Passive suspensions

1. **Introduction.** In automobiles design, there have been focuses on various important factors; the ride quality becomes one of the most challenging factors to improve ride comfort for driver and passengers. Many attempts are to design an effective and efficient vehicle suspension system which is capable of isolating road disturbance, improving driving and riding comfort, and enhancing vehicle's performance.

Recently published reports in the literature are mostly related to design in different kinds of passive, active, and semi-active vehicle suspensions via various control strategies. In the past two decades, the control design techniques for the vehicle suspension systems have drawn researchers' attention. However, the particular interest is the use of an advanced nonlinear control technique to achieve performance requirement for vehicle suspensions which include: i) isolating passengers from vibration and shock occurring from road roughness; ii) suppressing the hop of the wheels to maintain firm and uninterrupted contact of wheels to road; and iii) keeping suspension strokes within an allowable maximum [1].

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To the best knowledge of the authors, there are some relevant instance of nonlinear schemes employed in this field such as sliding mode control [2-4], backstepping control [5-7], immersion and invariance control [8], feedback linearization control [9, 10], predictive control [11], and adaptive control [12-14].

In most engineering systems, disturbances are not practically avoidable which consequently degrade the desired control performances of the closed loop dynamics. Such disturbances include external disturbances, parametric uncertainties and unknown nonlinear terms. Therefore, it is necessary to include the disturbance dynamics in the desired control design method so as to get rid of the effects of aforementioned disturbances. A recent disturbance observer method is designed to compensate the effects of external disturbances and mismatched disturbances/uncertainties. This method is widely accepted and it was used to estimate the system disturbances. Currently, disturbance observer design combined with nonlinear control methods has been developed [15-22]. The disturbance observer-based control is a promising method due to its capability to reject external disturbance and improve robustness against uncertainties [15]. Besides that it is an effective means to handle external disturbances and system uncertainties [15-18] simultaneously. In addition, disturbance observer design method can be further used for several control systems [19-22]. In [19], an adaptive backstepping control combined with disturbance observer method was proposed for a class of nonlinear systems with multiple mismatched disturbances containing both single harmonic and constant disturbances. With the help of a combination of finite time integral sliding mode scheme and nonlinear disturbance observer technique, a composite anti-disturbance design [20] for a missile system was reported to obtain a good disturbance rejection performance and reject other types of disturbances. Kim et al. [21] proposed a nonlinear position tracking controller with a disturbance observer. The proposed controller was able to track the desired position despite the disturbance for electrohydraulic actuators. Recently, Kanchanaharuthai and Mujjalinvimut [22] have proposed a disturbance observer based backstepping control for power systems with external disturbances. The proposed method offered enhanced transient performances by suppressing system oscillations despite undesired disturbances. These published works indicate that the combination of advanced nonlinear control methods and the disturbance observer techniques has numerous advantages: it is a systematic method that ensures the closed-loop stability, offers superior transient performances, and has the rejection disturbance ability as compared with nonlinear control method alone.

For vehicle active suspension systems of which is the particular interest, less attention has been paid for the combination of the advanced nonlinear method with the disturbance observer design [23-26]. This is due to some auxiliary terms from disturbance observer design capable of eliminating the undesired effect from the road disturbances. In [23, 24], a disturbance observer based sliding mode control method has been applied to active suspension systems. The presented control law not only can reduce the acceleration of the sprung mass via sliding mode control, but also is able to estimate the effects of the uncertain, nonlinear spring and damper, load variation, and the unknown road disturbances, simultaneously. Even though the developed scheme is a promising and effective scheme, the resulting controller leads to the chattering issues in the system responses. In [25], a super-twisting controller of second-order sliding mode control has been developed for active suspensions. The proposed control method can improve the over control performance and reduce the chattering problem in the control input arising from the conventional sliding mode control [23, 24]. A disturbance observer based optimal control [26] for active suspensions has been proposed by using an LMI optimization technique. The proposed controller can minimize $\mathcal{L}_2$ gain of the closed-loop system from disturbances to regulated outputs.

This paper continues this line of investigation and concentrates on the design of nonlinear disturbance observer based backstepping-like control for an active quarter-car suspension system. According to the disturbance observer based control method [15-18], it can be observed that sliding mode method and backstepping method are often used to combine with disturbance observer design. However, although both methods have many advantages and can be successfully applied to numerous control systems, they have important disadvantages. The sliding mode control has the undesired effect arising from a discontinuous control signal, resulting in inevitable chattering issues. For backstepping, its obvious drawbacks are the problem of "explosion of complexity" and how to select a suitable virtual control used in each design step to find out the final controller. In order to avoid these drawbacks from two control strategies above, a backstepping-like control [27, 28] is used to combine with the disturbance observer design because the resulting control law has not the effect of chattering problems and does not require finding virtual control as used in backstepping.

Therefore, this paper deals with the design of a nonlinear feedback stabilizing control law on the basis of a backstepping-like control combining with the disturbance observer design to improve the ride quality of driver and passengers in spite of the effects of road disturbances.

As the above discussion, the followings are the major contributions of this work.

- The design procedure based on a combination of a nonlinear disturbance observer technique and backstepping-like control design is proposed to improve the ride quality for active suspension in the presence of inevitable road disturbances which has not yet been investigated.
- Although there is the road disturbance, the overall closed-loop system is input-to-state stable.
- In the absence of disturbances, the proposed scheme has the property of faster nominal performance recovery as compared to a backstepping-like control without the disturbance observer and passive suspensions (uncontrolled system). Further, the overall closed-loop system is asymptotically stable.
- The proposed design also offers better dynamic performances and a satisfactory disturbance rejection ability, thereby leading to ride-comfort improvement for driver and passengers.

The latter part of the paper is organized as follows. Section 2 is a brief presentation of dynamic model of an active quarter-car suspension incorporated with assumption and two significant lemmas, and the problem statement. Section 3 is about controller design and stability analysis. Simulation results are discussed in Section 4 and finally the paper is concluded in Section 5.

