2603

A Simplified Approach to Water Influx Calculations–Finite Aquifer Systems

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Introduction

All gas and oil reservoirs are associated to varying extents with formation waters. The inclusion of the effects of expansion or invasion of this water into oil and gas reservoirs has taken many forms, from recognizing the effects of the expansion of the connate water[1] within the gas or oil reservoir itself, to calculating water influx or efflux across a boundary (with the boundary usually being that of an oil or gas reservoir).

There are four currently popular methods used for calculating water influx into reservoirs. They are:

1. Schilthuis, steady state[1-3]
2. Hurst Simplified, unsteady state[1,2]
3. Resistance or Influence Function, unsteady state[4-6]
4. van Everdingen-Hurst Radial, unsteady state[7]

The first three methods have proved useful for predicting water drive performance after sufficient historical data have been obtained to fix the necessary influx constants. With what some consider to be disappointing results,[1,8] the van Everdingen-Hurst Radial method is often used with geological and core data when little or no performance history is available. It has also been used to predict reservoir performance after enough historical data have been accumulated to develop values of the influx constants, t_D and C.

In an attempt to include geometries other than radial, derivations for both limited and infinite systems have been made to cover linear,[7,9,10] spherical,[11] elliptical,[12] thick-sand,[13] and wedge-shaped[18] reservoir-aquifer models.

The many rigorous geometrical representations that have been developed cannot readily handle the effect of interference between reservoirs. Electric analyzer studies of the Smackover Limestone aquifer in Arkansas by Bruce,[14] of the Woodbine aquifer in East Texas by Rumble *et al.*,[15] and of the Ellenberger in West Texas by Moore and Truby[16] have shown that reservoirs sharing a common aquifer can severely interfere with each other, and that, for individual reservoirs in a common aquifer, water drive performance calculations that do not consider interference can be greatly in error.

Mortada[17] developed a mathematical method with which to handle interference in a basically infinite radial aquifer system. The method has been applied to field cases.[18,19] Coats concluded from his own study that, "In predicting the pressure-volume behavior of gas reservoirs situated on the common aquifer the effect of interference from other reservoirs on the common aquifer must be accounted for."

Another aquifer problem more recently presented in the literature[20] is that of flank water injection for pressure maintenance, either to initiate or to supplement edge-water influx. A case history[21] shows that we need to be able to study the effects of injecting water into the aquifer instead of merely including it in the hydrocarbon material balance equation.

This approach to water influx calculations offers a useful and flexible method of forecasting and analyzing the performance of water drive reservoirs. The separation of the water influx problem into a rate equation and a material balance equation, not requiring superposition, makes the concepts and calculations quite simple and easy to apply.

Little wonder that efforts[17, 22, 23] have been made to simplify the water drive performance prediction methods, even to the point frequently of using the infinite solution without trying to define fairly clearly the limits and characteristics of the aquifer.

If we are to predict realistically the performance of water drive reservoirs, then, a simple method must be developed that can *readily* handle all the basic geometries, interference from other reservoirs, and water injection and production from the aquifer; the method should also be flexible enough that it can be further improved or added to as a problem requires.

We shall present here an approach that utilizes the "stabilized", or pseudosteady-state aquifer productivity index and an aquifer material balance to represent the finite compressible system. Much of this has been treated in the literature in the form of solutions to individual well problems and reservoir material balance derivations. For some reason — possibly a concern for the *early* transient effects — any earlier efforts to extend this available technology to aquifer or water drive problems have not been reported.

We hope to develop the idea that this simplified approach is accurate enough for engineering purposes, especially for field production forecasting of times involving some 10 to 20 years, by comparing the PI-Aquifer Material Balance solution with the van Everdingen-Hurst solution through the use of example problems. Solutions mainly involve finding a reasonable rate equation for the problem, and considering the aquifer encroachable water volume represented in the material balance equation as being independent of geometry only to the extent that basic mensuration equations can be applied.

Basic Equations

The generalized rate equation for an aquifer without regard to geometry or defining a specific type of flow is:

$$q_w = J_w (\bar{p} - p_{wf})^n \quad , \quad \ldots \ldots \quad (1)$$

with n usually being represented as unity (1) when the flow obeys Darcy's law and is at pseudosteady state or steady state. J_w is defined as the productivity index (PI) of the aquifer and is analogous to the PI of an oil well or the gas well backpressure curve coefficient.

The aquifer material balance for a constant compressibility can be written in its simplest form as

$$\bar{p} = -\left(\frac{p_i}{W_{ei}}\right) W_e + p_i \quad , \quad \ldots \ldots \quad (2)$$

where $\bar{p}$ is the average aquifer pressure (shut-in), W_{ei} is the initial encroachable water in place at initial pressure p_i, and W_e is the cumulative water efflux from the aquifer or influx into a reservoir.

By combining Eqs. 1 and 2 (see Appendices A and B for complete derivation), we can obtain the equation expressing the instantaneous rate of water influx as a function of time, and the inner boundary pressure p_{wf}.

$$e_w(t) = \frac{J_w (p_i - p_{wf})}{e^{[(q_{wi})_{max}/W_{ei}]t}} . \quad \ldots \ldots \quad (3)$$

$(q_{wi})_{max}$ is defined as the initial open-flow potential of the aquifer, again analogous to the open-flow potential of an oil well or of a gas well. Fig. 1 is a graphical representation of the generalized rate equation expressed as Eq. 1 and the aquifer open-flow potential described above. Note that if we let W_{ei} become large, Eq. 3 reduces to the Schilthuis steady-state equation

$$e_w = J_w (p_i - p_{wf}) \quad \ldots \ldots \quad (4)$$

The final form of the cumulative water influx equation (given also in Appendix B)

$$W_e = \frac{W_{ei}}{p_i} (p_i - p_{wf}) \{1 - e^{-[(q_{wi})_{max}/W_{ei}]t}\} \quad \ldots \ldots \quad (5)$$

is not useful by itself because it cannot handle a changing inner boundary pressure p_{wf} while representing the aquifer pressure always at its initial value. Hurst[24] and others have handled this problem by the method of superposition.

We can rewrite the equation to represent the cumulative water influx over an interval of time Δt, then start the problem again after every time interval (as can be done for any material balance problem). With the aid of the aquifer material balance equation, we can redetermine a new aquifer shut-in pressure $\bar{p}_n$, then solve over a new time interval Δt. This re-evaluation of the aquifer shut-in pressure each time eliminates the need for superposition.

A significant point here is that we need not always go back to the initial pressure to start a water influx calculation. We can conveniently start it at any time provided we can obtain a value to represent the aquifer shut-in pressure.

The interval equation is

$$\Delta W_{en} = \frac{W_{ei}}{p_i} [\bar{p}_{(n-1)} - \bar{p}_{wfn}] \bullet \{1 - e^{-[(q_{wi})_{max}/W_{ei}]\Delta tn}\} \quad \ldots \quad (6)$$

The ratios W_{ei}/p_i and $(q_{wi})_{max}/W_{ei}$ can be further simplified to eliminate p_i from the expressions, which then do not need to be initiated again to new aquifer shut-in pressures. These forms are retained so as to keep their physical meanings.

The time interval is determined by

$$\Delta tn = t_n - t_{(n-1)} \quad ; \quad \ldots \ldots \quad (7)$$

and the average pressure

$$\bar{p}_{wfn} = \frac{p_{wf(n-1)} + p_{wf(n)}}{2} \quad \ldots \ldots \quad (8)$$

represents the constant pressure used at the reservoir-aquifer boundary during the time interval Δtn. Fig. 2 depicts this pressure-time relationship and the step curve that attempts to approximate it. This method of representing the average pressure, $\bar{p}_{wfn}$, is applicable to both past and future performance predictions.

To start the calculation again for the aquifer shut-in pressure $\bar{p}$, we will make use of the general aquifer material balance equation derived in Appendix B.

$$\bar{p} = -\left[\frac{W_e + \sum_{2}^{j} W_{ej} + (W_p - W_i)\,B_w}{W_{ei}}\right] p_i + p_i, \quad (9)$$

where $W_e = \sum_{1}^{n} W_{en}$, the total cumulative influx (to time tn) into the reservoir of interest. The term $\sum_{2}^{j} W_{ej}$ is the total cumulative influx into other reservoirs within the common aquifer and is further discussed under Aquifer Interference. All other terms have the conventional definition or have previously been defined.

The realistic water influx rate and cumulative water influx relationship during an interval of time Δt is depicted in Fig. 3 along with that which results from using a step-function constant pressure as an approximation in any water influx instantaneous rate equation.

Step-Function Solutions

It now appears that the simplification of the water influx problem is still none too simple. In reality, though, we have reduced the problem so that we can recognize that a simple time-incremented step-function solution using the rate equation $q_w = J_w(\bar{p} - p_{wf})$ to establish a constant rate over a time interval, and the aquifer material balance equation $\bar{p} = -\left(\frac{p_i}{W_{ei}}\right) W_e + p_i$ to evaluate the aquifer shut-in pressure after efflux from the aquifer, will give the analytical solutions to the problem when Δt is allowed to become small. A Δt of a month in a normal reservoir problem does reproduce these analytical solutions. (Constant rate steps over a Δt of 1 year for all cases of $r_a/r_r \geq 5$ reported in this study gave results identical with those obtained using Eq. 6.) Fig. 4 illustrates this straightforward step-function approach.

For a time interval Δtn, from $t_{(n-1)}$ to t_n, the working equation for the rate equation would be

$$q_w = J_w(\bar{p}_{(n-1)} - \bar{p}_{wfn}) \quad (10)$$

The cumulative efflux during the time interval Δtn would be

$$\Delta W_{en} = \Delta tn(q_w), \quad (11)$$

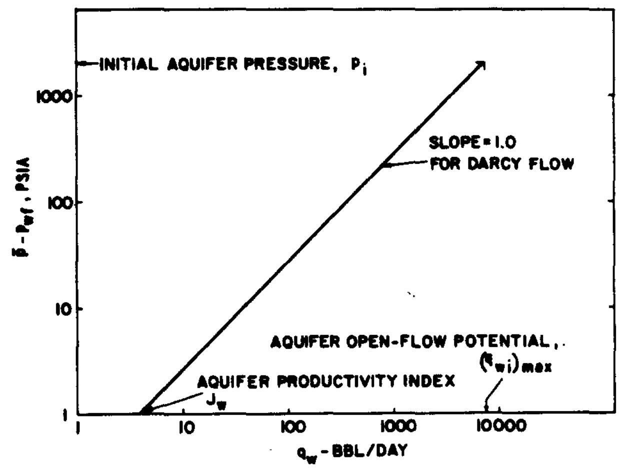

Fig. 1—Aquifer "backpressure curve" with open-flow potential, log-log plot.

