MEMS Tutorial: Pull-in voltage in electrostatic microactuators

In this tutorial, we cover the pull-in effect in electrostatic MEMS devices. Figure 1 shows the schematic of an electrostatic actuator that could be used for example as a tunable RF capacitor. When voltage is applied over the capacitance, electrostatic force will work to reduce the plate separation $d-x$. At small voltages, the electrostatic voltage is countered by the spring force $F_k = kx$ but as voltage is increased the plates will eventually snap together. Estimating this pull-in voltage U_P and the plate travel distance x_P before pull-in effect is required for the successful design of electrostatic actuators, switches, varactors, and sensors.

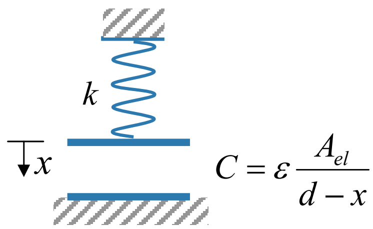


Figure 1. *Schematic of an electrostatic actuator. The plate is attached to a spring k. The capacitor capacitance C depends on the plate area A_{el} and gap $d-x$.*

To derive the expression for pull-in, we start by writing the total potential energy in the system:

$$E = -\frac{1}{2}\frac{\varepsilon A_{el}}{d-x}U^2 + \frac{1}{2}kx^2, \tag{1}$$

where the first term is the electrostatic potential of the deformable capacitor and of the voltage source and the second term is due to the mechanical energy stored in the spring. The force acting on the movable plate is obtained by deriving Equation (1):

$$F = -\frac{\partial E}{\partial x} = \frac{1}{2}\frac{\varepsilon A_{el}}{(d-x)^2}U^2 - kx. \tag{2}$$

At equilibrium, the electrostatic force and spring force cancels ($F = 0$) and Equation (2) gives:

$$kx = \frac{1}{2}\frac{\varepsilon A_{el}}{(d-x)^2}U^2. \tag{3}$$

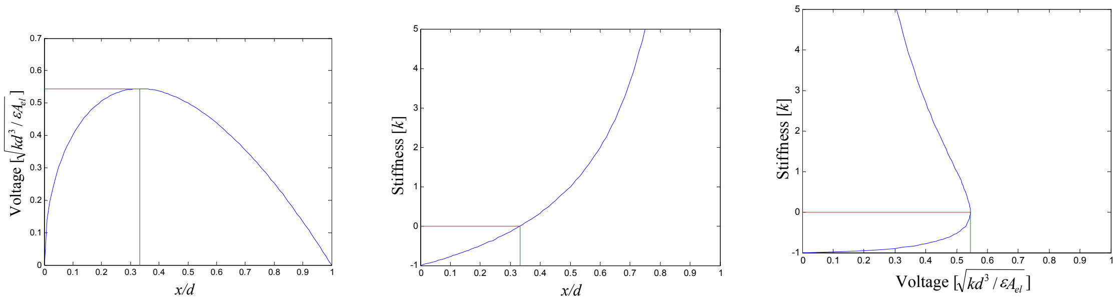
Equation (3) can be solved for the equilibrium plate position x as a function of applied voltage U as shown in Figure 2(a). Above the pull-in voltage V_P, Equation (3) has no solutions. The solution above the pull-in displacement (green line) are shown to be unstable in the following.

A simple expression for the pull-in point is obtained by deriving Equation (2) to obtain the *stiffness* of the system:

$$\frac{\partial F}{\partial x} = \frac{\varepsilon A_{el}}{(d-x)^3}U^2 - k. \tag{4}$$

Substituting Equation (3) gives the stiffness around the equilibrium point:

$$\frac{\partial F}{\partial x} = \frac{2kx}{(d-x)} - k. \tag{5}$$



(a) The equilibrium relationship between plate displacement x and voltage U.

(b) The stiffness as a function of equilibrium point x. Positive stiffness indicates unstable equilibrium point.

(c) The stiffness as a function of voltage U.

Figure 2. *Pull-in effect in plate capacitor. No equilibrium solutions are obtained above the pull-in voltage (red line). Above the pull-in displacement (green line), the actuator is unstable.*

With no applied voltage Equation (5) is simply $\frac{\partial F}{\partial x} = -k$; a small positive movement δx result in negative restoring force $\frac{\partial F}{\partial x}\delta x = -k\delta x$. Increasing the bias voltage U makes the stiffness less negative. The unstable point is given by $\frac{\partial F}{\partial x} = 0$ giving

$$x = \frac{1}{3}d. \tag{6}$$

Beyond this point the stiffness becomes positive as shown in Figure 2(b) and the system is unstable: a small positive movement δx result in positive force that increases x. Substituting Eq. (6) to Eq. (3) gives the pull-in voltage at which the system becomes unstable

$$U_P = \sqrt{\frac{8}{27}\frac{kd^3}{\varepsilon A_{el}}}. \tag{7}$$

Exercise:

Figure on the right shows a schematic of a MEMS device that has capacitors on both sides of the movable plate. Assume that equal voltage is applied over both plates. Note that the equilibrium point is always at $x = 0$. Derive the expression for the pull-in voltage. The correct answer is $V_P = \sqrt{kd^3/2\varepsilon A_{el}}$.