2. **Dynamic Model Description and Preliminaries.**

2.1. **Quarter suspension system model.** In this subsection, a quarter-car suspension model considered in this paper is shown in Figure 1. This system consists of two parts. One is the single wheel connected to the quarter portion of the car body through a combination of a linear spring (K_a), a linear damper (C_a) and an actuator force (u), respectively. The other is the tire that in this work is assumed to be a simple spring (K_t) without damping. Thus, with the help of Newton's second law, we can derive the motion equations of this system as follows:

$$\begin{cases} M_b\ddot{x}_s + K_a(x_s - x_w) + C_a(\dot{x}_s - \dot{x}_w) - u = 0 \\ M_{us}\ddot{x}_w + K_a(x_w - x_s) + C_a(\dot{x}_w - \dot{x}_s) + K_t(x_w - r) + u = 0, \end{cases} \tag{1}$$

where M_b and M_{us} denote the masses of car body and wheel, respectively. x_s and x_w represent the displacement of car body and wheel. K_a and K_t are the spring coefficients. C_a denotes the damper coefficient and r is the road disturbance.

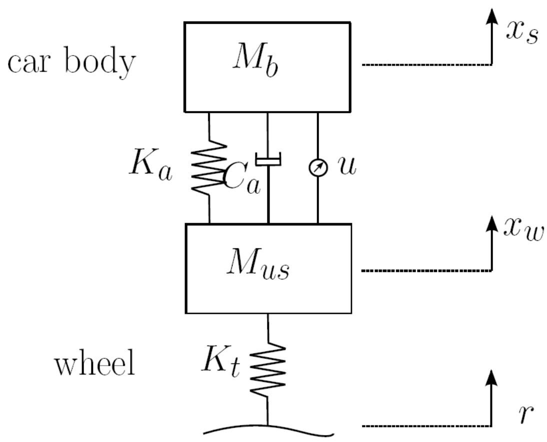

Figure 1. Quarter car model

In accordance with the result reported in [29], let us introduce the following state variables:

$$y_1 = M_{us}x_w + M_b x_s,\ y_2 = \dot{y}_1,\ y_3 = x_s - x_w,\ y_4 = \dot{y}_3. \tag{2}$$

Differentiating the state variables (2), we have the dynamic equations capable of governing the motions of active suspension systems with the road disturbances (r) expressed as an affine nonlinear system as follows:

$$\dot{y} = f(y) + g(y)u(y) + g_r(y)d(t), \tag{3}$$

with

$$\begin{cases} f(y) = \begin{bmatrix} f_1(u) \\ f_2(y) \\ f_3(y) \\ f_4(y) \end{bmatrix} = \begin{bmatrix} y_2 \\ \beta_1 y_1 + \beta_2 y_3 \\ y_4 \\ \beta_3 y_1 + \beta_4 y_3 + \beta_5 y_4 \end{bmatrix}, \\ g(y) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ g_4(y) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \left(\frac{1}{M_b} + \frac{1}{M_{us}}\right) \end{bmatrix},\ g_r(y) = \begin{bmatrix} 0 \\ g_{2r} \\ 0 \\ g_{4r}(y) \end{bmatrix} = \begin{bmatrix} 0 \\ K_t \\ 0 \\ -\frac{K_t}{M_{us}} \end{bmatrix}, \\ d(t) = d_1(t) = d_2(t) = r, \end{cases} \tag{4}$$

where $\beta_1 = -\frac{K_t}{M_b+M_{us}}$, $\beta_2 = \frac{K_t M_b}{M_b+M_{us}}$, $\beta_3 = \frac{K_t}{M_{us}(M_b+M_{us})}$, $\beta_4 = -\left[K_a\left(\frac{1}{M_b} + \frac{1}{M_{us}}\right) + \beta_3 M_b\right]$ and $\beta_5 = -C_a\left(\frac{1}{M_b} + \frac{1}{M_{us}}\right)$.

For the sake of simplicity, the active suspension system considered in (3) and (4) can be expressed as follows.

$$\begin{cases} \dot{y}_1 = y_2, \\ \dot{y}_2 = \beta_1 y_1 + \beta_2 y_3 + K_t d_1(t), \\ \dot{y}_3 = y_4, \\ \dot{y}_4 = f_4(y) + g_4(y)u + g_{4r}(y)d_2(t). \end{cases} \tag{5}$$

Assumption 2.1. *The road disturbance $d(t)$ and its first derivative are bounded. Additionally, the road disturbance considered throughout this work also satisfies the condition of* $\lim_{t\to+\infty} d_j(t) = \lim_{t\to+\infty} \dot{d}_j(t) = 0$, $(j = 1, 2)$.

2.2. **Preliminaries.** In this subsection, some important lemmas are mentioned as follows for convenience of the reader. Consider the following system

$$\dot{y} = f(t, y, u),\ y \in \mathbb{R}^n,\ u \in \mathbb{R}^m. \tag{6}$$

Definition 2.1. [31] *A continuous function $\alpha : [0, a) \to [0, +\infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It belongs to class $\mathcal{K}_\infty$ if $a = +\infty$ and $a(r) \to +\infty$ as $r \to +\infty$.*

Lemma 2.1. [31] *Let $V : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that*

$$\alpha_1(||y||) \le V(t, y) \le \alpha_2(||y||)$$
$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial y} f(t, y, u) \le -W_3(y),\ \forall ||x|| \ge \rho(||u||) > 0,$$

for all $(t, y, u) \in [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m$, where α_1 and α_2 are class $\mathcal{K}$ functions, ρ is a class $\mathcal{K}$ function, and $W_3(y)$ is a continuous positive definite function on $\mathbb{R}^n$. Then, system (6) is input-to-state stable (ISS).

Lemma 2.2. [31] *Consider the following system (6). If the following conditions are satisfied*

- *system $\dot{y} = f(t, y, u)$ is input-to-state stable,*
- $\lim_{t\to+\infty} u = 0$,

then the states of the system (5) will asymptotically converge to zero, that is, $\lim_{t\to+\infty} y(t) = 0$.

Problem statement: The objectives of this paper are to stabilize the active suspension system (5) with the external (road) disturbance d and to obtain more comfortable riding, which can be formulated as follows: with the help of the nonlinear disturbance observer-based backstepping-like control technique [16], find out, if possible, a stabilizing (state) feedback controller $u(y)$ and disturbance estimation $\hat{d}$ as follows:

$$\begin{cases} u = \phi\left(y, \hat{d}\right) \\ \dot{\hat{d}} = \varphi\left(y, u, \hat{d}\right) \end{cases} \tag{7}$$

such that the overall closed-loop systems (5) and (7) are input-to-state stable, where $\hat{d}$ is the estimate of d.