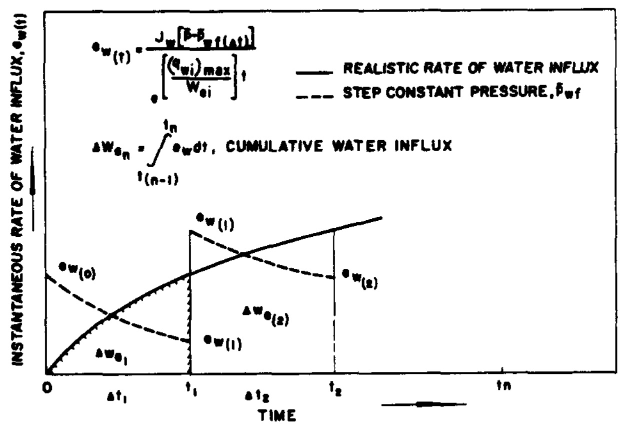

Fig. 3—Calculated rate of water influx using a step-constant pressure at the aquifer inner boundary compared with a realistic representation.

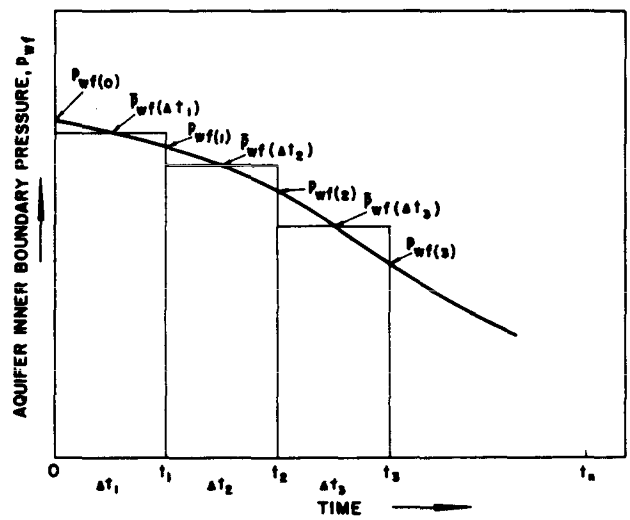

Fig. 2—Pressure-time relationship at aquifer inner boundary as a step-function approximation.

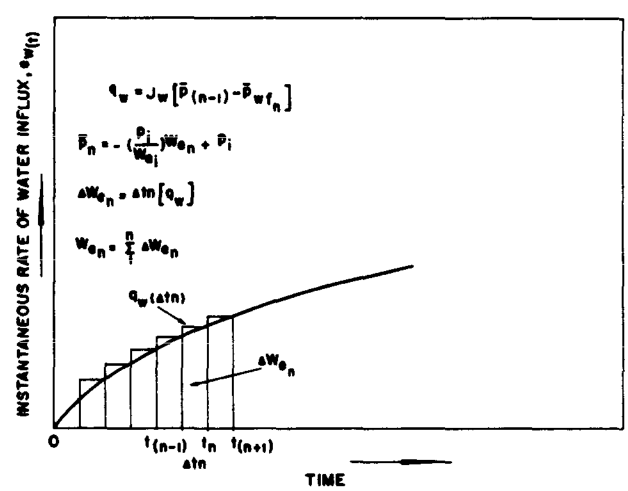

Fig. 4—A constant rate step-function approximation to water influx over short time intervals.

and the total cumulative efflux to time *tn* would be

$$W_{en} = \sum_{1}^{n} \Delta W_{en} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Then to update the aquifer average pressure for the next time interval,

$$\bar{p}_n = -\left(\frac{p_i}{W_{ei}}\right) W_{en} + p_i \quad . \quad . \quad . \quad . \quad (13)$$

Rate Equations

In all derivation methods that attempt to predict water influx and that assume a constant compressibility, it is necessary to start with the same volume of initial encroachable water in place for a given set of variables. Therefore, to predict water influx accurately with the PI-Aquifer Material Balance approach, we need only find a suitable rate equation.

Aquifer Productivity Index J_w

The aquifer productivity index J_w values used in this study were calculated from a stabilized backpressure equation for finite radial flow conditions ($\theta = 360°$). The early transient period was neglected. For the finite, slightly compressible, radial aquifers studied, we used the "stabilized" pseudosteady-state rate equation:

$$q_w = \frac{7.08\, kh\, (\bar{p} - p_{wf})}{\mu \left[\ln\left(\frac{r_a}{r_r}\right) - \frac{3}{4} \right]} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

We have then a productivity index for radial "stabilized" flow

$$J_w = \frac{7.08\, kh}{\mu \left[\ln\left(\frac{r_a}{r_r}\right) - \frac{3}{4} \right]} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The initial aquifer potential, $(q_{wi})_{max}$, then is

$$(q_{wi})_{max} = J_w\,(p_i - 0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The initial encroachable water in place, W_{ei}, for radial geometry ($\theta = 360°$) is determined by

$$W_{ei} = \frac{\pi}{5.61}\,(r_a^2 - r_r^2)\,\theta h\, c_t\, p_i \quad . \quad . \quad . \quad (17)$$

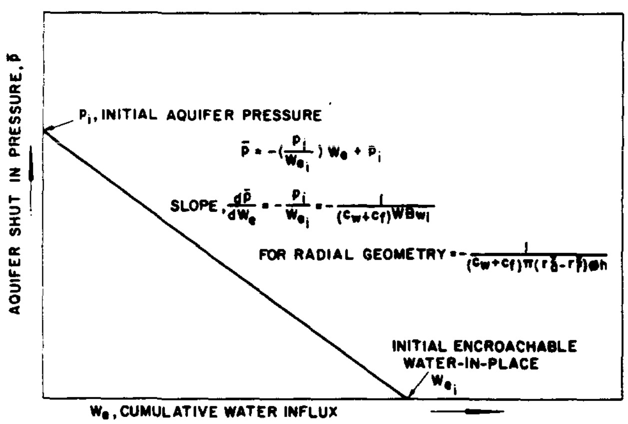

Fig. 5—Graphical representation of the aquifer material balance equation.

Table 4 summarizes the rate equations from which PI can be calculated for finite radial and linear systems for pseudostady-state and steady-state conditions. Also included in this table are the unsteady-state equations for radial and linear transient flow that can be used for a system that does not reach pseudosteady state or steady state during the period of interest (see Fig. 20). Note that the infinite radial flow equation given in Table 4 is nothing more than the Hurst Simplified water influx equation defined.

As in individual well problems, we could also introduce the concept of skin into the equations to allow theory to fit the observed data. Where changing the internal aquifer radius, r_r, would also cause a change in the aquifer volume, W_{ei}, the concept of skin would allow us to vary PI without changing r_r. This would take on special significance if we attempt to match historical influx data from a best combination of J_w-W_{ei} while trying to conform with the existing geometry of the system.

As a guide to the times at which pseudosteady state and steady state are reached in a radial system, we can use this equation[1] for pseudosteady state:

$$t_{ps} \cong \frac{0.02\, \mu\, c_t\, \phi\, r_a^2}{k} \; ; \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and this equation[1] for steady state:

$$t_s \cong \frac{0.04\, \mu\, c_t\, \phi\, r_a^2}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The equations for a linear system could be derived like those for the radial system. All units in the above equations are in terms of days, centipoises, psi^{-1}, feet, and darcies.

In estimating times, we must remember to consider the drainage boundaries that are established when there is interference from other reservoirs in the same aquifer.

Selection of Rate Equations

Fig. 21 lists some possible types of aquifer flow systems that could be used as a guide in selecting appropriate rate equations. Many problems can be expressed in terms of essentially linear or radial flow.

Fig. 21a describes a flow system that is obviously linear but whose distances between sealing faults describe the cross-sectional area to be used with the aquifer rate equation. In water influx calculations we are trying to describe the flow in the aquifer itself. The cross-sectional area at the aquifer-reservoir boundary is not necessarily applicable, especially after pseudosteady state or steady state has been established.

Fig. 21b describes flow in a long, narrow reservoir. That this type of flow could be classed as linear has been demonstrated by Havlena and Odeh[25] from an analysis of a gas reservoir 11 miles long and 1.5 miles wide. Their analysis, using the material balance as an equation of a straight line, indicated that the influx rate was proportional to the square root of time.

Fig. 21c is an extension of the concept developed by Fig. 21b but in an additional dimension. Bottom-water drive in a long, narrow field could be better approximated by radial flow in the vertical direction, with h being the length of the reservoir.

Fig. 21d would be represented by most engineers as a radial flow system of 180°, using a radius to flow equivalent to r_r. However, by redefining the system to consider the dashed lines to be the new boundaries and by treating the volume of water between the fault and the actual reservoir boundary as a part of the reservoir (so that the expansion of this portion of the aquifer would take place with no resistance to flow) we can readily see that it is, for practical purposes, a linear flow situation. This approach should give an optimistic answer, but not so optimistic as it would be if the problem were treated as a radial flow system.

Fig. 21e illustrates a reservoir located between two parallel sealing faults that terminate in a large aquifer. Flow into the reservoir would be linear, and there would be an essentially constant pressure at the outer boundary. This would require a steady-state approximation with the productivity index, J_w, being a function of the length of the sealing faults and the distance between.

Fig. 21f depicts a wedge sand. Solutions to this problem have been reported in the literature[9,13] in terms of an extension of linear flow. Turned on end, it can also be treated as radial flow, with an angle θ and the width represented by the distance h.