For the developed design procedure in the next section, a combination of the backstepping-like method and disturbance observer design will be presented to obtain a composite nonlinear controller (7). In comparison with the conventional backstepping-like method, the proposed approach will introduce the disturbance estimation terms into control variables. These terms are also used for compensating the external disturbances at each step, and the estimation error dynamics are included for the closed-loop stability analysis.

Remark 2.1. *In the literature, disturbance observer techniques are often combined with two main nonlinear control schemes: sliding mode and backstepping. The former depends upon selecting a suitable sliding surface, and to overcome the effect of disturbances, the obtained control law includes a discontinuous function, thereby resulting in the unavoidable chattering problem. Although the latter is an effective method, it is based on selecting* *recursively appropriate virtual inputs for lower dimension subsystems of the overall system and the Lyapunov functions are designed for each stable virtual controller* [30, 31]. *In addition, as the dynamic model of interest becomes higher, the appropriate selection of virtual control in each design step and the problem of "explosion of complexity" are more complicated. In this paper, by using backstepping-like method, we add and subtract some terms systematically in each design step, but such terms can be chosen easier than the virtual control in backstepping approach. For this work, we combine the advantage of the backstepping-like method with the disturbance rejection ability of disturbance observer design to mitigate the effect of road disturbances unavoidably arising in vehicle suspension systems.*

3. **Controller Design and Stability Analysis.** This section is aimed to determine the control laws for stabilizing the active quarter-car suspension system. The proposed design procedure comprises the following three parts.

- The first part introduces a nonlinear disturbance observer technique to online estimate the unknown, but bounded, disturbances and to compensate for the road disturbance.
- The second part proposes an approach consisting of the backstepping-like control method and the result of the disturbance estimator from the first part to find the desired controller in each design step.
- The last part shows that Lyapunov stability theorem is used to analyze the overall closed-loop system stability. Despite having the disturbance in the system, the results indicate that it is able to achieve both the system stability and the desired control performances by the obtained controller.

3.1. **Nonlinear disturbance observer design.** The aim of designing the disturbance observer is to estimate the road (external) disturbance and other uncertainties so that the effect of disturbances is removed and the whole system performance can be enhanced. The disturbance observer proposed in [15, 16] is used to estimate the disturbance and is applied with the control input.

Therefore, the nonlinear disturbance observer for the system (5) is designed as

$$\begin{cases} \hat{d}_i = \lambda_i(y_j - p_j), \ i = 1, 2, \ j = 2, 4, \\ \dot{p}_2 = \beta_1 y_1 + \beta_2 y_3 + K_t \hat{d}_1, \\ \dot{p}_4 = f_4(y) + g_4(y)u + g_{4r}\hat{d}_2, \end{cases} \tag{8}$$

where $\lambda_j > 0$ is a design parameter. Thus, based on (8) the disturbance estimation dynamics can be expressed in the following form:

$$\dot{\hat{d}}_j = \lambda_i(\dot{y}_j - \dot{p}_j) = \lambda(d_j - \hat{d}_j), \quad j = 1, 2. \tag{9}$$

Let us define the disturbance estimation error as $e_j = d_j - \hat{d}_j$, and the estimation error dynamics can be expressed as follows.

$$\dot{e}_j = -\lambda_j e_j + \dot{d}_j. \tag{10}$$

3.2. **Backstepping-like design.** According to the concept reported in [16], the stabilization problem for the system (5) is solved by designing a backstepping-like control. The design procedure is developed step by step as follows.

Step 1: We start from focusing on the first subsystem (5), and then a Lyapunov function candidate is chosen as

$$V_1 = \frac{1}{2}y_1^2. \tag{11}$$

Then the time derivative of V_1 along the system trajectories becomes

$$\dot{V}_1 = y_1\dot{y}_1 = y_1y_2 = -c_1y_1^2 + y_1(c_1y_1 + y_2), \tag{12}$$

where $c_1 > 0$ is a design parameter.

Step 2: From (12), it is observed that the second term can be neither positive nor negative. Thus, we can eliminate the result from the aforementioned equation by choosing the Lyapunov function candidate as:

$$V_2 = \frac{1}{2}y_1^2 + \frac{1}{2}(c_1y_1 + y_2)^2 + \frac{1}{2}e_1^2. \tag{13}$$

After calculating the derivative of (13), we have

$$\begin{aligned}\dot{V}_2 =& -c_1y_1^2 + y_1(c_1y_1 + y_2) + (c_1y_1 + y_2)(c_1\dot{y}_1 + \dot{y}_2) + e_1\left(-\lambda_1 e + \dot{d}_1\right)\\ =& -c_1y_1^2 + (c_1y_1 + y_2)\left((1 + \beta_1)y_1 + c_1y_2 + \beta_2y_3 + K_t\hat{d}_1\right) - \lambda_1e_1^2 + e_1\dot{d}_1\\ &+ (c_1y_1 + y_2)K_te_1. \end{aligned}\tag{14}$$

It is observed that the last term of (14) can be straightforwardly computed by using Young inequality as

$$(c_1y_1 + y_2)K_te_1 \le \frac{1}{4\epsilon_1}K_t^2(c_1y_1 + y_2)^2 + \epsilon_1e_1^2 = \hat{c}_2(c_1y_1 + y_2)^2 + \epsilon_1e_1^2. \tag{15}$$

After adding and subtracting $\bar{c}_2(c_1y_1 + y_2)$ where $\bar{c}_2 = c_2 + \hat{c}_2$, $c_2 > 0$, $\hat{c}_2 = \frac{K_t^2}{4\epsilon_1}$, $\epsilon_1 > 0$, into the equation above, we have

$$\begin{aligned}\dot{V}_2 =& -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 + (c_1y_1 + y_2)\Big(-\bar{c}_2(c_1y_1 + y_2) + (1 + \beta_1)y_1\\ &+ c_1y_2 + \beta_2y_3 + K_t\hat{d}_1\Big) - (\lambda_1 - \epsilon_1)e_1^2 + e_1\dot{d}_1\\ \le& -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 + (c_1y_1 + y_2)\mathcal{M} - (\lambda_1 - \epsilon_1)e_1^2 + e_1\dot{d}_1, \end{aligned}\tag{16}$$

where $\mathcal{M} = \gamma_1y_1 + \gamma_2y_2 + \gamma_3y_3 + K_t\hat{d}_1$, $\gamma_1 = c_1\bar{c}_2 + \beta_1 + 1$, $\gamma_2 = c_1 + \bar{c}_2$, $\gamma_3 = \beta_2$. In the same manner, it can be seen that the third term of (16) is not always negative. So, one needs to cancel this term.

Step 3: Let us define the Lyapunov function of Step 2 as

$$V_3 = V_2 + \frac{1}{2}\mathcal{M}^2. \tag{17}$$

Then the time derivative of V_3 along the system trajectories turns into as follows:

$$\begin{aligned}\dot{V}_3 =& -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 + (c_1y_1 + y_2)\mathcal{M}\\ &+ \mathcal{M}\left(\frac{\partial\mathcal{M}}{\partial y_1}\dot{y}_1 + \frac{\partial\mathcal{M}}{\partial y_2}\dot{y}_2 + \frac{\partial\mathcal{M}}{\partial y_3}\dot{y}_3 + \frac{\partial\mathcal{M}}{\partial\hat{d}_1}\dot{\hat{d}}_1\right) - (\lambda_1 - \epsilon_1)e_1^2 + e_1\dot{d}_1\\ =& -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 + \mathcal{M}\Bigg[c_1y_1 + y_2 + \frac{\partial\mathcal{M}}{\partial y_1}y_2 + \frac{\partial\mathcal{M}}{\partial y_2}\left(\beta_1y_1 + \beta_2y_3 + K_t\hat{d}_1\right)\\ &+ \frac{\partial\mathcal{M}}{\partial y_3}y_4\Bigg] - (\lambda_1 - \epsilon_1)e_1^2 + e_1\dot{d}_1 + \mathcal{M}\left(\frac{\partial\mathcal{M}}{\partial y_2}K_t + \frac{\partial\mathcal{M}}{\partial\hat{d}_1}\lambda_1\right)e_1. \end{aligned}\tag{18}$$

On the basis of Young inequality, the last term in (18) can be directly computed as

$$\mathcal{M}\left(\frac{\partial\mathcal{M}}{\partial y_2}K_t + \frac{\partial\mathcal{M}}{\partial\hat{d}_1}\lambda_1\right)e_1 \le \frac{1}{4\epsilon_1}\left(\frac{\partial\mathcal{M}}{\partial y_2}K_t + \frac{\partial\mathcal{M}}{\partial\hat{d}_1}\lambda_1\right)^2\mathcal{M}^2 + \epsilon_1e_1^2 = \hat{c}_3\mathcal{M}^2 + \epsilon_1e_1^2. \tag{19}$$

Then substituting (19) into the last term of (18) together with adding and subtracting $\bar{c}_3\mathcal{M}$, where $\bar{c}_3 = c_3 + \hat{c}_3$, $c_3 > 0$, $\hat{c}_3 = \frac{1}{4\epsilon_1}\left(\frac{\partial\mathcal{M}}{\partial y_2}K_t + \frac{\partial\mathcal{M}}{\partial\hat{d}_1}\lambda_1\right)^2$, $\epsilon_1 > 0$, into (18); therefore, we will obtain

$$\dot{V}_3 \leq -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 - c_3\mathcal{M}^2 + \mathcal{M}\mathcal{P} - (\lambda_1 - 2\epsilon_1)e_1^2 + e_1\dot{d}_1, \tag{20}$$

where $\mathcal{P} = \bar{c}_3\mathcal{M} + \alpha_1y_1 + \alpha_2y_2 + \alpha_3y_3 + \alpha_4y_4 + \alpha_r\hat{d}_1$, $\alpha_1 = c_1 + \beta_1\frac{\partial\mathcal{M}}{\partial y_2}$, $\alpha_2 = 1 + \frac{\partial\mathcal{M}}{\partial y_1}$, $\alpha_3 = \beta_2\frac{\partial\mathcal{M}}{\partial y_2}$, $\alpha_4 = \frac{\partial\mathcal{M}}{\partial y_3}$ and $\alpha_r = K_t\frac{\partial\mathcal{M}}{\partial\hat{d}_1}$.

Step 4: From (20), it is obvious that the fourth term can either be positive or negative. In order to eliminate it, the Lyapunov function is applied in (21).

$$V_4 = V_3 + \frac{1}{2}\mathcal{P}^2 + \frac{1}{2}e_2^2. \tag{21}$$

Based on (20), the derivative of (21) becomes

$$\begin{aligned}
\dot{V}_4 =& -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 - c_3\mathcal{M}^2 + \mathcal{P}\left(\mathcal{M} + \dot{\mathcal{P}}\right) - (\lambda_1 - 2\epsilon_1)e_1^2 - \lambda_2e_2^2 + \sum_{i=1}^{2}e_i\dot{d}_i \\
=& -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 - c_3\mathcal{M}^2 + \mathcal{P}\Bigg[\mathcal{M} + \mathcal{M}_1y_2 + \mathcal{M}_2\left(\beta_1y_1 + \beta_2y_3 + K_t\hat{d}_1\right) \\
&+ \mathcal{M}_3y_4 + \frac{\partial\mathcal{P}}{\partial y_4}\left(f_4(y) + g_4(y)u(y) + g_{4r}\hat{d}_2\right)\Bigg] - (\lambda_1 - 2\epsilon_1)e_1^2 - \lambda_2e_2^2 + \sum_{i=1}^{2}e_i\dot{d}_i \\
&+ \mathcal{P}\left(\mathcal{M}_2 + \mathcal{M}_{\hat{d}_1}\lambda_1\right)e_1 + \mathcal{P}\left(\frac{\partial\mathcal{P}}{\partial y_4}g_{4r}\right)e_2,
\end{aligned} \tag{22}$$

where $\mathcal{M}_i = \bar{c}_3\frac{\partial\mathcal{M}}{\partial y_i} + \frac{\partial\mathcal{P}}{\partial y_i}$, $i = 1, 2, 3$ and $\mathcal{M}_{\hat{d}_1} = \bar{c}_3\frac{\partial\mathcal{M}}{\partial\hat{d}_1} + \frac{\partial\mathcal{P}}{\partial\hat{d}_1}$.