These illustrations are given only as a guide to show that many reservoir-aquifer systems can be defined in terms of radial or linear flow. Both the simplified method and the van Everdingen-Hurst solutions are applicable if we view the problems in terms of finding the proper representation of a rate equation. However, the simplified method allows us to use different dimensions or geometries when defining the aquifer productivity index and the aquifer volume for a given problem.

Aquifer Interference

By separating the water influx problem as we have into a rate equation and a material balance equation, we can examine each individually as to its effect on interference. Consider an aquifer of radius r containing two similar fields, A and B (they need not be similar when applying the simplified method). We assume Field A has been producing long enough to reach steady state. Let the productivity index of Field A be $J_w = f(r)$. When Field B begins producing, the productivity index of Field A will *increase*, becoming $J_w = f(r/2)$. From the standpoint of the rate equation, the deliverability of the aquifer for Field A will be *increased* after Field B begins producing. As pointed out by Bruce[14] in his study of the Smackover aquifer, the interference effect is totally one of "competition among pools for the common water supply".

From the aquifer material balance standpoint, Field A would initially have an aquifer volume of W_{ei} bbl available to it for water influx. However, after Field B begins producing, the aquifer drainage volume available to Field A is reduced. It can be approximated by the basic relationship given by Matthews *et al.*[26] — "at (pseudo) steady state the drainage volumes in a bounded reservoir are proportional to the rates of withdrawal from each drainage volume."

$$W_{eiA}(t) = \frac{J_{wA}(\bar{p} - p_{wfA})}{J_{wA}(\bar{p} - p_{wfA}) + J_{wB}(\bar{p} - p_{wfB})} \cdot [W_{ei} - W_{eA}(t)] \, , \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where

$W_{eiA}(t)$ = encroachable water in place available to Field A at time t.

If for simplicity we are assuming equal inner boundary pressures and equal PI values for Fields A and B (equal influx rates),

$$W_{eiA}(t) = \frac{W_{ei}(t)}{2} \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

after Field B starts production and reaches pseudo-steady state. The transient time will now be shorter than the transient period of Field A producing alone.

In our aquifer material balance equation, the interference term for other reservoirs with respect to a given resevoir is given by the summation term $\sum_2^j W_{ej}$, which represents the sum of the cumulative influx into all other reservoirs in the common aquifer. This results in additional depletion, or decline in the average pressure of the common aquifer as a result of these fields' also having water influx.

The expanded expression is more easily visualized in the time-incremented step-function approach for a time interval Δt. The cumulative influx into all reservoirs from Fields 2 to j (the field of interest is Field 1) is

$$\Delta W_e(\Delta t) = J_{w(2)}[\bar{p} - p_{wf(2)}]\Delta t + J_{w(3)}[\bar{p} - p_{wf(3)}]\Delta t + \ldots + J_{w(j)}[\bar{p} - p_{wf(j)}]\Delta t \, . \quad . \quad . \quad (22)$$

Also, when handling the problem from a time-incremented standpoint, we could even, for completeness, include to some extent the change in compressibility of the total system by allowing each field, including all reservoirs within the common aquifer, to contribute to the total compressibility:

$$c_t = S_o c_o + S_g c_g + S_w c_w + c_f \, . \quad . \quad . \quad . \quad (23)$$

If we include all except the reservoir of interest (Reservoir 1) this becomes

$$c_t = \sum_2^j \left(\frac{N B_o c_o + G B_g c_g}{V_p} \right)_j + S_w c_w + c_f \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where V_p is the total pore volume of the aquifer and nonproducing fields. Muskat[27] points out that the indicated abnormally high compressibility, $c_t = 36 \times 10^{-6}$ psi^{-1} of the East Texas Woodbine aquifer could be due to gas fields or gas caps of oil fields distributed in the aquifer.

If we do not wish to include the compressibility of the other reservoirs within the aquifer, Eq. 24 reduces to the simple expression

$$c_t = c_w + c_f \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Water Injection into the Aquifer

The usual method of treating water injection for studying pressure maintenance is to include a water injection term in the hydrocarbon material balance equation. A form of the material balance equation for a gas reservoir is

$$G_pB_g = G(B_g - B_{gi}) + W_e + B_w(W_i - W_p) \quad (26)$$

The basic assumption here with respect to water injection is that all water injected is instantly available to the reservoir, which would be realistic if the water was injected uniformly throughout the reservoirs as in pattern waterflooding. However, when the purpose is to maintain pressure, we generally use a flank water injection, with the injection wells located in the aquifer.

A more realistic approach is to include a water injection term in the aquifer material balance equation so as to incorporate the effects of the resistance to flow across the reservoir-aquifer boundary. For high-permeability boundaries, the results would be essentially the same. However, where the permeability at the boundary is low, over a realistic time period little or no water may enter the reservoir. The option should be available, at least, to study it both ways or in combination. Eq. 9 includes water injection into the aquifer in such a manner that the total water influx, W_e is also a function of the water injected, $W_e = f(W_i)$.

In an interesting case history[21] of the Pegasus Ellenburger reservoir we are told of an attempt to maintain pressure by using flank water injection to supplement edgewater influx. The peripheral project failed to maintain pressure, resulting in very high pressures around the injection wells. Injection into the central producing area was required to halt the pressure decline. The water influx constants from the edgewater drive were established before water injection was begun. The PI-Aquifer Material Balance approach would have been more successful in predicting the final outcome.

Historical Data

There are two differing treatments of historical data from reservoirs subject to water drive. They are usually referred to as

1. The Material Balance as an Equation of a Straight Line,[25] and
2. The Resistance or Influence Function.[4,6]

They differ mainly in their primary objectives. The straight-line approach attempts to determine the original gas or oil in place using the historical data, whereas the resistance or influence-function approach fixes a best estimate of gas or oil in place and then attempts to determine a best fit of the data to arrive at a resistance or influence function $F(t)$ with which to predict future performance.

When the objective is to determine recoverable reserves, a precise value for oil or gas originally in place may not be justified because of the inaccuracies involved in arriving at reliable values for residual gas or oil saturation and sweep efficiency. If, however, in determining original in-place values the resulting influx coefficients, C and t_D, are to be used to make future performance predictions (reservoir pressures and producing rates) the two treatments will accomplish the same thing.

With the simplified procedure, where the problem has been separated into its two basic components — productivity index and aquifer material balance — we can approach the problem the way we would approach it to determine the resistance or influence function. For a gas reservoir:

1. We can fix a best estimate of gas in place, G.
2. Using the incremental form of the reservoir material balance equation for a time interval Δtn and two historical reservoir pressures, p_{wfn} and $p_{wf(n-1)}$, we can solve for a water influx volume

$$\Delta W_{en} = \Delta(G_pB_g) - G\Delta B_g + \Delta(W_pB_w) \quad (27)$$

Then the average influx rate during the time interval is

$$\bar{e}_w(\Delta tn) = \frac{\Delta W_{en}}{\Delta tn}, \quad (28)$$

which is represented at time $\frac{t_n + t_{n-1}}{2}$ (29)

3. We can plot the average influx rate $\bar{e}_w(\Delta tn)$ as a function of time.
4. We can calculate water influx rates as functions of time, using various combined values of aquifer productivity index and encroachable water in place. These rates of water influx are plotted with those calculated using the material balance equation.
5. We can select the best combination of $J_w - W_{ei}$ to fit the problem. Although a statistical approach

TABLE 1—HYPOTHETICAL GAS RESERVOIR AND AQUIFER PROPERTIES.

Gas Reservoir Properties	
Initial gas reservoir pressure, psia	2,000
Porosity, fraction	0.20
Pay thickness, ft	100
Water saturation, fraction PV	0.20
Initial formation volume factor for gas, scf/reservoir cf	154.26
Reservoir radius, ft	10,000
Gas gravity (to air)	0.700
Pseudo critical temperature for gas, °R	392
Pseudo critical pressure for gas, psia	668
Reservoir depth, ft	7,000
Reservoir temperature, °F	130
Initial gas-law deviation factor	0.780
Initial gas in place, Bcf	776
Rate of take (1 MMcf/D to 8.59 Bcf gas in place), Mscf/D	90,338
Total field wellhead potential, Mscf/D	250,000
Initial wellhead shut-in pressure, psia	1,600
Slope of wellhead backpressure curve	0.700
Line pressure, psia	200
Aquifer Properties	
Initial pressure in aquifer, psia	2,000
Permeability, md	10, 50, 100, 1,000
r_a/r_r	3, 5, 7, 10
r_a (using r_r = 10,000 ft), thousands of ft	30, 50, 70, 100
Porosity, fraction	0.20
Aquifer thickness, ft	100
Total compressibility for aquifer, 1/psi	6×10^{-6}
Viscosity of water, cp	0.50

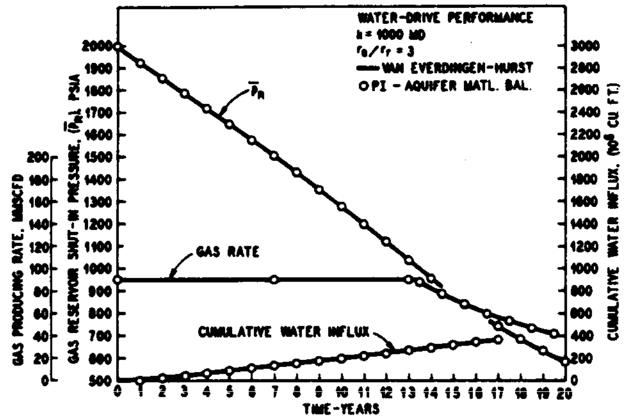

Fig. 6

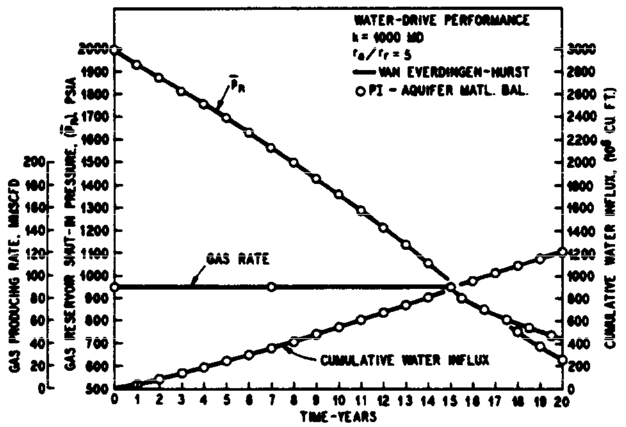

Fig. 7

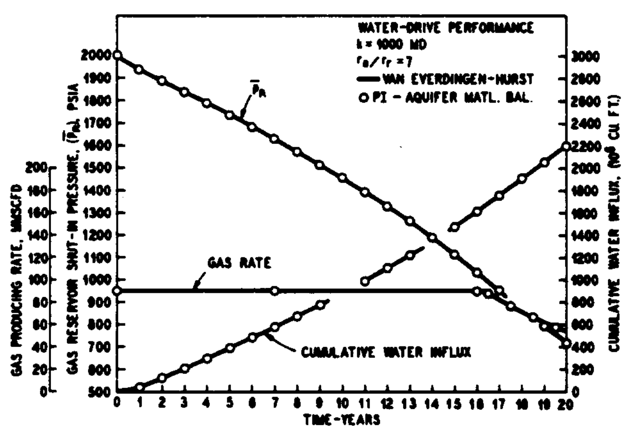

Fig. 8

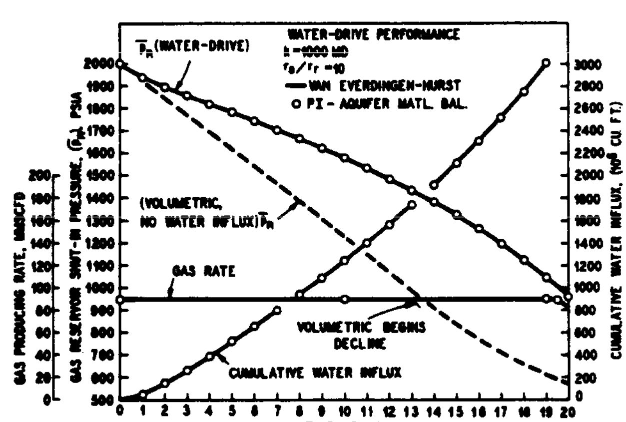

Fig. 9

could be used to make a selection, an engineer's analysis based on intimate knowledge of each data point and field history would be preferable.

A good starting point for J_w and W_{ei} should be based on the basic geometry of the reservoir-aquifer system being studied. For a strictly radial geometry the productivity index of the aquifer, J_w, the water in place, W_{ei}, and the original gas in place, G, are all functions of common variables in that $J_w = f(\ln r_r, \ln r_a)$; $W_{ei} = f(r_r^2, r_a^2)$; and $G = f(r_r^2)$. If aquifer interference occurs at a later time, J_w will change as a result of a change in drainage radius r_a, but only as the $\ln r_a(t)$, whereas the water in place $W_{ei}(t)$ will change as the square of $r_a(t)$, with the gas in place remaining the same. Therefore it is possible to have more than one value of water in place as a solution during the producing life of a field.

During the early times, before pseudosteady state is established, the aquifer productivity index, $J_w(t)$ plotted vs the $\ln t$ and $\sqrt{t}$ for radial and linear flow, respectively, should be straight lines.[4,6] No fixed value of J_w and $W_{ei}(t)$ exists during the early transient period.

Discussion of Results

The method chosen with which to illustrate a comparison between the PI-Aquifer Material Balance approach and the more rigorous solutions of van Everdingen-Hurst is a hypothetical gas reservoir surrounded by a finite radial aquifer. Using a gas reservoir does not require the introduction of variables such as k_g/k_o relationships that may later be suspected of contributing to some of the basic responses shown by the water drive performance.

The properties used for the gas reservoir and aquifer are listed in Table 1. So that the effect of the early transient period could be investigated, we chose a range of permeabilities and external radii of the aquifer. In each case, the aquifer inner boundary pressure was represented by the average pressure determined from the solution of the gas reservoir material balance. Values used for water viscosity and the total compressibility are typical of those often used in the literature for water influx calculations.

A typical gas withdrawal rate of take of 1 MMcf/D to 8.59 Bcf in place (1 MMcf/D to 7.3 Bcf recoverable with an 85-percent recovery factor) was used so as to obtain realistic water influx values. A more rapid gas withdrawal rate would result in less water influx for the same reservoir and aquifer properties used in this study. No attempt has been made to determine recoverable reserves at abandonment based on residual gas and sweep efficiencies. This could be handled, however, by the methods suggested by Agarwal *et al.*[28] All forecasts are carried out for a full period of 20 years, that time defined by a 1-to-7.3 rate of take. A constant field wellhead potential for the gas reservoir was used for all cases.

Figs. 6 through 10 illustrate the water drive performance for an aquifer with a permeability of 1,000 md at four different external aquifer radii — 30,000, 50,000, 70,000 and 100,000 ft (19 miles). In all cases, the PI-Aquifer Material Balance solutions match identically the gas producing rates, reservoir pressure, and cumulative water influx determined using the van Everdingen-Hurst solutions.

The simplified approach does not utilize superposition, whereas the van Everdingen-Hurst solution does. To investigate the effects of superposition when producing rates are varied severely, a variable producing rate situation was studied (Fig. 10). This was done for the largest aquifer radius. Excellent agreement was obtained for this 1,000-md permeability case.

Figs. 11 through 14 illustrate the water drive performance when the aquifer permeability is changed to 100 md. In these cases, the departure from the van Everdingen-Hurst solutions is quite small with respect to reservoir pressure and cumulative water influx and is well within engineering accuracy. The gas producing rates are identical.

Fig. 12 includes the additional points representing results using the van Everdingen-Hurst radial infinite solution. After early times, their solution departs from the $r_a/r_r = 10$ case about as much above the line as the simplified does below. What is interesting here is that within the limits of field data, it would be difficult to determine the actual extent of the aquifer. That is, we could easily maintain that the performance data indicates an infinite radial aquifer. There would be enough room to adjust the internal boundary pressures to force a fit to an infinite solution.

Fig. 14, showing the performance of the aquifer, illustrates why the cumulative water influx as calculated by the PI-Aquifer Material Balance method departs constantly from that calculated by the van Everdingen-Hurst method. The departure results, not unexpectedly, from a difference in influx rates during the early transient period. After this period, the influx rates agree quite well.

Figs. 15 and 16 illustrate the water drive performance for an aquifer of 50-md permeability. The departure of the cumulative water influx from that derived by the van Everdingen-Hurst solution is most pronounced for the aquifer-to-reservoir ratio of $r_a/r_r = 10$, an aquifer external radius of 100,000 ft. The constant departure indicates that the difference occurs as a result of the early transient period, as shown in Fig. 14. Still, the reservoir pressure and gas producing rate agree quite well.

Figs. 17 through 20 give the water drive perform-

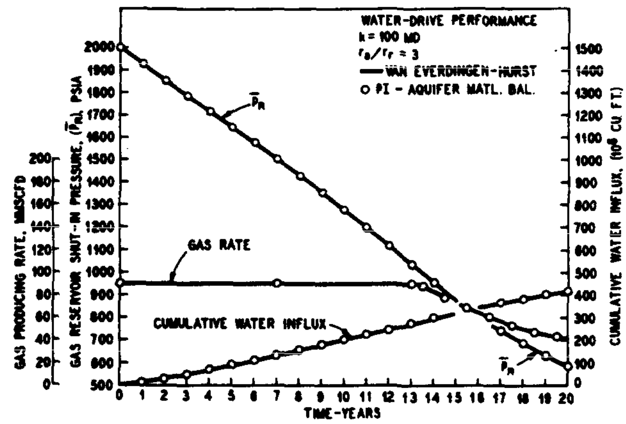

Fig. 11

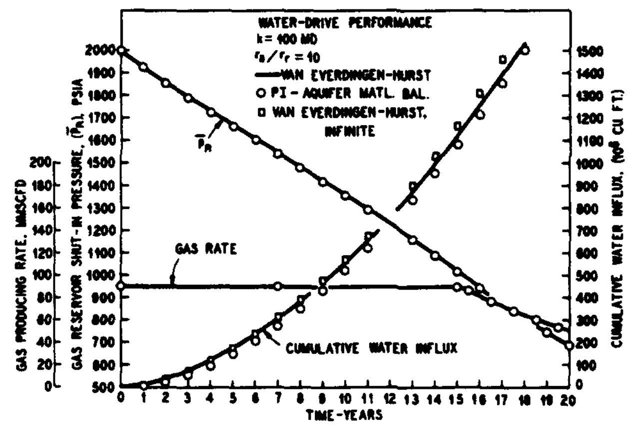

Fig. 12

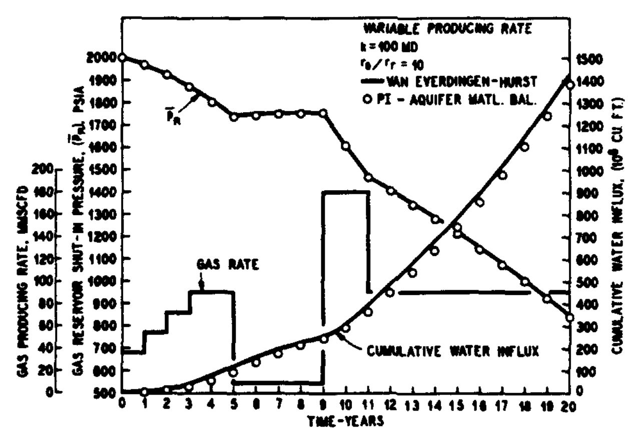

Fig. 13

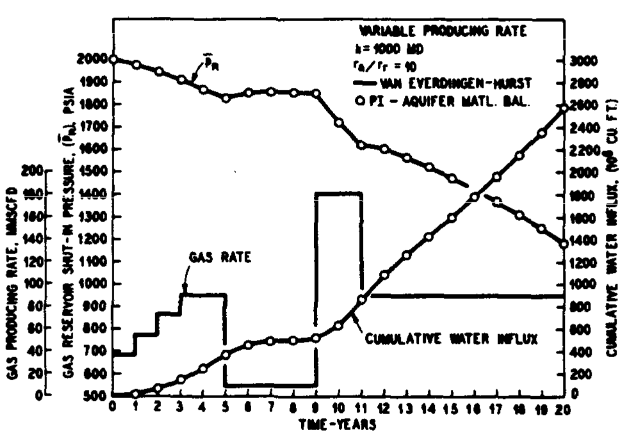

Fig. 10

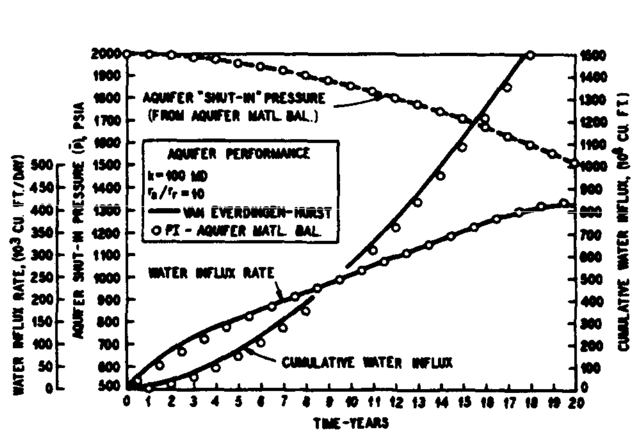

Fig. 14

ance using a 10-md aquifer permeability. In Fig. 17, $r_a/r_r = 3$, the cumulative water influx for the PI-Aquifer Material Balance solution is always greater, indicating that the van Everdingen-Hurst solution was dominated by linear flow, resulting in a lower influx rate than the radial flow determination.