From (22), the following suitable control law is selected so as to accomplish the desired control performance.

$$u = -\frac{1}{\frac{\partial\mathcal{P}}{\partial y_4}g_4(y)}\left[\bar{c}_4\mathcal{P} + \mathcal{M} + \mathcal{M}_1y_1 + \mathcal{M}_2f_2\left(y, \hat{d}\right) + \mathcal{M}_3y_4 + \frac{\partial\mathcal{P}}{\partial y_4}f_4\left(y, \hat{d}\right)\right], \tag{23}$$

where $\bar{c}_4 = c_4 + \hat{c}_{41} + \hat{c}_{42}$, $c_4 > 0$, $\hat{c}_{41} = \frac{1}{4\epsilon_1}\left(\mathcal{M}_2 + \mathcal{M}_{\hat{d}_1}\lambda_1\right)^2$ and $\hat{c}_{42} = \frac{1}{4\epsilon_2}\left(\frac{\partial\mathcal{P}}{\partial y_4}g_{4r}\right)^2$, $f_2\left(y, \hat{d}\right) = \beta_1y_1 + \beta_2y_3 + K_t\hat{d}_1$, and $f_4\left(y, \hat{d}\right) = f_4(y) + g_{4r}\hat{d}_2$.

Once the developed control law (23) is substituted into (22), it gives

$$\begin{aligned}
\dot{V}_4 =& -c_1y_1^2 - c_2(c_1y_1 + y_2)^2 - c_3\mathcal{M}^2 - \bar{c}_4\mathcal{P}^2 - (\lambda_1 - 2\epsilon_1)e_1^2 - \lambda_2e_2^2 + \sum_{i=1}^{2}e_i\dot{d}_i \\
&+ \mathcal{P}(\mathcal{M}_2 + \mathcal{M}_{\hat{d}_1}\lambda_1)e_1 + \mathcal{P}\left(\frac{\partial\mathcal{P}}{\partial y_4}g_{4r}\right)e_2.
\end{aligned} \tag{24}$$

Seemingly, with the help of Young inequality, the last two terms of (24) can be transformed into the following inequalities:

$$\mathcal{P}\left(\mathcal{M}_2 + \mathcal{M}_{\hat{d}_1}\lambda_1\right)e_1 \leq \frac{1}{4\epsilon_1}\left(\mathcal{M}_2 + \mathcal{M}_{\hat{d}_1}\lambda_1\right)^2\mathcal{P}^2 + \epsilon_1e_1^2 = \hat{c}_{41}\mathcal{P}^2 + \epsilon_1e_1^2, \tag{25}$$

$$\mathcal{P}\left(\frac{\partial\mathcal{P}}{\partial y_4}g_{4r}\right)e_2 \leq \frac{1}{4\epsilon_2}\left(\frac{\partial\mathcal{P}}{\partial y_4}g_{4r}\right)^2\mathcal{P}^2 + \epsilon_2e^2 = \hat{c}_{42}\mathcal{P}^2 + \epsilon_2e_2^2. \tag{26}$$

Combining the inequalities (25) and (26) with (24), it will then become

$$\dot{V}_4 \leq -c_1 y_1^2 - c_2(c_1 y_1 + y_2)^2 - c_3\mathcal{M}^2 - c_4\mathcal{P}^2 - (\lambda_1 - 3\epsilon_1)e_1^2 - (\lambda_2 - \epsilon_2)e_2^2 + \sum_{i=1}^{2} e_i \dot{d}_i. \quad (27)$$

The stability analysis of the closed-loop dynamics with the proposed control law (23) is presented in the next section.

Remark 3.1. *Note that on the basis of the idea reported in* [16] *the control law (23) consists of some auxiliary terms* $\hat{c}_2, \hat{c}_{i1}, \ldots, \hat{c}_{42}$ *introduced to deal with the crossing terms arising from the effect of disturbances, compensation errors, and system states. In contrast, these auxiliary terms are not introduced into the conventional backstepping-like scheme, thereby leading to unsatisfactory control performances.*

3.3. **Stability analysis.** In this subsection, the proposed control law (23) ensures the overall closed-loop stability of suspension systems (5). Therefore, the following theorem can be summarized.

Theorem 3.1. *Under Assumption 2.1, the nonlinear disturbance observer-based backstepping-like controller (23) can guarantee that the overall closed-loop system combining the system (5) with the disturbance observer error dynamics (10) and the presented control law is input-to-state stable. Besides, after the disturbance input vanishes, the origin of the system is asymptotically stable.*

Proof: To demonstrate the closed-loop stability of the presented control strategy, let us define the following Lyapunov function for the closed-loop system.

$$V_4 = \frac{1}{2}\left(y_1^2 + (c_1 y_1 + y_2)^2 + \mathcal{M}^2 + \mathcal{P}^2 + e_1^2 + e_2^2\right). \quad (28)$$

After computing the time derivative of the Lyapunov function candidate (28), the closed-loop system can be expressed as

$$\dot{V}_4 \leq -c_1 y_1^2 - c_2(c_1 y_1 + y_2)^2 - c_3\mathcal{M}^2 - c_4\mathcal{P}^2 - (\lambda_1 - 3\epsilon_1)e_1^2 - (\lambda_2 - \epsilon_2)e_2^2 + \sum_{i=1}^{2} e_i \dot{d}_i. \quad (29)$$

We choose $\lambda_1 = a_{01} + 3\epsilon_1$, $\lambda_2 = a_{02} + \epsilon_2$, $a_{0j} > 0$, $(j = 1, 2)$ to obtain

$$\begin{aligned}\dot{V}_4 &\leq -c_1 y_1^2 - c_2(c_1 y_1 + y_2)^2 - c_3\mathcal{M}^2 - c_4\mathcal{P}^2 - \sum_{j=1}^{2} a_{0j} e_j^2 + \sum_{j=1}^{2} e_j \dot{d}_j \\ &\leq -c_1 y_1^2 - c_2(c_1 y_1 + y_2)^2 - c_3\mathcal{M}^2 - c_4\mathcal{P}^2 - a_0\|e\|^2 + \|e\| \left\|\dot{d}\right\|, \end{aligned} \quad (30)$$

where $e = [e_1, e_2]^T$, $\dot{d} = \left[\dot{d}_1, \dot{d}_2\right]^T$, $a_0 = \min\{a_{01}, a_{02}\}$.