Fig. 19, $r_a/r_r = 7$, shows a continuously increasing departure of the cumulative water influx as a result of transient flow effects throughout. In these cases as in all previous cases, the gas producing rates agree. In all the 10-md aquifer permeability cases, the gas reservoir is behaving essentially as a volumetric reservoir.

From a check of the time it takes to establish pseudosteady state, it was found that the productivity index representing the fixed dimensions of 100,000 ft for r_a of the aquifer, could not become established during the 20-year period of the forecast. Therefore, the Hurst-Simplified (Defined) equation given in Table 4 was used. The results obtained using this equation are quite good (see Fig. 20). As in individual well forecasts, the Hurst-Simplified (Defined) equation could be used until pseudosteady state is established; then, after applying the material balance equation to determine the aquifer shut-in pressure, we could use the pseudosteady-state rate equation for the rest of the forecast.

Because the results presented in this study were based on finite aquifer systems, it would be appropriate to discuss briefly the terms "finite" and "infinite"

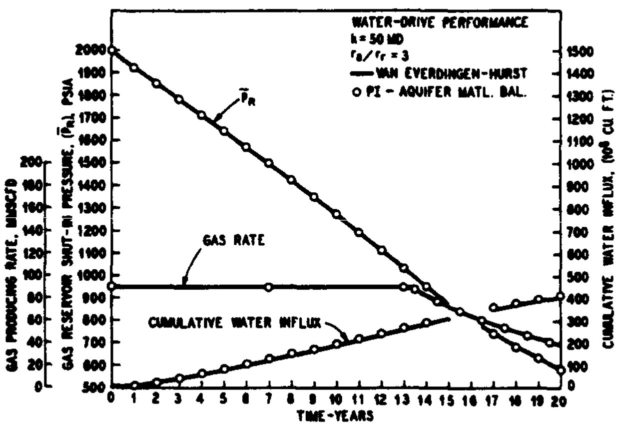

Fig. 15

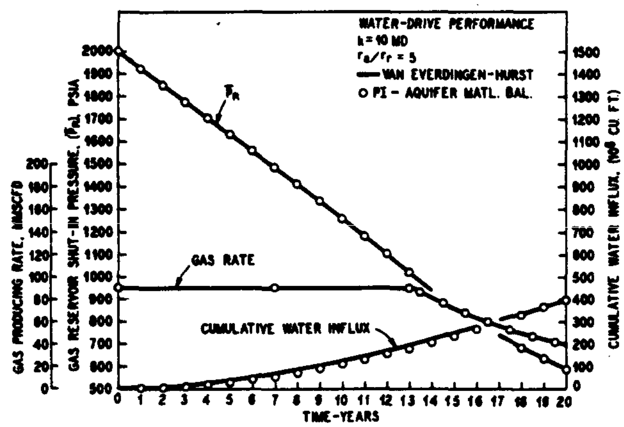

Fig. 18

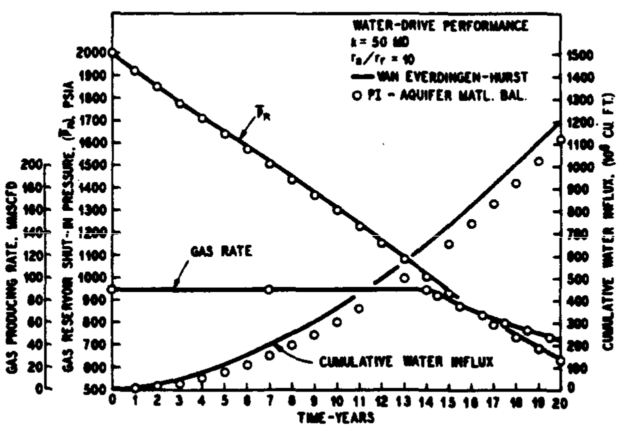

Fig. 16

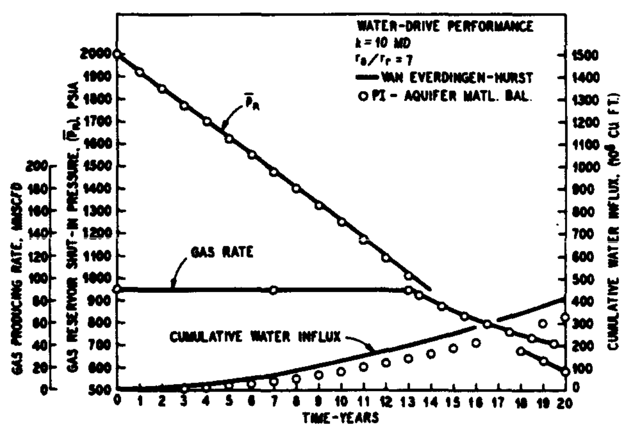

Fig. 19

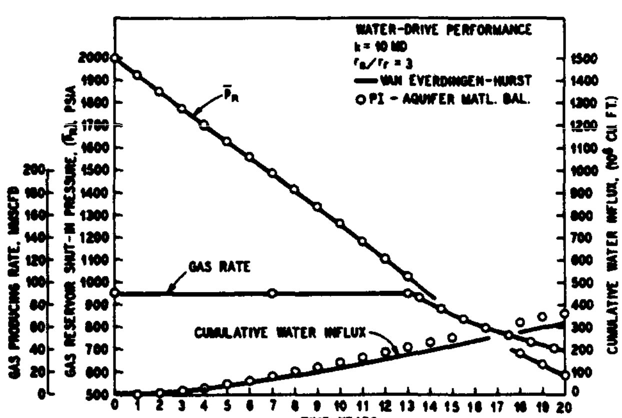

Fig. 17

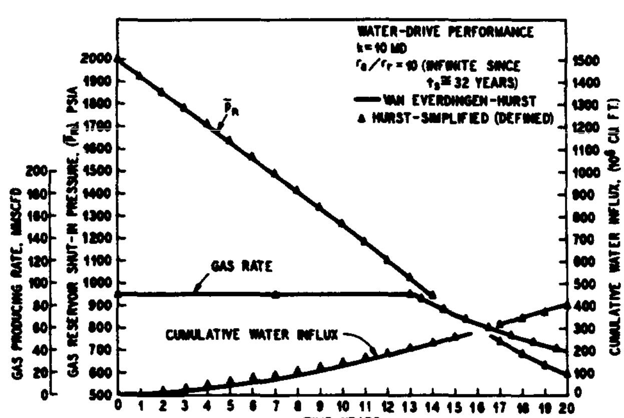

Fig. 20

TABLE 2—RADIAL FLOW WATER INFLUX VARIABLES USED FOR THE PI-AQUIFER MATERIAL BALANCE SOLUTIONS FOR A 20-YEAR FORECAST OF THE HYPOTHETICAL GAS RESERVIOR

			$k = 0.010$ darcy (pseudosteady state)			$k = 0.050$ darcy (pseudosteady state)		
r_e/r_r	Radius of Aquifer (ft)	Initial Encroach-able Water in Place (10^6 bbl)	J_w (B/D/psi)	Initial Potential (B/D)	Stabili-zation Time (years)	J_w (B/D/psi)	Initial Potential (B/D)	Stabili-zation Time (years)
3	30,000	107.52	40.620	81,240	3.0	203.10	406,200	0.6
5	50,000	322.56	16.477	32,954	8.2	82.385	164,770	1.6
7	70,000	645.12	11.840	23,680	16.1	59.200	118,400	3.2
10	100,000	1,330.56	9.1202	18,240	32.9*	45.601	91,202	6.6

			$k = 0.100$ darcy (pseudosteady state)			$k = 1.000$ darcy (pseudosteady state)		
r_e/r_r	Radius of Aquifer (ft)	Initial Encroach-able Water in Place (10^6 bbl)	J_w (B/D/psi)	Initial Potential (B/D)	Stabili-zation Time (years)	J_w (B/D/psi)	Initial Potential (B/D)	Stabili-zation Time (years)
3	30,000	107.52	406.20	812,400	0.3	4,062.0	8,124,000	0.03
5	50,000	322.56	164.77	329,540	0.8	1,647.7	3,295,400	0.08
7	70,000	645.12	118.40	236,800	1.6	1,184.0	2,368,000	0.16
10	100,000	1,330.56	91.202	182,400	3.3	912.02	1,824,000	0.33

*Stabilization time for this value of J_w exceeds duration of forecast.

as applied to water influx problems. "Finite", as used in this study, indicates only that finite dimensions were used for defining the aquifer productivity index, J_w, and the aquifer volume, W_{ei}. "Infinite", when applied to an aquifer, can take on at least three different meanings.

1. The aquifer volume W_{ei} is very large (infinite). This can result in a Schilthuis steady-state aquifer behavior.

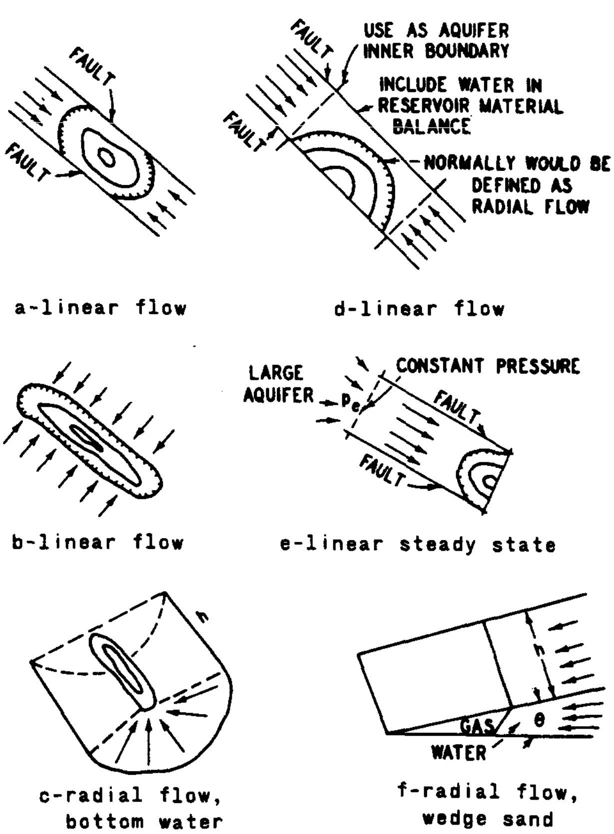

Fig. 21—Types of flow systems for rate equation.

2. The deliverability or productivity index, J_w, is very large (infinite). As a special case of water influx, an infinite productivity index is always assumed when the expansion of the water within the hydrocarbon reservoir itself is included in the reservoir material balance equation by the addition of a water compressibility term.

3. Transient flow exists during the entire period of interest, with the result that an infinite *solution* is applicable.

For the studies involving the largest aquifer radius used — 100,000 ft — the 10-md aquifer permeability case was the only one that could be classed as infinite — and then only because the infinite *solution* could be applied. Its volume of water influx was so insignificant as to cause the gas reservoir to behave like a volumetric reservoir. The 100-md case response as a finite aquifer (even with no transients being considered for the simplified solution) was such that it *appeared* to behave like an infinite aquifer solution (see Fig. 12). The term "infinite" when applied to water influx problems should always be qualified as to which of the above definitions is meant.

In review, the good results obtained with the PI-Aquifer Material Balance approach are suprising when we consider that the additional flow contributions from the early transient period have been omitted and that there exists the condition $r_r << r_a$, imposed in the derivation of the pseudosteady state radial flow equation. Variations of the constant in the term $[\ln (r_a/r_r) - 3/4]$ were studied by using $-1/2$ and -1, as well as some of the other suggested methods of expressing the inner boundary pressure, p_{wf}. In all cases, the results obtained were significantly poorer than those reported in this study.

Certainly in many cases the additional capabilities of the PI-Aquifer Material Balance, when properly included instead of omitted, can far outweigh any early transient effects omitted. In many cases where the transient is of long duration, as for the 10-md

TABLE 3—RADIAL FLOW WATER INFLUX VARIABLES USED FOR THE VAN EVERDINGEN-HURST SOLUTIONS FOR A 20-YEAR FORECAST OF THE HYPOTHETICAL GAS RESERVOIR

Aquifer Permeability (darcies)	Ratio of Dimensionless Time to Real Time (1/year)	Water Influx Constant (cu ft/psi)
0.010	0.38483	75,396
0.050	1.9242	75,396
0.100	3.8483	75,396
1.000	38.483	75,396

cases given in this study, it makes little difference whether the transient effects are included or not.

Example Calculations

An example of a water drive performance prediction for a gas reservoir using the PI-Aquifer Material Balance approach is given in detail in Table 5. The calculations were performed on a desk calculator using the simple trial-and-error procedure of iterative substitution. The iterative calculations are shown only for Years 1 and 20. During the period of constant producing rate, the second trial was always within 1 psi of the final answer. When the producing rate was limited by the backpressure curve, an additional iteration was required.

Conclusions

The PI-Aquifer Material Balance approach to water influx calculations offers a very useful and flexible method for forecasting and analyzing the performance of water drive reservoirs. The separation of the water influx problem into a rate equation and a material balance equation, not requiring the use of superposition, makes the concepts and calculations quite simple and easy to apply.

Acknowledgments

I wish to thank Phillips Petroleum Co. for permission to publish this paper. The assistance of M. W. Canon, who programmed the calculations and helped with all the computer runs, is gratefully acknowledged.

Nomenclature

b = width, ft
B_g = gas formation volume factor, reservoir bbl/scf
B_o = oil formation volume factor, reservoir bbl/surface bbl
B_w = water formation volume factor, reservoir bbl/surface bbl
c_f = formation (rock) compressibility, psi^{-1}
c_g = gas compressibility, psi^{-1}
c_o = oil compressibility, psi^{-1}
c_t = total or effective aquifer compressibility, psi^{-1}
c_w = water compressibility (includes the effect of dissolved gas), psi^{-1}
C_g = gas well backpressure curve coefficient (gas well productivity index)
e = natural logarithm base 2.71828
e_w = water influx or efflux rate, reservoir bbl/D
$\bar{e}_w(\Delta tn)$ = average influx or efflux rate during time interval (Δtn), reservoir bbl/D
G = initial gas in place, Bscf
G_p = cumulative gas production, Bscf
h = aquifer thickness, ft
i = subscript to denote initial value or conditions (except for cumulative water injected, W_i)
J_w = aquifer productivity index, reservoir bbl/D/psi
k = aquifer permeability, darcies
L = length, ft
n = exponent of backpressure curve, also used as a subscript to denote the end of an interval
$\bar{p}$ = average aquifer pressure (shut-in pressure), psia
$\bar{p}_{(n-1)}$ = average aquifer pressure (shut-in pressure) at the beginning of an interval, psia
p_e = external boundary pressure, psia
p_i = initial aquifer pressure, psia
$\bar{p}_R$ = gas reservoir average pressure (shut-in pressure), psia
p_{tf} = wellhead tubing flowing pressure, psia
p_{ts} = wellhead shut-in pressure, psia
p_{wf} = inner aquifer boundary pressure, psia
$\bar{p}_{wf}$ = a constant inner boundary pressure for a time interval (Δtn) (see Eq. 8), psia
PI = productivity index, reservoir bbl/D/psi
q_g = gas flow rate, Mscf/D
$\bar{q}_g$ = average gas flow rate during an interval, Mscf/D

TABLE 4—RADIAL AND LINEAR AQUIFER RATE EQUATIONS[2]

Type of Boundary	Radial Flow	Linear Flow
Finite—closed (no flow) at outer boundary	$q_w = \dfrac{7.08\,kh(\bar{p} - p_{wf})}{\mu\left[\ln\left(\frac{r_e}{r_r}\right) - \frac{3}{4}\right]}$	$q_w = \dfrac{3(1.127)\,kbh\,(\bar{p} - p_{wf})}{\mu L}$
Finite—constant pressure at outer boundary	$q_w = \dfrac{7.08\,kh\,(p_e - p_{wf})}{\mu\left[\ln\left(\frac{r_e}{r_r}\right)\right]}$	$q_w = \dfrac{1.127\,kbh\,(p_e - p_{wf})}{\mu L}$
Infinite	$q_w = \dfrac{7.08\,kh\,(p_i - p_{wf})}{\mu\left[\ln\sqrt{\frac{14.23\,kt}{\phi\mu c_t r_r^2}}\right]}$ [Hurst Simplified (Defined)]	$q_w = \dfrac{kbh\,(p_i - p_{wf})}{\mu\sqrt{\frac{6.33\,kt}{\phi\mu c_t}}}$

TABLE 5—EXAMPLE OF A WATER DRIVE PERFORMANCE PREDICTION FOR A GAS RESERVOIR USING THE PI-AQUIFER MATERIAL BALANCE APPROACH