According to Assumption 2.1, one can employ the term $-a_0\|e\|^2$ to dominate $\|e\| \left\|\dot{d}\right\|$ in (30); subsequently, we rewrite the foregoing inequality as

$$\dot{V}_4 \leq -c_1 y_1^2 - c_2(c_1 y_1 + y_2)^2 - c_3\mathcal{M}^2 - c_4\mathcal{P}^2 - (1-\theta)a_0\|e\|^2 - \theta a_0\|e\|^2 + \|e\| \left\|\dot{d}\right\|, \quad (31)$$

where $0 < \theta < 1$. Then,

$$\dot{V}_4 \leq -c_1 y_1^2 - c_2(c_1 y_1 + y_2)^2 - c_3\mathcal{M}^2 - c_4\mathcal{P}^2 - (1-\theta)a_0\|e\|^2, \ \forall \|e\| \geq \frac{\left\|\dot{d}\right\|}{a_0\theta}. \quad (32)$$

Thus, the conditions of Lemmas 2.1 and 2.2 are satisfied with $\alpha_1(r) = c_1 r^2$, $\alpha_2(r) = c_2 r^2$, and $\rho(r) = (1/a_0\theta)r$, and we can conclude that the overall closed-loop system is input-to-state stable. Furthermore, when the road disturbance input vanishes from the system, it implies that both d and $\hat{d}$ converge to zero together with $\dot{d} \to 0$. With the help of Lyapunov stability theory [31], it is not difficult to indicate that $\lim_{t\to+\infty} y_1 = 0$, $\lim_{t\to+\infty}(c_1 y_1 + y_2) = 0$, $\lim_{t\to+\infty} \mathcal{M} = \lim_{t\to+\infty}\left(\gamma_1 y_1 + \gamma_2 y_2 + \gamma_3 y_3 + K_t \hat{d}_1\right) = 0$, and $\lim_{t\to+\infty} \mathcal{P} = \lim_{t\to+\infty}\left(\bar{c}_3\mathcal{M} + \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 + \alpha_4 y_4 + \alpha_r \hat{d}_1\right) = 0$ as well. After combining Lemma 2.2 with Assumption 2.1, it follows that all trajectories of y_i, $(i = 1, 2, 3, 4)$ and e_j, $(j = 1, 2)$ of the closed-loop dynamics converge to zero. This means that $y_i \to 0$ and $e_j \to 0$ as $t \to +\infty$. This completes the proof.

4. **Simulation Results.** In this section, simulation results of active suspensions from the proposed controller are discussed to indicate the effectiveness of the developed strategy. The performance of the proposed control scheme is evaluated and verified in the MATLAB environment.

To carry out the simulation, the physical parameters, the control parameters, and the initial condition under consideration are given as follows.

- The physical parameters used in the simulation are the same as those used in [7] as follows:

$$M_b = 290 \text{ kg}, \ M_{us} = 59 \text{ kg}, \ K_a = 16,812 \text{ N/m},$$
$$K_t = 190,000 \text{ N/m}, \ C_a = 1,000 \text{ N/(m/sec)}.$$

- The controller parameters are set as $\epsilon_j = 0.001$, $c_i = 40$, $\lambda_j = 200$, $(j = 1, 2, \ i = 1, 2, 3, 4)$.
- The initial states of the system are selected as $y_0 = [0, 0, 0, 0]^T$, $p = [0, 0]^T$, $\hat{d}_0 = [0, 0]^T$, and $e = [0, 0]^T$.

To illustrate the performance achieved by the proposed controller, we select the road disturbance as an isolated bump in a smooth road surface represented in the following form:

$$r = d_1(t) = d_2(t) = \begin{cases} 0.025(1 - \cos(8\pi t)), & 0.5 \le t < 0.75, \\ 0, & \text{otherwise} \end{cases}.$$

The time domain simulations are carried out to investigate the system stability enhancement and the dynamic performance of the designed controller, as given in (23), in the system in the presence of the road disturbance. To evaluate the effectiveness of the proposed controller (nonlinear disturbance observer-based backstepping-like controller), both an uncontrolled system and a conventional backstepping-like controller are used in the simulation study for the purpose of comparison.

- The uncontrolled system is set as $u = 0$.
- The conventional backstepping-like controller is designed as

$$u = -\frac{1}{\alpha_4 g_4(y)}(c_4\mathcal{P} + \mathcal{M} + c_3\mathcal{M} + \alpha_1 y_2 + \alpha_2 f_2(y) + \alpha_3 y_4 + \alpha_4 f_4(y)), \tag{33}$$

where $\mathcal{M} = \sum_{i=1}^{3} \gamma_i y_i$, $\gamma_1 = c_1 c_2 + 1 + \beta_1$, $\gamma_2 = c_1 + c_2$, $\gamma_3 = \beta_2$, $\mathcal{P} = c_3\mathcal{M} + \sum_{j=1}^{4} \alpha_j y_j$, $\alpha = c_1 + \gamma_2\beta_2$, $\alpha_2 = 1 + \gamma_1$, $\alpha_3 = \gamma_2\beta_2$, $\alpha_4 = \gamma_3$. $c_j > 0$ are design parameters. The controller parameters of this scheme are chosen as $c_j = 40$, $(j = 1, 2, 3, 4)$.

The simulation results are presented and discussed as follows. Time trajectories of the body acceleration, body travel, suspension travel, together with wheel travel under three controllers are presented in Figures 2(a)-2(d).