RESERVOIR MATERIAL BALANCE CALCULATIONS

n	TIME t	$\Delta t_n = (t_n - t_{n-1})$	q_{gc} or $\bar{q}_g$	ΔG_p	G_p	$G - G_p$	$G - \left(\frac{W_e}{B_{gi}}\right)'$*	$\left[\frac{G - G_p}{G - \frac{W_e}{B_{gi}}}\right]$	$\frac{\bar{p}_n}{Z}$	$Z(\bar{p})$**	$\bar{p}_n$ and p_{wf}	$p_{ts} = \bar{p}_n/\sqrt{e^x}$***	$q_g = C_g(p_{ts}^2 - p_{tf}^2)^n$	$\bar{q}_g = \frac{(q_{gn-1} + q_{gn})}{2}$
		$(2)_n - (2)_{n-1}$	90,330 or (15)	(3) x (4) x 10^{-6}	Σ (5)	776.000 − (6)	776.000 − (22)	(7) ÷ (8)	2565 x (9)	USING (10)	(10) x (11)	(12) ÷ $\sqrt{e^x}$	$1029.4 \times \left(\frac{p_{ts}^2 - 200^2}{1000}\right)^{0.7}$	$\frac{(14)_{n-1} + (14)_n}{2}$
(1)	DAYS (2)	DAYS (3)	Mscfd (4)	Bscf (5)	Bscf (6)	Bscf (7)	Bscf (8)	Bscf (9)	PSIA (10)	(11)	PSIA (12)	PSIA (13)	Mscfd (14)	Mscfd (15)
1	0	0	0	0	0	776.000	776.000	1.00000	2565	0.780	2000	1008	>90,330	
1 a	365	365	90,330	32.973	32.973	743.027	776.000	0.95751	2456	0.782	1921			
b					32.973	743.027	774.000	0.95000	2400	0.783	1926			
c					32.973	743.027	774.950	0.95000	2460	0.783	1925	1535	>90,330	
2	730	CONSTANT Δt FOR THIS EXAMPLE	90,330	32.973	65.946	710.054	771.916	0.91986	2350	0.786	1856	1402	>90,330	FOR RATES LIMITED BY BACK-PRESSURE CURVE
3	1095		90,330	32.973	98.919	677.081	767.806	0.88200	2284	0.790	1780	1431	>90,330	
4	1460		90,330	32.973	131.892	644.108	760.806	0.84674	2172	0.794	1725	1382	>90,330	
5	1825		90,330	32.973	164.865	611.135	752.824	0.81179	2082	0.798	1661	1333	>90,330	
6	2190		90,330	32.973	197.838	578.112	743.540	0.77750	1994	0.803	1601	1286	>90,330	
7	2555		90,330	32.973	230.811	545.189	733.085	0.74371	1907	0.808	1541	1242	>90,330	
8	2920		90,330	32.973	263.784	512.216	721.302	0.71004	1821	0.812	1479	1194	>90,330	
9	3285		90,330	32.973	296.757	479.243	708.580	0.67633	1735	0.819	1421	1150	>90,330	
10	3650		90,330	32.973	329.730	446.270	694.780	0.64230	1648	0.824	1360	1108	>90,330	
11	4015		90,330	32.973	362.703	413.297	678.734	0.60883	1560	0.830	1295	1062	>90,330	
12	4380		90,330	32.973	395.676	380.324	663.785	0.57303	1470	0.837	1230	1001	>90,330	
13	4745		90,330	32.973	428.649	347.351	648.550	0.53723	1378	0.843	1162	948	>90,330	
14	5110		90,330	32.973	461.622	314.378	628.388	0.50036	1283	0.860	1081	882	>90,330	
15	5475		90,330	32.973	494.595	281.405	608.837	0.46212	1185	0.860	1018	834	>90,330	
16	5840		88,362	32.252	526.847	249.153	588.482	0.42344	1086	0.868	943	775	88,388	88,362
17	6205		81,570	29.776	556.623	219.377	566.001	0.38704	993	0.877	871	717	78,770	81,570
18	6570		72,520	26.473	583.096	192.904	544.320	0.35430	900	0.886	804	664	68,288	72,520
19	6935		64,502	23.543	606.639	169.361	521.152	0.32487	834	0.892	744	616	60,717	64,502
20 a	7300	365	60,717	22.182	628.881	147.199	521.152	0.28245	724	0.903	654	543	48,744	56,230
b			56,230	20.158	626.798	148.202	487.008	0.30021	770	0.888	692	574	54,388	57,500
c			57,500	20.980	627.629	148.371	487.534	0.29821	765	0.900	688	570	53,775	57,246
d			57,246	20.895	627.534	148.466	487.470	0.29844	765	0.900	688	570	53,775	57,246

PI – AQUIFER MATERIAL BALANCE CALCULATIONS (AQUIFER K = 100 md; r_a/r_r = 10)

n	$\bar{p}_{wf} = \frac{(p_{wf\,n-1} + p_{wf\,n})}{2}$	$\bar{p}_{n-1}$	$(\bar{p}_{n-1} - \bar{p}_{wf})$	$1 - e^{-\left[\frac{(q_{wi})\max}{W_{ei}}\right]\Delta t_n}$	ΔW_e Eq. 6	W_e	$\frac{W_e}{B_{gi}}$	$p_i - \bar{p} = \left(\frac{p_i}{W_{ei}}\right) W_e$	$\bar{p}_n$
	$\frac{(12)_{n-1} + (12)_n}{2}$	$(24)_{n-1}$	(17) − (16)	0.04001	665,200 x (18) x (19)	Σ (20)	(21) ÷ 1.1545 x 10^6	1.50312 x 10^{-6} x (21)	2000.0 − (23)
(1)	PSIA (16)	PSIA (17)	PSI (18)	(19)	RES BBL (20)	RES BBL (21)	Bscf (22)	PSI (23)	PSIA (24)
1	2000.0				0	0	0	0	2000.0
1 a	1960.5	2000.0	39.5	0.04001	1,282,844	1,282,844	1.111		
b	1963.0	2000.0	37.0		1,201,464	1,201,464	1.041		
c	1962.5	2000.0	37.5		1,217,700	1,217,700	1.056	1.8	1998.2
2	1890.0	1998.2	108.2	CONSTANT FOR A CONSTANT Δt	3,513,470	4,731,170	4.080	7.1	1992.9
3	1822.0	1992.9	170.9		5,549,406	10,280,835	8.906	15.5	1984.5
4	1757.0	1984.5	227.5		7,387,300	17,668,015	15.304	26.6	1973.4
5	1693.0	1973.4	280.4		9,105,149	26,773,164	23.190	40.2	1958.8
6	1631.0	1958.8	328.8		10,878,704	37,448,968	32.438	58.3	1943.7
7	1571.0	1943.7	372.7		12,102,314	49,552,272	42.921	74.5	1925.5
8	1510.0	1925.5	415.5		13,402,118	63,044,388	54.080	94.8	1905.2
9	1450.0	1905.2	455.2		14,781,254	77,825,642	67.411	117.0	1883.0
10	1380.5	1883.0	483.5		16,024,932	93,850,574	81.291	141.1	1858.9
11	1320.5	1858.9	532.4		17,280,883	111,130,867	96.288	167.1	1832.8
12	1262.5	1832.9	570.4		18,522,829	129,880,896	112.380	194.9	1805.1
13	1196.0	1805.1	608.1		19,778,885	149,438,381	129.441	224.8	1775.4
14	1128.5	1775.4	648.9		21,071,881	170,510,472	147.882	256.3	1743.7
15	1054.5	1743.7	688.2		22,378,782	192,888,174	167.077	288.9	1710.1
16	988.5	1710.1	728.6		23,881,571	216,581,745	187.588	326.5	1674.5
17	907.0	1674.5	767.5		24,922,288	241,504,886	209.185	363.8	1637.0
18	837.5	1637.0	799.5		25,861,304	267,405,388	231.672	402.0	1598.0
19	774.0	1598.0	824.0		26,758,828	294,222,297	254.840	442.3	1567.7
20 a	688.0	1567.7	858.7	0.04001	27,883,788	322,106,883	278.888		
b	718.0	1567.7	838.1		27,288,730	321,488,835	278.488		
c	716.0	1567.7	841.7		27,331,882	321,563,979	278.522		
d	716.0	1567.7	841.7		27,331,882	321,553,979	278.522	483.3	1516.7

EQUATIONS:

$$\frac{\bar{p}_n}{\bar{Z}} = \frac{\bar{p}_i}{Z_i}\left[\frac{G - G_p}{G - \left(\frac{W_e}{B_{gi}}\right)}\right]$$; GAS RESERVOIR MATERIAL BALANCE EQUATION WITH WATER INFLUX.

$$\Delta W_e = \frac{W_{ei}}{p_i}(\bar{p}_{n-1} - \bar{p}_{wf})\left\{1 - e^{-\left[\frac{(q_{wi})\max}{W_{ei}}\right]\Delta t_n}\right\}$$; EQUATION 6 IN PAPER.

$$q_g = C_g(p_{ts}^2 - p_{tf}^2)^n$$; TOTAL FIELD WELLHEAD BACK-PRESSURE CURVE EQUATION.

$$\bar{p} = -\left(\frac{p_i}{W_{ei}}\right) + p_i$$; EQUATION 2 IN PAPER.

$$p_{ts} = \bar{p}_n / \sqrt{e^x}$$ WHERE $x = 0.0375\,\gamma_g\,D / \bar{T}\bar{Z}$

NOTES:

*THE PRIMED QUANTITY IS OBTAINED FROM THE PREVIOUS TRAIL WITHIN A TIME INTERVAL.
** GAS DEVIATION FACTORS OBTAINED FROM REFERENCE 30.
*** A PLOT OF $\bar{p}_n$ VS $\sqrt{e^x}$ FOR A GIVEN RESERVOIR IS USED FOR THIS CALCULATION.

ALL BASIC DATA FOR THIS PROBLEM APPEAR IN TABLES 1 AND 2.

q_{gc} = constant gas flow rate, Mscf/D
$(q_{wi})_{max}$ = initial open-flow potential of the aquifer, reservoir bbl/D
r_a = external radius of aquifer, ft
r_r = internal radius of aquifer, ft
t = time, days
Δtn = time interval n
t_{ps} = time to establish pseudosteady state, days
t_s = time to establish steady state, days
V_p = pore volume
W = initial water in place, surface bbl
W_e = cumulative water influx into a reservoir or efflux from the aquifer, reservoir bbl
ΔW_{en} = cumulative water influx or efflux during an interval, reservoir bbl
W_{ej} = cumulative water influx into reservoir (j) within the common aquifer, reservoir bbl
W_{ei} = initial *encroachable* water in place at pressure p_i, reservoir bbl
$W_{ei}(t)$ = encroachable water in place at time (t), reservoir bbl
W_i = cumulative water injected, surface bbl
W_p = cumulative water produced, surface bbl
z = gas deviation factor
ϕ = porosity, fraction
μ = viscosity of water, cp

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APPENDIX A

Aquifer Material Balance

A material balance equation may be developed for a finite aquifer system as follows.