From all these figures, it is overall observed that time trajectories eventually tend to their steady-state respectively as time goes to infinity after the road disturbance vanishes. Clearly, the oscillations in bump responses are slowly damped by the uncontrolled system (passive suspensions). On the other hand, the active controller (the proposed controller and the conventional backstepping-like controller) improves significantly, that is, better transient dynamic performance on ride comfort (lower peak and shorter setting time in the car body acceleration), and suspension travel. Obviously, for the proposed design, the body acceleration is reduced by almost 40%, and the body travel by almost 90% as compared with passive suspensions. It is known well that the car body acceleration is directly associated with the ride quality. Though the road disturbance exists in the system, the active control can achieve satisfactory disturbance rejection ability and clearly improve the ride quality. From Figure 2(a), with an inclusion of the road disturbance, the developed control law not only exhibits a fast response to mitigate the adverse effect, but also provides the desired control performance such as small overshoot magnitudes and rapidly suppressing system oscillations. In addition, the conventional backstepping-like control offers the control performance superior to the uncontrolled system. However, it still has a poor disturbance rejection performance and brings undesired control performance, such as an unsatisfactory overshoot magnitude in the car body acceleration

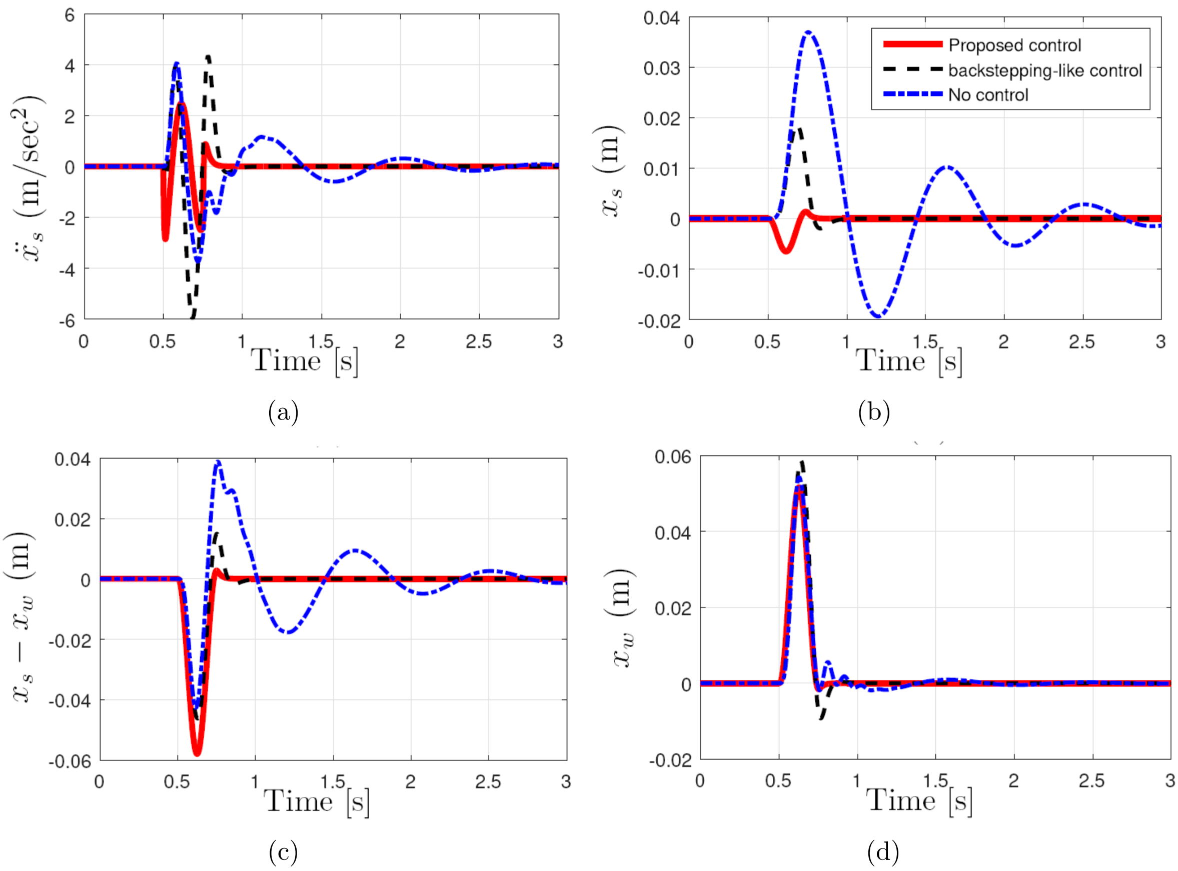

FIGURE 2. Controller performance: (a) body acceleration ($\ddot{x}_s$), (b) body travel (x_s), (c) suspension travel ($x_s - x_w$), (d) wheel travel (x_w) (Solid: the proposed control, Dashed: backstepping-like control, Dotted: no control)

and slowly suppressing system oscillations as compared with the developed scheme are observed.

Figures 2(b)-2(d) indicate that the body travel, suspension travel, and wheel travel from the proposed design are less than those from both the conventional backstepping-like control and uncontrolled system. Besides, it can be seen that during $0 \leq t < 0.5$ s and $t > 0.75$ s when there is no road disturbance, the proposed strategy has the property of rapidly nominal performance recovery. Even if the conventional backstepping-like control and uncontrolled system has also this property, both provide rather undesirable transient control performances. Thus, the presented scheme provides significant improvement in the car body acceleration over both the conventional backstepping-like control and passive suspensions. This is due to an inclusion of the disturbance observer strategy utilized for estimating the inevitably road disturbance.