Total Volume at Pressure $\bar{p}$

$$\left[\left(\begin{array}{c}\text{Volume of}\\ \text{Initial Contents}\\ \text{at } \bar{p}\end{array}\right)+\left(\begin{array}{c}\text{Volume of all Injected}\\ \text{and Encroached Fluids}\\ \text{at } \bar{p}\end{array}\right)\right]$$

Pore Volume at Pressure $\bar{p}$

$$\begin{array}{cc}\text{Original Pore Volume} & \text{Lost Pore Volume}\end{array}$$

$$-\left[\left(\begin{array}{c}\text{Volume of Initial}\\ \text{Contents}\\ \text{at } p_i\end{array}\right)-\left(\begin{array}{c}\text{Loss of}\\ \text{Pore Volume}\\ \text{at } \bar{p}\end{array}\right)\right]$$

Total Voidage Volume at Pressure $\bar{p}$

$$=\left[\left(\begin{array}{c}\text{Volume Effluxed and Produced}\\ \text{at } \bar{p}\end{array}\right)\right] \quad . \quad . \quad \text{(A-1)}$$

In the algebraic form using the standard AIME nomenclature,

$$\{[W\,B_w]+[W_i\,B_w]\}-\{[W\,B_{wi}]-[c_f(p_i-\bar{p})W\,B_{wi}]\}$$
$$=\{[W_e]+[W_p\,B_w]\}\; . \; . \; . \; . \; . \; . \quad \text{(A-2)}$$

Dividing through by B_{wi},

$$\left\{\left[W\frac{B_w}{B_{wi}}\right]+\left[W_i\frac{B_w}{B_{wi}}\right]\right\}-\left\{W-\left[c_f(p_i-\bar{p})W\right]\right\}$$
$$=\left\{\left[\frac{W_e}{B_{wi}}\right]+\left[W_p\frac{B_w}{B_{wi}}\right]\right\}. \quad \text{(A-3)}$$

Substituting

$$\frac{B_w}{B_{wi}}=1+c_w(p_i-\bar{p}), \quad \text{(A-4)}$$

we obtain

$$W[1+c_w(p_i-\bar{p})]-W[1-c_f(p_i-\bar{p})]$$
$$=\frac{1}{B_{wi}}[W_e+B_w(W_p-W_i)], \quad \text{(A-5)}$$

or

$$W B_{wi}\{[1+c_w(p_i-\bar{p})]-[1-c_f(p_i-\bar{p})]\}$$
$$=W_e+B_w(W_p-W_i); \quad \text{(A-6)}$$

collecting terms,

$$W B_{wi}[(c_w+c_f)(p_i-\bar{p})]=W_e+(W_p-W_i)B_w. \quad \text{(A-7)}$$

Rearranging Eq. A-7 we have

$$\bar{p}=-\left[\frac{W_e+(W_p-W_i)B_w}{(c_w+c_f)W B_{wi}}\right]+p_i. \quad \text{(A-8)}$$

To further generalize the equation to include interference effects of other reservoirs in a common aquifer,

$$\bar{p}=-\left[\frac{W_e+\sum_{2}^{j}W_{ej}+(W_p-W_i)B_w}{(c_w+c_f)W B_{wi}}\right]+p_i, \quad \text{(A-9)}$$

where W_e represents the cumulative water influx for the reservoir of interest, and W_{ej} represents cumulative water influx into reservoir (j) within the common aquifer. The water compressibility can be considered then as effective compressibility, which includes the compressibility of the other nonproducing hydrocarbon reservoirs.

Eq. A-9 is the general equation, but to simplify the further derivation we will set the interference, water production, and water injection terms to zero; that is, $\Sigma W_{ej}=0$, $W_p=0$, and $W_i=0$.

We then have

$$\bar{p}=-\left[\frac{1}{(c_w+c_f)W B_{wi}}\right]W_e+p_i. \quad \text{(A-10)}$$

Defining $[(c_w+c_f)W B_{wi}]\,p_i=W_{ei}$, as the initial encroachable water in place, we can write for the aquifer material balance equation

$$\bar{p}=-\left(\frac{p_i}{W_{ei}}\right)W_e+p_i, \quad \text{(A-11)}$$

which can be represented in graphical form as shown in Fig. 5.

Note that the term W_{ei} is *not* total water in place, $W B_{wi}$ (which represents the total aquifer pore volume). The aquifer will still be 100-percent saturated with water when all the aquifer pressure is depleted; that is, when $\bar{p}=0$. Note, too, that the determination of W_{ei}, the initial encroachable water in place, is not basically geometry-dependent except to the extent that fundamental mensuration rules can be applied. Isopachous planimetry would be the most rigorous of all approaches.

APPENDIX B

Water Influx Equations

Aquifer Rate Equation

The aquifer rate equation independent of geometry is

$$q_w=J_w(\bar{p}-p_{wf})^{1.0} \quad \text{(B-1)}$$

The aquifer rate equation when graphically depicted is analogous to the productivity index curve of the oil wells and to the backpressure curve of the gas wells (see Fig. 1).

The rate-time relationship for water influx against an increasing Δp is shown graphically in Figs. 3 and 4.

The cumulative influx into the reservoir or efflux from the aquifer is determined by

$$W_e=\int_0^t q_w\,dt \quad \text{(B-2)}$$

Differentiating we have

$$dW_e=q_w\,dt, \quad \text{(B-3)}$$

or

$$\frac{dW_e}{dt}=q_w. \quad \text{(B-4)}$$

Using the aquifer rate equation, we obtain

$$q_w=J_w(\bar{p}-p_{wf}); \quad \text{(B-1)}$$

then

$$\frac{dW_e}{dt}=J_w(\bar{p}-p_{wf}). \quad \text{(B-5)}$$

At initial conditions we can define the maximum capacity or initial open-flow potential of the aquifer, when $p_{wf}=0$, as

$$(q_{wi})_{max}=J_w(p_i), \quad \text{(B-6)}$$

or

$$J_w=\frac{(q_{wi})_{max}}{p_i}. \quad \text{(B-7)}$$

Therefore,

$$q_w=\frac{(q_{wi})_{max}}{p_i}(\bar{p}-p_{wf}). \quad \text{(B-8)}$$

Then

$$\frac{dW_e}{dt}=\frac{(q_{wi})_{max}}{p_i}(\bar{p}-p_{wf}), \quad \text{(B-9)}$$

or

$$dW_e = \frac{(q_{wi})_{\max}}{p_i}(\bar{p} - p_{wf})dt \; . \quad . \quad . \quad . \quad . \qquad \text{(B-10)}$$

From the material balance equation slope,

$$\frac{d\bar{p}}{dW_e} = -\left(\frac{p_i}{W_{ei}}\right) , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-11)}$$

or

$$d\bar{p} = -\left(\frac{p_i}{W_{ei}}\right) dW_e \; . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-12)}$$

Combining Eqs. B-10 and B-12,

$$d\bar{p} = -\frac{p_i}{W_{ei}}\left[\frac{(q_{wi})_{\max}}{p_i}(\bar{p} - p_{wf})dt\right] \; . \quad . \qquad \text{(B-13)}$$

Simplifying and separating variables,

$$\frac{d\bar{p}}{(\bar{p} - p_{wf})} = -\frac{(q_{wi})_{\max}}{W_{ei}} dt \; . \quad . \quad . \quad . \quad . \qquad \text{(B-14)}$$

Then

$$\int_{p_i}^{\bar{p}} \frac{d\bar{p}}{(\bar{p} - p_{wf})} = -\frac{(q_{wi})_{\max}}{W_{ei}} \int_0^t dt \; . \quad . \quad . \quad . \qquad \text{(B-15)}$$

Rearranging and changing limits on p, we obtain

$$\frac{(q_{wi})_{\max}}{W_{ei}} \int_0^t dt = \int_{\bar{p}}^{p_i} \frac{d\bar{p}}{(\bar{p} - p_{wf})} \; . \quad . \quad . \quad . \quad . \qquad \text{(B-16)}$$

Integrating between limits gives us

$$\left[\frac{(q_{wi})_{\max}}{W_{ei}}\right] t = \ln\left[\frac{p_i - p_{wf}}{\bar{p} - p_{wf}}\right] , \quad . \quad . \quad . \qquad \text{(B-17)}$$

which can be expressed as

$$\frac{p_i - p_{wf}}{\bar{p} - p_{wf}} = e^{[(q_{wi})_{\max}/W_{ei}]t} \; ; \quad . \quad . \quad . \qquad \text{(B-18)}$$

but

$$q_w = J_w(\bar{p} - p_{wf}) \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-19)}$$

or

$$\frac{q_w}{J_w} = (\bar{p} - p_{wf}) \; . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-20)}$$

Therefore,

$$\frac{J_w(p_i - p_{wf})}{q_w} = e^{[(q_{wi})_{\max}/W_{ei}]t} \quad . \quad . \quad . \qquad \text{(B-21)}$$

Now, defining $e_w = q_w$,

$$e_{w(t)} = \frac{J_w(p_i - p_{wf})}{e^{[(q_{wi})_{\max}/W_{ei}]t}} \, , \quad . \quad . \quad . \quad . \qquad \text{(B-22)}$$

which is the final form expressing the instantaneous rate of water influx as a function of time and the internal boundary pressure, (p_{wf}). The equation is quite general and totally independent of geometry, and will use any consistent set of units.

Cumulative Water Influx Equation

Now we can derive the more useful cumulative water influx equation. If we combine the equations

$$W_e = \int_0^t e_w \, dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-2)}$$

and

$$e_w = \frac{J_w(p_i - p_{wf})}{e^{[(q_{wi})_{\max}/W_{ei}]t}} \, , \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-23)}$$

then

$$W_e = \int_0^t \frac{J_w(p_i - p_{wf})}{e^{[(q_{wi})_{\max}/W_{ei}]t}} \, dt \, , \quad . \quad . \qquad \text{(B-24)}$$

or

$$W_e = J_w(p_i - p_{wf}) \int_0^t e^{-[(q_{wi})_{\max}/W_{ei}]t} \, dt \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-25)}$$

Eq. B-25, when integrated between limits

$$W_e = J_w(p_i - p_{wf}) \left\{ \frac{e^{-[(q_{wi})_{\max}/W_{ei}]t}}{-\left[\frac{(q_{wi})_{\max}}{W_{ei}}\right]} \right\}_0^t , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-26)}$$

gives

$$W_e = \frac{J_w(p_i - p_{wf})}{\left[\frac{(q_{wi})_{\max}}{W_{ei}}\right]} \{1 - e^{-[(q_{wi})_{\max}/W_{ei}]t}\} \; ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-27)}$$

but

$$(q_{wi})_{\max} = J_w(p_i) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-6)}$$

Substituting and rearranging, we arrive at the final form of the cumulative water influx equation.

$$W_e = \frac{W_{ei}}{p_i}(p_i - p_{wf}) \; \{1 - e^{-[(q_{wi})_{\max}/W_{ei}]t}\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(B-28)}$$

It is interesting to note that both the instantaneous water influx rate equation and the cumulative influx equation are identical in form with equations derived by Russell and Prats[29] for predicting the performance of layered reservoirs. Their results and conclusions should be directly applicable when the simplified water influx approach is used. **JPT**

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