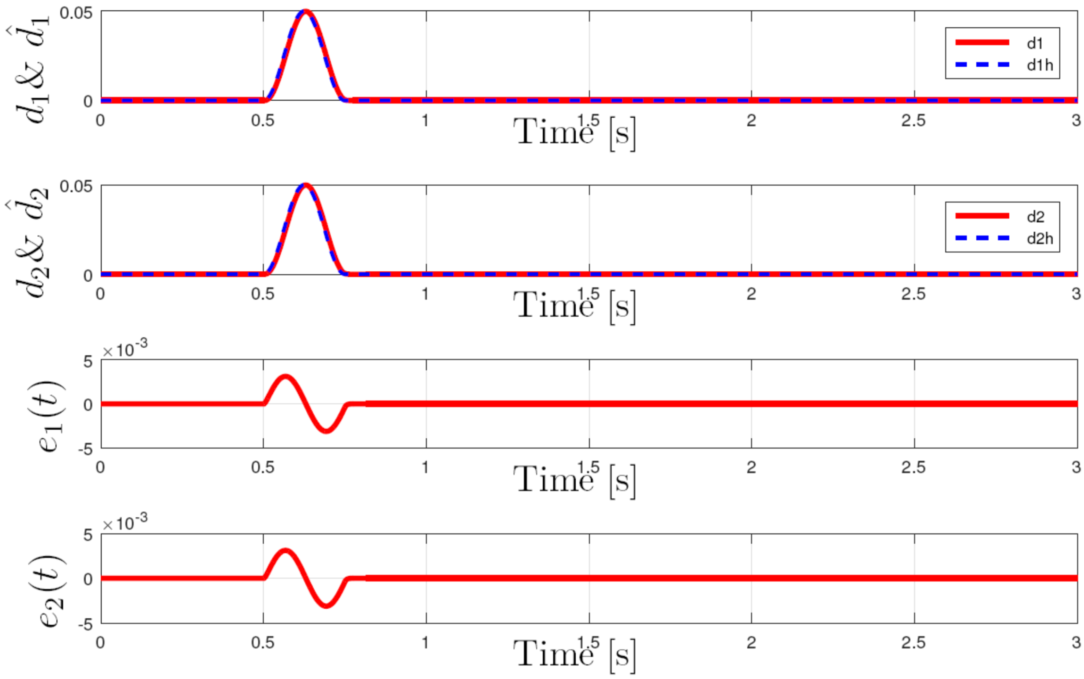

FIGURE 3. Road disturbance input (d_1 & d_2) and disturbance estimation ($\hat{d}_1$ & $\hat{d}_2$)

Figure 3 demonstrates time histories of road input disturbances and the estimate value of disturbances. This indicates that the developed disturbance observer effectively estimates the road disturbances and quick approaches to the disturbances with very fast convergence rates without oscillations. Thus, it can be concluded that the disturbance observer design used has a reference tracking performance because the road disturbance is compensated by the estimated value in the control input signal. Figure 4 shows the control input for the two controllers used to stabilize the closed-loop system. It is also evident that the control energy of the proposed control is clearly less than that of the conventional backstepping-like control.

From the above simulation results, it is evident that a combination of the backstepping-like control and the disturbance observer scheme applied to the active quarter-car suspension system provides the following advantages over the conventional backstepping-like control and passive suspensions.

- The developed design has the potential to improve the ride quality such as much overshoot magnitude reduction in the body acceleration and the body travel.

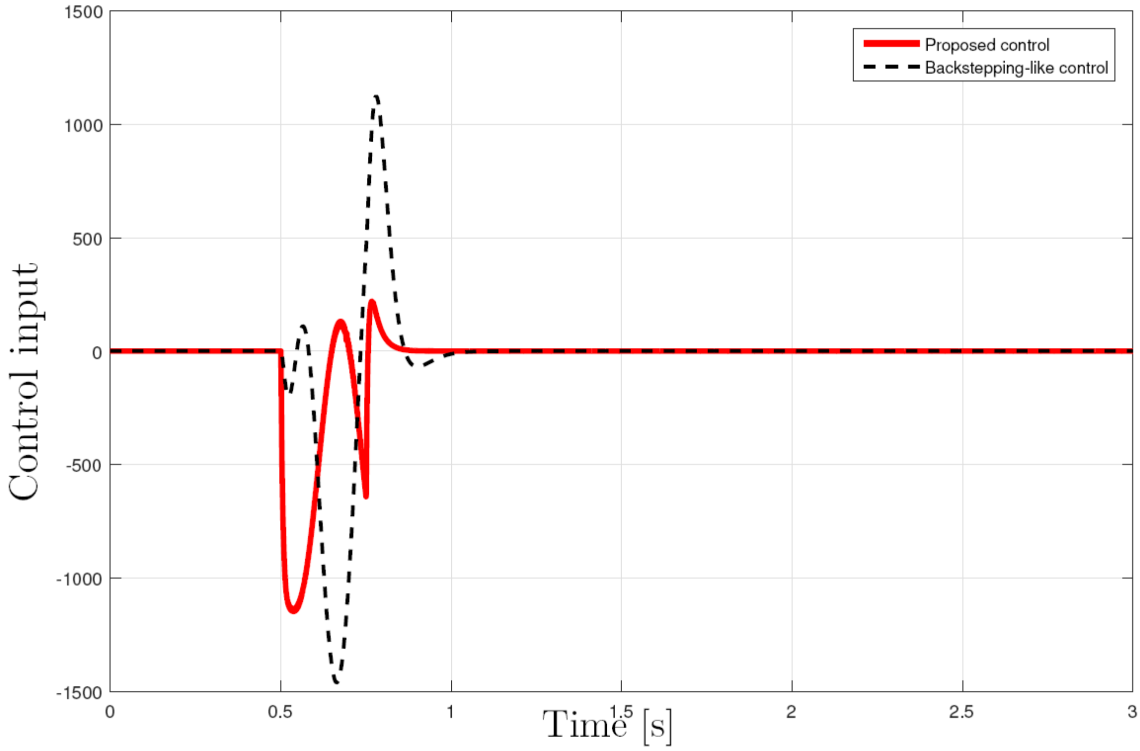

FIGURE 4. Control input ($u(t)$)

- The proposed strategy offers highly satisfactory transient performances and reject effectively unavoidable disturbances.
- The design procedure adds some auxiliary terms into each step of finding the desired controller. These terms are utilized to mitigate the adverse effect arising from the road disturbance. On the other hand, these terms are not included in the backstepping-like method alone, thereby leading to unsatisfactory control performances.

5. **Conclusion.** In this paper, a nonlinear disturbance observer-based backstepping-like control strategy is proposed for active quarter car suspension systems. The developed scheme offers satisfactory disturbance rejection performance and achieves improved dynamic performances. Closed-loop stability analysis with the result of the road disturbance is investigated. To validate the proposed scheme, the simulation results have indicated that the developed control method provides an improved transient performance and capability of rejecting road disturbances rapidly superior to the backstepping-like control and uncontrolled system. In particular, the presented control law is able to suppress rapidly the system oscillations and effectively reject inescapable disturbances. Consequently, it provides an opportunity to significantly improve the ride comfort of the driving passenger. Future study will be devoted to an inclusion of the effect of hydraulic actuator dynamics into our design procedure.

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